

# 4339 - ASSET PRICING AND INVESTMENTS

## ASSIGNMENT 2

### Exercises

1. Consider an economy with risky assets and a risk-free asset in which the CAPM holds.

Portfolio	Expected return	Standard deviation
Risk-free ( $r_f$ )	2%	0%
Market ( $r_M$ )	8%	20%

- (a) Compute the  $\beta$  of an asset with standard deviation 10% and correlation 0.5 with the market.
  - (b) An investor chooses a portfolio with 10% standard deviation. What must be her/his risk-aversion coefficient  $a$  according to the CAPM?
2. In practice, when applying the CAPM, we would often use a U.S. stock index (e.g., S&P 500 or CRSP market-weighted portfolio) as a proxy for the market portfolio of U.S. stocks, some international stock index for international stocks, etc. This question is designed to make you think about under what conditions these are valid approximations compared to using the global market portfolio of all assets, which is the implication of the plain vanilla CAPM.

Consider a setting with two broad asset classes  $E$  (stocks) and  $D$  (corporate bonds). There are two investors, a mutual fund and a life-insurance company. Let the covariance matrix of returns be denoted by the following matrix with block structure:

$$\Sigma = \begin{pmatrix} \Sigma_E & \Sigma_{ED} \\ \Sigma_{ED}^t & \Sigma_D \end{pmatrix}$$

Describe whether a version of the CAPM holds in the following setups, and what the relevant market portfolio is. Please justify your answer economically, but you do not need to provide a formal proof.

- (a) *Frictionless Benchmark:* Both the mutual fund and life-insurance company can freely invest in both stock- and equity markets.

- (b) *Fund (demand-side) segmentation:* The mutual fund's mandate restricts its portfolio to equities only. The life-insurance company faces regulations that require it to be invested in bonds only.

*Hint:* What are demand and supply for the risky asset portfolios in stocks and bonds? What is the relevant market portfolio in each asset class?

- (c) \* *Asset (supply-side) segmentation & imperfect fund segmentation:* The mutual fund is now a mixed fund that can invest in both stocks and bonds, but the life-insurance company is still restricted to bonds only. Additionally, the covariance matrix takes a special form where  $\Sigma_{ED} = \mathbf{0}$ , that is, all equities are uncorrelated with all corporate bonds.

*Hint:* Because of the special covariance matrix structure with  $\Sigma_{ED} = \mathbf{0}$ , the inverse of the covariance matrix is simply:

$$\Sigma^{-1} = \begin{pmatrix} \Sigma_E^{-1} & \mathbf{0} \\ \mathbf{0} & \Sigma_D^{-1} \end{pmatrix}$$

- (d) \* *Imperfect fund segmentation:* Like in the previous part, but now  $\Sigma_{ED} \neq \mathbf{0}$ .

*Hint:* In this case, do all investors investing in bonds  $D$  want to hold the same portfolio of risky assets? What does that do to the tangency portfolio = market portfolio equilibrium argument?

- (e) Comment on how segmentation affects how we should think about applying the CAPM. Is it always a bad idea to use a U.S. stock index as a proxy for the market portfolio?

## Case Study Questions

*To solve these questions, 2 data files are provided.*

- *Riskfree.txt* contains monthly returns for the 1 month T-Bill rate
- *StockPrice.txt* contains monthly prices for the stock index and individual stocks of Doleo and Thyssenkrupp.

*The time period for risky assets starts one month earlier so you can compute monthly returns over the same period when the T-Bill rate is available.*

### 1. Changing portfolio weights (qualitative questions)

- (a) What are the motivation(s) of Schumpeter's manager for changing portfolio weights over time relative to the 60/40 benchmark for stocks/T-Bills?

- (b) Another manager argues that varying the weight on the stock index relative to T-bills should not change the reward to risk ratio. Explain this claim using an argument seen in class.
2. As of April 2014, Schumpeter invests 65% in the stock index and 35% in T-Bills. The manager considers allocating 1% away from bonds into stocks.
- (a) Compute the annualized standard deviation of the *excess* returns over the T-Bill rate for each of the three following risky assets: the stock index, Doleo, and ThyssenKrupp. To estimate the annualized standard deviation, compute the standard deviation of monthly excess returns and multiply by  $\sqrt{12}$ .
  - (b) Based on historical data, compute the standard deviation of a portfolio matching the current fund allocation: 35% in bonds and 65% in stocks.
  - (c) Repeat the exercise above for 3 different portfolios with 34% in T-Bills and either (i) 66% in the stock index, (ii) 65% in the stock index and 1% in Doleo, (ii) 65% in the stock index and 1% in ThyssenKrupp.
  - (d) Comment on your findings in question 2c in light of the results from question 2a. You can use a simple risk decomposition.
3. To understand the results from your analysis, you propose to estimate the  $\beta$  of Doleo and Thyssenkrupp.
- (a) Run a regression for each individual stock of its excess return on the stock index excess return. Report the  $\beta$  estimates and the plots of the security characteristic lines. *You may adapt the Python code from `indexmodel.ipynb`.*
  - (b) How do these results help explain the findings of question 2?