Midterm Solutions

CSEE W3827 - Fundamentals of Computer Systems — Oct. 22, 2018
Fall 2018 — Prof. Rubenstein

This midterm contains 5 pages: a question 0 and 3 other questions, and totals 100 points. To get full credit you must answer all questions. **NO BOOKS, NOTES OR ELECTRONIC DEVICES PERMITTED!** The time allowed is 75 minutes.

Please answer all questions in the blue book, using a **separate** page for each question. **Show all work! We are not just looking for the right answer, but also how you reached the right answer. Right answers without work get no credit!!!**

YOU MAY KEEP YOUR COPY OF THE QUESTIONS.
Some advice:

- Be sure to leave some time to work on each problem. The right answer to each problem does not require a very long answer.
- Be sure to start every problem. And take some time to think about how to set the problem up before you start writing.

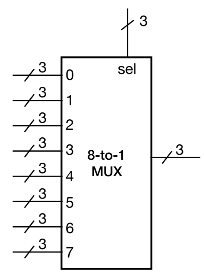
0. (5 pts) Do the following in the blue book:
   (a) CLEARLY write your name **and UNI** on the front cover.
   (b) start each numbered question's solution on a new page. So question 2 should start on a new page, question 3 on a new page, etc.
   (c) label solutions (e.g., 2a, 2b, 2c or 2a, b, c)

1. (30 pts) A k-bit incrementer circuit takes in a k-bit value $X = X_{k-1}X_{k-2}\cdots X_1X_0$ and (unsigned binary) adds 1 to it to produce $Y = Y_{k-1}Y_{k-2}\cdots Y_1Y_0$. For instance, for $k = 8$, if $X = 00110111$, the incremented value would be $Y = 00111000$. If X is all 1's, then Y should be all 0's (we don't worry about overflow).

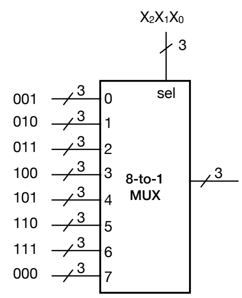
   (a) (15 pts) Describe the value of the ith output bit algebraically for any $i > 1$ as a function of the input bits $\{X_j\}$.

   Answer: If every bit to the right (all less significant) are 1, then $Y_i = \overline{X_i}$. Otherwise $Y_i = X_i$. Thus $Y_i = X_i \oplus X_{i-1}X_{i-2}\cdots X_0$.

   (b) (15 pts) Build a 3-bit incrementer using a 3-bit 8-to-1 MUX, as pictured. Show how to increment an input $X = X_2X_1X_0$.



   Answer: Simply feed X into the selector, and let the ith input to the MUX have the 3-bit value representing $i + 1$ (mod 8).



2. (30 pts) $X = X_{k-1}X_{k-2}\cdots X_1X_0$ is a k-bit number provided in 1's complement representation. In this problem, you are to convert the number to 2-s complement representation.

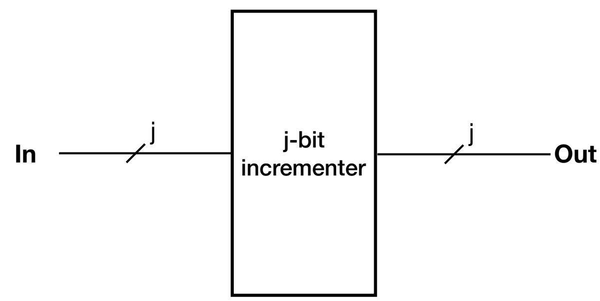
(a) (10 pts) Describe the algorithm (i.e., in simple pseudocode) where you can reference the entire input X as a variable or the individual bits $\{X_i\}$ in your pseudocode. Demonstrate the correctness of your solution when $k = 4$ for two cases: $X = -1$ and $X = 1$.

Answer: Intuitively, if $X_{k-1} = 0$, then return X. Else (if $X_{k-1} = 1$), return $X + 1$ (where "+" is addition).

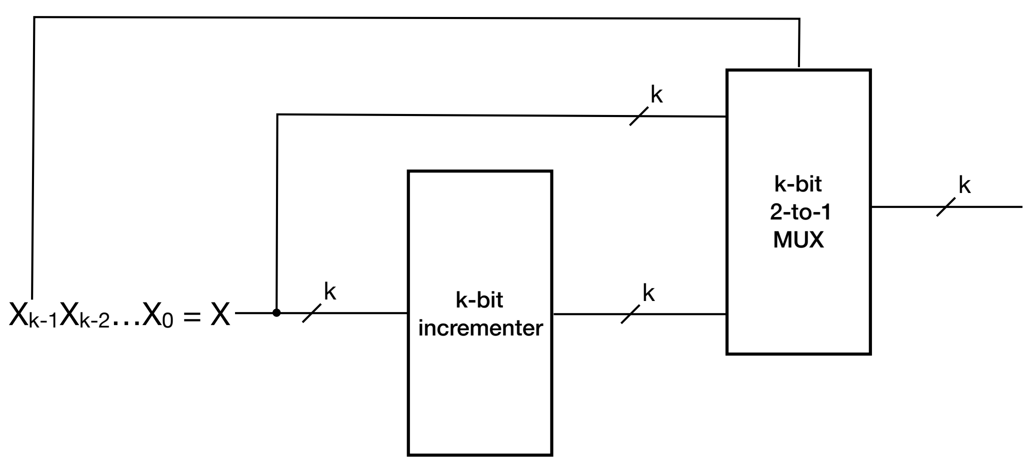
(b) (10 pts) Design a circuit that converts X into 2's-complement form. You may use any of the basic gates (AND, OR, NOT, XOR) or standard circuitry (MUX,Decoder, Enabler, half-adder, full-adder, ripple-carry adder) needed to design the conversion circuit.

Answer: There may be several ways to do this, but the easiest is to use a ripple-carry adder and add X to 0 with the $C_{in} = X_{k-1}$.

(c) (10 pts) Implement the converter using a multi-bit incrementer (i.e., described in problem 1, and depicted below as a j-bit incrementer), a multi-bit 2-to-1 MUX, and any basic gates (AND, OR, NOT, XOR) you additionally require.

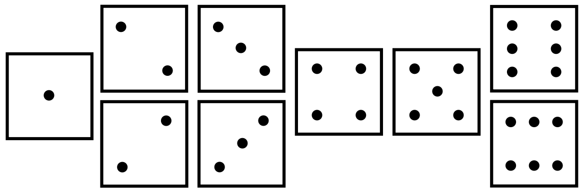


Answer:

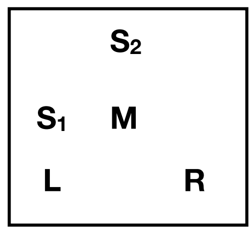


A k-bit 2-1 MUX takes in as input X on its 0-input and $X+1$ on its 1-input. The selector is X_{k-1}, such that if $X_{k-1} = 0$, then X is output, and if $X_{k-1} = 1$, then $X + 1$ is output.

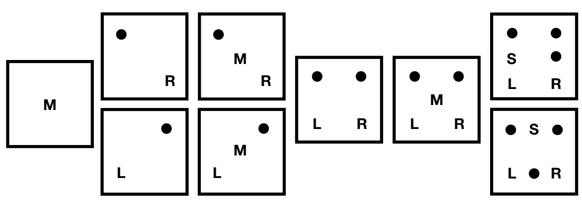
3. (35 pts) Professor Rubenstein's continuing obsession with dice games has him (making you) build a device that can scan the number of dots on the top of a 6-sided die to determine its current rolled value. The scanner is placed parallel to the die, such that the possible orientations are possible for each value. 2, 3 and 6 have two possible orientations depending on the rotation, and other values have only a single orientation.



To read the die, 5 scanners are employed. Each scanner views a particular part of the upward face, and outputs a 1 when the portion of the face viewed contains a dot. These five outputs are labeled M (for middle scan), L (for bottom left scan), R (for bottom right scan), and S_1 and S_2 (for scans that can pick up when the die shows a six). The values provided to the circuit designer are M, L, R, and $S = S_1 + S_2$.



You are to help design a circuit that takes inputs M, L, R and S, and outputs a 3-bit unsigned binary, ABC, equal to the value of the current roll. Each configuration above produces a unique assignment of M, L, R and S. For instance, if only M is 1, then the value depicted on the die must be 1. If only $M = 1$ and $L = 1$, the die value is 3. If only $M = 1$ and $R = 1$, the die value is also 3, just rotated the other way. The figure below shows, for each orientation listed in the top figure, which input values equate to 1 for that orientation.



Provide simplified (in SoP form) expressions of the 3-bit binary die value in terms of these 4 scanner values M, L, R, S.

Answer: The following truth table shows the 16 possible configurations of M, L, R, and S, and the corresponding value of the die, and the corresponding unsigned binary representation:

M	L	R	S	val	A	B	C
0	0	0	0	N/A	X	X	X
0	0	0	1	N/A	X	X	X
0	0	1	0	2	0	1	0
0	0	1	1	N/A	X	X	X
0	1	0	0	2	0	1	0
0	1	0	1	N/A	X	X	X
0	1	1	0	4	1	0	0
0	1	1	1	6	1	1	0
1	0	0	0	1	0	0	1
1	0	0	1	N/A	X	X	X
1	0	1	0	3	0	1	1
1	0	1	1	N/A	X	X	X
1	1	0	0	3	0	1	1
1	1	0	1	N/A	X	X	X
1	1	1	0	5	1	0	1
1	1	1	1	N/A	X	X	X

The corresponding K-maps and solutions for A,B,C are:

A:

		S	S		
	X	X	X	0	
	0	X	1	1	L
M	0	X	X	1	L
M	0	X	X	0	
			R	R	

$A = LR$

B:

		S	S		
	X	X	X	1	
	1	X	1	0	L
M	1	X	X	0	L
M	0	X	X	1	
			R	R	

$B = S + R\bar{L} + L\bar{R} = S + (L \oplus R)$

C:

		S	S		
	X	X	X	0	
	0	X	0	0	L
M	1	X	X	1	L
M	1	X	X	1	
			R	R	

$C = M$