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Tugas 5

Studi Kasus 5

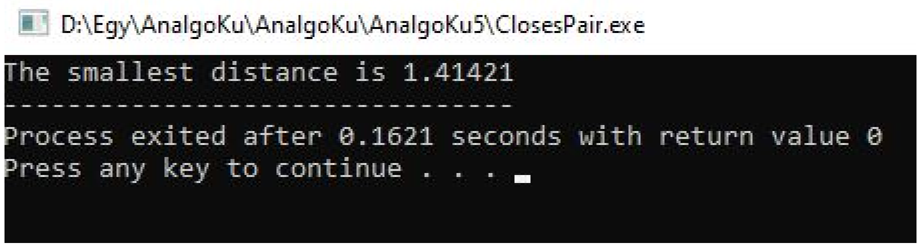
Program :

```
#include <bits/stdc++.h>
using namespace std;

// A structure to represent a Point in 2D plane
class Point
{
	public:
	int x, y;
};

/* Following two functions are needed for library function qsort().
Refer: http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */

// Needed to sort array of points
// according to X coordinate
int compareX(const void* a, const void* b)
{
	Point *p1 = (Point *)a, *p2 = (Point *)b;
	return (p1->x - p2->x);
}

// Needed to sort array of points according to Y coordinate
int compareY(const void* a, const void* b)
{
	Point *p1 = (Point *)a, *p2 = (Point *)b;
	return (p1->y - p2->y);
}
```

```
// A utility function to find the
// distance between two points
float dist(Point p1, Point p2)
{
	return sqrt( (p1.x - p2.x)*(p1.x - p2.x) +
				(p1.y - p2.y)*(p1.y - p2.y)
			);
}

// A Brute Force method to return the
// smallest distance between two points
// in P[] of size n
float bruteForce(Point P[], int n)
{
	float min = FLT_MAX;
	for (int i = 0; i < n; ++i)
		for (int j = i+1; j < n; ++j)
			if (dist(P[i], P[j]) < min)
				min = dist(P[i], P[j]);
	return min;
}

// A utility function to find
// minimum of two float values
float min(float x, float y)
{
	return (x < y)? x : y;
}


// A utility function to find the
// distance beween the closest points of
// strip of given size. All points in
// strip[] are sorted accordint to
// y coordinate. They all have an upper
// bound on minimum distance as d.
// Note that this method seems to be
// a O(n^2) method, but it's a O(n)
```

```
// method as the inner loop runs at most 6 times
float stripClosest(Point strip[], int size, float d)
{
	float min = d; // Initialize the minimum distance as d

	qsort(strip, size, sizeof(Point), compareY);

	// Pick all points one by one and try the next points till the difference
	// between y coordinates is smaller than d.
	// This is a proven fact that this loop runs at most 6 times
	for (int i = 0; i < size; ++i)
		for (int j = i+1; j < size && (strip[j].y - strip[i].y) < min; ++j)
			if (dist(strip[i],strip[j]) < min)
				min = dist(strip[i], strip[j]);

	return min;
}

// A recursive function to find the
// smallest distance. The array P contains
// all points sorted according to x coordinate
float closestUtil(Point P[], int n)
{
	// If there are 2 or 3 points, then use brute force
	if (n <= 3)
		return bruteForce(P, n);

	// Find the middle point
	int mid = n/2;
	Point midPoint = P[mid];

	// Consider the vertical line passing
	// through the middle point calculate
	// the smallest distance dl on left
	// of middle point and dr on right side
	float dl = closestUtil(P, mid);
	float dr = closestUtil(P + mid, n - mid);

	// Find the smaller of two distances
```

```
	float d = min(dl, dr);

	// Build an array strip[] that contains
	// points close (closer than d)
	// to the line passing through the middle point
	Point strip[n];
	int j = 0;
	for (int i = 0; i < n; i++)
		if (abs(P[i].x - midPoint.x) < d)
			strip[j] = P[i], j++;

	// Find the closest points in strip.
	// Return the minimum of d and closest
	// distance is strip[]
	return min(d, stripClosest(strip, j, d) );
}

// The main functin that finds the smallest distance
// This method mainly uses closestUtil()
float closest(Point P[], int n)
{
	qsort(P, n, sizeof(Point), compareX);

	// Use recursive function closestUtil()
	// to find the smallest distance
	return closestUtil(P, n);
}

// Driver code
int main()
{
	Point P[] = {{2, 3}, {12, 30}, {40, 50}, {5, 1}, {12, 10}, {3, 4}};
	int n = sizeof(P) / sizeof(P[0]);
	cout << "The smallest distance is " << closest(P, n);
	return 0;
}
```



2. Tentukan rekurensi dari algoritma tersebut, dan selesaikan rekurensinya menggunakan metode recursion tree untuk membuktikan bahwa algoritma tersebut memiliki Big-O (n lg n)

Biarkan kompleksitas waktu dari algoritma di atas menjadi T(n). Mari kita asumsikan bahwakita menggunakan algoritma pengurutan O (nLogn). Algoritma di atas membagi semua titik dalam dua set dan secara rekursif memanggil dua set. Setelah membelah, ia menemukan strip dalam waktu O (n), mengurutkan strip dalam waktu O (nLogn) dan akhirnya menemukantitik terdekat dalam strip dalam waktu O (n)

T (n) = 2T (n / 2) + O (n) + O (nLogn) + O (n)
T (n) = 2T (n / 2) + O (nLogn)
T (n) = T
(n x Logn x Logn)

Studi Kasus 6

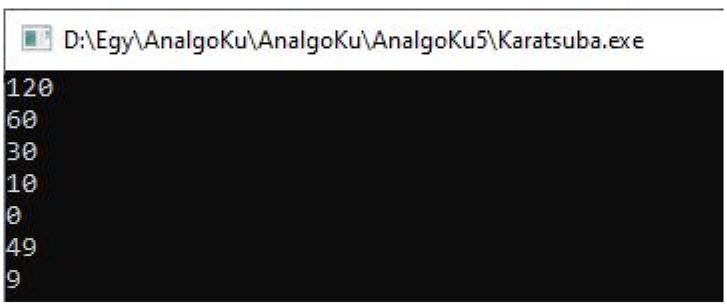
Program :

```
#include<iostream>
#include<stdio.h>

using namespace std;

// FOLLOWING TWO FUNCTIONS ARE COPIED FROM http://goo.gl/q0OhZ
// Helper method: given two unequal sized bit strings, converts them to
// same length by adding leading 0s in the smaller string. Returns the
// the new length
int makeEqualLength(string &str1, string &str2)
{
	int len1 = str1.size();
```

```
	int len2 = str2.size();
	if (len1 < len2)
	{
		for (int i = 0 ; i < len2 - len1 ; i++)
			str1 = '0' + str1;
		return len2;
	}
	else if (len1 > len2)
	{
		for (int i = 0 ; i < len1 - len2 ; i++)
			str2 = '0' + str2;
	}
	return len1; // If len1 >= len2
}

// The main function that adds two bit sequences and returns the addition
string addBitStrings( string first, string second )
{
	string result; // To store the sum bits

	// make the lengths same before adding
	int length = makeEqualLength(first, second);
	int carry = 0; // Initialize carry

	// Add all bits one by one
	for (int i = length-1 ; i >= 0 ; i--)
	{
		int firstBit = first.at(i) - '0';
		int secondBit = second.at(i) - '0';

		// boolean expression for sum of 3 bits
		int sum = (firstBit ^ secondBit ^ carry)+'0';

		result = (char)sum + result;

		// boolean expression for 3-bit addition
		carry = (firstBit&secondBit) | (secondBit&carry) | (firstBit&carry);
	}
```

```
        // if overflow, then add a leading 1
        if (carry) result = '1' + result;

        return result;
}

// A utility function to multiply single bits of strings a and b
int multiplyiSingleBit(string a, string b)
{ return (a[0] - '0')*(b[0] - '0'); }

// The main function that multiplies two bit strings X and Y and returns
// result as long integer
long int multiply(string X, string Y)
{
        // Find the maximum of lengths of x and Y and make length
        // of smaller string same as that of larger string
        int n = makeEqualLength(X, Y);

        // Base cases
        if (n == 0) return 0;
        if (n == 1) return multiplyiSingleBit(X, Y);

        int fh = n/2; // First half of string, floor(n/2)
        int sh = (n-fh); // Second half of string, ceil(n/2)

        // Find the first half and second half of first string.
        // Refer http://goo.gl/lLmgn for substr method
        string Xl = X.substr(0, fh);
        string Xr = X.substr(fh, sh);

        // Find the first half and second half of second string
        string Yl = Y.substr(0, fh);
        string Yr = Y.substr(fh, sh);

        // Recursively calculate the three products of inputs of size n/2
        long int P1 = multiply(Xl, Yl);
        long int P2 = multiply(Xr, Yr);
        long int P3 = multiply(addBitStrings(Xl, Xr), addBitStrings(Yl, Yr));
```

```
        // Combine the three products to get the final result.
        return P1*(1<<(2*sh)) + (P3 - P1 - P2)*(1<<sh) + P2;
}

// Driver program to test aboev functions
int main()
{
        printf ("%ld\n", multiply("1100", "1010"));
        printf ("%ld\n", multiply("110", "1010"));
        printf ("%ld\n", multiply("11", "1010"));
        printf ("%ld\n", multiply("1", "1010"));
        printf ("%ld\n", multiply("0", "1010"));
        printf ("%ld\n", multiply("111", "111"));
        printf ("%ld\n", multiply("11", "11"));
}
```



2. Rekurensi dari algoritma tersebut adalah T (n) = 3T (n / 2) + O (n), dan selesaikan rekurensinya menggunakan metode substitusi untuk membuktikan bahwa algoritma tersebut memiliki Big-O (n lg n)

- Let's try divide and conquer.
  - Divide each number into two halves.
    - $x = x_H r^{n/2} + x_L$
    - $y = y_H r^{n/2} + y_L$
  - Then:
    $$xy = (x_H r^{n/2} + x_L)\, y_H r^{n/2} + y_L$$
    $$= x_H y_H r^n + (x_H y_L + x_L y_H) r^{n/2} + x_L y_L$$
  - Runtime?
    - T(n) = 4 T(n/2) + O(n)
    - T(n) = O(n^2)
- Instead of 4 subproblems, we only need 3 (with the help of clever insight).
- Three subproblems:
  - $a = x_H y_H$
  - $d = x_L y_L$
  - $e = (x_H + x_L)(y_H + y_L) - a - d$
- Then $xy = a r^n + e r^{n/2} + d$
- T(n) = 3 T(n/2) + O(n)
- $T(n) = O(n^{\log 3}) = O(n^{1.584...})$

Studi Kasus 7

Program :

2. Relasi rekurensi untuk algoritma rekursif di atas dapat ditulis seperti di bawah ini. C adalah konstanta. T (n) = 4T (n / 2) + C. Selesaikan rekurensi tersebut dengan Metode Master

Kompleksitas Waktu:
Relasi perulangan untuk algoritma rekursif di atas dapat ditulis seperti di bawah ini. C adalah konstanta.
T (n) = 4T
(n / 2) + C
Rekursi di atas dapat diselesaikan dengan menggunakan Metode Master dan kompleksitas waktu adalah O (n2)