ME 388F Homework 6

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1 First and Last Collision Transport

(a) See below for treatment of the source.

(b) Our past implementation allowed the introduction of multiple materials in our domain through the use of a list. To that end, our Monte Carlo and discrete ordinates simulations take as inputs the transport cross sections $[2,2,10]$ and scattering cross sections $[1.99,1.8,2]$.

(c) Because of the use of divided differences in our discrete ordinates code, we use an equal mesh cell size throughout the domain to accurately determine absorption processes.

(d) Given the uncollided flux satisfies the transport equation

$$\mu \frac{d\psi}{dx} + \Sigma_t \psi = 0 \tag{1}$$

we can solve exactly for the $\mu = 1$ flux as

$$\psi(x, \mu = 1) = \begin{cases} e^{-2x} & 0 \le x \le 40 \\ e^{-80} e^{-10(x-40)} & 40 \le x \le 50 \end{cases} \tag{2}$$

throughout the simulation domain. We can then average this angular flux across the cells in the domain given the coordinates of our mesh.

(e) Given the angular flux given above, which matches the angular flux since no other angle is present, the resulting source is

$$Q = \frac{1}{2}\Sigma_s \phi = \begin{cases} 0.995e^{-2x} & 0 \le x \le 35 \\ 0.9e^{-2x} & 35 \le x \le 40 \\ e^{-80} e^{-10(x-40)} & 40 \le x \le 50 \end{cases} \tag{3}$$

This source may be implemented directly in our discrete ordinates code. To implement this as a particle source position in Monte Carlo, we compute an unnormalized cumulative distribution function as

$$CDF = \begin{cases} 0.995(\frac{1}{2} - \frac{1}{2}e^{-2x}) & 0 \le x \le 35 \\ 0.995(\frac{1}{2} - \frac{1}{2}e^{-70}) + 0.9e^{-70}(\frac{1}{2} - \frac{1}{2}e^{-2(x-35)}) & 35 \le x \le 40 \\ 0.995(\frac{1}{2} - \frac{1}{2}e^{-70}) + 0.9e^{-70}(\frac{1}{2} - \frac{1}{2}e^{-2(40-35)}) + e^{-80}(\frac{1}{10} - \frac{1}{10}e^{-10(x-40)}) & 40 \le x \le 50 \end{cases} \tag{4}$$

Then we use the maximum of the CDF over our domain to rescale a uniformly distributed random number $\tilde{r}$ and solve for the corresponding initial location x_0 through $\tilde{r} = CDF(x_0)$.

We consider this as an isotropic source in both the discrete ordinates and Monte Carlo simulation codes, and run as usual with this source to obtain a scalar flux.

(f) We are able to obtain this directly using the scattering cross sections in each cell.

(g) Given the transport cross sections, the weight that a mesh cell located at x_i will have on the detector is given by

$$\frac{\psi(1000,\mu=1)}{\psi(x,\mu=1)} = \frac{\psi(50,\mu=1)}{\psi(x,\mu=1)} = \begin{cases} e^{-180+2x} & 0 \leq x \leq 40 \\ e^{-500+10x} & 40 \leq x \leq 50 \end{cases} \tag{5}$$

(h) We now integrate the weight above times the last collision source to obtain the detector source, summing over the mesh cells and multiplying by the cell width (to account for using more cells).

The main results of our simulations are shown in Figure 1. The discrete ordinates code was run with 64 discrete ordinates. To obtain the collided source, vacuum boundary conditions were used (with internal source given by the collided flux).

On weighting, we find detector values of -1.7×10^{-28} for discrete ordinates with 1000 cells, 4.3×10^{-50} for discrete ordinates with 4000 cells, 8.2×10^{-49} for Monte Carlo with 1000 cells and 10000 histories, and 4.3×10^{-50} for Monte Carlo with 4000 cells and 40000 histories. For the shown simulations with no collided flux, the discrete ordinate simulation had a detector value of -1.3×10^{-28} and Monte Carlo simulation had 1.5×10^{-51}.

These results are miniscule, which we attribute to the fact that the particles are absorbed very far away from the detector. The scalar flux values closest to the detector, as seen in Figure 2, fall far below the threshold for the source iteration method to determine the scalar flux (10^{-10}). We also see from this plot that there are no Monte Carlo tallies close to the detector. Consequently, we expect a flux of zero at the detector.

Further Questions:

- We have used both S_{64} and Monte Carlo simulations to obtain these results. We see that the collided flux simulations yield qualitatively similar results in both simulation types. However, it seems that there is a normalization difference that causes the collided fluxes in Monte Carlo to be about 30% smaller than the deterministic code.

- On using a mesh four times smaller, our discrete ordinates simulation finds the same scalar flux. This suggests that our discrete ordinates code has converged to a solution. The Monte Carlo simulation with the four times smaller mesh and four times as many particle histories shows a very similar, although not identical, scalar flux.

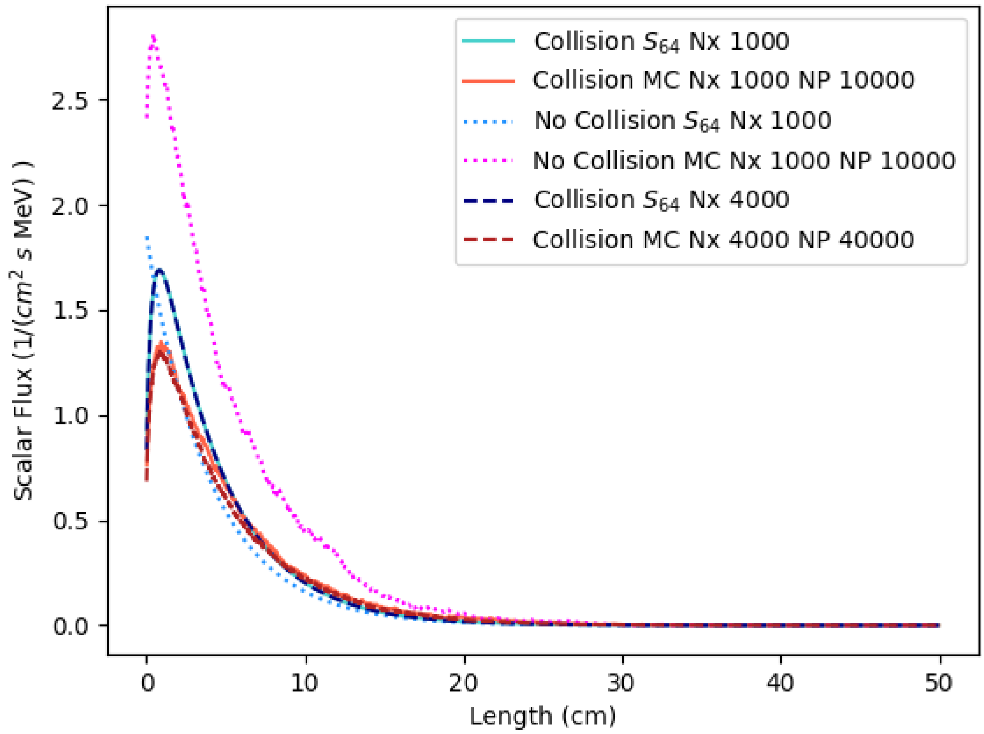

Figure 1: Scalar fluxes across several simulations to find the value at the detector.

- One way that I propose to do this without using the first and last collision sources is just running the simulation as usual with a left boundary source. We see that in discrete ordinate simulations that there is an exponential decrease in the scalar flux, meaning that the peak occurs farther from the detector so the scalar flux has a smaller impact. In Monte Carlo simulations, we do see a peak that forms before the peak in the collision source simulations, though via a normalization issue the scalar flux is larger.
- A weight window method has not been implemented at this time.

2 References

All code used was provided in the s25me388f_hansen GitHub repository. In this assignment, I used the code from past assignments in addition to lecture material on first and last collision transport. I also consulted with Max Hoffing and Abdullah Aljuaid to discuss the shape of the scalar fluxes obtained from using the first and last collision source.

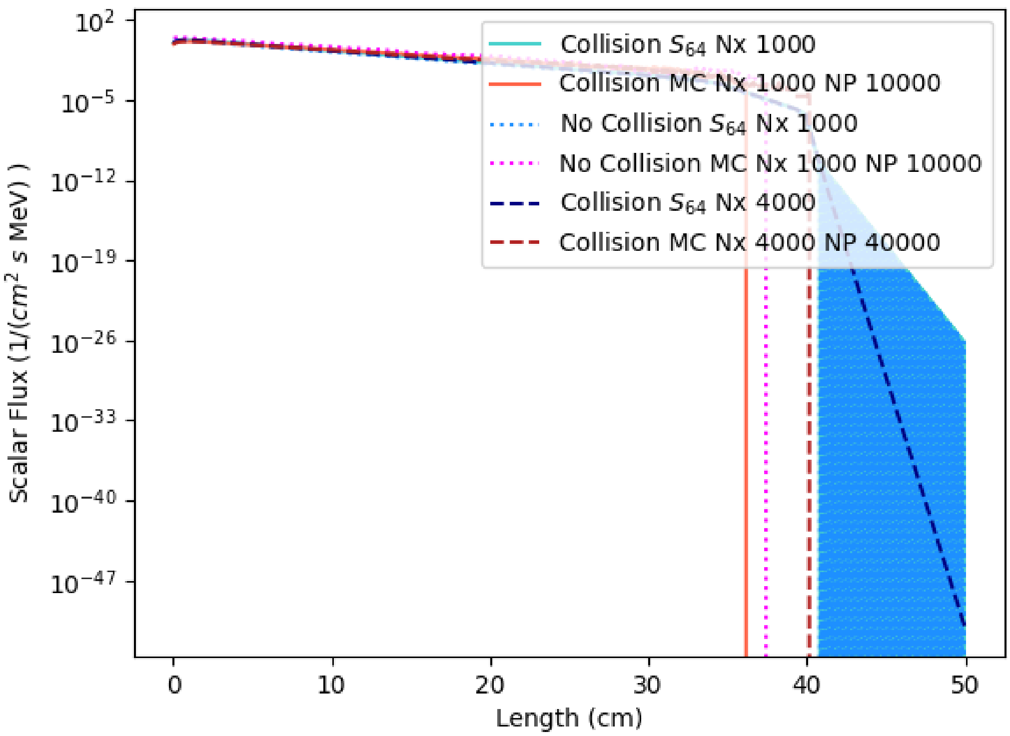

Figure 2: Log of scalar flux values for simulations. We see a strong decay closest to the detector, suggesting that the detector flux is near zero.