Lab 2: The pendulum

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Introduction

The objective of this lab was to learn about dynamics of systems which have a non linear force law and also to learn how to use python to solve coupled ordinary differential equations. In exercise 1 the python script was used to solve the linear pendulum equation, by the use of the trapezoidal rule. The trapezoidal rule was also used tin exercise 2 but in this case it was used to solve the non linear pendulum equation. In exercise 3 the non linear pendulum equation was again solved but now the Runge-Kutta algorithm was used in place of the trapezoidal rule. In exercise 4 and 5 the Runge-Kutta algorithm was again used, in exercise 4 for a damped non linear pendulum and in exercise 5 for a damped, driven non linear pendulum.

Methodology

Exercise 1 : Solving Linear pendulum equation

The following equation shows the motion of the pendulum

$$\frac{\mathrm{d}^2\theta}{\mathrm{dt}^2} = \frac{-g}{\mathrm{L}}\sin\theta \quad \text{(equation 1)}$$

For the simple pendulum, if it only being used for small angles, sin(θ) can be approximated to equal θ. This means that equation 1 can be simplified to the following:

$$\frac{\mathrm{d}^2\theta}{\mathrm{dt}^2} = \frac{-g}{\mathrm{L}}\theta \quad \text{(equation 2)}$$

This simplification allows the equation to be solved analytically.

$$f(\theta,\omega,t) = -\frac{g}{L}\sin(\theta) - k\omega + Acos(\varphi t) \text{ (equation 4)}$$

Equation 4 was coded into python with sin(θ) replaced with θ as we are using small angles. Theta was initialised to 0.2, and omega as 0.0. Time t was initialised to zero, and dt, the change in time to 0.01. nsteps was set to 0 as well. g/L was set to 1. K and A were set to zero and phi was set to 0.66667. This made equation 4 simply a function of theta as seen in equation 5, which then applies to a linear non damped oscillator

$$f(\theta) = -\frac{g}{L}\theta \quad \text{(equation 5)}$$

The code given for the trapezoid rule was added in a for loop for nsteps between 0 and 1000. The values of theta and omega against nsteps was made, with the x axis having a range of [0,500] and the y axis [-π,π]. The script was then used for a variety of initial condition for theta, (0.2, 1.0, 3.14, 0.0) and for omega (0.0, 0.0, 0.0, 1.0), which was also plotted.

Exercise 2: Solving non liner pendulum equation

In exercise 2 as this is for a nonlinear pendulum sin(θ) was put back into the equation and as there is still no damping or driving the equation is as follows:

$$f(\theta) = -\frac{g}{L}\sin(\theta) \text{ (equation 6)}$$

The code from part 1 was copied and sin(θ) replaced θ, using np.sin, solving the non linear pendulum equation using the trapezoidal rule. Four graphs were made with the same initial conditions, for theta, (0.2, 1.0, 3.14, 0.0) and for omega (0.0, 0.0, 0.0, 1.0).

Exercise 3: Change the numerical integration algorithm

In exercise 2 the non linear pendulum method was again solved, but this time the trapezoidal rule was replaced by the Runge-Kutta algorithm. The trapezoidal rule is accurate to the Δt order, but the Runge-Kutta algorithm is accurate to the Δt^2 order, as the Taylor series expansion for the trapezoidal rule is carried out at the beginning of the time step, whereas the Runge-Kutta algorithm has the Taylor series carried out about the middle of the time step. The code from part 2 was copied and equation 6 was again used. In the for loop the code for the trapezoidal rule was replaced by the code for the Runge-Kutta algorithm. A comparison was made between the Runge-Kutta algorithm and the trapezoidal rule, using initial values theta= 3.14 and omega = 0.0. The results were then graphed.

Exercise 4: The damped non linear pendulum

In exercise 4 the behaviour of a damped non linear pendulum was simulated, . The code from part 3 was copied, so the Runge-Kutta algorithm was used, but now the pendulum was damped. This meant changing the initial variable of k from 0.0 to 0.5 in equation 4. This enabled the damping and meant the formula changed to the following equation:

$$f(\theta, \omega) = -\frac{g}{L}\sin(\theta) - k\omega \text{ (equation 7)}$$

A plot was made of theta vs time and omega vs time as well. Theta was initialised to 3.0 and omega was initialised to 0.0.

Exercise 5: The damped, driven non linear pendulum

In exercise 5 a driving force was added to the damped, driven pendulum. The driving force has a period 2θ/ϕ and a frequency of ϕ. If the motion has the same period as the driving force or an integer multiple, the motion is periodic, if not the motion is a aperiodic. As there is both damping and a driving force chaotic motion can be found.

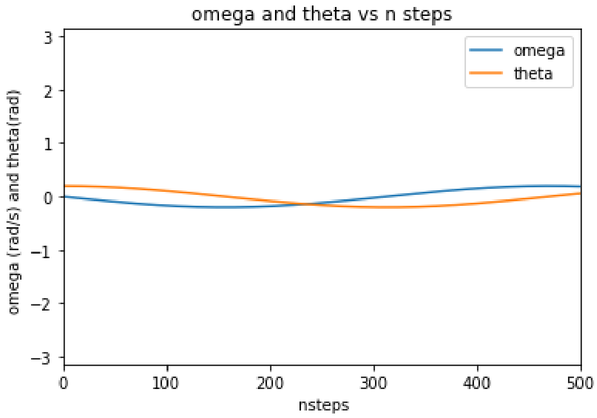
As there is now both is now damping and chaos, the full equation 4 can be used:

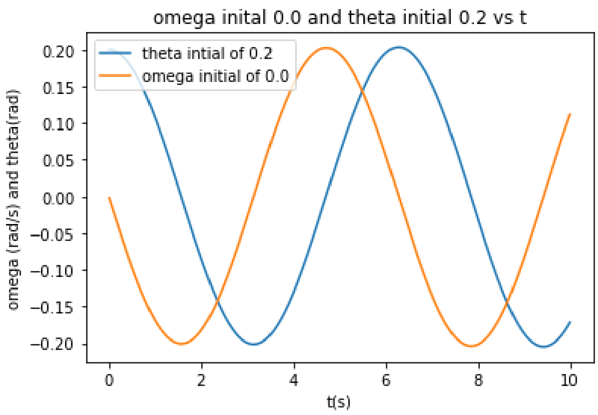
$$f(\theta, \omega, t) = -\frac{g}{L}\sin(\theta) - k\omega + Acos(\varphi t) \text{ (equation 4)}$$

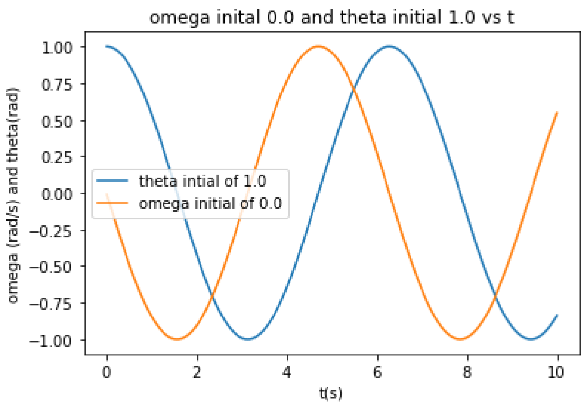
A was set to 0.9, enabling the sinusoidal driving force. A variable iteration-number was initialised at zero and another variable transient was initialised at 5000. The iteration number was set to increase by 1 every time theta and omega updated. Once the iteration number was greater than the transient, theta vs omega was plotted. This is a phase plot , which for a linear pendulum should be a circle, and for a damped, driven non linear pendulum is a loop.

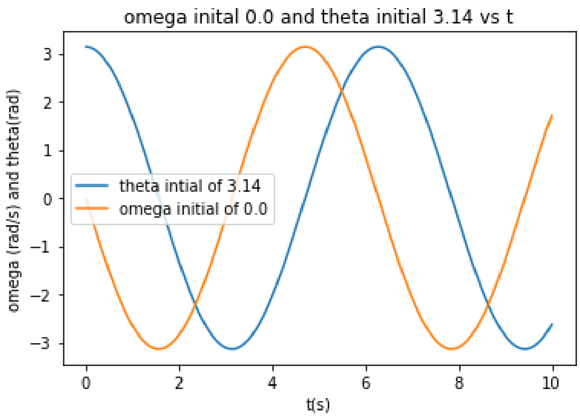
Results

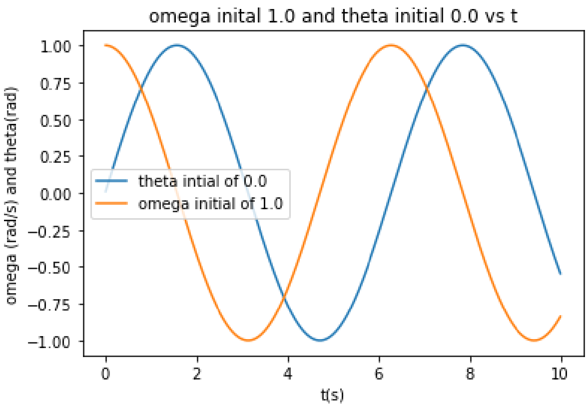
Exercise 1







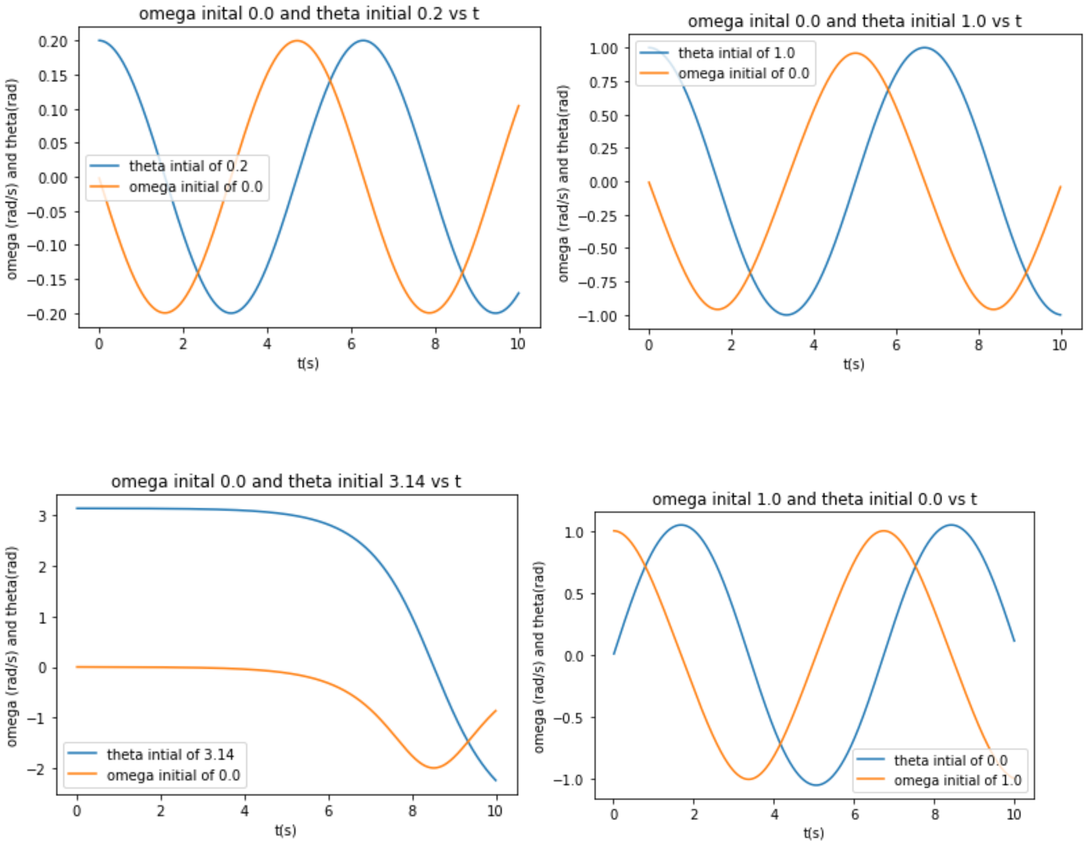




(graphs of linear pendulum exercise 1)

From these graphs of the linear pendulum we can see that the linear pendulum motion is sinusoidal, which would imply that it motion is simple harmonic.

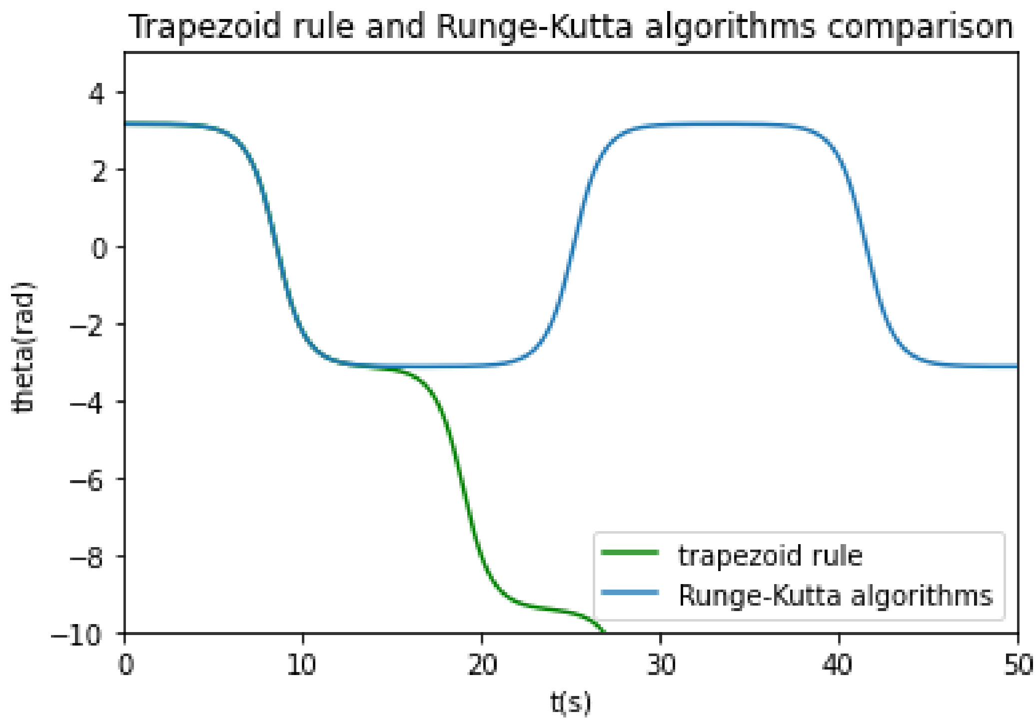
Exercise 2



(figure 2: graphs of exercise 2 non linear pendulum)

For these graphs of the non linear pendulum they are sinusoidal and almost identical to the graphs in part 1, except when the initial conditions have theta set to 3.14. This is because for the initial conditions of theta set to 0.0, 0.2 and 1.0 sin(θ) can be approximated to θ as the theta value is low enough but for the theta value of 3.14 the approximation doesn't hold as sin of 3.14 is approximately 0 which does not equal to 3.14. This is the reason for the variation of these graphs.

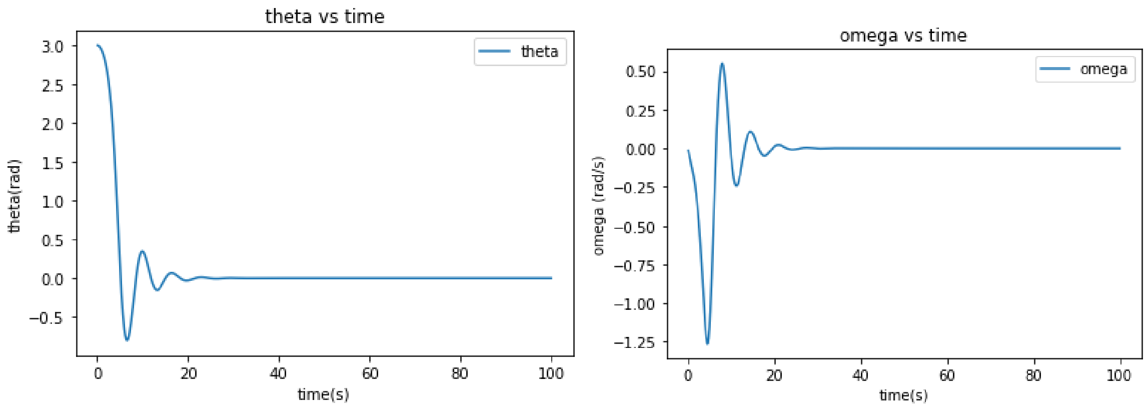
Exercise 3



(Figure 3: graph of trapezoidal rule and Runge-Kutta algorithms comparison)

The Runge-Kutta algorithm was able to stimulate the non linear pendulum along with the trapezoidal rule. From figure three a direct comparison is able to be seen between the trapezoidal rule and the Runge-Kutta algorithm. It is clear that after one time one full rotation of theta the trapezoidal rule begins to fail, where as the Runge-Kutta algorithm is able to continue past into any number of rotations.

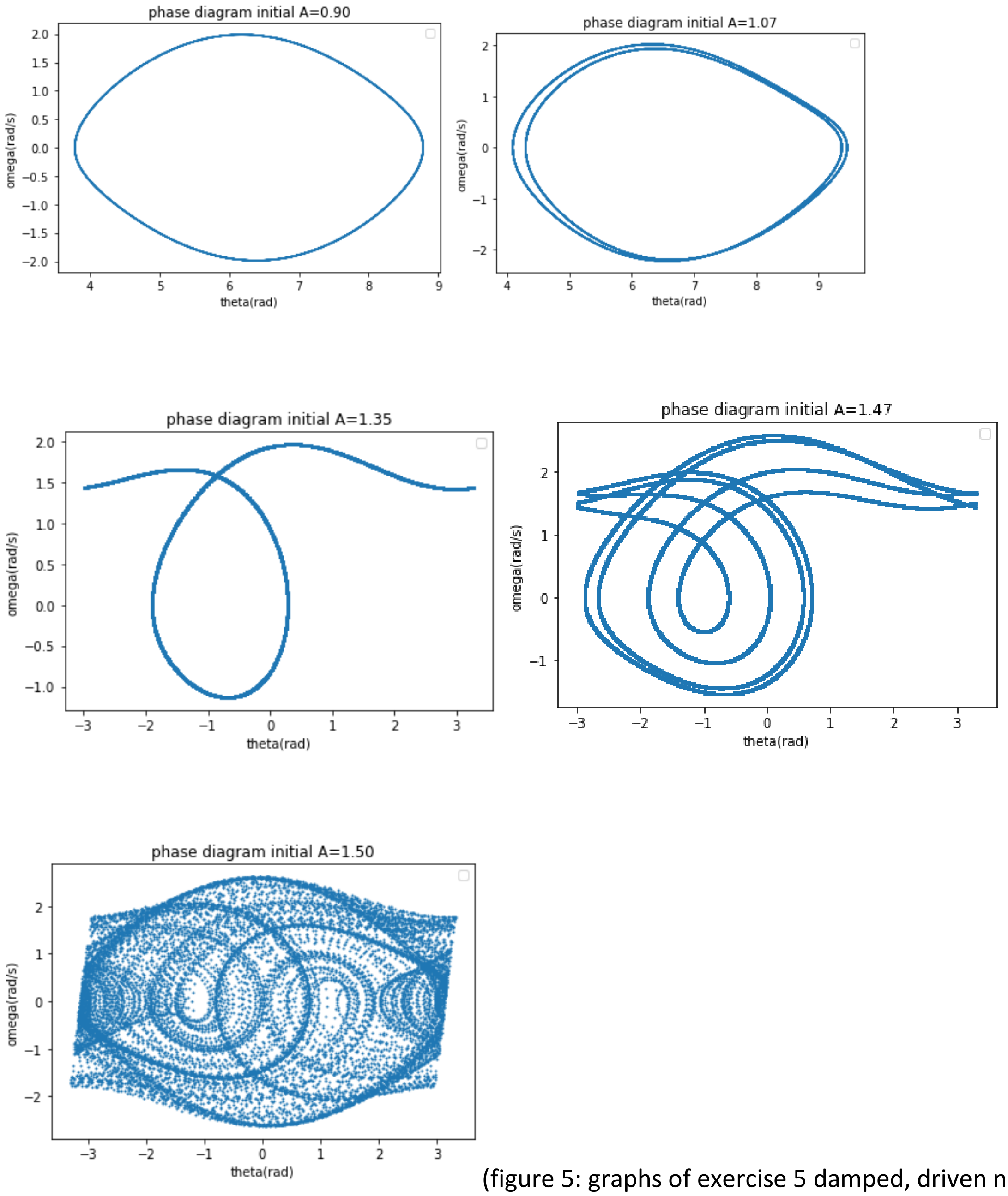
Exercise 4



(figure 4: graphs of theta vs time and omega vs time)

From figure 4 it can be seen that the angle reached by the pendulum gradually decreases as the dampening has been added and that it the angle eventually reaches zero implying the motion of the pendulum has stopped. Similarly omega, the angular frequency, also starts to gradually decrease and eventually reaches zero due to the addition of the dampening, again implying the pendulum motion has finished. This is what would be expected of a dampened pendulum.

Exercise 5



(figure 5: graphs of exercise 5 damped, driven non linear pendulum)

From figure 5 we can see the motion of the damped, non linear, driven pendulum with different initial amplitudes and set k and ϕ, frequency of driving force, at 0.5 and 0.66667.

For The amplitude initially set at 0.90 the graph shows the motion of a closed loop, implying one cycle completed by the pendulum

For The amplitude initially set at 1.07, the graph shows the motion of two closed loops, implying two cycles completed by the pendulum.

For The amplitude initially set at 1.35, the graph shows that the motion is periodic as there are closed loops but the motion is started to get less simple

For The amplitude initially set at 1.47, the graph shows periodic motion as closed loops are visible but again more complex

For The amplitude initially set at 1.50, the graph shows chaotic motion as there are no more closed loops, which means the motion is no longer periodic.

Figure 4 shows how a slight change in amplitude can make a periodic pendulum become aperiodic and chaotic.

Conclusion

In conclusion we were able to use the trapezoidal rule to solve coupled differential equations in exercise 1 and 2, and using the Runge-Kutta method for exercise 3,4 and 5. By comparing exercise 1 and 2 the difference between the linear pendulum and non linear pendulum was shown by the difference in the $\sin(\theta)$ to θ approximation for the theta value of 3.14. By comparing exercise 2 and 3 the Runge-Kutta method was found to be superior than the trapezoid method after one rotation. In exercise 4 the addition of the dampening caused the theta value and omega values to decrease and reach, showing the stopping of the pendulum motion. In exercise 5 the increase of the initial amplitude causes the motion of the pendulum to become more complex until it eventually became chaotic.