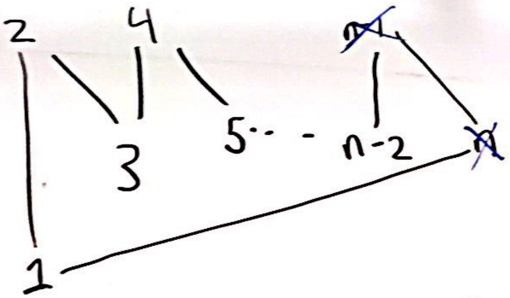
$P =$



$a_n :=$ antichains of P

$b_n :=$ order filters of P

n odd

$a_n = a_{n-1} + a_{n-2}$

Since there is a bijection from {order filters of P} $\to$ {antichains of P}, we can consider antichains of P.

There are two possible cases for any antichain of P:

Case 1: n is in the antichain

Any antichain of P that includes n cannot also include $n-1$ or 1, because n is comparable to both 1 and $n-1$. Thus, we form an antichain from a poset with the structure above with $n-3$ elements, of which there are exactly a_{n-3} antichains.

Case 2: n is not in the antichain

If n is not in the antichain, then we consider antichains formed from the poset with the same structure as P with $n-1$ elements.

We have two possible sub-cases:

- Case A: 1 is in the antichain

If 1 is in the antichain, we can't have 2 or $n-1$ in the antichain because 1 is comparable to both 2 and $n-1$. Thus, we form an antichain from a poset with the structure above with $n-4$ elements, of which there are exactly a_{n-4} antichains.

- Case B: 1 is not in the antichain

If 1 is not in the antichain, then we form an antichain from a poset with the same structure above with $n-2$ elements of which there are exactly a_{n-2} antichains.

Combining all the cases, we have $b_n = a_{n-2} + \underbrace{a_{n-3} + a_{n-4}}_{a_{n-2}} = 2a_{n-2}$. $\square$