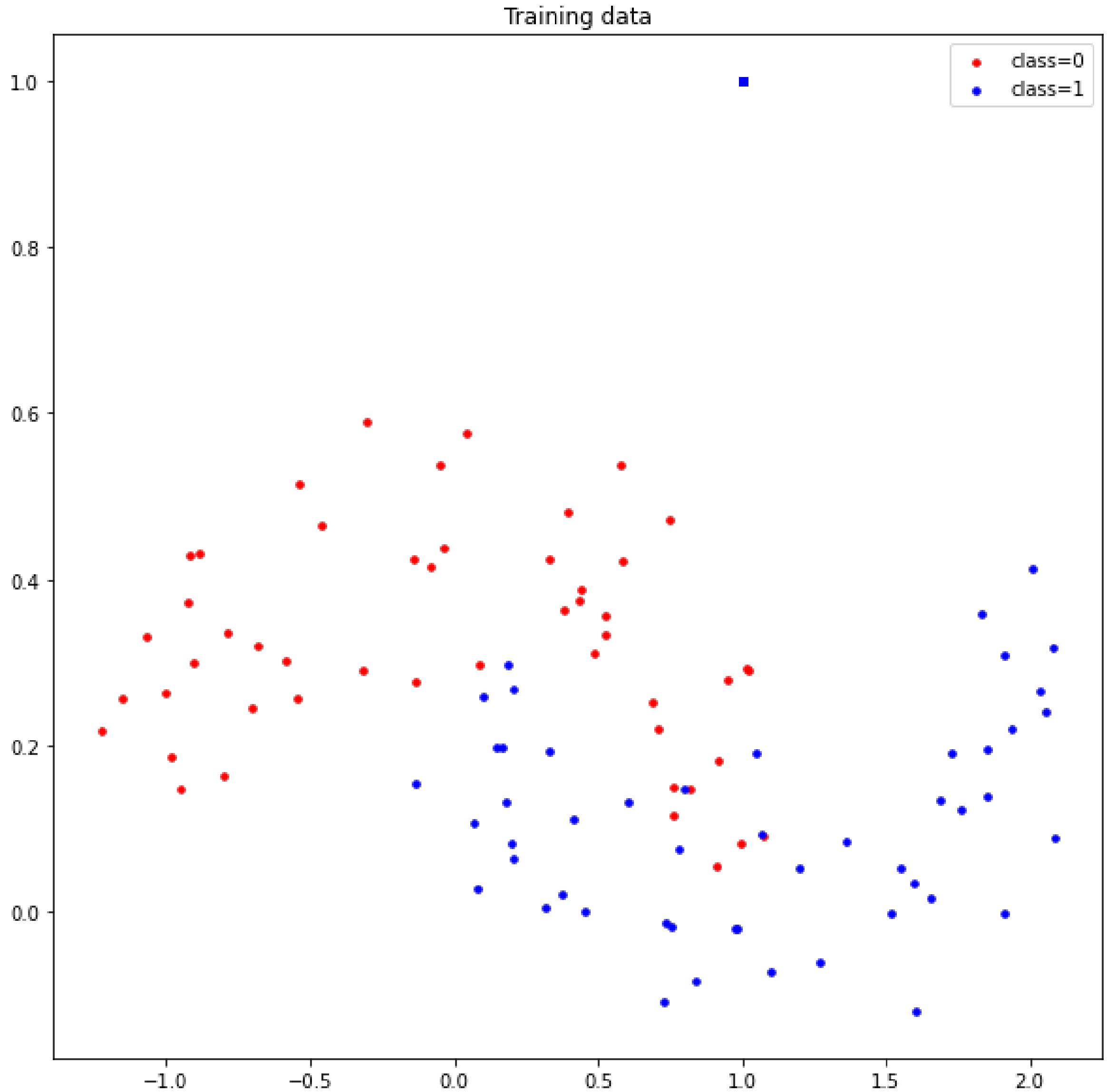
Load Data

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

# import data with numpy
data_train  = np.loadtxt('training.txt', delimiter=',')
data_test   = np.loadtxt('testing.txt', delimiter=',')

# number of training data
number_data_train   = data_train.shape[0]
number_data_test    = data_test.shape[0]

# training data
x1_train            = data_train[:,0] # feature 1
x2_train            = data_train[:,1] # feature 2
idx_class0_train    = (data_train[:,2]==0) # index of class0
idx_class1_train    = (data_train[:,2]==1) # index of class1

# testing data
x1_test             = data_test[:,0] # feature 1
x2_test             = data_test[:,1] # feature 2
idx_class0_test     = (data_test[:,2]==0) # index of class0
idx_class1_test     = (data_test[:,2]==1) # index of class1
```

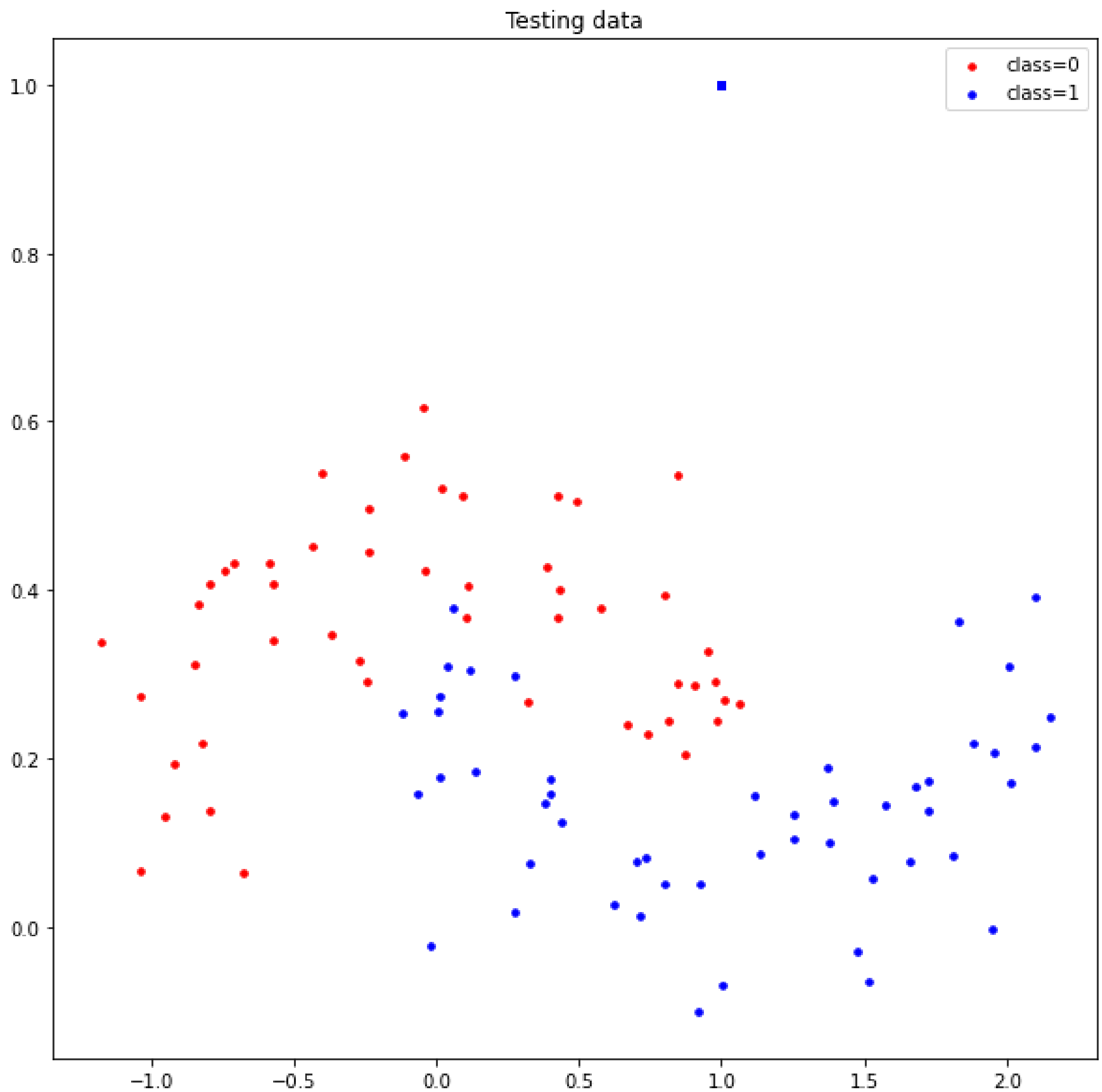
In [7]:
```
number_data_test
```

Out[7]: 200

In [2]:
```
plt.figure(1,figsize=(10,10))
plt.scatter(  x1_train[idx_class0_train], x2_train[idx_class0_train], s=50, c='r', marker='.', label='class=0')
plt.scatter(  x1_train[idx_class1_train], x2_train[idx_class1_train], s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```



In [5]:
```
plt.figure(1,figsize=(10,10))
plt.scatter(  x1_test[idx_class0_test], x2_test[idx_class0_test], s=50, c='r', mark
er='.', label='class=0')
plt.scatter(  x1_test[idx_class1_test], x2_test[idx_class1_test], s=50, c='b', mark
er='.', label='class=1')
plt.title('Testing data')
plt.legend()
plt.show()
```



equation

In [252]:

```
# sigmoid function
def sigmoid(z):
    overflow=(z>-709).flatten()
    sigmoid_f= np.zeros(z.shape)
    sigmoid_f[overflow] =1/(1+np.exp(-z[overflow]))
    return sigmoid_f


# predictive function definition
def f_pred(X,w):
    p = sigmoid(np.dot(X,w))
    return p


# loss function definition
def loss_logreg(X,y,w,lambda_):
    loss=np.mean(-y*np.log(f_pred(X,w)+1e-30)-(1-y)*np.log(1-f_pred(X,w)+1e-30))+lambda_*np.sum(w*w)/2
    return loss


# gradient function definition
def grad_loss(X,y,w,lambda_):
    n = len(y)
    grad =  np.dot(X.T,(f_pred(X,w)-y))/n*2+lambda_*w
    return grad


# gradient descent function definition
def grad_desc(X_train, X_test, y_train, y_test , w_init, tau, max_iter,lambda_):

    L_iters_train = np.zeros([max_iter])# record the loss values
    L_iters_test = np.zeros([max_iter])
    w = w_init # initialization

    for i in range(max_iter): # loop over the iterations
        grad_f = grad_loss(X_train,y_train,w,lambda_) # gradient of the loss
        w =  w - tau* grad_f # update rule of gradient descent
        L_iters_train[i] =  loss_logreg(X_train,y_train,w,lambda_)# save the current loss value
        L_iters_test[i] =  loss_logreg(X_test,y_test,w,lambda_)# save the current loss value
    return w, L_iters_train, L_iters_test
```

In [194]:

```
print(w_init.shape)
X_train.shape
```

```
(100, 1)
```

Out[194]:

```
(200, 100)
```

In [ ]:

In [196]:
```
z=np.dot(X_train,w_init)
overflow=(z>-709).flatten()
print(overflow.shape)
sigmoid_f= np.zeros(z.shape)
sigmoid_f.shape
sigmoid_f[overflow] =1/(1+np.exp(-z[overflow]))
```

```
(200,)
```

define function

In [229]:
```
def makeArray(x1,x2):
    X = np.ones([number_data_test,100])

    for i in range(10):
        for j in range(10):
            X[:,10*i+j]=X[:,10*i+j]*np.power(x1,i)*np.power(x2,j)
    return X

def makeArray2(x1,x2):
    X = np.ones([2500,100])

    for i in range(10):
        for j in range(10):
            X[:,10*i+j]=X[:,10*i+j]*np.power(x1,i)*np.power(x2,j)
    return X
```

In [230]:
```
X_train=makeArray(x1_train,x2_train)
X_test=makeArray(x1_test,x2_test)
```

In [231]:
```
y_train = data_train[:,2].reshape(-1,1)
y_test = data_test[:,2].reshape(-1,1)
```

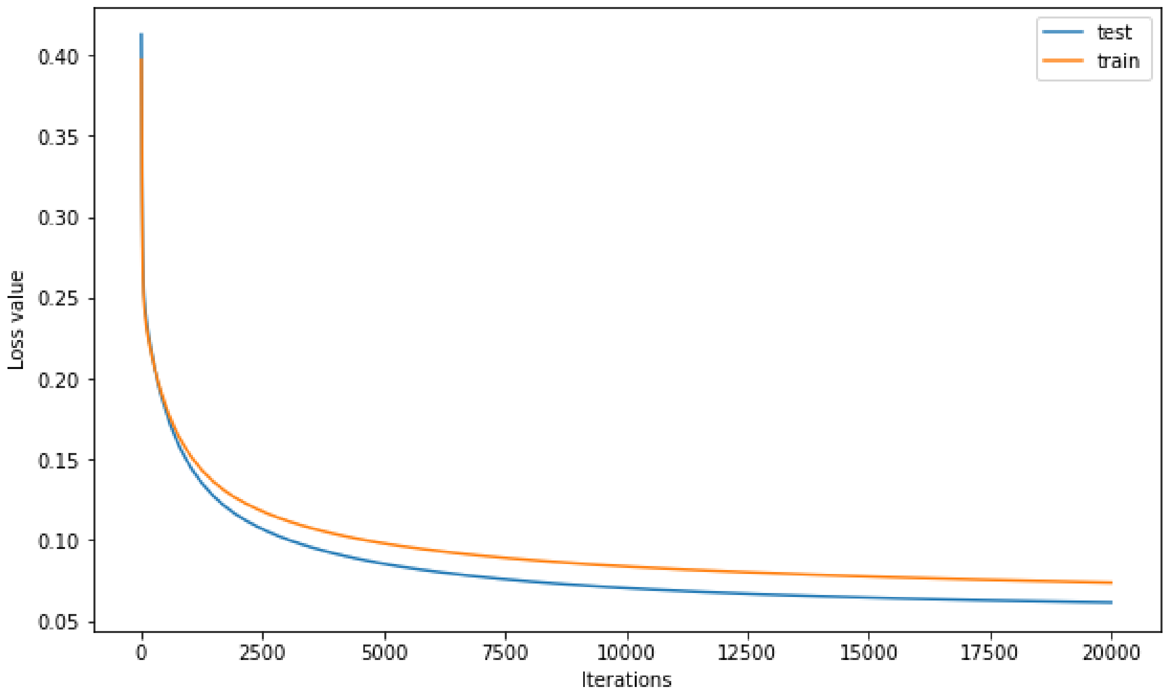
In [232]:
```
len(y_test)
lambda_=0.00001
```

In [233]:
```
w_init=np.ones([100,1])*0.3
```

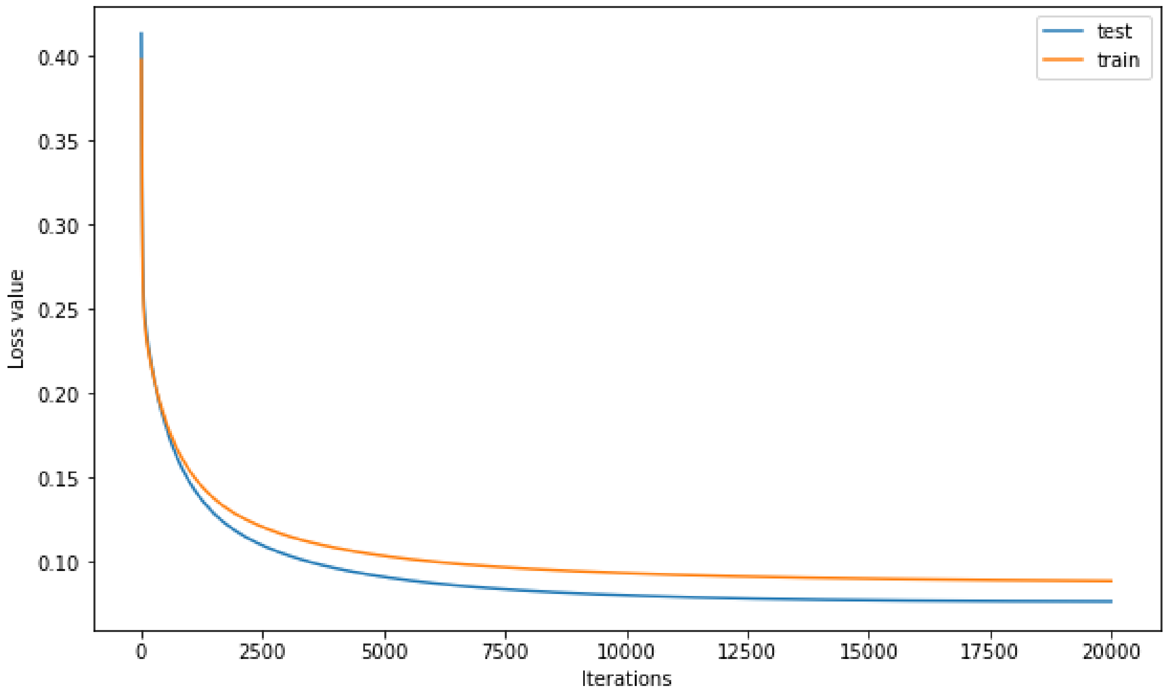
In [234]:
```
np.sum(w_init*w_init)
```

Out[234]:
```
8.999999999999998
```

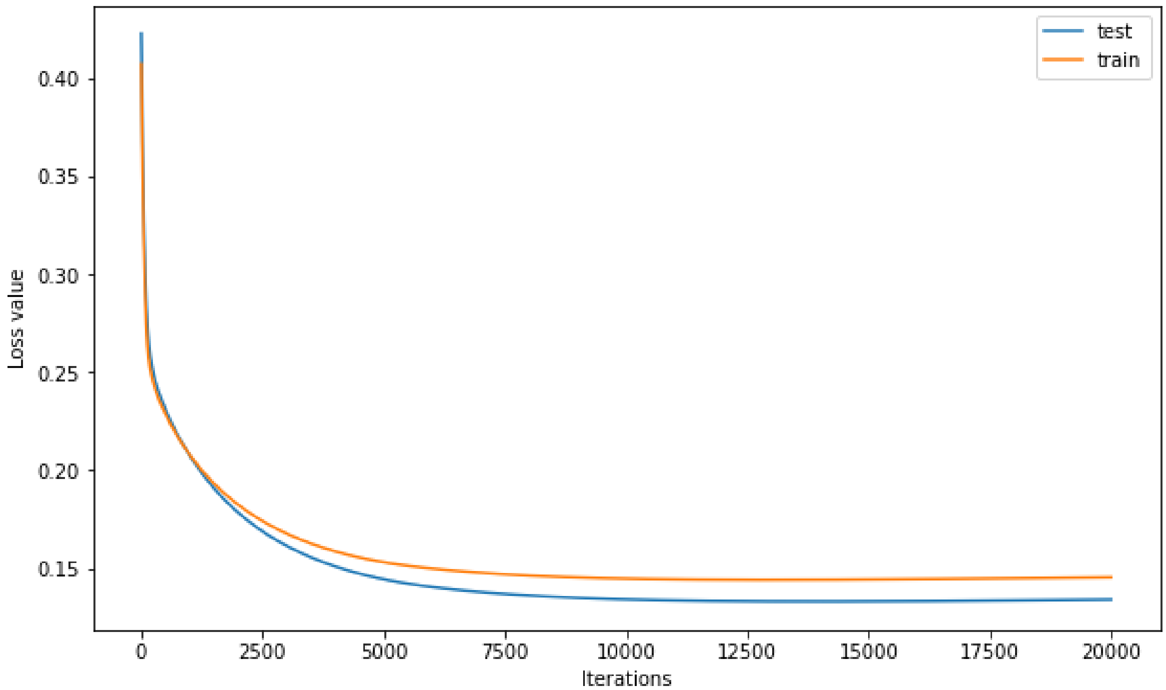
In [253]:
```
w_e5, L_iters_train_e5, L_iters_test_e5 = grad_desc(X_train, X_test, y_train, y_tes
t , w_init, tau=0.1, max_iter=20000, lambda_=0.00001)
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e5, label="test")
plt.plot(L_iters_train_e5, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



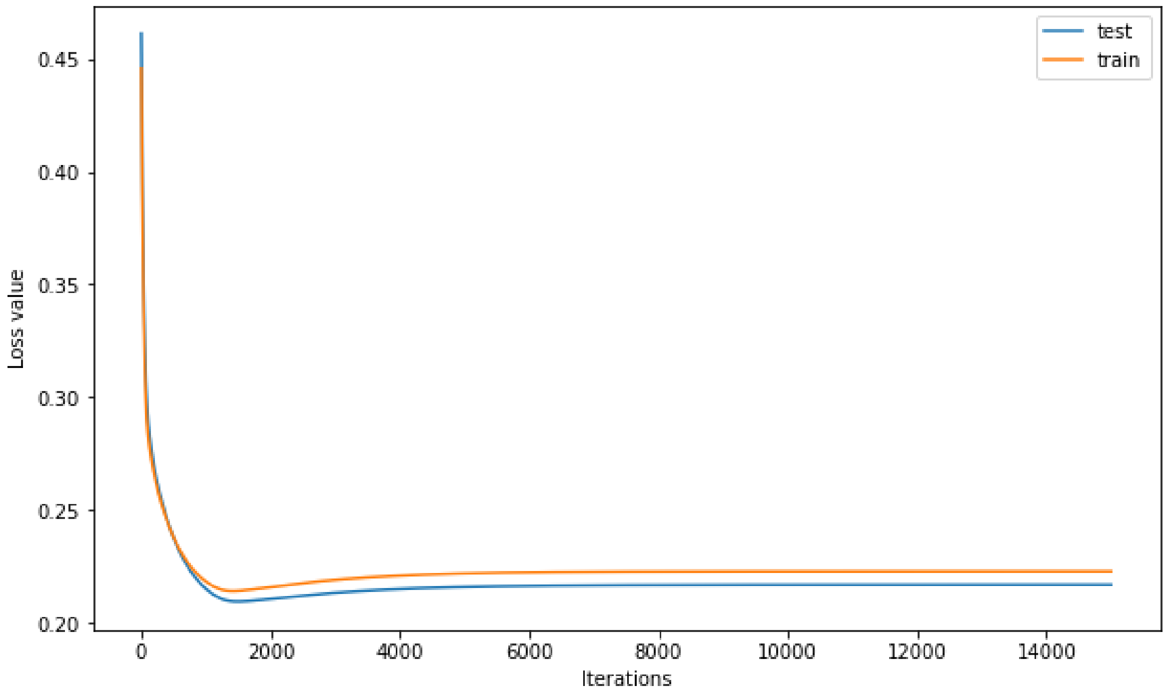
In [254]:
```
w_e4, L_iters_train_e4, L_iters_test_e4 = grad_desc(X_train, X_test, y_train, y_tes
t , w_init, tau=0.1, max_iter=20000, lambda_=0.0001)
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e4, label="test")
plt.plot(L_iters_train_e4, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



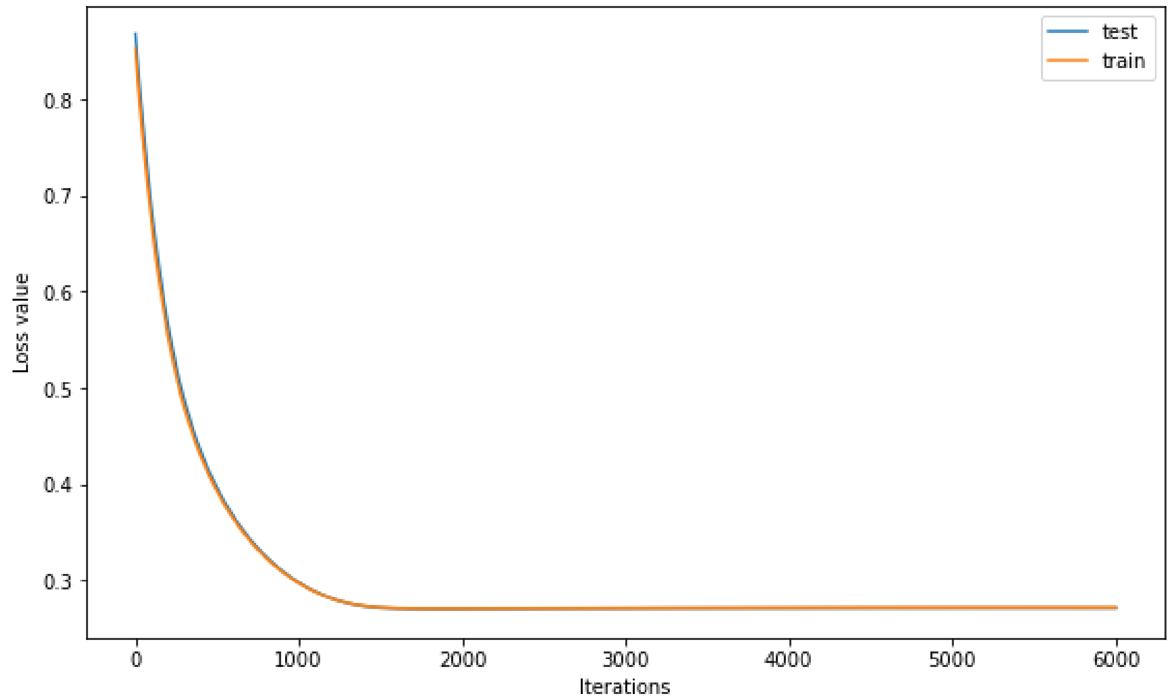
In [265]:
```
w_e3, L_iters_train_e3, L_iters_test_e3 = grad_desc(X_train, X_test, y_train, y_tes
t , w_init, tau=0.03, max_iter=20000, lambda_=0.001)
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e3, label="test")
plt.plot(L_iters_train_e3, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



In [269]:
```
w_e2, L_iters_train_e2, L_iters_test_e2 = grad_desc(X_train, X_test, y_train, y_tes
t , w_init, tau=0.05, max_iter=15000, lambda_=0.01)
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e2, label="test")
plt.plot(L_iters_train_e2, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



In [270]:
```
w_e1, L_iters_train_e1, L_iters_test_e1 = grad_desc(X_train, X_test, y_train, y_tes
t , w_init, tau=0.01, max_iter=6000, lambda_=0.1)
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e1, label="test")
plt.plot(L_iters_train_e1, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



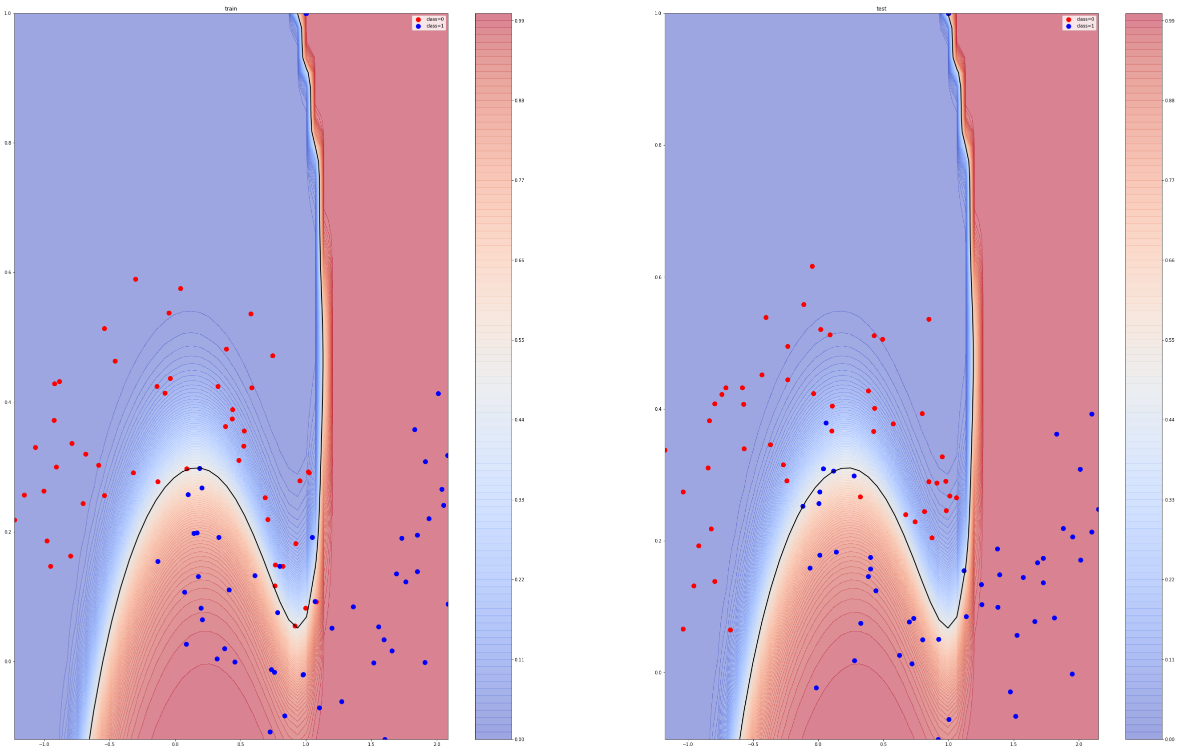
In [275]:
```
def makeArray2(x1,x2):
    X = np.ones([2500,100])

    for i in range(10):
        for j in range(10):
            X[:,10*i+j]=X[:,10*i+j]*np.power(x1,i)*np.power(x2,j)
    return X
```

In [276]:

Out[276]:
```
array([[1.62480369e-11],
       [2.12908866e-08],
       [3.54074550e-06],
       ...,
       [1.00000000e+00],
       [1.00000000e+00],
       [1.00000000e+00]])
```

In [350]:
```
def plotColorMap(w):
    plt.figure(figsize=(48,30))
    plt.subplot(1,2,1)
    x1_min, x1_max = x1_train.min(), x1_train.max() # min and max of grade 1
    x2_min, x2_max = x2_train.min(), x2_train.max() # min and max of grade 2

    xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max
))  # create meshgrid
    x1_1=xx1.reshape(-1)
    x2_1=xx2.reshape(-1)

    X2_train=makeArray2(x1_1,x2_1)
    p=f_pred(X2,w).reshape(50,50)


    ax = plt.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.5)
    cbar = plt.colorbar(ax)
    cbar.update_ticks()

    plt.contour(xx1, xx2, p, levels=[0.5], linewidths=2, colors='k')
    plt.scatter(  x1_train[idx_class0_train], x2_train[idx_class0_train], s=400, c=
'r', marker='.', label='class=0')
    plt.scatter(  x1_train[idx_class1_train], x2_train[idx_class1_train], s=400, c=
'b', marker='.', label='class=1')

    plt.legend()
    plt.title('train')

    plt.subplot(1,2,2)

    x1_min, x1_max = x1_test.min(), x1_test.max() # min and max of grade 1
    x2_min, x2_max = x2_test.min(), x2_test.max() # min and max of grade 2

    xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max
))  # create meshgrid
    x1_1=xx1.reshape(-1)
    x2_1=xx2.reshape(-1)

    X2_train=makeArray2(x1_1,x2_1)
    p=f_pred(X2,w).reshape(50,50)

    ax = plt.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.5)
    cbar = plt.colorbar(ax)
    cbar.update_ticks()

    plt.scatter(  x1_test[idx_class0_test], x2_test[idx_class0_test], s=400, c='r',
marker='.', label='class=0')
    plt.scatter(  x1_test[idx_class1_test], x2_test[idx_class1_test], s=400, c='b',
marker='.', label='class=1')
    plt.contour(xx1, xx2, p, levels=[0.5], linewidths=2, colors='k')
    plt.legend()
    plt.title('test')
    plt.show()
```

In [351]:
```
plotColorMap(w_e5)
```



In [296]:
```
x1_min
```

Out[296]: -1.2278

In [359]:
```
def accuracy(w,X,idx_class1):
    y_pred=f_pred(X,w)
    idx_class1_pred = (y_pred>=0.5 ).reshape(-1)
    acc=idx_class1_pred==idx_class1
    return acc.mean()
```

In [362]:
```
accuracy(w_e1,X_train,idx_class1_train)
```

Out[362]: 0.855

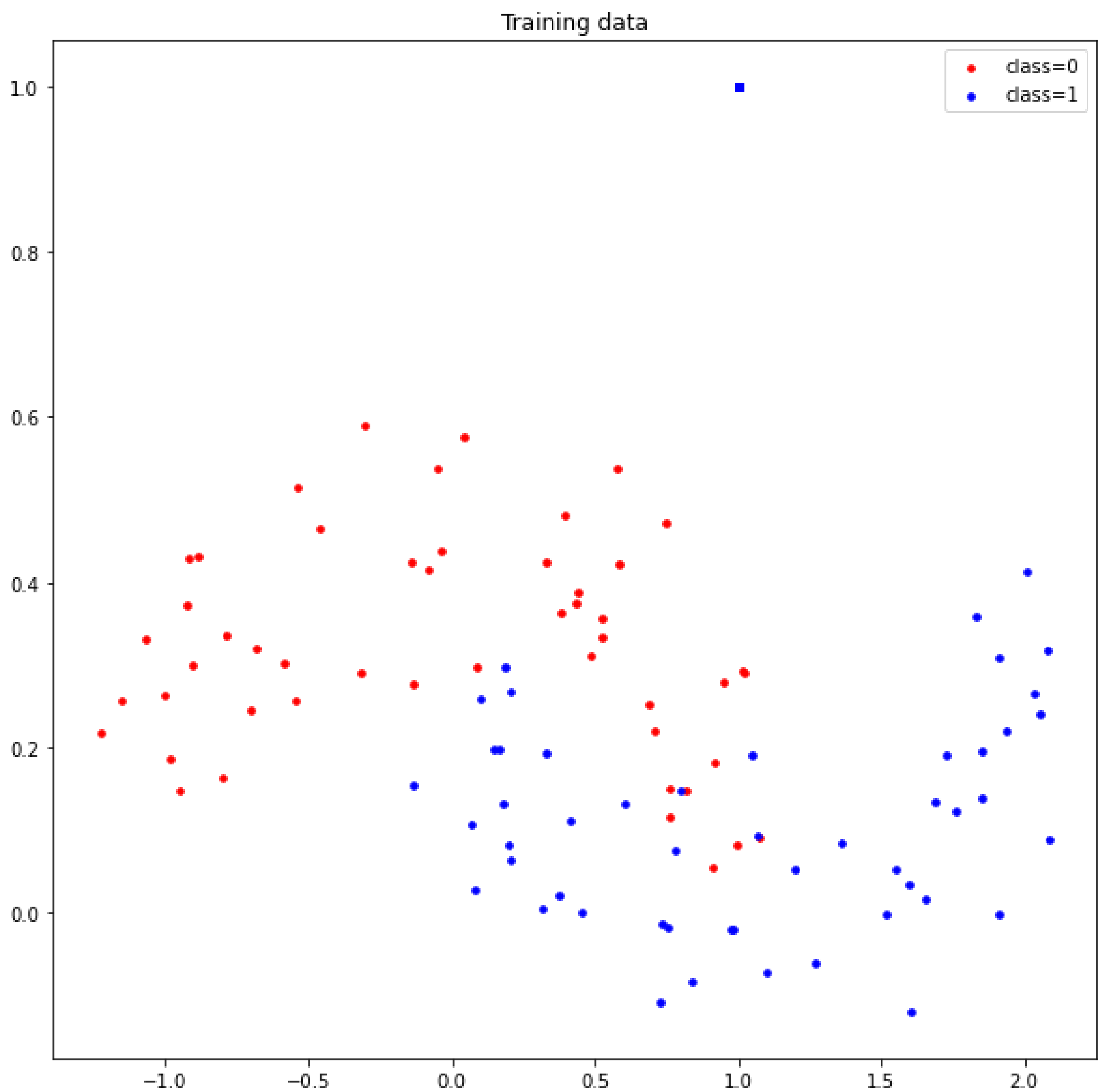
In [374]:
```
w_array=[w_e5,w_e4,w_e3,w_e2,w_e1]
lambda_array=[0.00001,0.0001,0.001,0.01,0.1]
for i,w in zip(lambda_array,w_array):
    print("lambda = ",i,"accuracy = ", accuracy(w,X_train,idx_class1_train),"%")
```

```
lambda =  1e-05 accuracy =  0.985 %
lambda =  0.0001 accuracy =  0.98 %
lambda =  0.001 accuracy =  0.975 %
lambda =  0.01 accuracy =  0.94 %
lambda =  0.1 accuracy =  0.855 %
```

Output using the dataset

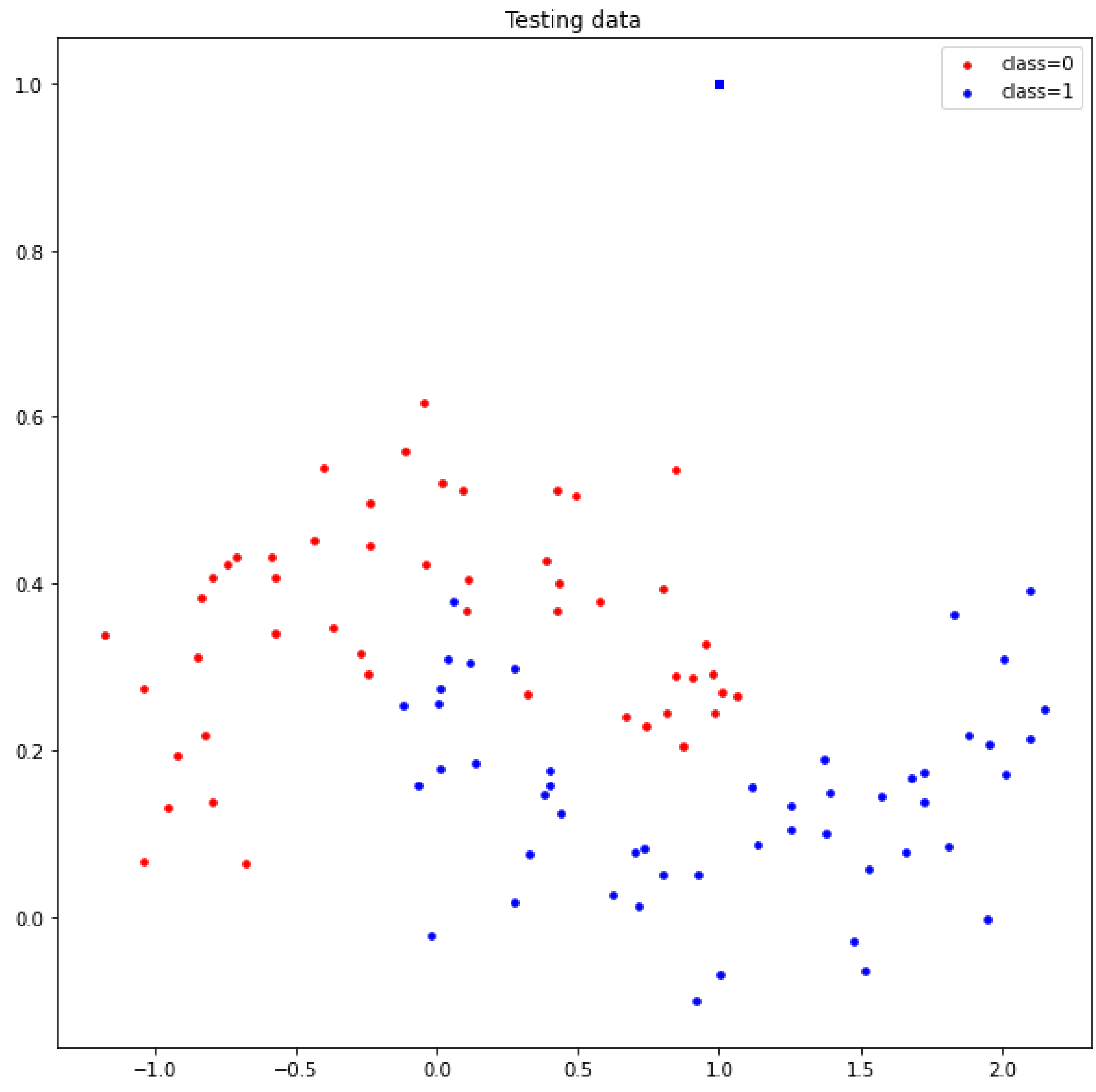
1. Plot the training data [0.5pt]

In [4]:
```
plt.figure(1,figsize=(10,10))
plt.scatter(  x1_train[idx_class0_train], x2_train[idx_class0_train], s=50, c='r',
marker='.', label='class=0')
plt.scatter(  x1_train[idx_class1_train], x2_train[idx_class1_train], s=50, c='b',
marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```



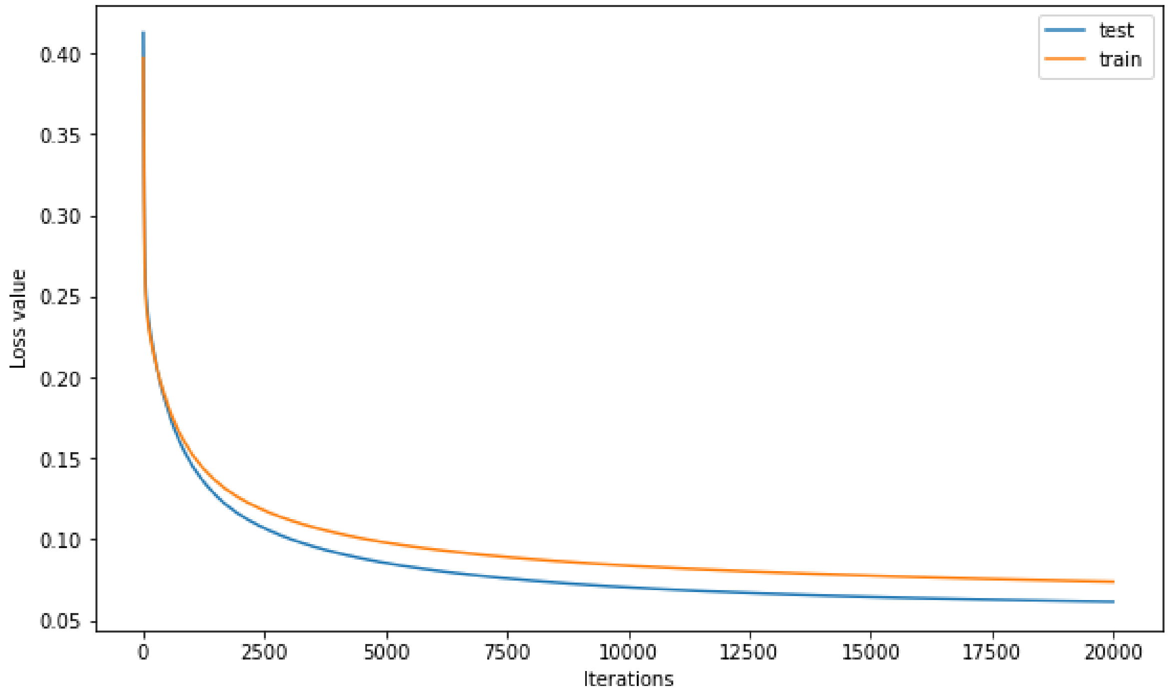
2. Plot the testing data [0.5pt]

In [8]:
```
plt.figure(1,figsize=(10,10))
plt.scatter(  x1_test[idx_class0_test], x2_test[idx_class0_test], s=50, c='r', marker='.', label='class=0')
plt.scatter(  x1_test[idx_class1_test], x2_test[idx_class1_test], s=50, c='b', marker='.', label='class=1')
plt.title('Testing data')
plt.legend()
plt.show()
```



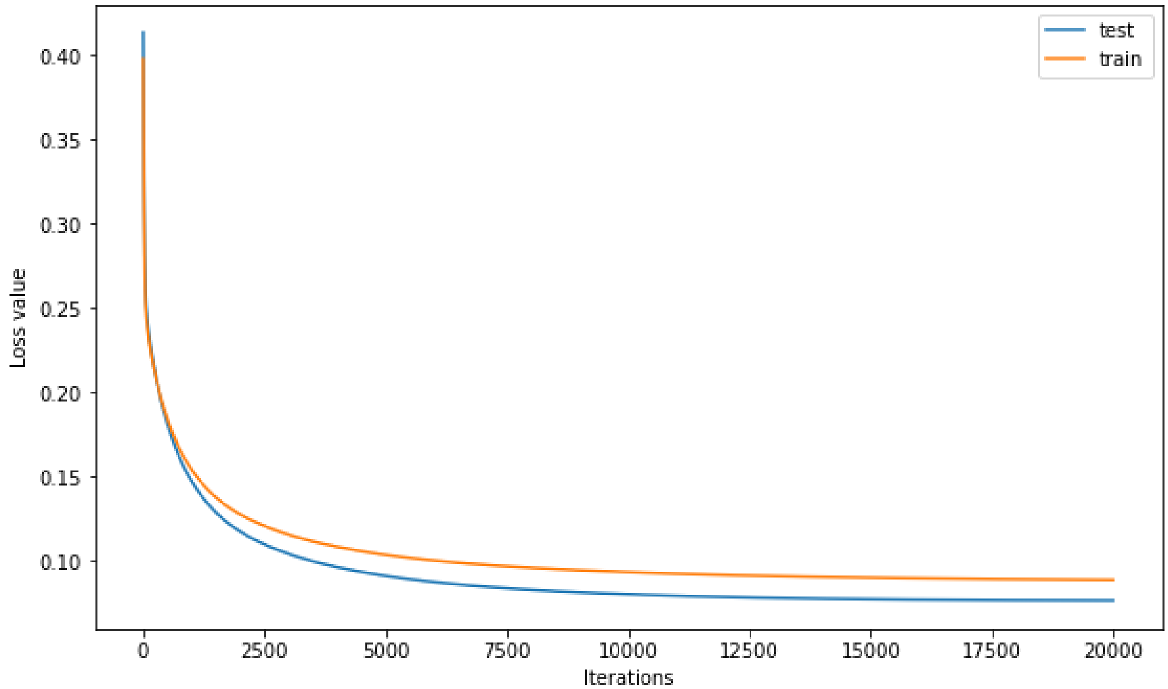
3. Plot the learning curve with λ=0.00001

In [383]:
```
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e5, label="test")
plt.plot(L_iters_train_e5, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



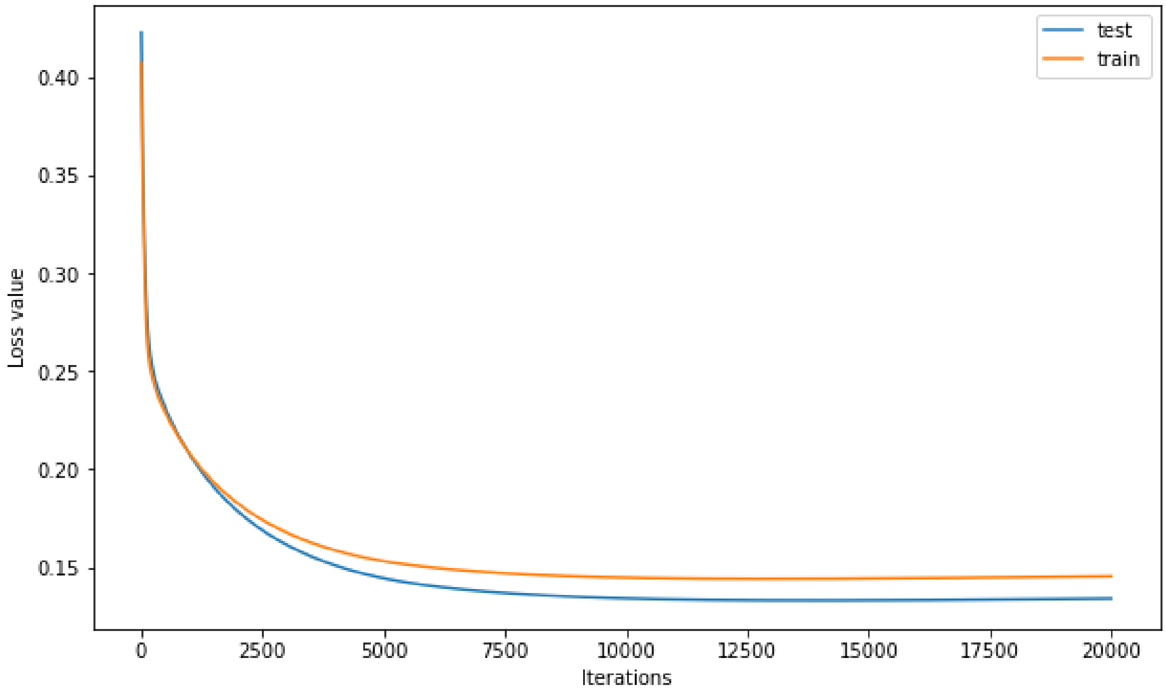
4. Plot the learning curve with λ=0.0001 [1pt]

In [384]:
```
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e4, label="test")
plt.plot(L_iters_train_e4, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



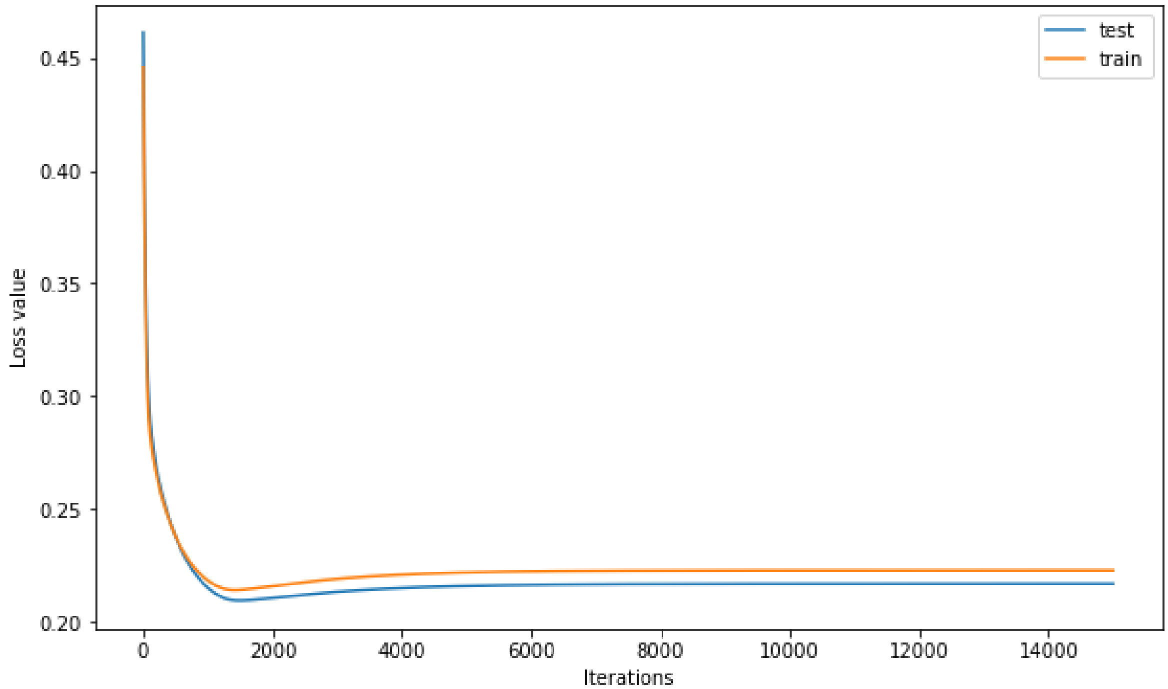
5. Plot the learning curve with λ=0.001 [1pt]

In [385]:
```
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e3, label="test")
plt.plot(L_iters_train_e3, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



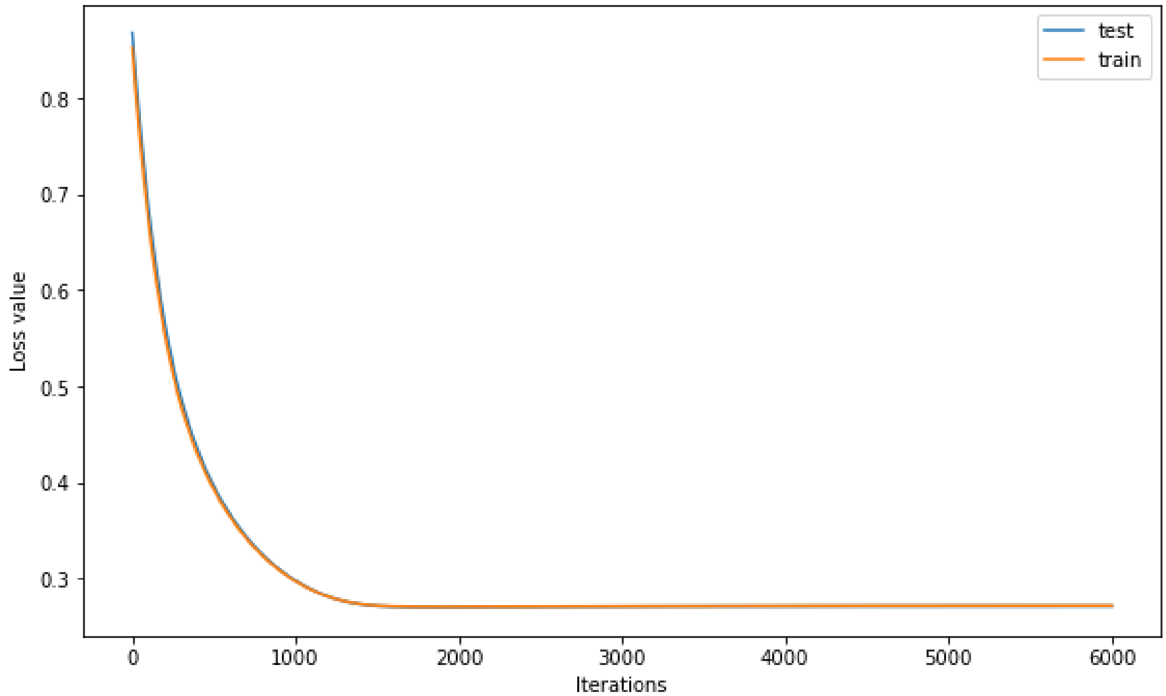
6. Plot the learning curve with λ=0.01 [1pt]

In [386]:
```
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e2, label="test")
plt.plot(L_iters_train_e2, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



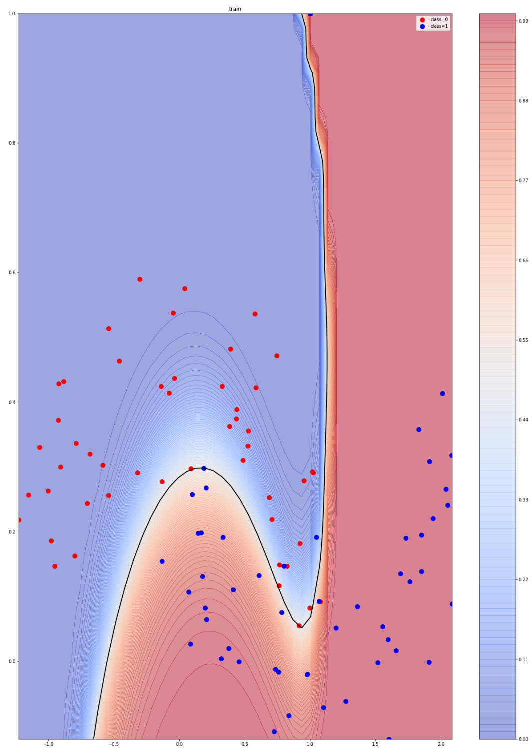
7. Plot the learning curve with λ=0.1 [1pt]

In [387]:
```
plt.figure(4, figsize=(10,6))
plt.plot(L_iters_test_e1, label="test")
plt.plot(L_iters_train_e1, label="train")
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.legend()
plt.show()
```



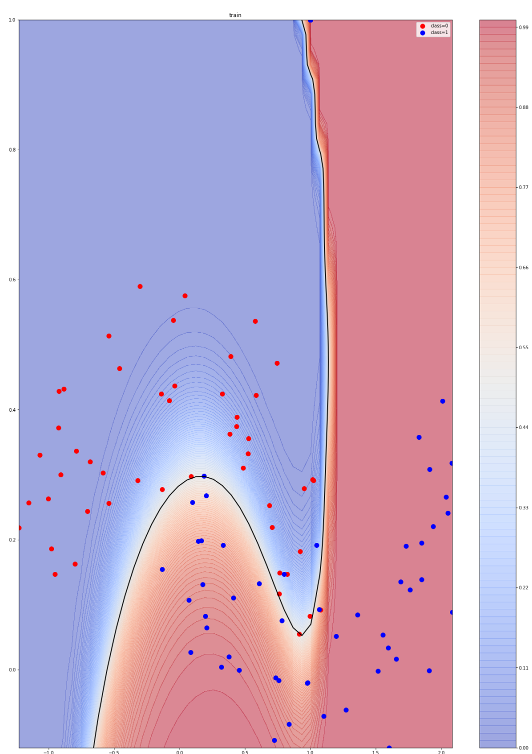
8. Plot the probability map of the obtained classifier with λ=0.00001[1pt]

In [352]:
```
plotColorMap(w_e5)
```



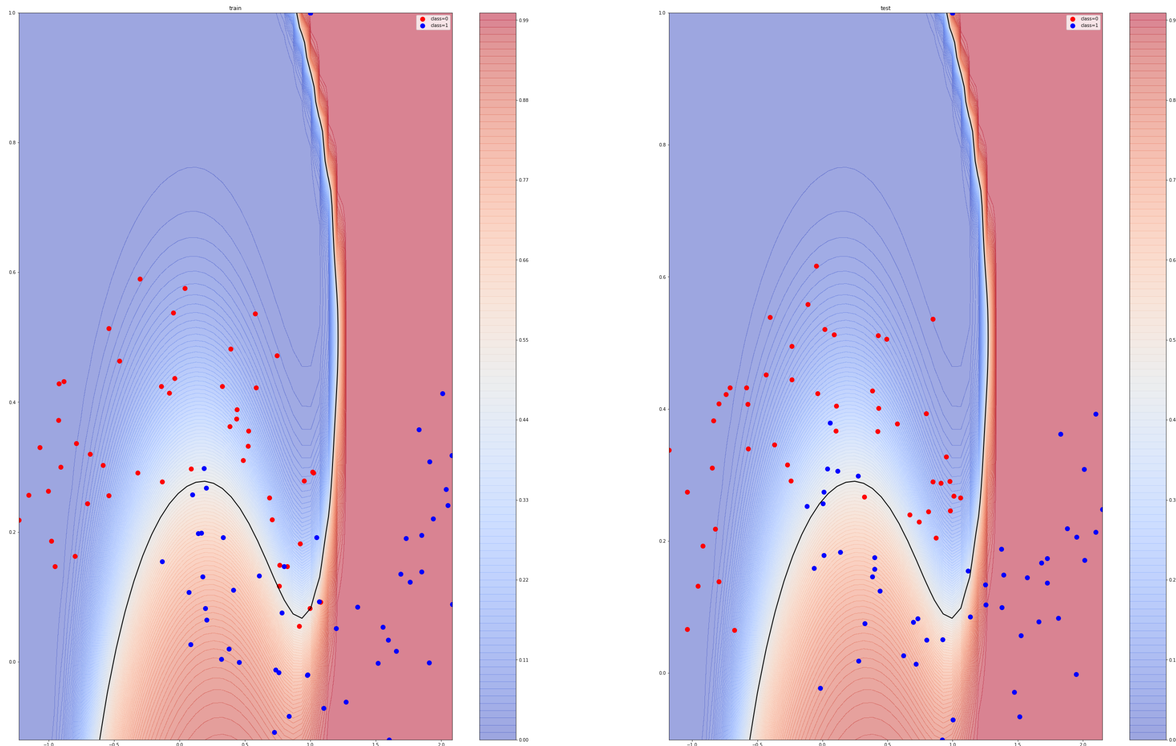
9. Plot the probability map of the obtained classifier with λ=0.0001[1pt]

In [353]:
```
plotColorMap(w_e4)
```



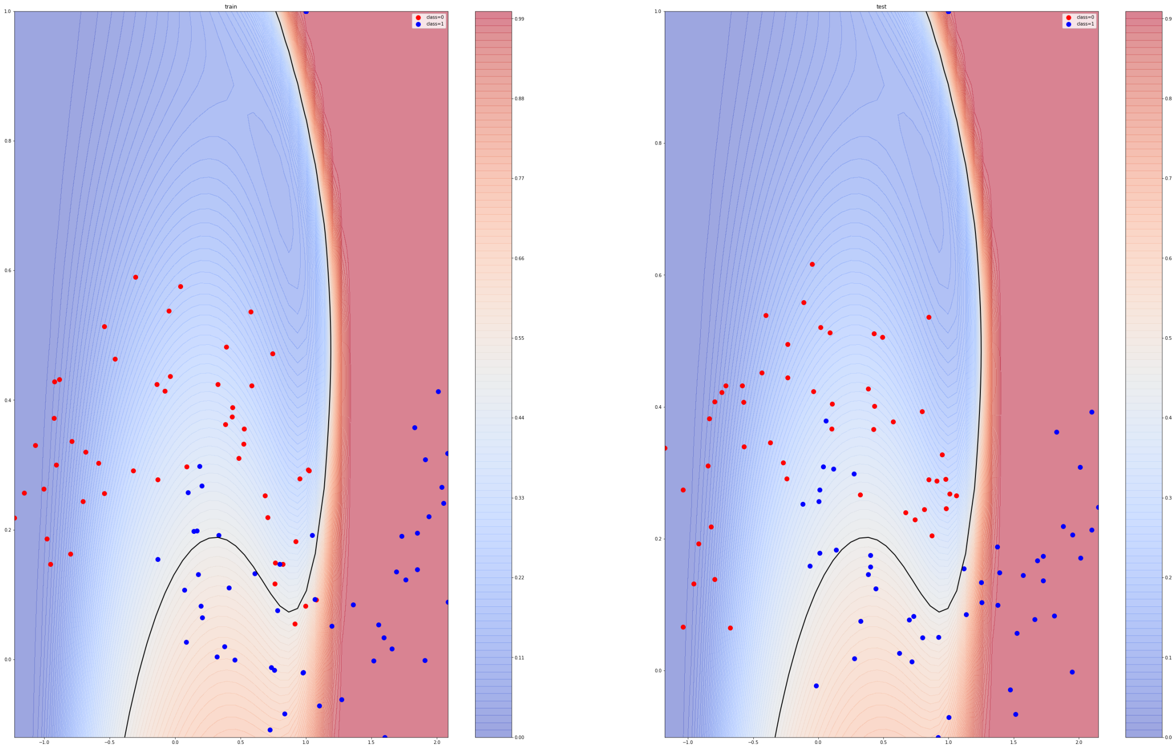
10. Plot the probability map of the obtained classifier with λ=0.001[1pt]

In [354]:
```
plotColorMap(w_e3)
```



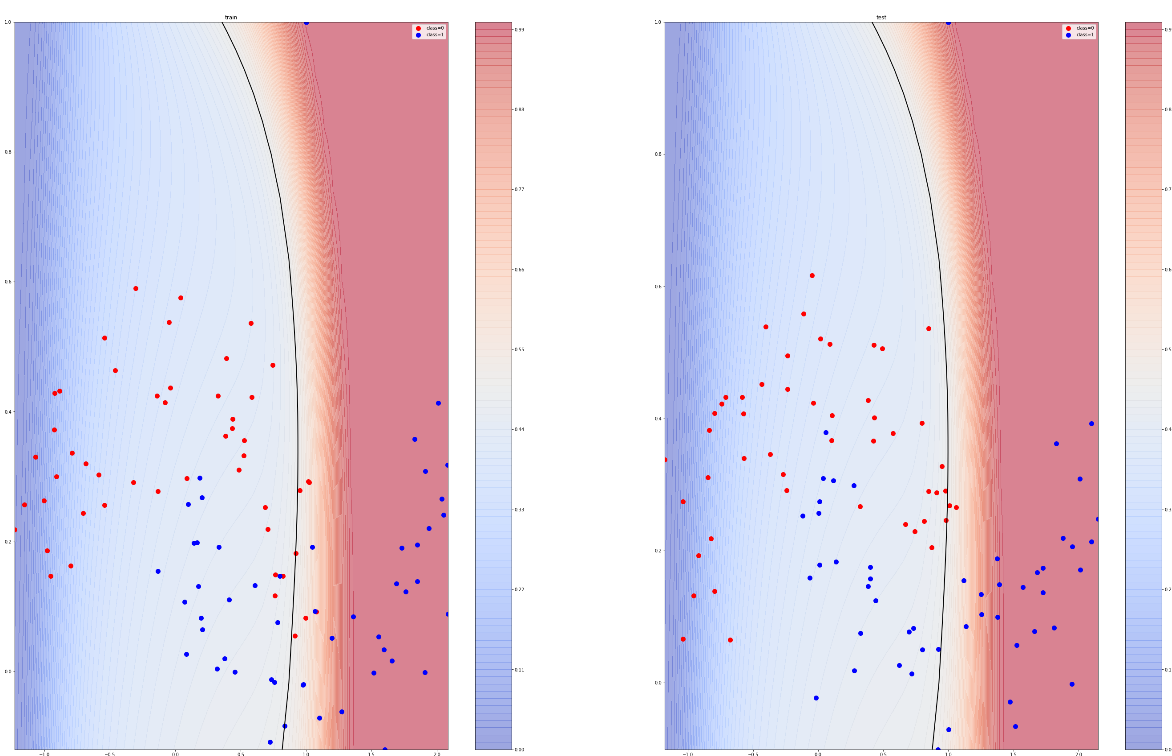
11. Plot the probability map of the obtained classifier with λ=0.01[1pt]

In [355]:
```
plotColorMap(w_e2)
```



12. Plot the probability map of the obtained classifier with λ=0.1[1pt]

In [356]:
```
plotColorMap(w_e1)
```



13. Print the final training accuracy with the given regularization parameters [2.5pt]

In [376]:
```
for i,w in zip(lambda_array,w_array):
    print("lambda = ",i,"accuracy = ", accuracy(w,X_train,idx_class1_train),"%")
```

```
lambda =  1e-05 accuracy =  0.985 %
lambda =  0.0001 accuracy =  0.98 %
lambda =  0.001 accuracy =  0.975 %
lambda =  0.01 accuracy =  0.94 %
lambda =  0.1 accuracy =  0.855 %
```

14. Print the final testing accuracy with the given regularization parameters [2.5pt]

In [375]:
```
for i,w in zip(lambda_array,w_array):
    print("lambda = ",i,"accuracy = ", accuracy(w,X_test,idx_class1_test),"%")
```

```
lambda =  1e-05 accuracy =  0.975 %
lambda =  0.0001 accuracy =  0.975 %
lambda =  0.001 accuracy =  0.965 %
lambda =  0.01 accuracy =  0.95 %
lambda =  0.1 accuracy =  0.865 %
```