Different DNS formulations

Exact instantaneous equations or DNS formulations: Approach 1 (multi-domain)

Summary:
Single-phase equations for each phase, Eqs. (1.3)
+
Jump conditions (the balance of transports) at the interface, Eqs. (2.3)
+
A model for interface velocity in Eq. (3.3)

For each phase, k, the continuity, momentum, scalars ($x \in k^{\text{th}}$ phase):

$$\frac{\partial \rho_k}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j}\right) = 0$$
$$\frac{\partial}{\partial t}\left(\rho_k U_{k,i}\right) + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j} U_{k,i}\right) = \frac{\partial \sigma_{k,ij}}{\partial x_j} + \rho_k g_i \qquad ; \sigma_{k,ij} = -p_k \delta_{ij} + \tau_{k,ij}$$
$$\frac{\partial}{\partial t}\left(\rho_k Q_k\right) + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j} Q_k\right) = -\frac{\partial J_{Q_k,j}}{\partial x_j} + \rho_k S_{Q_k} \tag{1.3}$$

Jump conditions (the balance of interface transport) at the interface:

$$\sum_{k=1}^{2} \dot{m}^{(I_k)} = 0; \qquad \dot{m}''^{(I_k)} = -\rho_k\left(\vec{U}_k - \vec{U}^{(I_k)}\right).\hat{n}^{(I_k)}$$
$$-\sum_{k=1}^{2}\left(\underline{\sigma}_k.\hat{n}^{(I_k)} + \dot{m}''^{(I_k)}\vec{U}_k\right) + \vec{f}_\sigma = 0 \tag{2.3}$$
$\dots$

Interface location closure:

$$\frac{d\vec{x}^{(I_k)}}{dt} = \vec{U}^{(I_k)} \tag{3.3}$$
A simple model (no breakup, evaporation, etc.): $\vec{U}^{(I_k)} = \lim_{\vec{x}\to\vec{x}^{(I_k)}} \vec{U}_k$

Exact instantaneous equations or DNS formulations: Approach 2 (multi-fluid)

Summary:
Equations for the generalized variable of each phase, Eqs. (5.3)
+
The indicator functions advection, Eqs. (4.3)
+
Models for interface transfer source terms, $S_m^{(I_k)}$, $S_{U_i}^{(I_k)}$, and $S_Q^{(I_k)}$ satisfying jump condition constraints, Eqs. (2.3)

Comments:
- Based on the

For each phase, k, the indicator function advection:

$$\frac{\partial \chi_k}{\partial t} + U_{k,j}\frac{\partial \chi_k}{\partial x_j} = \overbrace{\left(U_{k,j} - U_j^{(I_k)}\right)\frac{\partial \chi_k}{\partial x_j}}^{\substack{\text{volume flow rate to phase k} \\ \text{across the interface } I_k \\ S_m^{(I_k)}/\rho_k}} \tag{4.3}$$

For each phase, k, the continuity, momentum, scalars all over the domain:

$$\frac{\partial}{\partial t}\left(\rho_k \chi_k\right) + \frac{\partial}{\partial x_j}\left(\rho_k U_{k,j}\chi_k\right) = S_m^{(I_k)} \tag{5.3}$$

generalized variables $Q_k\chi_k$, see references [1] (section 2.3), [2] (chap 3), and [3] for the derivation of the equations	$$\frac{\partial}{\partial t}(\rho_k U_{k,i}\chi_k)+\frac{\partial}{\partial x_j}(\rho_k U_{k,j}U_{k,i}\chi_k)=\frac{\partial}{\partial x_j}(\sigma_{k,ij}\chi_k)+\rho_k\chi_k g_i+\overbrace{S_{U_i}^{(I_k)}}^{\substack{\text{momentum transfer due}\\\text{to pressure and viscous}\\\text{stress at the interface } I_k}}+\overbrace{U_{k,i}S_m^{(I_k)}}^{\substack{\text{momentum}\\\text{transfer by}\\\text{phase change}}}$$ $$\frac{\partial}{\partial t}(\rho_k Q_k\chi_k)+\frac{\partial}{\partial x_j}(\rho_k U_{k,j}Q_k\chi_k)=-\frac{\partial}{\partial x_j}(J_{Q_k,j}\chi_k)+\rho_k\chi_k S_{Q_k}+S_Q^{(I_k)}+Q_k S_m^{(I_k)}$$

<table>
<tr><th colspan="2">Exact instantaneous equations or DNS formulations: Approach 3 (one-fluid)</th></tr>
<tr><td rowspan="6">Summary:
Equations for the generalized variable (7.3)
+
The indicator function Eqs. (6.3)
+
Models for interface transfer source terms, including $S_\chi^{(I_k)}$, surface tension, f_σ, [4] (appendix A, section 2.4.2), and $S_{Qm}^{(I)}$ satisfying jump condition constraints (2.3)

Comments:
- Based on the generalized variable $Q=\sum_{k=1}^{N}Q_k\chi_k$, see reference [3]</td><td>For each phase, k, the continuity, momentum, scalars all over the domain:</td></tr>
<tr><td>$$\frac{\partial}{\partial t}(\rho_k\chi_k)+\frac{\partial}{\partial x_j}(\rho_k U_j\chi_k)=S_m^{(I_k)} \qquad (6.3)$$</td></tr>
<tr><td>The one-fluid continuity, momentum, scalars all over the domain:</td></tr>
<tr><td>$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_j}(\rho U_j)=0\quad ;(\sum_{k=1}^{N}S_m^{(I_k)}=0)$$ $$\frac{\partial}{\partial t}(\rho U_i)+\frac{\partial}{\partial x_j}(\rho U_i U_j)=\frac{\partial\sigma_{ij}}{\partial x_j}+\rho g_i+\overbrace{\sum_{k=1}^{N}\left[S_{U_i}^{(I_k)}+U_{k,i}S_m^{(I_k)}\right]}^{\substack{\text{surface tension force}\\ F_{\sigma,i}}} \qquad (7.3)$$ $$\frac{\partial}{\partial t}(\rho Q)+\frac{\partial}{\partial x_j}(\rho U_j Q)=-\frac{\partial J_{Q,j}}{\partial x_j}+\rho S_Q+\overbrace{\sum_{k=1}^{N}\left[S_Q^{(I_k)}+Q_k S_m^{(I_k)}\right]}^{S_{Qm}^{(I)}}$$</td></tr>
<tr><td>The secondary generalized properties, like density, viscosity, etc.:</td></tr>
<tr><td>$$Q=\sum_{k=1}^{N}Q_k\chi_k \qquad (8.3)$$</td></tr>
</table>

References

1. Michaelides, E., C.T. Crowe, and J.D. Schwarzkopf, *Multiphase flow handbook*. 2016: CRC Press.
2. Naud, B., *PDF modeling of turbulent sprays and flames using a particle stochastic approach.* 2003.
3. Kataoka, I., *Local instant formulation of two-phase flow.* International Journal of Multiphase Flow, 1986. **12**(5): p. 745-758.
4. Tryggvason, G., R. Scardovelli, and S. Zaleski, *Direct Numerical Simulations of Gas-Liquid Multiphase Flows*. 2011: Cambridge University Press.