Lab Work - Week 2

Math 105B Lab

Summer Session II - 2022

1 Hermite Interpolation

CODING ASSIGNMENT 4 - Computing Hermite Polynomial Coefficients

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Produce a function with the following specifications:

NAME:	hermiteInterp_########
INPUT:	X,Y,YPrime
OUTPUT:	Q
DESCRIPTION:	Given data X= $(x_1, ..., x_n)$, Y= $(y_1, ..., y_n) = (f(x_1), ..., f(x_n))$, and YPrime= $(f'(x_1), \ldots, f'(x_n))$, this function computes the vector Q, consisting of the $2n$ Hermite polynomial coefficients.
PSUEDOCODE:	Pg. 139

Useful MATLAB concepts/functions

- Recall MATLAB 1-indexes, so you need to re-interpret the indexing when implementing the pseudocode in MATLAB.

EXERCISES

1. Use a Hermite polynomial to interpolate the function $f(x) = e^{0.1x^2}$ given below in tabular form, at the point $x = 1.25$. Use $H_5(1.25)$ with nodes x_0, x_1, x_2 and $H_3(1.25)$, with nodes x_0, x_1.

x	f(x)	f'(x)
$x_0 = 1$	1.105170918	0.2210341836
$x_1 = 2$	1.491824698	0.5967298792
$x_2 = 3$	2.459603111	1.475761867

2. What is your estimate of the errors? Justify your answer. Find error bounds for these approximations.

2 Cubic Spline Interpolation

CODING ASSIGNMENT 5 - Natural Cubic Spline Interpolation

Produce a function with the following specifications:

NAME:	naturalCubicSpline_########
INPUT:	X, Y
OUTPUT:	a,b,c,d
DESCRIPTION:	Given data X= $(x_1, ..., x_n)$ and Y= $(y_1, ..., y_n) = (f(x_1), ..., f(x_n))$, this function computes the vectors $a = (a_1, \ldots, a_{n-1})$, $b = (b_1, \ldots, b_{n-1}), \ldots, d = (d_1, \ldots, d_{n-1})$, the cubic spline interpolant polynomial coefficients.
PSEUDOCODE:	pg. 148

Useful MATLAB concepts/functions

- To reverse index a for loop we can use an intermediate variable, such as in the following code:

```
for j=1:n
    i=n-j+1;
    % Code using the reversed index i goes here.
end
```

EXERCISES

1. Compute and graph the natural cubic spline interpolant for the data shown in the table below. To plot, use these points, together with the uniformly spaced points given in problem 3 below.

x	f(x)
-2.4061	-0.3984
-1.0830	-0.7611
-0.6440	-0.9688
-0.4068	-0.9791
-0.2448	-0.7899
-0.1158	-0.4397
0	0
0.1158	0.4397
0.2448	0.7899
0.4068	0.9791
0.6440	0.9688
1.0830	0.7611
2.4061	0.3984

2. **(Participation Assessment)** Use your Lagrange interpolation algorithm from computer assignment 1 to compute the corresponding Lagrange polynomial interpolant, at the same points as listed in problem 3. How does the result compare to that for the cubic spline? Explain your results.

3. The function given the table above is: $f(x) = \frac{x}{1/4+x^2}$. Use the cubic spline interpolant you found above to approximate the function at the uniformly spaced grid points:

   z=linspace(-2,2,10).

   What is the maximum error for the cubic spline interpolant? How does the result compare to that predicted by the theoretical error estimate?