Supervised classification - improving capacity learning

0. Import library

Import library

In [1]:

```
# Import libraries

# math library
import numpy as np

# visualization library
%matplotlib inline
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('png2x','pdf')
import matplotlib.pyplot as plt

# machine learning library
from sklearn.linear_model import LogisticRegression

# 3d visualization
from mpl_toolkits.mplot3d import axes3d

# computational time
import time

import math
```

1. Load and plot the dataset (dataset-noise-02.txt)

The data features for each data i are $x_i = (x_{i(1)}, x_{i(2)})$.

The data label/target, y_i, indicates two classes with value 0 or 1.

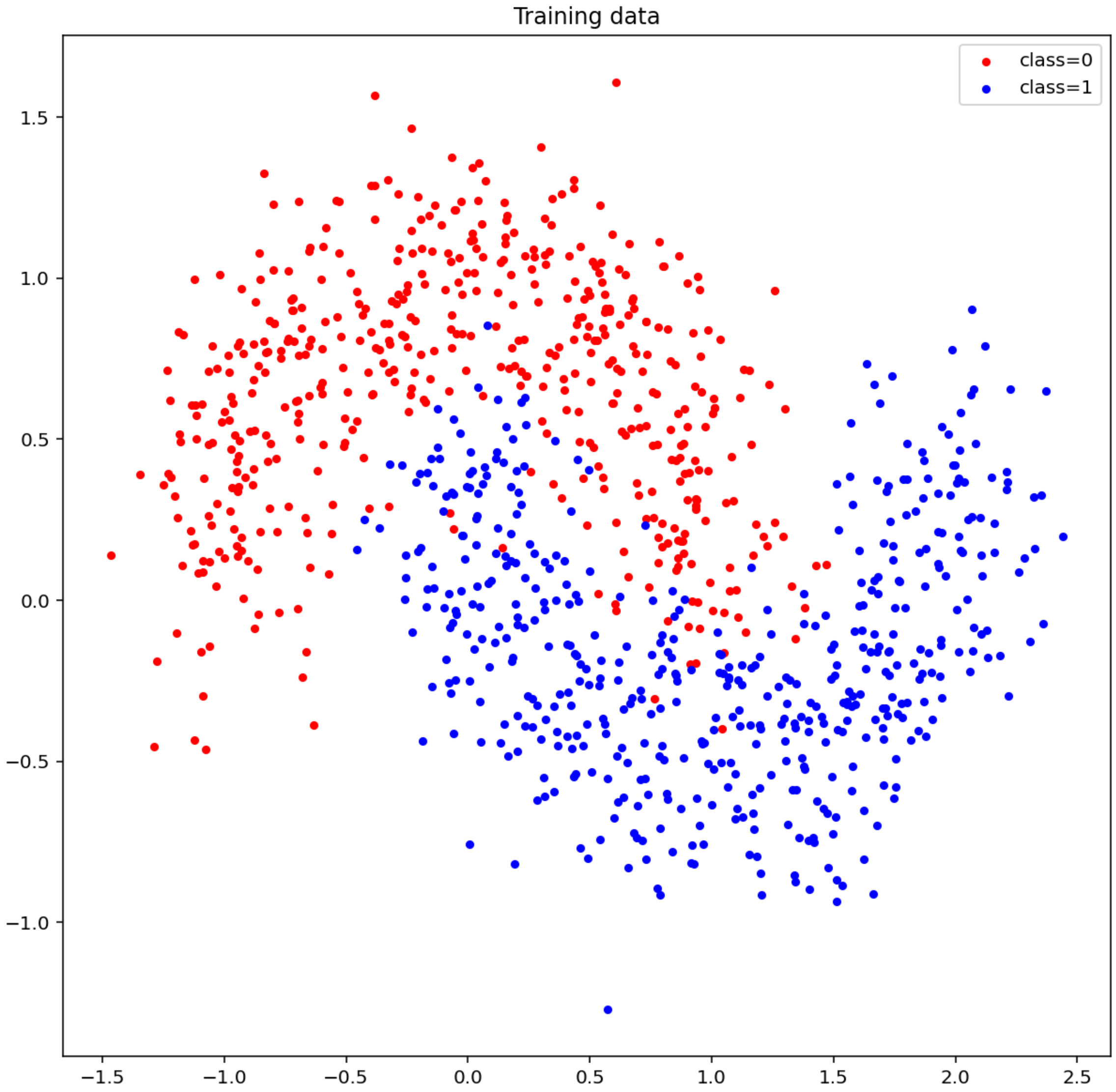
Plot the data points.

You may use matplotlib function `scatter(x,y)` .

In [2]:

```
# import data with numpy
data = np.loadtxt('dataset-b.txt', delimiter=',')

# number of training data
n = data.shape[0]
print('Number of the data = {}'.format(n))
print('Shape of the data = {}'.format(data.shape))
print('Data type of the data = {}'.format(data.dtype))

# plot
x1 = data[:,0] # feature 1
x2 = data[:,1] # feature 2
idx = data[:,2] # label

idx_class0 = (idx==0)  # index of class0
idx_class1 =  (idx==1) # index of class1

plt.figure(1,figsize=(10,10))
plt.scatter(  x1[idx_class0], x2[idx_class0], s=50, c='r', marker='.', label='class=0')
plt.scatter(  x1[idx_class1], x2[idx_class1], s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```

```
Number of the data = 1000
Shape of the data = (1000, 3)
Data type of the data = float64
```



2. Define a logistic regression loss function and its gradient

In [5]:

```
# sigmoid function
def sigmoid(z):
    sigmoid_f =1/(1+np.exp(-z))
    return sigmoid_f


# predictive function definition
def f_pred(X,w):
    p = sigmoid(np.dot(X,w))
    return p


# loss function definition
def loss_logreg(y_pred,y):
    loss=-y*np.log(y_pred)-(1-y)*np.log(1-y_pred)
    return np.mean(loss)


# gradient function definition
def grad_loss(y_pred,y,X):
    n = len(y)
    grad =  np.dot(X.T,(y_pred-y))/n*2
    return grad


# gradient descent function definition
def grad_desc(X, y , w_init, tau, max_iter):

    L_iters = np.zeros([max_iter]) # record the loss values
    w = w_init # initialization
    for i in range(max_iter): # loop over the iterations
        y_pred = f_pred(X,w) # linear predicition function
        grad_f = grad_loss(y_pred,y,X) # gradient of the loss
        w =  w - tau* grad_f # update rule of gradient descent
        L_iters[i] =  loss_logreg(y_pred,y)# save the current loss value

    return w, L_iters
```

3. define a prediction function and run a gradient descent algorithm

The logistic regression/classification predictive function is defined as:

$$p_w(x) = \sigma(Xw)$$

The prediction function can be defined in terms of the following feature functions f_i as follows:

$$X = \begin{bmatrix} f_0(x_1) & f_1(x_1) & f_2(x_1) & f_3(x_1) & f_4(x_1) & f_5(x_1) & f_6(x_1) & f_7(x_1) & f_8(x_1) & f_9(x_1) \\ f_0(x_2) & f_1(x_2) & f_2(x_2) & f_3(x_2) & f_4(x_2) & f_5(x_2) & f_6(x_2) & f_7(x_2) & f_8(x_2) & f_9(x_2) \\ \vdots \\ f_0(x_n) & f_1(x_n) & f_2(x_n) & f_3(x_n) & f_4(x_n) & f_5(x_n) & f_6(x_n) & f_7(x_n) & f_8(x_n) & f_9(x_n) \end{bmatrix}$$

where $x_i = (x_i(1), x_i(2))$ and you can define a feature function f_i as you want.

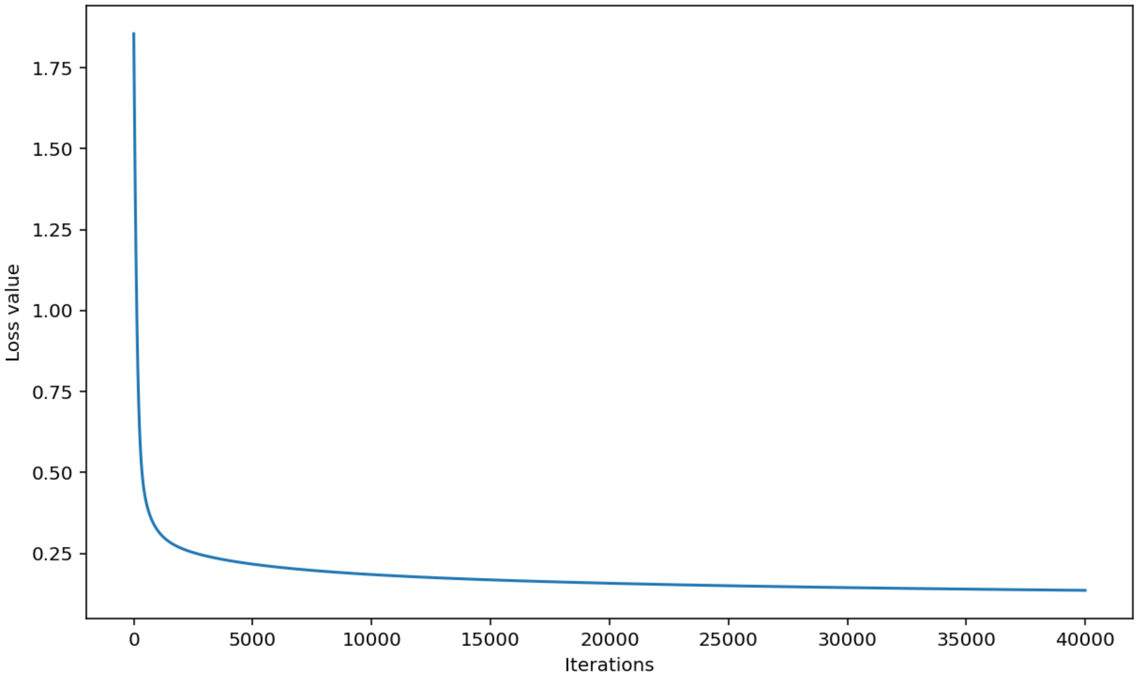
You can use at most 10 feature functions f_i, $i = 0, 1, 2, \cdots, 9$ in such a way that the classification accuracy is maximized. You are allowed to use less than 10 feature functions.

Implement the logistic regression function with gradient descent using a vectorization scheme.

In [6]:

```
# construct the data matrix X, and label vector y
n = data.shape[0]
X = np.ones([n,10])
X[:,1]=x1
X[:,2]=x2
X[:,3]=x1*x1
X[:,4]=x1*x2
X[:,5]=x2*x2
X[:,6]=np.power(x1,3)
X[:,7]=np.power(x2,3)
X[:,8]=x1*x1*x2
X[:,9]=x1*x2*x2
y = data[:,2][:,None] # label
print(y.shape)


# run gradient descent algorithm
start = time.time()
w_init = np.ones([10,1])
tau =1e-2; max_iter = 40000
w, L_iters = grad_desc(X,y,w_init,tau,max_iter)
print('Time=',time.time() - start)
print(L_iters[-1])
print(w)


# plot
plt.figure(4, figsize=(10,6))
plt.plot(L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```

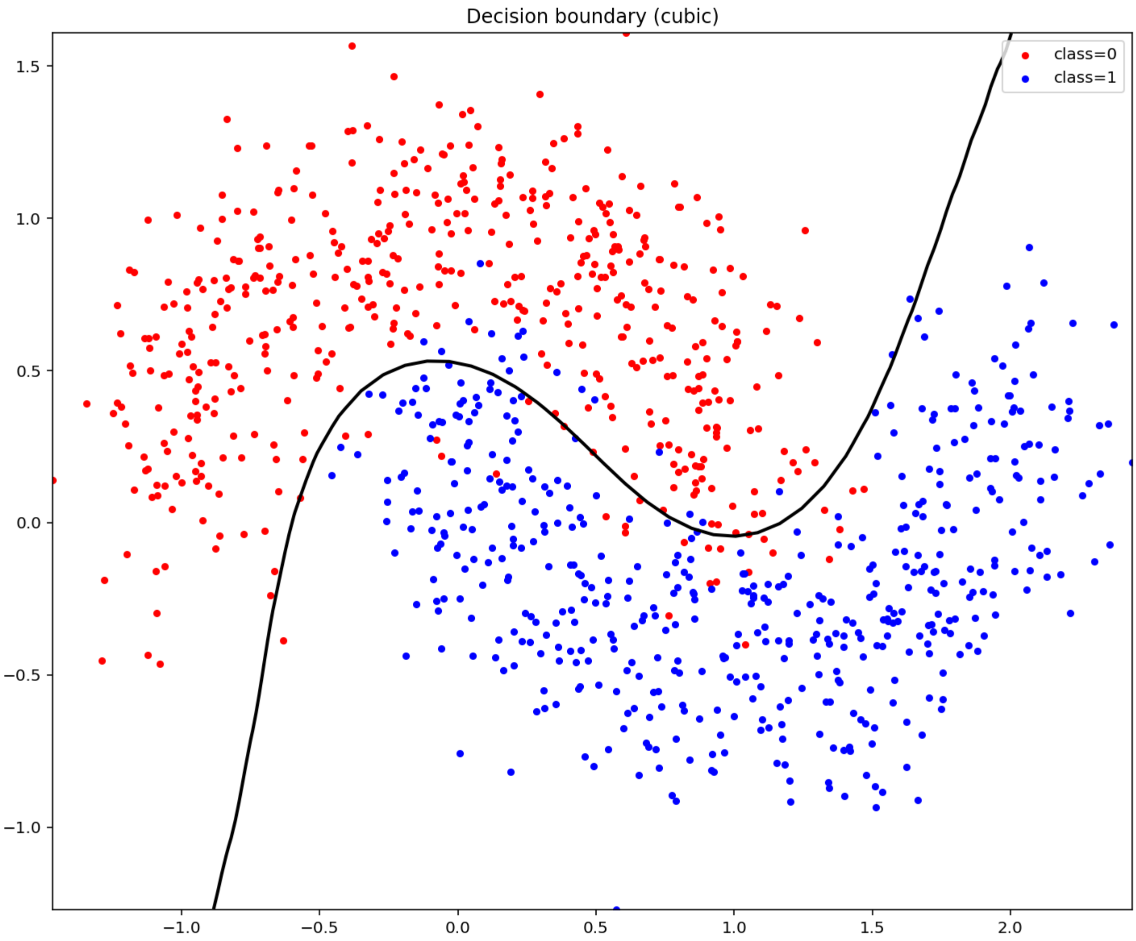
```
(1000, 1)
Time= 3.692631721496582
0.13615812340394673
[[ 2.76872516]
 [-1.20676566]
 [-4.25781319]
 [-6.74167132]
 [-1.22325886]
 [-0.65452833]
 [ 4.91283126]
 [-2.39159561]
 [-0.42636867]
 [ 2.38082637]]
```



4. Plot the decisoin boundary

In [9]:

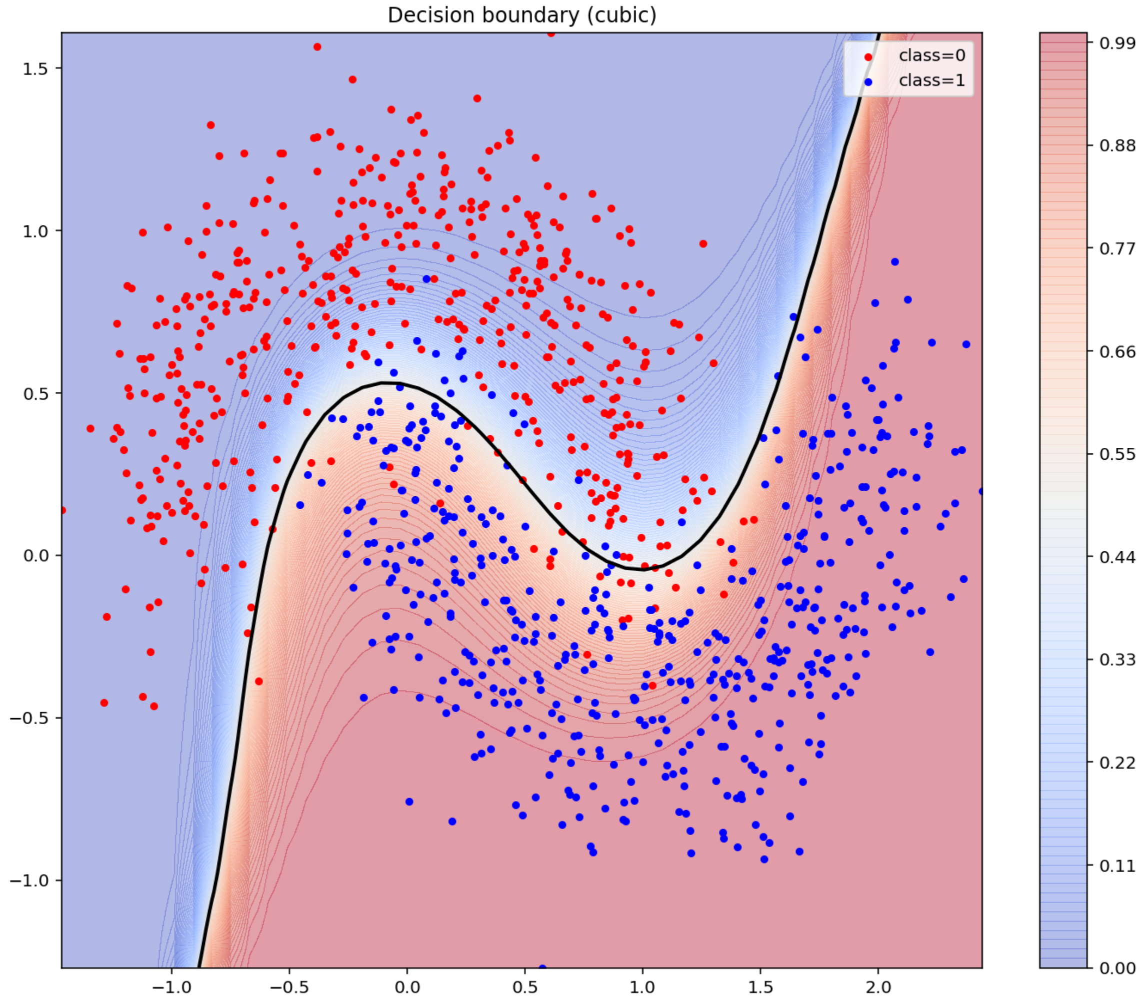
```
# compute values p(x) for multiple data points x
x1_min, x1_max = x1.min(), x1.max() # min and max of grade 1
x2_min, x2_max = x2.min(), x2.max() # min and max of grade 2
xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max))  # create meshg
rid
x1_1=xx1.reshape(-1)
x2_1=xx2.reshape(-1)
X2 = np.ones([2500,10])
X2[:,1]=x1_1
X2[:,2]=x2_1
X2[:,3]=x1_1*x1_1
X2[:,4]=x1_1*x2_1
X2[:,5]=x2_1*x2_1
X2[:,6]=np.power(x1_1,3)
X2[:,7]=np.power(x2_1,3)
X2[:,8]=x1_1*x1_1*x2_1
X2[:,9]=x1_1*x2_1*x2_1

p = f_pred(X2,w )
p = p.reshape(50,50)

# plot
plt.figure(4,figsize=(12,10))

#ax = plt.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.6)
#cbar = plt.colorbar(ax)
#cbar.update_ticks()

plt.scatter( x1[idx_class0], x2[idx_class0], s=50, c='r', marker='.', label='class=0')
plt.scatter(x1[idx_class1], x2[idx_class1] , s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, levels=[0.5], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (cubic)')
plt.show()
```



5. Plot the probability map

In [10]:

```
# plot
plt.figure(4,figsize=(12,10))

ax = plt.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.4)
cbar = plt.colorbar(ax)
cbar.update_ticks()

plt.scatter( x1[idx_class0], x2[idx_class0], s=50, c='r', marker='.', label='class=0')
plt.scatter(x1[idx_class1], x2[idx_class1] , s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, levels=[0.5], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (cubic)')
plt.show()
```



6. Compute the classification accuracy

The accuracy is computed by:

$$\text{accuracy} = \frac{\text{number of correctly classified data}}{}$$

In [15]:

```
# compute the accuracy of the classifier
n = data.shape[0]


y_pred = f_pred(X,w )
y_pred
idx_class1_pred = (y_pred>=0.5 ).reshape(-1)
acc=idx_class1_pred==idx_class1

print('total number of correctly classified data = ', acc.sum())
print('accuracy(%) = ', acc.mean())
```

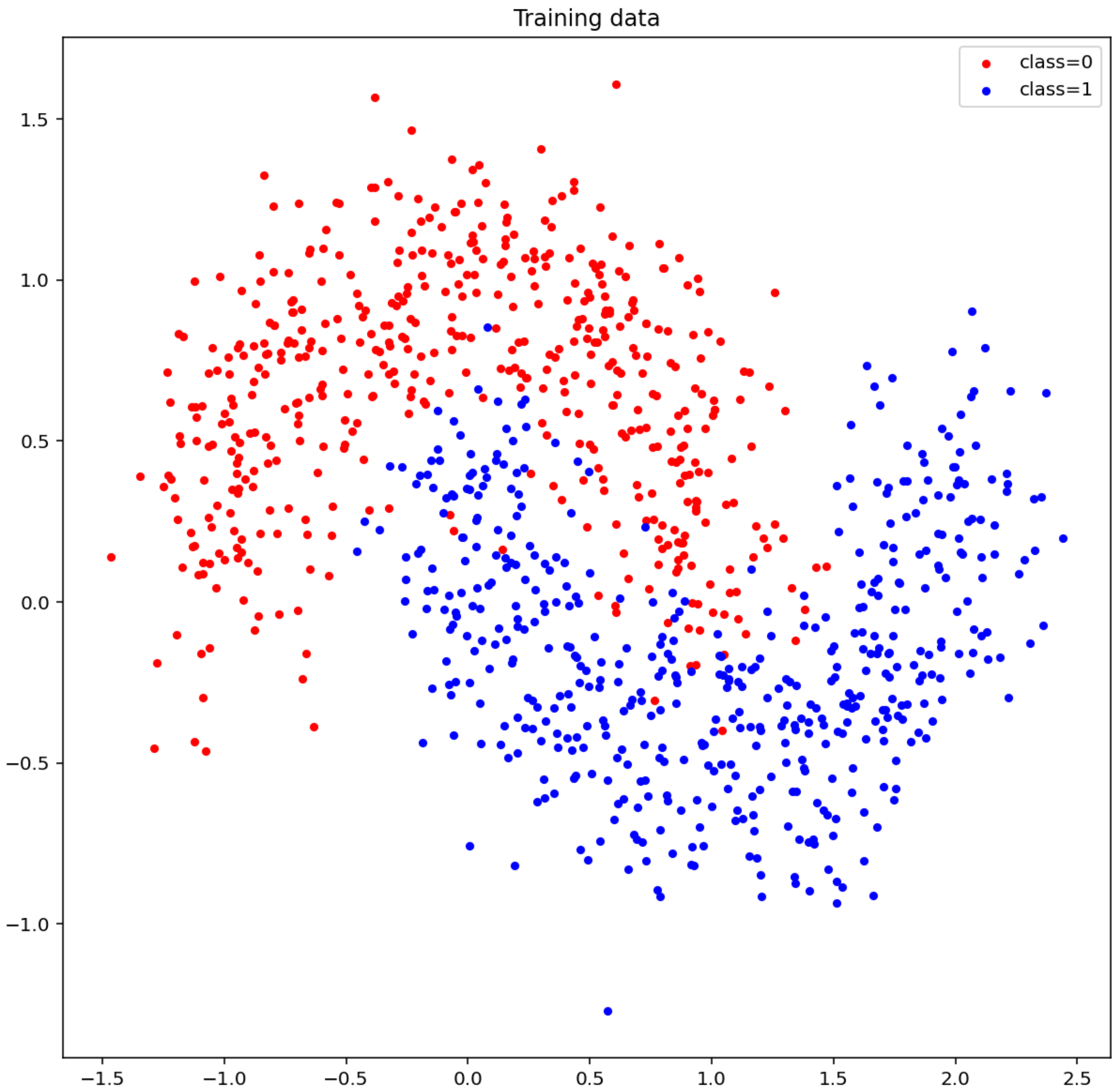
```
total number of correctly classified data =  953
accuracy(%) =  0.953
```

Output using the dataset (dataset-noise-02.txt)

1. Visualize the data [1pt]

In [3]:

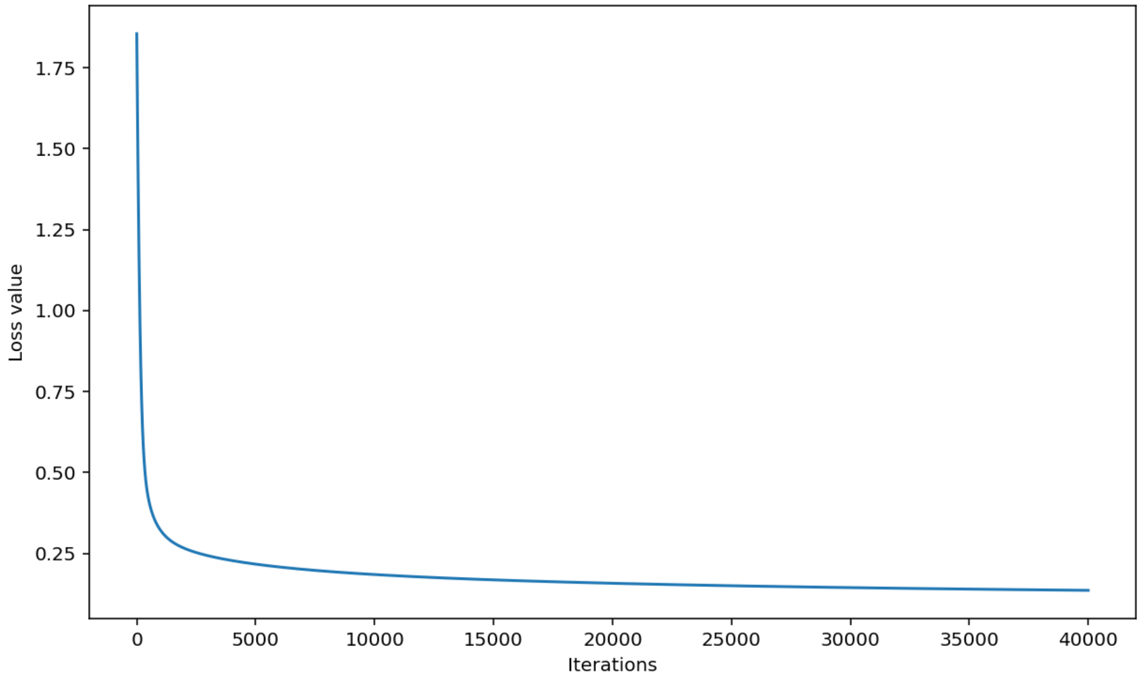
```
plt.figure(1,figsize=(10,10))
plt.scatter(  x1[idx_class0], x2[idx_class0], s=50, c='r', marker='.', label='class=0')
plt.scatter(  x1[idx_class1], x2[idx_class1], s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```



2. Plot the loss curve obtained by the gradient descent until the convergence [2pt]

In [7]:

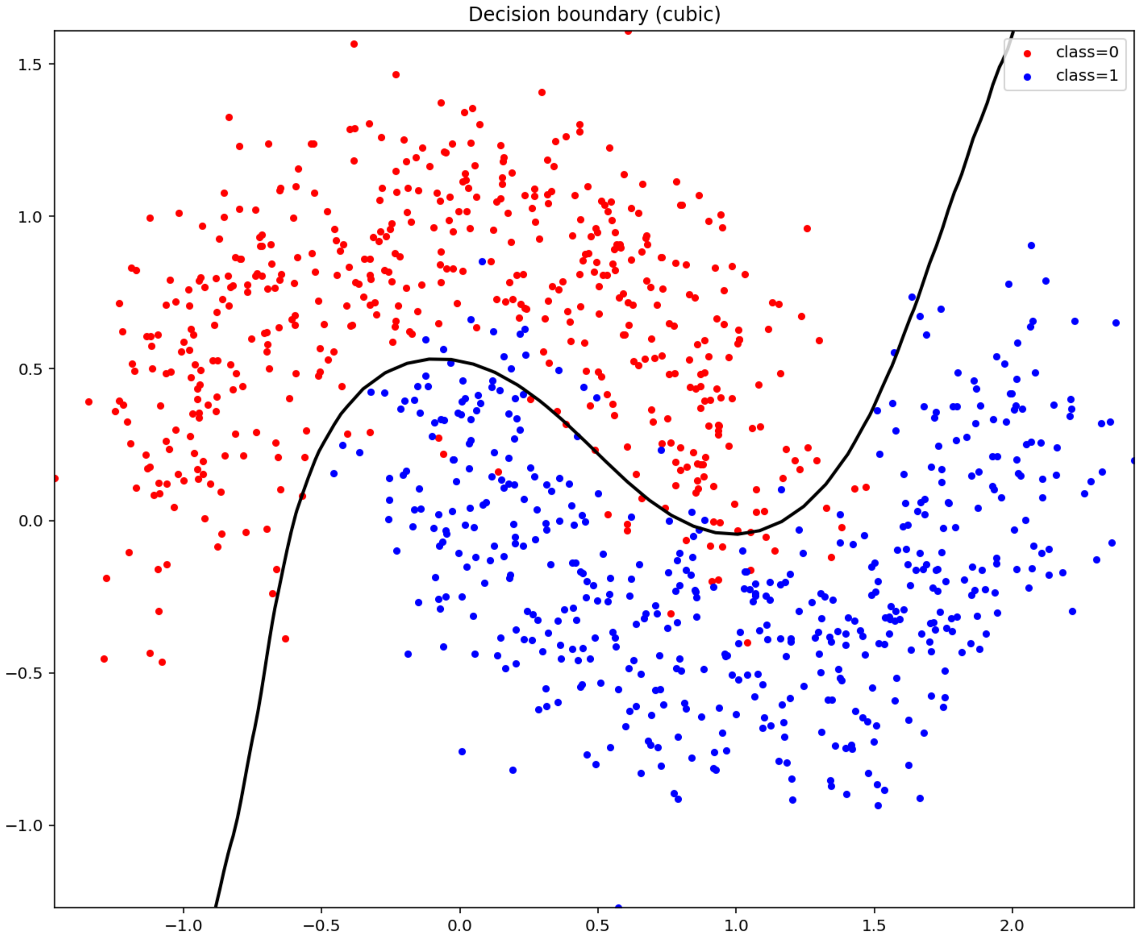
```
plt.figure(4, figsize=(10,6))
plt.plot(L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```



3. Plot the decisoin boundary of the obtained classifier [2pt]

In [12]:

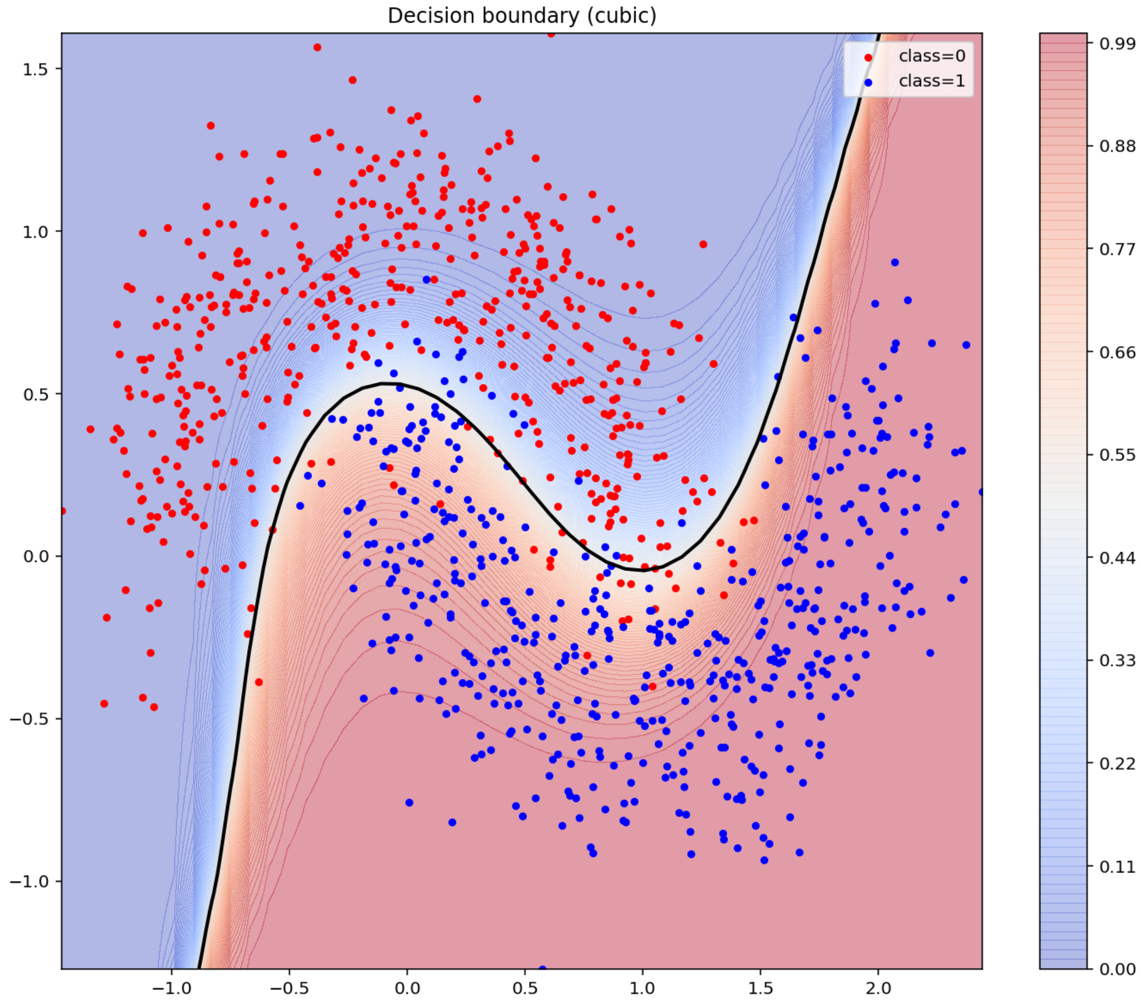
```
plt.figure(4,figsize=(12,10))
plt.scatter( x1[idx_class0], x2[idx_class0], s=50, c='r', marker='.', label='class=0')
plt.scatter(x1[idx_class1], x2[idx_class1] , s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, levels=[0.5], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (cubic)')
plt.show()
```



4. Plot the probability map of the obtained classifier [2pt]

In [13]:

```
plt.figure(4,figsize=(12,10))

ax = plt.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.4)
cbar = plt.colorbar(ax)
cbar.update_ticks()

plt.scatter( x1[idx_class0], x2[idx_class0], s=50, c='r', marker='.', label='class=0')
plt.scatter(x1[idx_class1], x2[idx_class1] , s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, levels=[0.5], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (cubic)')
plt.show()
```



5. Compute the classification accuracy [1pt]

In [17]:

```
print('total number of data =', n)
print('total number of correctly classified data = ', acc.sum())
print('accuracy(%) = ', acc.mean())
```

```
total number of data = 1000
total number of correctly classified data =  953
accuracy(%) =  0.953
```

In [ ]:

```
```