Estimating Nominal GDP using the Exponential Decay Function

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The following method estimates nominal gross domestic product using the exponential decay function. The economy is characterized using two parameters which are estimated using variables lagged by three periods.

Consider $100 being deposited into a bank account where the cash in the account is used to pay for goods or services at a constant percentage rate over time. Over a long enough time period the cash will result in $100 of nominal production. During the lifespan of the account the balance in the account will be loaned out by the bank for investment in the economy.

Consider a fixed period of time (one month) where funds are spent at a constant rate and where simultaneously the bank is making loans. The rate of decay function is $x(t)$, where x is the value at time t, k is the decay parameter, and x_0 is the value of x at time zero,

$$x(t) = x_0 \cdot e^{-k \cdot t}$$

This is a function of time with one parameter k. If instead time is replaced with the parameter c the decay function representing the balance in the account after one month is,

$$x(\text{after one month}) = x_0 \cdot e^{-k \cdot c}$$

After one month the amount spent and loaned from the account could be determined by solving the following two equations for c and k,

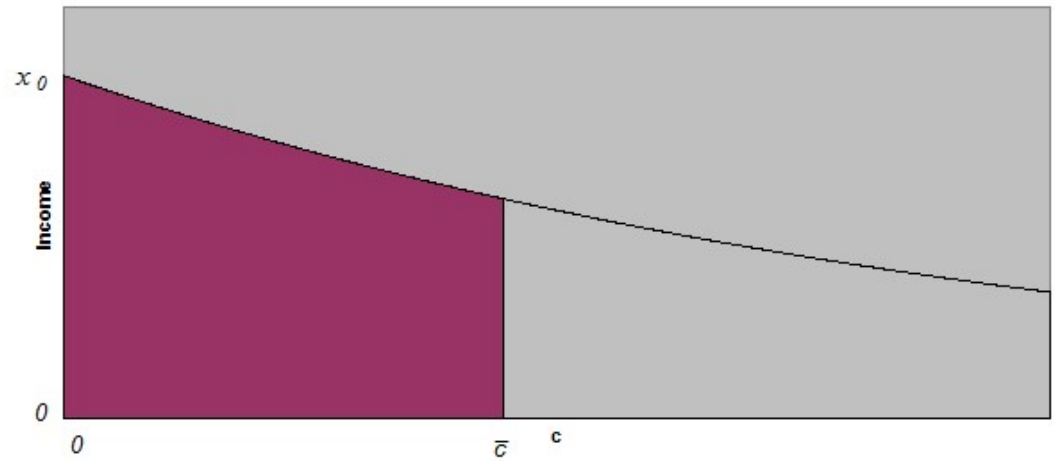
$$\begin{aligned} \text{Amount spent after one month} &= x_0 - x(\text{after one month}) \\ &= x_0 - x_0 \cdot e^{-k \cdot c} \\ &= x_0 \cdot (1 - e^{-k \cdot c}) \end{aligned}$$

$$\begin{aligned} \text{Amount loaned after one month} &= \int_0^c x_0 \cdot e^{-k \cdot c}\, dc \\ &= (x_0 / k) \cdot (1 - e^{-k \cdot c}) \\ &= \text{Amount spent after one month} / k \end{aligned}$$

In this case since funds are simultaneously being spent and loaned, k represents the ratio of funds spent to amount loaned. As these equations describe the use of funds for production and investment they can be used to model the economy.

Note that at each moment in time there is income earned or generated by the members of the economy. The activity of the economic system will be determined by the

parameters c and k and by the amount of existing funds available for consumption and investment. Consider the following graph which represents the income x_0 and its value after it decays into consumption and investment after $\bar{c}$ units,



For income x_0 generated at the first moment of the period, the investment it generates over the period is the area under the curve from 0 to $\bar{c}$. Therefore, if x_0 is generated at each moment of the period the total investment generated for the period is found by summing each integral from 0 to c for each value of c from $\bar{c}$ to 0.

Spending generated over $\bar{c}$ units of the period from the income earned at the first moment of the period

$$= k \cdot (\text{total income for period} / \bar{c}) \cdot \int_0^{\bar{c}} e^{-k \cdot c}\, dc$$
$$= (\text{total income} / \bar{c}) \cdot (1 - e^{-k \cdot \bar{c}})$$

Because this is an exponential integral which is not defined for $\bar{c} = 0$, it is necessary to assume the income generated at every moment in the period is the same (this assumption also means income (and therefore consumption and investment) is jumping between periods). In which case the first $\bar{c}$ in the above equation is constant and integration will give the expression.

Spending generated from all income earned over the period

$$= (\text{total income} / \bar{c}) \cdot \int_0^{\bar{c}} (1 - e^{-k \cdot c})\, dc$$
$$= (\text{total income} / \bar{c}) \cdot (\bar{c} + (1 / k) \cdot (e^{-k \cdot \bar{c}} - 1))$$

And investment is,

Investment generated from all income earned over the period

$$= \text{Spending generated from all income earned over the period} / k$$
$$= (\text{total income} / (\bar{c} \cdot k)) \cdot (\bar{c} + (1 / k) \cdot (e^{-k \cdot \bar{c}} - 1))$$

Therefore k and $\bar{c}$ for each period can be determined by solving the spending and investment equations above where each period,

Total Income = Consumption + Government Expenditure + Investment + Exports - Imports

Spending = Consumption + Government Expenditure + Imports

Investment = Investment

Alternatively, since k is the ratio of spending to investment, k can be directly inserted into the spending equation above and $\bar{c}$ can be calculated.

Once the unspent income in a period is determined the amount which remains after subsequent periods of spending can be calculated by multiplying by the decay function,

Income from period j available to be used for spending in period i
$= Unspent\ income\ from\ period\ j \cdot \prod e^{-k_m \cdot \bar{c}_m}$
$= (GDP_j / (k_j \cdot \bar{c}_j)) \cdot (1 - e^{-k_j \cdot \bar{c}_j}) \cdot \prod e^{-k_m \cdot \bar{c}_m}$

For every period m between $j+1$ and i.

Unspent income along with the income generated at each moment of the current period form the funds available to be used for production or investment.

To summarize, values for k for each period can be calculated by taking the ratio of spending to investment. Funds available for spending and investment for a given period can be determined by taking all unspent income from previous periods and adding them to income for the current period. The spending and investment equations for the current period can then be applied and a value for $\bar{c}$ for the current period can be backed out to match the current period's spending and investment figures.

To produce a forecast for next periods economic activity note that the amount of funds spent next period is given by the equation,

$Consumption_{t+1} + Government\ Expenditure_{t+1} + Imports_{t+1} =$
$Spending\ from\ income\ generated\ in\ period_{t+1} + \sum_{i=j}^{t} (Spending\ from\ income\ generated\ in\ period\ i)$

Where j is the first period of the sample. The second term on the right hand side will be a function of figures already known except for $\bar{c}_{t+1}$ and k_{t+1}. The first term on the right hand side will be a function of $\bar{c}_{t+1}$ and k_{t+1} and also of $Consumption_{t+1}$, $Investment_{t+1}$, $Government\ Expenditure_{t+1}$, $Imports_{t+1}$ and $Exports_{t+1}$. Therefore estimates for $Consumption_{t+1}$and $Investment_{t+1}$ could be formed using estimates for k_{t+1} and $\bar{c}_{t+1}$ if we first form estimates for $Government\ Expenditure_{t+1}$, $Imports_{t+1}$ and $Exports_{t+1}$. In the appendix the code and figures provided have no special treatment for estimating *Government Expenditure*, *Imports* or *Exports*; the period change is simply

estimated using a regression and added to previous period's value to form an estimate for the current period.

Let *C, I, G, M* and *E* denote *Consumption, Investment, Government Expenditure, Imports* and *Exports*. The equation above can be written as,

$$C_{t+1} + G_{t+1} + M_{t+1} = (1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (C_{t+1} + G_{t+1} + I_{t+1} + E_{t+1} - M_{t+1}) + \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}})$$

$$C_{t+1} + G_{t+1} + M_{t+1} = (1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (C_{t+1} + G_{t+1} + ((C_{t+1} + M_{t+1} + G_{t+1})/k) + E_{t+1} - M_{t+1}) + \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}})$$

Where the expression $\prod e^{-k_m \cdot \bar{c}_m}$ is from $i+1$ to t and $\bar{c}$ and k are the parameters for period $t+1$.

Solving for $Consumption_{t+1}$ gives,

$$C_{t+1} = ((1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (E_{t+1} + M_{t+1} + G_{t+1} + ((M_{t+1} + G_{t+1}) / k)) - M_{t+1} - G_{t+1} + \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}})) / (1 - (1 + (e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (1 + (1 / k)))$$

It is not immediately clear *Consumption* is increasing in $\bar{c}$ due to the expression,

$$1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))$$

However, this expression is increasing in $\bar{c}$ when,

$$\bar{c} \cdot k > ln\ (1 + \bar{c} \cdot k)$$

Which holds for the relevant range where $\bar{c} \cdot k$ *is* greater than zero.

An aspect of this model is that $\bar{c}$ does not have to increase for consecutive periods to exhibit increasing levels of total activity. For example, assume the economy has no *Government Expenditure*, *Exports*, and *Imports* so that the equation above for *Consumption* becomes,

$$C_{t+1} = \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}}) / (1 - (1 + (e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (1 + (1 / k)))$$

For simplification let the expressions in C_{t+1} be defined in the following way, where the funds available at the beginning of period $t+1$ are,

$$A(\bar{c}_{t+1}) = \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m})$$

For m from $i+1$ to t, and,

$$B(\bar{c}_{t+1}) = 1 - (1 + (e^{-k_{t+1} \cdot \bar{c}_{t+1}} - 1) / (\bar{c}_{t+1} \cdot k_{t+1})) \cdot (1 + (1 / k_{t+1}))$$

Then,

$$C_{t+1} = A(\bar{c}_{t+1}) \cdot (1 - e^{-k_{t+1} \cdot \bar{c}_{t+1}}) / B(\bar{c}_{t+1})$$

If C_{t+1} and C_t have the same value then dividing one by the other results in the expression,

$$A(\bar{c}_{t+1}) / A(\bar{c}_t) = (1 - e^{-k_t \cdot \bar{c}_t}) \cdot B(\bar{c}_{t+1}) / (1 - e^{-k_{t+1} \cdot \bar{c}_{t+1}}) \cdot B(\bar{c}_t)$$

If k is the same for periods t and $t+1$ and if more funds are available at the beginning of period $t+1$ than were available at the beginning of t, then since $(1 - e^{-k_{t+1} \cdot \bar{c}_{t+1}})$ is increasing in $\bar{c}$ and $B(\bar{c}_{t+1})$ is decreasing in $\bar{c}$, $\bar{c}_{t+1}$ would need to be less than $\bar{c}_t$ if *Consumption* was unchanged from period t to $t+1$.

A scatter plot of k and $\bar{c}$ are presented in the appendix[1]. Examining the series shows that k and $\bar{c}$ have trended in a zig-zag pattern from 1970 to the present. The changes in the trending of k and $\bar{c}$ occur in 2003, 2006, and 2008. I believe this pattern is due to the relative increase of *Imports* to *Exports* as during these periods *Imports* exceeded *Exports* by 4% to 5% relative to *GDP*. This trend between *Imports* and *Exports* would cause k to increase and would require $\bar{c}$ to increase as less funds would be available.

Generally k and $\bar{c}$ are inversely related and were modeled through the relationship,

$$k = v / c$$

The parameters v and c were estimated and used to form an estimate for k.

As mentioned above a decrease in $\bar{c}$ is not necessarily indicative of a decrease in total activity due to the amount of funds available at the beginning of the period. However, the general movement of k and $\bar{c}$ can be interpreted in the following way. Moving horizontally in the k - $\bar{c}$ plane holds the economic activity of the system constant; the total of spending and investment will be unchanged, only the ratio of activity between the spending and investment will change. Moving vertically means the ratio between spending and investment is unchanged as the total activity of the system changes. Moving diagonally from the upper left to lower right corner results in spending being relatively unchanged; as k increases the ratio of spending to investment decreases and to hold consumption constant c decreases.

For each period c and k equate the expressions for spending and investment to their actual values and may change between periods for a number of reasons.

If the savings rate decreases and the rate of loans continues to the match the rate of savings then the total amount of currency being circulated is unchanged and only k would increase.

[1] Figure 14, page 11

If more money is printed and loaned investment would immediately increase while spending would be unchanged. This would cause the system to move diagonally to the upper left. If the savings rate is unchanged then eventually the ratio between spending and investment would return to its original value and the system would move right horizontally with *c* unchanged. Overall given enough time the only change to the system would be an increase in *c* if no other changes occurred.

If the rate that loans are issued decreases this would cause the amount of money in circulation to decrease. After the initial decrease to investment, if spending is reduced to preserve the ratio between spending and investment then *k* will be unchanged and the only effect will be that *c* has decreased. If spending is unchanged then the effective savings rate will have decreased and the system would move diagonally to the bottom right corner.

In the appendix are regression outputs for *c, v, Exports, Imports* and *Government Expenditure*[2]. The data range is from Q3 1969 to Q2 2020 and only forecasts for 1985 Q1 to Q2 2020 are generated. The parameters to be estimated are; *c*, *v*, and the change in *G*, *E* and *M*. In addition to the time variable, binary variables for Q4 2008 and Q1 2020, and three trend variables noted above, the other variables used for estimation (and their three period lagged values) were,

- Inflation
- US_CANExchangeRate
    - US/CAD exchange rate
- NetSaving
    - Net Saving
- a_line
    - Parameter *v*
- InterestPaymentSales_Ratio
    - Calculated as the product of investment and the interest rate divided by the sum of exports and consumption
- NetSavingIncome_Ratio
    - Calculated as Net Saving divided by GDP
- Tau_c
    - Calculated as *c* divided by the negative log of 1 divided by 1 plus *k* divided by *k*
- Cs_cn
    - The difference between *c* and the value of *c* that would hold consumption constant (it would be the previous periods value of *c* but in application it is backed out due to changes in exports, imports, and government expenditure and also because when calculating available funds unused income from 35 or more periods ago is discarded for computing purposes)
- a_line_an
    - The difference between *v* and the *v* required to hold consumption constant
- MoneySupply_relative_change

[2] The difference between *c* and *v* that would hold *Consumption* and *Investment* constant are the dependent variables estimated.

    - Percent change in money supply
- k_relative_change
    - Percent change in *k*
- c_s_relative_change
    - Percent change in *c*
- Yconetofour
    - The slope of the yield curve for t-bills from one to four years
- ycfivetothirty
    - The slope of the yield curve for t-bills from five to thirty years
    - The maximum length of t-bills changes over time; from 1961-1971 it is 7 years, from 1971 to 1981 it is 15 years, from 1981 to 1985 it is 20 years, and from 1981 to the present it is 30 years
- yconetofour_ns
    - The slope of the yield curve for t-bills from one to four years multiplied by NetSaving
- ycfivetothirty_ns
    - The slope of the yield curve for t-bills from five to thirty years multiplied by NetSaving

Please see the appendix for regression and forecast charts. There are a couple of things to note.

- When estimating the model the most important variables were Net Savings, Net Savings Income Ratio and the Yield Curve variables.
- Included is code that generates the data, regression outputs and a spreadsheet of the generated data reported in the appendix. Note that the code and spreadsheet have fitted values for the most recent forecast period Q3 2020. These fitted values for Q3 2020 are just the actual values for Q2 2020.

Overall this is a simple model for forecasting an economic system. Its strength is that it requires no equilibrium, optimization or simplifying assumptions.

Appendix

Figure 1

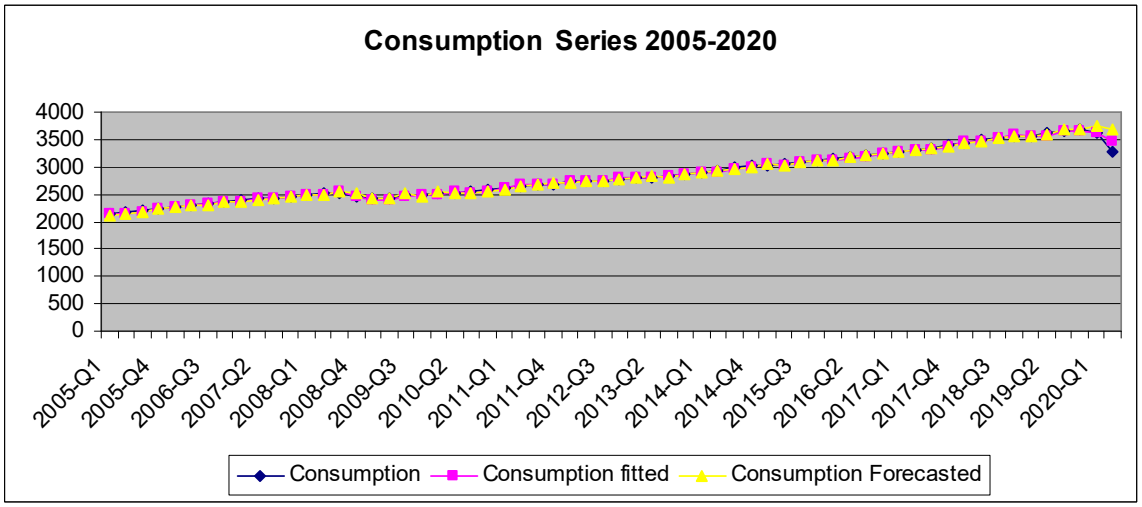


Figure 2

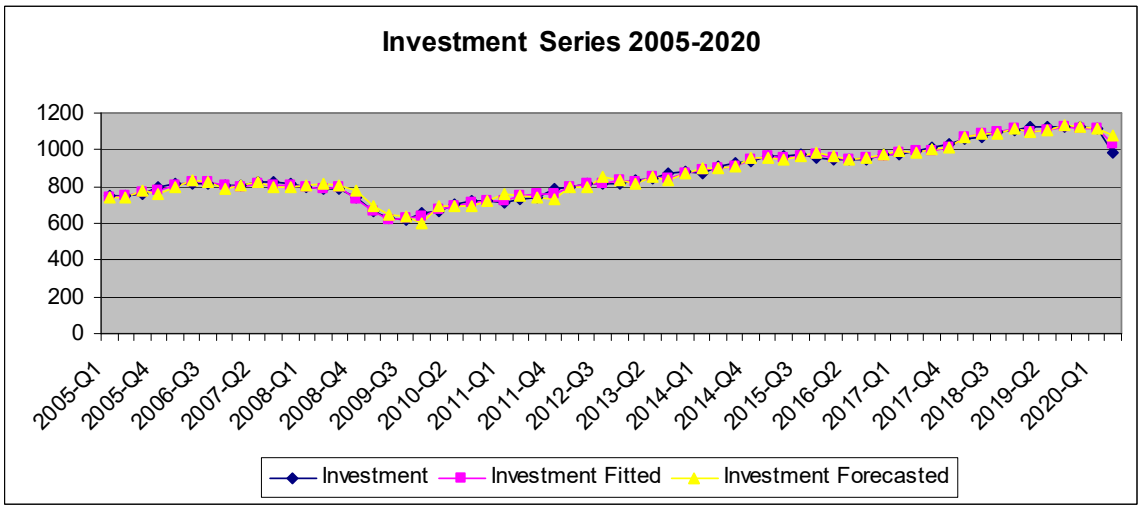


Figure 3

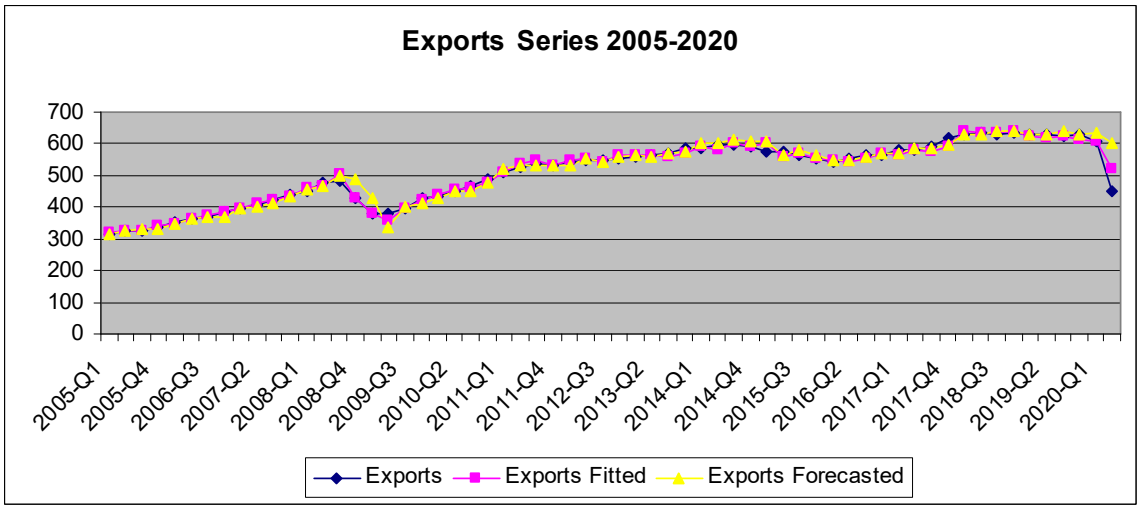


Figure 4

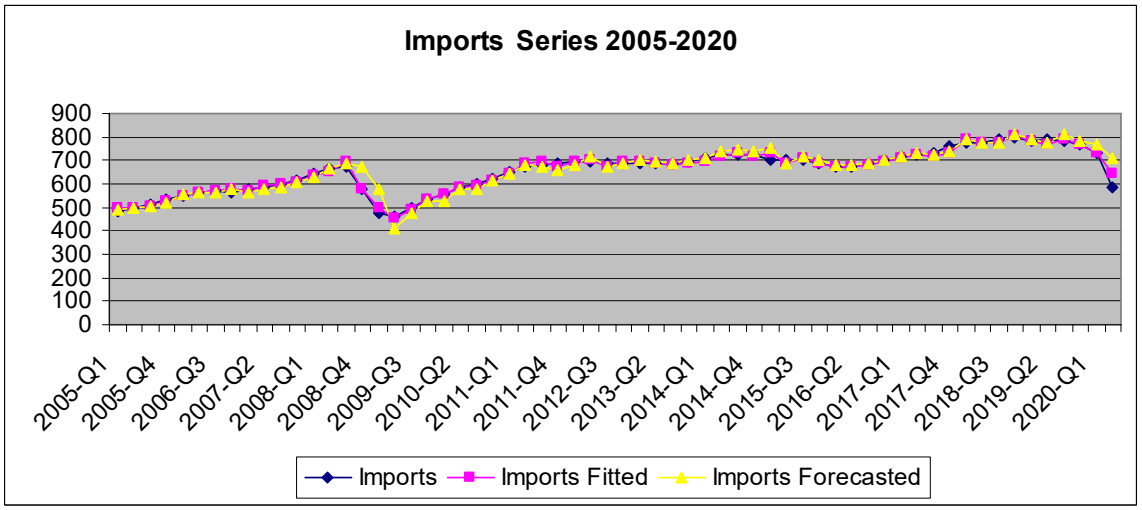


Figure 5

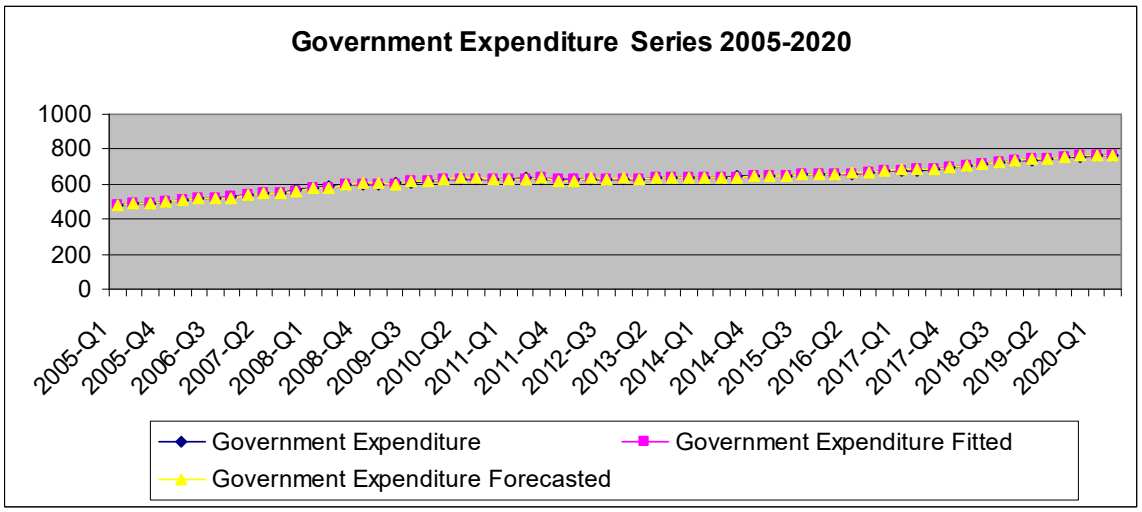


Figure 6

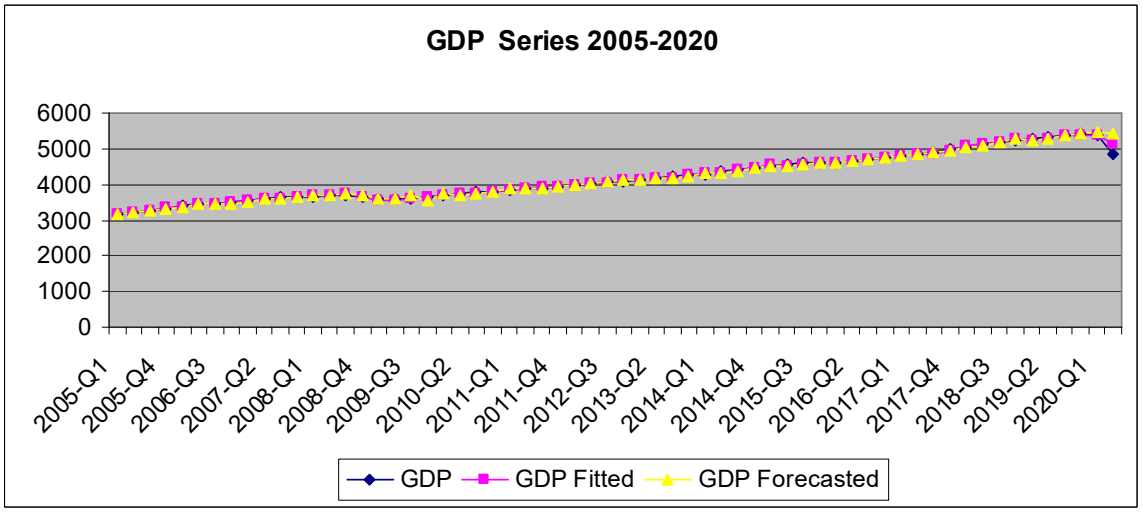


Figure 7

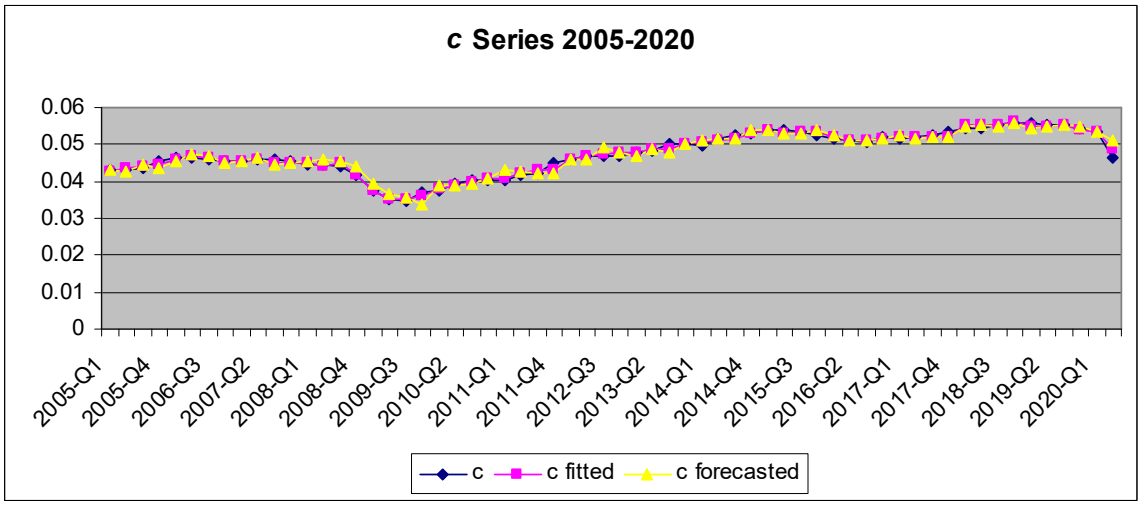


Figure 8

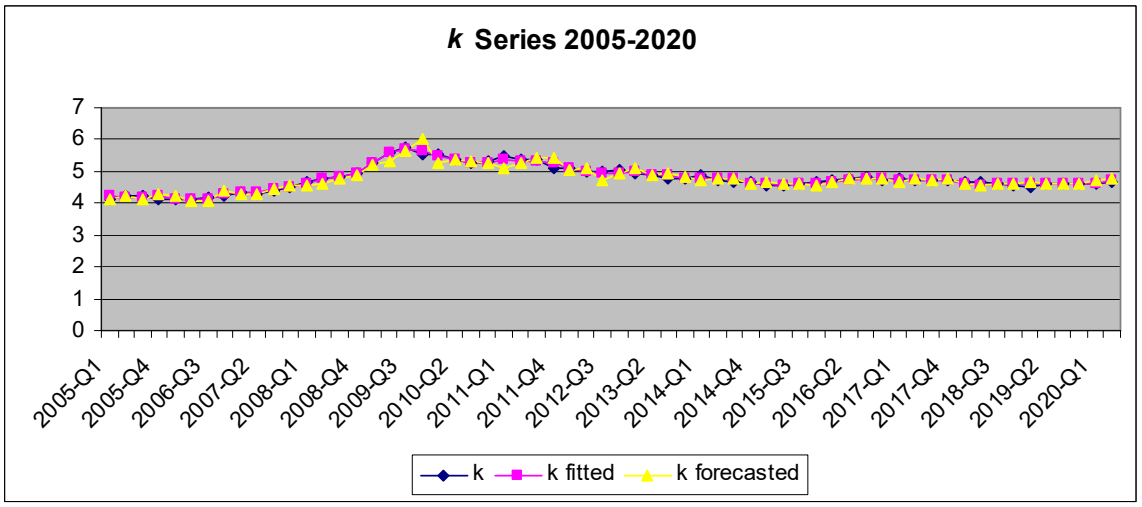


Figure 9

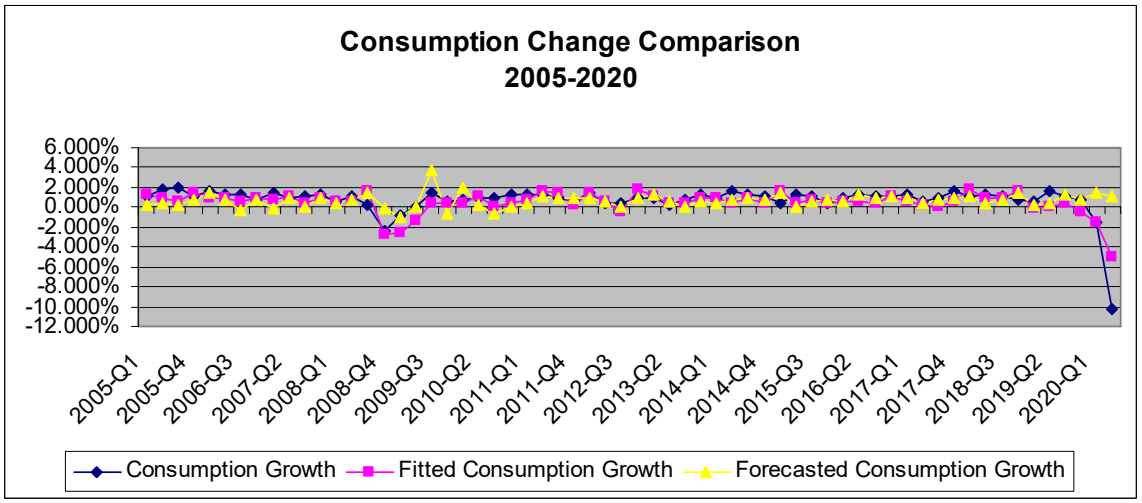


Figure 10

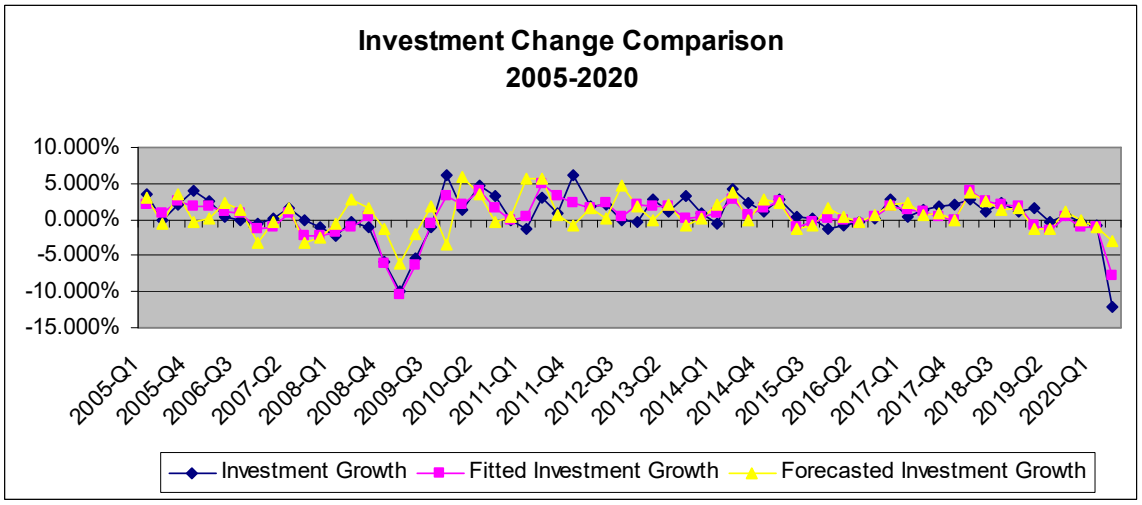


Figure 11

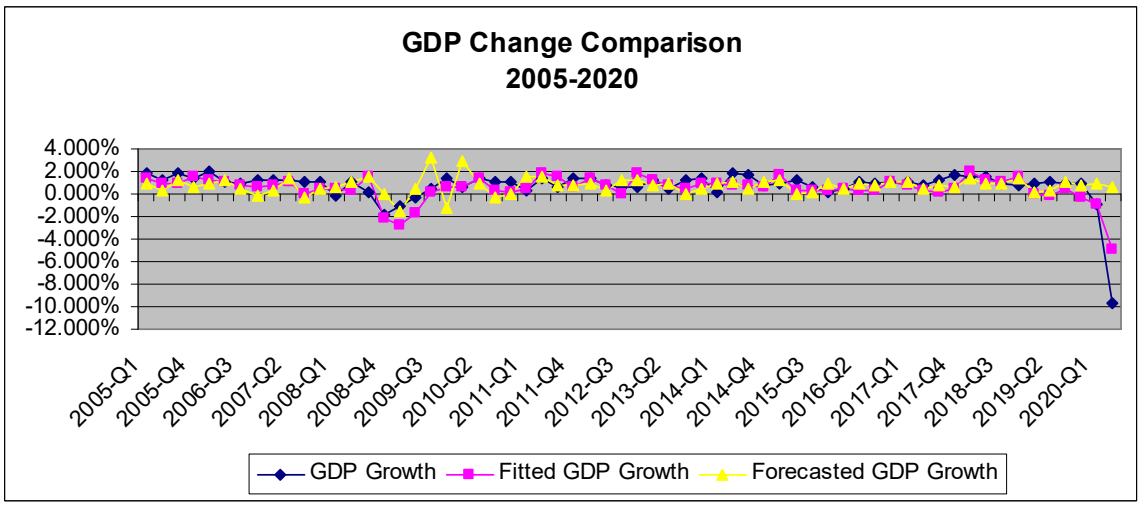


Figure 12

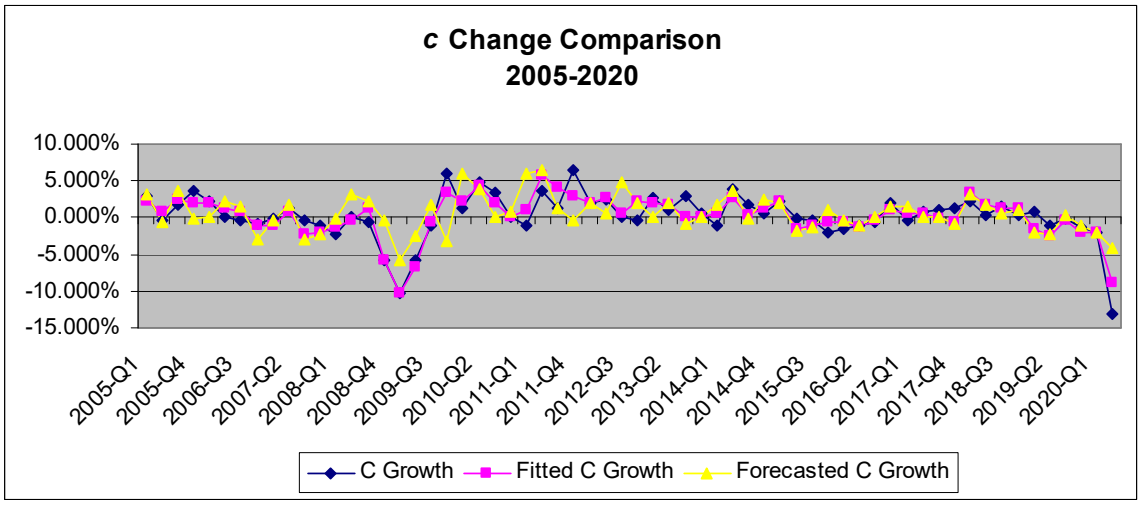


Figure 13

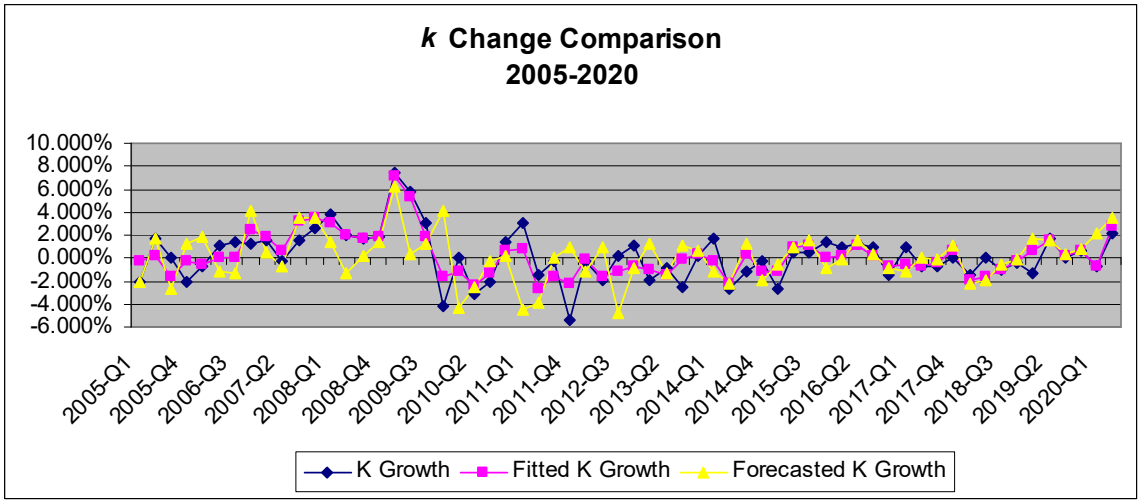
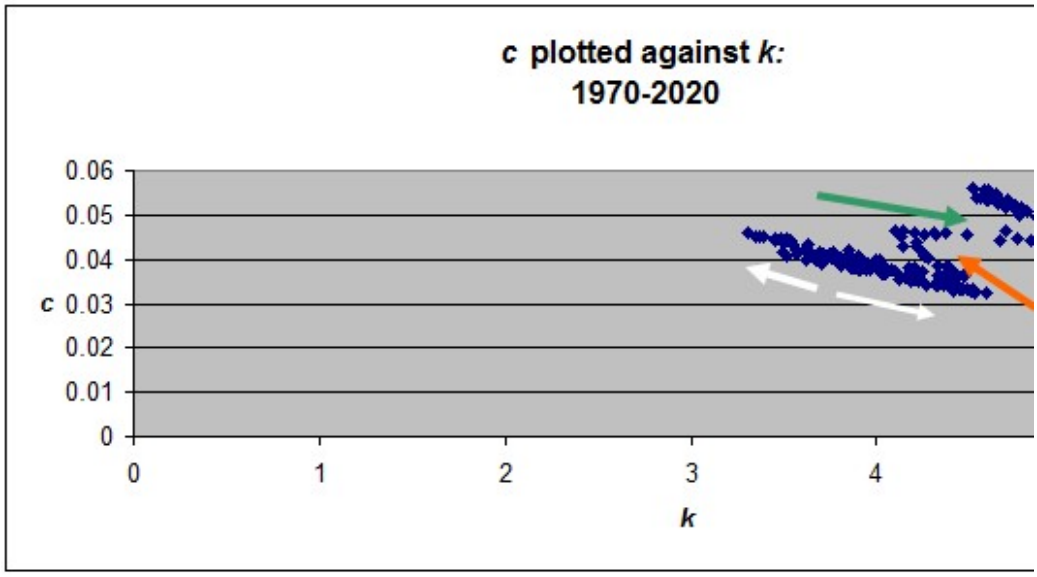


Figure 14



c Regression Output

```
Call:
lm(formula = cs_cn_r ~ time_r + time_binary_2008_Q4 + time_binary_2020_Q1 +
    c_k_change_175 + c_k_change_186 + c_k_change_200 + Inflation_L1_r +
    Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r + NetSaving_L2_r +
    NetSaving_L3_r + MoneySupply_relative_change_L1_r + MoneySupply_relative_change_L2_$
    MoneySupply_relative_change_L3_r + InterestPaymentSales_Ratio_L1_r +
    InterestPaymentSales_Ratio_L2_r + InterestPaymentSales_Ratio_L3_r +
    NetSavingIncome_Ratio_L1_r + NetSavingIncome_Ratio_L2_r +
    NetSavingIncome_Ratio_L3_r + tau_c_L1_r + tau_c_L2_r + tau_c_L3_r +
    k_relative_change_L1_r + k_relative_change_L2_r + k_relative_change_L3_r +
    c_s_relative_change_L1_r + c_s_relative_change_L2_r + c_s_relative_change_L3_r +
    cs_cn_L1_r + cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r +
    a_line_L3_r + a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r +
    US_CANExchangeRate_L1_r + US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r +
    ycfivetothirty_L1_r + ycfivetothirty_L2_r + ycfivetothirty_L3_r +
    yconetofour_L1_r + yconetofour_L2_r + yconetofour_L3_r +
    yconetofour_ns_L1_r + yconetofour_ns_L2_r + yconetofour_ns_L3_r +
    ycfivetothirty_ns_L1_r + ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r,
    data = subset(R_US_df2_r, time_r > 39 & time_r < 242))

Residuals:
       Min         1Q     Median         3Q        Max
-2.571e-03 -4.460e-04 -5.275e-05  4.365e-04  2.416e-03

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                     -8.532e-04  2.967e-03  -0.288 0.774059
time_r                          -1.770e-05  1.523e-05  -1.162 0.247131
time_binary_2008_Q4             -2.486e-03  1.069e-03  -2.325 0.021445 *
time_binary_2020_Q1             -4.658e-05  1.048e-03  -0.044 0.964612
c_k_change_175                   1.142e-03  6.529e-04   1.748 0.082470 .
c_k_change_186                  -6.834e-04  6.470e-04  -1.056 0.292550
c_k_change_200                   5.708e-04  8.661e-04   0.659 0.510919
Inflation_L1_r                  -3.870e-06  1.869e-04  -0.021 0.983508
Inflation_L2_r                  -1.688e-05  1.667e-04  -0.101 0.919485
Inflation_L3_r                   7.779e-05  1.696e-04   0.459 0.647250
NetSaving_L1_r                  -2.235e-05  1.248e-05  -1.791 0.075339 .
NetSaving_L2_r                   4.415e-05  1.833e-05   2.408 0.017290 *
NetSaving_L3_r                  -1.929e-05  1.312e-05  -1.470 0.143729
MoneySupply_relative_change_L1_r  1.513e-02  1.378e-02   1.098 0.274206
MoneySupply_relative_change_L2_r  1.413e-02  1.375e-02   1.027 0.305905
MoneySupply_relative_change_L3_r  2.261e-03  1.157e-02   0.195 0.845335
InterestPaymentSales_Ratio_L1_r   7.813e-04  7.198e-04   1.085 0.279526
InterestPaymentSales_Ratio_L2_r  -1.338e-03  1.012e-03  -1.322 0.188112
InterestPaymentSales_Ratio_L3_r   6.830e-04  7.130e-04   0.958 0.339675
NetSavingIncome_Ratio_L1_r        1.290e-01  2.709e-02   4.761 4.57e-06 ***
NetSavingIncome_Ratio_L2_r       -1.422e-01  3.911e-02  -3.636 0.000382 ***
NetSavingIncome_Ratio_L3_r        1.807e-02  2.744e-02   0.659 0.511199
tau_c_L1_r                        5.090e+00  2.723e+00   1.869 0.063576 .
tau_c_L2_r                       -7.636e+00  4.223e+00  -1.808 0.072603 .
tau_c_L3_r                        2.446e+00  2.564e+00   0.954 0.341583
k_relative_change_L1_r            3.820e-02  3.867e-02   0.988 0.324816
k_relative_change_L2_r           -2.847e-02  4.448e-02  -0.640 0.523073
k_relative_change_L3_r           -9.938e-02  3.467e-02  -2.867 0.004760 **
c_s_relative_change_L1_r         -9.807e-03  4.418e-02  -0.222 0.824647
c_s_relative_change_L2_r          6.929e-03  4.646e-02   0.149 0.881650
c_s_relative_change_L3_r         -1.049e-01  3.743e-02  -2.802 0.005760 **
cs_cn_L1_r                       -5.585e+00  3.275e+00  -1.705 0.090214 .
cs_cn_L2_r                        2.185e+00  3.107e+00   0.703 0.483030
cs_cn_L3_r                        2.710e-01  6.790e-01   0.399 0.690404
a_line_L1_r                      -4.324e+00  1.808e+00  -2.392 0.018035 *
a_line_L2_r                       7.873e+00  3.149e+00   2.501 0.013497 *
a_line_L3_r                      -3.494e+00  1.797e+00  -1.944 0.053772 .
a_line_an_L1_r                    3.006e+00  1.105e+00   2.721 0.007294 **
a_line_an_L2_r                   -2.376e+00  1.039e+00  -2.287 0.023625 *
a_line_an_L3_r                    3.691e-01  1.941e-01   1.902 0.059151 .
US_CANExchangeRate_L1_r           3.740e-03  2.269e-03   1.648 0.101427
US_CANExchangeRate_L2_r           4.527e-04  3.012e-03   0.150 0.880730
US_CANExchangeRate_L3_r          -2.020e-03  2.225e-03  -0.908 0.365411
ycfivetothirty_L1_r               5.646e-03  7.772e-03   0.726 0.468748
ycfivetothirty_L2_r              -5.892e-03  8.444e-03  -0.698 0.486430
ycfivetothirty_L3_r              -7.420e-03  7.573e-03  -0.980 0.328811
yconetofour_L1_r                 -1.994e-04  1.718e-03  -0.116 0.907743
yconetofour_L2_r                  4.664e-03  1.841e-03   2.534 0.012337 *
yconetofour_L3_r                 -1.729e-03  1.551e-03  -1.115 0.266595
yconetofour_ns_L1_r              -1.342e-05  2.339e-05  -0.574 0.566976
yconetofour_ns_L2_r              -3.366e-05  2.744e-05  -1.227 0.221914
yconetofour_ns_L3_r               1.285e-05  1.995e-05   0.644 0.520532
ycfivetothirty_ns_L1_r           -1.042e-04  9.888e-05  -1.054 0.293819
ycfivetothirty_ns_L2_r            3.633e-05  1.224e-04   0.297 0.767049
ycfivetothirty_ns_L3_r            1.801e-04  1.058e-04   1.702 0.090840 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0008797 on 147 degrees of freedom
Multiple R-squared:  0.6104,    Adjusted R-squared:  0.4673
F-statistic: 4.266 on 54 and 147 DF,  p-value: 1.663e-12
```

v Regression Output

```
Call:
lm(formula = a_line_an_r ~ time_r + time_binary_2008_Q4 + time_binary_2020_Q1 +
    c_k_change_175 + c_k_change_186 + c_k_change_200 + Inflation_L1_r +
    Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r + NetSaving_L2_r +
    NetSaving_L3_r + MoneySupply_relative_change_L1_r + MoneySupply_relative_change_L2_$
    MoneySupply_relative_change_L3_r + InterestPaymentSales_Ratio_L1_r +
    InterestPaymentSales_Ratio_L2_r + InterestPaymentSales_Ratio_L3_r +
    NetSavingIncome_Ratio_L1_r + NetSavingIncome_Ratio_L2_r +
    NetSavingIncome_Ratio_L3_r + tau_c_L1_r + tau_c_L2_r + tau_c_L3_r +
    k_relative_change_L1_r + k_relative_change_L2_r + k_relative_change_L3_r +
    c_s_relative_change_L1_r + c_s_relative_change_L2_r + c_s_relative_change_L3_r +
    cs_cn_L1_r + cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r +
    a_line_L3_r + a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r +
    US_CANExchangeRate_L1_r + US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r +
    ycfivetothirty_L1_r + ycfivetothirty_L2_r + ycfivetothirty_L3_r +
    yconetofour_L1_r + yconetofour_L2_r + yconetofour_L3_r +
    yconetofour_ns_L1_r + yconetofour_ns_L2_r + yconetofour_ns_L3_r +
    ycfivetothirty_ns_L1_r + ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r,
    data = subset(R_US_df2_r, time_r > 39 & time_r < 242))

Residuals:
       Min         1Q     Median         3Q        Max
-2.615e-03 -5.850e-04 -2.777e-05  6.249e-04  2.251e-03

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                      5.166e-03  3.430e-03   1.506  0.13411
time_r                           4.209e-05  1.761e-05   2.390  0.01810 *
time_binary_2008_Q4             -9.996e-03  1.236e-03  -8.089 2.07e-13 ***
time_binary_2020_Q1             -7.265e-03  1.211e-03  -5.997 1.49e-08 ***
c_k_change_175                   7.035e-04  7.547e-04   0.932  0.35282
c_k_change_186                   1.081e-03  7.479e-04   1.446  0.15036
c_k_change_200                   1.859e-03  1.001e-03   1.856  0.06540 .
Inflation_L1_r                  -5.704e-05  2.160e-04  -0.264  0.79213
Inflation_L2_r                  -5.951e-06  1.927e-04  -0.031  0.97540
Inflation_L3_r                   1.712e-05  1.961e-04   0.087  0.93054
NetSaving_L1_r                  -1.300e-06  1.443e-05  -0.090  0.92834
NetSaving_L2_r                  -1.776e-06  2.119e-05  -0.084  0.93333
NetSaving_L3_r                  -1.091e-05  1.517e-05  -0.720  0.47294
MoneySupply_relative_change_L1_r 3.916e-03  1.593e-02   0.246  0.80618
MoneySupply_relative_change_L2_r 3.406e-02  1.589e-02   2.143  0.03376 *
MoneySupply_relative_change_L3_r -2.103e-02 1.337e-02  -1.572  0.11801
InterestPaymentSales_Ratio_L1_r -4.313e-04  8.321e-04  -0.518  0.60500
InterestPaymentSales_Ratio_L2_r -9.374e-05  1.170e-03  -0.080  0.93623
InterestPaymentSales_Ratio_L3_r  4.189e-04  8.242e-04   0.508  0.61207
NetSavingIncome_Ratio_L1_r       4.472e-02  3.132e-02   1.428  0.15540
NetSavingIncome_Ratio_L2_r      -3.691e-02  4.522e-02  -0.816  0.41565
NetSavingIncome_Ratio_L3_r       1.302e-02  3.172e-02   0.410  0.68213
tau_c_L1_r                       5.874e+00  3.148e+00   1.866  0.06405 .
tau_c_L2_r                      -8.705e+00  4.882e+00  -1.783  0.07661 .
tau_c_L3_r                       3.011e+00  2.964e+00   1.016  0.31143
k_relative_change_L1_r          -3.585e-02  4.470e-02  -0.802  0.42382
k_relative_change_L2_r           6.636e-02  5.141e-02   1.291  0.19885
k_relative_change_L3_r          -3.436e-02  4.008e-02  -0.857  0.39274
c_s_relative_change_L1_r        -8.378e-02  5.107e-02  -1.640  0.10307
c_s_relative_change_L2_r         1.099e-01  5.371e-02   2.046  0.04251 *
c_s_relative_change_L3_r        -1.724e-02  4.327e-02  -0.399  0.69082
cs_cn_L1_r                      -5.925e+00  3.786e+00  -1.565  0.11970
cs_cn_L2_r                       2.306e+00  3.591e+00   0.642  0.52185
cs_cn_L3_r                      -5.081e-01  7.849e-01  -0.647  0.51847
a_line_L1_r                     -5.407e+00  2.090e+00  -2.587  0.01065 *
a_line_L2_r                      8.177e+00  3.640e+00   2.247  0.02615 *
a_line_L3_r                     -2.934e+00  2.077e+00  -1.412  0.15999
a_line_an_L1_r                   4.136e+00  1.277e+00   3.238  0.00149 **
a_line_an_L2_r                  -2.330e+00  1.201e+00  -1.940  0.05432 .
a_line_an_L3_r                   4.255e-01  2.243e-01   1.897  0.05982 .
US_CANExchangeRate_L1_r         -4.771e-03  2.623e-03  -1.819  0.07099 .
US_CANExchangeRate_L2_r          9.495e-04  3.482e-03   0.273  0.78545
US_CANExchangeRate_L3_r          1.409e-03  2.572e-03   0.548  0.58469
ycfivetothirty_L1_r              2.956e-03  8.985e-03   0.329  0.74260
ycfivetothirty_L2_r              1.593e-03  9.761e-03   0.163  0.87056
ycfivetothirty_L3_r              2.842e-03  8.754e-03   0.325  0.74595
yconetofour_L1_r                 3.787e-03  1.986e-03   1.907  0.05849 .
yconetofour_L2_r                -1.775e-03  2.128e-03  -0.834  0.40560
yconetofour_L3_r                -2.155e-03  1.793e-03  -1.202  0.23118
yconetofour_ns_L1_r             -3.889e-05  2.704e-05  -1.438  0.15245
yconetofour_ns_L2_r              2.614e-05  3.173e-05   0.824  0.41132
yconetofour_ns_L3_r              1.796e-05  2.306e-05   0.779  0.43730
ycfivetothirty_ns_L1_r           1.244e-06  1.143e-04   0.011  0.99133
ycfivetothirty_ns_L2_r          -7.104e-05  1.415e-04  -0.502  0.61638
ycfivetothirty_ns_L3_r           8.747e-05  1.223e-04   0.715  0.47553
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.001017 on 147 degrees of freedom
Multiple R-squared:  0.7423,    Adjusted R-squared:  0.6477
F-statistic: 7.842 on 54 and 147 DF,  p-value: < 2.2e-16
```

Exports Regression Output

```
Call:
lm(formula = Exports_current_change_r ~ time_r + time_binary_2008_Q4 +
    time_binary_2020_Q1 + c_k_change_175 + c_k_change_186 + c_k_change_200 +
    Inflation_L1_r + Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r +
    NetSaving_L2_r + NetSaving_L3_r + MoneySupply_relative_change_L1_r +
    MoneySupply_relative_change_L2_r + MoneySupply_relative_change_L3_r +
    InterestPaymentSales_Ratio_L1_r + InterestPaymentSales_Ratio_L2_r +
    InterestPaymentSales_Ratio_L3_r + NetSavingIncome_Ratio_L1_r +
    NetSavingIncome_Ratio_L2_r + NetSavingIncome_Ratio_L3_r +
    tau_c_L1_r + tau_c_L2_r + tau_c_L3_r + k_relative_change_L1_r +
    k_relative_change_L2_r + k_relative_change_L3_r + c_s_relative_change_L1_r +
    c_s_relative_change_L2_r + c_s_relative_change_L3_r + cs_cn_L1_r +
    cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r + a_line_L3_r +
    a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r + US_CANExchangeRate_L1_r +
    US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r + ycfivetothirty_L1_r +
    ycfivetothirty_L2_r + ycfivetothirty_L3_r + yconetofour_L1_r +
    yconetofour_L2_r + yconetofour_L3_r + yconetofour_ns_L1_r +
    yconetofour_ns_L2_r + yconetofour_ns_L3_r + ycfivetothirty_ns_L1_r +
    ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r, data = subset(R_US_df2_r,
    time_r > 39 & time_r < 242))

Residuals:
     Min       1Q   Median       3Q      Max
-17.8130  -2.6281  -0.1783   2.0206  17.2483

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                      4.255e+01  1.741e+01   2.444 0.015715 *
time_r                           1.577e-01  8.939e-02   1.764 0.079777 .
time_binary_2008_Q4             -5.446e+01  6.274e+00  -8.680 6.88e-15 ***
time_binary_2020_Q1             -2.442e+01  6.150e+00  -3.970 0.000112 ***
c_k_change_175                   2.622e+00  3.832e+00   0.684 0.494836
c_k_change_186                   9.831e+00  3.797e+00   2.589 0.010589 *
c_k_change_200                   5.008e+00  5.083e+00   0.985 0.326142
Inflation_L1_r                  -2.474e-01  1.097e+00  -0.226 0.821827
Inflation_L2_r                  -2.536e+00  9.782e-01  -2.593 0.010479 *
Inflation_L2_r                  -2.536e+00  9.782e-01  -2.593 0.010479 *
Inflation_L3_r                   3.731e-01  9.956e-01   0.375 0.708361
NetSaving_L1_r                   7.028e-02  7.324e-02   0.960 0.338802
NetSaving_L2_r                  -7.239e-02  1.076e-01  -0.673 0.502170
NetSaving_L3_r                  -3.091e-02  7.700e-02  -0.401 0.688707
MoneySupply_relative_change_L1_r -3.851e+01  8.088e+01  -0.476 0.634664
MoneySupply_relative_change_L2_r  6.940e+01  8.069e+01   0.860 0.391151
MoneySupply_relative_change_L3_r -1.330e+02  6.790e+01  -1.959 0.051951 .
InterestPaymentSales_Ratio_L1_r  -1.936e+00  4.224e+00  -0.458 0.647416
InterestPaymentSales_Ratio_L2_r   7.384e+00  5.937e+00   1.244 0.215620
InterestPaymentSales_Ratio_L3_r  -5.486e+00  4.184e+00  -1.311 0.191895
NetSavingIncome_Ratio_L1_r        4.274e+01  1.590e+02   0.269 0.788450
NetSavingIncome_Ratio_L2_r       -1.648e+02  2.296e+02  -0.718 0.473889
NetSavingIncome_Ratio_L3_r        6.294e+01  1.610e+02   0.391 0.696478
tau_c_L1_r                        8.764e+03  1.598e+04   0.548 0.584231
tau_c_L2_r                       -2.367e+04  2.478e+04  -0.955 0.341091
tau_c_L3_r                        1.531e+04  1.505e+04   1.017 0.310607
k_relative_change_L1_r           -1.217e+02  2.269e+02  -0.536 0.592680
k_relative_change_L2_r            1.121e+02  2.610e+02   0.430 0.668124
k_relative_change_L3_r           -6.105e+01  2.035e+02  -0.300 0.764555
c_s_relative_change_L1_r         -4.781e+02  2.593e+02  -1.844 0.067209 .
c_s_relative_change_L2_r          4.264e+02  2.727e+02   1.564 0.119980
c_s_relative_change_L3_r          1.658e+02  2.197e+02   0.755 0.451525
cs_cn_L1_r                       -1.738e+03  1.922e+04  -0.090 0.928071
cs_cn_L2_r                        9.829e+03  1.823e+04   0.539 0.590611
cs_cn_L3_r                       -5.807e+03  3.985e+03  -1.457 0.147215
a_line_L1_r                      -9.400e+03  1.061e+04  -0.886 0.377119
a_line_L2_r                       1.890e+04  1.848e+04   1.023 0.307974
a_line_L3_r                      -1.005e+04  1.055e+04  -0.953 0.342343
a_line_an_L1_r                    9.033e+03  6.484e+03   1.393 0.165669
a_line_an_L2_r                   -5.272e+03  6.097e+03  -0.865 0.388595
a_line_an_L3_r                    5.604e+02  1.139e+03   0.492 0.623384
US_CANExchangeRate_L1_r          -4.609e+01  1.332e+01  -3.461 0.000705 ***
US_CANExchangeRate_L2_r           2.263e+00  1.768e+01   0.128 0.898299
US_CANExchangeRate_L3_r           3.018e+01  1.306e+01   2.311 0.022213 *
ycfivetothirty_L1_r               3.694e+01  4.561e+01   0.810 0.419361
ycfivetothirty_L2_r              -2.360e+01  4.955e+01  -0.476 0.634562
ycfivetothirty_L3_r               3.321e+01  4.444e+01   0.747 0.456165
yconetofour_L1_r                  2.598e+00  1.008e+01   0.258 0.797013
yconetofour_L2_r                 -5.711e+00  1.080e+01  -0.529 0.597848
yconetofour_L3_r                 -1.064e+01  9.101e+00  -1.169 0.244391
yconetofour_ns_L1_r              -7.332e-02  1.373e-01  -0.534 0.594078
yconetofour_ns_L2_r               6.319e-02  1.611e-01   0.392 0.695393
yconetofour_ns_L3_r               5.932e-02  1.171e-01   0.507 0.613045
ycfivetothirty_ns_L1_r           -2.139e-01  5.803e-01  -0.369 0.712982
ycfivetothirty_ns_L2_r            4.545e-01  7.183e-01   0.633 0.527927
ycfivetothirty_ns_L3_r           -1.642e-01  6.208e-01  -0.265 0.791721
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.163 on 147 degrees of freedom
Multiple R-squared:  0.7333,    Adjusted R-squared:  0.6354
F-statistic: 7.487 on 54 and 147 DF,  p-value: < 2.2e-16
```

Imports Regression Output

```
Call:
lm(formula = Imports_current_change_r ~ time_r + time_binary_2008_Q4 +
    time_binary_2020_Q1 + c_k_change_175 + c_k_change_186 + c_k_change_200 +
    Inflation_L1_r + Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r +
    NetSaving_L2_r + NetSaving_L3_r + MoneySupply_relative_change_L1_r +
    MoneySupply_relative_change_L2_r + MoneySupply_relative_change_L3_r +
    InterestPaymentSales_Ratio_L1_r + InterestPaymentSales_Ratio_L2_r +
    InterestPaymentSales_Ratio_L3_r + NetSavingIncome_Ratio_L1_r +
    NetSavingIncome_Ratio_L2_r + NetSavingIncome_Ratio_L3_r +
    tau_c_L1_r + tau_c_L2_r + tau_c_L3_r + k_relative_change_L1_r +
    k_relative_change_L2_r + k_relative_change_L3_r + c_s_relative_change_L1_r +
    c_s_relative_change_L2_r + c_s_relative_change_L3_r + cs_cn_L1_r +
    cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r + a_line_L3_r +
    a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r + US_CANExchangeRate_L1_r +
    US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r + ycfivetothirty_L1_r +
    ycfivetothirty_L2_r + ycfivetothirty_L3_r + yconetofour_L1_r +
    yconetofour_L2_r + yconetofour_L3_r + yconetofour_ns_L1_r +
    yconetofour_ns_L2_r + yconetofour_ns_L3_r + ycfivetothirty_ns_L1_r +
    ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r, data = subset(R_US_df2_r,
    time_r > 39 & time_r < 242))

Residuals:
    Min      1Q  Median      3Q     Max
-32.329  -3.520   0.007   3.380  22.452

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                       4.922e+01  2.588e+01   1.902  0.05914 .
time_r                            2.430e-01  1.329e-01   1.828  0.06951 .
time_binary_2008_Q4              -8.876e+01  9.326e+00  -9.517  < 2e-16 ***
time_binary_2020_Q1              -3.432e+01  9.142e+00  -3.754  0.00025 ***
c_k_change_175                    1.081e+01  5.695e+00   1.899  0.05954 .
c_k_change_186                    8.556e+00  5.644e+00   1.516  0.13167
c_k_change_200                    8.177e+00  7.555e+00   1.082  0.28087
Inflation_L1_r                   -7.157e-01  1.630e+00  -0.439  0.66127
Inflation_L2_r                   -2.328e+00  1.454e+00  -1.601  0.11150
Inflation_L3_r                    3.158e-02  1.480e+00   0.021  0.98300
NetSaving_L1_r                    6.214e-02  1.089e-01   0.571  0.56898
NetSaving_L2_r                   -6.046e-03  1.599e-01  -0.038  0.96990
NetSaving_L3_r                   -6.062e-02  1.145e-01  -0.530  0.59718
MoneySupply_relative_change_L1_r -1.312e+02  1.202e+02  -1.092  0.27674
MoneySupply_relative_change_L2_r  2.109e+02  1.199e+02   1.758  0.08081 .
MoneySupply_relative_change_L3_r -1.535e+02  1.009e+02  -1.520  0.13054
InterestPaymentSales_Ratio_L1_r  -4.080e+00  6.279e+00  -0.650  0.51688
InterestPaymentSales_Ratio_L2_r   7.933e+00  8.825e+00   0.899  0.37021
InterestPaymentSales_Ratio_L3_r  -3.790e+00  6.220e+00  -0.609  0.54325
NetSavingIncome_Ratio_L1_r        1.153e+02  2.363e+02   0.488  0.62646
NetSavingIncome_Ratio_L2_r       -3.824e+02  3.412e+02  -1.121  0.26430
NetSavingIncome_Ratio_L3_r        2.104e+02  2.394e+02   0.879  0.38082
tau_c_L1_r                        6.167e+03  2.375e+04   0.260  0.79552
tau_c_L2_r                       -4.740e+04  3.684e+04  -1.287  0.20021
tau_c_L3_r                        4.080e+04  2.237e+04   1.824  0.07017 .
k_relative_change_L1_r           -7.947e+02  3.373e+02  -2.356  0.01979 *
k_relative_change_L2_r            7.140e+02  3.880e+02   1.840  0.06773 .
k_relative_change_L3_r           -2.854e+02  3.024e+02  -0.943  0.34699
c_s_relative_change_L1_r         -1.049e+03  3.854e+02  -2.721  0.00729 **
c_s_relative_change_L2_r          9.087e+02  4.053e+02   2.242  0.02646 *
c_s_relative_change_L3_r          2.968e+01  3.265e+02   0.091  0.92770
cs_cn_L1_r                       -1.087e+03  2.857e+04  -0.038  0.96970
cs_cn_L2_r                        4.478e+04  2.710e+04   1.652  0.10062
cs_cn_L3_r                       -8.160e+03  5.923e+03  -1.378  0.17040
a_line_L1_r                      -1.030e+04  1.577e+04  -0.653  0.51471
a_line_L2_r                       3.649e+04  2.747e+04   1.329  0.18607
a_line_L3_r                      -2.647e+04  1.568e+04  -1.689  0.09341 .
a_line_an_L1_r                    1.692e+04  9.638e+03   1.755  0.08129 .
a_line_an_L2_r                   -1.758e+04  9.063e+03  -1.940  0.05430 .
a_line_an_L3_r                    2.649e+03  1.693e+03   1.565  0.11981
US_CANExchangeRate_L1_r          -4.356e+01  1.980e+01  -2.201  0.02933 *
US_CANExchangeRate_L2_r           2.210e+01  2.627e+01   0.841  0.40167
US_CANExchangeRate_L3_r           1.184e+01  1.941e+01   0.610  0.54287
ycfivetothirty_L1_r               5.439e+01  6.780e+01   0.802  0.42369
ycfivetothirty_L2_r              -4.322e+00  7.366e+01  -0.059  0.95329
ycfivetothirty_L3_r               2.362e+01  6.606e+01   0.357  0.72124
yconetofour_L1_r                  1.187e+01  1.499e+01   0.792  0.42966
yconetofour_L2_r                 -1.302e+01  1.606e+01  -0.811  0.41874
yconetofour_L3_r                 -1.490e+01  1.353e+01  -1.101  0.27254
yconetofour_ns_L1_r              -8.019e-02  2.041e-01  -0.393  0.69492
yconetofour_ns_L2_r               1.324e-01  2.394e-01   0.553  0.58108
yconetofour_ns_L3_r               1.200e-01  1.740e-01   0.689  0.49163
ycfivetothirty_ns_L1_r           -9.358e-01  8.626e-01  -1.085  0.27973
ycfivetothirty_ns_L2_r            7.525e-01  1.068e+00   0.705  0.48210
ycfivetothirty_ns_L3_r           -3.698e-01  9.228e-01  -0.401  0.68922
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.674 on 147 degrees of freedom
Multiple R-squared:  0.7695,    Adjusted R-squared:  0.6849
F-statistic:  9.09 on 54 and 147 DF,  p-value: < 2.2e-16
```

Government Expenditure Regression Output

```
Call:
lm(formula = GovernmentExpenditure_current_change_r ~ time_r +
    time_binary_2008_Q4 + time_binary_2020_Q1 + c_k_change_175 +
    c_k_change_186 + c_k_change_200 + Inflation_L1_r + Inflation_L2_r +
    Inflation_L3_r + NetSaving_L1_r + NetSaving_L2_r + NetSaving_L3_r +
    MoneySupply_relative_change_L1_r + MoneySupply_relative_change_L2_r +
    MoneySupply_relative_change_L3_r + InterestPaymentSales_Ratio_L1_r +
    InterestPaymentSales_Ratio_L2_r + InterestPaymentSales_Ratio_L3_r +
    NetSavingIncome_Ratio_L1_r + NetSavingIncome_Ratio_L2_r +
    NetSavingIncome_Ratio_L3_r + tau_c_L1_r + tau_c_L2_r + tau_c_L3_r +
    k_relative_change_L1_r + k_relative_change_L2_r + k_relative_change_L3_r +
    c_s_relative_change_L1_r + c_s_relative_change_L2_r + c_s_relative_change_L3_r +
    cs_cn_L1_r + cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r +
    a_line_L3_r + a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r +
    US_CANExchangeRate_L1_r + US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r +
    ycfivetothirty_L1_r + ycfivetothirty_L2_r + ycfivetothirty_L3_r +
    yconetofour_L1_r + yconetofour_L2_r + yconetofour_L3_r +
    yconetofour_ns_L1_r + yconetofour_ns_L2_r + yconetofour_ns_L3_r +
    ycfivetothirty_ns_L1_r + ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r,
    data = subset(R_US_df2_r, time_r > 39 & time_r < 242))

Residuals:
    Min      1Q  Median      3Q     Max
-7.9787 -1.0767 -0.0616  1.0802  5.4939

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)
(Intercept)                      -2.245e+01  7.635e+00  -2.941 0.003804 **
time_r                            1.385e-01  3.920e-02   3.533 0.000549 ***
time_binary_2008_Q4              -1.086e+01  2.751e+00  -3.947 0.000122 ***
time_binary_2020_Q1              -3.766e+00  2.697e+00  -1.396 0.164791
c_k_change_175                   -6.039e-01  1.680e+00  -0.359 0.719826
c_k_change_186                   -5.584e-01  1.665e+00  -0.335 0.737834
c_k_change_200                   -6.172e+00  2.229e+00  -2.769 0.006347 **
Inflation_L1_r                    1.923e-02  4.810e-01   0.040 0.968163
Inflation_L2_r                   -8.062e-01  4.290e-01  -1.879 0.062186 .
Inflation_L3_r                    4.028e-01  4.366e-01   0.922 0.357799
NetSaving_L1_r                   -6.155e-02  3.212e-02  -1.916 0.057248 .
NetSaving_L2_r                    7.738e-03  4.719e-02   0.164 0.869969
NetSaving_L3_r                    6.407e-03  3.377e-02   0.190 0.849776
MoneySupply_relative_change_L1_r  7.123e+00  3.547e+01   0.201 0.841114
MoneySupply_relative_change_L2_r  4.677e+01  3.539e+01   1.322 0.188318
MoneySupply_relative_change_L3_r -9.695e+00  2.978e+01  -0.326 0.745196
InterestPaymentSales_Ratio_L1_r  -2.295e-01  1.853e+00  -0.124 0.901599
InterestPaymentSales_Ratio_L2_r   2.966e+00  2.604e+00   1.139 0.256531
InterestPaymentSales_Ratio_L3_r  -2.544e+00  1.835e+00  -1.386 0.167709
NetSavingIncome_Ratio_L1_r        1.108e+02  6.972e+01   1.589 0.114157
NetSavingIncome_Ratio_L2_r       -3.843e+01  1.007e+02  -0.382 0.703187
NetSavingIncome_Ratio_L3_r        9.301e+00  7.062e+01   0.132 0.895404
tau_c_L1_r                        1.573e+04  7.009e+03   2.244 0.026313 *
tau_c_L2_r                       -1.251e+04  1.087e+04  -1.151 0.251756
tau_c_L3_r                       -2.875e+03  6.599e+03  -0.436 0.663733
k_relative_change_L1_r            4.037e+01  9.952e+01   0.406 0.685598
k_relative_change_L2_r            8.147e-01  1.145e+02   0.007 0.994331
k_relative_change_L3_r            6.756e+01  8.923e+01   0.757 0.450158
c_s_relative_change_L1_r         -2.176e+02  1.137e+02  -1.914 0.057625 .
c_s_relative_change_L2_r         -3.056e+01  1.196e+02  -0.256 0.798643
c_s_relative_change_L3_r          8.377e+01  9.633e+01   0.870 0.385944
cs_cn_L1_r                       -1.300e+04  8.429e+03  -1.542 0.125278
cs_cn_L2_r                       -3.454e+03  7.996e+03  -0.432 0.666374
cs_cn_L3_r                       -6.938e+02  1.748e+03  -0.397 0.691925
a_line_L1_r                      -9.542e+03  4.653e+03  -2.051 0.042066 *
a_line_L2_r                       7.661e+03  8.103e+03   0.945 0.346020
a_line_L3_r                       1.707e+03  4.625e+03   0.369 0.712579
a_line_an_L1_r                    4.756e+03  2.843e+03   1.673 0.096511 .
a_line_an_L2_r                    9.958e+02  2.674e+03   0.372 0.710110
a_line_an_L3_r                   -5.670e+01  4.994e+02  -0.114 0.909774
US_CANExchangeRate_L1_r          -5.757e+00  5.840e+00  -0.986 0.325888
US_CANExchangeRate_L2_r           4.599e+00  7.752e+00   0.593 0.553879
US_CANExchangeRate_L3_r          -7.122e-01  5.726e+00  -0.124 0.901182
ycfivetothirty_L1_r               3.534e-01  2.000e+01   0.018 0.985928
ycfivetothirty_L2_r              -2.534e+01  2.173e+01  -1.166 0.245477
ycfivetothirty_L3_r               2.196e+01  1.949e+01   1.127 0.261728
yconetofour_L1_r                  4.489e+00  4.422e+00   1.015 0.311715
yconetofour_L2_r                  1.335e+00  4.738e+00   0.282 0.778536
yconetofour_L3_r                 -3.552e+00  3.991e+00  -0.890 0.374962
yconetofour_ns_L1_r              -5.928e-02  6.020e-02  -0.985 0.326385
yconetofour_ns_L2_r               5.356e-03  7.064e-02   0.076 0.939662
yconetofour_ns_L3_r               2.330e-02  5.133e-02   0.454 0.650625
ycfivetothirty_ns_L1_r           -3.844e-03  2.545e-01  -0.015 0.987969
ycfivetothirty_ns_L2_r            4.636e-01  3.150e-01   1.472 0.143231
ycfivetothirty_ns_L3_r           -3.923e-01  2.723e-01  -1.441 0.151751
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.264 on 147 degrees of freedom
Multiple R-squared:  0.6155,    Adjusted R-squared:  0.4743
F-statistic: 4.358 on 54 and 147 DF,  p-value: 7.846e-13
```