Sipser. pp. 80-82.

1. Show $A = \{0^n 1^n \mid n \geq 0\}$ is nonregular.

Assume for contradiction A is regular, so A satisfies the Pumping Lemma. In this case, we know that there exists a number p such that all strings $s \in A$, $|s| \geq p$, can be divided into three substrings $s = xyz$ s.t.

(1) $xy^iz \in A$ for all $i \geq 0$.
(2) $|y| > 0$
(3) $|xy| \leq p$.

Pick $s = 0^p 1^p$. $s \in A$, $|s| \geq p$.

By (3), $|xy| \leq p$, which means x and y are made up of 0's. By (2), $|y| > 0$, so y contains at least one zero.

$$s = \underbrace{0000\ldots\ldots}_{x}\underbrace{000}_{y}\underbrace{0111\ldots11111}_{z}$$

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Now consider $xy^2z = xyyz$, which is in A by (1)

$xyyz = 0^{p+|y|}1^p \notin A$.

Therefore A does not satisfy the pumping lemma, so our assumption leads to a contradiction and A is nonregular □

2. Show $B = \{0^{n^2} \mid n \geq 0\}$ is nonregular.

- Assume for contradiction that B is regular
  ↳ B satisfies the PL.
- This means $\exists\, p$ such that $s \in B$, $|s| \geq p$ can

be divided into x, y, z such that

(1) $xy^iz \in B$ for all $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$.

- 0^{p^2} will be our string s. $s \in B$, $|s| \geq p$.
  - x, y are all zeroes.
  - $|y| > 0$.
  - $|xy| \leq p \Rightarrow |y| \leq p$.
  - now let's consider $xyyz$.

$$xyyz = 0^{p^2 + |y|}$$

- $p^2 + 1 \leq p^2 + |y| \leq p^2 + p$
- $(p+1)^2 = p^2 + 2p + 1$
- $p^2 < \underline{p^2 + |y|} < (p+1)^2$

$\therefore$ $xyyz \notin B$. So B fails the PL, and B must be nonregular. $\square$