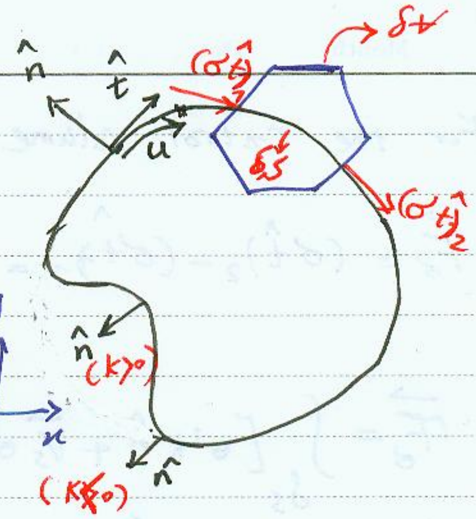
Subject:
Year. Month. Date. ( )

A. proof (2D):



parametric description $t \rightarrow$ is implied

$$\vec{x}(u) = x(u)\hat{i} + y(u)\hat{j}$$

if u is arc length:

$$\hat{t} = d\vec{x}/du \quad (1.3A)$$

$$\hat{n} = \frac{1}{k}\frac{d^2\vec{x}}{du^2} = \frac{1}{k}\frac{d\hat{t}}{du}, \quad k = |d^2\vec{x}/du^2| \quad (2.3A)$$

Exercise: show that

$$\hat{t} = -\frac{1}{k}\, d\hat{n}/du \quad (3.3A)$$

proof:

$$\hat{t}\cdot\hat{n} = 0 \longrightarrow \underbrace{\frac{d\hat{t}}{du}}_{\substack{\downarrow (2.3A)\\ k\hat{n}}}\cdot\hat{n} + \hat{t}\cdot\frac{d\hat{n}}{du} = 0 \longrightarrow \hat{t}\cdot\frac{d\hat{n}}{du} = -k$$

$$\hat{n}\cdot\hat{n} = 1 \longrightarrow 2\hat{n}\cdot d\hat{n}/du = 0 \longrightarrow \hat{n}\cdot\frac{d\hat{n}}{du} = 0$$

$$\longrightarrow \frac{d\hat{n}}{du} = -k\hat{t}$$

$$(3.3A) \longrightarrow k = -\hat{t}\cdot d\hat{n}/du = -\vec{\nabla}_s\cdot\hat{n} \; ; \quad \vec{\nabla}_s = \hat{t}\,\frac{d}{du} \quad (4.3A)$$

It can be shown that if $\hat{n}$ is defined (extended) as a volume field

$$\vec{\nabla}_s\cdot\hat{n} = \vec{\nabla}\cdot\hat{n} \quad (5.3A)$$

Therefore

$$k = -\vec{\nabla}\cdot\hat{n} \quad (6.3A)$$

proof: according to curvilinear coordinate relations.

$$\vec{\nabla} = \hat{t}\frac{\partial}{\partial u} + \underbrace{\vec{m}}_{\substack{\downarrow\\ \text{parallel to } \hat{n}}}\frac{\partial}{\partial n} \longrightarrow \vec{\nabla}\cdot\vec{n} = \underbrace{\hat{t}\cdot\frac{\partial\hat{n}}{\partial u}}_{\vec{\nabla}_s\cdot\hat{n}} + \cancel{\vec{m}\cdot\underbrace{\frac{d\hat{n}}{dn}}_{\vec{\omega}\times\vec{n}}} = \vec{\nabla}_s\cdot\vec{n}$$

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For the control volume δV

$$\vec{F}_\sigma = (\sigma\hat{t})_2 - (\sigma\hat{t})_1 = \int_{\delta S} \frac{\partial(\sigma\hat{t})}{\partial u}\,du = \int_{\delta S} \sigma \underbrace{\frac{\partial\hat{t}}{\partial u}}_{\substack{\downarrow \\ (2.3A) \\ k\hat{n}}} du + \int_{\delta S} \hat{t}\underbrace{\frac{\partial\sigma}{\partial u}}_{\substack{(4.3A) \\ \vec{\nabla}_s\sigma}} du$$

$$\vec{F}_\sigma = \int_{\delta S}\left[\sigma k\hat{n} + \vec{\nabla}_s\sigma\right]\frac{dS}{du} \xrightarrow{(0.3A)} \vec{f}_\sigma = \sigma k\hat{n} + \vec{\nabla}_s\sigma$$

$$\vec{F}_\sigma = \vec{f}_\sigma\,\delta_S$$

⊕ Interfacial area density

knowing

$$\chi_K = H(\varphi_K(\vec{x},t)) \xrightarrow[\text{rule}]{\text{chain}} \vec{\nabla}\chi_K = \frac{\partial H}{\partial\varphi_K}\vec{\nabla}\varphi_K$$

H → Heriside function

φ_K → level function

$\frac{\partial H}{\partial\varphi_K}$ → $\delta(\varphi_K)$

interface

$$\varphi_K(\vec{x},t) = 0 \longrightarrow \hat{n}^{(Ik)} = -\frac{\vec{\nabla}\varphi_K}{|\vec{\nabla}\varphi_K|}$$

$$\vec{\nabla}\chi_K = \underbrace{-\delta(\varphi_K)\,|\vec{\nabla}\varphi_K|}_{a_{Ik}\text{ : interfacial area density}}\,\hat{n}^{(Ik)} \xrightarrow{(16.3)} \delta_S = \overset{\substack{1D \\ \uparrow}}{\delta}(\varphi_K)\,|\vec{\nabla}\varphi_K| = a_{Ik}$$

PAPCO