4231 Homework 5

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Problem 1

Exercise 22.2-8 on Page 602: Diameter of a tree. (Aim for a linear-time algorithm.)

22.2-8

The ***diameter*** of a tree $T = (V, E)$ is defined as $\max_{u,v \in V} \sigma(u, v)$, that is, the largest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm.

Problem 2

Exercise 22.3-5 (c), 22.3-8, and 22.3-11 on Page 612. (For 22.3-11, you only need to give an example to show that the situation described here is possible. In both 22.3-8 and 22.3-11, the examples are very simple directed graphs.)

22.3-5

Show that edge (u, v) is

c. a cross edge if and only if $v.d < v.f < u.d < u.f$.

22.3-8

Give a counterexample to the conjucture that if a directed graph G contains a path from u to v, and if $u.d < v.d$ in a depth-first search of G, then v is a descendant of u in the depth-first forest produced.

22.3-11

Explain how a vertex u of a directed graph can end up in a depth-first tree containing only u, even though u has both incoming and outgoing edges in G.

Problem 3

Exercise 22.4-2 on Page 614.

22.4-2

Give a linear-time algorithm that takes as input a directed acyclic graph $G = (V, E)$ and two vertices s and t, and returns the number of simple paths from s to t in G. For example, the directed acyclic graph of Figure 1 contains exactly four simple paths from vertex p to vertex v: pov, $poryv$, $posryv$, and $psryv$. (Your algorithm needs only to count the simple paths, not list them.)

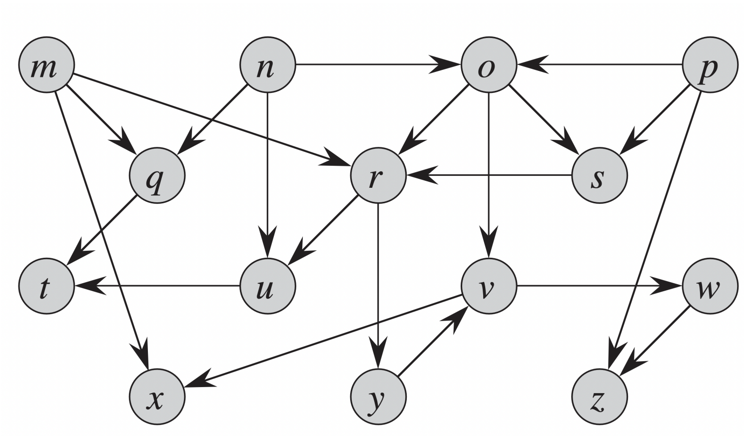

Figure 1: A dag for topological sorting.

Problem 4

Exercise 22.5-7 on Page 621.

22.5-7

A directed graph $G = (V, E)$ is ***semiconnected*** if, for all pairs of vertices $u, v \in V$, we have $u \leadsto v$ or $v \leadsto u$. Give an efficient algorithm to determine whether or not G is semiconnected. Prove that your algorithm is correct, and analyze its running time.

Problem 5

Exercise 23.1-3 and 23.1-8 on Page 629.

23.1-3

Show that if an edge (u, v) is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.

23.1-8

Let T be a minimum spanning tree of a graph G, and let L be the sorted list of the edge weights of T. Show that for any other minimum spanning tree T' of G, the list L is also the sorted list of edge weights of T'.