Benha University

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Report about the following:

"Mechanical Vibration project"

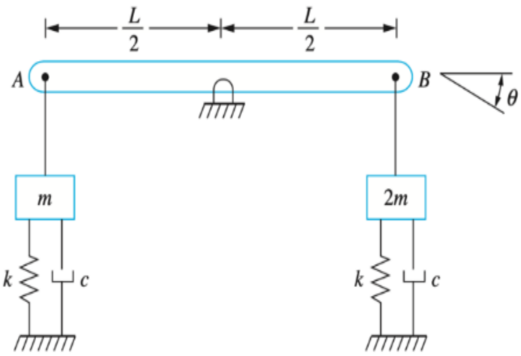
Dr : Mohamed Osama

➢ What this system is:-

You have:

1. a **slender bar** (mass m) supported at its middle
2. connected at both ends to:
   - sliding blocks
   - each block has a **spring** $k$and **damper** c

- left block mass = m
- right block mass = $2m$



Slender bar of mass m connected to blocks through rigid links at A and B

When the bar rotates a little by an angle θ, the ends move up/down and pull the springs and dampers.

mathematical solution

1. Kinetic energy:-

1-Bar :-

Moment of inertia of a slender bar about its center:

$$J = \frac{mL^2}{12}$$

Rotational kinetic energy:

$$T_{\text{bar}} = \frac{1}{2}J\dot{\theta}^2 = \frac{1}{2}\left(\frac{mL^2}{12}\right)\dot{\theta}^2$$

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lock at A (mass m)

Velocity:

$$\dot{y}_A = \frac{L}{2}\dot{\theta}$$

$$T_A = \frac{1}{2}m\left(\frac{L}{2}\dot{\theta}\right)^2$$

(c) Block at B (mass $2m$)

$$\dot{y}_B = -\frac{L}{2}\dot{\theta}$$

$$T_B = \frac{1}{2}(2m)\left(\frac{L}{2}\dot{\theta}\right)^2$$

Total kinetic energy

Total kinetic energy

$$T = T_{\text{bar}} + T_A + T_B$$

Compute:

$$T = \frac{1}{2}\left(\frac{mL^2}{12}\right)\dot{\theta}^2 + \frac{1}{2}m\left(\frac{L^2}{4}\right)\dot{\theta}^2 + \frac{1}{2}(2m)\left(\frac{L^2}{4}\right)\dot{\theta}^2$$

Factor:

$$T = \frac{1}{2}\left[\frac{m_{bar}L^2}{12} + \frac{m_aL^2}{4} + \frac{2m_aL^2}{4}\right]\dot{\theta}^2$$

Simplify inside brackets:

$$\frac{1}{12}m_{bar}L^2 + \frac{3}{12}m_aL^2 + \frac{6}{12}m_aL^2 = \frac{1}{12}L^2[m_{bar} + 9m_a]$$

So:

$$T = \frac{1}{2}\left(\frac{1}{12}L^2[m_{bar} + 9m_a]\right)\dot{\theta}^2 = \frac{1}{2}(J_{\text{eq}})\dot{\theta}^2$$

This means the equivalent rotational inertia is:

$$J_{\text{eq}} = \frac{1}{12}L^2[m_{bar} + 9m_a]$$

Potential energy (springs)

Spring at A:

$$U_A = \frac{1}{2} k \left(\frac{L}{2}\theta\right)^2$$

Spring at B:

$$U_B = \frac{1}{2} k \left(\frac{L}{2}\theta\right)^2$$

Total:

$$\frac{1}{2} k \left(\frac{L}{2}\theta\right)^2 + \frac{1}{2} k \left(\frac{L}{2}\theta\right)^2 = \frac{1}{2} k_{\text{eq}} \theta^2$$

So equivalent rotational stiffness:

$$k_{\text{eq}} = k \frac{L^2}{2}$$

Damping energy (viscous dampers)

$$U_A = \frac{1}{2} C \left(\frac{L}{2}\dot{\theta}\right)^2$$

$$U_B = \frac{1}{2} C \left(\frac{L}{2}\dot{\theta}\right)^2$$

$$\frac{1}{2} C \left(\frac{L}{2}\dot{\theta}\right)^2 + \frac{1}{2} C \left(\frac{L}{2}\dot{\theta}\right)^2 = \frac{1}{2} C_{\text{eq}} \dot{\theta}^2$$

$$C_{\text{eq}} = C \frac{L^2}{2}$$

EQ OF MOTION

Mathematically, it behaves like:

$$J_{\mathbf{eq}}\ddot{\theta} + c_{\mathbf{eq}}\dot{\theta} + k_{\mathbf{eq}}\theta = 0$$

$$\left(\frac{1}{12}L^2[m_{bar} + 9m_a]\right)\ddot{\theta} + \left(c\frac{L^2}{2}\right)\dot{\theta} + \left(k\frac{L^2}{2}\right)\theta = F_{(t)}$$

Given :-

- $\boldsymbol{m_a = .25kg}$
- $\boldsymbol{m_{bar} = .55kg}$
- K=163.5 N.m
- L=0.56m

If the system is **free UNDAMPED**

Undamped means: $C = 0$
Free motion means: $F(t) = 0$

__So the damping term disappears:-

$$\left(\frac{1}{12}L^2\left[m_{\text{bar}} + 9m_a\right]\right)\ddot{\theta} + \left(\frac{kL^2}{2}\right)\theta = 0$$

__Divide by $\frac{1}{12}L^2(m_{\text{bar}} + 9m_a)$:-

$$\ddot{\theta} + \frac{\frac{kL^2}{2}}{\frac{1}{12}L^2(m_{\text{bar}} + 9m_a)}\theta = 0$$

_Final free-undamped equation (for YOUR form):-

$$\boxed{\ddot{\theta} + \frac{6k}{m_{\text{bar}} + 9m_a}\theta = 0}$$

_and the natural frequency is:

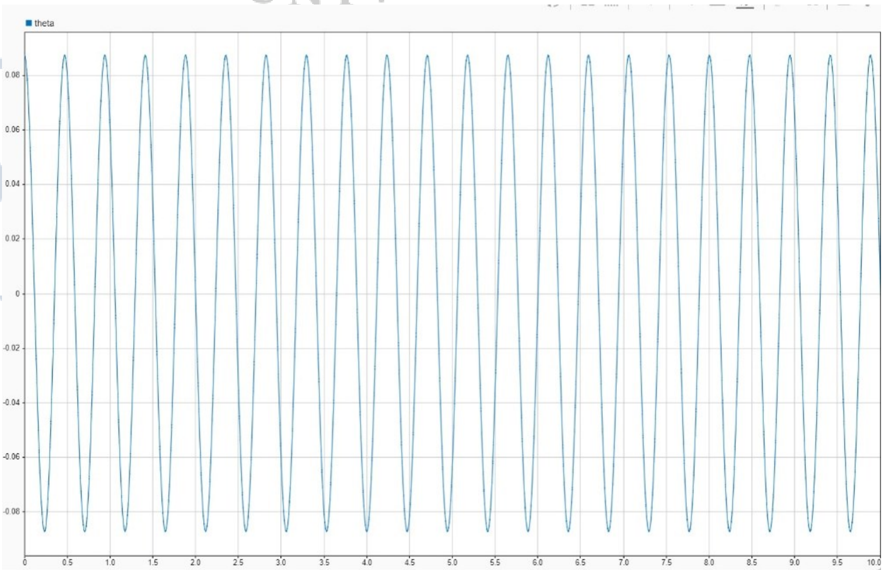
$\omega_n = \sqrt{\frac{6k}{m_{\text{bar}}+9m_a}} = \sqrt{\frac{6k}{m_{\text{bar}}+9m_a}} = 18.71\frac{rad}{s}$

General solution

$$\theta(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

Apply initial conditions: $\theta_0 = \pi/36$ ________ $\dot{\theta} = 0$

$$\theta(t) = \frac{\pi}{36}\cos(18.71t)$$



The system is free DAMPED

Underdamped (0 < ζ < 1):

__Initial condition $\theta_0 = 5°, \dot{\theta}_0 = 0$
using the previous parameters:

$$Ceq = 1.568, \zeta = 0.572, \omega_n = 18.71 \text{ rad/s}, \omega_d = 15.35 \text{ rad/s}, \zeta * \omega_n = 10.70$$

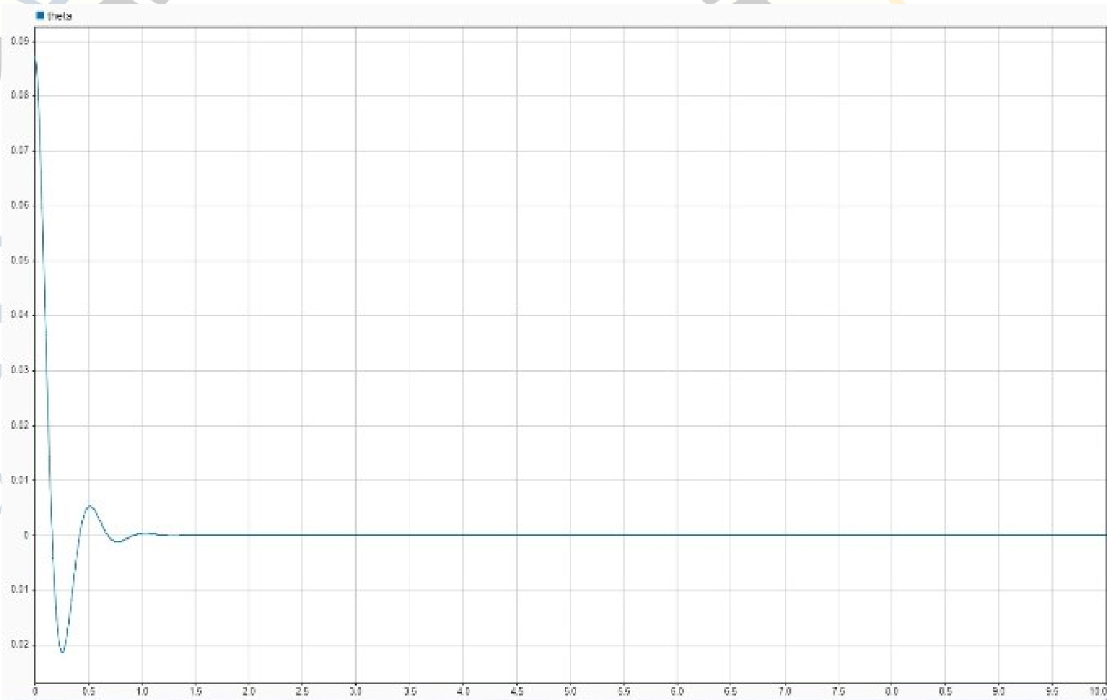
Step 1: Plug values into θ(t)
The general underdamped solution:

$$\boldsymbol{\theta(t) = e^{-\zeta\omega_n t}[\theta_0 \cos(\omega_d t) + \frac{\dot{\theta}_0 + \zeta\omega_n\theta_0}{\omega_d}\sin(\omega_d t)]}$$

$$\theta(t) = e^{-10.70t}\left[\frac{\pi}{36}\cos(15.35t) + \frac{0 + 10.70 \cdot \frac{\pi}{36}}{15.35}\sin(15.35t)\right]$$

Step 2: Final θ(t) in degrees

$$\boxed{\theta(t) = e^{-10.70t}\left[\frac{\pi}{36}\cos(15.35t) + 0.06\sin(15.35t)\right] \text{ degrees}}$$



Critically damped formula

For **critical damping** ($\zeta = 1$):

$$\theta(t) = (\theta_0 + (\dot{\theta}_0 + \omega_n \theta_0)t)e^{-\omega_n t}$$

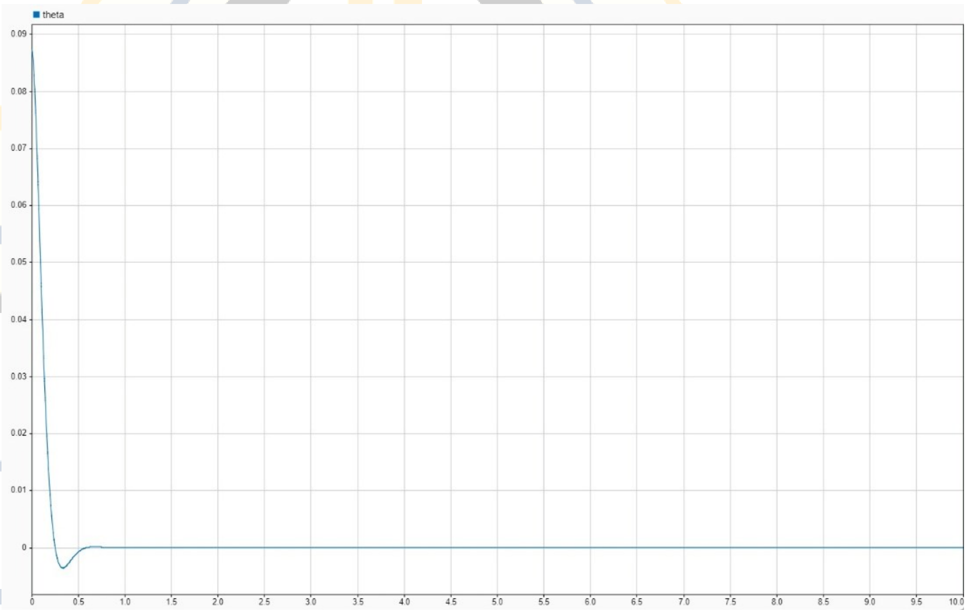
__Initial condition $\theta_0 = 5°, \dot{\theta}_0 = 0$

Plug in numbers

$$\dot{\theta}_0 + \omega_n \theta_0 = 0 + 18.71 \cdot \frac{\pi}{36} = 1.632$$

So the solution is:

$$\theta(t) = (\frac{\pi}{36} + 1.632\, t)\, e^{-18.71t}$$



overdamped case:

$$C_{\text{eq}} = 4.7, \zeta = 1.72, \omega_n = 18.71 \text{ rad/s}, \theta_0 = 5°, \dot{\theta}_0 = 0$$

For overdamped systems:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Plug numbers:

$$-\zeta\omega_n = -1.72 \cdot 18.71 \approx -32.16$$
$$\omega_n\sqrt{\zeta^2 - 1} = 18.71\sqrt{1.72^2 - 1} =\approx 26.16$$

So roots:

$$s_1 = -32.16 + 26.16 \approx -6.0$$
$$s_2 = -32.16 - 26.16 \approx -58.32$$

General overdamped solution

$$\theta(t) = Ae^{s_1 t} + Be^{s_2 t}$$

Step 4: Apply initial conditions

$$\theta(0) = \theta_0 \Rightarrow A + B = \frac{\pi}{36}$$
$$\dot{\theta}(0) = 0 \Rightarrow s_1 A + s_2 B = 0$$

Plug numbers:

$$-6A - 58.32B = 0 \Rightarrow A = -9.72B$$

Also: $A + B = \frac{\pi}{36} \Rightarrow -9.72B + B = \frac{\pi}{36} \Rightarrow$ B= -0.01

$$A = -9.72 * (-.01) = .0972$$

Final θ(t) equation

$$\theta(t) = 0.0972\, e^{-6.0t} - 0.01\, e^{-58.32t}$$

_Forced Undamped:-

Eq of motion :-

$$\left(\frac{1}{12}\mathrm{L}^2[\mathrm{m_{bar}} + 9\mathrm{m_a}]\right)\ddot{\theta} + \left(\mathrm{C}\frac{\mathrm{L}^2}{2}\right)\dot{\theta} + \left(\mathrm{k}\frac{\mathrm{L}^2}{2}\right)\theta = \mathrm{F}_{(t)}$$

Then the equation becomes:

$$\left(\frac{1}{12}\mathrm{L}^2[\mathrm{m_{bar}} + 9\mathrm{m_a}]\right)\ddot{\theta} + \left(\mathrm{k}\frac{\mathrm{L}^2}{2}\right)\theta =$$

For **undamped forced vibration**, the solution is:

$$\theta(t) = \theta_{\mathrm{h}}(t) + \theta_{\mathrm{p}}(t)$$

- $\theta_{\mathrm{h}}(t)$= homogeneous solution (free vibration):

$$\theta_{\mathrm{h}}(t) = \theta_0 \cos(\omega_n t) + \frac{\dot{\theta}_0}{\omega_n}\sin(\omega_n t)$$

- $\theta_{\mathrm{p}}(t)$= particular solution depends on $F(t)$.

Particular Solution: $\theta_{\mathrm{p}}(t)$= $\boldsymbol{\theta o}\beta\sin(\omega t)$
  - $F_0 = 50N$
  - $k_{eq} = \frac{163.5N}{m}$
  - $\omega_f = 5 \, {rad}/{s}$
  - ω_n = 18.71 rad/s

$\theta_0 = \frac{F_0}{k_{eq}} = \frac{50}{163.5}$= 0.3

r=$\frac{\omega_f}{\omega_n}$ =$\frac{5}{18.71}$ = 0.26

β=$\frac{1}{1-r^2}$ = 1.07

For Particular Solution: $\theta_{\mathrm{p}}(t)$= $\boldsymbol{\theta o}\beta\sin(\omega t)$ :::: $\theta_{\mathrm{p}}(t)$= 0.321 sin $(5t)$.

The standard solution is:-

$$\theta(t) = C_1\cos(\omega_n t) + C_2\sin(\omega_n t) + 0.321\sin(5t)$$

That means the initial condition is:

- $\theta(0) = 5°$
- $\dot{\theta}(0) = 0$

substitute $\boldsymbol{t = 0}$

$$\theta(0) = C_1\cos(0) + C_2\sin(0) + 0.321\sin(0)$$

$$\theta(0) = C_1(1) + C_2(0) + 0 = C_1$$

$$\boxed{C_1 = \frac{\pi}{36}}$$

Differentiate θ(t)

$$\dot{\theta}(t) = -5\omega_n sin\,(\omega_n t) + C_2\omega_n cos\,(\omega_n t) + 0.321(5)cos\,(5t)$$

Apply $\boldsymbol{t = 0}$***:-***

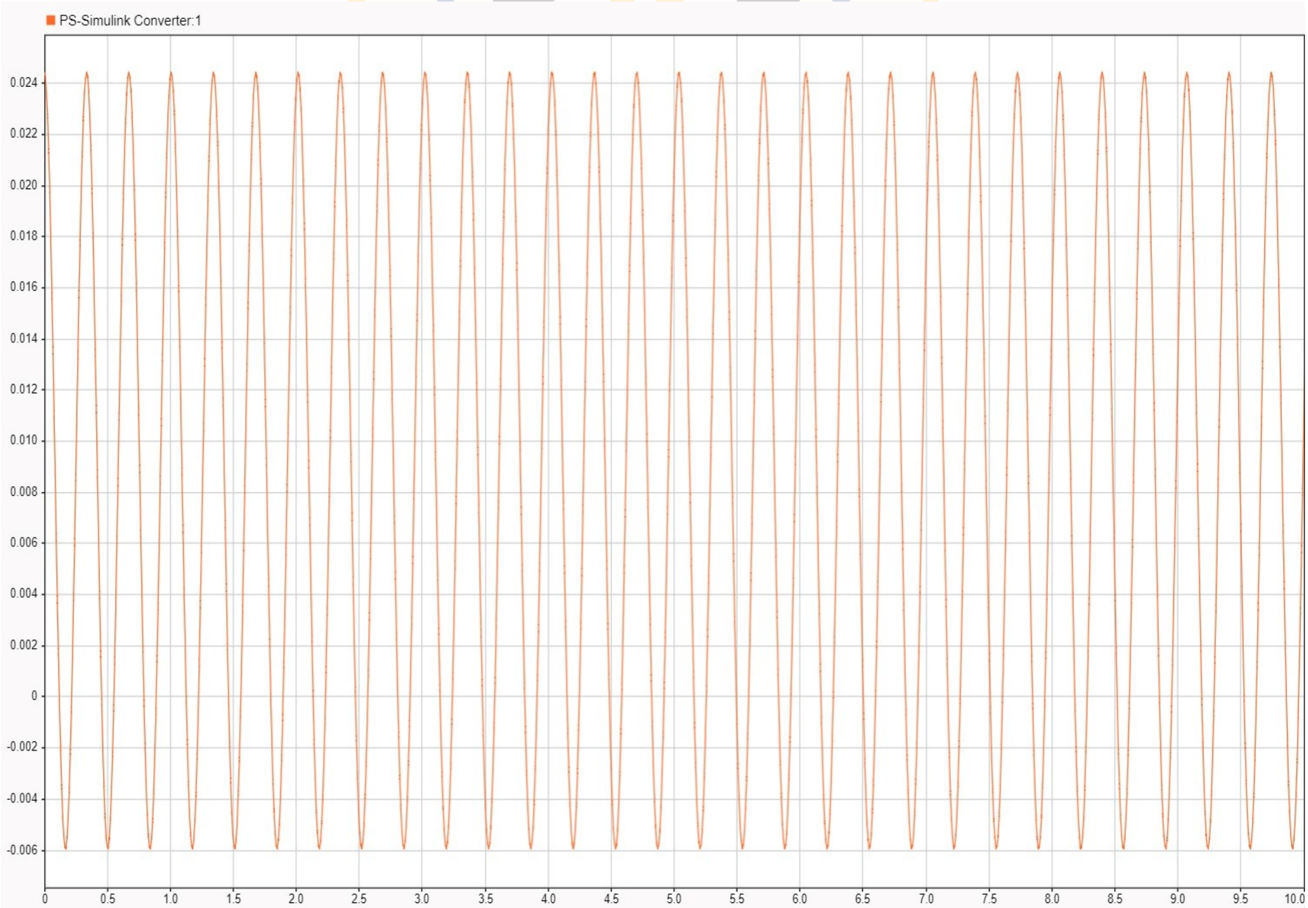
$$\dot{\theta}(0) = -5\omega_n(0) + C_2\omega_n(1) + 0.321(5)(1)$$

$$0 = C_2\omega_n + 1.605$$

$$C_2 = -\frac{1.605}{18.71} = -0.0858$$

and numerically (if $\omega_n = 18.71$*):*

$$\boxed{\theta(t) = \frac{\pi}{36}cos\,(18.71t) - 0.0858sin\,(18.71t) + 0.321sin\,(5t)}$$



Forced Damped:-

Equation of Motion: $Jo\ddot{\theta} + Ceq\dot{\theta} + Keq\theta = Mo\sin(\omega t)$

Solution : $\theta(t) = \theta h(t) + \theta p(t)$

- $F_0 = 50N$
- $k_{eq} = \frac{163.5N}{m}$
- $\omega_f = 5 \, {rad}/{s}$
- $\omega_n = 18.71$ rad/s
- $\theta_0 = \frac{F_0}{k_{eq}} = \frac{50}{163.5} = 0.3$
- r $= \frac{\omega_f}{\omega_n} = \frac{5}{18.71} = 0.26$
- $\beta = \frac{1}{\sqrt{(1-r^2)^2+(2r\zeta)^2}} = 1.06$
- $\psi = tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = 17.69°$
- $\omega_d = 15.35$

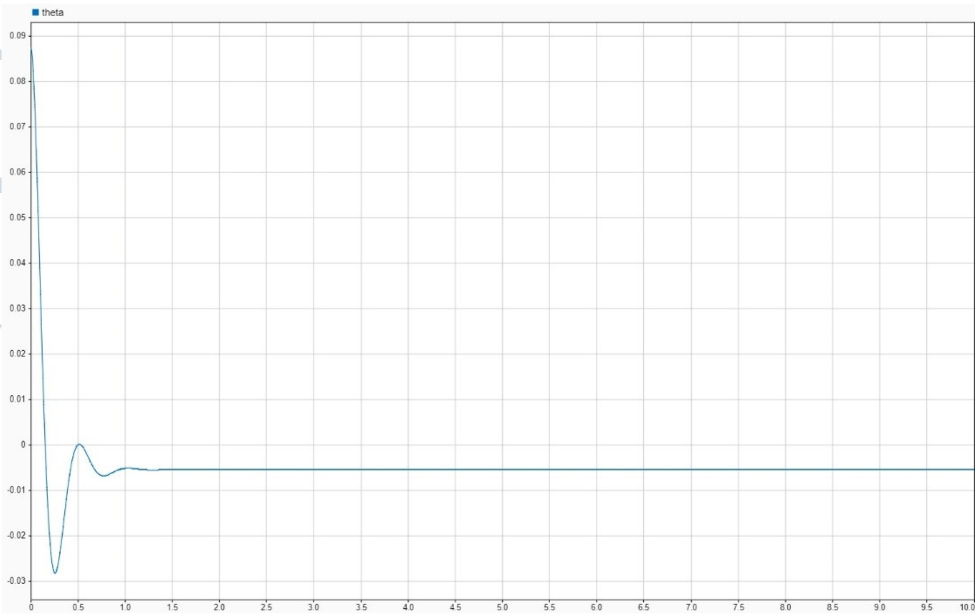
full solution for a damped, forced vibration system:

$$\theta(t) = e^{-\zeta\omega_n t}[a_1 cos\,(\omega_d t) + a_2 sin\,(\omega_d t)] \; + \; \theta_0\,\beta\,sin\,(\omega t - \psi)$$

initial conditions:-

$$\theta(0) = 5° = 0.0873\; rad, \dot{\theta}(0) = 0$$

$$\boxed{\theta(t) = e^{-10.70t}[0.1838\cos(15.35t) + 0.0294\sin(15.35t)] + 0.318\sin(5t - 17.69°)}$$



Critically Damped (ζ = 1):-

- $F_0 = 50N$
- $k_{eq} = \frac{163.5N}{m}$
- $\omega_f = 5 \, {rad}/{s}$
- $\omega_n = 18.71$ rad/s
- $\theta_0 = \frac{F_0}{k_{eq}} = \frac{50}{163.5} = 0.3$
- r $= \frac{\omega_f}{\omega_n} = \frac{5}{18.71} = 0.26$
- $\beta = \frac{1}{\sqrt{(1-r^2)^2 + (2r\zeta)^2}} = 0.94$
- $\psi = tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = 29.14°$
- $\omega_d = 0$

The form (critical damping) is:

$$\theta(t) = (a_1 + a_2 t)\, e^{-\omega_n t} \;+\; \theta_0 \beta \sin(\omega t - \psi)$$

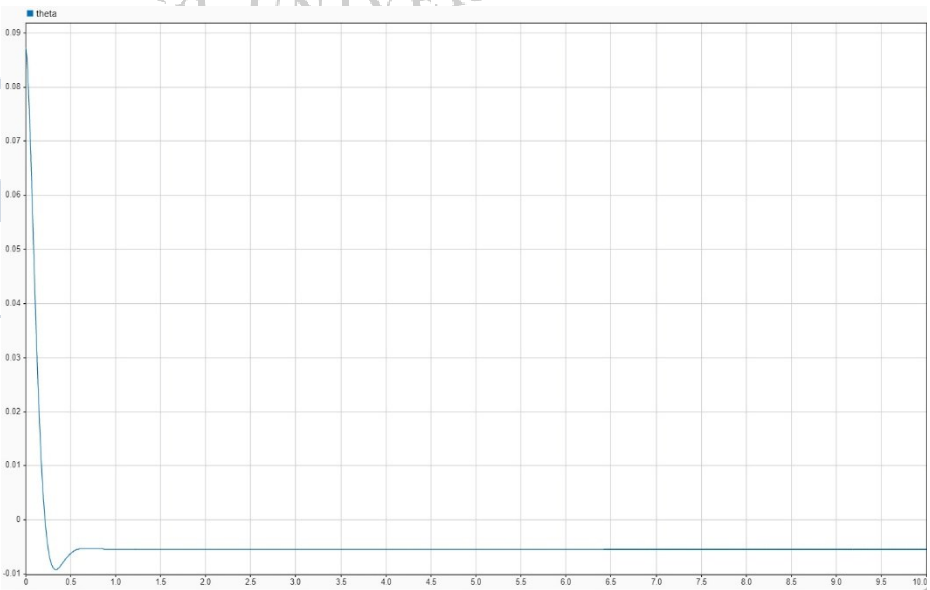
Initial conditions:-

$\theta(0) = 5° = 0.08727$ rad

$\dot{\theta}(0) = 0$

Final solution:-

$$\boxed{\theta(t) = (0.224 + 2.96\, t)\, e^{-18.71t} \;+\; 0.282 \sin(5t - 29.14°)}$$

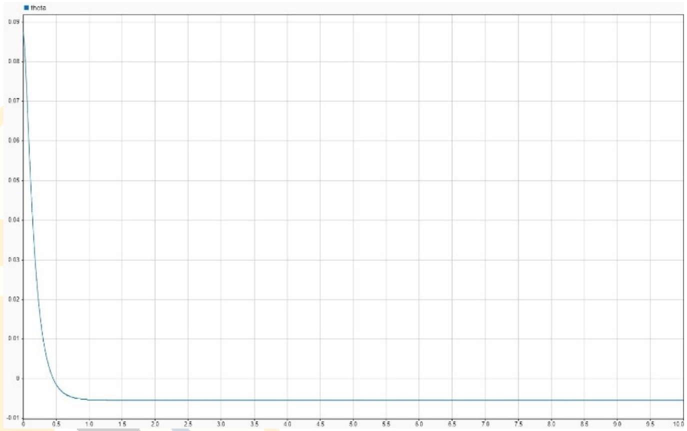


overdamped forced case:-

the general solution for overdamped motion is:-

$$\theta(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + \theta_0 \beta \sin(\omega t - \psi)$$

Given values



- $\zeta = 1.72 > 1 \rightarrow$ *overdamped*
- $\omega_n = 18.71\ rad/s$
- $\omega = 5\ rad/s$
- $\theta_0 = F_0/k_{eq} = 0.3$
- $\beta = 0.774$
- $\psi = 43.8°$

Initial conditions:-

$\theta(0) = 5° = 0.08727\ rad,$

$\dot{\theta}(0) = 0$

ind the roots $s_1$and s_2

For overdamped:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Compute:

$-\zeta\omega_n = -32.16$
$\sqrt{\zeta^2 - 1} = \sqrt{1.72^2 - 1} = 1.399$
$\omega_n\sqrt{\zeta^2 - 1} = 26.16$

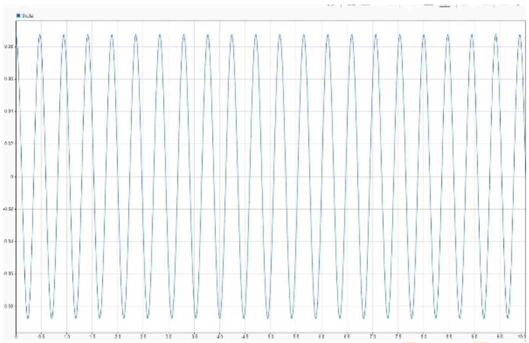
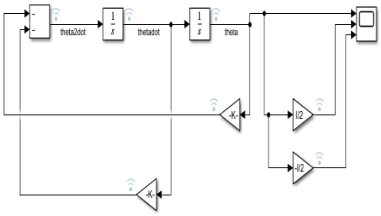
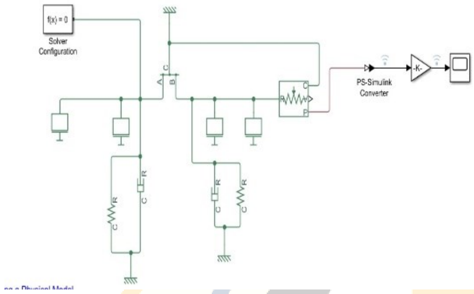
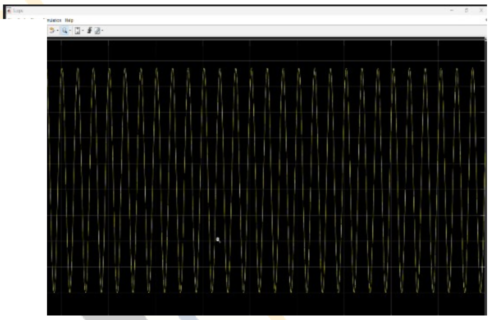
So roots:

$$s_1 = -32.16 + 26.16 = -6.0$$
$$s_2 = -32.16 - 26.16 = -58.32$$

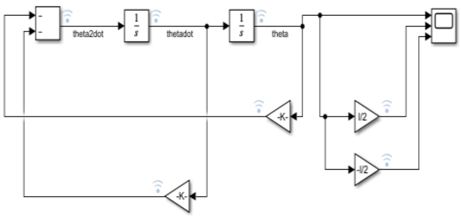
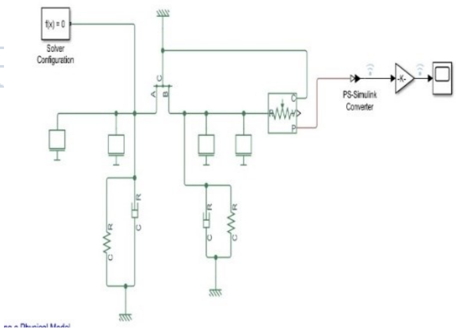
Final solution in :-

$$\boxed{\theta(t) = 0.2602\, e^{-6.0t} - 0.0124\, e^{-58.32t} + 0.232 sin\,(5t - 0.764)}$$

Free undamped:-

SIMULINK	 
SIMSCAPE	 

Free damped:- under damped

SIMULINK	 
SIMSCAPE	 

Free damped: critical :-

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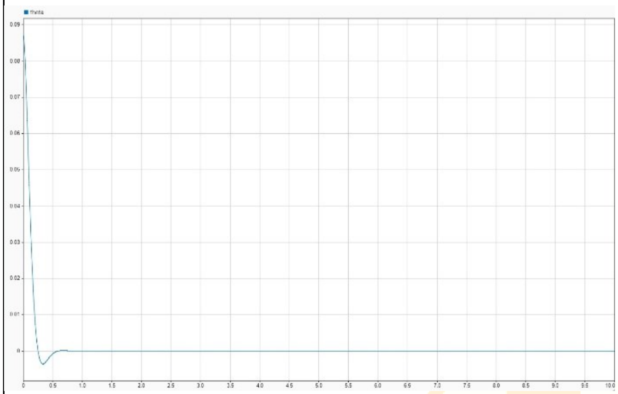
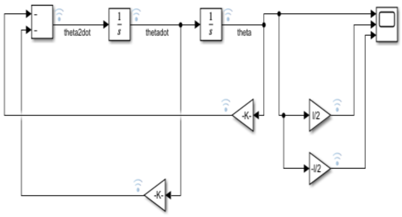
<tr><td>SIMULINK</td><td>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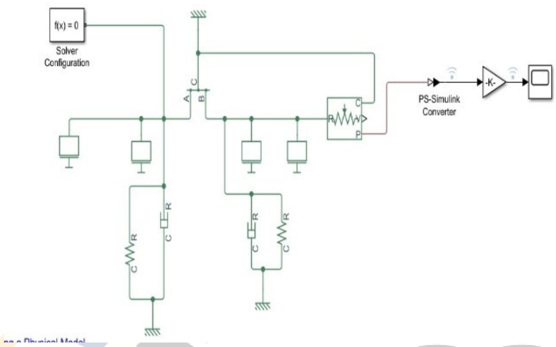
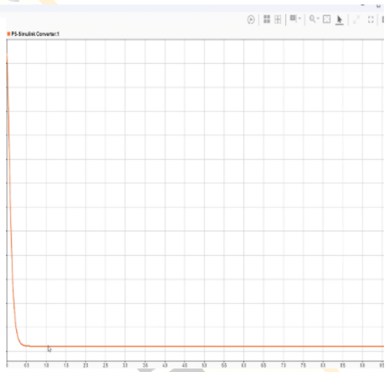
<tr><td>SIMSCAPE</td><td>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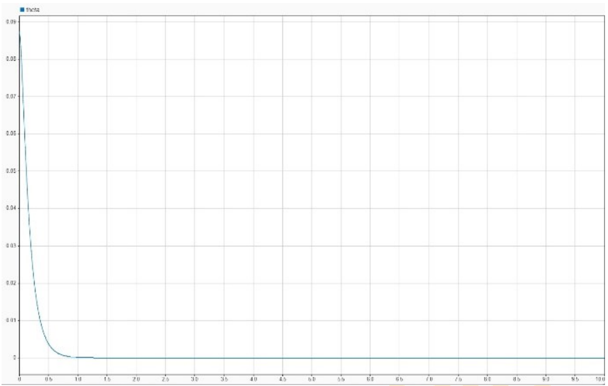
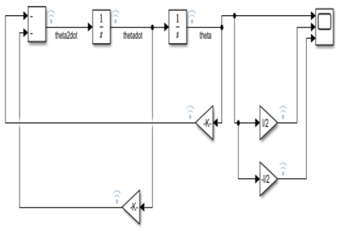
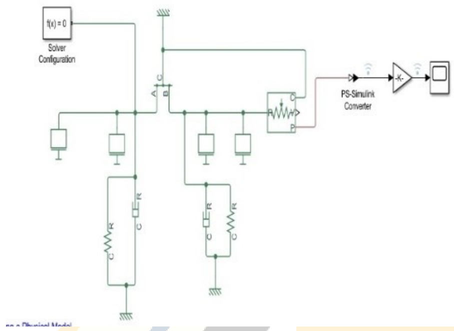
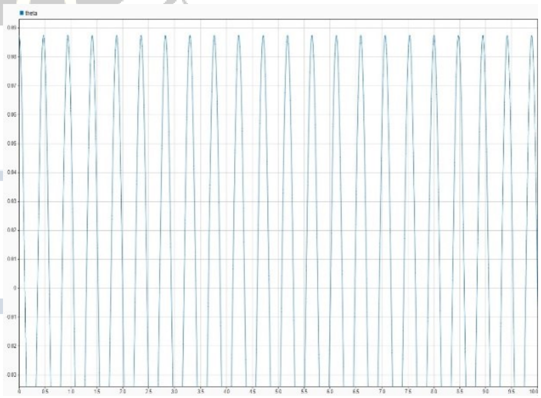
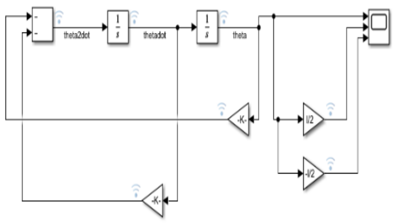
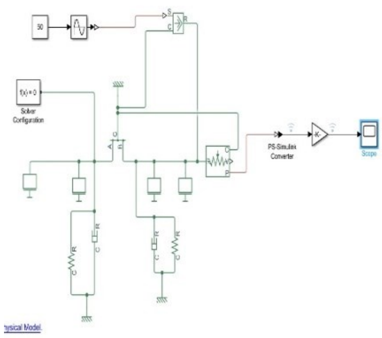
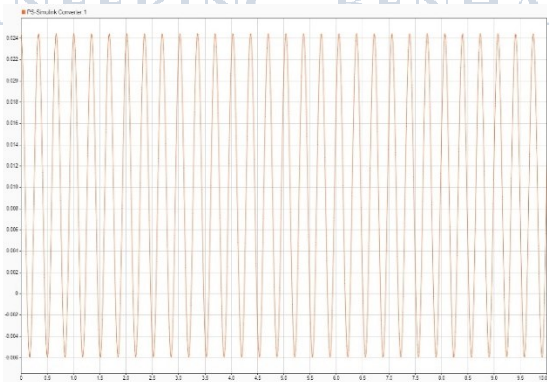
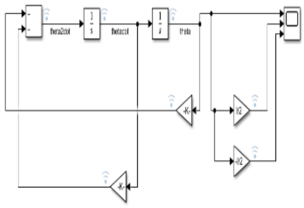
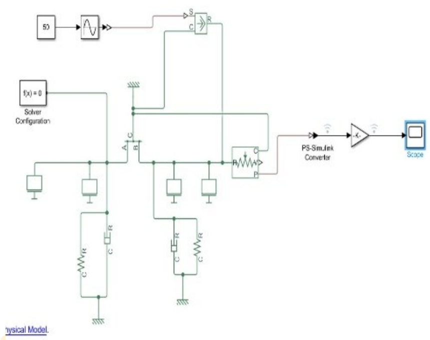
Free damped over damped :-

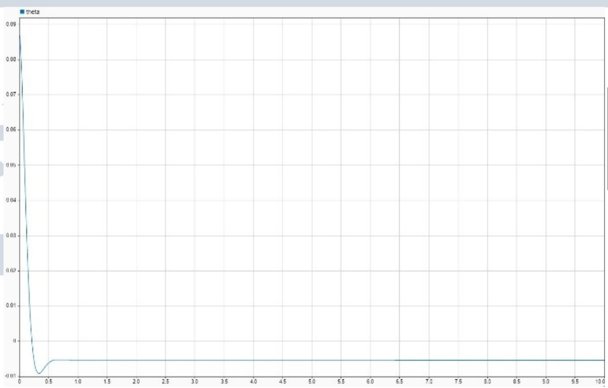
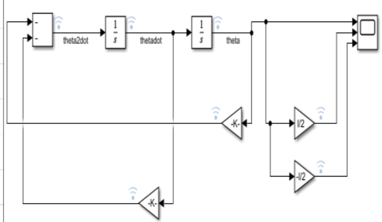
SIMULINK	 
SIMSCAPE	 

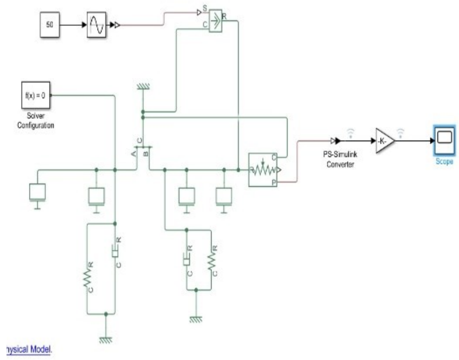
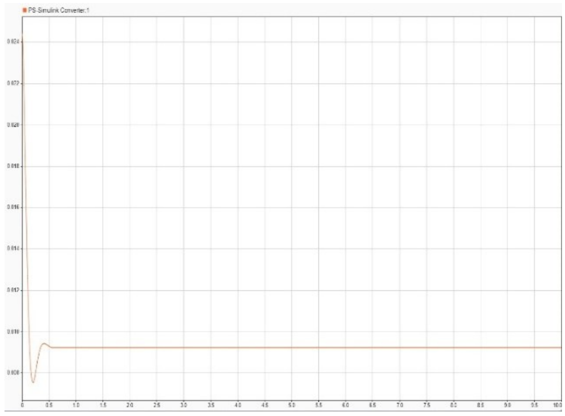
Forced undamped:-

SIMULINK	 
SIMSCAPE	 

Forced damped :- under damped :-

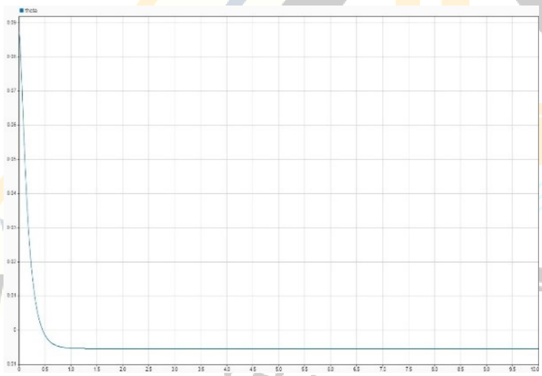
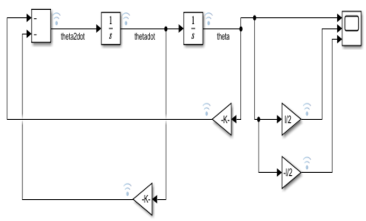
SIMULINK	 
SIMSCAPE	 

SIMULINK	 

SIMSCAPE	 

Forced damped :- critical damped :-

Forced damped :- over damped :-

SIMULINK	 
SIMSCAPE