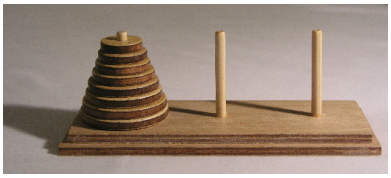
1. Consider the Tower of Hanoi puzzle.



   i.) Is it possible to solve this puzzle in $2^n - 1$ moves? Play this puzzle with your n paper disks, where $n = 1, 2, 3$, and 4.

   ii.) Prove (using induction) that it is possible to solve this puzzle in $2^n - 1$ moves, or give a counterexample (find a number M where you need more than $2^M - 1$ to solve the puzzle).

   Solution: See Gilbert-Vanstone 'An intro to mathematical thinking', the textbook for Math 2710 UConn course, page 94.

2. There are N students in this class. I want to make sure that everyone is paired with everyone else exactly once during the semester.

   a.) How many times should I schedule pair activities? Write the answer as an explicit (closed-form) formula.

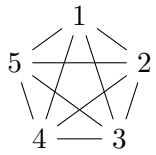
   b.) Prove your closed-form formula using induction.

3. To make sure everyone knows everyone, please have everyone shake hands or fist-bump with everyone in the group. Then count the number of occurrences.

   A.) If there are N people in a large group, how many handshakes will happen if everyone shakes hands exactly once with everyone in the group? Write the answer as an explicit (closed-form) formula.

   B.) Prove your closed-form formula using induction.

4. On the board, please draw the complete graph on 5 vertices



   Count the number of edges in this graph.

   I.) How many edges are there for the complete graph on N vertices? Write the answer as an explicit (closed-form) formula.

   II.) Prove your closed-form formula using induction.

   Solution: $\binom{n}{2}$ which is $\dfrac{n(n-1)}{2}$.

5. The following even numbers can be written as the sum of two primes:

$$6 = 3 + 3$$
$$8 = 3 + 5$$
$$10 = 3 + 7 = 5 + 5$$
$$12 = 7 + 5$$

a. Write each of the even numbers 50, 70, and 100 as the sum of two primes. Find as many ways as possible to write them as sums of two primes.

b. Is it possible to write every even number (larger than 2) as the sum of two primes?

c. Prove (using induction) that it is possible, or give a counterexample (find an even number $M > 2$ which cannot be written as the sum of two primes).

Solution: $100 = 3 + 97 = 11 + 89 = 17 + 83 = 29 + 71 = 41 + 59 = 47 + 53$.
This is the Goldbach conjecture.