

Vole Simulations

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April 10, 2017

To look at the sensitivity of the prey population to the model parameters, we can run the model for a variety of different parameters and look how the curve changes. For this simulation, we will set a variety of r values for population runs for vole populations.

Population growth rates

From the literature review (Sarah this is the average data from the model summary data), we constructed minimum, average, and maximum vole stage structured matrices. It is important to note that survival measurements are from *M. townsendii* and fertility measurements were from *M. californicus* because no studies on either species contained both measurements.

##	Juvenile	SubAdult	Adult
## Juvenile	0.00	0.00	3.45
## SubAdult	0.59	0.00	0.00
## Adult	0.00	0.72	0.80
##	Juvenile	SubAdult	Adult
## Juvenile	0.0	0.00	6.55
## SubAdult	0.8	0.00	0.00
## Adult	0.0	0.81	0.86

Parameters

We can use the `pred_prey` function created in our `owls` r-package, then use an ode solver to solve the derivatives. We will use a type two functional response to model the behavior of the prey population. A type two functional response is the simplest model to use in the wake of low data. The response will capture typical more realistic predator-prey relations. The differential equations are below:

$$\frac{dprey}{dt} = rN(1 - \frac{N}{K_{prey}}) - \frac{k_{max}N}{N + D}P$$
$$\frac{dpredator}{dt} = \beta PN - \delta P$$

The model requires multiple parameters: r = growth rate of prey pop (voles/season) This value is extracted from the population matrices using the `popbio` package to calculate λ , then $r = \ln(\lambda)$

K_{prey} = carrying capacity of prey (voles)

α = attack rate of predator (or capture efficiency; the larger α is, the more the per capita growth rate of the prey population is depressed by the addition of a single predator) (units=1/season) The attack rate was calculated from Derting and Cranford (1989). They found that, on average, it takes 1.1 attacks/vole - for this analysis, we assume the attack rate is the same for gophers as for voles. If there are 11 successful attacks per night the average number observed in observations recorded in Bunn et al (1982), then there should be 12.1 total attacks per night. If there is one foraging event per night, and 90 foraging events per season, then we expect 1089 attacks per season. However, this is if there are a significant amount of gophers across the landscape (scaling up from the Derting and Cranford densities, we get over 3000 voles/ha, an unreasonable number). Therefore, we would expect attack rate to decrease with density of the prey

population. In sensitivity analysis if the attack rate is less than three, there is coexistence between gopher and owl populations.

h = handling time The amount of time it takes an owl to kill and eat a single vole ($1/k_{\max}$) (season/vole)

β = assimilation efficiency (efficiency of turning voles into per capita growth) This number is calculated from field data and represents the fractional value of one vole to an owl producing 4.33 chicks.

δ = death rate of predator (#predator/season)

k_{\max} = maximum feeding rate the most gophers an owl can eat in a single time unit (voles/season). Calculated from empirical data.

D = half saturation constant ($1/(\alpha \cdot \text{handling time})$) The abundance of prey at which the feeding rate is half maximal (gopher)

State variables: N = starting population of prey P = starting population of predator

For this model, time is in terms of season (3 months), so each time interval is represented by a season and four seasons are equivalent to one year.

```
#Parameters
alpha=0.924
beta = 5.85e-4
delta=0.01
K_prey=1000
k_max=654
h=1/k_max
D=1/(alpha*h)
r=r_vole
parameters <- c(r, alpha, beta, delta, K_prey, k_max, D)

#State variables:
N=c(1000, 1000, 1000, 1000,
    500, 500, 500, 500,
    100, 100, 100, 100,
    50, 50, 50, 50,
    10, 10, 10, 10,
    2,2,2,2)

P=c(0.2, 0.6, 1.0, 2.0,
    0.2, 0.6, 1.0, 2.0,
    0.2, 0.6, 1.0, 2.0,
    0.2, 0.6, 1.0, 2.0,
    0.2, 0.6, 1.0, 2.0,
    0.2, 0.6, 1.0, 2.0)
state<-cbind(N, P)
times<- seq(0, 40, by=1)
```

Simulations, $K=1000$

We can then run simulations for these various lists of r values while varying predator density.

Then, plotting the prey density over time by each N .

Table 1: N init= 1000

time	N	P	r	K
40	773.6082592	0.2	0.39	1000
40	3.0257044	0.6	0.39	1000
40	0.0000015	1.0	0.39	1000
40	0.0000000	2.0	0.39	1000
40	876.7985490	0.2	0.67	1000
40	524.9124428	0.6	0.67	1000
40	0.0826662	1.0	0.67	1000
40	0.0000000	2.0	0.67	1000
40	974.0764538	0.2	3.00	1000
40	919.6273646	0.6	3.00	1000
40	861.0434434	1.0	3.00	1000
40	687.5266110	2.0	3.00	1000
40	987.1381143	0.2	6.00	1000
40	960.8054097	0.6	6.00	1000
40	933.5926619	1.0	6.00	1000
40	861.0434434	2.0	6.00	1000

Table 2: N init= 500

time	N	P	r	K
40	773.5859613	0.2	0.39	1000
40	1.5159708	0.6	0.39	1000
40	0.0000006	1.0	0.39	1000
40	0.0000000	2.0	0.39	1000
40	876.7985485	0.2	0.67	1000
40	524.8695079	0.6	0.67	1000
40	0.0427236	1.0	0.67	1000
40	0.0000000	2.0	0.67	1000
40	974.0764538	0.2	3.00	1000
40	919.6273646	0.6	3.00	1000
40	861.0434434	1.0	3.00	1000
40	687.5266110	2.0	3.00	1000
40	987.1381143	0.2	6.00	1000
40	960.8054097	0.6	6.00	1000
40	933.5926619	1.0	6.00	1000
40	861.0434434	2.0	6.00	1000

Table 3: N init= 100

time	N	P	r	K
40	773.3433766	0.2	0.39	1000
40	0.1754935	0.6	0.39	1000
40	0.0000001	1.0	0.39	1000
40	0.0000000	2.0	0.39	1000
40	876.7985456	0.2	0.67	1000
40	524.0642163	0.6	0.67	1000
40	0.0049258	1.0	0.67	1000
40	0.0000000	2.0	0.67	1000

time	N	P	r	K
40	974.0764538	0.2	3.00	1000
40	919.6273646	0.6	3.00	1000
40	861.0434434	1.0	3.00	1000
40	687.5266110	2.0	3.00	1000
40	987.1381143	0.2	6.00	1000
40	960.8054097	0.6	6.00	1000
40	933.5926619	1.0	6.00	1000
40	861.0434434	2.0	6.00	1000

Table 4: N init= 50

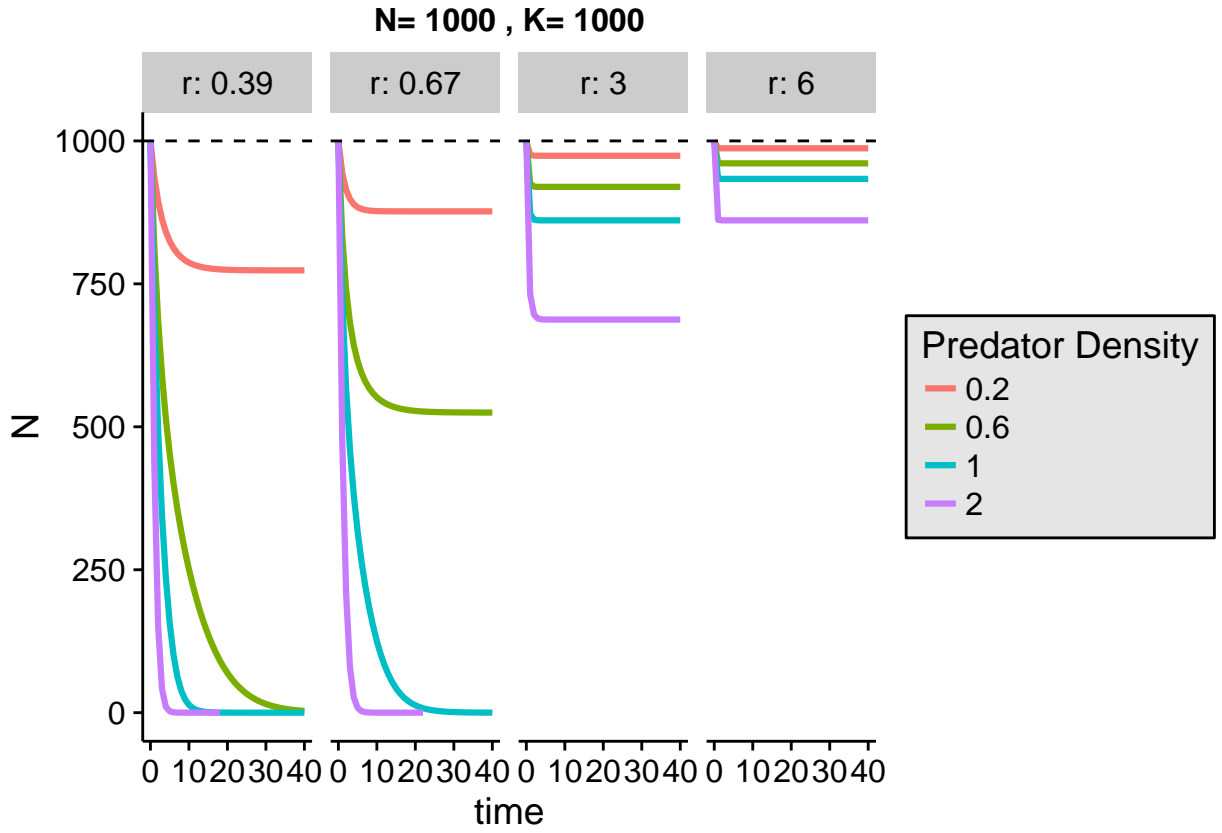
time	N	P	r	K
40	772.9579009	0.2	0.39	1000
40	0.0783790	0.6	0.39	1000
40	0.0000000	1.0	0.39	1000
40	0.0000000	2.0	0.39	1000
40	876.7985426	0.2	0.67	1000
40	522.0172079	0.6	0.67	1000
40	0.0021884	1.0	0.67	1000
40	0.0000000	2.0	0.67	1000
40	974.0764538	0.2	3.00	1000
40	919.6273646	0.6	3.00	1000
40	861.0434434	1.0	3.00	1000
40	687.5266110	2.0	3.00	1000
40	987.1381143	0.2	6.00	1000
40	960.8054097	0.6	6.00	1000
40	933.5926619	1.0	6.00	1000
40	861.0434434	2.0	6.00	1000

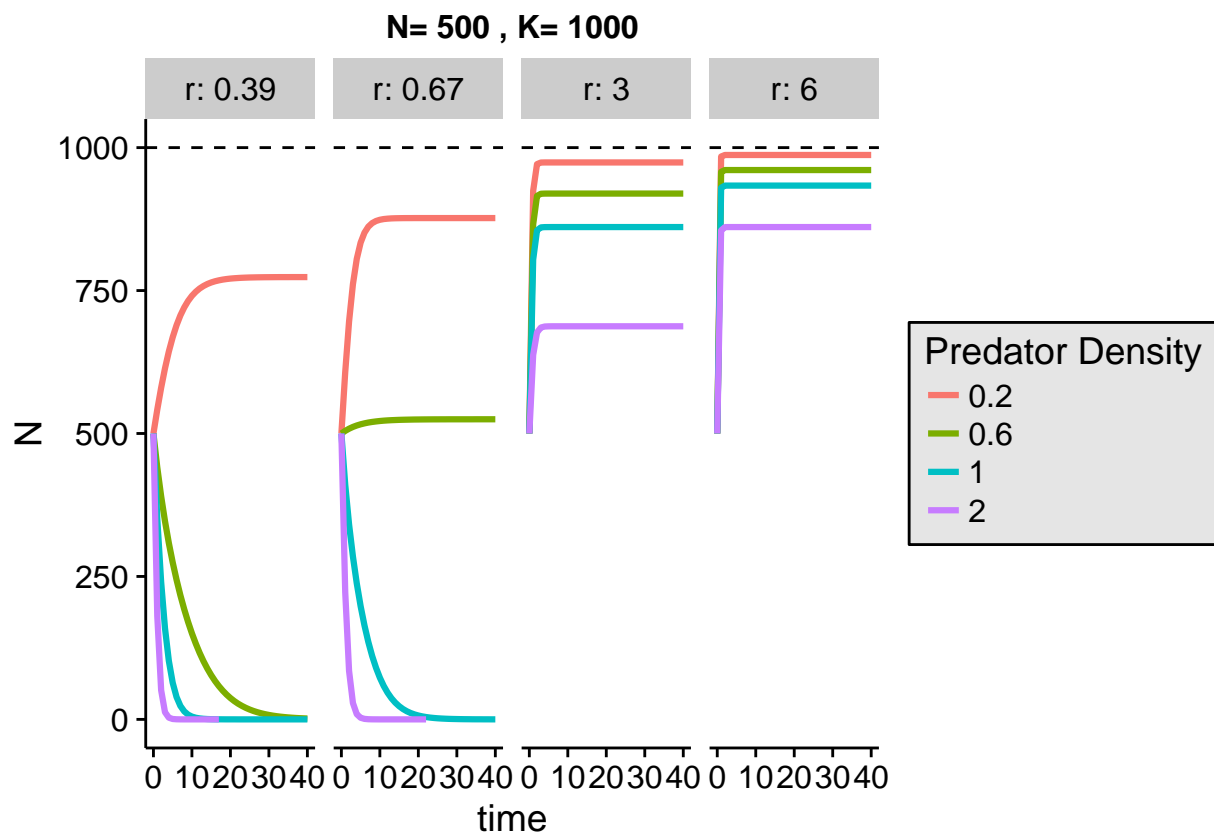
Table 5: N init= 10

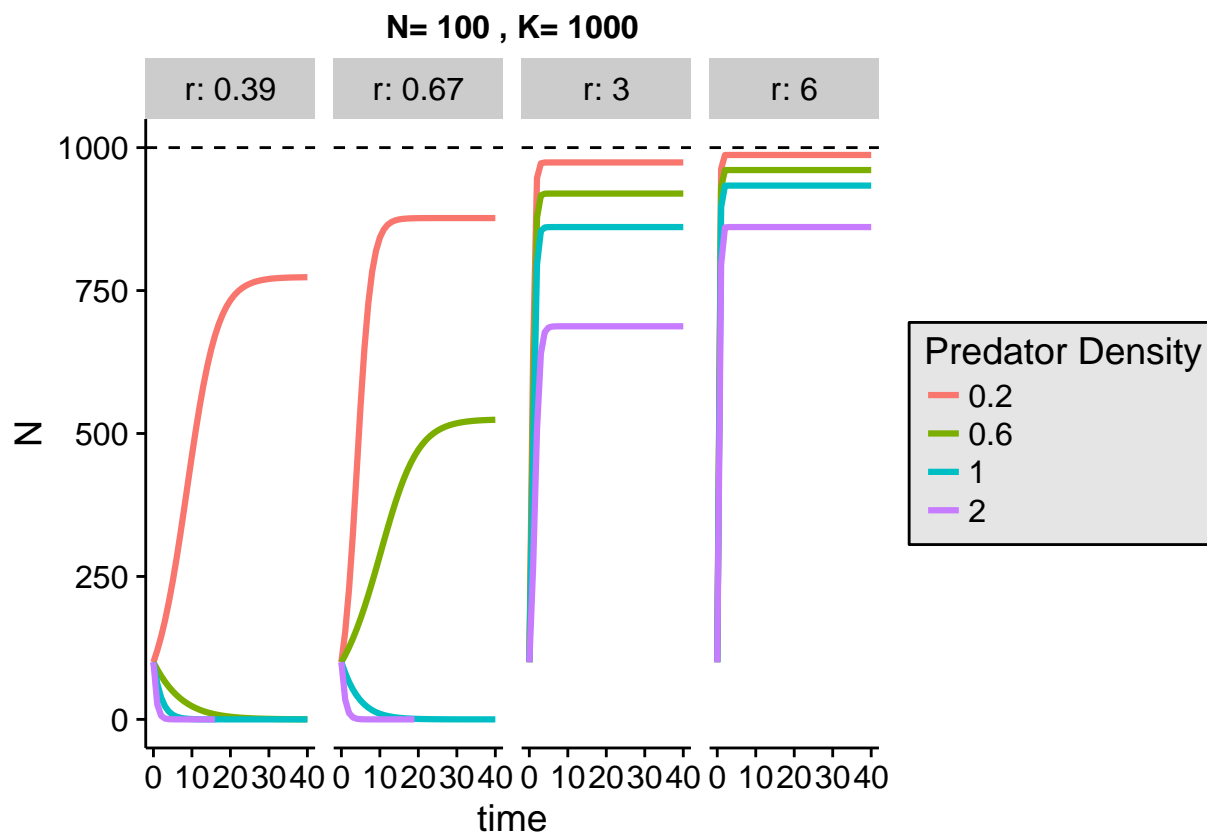
time	N	P	r	K
40	768.6820281	0.2	0.39	1000
40	0.0142702	0.6	0.39	1000
40	0.0000000	1.0	0.39	1000
40	0.0000000	2.0	0.39	1000
40	876.7985012	0.2	0.67	1000
40	476.0096370	0.6	0.67	1000
40	0.0003967	1.0	0.67	1000
40	0.0000000	2.0	0.67	1000
40	974.0764538	0.2	3.00	1000
40	919.6273646	0.6	3.00	1000
40	861.0434434	1.0	3.00	1000
40	687.5266110	2.0	3.00	1000
40	987.1381143	0.2	6.00	1000
40	960.8054097	0.6	6.00	1000
40	933.5926619	1.0	6.00	1000
40	861.0434434	2.0	6.00	1000

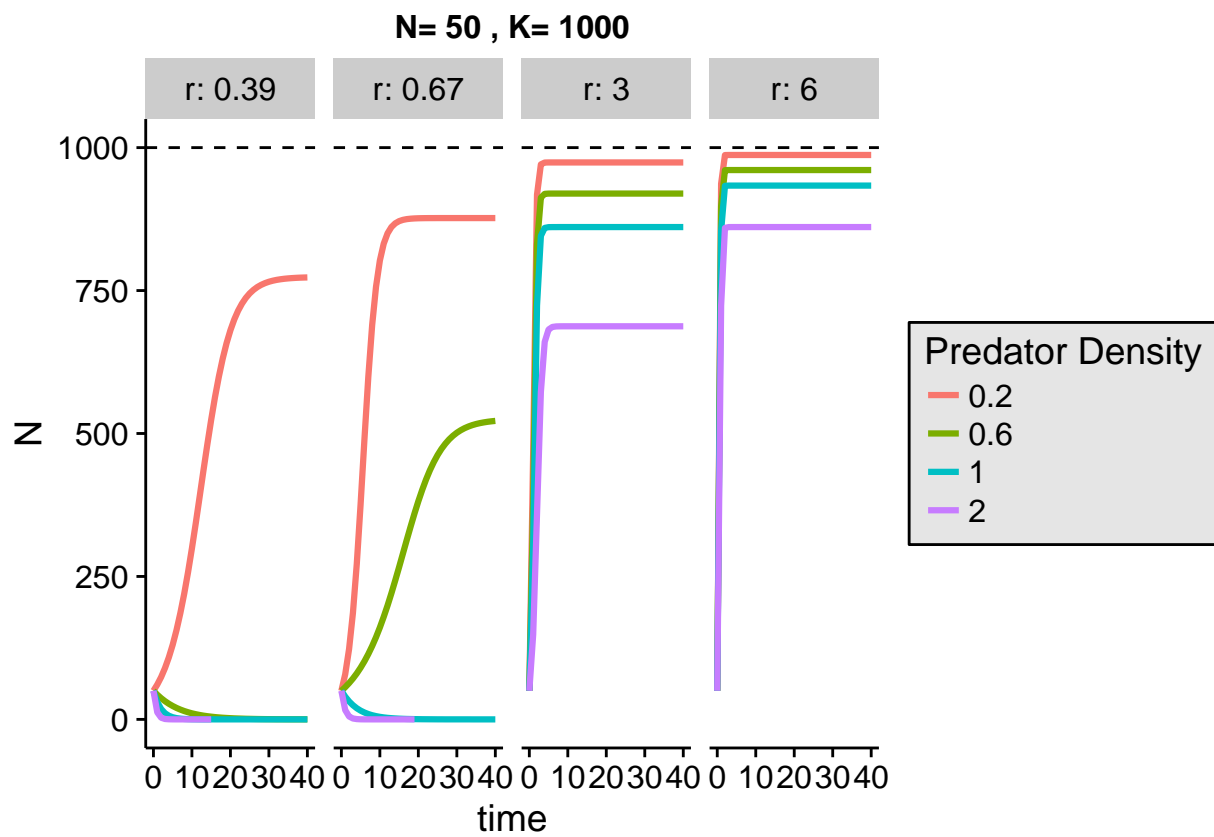
Table 6: N init= 2

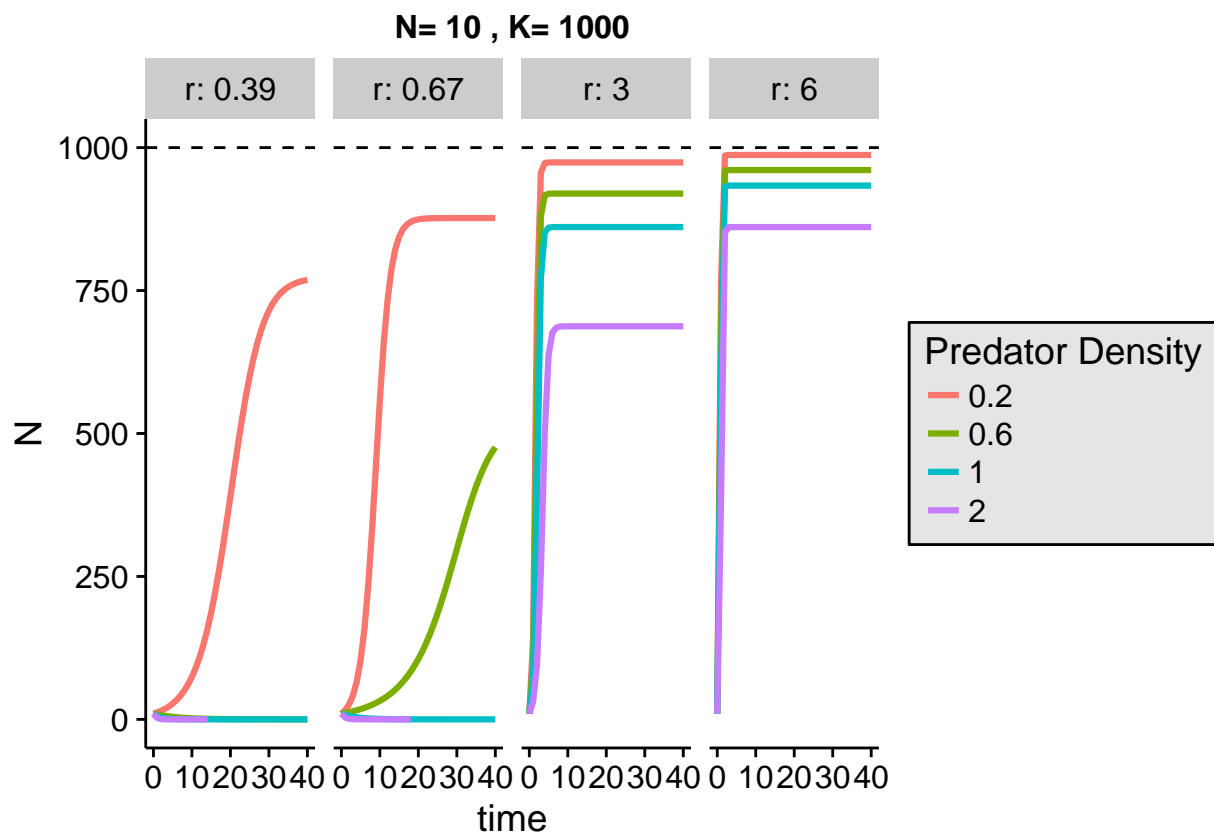
time	N	P	r	K
40	738.4220644	0.2	0.39	1000
40	0.0028002	0.6	0.39	1000
40	0.0000000	1.0	0.39	1000
40	0.0000000	2.0	0.39	1000
40	876.7982708	0.2	0.67	1000
40	208.8812500	0.6	0.67	1000
40	0.0000778	1.0	0.67	1000
40	0.0000000	2.0	0.67	1000
40	974.0764538	0.2	3.00	1000
40	919.6273646	0.6	3.00	1000
40	861.0434434	1.0	3.00	1000
40	687.5266110	2.0	3.00	1000
40	987.1381143	0.2	6.00	1000
40	960.8054097	0.6	6.00	1000
40	933.5926619	1.0	6.00	1000
40	861.0434434	2.0	6.00	1000

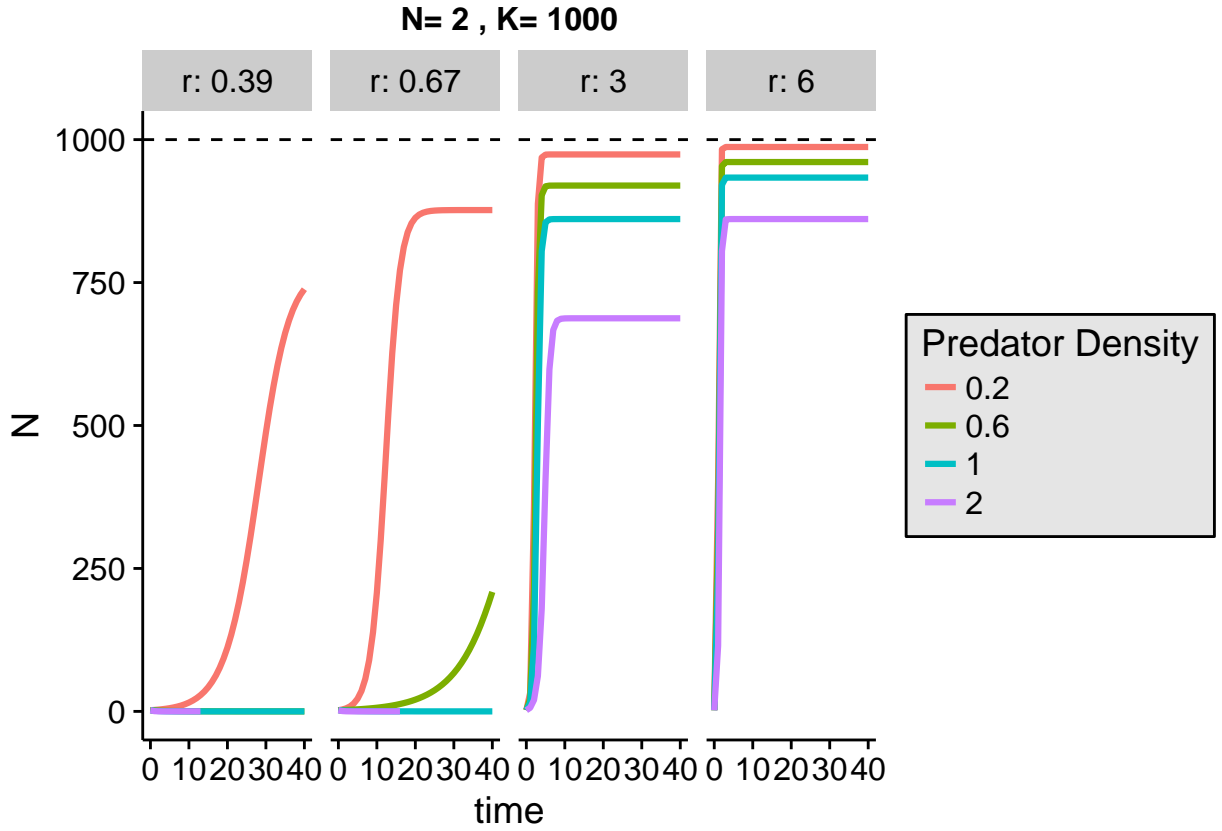












Simulations, K=500

Suppose we want to investigate what happens when the carrying capacity is lower, K=500

Table 7: N init= 1000

time	N	P	r	K
40	339.9754045	0.2	0.39	500
40	0.6110814	0.6	0.39	500
40	0.0000006	1.0	0.39	500
40	0.0000000	2.0	0.39	500
40	412.9003401	0.2	0.67	500
40	164.6925100	0.6	0.67	500
40	0.0155165	1.0	0.67	500
40	0.0000000	2.0	0.67	500
40	481.6724269	0.2	3.00	500
40	443.1783922	0.6	3.00	500
40	401.7623819	1.0	3.00	500
40	279.1058599	2.0	3.00	500
40	490.9068083	0.2	6.00	500
40	472.2900677	0.6	6.00	500
40	453.0513840	1.0	6.00	500
40	401.7623819	2.0	6.00	500

Table 8: N init= 500

time	N	P	r	K
40	339.9620223	0.2	0.39	500
40	0.4701182	0.6	0.39	500
40	0.0000003	1.0	0.39	500
40	0.0000000	2.0	0.39	500
40	412.9003398	0.2	0.67	500
40	164.6056571	0.6	0.67	500
40	0.0121963	1.0	0.67	500
40	0.0000000	2.0	0.67	500
40	481.6724269	0.2	3.00	500
40	443.1783922	0.6	3.00	500
40	401.7623819	1.0	3.00	500
40	279.1058599	2.0	3.00	500
40	490.9068083	0.2	6.00	500
40	472.2900677	0.6	6.00	500
40	453.0513840	1.0	6.00	500
40	401.7623819	2.0	6.00	500

Table 9: N init= 100

time	N	P	r	K
40	339.8374817	0.2	0.39	500
40	0.1354951	0.6	0.39	500
40	0.0000001	1.0	0.39	500
40	0.0000000	2.0	0.39	500
40	412.9003377	0.2	0.67	500
40	163.6614449	0.6	0.67	500
40	0.0036980	1.0	0.67	500
40	0.0000000	2.0	0.67	500
40	481.6724269	0.2	3.00	500
40	443.1783922	0.6	3.00	500
40	401.7623819	1.0	3.00	500
40	279.1058599	2.0	3.00	500
40	490.9068083	0.2	6.00	500
40	472.2900677	0.6	6.00	500
40	453.0513840	1.0	6.00	500
40	401.7623819	2.0	6.00	500

Table 10: N init= 50

time	N	P	r	K
40	339.6639920	0.2	0.39	500
40	0.0691853	0.6	0.39	500
40	0.0000000	1.0	0.39	500
40	0.0000000	2.0	0.39	500
40	412.9003359	0.2	0.67	500
40	162.2176659	0.6	0.67	500
40	0.0019050	1.0	0.67	500
40	0.0000000	2.0	0.67	500

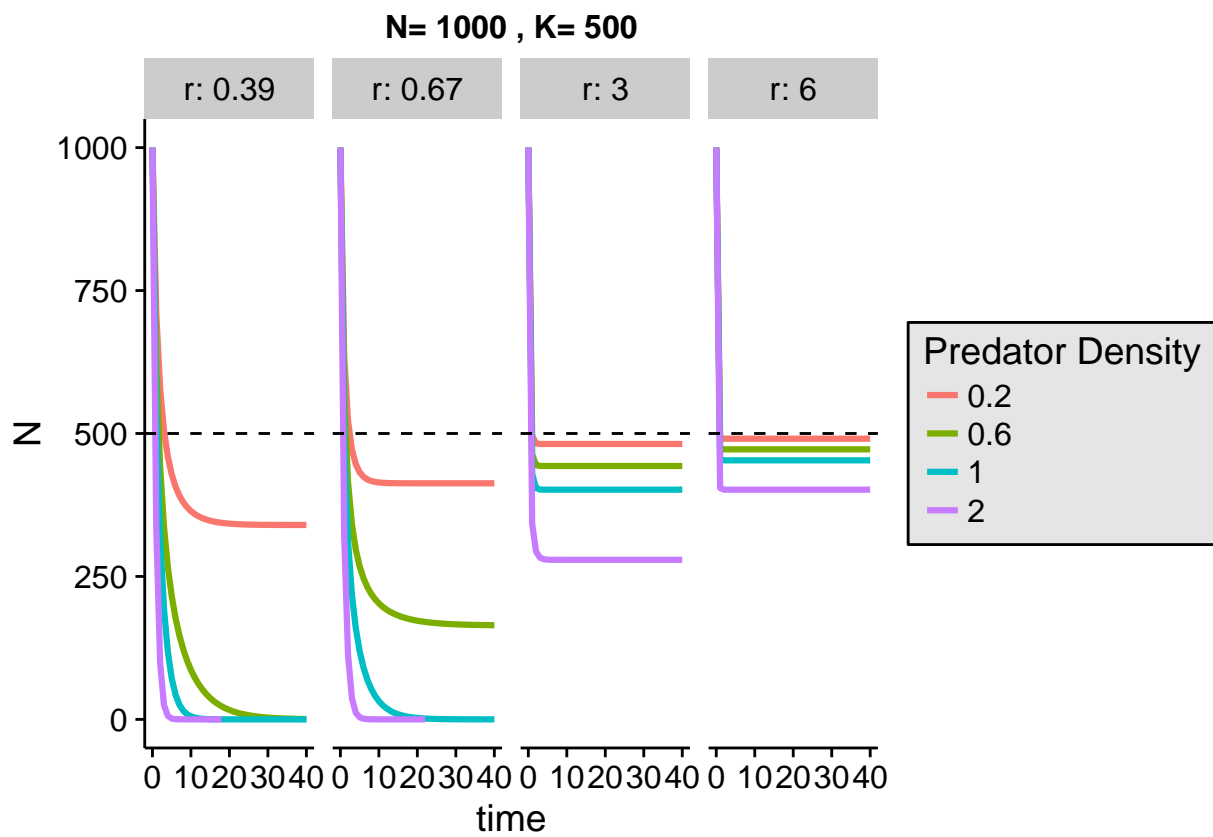
time	N	P	r	K
40	481.6724269	0.2	3.00	500
40	443.1783922	0.6	3.00	500
40	401.7623819	1.0	3.00	500
40	279.1058599	2.0	3.00	500
40	490.9068083	0.2	6.00	500
40	472.2900677	0.6	6.00	500
40	453.0513840	1.0	6.00	500
40	401.7623819	2.0	6.00	500

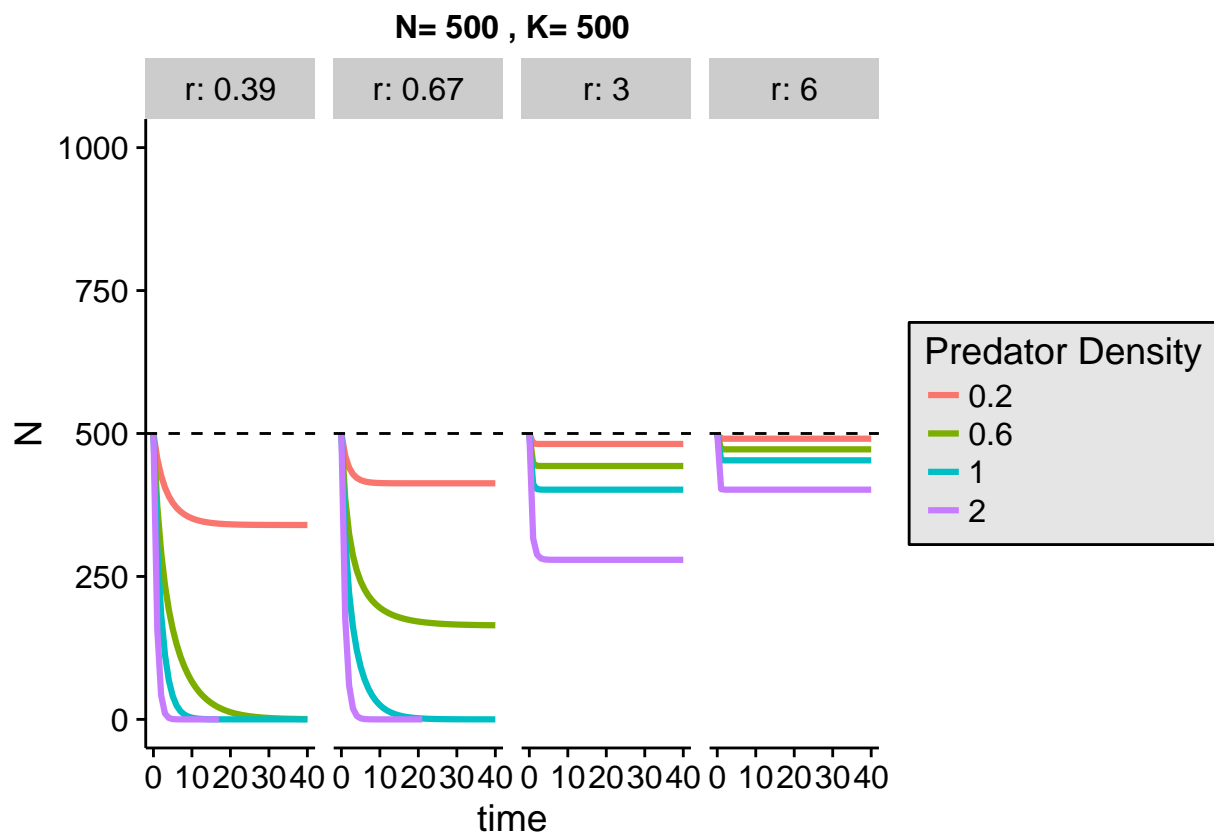
Table 11: N init= 10

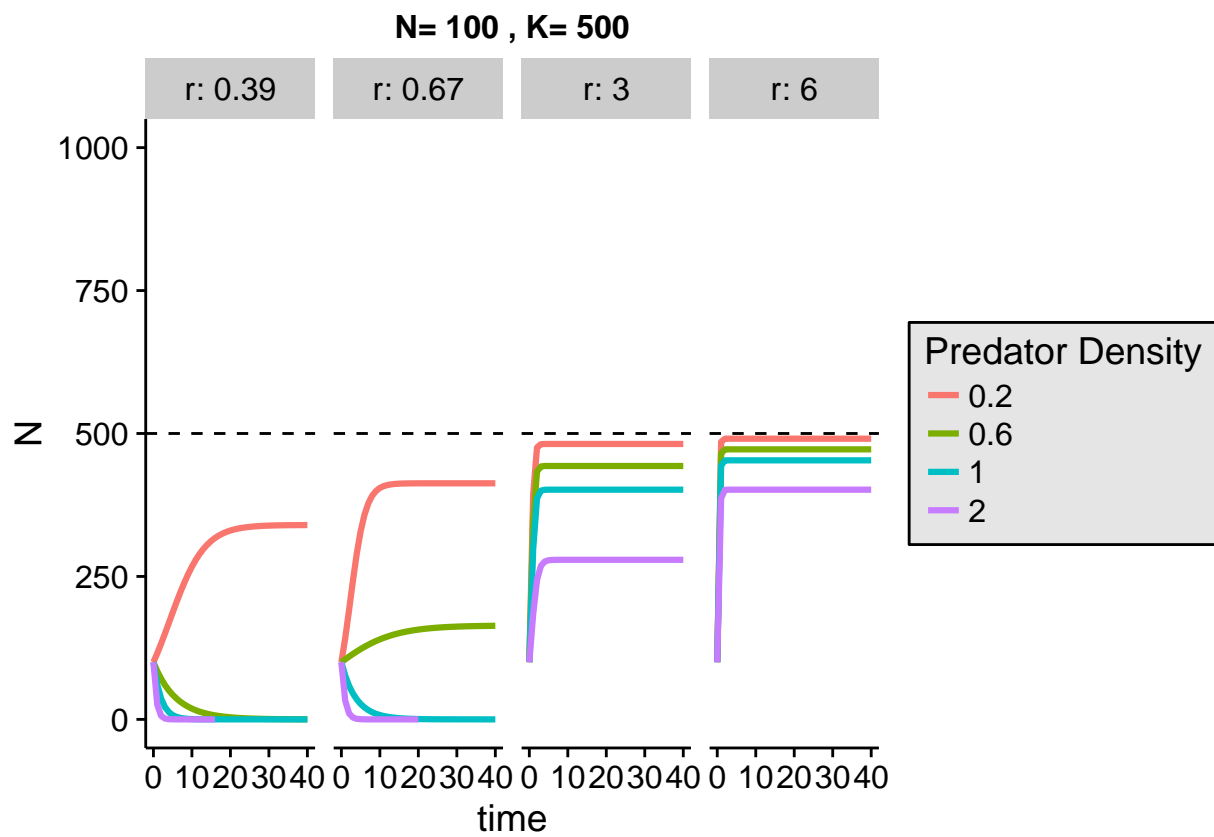
time	N	P	r	K
40	338.0834217	0.2	0.39	500
40	0.0139323	0.6	0.39	500
40	0.0000000	1.0	0.39	500
40	0.0000000	2.0	0.39	500
40	412.9003118	0.2	0.67	500
40	148.7665548	0.6	0.67	500
40	0.0003862	1.0	0.67	500
40	0.0000000	2.0	0.67	500
40	481.6724269	0.2	3.00	500
40	443.1783922	0.6	3.00	500
40	401.7623819	1.0	3.00	500
40	279.1058599	2.0	3.00	500
40	490.9068083	0.2	6.00	500
40	472.2900677	0.6	6.00	500
40	453.0513840	1.0	6.00	500
40	401.7623819	2.0	6.00	500

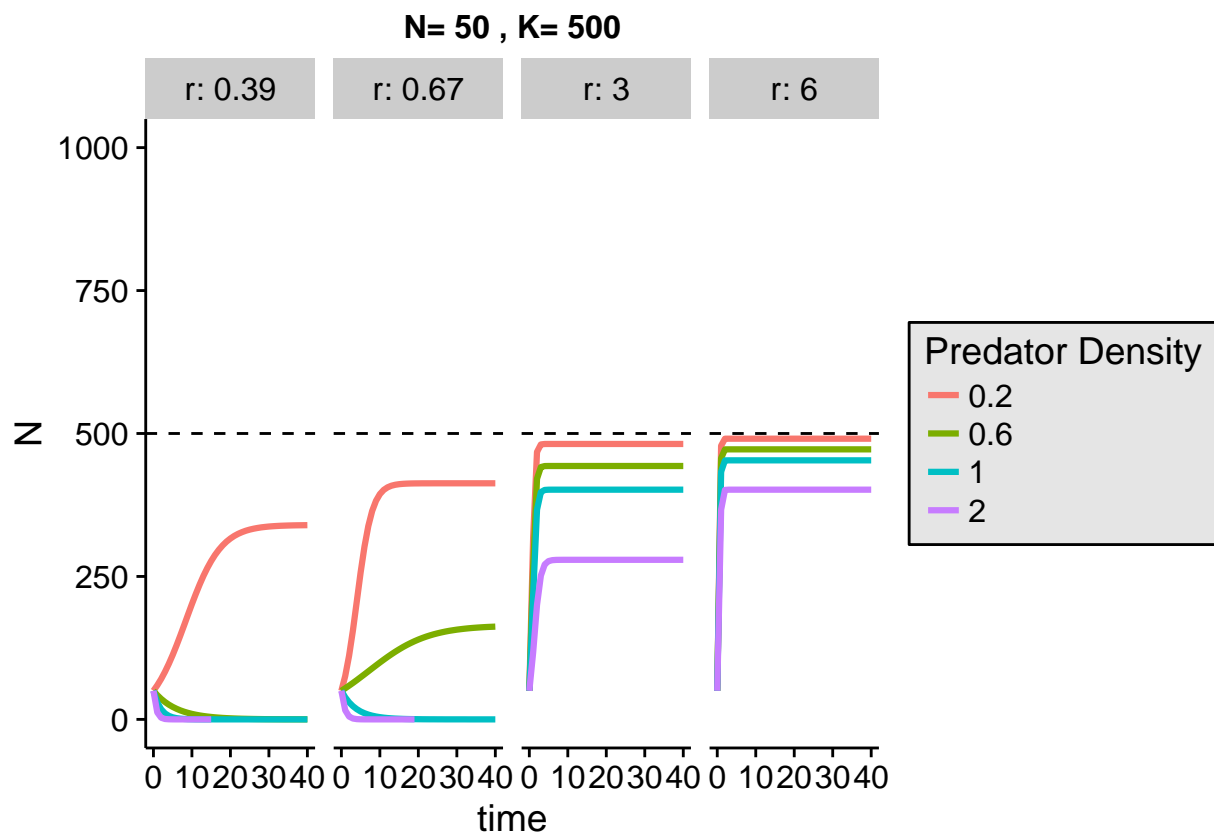
Table 12: N init= 2

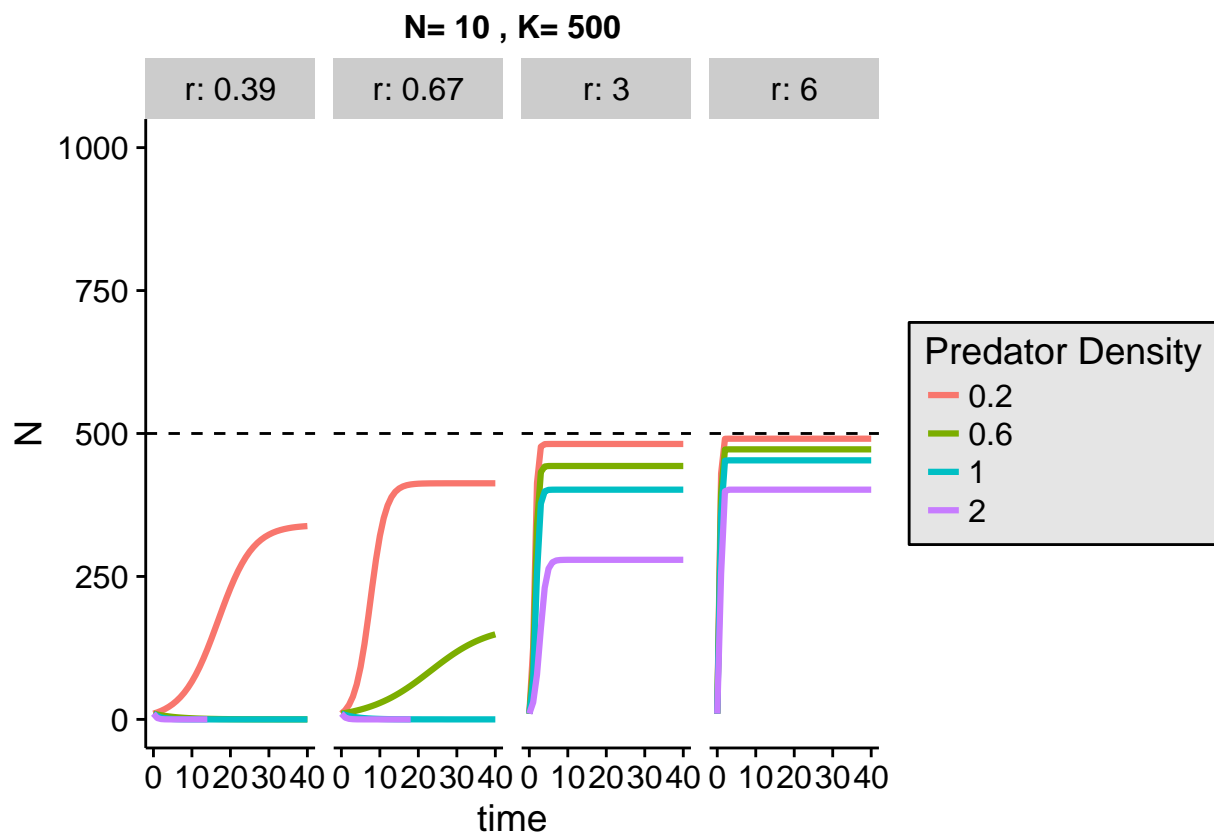
time	N	P	r	K
40	329.1476220	0.2	0.39	500
40	0.0027869	0.6	0.39	500
40	0.0000000	1.0	0.39	500
40	0.0000000	2.0	0.39	500
40	412.9001871	0.2	0.67	500
40	97.9620775	0.6	0.67	500
40	0.0000773	1.0	0.67	500
40	0.0000000	2.0	0.67	500
40	481.6724269	0.2	3.00	500
40	443.1783922	0.6	3.00	500
40	401.7623819	1.0	3.00	500
40	279.1058599	2.0	3.00	500
40	490.9068083	0.2	6.00	500
40	472.2900677	0.6	6.00	500
40	453.0513840	1.0	6.00	500
40	401.7623819	2.0	6.00	500

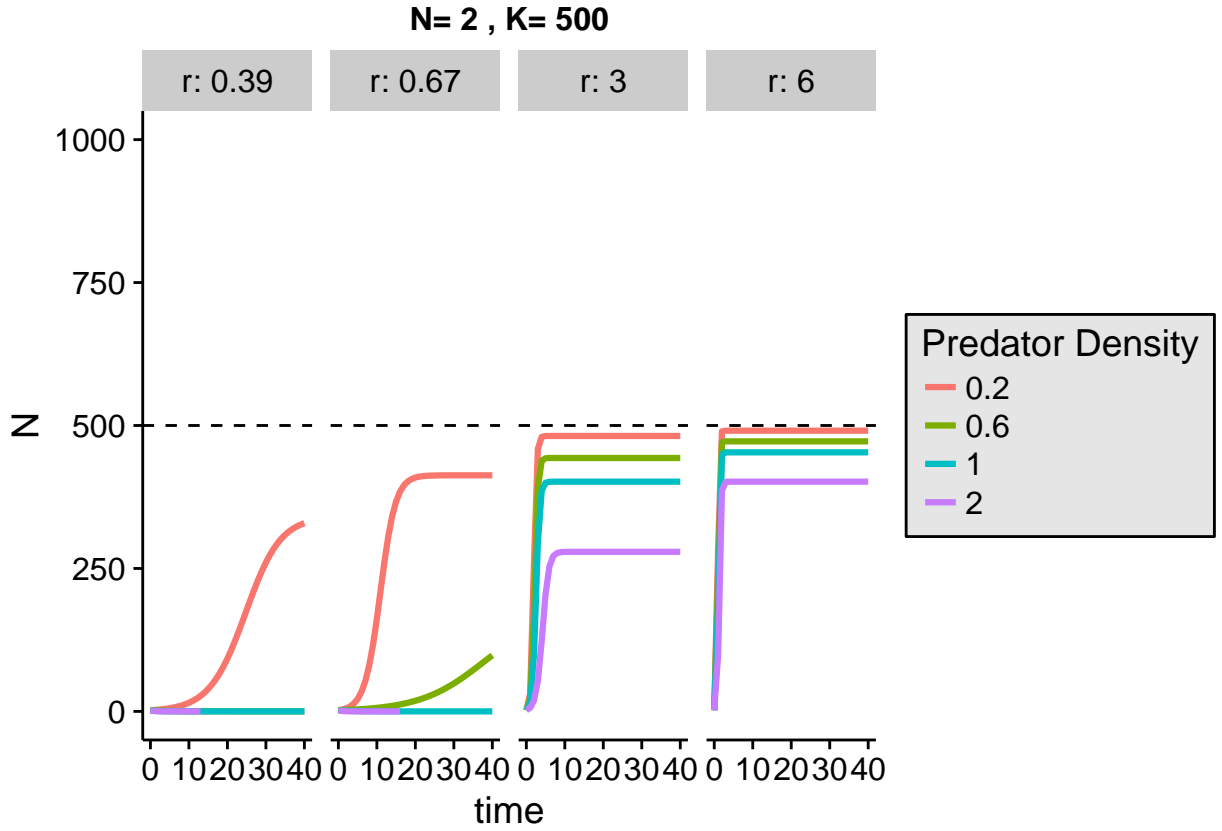












Simulations, K=100

Changing K to 100

Table 13: N init= 1000

time	N	P	r	K
40	56.1087386	0.2	0.39	100
40	0.0681389	0.6	0.39	100
40	0.0000001	1.0	0.39	100
40	0.0000000	2.0	0.39	100
40	75.0625678	0.2	0.67	100
40	19.6530757	0.6	0.67	100
40	0.0016995	1.0	0.67	100
40	0.0000000	2.0	0.67	100
40	94.5660182	0.2	3.00	100
40	83.4694376	0.6	3.00	100
40	72.0454644	1.0	3.00	100
40	41.8379886	2.0	3.00	100
40	97.2922094	0.2	6.00	100
40	91.8210460	0.6	6.00	100
40	86.2731706	1.0	6.00	100
40	72.0454644	2.0	6.00	100

Table 14: N init= 500

time	N	P	r	K
40	56.1079435	0.2	0.39	100
40	0.0652103	0.6	0.39	100
40	0.0000001	1.0	0.39	100
40	0.0000000	2.0	0.39	100
40	75.0625678	0.2	0.67	100
40	19.6495881	0.6	0.67	100
40	0.0016336	1.0	0.67	100
40	0.0000000	2.0	0.67	100
40	94.5660182	0.2	3.00	100
40	83.4694376	0.6	3.00	100
40	72.0454644	1.0	3.00	100
40	41.8379886	2.0	3.00	100
40	97.2922094	0.2	6.00	100
40	91.8210460	0.6	6.00	100
40	86.2731706	1.0	6.00	100
40	72.0454644	2.0	6.00	100

Table 15: N init= 100

time	N	P	r	K
40	56.1014413	0.2	0.39	100
40	0.0478237	0.6	0.39	100
40	0.0000000	1.0	0.39	100
40	0.0000000	2.0	0.39	100
40	75.0625677	0.2	0.67	100
40	19.6206851	0.6	0.67	100
40	0.0012301	1.0	0.67	100
40	0.0000000	2.0	0.67	100
40	94.5660182	0.2	3.00	100
40	83.4694376	0.6	3.00	100
40	72.0454644	1.0	3.00	100
40	41.8379886	2.0	3.00	100
40	97.2922094	0.2	6.00	100
40	91.8210460	0.6	6.00	100
40	86.2731706	1.0	6.00	100
40	72.0454644	2.0	6.00	100

Table 16: N init= 50

time	N	P	r	K
40	56.0932172	0.2	0.39	100
40	0.0356758	0.6	0.39	100
40	0.0000000	1.0	0.39	100
40	0.0000000	2.0	0.39	100
40	75.0625674	0.2	0.67	100
40	19.5840098	0.6	0.67	100
40	0.0009352	1.0	0.67	100
40	0.0000000	2.0	0.67	100

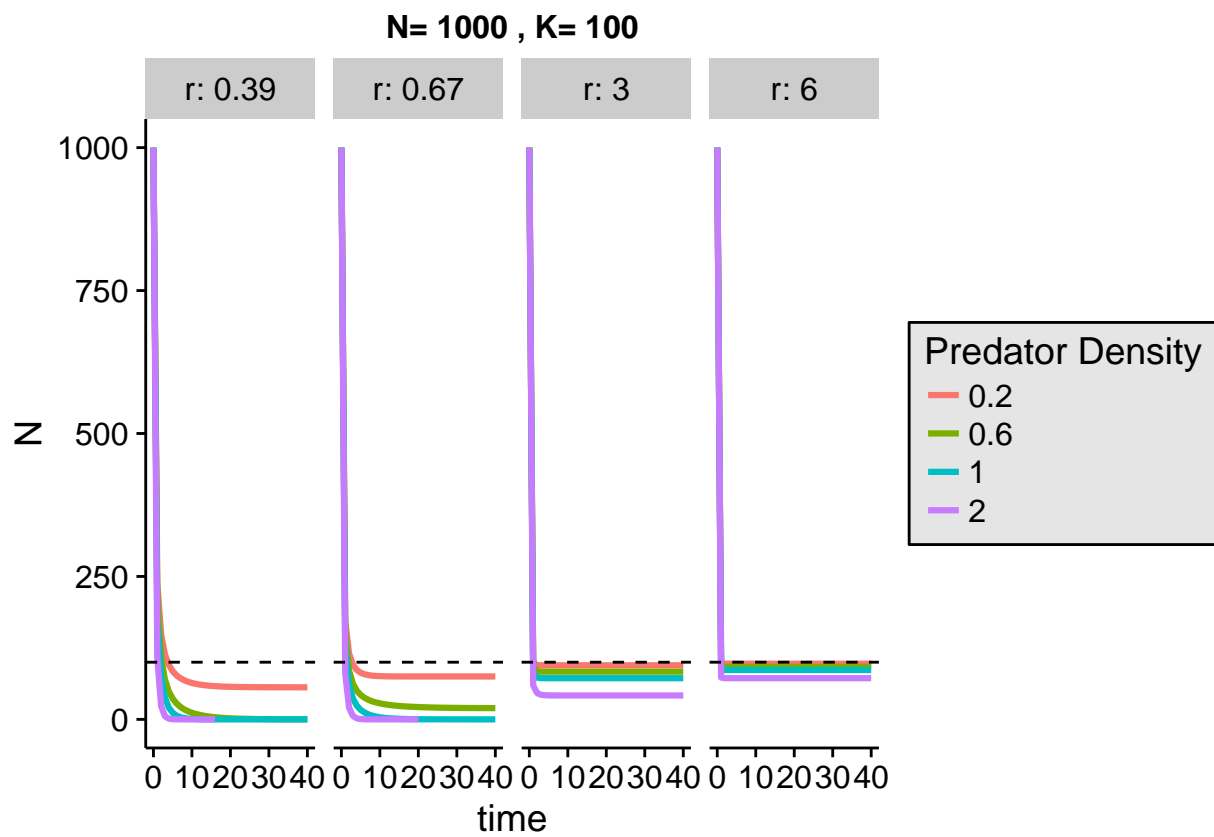
time	N	P	r	K
40	94.5660182	0.2	3.00	100
40	83.4694376	0.6	3.00	100
40	72.0454644	1.0	3.00	100
40	41.8379886	2.0	3.00	100
40	97.2922094	0.2	6.00	100
40	91.8210460	0.6	6.00	100
40	86.2731706	1.0	6.00	100
40	72.0454644	2.0	6.00	100

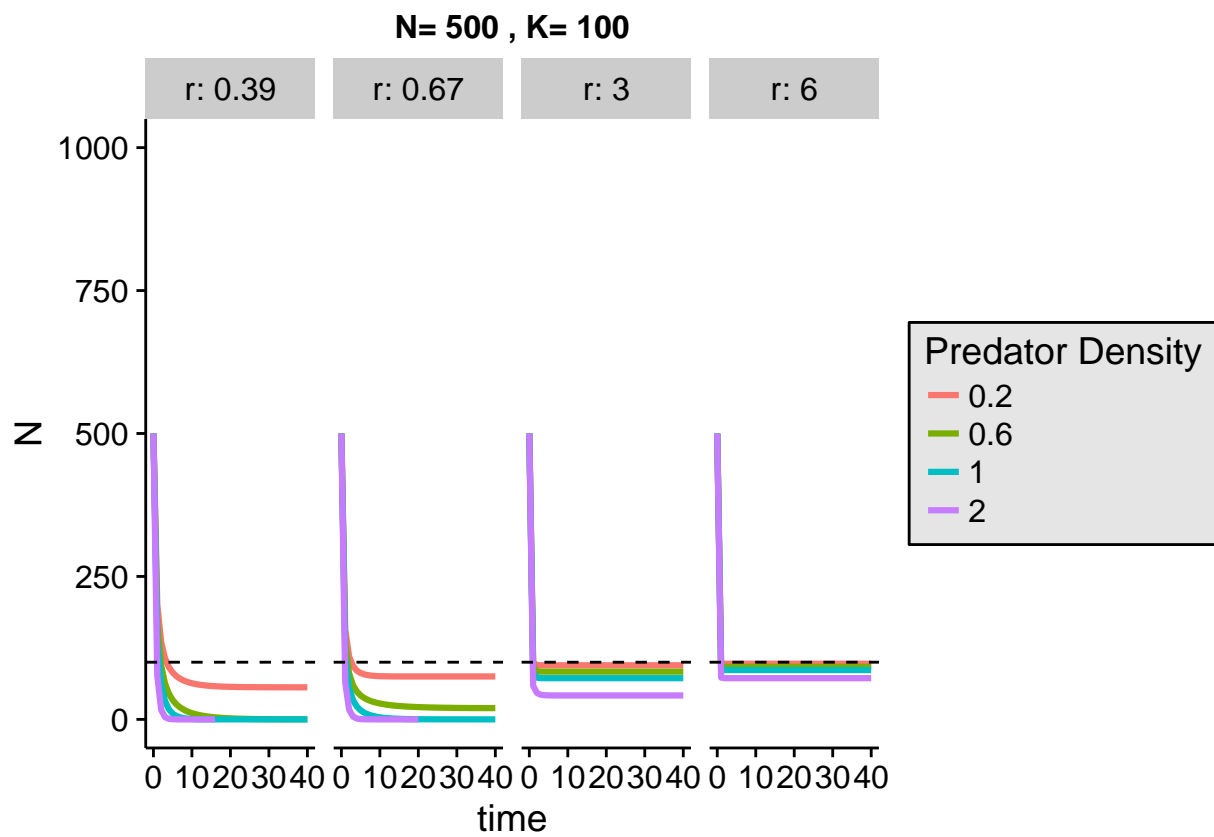
Table 17: N init= 10

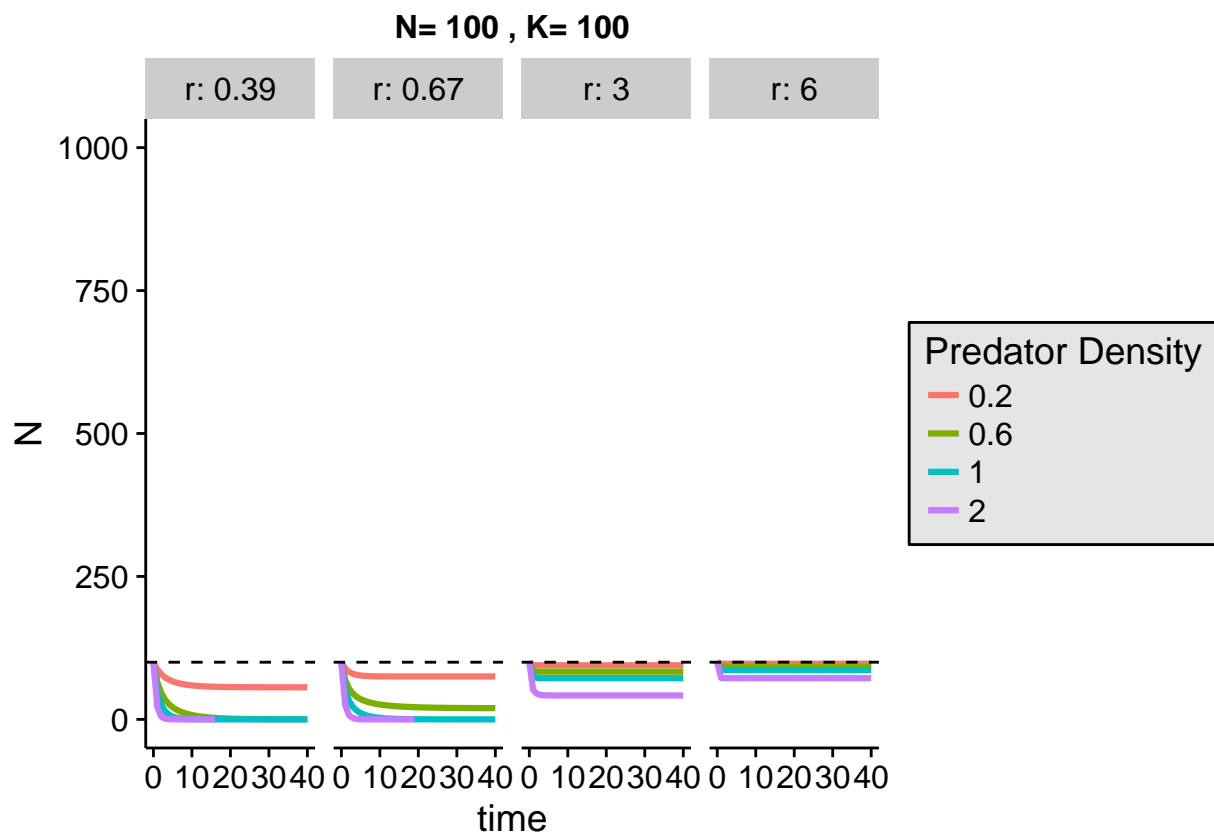
time	N	P	r	K
40	56.0268462	0.2	0.39	100
40	0.0117129	0.6	0.39	100
40	0.0000000	1.0	0.39	100
40	0.0000000	2.0	0.39	100
40	75.0625658	0.2	0.67	100
40	19.2920286	0.6	0.67	100
40	0.0003190	1.0	0.67	100
40	0.0000000	2.0	0.67	100
40	94.5660182	0.2	3.00	100
40	83.4694376	0.6	3.00	100
40	72.0454644	1.0	3.00	100
40	41.8379886	2.0	3.00	100
40	97.2922094	0.2	6.00	100
40	91.8210460	0.6	6.00	100
40	86.2731706	1.0	6.00	100
40	72.0454644	2.0	6.00	100

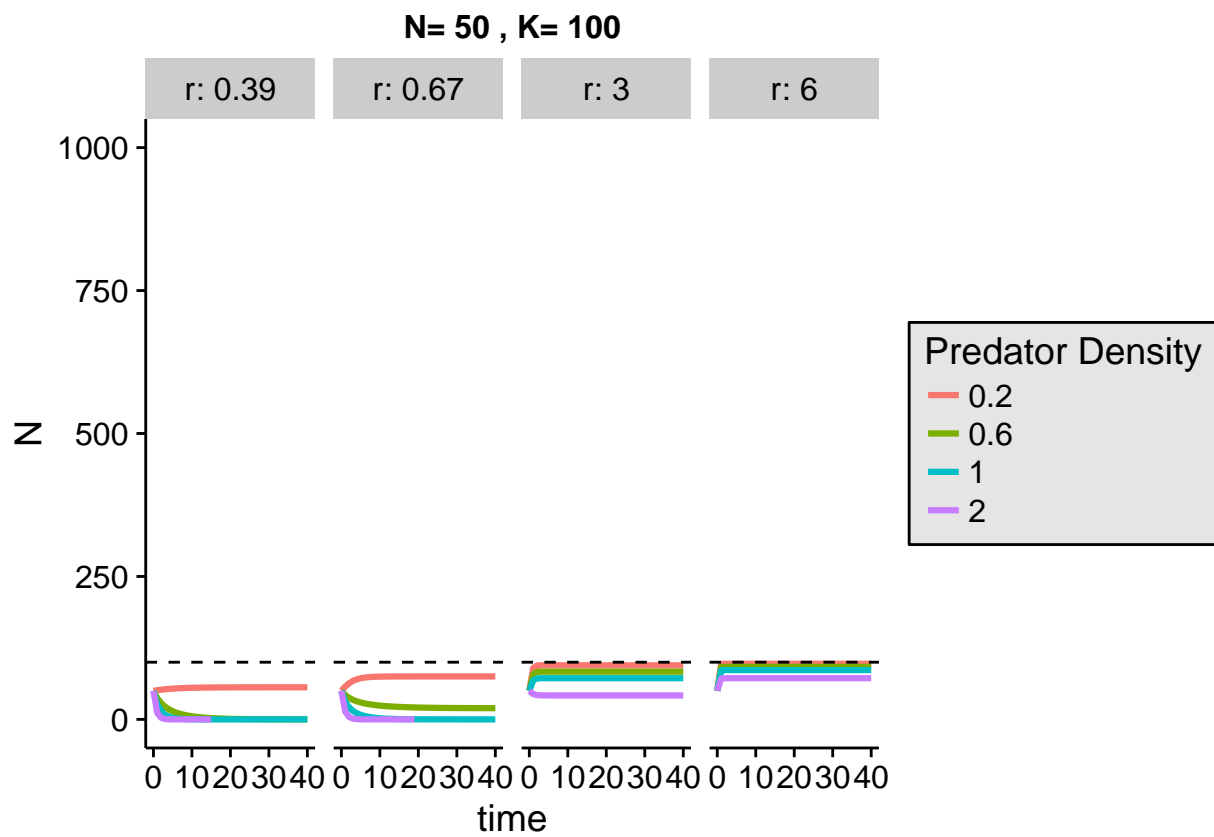
Table 18: N init= 2

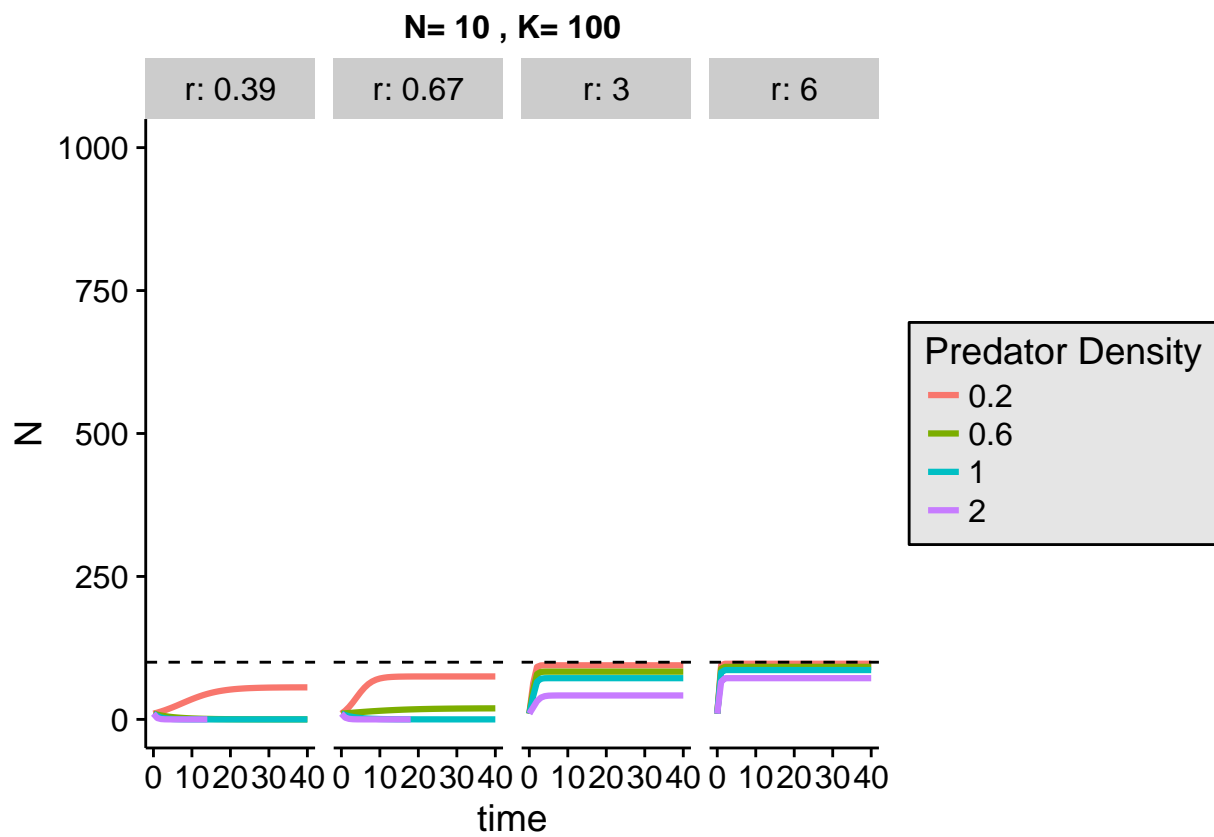
time	N	P	r	K
40	55.6943350	0.2	0.39	100
40	0.0026851	0.6	0.39	100
40	0.0000000	1.0	0.39	100
40	0.0000000	2.0	0.39	100
40	75.0625578	0.2	0.67	100
40	17.9437290	0.6	0.67	100
40	0.0000742	1.0	0.67	100
40	0.0000000	2.0	0.67	100
40	94.5660182	0.2	3.00	100
40	83.4694376	0.6	3.00	100
40	72.0454644	1.0	3.00	100
40	41.8379886	2.0	3.00	100
40	97.2922094	0.2	6.00	100
40	91.8210460	0.6	6.00	100
40	86.2731706	1.0	6.00	100
40	72.0454644	2.0	6.00	100

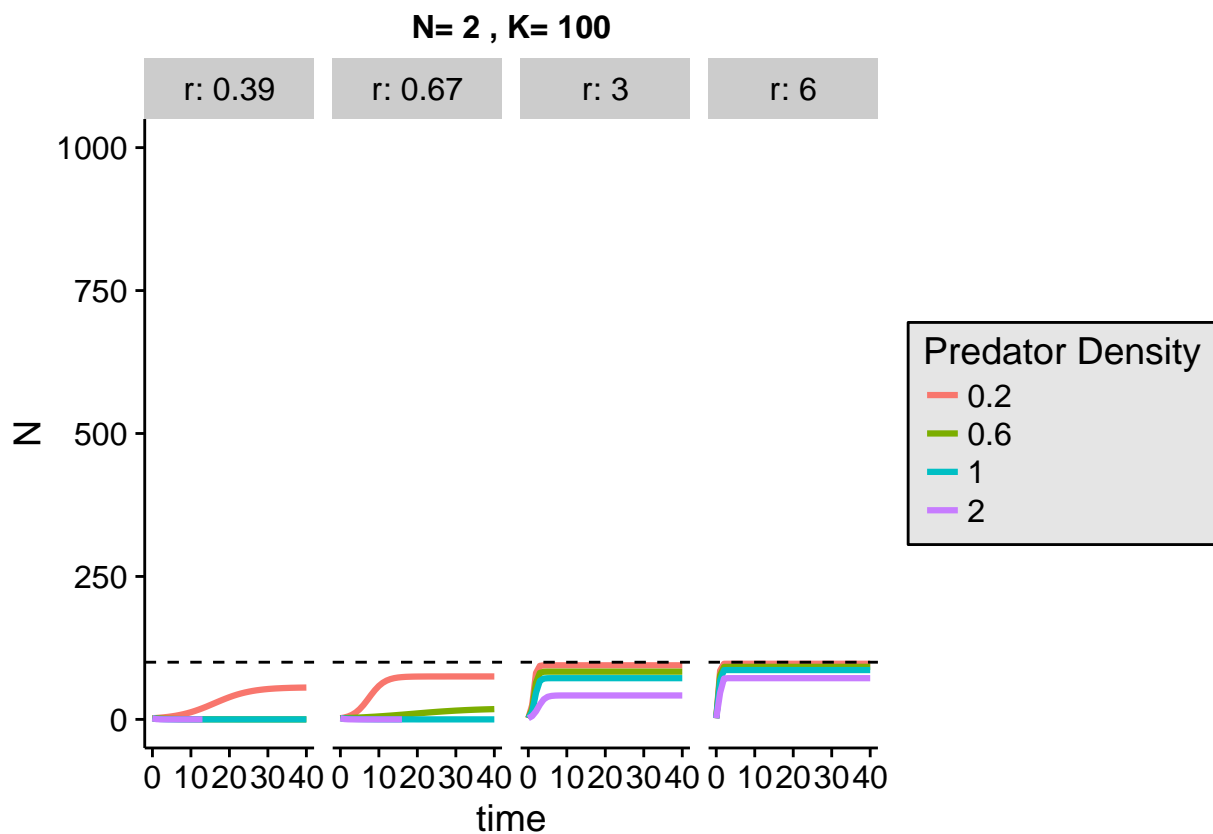












Simulations, K=50

Table 19: N init= 1000

time	N	P	r	K
40	27.1910915	0.2	0.39	50
40	0.0317032	0.6	0.39	50
40	0.0000000	1.0	0.39	50
40	0.0000000	2.0	0.39	50
40	36.8921731	0.2	0.67	50
40	9.2444898	0.6	0.67	50
40	0.0007904	1.0	0.67	50
40	0.0000000	2.0	0.67	50
40	47.1122172	0.2	3.00	50
40	41.2690721	0.6	3.00	50
40	35.3322003	1.0	3.00	50
40	20.0483867	2.0	3.00	50
40	48.5588703	0.2	6.00	50
40	45.6599767	0.6	6.00	50
40	42.7384714	1.0	6.00	50
40	35.3322003	2.0	6.00	50

Table 20: N init= 500

time	N	P	r	K
40	27.1908959	0.2	0.39	50
40	0.0310281	0.6	0.39	50
40	0.0000000	1.0	0.39	50
40	0.0000000	2.0	0.39	50
40	36.8921731	0.2	0.67	50
40	9.2436802	0.6	0.67	50
40	0.0007753	1.0	0.67	50
40	0.0000000	2.0	0.67	50
40	47.1122172	0.2	3.00	50
40	41.2690721	0.6	3.00	50
40	35.3322003	1.0	3.00	50
40	20.0483867	2.0	3.00	50
40	48.5588703	0.2	6.00	50
40	45.6599767	0.6	6.00	50
40	42.7384714	1.0	6.00	50
40	35.3322003	2.0	6.00	50

Table 21: N init= 100

time	N	P	r	K
40	27.1893131	0.2	0.39	50
40	0.0264027	0.6	0.39	50
40	0.0000000	1.0	0.39	50
40	0.0000000	2.0	0.39	50
40	36.8921731	0.2	0.67	50
40	9.2370957	0.6	0.67	50
40	0.0006697	1.0	0.67	50
40	0.0000000	2.0	0.67	50
40	47.1122172	0.2	3.00	50
40	41.2690721	0.6	3.00	50
40	35.3322003	1.0	3.00	50
40	20.0483867	2.0	3.00	50
40	48.5588703	0.2	6.00	50
40	45.6599767	0.6	6.00	50
40	42.7384714	1.0	6.00	50
40	35.3322003	2.0	6.00	50

Table 22: N init= 50

time	N	P	r	K
40	27.1873248	0.2	0.39	50
40	0.0222152	0.6	0.39	50
40	0.0000000	1.0	0.39	50
40	0.0000000	2.0	0.39	50
40	36.8921731	0.2	0.67	50
40	9.2288110	0.6	0.67	50
40	0.0005713	1.0	0.67	50
40	0.0000000	2.0	0.67	50

time	N	P	r	K
40	47.1122172	0.2	3.00	50
40	41.2690721	0.6	3.00	50
40	35.3322003	1.0	3.00	50
40	20.0483867	2.0	3.00	50
40	48.5588703	0.2	6.00	50
40	45.6599767	0.6	6.00	50
40	42.7384714	1.0	6.00	50
40	35.3322003	2.0	6.00	50

Table 23: N init= 10

time	N	P	r	K
40	27.171372	0.2	0.39	50
40	0.009768	0.6	0.39	50
40	0.000000	1.0	0.39	50
40	0.000000	2.0	0.39	50
40	36.892173	0.2	0.67	50
40	9.162756	0.6	0.67	50
40	0.000262	1.0	0.67	50
40	0.000000	2.0	0.67	50
40	47.112217	0.2	3.00	50
40	41.269072	0.6	3.00	50
40	35.332200	1.0	3.00	50
40	20.048387	2.0	3.00	50
40	48.558870	0.2	6.00	50
40	45.659977	0.6	6.00	50
40	42.738471	1.0	6.00	50
40	35.332200	2.0	6.00	50

Table 24: N init= 2

time	N	P	r	K
40	27.0916748	0.2	0.39	50
40	0.0025679	0.6	0.39	50
40	0.0000000	1.0	0.39	50
40	0.0000000	2.0	0.39	50
40	36.8921706	0.2	0.67	50
40	8.8453990	0.6	0.67	50
40	0.0000706	1.0	0.67	50
40	0.0000000	2.0	0.67	50
40	47.1122172	0.2	3.00	50
40	41.2690721	0.6	3.00	50
40	35.3322003	1.0	3.00	50
40	20.0483867	2.0	3.00	50
40	48.5588703	0.2	6.00	50
40	45.6599767	0.6	6.00	50
40	42.7384714	1.0	6.00	50
40	35.3322003	2.0	6.00	50

