HW #0

CSEE W3827 - Fundamentals of Computer Systems Spring 2022

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Assigned 1/18/2022

Modular arithmetic: computers are limited in the size of numbers they can store. These numbers thus often represent a modular value, so it's important to understand what a value is modulo some base value. Understanding modular arithmetic where negative values are also included is especially important when we talk about 2's complement form.

1. $x \pmod{y}$ equals the remainder when x is divided by y. For instance, $5 \pmod{3} = 2; 100 \pmod{5} = 0; 17 \pmod{20} = 17; 81 \pmod{2} = 1$. Compute
   (a) $8 \pmod{3}$
   (b) $6 \pmod{4}$
   (c) $1203912 \pmod{10}$
2. Note that if $x \pmod{y} = z \pmod{y}$, then $x - z = ky$ for some integer k. We can use this property to extend mod to negative numbers as well, i.e., $-4 \pmod{6} = 2$, $-1024912489 \pmod{10} = 1$. Compute
   (a) $-8 \pmod{10}$
   (b) $-1 \pmod{14}$
   (c) $-30 \pmod{15}$
3. Finally, we can alternately define $x \pmod{2y}$ to range from $-y$ to $y - 1$ instead of from 0 to $2y - 1$. So $4 \pmod{8} = -4$, $5 \pmod{8} = -3, 2 \pmod{8} = 2, 11 \pmod{8} = 3, 15 \pmod{8} = -1$, etc. Using this paradigm, compute
   (a) $3 \pmod{10}$
   (b) $7 \pmod{10}$
   (c) $5 \pmod{10}$
   (d) $-8 \pmod{10}$
   (e) $-49 \pmod{10}$

   Boolean Logic: All our circuitry that we develop will utilize basic boolean logic constructs (AND, OR, NOT), so it is important to have a clear understanding of these basic logical concepts.
4. Given that x AND y is TRUE if and only if x is true and y is true, x OR Y is TRUE if either x is true or y is true, or both, and x XOR y is TRUE only when either x is true or y is true (but not both), which of the following statements are TRUE?
   (a) You are a student in 3827 AND you live on Mars
   (b) You are a student in 3827 OR you live on Mars
   (c) You've been on Earth AND you drank water at least once
   (d) You live on Earth OR you drank water at least once
   (e) You live on Earth XOR you drank water at least once
   (f) (You live on Earth XOR you drank water at least once) XOR at some point in your life you had a parent.
   (g) It's NOT the case that you've spoken on a phone.
   (h) It's NOT the case that (you were born male AND you were born female)
   (i) (You were NOT born male) OR (You were NOT born female)
   (j) It's NOT the case that (you were born male OR you were born female)
   (k) (You were NOT born male) AND (You were NOT born female)

We will sometimes be performing arithmetic shift left and right operations, as well as building circuits that perform these operations, so it's important to understand what shifting is all about:

5. Imagine a row of students sitting in a classrom, where a '1' indicates a seat is occupied and '0' indicates a seat is empty. Suppose the row has the following seating sequence from left to right (from the Professor's perspective): 110101000111101

   (a) If the Professor asks all students to shift left (from the Professor's perspective) 3 seats, with any students being left without a seat to exit the row, what would the resulting seat arrangement look like?

   (b) Suppose instead a '1' meant the seat was occupied by a student wearing glasses and the '0' meant occupied by a student not wearing glasses. If instead, the Professor asked the students who were shifted and lost seats without seats to "wrap around" and maintain their same order among those wrapped around, what would would the resulting arrangement look like?

Early on, we will delve into how floating point numbers are represented in computer memory. Understanding scientific notation is immensely helpful.

6. Recall that a number is in scientific notation form when written as $X_0.X_1X_2...X_k \cdot 10^Y$, where X_0 is a non-0 digit[1], each X_i is a digit, there is only a single digit to the left of the decimal, and Y is any (negative or positive integer).

   (a) Write 475.52 using scientific notation

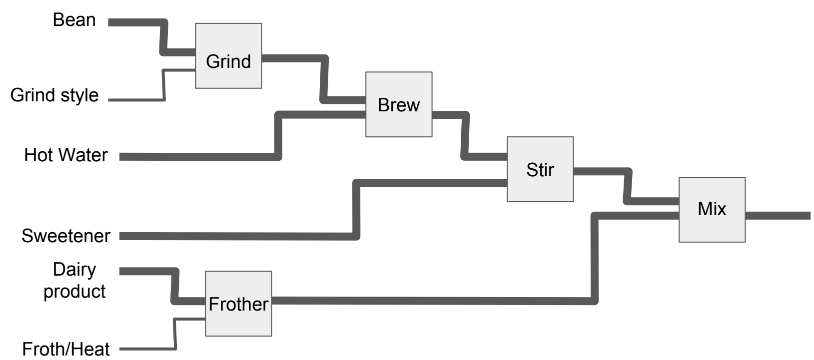
   (b) Write -0.0045 using scientific notation

   (c) Compute $1.45 \cdot 10^3 + 3.22 \cdot 10^2$ and represent the answer using scientific notation.

   (d) Compute $(7.02 \cdot 10^3) \cdot (4.1 \cdot 10^{-2})$ and represent the answer using scientific notation.

[1] except for the case where the value of the number being represented is actually 0.

There will be occasions where we will need to understand how changing inputs affects outputs as the new inputs flow through the circuitry toward the outputs.



7. The figure above shows an automated system for continually producing coffee for a coffee shop. A user selects a) the bean type (e.g., Colombian, French), a Grind Style (e.g., coarse, fine), a type of sweetener (e.g., sugar, stevia), a Dairy product (e.g., Cream, Almond Milk) and a froth style (cold, hot, frothed).

   The items are continually pumped into the system through pipes, and move quickly (i.e., 10 milliseconds) through these pipes to reach "combination phases" where they are combined in some fashion (ground, brewed, stirred, etc.) Each combination phase takes exactly 10 seconds. Note that some combination phases, such as grind and froth can be implemented in parallel.

   (a) If the inputs are set at time t, at what time does the desired type of coffee exit the system?

   (b) For a generalized system built of this type where there are quick-moving (10 ms) pipes and slower "combination phase" stations, some of which can run in parallel, describe how to calculate the time for inputs to reach the output.

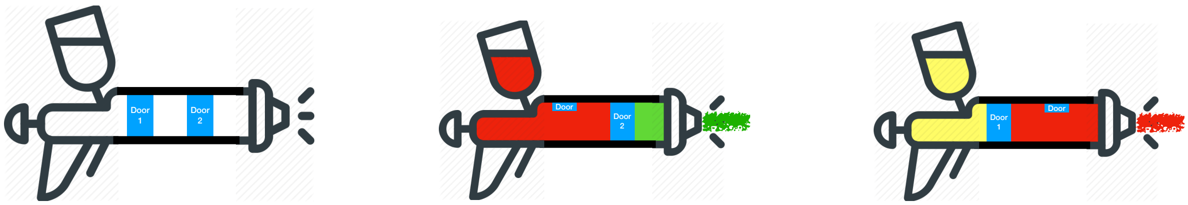
   (c) Suppose the system has been producing a certain type of coffee drink using Colombian beans and milk for the the past hour, and at time t the bean is changed from Colombian to French, and at time $t+15$ the dairy product is changed from milk to soy. Describe how the drink exiting the system changes with time.

   Understanding the relation between the number of (boolean) variables and the number of possible input combinations there can be is important, especially when building decoder circuits (and corresponding truth / excitiation tables), and discussing depth of circuitry.

8. Suppose a dinner has k courses. Each course is unique and offers two possible options. How many different dinner combinations are there?

9. Consider a tournament with N teams. Each round in the tournament, teams pair up and compete, with only the winner going to the next round. If the number of teams in any round is odd, then a team who cannot pair moves on automatically to the next round. How many rounds are there?

We will build flip-flop circuits that will clock (i.e., coordinate the time at which information flowing) the time when outputs from the circuit can change.

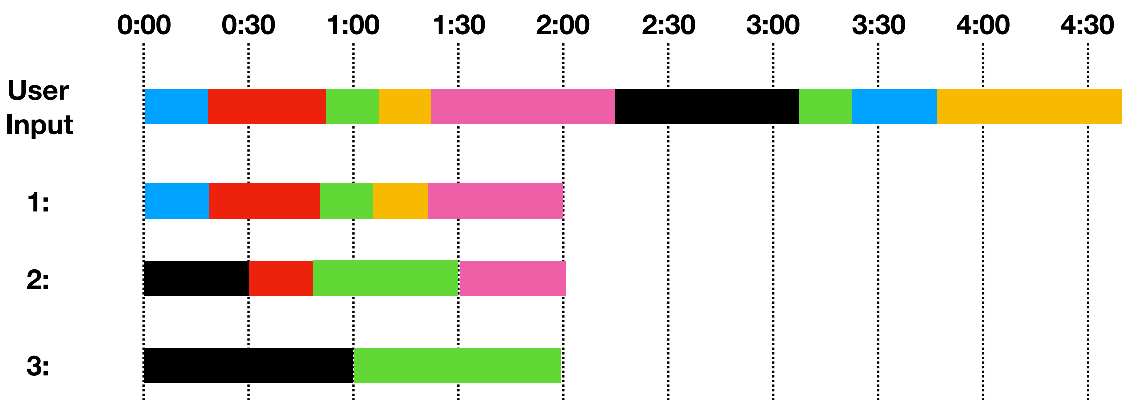


10. Consider a painting gun that works as follows:

   - A user loads paint into a first compartment, which flows into a second compartment, which flows into a third compartment, from which paint is sprayed from the gun. This is depicted above in the figure on the left.
   - There are two doors, 1 and 2. Closing Door 1 and opening Door 2 prevents paint from flowing between first and second compartments, and paint from the second compartment flows into the third and out the gun. Opening Door 1 and closing Door 2 causes paint to flow from first to second compartment, but paint in the second compartment cannot flow to the third, so whatever paint is in the third compartment comes out of the gun.

   Note: you should assume that when a door opens, the paint in the left compartment immediately replaces any existing paint in the right compartment.

   Suppose every thirty seconds the doors swing, such that from the 0- to 30-second mark, door A is closed and door B is open, and that from the 30- to 60-second mark, door A is open and door B is closed.



   (a) Consider the timeline above where the color of paint loaded by the user is depicted over time. Show what color is in compartments 1,2 and 3 (and hence out the gun) during these times. We assume compartments 2 and 3 initially have black, and the initial process is depicted.

   (b) What can be generally said about the color output by the gun over time with respect to user input over time?

In the second half of the course, we will explore the interface between circuitry and code. These questions should get you thinking about what "state" the computer must keep during execution of high level code, especially with regard to arrays, overloaded variable names, and recursive calls.

11. Consider the following code snippet:

```
x = 0;
y = 100;
while (x < y) {
   if (x==0){
      A[x] = 0;
   }
   else A[x] = A[x-1] + x;
   x = x + 1;
}
```

Suppose each variable is of type "integer". How much memory in terms integers (without optimization) is needed to store the data in the above program? Do not include memory that might be needed to store a temporary value for which a variable is not explicitly defined.

12. Consider the following code snippet:

```
call_me(int x) {
  int x,y; /* make space for local variables x and y */
  if (x==1){
     return 1;
  }
  y = 5;
  y = y + call_me(x-1) + x*x;
  return y;
}

main(){
  print call_me(100);
}
```

Same question as above. Again, assume that there are no optimizations.

13. Consider the following code snippet:

```
int call_me(int x) {
   x = x + 3;
   return x;
}

int main(){
   int y = 5;
   int x = 7;
   int z = y + call_me(x);
   int q = z + x;
```

```
   printf("The value of q is %d\n", q);
}
```

What value is printed for the variable q?