

# Shunt Fault Calculator for Power Systems using Rake Equivalent for Z-Bus Construction

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**Abstract**— a program written in MATLAB was created which can simulate non-simultaneous Three Phase Fault, Single Line to Ground Fault, Line to Line Fault, and Double Line to Ground Fault. It could be introduced with a fault impedance and can simulate change in symmetry. Its input can be edited on a spreadsheet and can analyze data such as generator, motor, line impedance, utility, 2 winding and 3 winding transformer. Phase shifts due to transformer configuration (e.g. Wye to Delta) is also incorporated in the program.

## I. INTRODUCTION

Fault studies are important in analyzing power systems. From the current condition of the system to its development projects, calculating fault currents is necessary to evaluate the performance of protective devices and carefully size incoming devices. Since power systems could range from thousand to hundred thousand of buses and equipment, a power engineer will have a problem in simulating enormous power systems. Thus a MATLAB code is generated to answer these limitations of a power engineer. The code will give important description of the current system such as the Z-Bus Matrix and the aftermath of a faulted bus such as the network's line currents and bus voltages.

## II. INPUT

The input data of the program is the excel file FaultStudyData.xls. The input data conforms to the format described in [1]. Additional details of the input data are described accordingly.

All parameters of the input data should *strictly* be either real or complex number. For the configuration of the transformer: '1' refers wye to ground connection, while '2' refers to delta connection. Other configuration may be added (e.g. wye to neutral, zigzag, etc.) for future development of the program.

The Face of the Clock (FOC) accepts from 0 to 11 and assumes that phase shift describes how much the low/aux. side of the transformer lags the high side.

The steady state reactance of the motors and generators are used (initially) for the fault calculation.

The program don't have error handling capability hence the user must strictly follow the mnemonics used in the code, for example in change of symmetry phaseA = '1'. If user put invalid data he might need to rerun the program.

The function for reading the data in the spreadsheet might give an error, which is caused by xlsread that read spaces as valid data. If this happens user must delete rows below the valid data in the spread sheet and rerun the program.

## III. ALGORITHM

The program can be divided to six procedures.

The first one is the knowing the maximum number of bus in the spreadsheet 'FaultStudyData.xls'. Fig.1 describes the procedure for determining the number of buses. Since most equipment in power systems are connected by either a line or a transformer, the input to the function are transformers and lines only.

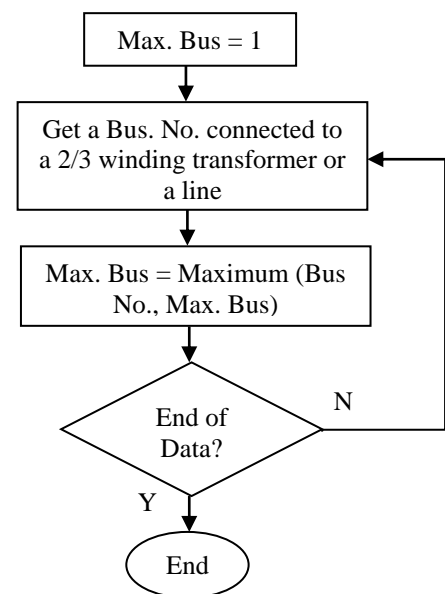


Fig. 1 Maximum Bus Algorithm

The second procedure is getting the Base Values of all the buses in the power system. The algorithm is shown in Fig. 2 and Fig 3.

'Pre Bus' refers to the bus used for computing the base values of other buses connected to it thru certain branches. 'Pre kV' is the base kV of the 'Pre Bus'. 'Post Bus' and 'Post kV' refers to the buses and their base kV which will have computed

base values according to the base values of 'Pre Bus'. 'Sbase' refers to the solitary base MVA of the power system. Nf/Ni refers to transformer ratio or '1' if it corresponds to a line.

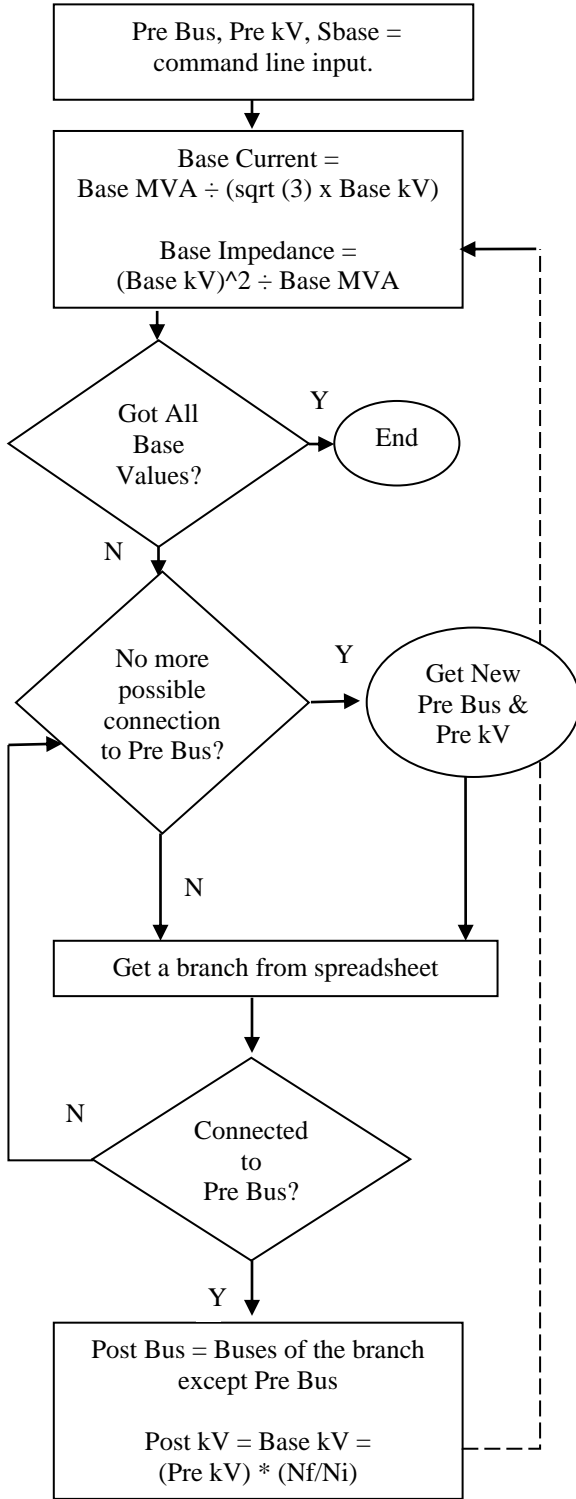


Fig. 2 Base Values Computation Part 1

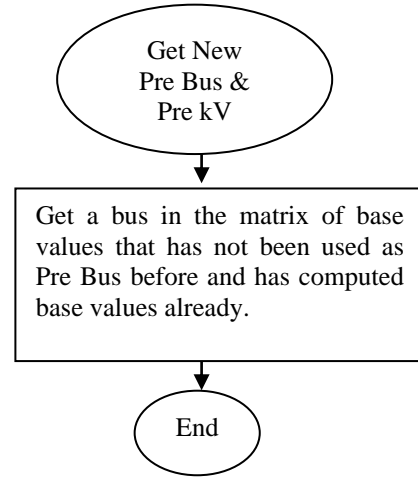


Fig. 3 Base Values Computation Part 2

The third procedure involves getting the impedance table of the system. The impedance table is just a collection of per unit values of the data from the spreadsheet. Hence their values are just the impedance of the branch divided by its corresponding base impedance.

The computation for the impedance of different types of branches can be found in [2].

Regarding the three winding transformer after calculating the values of  $Z_h$ ,  $Z_x$ , and  $Z_y$  (p.u. values of impedance of transformer after converting it to a wye connected impedance), additional bus is created for its pseudo-neutral node.

The zero impedance table can be constructed by replacing the nodes of transformers that have a delta connection with reference bus '0'.

The values computed from the impedance table will be used for the Zbus building and fault calculation.

The fourth procedure involves calculating the phase shift for every bus caused by the configuration of the transformers. The bus with zero phase shift in the code is the faulted bus.

The faulted bus will have '0' FOC and will be considered as an OLDBUS. FOCs of buses (NEWBUS) connected to the faulted bus via some branches are computed using Eq. 1 if it's on the low side of the transformer and Eq. 2 if it's on the high side of the transformer.  $FOC_{NEW-OLD}$  is the original FOC of the transformer describing how much the low side lags the high side.  $FOC_{NEW-OLD} = 1$  if the associated branch is a line.

FOC of succeeding buses are computed in the similar fashion with the OLDBUS replaced by buses that has FOC values already.

$$FOC_{NEWBUS} = FOC_{OLDBUS} + FOC_{NEW-OLD} \quad (1)$$

$$FOC_{NEWBUS} = FOC_{OLDBUS} - FOC_{NEW-OLD} \quad (2)$$

The fifth procedure involves the computation for the Zbus of the Positive, Negative, Zero Impedance Table.

The construction of the Zbus is straightforward and just needs a close monitoring of the existing old buses that had

been included in the Zbus. The step by step procedure can be found in [3]. The utilities and motors are treated as generator for the z bus building.

To avoid a line or transformer that connects to two new buses during the Zbus creation, the impedance table is arranged such that the elements connected to ground are read first and the other branches are arranged such that there is always an oldbus connected to them.

Since the numbering of the buses by the user may pose a problem during Zbus construction, each bus added is assigned with a sequence number corresponding to the iteration it was added to the Zbus. For example a generator that is connected to Bus 9 is read first by the algorithm hence it is assigned with a sequence number '1'. These sequence numbers will be used to arrange the completed Zbus such that each row and column number of the Zbus matrix will correspond to a Bus Number of the power system.

The sixth and last procedure is the fault calculation. All of the formulas needed for the 4 types of fault can be reviewed in [3] and [4]. The fault calculation starts with the positive, negative, and zero sequence values of the bus voltages and line currents. The phase values are then calculated from the sequence values by using the 3 by 3 - [A] matrix.

The only noticeable chunks of code included in the program are the inclusion of phase shift and change in symmetry.

The equivalent positive and negative sequence after applying phase shift are shown in Eq. 3 and Eq. 4 respectively. These equations are also true for the computation of the line currents.

$$V_{A1}' = V_{A1} * \exp(\sqrt{-1}*(-1)*FOC*\pi/6) \quad (3)$$

$$V_{A2}' = V_{A2} * \exp(\sqrt{-1}*(+1)*FOC*\pi/6) \quad (4)$$

For the change in symmetry, depending on the faulted phase for SLGF and the un-faulted phase of LLF and DLGF the positive and negative sequence currents are calculated using Eq. 5 to 7.

Eq. 5 and Eq. 6 are used for SLGF at phase B and LLF/DLGF at phase A and C.

Eq. 7 and Eq.8 are used for SLGF at phase C and LLF/DLGF at phase A and B.

$$I_{A1}' = I_{A1} / (1<240 \text{ deg}) \quad (5)$$

$$I_{A2}' = I_{A2} / (1<120 \text{ deg}) \quad (6)$$

$$I_{A1}' = I_{A1} / (1<120 \text{ deg}) \quad (7)$$

$$I_{A2}' = I_{A2} / (1<240 \text{ deg}) \quad (8)$$

#### IV. RESULTS

The table of base values displays the equivalent base values of the buses with respect to the initial base kV and base MVA entered by the user at the command line. It includes the line to line voltage in kV, line current in kA, and base impedance in ohms.

The sequence impedances are shown afterwards which are the per unit resistance and reactance connecting the 'From Bus' and 'To Bus'. The zero bus refers to the ground and the additional buses represent the neutral node of transformers after converting them to wye connected impedances.

Elements in the Z-bus Matrices are in per unit and are magnitudes only. Each row and column number corresponds to a bus number in the power system. For example the element in row 4 column 4 corresponds to the self-impedance of bus 4(or equivalent impedance as seen from bus 4).

The fault current in p.u. and kA of the faulted bus is shown after the Zbus Matrix.

Bus voltages for each phase during fault are expressed in p.u. as well as in line to line kV. The phase angles correspond to line to neutral voltages of the phases.

Values of the line currents are computed under the assumption that the currents are emanating from the 'From Bus' and going to the 'To Bus'. The currents entering the fictional buses of the three winding transformer can be interpreted as line currents emanating from the 'From Bus' and going to the transformer. Each fictional bus number corresponds to the arrangement of the three winding transformers in the input data. For example if there are only 9 buses in the system, a fictional bus number of 10 corresponds to the first three winding transformer entered in the excel file.

#### V. SAMPLE RUNS

The test system discussed and analyzed in [5] is shown in Fig. 4

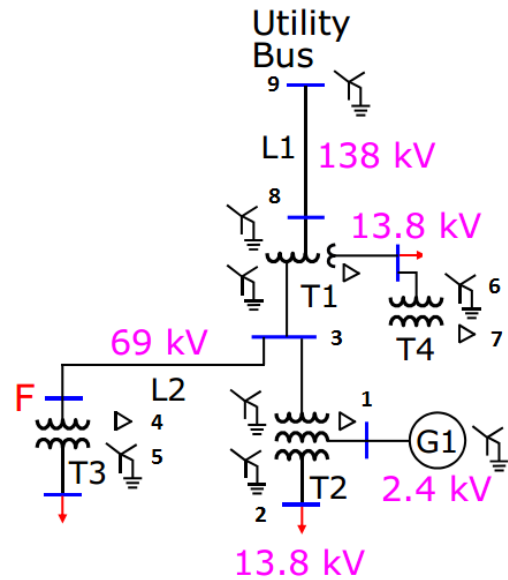


Fig. 4 Test System

Using the data in [5], the created fault simulator is compared with the output of ETAP 12.6.0 in Table I to VIII. Outputs of ETAP are shown in figs. 5 to 8.

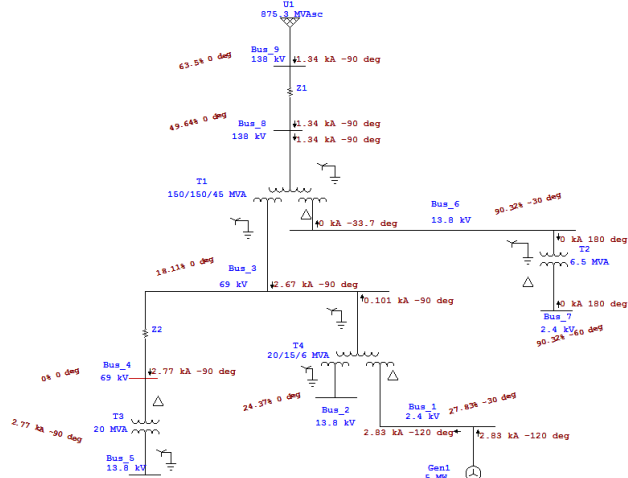


Fig. 5 ETAP Result for Three Phase Fault at bus 4

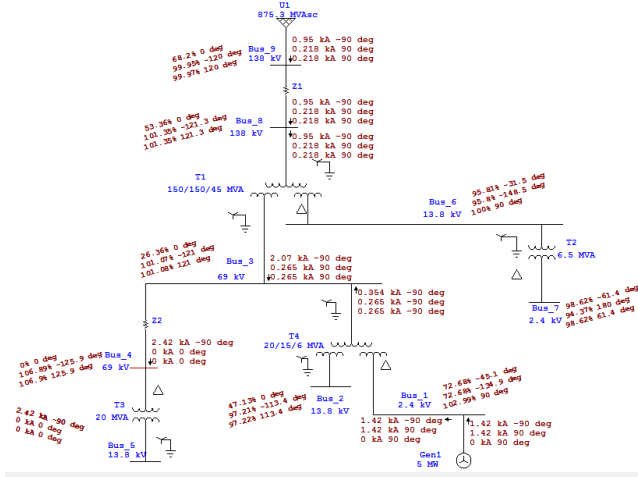


Fig. 6 ETAP Result for Single Line to Ground Fault at bus 4

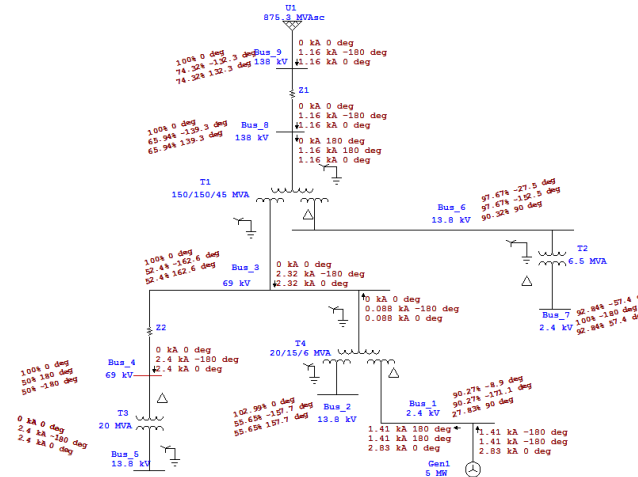


Fig. 7 ETAP Result for Line to Line Fault at bus 4

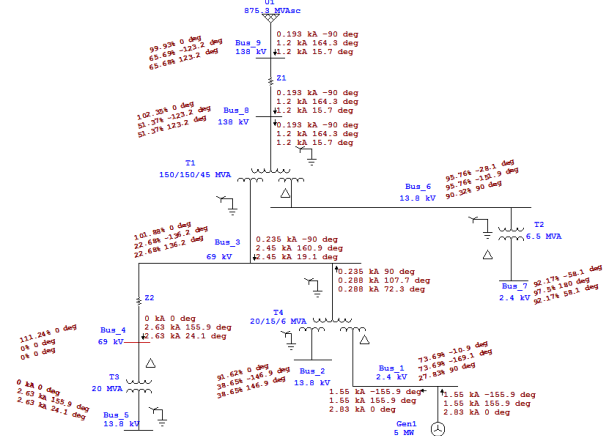


Fig. 8 ETAP Result for Double Line to Ground Fault at bus 4

TABLE I. PHASE A BUS VOLTAGES FOR THREE PHASE FAULT AT BUS 4

Three Phase Fault [Phase A]				
Bus No.	MATLAB		ETAP	
	Bus kV (p.u.)	Angle (deg.)	Bus kV (p.u.)	Angle (deg.)
1	27.02%	-30	27.83%	-30
2	23.66%	0	24.37%	0
3	18.11%	0	18.11%	0
4	0.00%	0	0.00%	0
5	0.00%	0	NA	NA
6	90.32%	-30	90.32%	-30
7	90.32%	-60	90.32%	-60
8	49.64%	0	49.64%	0
9	63.50%	0	63.50%	0

TABLE II. PHASE A LINE CURRENTS FOR THREE PHASE FAULT AT BUS 4

Three Phase Fault [Phase A]					
From Bus	To Bus	MATLAB		ETAP	
		Iline (kA)	Angle (deg.)	Iline (kA)	Angle (deg.)
0	1	2.83	-120	2.83	-120
0	9	1.34	-90	1.34	-90
1	11	2.83	-120	2.83	-120
2	11	0	0	0	0
3	4	2.77	-90	2.77	-90
3	10	2.67	90	2.67	90
3	11	0.101	90	0.101	90
4	5	0	0	0	0
6	7	0	0	0	180
6	10	0	0	0	-33.7
8	9	1.34	90	1.34	90
8	10	1.34	-90	1.34	90

TABLE III. PHASE A BUS VOLTAGES FOR SINGLE LINE TO GROUND FAULT AT BUS 4

Single Line to Ground Fault [Phase A]				
Bus No.	MATLAB		ETAP	
	Bus kV (p.u.)	Angle (deg.)	Bus kV (p.u.)	Angle (deg.)
1	70.57%	-45.11	72.68%	-45.1
2	45.75%	0	47.13%	0
3	26.36%	0	26.36%	0
4	0.00%	180	0.00%	0
5	61.72%	-54.11	NA	NA
6	95.81%	-31.46	95.81%	-31.5
7	98.62%	-61.42	98.62%	-61.4
8	53.36%	0	53.36%	0
9	68.20%	0	68.20%	0

TABLE IV. PHASE A LINE CURRENTS FOR SINGLE LINE TO GROUND FAULT AT BUS 4

Single Line to Ground Fault [Phase A]					
From Bus	To Bus	MATLAB		ETAP	
		Iline (kA)	Angle (deg.)	Iline (kA)	Angle (deg.)
0	1	1.425	-90	1.42	-90
0	9	0.95	-90	0.95	-90
1	11	1.425	-90	1.42	-90
2	11	0	0	0	0
3	4	2.423	-90	2.42	-90
3	10	2.07	90	2.07	90
3	11	0.354	90	0.354	90
4	5	0	0	0	0
6	7	0	0	0	0
6	10	0	0	0	0
8	9	0.95	90	0.95	90
8	10	0.95	-90	0.95	-90

TABLE V. PHASE B BUS VOLTAGES FOR LINE TO LINE FAULT AT BUS 4

Line to Line Fault [Phase B]				
Bus No.	MATLAB		ETAP	
	Bus kV (p.u.)	Angle (deg.)	Bus kV (p.u.)	Angle (deg.)
1	87.65%	-171.13	90.27%	-171.1
2	54.04%	-157.72	55.65%	-157.7
3	52.40%	-162.58	52.40%	-162.6
4	50.00%	180	50.00%	180
5	86.60%	180	NA	NA
6	97.67%	-152.46	97.67%	-152.5
7	100.00%	180	100.00%	-180
8	65.94%	-139.31	65.94%	-139.3
9	74.32%	-132.28	74.32%	-132.3

TABLE VI. PHASE B LINE CURRENTS FOR LINE TO LINE FAULT T BUS 4

Line to Line Fault [Phase B]					
From Bus	To Bus	MATLAB		ETAP	
		Iline (kA)	Angle (deg.)	Iline (kA)	Angle (deg.)
0	1	1.413	180	1.41	-180
0	9	1.158	-180	1.16	-180
1	11	1.413	-180	1.41	180
2	11	0	0	0	0
3	4	2.403	180	2.4	-180
3	10	2.315	0	2.32	0
3	11	0.088	0	0.088	0
4	5	0	0	0	0
6	7	0	0	0	0
6	10	0	0	0	0
8	9	1.158	0	1.16	0
8	10	1.158	180	1.16	180

TABLE VII. PHASE B BUS VOLTAGES FOR DOUBLE LINE TO GROUND FAULT AT BUS 4

Double Line to Ground Fault [Phase B]				
Bus No.	MATLAB		ETAP	
	Bus kV (p.u.)	Angle (deg.)	Bus kV (p.u.)	Angle (deg.)
1	71.56%	-169.12	73.69%	-169.1
2	37.53%	-146.91	38.65%	-146.9
3	22.68%	-136.24	22.68%	-136.2
4	0.00%	104.04	0.00%	0
5	64.22%	180	NA	NA
6	95.76%	-151.86	95.76%	-151.9
7	97.50%	180	97.50%	180
8	51.37%	-123.2	51.37%	-123.2
9	65.68%	-123.15	65.69%	-123.2

TABLE VIII. PHASE B LINE CURRENTS FOR DOUBLE LINE TO GROUND FAULT AT BUS 4

Double Line to Ground Fault [Phase B]					
From Bus	To Bus	MATLAB		ETAP	
		Iline (kA)	Angle (deg.)	Iline (kA)	Angle (deg.)
0	1	1.548	155.89	1.55	155.9
0	9	1.202	164.33	1.2	164.3
1	11	1.548	155.89	1.55	155.9
2	11	0	0	0	0
3	4	2.633	155.89	2.63	155.9
3	10	2.45	-19.08	2.45	-19.1
3	11	0.288	-72.3	0.288	-72.3
4	5	0	0	0	0
6	7	0	0	0	0
6	10	0	0	0	0
8	9	1.202	-15.67	1.2	-15.7
8	10	1.202	164.33	1.2	164.3

Sample output of the program for a double line ground fault at bus 4 are shown below.

### Table of Base Values ###			
Bus Number	Base kV	Base kA	Base Ohm
1.0000	2.4716	23.3590	0.0611
2.0000	14.2119	4.0624	2.0198
3.0000	69.0000	0.8367	47.6100
4.0000	69.0000	0.8367	47.6100
5.0000	14.2119	4.0624	2.0198
6.0000	13.8000	4.1837	1.9044
7.0000	2.4000	24.0563	0.0576
8.0000	138.0000	0.4184	190.4400
9.0000	138.0000	0.4184	190.4400

### Table of POSITIVE Impedance in per unit ###			
From Bus	To Bus	Resistance	Reactance
0	1.0000	0	6.0344
0	9.0000	0.0000	0.1142
1.0000	11.0000	0	0.2781
2.0000	11.0000	0	-0.0409
3.0000	4.0000	0	0.0546
3.0000	10.0000	0	0.2260
3.0000	11.0000	0	0.4589
4.0000	5.0000	0	0.3771
6.0000	7.0000	0	0.9000
6.0000	10.0000	0	0.5940
8.0000	9.0000	0	0.0434
8.0000	10.0000	0	-0.1273

### POSITIVE Zbus Matrix(Magnitude) ###						
Columns 1 through 7						
0.8529	0.6141	0.2201	0.2201	0.2201	0.0260	0.0260
0.6141	0.6015	0.2302	0.2302	0.2302	0.0272	0.0272
0.2201	0.2302	0.2469	0.2469	0.2469	0.0292	0.0292
0.2201	0.2302	0.2469	0.3016	0.3016	0.0292	0.0292
0.2201	0.2302	0.2469	0.3016	0.6787	0.0292	0.0292
0.0260	0.0272	0.0292	0.0292	0.0292	0.6242	0.6242
0.0260	0.0272	0.0292	0.0292	0.0292	0.6242	1.5242
0.1353	0.1416	0.1519	0.1519	0.1519	0.1569	0.1569
0.0981	0.1026	0.1101	0.1101	0.1101	0.1138	0.1138
0.0260	0.0272	0.0292	0.0292	0.0292	0.0302	0.0302
0.6141	0.6424	0.2302	0.2302	0.0272	0.0272	0.0272
Columns 8 through 11						
0.1353	0.0981	0.0260	0.6141			
0.1416	0.1026	0.0272	0.6424			
0.1519	0.1101	0.0292	0.2302			
0.1519	0.1101	0.0292	0.2302			
0.1519	0.1101	0.0292	0.2302			
0.1569	0.1138	0.0302	0.0272			
0.1569	0.1138	0.0302	0.0272			
0.1541	0.1117	0.1569	0.1416			
0.1117	0.1124	0.1138	0.1026			
0.1569	0.1138	0.0302	0.0272			
0.1416	0.1026	0.0272	0.6424			

### Table of Bus Voltages for Phase B ###			
Bus Number	P.U.	kV	Angle_deg
1.0000	0.7156	1.7687	-169.1162
2.0000	0.3753	5.3335	-146.9086
3.0000	0.2268	15.6471	-136.2424
4.0000	0.0000	0.0000	104.0362
5.0000	0.6422	9.1275	180.0000
6.0000	0.9576	13.2143	-151.8598
7.0000	0.9750	2.3400	180.0000
8.0000	0.5137	70.8901	-123.1957
9.0000	0.6568	90.6401	-123.1540

###		Phase B:			###
From Bus	To Bus	P.U.	kA	Angle_deg	
0	1.0000	0.0662	1.5475	155.8880	
0	9.0000	2.8740	1.2024	164.3283	
1.0000	11.0000	0.0662	1.5475	155.8880	
2.0000	11.0000	0	0	0	
3.0000	4.0000	3.1464	2.6328	155.8880	
3.0000	10.0000	2.9280	2.4500	-19.0800	
3.0000	11.0000	0.3446	0.2883	-72.3038	
4.0000	5.0000	0	0	0	
6.0000	7.0000	0	0	0	
6.0000	10.0000	0	0	0	
8.0000	9.0000	2.8740	1.2024	-15.6717	
8.0000	10.0000	2.8740	1.2024	164.3283	

As seen from Tables I-VIII, results of ETAP and the created program in MATLAB are the same except for the bus voltages at 1 and 2 which have a difference of 0.01 to 0.02 p.u. The probable cause of this difference is the prefault voltage of approximately 1.02 p.u. in buses 1 and 2. This can be confirmed in ETAP by running a load flow.

Based on these comparisons we can say that the created fault simulator produces accurate results.

## VI. SUMMARY OF ACCOMPLISHMENTS

The fault simulator created in MATLAB have the following capabilities.

- Receive input from the command line and the spreadsheet 'FaultStudyData.xls'
- Gets the equivalent base values of a power system.
- Gets the per unit Impedance Table of the power system.
- Can model zero sequence networks especially for transformers with different types of connections.
- Could accept and model data of power system elements such as generator, motor, line, 2 winding transformer, 3 winding transformer, and utility.
- Creates Z bus matrices for the positive, negative and zero networks.
- Can accept phase shift adjustments based on the configuration of the transformers.
- Can simulate three phase, single line to ground, line to line, and double line to ground faults.
- Can simulate change in symmetry for the 4 types of shunt faults.

## ACKNOWLEDGMENT

I would like to give thanks to the following:

Prof. Wali and Prof. Jordan for their support and notes that helped me in creating the program and validating the results of my program though hand calculation.

ETAP for their trustworthy simulator that helped me to validate the results of my program.

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