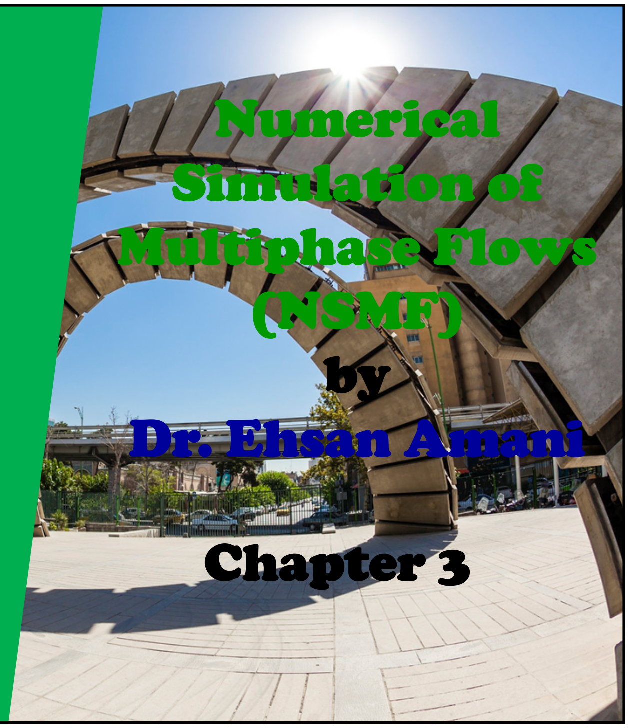


Numerical Simulation of Multiphase Flows (NSMF)

by

Dr. Ehsan Amani

Chapter 3

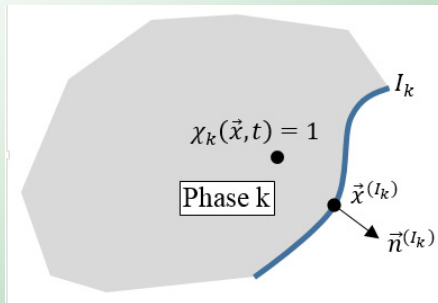
Instantaneous equations of multiphase flows

- A short introduction to tensorial notation

⇨ Lecture Notes: III.1

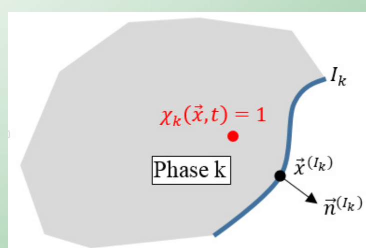
Chap 3 By E. Amani

DNS formulations



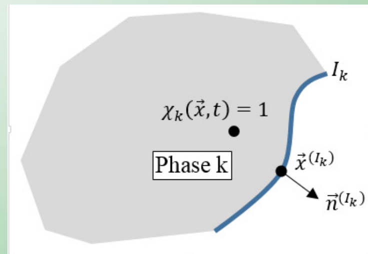
- Approach 1: multi-domain
- Approach 2: multi-fluid
- Approach 3: one-fluid

DNS approach 1: multi-domain



DNS approach 1: multi-domain

- For each phase, k, the continuity, momentum, scalars ($x \in k^{\text{th}}$ phase)



$$\frac{\partial \rho_k}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j}\right) = 0$$

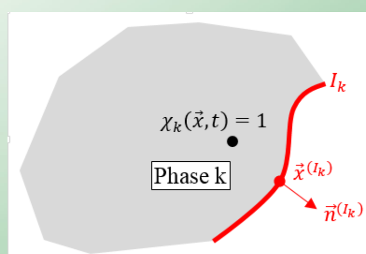
$$\frac{\partial}{\partial t}\left(\rho_k U_{k,i}\right) + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j} U_{k,i}\right) = \frac{\partial \sigma_{k,ij}}{\partial x_j} + \rho_k g_i \; ; \sigma_{k,ij} = -p_k \delta_{ij} + \tau_{k,ij}$$

$$\frac{\partial}{\partial t}\left(\rho_k Q_k\right) + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j} Q_k\right) = -\frac{\partial J_{Q_k,j}}{\partial x_j} + \rho_k S_{Q_k}$$

Chap 3 **By E. Amani**

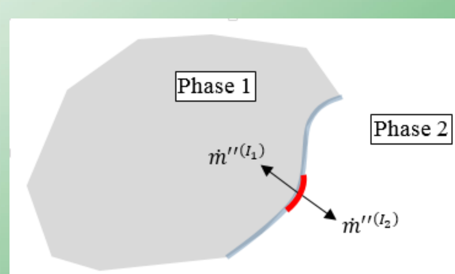
DNS approach 1: multi-domain

- Jump conditions (the balance of transports) at the interface:



$$\dot{m}''^{(I_k)} = -\rho_k\left(\vec{U}_k - \vec{U}^{(I_k)}\right).\hat{n}^{(I_k)}$$

$$\sum_{k=1}^{2} \dot{m}''^{(I_k)} = 0$$

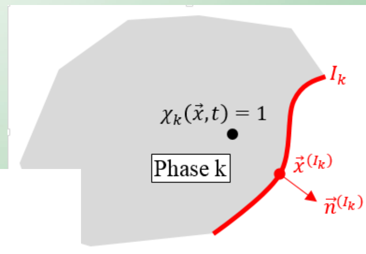


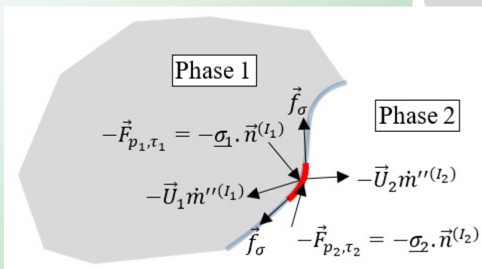
Mass balance per unit interface area ◄

Chap 3 **By E. Amani**

DNS approach 1: multi-domain

- Jump conditions (the balance of transports) at the interface:





momentum balance per unit interface area ◀

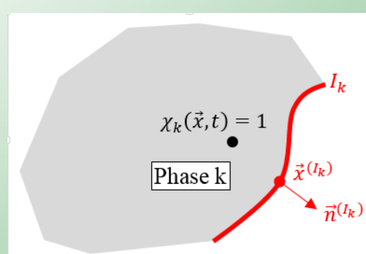
$$-\sum_{k=1}^{2}\left(\underline{\sigma}_k.\hat{n}^{(I_k)}+\dot{m}''^{(I_k)}\vec{U}_k\right)+\vec{f}_\sigma=0$$

⇨ **Energy equation jump condition? (HW)**

Chap 3 **By E. Amani**

DNS approach 1: multi-domain

- Interface location closure:



$$\frac{d\vec{x}^{(I_k)}}{dt}=\vec{U}^{(I_k)}$$

+ A simple model (no breakup, evaporation, etc.):

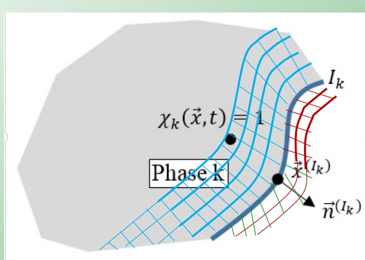
$$\vec{U}^{(I_k)}=\lim_{\vec{x}\to\vec{x}^{(I_k)}}\vec{U}_k$$

Chap 3 **By E. Amani**

DNS approach 1: multi-domain

- Features:

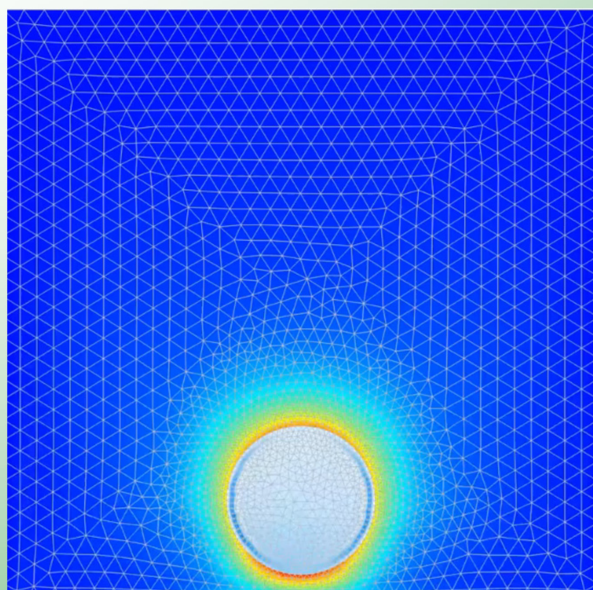
- not defined in all space,
- needs interface-fitted dynamic mesh,



Chap 3 By E. Amani

DNS approach 1: multi-domain

Rising bubble benchmark ►



Chap 3 By E. Amani

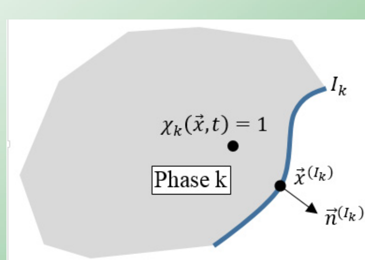
DNS approach 1: multi-domain

- Features:

- not defined in all space,
- needs interface-fitted dynamic mesh,
- highest accuracy (does not need approximation of the delta function)
- highest computational costs

DNS approach 2: multi-fluid

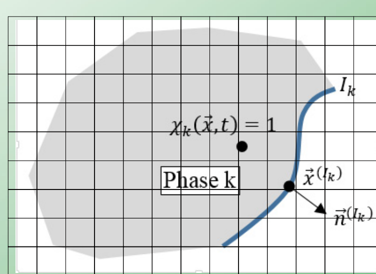
- based on the generalized variables $Q_k \chi_k$
- see references [3] (section 2.3), [Naud 2003] (chap 3), and [Kataoka 1986] for the derivation of the equations
- derivation using indicator function advection equation



$$\frac{\partial \chi_k}{\partial t} + U_j^{(I_k)} \frac{\partial \chi_k}{\partial x_j} = 0$$

DNS approach 2: multi-fluid

- For each phase, k, the indicator function advection:



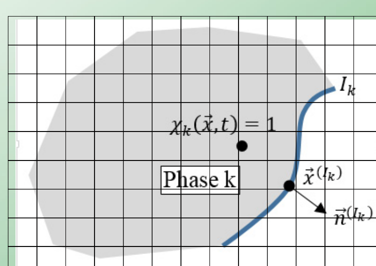
volume flow rate to phase k across the interface I_k

$$\frac{\partial \chi_k}{\partial t} + U_j^{(I_k)} \frac{\partial \chi_k}{\partial x_j} = 0 \quad \Rightarrow \quad \frac{\partial \chi_k}{\partial t} + U_{k,j}\frac{\partial \chi_k}{\partial x_j} = \overbrace{\left(U_{k,j}-U_j^{(I_k)}\right)\frac{\partial \chi_k}{\partial x_j}}^{S_m^{(I_k)}/\rho_k}$$

Chap 3 By E. Amani

DNS approach 2: multi-fluid

- For each phase, k, the continuity, momentum, scalars all over the domain:



mass transfer rate to phase k across the interface I_k

$$\frac{\partial}{\partial t}(\rho_k \chi_k) + \frac{\partial}{\partial x_j}(\rho_k U_{k,j}\chi_k) = \overbrace{\rho_k\left(U_{k,j}-U_j^{(I_k)}\right)\frac{\partial \chi_k}{\partial x_j}}^{S_m^{(I_k)}}$$

Chap 3 By E. Amani

DNS approach 2: multi-fluid

- For each phase, k, the continuity, momentum, scalars all over the domain:

mass transfer rate to phase k across the interface I_k

$$\frac{\partial}{\partial t}(\rho_k \chi_k) + \frac{\partial}{\partial x_j}(\rho_k U_{k,j} \chi_k) = \overbrace{\rho_k \left(U_{k,j} - U_j^{(I_k)}\right) \frac{\partial \chi_k}{\partial x_j}}^{S_m^{(I_k)}}$$

momentum transfer due to pressure and viscous stress at the interface I_k

momentum transfer by phase change

$$\frac{\partial}{\partial t}(\rho_k U_{k,i} \chi_k) + \frac{\partial}{\partial x_j}(\rho_k U_{k,j} U_{k,i} \chi_k) = \frac{\partial}{\partial x_j}(\sigma_{k,ij} \chi_k) + \rho_k \chi_k g_i + \overbrace{\left(-\sigma_{k,ij} \frac{\partial \chi_k}{\partial x_j}\right)}^{S_{U_i}^{(I_k)}} + \overbrace{U_{k,i} S_m^{(I_k)}}$$

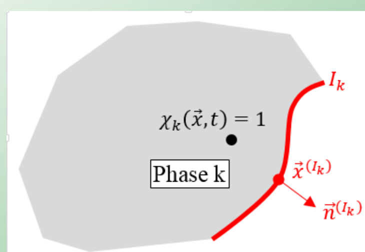
$$\frac{\partial}{\partial t}(\rho_k Q_k \chi_k) + \frac{\partial}{\partial x_j}(\rho_k U_{k,j} Q_k \chi_k) = -\frac{\partial}{\partial x_j}(J_{Q_k,j} \chi_k) + \rho_k \chi_k S_{Q_k} + \overbrace{\left(J_{Q_k,j} \frac{\partial \chi_k}{\partial x_j}\right)}^{S_Q^{(I_k)}} + Q_k S_m^{(I_k)}$$

Chap 3 By E. Amani

DNS approach 2: multi-fluid

- Closures:

Models for interface transfer source terms, $S_m^{(I_k)}$, $S_{U_i}^{(I_k)}$, and $S_Q^{(I_k)}$ satisfying jump condition constraints



Chap 3 By E. Amani

DNS approach 2: multi-fluid

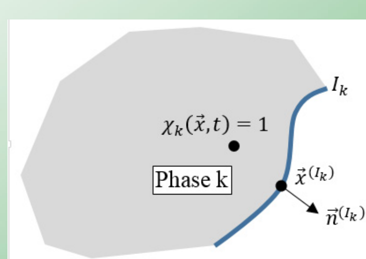
- Features:

- defined in all space (does not need interface-fitted mesh),
- the interface transfer source terms appears at the interface only and are zero inside the phases,
- interface transfer source terms can be described by delta functions [Naud 2003],
- the numerics needs approximation of the delta function
- the basis of the Eulerian-Lagrangian formulation

Chap 3 By E. Amani

DNS approach 3: one-fluid

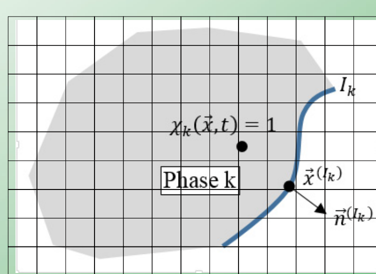
- based on the generalized variable $Q = \sum_{k=1}^{N} Q_k \chi_k$, see reference [Kataoka 1986]
- the governing equations are derived using $\sum_{k=1}^{N}(5.3)$ applying the jump condition constraints



Chap 3 By E. Amani

DNS approach 3: one-fluid

- The one-fluid continuity, momentum, scalars all over the domain:



$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_j}(\rho U_j)=0 \quad ;(\sum_{k=1}^{N} S_m^{(I_k)}=0)$$

$$\frac{\partial}{\partial t}(\rho U_i)+\frac{\partial}{\partial x_j}(\rho U_i U_j)=\frac{\partial \sigma_{ij}}{\partial x_j}+\rho g_i+\overbrace{\sum_{k=1}^{N}\left[S_{U_i}^{(I_k)}+U_{k,i}S_m^{(I_k)}\right]}^{\substack{\text{surface tension force}\\ F_{\sigma,i}}}$$

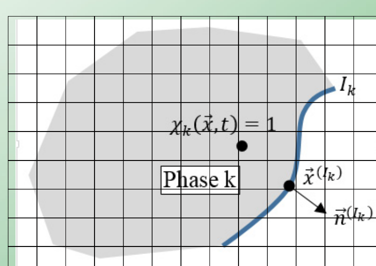
$$\frac{\partial}{\partial t}(\rho Q)+\frac{\partial}{\partial x_j}(\rho U_j Q)=-\frac{\partial J_{Q,j}}{\partial x_j}+\rho S_Q+\overbrace{\sum_{k=1}^{N}\left[S_Q^{(I_k)}+Q_k S_m^{(I_k)}\right]}^{S_{Qm}^{(I)}}$$

Chap 3 **By E. Amani**

DNS approach 3: one-fluid

- The indicator function advection for each phase k:

$$\frac{\partial \chi_k}{\partial t}+U_j\frac{\partial \chi_k}{\partial x_j}=\overbrace{\left(U_j-U_j^{(I_k)}\right)\frac{\partial \chi_k}{\partial x_j}}^{S_\chi^{(I_k)}}$$



  - Can be recast as: $\frac{\partial}{\partial t}(\rho_k \chi_k)+\frac{\partial}{\partial x_j}(\rho_k \chi_k U_j)=S_m^{(I_k)}$
- The secondary generalized properties, like density, viscosity, etc.:

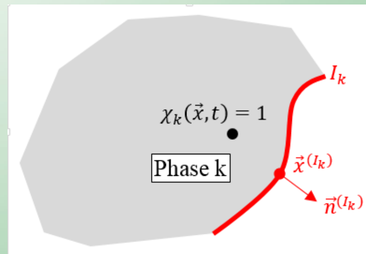
$$Q=\sum_{k=1}^{N} Q_k \chi_k$$

Chap 3 **By E. Amani**

DNS approach 3: one-fluid

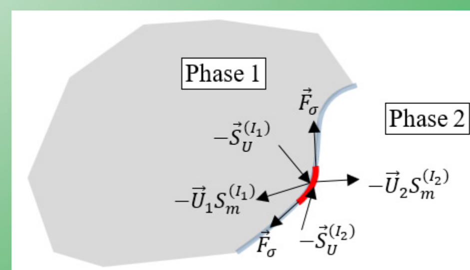
- Closures:

Models for interface transfer source terms, including $S_\chi^{(I_k)}$, surface tension, f_σ, and $S_{Qm}^{(I)}$ satisfying jump condition constraints



$$-\sum_{k=1}^{N}\left[S_{U_i}^{(I_k)}+U_{k,i}S_m^{(I_k)}\right]+F_{\sigma,i}=0$$

momentum balance per unit fluid volume ▶



DNS approach 3: one-fluid

- Features:

- defined in all space (does not need interface-fitted mesh),
- the interface transfer source terms appears at the interface only and are zero inside the phases,
- the surface tension force is described using a delta function and surface curvature, see reference [1] (appendix A, section 2.4.2),
- the numerics needs approximation of the delta function
- simpler and more widely used for DNS than approach 2

DNS approach 3: one-fluid

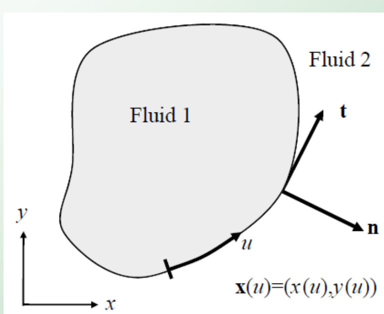
- Example: VOF method

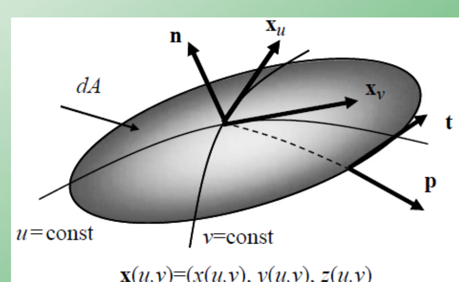
Spray atomization ▲

Chap 3 By E. Amani

DNS approach 3: one-fluid

- Details of formulation
  - The parametric description of an interface:





⇨ **Lecture Notes: III.3**

Chap 3 By E. Amani

Appendices

- Summary of DNS approaches: "chap3-DifferentDNSFormulationsSummary.pdf"
- Optional
  - The proofs in 2D (intermediate): "chap3-2DProofs.pdf"
  - The proofs in 3D (advanced): "chap3-3DProofs.pdf"
- HW#3 (optional)
  - Practice of using the tensorial notation
  - Derivation of some important relations