Fast and Somewhat Accurate Algorithms (Team 4)

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IMA: Modeling in Industry Final Presentation

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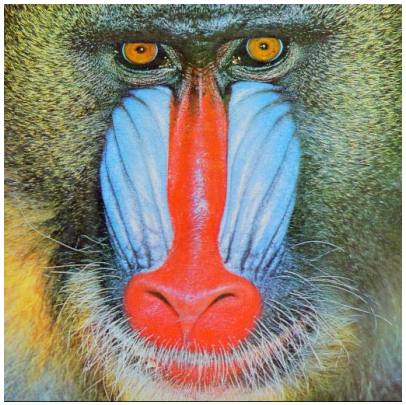
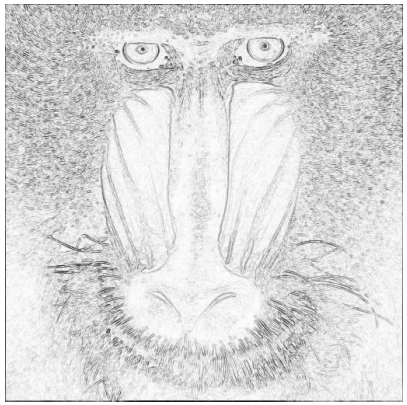
Outline





Problem

We study the tradeoff between accuracy and system complexity as measured by processing speed and hardware complexity.

Reduce processing time using look-up tables (LUT)

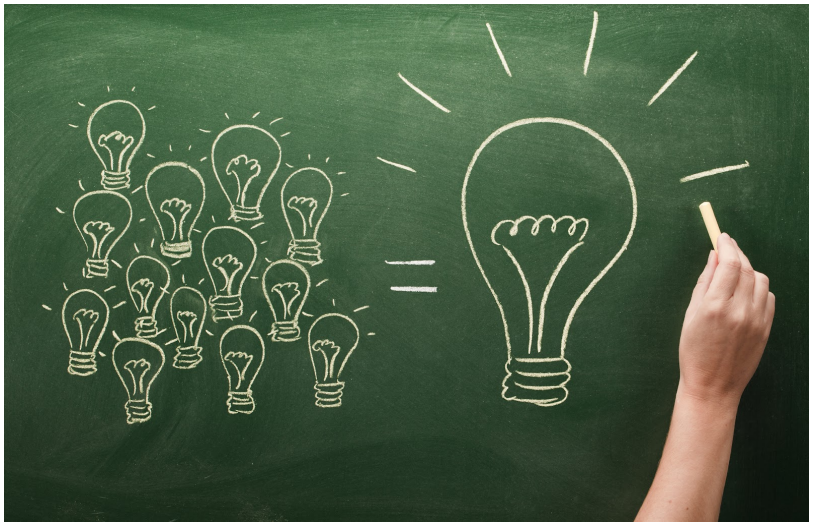
IDEA

Prior to applying filter, pre-compute convolutions for all possible square matrices and store them in a multi-dimensional table.

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix} * K \rightarrow LUT(x_1, x_2, \ldots, x_9)$$

- Save time by retrieving the values in $LUT(x_1, \ldots, x_9)$ (in place of direct algebraic computations).
- Challenge: a full-size LUT for a 3×3 kernel requires 2^{72} bytes (assuming each pixel is 8 bits).

Some Strategies to reduce LUTs' size



1. LUT and Truncation
2. Matrix Decomposition and Approximate decomposition

Truncation to reduce LUT's size

Example: truncating by 3 digits.
Replace

$$\left.\begin{array}{rl} 40 = & 00101\underline{000} \\ & 00101\underline{001} \\ & 00101\underline{010} \\ & 00101\underline{011} \\ & 00101\underline{100} \\ & 00101\underline{101} \\ & 00101\underline{110} \\ 47 = & 00101\underline{111} \end{array}\right\} \text{ with } \longrightarrow 00101\underline{100}$$

to do the kernel convolution, and pre-store the results in the LUT with index 00101.

Decomposing rank 1 matrix

IDEA

If the kernel $(n \times n)$ matrix K has rank 1, then

$$K = cb^T$$

where $c, b \in \mathbb{R}^n$. For example, the low pass filter kernel.

Emily writes: the person doing this slide can also mention quickly the existence of other commonly-used rank-1 filters. The gradient filters such as Sobel gradient filter (an edge-detection filter) is also of rank 1 and can be decomposed similarly.

$$K = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Applying a filter K to an Image then becomes

$$\text{IMAGE} * K = \text{IMAGE} * (cb^T) = (\text{IMAGE} * c) * b^T$$

Decomposing rank 1 matrix

An example: blurring filter with kernel

$$K = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} = c \cdot c^T$$

Truncation matrix:

$$T_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$$

By symmetry, only one look up table is needed.

1. Step 1: Generate look up table for $c = [1 \;\; 2 \;\; 1]^T$.
2. Step 2: Convolve the image with c.
3. Step 3: Convolve transpose of the image with c.

(a) Original image

(b) Low pass, direct

(c) Low pass, LUT with T_1

(d) Low pass, LUT with T_2

Figure: Comparison of filtered images

An Approximate Decomposition

IDEA

The more zeros in our filter the smaller the LUT. Can we decompose the (3× 3) filter K in a better way? Consider:

$$K = A * B$$

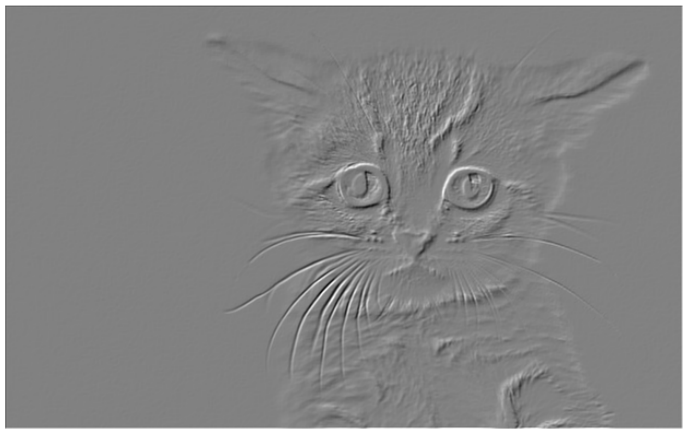
where

$$A = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ e & f & 0 \end{bmatrix}, B = \begin{bmatrix} g & h & i \\ j & k & l \\ 0 & 0 & 0 \end{bmatrix}$$

- Size of the LUT reduces from 2^{72} bytes to 2^{49} bytes before any truncation.
- We can solve the minimization problem

$$\min(\|K - A * B\|_F + TotalEdgeSum)$$

Approximate Decomposition Example



$$K = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, A = \begin{bmatrix} -0.000 & -0.000 & 0.000 \\ -0.000 & 1.702 & 0 \\ 0.000 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} -0.587 & -0.587 & 0 \\ -0.587 & 0 & 0.587 \\ 0 & 0.587 & 0.587 \end{bmatrix}$$

- $\|K - A_T * B_T\|_F < 3.04 * 10^{-5}$

	Emboss	Low-Pass	Averaging
PSNR	42.0500	71.0895	
l_2 - error	0.0079	2.7895e-04	
max - error	0.0144	6.1630e-04	

Table: Comparison of errors

The errors are measured as such

$$\text{PSNR(A,B)} = 20 \cdot \log_{10}\left(\frac{\text{MAX}_B}{\sqrt{\text{MSE}}}\right)$$

$$l_2\text{-error(A,B)} = \sqrt{\frac{\sum_{i,j}(a_{ij}-b_{ij})^2}{h \times w}}$$

$$l_\infty\text{-error(A,B)} = \max_{i,j}|a_{ij} - b_{ij}|$$

Decomposing a Kernel Matrix by Rows

- A kernel matrix $K_{n\times n}$ of *any* rank can always be decomposed as

$$K = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} * [r_1] + \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} * [r_2] + \cdots + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} * [r_n],$$

where r_i is the ith row of K.

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- Compare this to a single LUT, which requires only one look-up operation, but the table contains 2^{8n^2} entries!
- Even when $n = 3$, this is the difference between one 4-zebibyte table (≈1% of one mole of bytes) and three 16MB tables!

Decomposing a Kernel Matrix by Rows

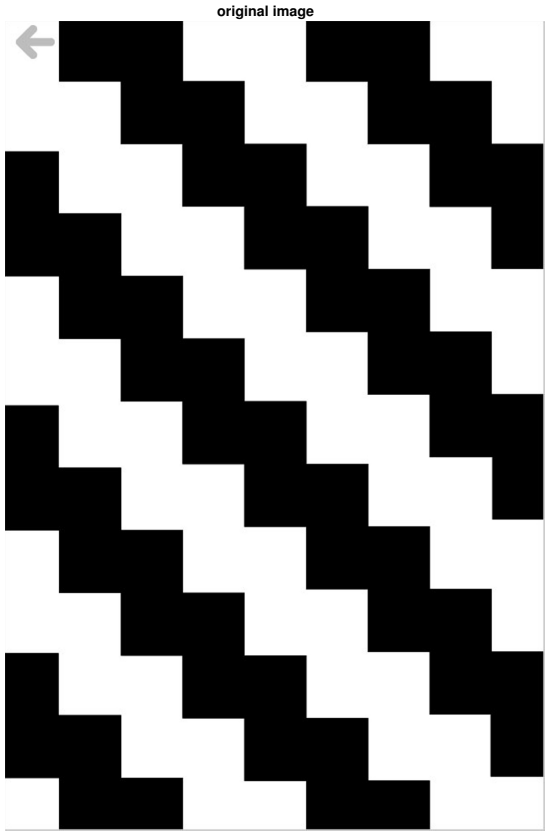

Figure: Original Image

Decomposing a Kernel Matrix by Rows

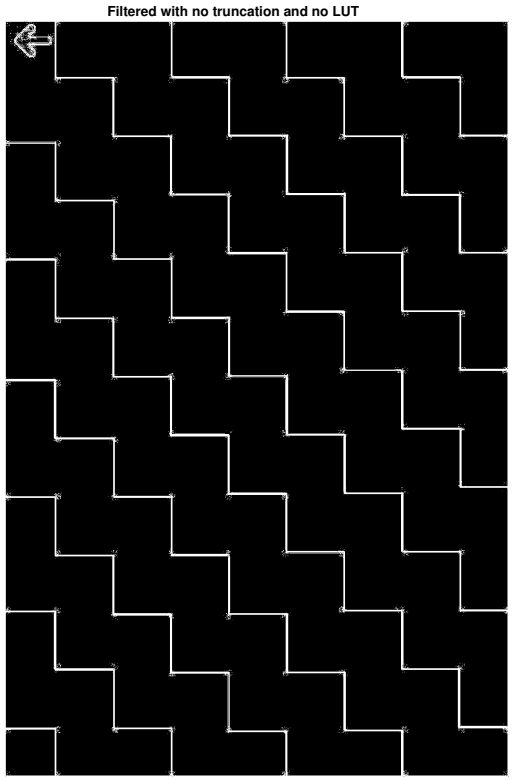

Figure: Image after HP filter

Decomposing a Kernel Matrix by Rows

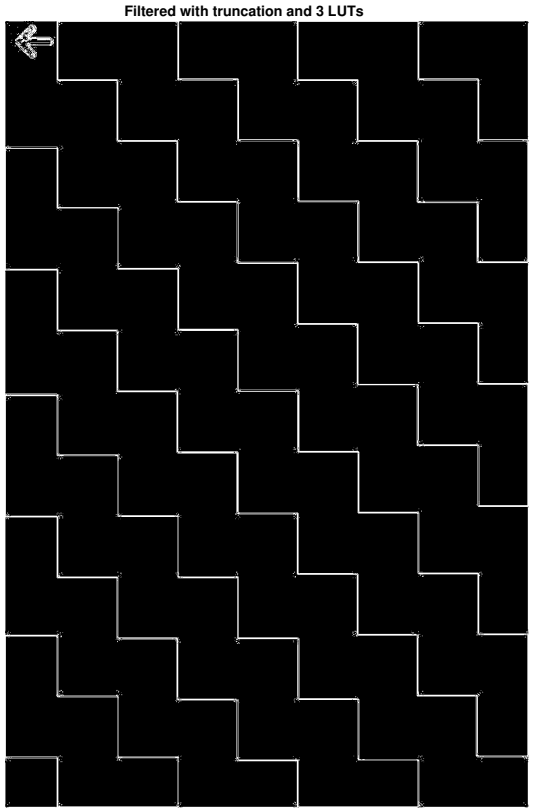

Figure: Image after HP filter with 3 LUT's containing 16, 4096, and 16 entries

Truncation and Blurring Filter (using Edge-detection filter)

Observation

1. The truncation method can be used to reduce running time.
2. Truncation makes some errors such as staircase effects for blurring, but is decent for edge-detecting.

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Goal

Take advantage of the truncation method avoiding staircase effects!

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IDEA

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$$L \approx I - H(T)$$

Truncation and Blurring Filter (using Edge-detection filter)

Let

$$L = \frac{1}{8}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } T = \begin{bmatrix} 8 & 6 & 8 \\ 6 & 4 & 6 \\ 8 & 6 & 8 \end{bmatrix}.$$

(a) Low-pass filter, L (b) Truncation, $L(T)$ (c) New Idea, $I - H(T)$

Figure: Comparison of filtered images

The table shows us how $L(T_i)$ and $I - H(T_i)$ are different from the low-pass filter $L = I - H$ for each i. $L(T_i)$ is Old and $I - H(T_i)$ is New in the table.

	T_1		T_2		T_3	
	Old	New	Old	New	Old	New
PSNR	38.12	39.58	30.80	32.84	22.59	25.59
l_2 - error	3.06	2.80	7.40	5.87	18.87	13.52
l_∞ - error	8.42	7.40	19.64	15.81	43.35	35.70

Table: Comparison of errors

Further Directions

1. Refine convolution decomposition
2. SVD (singular value decomposition)

Thank you!