A FLUX TUBE MODEL FOR HADRONS

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We have constructed a potential model for hadrons based on the strong coupling flux tube limit of QCD. The model, with quark and string-like degrees of freedom, offers a simple and unified framework in which not only ordinary mesons and baryons but also multiquark states, hermaphrodites, and pure glue states can be discussed. We comment on the relation of this model to the two-body $F_i \cdot F_j$ potential model for pure quark states. We also present the expected quantum numbers and estimated masses of the low-lying hermaphrodite and pure glue states.

Introduction. The nonrelativistic quark model with two-body $F_i \cdot F_j$ colour-dependent confinement and one-gluon exchange potentials has been notoriously successful in describing the spectroscopy, static properties, and decay amplitudes of mesons and baryons [‡1]. While that model is immediately applicable to the study of multiquark systems, it cannot, unfortunately, be used to study the gluonic degrees of freedom in hadron physics. We present here a flux tube model for hadrons based on the strong coupling limit of QCD which is in principle applicable to all hadrons and which reduces approximately to the $F_i \cdot F_j$ potential model in the meson and baryon sectors, and possibly also in the multiquark sector. This model, with its parameters fixed from mesons and baryons, makes predictions for hermaphrodite [‡2] and pure glue states quite different from those of other models.

The eigenstates of the strong coupling limit of (lattice) QCD consist of quarks on lattice sites connected by arbitrary paths of flux links, or, in the absence of quarks, simply of arbitrary closed loops of flux [2]. In all cases the colour indices of the quark fields q_i on the lattice sites and the flux links U_{mn} which straddle lattice sites are contracted to colour singlets on each site, thereby guaranteeing local colour gauge invariance. Thus the degrees of freedom in the strong coupling limit are quarks and flux tubes, rather than quarks and gluons.

In our model we assume:

(1) The flux tube picture survives departures from the strong coupling limit.

(2) The flux tube can be assigned a constant mass per unit length b_0 and treated as a quantum string.

(3) For fixed quark positions (if any) the quantum states of the system consist of (mixtures of) the various allowed flux tube topologies in various states of excitation of their normal modes.

(4) Flux tube modes with scales below some critical size $\lambda_0 \sim 1$ fm do not exist.

In order to make the model useful we make several further working hypotheses:

(5) Low-lying flux tube modes can be treated in a nonrelativistic, small oscillations approximation.

(6) Flux tube dynamics allow an adiabatic treatment of the flux tubes in the presence of quark motion.

In this picture, ordinary mesons and baryons correspond to quarks moving in an effective potential generated by the (adiabatically varying) ground state of the flux tubes; multiquark states are more complicated since they must normally make transitions between adiabatic

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[‡1] For a recent review, see, for example, ref. [1]. F_i is the colour matrix $\frac{1}{2}\lambda_i$ ($-\frac{1}{2}\lambda_i^*$) of quark (antiquark) i.

[‡2] A state with both quark and gluonic degrees of freedom in evidence.

surfaces; hermaphrodites are quark states in which the flux tubes are either topologically or modally excited; and pure glue states are closed string configurations, the lowest-lying of which is a glueloop. In the next section we specify more fully the model which emerges from our assumptions. We then briefly discuss the application of the model to each of these sectors.

The model. We construct the model hamiltonian appropriate to any given situation by considering all topologically allowed flux tube configurations for a given fixed arrangement of any quarks present[‡3]. The string hamiltonian appropriate to each flux tube configuration S is then quantized in the approximations stated above. The various allowed states of the string S_n correspond to a band of excitations built on that string topology. If there are no quarks present then these states are pure glue states. If there are quarks, then these states define a set of quark string potential functions $V_{S_n}(r_1, ..., r_k)$ of the fixed quark positions r_i. As the quark positions are slowly varied, the string states will evolve adiabatically to define a set of adiabatic surfaces analogous to molecular potentials built on various electronic states. As in the molecular case, these surfaces will not normally intersect since as two surfaces approach each other they will mix and repel. A given adiabatic surface may therefore correspond to various string topologies and excitations as the r_i's are varied.

Ordinary mesons and baryons are distinguished in the model by being the only quark-containing states for which it is plausible that a treatment based on a single lowest adiabatic surface is appropriate. The reason is that, as we shall see below, in these cases the next adiabatic surface is much higher in energy. The simplest hermaphroditic states correspond to exciting mesons and baryons onto these other adiabatic surfaces.

For multiquark systems a treatment based on only the lowest adiabatic surface would appear to be inadequate. In this case the lowest potential surfaces corresponding to different flux tube topologies cross in the absence of mixing. There will therefore in general be secondary adiabatic surfaces close in energy to the lowest-lying one.

‡3 An allowed flux tube topology consists of an oriented string topology with a unit of three-flux leaving (entering) each quark (antiquark) and with the strings (which may be multiple and/or disjoint) either continuous or forming ϵ_{ijk} junctions.

Mesons and meson hermaphrodites. The simplest meson flux tube topology is, of course, just a single string. Qualitative arguments indicate that other string topologies will lie above the mode excitations of this simple string, so we begin by considering its quantization. In the approximations being considered here, we simply obtain (ignoring for now the string moment of inertia)

$$H = p^2/2\mu + b_0 r + \sum_{i,n} \frac{1}{b_0 r}(\pi_n^i \pi_n^i + \tfrac{1}{4} n^2\pi^2 b_0^2 a_n^i a_n^i) , \tag{1}$$

where the a_n^i are defined by

$$y(\xi, t) = \sum_{i,n} \hat{e}_i a_n^i(t) \sin n\pi\xi , \tag{2}$$

where $\hat{e}_i$ (i = 1, 2) is a vector transverse to the interquark separation $\boldsymbol{r}$, $\xi \boldsymbol{r}$ is the equilibrium string position relative to the antiquark, μ is the $q\bar{q}$ reduced mass, and π_n^i is the momentum conjugate to a_n^i. The adiabatic potentials built on this flux tube topology are thus characterized by mode occupation numbers. The lowest adiabatic surface has all of the allowed modes of the flux tube in their ground states and these alone would give as $r \to \infty$

$$V_0(r) \sim b_0 r + 2 \sum_{n=1}^{2r/\lambda_0} (n\pi/2r) \sim (b_0 + 2\pi/\lambda_0^2)r . \tag{3}$$

[The upper limit $n_{max} = 2r/\lambda_0$ arises via our assumption (4).] There is, however, probably a missing term in (3): while the condensation of the string allows some (soft) zero-point oscillations to exist, it is natural to assume that it excludes (in the spirit of the Casimir effect) some weak-coupling gluonic modes of a comparable wavelength, giving an additional term $-b_g r$ in (3). The net effect would be to produce a "renormalized" string potential $br + c$ (with $b \simeq b_0 + 2\pi/\lambda_0^2 - b_g$) which we associate with the phenomenological linear potential model with $b \simeq 0.18$ GeV^2 and $c \simeq -0.7$ GeV.

The next adiabatic surface will be based on a string with one transverse excitation in the n = 1 mode. Such a state will have $\sigma = \pm 1$ unit of angular momentum about $\boldsymbol{r}$ and will lead to an effective $q\bar{q}$ radial hamiltonian

$$H_1 = -(1/2\mu)\,\partial^2/\partial r^2 + (l^2 + l - 1)/2\mu r^2 + br + \pi/r + c + V_{sr}\,, \tag{4}$$

where l is the total angular momentum and $V_{sr}(r)$ is the short-range potential from one-gluon exchange. The presence of the centrifugal barrier term [this term follows from calculating the expectation value of $L^2_{quark} = (L - L_{string})^2$; note that $l \geqslant |\sigma|$] gives hermaphrodites a very large excitation energy relative to ground state mesons: in addition to gaining the mode excitation energy, they gain a centrifugal repulsion and lose their attractive short-range interactions (the large Coulomb term $-4\alpha_s/3r$ as well as the $S = 0$ hyperfine interaction). We estimate the lowest non-strange state on this surface to be at 1.8 GeV. This state is actually doubly degenerate ($\sigma = \pm 1$) corresponding to a parity doubling and so, when combined with spin we expect *eight* nearly degenerate nonets with $J^{PC} = 0^{\pm\mp}$, $1^{\pm\mp}$, $2^{\pm\mp}$, and $1^{\pm\pm}$. Note that the equilibrium radii of the analogous states for very heavy quarks may be less than λ_0 so that these particular states might not exist.

Normal meson decay by string breaking may make any or all of these states quite broad. In addition, those states without exotic quantum numbers can decay by a mechanism roughly analogous to pre-dissociation in molecules: deviations from the adiabatic limit (which could easily be substantial) can mix these states directly with nearby normal $q\bar{q}$ excited states of the lowest adiabatic surfaces. It would therefore seem to be the best strategy to search for the exotic 0^{+-}, 1^{-+}, or 2^{+-} nonets since they will neither be hidden nor additionally broadened by the "background" of ordinary excited mesons. Finally we note that the internal structure of these hermaphroditic states is very different from those of models with constitutent gluons [‡4] and their masses are predicted to be considerably higher.

Topological meson hermaphrodites are mentioned below.

[‡4] At the most basic level our states differ in their colour properties. They also differ in detail: e.g., our $q\bar{q}$ wave function vanishes at $r = 0$.

Baryons and baryon hermaphrodites. The lowest-lying qqq flux tube configuration is either an angularly symmetric three string junction or, if one of the angles formed by the qqq triangle is greater than 120°, two strings joining the three quarks at an elbow junction. Qualitative arguments once again indicate that other topologies and mode excitations are high-lying so that it is sensible to build a spectroscopy on the lowest adiabatic surface. It has recently been shown [3] that this surface has a spectrum similar to that of the two-body $\boldsymbol{F}_i \cdot \boldsymbol{F}_j$ potential; this provides some justification for the phenomenological success of the latter for baryons [1].

Baryon hermaphrodites, based on either topological or modal flux tube excitations, can be discussed along the same lines as meson hermaphrodites, although the discussion is, of course, considerably more difficult. We believe that the mode excitations will have a character quite similar to that in mesons and we estimate the masses of the lowest non-strange baryons of this type to be about 2.2 GeV with the quantum numbers of an $L^P = 1^{\pm}$ 70-plet of SU(6), i.e., one $N\frac{5}{2}^{\pm}$, two $N\frac{3}{2}^{\pm}$, two $N\frac{1}{2}^{\pm}$, and $\Delta\frac{3}{2}^{\pm}$ and $\Delta\frac{1}{2}^{\pm}$. The remarks made earlier on the widths of hermaphrodite mesons apply equally well here; unfortunately, none of these quantum numbers will particularly stand out in this mass region.

It seems likely to us that the lowest-lying topological excitations in both mesons and baryons will be higher in mass than the lowest mode excitations. Such states would of course have the quantum numbers of the ground states and so could be confused with (or mixed with) ordinary radial excitations.

Pure flux tube states. While apparently simpler structurally than the quark-containing sectors, the absence of the static quark limit leads to extra uncertainties in the application of our model to pure glue states. It is, however, natural to suppose that the simplest possible topology, that of a torus, will form the basis of the lowest-lying of these states and we accordingly assume that we can describe such a glueloop by an angle-dependent displacement in cylindrical coordinates (z, ρ, ϕ) [‡5]

[‡5] We take $+\hat{z}$ to define the orientation of the loop's flux lines by the right-hand rule.

$$\rho(\phi, t) = \hat{\boldsymbol{\rho}} \left(\rho_0(t) + \sum_{n=2} \alpha_n^\rho(t) \sin n\phi + \sum_{n=2} \beta_n^\rho(t) \cos n\phi \right) + \hat{\boldsymbol{z}} \left(\sum_{n=2} \alpha_n^z(t) \sin n\phi + \sum_{n=2} \beta_n^z(t) \cos n\phi \right) . \quad (5)$$

With the usual approximations this leads to the hamiltonian

$$H = \pi_0^2/4\pi b_0 \rho_0 + 2\pi b_0 \rho_0 + \sum_{i,n} [\pi_i^2/2\pi b_0 \rho_0 + (n^2 \pi b_0/2\rho_0)\gamma_i^2] , \quad (6)$$

where $i = 1, 2, 3, 4$ corresponding to the four independent modes for each n. If we consider the hamiltonian of the transverse oscillations for fixed ρ_0 then we see that its modes have a frequency $\omega_n^i = n/\rho_0$ so that with all the allowed oscillators in their ground states we would find an effective potential energy in ρ_0 as $\rho_0 \to \infty$

$$V_0(\rho_0) \sim 2\pi b_0 \rho_0 + 4 \sum_{n=2}^{2\pi\rho_0/\lambda_0} (n/2\rho_0) \sim 2\pi(b_0 + 2\pi/\lambda_0^2)\rho_0 , \quad (7)$$

in exact analogy to eq. (3). With the inclusion of Casimir effects once again it is natural to assume that the hamiltonian appropriate to ρ_0 is just

$$H = \pi_0^2/2\pi b \rho_0 + 2\pi b \rho_0 + c , \quad (8)$$

with b and c as in mesons ‡6. With the change of variables to $\zeta = \rho_0^{3/2}$ this becomes an ordinary one-dimensional Schrodinger problem with the boundary condition that $\psi(\zeta) = 0$ at $\zeta = 0$. This leads to a ground state glueloop with a mass of about 1.5 GeV and a mean radius of about 0.3 fm, and a first radial excitation at about 2.7 GeV.

These ground state glueloops will form the basis of both rotational and vibrational bands of states. We expect 0^{++} states at the masses mentioned above, rotational states with $J^{PC} = 1^{+-}$ at 2.2 GeV and 3.1 GeV, with $J^{PC} = 2^{++}$ at 2.7 GeV, and with $J^{PC} = 3^{+-}$ at 3.2 GeV. The lowest lying vibrational states are four quadrupole excitations with $|\sigma| = 2$ units of angular momentum about $\hat{z}$. These lead to $J^{PC} = 2^{++}$, 2^{+-}, 2^{-+}, and 2^{--} states at about 3.2 GeV. The existence of topological mixing (through which the glueloops with orientations $+\hat{z}$ and $-\hat{z}$ can convert into one another for small ρ_0) should have only minor effects on states with non-vacuum quantum numbers, but we cannot rule out the possibility that such effects may be important for our 0^{++} states ‡7.

‡6 If the constant c in glueloops differs from that in mesons, our entire glueloop spectrum is, of course, shifted by this difference.

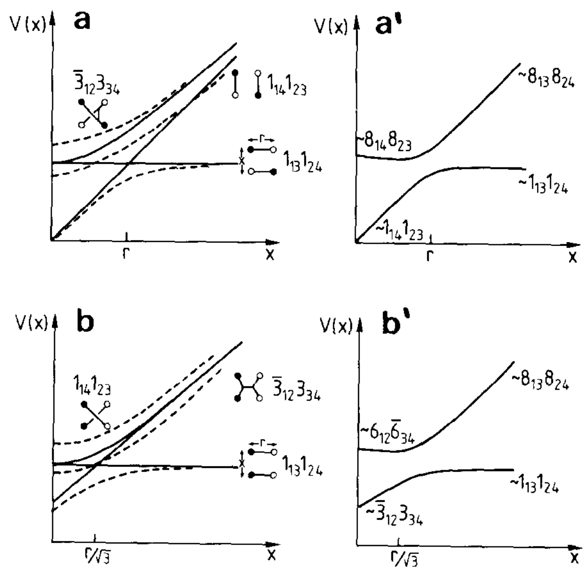

Fig. 1. The adiabatic potentials of the flux tube model and of the two body potential for two $qq\bar{q}\bar{q}$ arrangements: (a) and (b) show the potentials of the model of the text before (shown by solid curves) and after (shown schematically by dashed curves) topological mixing, while (a') and (b') show the related potentials of the two-body $\boldsymbol{F}_i \cdot \boldsymbol{F}_j$ model.

Multiquark states. The use of the two-body $\boldsymbol{F}_i \cdot \boldsymbol{F}_j$ potential in multiquark systems would seem particularly unwarranted so that application of our model to this sector is of special interest. Preliminary indications, however, are that the $\boldsymbol{F}_i \cdot \boldsymbol{F}_j$ model may have been an unexpectedly reasonable one, at least for low-lying multiquark states. We discuss here the $qq\bar{q}\bar{q}$ system as a prototype.

We have already mentioned the existence in general

‡7 Such mixing, if present, would lower their predicted masses. At the moment, we doubt if the mixing is very strong.

of secondary low-lying adiabatic surfaces in multiquark systems. This is illustrated in fig. 1 which shows one-dimensional slices through the potential surfaces associated with the planar quark arrangements and flux tube topologies shown. We have labelled the states with the quark colour clusterings they would have in the limit $U_{ij} \to \delta_{ij}$ in order to make a correspondence with the (non-gauge-invariant) two-body potential model. While the actual situation will be more complicated, we have assumed that in their ground states the energies of the various string topologies (before topological mixing) are given by bl where l is the total string length (solid curves). We see that after topological mixing (shown schematically by dashed curves) the two lowest-lying adiabatic curves are quite similar to the two adiabatic curves of the two-body potential. Quite apart from this partial coincidence of the two models in this example, we believe that this discussion shows that it is possible, at least in principle, to make a realistic flux tube model for multiquark systems. One immediate consequence of the model, incidentally, is the absence of the long range van der Waals-like interactions between colour singlets which plague the $\boldsymbol{F}_i \cdot \boldsymbol{F}_j$ model [4].

Relation to relativistic string models. Although relativistic string models are beset with quantization difficulties, we have tried to compare with them [5–8]. We find, among other things, that:

(1) the usual relativistic string lagrangians [6,7] reduce to ours in appropriate limits [‡8],

(2) the classical relativistic dumbell trajectory [6] leads to an E versus J relation numerically similar to that which follows from (1).

(3) the breathing modes of the closed string [7], if quantized under the same assumptions as those above, lead to a solvable hamiltonian with masses $M_n = [4\pi b(2n + \frac{3}{2})]^{1/2}$, not far from those of (8).

Despite these similarities, we have been unable to establish a one-to-one correspondence between the relativistic string states and ours.

[‡8] Note, however, that our string has a flux direction associated with it.

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