From Hydrogen to Supernovae: Nuclear Evolution in Stars

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Introduction

Nuclear fusion is the reaction where light nuclei fuse together to form heavier elements, thereby releasing massive amounts of energy. It is the process by which stars fuel themselves throughout their evolution, from protostars to neutron stars. There are many different fusion processes which can occur, and understanding them could be the key to sustainable energy production on Earth. This poster will primarily look at the fusion processes which occur in stars, white dwarfs and neutron stars, and finally how fusion could be achieved on Earth.

Proton-Proton Chain Reaction

When the core of a protostar heats up to 10^7 K, the pp-chain becomes the primary energy source. Two protons fuse to form a duetron. One of the protons will experience beta plus decay.[1]

$$p + p \rightarrow^2_1 D + e^+ + v_e$$

The resultant positron will annihilate with an electron into two gamma rays and the complete equation can be seen.

$$p + p + e^- \rightarrow^2_1 D + v_e + 1.442MeV$$

This reaction is quite slow due to the fact is it started by weak nuclear forces. On average a proton stays in the core of the sun for $1x10^9$ years. Due to the time taken it has so far been experimentally impossible to calculate the cross section[2].

A fast reaction is now initiated by the strong nuclear force. [2].

$${}^2_1D +^1_1 H \rightarrow^3_2 He + \gamma + 5.493MeV$$

In our sun deuterium only exists for approximately one second before fusing with a proton[1]. The overall reaction is

$$4^1H^+ + 2e^- \rightarrow^4 He^{2+} + 2v_e$$

This releases 26.73 MeV of energy, not accounting for what is lost to neutrinos[2]

P-P I Chain

The first branch of the p-p chain produces 83.3% of ^4He[1]. The equation for the p-p I branch is as follows:

$${}^3_2He +^3_2 He \rightarrow {}^4_2He + 2^1_1H + 12.859MeV$$

This branch is dominant between temperatures of 10-18MK[3].

P-P II Chain

The p-p II branch is the source of 16.68% of ^4He created in the p-p chain. This section includes the following three reactions:

$$\begin{aligned}
{}^3_2He +^4_2 He &\rightarrow {}^7_4Be + \gamma + 1.59MeV \\
{}^7_4Be +^1_1 H &\rightarrow^8_5 B + \gamma \\
{}^8_5B &\rightarrow^8_4 Be + e^+ + v_e \\
{}^8_4Be &\rightarrow 2^4_2He
\end{aligned}$$

This branch is dominant with temperatures between 14-23MK[1].

Other types of Nuclear Fusion in Stars

The CNO Cycle

The Carbon-Nitrogen-Oxygen Cycle (CNO cycle) is a form of nuclear fusion that starts to dominate over the p-p chain in stars over $1.3M_\odot$. In the cycle, hydrogen repeatedly fuses with C-12 until it forms an unstable isotope of oxgyen, which decays and emits an α-particle. There are actually many different CNO cycles, but they all have the same net result:

$$4^1_1\mathrm{H} \rightarrow {}^4_2\mathrm{He} + 2e^+ + 2\nu_e + 3\gamma + E$$

There are two different types of CNO cycles: Hot CNO cycles and Cold CNO cycles. The difference between them is that at higher temperatures, the timescale for proton capture becomes shorter than that for beta decay, allowing for nucleosynthesis pathways that are otherwise inaccessible. These pathways are denoted as Hot CNO cycles. The minimum temperature needed for a HCNO cycle to occur is about 0.1GK, whereas they tend to dominate over Cold CNO cycles at about 0.5GK. Examples of where this occurs are in supernovae and X-ray bursts[3].

The Triple Alpha Process and Beyond

The triple alpha process is different from the CNO cycle and the p-p chain in that it involves the fusion of helium rather than hydrogen. In the process, three He-4 nuclei fuse to form C-12. The nuclear equation for the reaction is given by:

$$\begin{aligned}
{}^4_2\mathrm{He} +^4_2 \mathrm{He} &\rightarrow^8_4 \mathrm{Be} - 0.0918MeV \\
{}^8_4\mathrm{Be} +^4_2 \mathrm{He} &\rightarrow^{12}_6 \mathrm{C} + 7.367MeV
\end{aligned}$$

The triple alpha process is highly unstable due to the small half-life of Be-8 decay back into two He-4 nuclei, i.e., another He-4 has to fuse with Be-8 within this timeframe ($8.19x10^{-17}s$)[4]. Central temperatures of 10^8K are necessary to overcome the Be-8 barrier[5]. Be-8 then continues to fuse with C-12 until it reaches Ni-56, which subsequently decays to Fe-56. This level of fusion only occurs in stars above $10M_\odot$[6] and supernovae.

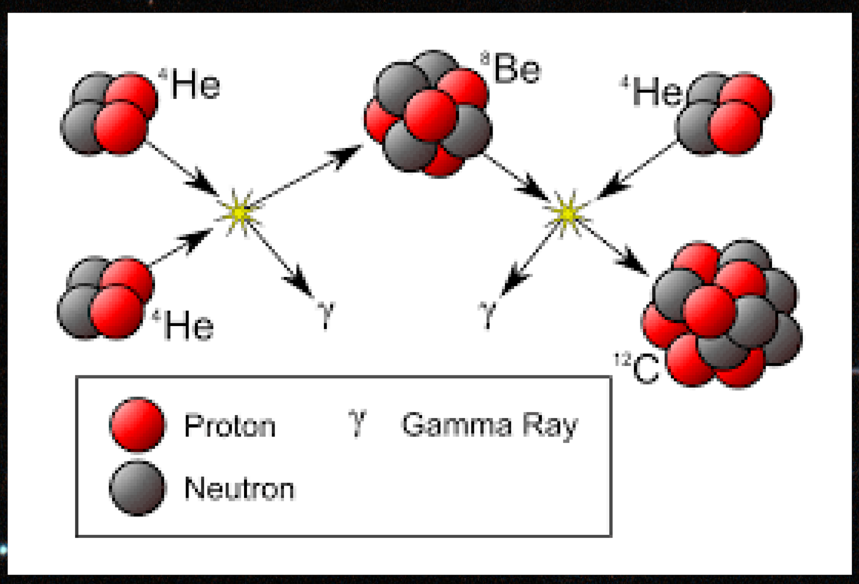

Figure 1. The Triple Alpha Process, Reproduced from [7]

High Density Regime

Degeneracy Pressure

Degeneracy Pressure is a purely Quantum Mechanical effect, occurring when a large amount of identical particles packed together with such high density that gravity is trying to push each of the particles into the same quantum state. For instance take a particle in state m and a particle in state n, we say $|mn\rangle = |m\rangle \otimes |n\rangle$ and vice versa and define the permutation operator $\hat{P}\,|mn\rangle = |nm\rangle$.

$$\begin{gathered}
\hat{P}\,|mn\rangle = |nm\rangle \implies \hat{P}^2\,|mn\rangle = |mn\rangle \implies \hat{P} = \pm 1 \\
|mn\rangle \pm |nm\rangle = 0
\end{gathered} \tag{1}$$

For the negative case Eq. 1 shows the **Pauli Exclusion Principle**, where the identical particles are fermions. [8] It can be found in White Dwarfs in the form of Electron Degeneracy Pressure, where the pressure $P \propto \rho_e^{5/3}$, and in the form of Neutron Degeneracy Pressure found in Neutron stars. These forces keep the stars from collapsing up to the Chandrasekhar and Tolman-Oppenheimer-Volkoff limits respectively. [9]

Pycnonuclear Fusion

Under high density, atoms arrange themselves into the lowest energy state crystal lattice. Their natural oscillations, known as the *zero-point oscillations*, can be of frequency ω such that $\hbar\omega > E_C$, the Coulomb barrier, and so their nuclei can fuse [10].
It's hypothesised that White Dwarfs undergo pycnonuclear fusion, turning their cores into ^{56}Fe and becoming Black Dwarfs [11]. Whereas, pycnonuclear fusion in Neutron Stars acts a major heat source. Through a more Quantum Mechanical lens, pycnonuclear fusion manifests as the overlap of wave-functions, seen in Fig. 2. [12]

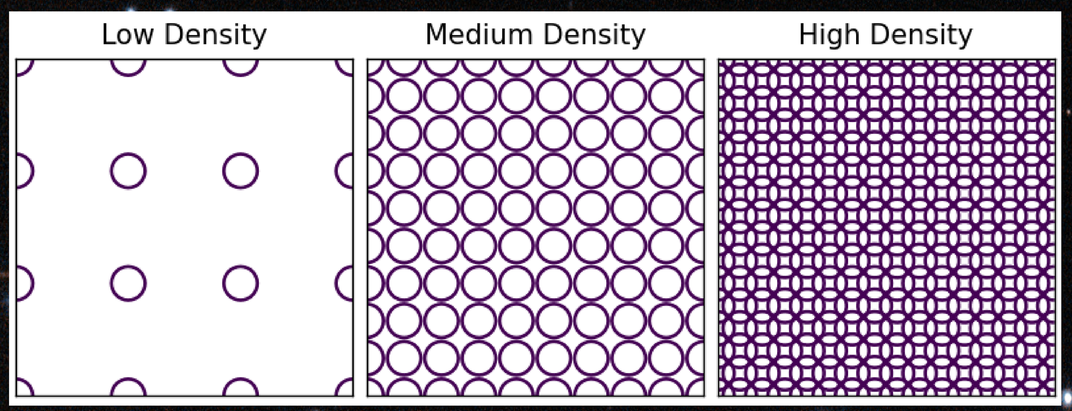

Figure 2. A crude demonstration of the overlapping of spherical orbital wave functions.

Fusion on Earth

Fusion power on Earth is one of the next big goals for physicists. Achieving it would mean a clean source of energy with power outputs many times greater than the current fission reactors we have. However, it goes without saying that this is no easy task. While there are many types of fusion reactors, a question one might ask is what might the requirements be for successful, self-sustained fusion? The metric used is known as the Lawson criterion, developed by John D. Lawson in 1955. The Lawson criteria is specified in terms of three important parameters:

- The density of the plasma n.
- The time confinement τ_E (measures the rate at which the system loses energy to the environment).
- The temperature of the plasma, T.

product of these quantities is the minimum requirement for a successful fusion reaction, known as the triple product, $n\tau_E T$. As it happens, these conditions are quite tricky, and the plasma density and temperature cannot be too high[13].
The reaction between deuterium and tritium is the most favourable of potential fusion reactions:

$${}^2_1\mathrm{D} +^3_1 \mathrm{T} \rightarrow^4_2 \mathrm{He} + n$$

. The Lawson Criteria can be given in terms of other parameters too.

$$n\tau_E T \geq \frac{12T^2}{E < \sigma v >} \tag{2}$$

Where $< \sigma v >$ is the average fusion cross section and E is the energy released per reaction. For the D-T reaction, it turns out the minimum temperature is $14keV$ (it is often convenient to measure temperature in units of eV instead of K. To do this, we can use the equation $E = kT$ where k is Boltzmann's constant).
At this temperature, the average fusion cross section can be approximated as $< \sigma v > \approxeq 1.1 \times 10^{-24} T^2 m^3 s^{-1}$, where T is expressed in keV[14]. Using Eq. 2 and expressing all the temperature and energies in keV:

$$\begin{aligned}
n\tau_E T &\geq \frac{(12)(14)^2}{(3.5 \times 10^3)(1.1 \times 10^{-24})(14)^2}\, keV\, s\, m^{-3} \\
n\tau_E T &\geq 3.12 \times 10^{21} keV\, s\, m^{-3}
\end{aligned}$$

Some reactors have come close: JT-60 in Japan reported a value of $1.5 \times 10^{21} keV\, m^{-3}\, s$ [15]. Note that while some reactors have already achieved the breakeven energy (this is given by the energy gain factor Q i.e. if $Q = 1$ means producing more energy than required to heat the plasma), a value higher than the triple product is still required for a sustained reaction. [16]

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