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# *4D GEMEOS Spatially-Extended Photon-Electron Soliton Gauge Group*

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General information field theory theorem and proof is shown for QED 0D⟶4D spacetime representations of photon and electron energy, mass, and angular momentum observables based on wavelength, energy level *n*, multivariable calculus, and the observables experimental values. ForAll wavelengths, a 4D cylindrical coordinate integration computationally reproduces the photon energy observable, holding ForAll wavelengths with a spin-conversion to computationally dualistic mass density moment of inertia times angular velocity integration rendering the angular momentum observable. ForAll electron energy levels *n* a free space monopole 4D spherical coordinate parameterization of the Bohr radii, with conversion to computationally dualistic mass density integration, computationally reproduces the electron mass observable, holding ForAll *n* with a moment of inertia times angular velocity integration rendering the angular momentum observable. ForAll wavelengths and energy levels the 4D photon and electron integrations range compressive through rarefactive of central cosmological constant vacuum energy momentum density , spanning all the factors in the relativistic energy equation to all 31 and 34 decimal places, in all cases greater than zero hence computationally reproducing a solution of the Yang-Mills-Navier-Stokes Millennium prize problems and a basis for a 4D spacetime human-ai experience worldview.

## 4D Spacetime Quantization

> "Physical objects are not in space, but these objects are spatially extended. In this way the concept 'empty space' loses its meaning." Albert Einstein 1952 [1].

QED models the photon $\gamma$ and electron $e^-$ (positron $e^+$) *ab initio* as zero-dimensional (0D) Dirac delta functional $\delta$ mathematical point particles $\delta_\gamma$ and $\delta_e$. However useful to the scientific method said theoretical first approximation has proven to be though, an imaginary invisible 0D mathematical point $\delta_\gamma$ conveys no information content towards the 4D antenna engineering spatially-extended wavelength $\lambda$ parameterizations and their characteristic far field radiation patterns. Further, the 4D physical dimensions and chemical properties of molecular dynamics interactions are, for the most part, properties of their electron clouds. Regarding which an imaginary invisible mathematical point $\delta_e$ conveys no information content towards the 4D density functional theory (DFT) continuum mechanical spatially-extended energy densities.

Consequently, many researchers are attracted to the logic, as stated by Wolfram, that since electrons have zero size and no substructure they must be made up of more fundamental elements [2]. The present 4D spacetime quantization information is based on the observed 4D flat universal spacetime energy density distribution of $\delta_{\text{Pa}}$ mathematical points, with time the fourth dimension of length from $t_{-\infty} \to t_\infty$, in units of pascals, being the fundamental mathematical physics object(s) upon which the present QED 0D⟶4D photon cylindrical coordinate and electron spherical coordinate integrations range from compressive through rarefactive of the central cosmological constant vacuum energy momentum density $\Lambda_{\text{Pa}}$.

The Einstein field equations of 1915 established the total field formal frame

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\,\pi\, T_{\mu\nu}, \tag{1}$$

wherein the Einstein curvature tensor $G_{\mu\nu}$, metric tensor $g_{\mu\nu}$, stress energy momentum density tensor $T_{\mu\nu}$, are in geometrized units with Newton's gravitational constant $G$ = the speed of light $c$ =1. For an electromagnetic field, in the otherwise "empty space" of $\Lambda$, the time-time matrix element of $T_{\mu\nu}$ is the relativistic mass density $T_{00} = \frac{1}{2\,c^2}(\epsilon_0\, E^2 + \frac{1}{\mu_0}\, B^2)$ in which $E$ and $B$ are the electric and magnetic fields respectively. So that for a free "empty spacetime" point singularity one can write $\Lambda_{\text{Pa}} = \Lambda_{T_{00}} = \delta_{T_{00}}$, regarding which multiplying by $c^2$ renders the electromagnetic stress energy momentum density tensor

$$T_{\mu\nu} = \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0\, E^2 + \frac{1}{\mu_0}\, B^2\right) & S_x / c & S_y / c & S_z / c \\ S_x / c & -\sigma_{\text{xx}} & -\sigma_{\text{xy}} & -\sigma_{\text{xz}} \\ S_y / c & -\sigma_{\text{yx}} & -\sigma_{\text{yy}} & -\sigma_{\text{yz}} \\ S_z / c & -\sigma_{\text{zx}} & -\sigma_{\text{zy}} & -\sigma_{\text{zz}} \end{pmatrix}, \tag{2}$$

where $\mathbf{S} = \frac{1}{\mu_0}\, \mathbf{E} \times \mathbf{B}$ is the Poynting energy flux vector, and $\sigma_{\text{ij}}$ are the Maxwell stress tensor components.

The reader will note in particular, in that $T_{\mu\nu}$ is expressed and measured in pressure units, e.g., pascals along the trace of $\sigma_{\mathrm{ij}}$, and free space has energy density value $\Lambda_{\mathrm{Pa}}$, the computational duality exists wherein both $\Lambda_{\mathrm{J}}$ energy density and $\Lambda_{\mathrm{kg}}$ mass density are expressed by the electromagnetic stress tensor form of $T_{\mu\nu}$ with the same units of pascals. Considering of course one never actually measures the gravitational or electromagnetic forces completely separately; including in the high-energy synchrotron collisions which generally are energized, controlled, and measured, to the furthest technological extent possible, by means of the electromagnetic stress energy momentum density tensor $T_{\mu\nu}$.

Recall further the original objection by mathematicians to the physical representation of the electron by the Dirac delta functional singularity $\delta_{\mathrm{e}}$, for the reason of course that $\delta$ is not actually a function, hence termed a functional since it only makes sense inside of an integral. Such as, for example, in every case of the present QED 0D⟶4D spacetime $\int \delta_{T_{00}}$ integrations representing spatially-extended expansions of the original Dirac mathematical point particle singularities $\delta_{\gamma}$ and $\delta_{\mathrm{e}}$.

## ■ Theorem

$$\forall\lambda \text{ Eq.}(6) == \frac{hc}{\lambda} \longrightarrow \text{Eq.}(9) == \hbar \cup \forall n \text{ Eq.}(13) == m_{\mathrm{e}} \longrightarrow \text{Eq.}(15) == \frac{\hbar}{2}$$

*ForAll wavelengths $\lambda$ Eq.(6) renders the Einstein-Planck photon energy observable $E = hc/\lambda$ ⟶ holding ForAll $\lambda$ with the computationally dualistic conversion to mass density moment of inertia $I \times$ angular velocity $\omega_{yy}$ integration of Eq.(9) rendering the angular momentum observable $\hbar$. In **U**nion with, ForAll energy levels n the free space electron monopole spherical coordinate integration Eq.(13) computationally dualistic conversion to mass density renders the electron mass invariant observable $\mathbf{m_e}$= $9.109 \times 10^{-31}$ kg* **⟶** *holding ForAll n with the mass density moment of inertia $I \times$ angular velocity $\omega_{zz}$ integration of Eq.(15) rendering the angular momentum invariant observable $\hbar/2$.*

## ■ Lemma

*The values of the cosmological constant vacuum energy momentum density $\Lambda_{Pa}$ are assumed to be the computationally dualistic values calculated by Baez and Tatom, wherein*

$$\text{energy density}\,\Lambda_J \approx 6 \times 10^{-10}\, J\, m^{-3} \text{ and mass density}\,\Lambda_{\mathrm{kg}} = \Lambda_J / c^2 \approx 7 \times 10^{-27}\, \mathrm{kg}\, m^{-3} [3]. \tag{3}$$

## Einstein-Planck Photon Energy Observable

*ForAll wavelengths* $\forall\lambda$ *the spatially-extended volume of the cylindrical coordinate transverse lemniscate expansion of the Poynting energy flux vector* $\boldsymbol{S} = 1 / \mu_0\, \boldsymbol{E} \times \boldsymbol{B}$*, over one wavelength*

$$4 \int_0^{\lambda} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\frac{\lambda}{4}\sqrt{\cos(2\theta)}} \left| \sin\left(\left(\frac{2\pi}{\lambda}\right) T_{yy}\right) \right| \left(1 - \left(\frac{r}{\frac{\lambda}{4}\sqrt{\cos(2\theta)}}\right)^2\right) r\, d\theta\, dT_{yy} = \frac{\lambda^3}{8\pi}, \tag{4}$$

*wherein time integrates along the* $T_{yy}$ *axis of propagation of the transverse travelling wave, is integrated throughout via the maximum energy density* $\rho_0^{\lambda\ max}$*, which at r=0 is twice the average energy density* $\rho^{\lambda\ avg}$*, according to*

$$\rho_0^{\lambda\,\max} = \rho_0^{\lambda} = 2 \times \rho^{\lambda\,\text{avg}} = \times\, 2\left(\frac{hc}{\lambda}\right) \Big/ \left(\frac{\lambda^3}{8\pi}\right) J\, m^{-3}. \tag{5}$$

*Thus rendering a 4D spacetime volumetric expansion, of the* $\delta_\gamma$ *representation of the Einstein-Planck photon energy observable*

$$4 \int_0^{\lambda} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\frac{\lambda}{4}\sqrt{\cos(2\theta)}} \rho_0^{\lambda} \left| \sin\left(\left(\frac{2\pi}{\lambda}\right) T_{yy}\right) \right| 1 - \left(\frac{r}{\frac{\lambda}{4}\sqrt{\cos(2\theta)}}\right)^2 r\, d\theta\, dT_{yy} == \frac{hc}{\lambda}. \tag{6}$$

Hence ForAll wavelengths of the electromagnetic spectrum Eq. (5) renders

```
h = QuantityMagnitude[ h , "Joules" "Seconds"];
c = QuantityMagnitude[ c , "Meters" "Seconds"^-1];
PhotonEnergy =
 N[ForAll[λ, UniformDistribution[{1.*^-12, 1.*^4}],
   ∫_0^λ ∫_(-π/4)^(π/4) ∫_0^(1/4 λ Sqrt[Cos[2 θ]]) Abs[Sin[(2 π/λ) y]] 2 ((2*h*c/λ)/(λ^3/(12 π)))
     (1 - (r/(1/4 λ Sqrt[Cos[2 θ]]))) r ⅆr ⅆθ ⅆy == (h * c)/λ]]

True
```

## Einstein-Planck Photon Angular Momentum Observable

*ForAll wavelengths* $\forall\lambda$ *the computationally dualistic conversion to mass density*

$$\mu_0^{\lambda\,\max} = \mu_0^\lambda = \rho_0^\lambda \,/\, c^2 = \left(2\,(hc\,/\,\lambda)\,/\,\left(\lambda^3\,/\,8\,\pi\right)\right)/\,c^2\;kg\;m^{-3}, \tag{7}$$

*for the moment of inertia integration I throughout the volume of Eq.(3), times the transverse spin angular velocity*

$$\omega_{\text{Transverse}} \;=\; \omega_{\text{yy}} \;=\; \left(\frac{c}{\lambda}\right)\left(\frac{h}{I\left(\frac{2\,\pi\,c}{\lambda}\right)}\right)\text{rad s}^{-1}, \tag{8}$$

*renders a 4D spacetime representation of the original Einstein-Planck photon* $\gamma$ *angular momentum* $I\omega$ *observable*

$$\int_0^\lambda\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{\frac{1}{4}\lambda\,\sqrt{\cos(2\,\theta)}} \mu_0^\lambda \left|\sin\left(\left(\frac{2\,\pi}{\lambda}\right)T_{\text{yy}}\right)\right|\left(1-\left(\frac{r}{\frac{1}{4}\,\lambda\,\sqrt{\cos(2\,\theta)}}\right)^2\right)\omega_T\,r\,dr\,d\theta\,dT_{\text{yy}} \;=\; \hbar. \tag{9}$$

Hence ForAll wavelengths of the electromagnetic spectrum Eq.(8) renders

```
h = QuantityMagnitude[h, "Joules" "Seconds"];
hbar = QuantityMagnitude[h/(2 π), "Joules" "Seconds"];
c = QuantityMagnitude[c, "Meters" "Seconds"^-1];
N[ForAll[λ, UniformDistribution[{1.*^-12, 1.*^4}],
```

$$\int_0^{\lambda}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{\frac{1}{4}\lambda\sqrt{\mathrm{Cos}[2\,\theta]}} 2\left(\frac{\frac{2*h*c}{\lambda}}{\frac{\lambda^3}{8\,\pi}}\right) \Big/ \mathbf{c}^2\,\mathbf{Abs}\left[\mathbf{Sin}\left[\left(\frac{2\,\pi}{\lambda}\right)\mathbf{y}\right]\right] \left(1-\left(\frac{\mathbf{r}}{\frac{1}{4}\lambda\sqrt{\mathrm{Cos}[2\,\theta]}}\right)^2\right) \left(\frac{\mathbf{c}}{\lambda}\left(\mathbf{h}\Big/\left(\int_0^{\lambda}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{\frac{1}{4}\lambda\sqrt{\mathrm{Cos}[2\,\theta]}} 2\left(\frac{\frac{2*h*c}{\lambda}}{\frac{\lambda^3}{8\,\pi}}\right) \Big/ \mathbf{c}^2\,\mathbf{Abs}\left[\mathbf{Sin}\left[\left(\frac{2\,\pi}{\lambda}\right)\mathbf{y}\right]\right] \left(1-\left(\frac{\mathbf{r}}{\frac{1}{4}\lambda\sqrt{\mathrm{Cos}[2\,\theta]}}\right)^2\right)\left(\frac{2\,\pi\,\mathbf{c}}{\lambda}\right)\mathbf{r}\,d\mathbf{r}\,d\theta\,d\mathbf{y}\right)\right)\right) \mathbf{r}\,d\mathbf{r}\,d\theta\,d\mathbf{y} == \mathbf{hbar}]]$$

```
True
```

## Electron Rest Mass Observable

*ForAll energy levels ∀n the free space electron monopole spherical coordinate volumetric expansion*

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{r_n} r^2 \sin(\phi)\, dr\, d\phi\, d\theta \;=\; \frac{4\pi r_n^3}{3}, \tag{10}$$

*of the 0D Dirac delta functional $\delta_e$ representation of the electron's computationally dualistic electron observables, of rest energy $8.187\times 10^{-14}$ J and rest mass $m_e$ = $9.109\times 10^{-31}$kg, is parameterized by the Bohr radius $a_0$= $5.292\times 10^{-11}$m, according to*

$$r_n = n^2 a_0 \sqrt{2}\ , \tag{11}$$

*ranging dynamically according to its maximum mass density being four times its average mass density*

$$\mu_{@r=0}^{n\,\max} = \rho_{@r=0}^{n\,\max} / c^2 = 4\left(\left(m_e c^2\right)/\left(4\pi r_n^3/3\right)\right)/c^2 \text{ kg } m^{-3}, \tag{12}$$

*so that the mass density function is at a maximum $\mu_0^n$ at $r$ = 0 and falls to zero at $r = n^2 a_0 \sqrt{2}$, according to the integrand factor $1 - (r / r_n)$ rendering the electron mass observable*

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{r_n} \mu_0^n \left(1 - \frac{r}{r_n}\right) r^2 \sin(\phi)\, dr\, d\phi\, d\theta \;=\; m_e. \tag{13}$$

Hence ForAll electron energy levels Eq. (13) renders

```
c = QuantityMagnitude[ c , "Meters" "Seconds"^-1];
eEnergy = QuantityMagnitude[ mₑ c² , "Joules"];
me = QuantityMagnitude[ mₑ , "Kilograms"];
bohr = QuantityMagnitude[ a₀ , "Meters"];
N[ForAll[n, UniformDistribution[{1, 10000}],
  ∫_0^(2π) ∫_0^π ∫_0^(√2*n²*bohr) ((4*eEnergy / ((4 π (n²*bohr*√2)³)/3)) / c²) (1 - r / (√2 * n² * bohr))
     r² Sin[ϕ] ⅆr ⅆϕ ⅆθ == me]]
```

```
True
```

## Electron Angular Momentum Observable

*ForAll energy levels ∀n Eq. (12) times the spin angular velocity*

$$\omega_{zz} = \left(hc / 8\,\pi^2\, r_n\right) \Big/ \int_0^{2\pi}\int_0^{\pi}\int_0^{r_n} \mu_0^n\left(1-\frac{r}{r_n}\right)\left(\frac{c}{2\,\pi \mathrm{r}_n}\right) r^2 \sin(\phi)\, dr\, d\phi\, d\theta \quad \mathrm{rad\ s}^{-1}, \tag{14}$$

*wherein time integrates along the $T_{zz}$ axis of Eq.(2), renders the 4D electron angular momentum observable*

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{r_n} \mu_0^n\left(1-\frac{r}{r_n}\right)\omega_{zz}\, r^2 \sin(\phi)\, dr\, d\phi\, d\theta == \frac{\hbar}{2}. \tag{15}$$

Hence ForAll electron energy levels Eq. (14) renders

```
hbar = QuantityMagnitude[h/(2 π), "Joules" "Seconds"];
c = QuantityMagnitude[c, "Meters" "Seconds"^-1];
eEnergy = QuantityMagnitude[me c^2, "Joules"];
me = QuantityMagnitude[me, "Kilograms"];
bohr = QuantityMagnitude[a0, "Meters"];
N[ForAll[n, UniformDistribution[{1, 10 000}],
```

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{2}*n^2*bohr}\left(\frac{\frac{4*eEnergy}{\left(4\,\pi\,\left(n^2*bohr*\sqrt{2}\right)^3\right)/3}}{c^2}\right)\left(1-\frac{r}{\sqrt{2}*n^2*bohr}\right)\left(\frac{c}{2\,\pi*\sqrt{2}*n^2*bohr}*\left(\frac{hbar}{2}\middle/\left(\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{2}*n^2*bohr}\left(\frac{\frac{4*eEnergy}{\left(4\,\pi\,\left(n^2*bohr*\sqrt{2}\right)^3\right)/3}}{c^2}\right)\left(1-\frac{r}{\sqrt{2}*n^2*bohr}\right)\left(\frac{c}{2\,\pi*\sqrt{2}*n^2*bohr}\right)r^2\,Sin[\phi]\,dr\,d\phi\,d\theta\right)\right)\right)r^2\,Sin[\phi]\,dr\,d\phi\,d\theta == \frac{hbar}{2}]]$$

```
True
```

## ■ Proof

ForAll wavelengths $\forall\lambda$ the cylindrical coordinate photon energy integration function Eq.(6) == $hc/\lambda \longrightarrow$ holding $\forall\lambda$ with the 4D photon angular momentum I$\omega$ integration function Eq.(9) == $\hbar$ in Union with ForAll energy levels $\forall n$ the spherical coordinate electron mass integration function Eq.(13) == $m_e \longrightarrow$ holding $\forall n$ with the 4D electron angular momentum I$\omega$ integration function Eq.(15) == $\hbar/2$, renders

```
Proof = DynamicModule[
  {h = QuantityMagnitude[Quantity[1, "PlanckConstant"],
     "Joules" "Seconds"],
   hbar = QuantityMagnitude[Quantity[1, "PlanckConstant"]/(2 π),
     "Joules" "Seconds"],
   c = QuantityMagnitude[Quantity[1, "SpeedOfLight"],
     "Meters"/"Seconds"],
   eEnergy = QuantityMagnitude[
     Quantity[1, "ElectronMass" "SpeedOfLight"^2], "Joules"],
   me = QuantityMagnitude[Quantity[1, "ElectronMass"],
     "Kilograms"],
   bohr = QuantityMagnitude[Quantity[1, "BohrRadius"],
     "Meters"]},
```

$$\forall_{\{\lambda,n\},\text{UniformDistribution}[\{\{1.*\text{^}-12,10000.\},\{1,10000\}\}]}\left(\int_0^{\lambda}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{\frac{1}{4}\lambda\sqrt{\text{Cos}[2\,\theta]}}\frac{2\,r\,(2\,c\,h)\,\text{Abs}\left[\text{Sin}\left[\frac{(2\,\pi)\,y}{\lambda}\right]\right]\left(1-\left(\frac{r}{\frac{1}{4}\lambda\sqrt{\text{Cos}[2\,\theta]}}\right)^2\right)}{\frac{\lambda\,\lambda^3}{8\,\pi}}\right.$$

$$\left.dr\,d\theta\,dy == \frac{c\,h}{\lambda} \Rightarrow\right.$$

$$\int_0^{\lambda}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{\frac{1}{4}\lambda\sqrt{\text{Cos}[2\,\theta]}} 2\left(\frac{\frac{2*h*c}{\lambda}}{\frac{\lambda^3}{8\,\pi}}\right)\Big/\ c^2\,\text{Abs}\left[\text{Sin}\left[\left(\frac{2\,\pi}{\lambda}\right)y\right]\right]$$

$$\left(1-\left(\frac{r}{\frac{1}{4}\,\lambda\,\sqrt{\mathrm{Cos}[2\,\theta]}}\right)^2\right)$$

$$\left(\frac{c}{\lambda}\left(h\Big/\left(\int_0^{\lambda}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{\frac{1}{4}\lambda\sqrt{\mathrm{Cos}[2\,\theta]}} 2\left(\frac{\frac{2*h*c}{\lambda}}{\frac{\lambda^3}{8\,\pi}}\right)\Big/\,c^2\right.\right.\right.$$

$$\mathrm{Abs}\left[\mathrm{Sin}\left[\left(\frac{2\,\pi}{\lambda}\right)y\right]\right]\left(1-\left(\frac{r}{\frac{1}{4}\,\lambda\,\sqrt{\mathrm{Cos}[2\,\theta]}}\right)^2\right)$$

$$\left.\left.\left.\left(\frac{2\,\pi\,c}{\lambda}\right)r\,dr\,d\theta\,dy\right)\right)\right)r\,dr\,d\theta\,dy == \mathrm{hbar}\;\&\&$$

$$\int_0^{2\,\pi}\int_0^{\pi}\int_0^{\sqrt{2}\,\mathrm{bohr}\,n^2}\frac{(4\,\mathrm{eEnergy})\,r^2\,\mathrm{Sin}[\phi]\left(1-\frac{r}{\sqrt{2}\,\mathrm{bohr}\,n^2}\right)}{\frac{1}{3}\,c^2\left(4\,\pi\left(\sqrt{2}\,\mathrm{bohr}\,n^2\right)^3\right)}$$

$$dr\,d\phi\,d\theta == \mathrm{me} \Rightarrow$$

$$\int_0^{2\,\pi}\int_0^{\pi}\int_0^{\sqrt{2}\,\mathrm{bohr}\,n^2}\left(\left((4\,\mathrm{eEnergy})\,r^2\,(c\,\mathrm{hbar})\,\mathrm{Sin}[\right.\right.$$

$$\left.\left.\phi]\left(1-\frac{r}{\sqrt{2}\,\mathrm{bohr}\,n^2}\right)\right)\Big/\right.$$

$$\left(\frac{1}{3}\left(c^2\left(4\,\pi\left(\sqrt{2}\,\mathrm{bohr}\,n^2\right)^3\right)\right)\right.$$

$$\left(\left(2\,\pi\,\sqrt{2}\,\mathrm{bohr}\,n^2\right)\right.$$

$$\left(2\int_0^{2\,\pi}\int_0^{\pi}\int_0^{\sqrt{2}\,\mathrm{bohr}\,n^2}\left(\left(c\,(4\,\mathrm{eEnergy})\,r^2\right.\right.\right.$$

$$\left.\left.\mathrm{Sin}[\phi]\left(1-\frac{r}{\sqrt{2}\,\mathrm{bohr}\,n^2}\right)\right)\Big/\right.$$

$$\left(\frac{1}{3}\left(2\,\pi\,\sqrt{2}\,\mathrm{bohr}\,n^2\right)\right.$$

$$\left.\left.\left.\left(c^2\left(4\,\pi\left(\sqrt{2}\,\mathrm{bohr}\,n^2\right)^3\right)\right)\right)\right)$$

$$\left.\left.\left.\left.\left.\mathbb{d}\mathbf{r}\,\mathbb{d}\phi\,\mathbb{d}\theta\right)\right)\right)\right)\mathbb{d}\mathbf{r}\,\mathbb{d}\phi\,\mathbb{d}\theta == \frac{\mathbf{hbar}}{\mathbf{2}}\right)\Big]$$

```
UniformDistribution[{{1. × 10^-12, 1.000 × 10^4}, {1, 10 000}}] ⇒
 True ⇒ True && True ⇒ True
```

□

## Conclusion

We have revisited the first principles of modern physics and expanded a local stress tensor operator on $T_{\mu\nu}$ to render QED 0D⟶4D spacetime constructive quantum field theory representations of a photon-electron soliton gauge group. The first two matrix elements of which gauge group are the Eq.(9) photon angular momentum operator $-\mathrm{I}\omega_{\lambda}^{\gamma}\left|\sigma_{\mathrm{yy}}\right\rangle$ and the Eq.(15) electron angular momentum operator $-\mathrm{I}\omega_{n}^{\mathrm{e}}\left|\sigma_{\mathrm{zz}}\right\rangle$, wherein the $Y_m^l$ term indicates the conjecture for the further smooth operator product expansion of the local Schwinger-Dirac-Einstein-Maxwell stress tensor to render the full Laplacian spherical harmonics along the trace of

$$T_{\mu\nu} = \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) & S_x / c & S_y / c & S_z / c \\ S_x / c & Y_m^l & -\sigma_{\mathrm{xy}} & -\sigma_{\mathrm{xz}} \\ S_y / c & -\sigma_{\mathrm{yx}} & -\mathrm{I}\omega_{\lambda}^{\gamma}\left|\sigma_{\mathrm{yy}}\right\rangle & -\sigma_{\mathrm{yz}} \\ S_z / c & -\sigma_{\mathrm{zx}} & -\sigma_{\mathrm{zy}} & -\mathrm{I}\omega_{n}^{\mathrm{e}}\left|\sigma_{\mathrm{zz}}\right\rangle \end{pmatrix}. \tag{16}$$

Thus $\forall \lambda n$ the Theorem self-adjoint group operation total field dynamics ranges compressive through rarefactive of the central cosmological constant vacuum energy density $\Lambda_{\mathtt{Pa}}$ spanning all the factors in the relativistic energy equation $E^2 = \left(m_0\, c^2\right)^2 + (pc)^2$ in all instances > 0 establishing a computationally reproducible basis for a 4D spacetime human-ai experience worldview.

Hence thesis success.

qed

## References


[1] A. Einstein, *Relativity: The Special and the General Theory*, Indirapuram: Samaira Book Publishers, 2017 Note to the Fifteenth Edition.

[2] S. Wolfram, “A New Kind of Science.” (May 16, 2017) www.wolframscience.com/nks/p525--elementary-particles/

[3] J. Baez and F. B. Tatom, “What’s the Energy Density of the Vacuum?”(2011) math.ucr.edu/home/baez/vacuum.html


### About the Author


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