# Supervised Logistic Regression for Classification

## 0. Import library

In [26]:

```
# Import libraries

# math library
import numpy as np

# visualization library
%matplotlib inline
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('png2x','pdf')
import matplotlib.pyplot as plt

# machine learning library
from sklearn.linear_model import LogisticRegression

# 3d visualization
from mpl_toolkits.mplot3d import axes3d

# computational time
import time
```

## 1. Load dataset

The data features $x_i = (x_{i(1)}, x_{i(2)})$ represent 2 exam grades $x_{i(1)}$ and $x_{i(2)}$ for each student $i$.

The data label $y_i$ indicates if the student $i$ was admitted (value is 1) or rejected (value is 0).

In [27]:

```
# import data with numpy
data = np.loadtxt('dataset.txt', delimiter=',')

# number of training data
n = data.shape[0]
print('Number of training data=',n)
```

```
Number of training data= 100
```

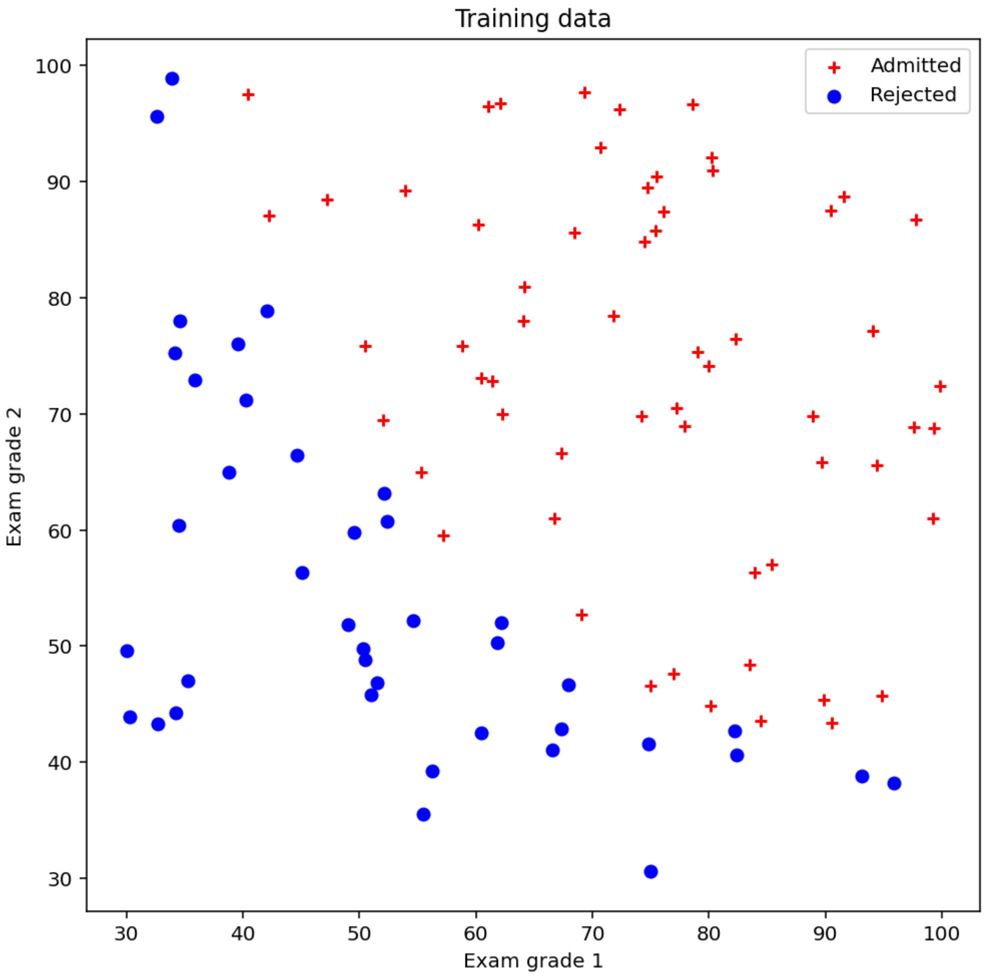
## 2. Explore the dataset distribution

Plot the training data points.

You may use matplotlib function `scatter(x,y)`.

In [28]:

```
x1 = data[:,0] # exam grade 1
x2 = data[:,1] # exam grade 2
idx_admit = (data[:,2]==1) # index of students who were admitted
idx_rejec = (data[:,2]==0) # index of students who were rejected

plt.figure(figsize=(8,8))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )
plt.title('Training data')
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Admitted","Rejected"])
plt.show()
```



In [29]:

```
x2
```

Out[29]:

```
array([78.02469282, 43.89499752, 72.90219803, 86.3085521 , 75.34437644,
       56.31637178, 96.51142588, 46.55401354, 87.42056972, 43.53339331,
       38.22527806, 30.60326323, 76.4819633 , 97.71869196, 76.03681085,
       89.20735014, 52.74046973, 46.67857411, 92.92713789, 47.57596365,
       42.83843832, 65.79936593, 48.85581153, 44.2095286 , 68.97235999,
       69.95445795, 44.82162893, 38.80067034, 50.25610789, 64.99568096,
       72.80788731, 57.05198398, 63.12762377, 69.43286012, 71.16774802,
       52.21388588, 98.86943574, 80.90806059, 41.57341523, 75.23772034,
       56.30804622, 46.85629026, 65.56892161, 40.61825516, 45.82270146,
       52.06099195, 70.4582    , 86.72782233, 96.76882412, 88.69629255,
       74.16311935, 60.999031  , 43.39060181, 60.39634246, 49.80453881,
       59.80895099, 68.86157272, 95.59854761, 69.82457123, 78.45356225,
       85.75993667, 47.02051395, 39.26147251, 49.59297387, 66.45008615,
       41.09209808, 97.53518549, 51.88321182, 92.11606081, 60.99139403,
       43.30717306, 78.03168802, 96.22759297, 73.0949981 , 75.85844831,
       72.36925193, 88.475865  , 75.80985953, 42.50840944, 42.71987854,
       69.8037889 , 45.6943068 , 66.58935318, 59.51428198, 90.9601479 ,
       85.5943071 , 78.844786  , 90.424539  , 96.64742717, 60.76950526,
       77.15910509, 87.50879176, 35.57070347, 84.84513685, 45.35828361,
       48.3802858 , 87.10385094, 68.77540947, 64.93193801, 89.5298129 ])
```

## 3. Sigmoid/logistic function

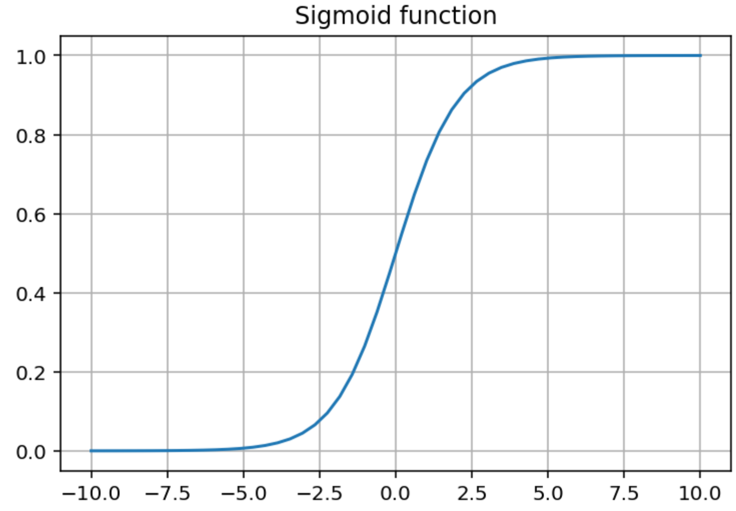
$$\sigma(\eta) = \frac{1}{1 + \exp^{-\eta}}$$

Define and plot the sigmoid function for values in [-10,10]:

You may use functions `np.exp` , `np.linspace` .

In [30]:

```
def sigmoid(z):

    sigmoid_f =1/(1+np.exp(-z))

    return sigmoid_f


# plot
x_values = np.linspace(-10,10)

plt.figure(2)
plt.plot(x_values,sigmoid(x_values))
plt.title("Sigmoid function")
plt.grid(True)
```




# 4. Define the prediction function for the classification

**The prediction function is defined by:**

$$p_w(x) = \sigma(w_0 + w_1 x_{(1)} + w_2 x_{(2)}) = \sigma(w^T x)$$

**Implement the prediction function in a vectorised way as follows:**

$$X = \begin{bmatrix} 1 & x_{1(1)} & x_{1(2)} \\ 1 & x_{2(1)} & x_{2(2)} \\ \vdots \\ 1 & x_{n(1)} & x_{n(2)} \end{bmatrix} \quad \text{and} \quad w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} \quad \Rightarrow \quad p_w(x) = \sigma(Xw) = \begin{bmatrix} \sigma(w_0 + w_1 x_{1(1)} + w \\ \sigma(w_0 + w_1 x_{2(1)} + w \\ \vdots \\ \sigma(w_0 + w_1 x_{n(1)} + w \end{bmatrix}$$

Use the new function `sigmoid`.

In [31]:

```
n = x1.size
X = np.stack([np.ones(n),x1,x2],axis=1)

# parameters vector
w = np.array([[0.3],[0],[0]])
w =np.array([-10,0.1,-0.2])[:,None]
# predictive function definition
def f_pred(X,w):

    p = sigmoid(np.dot(X,w).reshape(-1,1))

    return p
def f_pred2(X,w):

    f = np.matmul(X,w)

    return f
y_pred = f_pred(X,w)
y_pred
y_pred2=f_pred2(X,w)
y_pred2
```

Out[31]:

```
array([[-22.1425726 ],
       [-15.75032843],
       [-20.99569873],
       [-21.24345048],
       [-17.16560168],
       [-16.75494661],
       [-23.19161872],
       [-11.80832815],
       [-19.87423527],
       [-10.26339667],
       [ -8.0589001 ],
       [ -8.61928681],
       [-17.06568732],
       [-22.60727952],
       [-21.25352826],
       [-22.44436481],
       [-13.64107954],
       [-12.54102927],
       [-21.51927662],
       [-11.81731436],
       [-11.83048491],
       [-14.19219561],
       [-14.71768348],
       [-15.42069962],
       [-16.00206285],
       [-17.76379022],
       [-10.94530771],
       [ -8.44869519],
       [-13.86820098],
       [-19.12055581],
       [-18.42364852],
       [-12.86994486],
       [-17.41472678],
       [-18.68203155],
       [-20.20986023],
       [-14.97926662],
       [-26.38233714],
       [-19.76391323],
       [-10.83575775],
       [-21.62918006],
       [-12.87136988],
       [-14.21648603],
       [-13.66944754],
       [ -9.88677566],
       [-14.05976511],
       [-14.18993081],
       [-16.37233651],
       [-17.56840454],
       [-23.14645845],
       [-18.58276106],
       [-16.83814208],
       [-12.27255351],
       [ -9.62344895],
       [-18.62681711],
       [-14.93225815],
       [-17.00312248],
       [-14.00775115],
       [-25.86198951],
       [-16.54004511],
```

```
       [-18.51106624],
       [-19.61242619],
       [-15.87549151],
       [-12.22691275],
       [-16.91271253],
       [-18.82319106],
       [-11.56233017],
       [-25.461282  ],
       [-15.46938604],
       [-20.39525476],
       [-15.52360695],
       [-15.38915131],
       [-19.20240556],
       [-22.01086917],
       [-18.57321105],
       [-19.28759404],
       [-14.49106461],
       [-22.96874609],
       [-20.11615593],
       [-12.45612626],
       [-10.32130955],
       [-15.06936814],
       [ -9.65541069],
       [-16.58594489],
       [-16.17898576],
       [-20.15535398],
       [-20.27200924],
       [-21.56141175],
       [-20.5371376 ],
       [-21.465943  ],
       [-16.91910065],
       [-16.02238791],
       [-18.45690326],
       [-11.56592458],
       [-19.51975813],
       [-10.08707605],
       [-11.32714088],
       [-23.19460011],
       [-13.82358101],
       [-17.45238585],
       [-20.42837328]])
```

In [32]:

```
X = np.stack([np.ones(n),x1,x2],axis=1)
X.shape
x1.size
y_pred.shape
```

Out[32]:

```
(100, 1)
```

# 5. Define the classification loss function

**Mean Square Error**

$$L(w) = \frac{1}{n}\sum_{i=1}^{n}\left(\sigma(w^T x_i) - y_i\right)^2$$

**Cross-Entropy**

$$L(w) = \frac{1}{n}\sum_{i=1}^{n}\left(-y_i \log(\sigma(w^T x_i)) - (1 - y_i)\log(1 - \sigma(w^T x_i))\right)$$

**The vectorized representation is for the mean square error is as follows:**

$$L(w) = \frac{1}{n}\Big(p_w(x) - y\Big)^T \Big(p_w(x) - y\Big)$$

**The vectorized representation is for the cross-entropy error is as follows:**

$$L(w) = \frac{1}{n}\Big( - y^T \log(p_w(x)) - (1 - y)^T \log(1 - p_w(x))\Big)$$

where

$$p_w(x) = \sigma(Xw) = \begin{bmatrix} \sigma(w_0 + w_1 x_{1(1)} + w_2 x_{1(2)}) \\ \sigma(w_0 + w_1 x_{2(1)} + w_2 x_{2(2)}) \\ \vdots \\ \sigma(w_0 + w_1 x_{n(1)} + w_2 x_{n(2)}) \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

You may use numpy functions `.T` and `np.log`.

In [33]:

```
def mse_loss(label, h_arr):  # mean square error
    temp=h_arr-label
    L=temp*temp
    return np.mean(L)

def ce_loss(label, h_arr):  # cross-entropy error
#     L=np.dot(-y.T,np.log(h_arr))-np.dot((1-y).T,np.log(1-h_arr))
    L=-label*np.log(h_arr)-(1-label)*np.log(1-h_arr)
    return np.mean(L)
```

In [43]:

```
y = data[:,2][:,None]
# temp=y-y_pred
# temp*temp
# print(np.sum(temp*temp))
print(ce_loss(y,y_pred))
sadtry = -y*np.log(y_pred)-(1-y)*np.log(1-y_pred)
sadtry
```

```
10.391644147841939
```

Out[43]:

```
array([[2.41881626e-10],
       [1.44450565e-07],
       [7.61524510e-10],
       [2.12434505e+01],
       [1.71656017e+01],
       [5.28955493e-08],
       [2.31916187e+01],
       [1.18083356e+01],
       [1.98742353e+01],
       [1.02634316e+01],
       [3.16224479e-04],
       [1.80572699e-04],
       [1.70656874e+01],
       [2.26072795e+01],
       [5.88450511e-10],
       [2.24443648e+01],
       [1.36410807e+01],
       [3.57683917e-06],
       [2.15192766e+01],
       [1.18173217e+01],
       [7.27920759e-06],
       [1.41921963e+01],
       [4.05687094e-07],
       [2.00851557e-07],
       [1.60020630e+01],
       [1.77637902e+01],
       [1.09453254e+01],
       [2.14156764e-04],
       [9.48673209e-07],
       [4.96647347e-09],
       [1.84236485e+01],
       [1.28699474e+01],
       [2.73451462e-08],
       [1.86820316e+01],
       [1.67097214e-09],
       [3.12310867e-07],
       [3.48576723e-12],
       [1.97639132e+01],
       [1.96827569e-05],
       [4.04172251e-10],
       [1.28713724e+01],
       [6.69666265e-07],
       [1.36694487e+01],
       [5.08413236e-05],
       [7.83287911e-07],
       [6.87687610e-07],
       [1.63723366e+01],
       [1.75684046e+01],
       [2.31464584e+01],
       [1.85827611e+01],
       [1.68381421e+01],
       [1.22725582e+01],
       [9.62351511e+00],
       [8.13722104e-09],
       [3.27342663e-07],
       [4.12703094e-08],
       [1.40077520e+01],
       [5.86519722e-12],
       [1.65400452e+01],
```

```
       [1.85110663e+01],
       [1.96124262e+01],
       [1.27456390e-07],
       [4.89686575e-06],
       [4.51754269e-08],
       [6.68639446e-09],
       [9.51791327e-06],
       [2.54612820e+01],
       [1.91307044e-07],
       [2.03952548e+01],
       [1.55236071e+01],
       [2.07289097e-07],
       [1.92024056e+01],
       [2.20108692e+01],
       [1.85732111e+01],
       [1.92875940e+01],
       [1.44910651e+01],
       [2.29687461e+01],
       [2.01161559e+01],
       [3.89378757e-06],
       [3.29234295e-05],
       [1.50693684e+01],
       [9.65547476e+00],
       [1.65859450e+01],
       [1.61789859e+01],
       [2.01553540e+01],
       [2.02720092e+01],
       [4.32511693e-10],
       [2.05371376e+01],
       [2.14659430e+01],
       [4.48877605e-08],
       [1.60223880e+01],
       [1.84569033e+01],
       [9.48376354e-06],
       [1.95197581e+01],
       [1.00871177e+01],
       [1.13271529e+01],
       [2.31946001e+01],
       [1.38235820e+01],
       [1.74523859e+01],
       [2.04283733e+01]])
```

# 6. Define the gradient of the classification loss function

**Given the mean square loss**

$$L(w) = \frac{1}{n}\Big(p_w(x) - y\Big)^T \Big(p_w(x) - y\Big)$$

The gradient is given by

$$\frac{\partial}{\partial w}L(w) = \frac{2}{n}X^T\Big((p_w(x) - y) \odot (p_w(x) \odot (1 - p_w(x))\Big)$$

**Given the cross-entropy loss**

$$L(w) = \frac{1}{n}\Big( - y^T \log(p_w(x)) - (1 - y)^T \log(1 - p_w(x))\Big)$$

The gradient is given by

$$\frac{\partial}{\partial w}L(w) = \frac{2}{n}X^T(p_w(x) - y)$$

**Implement the vectorized version of the gradient of the classification loss function**

In [35]:

```
# gradient function definition(Cross Engropy)
def loss_logreg(y_pred,y):

    n = len(y)
    loss = np.dot(X.T,(y_pred-y))/n*2

    return loss


# Test loss function
y = data[:,2][:,None] # label
y_pred = f_pred(X,w) # prediction

loss = loss_logreg(y_pred,y)
loss
```

Out[35]:

```
array([[ -1.19997748],
       [-89.66073498],
       [-88.74679614]])
```

In [36]:

```
# gradient function (MeanSquareError)
def gradient_mse(y_pred,y):

    grad = np.matmul(X.T,y_pred-y)/y.size*2

    return grad
```

In [37]:

```
X.shape
w
```

Out[37]:

```
array([[-10. ],
       [  0.1],
       [ -0.2]])
```

# 7. Implement the gradient descent algorithm

**Vectorized implementation for the mean square loss:**

$$w^{k+1} = w^k - \tau \frac{2}{n} X^T \Big( (p_w(x) - y) \odot (p_w(x) \odot (1 - p_w(x)) \Big)$$

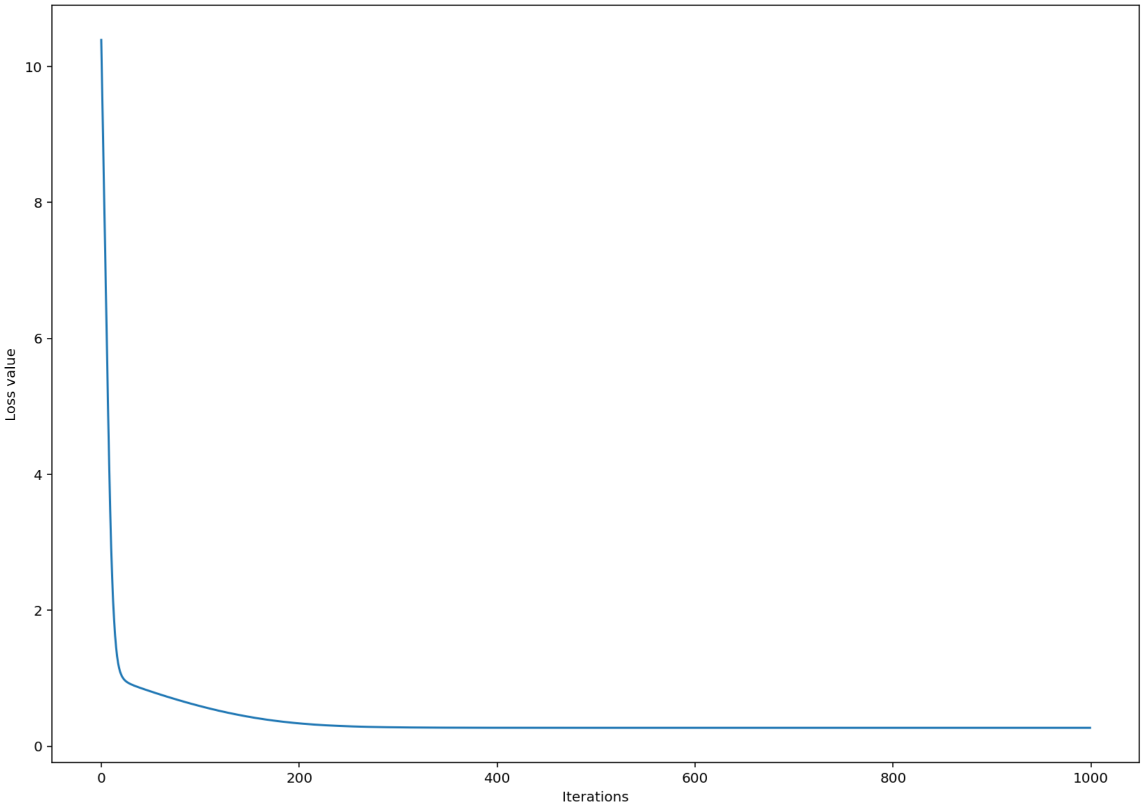
**Vectorized implementation for the cross-entropy loss:**

$$w^{k+1} = w^k - \tau \frac{2}{n} X^T (p_w(x) - y)$$

**Plot the loss values $L(w^k)$ w.r.t. iteration $k$ the number of iterations for the both loss functions.**

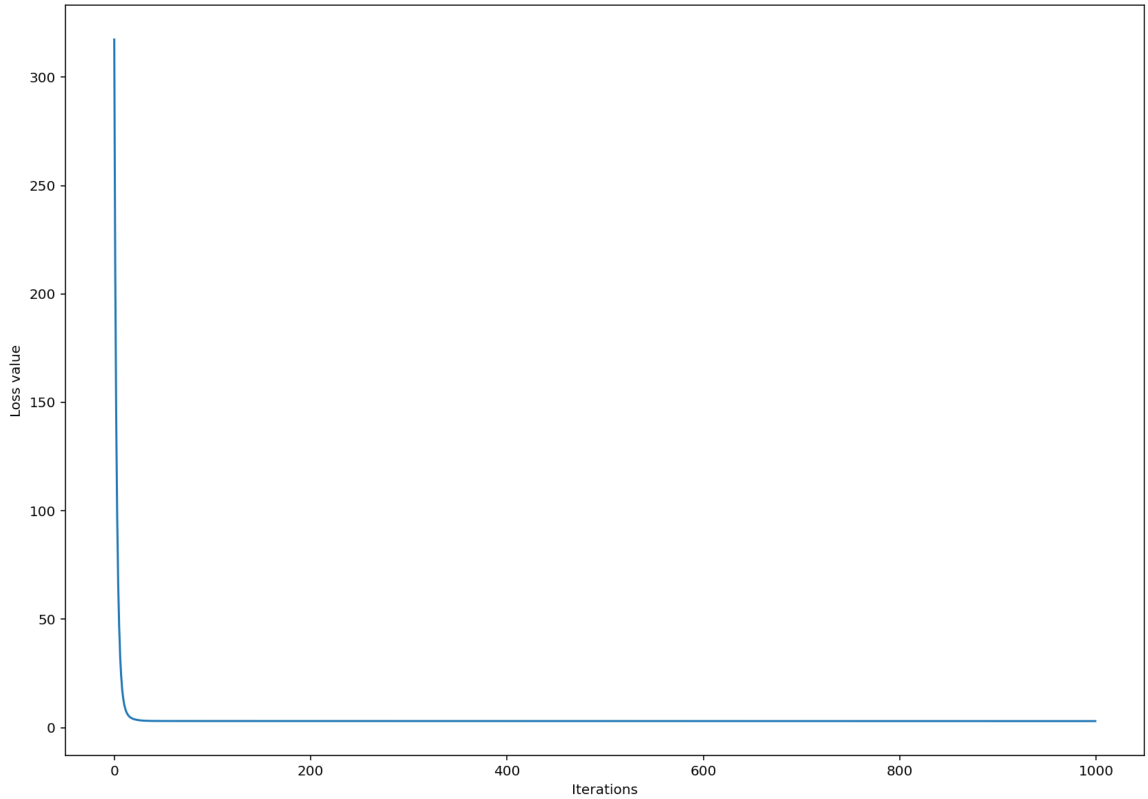
In [52]:

```
# cross_entropy gradient descent function definition
def cross_grad_desc(X, y , w_init=np.array([0,0,0])[:,None] ,tau=1e-4, max_iter=500):

    L_iters = np.zeros([max_iter]) # record the loss values
    w_iters = np.zeros([max_iter,2]) # record the loss values
    w = w_init # initialization

    for i in range(max_iter): # loop over the iterations

        y_pred = f_pred(X,w) # linear predicition function
        #grad_f = grad_loss(y_pred,y,X) # gradient of the loss
        grad_f = loss_logreg(y_pred,y) # gradient of the loss
        w = w - tau* grad_f # update rule of gradient descent

#         print(i)
#         print(y_pred)
        L_iters[i] = ce_loss(y,y_pred) # save the current loss value
#         print(L_iters[i])
        w_iters[i,:] = w[0],w[1] # save the current w value

    return w, L_iters, w_iters


# run gradient descent algorithm
start = time.time()
w_init = np.array([-10,0.1,-0.2])[:,None]
tau = 1e-4; max_iter = 1000
w, L_iters, w_iters = cross_grad_desc(X,y,w_init,tau,max_iter)


# plot
plt.figure(figsize=(14,10))
plt.plot(L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```


Loss value
Iterations
0
2
4
6
8
10
0
200
400
600
800
1000

In [39]:

```
# mse gradient descent function definition
def mse_grad_desc(X, y , w_init=np.array([0,0,0])[:,None] ,tau=1e-4, max_iter=500):

    L_iters = np.zeros([max_iter]) # record the loss values
    w_iters = np.zeros([max_iter,2]) # record the loss values
    w = w_init # initialization

    for i in range(max_iter): # loop over the iterations

        y_pred = f_pred2(X,w) # linear predicition function
        #grad_f = grad_loss(y_pred,y,X) # gradient of the loss
        grad_f = gradient_mse(y_pred,y) # gradient of the loss
        w = w - tau* grad_f # update rule of gradient descent

#         print(i)
#         print(y_pred)
        L_iters[i] = mse_loss(y,y_pred) # save the current loss value
#         print(L_iters[i])
        w_iters[i,:] = w[0],w[1] # save the current w value

    return w, L_iters, w_iters


# run gradient descent algorithm
start = time.time()
w_init = np.array([-10,0.1,-0.2])[:,None]
tau = 1e-4; max_iter = 1000
w, L_iters, w_iters = mse_grad_desc(X,y,w_init,tau,max_iter)


# plot
plt.figure(figsize=(14,10))
plt.plot(L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```




# 8. Plot the decision boundary

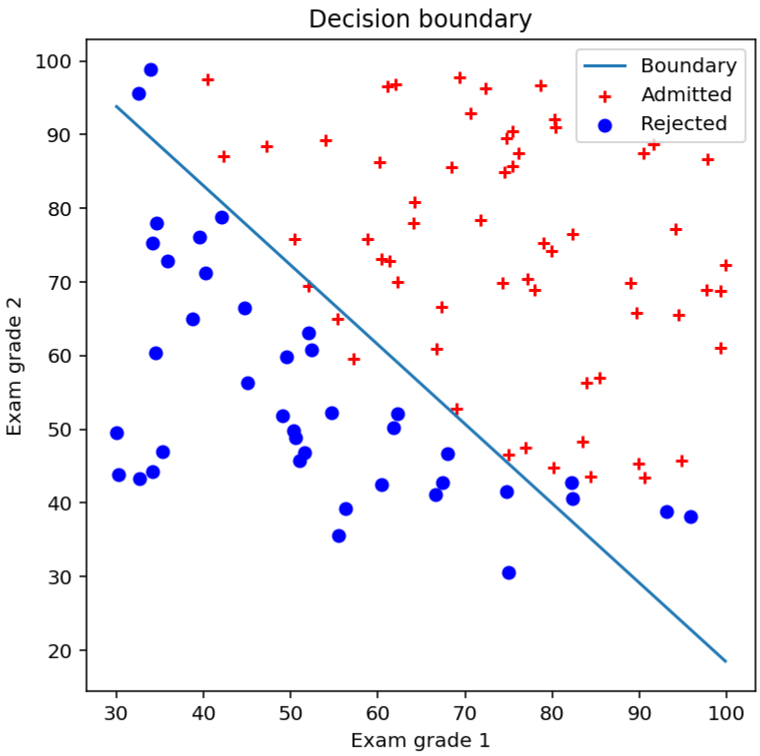
The decision boundary is defined by all points

$$x = (x_{(1)}, x_{(2)}) \quad \text{such that} \quad p_w(x) = 0.5$$

You may use numpy and matplotlib functions `np.meshgrid` , `np.linspace` , `reshape` , `contour` .

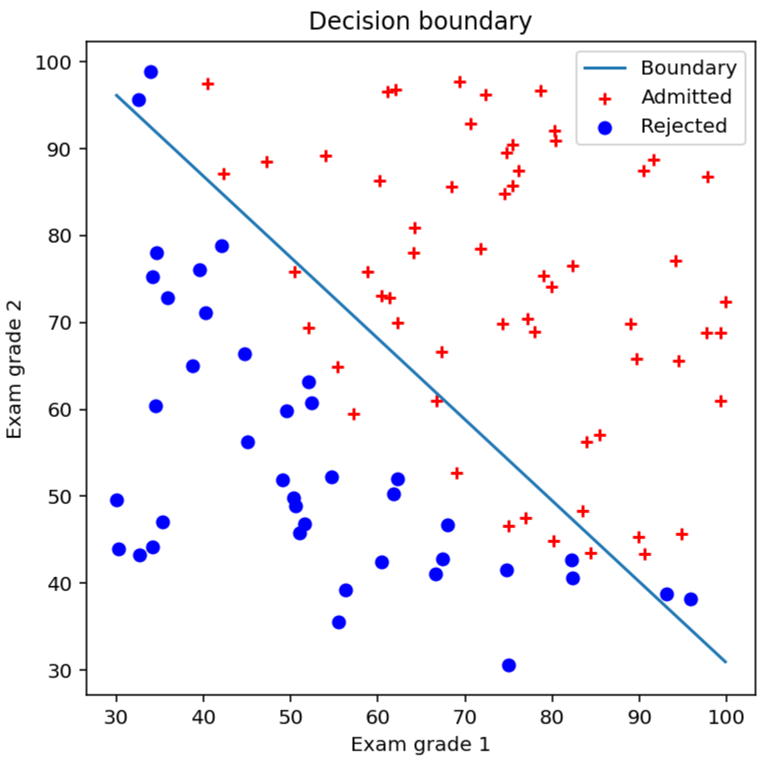
In [197]:

```
## Cross Entropy

# compute values p(x) for multiple data points x
x1_min, x1_max = X[:,1].min(), X[:,1].max() # min and max of grade 1
x2_min, x2_max = X[:,2].min(), X[:,2].max() # min and max of grade 2
# xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max)) # create mesh
grid
# X2 = np.ones([np.prod(xx1.shape),3])
# X2[:,1] = xx1.reshape(-1)
# X2[:,2] = xx2.reshape(-1)
# p = f_pred(X2,w)
# p = p.reshape(50,50)
x_line=np.linspace(x1_min, x1_max)
y_line=-(w[0]+w[1]*x_line)/w[2]

# plot
plt.figure(4,figsize=(6,6))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )
plt.plot(x_line,y_line)
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Boundary","Admitted","Rejected"])
plt.title('Decision boundary')
plt.show()
```



In [257]:

```
## MSE

# compute values p(x) for multiple data points x
x1_min, x1_max = X[:,1].min(), X[:,1].max() # min and max of grade 1
x2_min, x2_max = X[:,2].min(), X[:,2].max() # min and max of grade 2
x_line=np.linspace(x1_min, x1_max)
y_line=-(w[0]+w[1]*x_line)/w[2]

# plot
plt.figure(4,figsize=(6,6))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )
plt.plot(x_line,y_line)
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Boundary","Admitted","Rejected"])
plt.title('Decision boundary')
plt.show()
```




In [192]:

```
x_line=np.linspace(x1_min, x1_max)
y_line=-(w[0]+w[1]*x_line)/w[2]
```

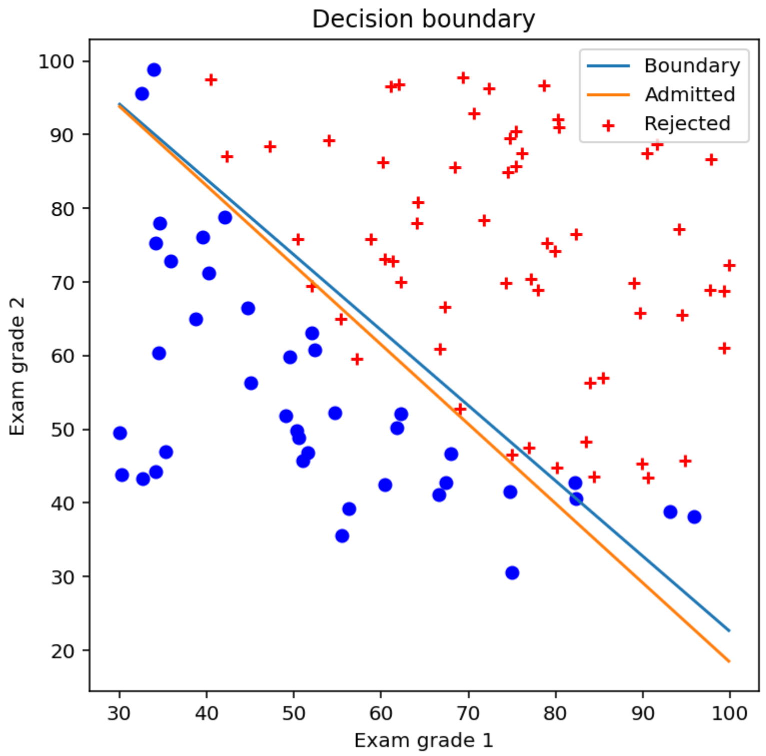
# 9. Comparison with Scikit-learn logistic regression algorithm with the gradient descent with the cross-entropy loss

---

You may use scikit-learn function `LogisticRegression(C=1e6)` .

In [58]:

```
# run logistic regression with scikit-learn
start = time.time()
logreg_sklearn = LogisticRegression(C=1e6)# scikit-learn logistic regression
logreg_sklearn.fit(data[:,[0,1]],data[:,2]) # learn the model parameters

# compute loss value
b=logreg_sklearn.intercept_
w_sklearn = logreg_sklearn.coef_

# plot
plt.figure(4,figsize=(6,6))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )

plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')

x1_min, x1_max = X[:,1].min(), X[:,1].max() # min and max of grade 1
x2_min, x2_max = X[:,2].min(), X[:,2].max() # min and max of grade 2
x_line=np.linspace(x1_min, x1_max)
y_line=-(b[0]+w_sklearn[0][0]*x_line)/w_sklearn[0][1]


plt.plot(x_line,y_line)
y_line2=-(w[0]+w[1]*x_line)/w[2]
plt.plot(x_line,y_line2)

plt.title('Decision boundary (black with gradient descent and magenta with scikit-learn)')
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Boundary","Admitted","Rejected"])
plt.title('Decision boundary')
plt.show()
```



In [48]:

```
start = time.time()
logreg_sklearn = LogisticRegression(C=1e6)# scikit-learn logistic regression
logreg_sklearn.fit(data[:,[0,1]],data[:,2]) # learn the model parameters
# logreg_sklearn.fit(X,data[:,2])
logreg_sklearn.coef_
logreg_sklearn.predict(data[:,[0,1]])
w_sklearn = logreg_sklearn.coef_
logreg_sklearn.intercept_
```

Out[48]:

```
array([-25.16138556])
```

In [53]:

```
w
```

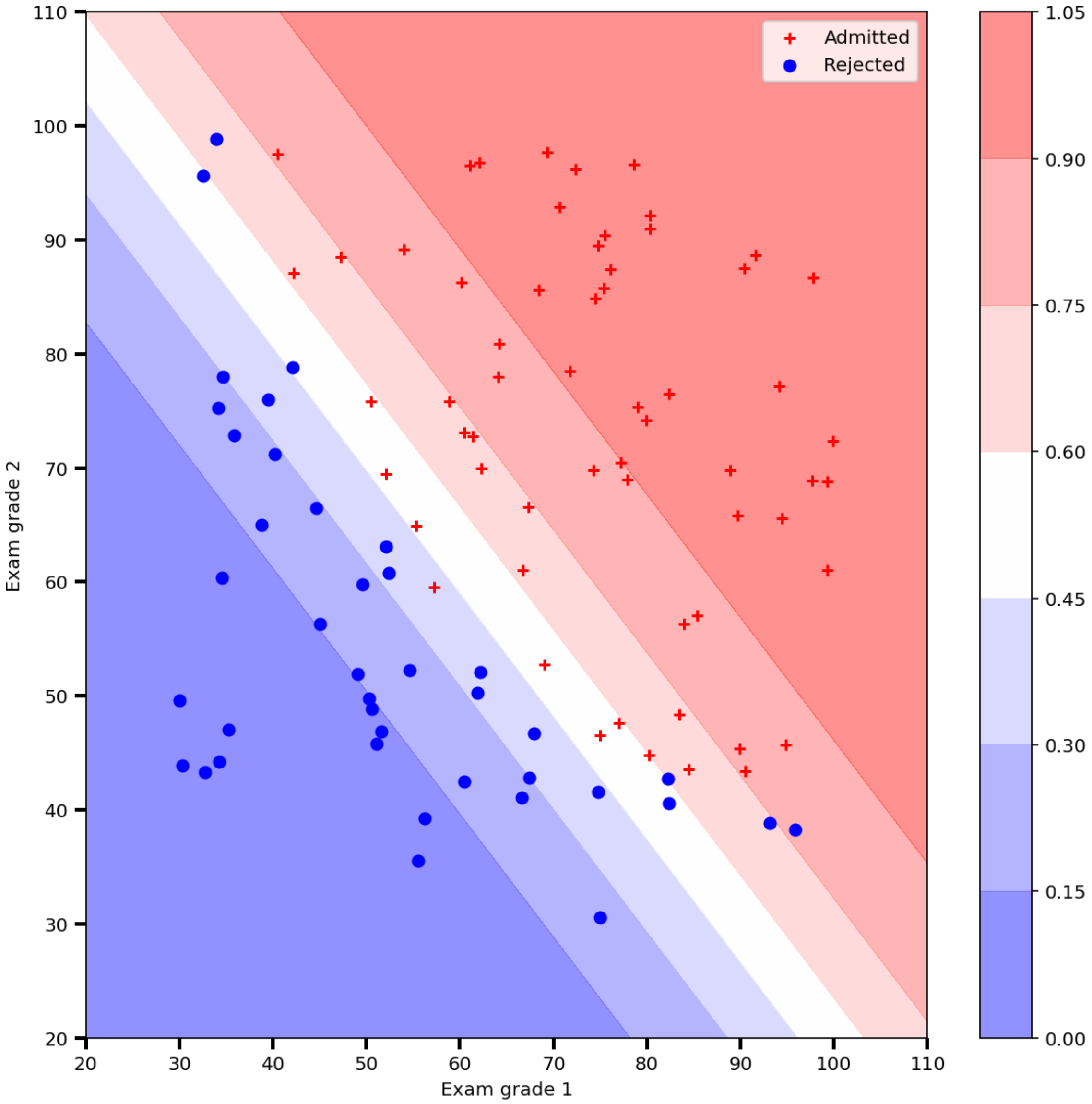
Out[53]:

```
array([[-10.00125246],
       [  0.08547625],
       [  0.07922129]])
```

## 10. Plot the probability map

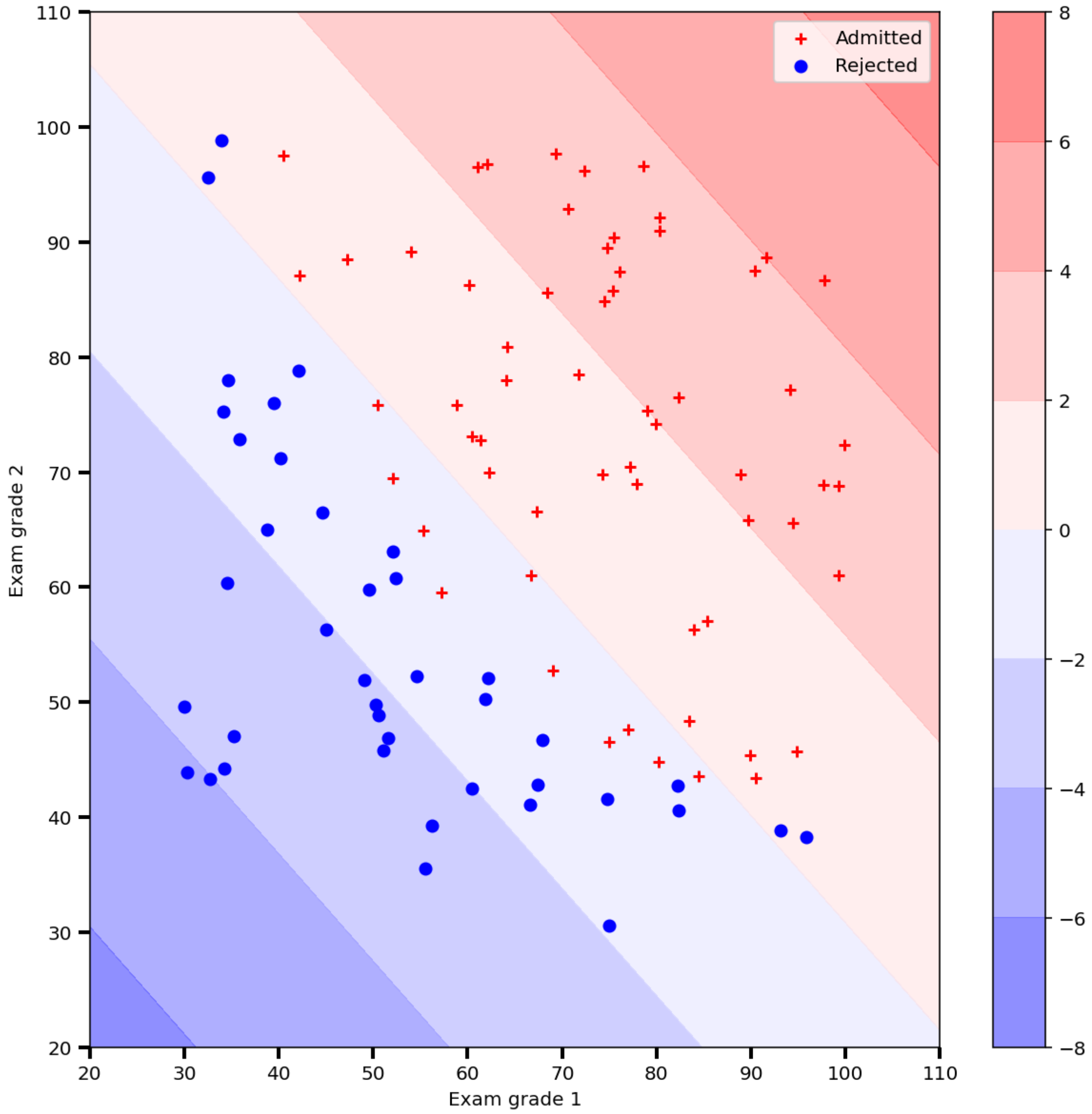
In [248]:

```
#cross entropy
num_a = 110
grid_x1 = np.linspace(20,110,num_a)
grid_x2 = np.linspace(20,110,num_a)

score_x1, score_x2 = np.meshgrid(grid_x1,grid_x2)

X_temp = np.ones([np.prod(score_x1.shape),3])
X_temp[:,1]=score_x1.reshape(-1)
X_temp[:,2]=score_x2.reshape(-1)
Z = f_pred(X_temp,w).reshape(110,110)
# xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max)) # create mesh
grid
# X2 = np.ones([np.prod(xx1.shape),3])
# X2[:,1] = xx1.reshape(-1)
# X2[:,2] = xx2.reshape(-1)
# p = f_pred(X2,w)
# p = p.reshape(50,50)
# for i in range(len(score_x1)):
#     for j in range(len(score_x2)):

#             predict_prob = sigmoid( )

#             Z[j, i] = predict_prob

            # actual plotting example
fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111)
ax.tick_params(length=6, width=2, grid_alpha=0.5)
ax.set_xlabel('Exam grade 1')
ax.set_ylabel('Exam grade 2')

ax.set_xlim(20, 110)
ax.set_ylim(20, 110)

cf = ax.contourf(score_x1,score_x2,Z,cmap="bwr",alpha=0.5)
ax.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+",label="Admitted")
ax.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o",label="Rejected" )
cbar = fig.colorbar(cf)
cbar.update_ticks()
plt.legend()
plt.show()
```


Admitted
Rejected
Exam grade 1
Exam grade 2
20
30
40
50
60
70
80
90
100
110
1.05
0.90
0.75
0.60
0.45
0.30
0.15
0.00

In [40]:

```
##MSE
num_a = 110
grid_x1 = np.linspace(20,110,num_a)
grid_x2 = np.linspace(20,110,num_a)

score_x1, score_x2 = np.meshgrid(grid_x1,grid_x2)

X_temp = np.ones([np.prod(score_x1.shape),3])
X_temp[:,1]=score_x1.reshape(-1)
X_temp[:,2]=score_x2.reshape(-1)
Z = f_pred2(X_temp,w).reshape(110,110)

fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111)
ax.tick_params(length=6, width=2, grid_alpha=0.5)
ax.set_xlabel('Exam grade 1')
ax.set_ylabel('Exam grade 2')

ax.set_xlim(20, 110)
ax.set_ylim(20, 110)

cf = ax.contourf(score_x1,score_x2,Z,cmap="bwr",alpha=0.5)
ax.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+",label="Admitted")
ax.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o",label="Rejected" )
cbar = fig.colorbar(cf)
cbar.update_ticks()
plt.legend()
plt.show()
```


Admitted
Rejected
Exam grade 1
Exam grade 2
110
100
90
80
70
60
50
40
30
20
8
6
4
2
0
−2
−4
−6
−8

In [210]:

```
num_a = 110
grid_x1 = np.linspace(20,110,num_a)
grid_x2 = np.linspace(20,110,num_a)

score_x1, score_x2 = np.meshgrid(grid_x1,grid_x2)

Z = np.zeros(score_x1.shape)
Z.shape
X_temp = np.ones([np.prod(score_x1.shape),3])
X_temp[:,1]=score_x1.reshape(-1)
X_temp[:,2]=score_x2.reshape(-1)
Z = f_pred(X_temp,w).reshape(110,110)
Z.shape
Z
```

Out[210]:

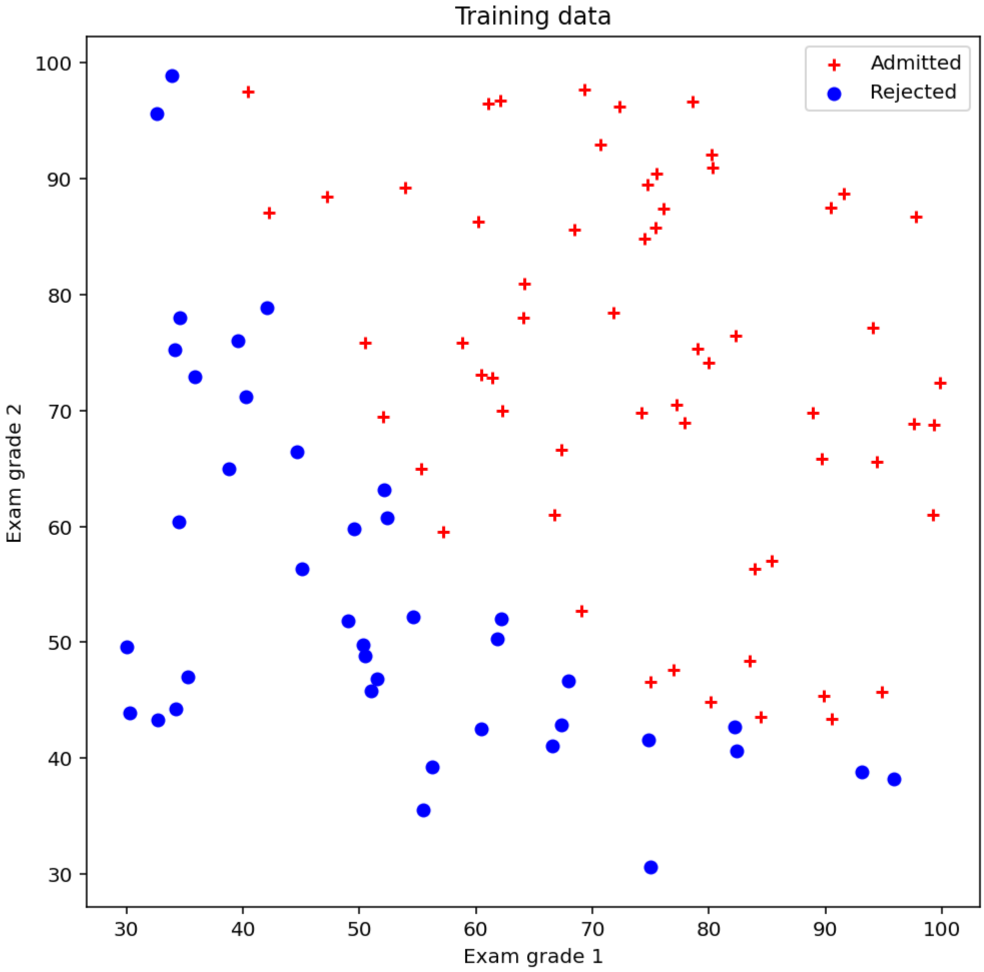
```
array([[0.00122047, 0.0013096 , 0.00140523, ..., 0.69939273, 0.71401906,
        0.72821029],
       [0.00130286, 0.001398  , 0.00150008, ..., 0.7129633 , 0.7271869 ,
        0.74096176],
       [0.00139081, 0.00149236, 0.00160132, ..., 0.7261611 , 0.73996924,
        0.75331763],
       ...,
       [0.57243439, 0.58961332, 0.60657641, ..., 0.99960786, 0.99963457,
        0.99965947],
       [0.58836306, 0.60534323, 0.62207289, ..., 0.99963268, 0.9996577 ,
        0.99968102],
       [0.60410872, 0.62085792, 0.63732393, ..., 0.99965593, 0.99967937,
        0.99970121]])
```

# Output results

## 1. Plot the dataset in 2D cartesian coordinate system (1pt)

In [18]:

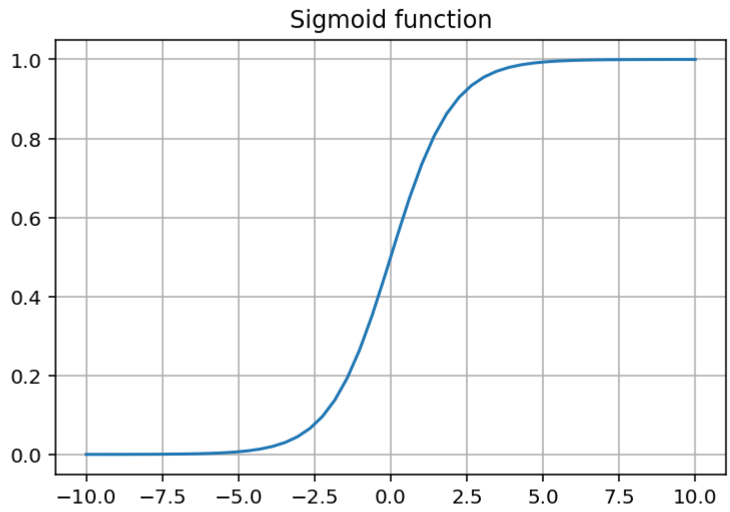
```
plt.figure(figsize=(8,8))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )
plt.title('Training data')
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Admitted","Rejected"])
plt.show()
```




## 2. Plot the sigmoid function (1pt)

In [20]:

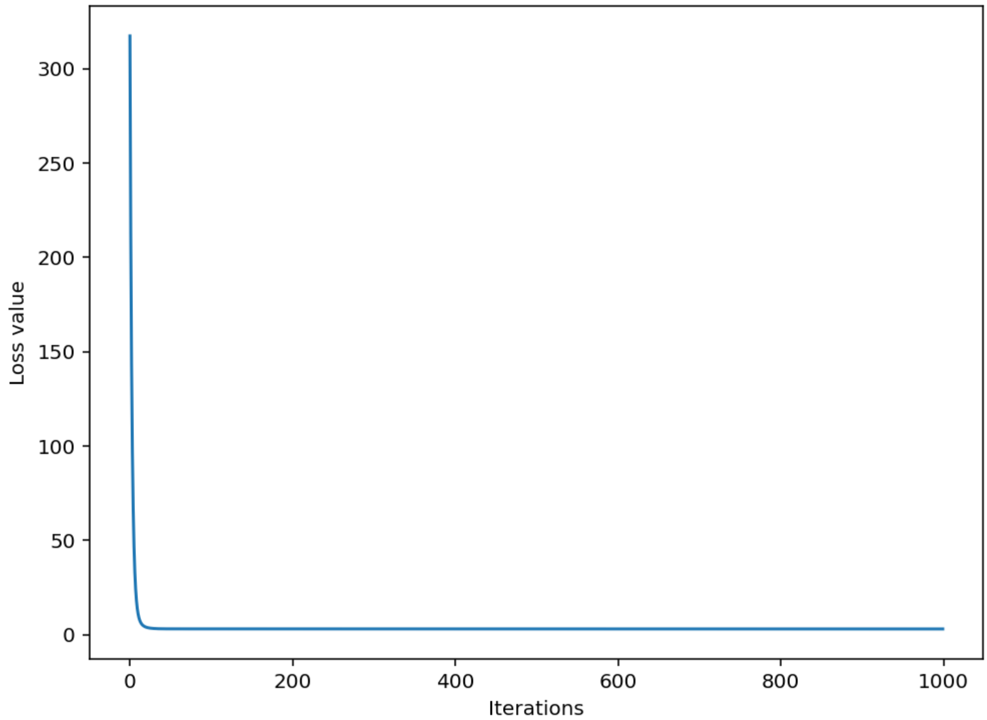
```
plt.figure(2)
plt.plot(x_values,sigmoid(x_values))
plt.title("Sigmoid function")
plt.grid(True)
```




## 3. Plot the loss curve in the course of gradient descent using the mean square error (2pt)

In [256]:

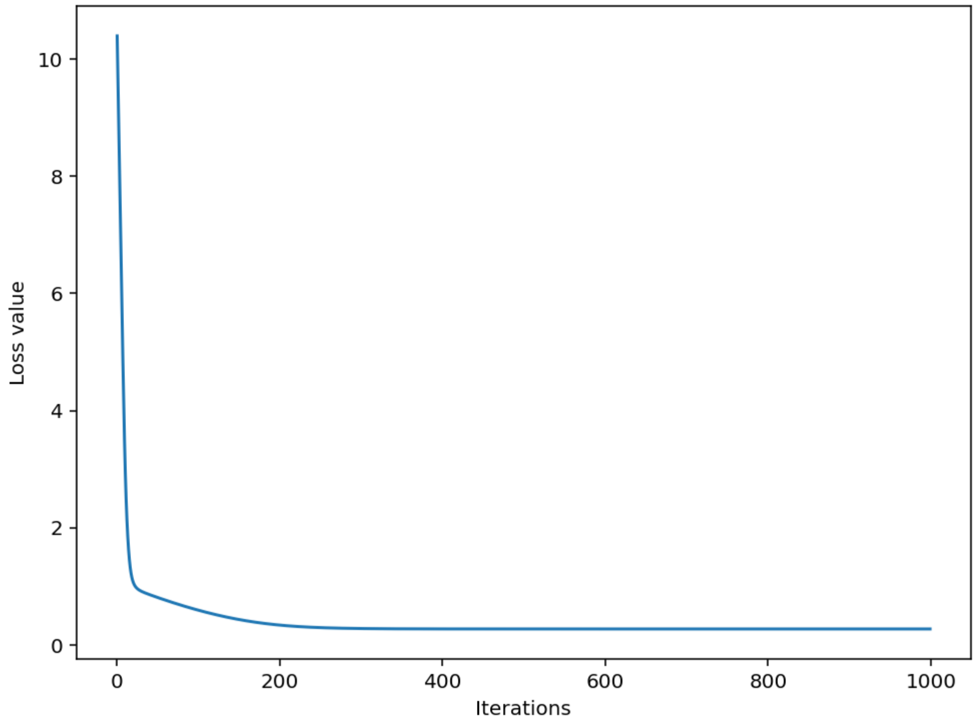
```
plt.figure(figsize=(8,6))
plt.plot(L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```



# 4. Plot the loss curve in the course of gradient descent using the cross-entropy error (2pt)

In [174]:

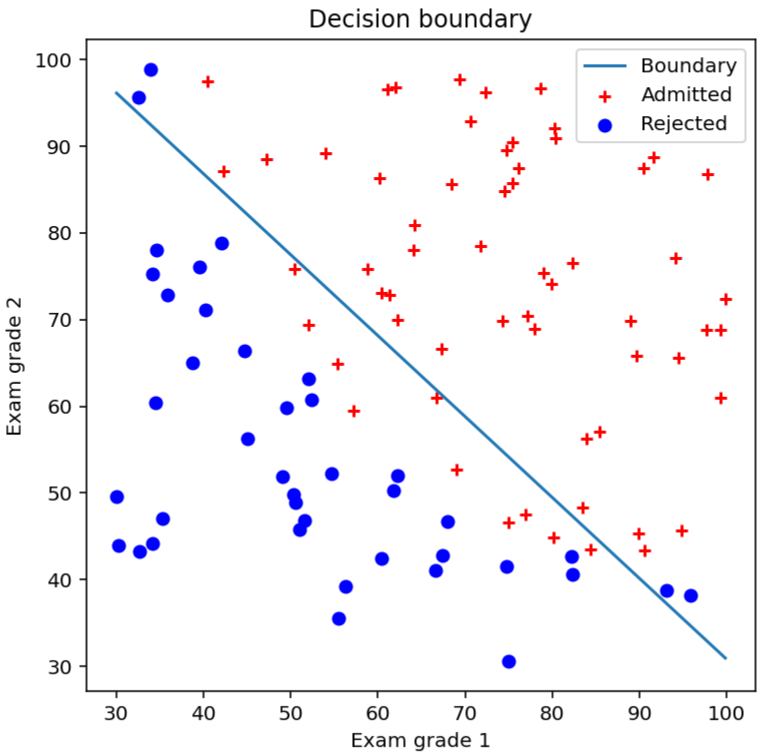
```
# plot
plt.figure(figsize=(8,6))
plt.plot(L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```




# 5. Plot the decision boundary using the mean square error (2pt)

In [258]:

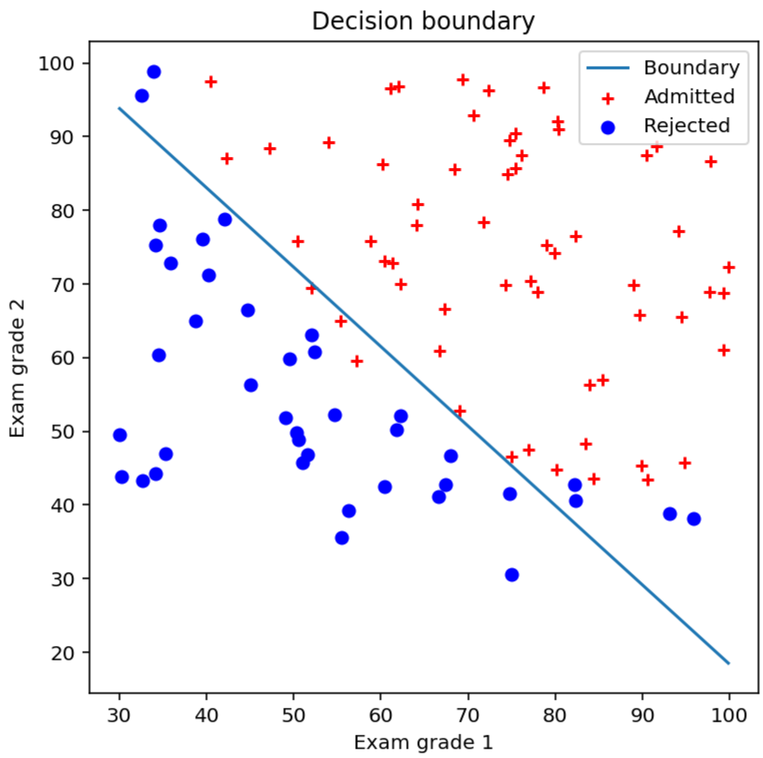
```
plt.figure(4,figsize=(6,6))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )
plt.plot(x_line,y_line)
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Boundary","Admitted","Rejected"])
plt.title('Decision boundary')
plt.show()
```




# 6. Plot the decision boundary using the cross-entropy error (2pt)

In [198]:

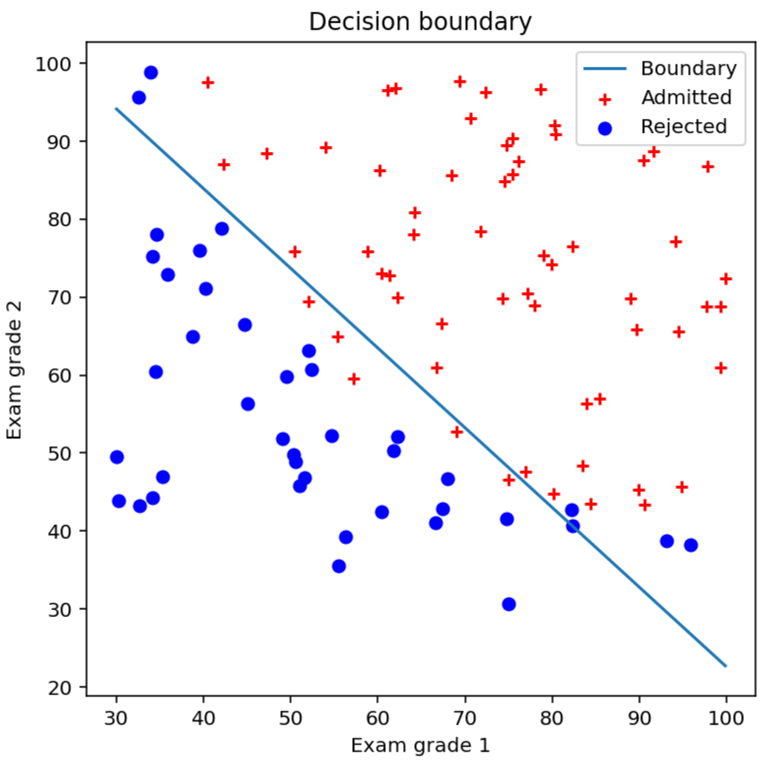
```
plt.figure(4,figsize=(6,6))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )
plt.plot(x_line,y_line)
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Boundary","Admitted","Rejected"])
plt.title('Decision boundary')
plt.show()
```




# 7. Plot the decision boundary using the Scikit-learn logistic regression algorithm (2pt)

In [57]:

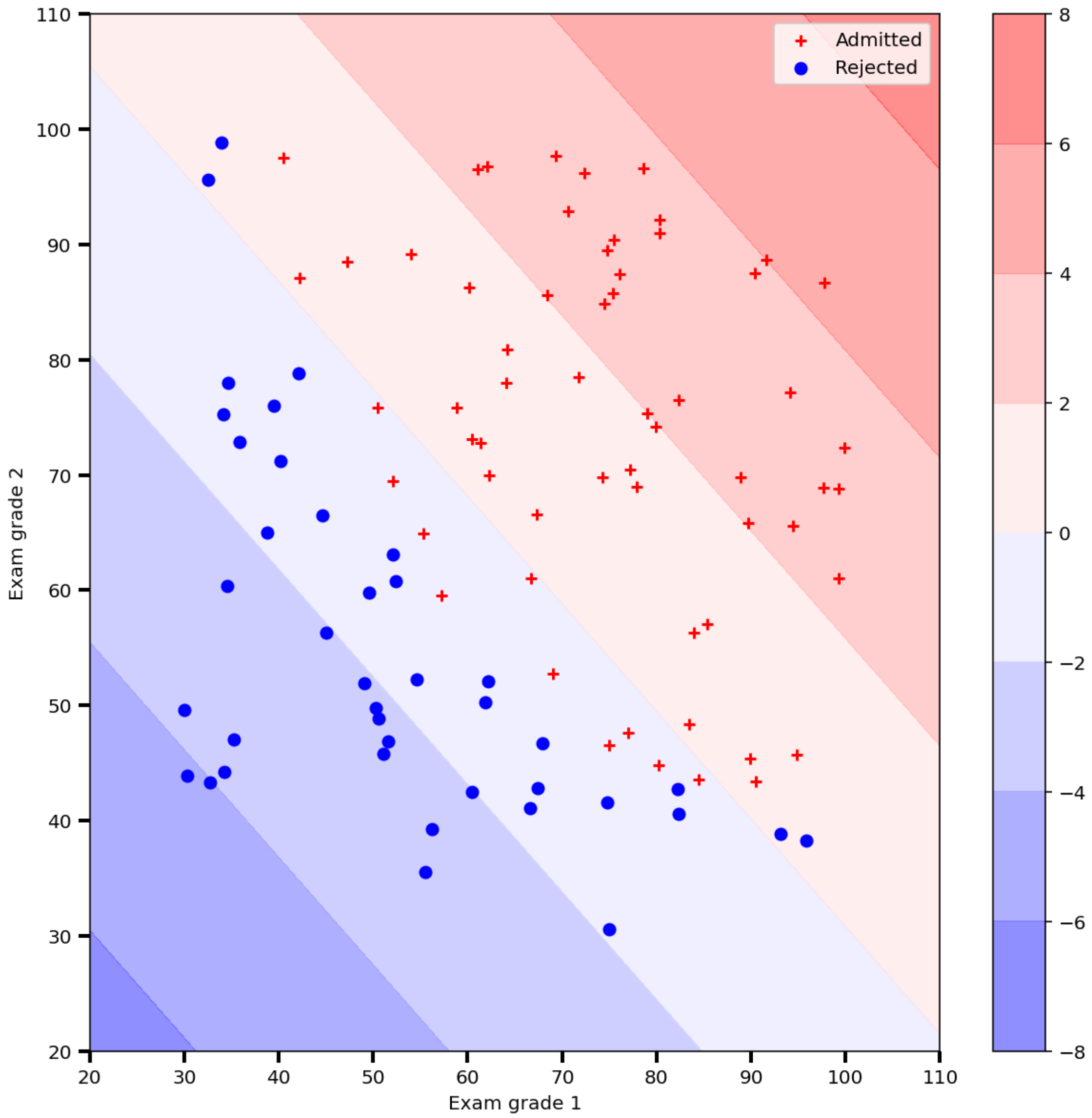
```
plt.figure(4,figsize=(6,6))
plt.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+")
plt.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o" )

plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')

x1_min, x1_max = X[:,1].min(), X[:,1].max() # min and max of grade 1
x2_min, x2_max = X[:,2].min(), X[:,2].max() # min and max of grade 2
x_line=np.linspace(x1_min, x1_max)
y_line=-(b[0]+w_sklearn[0][0]*x_line)/w_sklearn[0][1]


plt.plot(x_line,y_line)

plt.title('Decision boundary (black with gradient descent and magenta with scikit-learn)')
plt.xlabel('Exam grade 1')
plt.ylabel('Exam grade 2')
plt.legend(["Boundary","Admitted","Rejected"])
plt.title('Decision boundary')
plt.show()
```




## 8. Plot the probability map using the mean square error (2pt)

In [42]:

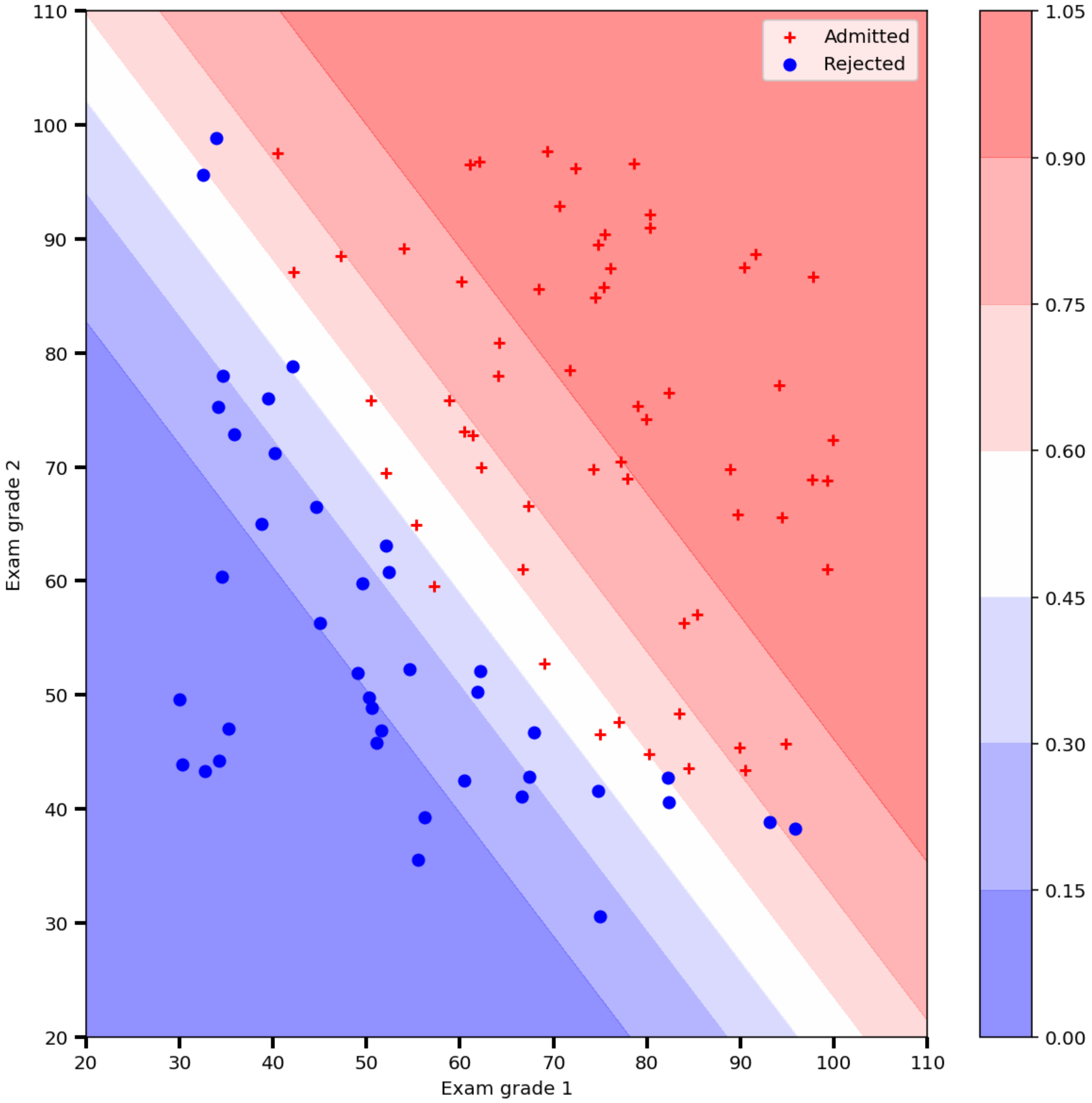
```
fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111)
ax.tick_params(length=6, width=2, grid_alpha=0.5)
ax.set_xlabel('Exam grade 1')
ax.set_ylabel('Exam grade 2')

ax.set_xlim(20, 110)
ax.set_ylim(20, 110)

cf = ax.contourf(score_x1,score_x2,Z,cmap="bwr",alpha=0.5)
ax.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+",label="Admitted")
ax.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o",label="Rejected" )
cbar = fig.colorbar(cf)
cbar.update_ticks()
plt.legend()
plt.show()
```




# 9. Plot the probability map using the cross-entropy error (2pt)

In [249]:

```
fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111)
ax.tick_params(length=6, width=2, grid_alpha=0.5)
ax.set_xlabel('Exam grade 1')
ax.set_ylabel('Exam grade 2')

ax.set_xlim(20, 110)
ax.set_ylim(20, 110)

cf = ax.contourf(score_x1,score_x2,Z,cmap="bwr",alpha=0.5)
ax.scatter(x=x1[idx_admit],y=x2[idx_admit],c="red",marker="+",label="Admitted")
ax.scatter(x1[idx_rejec],x2[idx_rejec],c="blue",marker="o",label="Rejected" )
cbar = fig.colorbar(cf)
cbar.update_ticks()
plt.legend()
plt.show()
```




In [ ]: