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Announcements

Announcements: Upcoming Assessments

- For HW3, do not hand in “Warmup Problems” – answers are on CourseWorks
- HW3 is due on Friday 2/18
- HW4 is due on Friday 2/25
- HW1 is graded
- Will have HW2 graded soon

Announcements: Homework 1 and Homework Submission

- Common errors from Homework 1:
 - Detecting when overflow happens (last two carries don't match)
 - Missing bits in question 5 (floating point number) – group bits in 4
 - Misinterpretation of signed magnitude/1's complement/2's complement
- Submitting homework on Gradescope:
 - Be sure you submit to the right TA and right homework number
 - There are a lot of assignments, so just search for text

Announcements: Feedback

- Form: <https://forms.gle/cnUmKVNYN7WvRbHA6>
- From feedback since last time:
 - “...it would be nice if you asked for HW problems after 15 mins of going through the important concepts rather than spending 40+ mins on the important concepts and then remaining time on the hw.”
 - Will start taking HW problems earlier and cover concepts as we go
 - Will denote notable concepts with “NC”
 - “It also would be nice if when going through the concepts you do some example problems that are similar to the homework.”
 - Coming up with questions is always hard, so I will probably default to using the homework questions to show concepts I am referring to

Homework 3 Warmup

Homework 3 Warmup: Problem 1

Build a MUX from a Decoder and some AND and OR gates.

- NC: Symmetry of inputs and circuit logic

Homework 3 Warmup: Problem 2

A MUX-with-Enable is a MUX with one additional selector input, E . When $E = 1$, the MUX-with-Enable behaves like a traditional MUX. When $E = 0$, the MUX-with-Enable is disabled and outputs 0. Build a MUX-with-Enable from a MUX (without enable) and some AND gates.

Homework 3 Warmup: Problem 3

A 1-to- 2^k DEMUX takes one data input I and a k -bit selector S as input and outputs 0 on each of $2^k - 1$ outputs, and outputs I on the j th output where j is the unsigned binary value represented by S . Construct a DEMUX from a decoder and a bunch of AND gates.

Homework 3 Warmup: Problem 4

Use five 2-to-1 MUXs and as many NOT gates as needed, build a circuit that takes a 5-bit value and a selector input S and returns

- when $S = 0$, the original number is returned
- when $S = 1$, the 1's complement of the number is returned.
- NC: MUXes as if/then/else statements

Homework 3 Warmup: Problem 5

Build a circuit using two 2-to-1 MUXs that takes a 2 bit-input B_1B_0 and a 1-bit selector S and returns B_1B_0 when $S = 0$ and returns B_0B_1 (i.e., switches the bit-order) when $S = 1$.

- NC: MUXes as if/then/else statements

Homework 3 Warmup: Problem 6

Solve problem 3 of the “Harder Problems” using four 16-to-1 MUXs, one MUX for each output NSH, NSL, EWH, EWL. Now solve using four 8-to-1 MUXs.

- NC: MUXes as brute-force solution

Homework 3 Warmup: Problem 7

Design a circuit that receives a k -bit string $A = A_{k-1}A_{k-2} \dots A_1A_0$ and, using $k - 2$ 4-to-1 MUXs, outputs k -bit string $B = B_{k-1}B_{k-2} \dots B_1B_0$ with the following properties

- $B_0 = A_0, B_{k-1} = A_{k-1}$
- $B_i = A_i$, whenever $A_{i+1} \neq A_{i-1}, 0 < i < k - 1$
- $B_i = A_{i-1}$, whenever $A_{i+1} = A_{i-1}, 0 < i < k - 1$
- NC: MUXes as if/then/else statements

Homework 3 Harder Problems

Homework 3 Harder Problems: Problem 1

Construct a 4-to-16 line decoder with an enable input using five 2-to-4 line decoders with enable inputs (Hint: Start at the outputs: If all that is being used is decoders, then how many decoders are connected directly to outputs?)

- NC: Symmetry of circuit
- NC: matching combinational circuits to number of inputs/outputs
- NC: partitioning of input (i.e. $l_3l_2l_1l_0$ into l_3l_2 and l_1l_0)

Homework 3 Harder Problems: Problem 2

A combinatorial circuit is specified by the following three Boolean functions:

$$F = X + \bar{Y} + \bar{X}YZ$$

Design the circuit with a decoder and external OR gates.

- NC: decoder as “cases” or minterms

Homework 3 Harder Problems: Problem 3

A traffic light controller receives a 4-bit input that changes every 5 seconds. The input sequence is a simple counter that counts from binary 0 to binary 15 and then starts again from binary 0. These signals go to 4 outputs, NSH, NSL, EWH, EWL. The first two outputs NSH and NSL respectively represent the high and low bits that are fed to the lamps that face in the North-south direction. The other two outputs EWH and EWL respectively represent the high and low bits that are fed to the lamps that face in the East-West direction. The following table indicates how setting the high and low bits determines the color of the lamp:

High	Low	Lamp Color
0	1	Green
0	1	Yellow
1	0	Red
1	1	Red

Homework 3 Harder Problems: Problem 3 (cont.)

Each light should be green for 30 seconds, yellow for 5 seconds, and then red for 45 seconds. There should be two 5 second intervals when both lights are red. Assume that for the interval where the input is 0000, the North-South light has just turned green, and the East-West light has been red for 5 seconds already.

Design the circuitry that feeds from $ABCD$ to the two outputs. You may represent your answer as algebraic equations (make sure to simplify).

- NC: at least one similar word problem (probably with FSMs) will be on exam

Homework 3 Harder Problems: Problem 4

The 01-swap operation, b , on a binary string S , permutes all occurrences of 01 within the original string to 10 (the process is not recursive). For instance:

- $b(0) = 0, b(1) = 1, b(00) = 00, b(01) = 10, b(11) = 11$
- $b(000) = 000, b(001) = 010, b(010) = 100, b(100) = 100$
- $b(0\ 01\ 01\ 1\ 01\ 0) = 0\ 10\ 10\ 1\ 10\ 0$ (spacing added for clarity)

Design a circuit using 2:1 multiplexers that can be used to perform the 01-swap on a k -bit string $S = S_{k-1}S_{k-2} \dots S_0$. where each S_i is a bit.

Homework 3 Harder Problems: Problem 4 (cont.)

- Ⓐ First, show the circuit, built using the 2:1 MUX, whose output is the i th bit of $b(S)$ where $0 < i < k - 1$. YOU DO NOT NEED ANY AND, OR, OR NOT GATES, only a single 2:1 MUX. This is somewhat challenging, so think what input information you need.
- Ⓑ Use contraction to solve the edge cases when $i = 0, k - 1$. You do not have to simplify the internals of the MUX, just explain why you “contracted” as you did.
- NC: solving parts of larger question will appear on exam
- NC: contraction is setting certain inputs to constants and/or getting rid of unnecessary components

Homework 3 Harder Problems: Problem 5

The isolated-1-shift-left operation (i1sl for short) applied to a binary string S moves a 1 bit by one position to the left in the solution if the bit is not adjacent to any other 1's (i.e., 0's on both sides, or if is the least-significant bit, a 0 immediately above it).

For instance, the following show application of i1sl to various strings:

- $010 \rightarrow 100, 011 \rightarrow 011, 01010 \rightarrow 10100$
- $011010 \rightarrow 011100, 011100 \rightarrow 011100$
- Build a simplified circuit (using only AND, OR, NOT gates) whose output is the i th bit of the i1sl, where $1 < i < k - 1$ (there's a hint here about what inputs are needed). You may leave your answer in algebraic (SoP) form in terms of the S_j . Note that the i th bit can be determined by only looking at a few of the S_j .

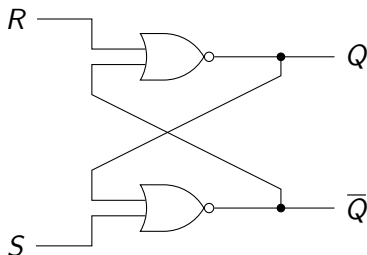
Homework 3 Harder Problems: Problem 5 (cont.)

- ⓑ Suppose it is known that the input string will never contain 3 consecutive 1's. Draw the simplified circuit.
- ⓒ Suppose the input string might contain 3 1's, but that no consecutive 4 bits contain 3 0's (i.e., 0000, 0001, 0010, 0100, or 1000 never appear as a substring), nor does 0101 ever appear (1010 might still appear though, e.g., 111011011010). Draw the simplified circuit.
- ⓓ Use contraction to design the circuits for the outputs of the $k - 1$ th, 1st, 0th bits.
- NC: contraction is setting certain inputs to constants and/or getting rid of unnecessary components

Homework 4 Topics

Homework 4 Topics: SR Latch

- Latches just hold state – can be changed by changing the values of the inputs
- SR latch has inputs of “Set” (S) and “Reset” (R)
- SR latch has outputs of Q and \bar{Q}



- For understanding behavior of circuit for given (R, S) , think about when NOR operation gives 0
 - Or more generally, passing a constant to a gate can make its output constant as well

Homework 4 Topics: D Latch

- Basically SR latch with some more circuitry to avoid “illegal” combinations of inputs (e.g. when $R = 1, S = 1$ what does the SR latch do? How can it “set” $Q = 1$ and “reset” $Q = 0$ at the same time?)
- D latch has inputs D and C (control, which ensures $S \neq R$ in the underlying SR latch)
 - D is the value to store/write if $C = 1$
 - C can also be E for enable


Homework 4 Topics: Clocking

- Clock signal oscillates between 0 and 1 with a fixed period
 - “Rising edge” of clock signal is the transition from “low” to “high” (i.e. $0 \rightarrow 1$)



- Kind of like a metronome, gives the circuit a sense of time on which it can operate
- Why should I care about clock? It determines the rate at which instructions are executed

Apple M1



Apple M1 chip

General information

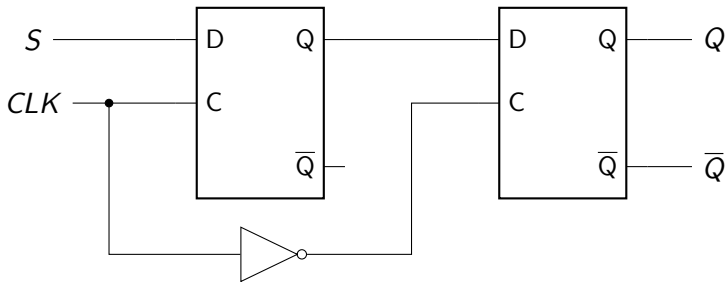
Launched	November 10, 2020 ^[1]
Designed by	Apple Inc.
Common manufacturer(s)	TSMC
Product code	APL1102 ^[2]

Performance

Max. CPU clock rate	3.2 GHz ^[1]
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Homework 4 Topics: Flip-Flops

- Latches do not support clocking individually
- Connecting latches in series (and ensuring that the clock inputs are complemented or staggered) acts as a flip-flop
- Example of D Flip-Flop:



- Left D latch is updated when $CLK = 1$ (new inputs read to left D latch), right D latch is updated when $CLK = 0$ (load new inputs to right D latch)

Homework 4 Topics: Flip-Flop Behavior

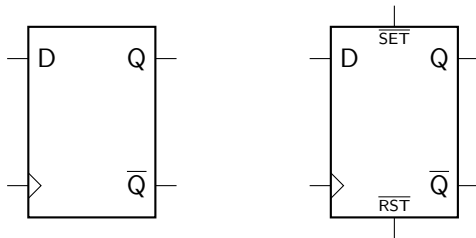
- For the D flip-flop:

$D(t)$	$Q(t + 1)$
0	0
1	1

- The input t is time, or more specifically, the t th clock cycle
 - A clock cycle is one period of the clock
- In other words, the input D at clock cycle t becomes the output Q at clock cycle $t + 1$ (the next clock cycle)

Homework 4 Topics: Flip-Flop Abstraction

- The two latch circuit from earlier is abstracted and is a D flip-flop (D since it has the same inputs as a D latch) (left circuit)



- The triangular input (bottom left) is for the clock signal
- How do we initialize the values of a flip-flop?
 - Use set and reset inputs (right circuit), implementation is not necessary to know (we are not really worried about initialization in this course)