Homework 4. CS 511.

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Problem 1

1. $\Phi = (\phi \vee \psi_1) \wedge (\phi \vee \psi_2) \wedge (\phi \vee \psi_3)$

   $\Psi = \exists y(y \leftrightarrow \phi) \wedge (y \vee \psi_1) \wedge (y \vee \psi_2) \wedge (y \vee \psi_3)$

   Ψ only holds true if there exists some formula y, which holds true if and only if ϕ holds true. In such a case, this means y is logically equivalent to ϕ, and thus substituting y for ϕ in the rest of the formula will result in a logically equivalent formula to Φ.

2. $\Phi = \theta(\phi_1, \psi_1) \wedge \theta(\phi_2, \psi_2) \wedge \theta(\phi_3, \psi_3)$

   $\Psi = \forall x \forall y(\vee_{1,2,3}(x \leftrightarrow \phi_i) \wedge (y \leftrightarrow \psi_i)) \rightarrow \theta(x, y)$

   If Φ holds true, then we know for any substitutions on ϕ_i or $\psi_i, 1 \leq i \leq 3$, which is exactly what is happening in Ψ. Ψ says that if for all x and y, x is logically equivalent to ϕ_i y is logically equivalent to $\psi_i 1 \leq i \leq 3 then \theta(x, y)$ must hold true. Which can only be the case since Φ is the conjunction of $\theta(\phi_i, \psi_i) 1 \leq i \leq 3$. Going the other way if Ψ holds true, then Φ must also hold true since Ψ is defining the substitution for ϕ_i and ψ_i, so if for all x and y, x is logically equivalent to ϕ_i y is logically equivalent to $\psi_i 1 \leq i \leq 3 then \theta(x, y)$ must hold true, then ϕ must also hold true. Therefore, $\Phi \leftrightarrow \Psi$.

Problem 2 2.3.3

a $\forall x, x \in \nVdash, \nvDash, \nVDash x < 4$

b $\forall x \forall y, x, y \in \nVdash, \nvDash, \nVDash, x < y$

c $\{\exists y, y \in \mathbb{N} y > 1, \exists y, y \in \mathbb{N} y > 2, ...\}$

Problem 3 Murder Mystery

Problems 5 code uploaded to this repository: `https://github.com/ehchao88/CS511_HW4`

Problem 4 Schubert's Steamroller

$\forall x(Killed(x, A) \rightarrow LivesIn(x, D))$

$\exists x(Killed(x, A))$

$LivesIn(A, D)$

$LivesIn(B, D)$

$LivesIn(C, D)$

$\forall x \forall y(Killed(x, y) \rightarrow Hates(x, y) \wedge RicherThan(y, x))$

$\forall x(Hates(A, x) \rightarrow \neg Hates(c, x))$

$Hates(A, C)$

$\forall x(RicherThan(A, x) \rightarrow Hates(B, x)$

$\forall x(Hates(A, x) \rightarrow Hates(B, x))$

$\forall x \exists y(\neg(Hates(x, y))$

$\forall x(Hates(A, x) \leftrightarrow (\neg(x = B) \wedge \neg(x = A)))$

$\neg(A = B)$

Problem 6 code uploaded to this repository: `https://github.com/ehchao88/CS511_HW4`