

# CSEE 3827: Fundamentals of Computer Systems, Spring 2022

## Lecture 10

Prof. Dan Rubenstein ([danr@cs.columbia.edu](mailto:danr@cs.columbia.edu))

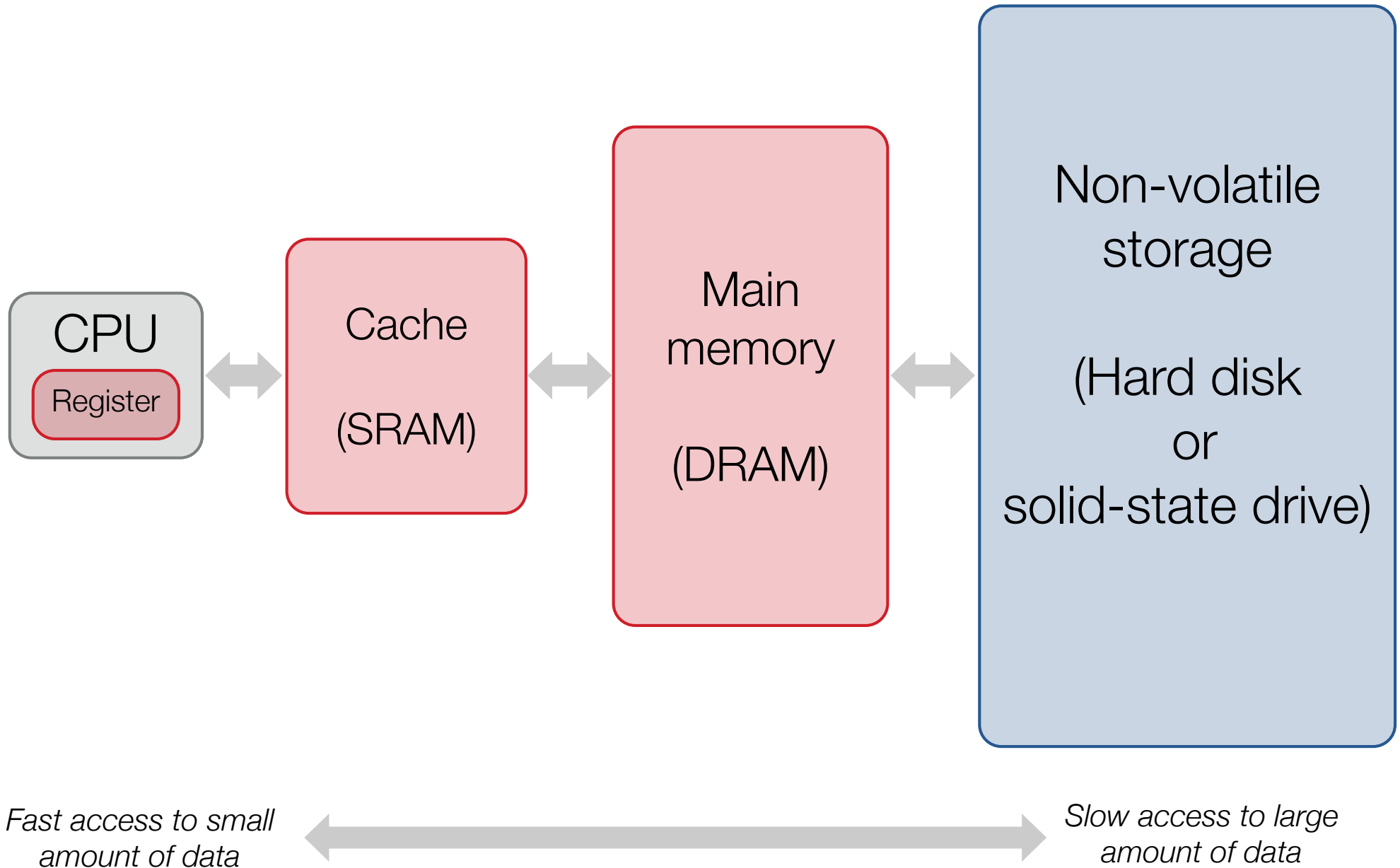
# Agenda (M&K 7.1-7.4)

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- Memory

# Storage hierarchy

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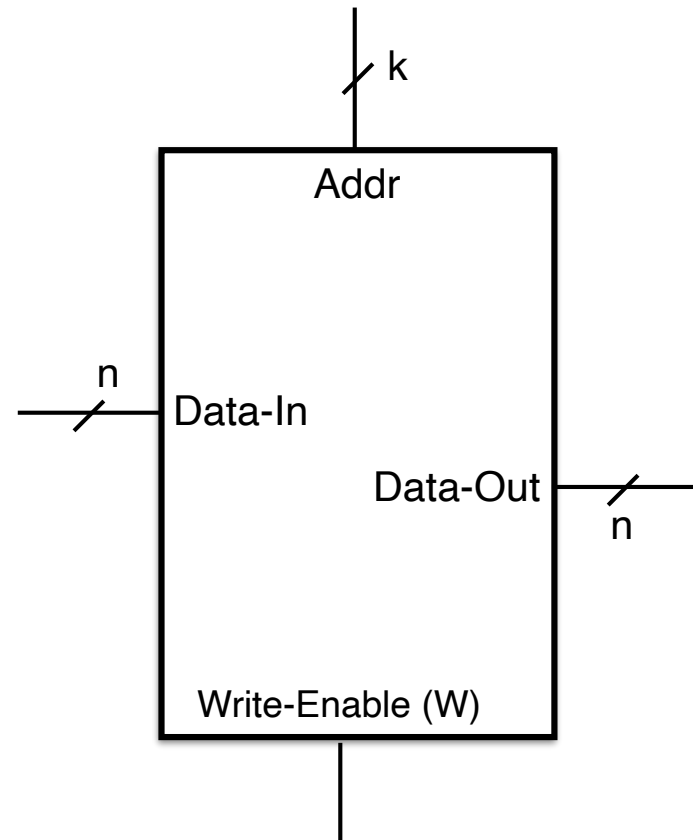


# Using Memory

# General Memory interface

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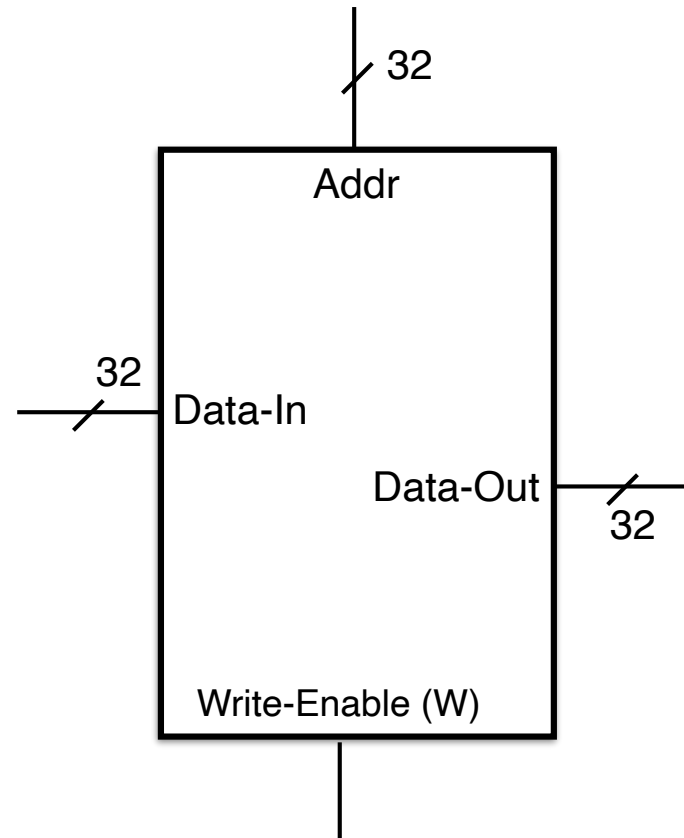
- k-bit addresses ( $2^k$  addresses), n-bit data
- **Read from memory:**
  - Specify a (k-bit) address, A
  - Data-out outputs the n-bit contents in memory at address, A
- **Write to memory:**
  - Specify a (k-bit) address, A
  - Through Data-In, supply n-bit data to write at address A
  - set Write-Enable to 1
- Careful, if wanting to read and Write-Enable set to 1, will also be writing.



# MIPS-specific

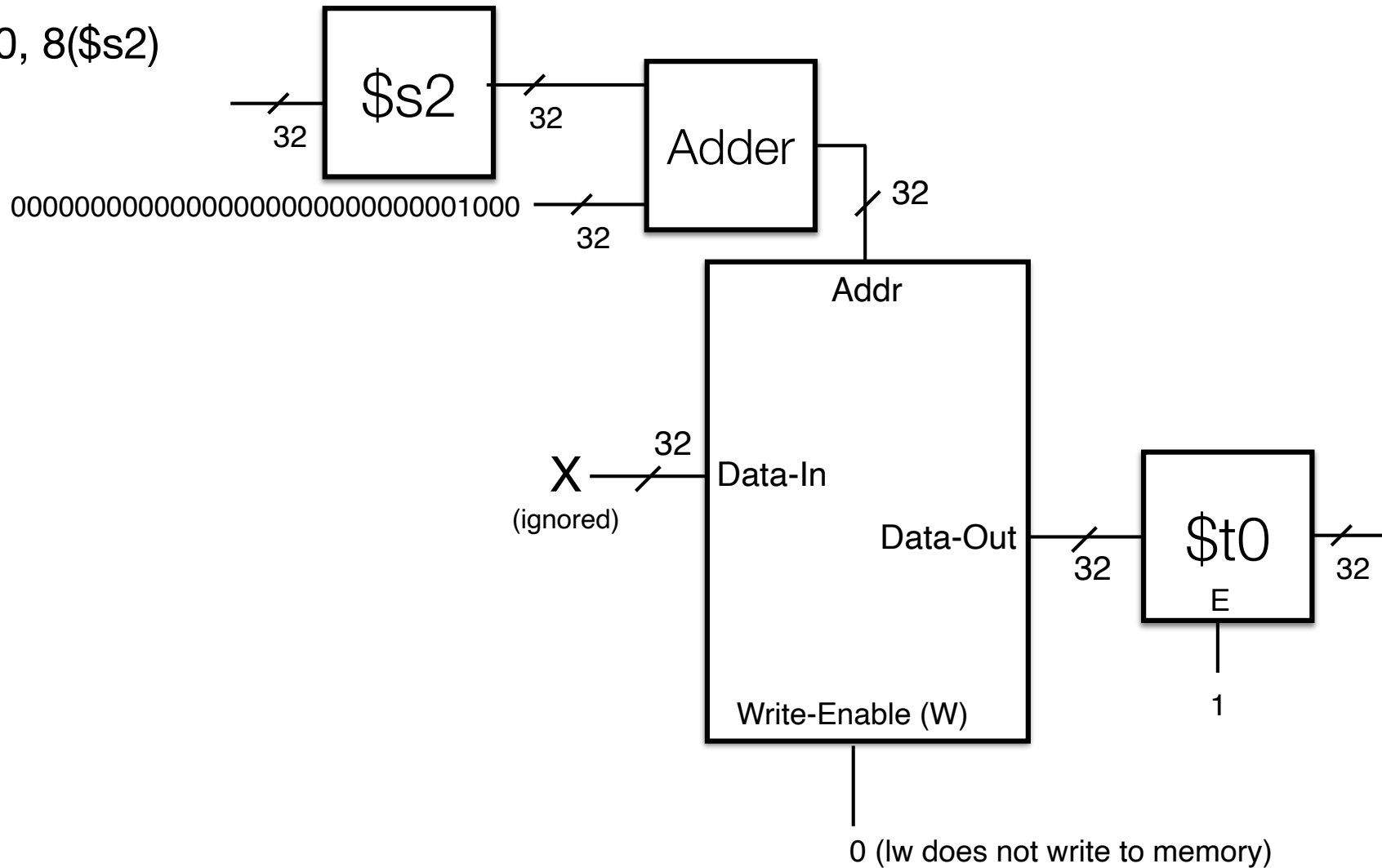
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- $k=32$ : 32-bit addresses ( $2^{32}$  byte-size addressable slots)
- $n=32$ : we access memory 32-bits at a time
- 32-bit address always multiple of 4 (2 least significant bits always “00”)



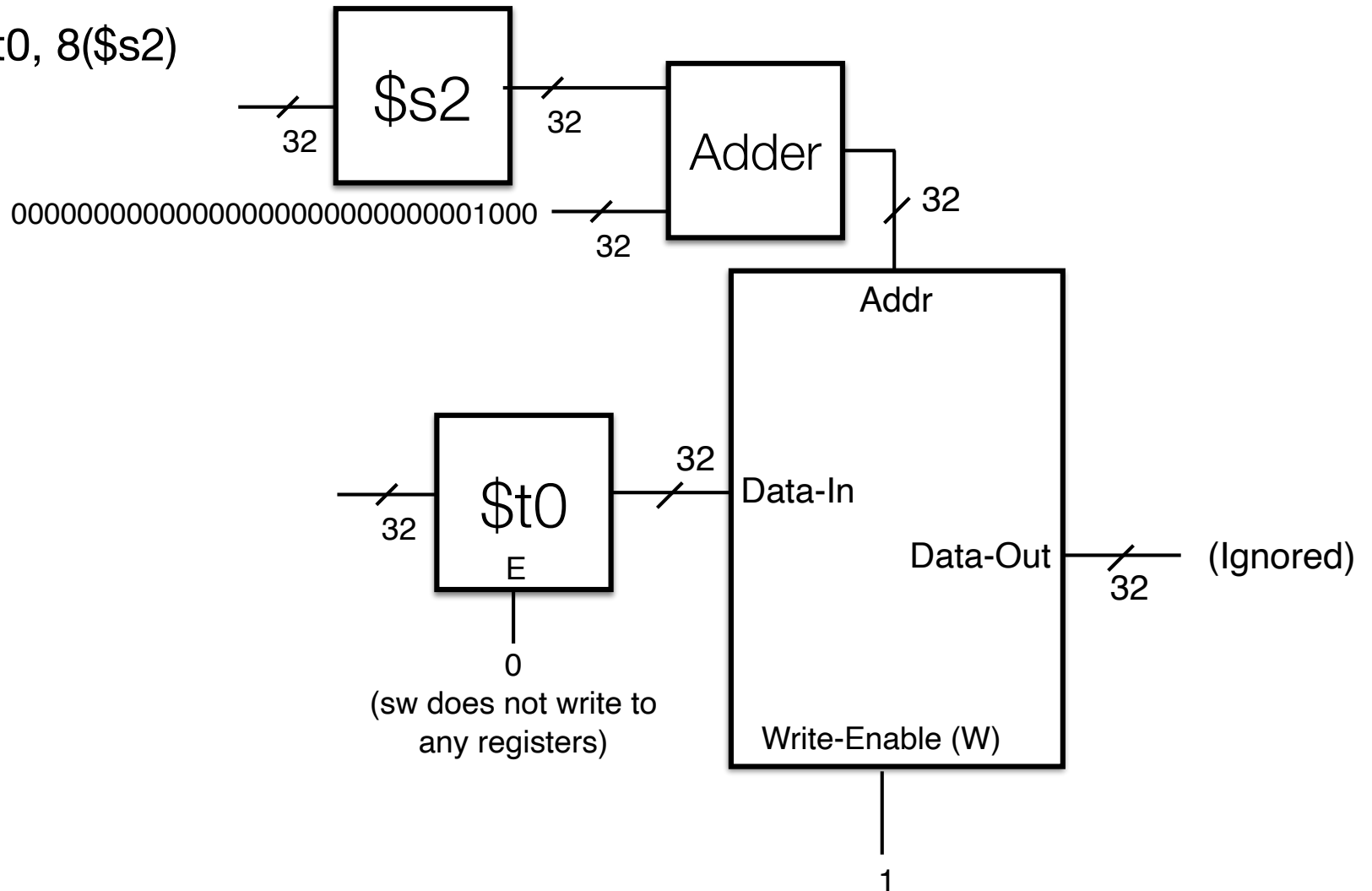
# MIPS-specific read example

- e.g., lw \$t0, 8(\$s2)



# MIPS-specific read example

- e.g., `sw $t0, 8($s2)`

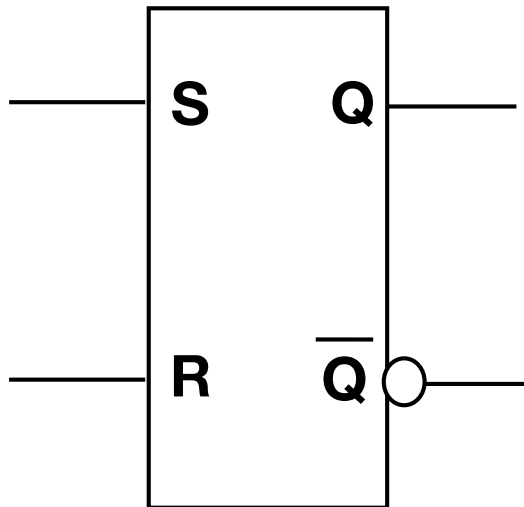




# Building Memory

# Recall: SR Latch

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S	R	Q	$\overline{Q}$
0	0	Q	$\overline{Q}$
0	1	0	1
1	0	1	0
1	1	0	0

Hold value

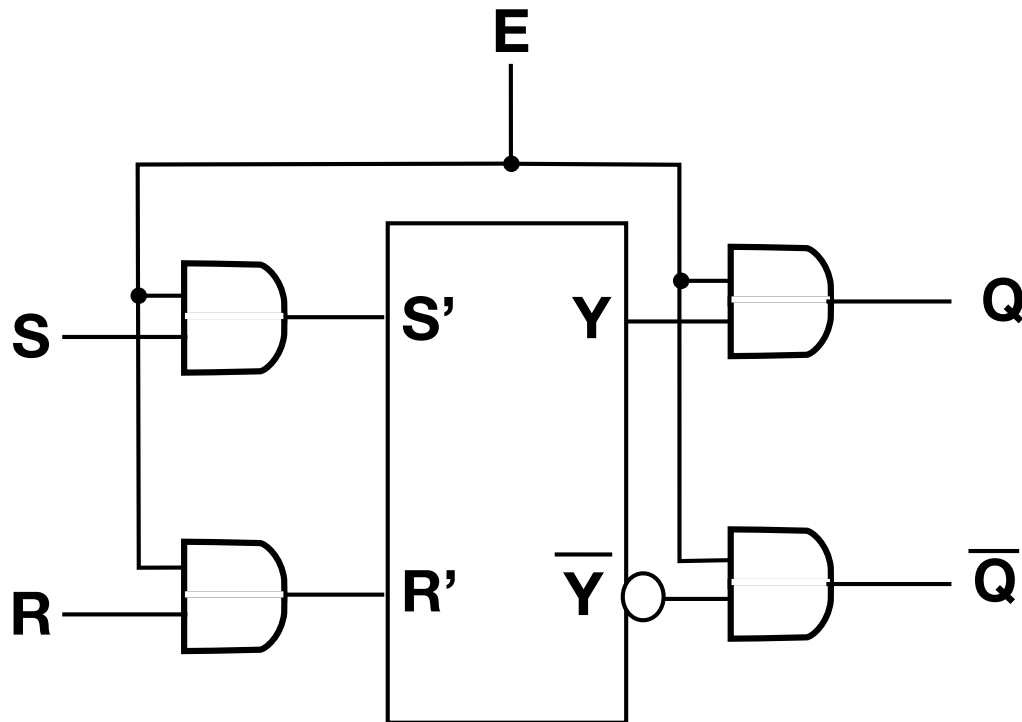
Reset

Set

# Single (1-bit) Memory Cell

# Memory “Cell”

- memory storage of a single bit
- built from SR-latch and some enablers (AND gates)



E	S	R	Y	$\bar{Y}$	Q	$\bar{Q}$
0	X	X	Y	$\bar{Y}$	0	0
1	0	0	Y	$\bar{Y}$	Y	$\bar{Y}$
1	0	1	0	1	0	1
1	1	0	1	0	1	0
1	1	1	0	0	0	0

Disable Inputs and  
Zero Outputs

Hold Value

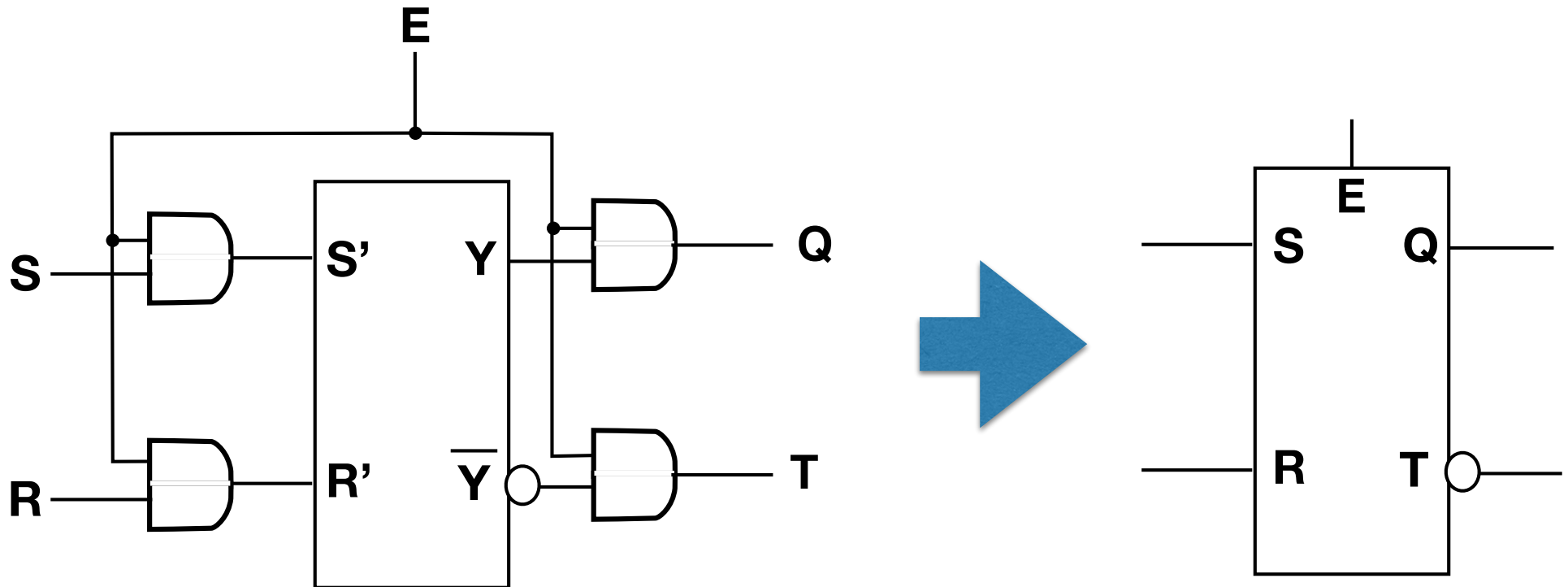
Reset (to 0)

Set (to 1)

Note: Setting  $E=0$  does not erase value in latch, just “zeros” the value being output

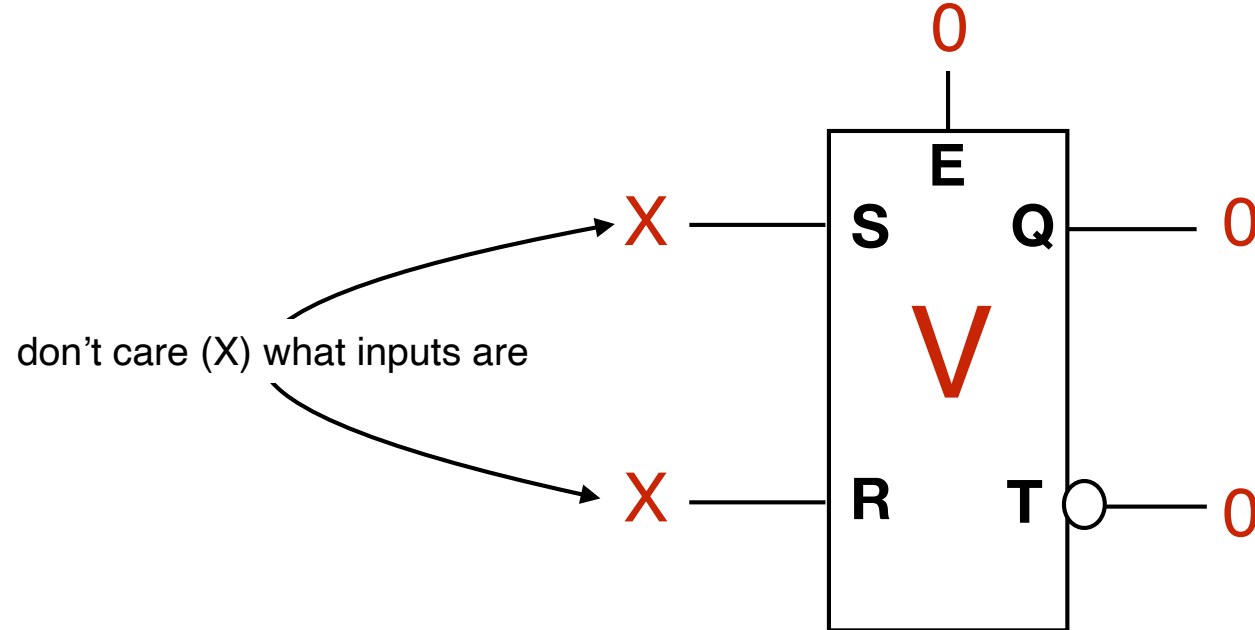
# Memory “Cell”

- memory storage of a single bit
- built from SR-latch and some enablers (AND gates)



# Disable a Cell

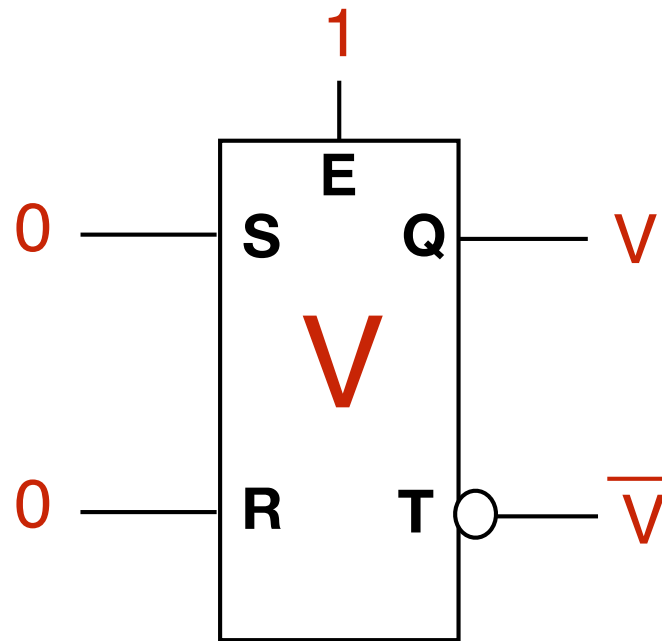
- Set  $E=0$ ,  $S=X$ ,  $R=X$
- Output not 0'd, cell (1-bit) storage unchanged



# Read from a Cell

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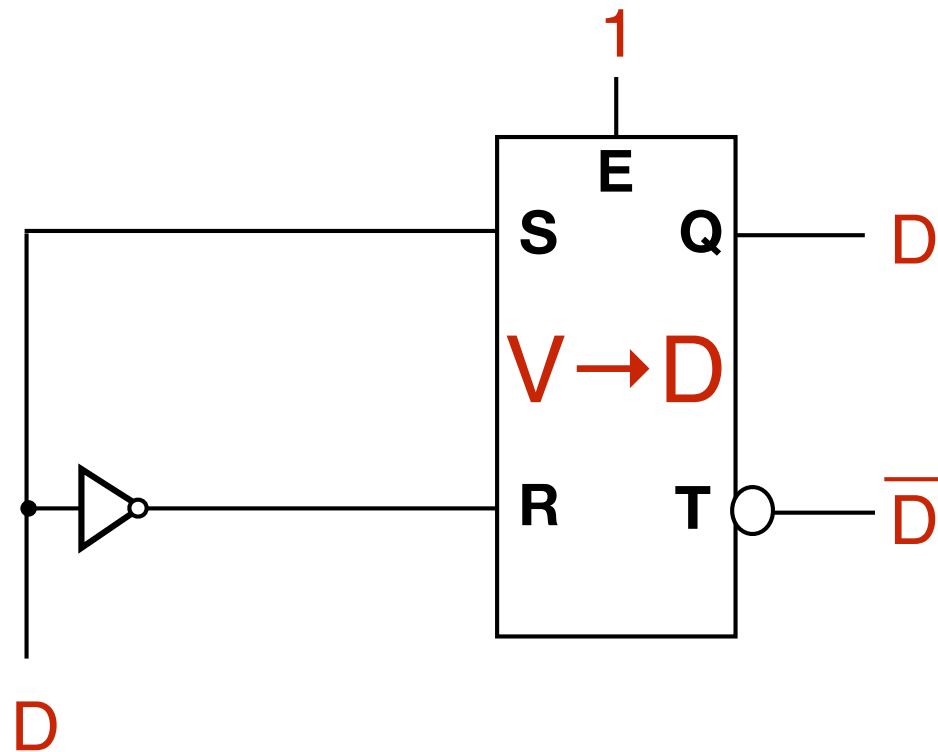
- Set  $E=1$ ,  $S=R=0$
- Output not 0'd, cell (1-bit) storage unchanged



# Write to a Cell

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- To write a data bit  $D$  into the cell
  - Enable the cell ( $E=1$ )
  - Set  $S=D$ ,  $R=\overline{D}$

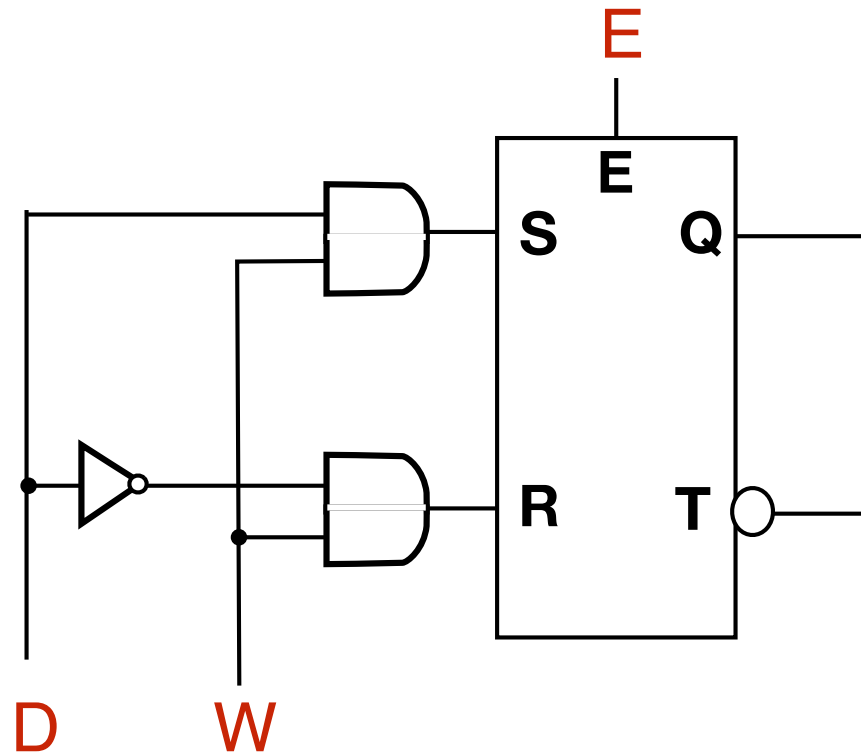




# Reading or Writing

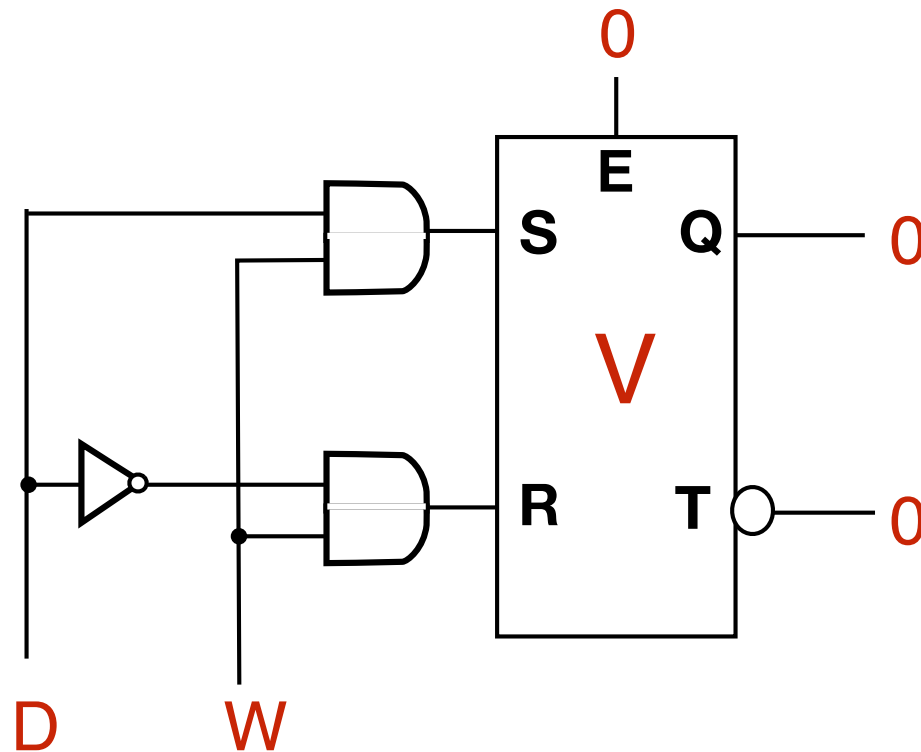
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- $E=0$ , ignore all inputs (outputs both 0)
- $E=1$ 
  - $W=0$ : output bit stored in cell
  - $W=1$ , write  $D$  into cell (also the output of the cell)



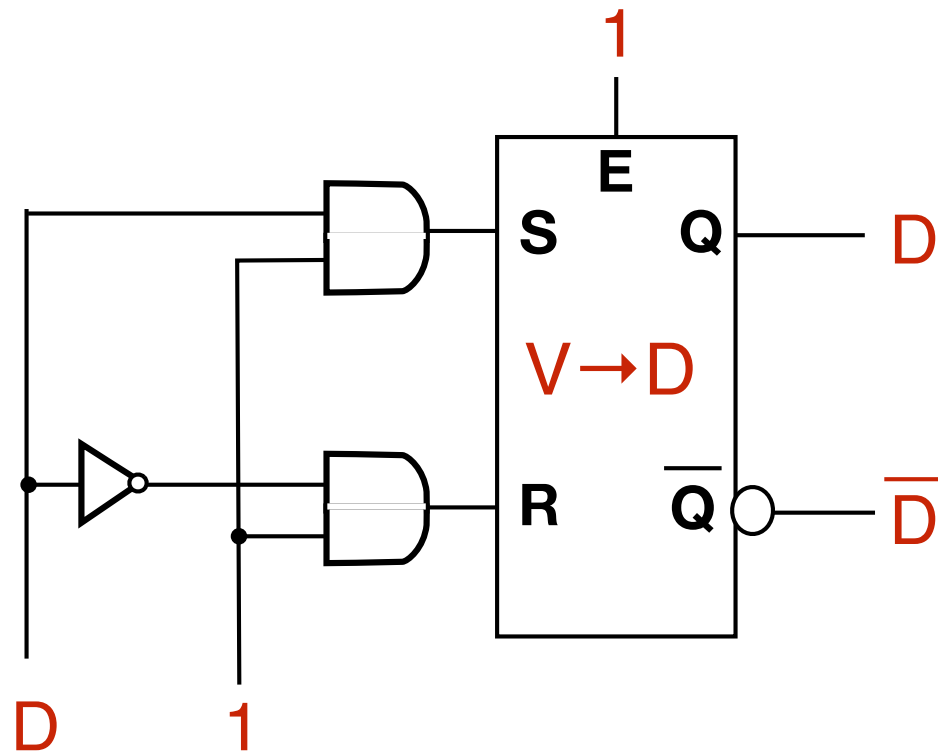
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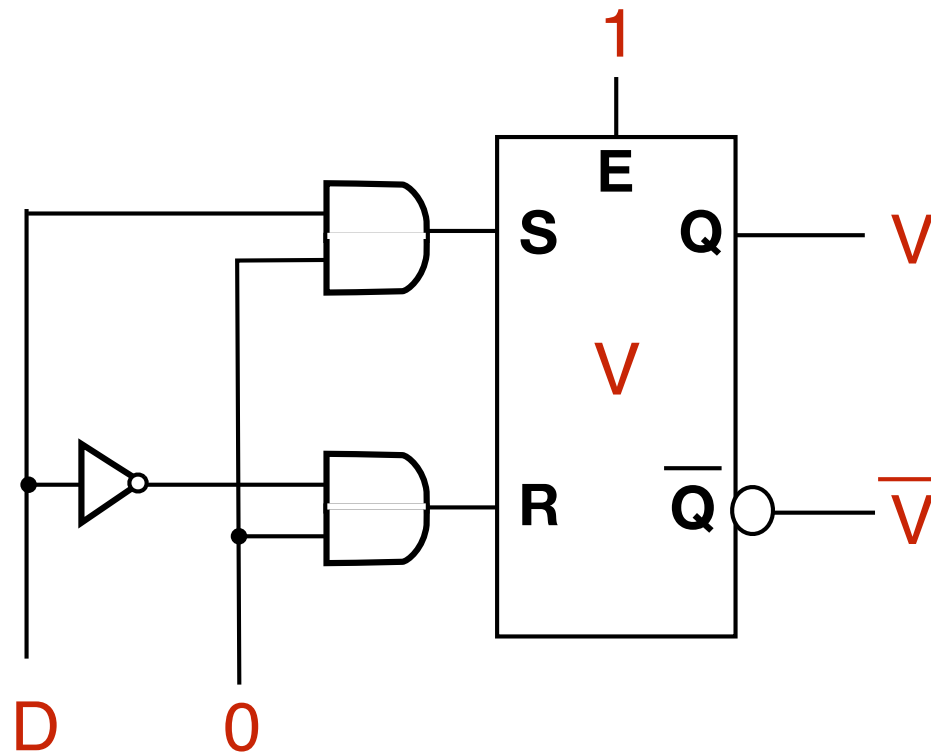
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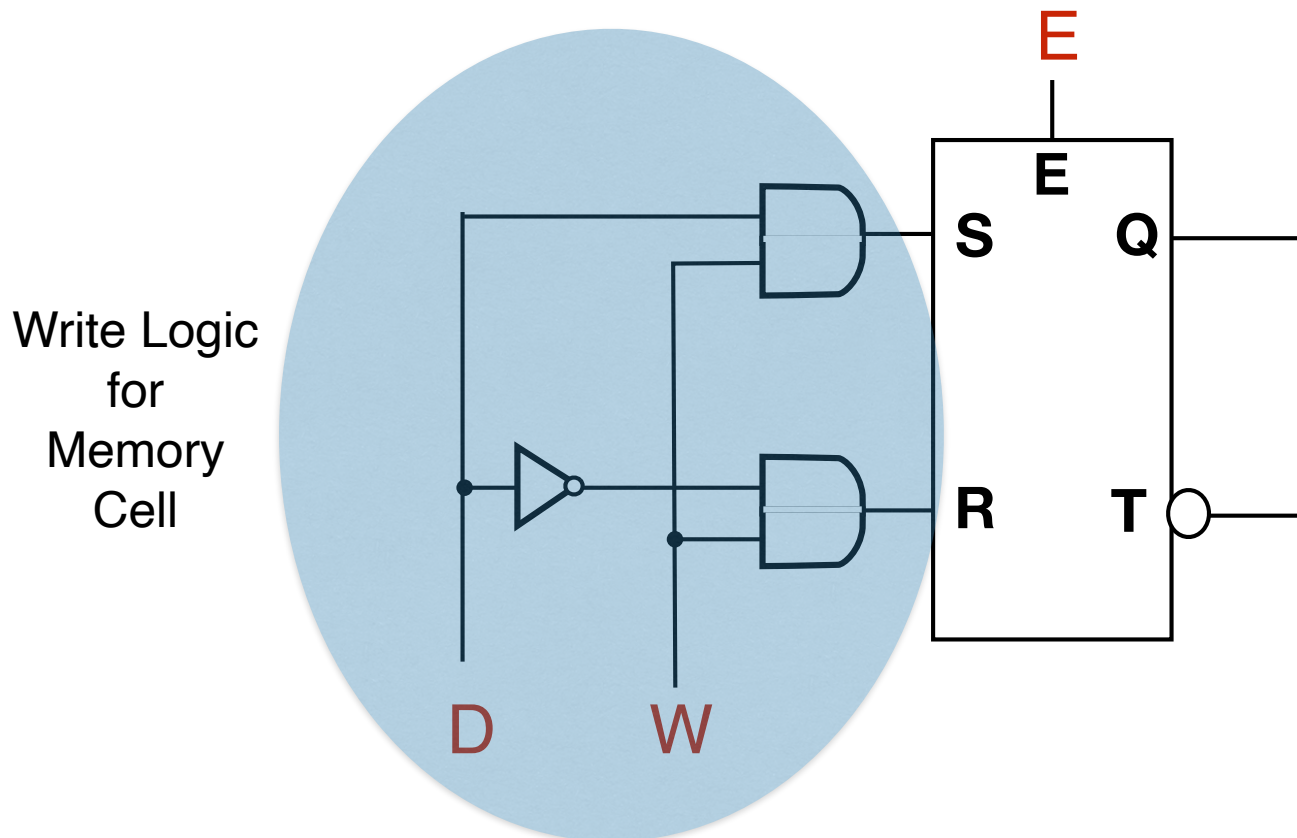
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**Memory Word**  
**(MIPS: 32 bits)**

# Word of MIPS Memory

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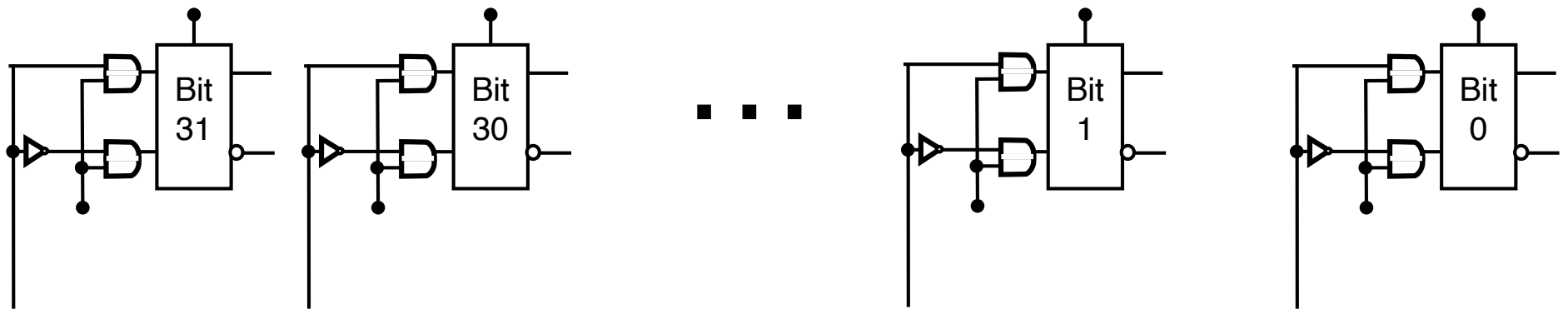
- memory storage of a single bit
- built from SR-latch and some enablers (AND gates)



32 latches w/ Enable

# Word of MIPS Memory

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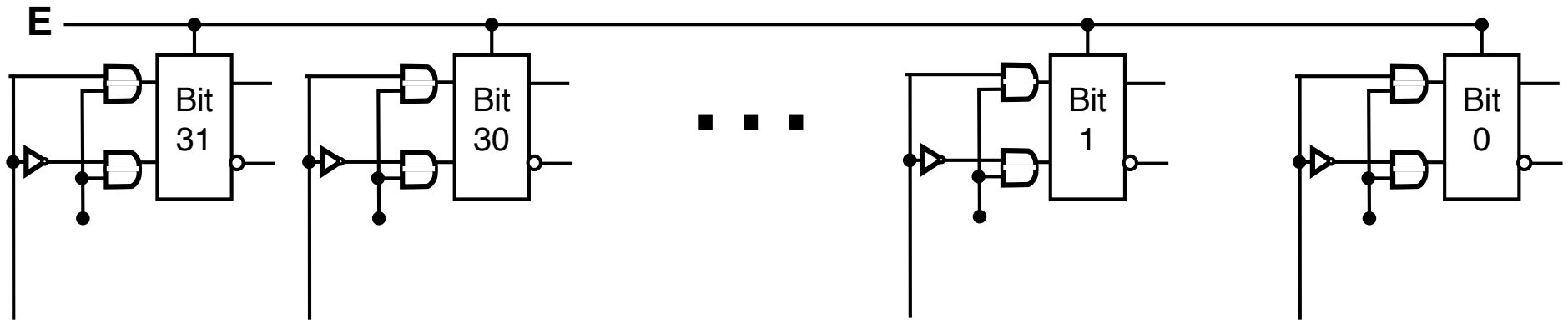
Attach Write Logic to each



# Word of MIPS Memory

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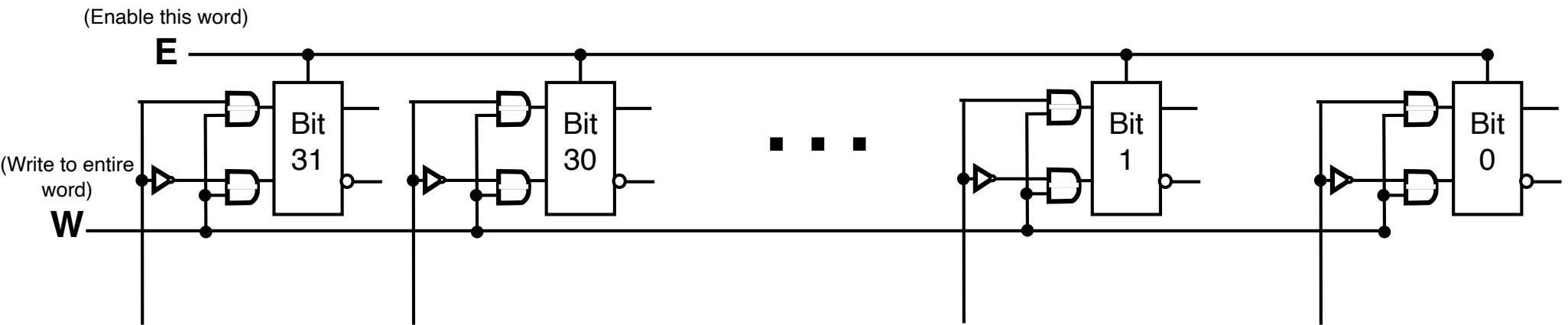
(Enable this word)



Attach to same enable signal  
(Enable all 32 or none)

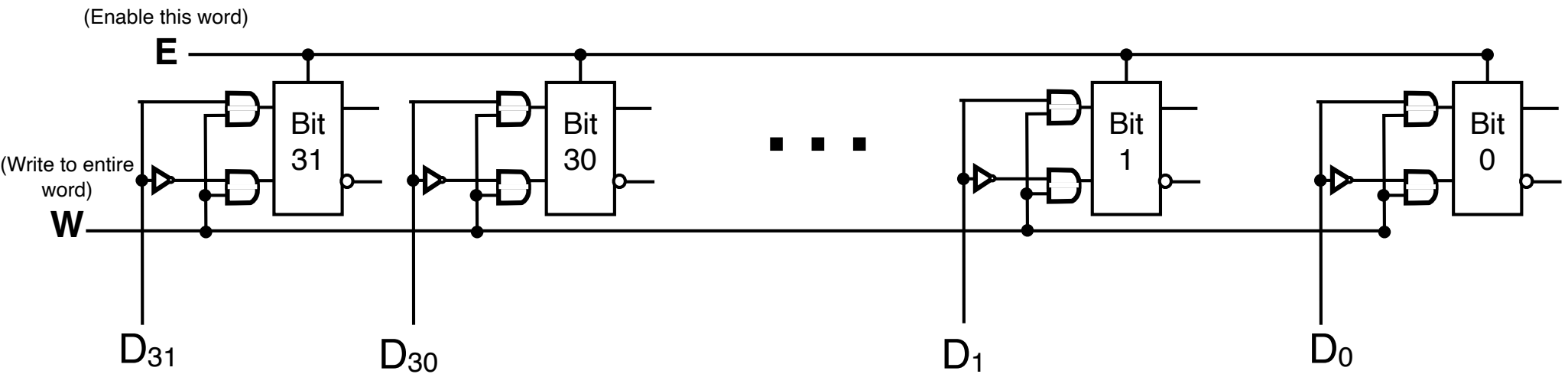
# Word of MIPS Memory

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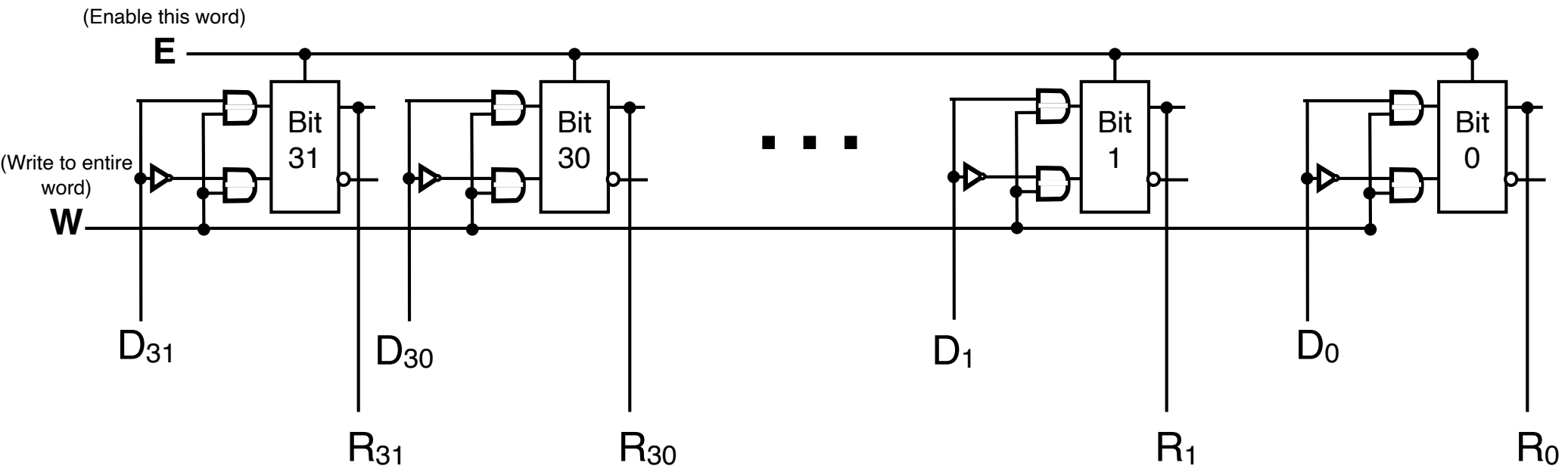
Attach to same write signal  
(write to all 32 (when enabled) or none)

# Word of MIPS Memory



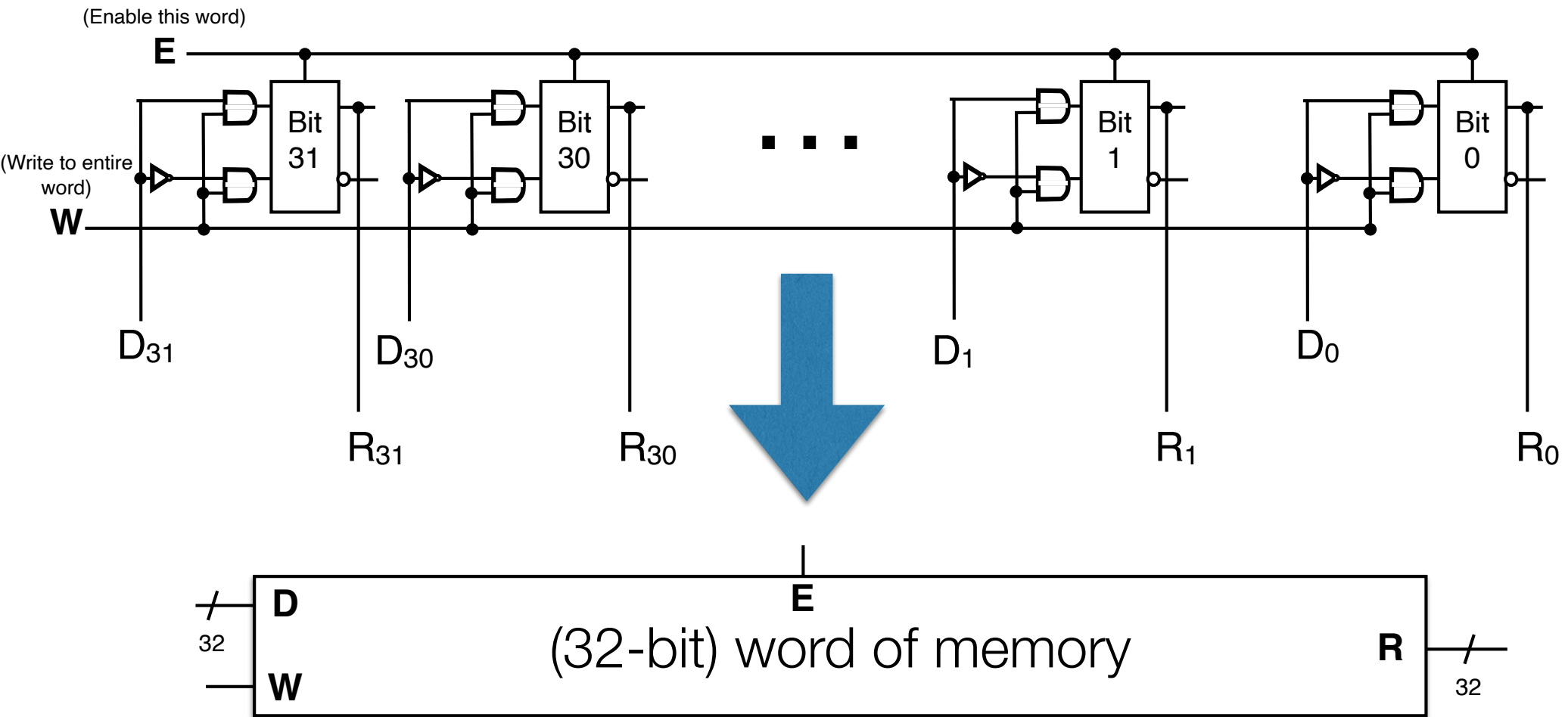
Attach 32-bit Data-In signal, D, bit-by-bit  
to corresponding latches

# Word of MIPS Memory



Attach 32-bit Data-Out signal, **R**, bit-by-bit  
to corresponding latches

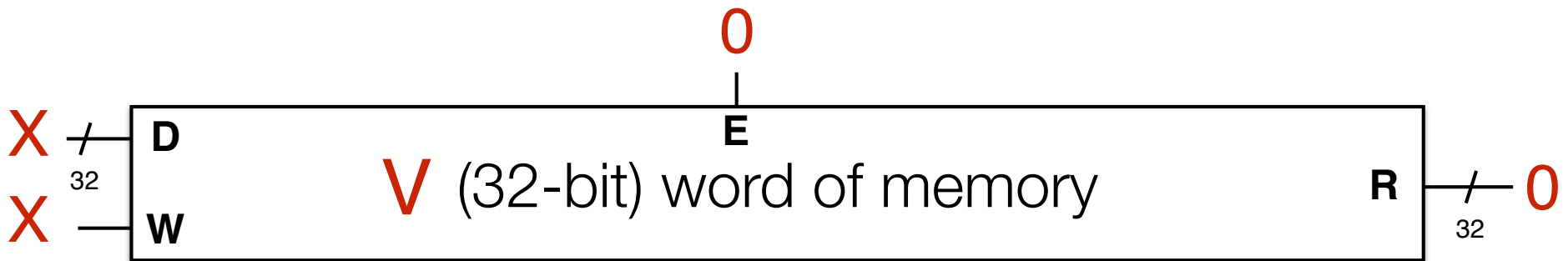
# Word of MIPS Memory



# Word of MIPS Memory

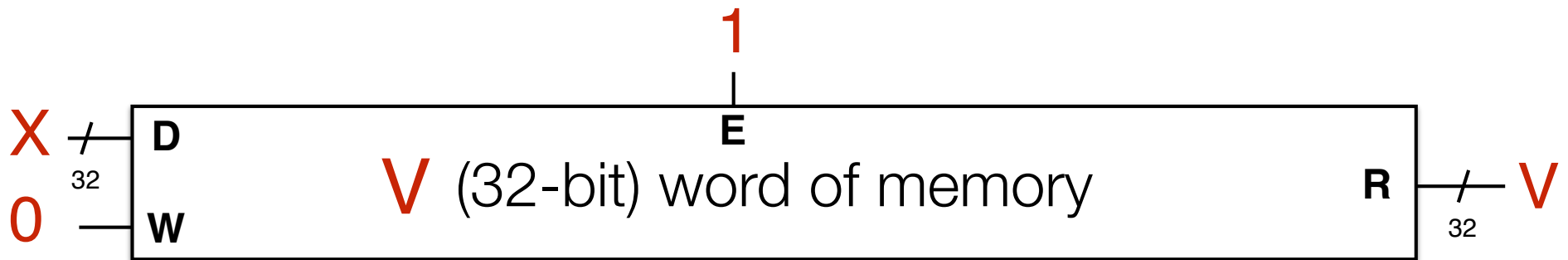
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- E=0: don't write to this word of memory, output all 0's



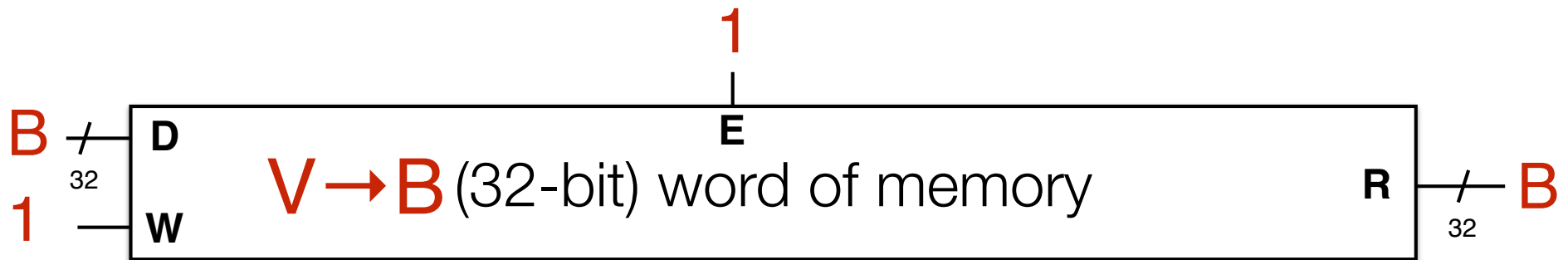
# Word of MIPS Memory

- E=1, W=0: read from this word of memory, but don't write



# Word of MIPS Memory

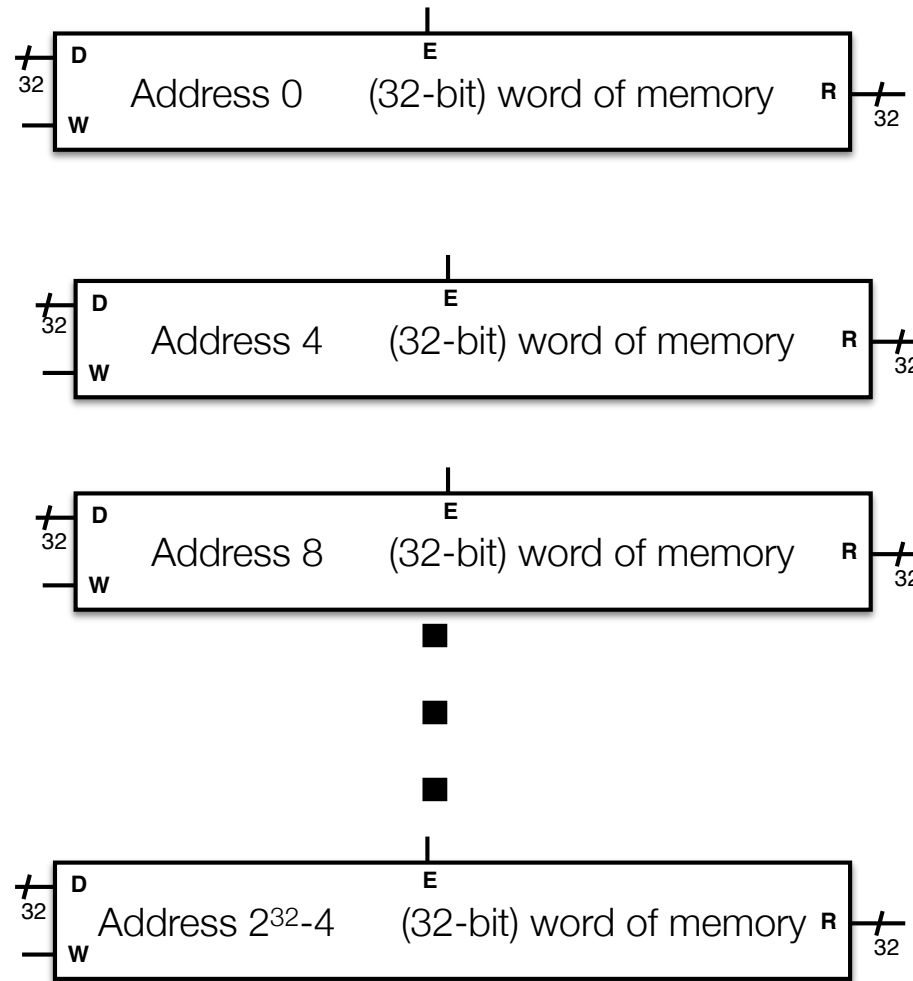
- E=1, W=1: write input at D into memory (read as well)





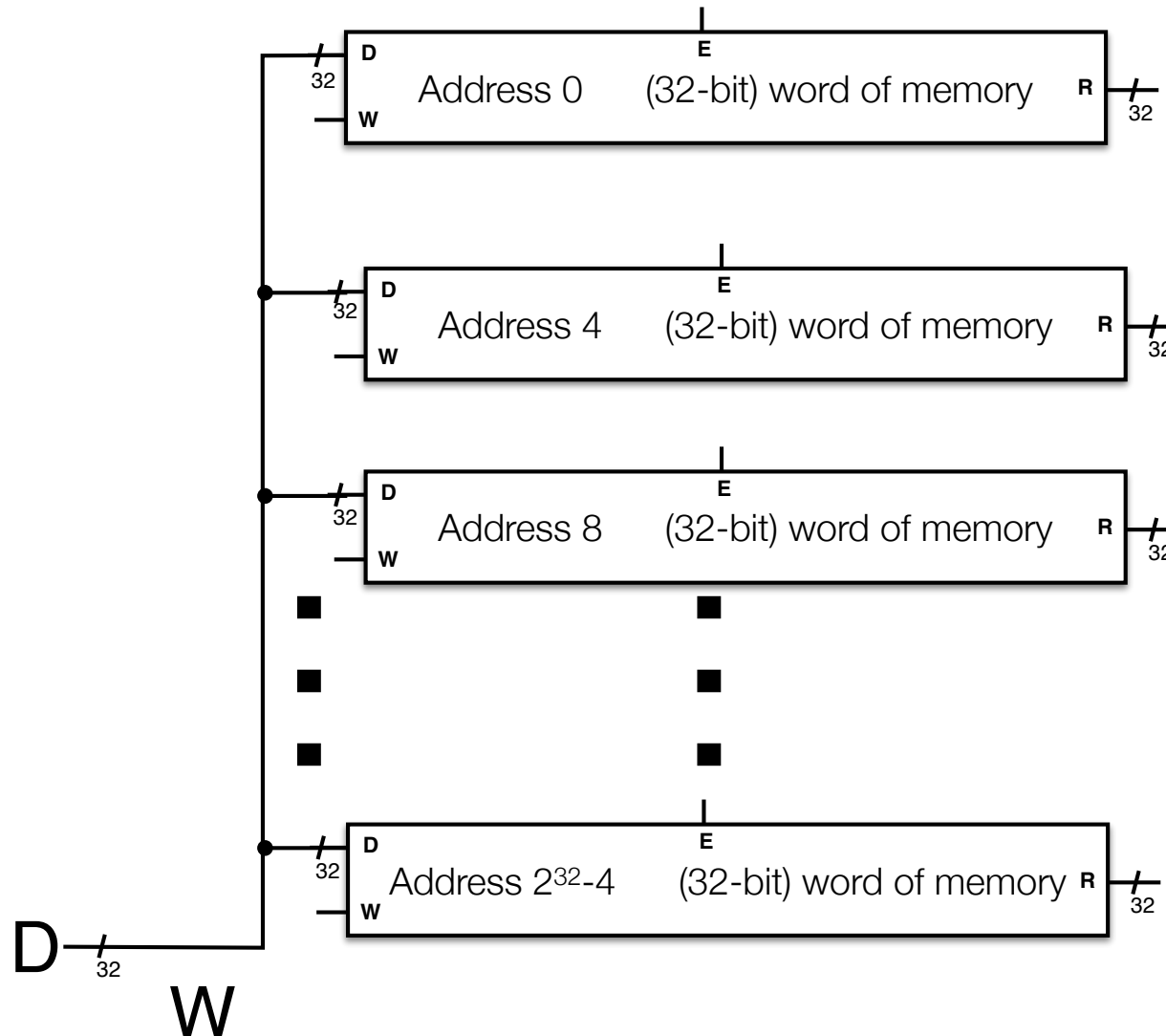
# $2^{32}$ bytes = $2^{30}$ words of Addressable Memory

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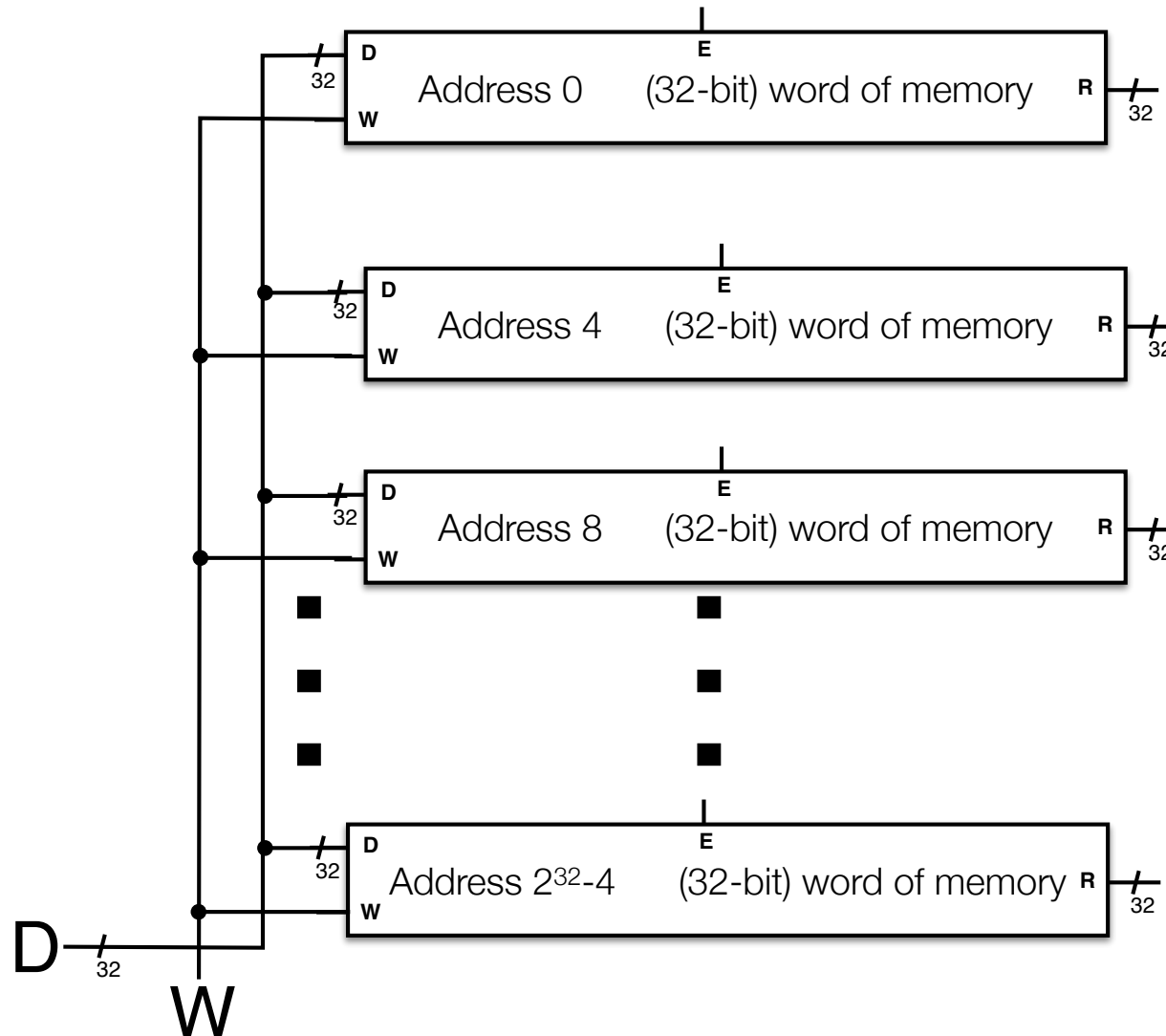
[30 bits describing address] 00



- Each address is really described by 30 bits (since the 2 lowest-order bits of the 32-bit address are 00)
- Data Input to memory, **D**, fed into every word address

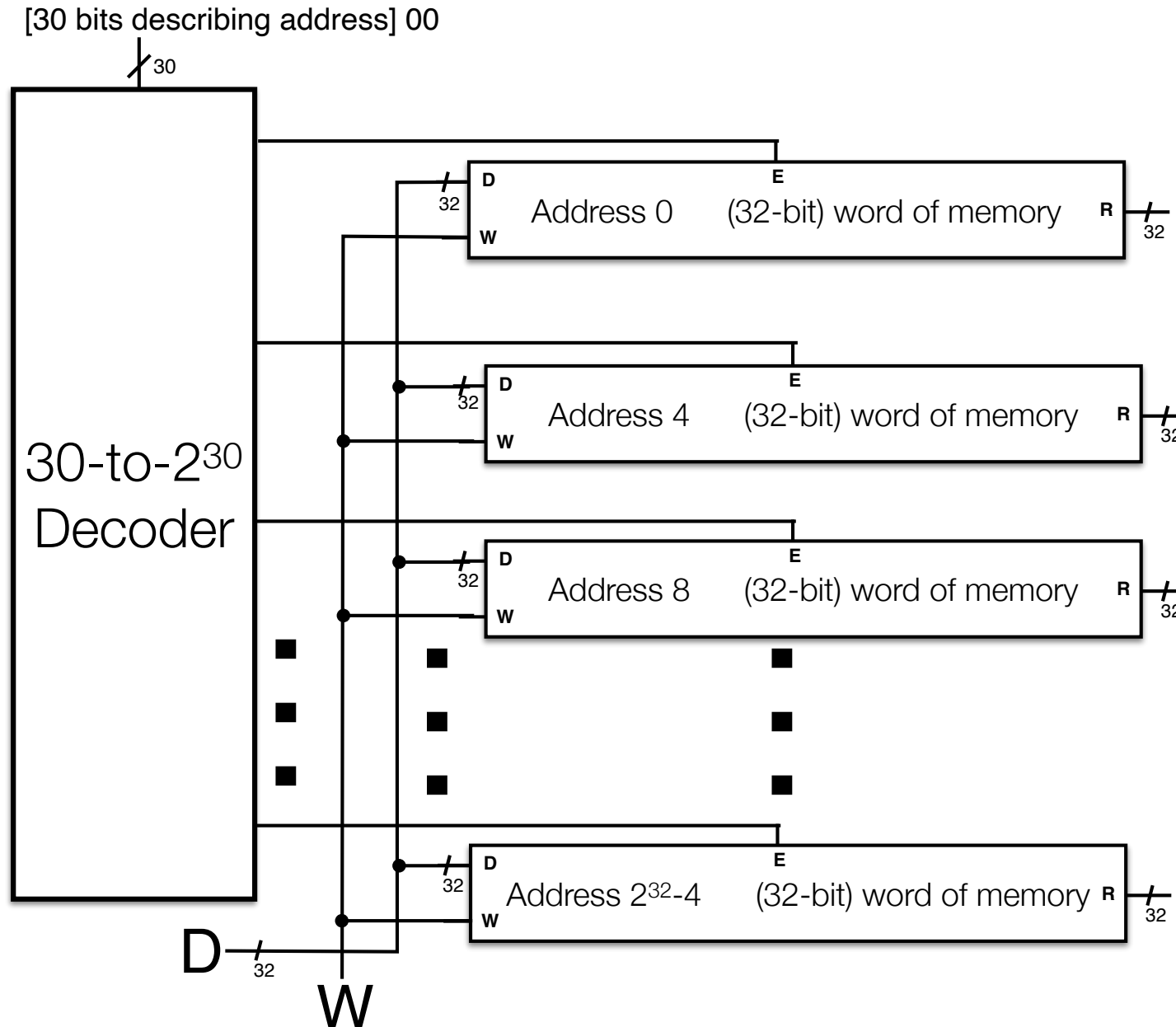
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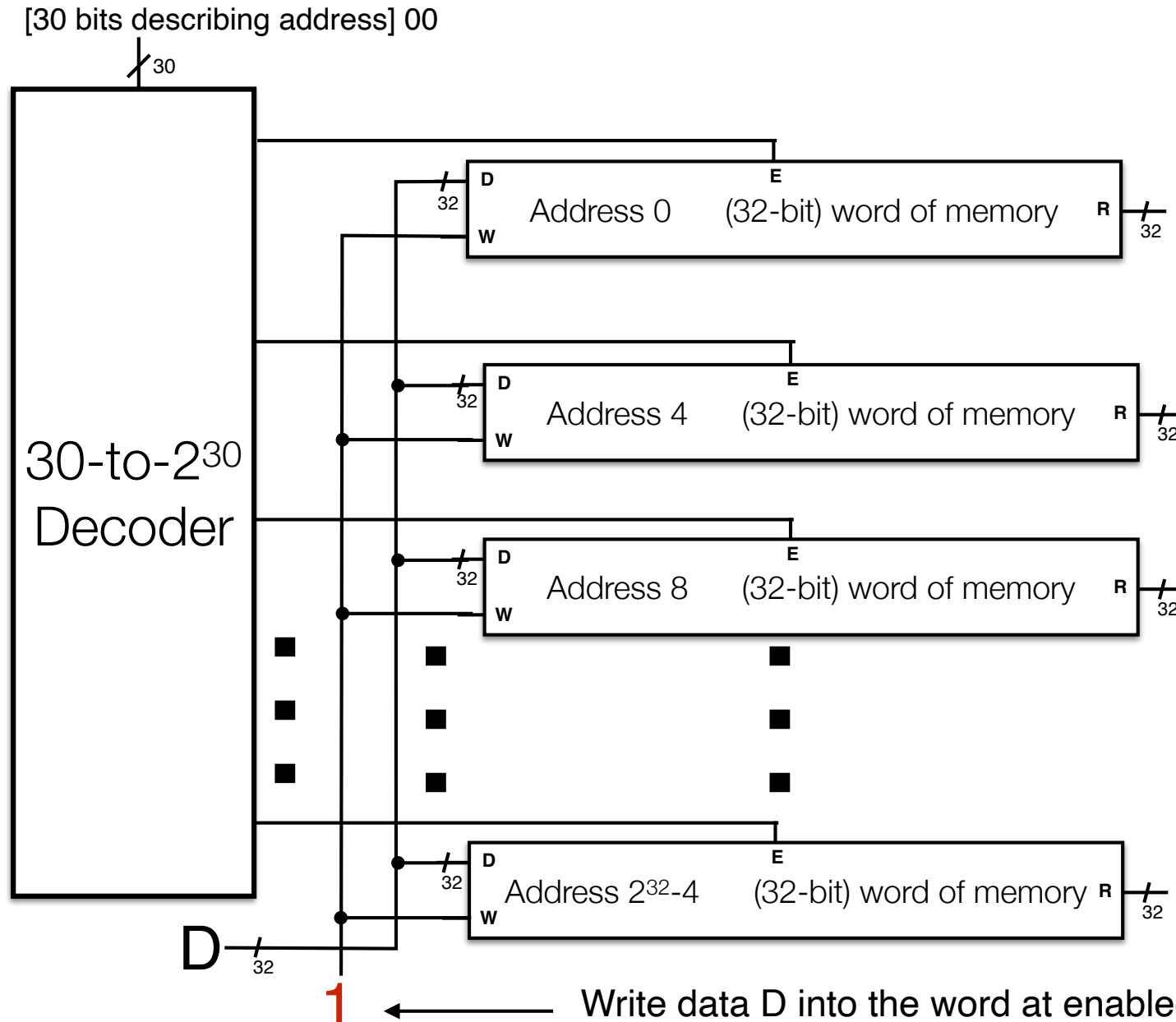
- Each address is really described by 30 bits (since the 2 lowest-order bits of the 32-bit address are 00)
- Data Input to memory, **D**, fed into word at each address
- Write Enable, **W**, fed into word at each address

# $2^{32}$ bytes = $2^{30}$ words of Addressable Memory



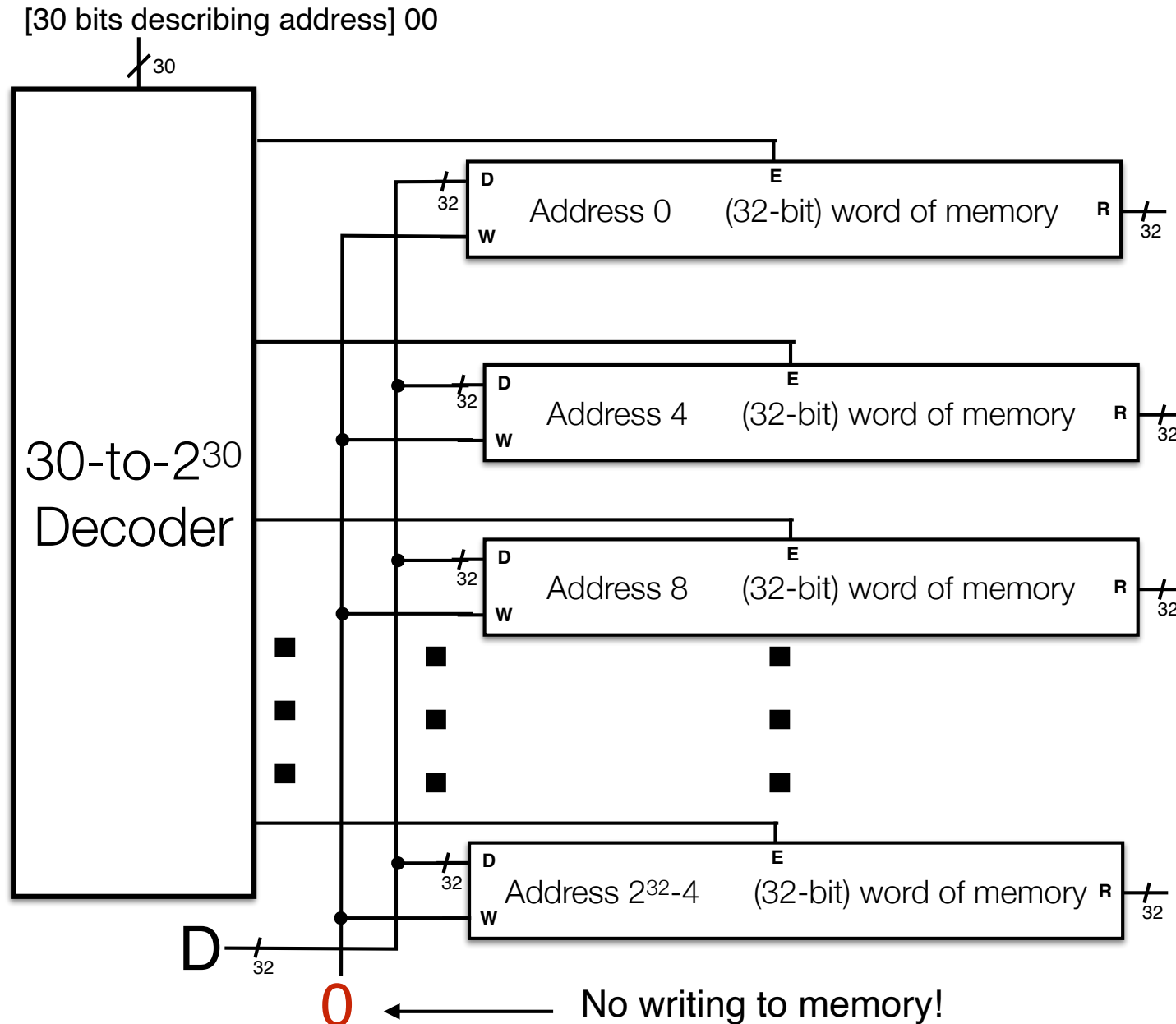
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- Decoder used to enable only 1 word

# $2^{32}$ bytes = $2^{30}$ words of Addressable Memory



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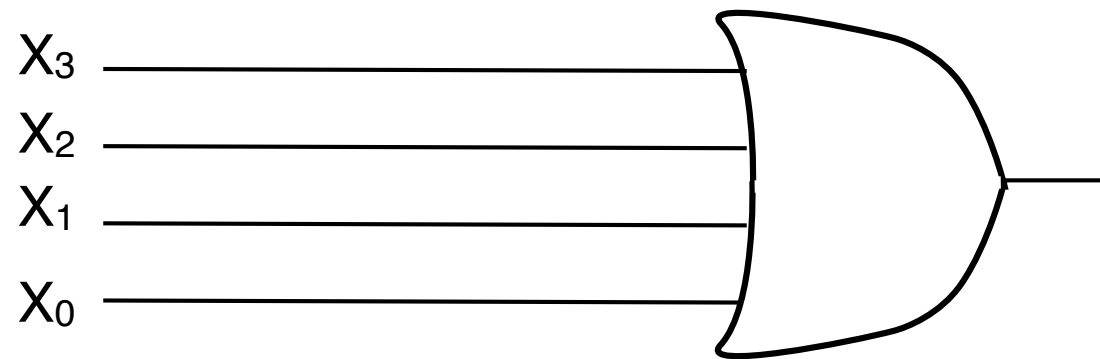


- Each address is really described by 30 bits (since the 2 lowest-order bits of the 32-bit address are 00)
- Data Input to memory, D, fed into word at each address
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# RECALL: Merge-with-known-0's

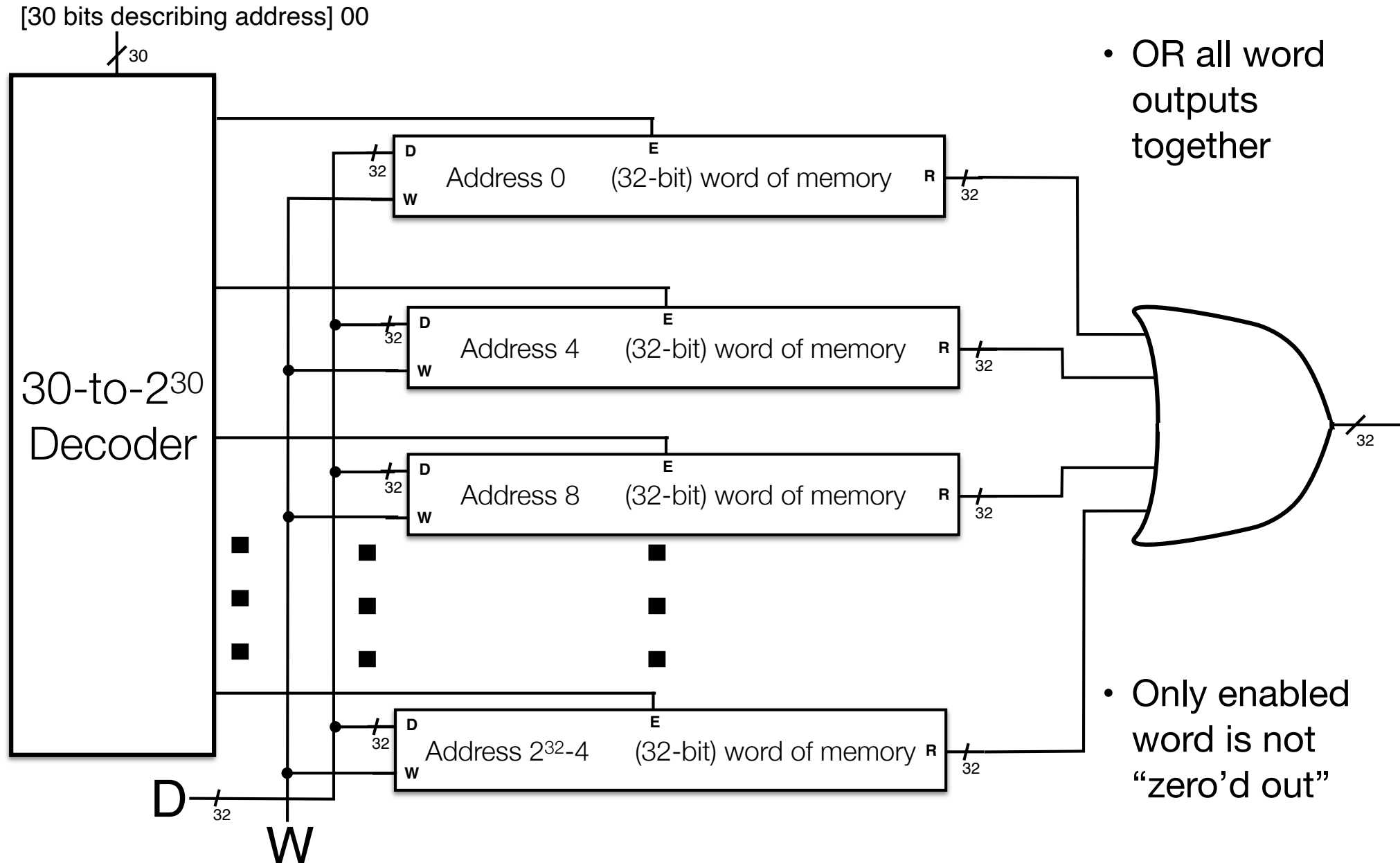
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- Suppose have  $m$  input signals
- All of them = 0 except maybe one  $X_j$ .
- Circuit to retrieve the value of the one unknown



- If all  $X_i=0$  for  $i \neq j$ , then the OR over all  $X_i$  (including  $X_j$ ) =  $X_j$

# $2^{32}$ bytes = $2^{30}$ words of Addressable Memory

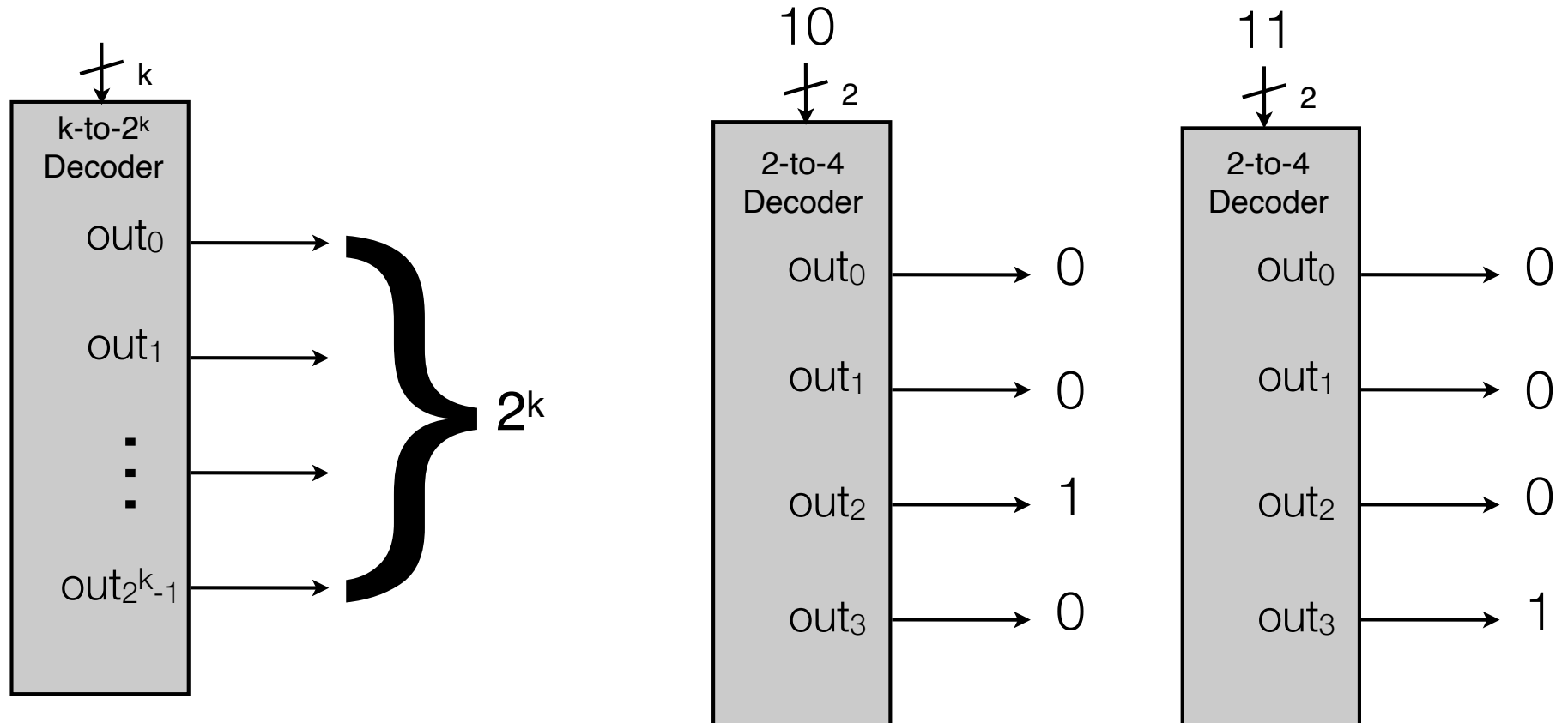




# Coincident Selection

# Recall: Decoder

- No “DATA” inputs, just a k-bit “selector” input
- $2^k$  1-bit outputs
- Selector input (i) chooses which output = 1, all other outputs = 0



# Pick along 1 Dimension: Use a decoder

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Suppose have  $2^k$  “items” and want to choose 1  
(here  $k=3$ )

# Pick along 1 Dimension: Use a decoder

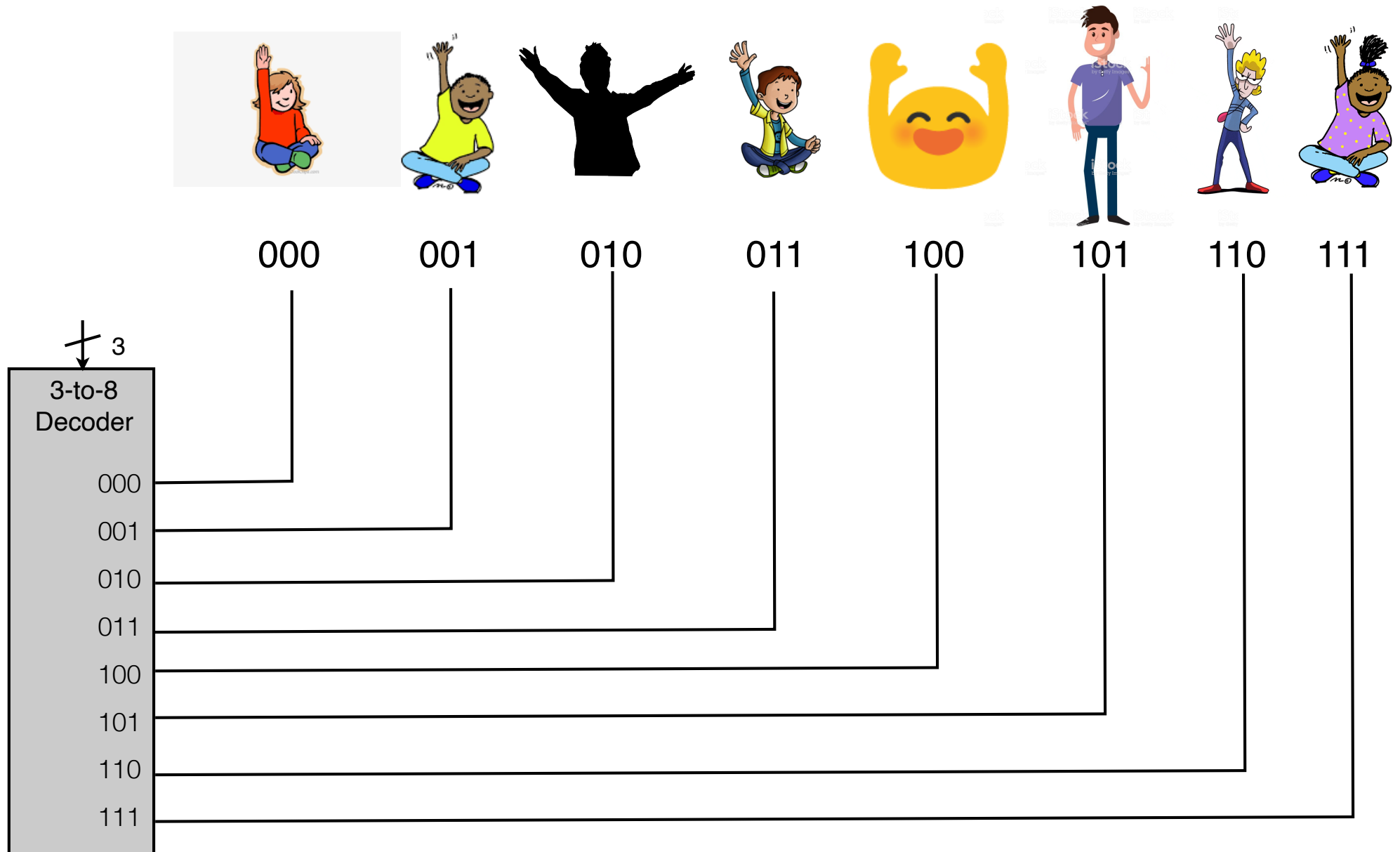
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Suppose have  $2^k$  “items” and want to choose 1  
(here  $k=3$ )

Assign a number to each “item”

# Pick along 1 Dimension: Use a decoder



# 1-Dimensional Decoding Summary

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- To enable 1 of  $2^k$  addresses:
  - 1 Decoder
  - k selector “pins” on decoder
  - $2^k$  output “pins”

# Pick along 2 Dimensions: Use 2 decoders

Column 0



Column 1



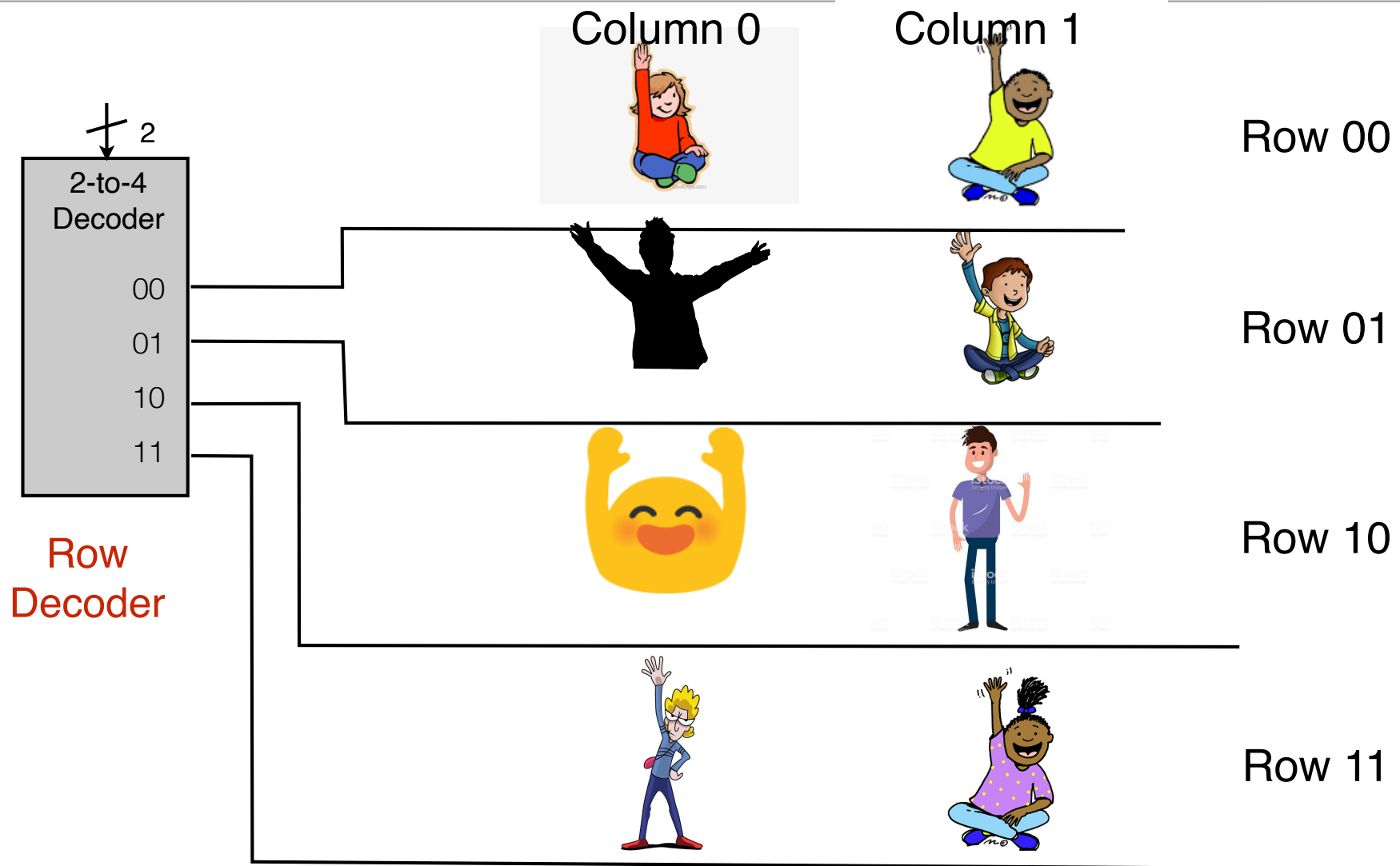
Row 00

Row 01

Row 10

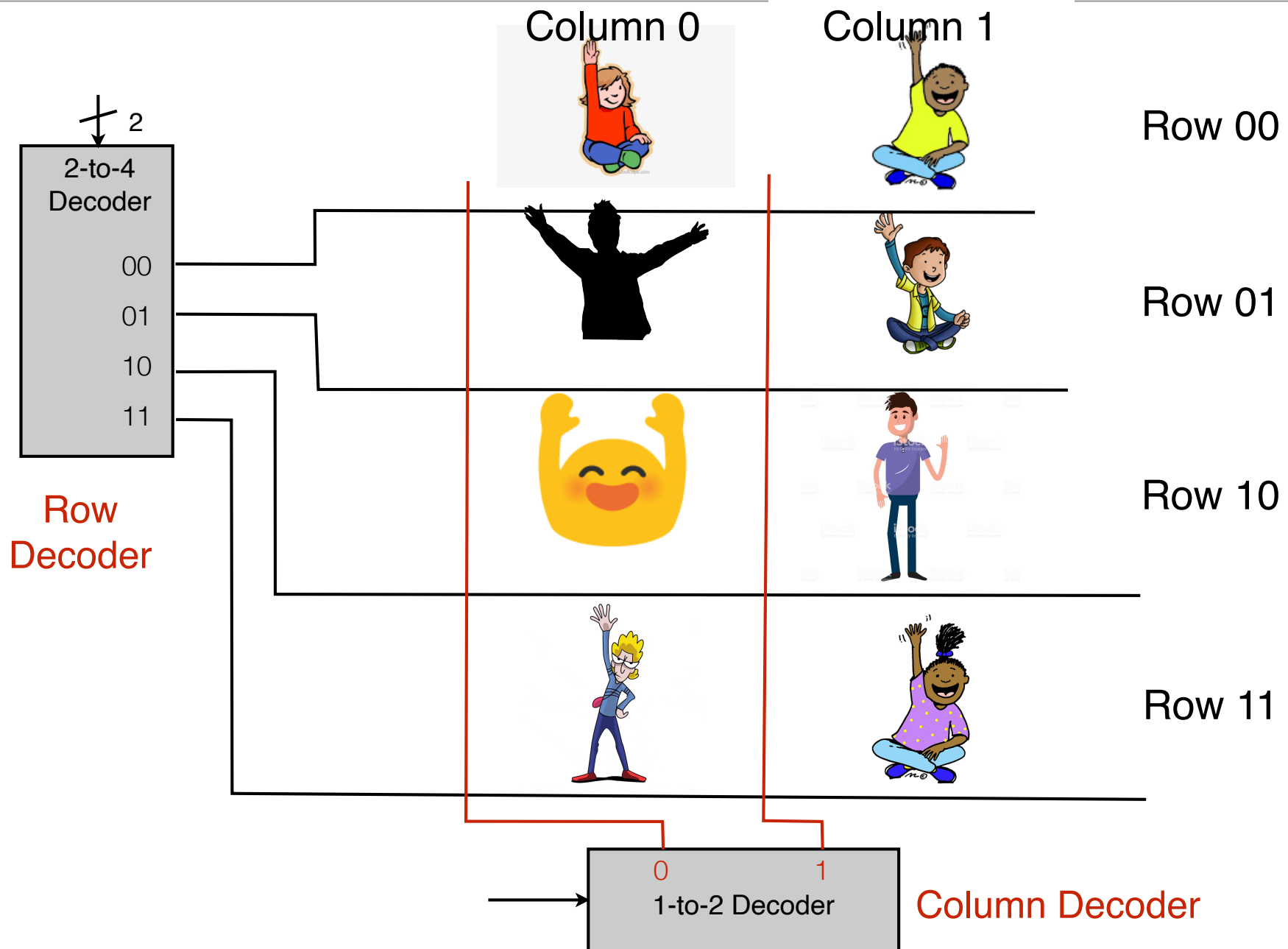
Row 11

# Pick along 2 Dimensions: Use 2 decoders

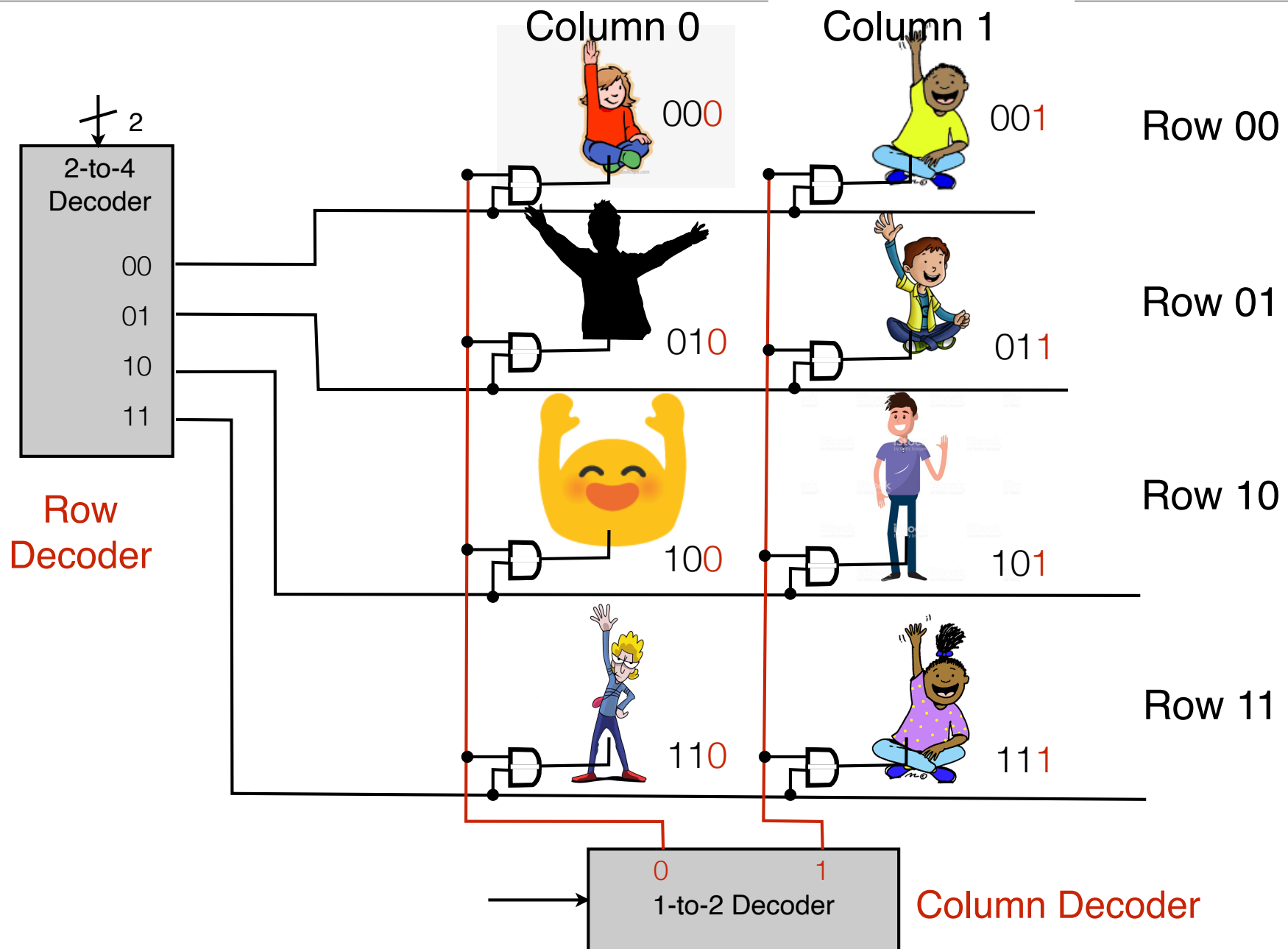




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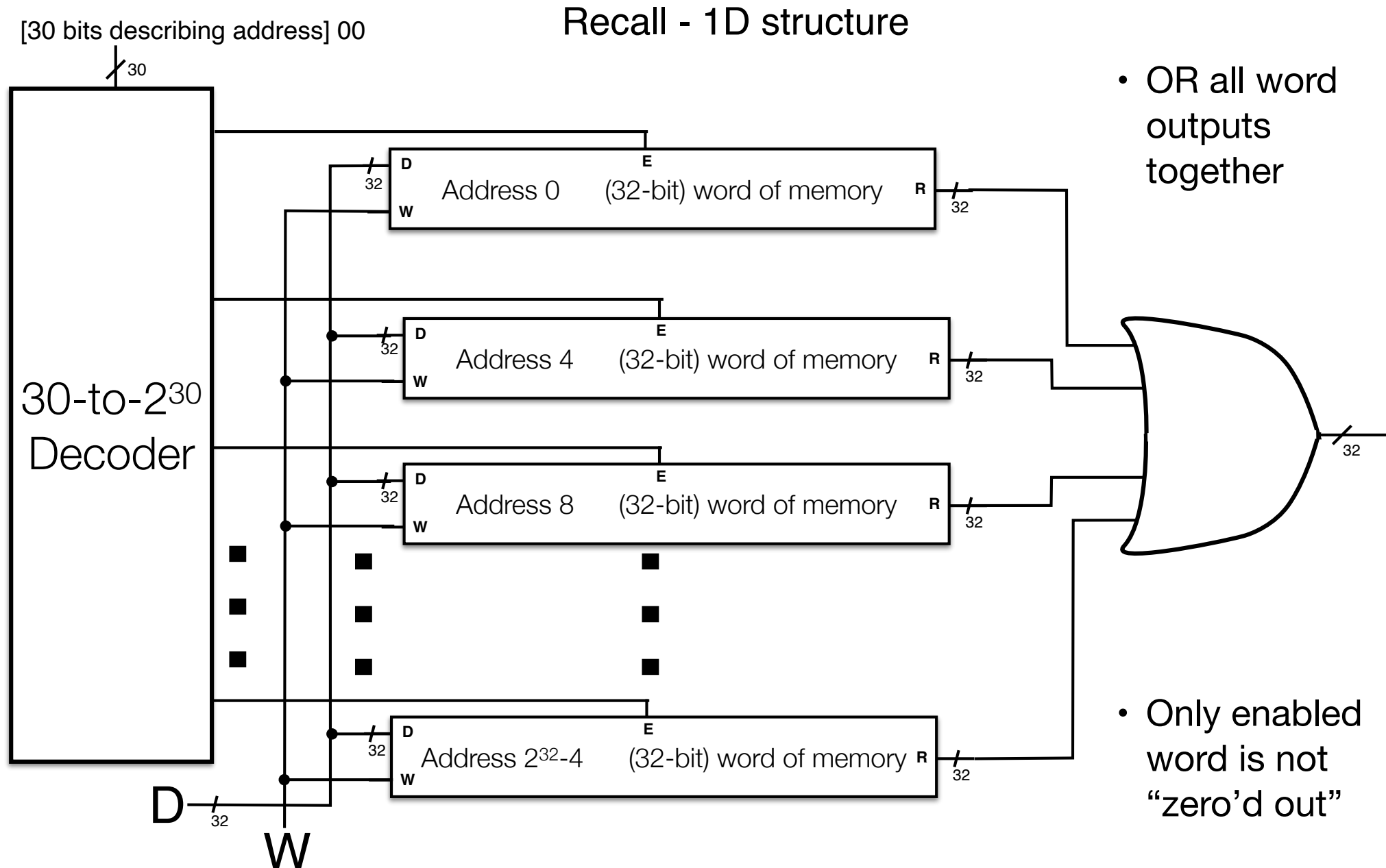
# 1-D v. 2-D Decoding Comparison Summary

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- 1D
  - To enable 1 of  $2^k$  addresses:
    - 1 Decoder
    - $k$  selector “pins” on decoder
    - $2^k$  output “pins”
    - e.g.,  $k=10$ , 1024 output pins
- 2D
  - To enable 1 of  $2^k$  addresses:
    - 2 Decoders
    - $k_1$  and  $k_2$  selector “pins” on decoder with  $k_1+k_2=k$
    - $2^{k_1} + 2^{k_2}$  output “pins”
    - e.g.,  $k_1=5, k_2=5$ , 64 output pins

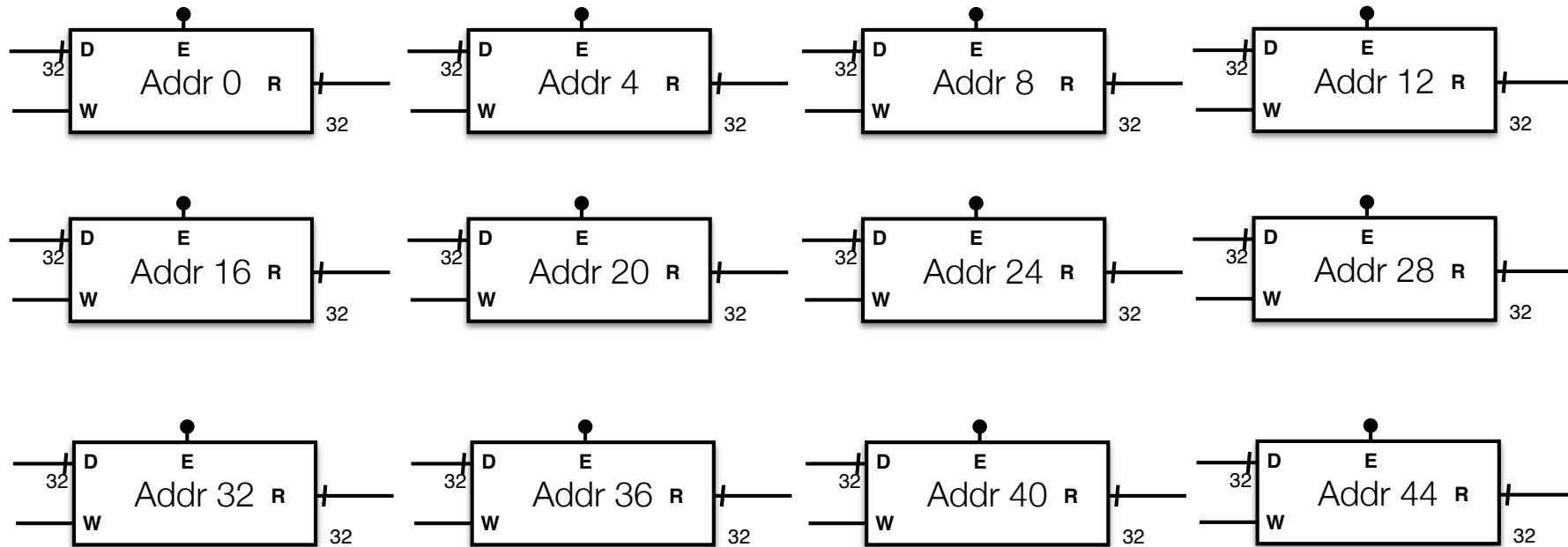
2D has Significantly Less Decoder Logic

# Modifying memory for coincident selection...



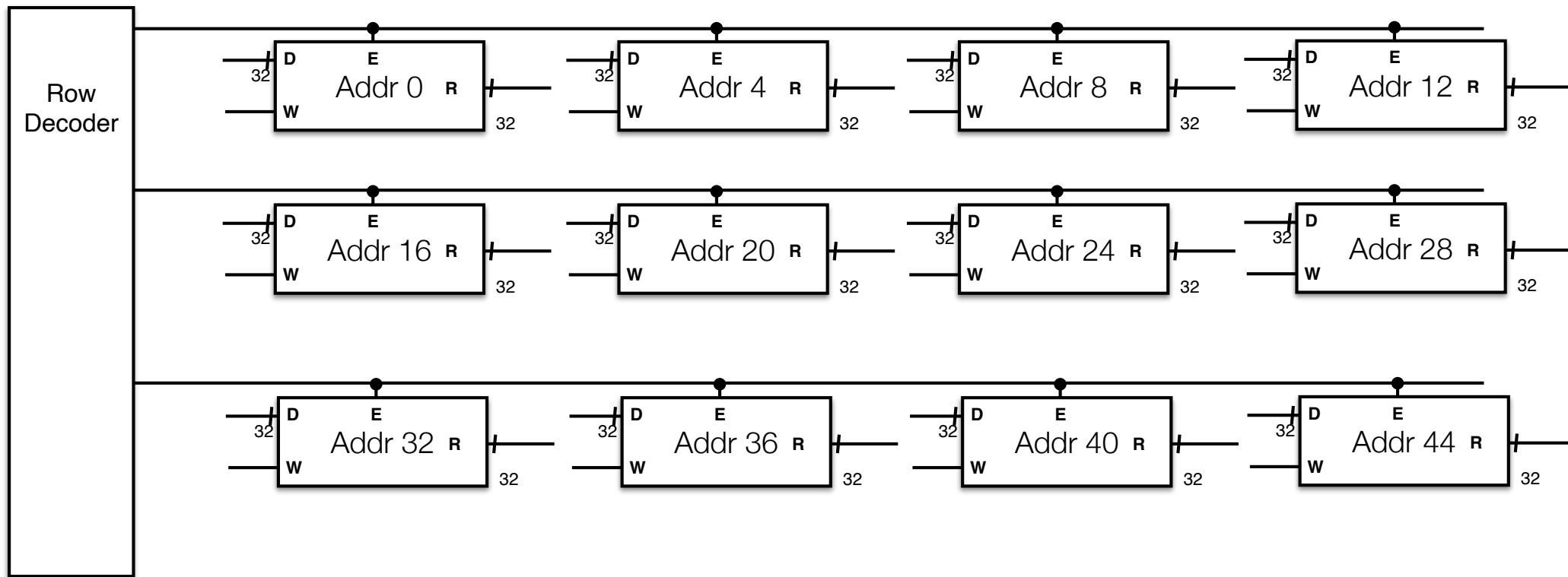
# Layout words of memory into rows and columns

- Arrange memory words into 2-dimensional grid



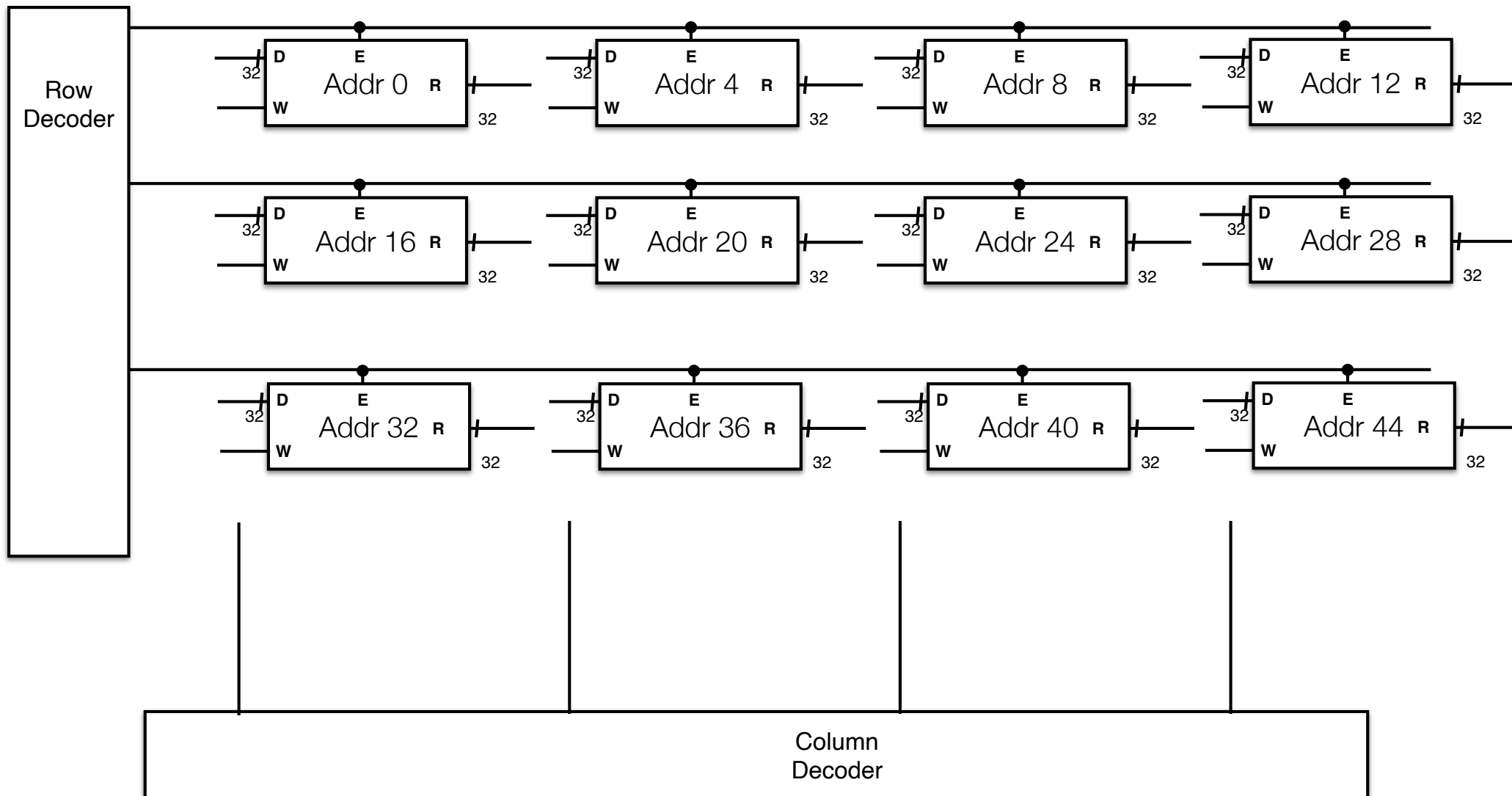
# Layout words of memory into rows and columns

- Row decoder enables 1 row of addresses for use



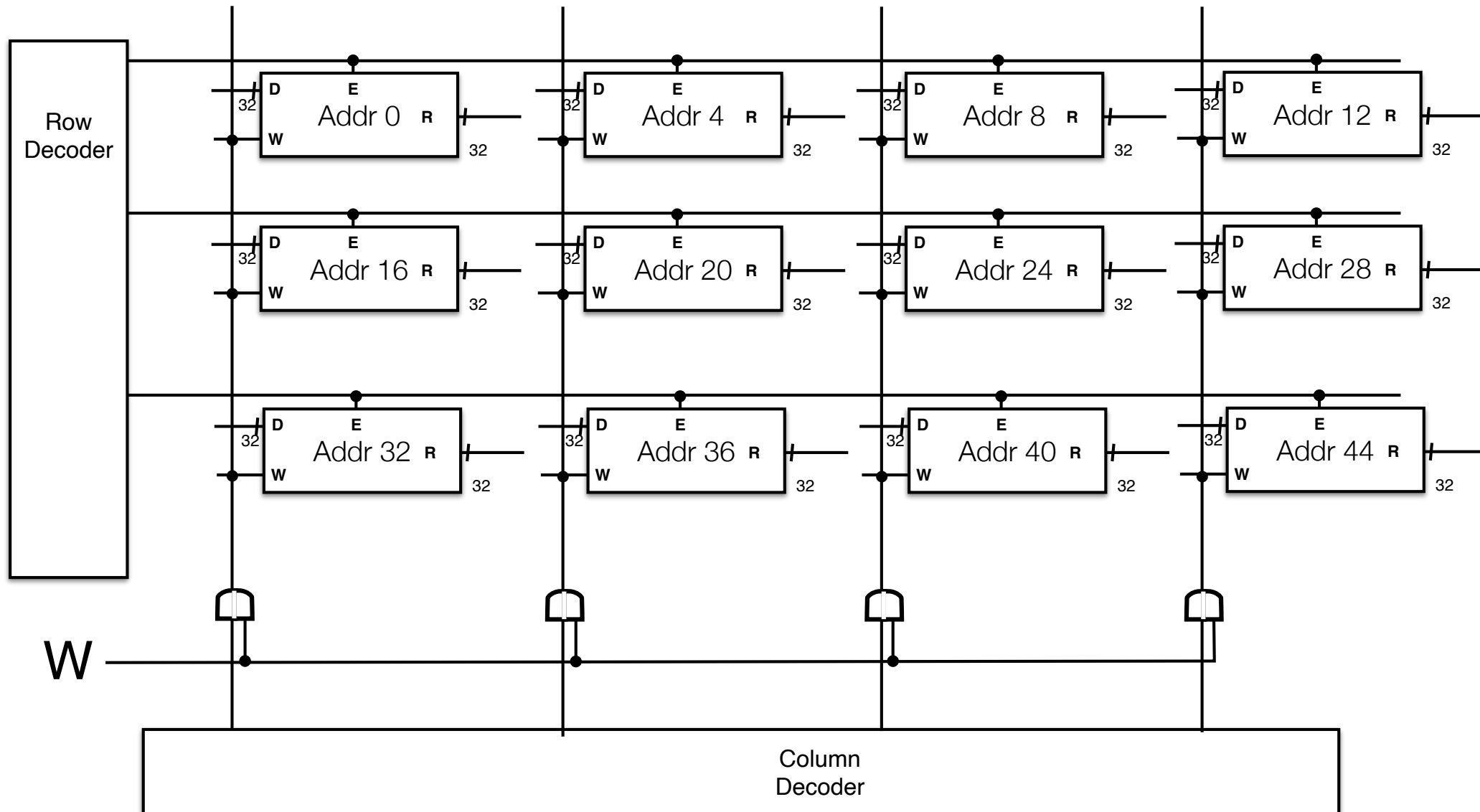
# Layout words of memory into rows and columns

- Column decoder will select a column, but how?



# Layout words of memory into rows and columns

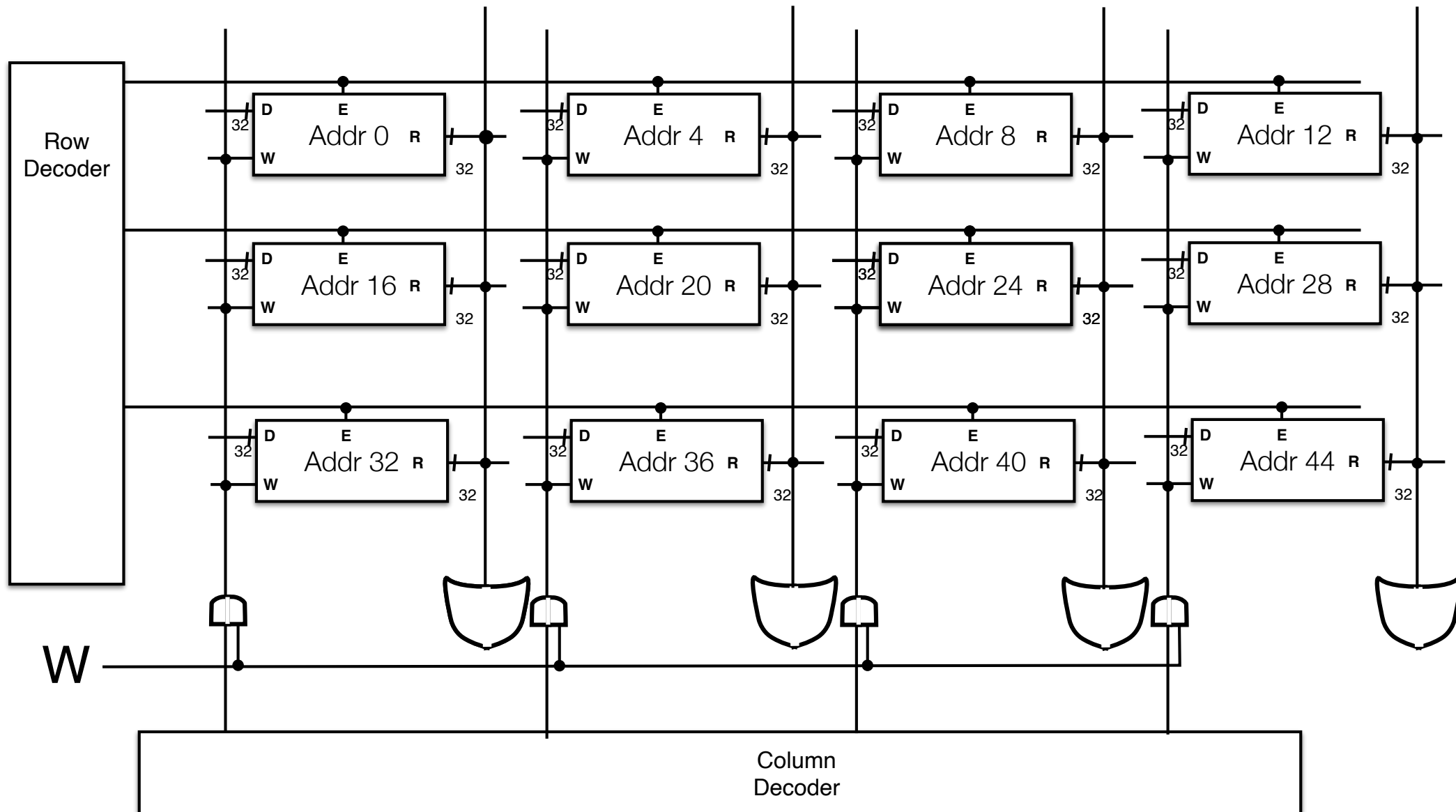
- For write: AND decoder output with W input





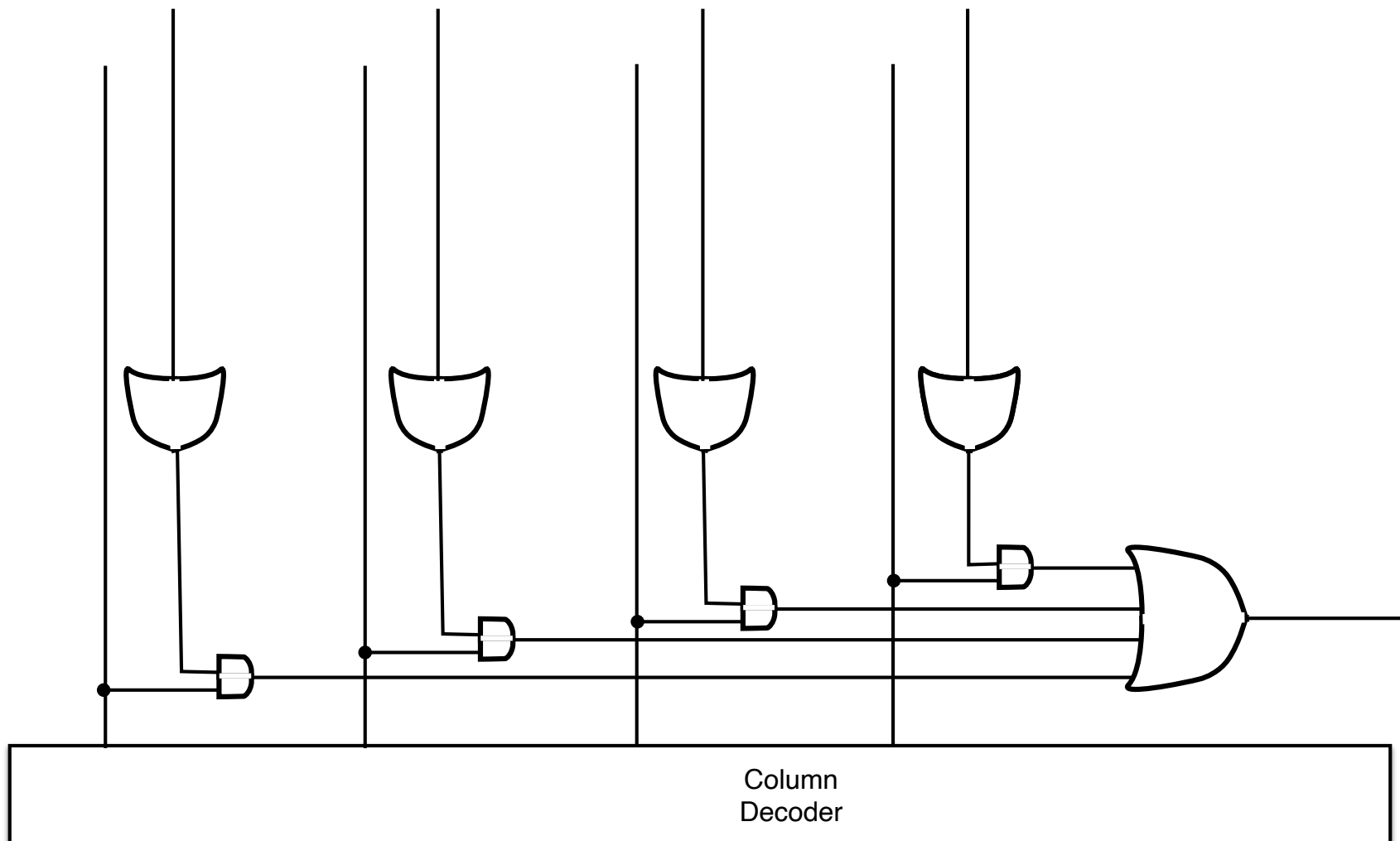
# Layout words of memory into rows and columns

- For read, take output of each column (only 1 row enabled)...



# Layout words of memory into rows and columns

- Choose the column to enable also



# Using Multiple Memory Chips to Scale Memory

# Two ways to Scale Memory

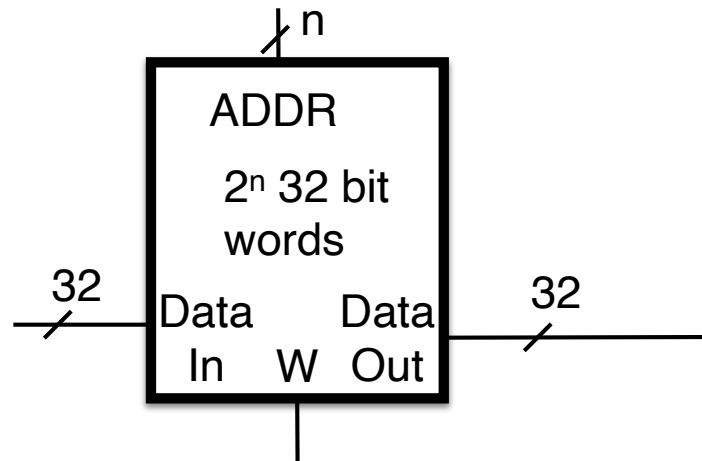
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- Given memory chips with  $2^n$  addresses and  $k$ -bit word size
  - Suppose want more addresses (e.g.,  $2^{n+i}$ )
  - Suppose want larger word size (e.g.,  $mk$  for some integer  $m > 1$ )

# Multi-chip memories: extend wordsize

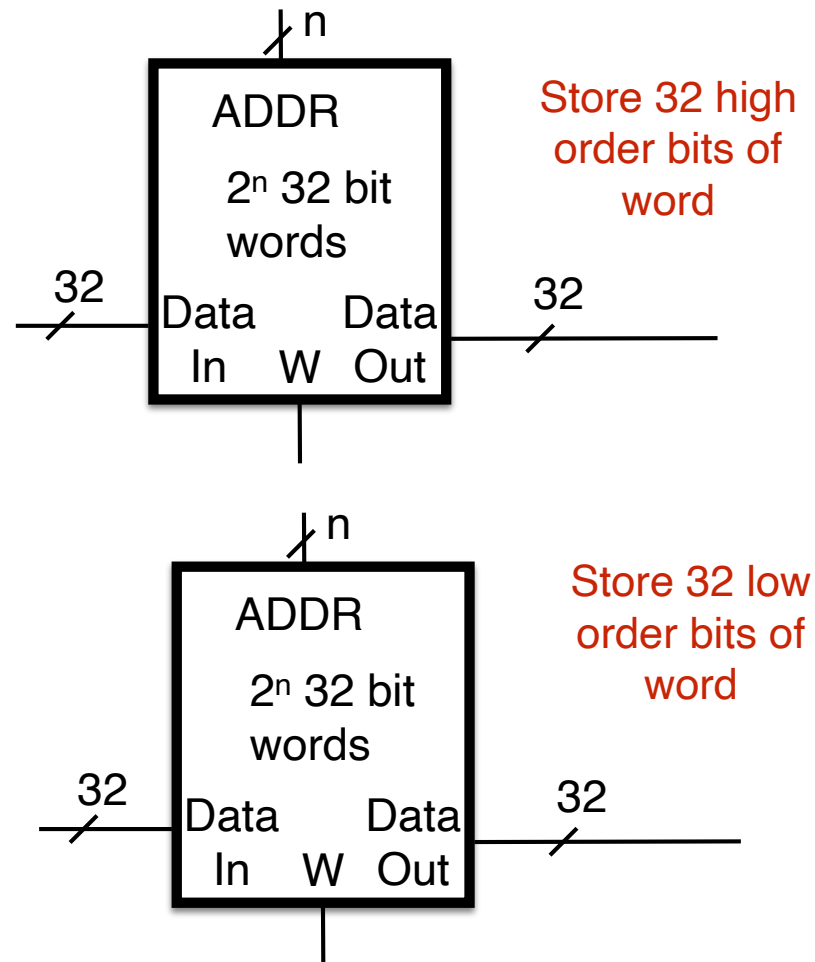
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- Extend wordsize by storing full word in parallel across multiple chips
- e.g., build 64-bit memory storing  $2^n$  words from two 32-bit  $2^n$ -word chips



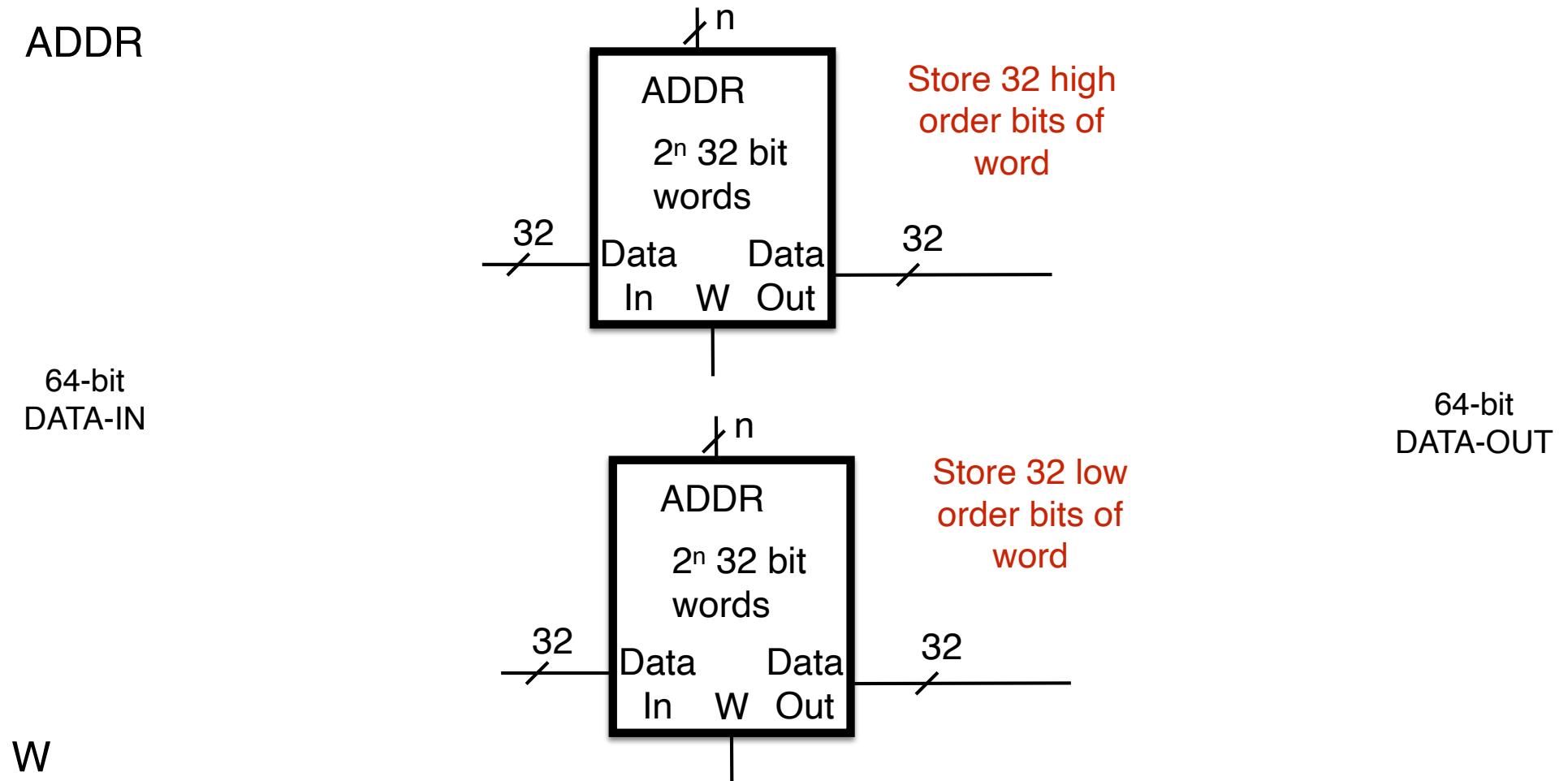
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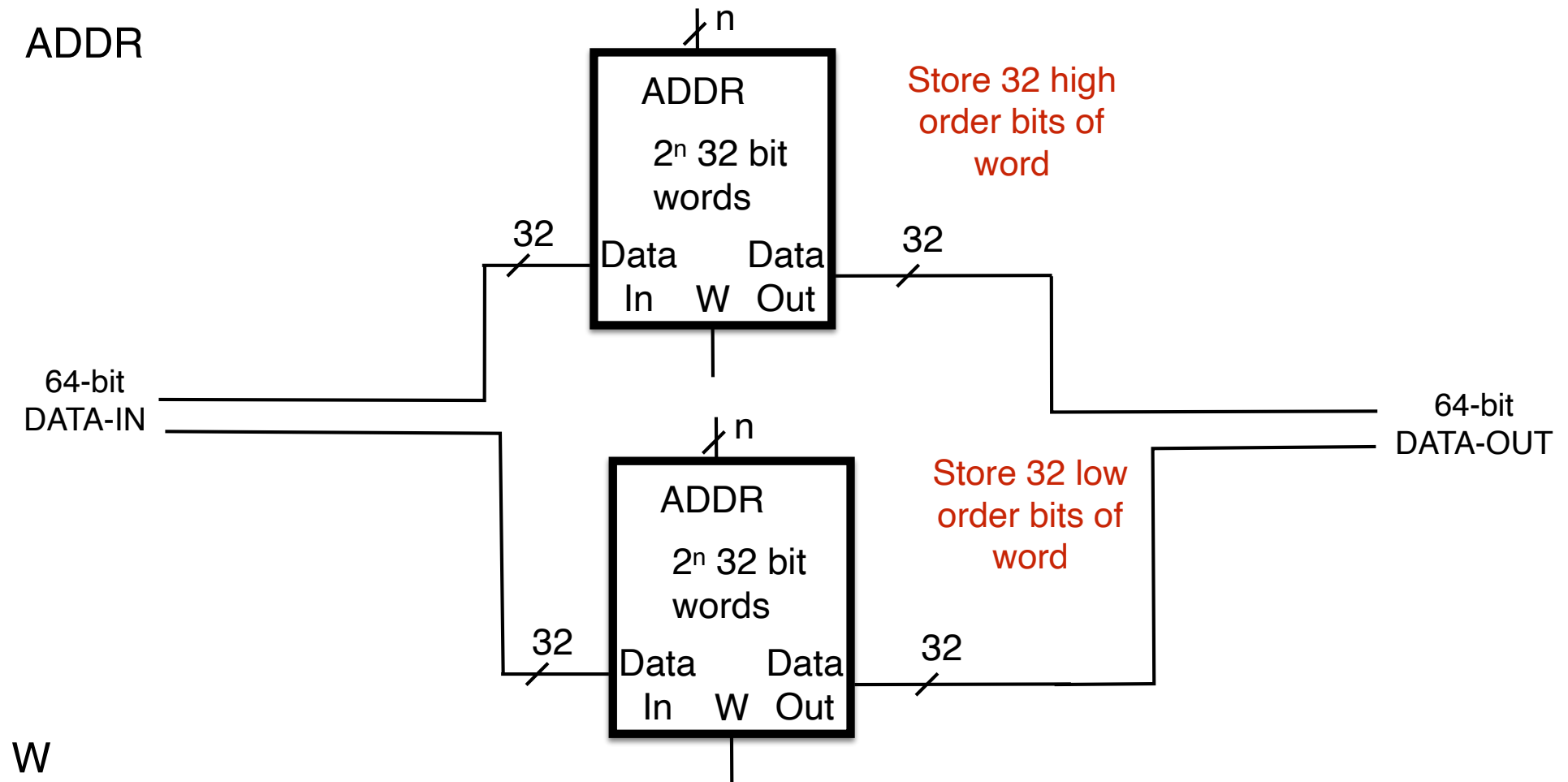
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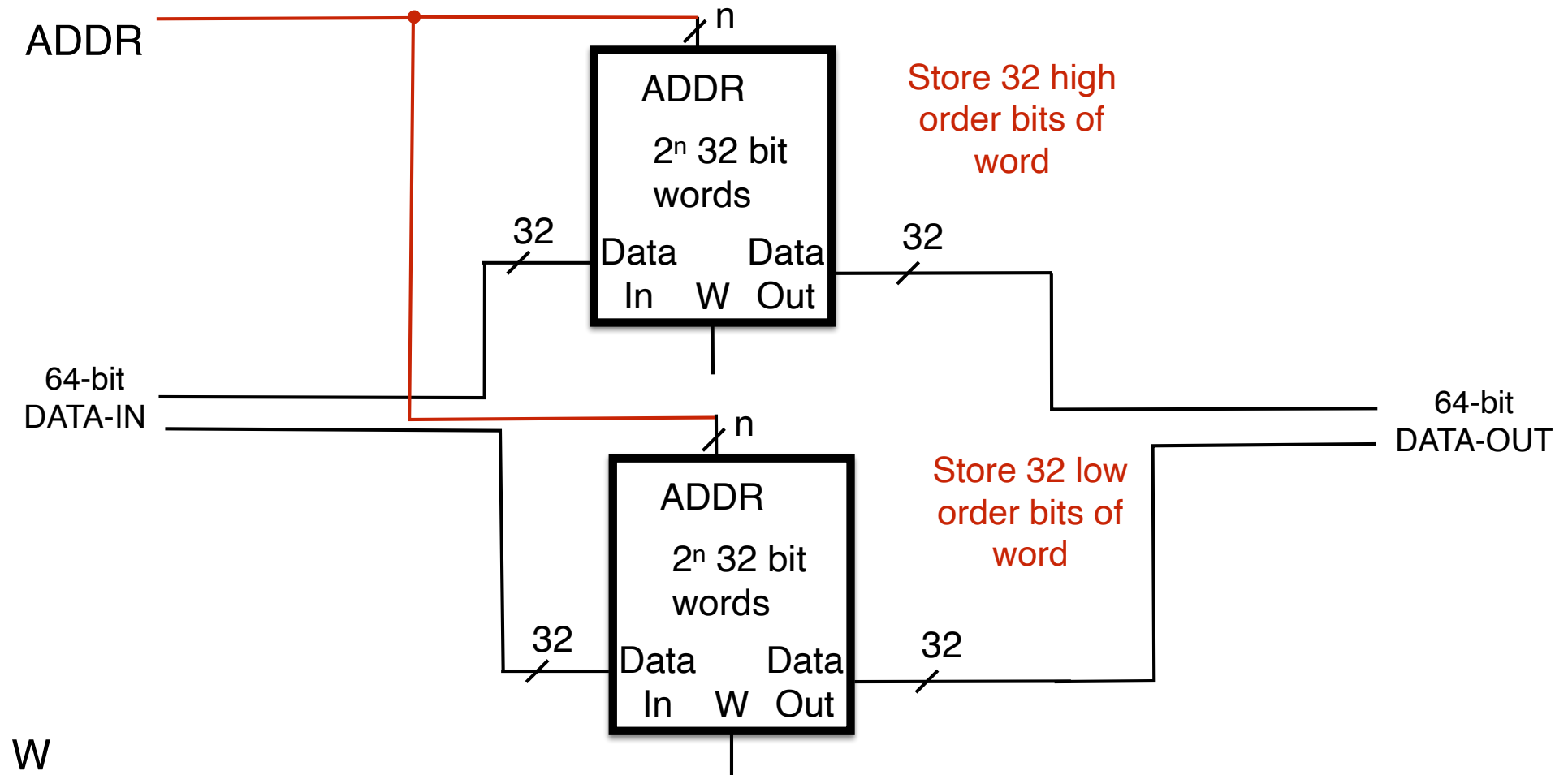
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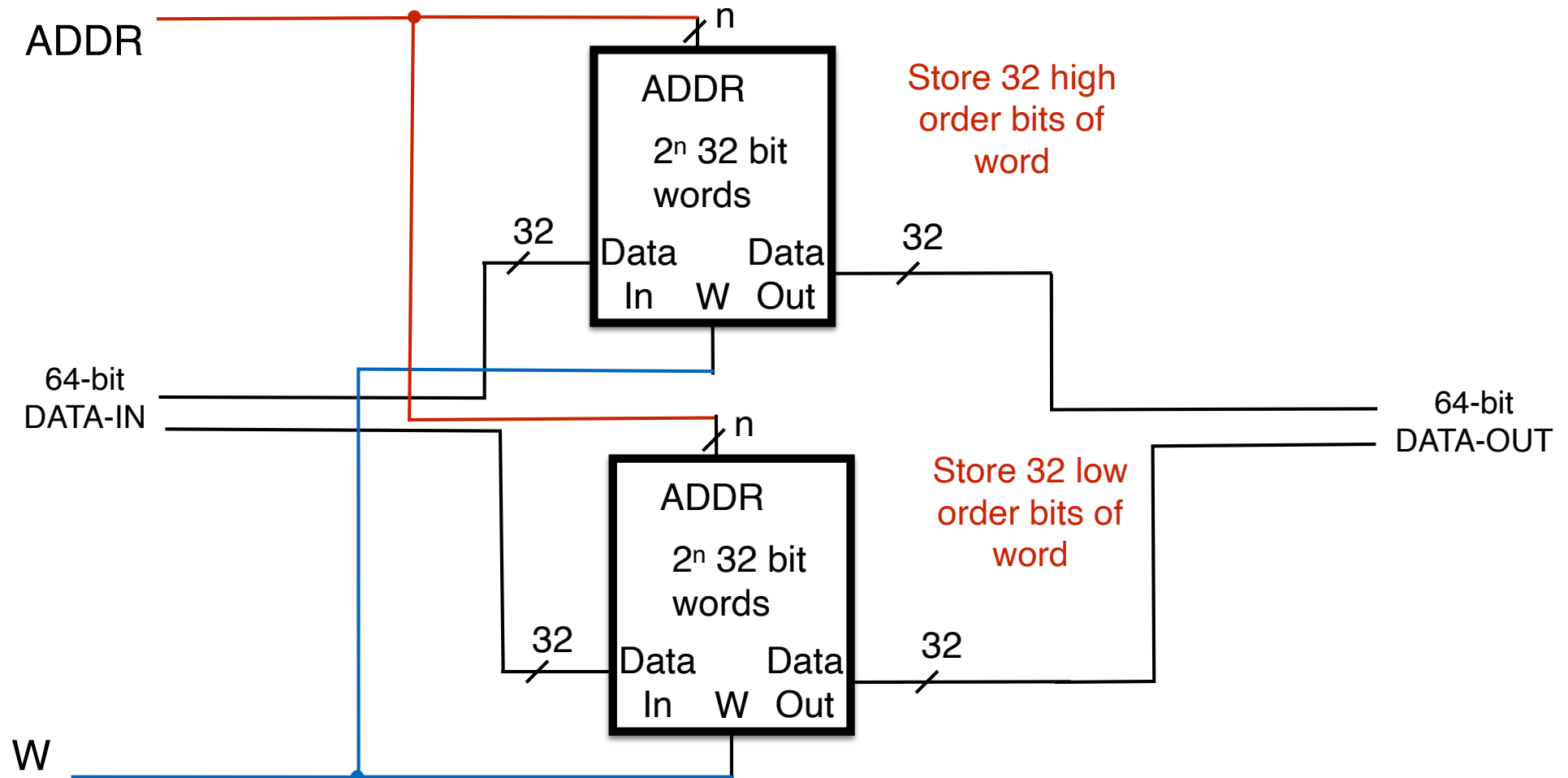
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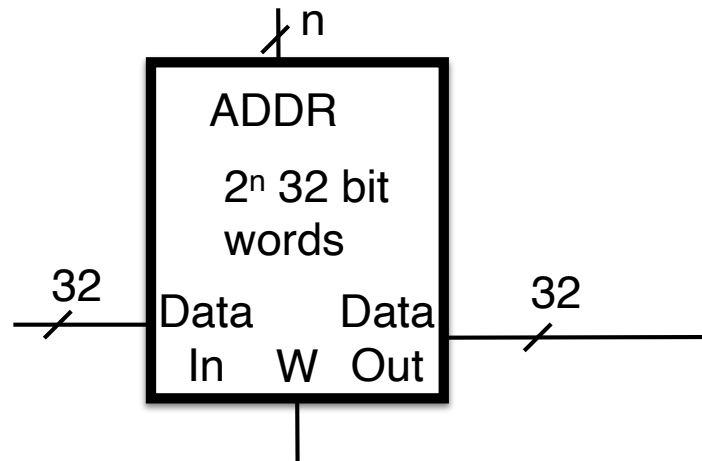
- Extend wordsize by storing full word in parallel across multiple chips
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# Multi-chip memories: extend address space

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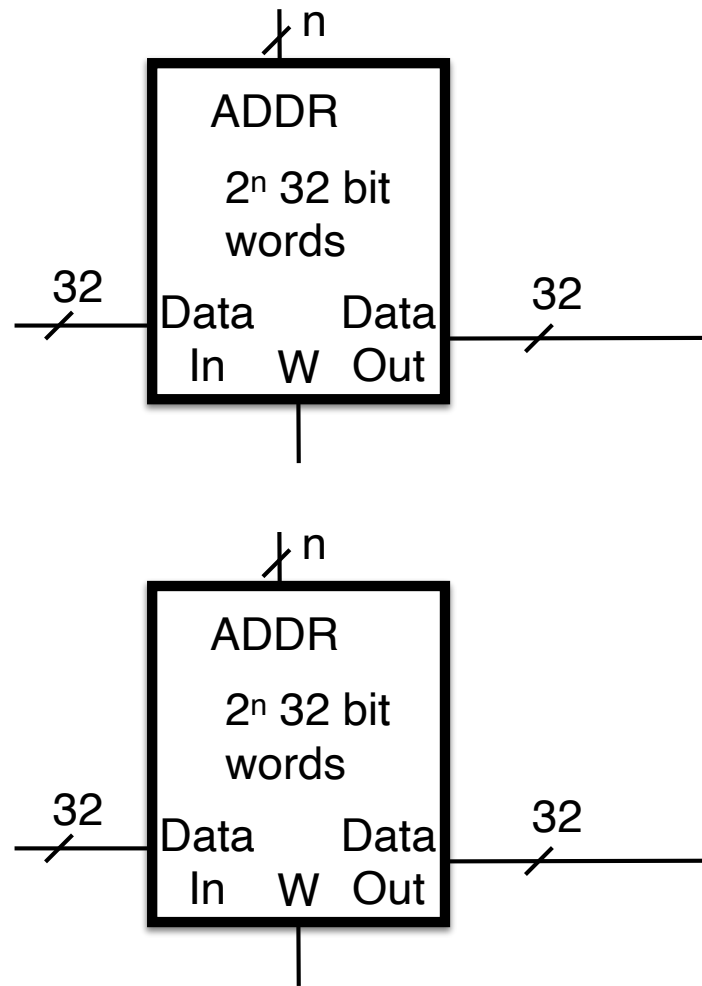
- Suppose each chip stores  $2^n$  32-bit words (has  $2^n$  n-bit addresses)



# Multi-chip memories: extend address space

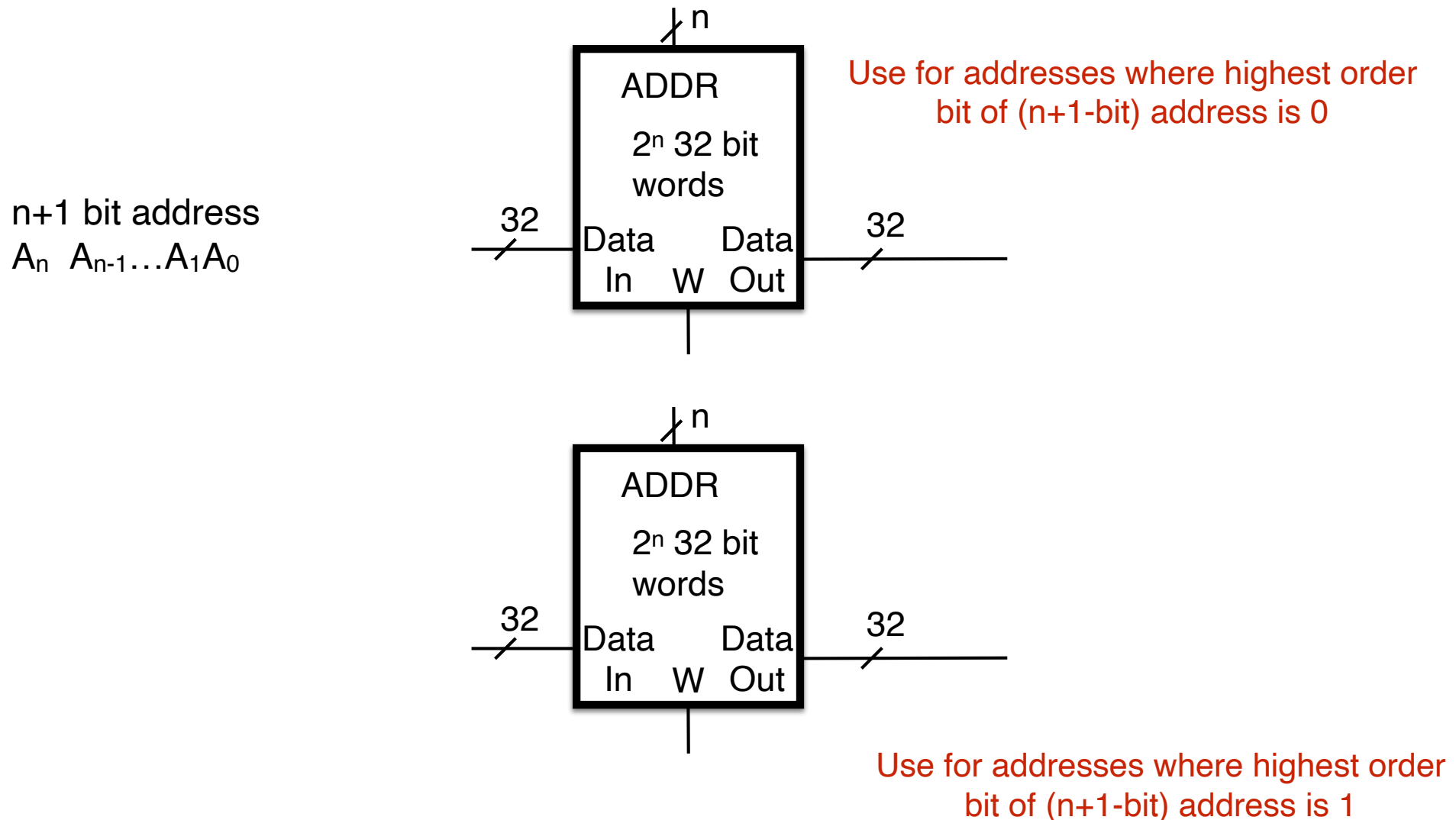
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- Suppose each chip stores  $2^n$  32-bit words (has  $2^n$  n-bit addresses)
- 2 of these chips can store  $2^{n+1}$  32-bit words ( $2^{n+1}$  (n+1)-bit addresses)



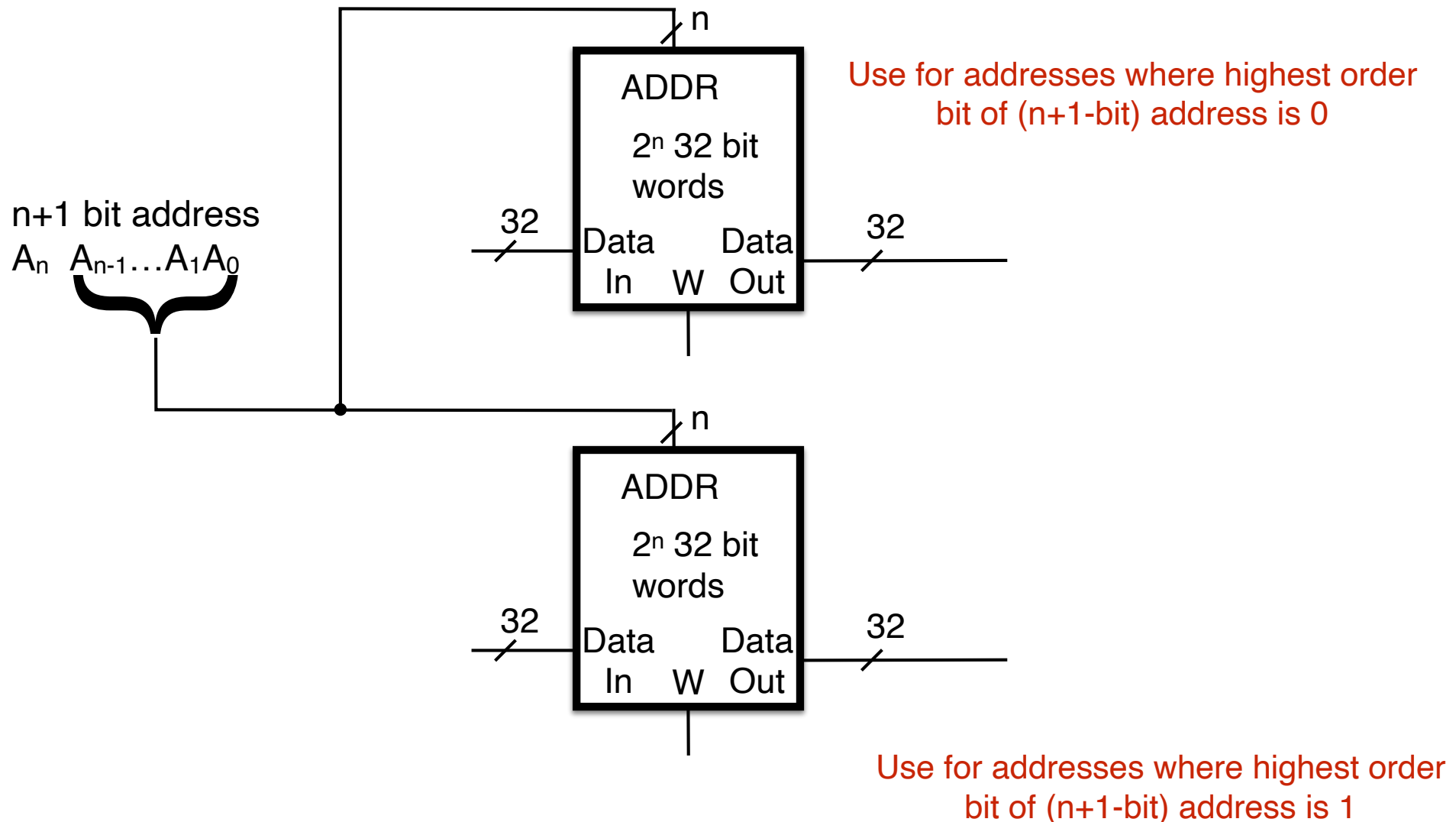
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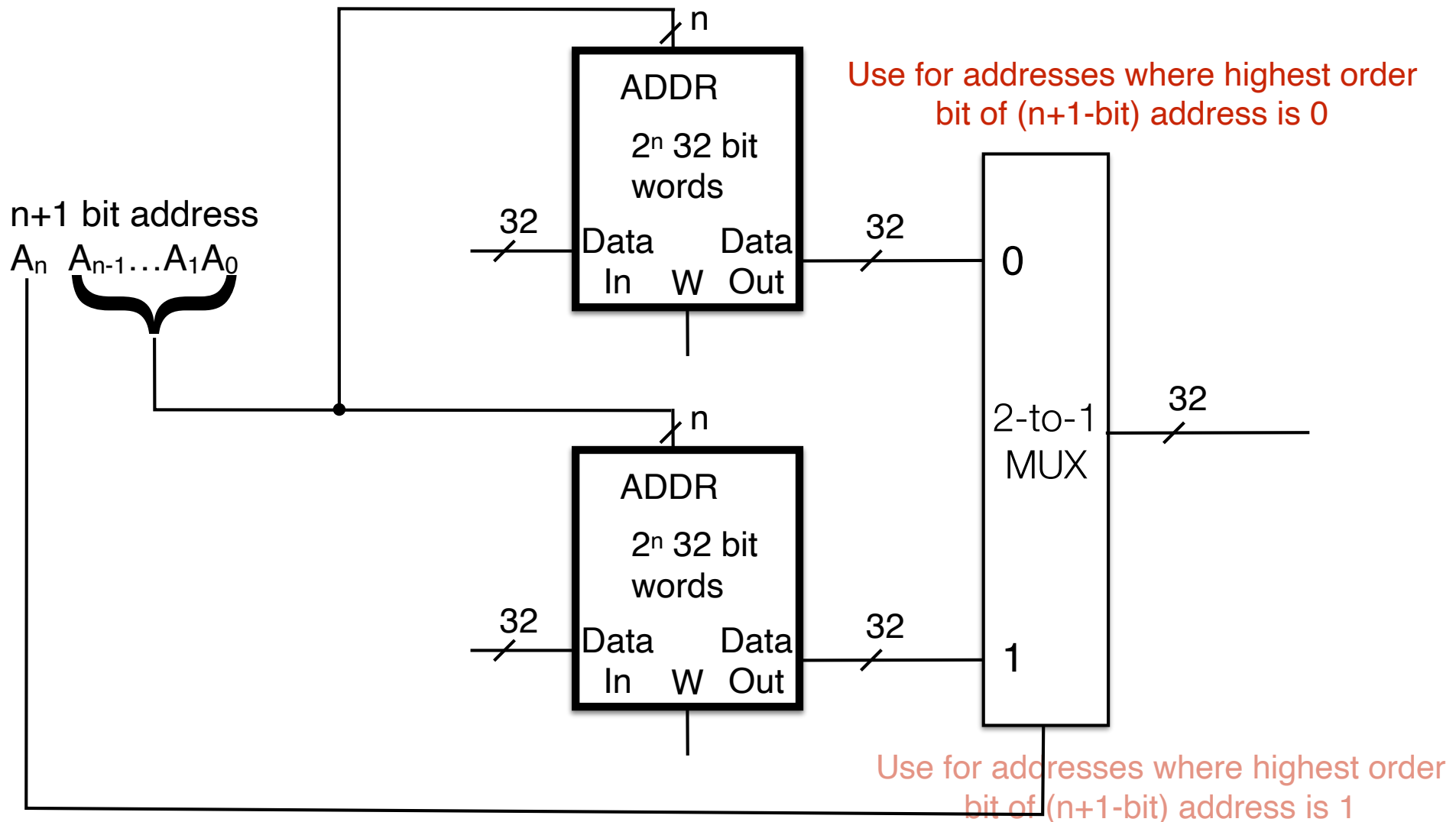
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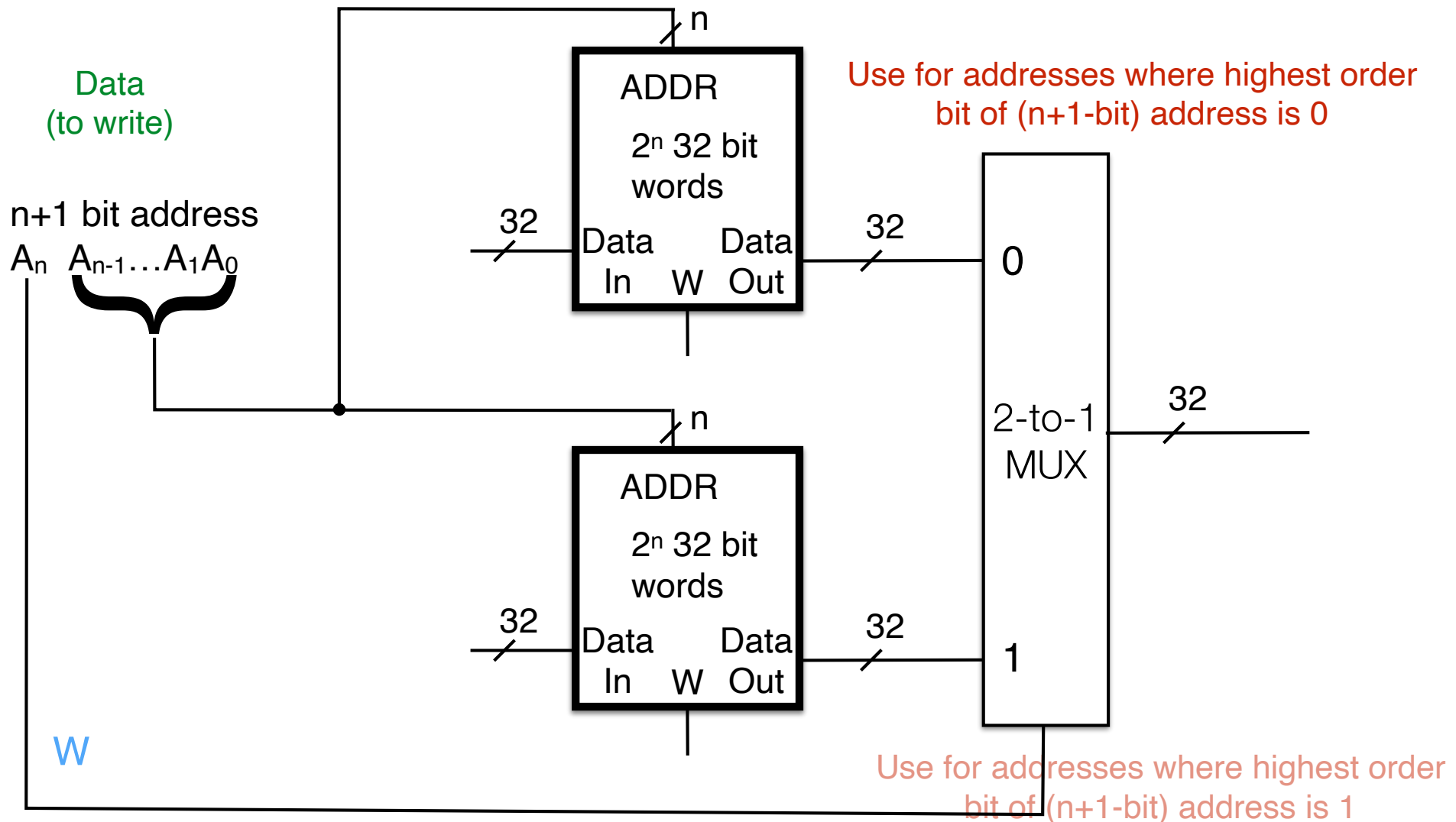
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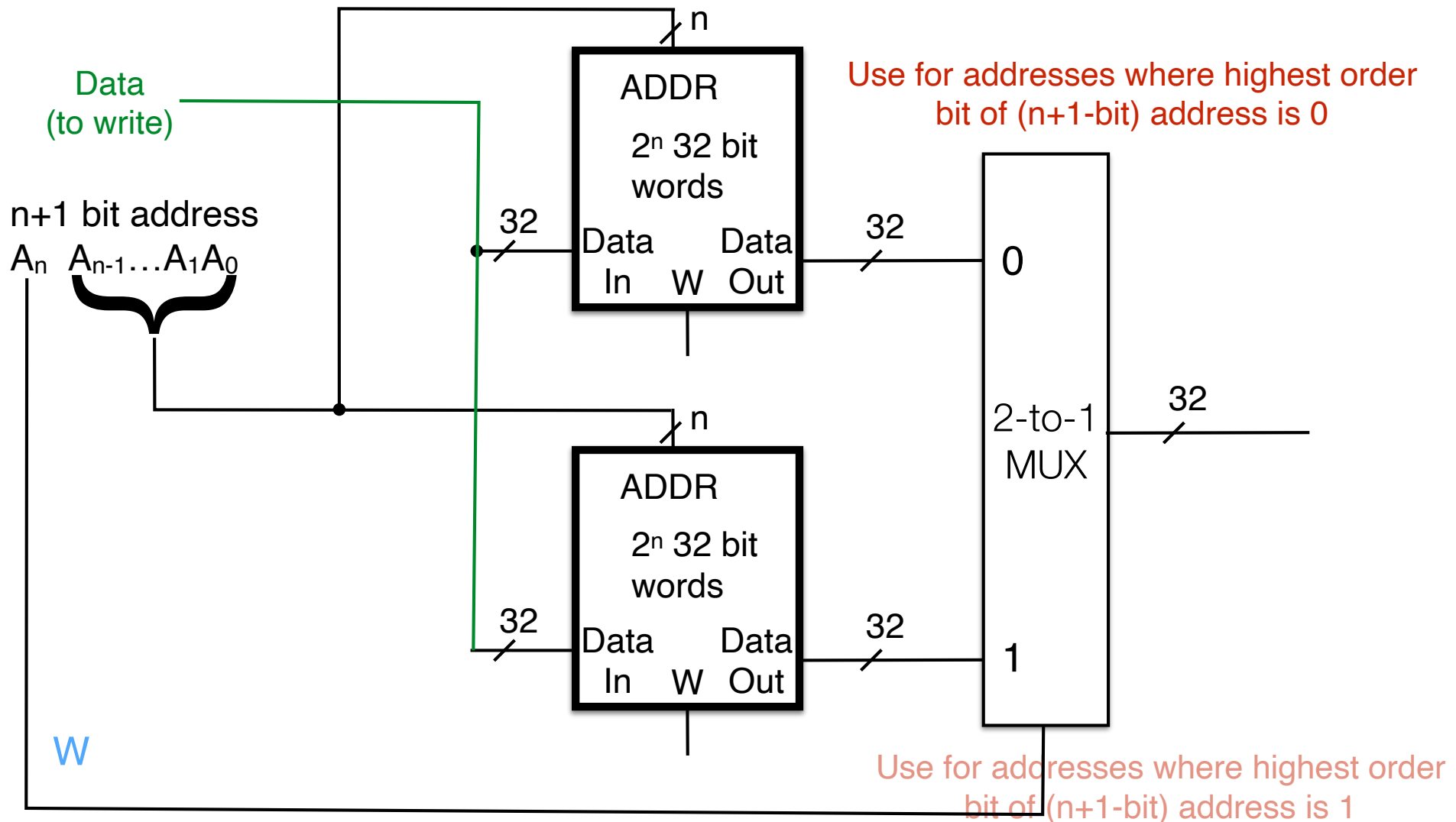
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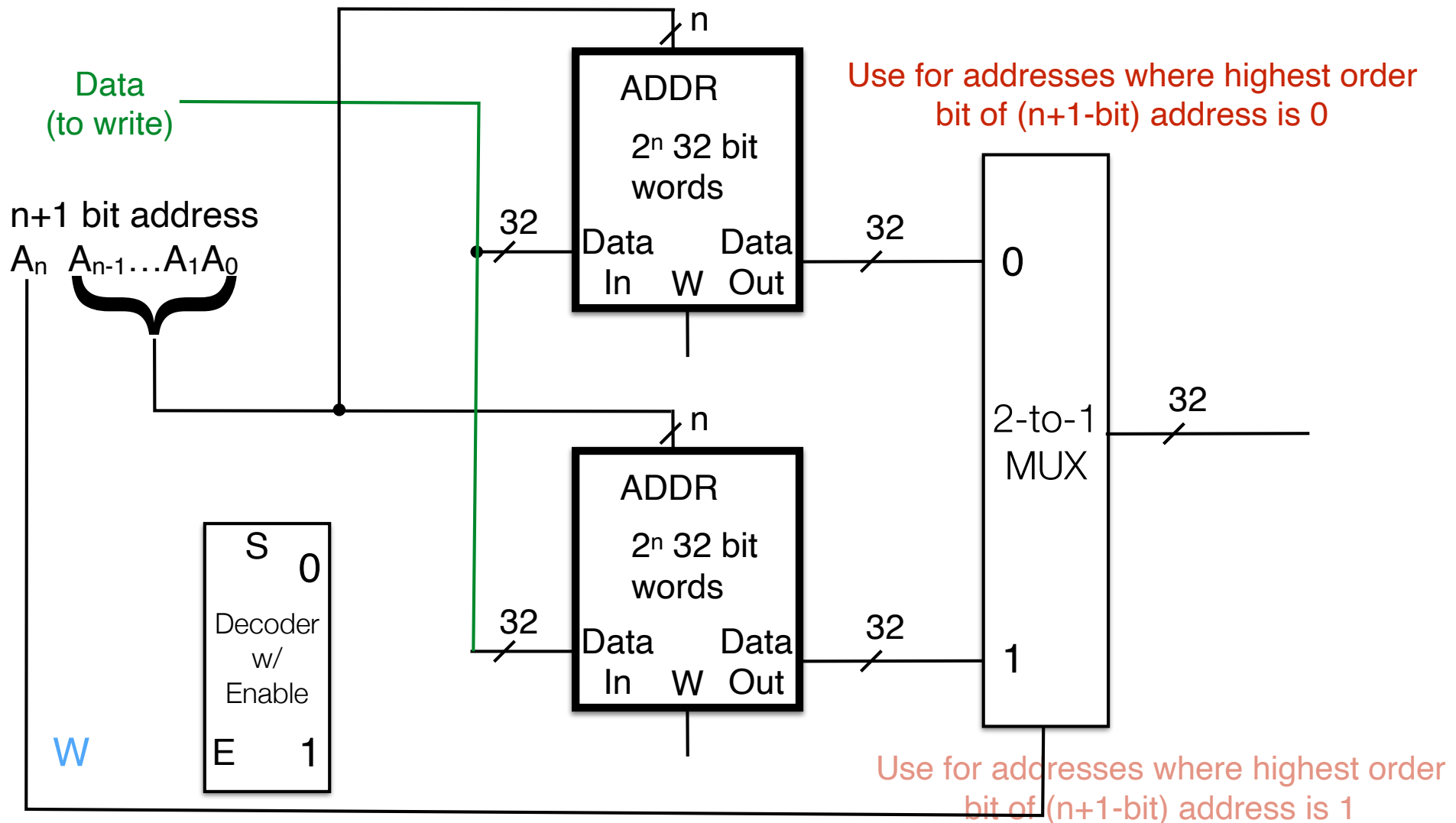
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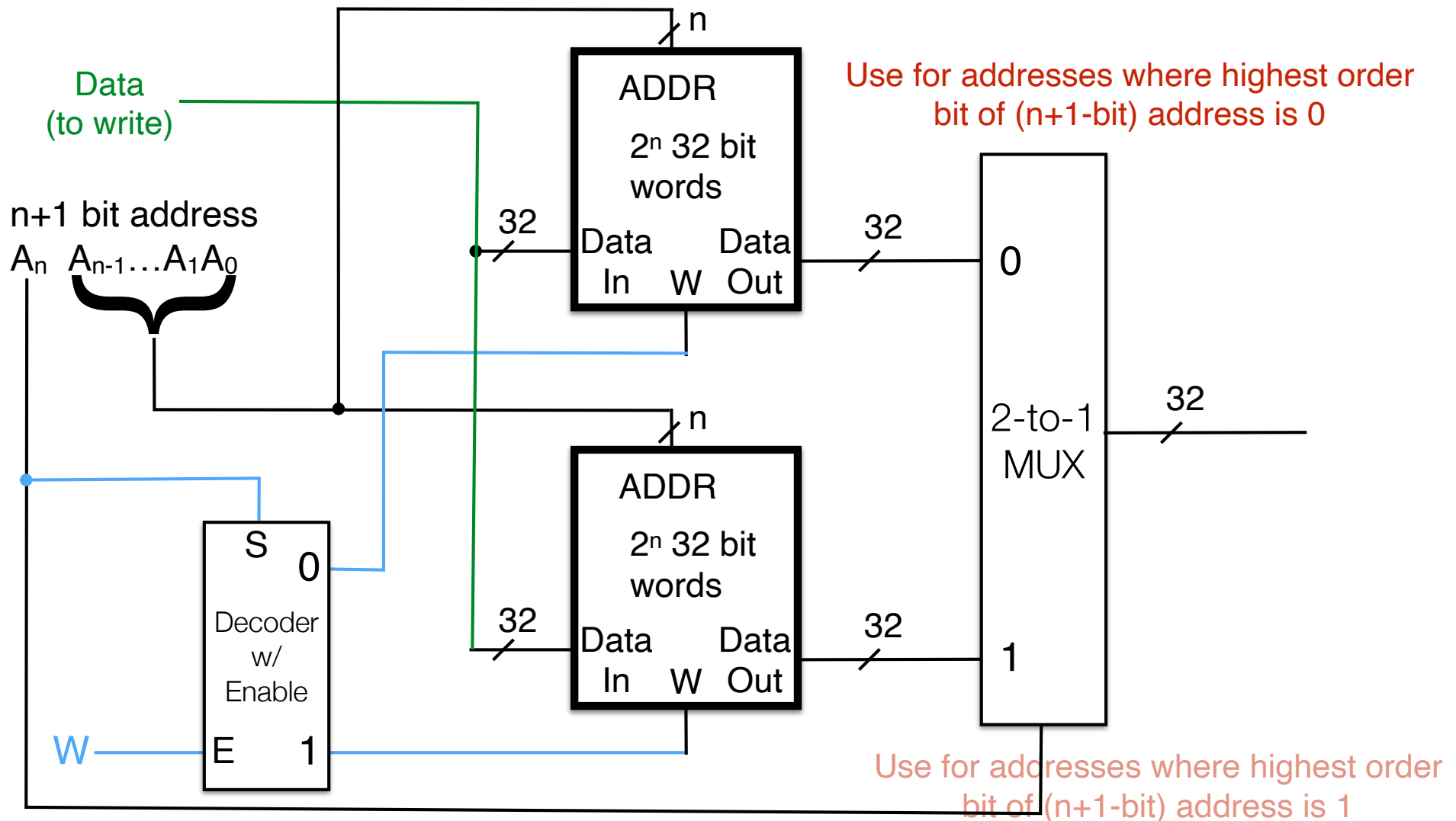
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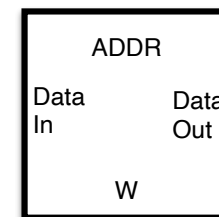
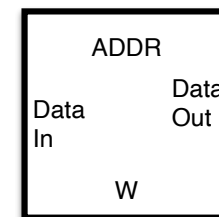
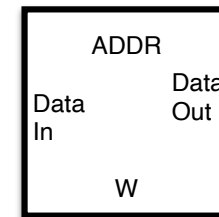
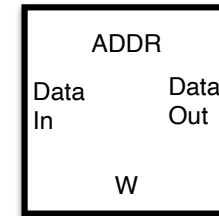


One more example:  
quadruple address  
space

# Extending address beyond 2 chips

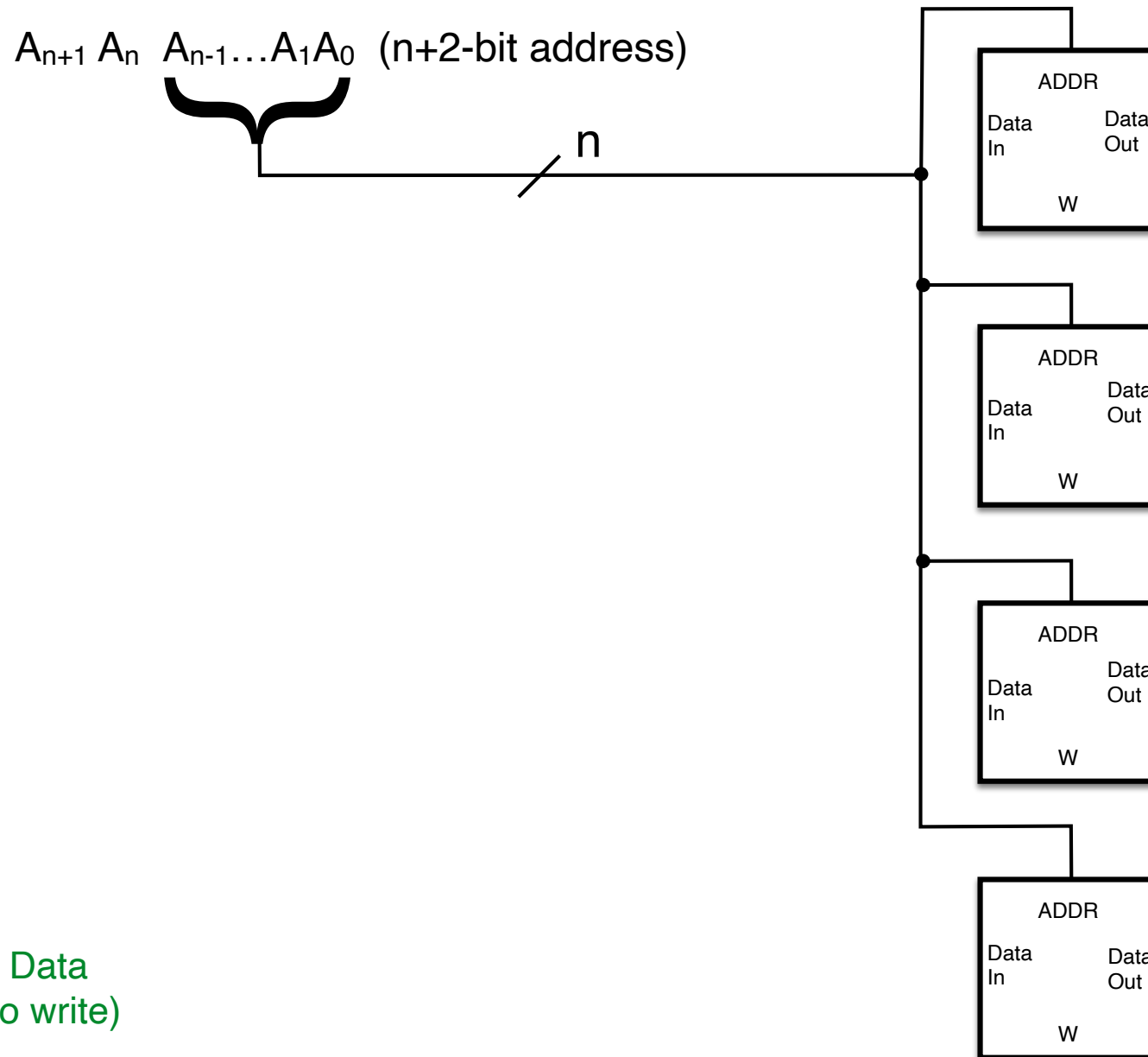
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$A_{n+1} A_n A_{n-1} \dots A_1 A_0$  (n+2-bit address)

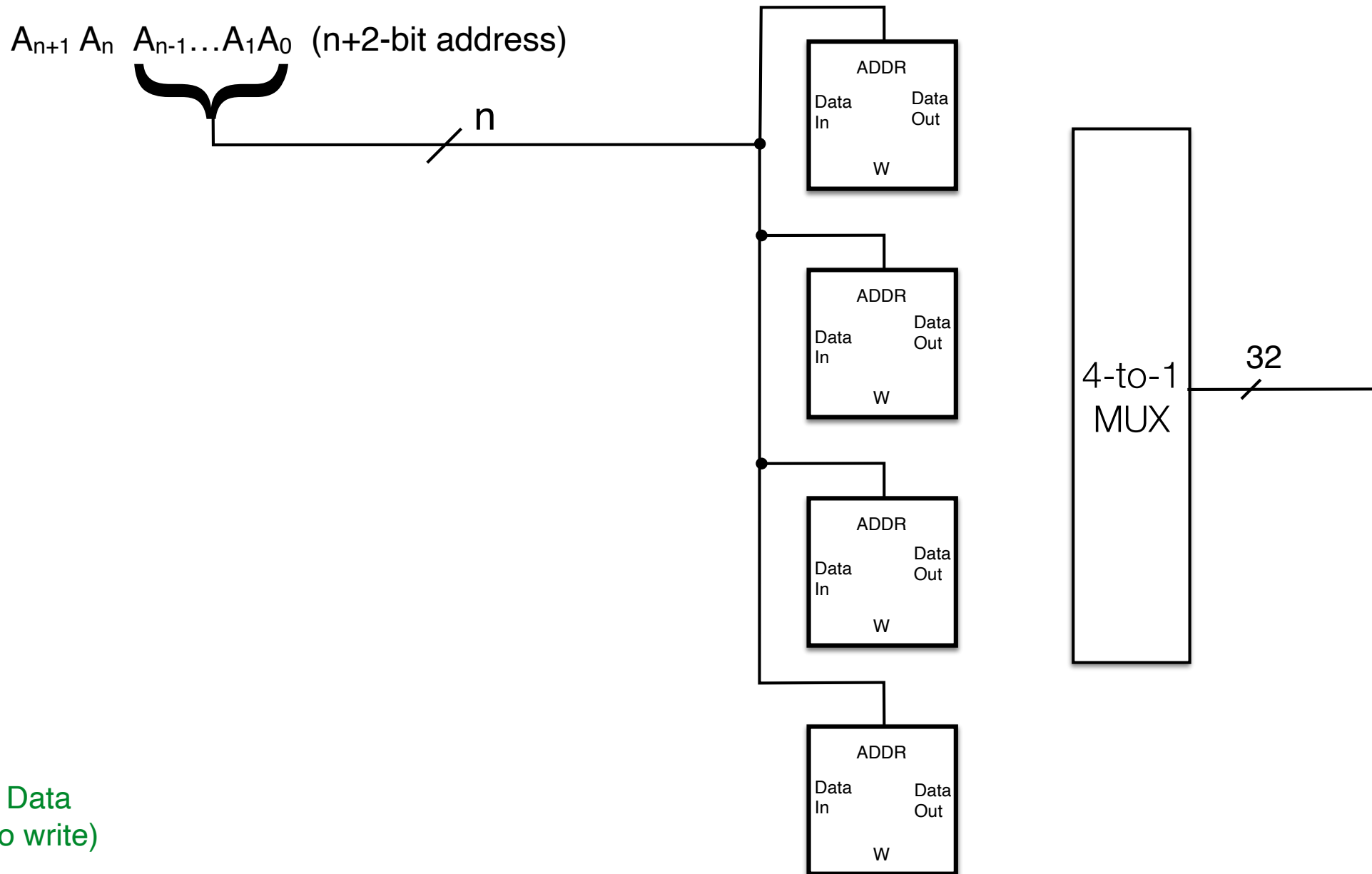


Data  
(to write)

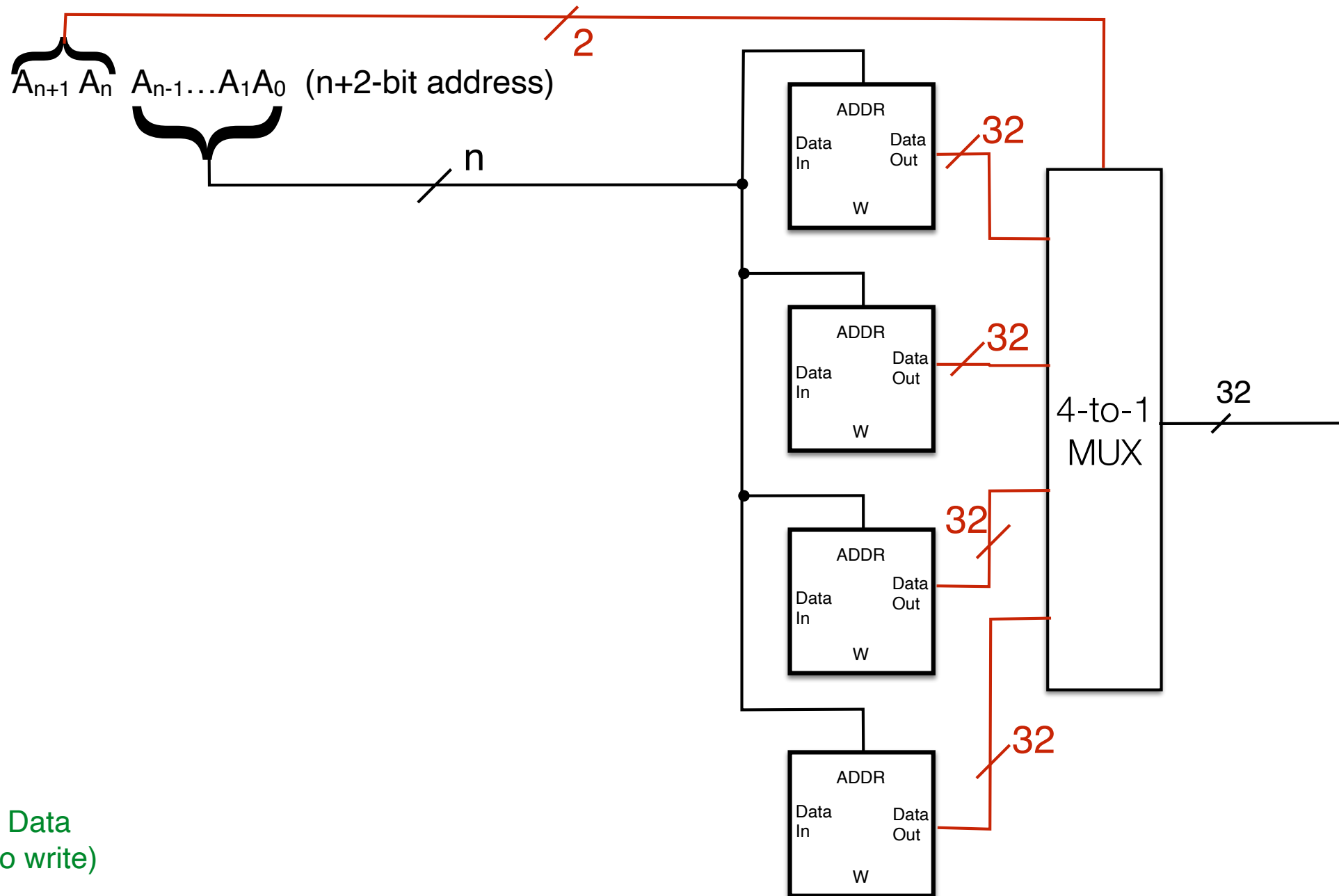
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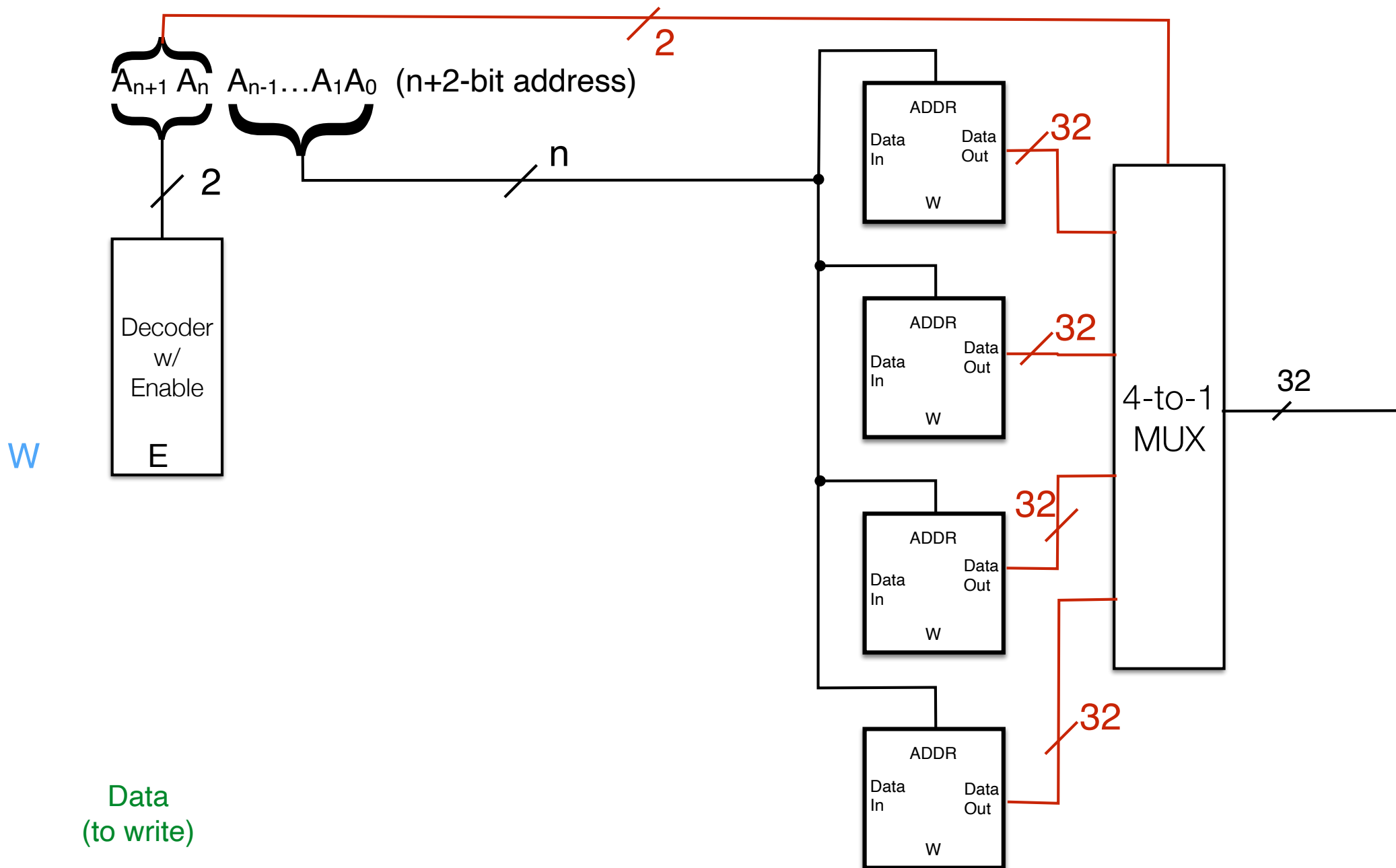


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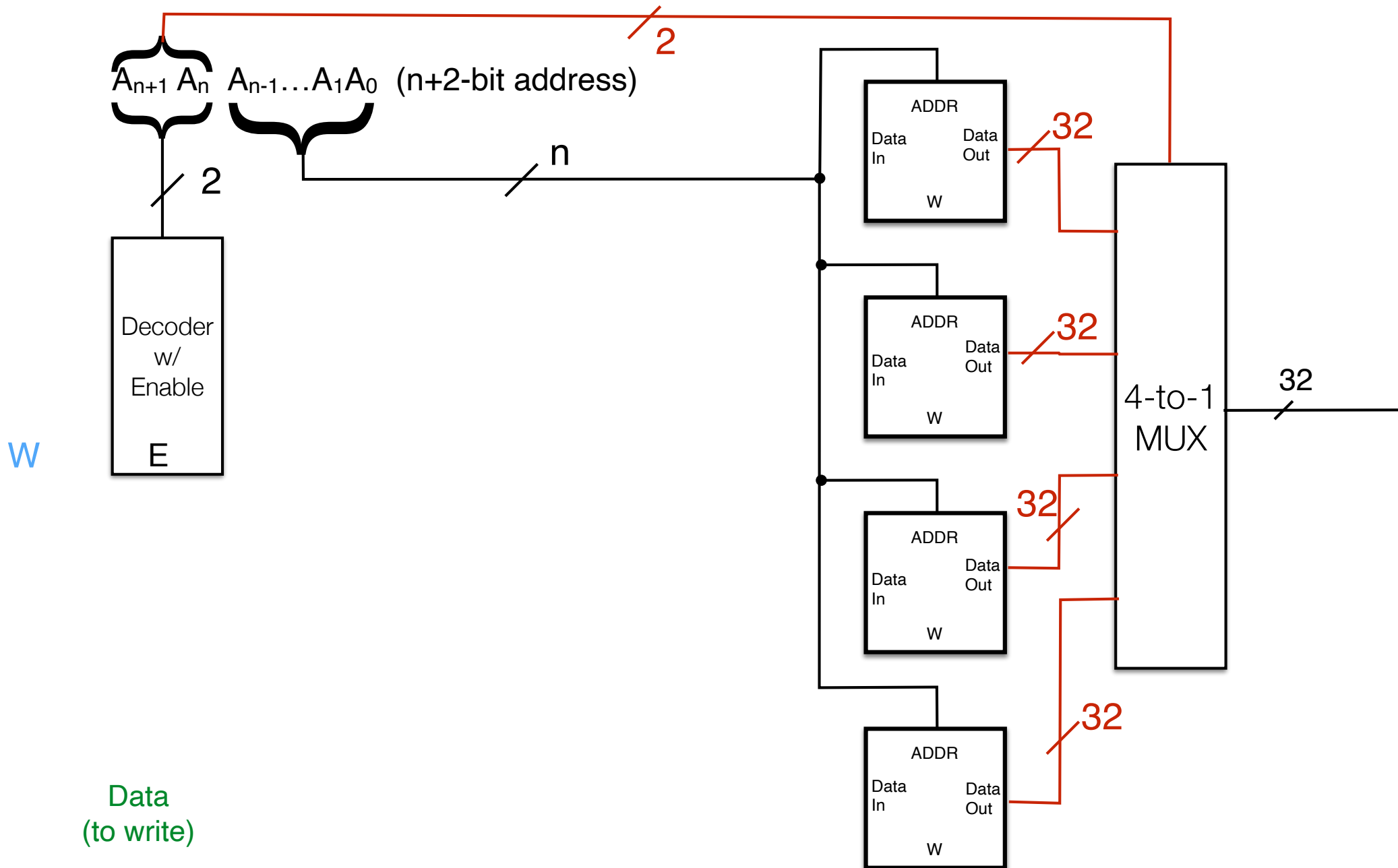




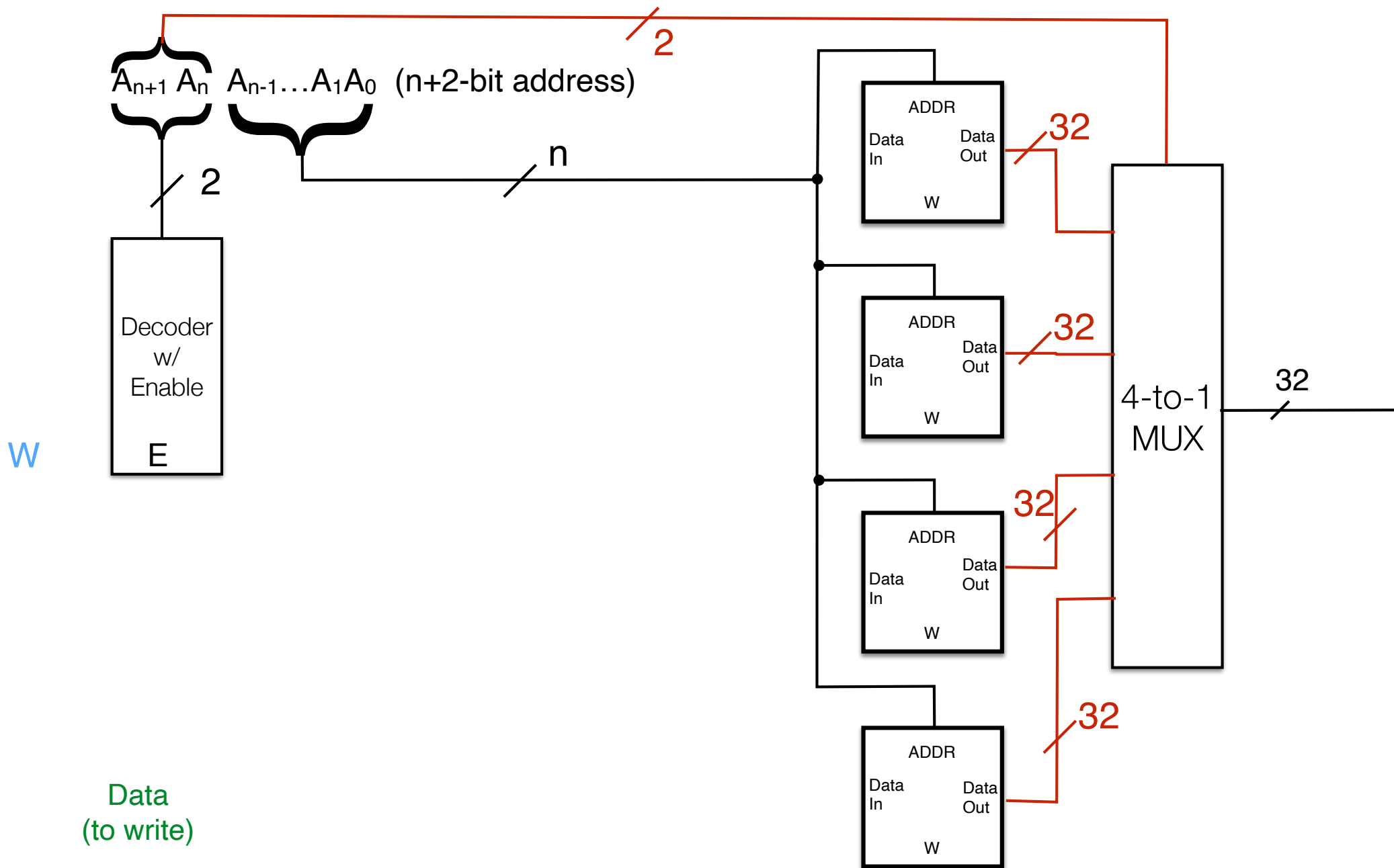
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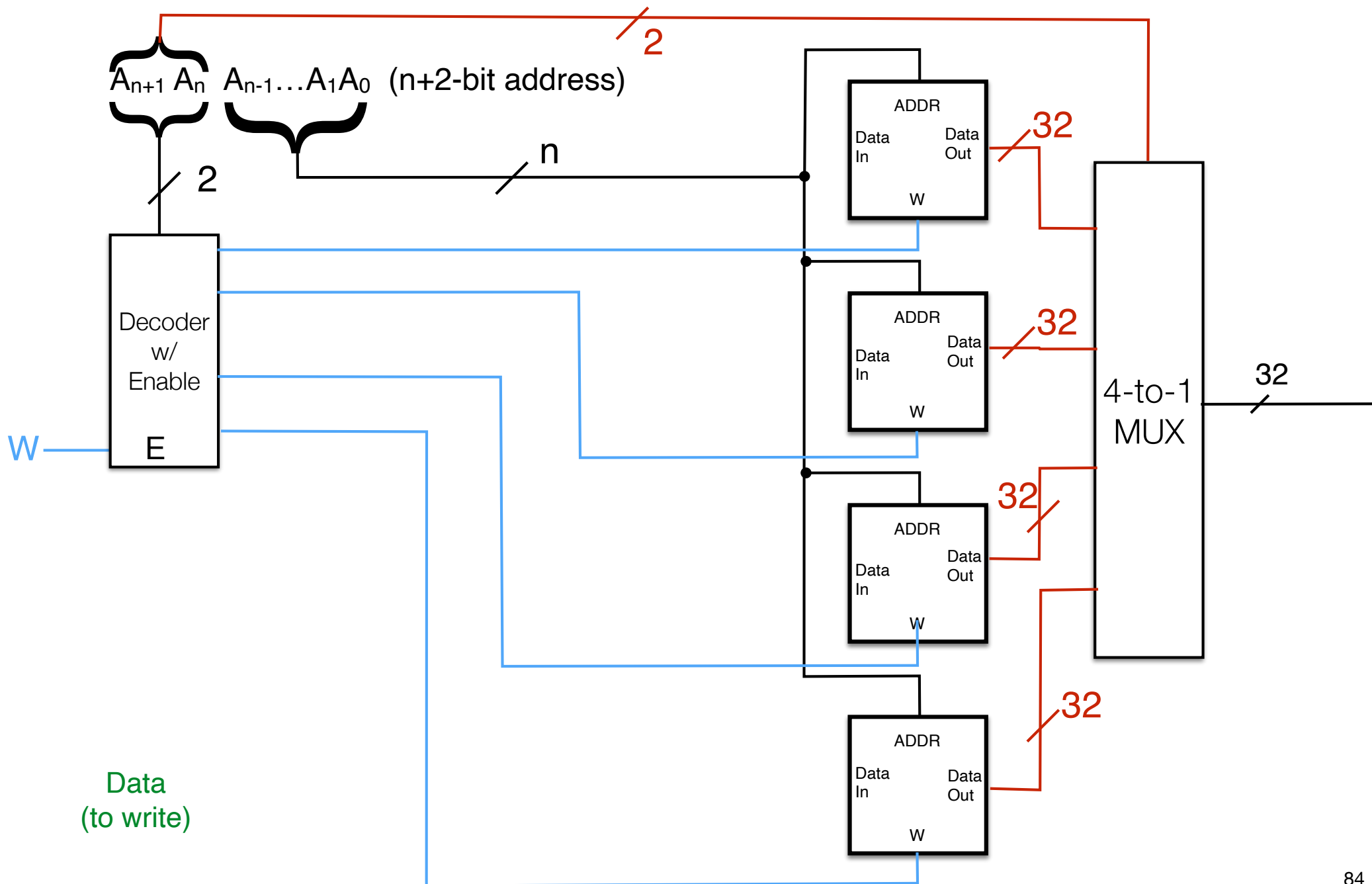
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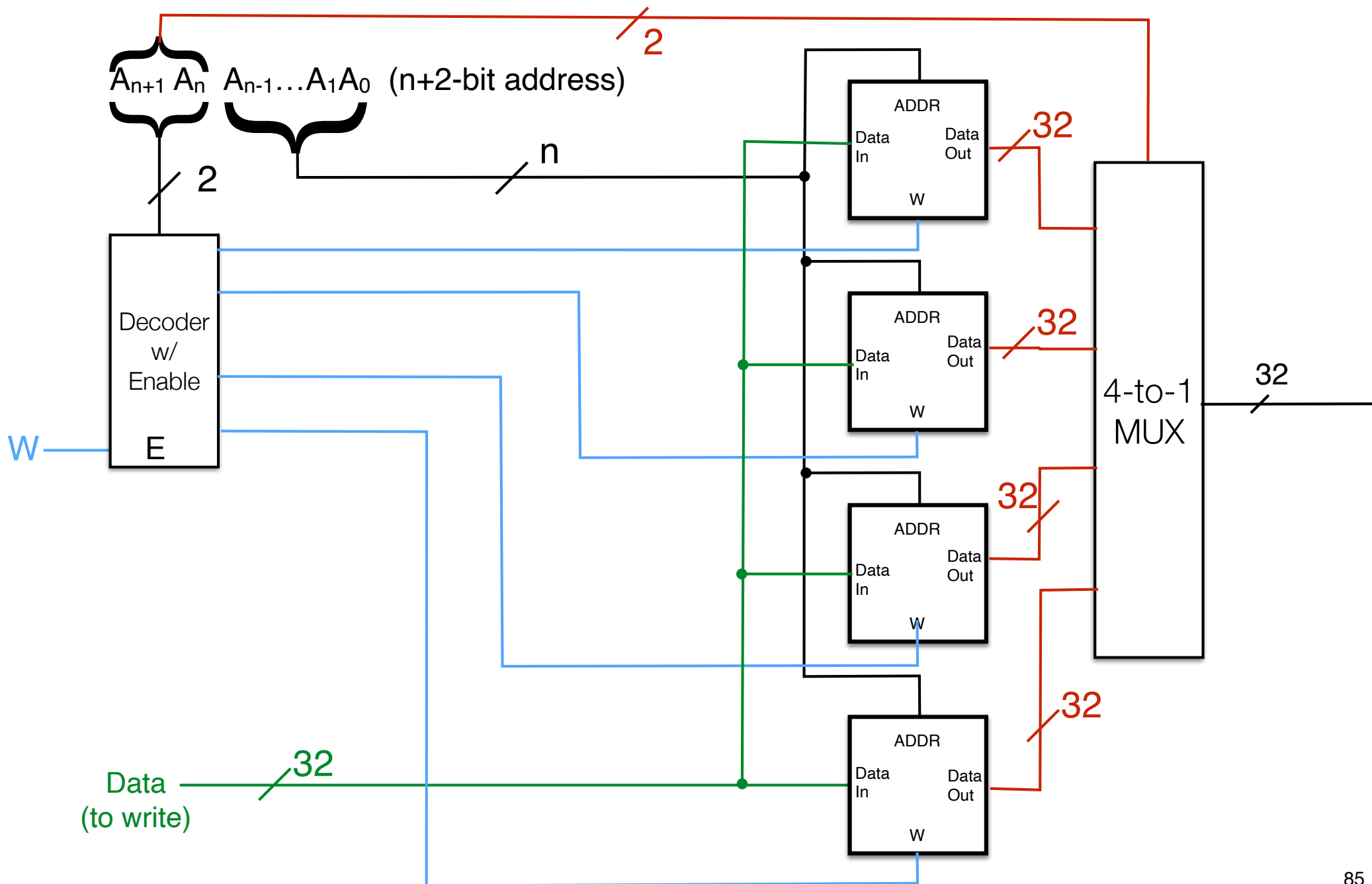
# Extending address beyond 2 chips



# Extending address beyond 2 chips



# Extending address beyond 2 chips



# Some Timing Issues with Memory

# Memory Timing: Write example

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- Even though memory not “on the clock”, timing still an issue:
  - inputs to memory are “on the clock”
  - Real memory takes several clock cycles to access (will deal with this later via cache)
  - must first be properly enabled for reading or writing before data is transferred
    - Write:
      - Appropriate address chosen & Data to write fed in
      - Wait for address & Data to stabilize
      - Activate write
    - Read:
      - Appropriate address chosen
      - Wait for address & corresponding data out to stabilize

# Memory Timing: Write example

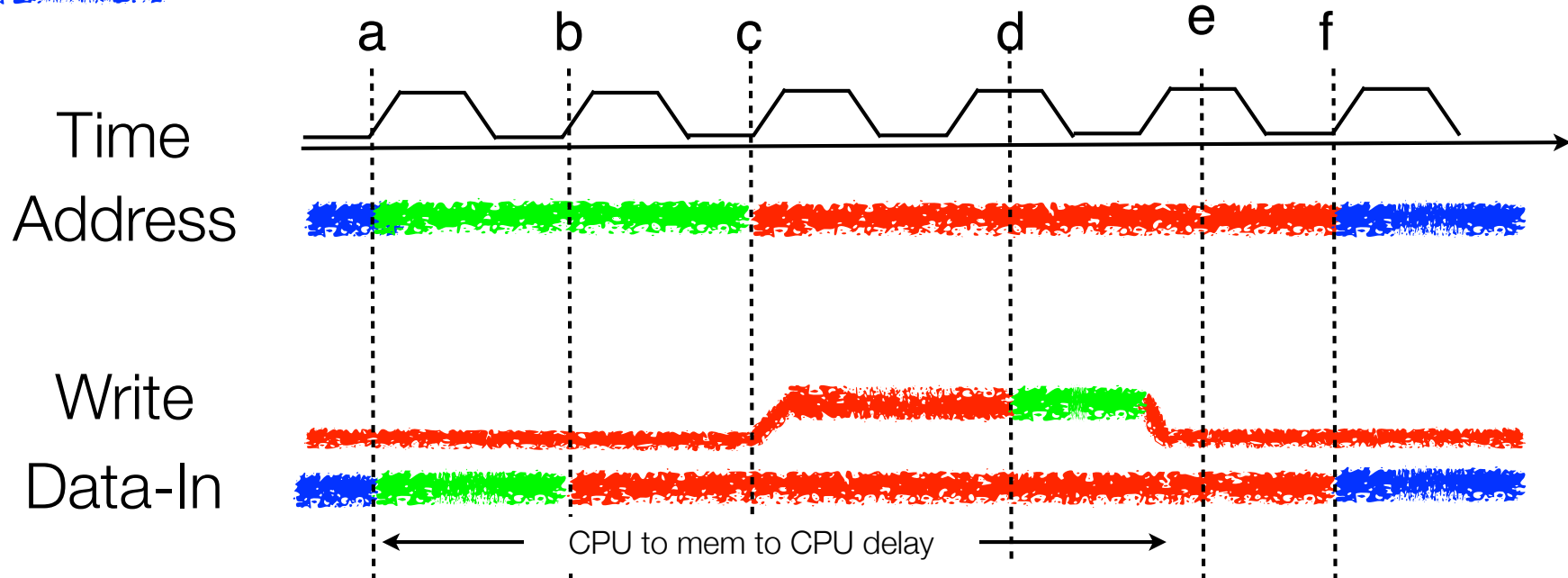
- Suppose:
  - 2 clock cycles to set address
  - 1 clock cycle to prepare data for writing
  - Write needs 1.5 clock cycles to ensure writing complete

- a. Write request (address + data to write) flows to chip
- b. Data to write stabilized
- c. Address stabilized
- d. Memory with new data stabilized
- e. Memory “frozen” (no writing)
- f. New instruction, new data

 Correct Value arrives

 Correct value “stabilized” in circuit

 Not correct value for current write





# Memory Timing: Read example

- Suppose:
    - 2 clock cycles to set address
    - 1 clock for data to flow from address to Data Out
- a. Read request (address) flows to chip  
b. Address stable, data from address flowing out  
c. Data Out stable  
d. New instruction, new data (read further at own risk...)

