Coupled Pendulum

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Abstract

This experiment was conducted to investigate different physical phenomena associated with coupled pendulums. The experiment was split up into four parts. In part 1 the mode frequency was found for mode 1(when the pendulums were oscillating in opposite directions) and mode 2 (when the pendulums were oscillating in the same direction, using both a stopwatch and videocom motion software. Using the stopwatch mode 1 frequency was found to be 0.64Hz, the same as using videocom motion, and the frequency of mode 2 was found to be 0.61Hz using the stopwatch and 0.59Hz using videocom. Also in part one the uncoupled frequency was found to be 0.6Hz. In part 2 the superposition of the modes was investigated and the pendulum were found to follow the following equations $x_A = Acos(\omega_1 t) + Acos(\omega_2 t)$ And $x_B = Acos(\omega_1 t) + Acos(\omega_2 t)$. in part three the pendulums were uncoupled using normal coordinates the frequency of the normal equations of motion were found to match up with the mode frequencies. In part four the affects of the hight of the clamps and width of the pendulums on the mode frequencies were tested. Only the height of the claps were found to have an affect, as the increase in height lowered the difference in the mode frequencies.

Introduction

Coupling is when the position of one pendulum affects the acceleration of the other pendulum

$$\ddot{x}_A = \alpha x_A + \beta x_B$$

(1)

$$\ddot{x}_B = \gamma x_A + \delta x_B$$

(2)

Where

$$x_A = A_1 \cos(\omega_1 t + \varepsilon_1) + A_2 \cos(\omega_2 \mathrm{t} + \varepsilon_2)$$

(3)

$$x_B = B_1 \cos(\omega_1 t + \varepsilon_1) + B_2 \cos(\omega_2 \mathrm{t} + \varepsilon_2)$$

(4)

As seen from equations 3 an 4 , the general motion of the pendulums is the superposition of simple harmonic oscillators, where ω_1 and ω_2 are the mode frequencies, which are the mode frequencies. The mode is when both pendulums go through their own pints of equilibrium simultaneously and oscillate at the same frequency.

When the pendulums are in one of their modes equations 3 and 4 are just

$$x_A = Acos(\omega_0 t + \varepsilon)$$

(5)

$$x_B = Bcos(\omega_0 t + \varepsilon)$$

(6)

Therefore there is no exchange in energy between pendulums, as either A_1=B_1=0 or A_2=B_2=0.

If these values of x_a and x_b are subbed into equation 1

$$\ddot{x}_A = \alpha Acos(\omega_0 t + \varepsilon) + \beta Bcos(\omega_0 t + \varepsilon)$$

(7)

$$\omega\text{\^{}}2 = (1/2)(-(\alpha + \delta) \pm \{(\alpha - \delta)^2 + 4\beta\gamma\}^{\frac{1}{2}}$$

(8)

And $\alpha=\delta=-\omega_k^2$, $\beta=\gamma=c^2$

So $\omega_1^2 = \omega_k^2 + c^2$

And $\omega_2^2 = \omega_k^2 - c^2$

As the pendulums are swinging in phase, $\omega_2^2 = w_0^2$

Therefore $\omega_k^2 = c^2 + w_0^2$

And $\omega_1^2 = w_0^2 + 2c^2$

The angular frequency of coupled oscillators is half of the sum of the individual angular frequencies

$$\frac{\omega_1 + \omega_2}{2}$$

(9)

And the beat frequency is the difference of the two : $\omega_{Beat} = \omega_1 - \omega_2$

$$q_1 = Ccos(\omega_1 t)$$

(10)

$$q_2 = Dcos(\omega_2 t)$$

(11)

$$x_A = \left(\frac{1}{2}\right)(q_1 + q_2) = \left(\frac{1}{2}\right)Ccos(\omega_1 t) + (\frac{1}{2})Dcos(\omega_2 t)$$

(12)

$$x_B = \left(\frac{1}{2}\right)(q_1 - q_2) = \left(\frac{1}{2}\right)Ccos(\omega_1 t) - (\frac{1}{2})Dcos(\omega_2 t)$$

(13)

$$x_A = A_0 = \left(\frac{1}{2}\right) C + \left(\frac{1}{2}\right) D$$

(14)

$$x_B = 0 = \left(\frac{1}{2}\right) C - \left(\frac{1}{2}\right) D$$

(15)

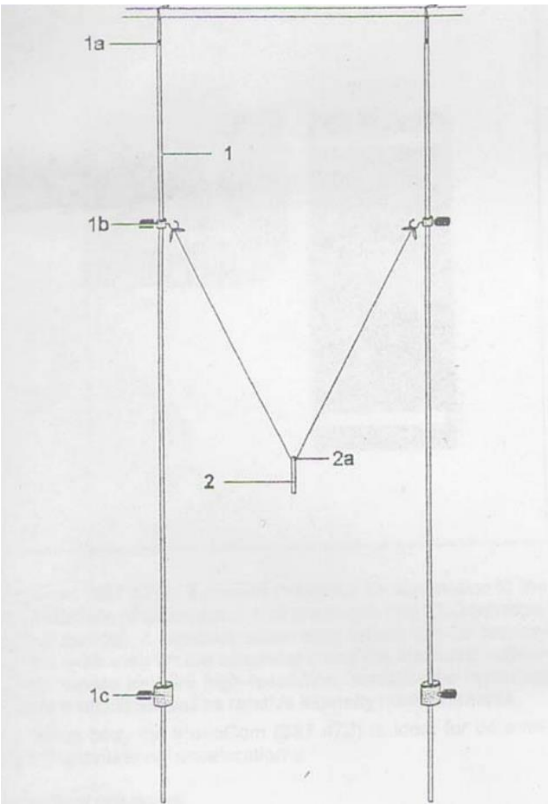
Therefore $C = A_0 = D$

Therefore $x_A = \left(\frac{1}{2}\right) A_0\left(cos(\omega_1 t) + cos(\omega_2 t)\right) = A_0 \cos\left(\frac{\omega_2 - \omega_1}{2}\right) \cos\left(\frac{(\omega_2 + \omega_1)}{2}\right)$ (16)

$x_B = \left(\frac{1}{2}\right) A_0\left(cos(\omega_1 t) - cos(\omega_2 t)\right) = A_0 \sin\left(\frac{\omega_2 - \omega_1}{2}\right) \sin\left(\frac{(\omega_2 + \omega_1)}{2}\right)$ (17)

$\frac{|\omega_1 + \omega_2|}{2} \gg \frac{|\omega_1 - \omega_2|}{2}$ motion varies with frequency $\frac{\omega_1 + \omega_2}{2}$ and $\frac{2\pi}{|\omega_1 - \omega_2|}$ therefore beat frequency is $\omega_1 - \omega_2$

Intro



(figure 1)

This experiment was split into four parts. In part 1 the mode frequency of the pendulum was found in two ways, using a stopwatch and then with videocom motions software. Using the stopwatch, the pendulums were set swinging and the amount of oscillations was divided by the time taken. This was done for both modes. Using videocom motions, the motion of oscillations for both modes over 80 seconds was taken. The Fourier transform of the wave motion then gave the frequencies of the modes. Finally in part 1, one of the pendulum was removed and the frequency was found using videocom motions and compared to the previous frequencies

In section 2 one of the pendulums was pulled aside to show how the pendulums oscillate with the equations of motion $x_A = Acos(\omega_1 t) + Acos(\omega_2 t)$ (20)

And $x_B = Acos(\omega_1 t) + Acos(\omega_2 t)$ (21)

. to prove this; when pendulum A was oscillating at its maximum displacement pendulum B was seen to be oscillating at its minimum, as seen in the graph, both pendulums had to have the same max amplitude. Also it was shown how each oscillator oscillates with a beat frequency of $\omega_{Beat} = \omega_1 - \omega_2$ which equals zero as when the beat frequency is zero the pendulums will stay in this cycle of motion.

In section 3 the motion of the two pendulums were decoupled digitally, using normal co-ordinates and then compared with the manually decoupled pendulum.

In section 4 aspects of the apparatus were changed to analyse what were the variable that affected the coupling of the pendulum. The distance between the two pendulums and the height of the mass connecting them were varied and their affects on the coupling was measured.

Experimental procedure

Initially the apparatus was set up in figure 1. The pendulums wee separated at a distance of 20cm. The clamps were attached 20cm above the masses. The infrared camera was set up, perpendicularly, at a suitable distance from the pendulum to track the motion of oscillation of the two pendulums. Video com motions was configured to 80fps and the default positions of the pendulums were configured.

In part 1 when the mode frequencies were found using a stopwatch, the pendulums were set oscillating in mode 1, in the same direction, and the time for 15 oscillations was taken, which 15 was then divided by to find the frequency. This was repeated twice and the average frequency was taken. The same steps were used to calculate the frequency when the pendulums were set oscillating in mode 2, in opposite directions.

Then videocom motions recorded the pendulums oscillating in mode for 80 seconds. The Fourier transform tab of the waveform was then used to obtain the frequency of the oscillations. These steps were used for the mode frequencies for both opposite and the same direction oscillations

Finally in part one, the mass connection the pendulums was disconnected as to decouple the pe. Then one of the pendulums was set oscillating and videocom motions was again used to track the motion. Then the Fourier transform was used to find the frequency.

In part 2 one of the pendulums was pulled to the side and set oscillating, not in mode with the other, and the superposition of the two modes was investigated, and the following equations were proved:
$x_A = Acos(\omega_1 t) + Acos(\omega_2 t)$ (20)

And $x_B = Acos(\omega_1 t) + Acos(\omega_2 t)$ (21)

In part 3 two equations were used to acquire the normal coordinates of the pendulums: s1+s2 and s2-s1. These equations were put into the formula section in videocom motions. The pendulums were then set to oscillate in both modes, and the frequency of the decoupled pendulum was found.

In part 4 the oscillations were measured with the mass coupling set to different height between the pendulums. The heights chosen were 10cm, 27 cm and 30cm,. This was also done for varying widths between the pendulum. The widths used were 12cm, 22cm and 20cm. The beat frequencies for each of these set ups was found. The set up which had a variable change related to a change in beat frequency was the variable that affected the coupling.

Results

Part 1 mode 1 (using stopwatch)

Oscillations	Time(s) ±0.005	Frequency(Hz)
15	23.36	0.64+-0.002
15	23.44	0.64+-0.002
15	23.52	0.64+-0.002

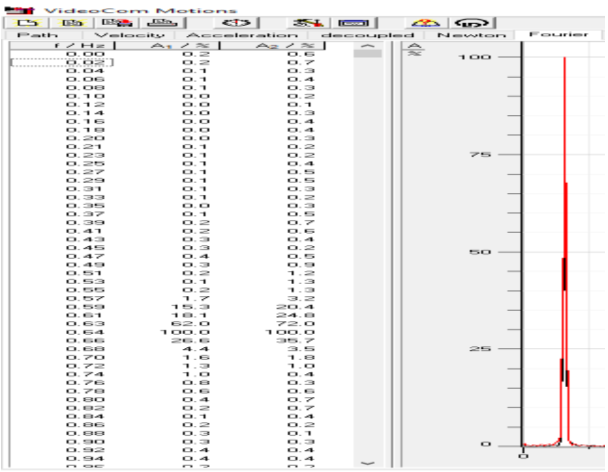
Therefore the average frequency for mode 1 was 0.64Hz+-0.002

Part 1 mode 2 (using stopwatch)

oscillations	time (s) ±0.005	frequency (Hz)
15	24.24	0.62+-0.002
15	24.58	0.61+-0.002
15	24.96	0.6+-0.002

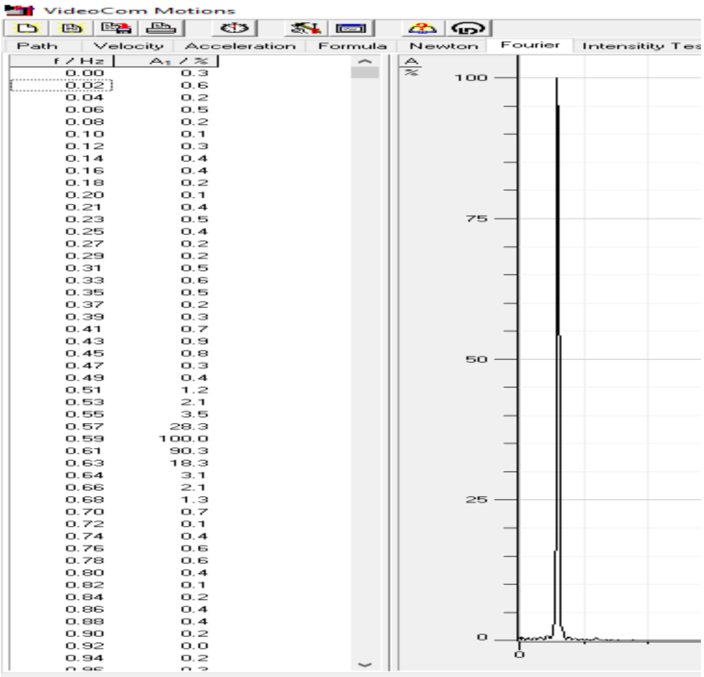
Therefore the average frequency for mode 2 was 0.61+-0.002

Part 1 mode 1 (videocom motions)



(figure 2)the mode 1 frequency was found to be 0.64Hz+-0.005, which is the same as was found using the stopwatch.

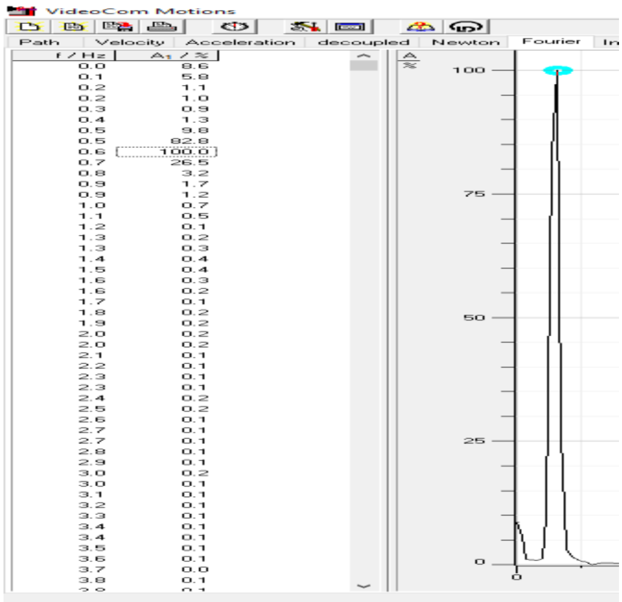
Part 1 mode 2 (using videocom motions)



f / Hz	A_1 / %
0.00	0.3
0.02	0.6
0.04	0.2
0.06	0.5
0.08	0.2
0.10	0.1
0.12	0.3
0.14	0.4
0.16	0.4
0.18	0.2
0.20	0.1
0.21	0.4
0.23	0.5
0.25	0.4
0.27	0.2
0.29	0.2
0.31	0.5
0.33	0.6
0.35	0.5
0.37	0.2
0.39	0.3
0.41	0.7
0.43	0.9
0.45	0.8
0.47	0.3
0.49	0.4
0.51	1.2
0.53	2.1
0.55	3.5
0.57	28.3
0.59	100.0
0.61	90.3
0.63	18.3
0.64	3.1
0.66	2.1
0.68	1.3
0.70	0.7
0.72	0.1
0.74	0.4
0.76	0.6
0.78	0.6
0.80	0.4
0.82	0.1
0.84	0.2
0.86	0.4
0.88	0.4
0.90	0.2
0.92	0.0
0.94	0.2

(figure 3)the mode 2 frequency was found to be 0.59Hz±0.005

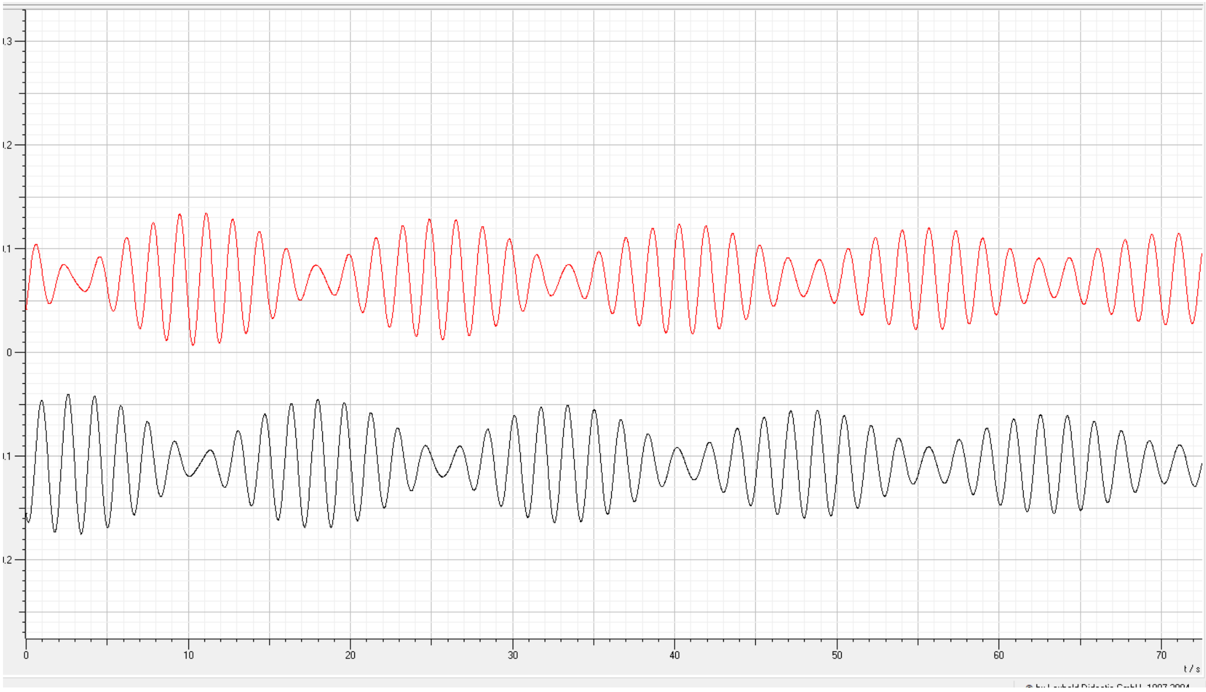
Part 1 uncoupled



f / Hz	A_1 / %
0.0	8.6
0.1	5.8
0.2	1.1
0.2	1.0
0.3	0.9
0.4	1.3
0.5	9.8
0.5	82.8
0.6	100.0
0.7	26.5
0.8	3.2
0.9	1.7
0.9	1.2
1.0	0.7
1.1	0.5
1.2	0.1
1.3	0.2
1.3	0.3
1.4	0.4
1.5	0.4
1.6	0.3
1.6	0.2
1.7	0.1
1.8	0.2
1.9	0.2
2.0	0.2
2.0	0.2
2.1	0.1
2.2	0.1
2.3	0.1
2.3	0.1
2.4	0.2
2.5	0.2
2.6	0.1
2.7	0.1
2.7	0.1
2.8	0.1
2.9	0.1
3.0	0.2
3.0	0.1
3.1	0.1
3.2	0.1
3.3	0.1
3.4	0.1
3.4	0.1
3.5	0.1
3.6	0.1
3.7	0.0
3.8	0.1

(figure 4) the uncoupled frequency was found to be 0.6±0.05Hz

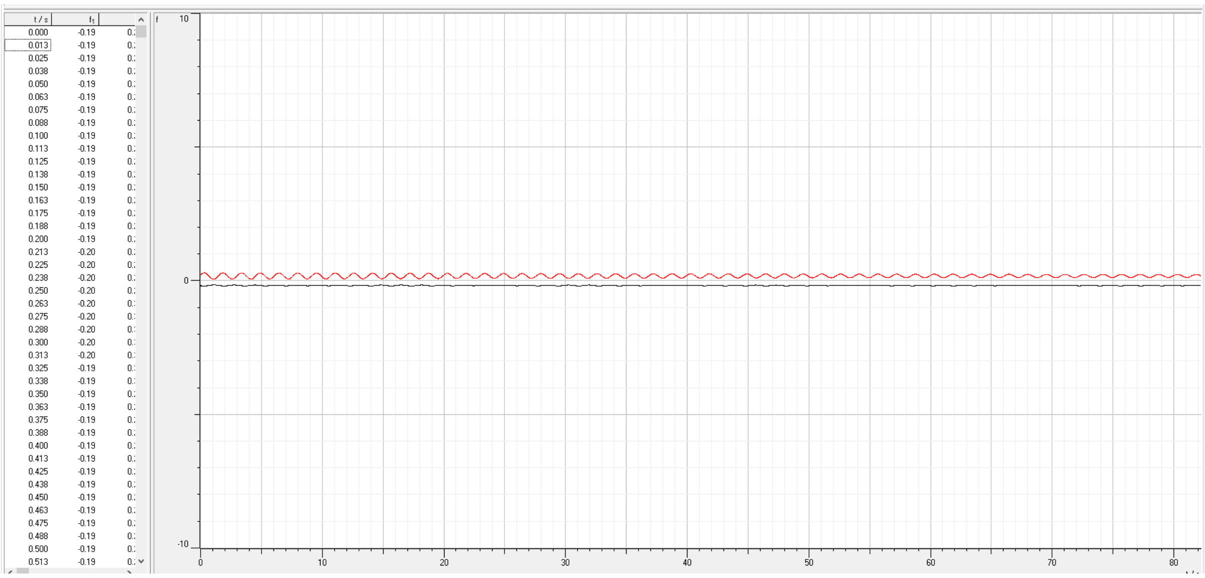
Part 2



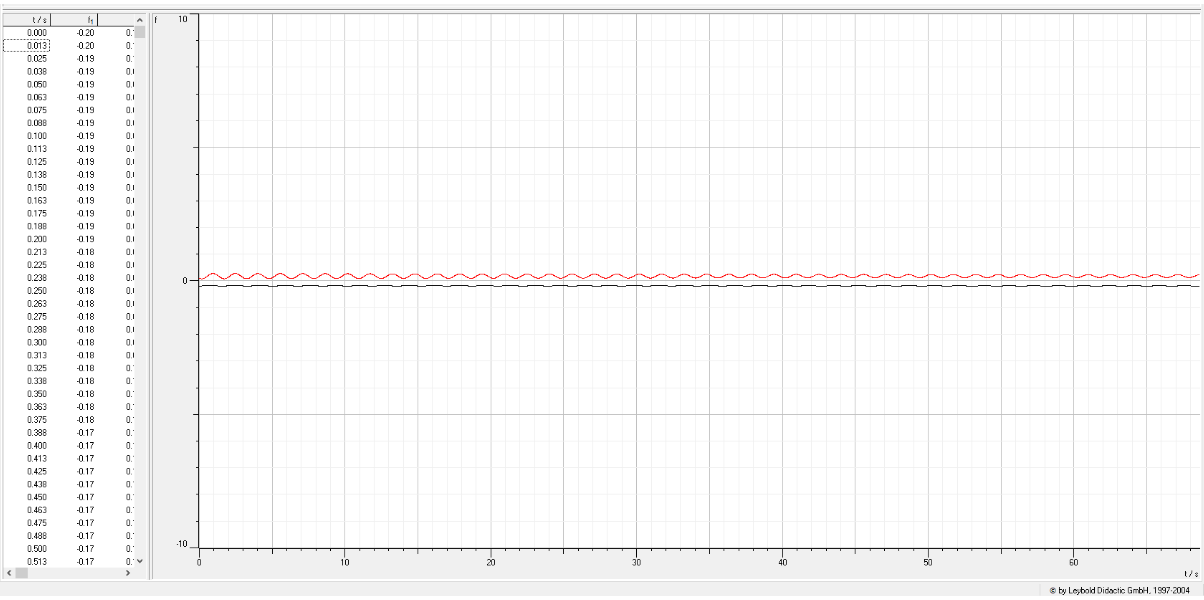
(figure 5)

The amplitudes are seen to be approximately the same size. When pendulum A is oscillating at its max displacement pendulum B is oscillating at its minimum. The beat frequency was found to be zero as both mode frequencies were found to be 0.6±0.05Hz

Part 3



(figure 6)



(figure 7)

The frequencies of the normal equations of motion were found to be 0.64±0.005Hz and 0.59±0.005Hz, which correspond to the mode 1 and mode 2 frequencies of the coupled pendulum.

Part 4

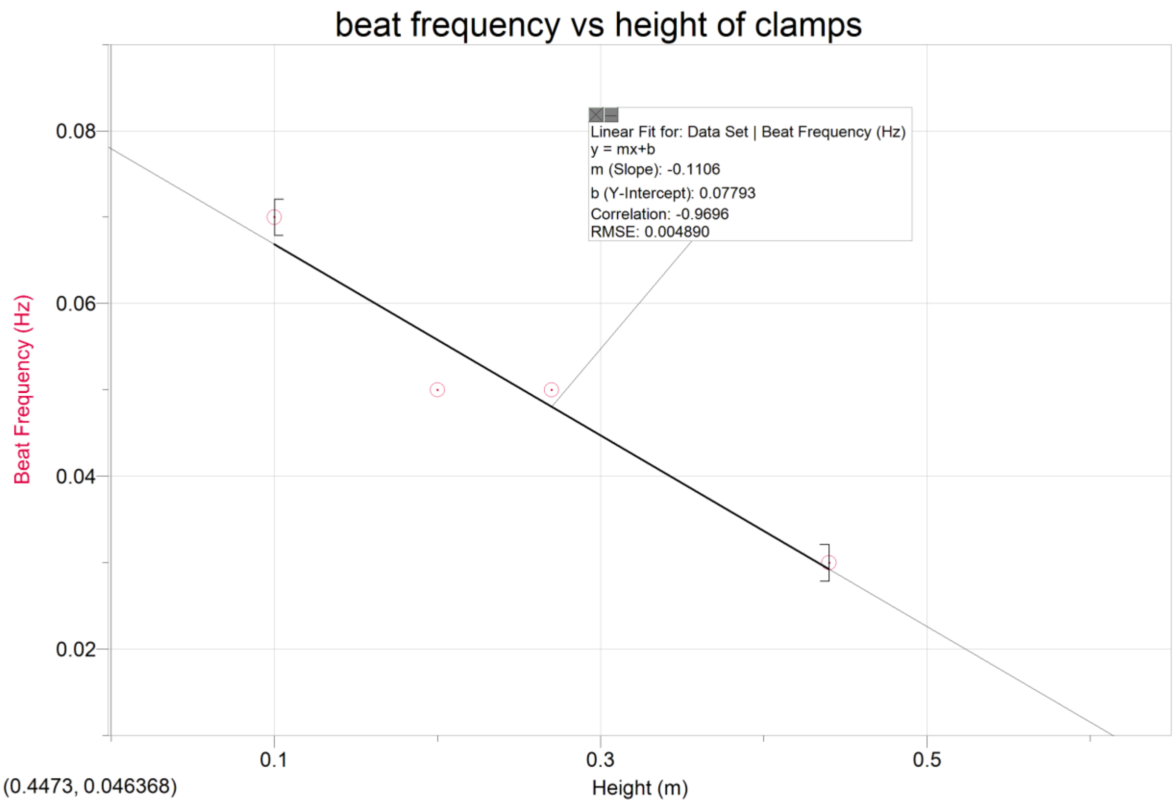
Change in height of clamp:

height(cm) ±0.5	mode 1 freq±0.005	mode 2 freq±0.005	beat frequency±0.007
10	0.66	0.59	0.07
20	0.64	0.59	0.05
27	0.64	0.59	0.05
44	0.62	0.59	0.03

Change in width of pendulums :

width (cm)±0.5	mode 1 freq±0.005	mode 2 freq±0.005	beat frequency±0.007
12	0.64	0.59	0.05
18	0.64	0.59	0.05
20	0.64	0.59	0.05
22	0.64	0.59	0.05

By analysing the results it was found that only the height of the clamp and not the width affected the coupling.



(figure 8)

Analysis of error

The error for the frequency was found in part 1 was found using the following formula $\sqrt{(\frac{\Delta t^2}{t})}$

The error for the beat frequency in part 4 was found using the following formula $\sqrt{(\Delta\omega_1)^2 + (\Delta\omega_2)^2}$

Sources of error could have occurred when using the stopwatch, du to human error, and when setting the pendulums oscillating by unknowingly using a

Conclusion

In conclusion the mode frequencies were found to be 0.64+-0.002Hz for mode 1, using the stopwatch and videocom, and 0.61+-0.002Hz for mode 2 using the stopwatch and 0.59+-0.005Hz using video motions. The uncoupled frequency was found to be 0.59+-0.005Hz. In part 2 the equations of motion (20) and (21) for the pendulums were found to be true as the both pendulums had the same max amplitude, when pendulum A was oscillating at its max displacement, pendulum B was oscillating with its minimum, and the beat frequency was found to be zero as both mode frequencies were found to be 0.6±0.05Hz. In part 3 the normal motion frequencies were found to correspond to the beat frequencies. In part 4 the increase of height of the clamps increased the coupling while the varying of the width had no affect on the coupling.

Appendix

$\ddot{x}_A = \alpha x_A + \beta x_B$ Position of pendulum A: where α and β are constants

$\ddot{x}_B = \gamma x_A + \delta x_B$ position of pendulum B: where γ and δ are constants

(figure 1)apparatus :1 pendulum, 1a point where pendulum is attached, 1b clamp, 1c bob,2 coupling mass, 2a attachment point of coupling mass.

(figure 2) part 1 mode 1 frequency using videocom motions Fourier transform

(figure 3) part 1 mode 2 frequency using videocom motions Fourier transform

(figure 4) part 1 uncoupled frequency Fourier transform

(figure 5) part 2 superposition of modes wave patterns

(figure 6) part 3 uncoupled digitally waveform mode 1

(figure 7) part 3 uncoupled digitally waveform mode 2

(figure 8) part 4 graph of beat frequency vs height of clamps