Eric Trinh

Project #3 – Algorithmic Experiments of Real-World Phenomena

In this report write-up, we will be looking certain graph algorithms to determine properties of models of real-world networks. For this experiment, we will be looking at the diameters, clustering coefficients, and degree distributions of both Endocrania and Barabasi-Albert graph models of increasing sizes.

Generation of Erdos-Renyi Graphs --

Erdos-Renyi graphs are characterized by a Poisson degree distribution as the size becomes large. Regarding the generation of such graphs, each pair of nodes has equal probability of being connected. The following is the pseudocode taken from lecture that I used to implement my Erdos-Renyi graph generation.

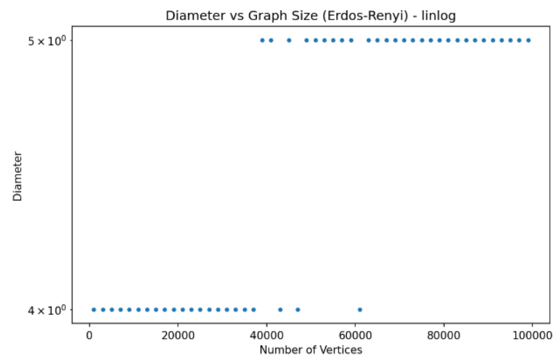
Faster Erdös-Rényi G(n,p) Generation

- The above algorithm for generating G(n,p) is slow if p is small, because most of the bits are 0.
- Probability of having k-1 0's then a 1 is $(1-p)^{k-1}p$
- Faster O(n+m)-time algorithm skips over runs of 0's:

ALG. 1: $\mathcal{G}(n,p)$
Input: number of vertices n, edge probability $0<p<1$
Output: $G=(\{0,\ldots,n-1\},E)\in\mathcal{G}(n,p)$
$E\leftarrow\emptyset$
$v\leftarrow 1;\ w\leftarrow -1$
while $v<n$ **do**
  draw $r\in[0,1)$ uniformly at random
  $w\leftarrow w+1+\lfloor\log(1-r)/\log(1-p)\rfloor$
  while $w\geqslant v$ **and** $v<n$ **do**
    $w\leftarrow w-v;\ v\leftarrow v+1$
  if $v<n$ **then** $E\leftarrow E\cup\{v,w\}$

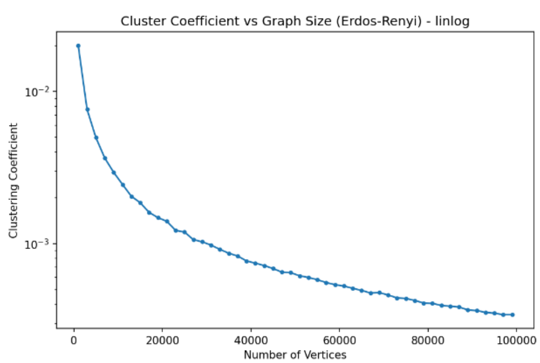
Diameter (Erdos-Renyi) --

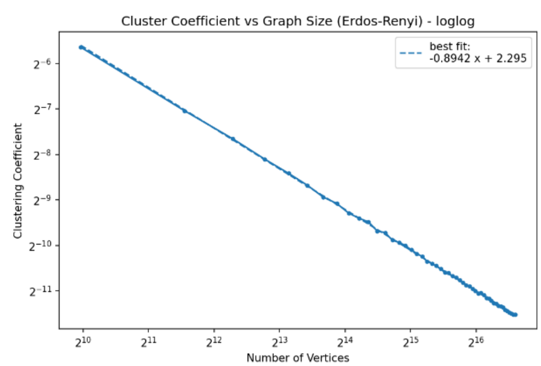
Based on the data, we can assume that the diameter does not increase with the size of the graph.



Clustering Coefficient (Erdos-Renyi) --

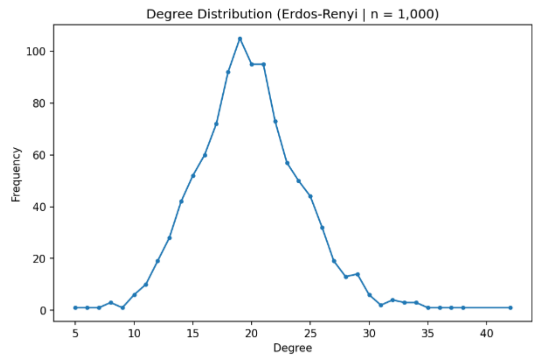
We can see from the plots that in a Erdos-Renyi graph, that clustering coefficient, a value that measures the degree to which nodes cluster together, decreases with increasing sizes of graphs. The relationship between the clustering coefficient and the size of a graph could be said to be an exponential relationship. This also means that the clustering coefficient decreases according to log(n) it has a negative linear slope in the loglog plot.

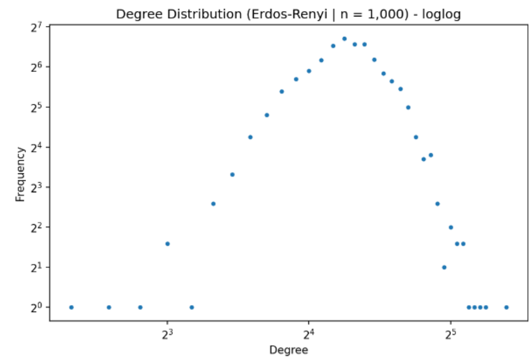


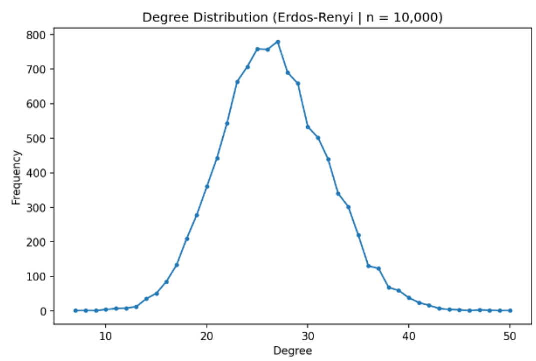


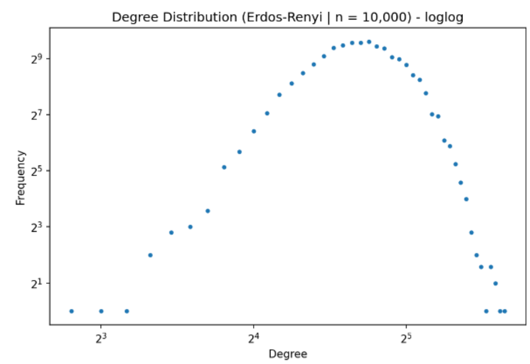
Degree Distribution (Erdos-Renyi) --

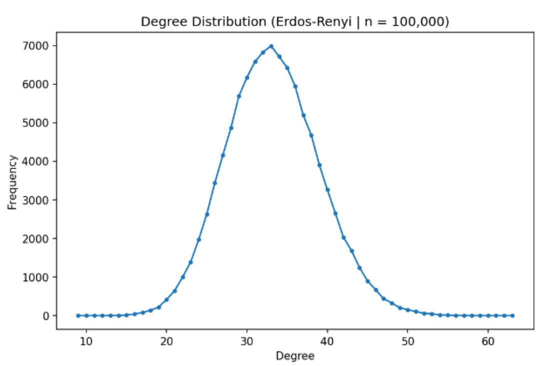
Based on the data collected, we were able to confirm that the degree distribution of Erdos-Renyi graphs follows a rough Poisson distribution when the size is large. Hence, the degree distribution of Erdos-Renyi graphs does not follow a power-law.

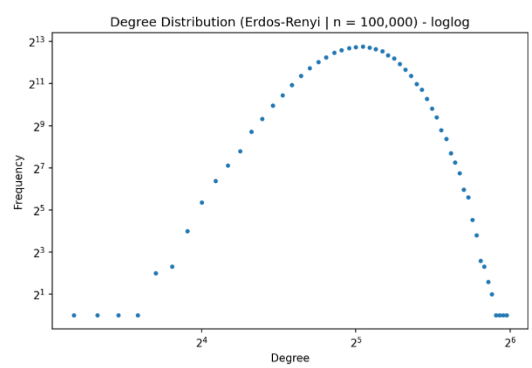












Generation of Barabasi-Albert Graphs --

Barabbas-Albert graphs are characterized by power law degree distribution. The idea in the generation of such Barabasi-Albert graphs is that it is more likely that heavily connected nodes are connected with new nodes. This is like a rich-get-richer process. The following is the pseudocode taken from lecture that I used to implement my Erdos-Renyi graph generation.

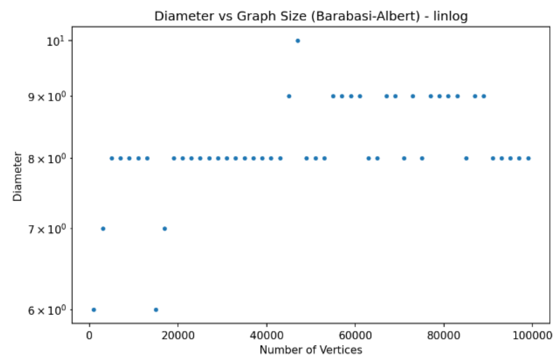
Faster Barabasi-Albert (BA) Algorithm

- Let d be the parameter for the BA algorithm

ALG. 5: preferential attachment
Input: number of vertices n
minimum degree $d \geqslant 1$
Output: scale-free multigraph
$G = (\{0, \ldots, n-1\}, E)$
M: array of length $2nd$ // M is an array of edges chosen so far.
for $v = 0, \ldots, n-1$ **do**
for $i = 0, \ldots, d-1$ **do**
$M[2(vd+i)] \leftarrow v$ // Each vertex v appears d_v times in M.
draw $r \in \{0, \ldots, 2(vd+i)\}$ uniformly at random
$M[2(vd+i)+1] \leftarrow M[r]$
$E \leftarrow \emptyset$
for $i = 0, \ldots, nd-1$ **do**
$E \leftarrow E \cup \{M[2i], M[2i+1]\}$

Diameter (Barabasi-Albert) ---

Based on the data, we can assume that the diameter does not increase with the size of the graph.



Clustering Coefficient (Barabasi-Albert) ---

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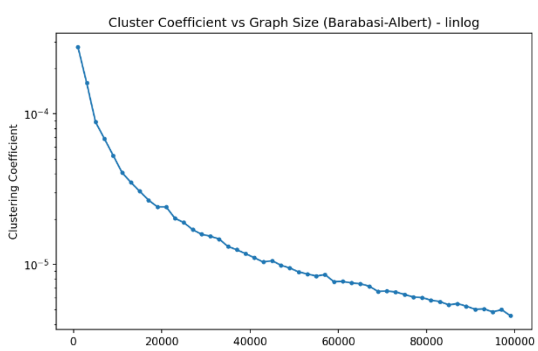

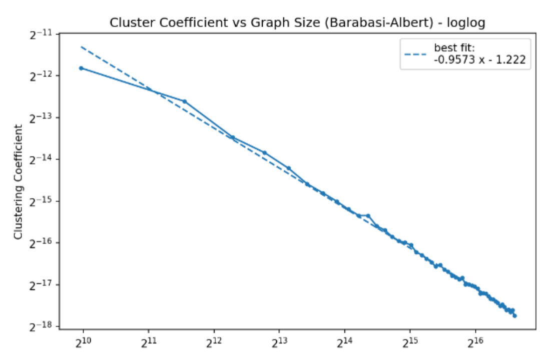

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Diameter Distribution (Barabasi-Albert) --

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