Optimization Project Proposal

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Introduction

An application of convex optimization can be found in economics in the form of portfolio optimization, where one aims to maximize the returns of a portfolio of securities while minimizing the risk associated. In our project, we aim to investigate the Markowitz and Mean-Absolute Deviation (MAD) models for portfolio optimization, implement both algorithms, compare their computational efficiency, and evaluate portfolio performance results in real market data, placing special focus on the mathematical theory behind the MAD model.

The Markowitz model—also called the mean-variance model—is the foundation of modern portfolio theory and expresses the objective as a mathematical optimization problem. The expected return of the portfolio is framed in the context of a mean, while its volatility and risk are captured through variance. These two key principles, along with other rational assumptions such as risk aversion and consumption preference, transform portfolio optimization into a quadratic programming problem. When constrained at a given expected return level, one can optimize by minimizing variance (and thus volatility). Conversely, one can optimize by maximizing expected return (i.e. minimizing negative expected return) when falling under constraints of volatility. Though groundbreaking, a significant drawback of the Markowitz model is its use of an L_2 risk function which necessitates calculating a potentially large, dense covariance matrix that can be computationally expensive when applied to real data.

Mean-Absolute Deviation (MAD) Model

With this in mind, Konno and Yamazaki developed the Mean-Absolute Deviation (MAD) model in 1991 to build upon the previous Markowitz model, notably by replacing the variance proxy for volatility with absolute deviation. Indeed by using absolute deviation instead of variance, the runtime for numeric computations is significantly reduced because the optimization becomes a linear programming problem rather than a quadratic problem. Yet another important contribution made by the MAD model is no longer requiring the returns of stocks to be normally distributed. In fact, the MAD model generates comparable returns to the Markowitz model under normally distributed returns, yet is able to be applied to non-normal returns as well. Importantly, the MAD model was tested on Japanese stock market returns data, which notably demonstrated returns much less normally distributed than the American stock market.

In our review, we first aim to expand on the findings in the Markowitz paper by considering higher dimensions and thus allow for a greater feasible set (the original paper analyzed only 3 securities due to computational limits at the time). We similarly try to recreate the findings of the MAD model paper in high-dimensional spaces as well. Next, we aim to empirically show the greater computational load of Markowitz's L_2 risk over MAD's L_1 risk metric by computing runtimes over a range of portfolio sizes and comparing their theoretical big-O complexities. We finally compare the two models by testing them on present-day returns data not only in American stock markets but also overseas markets. It is especially relevant to do cross-market analyses because the original MAD paper focused on the Japanese market, while Markowitz focused on the American market. Finally, we consider the motivations for further extensions upon the Markowitz and MAD models, including the addition of derivative instruments (like options), consideration of transaction costs and interest rates, cardinality restrictions, and short selling.

Citations

Hiroshi Konno, Hiroaki Yamazaki, (1991) Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market. Management Science 37(5):519-531.

Markowitz, Harry. "Portfolio Selection." The Journal of Finance, vol. 7, no. 1, 1952, pp. 77–91. JSTOR, https://doi.org/10.2307/2975974. Accessed 8 Oct. 2023.