```
> # ----------------------------------
> #   Eigenvalues & Eigenvectors
> # ----------------------------------
>
>  A <- matrix(c(1.96,.72,.72,1.54),2,2,byrow=T)
>  A
     [,1] [,2]
[1,] 1.96 0.72
[2,] 0.72 1.54
>
>
>  EA <- eigen(A)
>  EA
$values
[1] 2.5 1.0

$vectors
     [,1] [,2]
[1,] -0.8  0.6
[2,] -0.6 -0.8

>
>  EA$values
[1] 2.5 1.0
>
>  eigen(A)$values
[1] 2.5 1.0
>
```

```
>
> # --------------------------------
> #   Singular Value Decomposition
> # --------------------------------
>
>
>    A <- matrix(c(2,0,1,1,0,2,1,1,1,1,1,1),
+           ncol=4,byrow=T)
>    A
     [,1] [,2] [,3] [,4]
[1,]    2    0    1    1
[2,]    0    2    1    1
[3,]    1    1    1    1
>
>
>    svdA <- svd(A)
>    svdA
$d
[1] 3.464102e+00 2.000000e+00 2.117465e-16

$u
          [,1]          [,2]       [,3]
[1,] 0.5773503  7.071068e-01 -0.4082483
[2,] 0.5773503 -7.071068e-01 -0.4082483
[3,] 0.5773503 -3.924812e-17  0.8164966

$v
     [,1]          [,2] [,3]
[1,]  0.5  7.071068e-01 -0.5
[2,]  0.5 -7.071068e-01 -0.5
[3,]  0.5 -2.425902e-16  0.5
[4,]  0.5 -2.425902e-16  0.5

>
>
>   svdA$u %*% t(svdA$u)
             [,1]          [,2]          [,3]
[1,] 1.000000e+00  1.034058e-16  1.523304e-17
[2,] 1.034058e-16  1.000000e+00 -2.974780e-16
[3,] 1.523304e-17 -2.974780e-16  1.000000e+00
```

```
>   t(svdA$v) %*% svdA$v
              [,1]          [,2]         [,3]
[1,]  1.000000e+00 -2.196052e-16 5.551115e-17
[2,] -2.196052e-16  1.000000e+00 9.047667e-17
[3,]  5.551115e-17  9.047667e-17 1.000000e+00
>
>
>   svdA$u%*%diag(svdA$d)%*%t(svdA$v)
             [,1]         [,2] [,3] [,4]
[1,] 2.000000e+00 6.349715e-17    1    1
[2,] 6.915755e-16 2.000000e+00    1    1
[3,] 1.000000e+00 1.000000e+00    1    1
>
>
>   diag(svdA$d) %*%  diag(svdA$d)
     [,1] [,2]         [,3]
[1,]   12    0 0.000000e+00
[2,]    0    4 0.000000e+00
[3,]    0    0 4.483656e-32
>
>
>   eigen(A%*%t(A))$values
[1] 1.200000e+01 4.000000e+00 1.421085e-14
>
>
>   eigen(t(A)%*%A)$values
[1] 1.200000e+01 4.000000e+00 8.881784e-15 3.903128e-18
>
>
> #----------------------------------------
> #  An example where the singular values
> #  are the eigenvalues
> #----------------------------------------
>
>
>  A <- matrix(c(1.96,.72,.72,1.54),2,2,byrow=T)
>  A
     [,1] [,2]
[1,] 1.96 0.72
[2,] 0.72 1.54
```

```
>  svdA <- svd(A)
>  svdA
$d
[1] 2.5 1.0

$u
     [,1] [,2]
[1,] -0.8 -0.6
[2,] -0.6  0.8

$v
     [,1] [,2]
[1,] -0.8 -0.6
[2,] -0.6  0.8

>
>
> #---------------------------------------
> #  Trace and determinant of a matrix
> #---------------------------------------
>
>    A <- matrix(c(1,1, 1,
+                  2,5,-1,
+                  0,1, 1),3,3,byrow=T)
>    A
     [,1] [,2] [,3]
[1,]    1    1    1
[2,]    2    5   -1
[3,]    0    1    1
>
>
>  traceA <- sum(diag(A))
>  traceA
[1] 7
>
>
>  eigenA <- eigen(A)
>  eigenA
$values
[1] 5.336912+0.0000000i 0.831544+0.6578603i 0.831544-0.6578603i
```

```
$vectors
             [,1]                  [,2]                  [,3]
[1,] 0.2664919+0i -0.7907284+0.0000000i -0.7907284+0.0000000i
[2,] 0.9391936+0i  0.3390721-0.0574885i  0.3390721+0.0574885i
[3,] 0.2165581+0i -0.2058692-0.4627004i -0.2058692+0.4627004i

>
>
>  traceA <- sum(eigenA$values)
>  traceA
[1] 7+0i
>
>  Re(traceA)
[1] 7
>
>
>  detA <- Re(prod(eigenA$values))
>  detA
[1] 6
>
>
> #  An example where the eigenvalues
> #  are real numbers
>
>  A <- matrix(c(1,1, 1,
+               2,5,-1,
+               0, 1, -1),3,3,byrow=T)
>  A
     [,1] [,2] [,3]
[1,]    1    1    1
[2,]    2    5   -1
[3,]    0    1   -1
>
>
>  eigenA <- eigen(A)
>  eigenA
$values
[1]  5.372281e+00 -3.722813e-01  3.626284e-16
```

```
$vectors
          [,1]       [,2]       [,3]
[1,] 0.2529080 -0.7087331 -0.8164966
[2,] 0.9557928  0.3750693  0.4082483
[3,] 0.1499922  0.5975118  0.4082483

>
>
>  traceA <- sum(eigenA$values)
>  traceA
[1] 5
>
>
>  detA <- Re(prod(eigenA$values))
>  detA
[1] -7.252569e-16
>
>
> #-------------------------------------------
> #  Eigenvalues of a square symmetric matrix
> #-------------------------------------------
>
>  A<-matrix(c(4,2,-1,2,6,-4,-1,-4,9),3,3,byrow=T)
>  A
     [,1] [,2] [,3]
[1,]    4    2   -1
[2,]    2    6   -4
[3,]   -1   -4    9
>
>  EA <- eigen(A)
>  EA
$values
[1] 12.245772  4.433349  2.320879

$vectors
           [,1]      [,2]       [,3]
[1,] -0.2347350 0.7321107 -0.6394634
[2,] -0.5764345 0.4248579  0.6980108
[3,]  0.7827022 0.5324563  0.3222848
```

```
>  SVDA <- svd(A)
>  SVDA
$d
[1] 12.245772  4.433349  2.320879

$u
           [,1]      [,2]       [,3]
[1,] -0.2347350 0.7321107 -0.6394634
[2,] -0.5764345 0.4248579  0.6980108
[3,]  0.7827022 0.5324563  0.3222848

$v
           [,1]      [,2]       [,3]
[1,] -0.2347350 0.7321107 -0.6394634
[2,] -0.5764345 0.4248579  0.6980108
[3,]  0.7827022 0.5324563  0.3222848

>
>
> #-----------------------------------------
> #  An example of a square symmetric matrix
> #  that is not positive definite
> #-----------------------------------------
>
>  W<-matrix(c(4,2,-1,2,6,-4,-1,-4,-9),3,3,byrow=T)
>  W
     [,1] [,2] [,3]
[1,]    4    2   -1
[2,]    2    6   -4
[3,]   -1   -4   -9
>
>
>  EW <- eigen(W)
```

```
>  EW
$values
[1]   8.151345   2.865783 -10.017128

$vectors
           [,1]       [,2]       [,3]
[1,] -0.4665008  0.8838166 0.03528860
[2,] -0.8550024 -0.4607943 0.23799069
[3,]  0.2266009  0.0808510 0.97062616

>
>
>  t(EW$vectors)%*%EW$vectors
              [,1]          [,2]          [,3]
[1,]  1.000000e+00 -5.981747e-17 -3.102173e-17
[2,] -5.981747e-17  1.000000e+00  6.118966e-18
[3,] -3.102173e-17  6.118966e-18  1.000000e+00
>
>
>  SVDW <- svd(W)
>  SVDW
$d
[1] 10.017128  8.151345  2.865783

$u
           [,1]       [,2]       [,3]
[1,] 0.03528860 -0.4665008  0.8838166
[2,] 0.23799069 -0.8550024 -0.4607943
[3,] 0.97062616  0.2266009  0.0808510

$v
            [,1]       [,2]       [,3]
[1,] -0.03528860 -0.4665008  0.8838166
[2,] -0.23799069 -0.8550024 -0.4607943
[3,] -0.97062616  0.2266009  0.0808510



>
>
```

```
> #-------------------------------------
> #  Inverse of a matrix
> #-------------------------------------
>
>   A <- matrix(c(1.96,.72,.72,1.54),2,2,byrow=T)
>
>  Ainv <- solve(A)
>  Ainv
       [,1]   [,2]
[1,]  0.616 -0.288
[2,] -0.288  0.784
>
>
>  A%*%Ainv
              [,1]         [,2]
[1,]  1.000000e+00 1.908196e-17
[2,] -9.240113e-17 1.000000e+00
>
>
> #  Use the spectral decomposition
> #  to compute the inverse of a matrix
>
>  Aev<-eigen(A)$vectors
>  Aeval<-eigen(A)$values
>  Ainv2<-Aev%*%solve(diag(Aeval))%*%t(Aev)
>  Ainv2
       [,1]   [,2]
[1,]  0.616 -0.288
[2,] -0.288  0.784
>
>
> #-----------------------------------------
> #    Solutions to linear equations
> #-----------------------------------------
>
>  x<-c(1,1)
>  x
[1] 1 1
>
>  b<-solve(A,x)
```

```
>  b
[1] 0.328 0.496
>
>  A%*%Ainv
              [,1]         [,2]
[1,]  1.000000e+00 1.908196e-17
[2,] -9.240113e-17 1.000000e+00
```