Project

Johanna Günther

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1.94

Use induction to prove Pascal's formula.

Pascal's formula

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

Using induction in n.

Base case:

$\binom{0}{0} \overset{\text{i*}}{=} 1$

$$\binom{0}{0} = \frac{0!}{0!(0-0)!} = \frac{1}{1 \cdot 1} = 1$$

1 Induction hypothesis

$\forall m, n \in \mathbf{N}$ and $0 < m \leq n$
When $m = 0$ it is by defenition* $= 1$ and then the formula works.

Assume:

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

*By defenition of a binom

Inductive step:

$$Prove: \binom{n+1}{m} = \frac{(n+1)!}{m!((n+1)-m)!}$$

$$\begin{aligned}
\binom{n+1}{m} &\overset{\text{ii*}}{=} \binom{n}{m-1} + \binom{n}{m} \\
&\overset{\text{IH}}{=} \frac{n!}{(m-1)!(n-(m-1))!} + \frac{n!}{m!(n-m)!} \\
&= \frac{n!}{(m-1)!(n-m+1)!} + \frac{n!}{m(m-1)!(n-m)!} \\
&= \frac{n!}{(m-1)!(n-m+1)(n-m)!} + \frac{n!}{m(m-1)!(n-m)!} \\
&= \frac{n!}{(m-1)!(n-m)!}\left(\frac{1}{n-m+1} + \frac{1}{m}\right) \\
&= \frac{n!}{(m-1)!(n-m)!}\left(\frac{m}{m(n-m+1)} + \frac{n-m+1}{m(n-m+1)}\right) \\
&= \frac{n!}{(m-1)!(n-m)!} \cdot \frac{m+n-m+1}{m(n-m+1)} \\
&= \frac{n!(n+1)}{m(m-1)!(n-m+1)(n-m)!} \\
&= \frac{(n+1)!}{m!(n-m+1)!} \\
&= \frac{(n+1)!}{m!((n+1)-m)!}
\end{aligned}$$

Since we proved that the formula works for the base case $\binom{0}{0}$ we can now conclude that it works for all numbers $n \in \mathbf{N}$.When $m \in \mathbf{N} : 0 < m \leq\ n$.

Discuss

a.

How you found your solution.

I am not quite sure how I found my solution. I think I found it by thinking about the concept of proof by induction. However we have also discussed it during a seminar which I attended so I might have just remembered how to do it from there.

b.

If you understand the solution.

I think I understand the solution. Even though I understand how proof by induction works it feels a bit unproven. However I am not sure if I understand why I can use the formula for $\binom{n}{m-1}$. How do I know that it is going to work the same when $m = m - 1$ or am I just assuming that as well? In that case why do I not have to prove that as well? We have discussed it in mentor meeting now so I think I get it.

c.

What your current approach to solving challenging problems are.

My current approach to solving difficult problems are to think about them. Then read the appropriate chapter and discuss with someone else.

d.

If you find working on challenging mathematical problems enjoyable or not.

I find working on mathematical problems enjoyable. Usually I think I prefer when I do not have to count (only using constants and variables as letters) although it is annoying having a lot to wright.

Approved by: Marko Proloscic