Estimating Nominal GDP using the Exponential Decay Function

Written by: Andreas Bruegger
a.s.bruegger@gmail.com

The following method estimates nominal gross domestic product using the exponential decay function. The economy is characterized using two parameters which are estimated using variables lagged by three periods.

Consider $100 being deposited into a bank account where the cash in the account is used to pay for goods or services at a constant percentage rate over time. Over a long enough time period the cash will result in $100 of nominal production. During the lifespan of the account the balance in the account will be loaned out by the bank for investment in the economy.

Consider a fixed period of time (one month) where funds are spent at a constant rate and where simultaneously the bank is making loans. The rate of decay function is $x(t)$, where x is the value at time t, k is the decay parameter, and x_0 is the value of x at time zero,

$$x(t) = x_0 \cdot e^{-k \cdot t}$$

This is a function of time with one parameter k. If instead time is replaced with the parameter c the decay function representing the balance in the account after one month is,

$$x(\text{after one month}) = x_0 \cdot e^{-k \cdot c}$$

After one month the amount spent and loaned from the account could be determined by solving the following two equations for c and k,

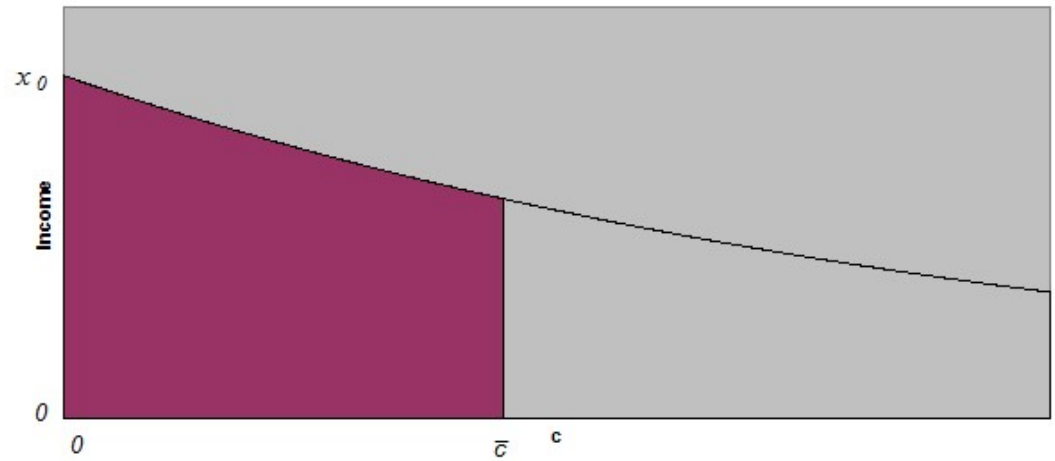
$$\begin{aligned} \text{Amount spent after one month} &= x_0 - x(\text{after one month}) \\ &= x_0 - x_0 \cdot e^{-k \cdot c} \\ &= x_0 \cdot (1 - e^{-k \cdot c}) \end{aligned}$$

$$\begin{aligned} \text{Amount loaned after one month} &= \int_0^c x_0 \cdot e^{-k \cdot c}\, dc \\ &= (x_0 / k) \cdot (1 - e^{-k \cdot c}) \\ &= \text{Amount spent after one month} / k \end{aligned}$$

In this case since funds are simultaneously being spent and loaned, k represents the ratio of funds spent to amount loaned. As these equations describe the use of funds for production and investment they can be used to model the economy.

Note that at each moment in time there is income earned or generated by the members of the economy. The activity of the economic system will be determined by the

parameters c and k and by the amount of existing funds available for consumption and investment. Consider the following graph which represents the income x_0 and its value after it decays into consumption and investment after $\bar{c}$ units,



For income x_0 generated at the first moment of the period, the investment it generates over the period is the area under the curve from 0 to $\bar{c}$. Therefore, if x_0 is generated at each moment of the period the total investment generated for the period is found by summing each integral from 0 to c for each value of c from $\bar{c}$ to 0.

Spending generated over $\bar{c}$ units of the period from the income earned at the first moment of the period

$$= k \cdot (\text{total income for period} / \bar{c}) \cdot \int_0^{\bar{c}} e^{-k \cdot c}\, dc$$
$$= (\text{total income} / \bar{c}) \cdot (1 - e^{-k \cdot \bar{c}})$$

Because this is an exponential integral which is not defined for $\bar{c} = 0$, it is necessary to assume the income generated at every moment in the period is the same (this assumption also means income (and therefore consumption and investment) is jumping between periods). In which case the first $\bar{c}$ in the above equation is constant and integration will give the expression.

Spending generated from all income earned over the period

$$= (\text{total income} / \bar{c}) \cdot \int_0^{\bar{c}} (1 - e^{-k \cdot c})\, dc$$
$$= (\text{total income} / \bar{c}) \cdot (\bar{c} + (1 / k) \cdot (e^{-k \cdot \bar{c}} - 1))$$

And investment is,

Investment generated from all income earned over the period

$$= \text{Spending generated from all income earned over the period} / k$$
$$= (\text{total income} / (\bar{c} \cdot k)) \cdot (\bar{c} + (1 / k) \cdot (e^{-k \cdot \bar{c}} - 1))$$

Therefore k and $\bar{c}$ for each period can be determined by solving the spending and investment equations above where each period,

Total Income = Consumption + Government Expenditure + Investment + Exports - Imports

Spending = Consumption + Government Expenditure + Imports

Investment = Investment

Alternatively, since k is the ratio of spending to investment, k can be directly inserted into the spending equation above and $\bar{c}$ can be calculated.

Once the unspent income in a period is determined the amount which remains after subsequent periods of spending can be calculated by multiplying by the decay function,

Income from period j available to be used for spending in period i
$= Unspent\ income\ from\ period\ j \cdot \prod e^{-k_m \cdot \bar{c}_m}$
$= (GDP_j / (k_j \cdot \bar{c}_j)) \cdot (1 - e^{-k_j \cdot \bar{c}_j}) \cdot \prod e^{-k_m \cdot \bar{c}_m}$

For every period m between $j+1$ and i.

Unspent income along with the income generated at each moment of the current period form the funds available to be used for production or investment.

To summarize, values for k for each period can be calculated by taking the ratio of spending to investment. Funds available for spending and investment for a given period can be determined by taking all unspent income from previous periods and adding them to income for the current period. The spending and investment equations for the current period can then be applied and a value for $\bar{c}$ for the current period can be backed out to match the current period's spending and investment figures.

To produce a forecast for next periods economic activity note that the amount of funds spent next period is given by the equation,

$Consumption_{t+1} + Government\ Expenditure_{t+1} + Imports_{t+1} =$
$Spending\ from\ income\ generated\ in\ period_{t+1} + \sum_{i=j}^{t} (Spending\ from\ income\ generated\ in\ period\ i)$

Where j is the first period of the sample. The second term on the right hand side will be a function of figures already known except for $\bar{c}_{t+1}$ and k_{t+1}. The first term on the right hand side will be a function of $\bar{c}_{t+1}$ and k_{t+1} and also of $Consumption_{t+1}$, $Investment_{t+1}$, $Government\ Expenditure_{t+1}$, $Imports_{t+1}$ and $Exports_{t+1}$. Therefore estimates for $Consumption_{t+1}$and $Investment_{t+1}$ could be formed using estimates for k_{t+1} and $\bar{c}_{t+1}$ if we first form estimates for $Government\ Expenditure_{t+1}$, $Imports_{t+1}$ and $Exports_{t+1}$. In the appendix the code and figures provided have no special treatment for estimating *Government Expenditure*, *Imports* or *Exports*; the period change is simply

estimated using a regression and added to previous period's value to form an estimate for the current period.

Let *C, I, G, M* and *E* denote *Consumption, Investment, Government Expenditure, Imports* and *Exports*. The equation above can be written as,

$$C_{t+1} + G_{t+1} + M_{t+1} = (1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (C_{t+1} + G_{t+1} + I_{t+1} + E_{t+1} - M_{t+1}) + \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}})$$

$$C_{t+1} + G_{t+1} + M_{t+1} = (1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (C_{t+1} + G_{t+1} + ((C_{t+1} + M_{t+1} + G_{t+1})/k) + E_{t+1} - M_{t+1}) + \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}})$$

Where the expression $\prod e^{-k_m \cdot \bar{c}_m}$ is from $i+1$ to t and $\bar{c}$ and k are the parameters for period $t+1$.

Solving for $Consumption_{t+1}$ gives,

$$C_{t+1} = ((1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (E_{t+1} + M_{t+1} + G_{t+1} + ((M_{t+1} + G_{t+1}) / k)) - M_{t+1} - G_{t+1} + \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}})) / (1 - (1 + (e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (1 + (1 / k)))$$

It is not immediately clear *Consumption* is increasing in $\bar{c}$ due to the expression,

$$1 + ((e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))$$

However, this expression is increasing in $\bar{c}$ when,

$$\bar{c} \cdot k > ln\,(1 + \bar{c} \cdot k)$$

Which holds for the relevant range where $\bar{c} \cdot k$ *is* greater than zero.

An aspect of this model is that $\bar{c}$ does not have to increase for consecutive periods to exhibit increasing levels of total activity. This is because total funds available for production and investment increases each period by the addition of income which is not used for production. For example, assume the economy has no *Government Expenditure*, *Exports*, and *Imports* so that the equation above for *Consumption* becomes,

$$C_{t+1} = \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m}) \cdot (1 - e^{-k\cdot\bar{c}}) / (1 - (1 + (e^{-k\cdot\bar{c}} - 1) / (\bar{c} \cdot k))) \cdot (1 + (1 / k)))$$

For simplification let the expressions in C_{t+1} be defined in the following way, where the funds available at the beginning of period $t+1$ are,

$$A(\bar{c}_{t+1}) = \sum_{i=j}^{t} ((GDP_i / (k_i \cdot \bar{c}_i)) \cdot (1 - e^{-k_i \cdot \bar{c}_i}) \cdot \prod e^{-k_m \cdot \bar{c}_m})$$

For m from $i+1$ to t, and,

$$B(\bar{c}_{t+1}) = 1 - (1 + (e^{-k_{t+1} \cdot \bar{c}_{t+1}} - 1) / (\bar{c}_{t+1} \cdot k_{t+1})) \cdot (1 + (1 / k_{t+1}))$$

Then,

$$C_{t+1} = A(\bar{c}_{t+1}) \cdot (1 - e^{-k_{t+1} \cdot \bar{c}_{t+1}}) / B(\bar{c}_{t+1})$$

If C_{t+1} and C_t have the same value then dividing one by the other results in the expression,

$$A(\bar{c}_{t+1}) / A(\bar{c}_t) = (1 - e^{-k_t \cdot \bar{c}_t}) \cdot B(\bar{c}_{t+1}) / (1 - e^{-k_{t+1} \cdot \bar{c}_{t+1}}) \cdot B(\bar{c}_t)$$

If k is the same for periods t and $t+1$ and if more funds are available at the beginning of period $t+1$ than were available at the beginning of t, then since $(1 - e^{-k_{t+1} \cdot \bar{c}_{t+1}})$ is increasing in $\bar{c}$ and $B(\bar{c}_{t+1})$ is decreasing in $\bar{c}$, $\bar{c}_{t+1}$ would need to be less than $\bar{c}_t$ if *Consumption* was unchanged from period t to $t+1$.

A scatter plot of k and $\bar{c}$ are presented in the appendix[1]. Examining the series shows that k and $\bar{c}$ have trended in a zig-zag pattern from 1970 to the present. The changes in the trending of k and $\bar{c}$ occur in 2003, 2006, and 2008. I believe this pattern is due to the relative increase of *Imports* to *Exports* as during these periods *Imports* exceeded *Exports* by 4% to 5% relative to *GDP*. This trend between *Imports* and *Exports* would cause k to increase and would require $\bar{c}$ to increase as less funds would be available.

Generally k and $\bar{c}$ are inversely related and were modeled through the relationship,

$$k = v / c$$

The parameters v and c were estimated and used to form an estimate for k.

As mentioned above a decrease in $\bar{c}$ is not necessarily indicative of a decrease in total activity due to the amount of funds available increasing at the beginning of the period. However, the general movement of k and $\bar{c}$ can be interpreted in the following way. Moving horizontally in the k - $\bar{c}$ plane holds the economic activity of the system constant; the total of spending and investment will be unchanged, only the ratio of activity between the spending and investment will change. Moving vertically means the ratio between spending and investment is unchanged as the total activity of the system changes. Moving diagonally from the upper left to lower right corner results in spending being relatively unchanged; as k increases the ratio of spending to investment decreases and to hold consumption constant c decreases.

For each period c and k equate the expressions for spending and investment to their actual values and may change between periods for a number of reasons.

[1] Figure 14, page 11

If the savings rate decreases and the rate of loans continues to the match the rate of savings then the total amount of currency being circulated is unchanged and only *k* would increase.

If more money is printed and loaned investment would immediately increase while spending would be unchanged. This would cause the system to move diagonally to the upper left. If the savings rate is unchanged then eventually the ratio between spending and investment would return to its original value and the system would move right horizontally with *c* unchanged. Overall given enough time the only change to the system would be an increase in *c* if no other changes occurred.

If the rate that loans are issued decreases this would cause the amount of money in circulation to decrease. After the initial decrease to investment, if spending is reduced to preserve the ratio between spending and investment then *k* will be unchanged and the only effect will be that *c* has decreased. If spending is unchanged then the effective savings rate will have decreased and the system would move diagonally to the bottom right corner.

In the appendix are regression outputs for *c, v, Exports, Imports* and *Government Expenditure*[2]. The data range is from Q3 1969 to Q2 2020 and only forecasts for 1985 Q1 to Q2 2020 are generated. The parameters to be estimated are; *c*, *v*, and the change in *G*, *E* and *M*. In addition to the time variable, binary variables for Q1 2009 and Q1 2020, and three trend variables noted above, the other variables used for estimation (and their three period lagged values) were,

- Inflation
- US_CANExchangeRate
    - US/CAD exchange rate
- NetSaving
    - Net Saving
- a_line
    - Parameter *v*
- InterestPaymentSales_Ratio
    - Calculated as the product of investment and the interest rate divided by the sum of exports and consumption
- NetSavingIncome_Ratio
    - Calculated as Net Saving divided by GDP
- Tau_c
    - Calculated as *c* divided by the negative log of 1 divided by 1 plus *k* divided by *k*
- Cs_cn
    - The difference between *c* and the value of *c* that would hold consumption constant (it would be the previous periods value of *c* but in application it is backed out due to changes in exports, imports, and government expenditure and also because when calculating available funds unused income from 35 or more periods ago is discarded for computing purposes)

[2] The difference between *c* and *v* that would hold *Consumption* and *Investment* constant are the dependent variables estimated.

- a_line_an
    - The difference between *v* and the *v* required to hold consumption constant
- MoneySupply_relative_change
    - Percent change in money supply
- k_relative_change
    - Percent change in *k*
- c_s_relative_change
    - Percent change in *c*
- Yconetofour
    - The slope of the yield curve for t-bills from one to four years
- ycfivetothirty
    - The slope of the yield curve for t-bills from five to thirty years
    - The maximum length of t-bills changes over time; from 1961-1971 it is 7 years, from 1971 to 1981 it is 15 years, from 1981 to 1985 it is 20 years, and from 1981 to the present it is 30 years
- yconetofour_ns
    - The slope of the yield curve for t-bills from one to four years multiplied by NetSaving
- ycfivetothirty_ns
    - The slope of the yield curve for t-bills from five to thirty years multiplied by NetSaving

Please see the appendix for regression and forecast charts. There are a couple of things to note.

- When estimating the model the most important variables were Net Savings, Net Savings Income Ratio and the Yield Curve variables.

Overall this is a simple model for forecasting an economic system. Its strength is that it requires no equilibrium, optimization or simplifying assumptions.

Appendix

Figure 1

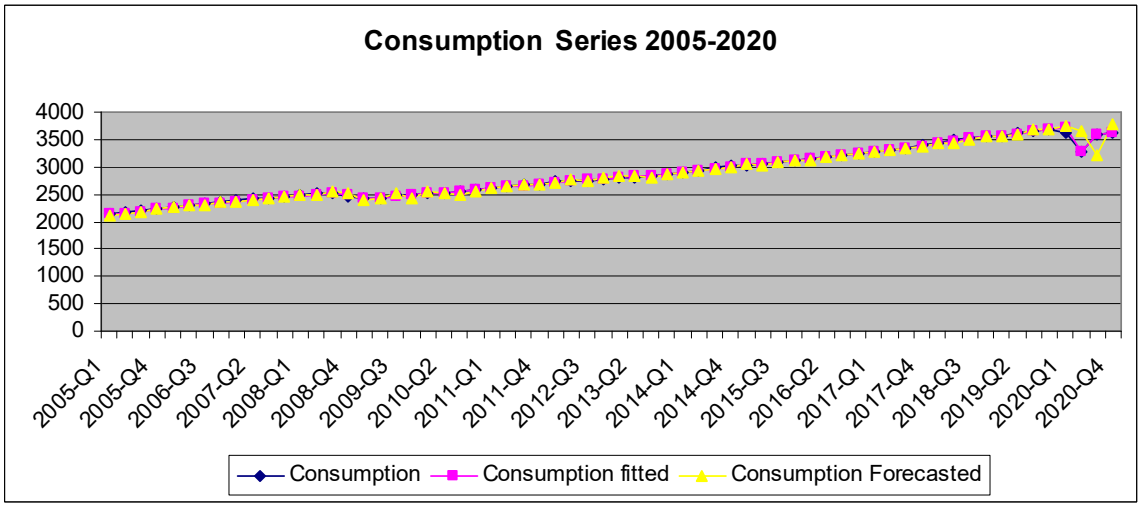


Figure 2

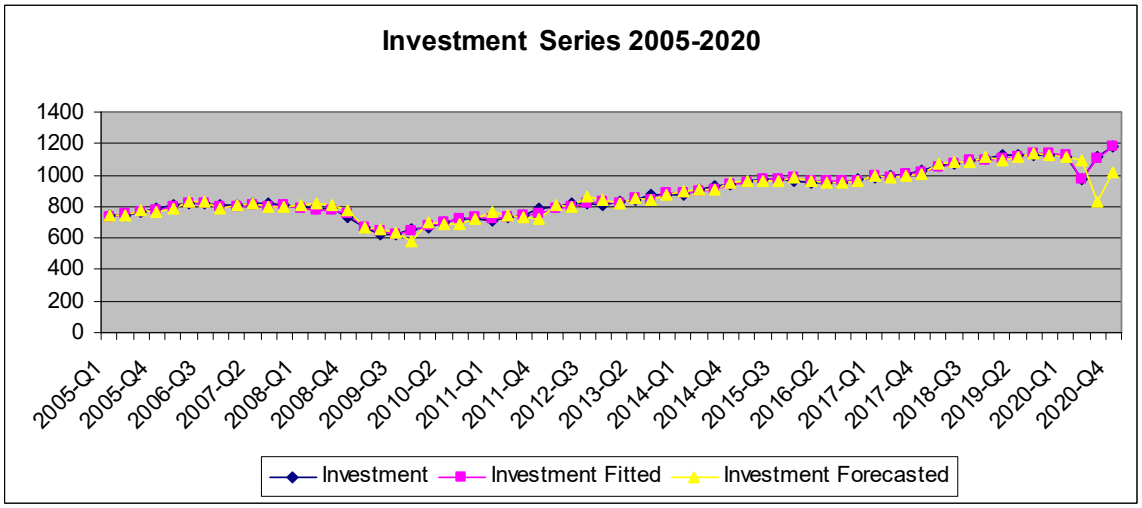


Figure 3

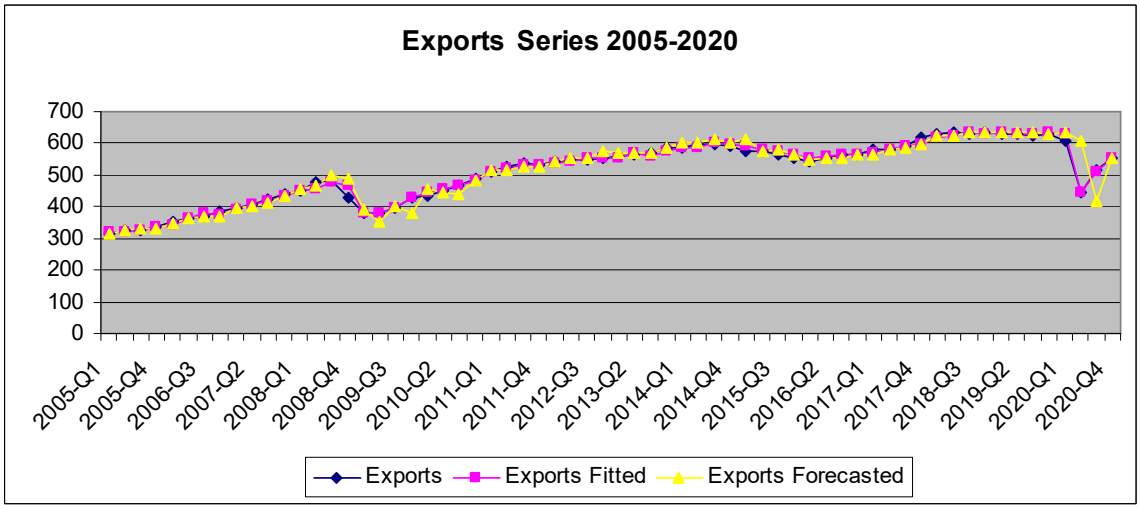


Figure 4

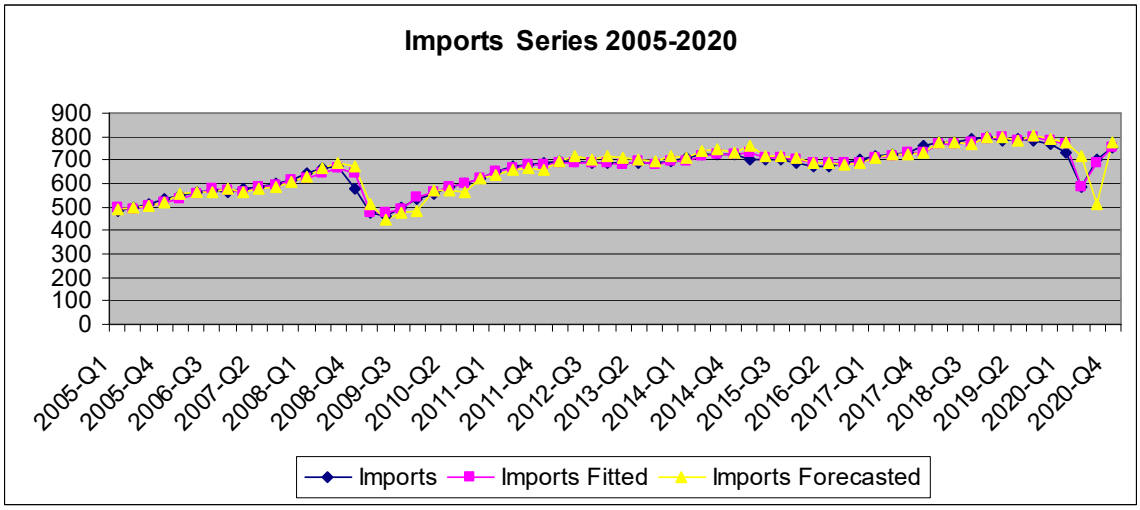


Figure 5

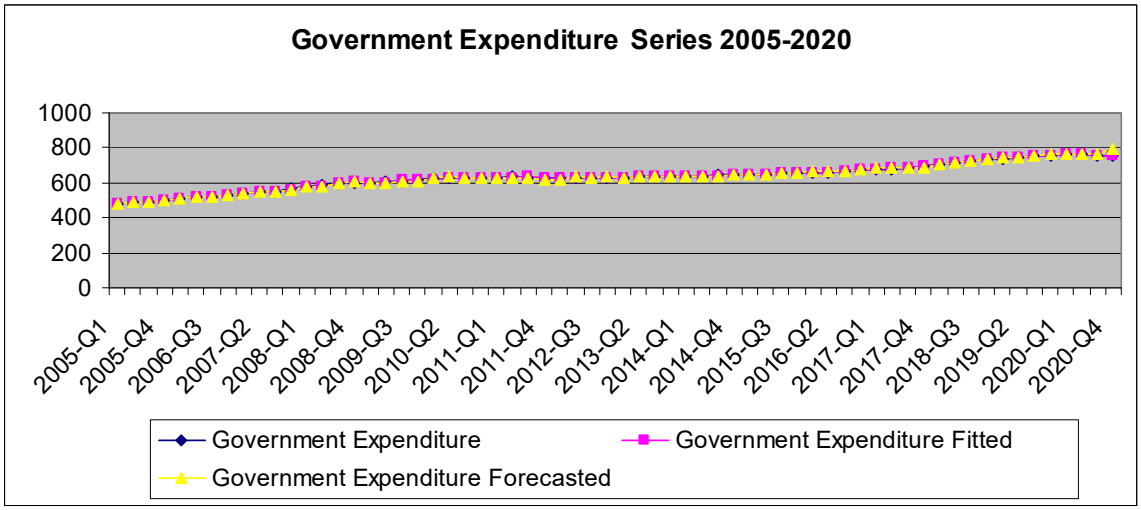


Figure 6

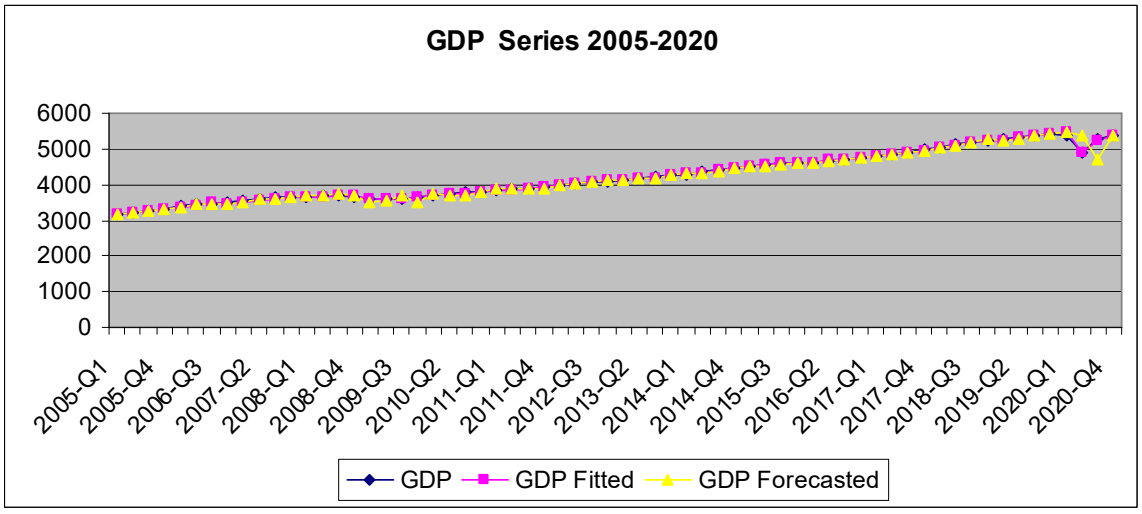


Figure 7

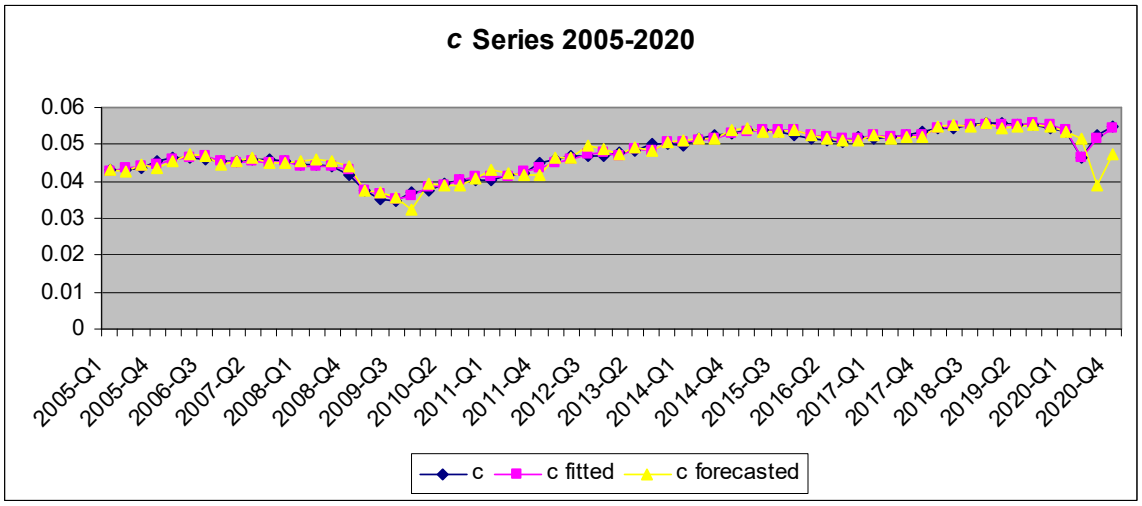


Figure 8

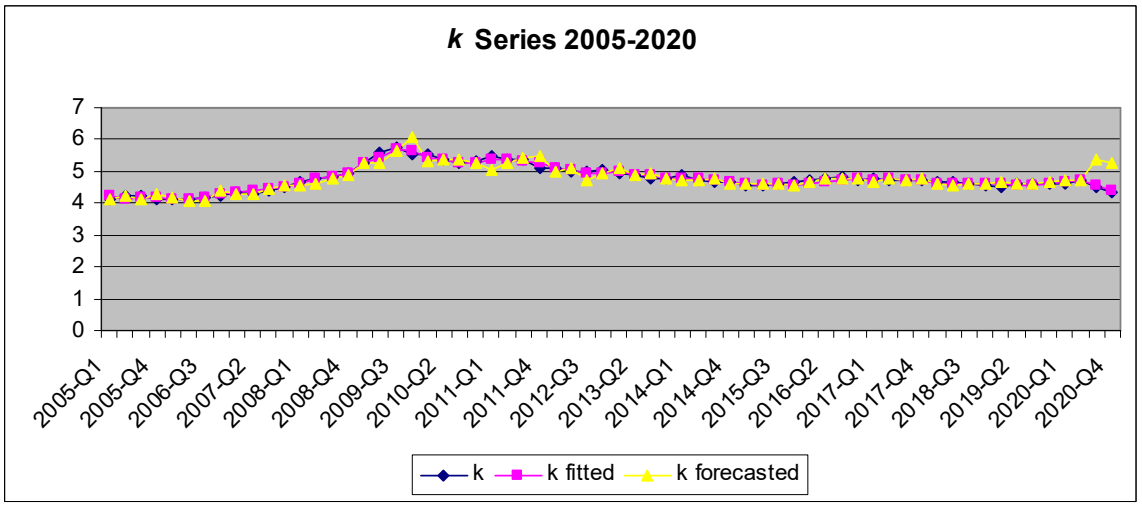


Figure 9

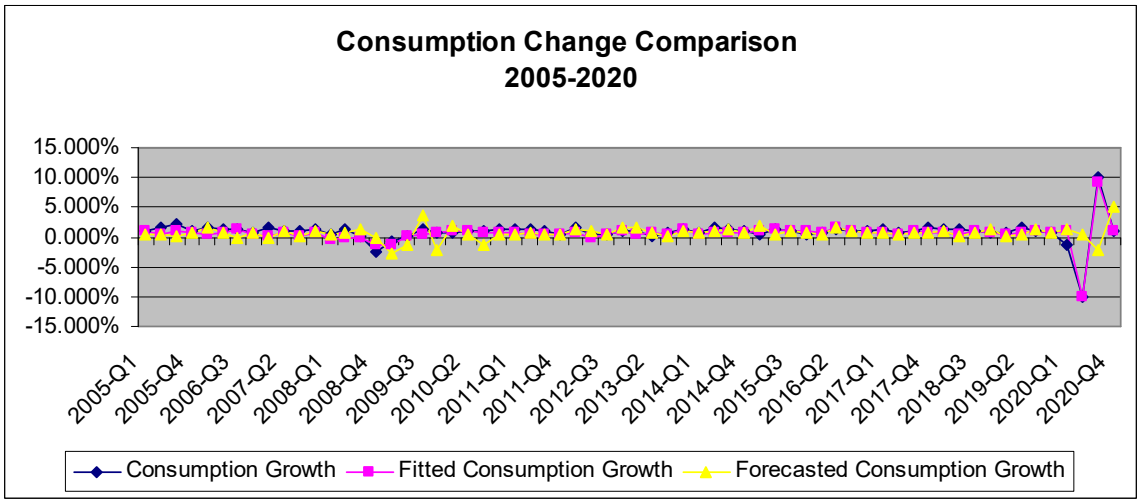


Figure 10

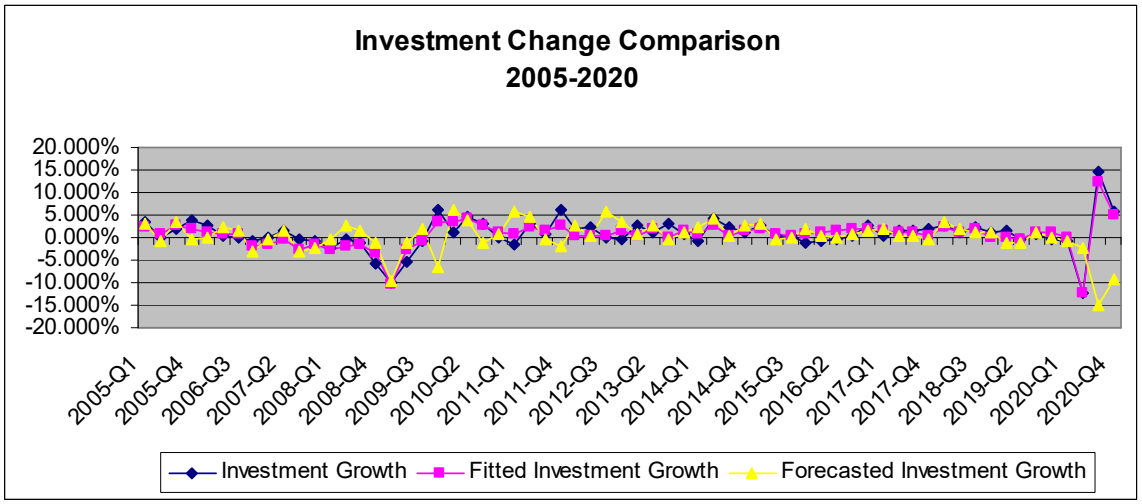


Figure 11

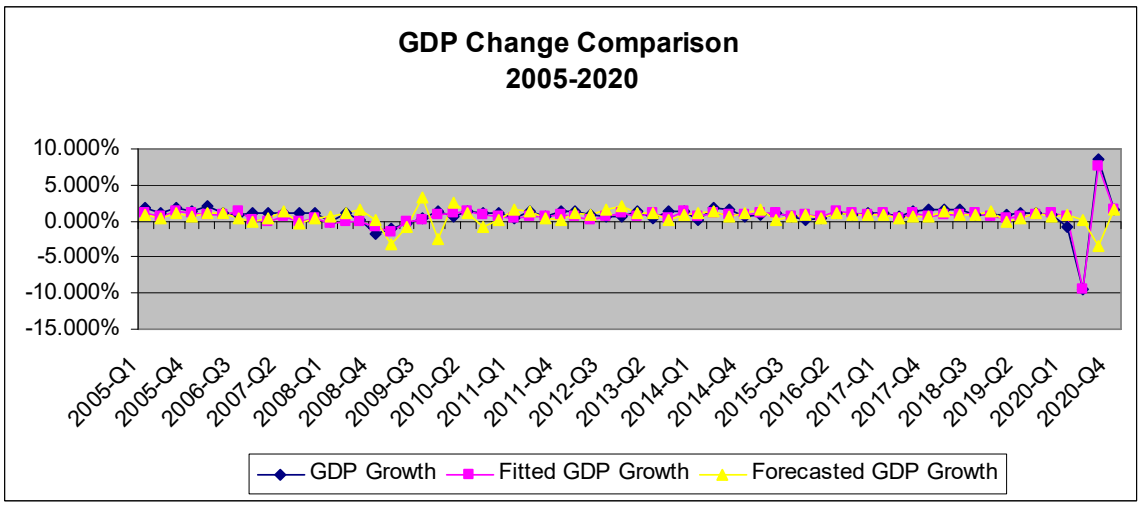


Figure 12

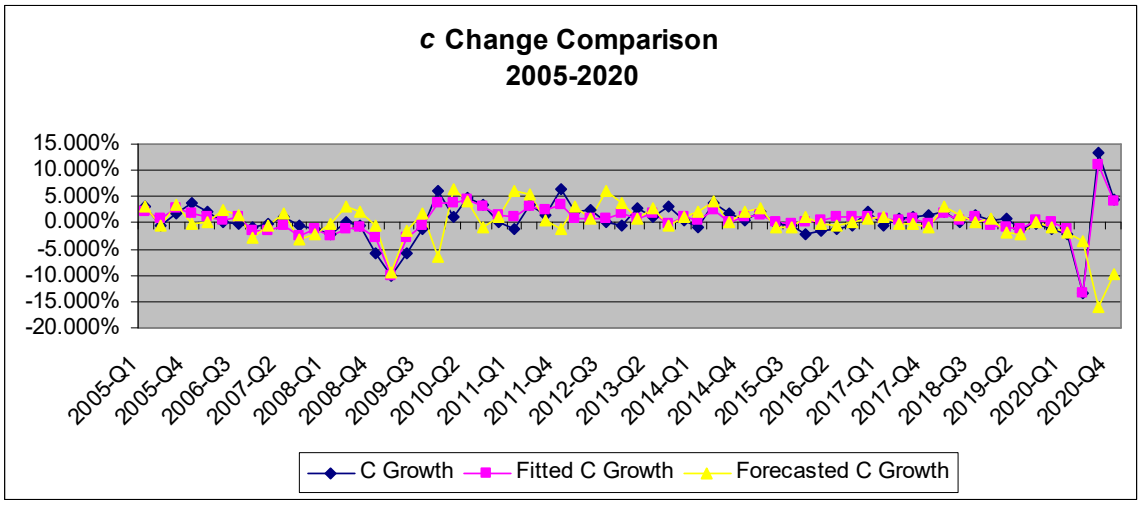


Figure 13

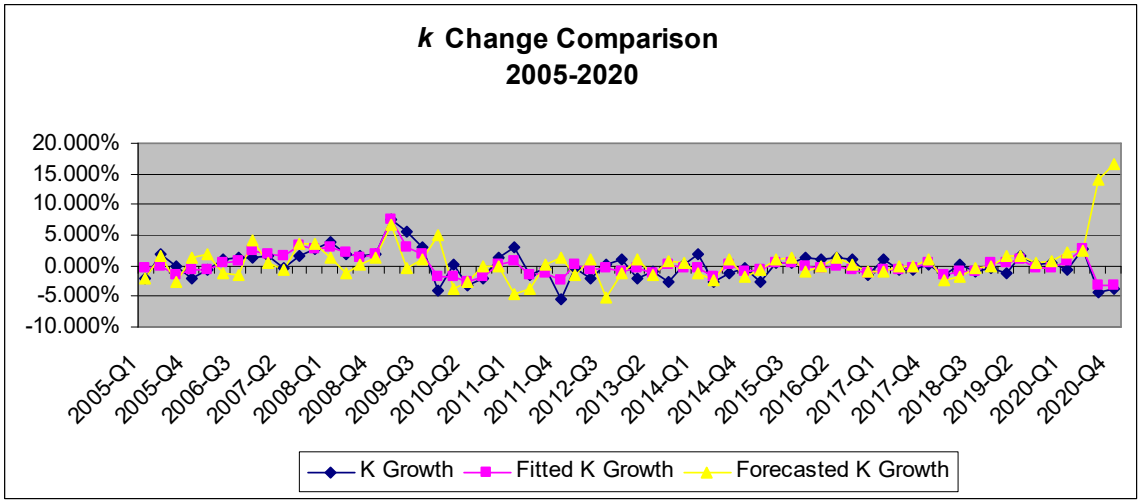
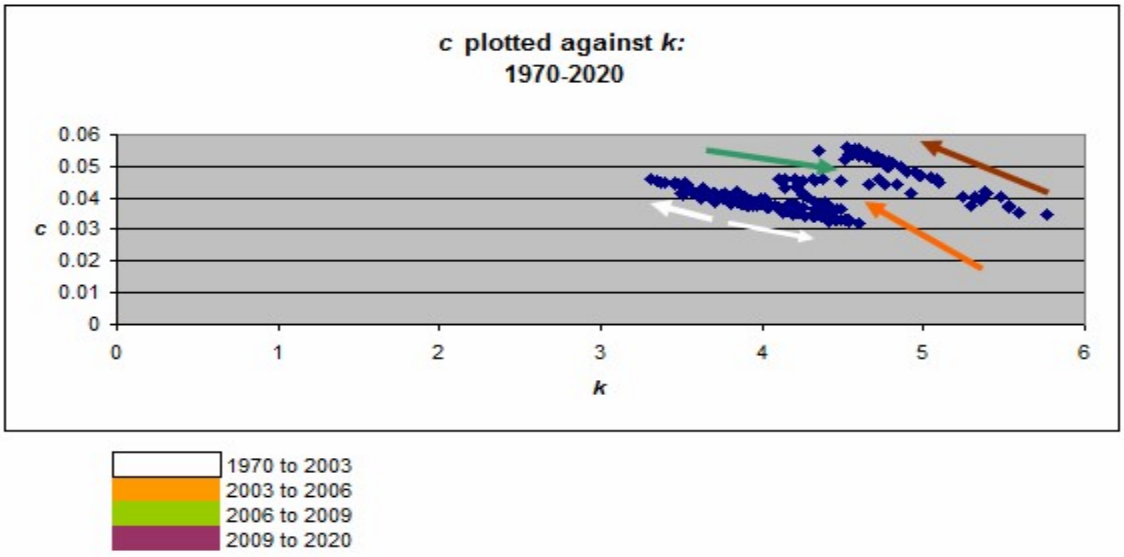


Figure 14



c Regression Output

```
Call:
lm(formula = cs_cn_r ~ time_r + time_binary_2008_Q4 + time_binary_2020_Q1 +
    c_k_change_175 + c_k_change_186 + c_k_change_200 + Inflation_L1_r +
    Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r + NetSaving_L2_r +
    NetSaving_L3_r + MoneySupply_relative_change_L1_r + MoneySupply_relative_change_L2_r +
    MoneySupply_relative_change_L3_r + InterestPaymentSales_Ratio_L1_r +
    InterestPaymentSales_Ratio_L2_r + InterestPaymentSales_Ratio_L3_r +
    NetSavingIncome_Ratio_L1_r + NetSavingIncome_Ratio_L2_r +
    NetSavingIncome_Ratio_L3_r + tau_c_L1_r + tau_c_L2_r + tau_c_L3_r +
    k_relative_change_L1_r + k_relative_change_L2_r + k_relative_change_L3_r +
    c_s_relative_change_L1_r + c_s_relative_change_L2_r + c_s_relative_change_L3_r +
    cs_cn_L1_r + cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r +
    a_line_L3_r + a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r +
    US_CANExchangeRate_L1_r + US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r +
    ycfivetothirty_L1_r + ycfivetothirty_L2_r + ycfivetothirty_L3_r +
    yconetofour_L1_r + yconetofour_L2_r + yconetofour_L3_r +
    yconetofour_ns_L1_r + yconetofour_ns_L2_r + yconetofour_ns_L3_r +
    ycfivetothirty_ns_L1_r + ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r,
    data = subset(R_US_df2_r, time_r > 39 & time_r < 244))

Residuals:
      Min        1Q    Median        3Q       Max
-0.0025909 -0.0004913  0.0000000  0.0005706  0.0024298

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                      2.203e-03  3.033e-03   0.726 0.468684
time_r                          -1.994e-06  1.522e-05  -0.131 0.895935
time_binary_2008_Q4             -2.566e-03  1.322e-03  -1.941 0.054173 .
time_binary_2020_Q1             -7.122e-03  1.237e-03  -5.758 4.70e-08 ***
c_k_change_175                   1.176e-03  6.801e-04   1.728 0.085994 .
c_k_change_186                  -8.050e-04  6.842e-04  -1.177 0.241200
c_k_change_200                   1.566e-03  8.812e-04   1.777 0.077564 .
Inflation_L1_r                   2.093e-05  2.039e-04   0.103 0.918397
Inflation_L2_r                   1.066e-05  1.749e-04   0.061 0.951483
Inflation_L3_r                   2.015e-05  1.794e-04   0.112 0.910743
NetSaving_L1_r                  -3.667e-05  1.257e-05  -2.917 0.004083 **
NetSaving_L2_r                   6.946e-05  1.826e-05   3.803 0.000208 ***
NetSaving_L3_r                  -3.234e-05  1.380e-05  -2.344 0.020414 *
MoneySupply_relative_change_L1_r 3.715e-02  1.359e-02   2.733 0.007042 **
MoneySupply_relative_change_L2_r 3.861e-03  1.466e-02   0.263 0.792621
MoneySupply_relative_change_L3_r 7.344e-03  1.244e-02   0.591 0.555705
InterestPaymentSales_Ratio_L1_r  1.151e-03  7.698e-04   1.495 0.137031
InterestPaymentSales_Ratio_L2_r -1.782e-03  1.066e-03  -1.672 0.096690 .
InterestPaymentSales_Ratio_L3_r  7.025e-04  7.535e-04   0.932 0.352624
NetSavingIncome_Ratio_L1_r       1.335e-01  2.834e-02   4.711 5.60e-06 ***
NetSavingIncome_Ratio_L2_r      -1.657e-01  4.068e-02  -4.072 7.53e-05 ***
NetSavingIncome_Ratio_L3_r       3.454e-02  2.875e-02   1.201 0.231506
tau_c_L1_r                       4.867e+00  2.870e+00   1.696 0.092022 .
tau_c_L2_r                      -6.735e+00  4.452e+00  -1.513 0.132493
tau_c_L3_r                       1.837e+00  2.710e+00   0.678 0.499046
k_relative_change_L1_r           1.093e-01  3.774e-02   2.897 0.004331 **
k_relative_change_L2_r          -2.317e-02  4.688e-02  -0.494 0.621856
k_relative_change_L3_r          -9.473e-02  3.643e-02  -2.600 0.010254 *
c_s_relative_change_L1_r         8.257e-02  4.128e-02   2.000 0.047296 *
c_s_relative_change_L2_r         1.191e-02  4.913e-02   0.242 0.808815
c_s_relative_change_L3_r        -1.060e-01  3.946e-02  -2.687 0.008039 **
cs_cn_L1_r                      -5.791e+00  3.452e+00  -1.677 0.095552 .
cs_cn_L2_r                       1.506e+00  3.284e+00   0.459 0.647132
cs_cn_L3_r                       4.031e-01  7.067e-01   0.570 0.569276
a_line_L1_r                     -4.331e+00  1.910e+00  -2.268 0.024755 *
a_line_L2_r                      6.821e+00  3.316e+00   2.057 0.041460 *
a_line_L3_r                     -2.492e+00  1.886e+00  -1.321 0.188398
a_line_an_L1_r                   2.496e+00  1.162e+00   2.148 0.033308 *
a_line_an_L2_r                  -1.585e+00  1.085e+00  -1.462 0.145982
a_line_an_L3_r                   2.815e-01  2.022e-01   1.392 0.165906
US_CANExchangeRate_L1_r          1.499e-03  2.312e-03   0.648 0.517855
US_CANExchangeRate_L2_r          1.048e-03  3.184e-03   0.329 0.742445
US_CANExchangeRate_L3_r         -2.267e-03  2.355e-03  -0.963 0.337160
ycfivetothirty_L1_r              3.411e-03  8.269e-03   0.412 0.680574
ycfivetothirty_L2_r              1.048e-03  8.825e-03   0.119 0.905646
ycfivetothirty_L3_r             -1.214e-02  7.929e-03  -1.531 0.127987
yconetofour_L1_r                -1.235e-03  1.792e-03  -0.689 0.491865
yconetofour_L2_r                 4.334e-03  1.914e-03   2.265 0.024963 *
yconetofour_L3_r                -8.980e-04  1.618e-03  -0.555 0.579709
yconetofour_ns_L1_r              2.571e-06  2.432e-05   0.106 0.915944
yconetofour_ns_L2_r             -3.274e-05  2.873e-05  -1.139 0.256360
yconetofour_ns_L3_r              6.786e-06  2.081e-05   0.326 0.744862
ycfivetothirty_ns_L1_r          -8.120e-05  1.050e-04  -0.773 0.440632
ycfivetothirty_ns_L2_r          -7.201e-05  1.292e-04  -0.557 0.578165
ycfivetothirty_ns_L3_r           2.396e-04  1.121e-04   2.138 0.034165 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0009306 on 149 degrees of freedom
Multiple R-squared:  0.6594,    Adjusted R-squared:  0.536
F-statistic: 5.343 on 54 and 149 DF,  p-value: 2.76e-16
```

Note: "time_binary_2008_Q4" was miscoded, it is for 2009 Q1 instead of 2008 Q4

v Regression Output

```
Call:
lm(formula = a_line_an_r ~ time_r + time_binary_2008_Q4 + time_binary_2020_Q1 +
    c_k_change_175 + c_k_change_186 + c_k_change_200 + Inflation_L1_r +
    Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r + NetSaving_L2_r +
    NetSaving_L3_r + MoneySupply_relative_change_L1_r + MoneySupply_relative_change_L2_r +
    MoneySupply_relative_change_L3_r + InterestPaymentSales_Ratio_L1_r +
    InterestPaymentSales_Ratio_L2_r + InterestPaymentSales_Ratio_L3_r +
    NetSavingIncome_Ratio_L1_r + NetSavingIncome_Ratio_L2_r +
    NetSavingIncome_Ratio_L3_r + tau_c_L1_r + tau_c_L2_r + tau_c_L3_r +
    k_relative_change_L1_r + k_relative_change_L2_r + k_relative_change_L3_r +
    c_s_relative_change_L1_r + c_s_relative_change_L2_r + c_s_relative_change_L3_r +
    cs_cn_L1_r + cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r +
    a_line_L3_r + a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r +
    US_CANExchangeRate_L1_r + US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r +
    ycfivetothirty_L1_r + ycfivetothirty_L2_r + ycfivetothirty_L3_r +
    yconetofour_L1_r + yconetofour_L2_r + yconetofour_L3_r +
    yconetofour_ns_L1_r + yconetofour_ns_L2_r + yconetofour_ns_L3_r +
    ycfivetothirty_ns_L1_r + ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r,
    data = subset(R_US_df2_r, time_r > 39 & time_r < 244))

Residuals:
       Min         1Q     Median         3Q        Max
-0.0065804 -0.0007477  0.0000527  0.0008014  0.0030839

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                      1.474e-02  4.634e-03   3.181 0.001789 **
time_r                           4.816e-05  2.325e-05   2.071 0.040040 *
time_binary_2008_Q4             -5.551e-03  2.020e-03  -2.748 0.006747 **
time_binary_2020_Q1             -2.784e-02  1.890e-03 -14.733  < 2e-16 ***
c_k_change_175                   1.847e-03  1.039e-03   1.777 0.077609 .
c_k_change_186                   1.055e-03  1.045e-03   1.010 0.314326
c_k_change_200                   4.598e-03  1.346e-03   3.415 0.000821 ***
Inflation_L1_r                  -9.377e-05  3.116e-04  -0.301 0.763895
Inflation_L2_r                  -4.424e-05  2.672e-04  -0.166 0.868696
Inflation_L3_r                  -1.572e-04  2.741e-04  -0.574 0.567091
NetSaving_L1_r                  -4.888e-05  1.921e-05  -2.545 0.011946 *
NetSaving_L2_r                   8.005e-05  2.791e-05   2.869 0.004720 **
NetSaving_L3_r                  -4.296e-05  2.108e-05  -2.038 0.043311 *
MoneySupply_relative_change_L1_r  4.415e-02  2.077e-02   2.126 0.035177 *
MoneySupply_relative_change_L2_r  4.703e-03  2.240e-02   0.210 0.833995
MoneySupply_relative_change_L3_r -1.521e-02  1.900e-02  -0.800 0.424775
InterestPaymentSales_Ratio_L1_r   2.516e-04  1.176e-03   0.214 0.830883
InterestPaymentSales_Ratio_L2_r  -4.922e-04  1.629e-03  -0.302 0.762921
InterestPaymentSales_Ratio_L3_r   1.372e-04  1.151e-03   0.119 0.905305
NetSavingIncome_Ratio_L1_r        7.406e-02  4.330e-02   1.710 0.089307 .
NetSavingIncome_Ratio_L2_r       -1.210e-01  6.216e-02  -1.946 0.053540 .
NetSavingIncome_Ratio_L3_r        5.996e-02  4.393e-02   1.365 0.174317
tau_c_L1_r                        6.829e+00  4.385e+00   1.557 0.121483
tau_c_L2_r                       -9.836e+00  6.803e+00  -1.446 0.150314
tau_c_L3_r                        3.247e+00  4.141e+00   0.784 0.434232
k_relative_change_L1_r            1.037e-01  5.766e-02   1.798 0.074236 .
k_relative_change_L2_r            1.029e-01  7.163e-02   1.437 0.152876
k_relative_change_L3_r            9.267e-03  5.566e-02   0.166 0.868006
c_s_relative_change_L1_r          7.600e-02  6.308e-02   1.205 0.230132
c_s_relative_change_L2_r          1.488e-01  7.506e-02   1.982 0.049323 *
c_s_relative_change_L3_r         -2.067e-02  6.028e-02  -0.343 0.732182
cs_cn_L1_r                       -7.527e+00  5.275e+00  -1.427 0.155715
cs_cn_L2_r                        2.833e+00  5.018e+00   0.565 0.573132
cs_cn_L3_r                        7.048e-01  1.080e+00   0.653 0.514911
a_line_L1_r                      -5.446e+00  2.918e+00  -1.867 0.063936 .
a_line_L2_r                       6.451e+00  5.067e+00   1.273 0.204954
a_line_L3_r                      -1.250e+00  2.882e+00  -0.434 0.665093
a_line_an_L1_r                    2.745e+00  1.775e+00   1.547 0.124100
a_line_an_L2_r                   -8.365e-01  1.657e+00  -0.505 0.614516
a_line_an_L3_r                   -1.370e-02  3.089e-01  -0.044 0.964693
US_CANExchangeRate_L1_r          -8.142e-03  3.533e-03  -2.305 0.022575 *
US_CANExchangeRate_L2_r           2.778e-03  4.865e-03   0.571 0.568932
US_CANExchangeRate_L3_r           8.948e-04  3.598e-03   0.249 0.803913
ycfivetothirty_L1_r              -4.490e-04  1.263e-02  -0.036 0.971696
ycfivetothirty_L2_r               2.607e-02  1.348e-02   1.934 0.055051 .
ycfivetothirty_L3_r              -1.377e-02  1.212e-02  -1.137 0.257438
yconetofour_L1_r                  1.875e-04  2.738e-03   0.068 0.945481
yconetofour_L2_r                 -3.450e-03  2.924e-03  -1.180 0.239949
yconetofour_L3_r                  3.114e-05  2.472e-03   0.013 0.989967
yconetofour_ns_L1_r               1.404e-05  3.715e-05   0.378 0.706098
yconetofour_ns_L2_r               3.223e-05  4.390e-05   0.734 0.464032
yconetofour_ns_L3_r               1.897e-06  3.180e-05   0.060 0.952515
ycfivetothirty_ns_L1_r            3.282e-06  1.605e-04   0.020 0.983707
ycfivetothirty_ns_L2_r           -3.765e-04  1.974e-04  -1.907 0.058451 .
ycfivetothirty_ns_L3_r            2.498e-04  1.712e-04   1.459 0.146697
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.001422 on 149 degrees of freedom
Multiple R-squared:  0.8144,    Adjusted R-squared:  0.7472
F-statistic: 12.11 on 54 and 149 DF,  p-value: < 2.2e-16
```

Note: "time_binary_2008_Q4" was miscoded, it is for 2009 Q1 instead of 2008 Q4

Exports Regression Output

```
Call:
lm(formula = Exports_current_change_r ~ time_r + time_binary_2008_Q4 +
    time_binary_2020_Q1 + c_k_change_175 + c_k_change_186 + c_k_change_200 +
    Inflation_L1_r + Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r +
    NetSaving_L2_r + NetSaving_L3_r + MoneySupply_relative_change_L1_r +
    MoneySupply_relative_change_L2_r + MoneySupply_relative_change_L3_r +
    InterestPaymentSales_Ratio_L1_r + InterestPaymentSales_Ratio_L2_r +
    InterestPaymentSales_Ratio_L3_r + NetSavingIncome_Ratio_L1_r +
    NetSavingIncome_Ratio_L2_r + NetSavingIncome_Ratio_L3_r +
    tau_c_L1_r + tau_c_L2_r + tau_c_L3_r + k_relative_change_L1_r +
    k_relative_change_L2_r + k_relative_change_L3_r + c_s_relative_change_L1_r +
    c_s_relative_change_L2_r + c_s_relative_change_L3_r + cs_cn_L1_r +
    cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r + a_line_L3_r +
    a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r + US_CANExchangeRate_L1_r +
    US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r + ycfivetothirty_L1_r +
    ycfivetothirty_L2_r + ycfivetothirty_L3_r + yconetofour_L1_r +
    yconetofour_L2_r + yconetofour_L3_r + yconetofour_ns_L1_r +
    yconetofour_ns_L2_r + yconetofour_ns_L3_r + ycfivetothirty_ns_L1_r +
    ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r, data = subset(R_US_df2_r,
    time_r > 39 & time_r < 244))

Residuals:
    Min      1Q  Median      3Q     Max
-38.549  -2.798  -0.079   2.954  20.606

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                       8.547e+01  2.191e+01   3.901 0.000144 ***
time_r                            2.278e-01  1.099e-01   2.073 0.039915 *
time_binary_2008_Q4              -3.979e+01  9.553e+00  -4.165 5.25e-05 ***
time_binary_2020_Q1              -1.732e+02  8.935e+00 -19.390  < 2e-16 ***
c_k_change_175                    6.776e+00  4.913e+00   1.379 0.169917
c_k_change_186                    9.272e+00  4.942e+00   1.876 0.062617 .
c_k_change_200                    1.754e+01  6.366e+00   2.755 0.006597 **
Inflation_L1_r                   -1.185e+00  1.473e+00  -0.804 0.422396
Inflation_L2_r                   -2.743e+00  1.263e+00  -2.172 0.031457 *
Inflation_L3_r                   -4.397e-01  1.296e+00  -0.339 0.734859
NetSaving_L1_r                   -1.293e-01  9.081e-02  -1.424 0.156610
NetSaving_L2_r                    2.859e-01  1.319e-01   2.167 0.031825 *
NetSaving_L3_r                   -1.968e-01  9.967e-02  -1.975 0.050143 .
MoneySupply_relative_change_L1_r  1.510e+02  9.820e+01   1.538 0.126285
MoneySupply_relative_change_L2_r -5.952e+01  1.059e+02  -0.562 0.574966
MoneySupply_relative_change_L3_r -1.003e+02  8.983e+01  -1.116 0.266025
InterestPaymentSales_Ratio_L1_r   2.443e+00  5.561e+00   0.439 0.661084
InterestPaymentSales_Ratio_L2_r   4.763e+00  7.700e+00   0.618 0.537194
InterestPaymentSales_Ratio_L3_r  -7.326e+00  5.443e+00  -1.346 0.180380
NetSavingIncome_Ratio_L1_r        1.283e+02  2.047e+02   0.627 0.531921
NetSavingIncome_Ratio_L2_r       -4.824e+02  2.939e+02  -1.641 0.102843
NetSavingIncome_Ratio_L3_r        2.691e+02  2.077e+02   1.296 0.197069
tau_c_L1_r                        1.115e+04  2.073e+04   0.538 0.591487
tau_c_L2_r                       -2.456e+04  3.216e+04  -0.763 0.446406
tau_c_L3_r                        1.422e+04  1.958e+04   0.726 0.468740
k_relative_change_L1_r            5.870e+02  2.726e+02   2.153 0.032908 *
k_relative_change_L2_r            2.692e+02  3.387e+02   0.795 0.427876
k_relative_change_L3_r            1.711e+02  2.632e+02   0.650 0.516673
c_s_relative_change_L1_r          3.268e+02  2.982e+02   1.096 0.274949
c_s_relative_change_L2_r          5.830e+02  3.549e+02   1.643 0.102548
c_s_relative_change_L3_r          2.046e+02  2.850e+02   0.718 0.473910
cs_cn_L1_r                       -6.683e+03  2.494e+04  -0.268 0.789083
cs_cn_L2_r                        9.412e+03  2.372e+04   0.397 0.692114
cs_cn_L3_r                       -9.510e+02  5.105e+03  -0.186 0.852460
a_line_L1_r                      -9.811e+03  1.379e+04  -0.711 0.478059
a_line_L2_r                       1.019e+04  2.396e+04   0.425 0.671285
a_line_L3_r                      -1.375e+03  1.362e+04  -0.101 0.919722
a_line_an_L1_r                    2.907e+03  8.392e+03   0.346 0.729562
a_line_an_L2_r                    2.177e+03  7.836e+03   0.278 0.781520
a_line_an_L3_r                   -1.413e+03  1.460e+03  -0.967 0.334899
US_CANExchangeRate_L1_r          -6.109e+01  1.670e+01  -3.657 0.000353 ***
US_CANExchangeRate_L2_r           7.645e+00  2.300e+01   0.332 0.740100
US_CANExchangeRate_L3_r           2.729e+01  1.701e+01   1.604 0.110769
ycfivetothirty_L1_r               9.461e+00  5.973e+01   0.158 0.874367
ycfivetothirty_L2_r               1.118e+02  6.375e+01   1.753 0.081646 .
ycfivetothirty_L3_r              -5.978e+01  5.728e+01  -1.044 0.298341
yconetofour_L1_r                 -1.176e+01  1.294e+01  -0.909 0.364905
yconetofour_L2_r                 -1.635e+01  1.382e+01  -1.183 0.238769
yconetofour_L3_r                  4.064e-01  1.169e+01   0.035 0.972309
yconetofour_ns_L1_r               1.379e-01  1.757e-01   0.785 0.433677
yconetofour_ns_L2_r               1.150e-01  2.076e-01   0.554 0.580318
yconetofour_ns_L3_r              -2.773e-02  1.504e-01  -0.184 0.853911
ycfivetothirty_ns_L1_r            7.220e-02  7.587e-01   0.095 0.924313
ycfivetothirty_ns_L2_r           -1.246e+00  9.334e-01  -1.334 0.184094
ycfivetothirty_ns_L3_r            7.781e-01  8.096e-01   0.961 0.338064
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.723 on 149 degrees of freedom
Multiple R-squared:  0.8557,    Adjusted R-squared:  0.8035
F-statistic: 16.37 on 54 and 149 DF,  p-value: < 2.2e-16
```

Note: "time_binary_2008_Q4" was miscoded, it is for 2009 Q1 instead of 2008 Q4

Imports Regression Output

```
lm(formula = Imports_current_change_r ~ time_r + time_binary_2008_Q4 +
    time_binary_2020_Q1 + c_k_change_175 + c_k_change_186 + c_k_change_200 +
    Inflation_L1_r + Inflation_L2_r + Inflation_L3_r + NetSaving_L1_r +
    NetSaving_L2_r + NetSaving_L3_r + MoneySupply_relative_change_L1_r +
    MoneySupply_relative_change_L2_r + MoneySupply_relative_change_L3_r +
    InterestPaymentSales_Ratio_L1_r + InterestPaymentSales_Ratio_L2_r +
    InterestPaymentSales_Ratio_L3_r + NetSavingIncome_Ratio_L1_r +
    NetSavingIncome_Ratio_L2_r + NetSavingIncome_Ratio_L3_r +
    tau_c_L1_r + tau_c_L2_r + tau_c_L3_r + k_relative_change_L1_r +
    k_relative_change_L2_r + k_relative_change_L3_r + c_s_relative_change_L1_r +
    c_s_relative_change_L2_r + c_s_relative_change_L3_r + cs_cn_L1_r +
    cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r + a_line_L3_r +
    a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r + US_CANExchangeRate_L1_r +
    US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r + ycfivetothirty_L1_r +
    ycfivetothirty_L2_r + ycfivetothirty_L3_r + yconetofour_L1_r +
    yconetofour_L2_r + yconetofour_L3_r + yconetofour_ns_L1_r +
    yconetofour_ns_L2_r + yconetofour_ns_L3_r + ycfivetothirty_ns_L1_r +
    ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r, data = subset(R_US_df2_r,
    time_r > 39 & time_r < 244))

Residuals:
    Min      1Q  Median      3Q     Max
-63.427  -4.068  -0.231   4.930  27.693

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)
(Intercept)                       1.195e+02  3.389e+01   3.528 0.000558 ***
time_r                            4.174e-01  1.700e-01   2.455 0.015245 *
time_binary_2008_Q4              -7.775e+01  1.478e+01  -5.262 4.87e-07 ***
time_binary_2020_Q1              -1.585e+02  1.382e+01 -11.472  < 2e-16 ***
c_k_change_175                    1.631e+01  7.600e+00   2.147 0.033451 *
c_k_change_186                    6.921e+00  7.645e+00   0.905 0.366714
c_k_change_200                    2.914e+01  9.846e+00   2.960 0.003584 **
Inflation_L1_r                   -2.379e+00  2.279e+00  -1.044 0.298213
Inflation_L2_r                   -2.457e+00  1.954e+00  -1.258 0.210490
Inflation_L3_r                   -1.366e+00  2.005e+00  -0.681 0.496696
NetSaving_L1_r                   -2.823e-01  1.405e-01  -2.009 0.046296 *
NetSaving_L2_r                    6.112e-01  2.041e-01   2.995 0.003211 **
NetSaving_L3_r                   -3.636e-01  1.542e-01  -2.359 0.019624 *
MoneySupply_relative_change_L1_r  2.220e+02  1.519e+02   1.462 0.145942
MoneySupply_relative_change_L2_r -2.795e+01  1.638e+02  -0.171 0.864751
MoneySupply_relative_change_L3_r -6.446e+01  1.389e+02  -0.464 0.643407
InterestPaymentSales_Ratio_L1_r   4.586e+00  8.601e+00   0.533 0.594662
InterestPaymentSales_Ratio_L2_r   1.472e+00  1.191e+01   0.124 0.901838
InterestPaymentSales_Ratio_L3_r  -6.284e+00  8.419e+00  -0.746 0.456573
NetSavingIncome_Ratio_L1_r        2.473e+02  3.167e+02   0.781 0.436007
NetSavingIncome_Ratio_L2_r       -9.055e+02  4.546e+02  -1.992 0.048201 *
NetSavingIncome_Ratio_L3_r        5.728e+02  3.213e+02   1.783 0.076598 .
tau_c_L1_r                        8.124e+03  3.207e+04   0.253 0.800338
tau_c_L2_r                       -4.251e+04  4.975e+04  -0.854 0.394205
tau_c_L3_r                        3.485e+04  3.028e+04   1.151 0.251623
k_relative_change_L1_r            5.484e+02  4.217e+02   1.300 0.195478
k_relative_change_L2_r            9.250e+02  5.238e+02   1.766 0.079473 .
k_relative_change_L3_r            9.149e+01  4.071e+02   0.225 0.822476
c_s_relative_change_L1_r          5.270e+02  4.613e+02   1.143 0.255071
c_s_relative_change_L2_r          1.104e+03  5.489e+02   2.012 0.046005 *
c_s_relative_change_L3_r          1.034e+02  4.409e+02   0.235 0.814917
cs_cn_L1_r                       -8.585e+03  3.858e+04  -0.223 0.824189
cs_cn_L2_r                        3.924e+04  3.669e+04   1.069 0.286586
cs_cn_L3_r                       -6.596e+02  7.896e+03  -0.084 0.933538
a_line_L1_r                      -1.084e+04  2.134e+04  -0.508 0.612089
a_line_L2_r                       1.875e+04  3.706e+04   0.506 0.613534
a_line_L3_r                      -9.083e+03  2.107e+04  -0.431 0.667080
a_line_an_L1_r                    5.785e+03  1.298e+04   0.446 0.656497
a_line_an_L2_r                   -3.052e+03  1.212e+04  -0.252 0.801538
a_line_an_L3_r                   -6.267e+02  2.259e+03  -0.277 0.781825
US_CANExchangeRate_L1_r          -7.257e+01  2.584e+01  -2.809 0.005639 **
US_CANExchangeRate_L2_r           3.101e+01  3.558e+01   0.872 0.384864
US_CANExchangeRate_L3_r           6.080e+00  2.631e+01   0.231 0.817538
ycfivetothirty_L1_r               1.747e+00  9.239e+01   0.019 0.984937
ycfivetothirty_L2_r               2.282e+02  9.861e+01   2.314 0.022030 *
ycfivetothirty_L3_r              -1.354e+02  8.860e+01  -1.528 0.128567
yconetofour_L1_r                 -1.272e+01  2.002e+01  -0.635 0.526291
yconetofour_L2_r                 -2.979e+01  2.138e+01  -1.393 0.165591
yconetofour_L3_r                  4.242e+00  1.808e+01   0.235 0.814804
yconetofour_ns_L1_r               2.831e-01  2.717e-01   1.042 0.299030
yconetofour_ns_L2_r               2.047e-01  3.211e-01   0.637 0.524797
yconetofour_ns_L3_r              -2.374e-02  2.326e-01  -0.102 0.918826
ycfivetothirty_ns_L1_r           -3.350e-01  1.173e+00  -0.285 0.775687
ycfivetothirty_ns_L2_r           -2.277e+00  1.444e+00  -1.577 0.116905
ycfivetothirty_ns_L3_r            1.316e+00  1.252e+00   1.051 0.294907
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.4 on 149 degrees of freedom
Multiple R-squared:  0.7833,    Adjusted R-squared:  0.7048
F-statistic: 9.974 on 54 and 149 DF,  p-value: < 2.2e-16
```

Note: "time_binary_2008_Q4" was miscoded, it is for 2009 Q1 instead of 2008 Q4

Government Expenditure Regression Output

```
ıla = GovernmentExpenditure_current_change_r ~ time_r +
_binary_2008_Q4 + time_binary_2020_Q1 + c_k_change_175 +
_change_186 + c_k_change_200 + Inflation_L1_r + Inflation_L2_r +
lation_L3_r + NetSaving_L1_r + NetSaving_L2_r + NetSaving_L3_r +
eySupply_relative_change_L1_r + MoneySupply_relative_change_L2_r +
eySupply_relative_change_L3_r + InterestPaymentSales_Ratio_L1_r +
erestPaymentSales_Ratio_L2_r + InterestPaymentSales_Ratio_L3_r +
SavingIncome_Ratio_L1_r + NetSavingIncome_Ratio_L2_r +
SavingIncome_Ratio_L3_r + tau_c_L1_r + tau_c_L2_r + tau_c_L3_r +
slative_change_L1_r + k_relative_change_L2_r + k_relative_change_L3_r +
_relative_change_L1_r + c_s_relative_change_L2_r + c_s_relative_change_L3
on_L1_r + cs_cn_L2_r + cs_cn_L3_r + a_line_L1_r + a_line_L2_r +
ine_L3_r + a_line_an_L1_r + a_line_an_L2_r + a_line_an_L3_r +
CANExchangeRate_L1_r + US_CANExchangeRate_L2_r + US_CANExchangeRate_L3_r
ivetothirty_L1_r + ycfivetothirty_L2_r + ycfivetothirty_L3_r +
netofour_L1_r + yconetofour_L2_r + yconetofour_L3_r +
netofour_ns_L1_r + yconetofour_ns_L2_r + yconetofour_ns_L3_r +
ivetothirty_ns_L1_r + ycfivetothirty_ns_L2_r + ycfivetothirty_ns_L3_r,
a = subset(R_US_df2_r, time_r > 39 & time_r < 244))

ls:
      1Q  Median      3Q     Max
 -1.0970  0.0376  1.1434  5.7615

ients:
                              Estimate Std. Error t value Pr(>|t|)
ept)                        -1.669e+01  7.797e+00  -2.140  0.03396 *
                             1.125e-01  3.912e-02   2.875  0.00463 **
nary_2008_Q4                -4.750e-01  3.400e+00  -0.140  0.88907
nary_2020_Q1                -1.817e+00  3.180e+00  -0.572  0.56850
nge_175                      6.525e-01  1.748e+00   0.373  0.70954
nge_186                     -3.497e-01  1.759e+00  -0.199  0.84269
nge_200                     -4.857e+00  2.265e+00  -2.144  0.03365 *
on_L1_r                     -2.602e-01  5.243e-01  -0.496  0.62044
on_L2_r                     -1.085e+00  4.495e-01  -2.413  0.01704 *
on_L3_r                      4.459e-01  4.612e-01   0.967  0.33526
ıg_L1_r                     -6.635e-02  3.232e-02  -2.053  0.04183 *
ıg_L2_r                      2.251e-02  4.695e-02   0.479  0.63235
ıg_L3_r                      8.198e-03  3.547e-02   0.231  0.81753
pply_relative_change_L1_r -2.130e+01  3.495e+01  -0.610  0.54308
pply_relative_change_L2_r  5.347e+01  3.769e+01   1.419  0.15808
pply_relative_change_L3_r -1.429e+01  3.197e+01  -0.447  0.65553
:PaymentSales_Ratio_L1_r  -5.661e-01  1.979e+00  -0.286  0.77523
:PaymentSales_Ratio_L2_r   3.639e+00  2.740e+00   1.328  0.18618
:PaymentSales_Ratio_L3_r  -2.858e+00  1.937e+00  -1.475  0.14222
ıgIncome_Ratio_L1_r         1.098e+02  7.286e+01   1.507  0.13406
ıgIncome_Ratio_L2_r        -4.722e+01  1.046e+02  -0.452  0.65226
ıgIncome_Ratio_L3_r         2.439e+00  7.391e+01   0.033  0.97372
l_r                         1.613e+04  7.378e+03   2.186  0.03038 *
2_r                        -1.596e+04  1.145e+04  -1.394  0.16537
3_r                         8.736e+01  6.967e+03   0.013  0.99001
ive_change_L1_r             3.366e+01  9.702e+01   0.347  0.72911
ive_change_L2_r             7.050e+01  1.205e+02   0.585  0.55946
ive_change_L3_r             1.187e+02  9.366e+01   1.268  0.20689
ative_change_L1_r          -2.605e+02  1.061e+02  -2.454  0.01527 *
ative_change_L2_r           3.691e+01  1.263e+02   0.292  0.77049
ative_change_L3_r           1.125e+02  1.014e+02   1.109  0.26916
l_r                        -1.258e+04  8.875e+03  -1.417  0.15849
2_r                         2.952e+02  8.443e+03   0.035  0.97215
3_r                        -1.022e+02  1.817e+03  -0.056  0.95522
L1_r                       -9.349e+03  4.909e+03  -1.904  0.05878 .
L2_r                        8.607e+03  8.526e+03   1.010  0.31435
L3_r                        5.988e+02  4.848e+03   0.124  0.90187
an_L1_r                     4.569e+03  2.986e+03   1.530  0.12812
an_L2_r                     3.835e+02  2.789e+03   0.138  0.89079
an_L3_r                    -2.889e+02  5.197e+02  -0.556  0.57909

an_L1_r                     4.569e+03  2.986e+03   1.530  0.12812
an_L2_r                     3.835e+02  2.789e+03   0.138  0.89079
an_L3_r                    -2.889e+02  5.197e+02  -0.556  0.57909
xchangeRate_L1_r           -4.284e+00  5.944e+00  -0.721  0.47221
xchangeRate_L2_r            4.226e+00  8.186e+00   0.516  0.60644
xchangeRate_L3_r           -5.141e-01  6.053e+00  -0.085  0.93242
othirty_L1_r                5.375e-01  2.126e+01   0.025  0.97986
othirty_L2_r               -6.174e+00  2.269e+01  -0.272  0.78591
othirty_L3_r                7.161e+00  2.038e+01   0.351  0.72585
four_L1_r                   3.753e+00  4.606e+00   0.815  0.41653
four_L2_r                  -1.835e+00  4.920e+00  -0.373  0.70965
four_L3_r                  -1.978e+00  4.159e+00  -0.476  0.63508
four_ns_L1_r               -5.000e-02  6.251e-02  -0.800  0.42503
four_ns_L2_r                3.399e-02  7.387e-02   0.460  0.64615
four_ns_L3_r               -1.623e-03  5.351e-02  -0.030  0.97584
othirty_ns_L1_r             1.525e-02  2.700e-01   0.056  0.95502
othirty_ns_L2_r             3.124e-01  3.322e-01   0.940  0.34857
othirty_ns_L3_r            -3.088e-01  2.881e-01  -1.072  0.28555

 codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

l standard error: 2.392 on 149 degrees of freedom
e R-squared:  0.5918,    Adjusted R-squared:  0.4439
stic: 4.001 on 54 and 149 DF,  p-value: 1.289e-11
```

_binary_2008_Q4" was miscoded, it is for 2009 Q1 instead of 2008 Q4