PSO - Stands for Particle Swarm Optimization

The movement of the particles is given by

$$v_{t+1} = w_t v_t + c_1 r_1 (p_t^b - x_t) + c_2 r_2 (p^g - x_t) \quad \text{---} \quad (1)$$

$$x_{t+1} = x_t + v_{t+1} \quad \text{---} \quad (2)$$

t: iteration number ;

w_t : inertia weight

r_1, r_2: random number between 0 & 1;

c_1, c_2: correction factor

p_t^b: pbest position at t iteration;

p^g: gbest position

v_t, x_t: velocity and position at t iteration

Maximize $f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

where $-5 \leq x_1, x_2 \leq 5$

PSO Parameter Setting (used only for illustration)

Randomly chosen

Population Size = 5 $\quad c_1 = c_2 = 1.5 \quad$ Max. Iteration = 20

Dimension of the problem = 2 $\quad$ Inertia weight (w) = 0.9

Goal is to update these values ALWAYS

v (velocity) $\quad x$ (position) $\quad f(x) \quad p_i^b$ (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Iteration 1:

- Randomly chosen Velocity between 0 & 1

Goal is to update these values ALWAYS

v (velocity) $\quad x$ (position) $\quad f(x) \quad p_i^b$ (pbest)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Use $x = L + rand. * (U - L)$

lower rand. upper

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5

Goal is to update these values ALWAYS

v (velocity) $\quad x$ (position) $\quad f(x) \quad p_i^b$ (pbest)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix} \quad \begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix} \quad \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Use $x = L + rand. * (U - L)$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):

Goal is to update these values ALWAYS

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	------	------
0.1908	0.2228	4.0579	−2.2150	32.6168	------	------
0.3828	0.3232	−3.7301	0.4688	13.2071	------	------
0.3976	0.3547	4.1338	4.5751	48.6753	------	------
0.0934	0.3773	1.3236	4.6489	41.4537	------	------

p^g (gbest) = [------ ------]

Maximize $f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):
- Since there is NO previous iteration exists, so pbest is $p^b = x$

Goal is to update these values ALWAYS

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [------ ------]

Maximize $f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):
- Since there is NO previous iteration exists, so pbest is $p^b = x$
- Calculate gbest p^g

Maximum fitness value = 48.6753

Goal is to update these values ALWAYS

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [------ ------]

Maximize $f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):
- Since there is NO previous iteration exists, so pbest is $p^b = x$
- Calculate gbest p^g

Maximum fitness value = 48.6753

Goal is to update these values ALWAYS

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix} \quad (x_1, x_2)$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) = [4.1338 4.5751]

Maximize $f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 2:

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix} \quad (x_1, x_2)$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) = [4.1338 4.5751]

v (velocity) | x (position) | $f(x)$ | p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) = [------ ------]

Iteration 2:

Velocity update:

For 1st particle, 1st component

$$v_{11} = wv_{11} + c_1 r_1 (p_{11}^b - x_{11}) + c_2 r_2 (p^g - x_{11})$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix} \quad (x_1, x_2)$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) = [4.1338 4.5751]

v (velocity) | x (position) | $f(x)$ | p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \quad \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) = [------ ------]

Iteration 2:

Velocity update:

For 1st particle, 1st component

$$v_{11} = wv_{11} + c_1 r_1 (p_{11}^b - x_{11}) + c_2 r_2 (p^g - x_{11})$$

$$= 0.9 \times 0.2194 + (1.5)(0.5949)(3.1472 - 3.1472) + (1.5)(0.0855)(4.1338 - 3.1472)$$

$$= \mathbf{0.3240}$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position) — x_1 x_2

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

v (velocity)

$$\begin{bmatrix} 0.3240 & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

x (position)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Iteration 2:

Velocity update:

For 1st particle, 1st component

$$v_{11} = wv_{11} + c_1 r_1 (p_{11}^b - x_{11}) + c_2 r_2 (p^g - x_{11})$$

$$= 0.9 \times 0.2194 + (1.5)(0.5949)(3.1472 - 3.1472) + (1.5)(0.0855)(4.1338 - 3.1472)$$

$$= \mathbf{0.3240}$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position) — x_1 x_2

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

Position update:

$$x_{11} = v_{11} + x_{11}$$

$$= 3.4712 \in (-5,5)$$

$= 0.3240 + 3.1472$

v (velocity)

$$\begin{bmatrix} 0.3240 & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

x (position)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Iteration 2:

Velocity update:

For 1st particle, 1st component

$$v_{11} = wv_{11} + c_1 r_1 (p_{11}^b - x_{11}) + c_2 r_2 (p^g - x_{11})$$

$$= 0.9 \times 0.2194 + (1.5)(0.5949)(3.1472 - 3.1472) + (1.5)(0.0855)(4.1338 - 3.1472)$$

$$= \mathbf{0.3240}$$

Position update:

$$x_{11} = v_{11} + x_{11}$$

$$= 3.4712 \in (-5,5)$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position) — x_1 x_2

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

v (velocity)

$$\begin{bmatrix} 0.3240 & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Iteration 2:

Velocity update:

1st particle; 2nd component:

$$v_{12} = wv_{12} + c_1 r_1 (p_{12}^b - x_{12}) + c_2 r_2 (p^g - x_{12})$$

$$= (0.9)(0.2449) + (1.5)(0.5949)(-4.0246 + 4.0246) + (1.5)(0.0855)(4.5751 + 4.0246)$$

$$= 1.3233$$

Position update:

$$x_{12} = v_{12} + x_{12}$$

$$= -2.7013 \in (-5, 5)$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.1150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.1150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

v (velocity)

$$\begin{bmatrix} 0.3240 & 1.3233 \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Iteration 2:

Velocity update:

1st particle; 2nd component:

$$v_{12} = wv_{12} + c_1 r_1 (p_{12}^b - x_{12}) + c_2 r_2 (p^g - x_{12})$$

$$= (0.9)(0.2449) + (1.5)(0.5949)(-4.0246 + 4.0246) + (1.5)(0.0855)(4.5751 + 4.0246)$$

$$= 1.3233$$

Position update:

$$x_{12} = v_{12} + x_{12}$$

$$= -2.7013 \in (-5, 5)$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.1150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2071 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.1150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

v (velocity)

$$\begin{bmatrix} 0.3240 & 1.3233 \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & -2.7013 \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [\cdots \quad \cdots]$

Iteration 2:

Velocity update:

For 2nd particle; 1st component

$$v_{21} = wv_{21} + c_1 r_1 (p^b_{21} - x_{21}) + c_2 r_2 (p^g - x_{21})$$
$$= 0.1815$$

$$x_{21} = v_{21} + x_{21}$$
$$= 4.2394 \in (-5,5)$$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

x_1	x_2
3.1472	−4.0246
4.0579	−2.2150
−3.7301	0.4688
4.1338	4.5751
1.3236	4.6489

$f(x)$

31.9645
32.6168
13.2071
48.6753
41.4537

p_i^b (pbest)

3.1472	−4.0246
4.0579	−2.2150
−3.7301	0.4688
4.1338	4.5751
1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)

0.3240	1.3233
0.1815	------
------	------
------	------
------	------

x (position)

3.4712	−2.7013
4.2394	------
------	------
------	------
------	------

$f(x)$

------

p_i^b (pbest)

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------	------
------	------
------	------
------	------

p^g (gbest) = [------ ------]

Iteration 2:

Velocity update:

For 2nd particle; 1st component

$$v_{21} = wv_{21} + c_1 r_1 (p^b_{21} - x_{21}) + c_2 r_2 (p^g - x_{21})$$
$$= 0.1815$$

$$x_{21} = v_{21} + x_{21}$$
$$= 4.2394 \in (-5,5)$$

Similarly, for 2nd particle; 2nd component, we can get $v_{22} = 1.0714$;
$x_{22} = -1.1436$

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

x_1	x_2
3.1472	−4.0246
4.0579	−2.2150
−3.7301	0.4688
4.1338	4.5751
1.3236	4.6489

$f(x)$

31.9645
32.6168
13.2071
48.6753
41.4537

p_i^b (pbest)

3.1472	−4.0246
4.0579	−2.2150
−3.7301	0.4688
4.1338	4.5751
1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)

0.3240	1.3233
0.1815	1.0714
------	------
------	------
------	------

x (position)

3.4712	−2.7013
4.2394	−1.1436
------	------
------	------
------	------

$f(x)$

------

p_i^b (pbest)

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------	------
------	------
------	------
------	------

p^g (gbest) = [------ ------]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

x_1	x_2
3.1472	−4.0246
4.0579	−2.2150
−3.7301	0.4688
4.1338	4.5751
1.3236	4.6489

$f(x)$

31.9645
32.6168
13.2071
48.6753
41.4537

p_i^b (pbest)

3.1472	−4.0246
4.0579	−2.2150
−3.7301	0.4688
4.1338	4.5751
1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)

0.3240	1.3233
0.1815	1.0714
1.3531	0.8175
0.3578	0.3192
0.4445	0.3301

x (position)

3.4712	−2.7013
4.2394	−1.1436
−2.3770	1.2863
4.4916	4.8943
1.7681	4.9790

$f(x)$

------

p_i^b (pbest)

------	------
------	------
------	------
------	------
------	------

p^g (gbest) = [------ ------]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

$$f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

Iteration 1:

v (velocity)	x (position)	$f(x)$	p_i^b (pbest)
0.2194 0.2449	3.1472 −4.0246	31.9645	3.1472 −4.0246
0.1908 0.2228	4.0579 −2.2150	32.6168	4.0579 −2.2150
0.3828 0.3232	−3.7301 0.4688	13.2071	−3.7301 0.4688
0.3976 0.3547	4.1338 4.5751	48.6753	4.1338 4.5751
0.0934 0.3773	1.3236 4.6489	41.4537	1.3236 4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)	x (position)	$f(x)$	p_i^b (pbest)
0.3240 1.3233	3.4712 −2.7013	27.8600	------ ------
0.1815 1.0714	4.2394 −1.1436	31.0325	------ ------
1.3531 0.8175	−2.3770 1.2863	13.7537	------ ------
0.3578 0.3192	4.4916 4.8943	53.7063	------ ------
0.4445 0.3301	1.7681 4.9790	45.5655	------ ------

p^g (gbest) = [------ ------]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate gbest (p^g):

Iteration 1:

v (velocity)	x (position)	$f(x)$	p_i^b (pbest)
0.2194 0.2449	3.1472 −4.0246	31.9645	3.1472 −4.0246
0.1908 0.2228	4.0579 −2.2150	32.6168	4.0579 −2.2150
0.3828 0.3232	−3.7301 0.4688	13.2071	−3.7301 0.4688
0.3976 0.3547	4.1338 4.5751	48.6753	4.1338 4.5751
0.0934 0.3773	1.3236 4.6489	41.4537	1.3236 4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)	x (position)	$f(x)$	p_i^b (pbest)
0.3240 1.3233	3.4712 −2.7013	27.8600	------ ------
0.1815 1.0714	4.2394 −1.1436	31.0325	------ ------
1.3531 0.8175	−2.3770 1.2863	13.7537	------ ------
0.3578 0.3192	4.4916 4.8943	53.7063	------ ------
0.4445 0.3301	1.7681 4.9790	45.5655	------ ------

p^g (gbest) = [------ ------]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate gbest (p^g):

Since 53.7063 > 48.6753 & its position is 4th.

So updated gbest is [4.4916 4.8943]

Iteration 1:

v (velocity)	x (position)	$f(x)$	p_i^b (pbest)
0.2194 0.2449	3.1472 −4.0246	31.9645	3.1472 −4.0246
0.1908 0.2228	4.0579 −2.2150	32.6168	4.0579 −2.2150
0.3828 0.3232	−3.7301 0.4688	13.2971	−3.7301 0.4688
0.3976 0.3547	4.1338 4.5751	48.6753	4.1338 4.5751
0.0934 0.3773	1.3236 4.6489	41.4537	1.3236 4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)	x (position)	$f(x)$	p_i^b (pbest)
0.3240 1.3233	3.4712 −2.7013	27.8600	------ ------
0.1815 1.0714	4.2394 −1.1436	31.0325	------ ------
1.3531 0.8175	−2.3770 1.2863	13.7537	------ ------
0.3578 0.3192	4.4916 4.8943	53.7063	------ ------
0.4445 0.3301	1.7681 4.9790	45.5655	------ ------

p^g (gbest) = [4.4916 4.8943]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate gbest (p^g):

Since 53.7063 > 48.6753 & its position is 4th.

So updated gbest is [4.4916 4.8943]

Calculate pbest (p^b):

For 1st particle:

Since 27.8600 < 31.9645, so pbest position remains same

For 3rd particle:

Since 13.7537 < 13.2971, so updated pbest position

Iteration 1:

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2971	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	27.8600	3.1472	−4.0246
0.1815	1.0714	4.2394	−1.1436	31.0325	4.0579	−2.2150
1.3531	0.8175	−2.3770	1.2863	13.7537	−2.3770	1.2863
0.3578	0.3192	4.4916	4.8943	53.7063	4.4916	4.8943
0.4445	0.3301	1.7681	4.9790	45.5655	1.7681	4.9790

p^g (gbest) = [4.4916 4.8943]

Iteration 3:

Update the velocity & position:

$$v_{11} = w\, v_{11} + c_1 r_1 (p_{11}^b - x_{11}) + c_2 r_2 (p^g - x_{11})$$

And $x_{11} = v_{11} + x_{11}$

Iteration 2: results

v (velocity)

$$\begin{bmatrix} 0.3240 & 1.3233 \\ 0.1815 & 1.0714 \\ 1.3531 & 0.8175 \\ 0.3578 & 0.3192 \\ 0.4445 & 0.3301 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & -2.7013 \\ 4.2394 & -1.1436 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 27.8600 \\ 31.0325 \\ 13.7537 \\ 53.7063 \\ 45.5655 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

p^g (gbest) = [4.4916 4.8943]

v (velocity)

$$\begin{bmatrix} 0.1334 & 0.9843 \\ 0.0337 & 0.7826 \\ 2.0987 & 1.1985 \\ 0.3221 & 0.2873 \\ 0.7493 & 0.2862 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.6045 & -1.7170 \\ 4.2731 & -0.3611 \\ -0.2784 & 2.4848 \\ 4.8137 & 5.0000 \\ 2.5174 & 5.0000 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} \text{------} \\ \text{------} \\ \text{------} \\ \text{------} \\ \text{------} \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \text{------} & \text{------} \\ \text{------} & \text{------} \\ \text{------} & \text{------} \\ \text{------} & \text{------} \\ \text{------} & \text{------} \end{bmatrix}$$

p^g (gbest) = [------ ------]

Iteration 3:

Update the velocity & position:

Calculate the fitness value:

$$f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

Iteration 2: results

v (velocity)

$$\begin{bmatrix} 0.3240 & 1.3233 \\ 0.1815 & 1.0714 \\ 1.3531 & 0.8175 \\ 0.3578 & 0.3192 \\ 0.4445 & 0.3301 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & -2.7013 \\ 4.2394 & -1.1436 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 27.8600 \\ 31.0325 \\ 13.7537 \\ 53.7063 \\ 45.5655 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

p^g (gbest) = [4.4916 4.8943]

v (velocity)

$$\begin{bmatrix} 0.1334 & 0.9843 \\ 0.0337 & 0.7826 \\ 2.0987 & 1.1985 \\ 0.3221 & 0.2873 \\ 0.7493 & 0.2862 \end{bmatrix}$$

x (position) (x1) (x2)

$$\begin{bmatrix} 3.6045 & -1.7170 \\ 4.2731 & -0.3611 \\ -0.2784 & 2.4848 \\ 4.8137 & 5.0000 \\ 2.5174 & 5.0000 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 25.4710 \\ 30.0346 \\ 19.3259 \\ 56.7306 \\ 46.7851 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \text{------} & \text{------} \\ \text{------} & \text{------} \\ \text{------} & \text{------} \\ \text{------} & \text{------} \\ \text{------} & \text{------} \end{bmatrix}$$

p^g (gbest) = [------ ------]

Iteration 3:

Update the velocity & position:

Calculate the fitness value:

Iteration 2: results

Max

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	27.8600	3.1472	−4.0246
0.1815	1.0714	4.2394	−1.1436	31.0325	4.0579	−2.2150
1.3531	0.8175	−2.3770	1.2863	13.7537	−2.3770	1.2863
0.3578	0.3192	4.4916	4.8943	53.7063	4.4916	4.8943
0.4445	0.3301	1.7681	4.9790	45.5655	1.7681	4.9790

p^g (gbest) = [4.4916 4.8943]

Update gbest (p^g):

Since 56.7306 > 53.7063, so updated gbest is

Gbest = [4.8137 5.0000]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1334	0.9843	3.6045	−1.7170	25.4710	------	------
0.0337	0.7826	4.2731	−0.3611	30.0346	------	------
2.0987	1.1985	−0.2784	2.4848	19.3259	------	------
0.3221	0.2873	4.8137	5.0000	56.7306	------	------
0.7493	0.2862	2.5174	5.0000	46.7851	------	------

p^g (gbest) = [4.8137 5.0000]

Iteration 3:

Update the velocity & position:

Calculate the fitness value:

Iteration 2: results

Max

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	27.8600	3.1472	−4.0246
0.1815	1.0714	4.2394	−1.1436	31.0325	4.0579	−2.2150
1.3531	0.8175	−2.3770	1.2863	13.7537	−2.3770	1.2863
0.3578	0.3192	4.4916	4.8943	53.7063	4.4916	4.8943
0.4445	0.3301	1.7681	4.9790	45.5655	1.7681	4.9790

p^g (gbest) = [4.4916 4.8943]

Update gbest (p^g):

Since 56.7306 > 53.7063, so updated gbest is

Gbest = [4.8137 5.0000]

Update pbest (p^b):

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1334	0.9843	3.6045	−1.7170	25.4710	3.4712	−2.7013
0.0337	0.7826	4.2731	−0.3611	30.0346	4.2394	−1.1436
2.0987	1.1985	−0.2784	2.4848	19.3259	−0.2784	2.4848
0.3221	0.2873	4.8137	5.0000	56.7306	4.8137	5.0000
0.7493	0.2862	2.5174	5.0000	46.7851	2.5174	5.0000

p^g (gbest) = [4.8137 5.0000]

Iteration 4:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Since 58.000 > 56.7306

gbest = [5.0000 5.0000]

Update pbest (p^b):

Iteration 3: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1334	0.9843	3.6045	−1.7170	25.4710	3.4712	−2.7013
0.0337	0.7826	4.2731	−0.3611	30.0346	4.2394	−1.1436
2.0987	1.1985	−0.2784	2.4848	19.3259	−0.2784	2.4848
0.3221	0.2873	4.8137	5.0000	56.7306	4.8137	5.0000
0.7493	0.2862	2.5174	5.0000	46.7851	2.5174	5.0000

p^g (gbest) = [4.8137 5.0000]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1561	0.8690	3.7606	−0.8480	25.1798	3.6045	−1.7170
0.0696	0.6935	4.3427	0.3325	30.5409	4.3427	0.3325
2.5419	1.4012	2.2635	3.8860	34.4996	2.2635	3.8860
0.2899	0.2586	5.0000	5.0000	58.0000	5.0000	5.0000
0.9689	0.2576	3.4863	5.0000	49.6953	3.4863	5.0000

p^g (gbest) = [5.0000 5.0000]

Iteration 5:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Update pbest (p^b):

Since there is no change in the fitness value (58.000) and hence optimum will reach.

$x_1 = 5, \quad x_2 = 5$

$f(x_1, x_2) = 58$

Iteration 4: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1561	0.8690	3.7606	−0.8480	25.1798	3.6045	−1.7170
0.0696	0.6935	4.3427	0.3325	30.5409	4.3427	0.3325
2.5419	1.4012	2.2635	3.8860	34.4996	2.2635	3.8860
0.2899	0.2586	5.0000	5.0000	58.0000	5.0000	5.0000
0.9689	0.2576	3.4863	5.0000	49.6953	3.4863	5.0000

p^g (gbest) = [5.0000 5.0000]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1601	0.7567	3.9208	−0.0914	26.2152	3.9208	−0.0914
0.1469	1.2228	4.4896	1.5553	33.7933	4.4896	1.5553
2.6386	1.4040	4.9021	5.0000	57.3244	4.9021	5.0000
0.2609	0.2327	5.0000	5.0000	58.0000	5.0000	5.0000
1.0661	0.2319	4.5524	5.0000	55.0672	4.5524	5.0000

p^g (gbest) = [5.0000 5.0000]

x_1 x_2

Iteration 3:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Since 56.7306 > 53.7063, so updated gbest is

Gbest = [4.8137 5.0000]

Update pbest (p^b):

Iteration 2: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	27.8600	3.1472	−4.0246
0.1815	1.0714	4.2394	−1.1436	31.0325	4.0579	−2.2150
1.3531	0.8175	−2.3770	1.2863	13.7537	−2.3770	1.2863
0.3578	0.3192	4.4916	4.8943	53.7063	4.4916	4.8943
0.4445	0.3301	1.7681	4.9790	45.5655	1.7681	4.9790

p^g (gbest) = [4.4916 4.8943]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1334	0.9843	3.6045	−1.7170	25.4710	3.4712	−2.7013
0.0337	0.7826	4.2731	−0.3611	30.0346	4.2394	−1.1436
2.0987	1.1985	−0.2784	2.4848	19.3259	−0.2784	2.4848
0.3221	0.2873	4.8137	5.0000	56.7306	4.8137	5.0000
0.7493	0.2862	2.5174	5.0000	46.7851	2.5174	5.0000

p^g (gbest) = [4.8137 5.0000]

$|f(x) - f(x_{pre})| < \epsilon$

Summary:

Update the velocity & position:

$$v_{t+1} = w_t v_t + c_1 r_1 (p_t^b - x_t) + c_2 r_2 (p^g - x_t)$$

$$x_{t+1} = x_t + v_{t+1}$$

Calculate the fitness value: (Max / Min)

Calculate the objective function values f(X)

Update gbest (p^g):

Compare the fitness value

Update pbest (p^b):

Compare the individual particle fitness value & select the best one.