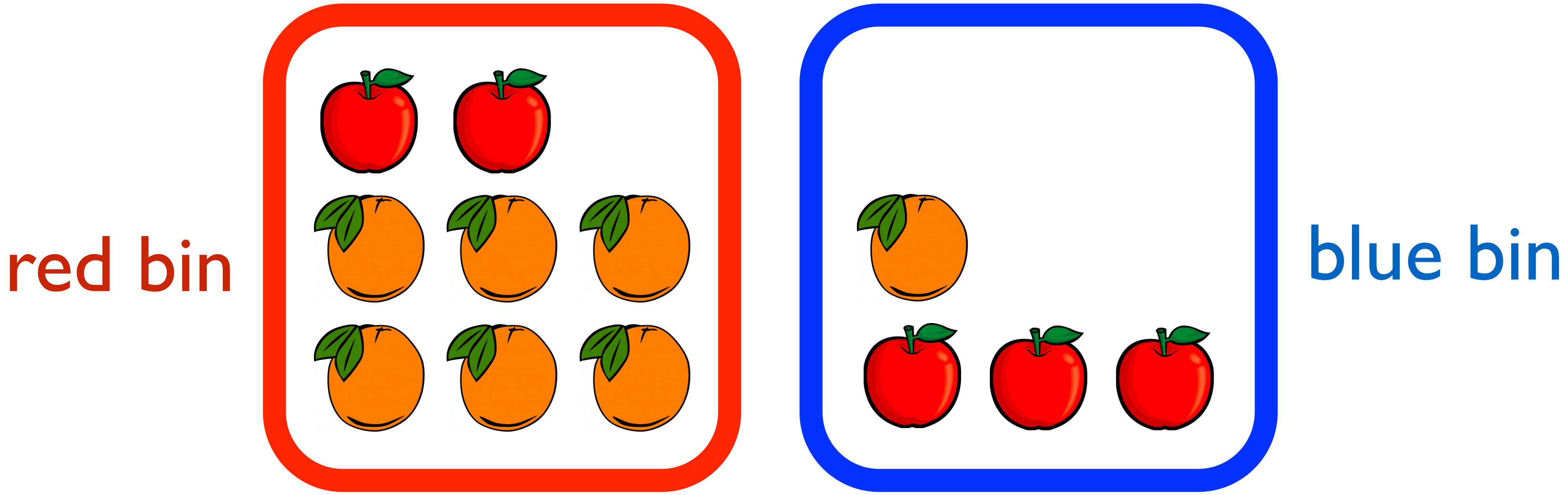
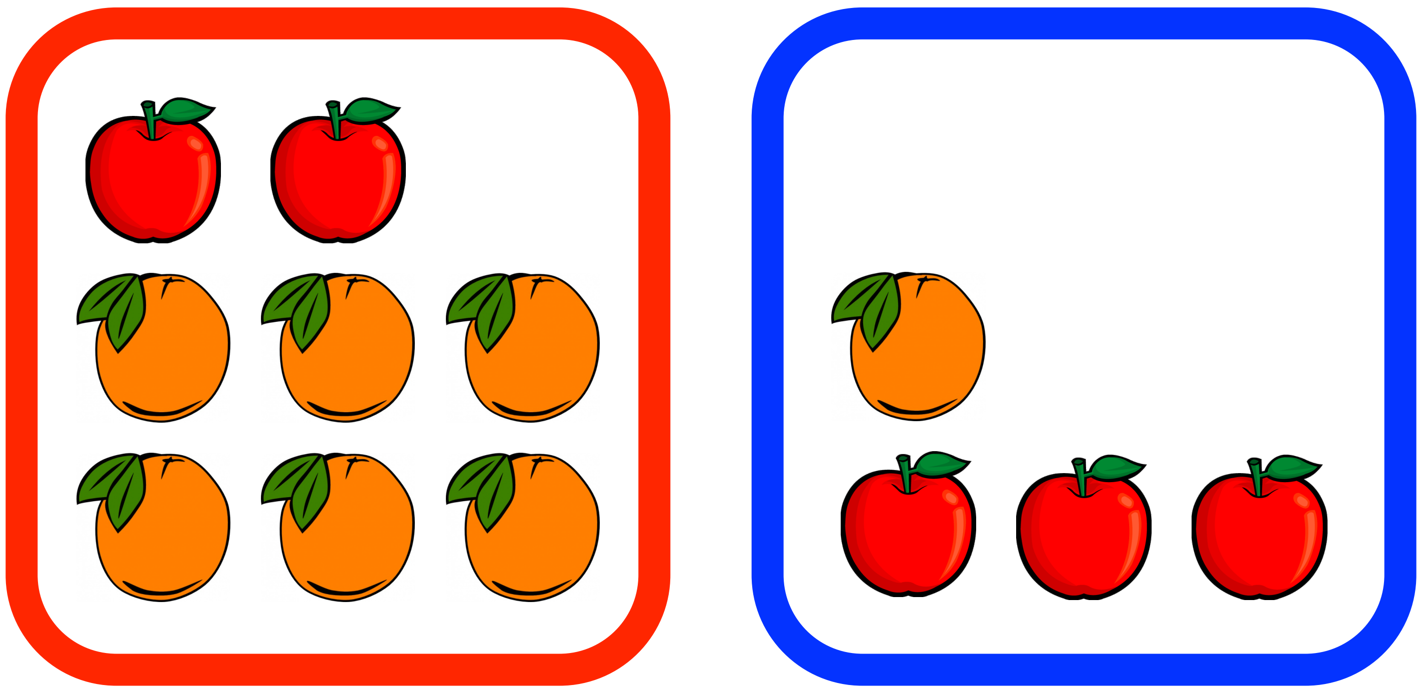
CS423: Probabilistic Programming

Posterior Inference, Basics of Anglican, and Importance Sampling

Hongseok Yang
KAIST


red bin
blue bin

red bin



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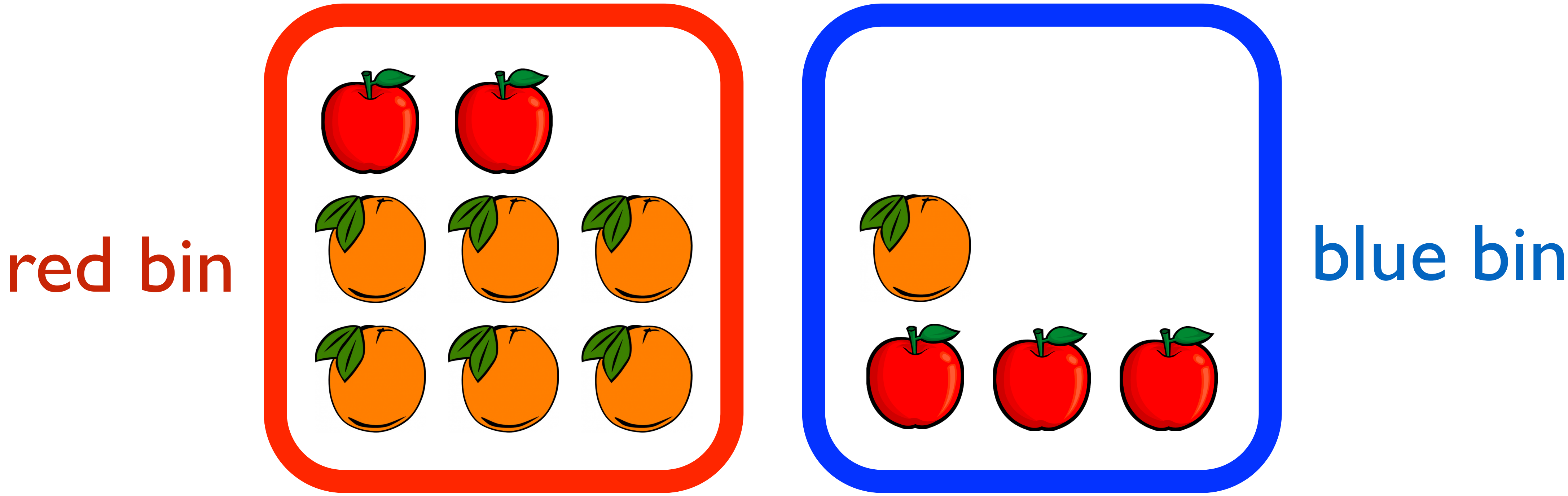

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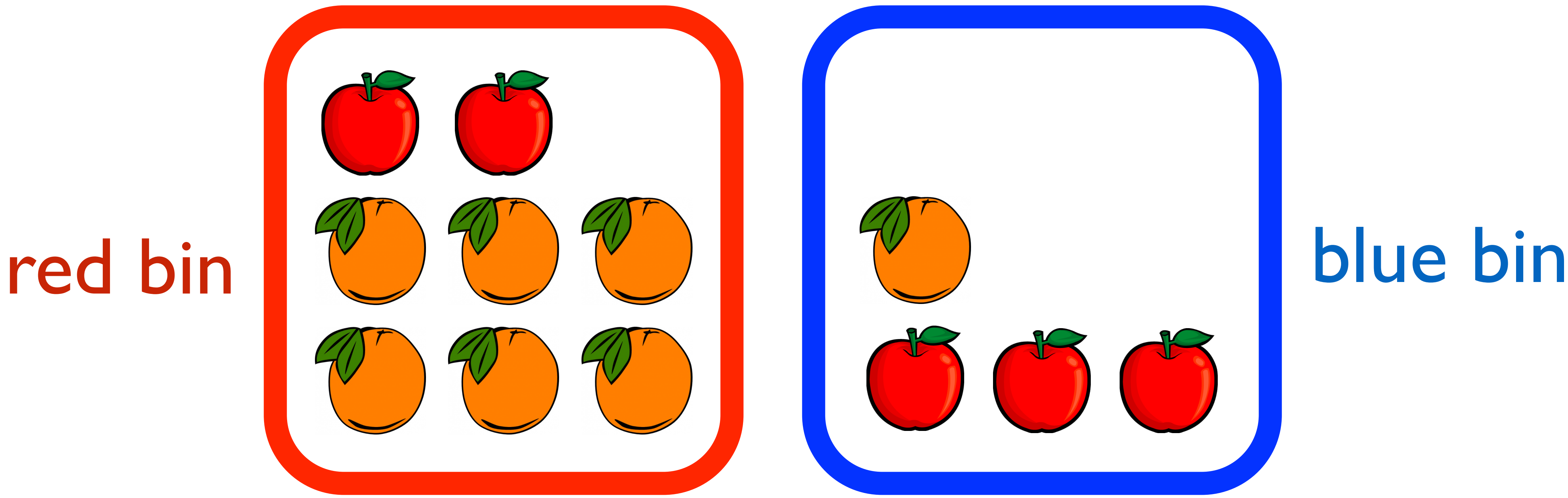

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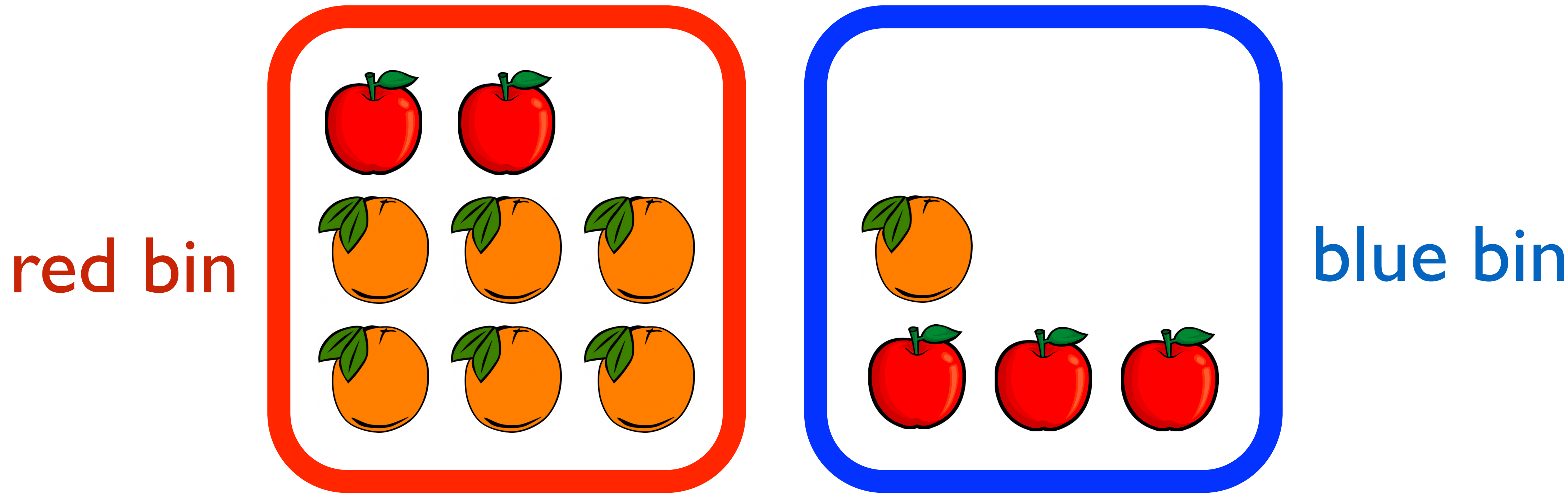
[Q] If I pick an orange, what is the probability that I picked the blue bin?

1) 5/6 2) 1/4 3) 5/8



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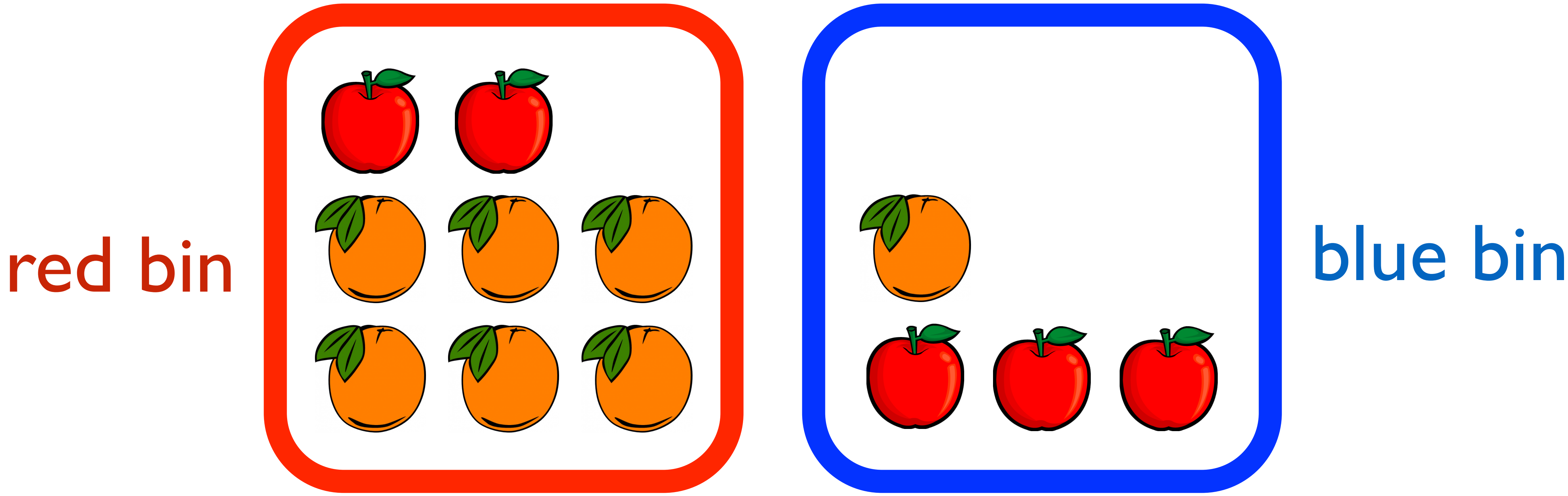

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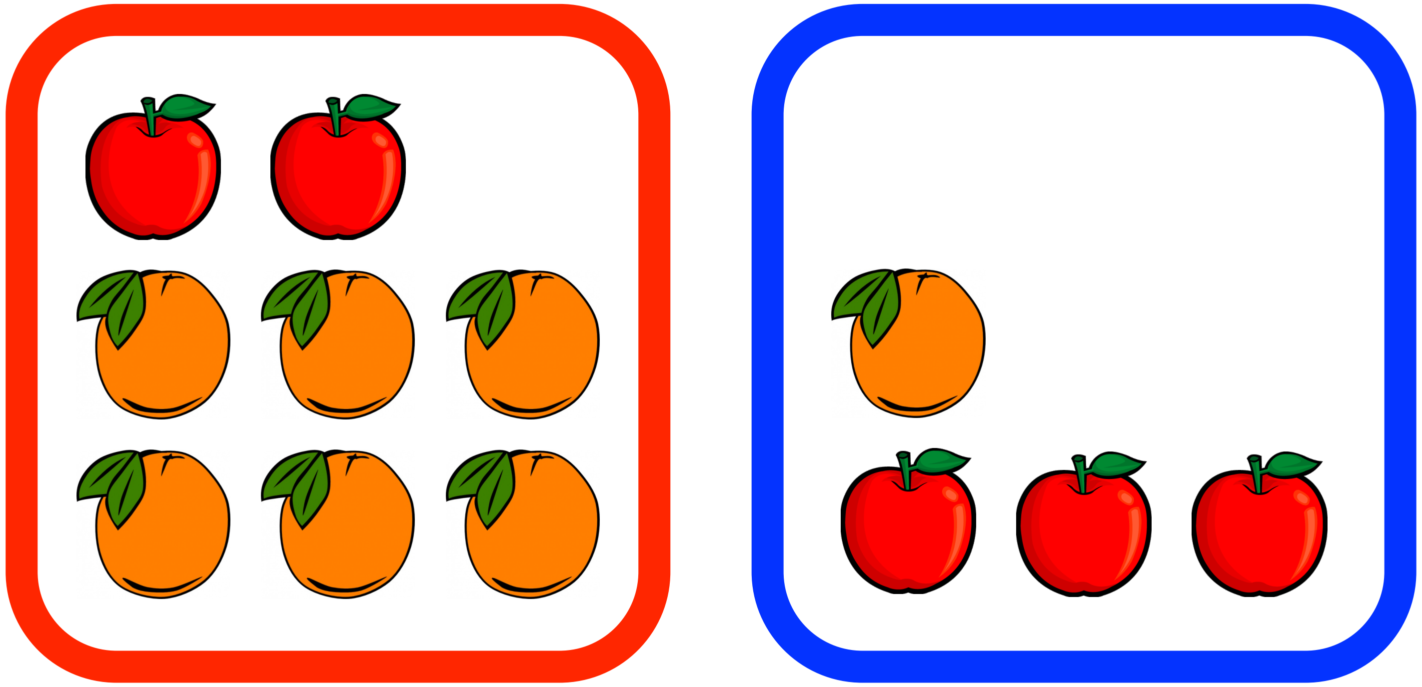

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$\boxed{p(\text{orange}|\text{red}) = 3/4 \qquad p(\text{orange}|\text{blue}) = 1/4}$

[Q

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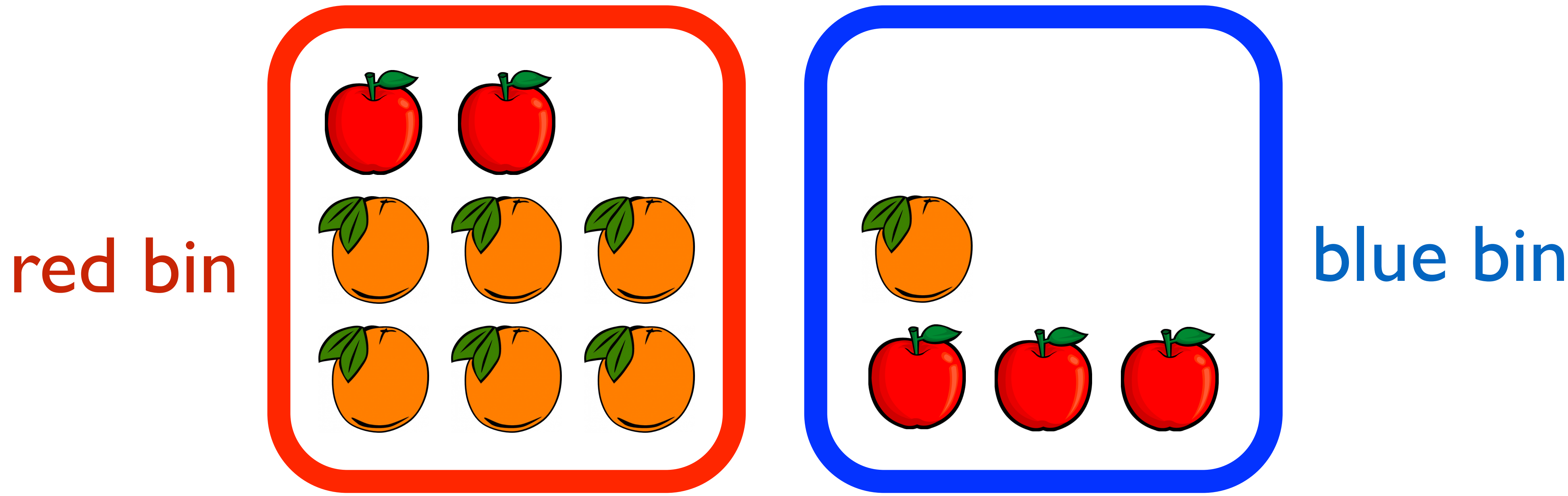

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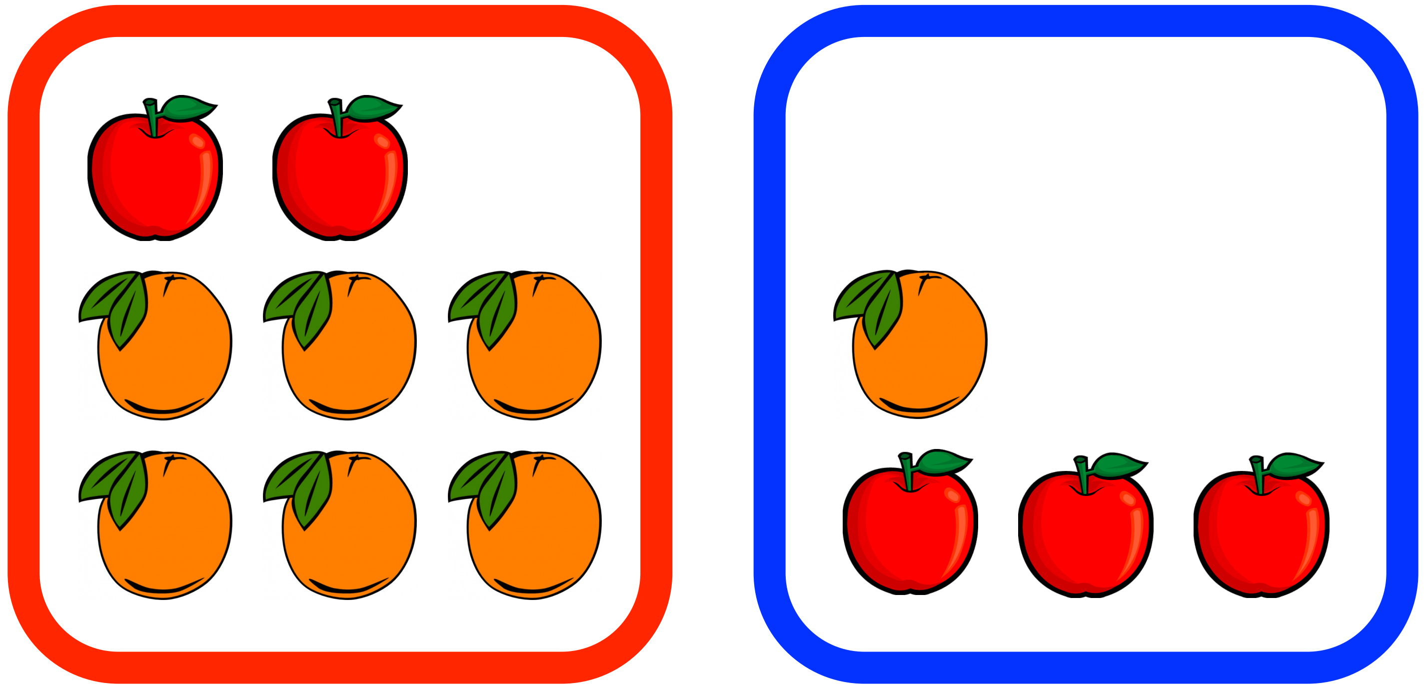

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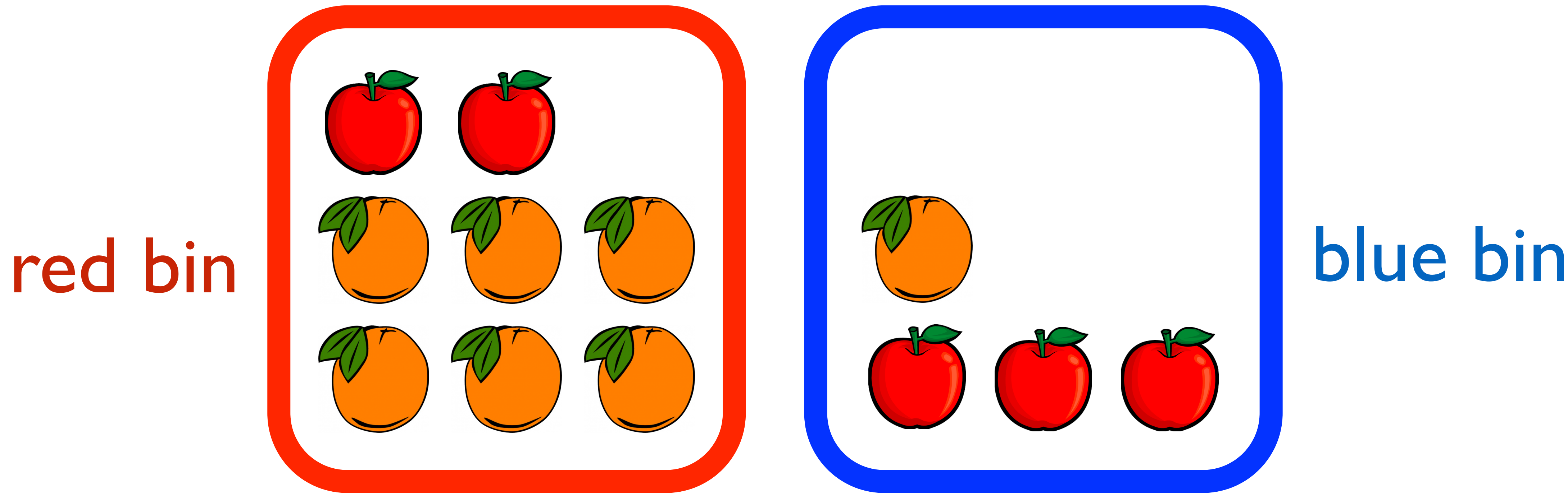

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Learning outcome

- Can describe prior, likelihood, posterior, Bayes' rule.
- Can solve the puzzle using Bayes' rule
- Can express/solve the puzzle in Anglican.
- Can explain importance sampling.

We will use discrete probabilities mostly.

Review of discrete probability, and posterior inference

- Consider random variables x,y,z,… having values in countable sets, such as {true,false} and $\mathbb{N}$.

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- Probability p assigns numbers between 0 and 1 for all possible value assignments of some variables.

$$p(x=0,y=0) = 1/24 \qquad p(x=0,y=1) = 3/24$$
$$p(x=1,y=0) = 5/24 \qquad p(x=1,y=1) = 15/24$$
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Enough.
Determines
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Conditional probability

$$p(x{=}v \mid y{=}w) =_{\text{def}} \frac{p(x{=}v,\, y{=}w)}{p(y{=}w)}$$

Says the prob. of x=v conditioned on y=w.

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[Lemma 2] (Bayes' rule)

$$p(x{=}v \mid y{=}w) = \frac{p(y{=}w \mid x{=}v) \times p(x{=}v)}{p(y{=}w)}$$

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In sloppy but simpler popular notation.

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[Q] Prove both lemmas.

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Bayes' rule

$$\underbrace{p(x \mid y)}_{\text{posterior distribution}} = \frac{\overbrace{p(y \mid x)}^{\text{likelihood}} \times \overbrace{p(x)}^{\text{prior distribution}}}{\underbrace{p(y)}_{\text{marginal likelihood}}}$$

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likelihood

prior distribution

marginal likelihood

posterior distribution

posterior $\propto$ likelihood $\times$ prior

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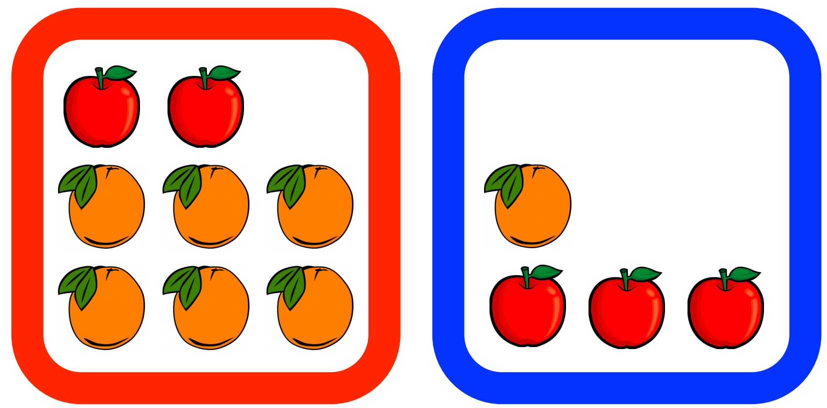
Puzzle again



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Exercise

A bag contains one ball, either white with prob 1/5 or black with prob 4/5. An additional white ball is put in, and the bag is shaken. Then, a ball is drawn, which proves to be white. What is now the chance of drawing a white ball?

Modified Ex 3.12 from MacKay's Info. Th. book

Posterior inference

- Computation of p(x|y) given i) p(y|x) and p(x) and ii) an observed value w of y.
- Bayes' rule and Req 2 give an algorithm:

$$p(x \mid y{=}w) = \frac{p(y{=}w \mid x) \times p(x)}{p(y{=}w)}$$

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Big sum for realistic models. Inefficient.

Approximate posterior inference

- Approximates posterior $p(x|y)$ using a set of samples or a simpler distribution.
- Commonly used in practice.
- Anglican implements many such algorithms.
- Often, the goal is to estimate $\mathbb{E}_{p(x|y)}[f(x)]$.

Expectation

$$\mathbb{E}_{p(x)}[f(x)] = \sum_x p(x)f(x)$$

1. Linearity:

$$\mathbb{E}_{p(x)}[\alpha f(x)+\beta g(x)] = \alpha\mathbb{E}_{p(x)}[f(x)]+\beta\mathbb{E}_{p(x)}[g(x)]$$

2. Independent random variables:

$$\mathbb{E}_{p(x)p(y)}[f(x)g(y)] = \mathbb{E}_{p(x)}[f(x)]\mathbb{E}_{p(y)}[g(y)]$$

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[Q] A coin with probability p of coming up heads. N coin tosses. Mean of the number of heads? Variance?

Conditioning and posterior inference in Anglican

Conditioning in Anglican

In Anglican, we condition a model by observed random variables using the observe construct:

`(observe` *distribution-object observed-value*`)`

Examples:

```
(observe (flip p) true)
```

```
(observe
  (categorical
    {:blue p, :red q, :green r})
  :blue)
```

[Q] Write an Anglican query for our puzzle using categorical distribution.

```
(defquery puzl [fruit]
  (let [bin
                                            ]
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  (let [bin (sample (categorical
                      {:red (/ 1 6),
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    ● ● ● ● ●


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  (let [bin (sample (categorical
                      {:red (/ 1 6),
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    (if (= bin :red)
      (observe (categorical
                 {:apple (/ 2 8),
                  :orange (/ 6 8)})
               fruit)
      (observe (categorical
                 {:apple (/ 3 4),
                  :orange (/ 1 4)})
               fruit))
    bin))
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Anglican function.
Performs inference.
Returns a lazy infinite list of Clojure maps.

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(def x (doquery :importance puzl [:orange]))
(println (first x))
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{:log-weight -1.3862943611198906,
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[A] Portion of (weighted) blue samples among all (weighted) samples.

Likelihood weighted importance sampling

[Goal] Estimate $\mathbb{E}_{p(x|y)}[f(x)]$ for a given f.

Likelihood weighted importance sampling

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posterior

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[Q1] Why is this a sensible algorithm?

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[Q1] Why is this a sensible algorithm?

[Q2] How to implement 1 & 2 for Anglican queries?

[Input] N and an Anglican query Q.

[Output] Weighted samples $(w_1,s_1),\ldots,(w_N,s_N)$.

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3. Return w and the result s of Q.

fruit = :orange

```
(defquery puzl [fruit]
  (let [bin (sample
              (categorical
                {:red (/ 1 6),
                 :blue (/ 5 6)}))]
    (if (= bin :red)
      (observe (categorical
                 {:apple (/ 2 8),
                  :orange (/ 6 8)})
               fruit)
      (observe (categorical
                 {:apple (/ 3 4),
                  :orange (/ 1 4)})
               fruit))
    bin))
```

fruit = :orange

w = 1.0

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(defquery puzl [fruit]
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fruit = :orange

w = 1.0 × 0.25

bin = :blue

fruit = :orange

$w = 1.0 \times 0.25$

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Thus, returns (0.25, :blue)

```
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fruit = :orange

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Thus, returns (0.25, :blue)

Other samples:

(0.75, :red)

(0.25, :blue)

(0.25, :blue)

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- Simple.
- Regarded as a semi-official semantics for Anglican and other probabilistic PLs.
- Log weight, not weight, used typically. Why?

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[Q] OK, but inefficient. Can you guess why? How can we improve it?

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[Q] Fill in ???

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   proposal $q(x)$

2. Compute weight $w_i = \cancel{p(y|x_i)}$ for each i.
   $p(y|x_i)p(x_i)/q(x)$

3. Return weighted avg. $(\sum_i w_i \times f(x_i)) / \sum_j w_j$.

[Q] Which q(x) is good?

Summary

- Learnt posterior inference using Bayes' rule in the context of discrete probabilities.
- In Anglican, we can condition using observe and perform posterior inference.
- Discussed the likelihood weighted importance sampling algorithm.