PSO - Stands for Particle Swarm

Optimization

The movement of the particles is given by
$$v_{t+1} = w_t v_t + c_1 r_1 \left(p_t^b - x_t \right) + c_2 r_2 \left(p^g - x_t \right) \qquad \bigcirc$$
 Position
$$x_{t+1} = x_t + v_{t+1} \qquad \bigcirc$$

t: iteration number ;

 w_t : inertia weight $\checkmark\!\!/$

 r_1, r_2 : random number between 0 & 1; c_1, c_2 : correction factor

 $\leftarrow p_t^b$: pbest position at t iteration;

 p^g : gbest position

 v_t, x_t : velocity and position at t iteration

Maximize
$$f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

where $-5 \le x_1, x_2 \le 5$

PSO Parameter Setting (used only for illustration)

Randomly chosen

Population Size = $\mathbf{5}$ $c_1 = c_2 = 1.5$

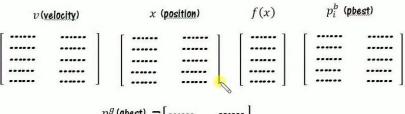
$$c_1 = c_2 = 1.5$$

Max. Iteration = 20

Dimension of the problem = 2

Inertia weight (w) = 0.9

Goal is to update these values ALWAYS



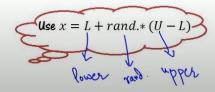
p^g (gbest) = [-----]

Iteration 1:

Randomly chosen Velocity
between 0 & 1

Goal is to update these values ALWAYS

$$p^g$$
 (gbest) = [-----]

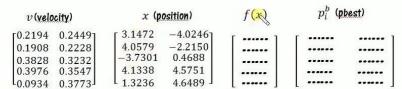


Iteration 1:

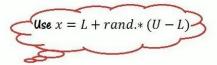
- Randomly chosen Velocity between 0 & 1
- → Randomly Initialize Position

 between -5 & 5

Goal is to update these values ALWAYS



$$p^g$$
 (gbest) = [-----]



Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position
 between -5 & 5
- La Calculate fitness values f(x):

Goal is to update these values ALWAYS

v (velocit	y)	x (po	(x_2)	f(x)	p_i^b (p)	est)
г0.2194 0	.24491	г 3.1472	-4.02461	[31.9645]	[•••••	•••••]
0.1908 0	.2228	4.0579	-2.2150	32.6168		
0.3828 0	.3232	-3.7301	0.4688	13.2071		
0.3976 0	.3547	4.1338	4.5751	48.6753		
L _{0.0934} 0	.3773	1.3236	4.6489	^L 41.4537 ^J	[

$$p^g$$
 (gbest) = [-----]

Maximize
$$f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):
- \clubsuit Since there is NO previous iteration exists, so phest is $p^b = x$

Goal is to update these values ALWAYS

v (velocity)	x (po	osition)	f(x)	p_i^b (pbest)
[0.2194 0.2449]	3.1472	$\begin{bmatrix} x_2 \\ -4.0246 \\ -2.2150 \\ 0.4688 \\ 4.5751 \\ 4.6489 \end{bmatrix}$	31.9645	3.1472	-4.0246
0.1908 0.2228	4.0579		32.6168	4.0579	-2.2150
0.3828 0.3232	-3.7301		13.2071	-3.7301	0.4688
0.3976 0.3547	4.1338		48.6753	4.1338	4.5751
0.0934 0.3773]	1.3236		41.4537	1.3236	4.6489

$$p^g$$
 (gbest) = [-----]

Maximize
$$f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):
- \clubsuit Since there is NO previous iteration exists, so pbest is $p^b=x$
- Laculate ghest p^g Maximum fitness value = 48.6753

Goal is to update these values ALWAYS

v (velocity)	x (position)	f(x)	p_i^b (obest)
0.2194 0.2449 0.1908 0.2228 0.3828 0.3232	$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \end{bmatrix}$	31.9645 32.6168 13.2071	3.1472 4.0579 -3.7301	$ \begin{array}{c} -4.\overline{0}246 \\ -2.2150 \\ 0.4688 \end{array} $
0.3976 0.3547 0.0934 0.3773	4.1338 4.5751 1.3236 4.6489	48.6753 41.4537	4.1338 1.3236	4.5751 4.6489

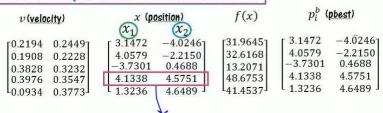
$$p^g$$
 (gbest) = [.....]

Maximize
$$f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

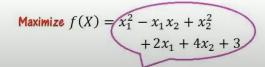


- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position
 between -5 & 5
- Calculate fitness values f(x):
- \bot Since there is NO previous iteration exists, so phest is $p^b = x$

Goal is to update these values ALWAYS

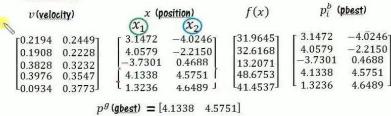


 p^g (gbest) = [4.1338 4.5751]



Iteration 2:

Iteration 1:





p^g (gbest) = [-----

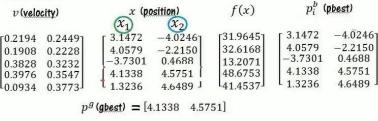
Iteration 2:

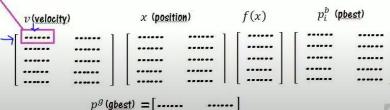
Velocity update:

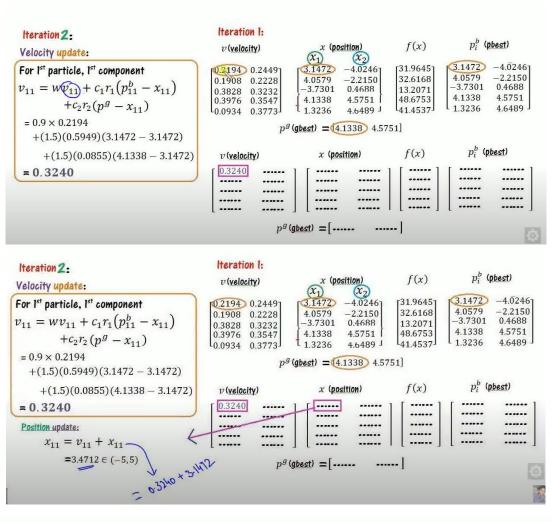
For 1st particle, 1st component

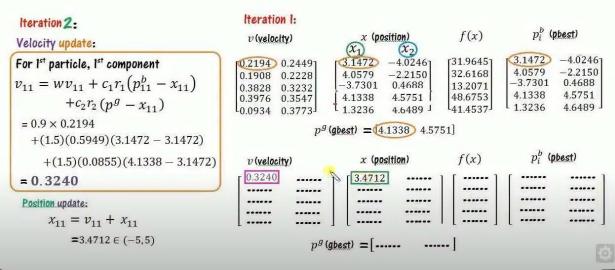
$$v_{11} = wv_{11} + c_1r_1(p_{11}^b - x_{11}) + c_2r_2(p^g - x_{11})$$

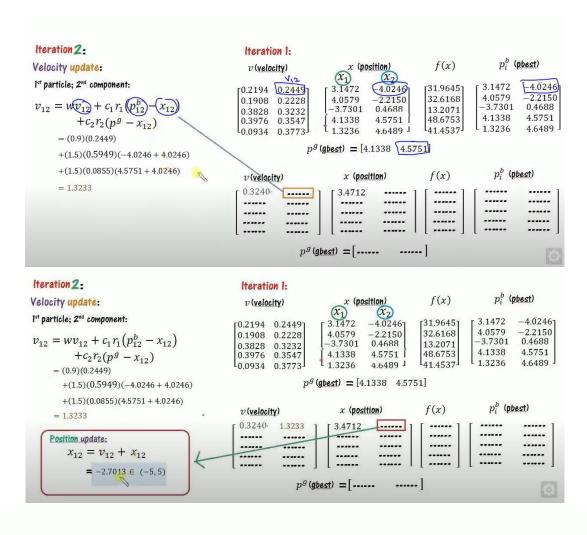
Iteration 1:

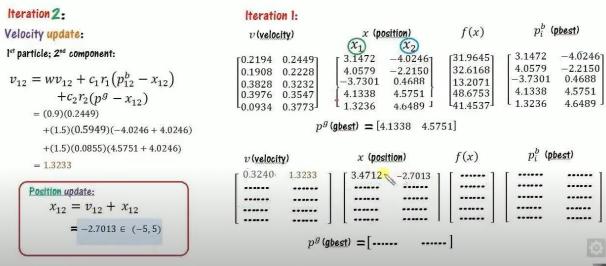


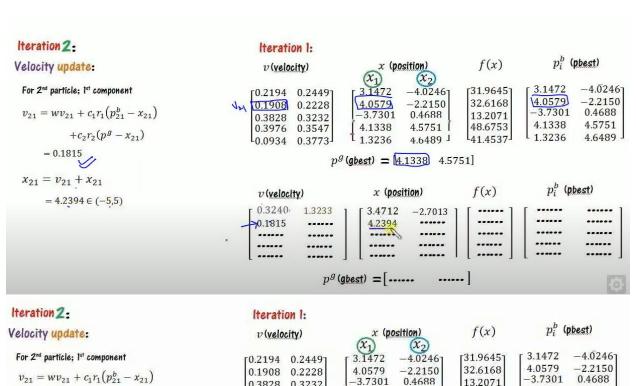


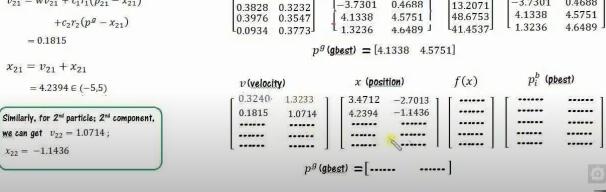


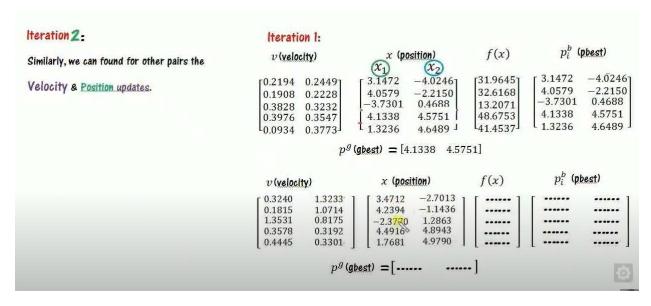












Iteration 2: Similarly, we can found for other pairs the Velocity & Position updates. Calculate the fitness values: $f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 1:

v (veloc	ity)	x (po	osition)	f(x)	p_i^b (pbest)
г0.2194	0.24491	г 3.1472	-4.02461	[31.9645]	3.1472	$-4.\overline{0}246$
0.22	0.2228	4.0579	-2.2150	32.6168	4.0579	-2.2150
0.3828	0.3232	-3.7301	0.4688	13.2071	-3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
$L_{0.0934}$	0.3773	1 1.3236	4.6489 J	[[] 41.4537]	1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocit	χ)	x (posit	ion) (v2)	f(x)	p_i^b (p	best)
г 0.3240	1.3233 1	[3.4712	-2.7013 1	[27.8600]	· · · · · ·	******
0.1815	1.0714	4.2394	-1.1436	31.0325		
1.3531	0.8175	-2.3770	1.2863	13.7537		
0.3578	0.3192	4.4916	4.8943	53.7063	*****	
0.4445	0.3301	1.7681	4.9790	L45.5655J		

$$p^g$$
 (gbest) = [-----]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate ghest (p^g) :

Iteration 1:

v (veloc	ity)	x (po	osition) (x_2)	f(x)	p_i^b (pbest)
0.1908 0.3828	0.2449 0.2228 0.3232 0.3547 0.3773	$\begin{bmatrix} 3.1472 \\ 4.0579 \\ -3.7301 \\ 4.1338 \\ 1.3236 \end{bmatrix}$	-4.0246 -2.2150 0.4688 4.5751 4.6489	31.9645 32.6168 13.2071 48.6783 41.4537	3.1472 4.0579 -3.7301 4.1338 1.3236	$ \begin{array}{c} -4.\overline{0}246 \\ -2.2150 \\ 0.4688 \\ 4.5751 \\ 4.6489 \end{array} $

$$p^g$$
 (gbest) = [4.1338 4.5751]

v (velocity)		x (positi	on)	f(x)	p_i^b (p)	best)
0.3240 0.1815 1.3531 0.3578 0.4445	1.3233 1.0714 0.8175 0.3192 0.3301	3.4712 4.2394 -2.3770 4.4916 1.7681	-2.7013 -1.1436 1.2863 4.8943 4.9790	27.8600 31.0325 13.7537 53.7063 45.5655		
	p^g (gbe	st) = [-1		53

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate gbest (p^g) :

Since 53.7063 > 48.6753 & its position is 4^{th} . So updated gbest is $[4.4916 \quad 4.8943]$

Iteration 1:

v (veloc	ity)	x (po	osition) x_2	f(x)	p_i^b (pbest)
0.1908 0.3828 0.3976	0.2449 0.2228 0.3232 0.3547 0.3773	3.1472 4.0579 -3.7301 4.1338 1.3236	-4.0246 -2.2150 0.4688 4.5751 4.6489	31.9645 32.6168 13.2971 48.6753 41.4537	3.1472 4.0579 -3.7301 4.1338 1.3236	-4.0246 ⁻ -2.2150 0.4688 4.5751 4.6489

$$p^g$$
 (gbest) = [4.1338 4.5751]

v (velocity	<u>(</u>)	x (positi	on)	f(x)	p_i^{ν} (p	best)
г 0.3240	1.3233 1	[3.4712	-2.7013	[27.8600 _]	•••••	•••••
0.1815	1.0714	4.2394	-1.1436	31.0325	*****	
1.3531	0.8175	-2.3770	1.2863	13.7537		
0.3578	0.3192	4.4916	4.8943	53.7063		
0.4445	0.3301	1.7681	4.9790	L45.5655J		

$$p^g$$
 (gbest) = [4.4916 4.8943]

Iteration 2: Iteration 1: x (position) x_1 x_2 x_3 x_4 x_4 x_2 x_4 p_i^b (pbest) f(x)v (velocity) Similarly, we can found for other pairs the (x_2) 3.1472 -4.0246731.9645 -4.02467[0.2194 0.2449] Velocity & Position updates. 32.6168 4.0579 -2.21500.1908 0.2228 4.0579 -2.2150-3.73010.4688 13.2971 48.6753 0.3828 0.3232 0.3976 0.3547 -3.73010.4688 Calculate the fitness values: 4.1338 4.5751 4.1338 4.5751 1 1.3236 41.4537 1.3236 4.6489 0.0934 0.3773 4.6489 J Calculate gbest (p^g) : p^g (gbest) = [4.1338 4.5751] Since 53.7063 > 48.6753 & its position is 4^{th} . So updated gbest is [4.4916 4.8943] p_i^b (pbest) f(x)v (velocity) x (position) Calculate phest (p^b) : 0.3240 1.3233 3.4712 -2.7013[27.8600] 3.1472 31.0325 13.7537 -1.14360.1815 1.0714 4.2394 4.0579 -2.2150For 1st particle: 1.2863 4.8943 1.2863 4.8943 1.3531 0.8175 -2.3770 -2.3770 53.7063 4.4916 Since 27.8600 < 31.9645, so phest position remains same 0.3578 0.3192 4.4916 1.7681 0.4445 0.3301 1.7681 4.9790 4.9790 For 3" particle: $(p^g(gbest)) = [4.4916 \ 4.8943]$ Since 13.7537 < 13.2971, so updated phest position

Iteration 3:

Update the velocity & position:

$$\begin{split} v_{11} &= w \; v_{11} + c_1 r_1 \big(p_{11}^b - x_{11} \big) \\ &+ c_2 r_2 \big(p^g - x_{11} \big) \end{split}$$
 And $x_{11} = v_{11} + x_{11}$

Iteration 2: results

v (velocity)	x (posit	x (position)		$p_i^{\scriptscriptstyle D}$ (pbest)	
0.3240 1.3233 0.1815 1.0714 1.3531 0.8175 0.3578 0.3192 0.4445 0.3301	3.4712	-2.7013	27.8600	3.1472	-4.0246
	4.2394	-1.1436	31.0325	4.0579	-2.2150
	-2.3770	1.2863	13.7537	-2.3770	1.2863
	4.4916	4.8943	53.7063	4.4916	4.8943
	1.7681	4.9790	45.5655	1.7681	4.9790

v(velocity)	x (position)	f(x)	p_i^b (pbest)
0.1334	0.98437	_[3.6045	-1.7170	[]	[•••••	•••••]
0.0337	0.7826	4.2731	-0.3611	•••••	•••••	
2.0987	1.1985	-0.2784	2.4848		•••••	•••••
0.3221	0.2873	4.8137	5.0000			
0.7493	0.2862	L 2.5174	5.0000	r 1	L	
	p^g	(gbest) =[.]		[6]

Iteration 3:

Update the velocity & position:

Calculate the fitness value:

$$f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

Iteration 2: results

v (velocity)	x (position)	f(x)	p_i^b (pbest)	
0.3240 1.3233 0.1815 1.0714 1.3531 0.8175 0.3578 0.3192 0.4445 0.3301	3.4712 -2.7013 4.2394 -1.1436 -2.3770 1.2863 4.4916 4.8943 1.7681 4.9790	27.8600 31.0325 13.7537 53.7063 45.5655	$\begin{bmatrix} 3.1472 \\ 4.0579 \\ -2.3770 \\ 4.4916 \\ 1.7681 \end{bmatrix}$	-4.02467 -2.2150 1.2863 4.8943 4.9790

$$\begin{bmatrix} 0.1334 & 0.9843 \\ 0.0337 & 0.7826 \\ 2.0987 & 1.1985 \\ 0.3221 & 0.2873 \\ 0.7493 & 0.2862 \end{bmatrix} \begin{bmatrix} 3.6045 & -1.7170 \\ 4.2731 & -0.3611 \\ -0.2784 & 2.4848 \\ 4.8137 & 5.0000 \\ 2.5174 & 5.0000 \end{bmatrix} \begin{bmatrix} 25.4710 \\ 30.0346 \\ 19.3259 \\ 56.7306 \\ 46.7851 \end{bmatrix} \begin{bmatrix} \cdots \\ p_i^b \text{ (pbest)} \end{bmatrix}$$

 p^g (gbest) = [-----



Update the velocity & position:

Calculate the fitness value:

Update ghest (p^g) :

Since 56.7306 > 53.7063, so updated ghest is Ghest = [4.8137 5.0000]



v (velocity)	x (position)	f(x)	$p_i^{\scriptscriptstyle D}$ (pbest)	
0.3240 1.3233 0.1815 1.0714 1.3531 0.8175 0.3578 0.3192 0.4445 0.3301	3.4712 -2.7013 4.2394 -1.1436 -2.3770 1.2863 4.4916 4.8943 1.7681 4.9790	[27.8600] 31.0325 ,13.7537 [53.7063] L45.5655	$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$	

v	velocity)	x (p	osition)	f(x)	p_i^b (obest)
г0.1334	0.98431	Г 3.6045	−1.7170 ₁	_[25.4710]	F	•••••]
0.0337	0.7826	4.2731	-0.3611	30.0346		
2.0987	1.1985	-0.2784	2.4848	19.3259	*****	
0.3221	0.2873	4.8137	5.0000	56:7306		
L _{0.7493}	0.2862	2.5174	5.0000	L46.7851J	L	******

 p^g (gbest) = [4.8137 5.0000]

Iteration 3:

Update the velocity & position:

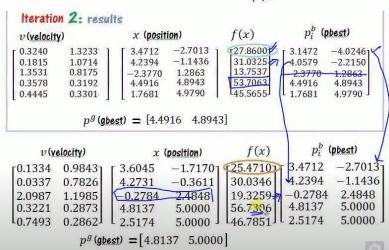
Calculate the fitness value:

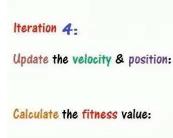
Update ghest (p^g) :

Since 56.7306 > 53.7063, so updated ghest is Ghest = [4.8137 5.0000]

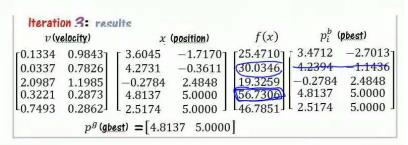
Update phest (p^b) :

Max





Update gbest
$$(p^g)$$
:
Since 58.000 > 56.7306
gbest = [5.0000 5.0000]
Update pbest (p^b) :



v (velocit	y)	x (pos	ition)	f(x)	p_i^b (pbe	st)
г0.1561	0.86907	_[3.7606	-0.8480 ₇	[25.1798]	_[3.6045	-1.7170
0.0696	0.6935	4.3427	0.3325	30.5409	4.3427	0.3325
2.5419	1.4012	2.2635	3.8860	34.4996	2.2635	3.8860
0.2899	0.2586	5.0000	5.0000	58.0000	5.0000	5.0000
L _{0.9689}	0.2576	13.4863	5.0000	L49.6953J	L3.4863	5.0000 J
	p^{g}	(gbest) = [5.0000 5.0	000]		100

Iteration 5:

Update the velocity & position:

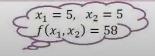
Calculate the fitness value:

Update ghest (p^g) :

Update phest (p^b) :

Since there is no change in the fitness

value (58.000) and hence optimum will reach.



v (v	elocity)	x	(position)	f(x)	p_i^b	(pbest)
0.1561	0.86907	[3.7606	-0.8480 ₁	[25.1798]	Γ3.6045	-1.7170
0.0696	0.6935	4.3427	0.3325	30.5409	4.3427	0.3325
2.5419	1.4012	2.2635	3.8860	34.4996	2.2635	3.8860
0.2899	0.2586	5.0000	5.0000	58.0000	5.0000	5.0000
0.9689	0.2576	L3.4863	5.0000 J	L49.6953J	L3.4863	5.0000

v (veloci	(X)	x (p	osition)	f(x)	p_i^b	(pbest)	
0.1601 0.1469 2.6386 0.2609 1.0661	0.7567 1.2228 1.4040 0.2327 0.2319	3.9208 4.4896 4.9021 5.0000 4.5524	-0.0914 1.5553 5.0000 5.0000 5.0000	[26.2152] 33.7933 57.3244 [58.0000] 55.0672]	3.9208 4.4896 4.9021 5.0000 4.5524	-0.0914 1.5553 5.0000 5.0000 5.0000	
		p^g (gbest)	= [5.0000	5.0000]			

