# Study of the reactions $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^0\eta$ at center-of-mass energies from threshold to 4.35 GeV using initial-state radiation

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We study the processes  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0\gamma$  and  $\pi^+\pi^-\pi^0\pi^0\eta\gamma$  in which an energetic photon is radiated from the initial state. The data were collected with the *BABAR* detector at SLAC. About 14 000 and 4700 events, respectively, are selected from a data sample corresponding to an integrated luminosity of 469 fb<sup>-1</sup>. The invariant mass of the hadronic final state defines the effective  $e^+e^-$  center-of-mass energy. From the mass spectra, the first precise measurement of the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  cross section and the first measurement ever of the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$  cross section are performed. The center-of-mass energies range from threshold to 4.35 GeV. The systematic uncertainty is typically between 10 and 13%. The contributions from  $\omega\pi^0\pi^0$ ,  $\eta\pi^+\pi^-$ , and other intermediate states are presented. We observe the  $J/\psi$  and  $\psi(2S)$  in most of these final states and measure the corresponding branching fractions, many of them for the first time.

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#### I. INTRODUCTION

Electron-positron annihilation events with initial-state radiation (ISR) can be used to study processes over a wide range of energies below the nominal  $e^+e^-$  center-ofmass (c.m.) energy  $(E_{\text{c.m.}})$ , as proposed in Ref. [1]. The possibility of exploiting ISR to make precise measurements of low-energy cross sections at high-luminosity  $\phi$ and B factories is discussed in Refs. [2–4], and motivates the studies described in this paper. Such measurements are of particular interest because of a ~3.5 standarddeviation discrepancy between the measured value of the muon anomalous magnetic moment  $(g_{\mu}-2)$  and the Standard Model value [5], where the Standard Model calculation requires input from experimental  $e^+e^-$  hadronic cross section data in order to account for hadronic vacuum polarization (HVP) terms. The calculation is most sensitive to the low-energy region, where the inclusive hadronic cross section cannot be measured reliably and a sum of exclusive states must be used. Not all accessible states have yet been measured, and new measurements will improve the reliability of the calculation. In addition, studies of ISR events at B factories are interesting in their own right, because they provide information on resonance spectroscopy for masses up to the charmonium region.

Studies of the ISR processes  $e^+e^- \to \mu^+\mu^-\gamma$  [6, 7] and  $e^+e^- \to X_h\gamma$ , using data from the BABAR experiment at SLAC, have been previously reported. Here  $X_h$  represents any of several exclusive hadronic final states. The  $X_h$  studied to date include: charged hadron pairs  $\pi^+\pi^-$  [7],  $K^+K^-$  [8], and  $p\bar{p}$  [9]; four or six charged mesons [10–12]; charged mesons plus one or two  $\pi^0$  mesons [11–15]; a  $K_S^0$  meson plus charged and neutral mesons [16]; and channels with  $K_L^0$  mesons [17]. The ISR events are characterized by good reconstruction efficiency and by well understood kinematics (see for example Ref. [13]), tracking, particle identification, and  $\pi^0$ ,  $K_S^0$ , and  $K_L^0$  reconstruction, demonstrated in above references.

This paper reports analyses of the  $\pi^+\pi^-3\pi^0$  and  $\pi^+\pi^-2\pi^0\eta$  final states produced in conjunction with a hard photon, assumed to result from ISR. While BABAR data are available at effective c.m. energies up to 10.58 GeV, the present analysis is restricted to energies below 4.35 GeV because of backgrounds from  $\Upsilon(4S)$  decays. As part of the analysis, we search for and observe in-

termediate states, including the  $\eta$ ,  $\omega$ ,  $\rho$ ,  $a_0(980)$ , and  $a_1(1260)$  resonances. A clear  $J/\psi$  signal is observed for both the  $\pi^+\pi^-3\pi^0$  and  $\pi^+\pi^-2\pi^0\eta$  channels, and the corresponding  $J/\psi$  branching fractions are measured. The decay  $\psi(2S) \to \pi^+\pi^-\pi^0\pi^0\pi^0$  is observed and its branching fraction is measured.

Previous measurements of the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$  cross section were reported by the M3N [18] and MEA [19] experiments, but with very limited precision, leading to a large uncertainty in the corresponding HVP contribution. The BABAR experiment previously measured the  $e^+e^- \to \eta\pi^+\pi^-$  reaction in the  $\eta \to \pi^+\pi^-\pi^0$  [14] and  $\eta \to \gamma\gamma$  [20] decay channels. Below, we present the measurement of  $e^+e^- \to \eta\pi^+\pi^-$  with  $\eta \to \pi^0\pi^0\pi^0$ : this process contributes to  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$ . There are no previous results for  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$ .

## II. THE BABAR DETECTOR AND DATASET

The data used in this analysis were collected with the BABAR detector at the PEP-II asymmetric-energy  $e^+e^-$  storage ring. The total integrated luminosity used is 468.6 fb<sup>-1</sup> [21], which includes data collected at the  $\Upsilon(4S)$  resonance (424.7 fb<sup>-1</sup>) and at a c.m. energy 40 MeV below this resonance (43.9 fb<sup>-1</sup>).

The BABAR detector is described in detail elsewhere [22]. Charged particles are reconstructed using the BABAR tracking system, which is comprised of the silicon vertex tracker (SVT) and the drift chamber (DCH), both located inside the 1.5 T solenoid. Separation of pions and kaons is accomplished by means of the detector of internally reflected Cherenkov light (DIRC) and energy-loss measurements in the SVT and DCH. Photons and  $K_L^0$  mesons are detected in the electromagnetic calorimeter (EMC). Muon identification is provided by the instrumented flux return.

To evaluate the detector acceptance and efficiency, we have developed a special package of Monte Carlo (MC) simulation programs for radiative processes based on the approach of Kühn and Czyż [23]. Multiple collinear soft-photon emission from the initial  $e^+e^-$  state is implemented with the structure function technique [24, 25], while additional photon radiation from final-state particles is simulated using the PHOTOS package [26]. The precision of the radiative simulation is such that it contributes less than 1% to the uncertainty in the measured hadronic cross sections.

We simulate  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$   $\pi^0\gamma$  events assuming production through the  $\omega(782)\pi^0\pi^0$  and  $\eta\rho(770)$  intermediate channels, with decay of the  $\omega$  to three pions and decay of the  $\eta$  to all its measured decay modes [27]. The two neutral pions in the  $\omega\pi^0\pi^0$  system are in an S-wave state and are described by a combination of phase space and  $f_0(980) \to \pi^0\pi^0$ , based on our study of the  $\omega\pi^+\pi^-$  state [14]. The simulation of  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta\gamma$  events is similarly based on two production channels: a

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phase space model, and a model with an  $\omega \pi^0 \eta$  intermediate state with a  $\pi^0 \eta$  S-wave system.

A sample of 100-200k simulated events is generated for each signal reaction and processed through the detector response simulation, based on the GEANT4 package [28]. These events are reconstructed using the same software chain as the data. Variations in detector and background conditions are taken into account.

For the purpose of background estimation, large samples of events from the main relevant ISR processes  $(2\pi\gamma, 3\pi\gamma, 4\pi\gamma, 5\pi\gamma, 2K\pi\gamma, \text{ and } \pi^+\pi^-\pi^0\pi^0\gamma)$  are simulated. To evaluate the background from the relevant non-ISR processes, namely  $e^+e^- \to q\bar{q} \ (q=u,d,s)$  and  $e^+e^- \to \tau^+\tau^-$ , simulated samples with integrated luminosities about twice that of the data are generated using the Jetset [29] and korals [30] programs, respectively. The cross sections for the above processes are known with an accuracy slightly better than 10%, which is sufficient for the present purposes.

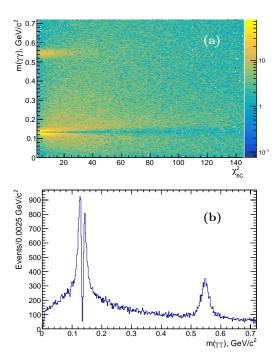


FIG. 1: (a) The invariant mass  $m(\gamma\gamma)$  of the third photon pair vs  $\chi^2_{2\pi 2\pi^0\gamma\gamma}$ . (b) The  $m(\gamma\gamma)$  distribution for  $\chi^2_{2\pi 2\pi^0\gamma\gamma} < 60$  and with additional selection criteria applied as described in the text.

## III. EVENT SELECTION AND KINEMATIC FIT

A relatively clean sample of  $\pi^+\pi^-3\pi^0\gamma$  and  $\pi^+\pi^-2\pi^0\eta\gamma$  events is selected by requiring that there be two tracks reconstructed in the DCH, SVT, or both, and seven or more photons, with an energy above

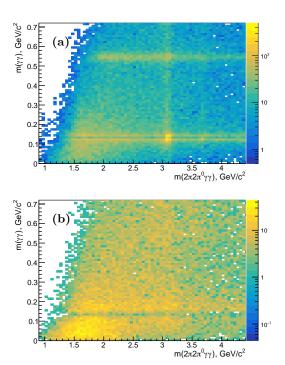


FIG. 2: (a) The third-photon-pair invariant mass  $m(\gamma\gamma)$  vs  $m(2\pi2\pi^0\gamma\gamma)$  for (a)  $\chi^2_{2\pi2\pi^0\gamma\gamma}<60$  and (b)  $60<\chi^2_{2\pi2\pi^0\gamma\gamma}<120$ .

0.02 GeV, in the EMC. We assume the photon with the highest energy to be the ISR photon, and we require its c.m. energy to be larger than 3 GeV.

We allow either exactly two or exactly three tracks in an event, but only two that extrapolate to within 0.25 cm of the beam axis and 3.0 cm of the nominal collision point along that axis. The reason a third track is allowed is to capture a relatively small fraction of signal events that contain a background track. The two tracks that satisfy the extrapolation criteria are fit to a vertex, which is used as the point of origin in the calculation of the photon directions.

We subject each candidate event to a set of constrained kinematic fits and use the fit results, along with charged-particle identification, to select the final states of interest and evaluate backgrounds from other processes. The kinematic fits make use of the four-momenta and covariance matrices of the initial  $e^+,\,e^-$ , and the set of selected tracks and photons. The fitted three-momenta of each track and photon are then used in further kinematical calculations.

Excluding the photon with the highest c.m. energy, which is assumed to arise from ISR, six other photons are combined into three pairs. For each set of six photons, there are 15 independent combinations of photon pairs. We retain those combinations in which the diphoton mass of at least two pairs lies within 35 MeV/ $c^2$  of the  $\pi^0$  mass  $m_{\pi^0}$ . The selected combinations are subjected to a fit in which the diphoton masses of the two pairs with

 $|m(\gamma\gamma)-m_{\pi^0}|<35~{\rm MeV}/c^2$  are constrained to  $m_{\pi^0}$ . In combination with the constraints due to four-momentum conservation, there are thus six constraints (6C) in the fit. The photons in the remaining ("third") pair are treated as being independent. If all three photon pairs in the combination satisfy  $|m(\gamma\gamma)-m_{\pi^0}|<35~{\rm MeV}/c^2$ , we test all possible combinations, allowing each of the three diphoton pairs in turn to be the third pair, i.e., the pair without the  $m_{\pi^0}$  constraint.

The above procedure allows us not only to search for events with  $\pi^0 \to \gamma \gamma$  in the third photon pair, but also for events with  $\eta \to \gamma \gamma$ .

The 6C fit is performed under the signal hypothesis  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma\gamma\gamma_{ISR}$ . The combination with the smallest  $\chi^2$  is retained, along with the obtained  $\chi^2_{2\pi 2\pi^0\gamma\gamma}$  value and the fitted three-momenta of each track and photon. Each selected event is also subjected to a 6C fit under the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{ISR}$  background hypothesis, and the  $\chi^2_{2\pi 2\pi^0}$  value is retained. The  $\pi^+\pi^-\pi^0\pi^0$  process has a larger cross section than the  $\pi^+\pi^-3\pi^0$  signal process and can contribute to the background when two background photons are present. Most events contain additional soft photons due to machine background or interactions in the detector material.

## IV. THE $\pi^+\pi^-3\pi^0$ FINAL STATE

## A. Additional selection criteria

The results of the 6C fit to events with two tracks and at least seven photon candidates are used to perform the final selection of the five-pion sample. We require the tracks to lie within the fiducial region of the DCH (0.45-2.40 radians) and to be inconsistent with being a kaon or muon. The photon candidates are required to lie within the fiducial region of the EMC (0.35-2.40 radians) and to have an energy larger than 0.035 GeV. A requirement that there be no charged tracks within 1 radian of the ISR photon reduces the  $\tau^+\tau^-$  background to a negligible level. A requirement that any extra photons in an event each have an energy below 0.7 GeV slightly reduces the multi-photon background.

Figure 1 (a) shows the invariant mass  $m(\gamma\gamma)$  of the third photon pair vs  $\chi^2_{2\pi 2\pi^0\gamma\gamma}$ . Clear  $\pi^0$  and  $\eta$  peaks are visible at small  $\chi^2$  values. We require  $\chi^2_{2\pi 2\pi^0\gamma\gamma} < 60$  for the signal hypothesis and  $\chi^2_{2\pi 2\pi^0} > 30$  for the  $2\pi 2\pi^0$  background hypothesis. This requirement reduces the contamination due to  $2\pi 2\pi^0$  events from 30% to about 1-2% while reducing the signal efficiency by only 5%.

Figure 1 (b) shows the  $m(\gamma\gamma)$  distribution after the above requirements have been applied. The dip in this distribution at the  $\pi^0$  mass value is a consequence of the kinematic fit constraint of the best two photon pairs to the  $\pi^0$  mass. Also, because of this constraint, the third photon pair is sometimes formed from photon candidates that are less well measured.

Figure 2 shows the  $m(\gamma\gamma)$  distribution vs the invariant mass  $m(2\pi 2\pi^0\gamma\gamma)$  for events (a) in the signal region  $\chi^2_{2\pi 2\pi^0\gamma\gamma} < 60$  and (b) in a control region defined by  $60 < \chi^2_{2\pi 2\pi^0\gamma\gamma} < 120$ . Events from the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  and  $\pi^+\pi^-2\pi^0\eta$  processes are clearly seen in the signal region, as well as  $J/\psi$  decays to these final states. In the control region no significant structures are seen and we use these events to evaluate background.

Our strategy to extract the signals for the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  and  $\pi^+\pi^-\pi^0\pi^0\eta$  processes is to perform a fit for the  $\pi^0$  and  $\eta$  yields in intervals of 0.05 GeV/ $c^2$  in the distribution of the  $\pi^+\pi^-2\pi^0\gamma\gamma$  invariant mass  $m(\pi^+\pi^-2\pi^0\gamma\gamma)$ .

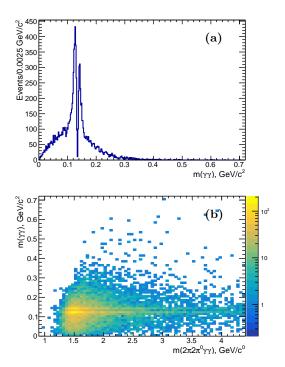


FIG. 3: The MC-simulated distribution for  $e^+e^- \to \eta \pi^+ \pi^-$  events of (a) the third-photon-pair invariant mass  $m(\gamma \gamma)$ , and (b)  $m(\gamma \gamma)$  vs  $m(\pi^+ \pi^- 2\pi^0 \gamma \gamma)$ .

## B. Detection efficiency

As mentioned in Sec. II, the model used in the MC simulation assumes that the five-pion final state results predominantly from  $\omega\pi^0\pi^0$  and  $\eta\pi^+\pi^-$  production, with  $\omega$  decays to three pions and  $\eta$  decays to all modes. As shown below, these two final states dominate the observed cross section.

The selection procedure applied to the data is also applied to the MC-simulated events. Figures 3 and 4 show (a) the  $m(\gamma\gamma)$  distribution and (b) the distribution of  $m(\gamma\gamma)$  vs  $m(2\pi2\pi^0\gamma\gamma)$  for the simulated  $\eta\pi^+\pi^-$  and  $\omega\pi^0\pi^0$  events, respectively. The  $\pi^0$  peak is not Gaus-

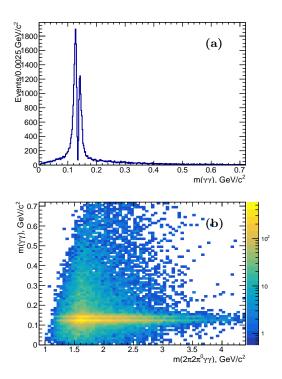


FIG. 4: The MC-simulated distribution for  $e^+e^- \to \omega \pi^0 \pi^0$  events of (a) the third-photon-pair invariant mass  $m(\gamma \gamma)$ , and (b)  $m(\gamma \gamma)$  vs  $m(\pi^+\pi^-2\pi^0\gamma\gamma)$ .

sian in either reaction and is broader for  $\eta\pi^+\pi^-$  events than for  $\omega\pi^0\pi^0$  events because the photon energies are lower. Background photons are included in the simulation. Thus these distributions include simulation of the combinatoric background that arises when background photons are combined with photons from the signal reactions.

The combinatoric background is subtracted using the data from the  $\chi^2$  control region. The method is illustrated using simulation in Fig. 5, which shows the  $m(\gamma\gamma)$  distribution with a bin width of 0.02 GeV/ $c^2$ . The dashed histograms show the simulated combinatoric background. The solid histograms show the simulated results from the signal region after subtraction of the simulated combinatoric background. The sum of three Gaussian functions with a common mean is used to describe the  $\pi^0$  signal shape. The fitted fit function is shown by the smooth curve in Fig. 5. We perform a fit of the  $\pi^0$  signal in every 0.05 GeV/ $c^2$  interval in the  $m(2\pi 2\pi^0 \gamma \gamma)$  invariant mass for the two different simulated channels.

Alternatively, for the  $\eta \pi^+ \pi^-$  events, we determine the number of events vs the  $m(2\pi 2\pi^0 \gamma \gamma)$  invariant mass by fitting the  $\eta$  signal from the  $\eta \to \pi^0 \pi^0 \pi^0$  decay: the simulated background-subtracted distribution is shown in Fig. 6(a). The fit function is again the sum of three Gaussian functions with a common mean.

Similarly, as an alternative for the  $\omega \pi^0 \pi^0$  events, the  $\omega$  mass peak can be used. The  $\omega$  mass peak in simula-

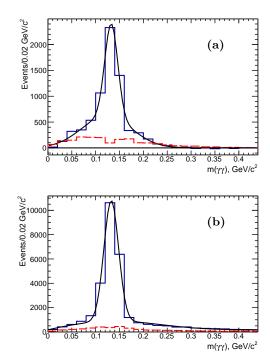


FIG. 5: The background subtracted MC-simulated  $m(\gamma\gamma)$  distribution for (a)  $e^+e^-\to\eta\pi^+\pi^-$  and (b)  $e^+e^-\to\omega\pi^0\pi^0$  events. The dashed histogram shows the simulated distribution from the  $\chi^2$  control region, used for subtraction. The fit function is described in the text.

tion is shown in Fig. 6(b), with three entries per event. We obtain the number of events by fitting  $m(\pi^+\pi^-\pi^0)$  in 0.05 GeV/ $c^2$  intervals of the  $m(\pi^+\pi^-2\pi^0\gamma\gamma)$  invariant mass. A Breit-Wigner (BW) function, convoluted with a Gaussian distribution to account for the detector resolution, is used to describe the  $\omega$  signal. A second-order polynomial is used to describe the background.

The mass-dependent detection efficiency is obtained by dividing the number of fitted MC events in each 0.05 GeV/ $c^2$  mass interval by the number generated in the same interval. Although the signal simulation accounts for all  $\eta$  decay modes, the efficiency calculation considers the signal  $\eta \to \pi^0 \pi^0 \pi^0$  decay mode only. This efficiency estimate takes into account the geometrical acceptance of the detector for the final-state photons and the charged pions, the inefficiency of the detector subsystems, and the event loss due to additional soft-photon emission from the initial and final states. Corrections that account for data-MC differences are discussed below.

The mass-dependent efficiencies from the  $\pi^0$  fit are shown in Fig. 7 by points for the  $\eta\pi^+\pi^-$  and by squares for the  $\omega\pi^0\pi^0$  intermediate states, respectively. The efficiencies determined from the  $\eta$  and  $\omega$  fits are shown in Fig. 7 by the triangles and upside-down triangles, respectively. These results are very similar to those obtained from the  $\pi^0$  fits.

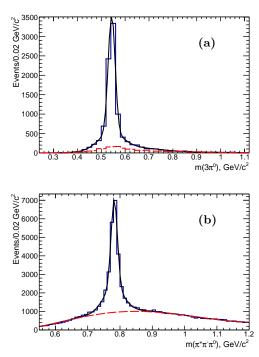


FIG. 6: (a) The background subtracted MC-simulated  $3\pi^0$  invariant mass for the  $e^+e^- \to \eta \pi^+\pi^-$  events. The dashed distribution is from the simulated  $\chi^2$  control region, used for background subtraction. (b) The  $\pi^+\pi^-\pi^0$  invariant mass for the MC-simulated  $e^+e^- \to \omega\pi^0\pi^0$  events (three entries per event). The solid curve shows the fit function used to obtain number of signal events. The dashed curve shows the fit function for the combinatorial background.

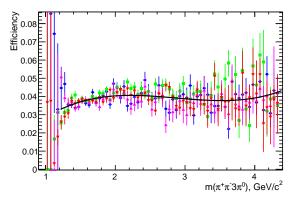


FIG. 7: The energy-dependent reconstruction efficiency for  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  events, determined using four different methods: see text. The curve shows the results of a fit to the average values, which is used in the cross section calculation.

From Fig. 7 it is seen that the reconstruction efficiency is about 4%, roughly independent of mass. By comparing the results of the four different methods used to evaluate the efficiency, we conclude that the overall acceptance does not change by more than 5% because of variations of the functions used to extract the number of events or

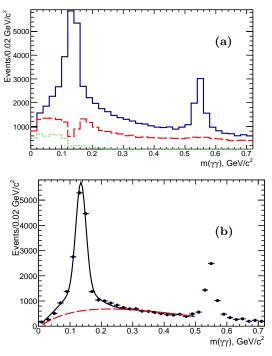


FIG. 8: (a) The third-photon-pair invariant mass  $m(\gamma\gamma)$  for data in the signal (solid) and  $\chi^2$  control (dashed) regions. The dotted histogram shows the estimated background from  $e^+e^-\to \pi^+\pi^-\pi^0\pi^0$ . (b) The  $m(\gamma\gamma)$  invariant mass for data after background subtraction. The curves are the fit results as described in the text.

the use of different models. This value is taken as an estimate of the systematic uncertainty in the acceptance associated with the simulation model used and with the fit procedure. We average the four efficiencies in each  $0.05~{\rm GeV}/c^2$  mass interval and fit the result with a third order polynomial function, shown in Fig. 7. The result of this fit is used for the cross section calculation.

## C. Number of $\pi^+\pi^-3\pi^0$ events

The solid histogram in Fig. 8 (a) shows the  $m(\gamma\gamma)$  data of Fig. 1 (b) binned in mass interval of 0.02 GeV/ $c^2$ . The dashed histogram shows the distribution of data from the  $\chi^2$  control region. The dotted histogram is the estimated remaining background from the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$  process. No evidence for a peaking background is seen in either of the two background distributions. We subtract the background evaluated using the  $\chi^2$  control region. The resulting  $m(\gamma\gamma)$  distribution is shown in Fig. 8 (b).

We fit the data of Fig. 8 (b) with a combination of a signal function, taken from simulation, and a background function, taken to be a third-order polynomial. The fit is performed in the  $m(\gamma\gamma)$  mass range from 0.0 to 0.5 GeV/ $c^2$ . The result of the fit is shown by the solid and dashed curves in Fig. 8 (b). In total  $14\,390\pm182$ 

events are obtained. Note that this number includes a relatively small peaking background component, due to  $q\bar{q}$  events, which is discussed in Sect. IV D. The same fit is applied to the corresponding  $m(\gamma\gamma)$  distribution in each 0.05 GeV/ $c^2$  interval in the  $\pi^+\pi^-2\pi^0\gamma\gamma$  invariant mass. The resulting number of  $\pi^+\pi^-3\pi^0$  event candidates as a function of  $m(\pi^+\pi^-3\pi^0)$ , including the peaking  $q\bar{q}$  background, is shown by the data points in Fig. 9.

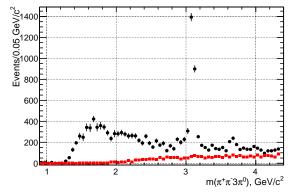


FIG. 9: The invariant mass distribution of  $\pi^+\pi^-3\pi^0$  events, obtained from the fit to the  $\pi^0$  mass peak. The contribution from non-ISR uds background is shown by squares.

## D. Peaking background

The major background producing a  $\pi^0$  peak following application of the selection criteria of Sect. IV.A is from non-ISR  $q\bar{q}$  events, the most important channel being  $e^+e^-\to\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$  in which one of the neutral pions decays asymmetrically, yielding a high energy photon that mimics an ISR photon. Figure 10 (a) shows the third-photon-pair invariant mass vs  $m(\pi^+\pi^-\pi^0\pi^0\gamma\gamma)$  for the non-ISR light quark  $q\bar{q}$  (uds) simulation: clear signals from  $\pi^0$  and  $\eta$  are seen. Figure 10(b) shows the projection plots for  $\chi^2_{2\pi^2\pi^0\gamma\gamma} < 60$  and  $60 < \chi^2_{2\pi^2\pi^0\gamma\gamma} < 120$ .

To normalize the uds simulation, we calculate the diphoton invariant mass distribution of the ISR candidate with all the remaining photons in the event. A  $\pi^0$  peak is observed, with approximately the same number of events in data and simulation, leading to a normalization factor of  $1.0 \pm 0.1$ . The resulting uds background is shown by the squares in Fig. 9: the uds background is negligible below 2 GeV/ $c^2$ , but accounts for more than half the total background for around 4 GeV/ $c^2$  and above.

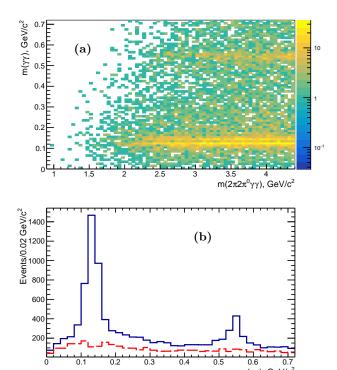


FIG. 10: (a) The third-photon-pair invariant mass vs  $m(\pi^+\pi^-\pi^0\pi^0\gamma\gamma)$  for the uds simulation. (b) The projection plot for (a) the signal region  $\chi^2_{2\pi2\pi^0\gamma\gamma}<60$  (solid histogram), and the control region  $60<\chi^2_{2\pi2\pi^0\gamma\gamma}<120$  (dashed histogram).

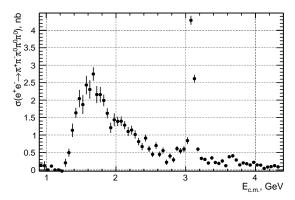


FIG. 11: The measured  $e^+e^-\to\pi^+\pi^-\pi^0\pi^0\pi^0$  cross section. The uncertainties are statistical only.

# E. Cross section for $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$

The  $e^+e^-\to\pi^+\pi^-\pi^0\pi^0\pi^0$  Born cross section is determined from

$$\sigma(2\pi 3\pi^0)(E_{\text{c.m.}}) = \frac{dN_{5\pi\gamma}(E_{\text{c.m.}})}{d\mathcal{L}(E_{\text{c.m.}})\epsilon_{5\pi}^{\text{corr}}\epsilon_{5\pi}^{\text{MC}}(E_{\text{c.m.}})(1+\delta_R)},$$
(1)

17	DLE 1. Summary	or the e	$\rightarrow n n n n$	7 CIOSS SE	ction measur	ement. The	uncertamne	s are statisti	car omy.
$E_{\mathrm{c.m.}},$	$GeV \qquad \sigma, \text{ nb}$	$E_{\rm c.m.},  {\rm GeV}$	$\sigma$ , nb	$E_{\rm c.m.},  {\rm GeV}$	$\sigma$ , nb	$E_{\rm c.m.},{\rm GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb
1.12	$0.00 \pm 0.02$	1.775	$2.20 \pm 0.23$	2.425	$0.92 \pm 0.10$	3.075	$4.36 \pm 0.13$	3.725	$0.29 \pm 0.05$
1.17	$0.00 \pm 0.03$	1.825	$2.03 \pm 0.17$	2.475	$0.61 \pm 0.09$	3.125	$2.66 \pm 0.11$	3.775	$0.15\pm0.04$
1.22	$-0.03 \pm 0.05$	1.875	$1.65 \pm 0.15$	2.525	$0.45\pm0.08$	3.175	$0.60\pm0.06$	3.825	$0.20\pm0.04$
1.27	$0.21 \pm 0.12$	1.925	$1.23 \pm 0.15$	2.575	$0.71\pm0.10$	3.225	$0.33\pm0.05$	3.875	$0.18\pm0.04$
1.32	$0.51 \pm 0.12$	1.975	$1.46 \pm 0.19$	2.625	$0.45 \pm 0.08$	3.275	$0.31 \pm 0.05$	3.925	$0.14\pm0.04$
1.37	$1.17 \pm 0.20$	2.025	$1.41 \pm 0.14$	2.675	$0.56\pm0.09$	3.325	$0.20\pm0.05$	3.975	$0.22\pm0.04$
1.42	$1.68 \pm 0.15$	2.075	$1.42 \pm 0.14$	2.725	$0.22\pm0.08$	3.375	$0.35\pm0.05$	4.025	$0.14\pm0.04$
1.47	$2.10 \pm 0.26$	2.125	$1.30 \pm 0.12$	2.775	$0.40 \pm 0.08$	3.425	$0.22 \pm 0.05$	4.075	$0.14 \pm 0.03$
1.52	$1.92 \pm 0.28$	2.175	$1.12 \pm 0.13$	2.825	$0.29 \pm 0.08$	3.475	$0.19\pm0.05$	4.125	$0.04 \pm 0.03$
1.57	$2.49 \pm 0.27$	2.225	$1.16 \pm 0.13$	2.875	$0.62 \pm 0.08$	3.525	$0.26 \pm 0.05$	4.175	$0.08 \pm 0.03$
1.62	$2.36 \pm 0.27$	2.275	$1.03 \pm 0.12$	2.925	$0.55\pm0.08$	3.575	$0.12\pm0.05$	4.225	$0.09 \pm 0.03$
1.67	$2.81 \pm 0.20$	2.325	$0.82 \pm 0.11$	2.975	$0.60 \pm 0.09$	3.625	$0.38 \pm 0.05$	4.275	$0.12\pm0.03$
1.72	$2.20 \pm 0.25$	2.375	$0.68 \pm 0.10$	3.025	$0.85\pm0.10$	3.675	$0.41\pm0.06$	4.325	$0.09\pm0.03$

TABLE I: Summary of the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  cross section measurement. The uncertainties are statistical only

where  $E_{\rm c.m.}$  is the invariant mass of the five-pion system;  $dN_{5\pi\gamma}$  is the background-subtracted number of selected five-pion events in the interval  $dE_{\rm c.m.}$ , and  $\epsilon_{5\pi}^{\rm MC}(E_{\rm c.m.})$  is the corresponding detection efficiency from simulation. The factor  $\epsilon_{5\pi}^{\rm corr}$  accounts for the difference between data and simulation in the tracking  $(1.0\pm1.0\%/{\rm per})$  track) [10] and  $\pi^0$  (3.0±1.0% per pion) [15] reconstruction efficiencies. The ISR differential luminosity,  $d\mathcal{L}$ , is calculated using the total integrated BABAR luminosity of 469 fb<sup>-1</sup> [13]. The initial- and final-state soft-photon emission is accounted for by the radiative correction factor  $(1+\delta_R)$ , which is close to unity for our selection criteria. The cross section results contain the effect of vacuum polarization because this effect is not accounted for in the luminosity calculation.

Our results for the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  cross section are shown in Fig. 11. The cross section exhibits a structure around 1.7 GeV with a peak value of about 2.5 nb, followed by a monotonic decrease toward higher energies. Because we present our data in bins of width 0.050 GeV/ $c^2$ , compatible with the experimental resolution, we do not apply an unfolding procedure to the data. Numerical values for the cross section are presented in Table I. The  $J/\psi$  region is discussed later.

#### F. Summary of the systematic studies

The systematic uncertainties, presented in the previous sections, are summarized in Table II, along with the corrections that are applied to the measurements.

The three corrections applied to the cross sections sum up to 12.5%. The systematic uncertainties vary from 10% for  $E_{\rm c.m.} < 2.5$  GeV to 50% for  $E_{\rm c.m.} > 3.5$  GeV. The largest systematic uncertainty arises from the fitting and background subtraction procedures. It is estimated by varying the background levels and the parameters of the functions used.

TABLE II: Summary of the systematic uncertainties in the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  cross section measurement.

Source	Correction	Uncertainty
Luminosity	_	1%
MC-data difference ISR		
Photon efficiency	+1.5%	1%
$\chi^2$ cut uncertainty	_	3%
Fit and background subtraction	_	7%
$E_{\rm c.m.} > 2.5  \mathrm{GeV}$	_	20%
$E_{\rm c.m.} > 3.5  \mathrm{GeV}$	_	50%
MC-data difference in track losses	+2%	2%
MC-data difference in $\pi^0$ losses	+9%	3%
Radiative corrections accuracy	_	1%
Acceptance from MC		
(model-dependent)	_	5%
Total (assuming no correlations)	+12.5%	10%
$E_{\rm c.m.} > 2.5  \mathrm{GeV}$		21%
$E_{\rm c.m.} > 3.5  {\rm GeV}$		50%

## G. Overview of the intermediate structures

The  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  process has a rich internal substructure. To study this substructure, we restrict events to  $m(\gamma\gamma) < 0.35~{\rm GeV}/c^2$ , eliminating the region populated by  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$ . We then assume that the  $m(\pi^+\pi^-2\pi^0\gamma\gamma)$  invariant mass can be taken to represent  $m(\pi^+\pi^-3\pi^0)$ .

Figure 12(a) shows the distribution of the  $\pi^0\pi^0\pi^0$  invariant mass. The distribution is seen to exhibit a prominent  $\eta$  peak, which is due to the  $e^+e^- \to \eta\pi^+\pi^-$  reaction. Figure 12(b) presents a scatter plot of the  $\pi^+\pi^-$  vs the  $3\pi^0$  invariant mass. From this plot, the  $\rho(770)\eta$  intermediate state is seen to dominate. Figure 12(c) presents a scatter plot of the  $3\pi^0$  invariant mass versus  $m(\pi^+\pi^-\pi^0\pi^0\gamma^0)$ .

The distribution of the  $\pi^+\pi^-\pi^0$  invariant mass (three entries per event) is shown in 13(a). A prominent  $\omega$  peak from  $e^+e^- \to \omega\pi^0\pi^0$  is seen. Some indications of  $\phi$  and  $J/\psi$  peaks are also present. The scatter plot in Fig. 13(b) shows the  $\pi^0\pi^0$  vs the  $\pi^+\pi^-\pi^0$  invariant mass. A scatter plot of the  $\pi^+\pi^-\pi^0$  vs the  $\pi^+\pi^-\pi^0\pi^0\gamma\gamma$  mass is shown

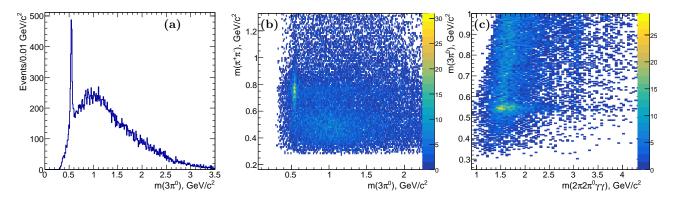


FIG. 12: (a) The  $\pi^0\pi^0\pi^0$  invariant mass. (b) The  $\pi^+\pi^-$  vs the  $\pi^0\pi^0\pi^0$  invariant mass. (c) The  $\pi^0\pi^0\pi^0$  invariant mass vs the five-pion invariant mass.

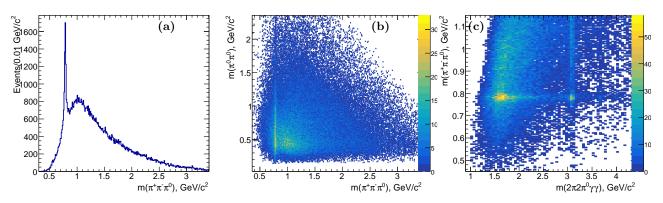


FIG. 13: (a) The  $\pi^+\pi^-\pi^0$  invariant mass (three combinations per event). (b) The  $\pi^0\pi^0$  vs the  $\pi^+\pi^-\pi^0$  invariant mass. (c) The  $\pi^+\pi^-\pi^0$  invariant mass vs the five-pion invariant mass.

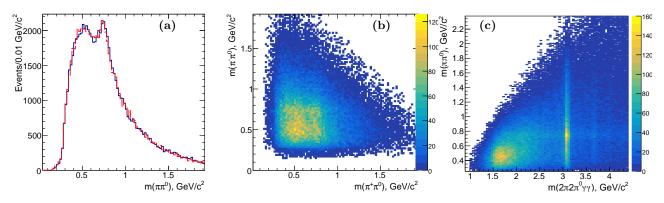


FIG. 14: (a) The  $\pi^+\pi^0$  (solid) and  $\pi^-\pi^0$  (dashed) invariant masses (three combinations per event). (b) The  $\pi^-\pi^0$  vs the  $\pi^+\pi^0$  invariant mass. (c) The  $\pi^\pm\pi^0$  invariant mass vs the five-pion invariant mass.

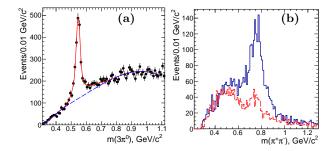


FIG. 15: (a) The  $3\pi^0$  invariant mass for data. The curves show the fit functions. The solid curve shows the  $\eta$  peak (based on MC simulation) plus the non- $\eta$  continuum background (dashed). (b) The  $\pi^+\pi^-$  invariant mass for events selected in the  $\eta$  peak region. The dashed histogram shows the continuum events in the  $\eta$ -peak sidebands.

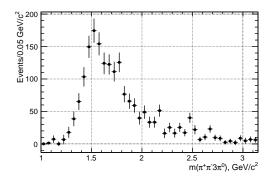


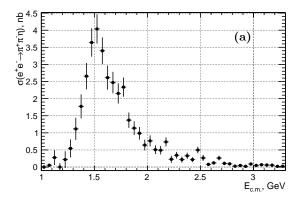
FIG. 16: The  $m(\pi^+\pi^-3\pi^0)$  invariant mass dependence of the selected data events for  $e^+e^- \to \eta\pi^+\pi^-, \eta \to 3\pi^0$ .

in Fig. 13(c). A clear signal for a  $J/\psi$  peak is seen.

Figure 14(a) shows the  $\pi^+\pi^0(\text{dotted})$  and  $\pi^-\pi^0(\text{solid})$  invariant masses (three entries per event). A prominent  $\rho(770)$  peak, corresponding to  $e^+e^- \to 3\pi\rho$ , is visible. The scatter plot in Fig. 14(b) shows the  $\pi^-\pi^0$  vs the  $\pi^+\pi^0$  invariant mass. An indication of the  $\rho^+\rho^-\pi^0$  intermediate state is visible. Figure 14(c) shows the  $\pi\pi^0$  invariant mass vs the five-pion invariant mass: a clear signal for the  $J/\psi$  and an indication of the  $\psi(2S)$  are seen.

# H. The $\eta \pi^+ \pi^-$ intermediate state

To determine the contribution of the  $\eta \pi^+ \pi^-$  intermediate state, we fit the events of Fig. 12(a) using a triple-Gaussian function to describe the signal peak, as in Fig. 6(a), and a polynomial to describe the background. The result of the fit is shown in Fig. 15(a). We obtain  $2102 \pm 112 \ \eta \pi^+ \pi^-$  events. The number of  $\eta \pi^+ \pi^-$  events as a function of the five-pion invariant mass is determined by performing an analogous fit of events in Fig. 12(c) in



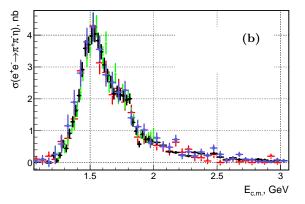


FIG. 17: (a) The energy dependent  $e^+e^- \to \eta \pi^+\pi^-$  cross section obtained in the  $2\pi 3\pi^0$  mode. (b) Comparison of the current results (squares) with previous measurements from BABAR in the  $\eta \to \pi^+\pi^-\pi^0$  (upside-down triangles) [14] and  $\eta \to \gamma\gamma$  modes (circles) [20]. Results from the SND experiment [32] are shown by triangles.

each 0.05 GeV/ $c^2$  interval of  $m(\pi^+\pi^-3\pi^0)$ . The resulting distribution is shown in Fig. 16.

The  $\pi^+\pi^-$  invariant mass distribution for events within  $\pm 0.7~{\rm GeV}/c^2$  of the  $\eta$  peak in Fig. 15(a) is shown in Fig. 15(b). A clear signal from  $\rho(770)$  is observed, supporting the statement that the reaction is dominated by the  $\rho(770)\eta$  intermediate state. The distribution of events from  $\eta$ -peak sidebands is shown by the dashed histogram.

Using Eq. (1), we determine the cross section for the  $e^+e^- \to \eta \pi^+\pi^-$  process. Our simulation takes into account all  $\eta$  decays, so the cross section results, shown in Fig. 17(a) and listed in Table III, correspond to all  $\eta$  decays. Systematic uncertainties in this measurement are the same as those listed in Table II. Figure 17(b) shows our measurement in comparison to our previous results [14, 20] and to those from the SND experiment [32]. These previous results are based on different  $\eta$  decay modes than that considered here. The different results are seen to agree within the uncertainties. Including the results of the present study, we have thus now measured

the  $e^+e^- \to \eta \pi^+\pi^-$  cross section in three different  $\eta$  decay modes.

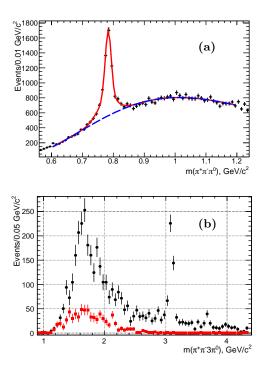


FIG. 18: (a) The  $\pi^+\pi^-\pi^0$  invariant mass for data. The solid curve shows the fit function for signal (based on MC-simulation) plus the combinatorial background (dashed curve). (b) The mass distribution of the  $\pi^+\pi^ 3\pi^0$  events in the  $\omega$  peak (circles) and estimated contribution from the  $\omega\pi^0$  background (squares).

## I. The $\omega \pi^0 \pi^0$ intermediate state

To determine the contribution of the  $\omega \pi^0 \pi^0$  intermediate state, we fit the events of Fig. 13(a) using a BW function to model the signal and a polynomial to model the background. The BW function is convoluted with a Gaussian distribution that accounts for the detector resolution, as described for the fit of Fig. 6(b). The result of the fit is shown in Fig. 18(a). We obtain  $3960 \pm 146$  $\omega \pi^0 \pi^0$  events. The number of the  $\omega \pi^0 \pi^0$  events as a function of the five-pion invariant mass is determined by performing an analogous fit of events in Fig. 13(c) in each 0.05 GeV/ $c^2$  interval of  $m(\pi^+\pi^-3\pi^0)$ . The resulting distribution is shown by the circle symbols in Fig. 18(b). We do not observe a clear  $f_0(980) \to \pi^0 \pi^0$  signal in the  $\pi^0\pi^0$  invariant mass, perhaps because of a large combinatorial background. In contrast, in our previous study of the  $e^+e^- \rightarrow \omega \pi^+\pi^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$  process [14], a clear  $f_0(980) \to \pi^+\pi^-$  signal was seen.

For the  $e^+e^- \to \omega \pi^0 \pi^0$  channel, there is a peaking background from  $e^+e^- \to \omega \pi^0 \to \pi^+\pi^-\pi^0\pi^0$ . A simulation of this reaction with proper normalization leads to

the peaking-background estimation shown by the square symbols in Fig. 18(b). This background is subtracted from the  $\omega \pi^0 \pi^0$  signal candidate distribution.

The  $e^+e^- \to \omega \pi^0 \pi^0$  cross section, corrected for the  $\omega \to \pi^+ \pi^- \pi^0$  branching fraction, is shown in Fig. 19 and tabulated in Table IV. The uncertainties are statistical only. The systematic uncertainties are about 10% for  $E_{\rm c.m.} < 2.4$  GeV, as discussed in Sec. IV F. No previous measurement exists for this process. The cross section exhibits a rise at threshold, a decrease at large  $E_{\rm c.m.}$ , and a clear resonance at around 1.6 GeV, possibly from the  $\omega(1650)$ . The measured  $e^+e^- \to \omega \pi^0 \pi^0$  cross section is around a factor of two smaller than that we observed for  $e^+e^- \to \omega \pi^+ \pi^-$  [14], as is expected from isospin symmetry.

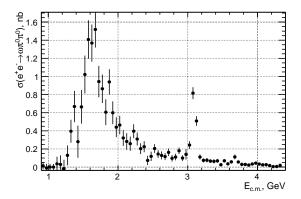


FIG. 19: The energy dependent  $e^+e^- \to \omega\pi^0\pi^0$  cross section in the  $\pi^+\pi^-3\pi^0$  mode.

# J. The $\rho(770)^{\pm}\pi^{\mp}\pi^{0}\pi^{0}$ intermediate state

A similar approach is followed to study events with a  $\rho^\pm$  meson in the intermediate state. Because the  $\rho$  meson is broad, a BW function is used to describe the signal shape. There are six  $\rho^\pm$  entries per event, leading to a large combinatoric background. To extract the contribution of the  $\rho^\pm\pi^\mp\pi^0\pi^0$  intermediate state we fit the events in Fig. 14(a) with a BW function to describe the signal and a polynomial to describe the background. The parameters of the  $\rho$  resonance are taken from Ref. [27]. The result of the fit is shown in Fig. 20(a). We obtain  $14\,894\pm501~\rho^\pm\pi^\mp\pi^0\pi^0$  events. The distribution of these events vs the five-pion invariant mass is shown by the square symbols in Fig. 21(a).

The circle symbols in Fig. 21(a) show the total number of  $\pi^+\pi^-3\pi^0$  events, repeated from Fig. 9. It is seen that the number of events with a  $\rho^{\pm}$  exceeds the total number of  $\pi^+\pi^-3\pi^0$  events, implying that there is more than one  $\rho^{\pm}$  per event, namely a significant production of  $e^+e^- \to \rho^+\rho^-\pi^0$ . To determine the rate of  $\rho^+\rho^-\pi^0$  events, we perform a fit to determine the number of  $\rho^+$  in intervals

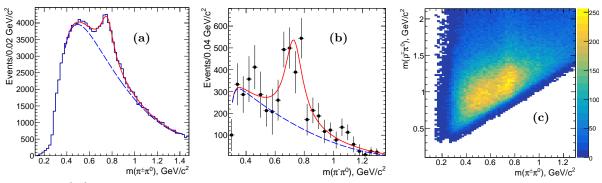


FIG. 20: (a) The  $\pi^{\pm}\pi^{0}$  invariant mass for data. The dashed curve shows the fit to the combinatorial background. The solid curve is the sum of the background curve and the BW function for the  $\rho^{\pm}$ . (b) The result of the  $\rho^{+}$  fit in bins of 0.04 GeV/ $c^{2}$  in the  $\rho^{-}$  mass. (c) Scatter plot of the  $\rho^{\pm}\pi^{0}$  invariant mass vs the  $\pi^{\mp}\pi^{0}$  invariant mass.

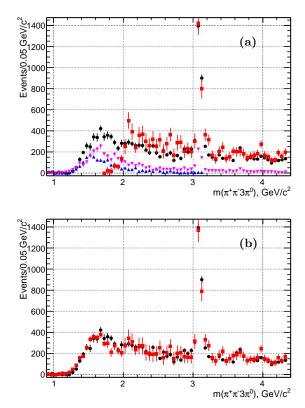


FIG. 21: (a) Number of events in bins of  $E_{\rm c.m.}$  from the  $\eta \pi^+ \pi^-$  (triangles),  $\omega \pi^0 \pi^0$  (upside-down triangles), and  $\rho \to \pi \pi^0$  (squares) intermediate states. The circles show the total event numbers obtained from the fit to the  $\pi^0$  peak. (b) The circles as are described for (a). The squares show the sums of event numbers with  $\eta$ ,  $\omega$  and the  $\rho$  contribution for correlated  $\rho^+ \rho^-$  production.

of 0.04 GeV/ $c^2$  in the  $\pi^-\pi^0$  distribution of Fig. 14(b). The result is shown in Fig. 20(b). Indeed, a significant  $\rho^+$  peak is observed.

The number of  $e^+e^- \to \rho^+\rho^-\pi^0$  events is determined

by fitting the data of Fig. 20(b) with the sum of a BW function and a polynomial. The sample is divided into three mass intervals:  $m(\pi^+\pi^-3\pi^0) < 2.5~{\rm GeV}/c^2$ ,  $2.5 < m(\pi^+\pi^-3\pi^0) < 3.0~{\rm GeV}/c^2$ , and  $m(\pi^+\pi^-3\pi^0) > 3.0~{\rm GeV}/c^2$ . For each mass interval we determine the number of  $\rho^+$  events. We find that the fraction of correlated  $\rho^+\rho^-$  events, relative to the total number of  $\pi^+\pi^-3\pi^0$  events with a  $\rho^\pm$ , decreases with the mass interval as  $0.49\pm0.05,\,0.37\pm0.07,\,{\rm and}\,0.23\pm0.10,\,{\rm respectively},\,{\rm where}$  the uncertainties are statistical. Thus, the  $\rho^+\rho^-\pi^0$  intermediate state dominates at threshold.

Intermediate states with either one or two  $\rho(770)$  are expected to be produced, at least in part, through  $e^+e^- \to \rho(1400,1700)^0\pi^0 \to a_1(1260)^\pm\pi^\mp\pi^0 \to \rho^\pm\pi^\mp\pi^0\pi^0$  and  $e^+e^- \to \rho^\pm a_1^\mp \to \rho^+\rho^-\pi^0$ , respectively. Figure 20(c) shows a scatter plot of the  $\rho^\pm\pi^0$  invariant mass vs the  $\pi^\mp\pi^0$  invariant mass. An indication of the  $a_1(1260)$  is seen, but it is not statistically significant.

## K. The sum of intermediate states

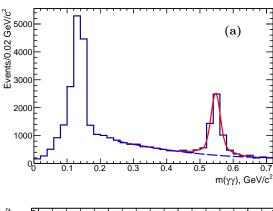
Figure 21(a) shows the number of  $\eta \pi^+ \pi^-$  (upsidedown triangles),  $\omega \pi^0 \pi^0$  (triangles), and  $\rho^{\pm} \pi^{\mp} \pi^0 \pi^0$ (square) intermediate state events, found as described in the previous sections, in comparison to the total number of  $\pi^+\pi^-3\pi^0$  events (circles) found from the fit to the  $\pi^0$  mass peak. The results for the  $\eta$  and  $\omega$  are repeated from Figs. 16 and 18, respectively. As noted above, a significant excess of events with a  $\rho$  is observed. Based on the results of our study of correlated  $\rho^+\rho^-$  production, we scale the number of events found from the fit to the rho peak so that it corresponds to the number of events with either a single  $\rho^{\pm}$  or with a  $\rho^{+}\rho^{-}$  pair. We then sum this latter result with the eta and omega curves in Fig. 21(a). The result of this sum is shown by the square symbols in Fig. 21(b). This summed curve is seen to be in agreement with the total number of  $\pi^+\pi^-3\pi^0$  events, shown by the circular symbols.

Note that below  $E_{\rm c.m.}=2$  GeV, the number of events is completely dominated by the  $\eta\pi^+\pi^-$  and  $\omega\pi^0\pi^0$  channels, so the cross section of the intermediate states with a  $\rho$  can be estimated as the difference between the total  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  cross section and the sum of the  $\eta\pi^+\pi^-$  and  $\omega\pi^0\pi^0$  contributions.

## V. THE $\pi^+\pi^-2\pi^0\eta$ FINAL STATE

## A. Determination of the number of events

The analogous approach to that described above for  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0$  events is used to study  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$  events. We fit the  $\eta$  signal in the third-photon-pair invariant mass distribution (cf., Fig. 1) with the sum of two Gaussians with a common mean, while the relatively smooth background is described by a second-order polynomial function, as shown in Fig. 22(a). We obtain  $4700 \pm 84$  events. Figure 22(b) shows the mass distribution of these events.



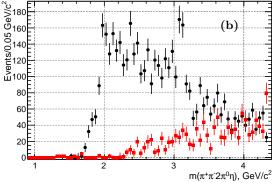


FIG. 22: (a) The third-photon-pair invariant mass for data. The dashed curve shows the fitted background. The solid curve shows the sum of background and the two-Gaussian fit function used to obtain the number of events with an  $\eta$ . (b) The invariant mass distribution for the  $\pi^+\pi^-2\pi^0\eta$  events obtained from the  $\eta$  signal fit. The contribution of the uds background events is shown by the squares.

## B. Peaking background

The major background producing an  $\eta$  peak is the non-ISR background, in particular  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0\eta$  when one of the neutral pions decays asymmetrically, producing a photon interpreted as ISR. The  $\eta$  peak from the uds simulation is visible in Fig. 10.

To normalize the uds simulation, we form the diphoton invariant mass distribution of the ISR candidate with all the remaining photons in the event. Comparing the number of events in the  $\pi^0$  peaks in data and uds simulation, we assign a scale factor of  $1.5 \pm 0.2$  to the simulation. We fit the  $\eta$  peak in the uds simulation in intervals of  $0.05 \text{ GeV}/c^2$  in  $m(\pi^+\pi^-\pi^0\pi^0\gamma^0)$ . The results are shown by the squares in Fig. 22 (b).

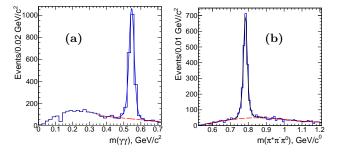


FIG. 23: (a) The third-photon-pair invariant mass for simulation of the  $e^+e^-\to\pi^+\pi^-\pi^0\pi^0\eta\gamma$  process. The dashed curve shows the fitted background. The solid curve shows the sum of background and the two-Gaussian fit function used to obtain the number of events with an  $\eta$ . (b) The  $\pi^+\pi^-\pi^0$  invariant mass for simulation. The solid curve shows a two-Gaussian fit function for the  $\omega$  signal plus the combinatorial background (dashed).

## C. Detection efficiency

We use simulated  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta\gamma$  events from the phase space model and with the  $\omega\pi^0\eta$  intermediate state to determine the efficiency. As for the data, we fit to find the  $\eta$  signal in the third photon pair in intervals of 0.05 GeV/ $c^2$  in  $m(\pi^+\pi^-\pi^0\pi^0\gamma\gamma)$ . The fit is illustrated in Fig. 23(a) using all  $\pi^+\pi^-\pi^0\pi^0\gamma\gamma$  candidates. The efficiency is determined as the ratio of the number of fitted events in each interval to the number generated in that interval. For the  $\omega\pi^0\eta$  intermediate channel, we also determine the efficiency using an alternative method, by fitting the  $\omega$  peak in the  $\pi^+\pi^-\pi^0$  invariant mass distribution, shown in Fig. 23(b).

The efficiencies obtained for the three methods are shown in Fig. 24. The circles and squares show the results from the fit to the  $\eta$  peak for the phase space and  $\omega \pi^0 \eta$  channels, respectively. The triangles show the results for the fit to the  $\omega$  peak. The efficiencies are calculated assuming the  $\eta \to \gamma \gamma$  mode only. The obtained efficiencies

are around 4%, similar to what is found for  $\pi^+\pi^-3\pi^0$  (Fig. 7). The results from the three methods are consistent with each other, and are averaged. The average is fit with a third-order polynomial, shown by the curve in Fig. 24. The result of the fit is used for the cross section determination.

We estimate the systematic uncertainty in the efficiency due to the fit procedure and the model dependence to be not more than 10%.

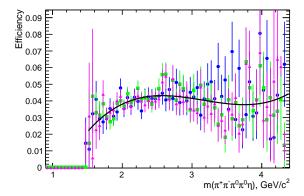


FIG. 24: The energy dependent detection efficiency, determined in three different ways: see text. The curve shows the fit to the average of the three and is used in the cross section determination.

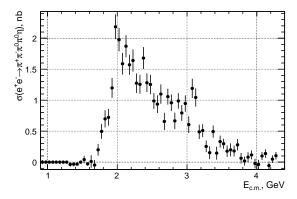


FIG. 25: Energy dependent cross section for  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$ . The uncertainties are statistical only.

# D. Cross section for $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$

The cross section for  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$  is determined using Eq. (1). The results are shown in Fig. 25 and listed in Table V. These are the first results for this process. The systematic uncertainties and corrections are the same as those presented in Table II except there is an increase in the uncertainty in the detection efficiency. The total systematic uncertainty for  $E_{\rm c.m.} < 2.5 \; {\rm GeV}$  is 13%.

## E. Overview of the intermediate structures

The  $\pi^+\pi^-2\pi^0\eta$  final state, like that for  $\pi^+\pi^-3\pi^0$ , has a rich substructure. Figure 26(a) shows the  $2\pi^0\eta$  invariant mass distribution for events selected by requiring  $|m(\gamma\gamma)-m(\eta)|<0.07$  GeV/ $c^2$  in Fig. 22(a). There is a small but clear signal for  $\eta(1285)$  production. The dotted histogram shows the background distribution, determined using an  $\eta$  sideband control region defined by  $0.07<|m(\gamma\gamma)-m(\eta)|<0.14$  GeV/ $c^2$ . Figure 26(b) shows a scatter plot of the  $\pi^+\pi^-$  invariant mass vs the  $2\pi^0\eta$  invariant mass. No structures are seen.

Figure 27(a) shows the  $\pi^+\pi^-\pi^0$  mass distribution (two entries per event). An  $\omega$  signal is clearly visible, as well as a bump close to 1 GeV/ $c^2$  corresponding to  $\phi \to \pi^+\pi^-\pi^0$ . The dotted histogram shows the estimate of the background, evaluated using the  $\eta$  sideband described above. The scatter plot in Fig. 27(b) shows the  $\pi^0\eta$  vs the  $\pi^+\pi^-\pi^0$  invariant mass. A clear correlation of  $\omega$  and  $a_0(980) \to \pi^0\eta$  production is seen. Figure 27(c) shows how  $\omega\pi^0\eta$  events are distributed over the  $\pi^+\pi^-2\pi^0\eta$  invariant mass.

Figure 28(a) presents the  $\pi^+\pi^0$  (solid) and  $\pi^-\pi^0$  (dotted) mass combinations (two entries per event) for the selected  $\pi^+\pi^-2\pi^0\eta$  events. Signals from the  $\rho^\pm$  are clearly visible, but they can also come from events with a  $\rho^+\rho^-$  pair. The fraction of  $\rho^+\rho^-$  events is extracted from the distribution in Fig. 28(b), where the  $\pi^+\pi^0$  vs the  $\pi^-\pi^0$  invariant mass is shown. Figure 28(c) displays the  $\pi^\pm\pi^0$  vs the  $\pi^+\pi^-2\pi^0\eta$  invariant mass.

# F. The $\omega \pi^0 \eta$ and $\phi \pi^0 \eta$ intermediate states

To determine the contribution of the  $\omega \pi^0 \eta$  and  $\phi \pi^0 \eta$  intermediate states, we fit the events in Fig. 27(a) with two Gaussian functions, one to describe the  $\omega$  peak and the other the  $\phi$  peak, and a polynomial function, which describes the background. The results of the fit are shown in Fig. 29(a). We obtain  $1676 \pm 22$  and  $269 \pm 68$  events for the  $\omega$  and  $\phi$ , respectively. The number of events as a function of the  $\pi^+\pi^-2\pi^0\eta$  invariant mass is determined by performing an analogous fit of events in Fig. 27(c) in intervals of 0.05 GeV/ $c^2$  in  $m(\pi^+\pi^-2\pi^0\eta)$ .

We select events within  $\pm 0.7~{\rm GeV}/c^2$  of the  $\omega$  peak in Fig. 29(a) and display the resulting  $\pi^0\eta$  invariant mass in Fig. 29(b). A very clear signal from the  $a_0(980)$  is observed, while no signal is seen in an  $\omega$  sideband defined by  $0.07 < |m(\pi^+\pi^-\pi^0) - m(\omega)| < 0.14~{\rm GeV}/c^2$ .

The obtained  $e^+e^- \to \omega \pi^0 \eta$  cross section, corrected for the  $\omega \to \pi^+\pi^-\pi^0$  branching fraction, is shown in Fig. 30 in comparison to previous results from SND [31]. The SND results, which are available only for energies below 2 GeV, are seen to lie systematically above our data. All systematic uncertainties discussed in section IV F are applied to the measured  $e^+e^- \to \omega \pi^0 \eta$  cross section, resulting in a total systematic uncertainty of 13% below 2.4 GeV. The results are presented in Table VI (statistical

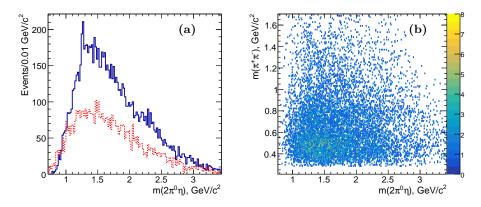


FIG. 26: (a) The  $2\pi^0\eta$  invariant mass of the selected  $\pi^+\pi^-2\pi^0\eta$  events (solid histogram), and the background determined from the  $\chi^2$  sideband (dotted histogram). (b) The  $\pi^+\pi^-$  vs the  $2\pi^0\eta$  mass for the selected events.

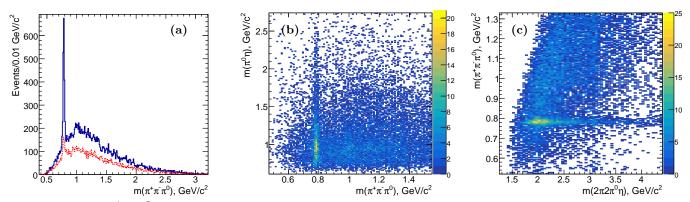


FIG. 27: (a) The  $\pi^+\pi^-\pi^0$  invariant mass with two entries per event (solid histogram) and the background estimate from the  $\eta$  sideband (dotted histogram). (b) The  $\pi^0\eta$  vs the  $\pi^+\pi^-\pi^0$  invariant mass. (c) The  $\pi^+\pi^-\pi^0$  invariant mass vs the  $\pi^+\pi^-2\pi^0\eta$  invariant mass.

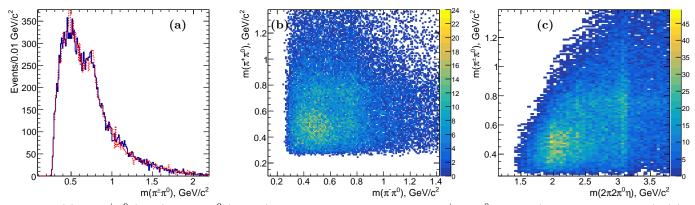


FIG. 28: (a) the  $\pi^+\pi^0$  (solid) and  $\pi^-\pi^0$  (dotted) invariant mass for the selected  $\pi^+\pi^-2\pi^0\eta$  events (two entries per event). (b) The  $\pi^-\pi^0$  vs the  $\pi^+\pi^0$  invariant mass for the selected events. (c) The  $\pi^\pm\pi^0$  invariant mass vs the  $\pi^+\pi^-2\pi^0\eta$  invariant mass.

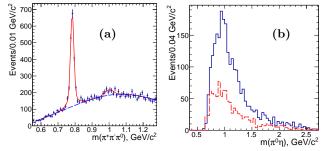


FIG. 29: (a) The  $\pi^+\pi^-\pi^0$  invariant mass for data. The dashed curve describes the non-resonant background. The solid curve shows the sum of the background and the fit functions for the  $\omega$  and  $\phi$  contributions, described in the text. (b) The  $\pi^0\eta$  invariant mass distribution for the events selected in the  $\omega$  peak (solid). The dashed histogram shows the distribution from the  $\omega$ -peak side band.

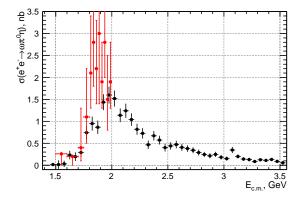


FIG. 30: The  $E_{\text{c.m.}}$  dependence of the  $e^+e^- \to \omega \pi^0 \eta$  cross section (circles) in comparison with the SND results [31] (squares).

uncertainties only) in bin widths of 0.05 GeV. Above 3.5 GeV, the cross section measurements are consistent with zero within the experimental accuracy.

# G. The $\rho(770)^{\pm}\pi^{\mp}\pi^{0}\eta$ intermediate state

The approach described in Sec. IV J is used to study events with a  $\rho^{\pm}$  meson in the intermediate state. We fit the events in Fig. 28(a) using a BW function to describe the  $\rho$  signal and a polynomial function to describe the background (four entries per event). The fit yields  $2908\pm202~\rho^{\pm}\pi^{\mp}\pi^{0}\eta$  events. The result of the fit is shown in Fig. 31(a). The distribution of these events vs the  $\pi^{+}\pi^{-}2\pi^{0}\eta$  invariant mass is shown by the squares in Fig. 32.

The size of our data sample is not sufficient to justify a sophisticated amplitude analysis, as would be needed to extract detailed information on all the intermediate

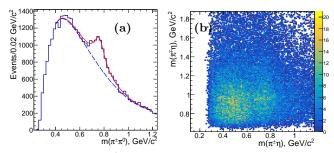


FIG. 31: (a) The  $\pi^{\pm}\pi^{0}$  invariant mass for data. The curves show the fit functions, described in the text. (b) The  $\pi^{\pm}\eta$  vs the  $\pi^{\mp}\pi^{0}$  invariant mass.

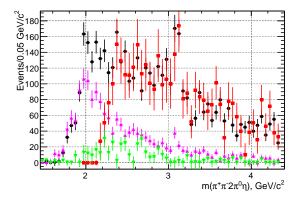


FIG. 32: Number of events in bins of  $E_{\rm c.m.}$  for inclusive  $\pi^+\pi^-2\pi^0\eta$  events (circles) and for the  $\omega\pi^0\eta$  (triangles),  $\phi\pi^0\eta$  (upside-down triangles), and  $\rho^\pm\pi^\mp\pi^0\eta$  (squares) intermediate states.

states. We can deduce that an intermediate  $a_0(980)\rho\pi$  state is present: a correlated bump at the  $a_0(980)$  and  $\rho$  invariant masses is seen in the scatter plot of Fig. 31(b), where the  $\pi^{\pm}\eta$  invariant mass is plotted vs the  $\pi^{\mp}\pi^0$  mass. Also, there is a contribution from  $\rho^+\rho^-\eta$ : a scatter plot of the  $\pi^{\pm}\pi^0$  vs the  $\pi^{\mp}\pi^0$  invariant mass is presented in Fig. 28(b), from which an enhancement corresponding to correlated  $\rho^+\rho^-$  production is visible.

## H. The sum of intermediate states

Figure 32 displays the number of events obtained from the fits described above to the  $\omega$  (triangles),  $\phi$  (upsidedown triangles), and  $\rho$  (square) peaks. The results are shown in comparison to the total number of  $\pi^+\pi^-2\pi^0\eta$  events (circles) obtained from the fit to the third photon pair invariant mass distribution. The sum of events from the intermediate states is seen to agree within the uncertainties with the total number of  $\pi^+\pi^-2\pi^0\eta$  events, except in the region around 2 GeV.

TABLE III: Summary of the  $e^+e^- \to \eta \pi^+\pi^-$  cross section measurement. The uncertainties are statistical only.

$E_{\mathrm{c.m.}}$	, GeV	$\sigma$ , nb	$E_{\rm c.m.},{\rm GeV}$	$\sigma$ , nb						
1.	075	$0.06 \pm 0.03$	1.475	$3.74 \pm 0.43$	1.875	$1.16 \pm 0.21$	2.275	$0.35 \pm 0.10$	2.675	$0.27 \pm 0.07$
1.	125	$0.29\pm0.23$	1.525	$4.14\pm0.44$	1.925	$1.00 \pm 0.19$	2.325	$0.22\pm0.09$	2.725	$0.11 \pm 0.05$
1.	175	$0.00\pm0.12$	1.575	$3.48\pm0.40$	1.975	$0.65\pm0.16$	2.375	$0.33\pm0.09$	2.775	$0.09 \pm 0.05$
1.	225	$0.23\pm0.25$	1.625	$2.67\pm0.36$	2.025	$0.78\pm0.16$	2.425	$0.22\pm0.07$	2.825	$0.03 \pm 0.04$
1.	275	$0.57\pm0.27$	1.675	$2.52\pm0.32$	2.075	$0.51\pm0.13$	2.475	$0.51\pm0.10$	2.875	$0.05 \pm 0.04$
1.	325	$1.15\pm0.34$	1.725	$2.20\pm0.30$	2.125	$0.50\pm0.13$	2.525	$0.27\pm0.09$	2.925	$0.02 \pm 0.04$
1.	375	$1.83 \pm 0.36$	1.775	$2.38\pm0.29$	2.175	$0.75\pm0.13$	2.575	$0.08\pm0.05$	2.975	$0.09 \pm 0.05$
1.	425	$2.74 \pm 0.40$	1.825	$1.39\pm0.23$	2.225	$0.23\pm0.11$	2.625	$0.12\pm0.06$	3.025	$0.05\pm0.05$

TABLE IV: Summary of the  $e^+e^- \to \omega \pi^0 \pi^0$  cross section measurement. The uncertainties are statistical only.

$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb								
1.125	$0.04 \pm 0.08$	1.775	$0.88 \pm 0.16$	2.425	$0.07 \pm 0.05$	3.075	$0.83 \pm 0.07$	3.725	$0.06 \pm 0.02$
1.175	$0.03 \pm 0.10$	1.825	$0.62 \pm 0.14$	2.475	$0.12\pm0.05$	3.125	$0.52 \pm 0.05$	3.775	$0.03 \pm 0.02$
1.225	$-0.02\pm0.10$	1.875	$0.96 \pm 0.14$	2.525	$0.21\pm0.05$	3.175	$0.11 \pm 0.03$	3.825	$0.03 \pm 0.01$
1.275	$0.13 \pm 0.11$	1.925	$0.61 \pm 0.13$	2.575	$0.15\pm0.04$	3.225	$0.08 \pm 0.02$	3.875	$0.02\pm0.01$
1.325	$0.41 \pm 0.13$	1.975	$0.45\pm0.11$	2.625	$0.13\pm0.04$	3.275	$0.08 \pm 0.02$	3.925	$0.03\pm0.02$
1.375	$0.69 \pm 0.18$	2.025	$0.47\pm0.10$	2.675	$0.12\pm0.04$	3.325	$0.07 \pm 0.02$	3.975	$0.04\pm0.01$
1.425	$0.29 \pm 0.18$	2.075	$0.33 \pm 0.09$	2.725	$0.17\pm0.04$	3.375	$0.06 \pm 0.02$	4.025	$0.03\pm0.01$
1.475	$0.68 \pm 0.19$	2.125	$0.29 \pm 0.09$	2.775	$0.10\pm0.04$	3.425	$0.07 \pm 0.02$	4.075	$0.02\pm0.01$
1.525	$1.05 \pm 0.21$	2.175	$0.26 \pm 0.08$	2.825	$0.11\pm0.04$	3.475	$0.03 \pm 0.02$	4.125	$0.03 \pm 0.01$
1.575	$1.44 \pm 0.22$	2.225	$0.40 \pm 0.08$	2.875	$0.18\pm0.04$	3.525	$0.07 \pm 0.02$	4.175	$0.02\pm0.01$
1.625	$1.40 \pm 0.21$	2.275	$0.31 \pm 0.07$	2.925	$0.10\pm0.03$	3.575	$0.04 \pm 0.02$	4.225	$0.01\pm0.01$
1.675	$1.55 \pm 0.20$	2.325	$0.21 \pm 0.06$	2.975	$0.14\pm0.06$	3.625	$0.06\pm0.02$	4.275	$0.01\pm0.01$
1.725	$0.96 \pm 0.18$	2.375	$0.23 \pm 0.06$	3.025	$0.25 \pm 0.04$	3.675	$0.11 \pm 0.03$	4.325	$0.02 \pm 0.01$

TABLE V: Summary of the  $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta$  cross section measurement. The uncertainties are statistical only.

$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb	$E_{\text{c.m.}},  \text{GeV}$	$\sigma$ , nb	$E_{\rm c.m.},{\rm GeV}$	$\sigma$ , nb
1.625	$0.01 \pm 0.10$	2.175	$1.59 \pm 0.16$	2.725	$1.07 \pm 0.13$	3.275	$0.26 \pm 0.09$	3.825	$0.02 \pm 0.07$
1.675	$-0.05\pm0.08$	2.225	$1.66\pm0.18$	2.775	$0.97\pm0.14$	3.325	$0.15\pm0.11$	3.875	$0.08 \pm 0.08$
1.725	$0.20 \pm 0.10$	2.275	$1.29 \pm 0.16$	2.825	$0.68\pm0.14$	3.375	$0.50\pm0.10$	3.925	$0.12 \pm 0.07$
1.775	$0.51 \pm 0.12$	2.325	$1.27\pm0.15$	2.875	$1.00 \pm 0.13$	3.425	$0.15\pm0.11$	3.975	$-0.02 \pm 0.08$
1.825	$0.71 \pm 0.14$	2.375	$1.70 \pm 0.18$	2.925	$0.81\pm0.13$	3.475	$0.34\pm0.10$	4.025	$-0.04 \pm 0.08$
1.875	$0.73 \pm 0.14$	2.425	$1.30 \pm 0.15$	2.975	$0.96 \pm 0.13$	3.525	$0.30\pm0.08$	4.075	$0.10 \pm 0.06$
1.925	$1.22 \pm 0.16$	2.475	$1.27 \pm 0.16$	3.025	$0.61 \pm 0.14$	3.575	$0.18\pm0.09$	4.125	$0.14 \pm 0.07$
1.975	$2.22 \pm 0.20$	2.525	$1.00\pm0.13$	3.075	$1.21 \pm 0.16$	3.625	$0.20\pm0.11$	4.175	$-0.06 \pm 0.07$
2.025	$2.01 \pm 0.19$	2.575	$0.95\pm0.15$	3.125	$1.06 \pm 0.15$	3.675	$0.18\pm0.09$	4.225	$0.05 \pm 0.06$
2.075	$1.61 \pm 0.18$	2.625	$1.11\pm0.16$	3.175	$0.50\pm0.12$	3.725	$0.28\pm0.09$	4.275	$0.10 \pm 0.06$
2.125	$1.90 \pm 0.18$	2.675	$0.67\pm0.14$	3.225	$0.52\pm0.11$	3.775	$0.06\pm0.09$	4.325	$0.04 \pm 0.06$

TABLE VI: Summary of the  $e^+e^- \to \omega\pi^0\eta$  cross section measurement. The uncertainties are statistical only.

$E_{\rm c.m.},  {\rm GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{ GeV}$	$\sigma$ , nb	$E_{\text{c.m.}}, \text{GeV}$	$\sigma$ , nb
1.525	$0.02 \pm 0.10$	2.125	$1.26 \pm 0.17$	2.725	$0.35 \pm 0.07$	3.325	$0.13 \pm 0.04$	3.925	$0.08 \pm 0.03$
1.575	$0.03 \pm 0.07$	2.175	$1.06 \pm 0.14$	2.775	$0.29 \pm 0.07$	3.375	$0.11\pm0.03$	3.975	$0.00 \pm 0.03$
1.625	$0.24 \pm 0.10$	2.225	$0.83 \pm 0.13$	2.825	$0.25\pm0.06$	3.425	$0.13\pm0.04$	4.025	$0.05 \pm 0.02$
1.675	$0.20 \pm 0.10$	2.275	$0.74\pm0.12$	2.875	$0.22\pm0.06$	3.475	$0.09\pm0.03$	4.075	$0.00 \pm 0.03$
1.725	$0.30 \pm 0.11$	2.325	$0.47\pm0.10$	2.925	$0.25\pm0.06$	3.525	$0.06\pm0.03$	4.125	$0.04\pm0.02$
1.775	$0.76 \pm 0.15$	2.375	$0.68\pm0.11$	2.975	$0.18\pm0.05$	3.575	$0.10\pm0.03$	4.175	$0.03 \pm 0.02$
1.825	$0.96 \pm 0.16$	2.425	$0.58\pm0.10$	3.025	$0.15\pm0.05$	3.625	$0.02\pm0.02$	4.225	$0.03 \pm 0.02$
1.875	$0.88\pm0.16$	2.475	$0.41 \pm 0.09$	3.075	$0.35\pm0.07$	3.675	$0.06\pm0.03$	4.275	$0.00 \pm 0.03$
1.925	$1.46 \pm 0.18$	2.525	$0.45 \pm 0.09$	3.125	$0.20\pm0.05$	3.725	$0.05\pm0.03$	4.325	$0.02 \pm 0.01$
1.975	$1.62 \pm 0.20$	2.575	$0.48 \pm 0.09$	3.175	$0.14\pm0.04$	3.775	$0.08\pm0.02$		
2.025	$1.54 \pm 0.19$	2.625	$0.41\pm0.08$	3.225	$0.13\pm0.04$	3.825	$0.04\pm0.03$		
2.075	$1.16 \pm 0.16$	2.675	$0.39 \pm 0.08$	3.275	$0.09 \pm 0.03$	3.875	$0.07 \pm 0.02$		

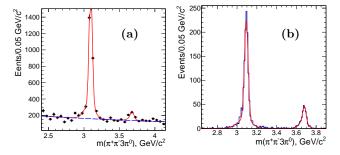


FIG. 33: (a) The  $\pi^+\pi^-3\pi^0$  mass distribution for ISR-produced  $e^+e^-\to\pi^+\pi^-\pi^0\pi^0$  events in the  $J/\psi$ – $\psi(2S)$  region. (b) The MC-simulated signals. The curves show the fit functions described in the text.

## VI. THE $J/\psi$ REGION

# A. The $\pi^+\pi^-3\pi^0$ final state

Figure 33(a) shows an expanded view of the  $J/\psi$  mass region from Fig. 9 for the five-pion data sample. Signals from  $J/\psi \to \pi^+\pi^-\pi^0\pi^0\pi^0$  and  $\psi(2S) \to \pi^+\pi^-\pi^0\pi^0\pi^0$  are clearly seen. The non-resonant background distribution is flat in this region.

The observed peak shapes are not purely Gaussian because of radiation effects and resolution, as is also seen in the simulated signal distributions shown in Fig. 33(b). The sum of two Gaussians with a common mean is used to describe them. We obtain  $2389 \pm 63~J/\psi$  events and  $177 \pm 27~\psi(2S)$  events. Using the results for the number of events, the detection efficiency, and the ISR luminosity, we determine the product:

$$B_{J/\psi \to 5\pi} \cdot \Gamma_{ee}^{J/\psi} = \frac{N(J/\psi \to \pi^+ \pi^- 3\pi^0) \cdot m_{J/\psi}^2}{6\pi^2 \cdot d\mathcal{L}/dE \cdot \epsilon^{\text{MC}} \cdot \epsilon^{\text{corr}} \cdot C}$$
(2)  
=  $(150 \pm 4 \pm 15) \text{ eV}$ ,

where  $\Gamma_{ee}^{J/\psi}$  is the electronic width,  $d\mathcal{L}/dE=180~\mathrm{nb}^{-1}/\mathrm{MeV}$  is the ISR luminosity at the  $J/\psi$  mass  $m_{J/\psi},~\epsilon^{\mathrm{MC}}=0.041$  is the detection efficiency from simulation with the corrections  $\epsilon^{corr}=0.88$ , discussed in Sec. IV F, and  $C=3.894\times10^{11}~\mathrm{nb\,MeV^2}$  is a conversion constant [27]. We estimate the systematic uncertainty for this region to be 10%, because no background subtraction is needed. The subscript "5 $\pi$ " for the branching fraction refers to the  $\pi^+\pi^-3\pi^0$  final state exclusively.

Using  $\Gamma_{ee}^{J/\psi}=5.55\pm0.14$  keV [27], we obtain  $B_{J/\psi\to 5\pi}=(2.70\pm0.07\pm0.27)\times 10^{-2}$ : no other measurements for this channel exist.

Using Eq.(2) and the result  $d\mathcal{L}/dE = 228 \text{ nb}^{-1}/\text{MeV}$  at the  $\psi(2S)$  mass, we obtain:

$$B_{\psi(2S)\to 5\pi} \cdot \Gamma^{\psi(2S)}_{ee} \ = \ (12.4 \pm 1.9 \pm 1.2) \ \text{eV} \ .$$

With  $\Gamma_{ee}^{\psi(2S)} = 2.34 \pm 0.06$  keV [27] we find  $B_{\psi(2S)\to 5\pi} = (5.2 \pm 0.8 \pm 0.5) \times 10^{-3}$ . For this channel also, no previous result exists.

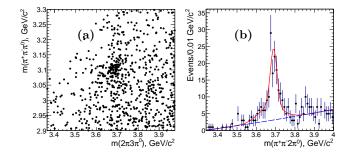


FIG. 34: (a) The three-pion combination closest to the  $J/\psi$  mass vs the five-pion mass. (b) The five-pion mass for the events with the three-pion mass in the  $\pm 50~{\rm MeV}/c^2$  interval around the  $J/\psi$  mass. The curves show the fit functions for all events (solid) and the contribution of the background (dashed).

The  $\psi(2S)$  peak partly corresponds to the decay chain  $\psi(2S) \to J/\psi \pi^0 \pi^0 \to \pi^+ \pi^- \pi^0 \pi^0 \pi^0$ , with  $J/\psi$  decay to three pions. We select the  $\pi^+ \pi^- \pi^0$  mass combination closest to the  $J/\psi$  mass. Figure 34(a) displays this  $\pi^+ \pi^- \pi^0$  mass vs the five-pion invariant mass. A clear signal from the above decay chain is seen. We select events in a  $\pm 0.05$  GeV/ $c^2$  window around the  $J/\psi$  mass and project the results onto  $m(\pi^+ \pi^- 3\pi^0)$ . The results are shown in Fig. 34(b). Performing a fit to this distribution yields  $142 \pm 21$   $\psi(2S) \to J/\psi \pi^0 \pi^0 \to \pi^+ \pi^- \pi^0 \pi^0 \pi^0$  events. In conjunction with the detection efficiency and ISR luminosity, this yields:

$$B_{\psi(2S)\to J/\psi\pi^0\pi^0} \cdot B_{J/\psi\to\pi^+\pi^-\pi^0} \cdot \Gamma_{ee}^{\psi(2S)} = (10.1 \pm 1.5 \pm 1.1) \text{ eV} .$$

With  $\Gamma_{ee}^{\psi(2S)}$  as stated above and  $B_{\psi(2S)\to J/\psi\pi^0\pi^0} = 0.1817 \pm 0.0031$  [27], we obtain  $B_{J/\psi\to\pi^+\pi^-\pi^0} = (2.29 \pm 0.28 \pm 0.23)\%$ , in agreement with our direct measurement  $B_{J/\psi\to\pi^+\pi^-\pi^0} = (2.18 \pm 0.19)\%$  [13] as well as with the PDG value  $B_{J/\psi\to\pi^+\pi^-\pi^0} = (2.11 \pm 0.07)\%$ . This gives us confidence that our normalization procedure is correct.

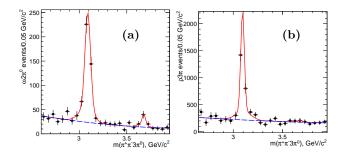


FIG. 35: (a) The five-pion mass for events with the three-pion combination in the  $\omega(782)$  mass region. (b) The five-pion mass for events with  $\pi^{\pm}\pi^{0}$  combination in the  $\rho(770)$  mass region. The curves show the fit functions described in the text.

## 1. The $\omega \pi^0 \pi^0$ intermediate state

The  $J/\psi \to \eta \pi^+ \pi^-$  branching fraction is very small, as we observed in our previous publication [20], and there is not a statistically significant signal in our sample, shown in Fig. 16. We do not attempt to extract a  $J/\psi$  branching fraction for this channel.

Figure 35(a) shows an expanded view of Fig. 18 with the  $\pi^+\pi^-3\pi^0$  mass distribution for events obtained by a fit to the  $\pi^+\pi^-\pi^0$  mass distribution. The two-Gaussian fit, implemented as discribed above, yields 398  $\pm$  29 and  $33\pm10$  events for the  $J/\psi$  and  $\psi(2S)$ , respectively. Using Eq.(2) we obtain:

$$\begin{array}{rl} B_{J/\psi\to\omega\pi^0\pi^0} \cdot B_{\omega\to\pi^+\pi^-\pi^0} \cdot \Gamma_{ee}^{J/\psi} &= \\ (24.9 \pm 1.8 \pm 2.5) \, \text{eV} \; , \\ B_{\psi(2S)\to\omega\pi^+\pi^-} \cdot B_{\omega\to\pi^+\pi^-\pi^0} \cdot \Gamma_{ee}^{\psi(2S)} &= \\ (2.3 \pm 0.7 \pm 0.2) \, \text{eV} \; . \end{array}$$

Using  $B_{\omega\to\pi^+\pi^-\pi^0}=0.891$  and the value of  $\Gamma_{ee}$  from Ref. [27], we obtain  $B_{J/\psi\to\omega\pi^0\pi^0}=(5.04\pm0.37\pm0.50)\times 10^{-3}$  and  $B_{\psi(2S)\to\omega\pi^0\pi^0}=(1.1\pm0.3\pm0.1)\times 10^{-3}$ . The value of  $B_{J/\psi\to\omega\pi^0\pi^0}$  listed in Ref. [27], based on the DM2 [33] result, is  $(3.4\pm0.8)\times 10^{-3}$ . There is no previous result for  $B_{\psi(2S)\to\omega\pi^0\pi^0}$ . Note that our result for  $B_{J/\psi\to\omega\pi^0\pi^0}$  is about a factor of two lower than our result  $B_{J/\psi\to\omega\pi^+\pi^-}=(9.7\pm0.9)\times 10^{-3}$  [14], as expected from isospin symmetry.

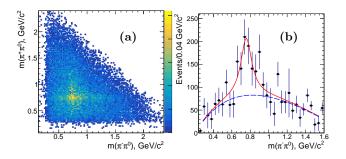


FIG. 36: (a) Scatter plot of the  $\pi^+\pi^0$  vs the  $\pi^-\pi^0$  invariant mass for the  $J/\psi$  region in Fig. 35(b). (b) Number of  $\pi^+\pi^0$  events in bins of 0.04 GeV/ $c^2$  in the  $\pi^-\pi^0$  mass. The curves show the fit functions for all events (solid) and the contribution of the background (dashed).

# 2. The $\rho^{\pm}\pi^{\mp}\pi^{0}\pi^{0}$ intermediate state

Figure 35(b) shows an expanded view of Fig. 21(a) (squares) for the  $\pi^+\pi^-3\pi^0$  mass, for events obtained from the fit to the  $\rho$  signal in the  $\pi^{\pm}\pi^0$  mass. The two-Gaussian fit yields 2299  $\pm$  201 and < 88 events at 90% C.L. for the  $J/\psi$  and  $\psi(2S)$ , respectively.

The obtained  $J/\psi \to \rho^{\pm}\pi^{\mp}\pi^{0}\pi^{0}$  result exceeds the total number of observed  $J/\psi$  events. This is because

of  $J/\psi$  decays to  $\rho^+\rho^-\pi^0$ . Figure 36(a) shows a scatter plot of the  $\pi^+\pi^0$  vs the  $\pi^-\pi^0$  invariant mass for 3051 events in a  $\pm 0.1$  GeV/ $c^2$  interval around the  $J/\psi$  peak of Fig. 35(b). To determine the rate of correlated  $\rho^+\rho^-$  production, we fit the  $\pi^+\pi^0$  invariant mass with a BW and combinatorial background function in intervals of 0.04 GeV/ $c^2$  in the  $\pi^-\pi^0$  mass distribution. The resulting distribution exibits a clear  $\rho$  peak, shown in Fig. 36(b), with a correlated  $\rho^+\rho^-$  yield of  $703\pm153$  events, corresponding to  $46\pm8\%$  of the  $\rho^\pm\pi^\mp\pi^0\pi^0$  events. Using this value we estimate the number of  $J/\psi$  decays to single- and double- $\rho$  to be  $1241\pm109\pm183$  and  $529\pm46\pm92$ , respectively. The second uncertainty is from the uncertainty in the fraction of  $\rho^+\rho^-$  events, given above. We obtain:

$$\begin{split} B_{J/\psi\to\rho^{\pm}\pi^{\mp}\pi^{0}\pi^{0}} \cdot \Gamma_{ee}^{J/\psi} &= (78 \pm 7 \pm 8 \pm 6) \, \text{eV} , \\ B_{J/\psi\to\rho^{+}\rho^{-}\pi^{0}} \cdot \Gamma_{ee}^{J/\psi} &= (33 \pm 3 \pm 3 \pm 3) \, \text{eV} . \end{split}$$

Dividing by the value of  $\Gamma_{ee}$  from Ref. [27] then yields:

$$B_{J/\psi \to \rho^{\pm} \pi^{\mp} \pi^{0} \pi^{0}} = (1.40 \pm 0.12 \pm 0.14 \pm 0.10) \times 10^{-2},$$
  
 $B_{J/\psi \to \rho^{+} \rho^{-} \pi^{0}} = (0.60 \pm 0.05 \pm 0.06 \pm 0.05) \times 10^{-2},$ 

where the third uncertainty is associated with the uncertainty arising from the procedure used to determine the correlated  $\rho^+\rho^-$  rate. No other measurements for these processes exist.

# B. The $\pi^+\pi^-2\pi^0\eta$ final state

Figure 37 shows an expanded view of Fig. 32, with a clear  $J/\psi$  signal seen in all three distributions: the inclusive  $\pi^+\pi^-2\pi^0\eta$  mass distribution (Fig. 37(a)) and the mass distributions for the  $\omega\pi^0\eta$  (Fig. 37(b)) and  $\rho^\pm\pi^\mp\pi^0\eta$  (Fig. 37(c)) intermediate states. Our fits yield  $203\pm29,\,27\pm14,\,$  and  $168\pm62$  events for the  $J/\psi$  decays into these final states, respectively. Only an upper limit with < 12 events at 90% C.L. is obtained for the  $\psi(2S)$  decay to  $\pi^+\pi^-2\pi^0\eta$ . We determine:

$$\begin{split} B_{J/\psi\to\pi^+\pi^-\pi^0\pi^0\eta} \cdot \Gamma_{ee}^{J/\psi} &= (12.8 \pm 1.8 \pm 2.0) \, \text{eV} \;, \\ B_{J/\psi\to\omega\pi^0\eta} \cdot B_{\omega\to3\pi} \cdot \Gamma_{ee}^{J/\psi} &= (1.7 \pm 0.8 \pm 0.3) \, \text{eV} \;, \\ B_{J/\psi\to\rho^\pm\pi^\mp\pi^0\eta} \cdot \Gamma_{ee}^{J/\psi} &= (10.5 \pm 4.1 \pm 1.6) \, \text{eV} \;, \\ B_{\psi(2S)\to\pi^+\pi^-\pi^0\pi^0\eta} \cdot \Gamma_{ee}^{\psi(2S)} &< 0.85 \, \text{eV} \; \text{at } 90\% \; \text{C.L..} \end{split}$$

Dividing by the appropriate  $\Gamma_{ee}$  value from Ref. [27], we find  $B_{J/\psi\to\pi^+\pi^-\pi^0\pi^0\eta}=(2.30\pm0.33\pm0.35)\times10^{-3},$   $B_{J/\psi\to\omega\pi^0\eta}=(3.4\pm1.6\pm0.6)\times10^{-4},$   $B_{J/\psi\to\rho^\pm\pi^\mp\pi^0\eta}=(1.9\pm0.7\pm0.3)\times10^{-3},$  and  $B_{\psi(2S)\to\pi^+\pi^-\pi^0\pi^0\eta}<3.5\times10^{-4}$  at 90% C.L.. There are no previous results for these final states.

## C. Summary of the charmonium region study

The rates of  $J/\psi$  and  $\psi(2S)$  decays to  $\pi^+\pi^-3\pi^0$ ,  $\pi^+\pi^-2\pi^0\eta$  and several intermediate final states have

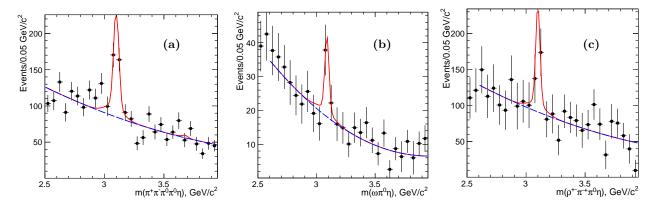


FIG. 37: The  $J/\psi$  region for the (a)  $\pi^+\pi^-2\pi^0\eta$ , (b)  $\omega\pi^0\eta$ , and (c)  $\rho^{\pm}\pi^{\mp}\pi^0\eta$  events. The curves show the fit functions described in the text.

TABLE VII: Sumr	mary of the $J/\psi$ and $\psi($	(2S) branching fractions.	
Measured	Measured	$J/\psi$ or $\psi(2S)$ Branching	Fraction $(10^{-3})$
Quantity	Value (eV)	Calculated, this work	PDG [27]
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \to \pi^+ \pi^- \pi^0 \pi^0 \pi^0}$	$150.0{\pm}4.0{\pm}15.0$	$27.0 \pm 0.7 \pm 2.7$	no entry
$\Gamma^{J/\psi}_{ee}\cdot \mathcal{B}_{J/\psi o\omega\pi^0\pi^0}\cdot \mathcal{B}_{\omega o3\pi}$	$24.8{\pm}1.8{\pm}2.5$	$5.04 \pm 0.37 \pm 0.50$	$3.4 {\pm} 0.8$
$\Gamma^{J/\psi}_{ee}\cdot \mathcal{B}_{J/\psi  o  ho^{\pm}\pi^{\mp}\pi^{0}\pi^{0}}$	$78.0 \pm 9.0 \pm 8.0$	$14.0 \pm 1.2 \pm 1.4$	no entry
$\Gamma_{ee}^{J/\psi}  {\cal B}_{J/\psi  ightarrow  ho^+  ho^- \pi^0}$	$33.0 \pm 5.0 \pm 3.3$	$6.0 \pm 0.9 \pm 0.6$	no entry
$\Gamma^{J/\psi}_{ee}\cdot {\cal B}_{J/\psi o\pi^+\pi^-\pi^0\pi^0\eta}$	$12.8 {\pm} 1.8 {\pm} 2.0$	$2.30 \pm 0.33 \pm 0.35$	no entry
$\Gamma^{J/\psi}_{ee} \cdot \mathcal{B}_{J/\psi  o \omega \pi^0 \eta} \cdot \mathcal{B}_{\omega  o 3\pi}$	$1.7 {\pm} 0.8 {\pm} 0.3$	$0.34{\pm}0.16{\pm}0.06$	no entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \to \rho^{\pm} \pi^{\mp} \pi^{0} \eta}$	$10.5{\pm}4.1{\pm}1.6$	$1.7 \pm 0.7 \pm 0.3$	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \to \pi^+\pi^-\pi^0\pi^0\pi^0}$	$12.4{\pm}1.8{\pm}1.2$	$5.2 \pm 0.8 \pm 0.5$	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \to J/\psi \pi^0 \pi^0} \cdot \mathcal{B}_{J/\psi \to 3\pi}$	$10.1{\pm}1.5{\pm}1.1$	$22.9 \pm 2.8 \pm 2.3$	$21.1 {\pm} 0.7$
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \to \omega \pi^0 \pi^0} \cdot \mathcal{B}_{\omega \to 3\pi}$	$2.3 {\pm} 0.7 {\pm} 0.2$	$1.1 \pm 0.3 \pm 0.1$	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \to \rho^{\pm} \pi^{\mp} \pi^{0} \pi^{0}}$	$<\!6.2$ at 90% C.L.	$<\!2.6$ at 90% C.L.	no entry
$\Gamma^{\psi(2S)}.\mathcal{B}$	<0.85 at 90% C.L.	<0.35 at 90% C.L.	no entry

TABLE VII: Summary of the  $J/\psi$  and  $\psi(2S)$  branching fractions

been measured. A small discrepancy with only one available current PDG value, measured by the DM2 experiment [33], is observed for the  $J/\psi \to \omega \pi^0 \pi^0$  decay rate. The measured products and calculated branching fractions are summarized in Table VII together with the available PDG values for comparison.

## VII. SUMMARY

The photon-energy and charged-particle momentum resolutions together with the particle identification capabilities of the BABAR detector permit the reconstruction of the  $\pi^+\pi^-3\pi^0$  and  $\pi^+\pi^-2\pi^0\eta$  final states produced at low effective center-of-mass energies via initial-state photon radiation in data collected in  $e^+e^-$  annihilation in the  $\Upsilon(4S)$  mass region.

The analysis shows that the effective luminosity and efficiency have been understood with 10–13% accuracy. The cross section measurements for the reaction  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0$  present a significant improvement on existing data. The  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\eta$  cross section has been measured for the first time.

The selected multi-hadronic final states in the broad range of accessible energies provide new information on hadron spectroscopy. The observed  $e^+e^- \to \omega \pi^0 \pi^0$  and  $e^+e^- \to \eta \pi^+\pi^-$  cross sections provide evidence of resonant structures around 1.4 and 1.7 GeV/ $c^2$ , which were previously observed by DM2 and interpreted as  $\omega(1450)$  and  $\omega(1650)$  resonances.

The initial-state radiation events allow a study of  $J/\psi$  and  $\psi(2S)$  production and a measurement of the corresponding products of the decay branching fractions and  $e^+e^-$  width for most of the studied channels, the majority of them for the first time.

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