Classical Field Theory

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1 Gauss's Law

Gauss's law states that the electric flux of any closed surface is equal to the charge withing that area
In Integral form, Gauss's law looks like the following:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \tag{1}$$

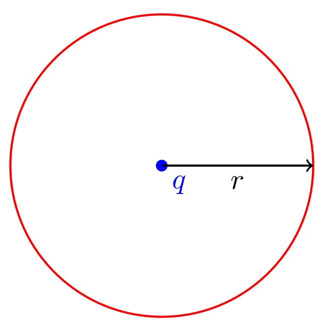
Q_{in} signifies we only take into account the charge within the gaussian surface

If we are working with a gaussian surface with uniform electric field we can take $\vec{E}$ outside of the integral then we end up with the following expression.

$$\vec{E} \cdot \vec{A} = \frac{Q_{in}}{\epsilon_0} \tag{2}$$

1.1 Spherical Symmetry

In the case of spherical symmetry, we can consider a point charge in three dimensions. We can draw a 2d boundary around the point charge.



In this case, using equation 2, and knowing that A in this case is the area of the sphere we end up with the result

$$\vec{E}(4\pi r^2) = \frac{q}{\epsilon_0}$$

1.1.1 Point charge in metal shell

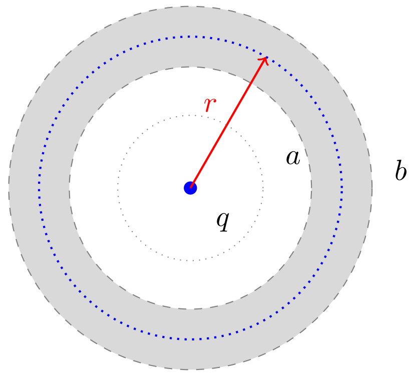


Figure 1: Point charge inside thick metal shell

here a marks the inner line of the metal shell and b marks the outer

1. for $r < a$
$$\vec{E}(4\pi r^2) = \frac{q}{\epsilon_0}$$
2. for $a < r < b$
$$\vec{E}(4\pi r^2) = 0$$
3. for $r > b$
$$\vec{E}(4\pi r^2) = \frac{q}{\epsilon_0}$$

1.2 Pre charged metall shell, conatining point charge

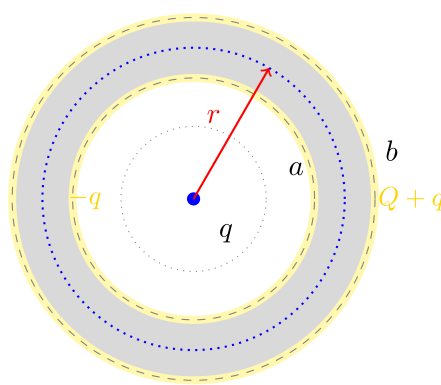


Figure 2: Point charge inside thick metal shell