

# Semantics III

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## Analyzing a Mandarin topic chain in PLA

Analysis tree for  $\llbracket \exists x(x = a \wedge Y^1 x \wedge P^1 x) \rrbracket^{\mathcal{M},g}$

$$\begin{array}{c}
 \exists x \quad \frac{(x = a)}{\{i \in c : g(x) = \llbracket a \rrbracket_{\mathcal{M}}\}} \quad \wedge \quad \frac{Y^1 x}{\{i \in c : g(x) \in \llbracket Y^1 \rrbracket_{\mathcal{M}}\}} \quad \wedge \quad \frac{P^1 x}{\{i \in c : g(x) \in \llbracket P^1 \rrbracket_{\mathcal{M}}\}} \\
 \hline
 = c_2 \\
 \hline
 c_2 \llbracket Y^1 x \rrbracket^{\mathcal{M},g} \\
 = \{j \in c_2 : g(x) \in \llbracket Y^1 \rrbracket_{\mathcal{M}}\} \\
 = \{i \in c : g(x) = \llbracket a \rrbracket_{\mathcal{M}} \& g(x) \in \llbracket Y^1 \rrbracket_{\mathcal{M}}\} \\
 \hline
 \{i \in c : g(x) = \llbracket a \rrbracket_{\mathcal{M}} \& g(x) \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \& g(x) \in \llbracket P^1 \rrbracket_{\mathcal{M}}\} \\
 \hline
 \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \& i \in c \llbracket x = a \wedge Y^1 x \wedge P^1 x \rrbracket^{\mathcal{M},g[x/d]}\} \\
 = \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \& i \in \{j \in c : d = \llbracket a \rrbracket_{\mathcal{M}} \& d \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \& d \in \llbracket P^1 \rrbracket_{\mathcal{M}}\}\} \\
 = \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \& d = \llbracket a \rrbracket_{\mathcal{M}} \& d \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \& d \in \llbracket P^1 \rrbracket_{\mathcal{M}}\}
 \end{array}$$

## Updating the null info. state

$$\begin{array}{l}
 \{\langle \rangle\} \llbracket \exists x(x = a \wedge Y^1 x \wedge P^1 x) \rrbracket^{\mathcal{M},g} \\
 = \{a : a = \llbracket a \rrbracket_{\mathcal{M}} \& a \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \& a \in \llbracket P^1 \rrbracket_{\mathcal{M}}\} \\
 = c_1
 \end{array}$$

Analysis tree for  $\llbracket \exists x(J^2 x \top_1 \wedge G^1 x) \rrbracket^{\mathcal{M},g}$

$$\begin{array}{c}
 \exists x \quad \frac{(J^2 x \top_1)}{\{i \in c : \langle g(x), i_1 \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}}\}} \quad \wedge \quad \frac{G^1 x}{\{i \in c : g(x) \in \llbracket G^1 \rrbracket_{\mathcal{M}}\}} \\
 \hline
 \{i \in c : \langle g(x), i_1 \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \& g(x) \in \llbracket G^1 \rrbracket_{\mathcal{M}}\} \\
 \hline
 \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \& i \in c \llbracket J^2 x \top_1 \wedge G^1 x \rrbracket^{\mathcal{M},g[x/d]}\} \\
 = \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \& i \in \{j \in c : \langle d, j_1 \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \& d \in \llbracket G^1 \rrbracket_{\mathcal{M}}\}\} \\
 = \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \& i \in c \& \langle d, i_1 \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \& d \in \llbracket G^1 \rrbracket_{\mathcal{M}}\}
 \end{array}$$

## Updating info. state $c_1$

$$\begin{array}{l}
 c_1 \llbracket \exists x(J^2 x \top_1 \wedge G^1 x) \rrbracket^{\mathcal{M},g} \\
 = \{j \cdot i : i \in c_1 \& \langle j, i_1 \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \& j \in \llbracket G^1 \rrbracket_{\mathcal{M}}\} \\
 = \{\langle j, a \rangle : a = \llbracket a \rrbracket_{\mathcal{M}} \& a \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \& a \in \llbracket P^1 \rrbracket_{\mathcal{M}} \& \langle j, a \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \& j \in \llbracket G^1 \rrbracket_{\mathcal{M}}\} \\
 = c_2
 \end{array}$$

**Analysis tree for  $\llbracket \exists x B^2 x \top_2 \rrbracket^{\mathcal{M}, g}$**

$$\begin{array}{l}
\exists x \quad B^2 x \top_2 \\
\hline
\{i \in c : \langle g(x), i_2 \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}}\} \\
\hline
\{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \ \& \ i \in c \llbracket B^2 x \top_2 \rrbracket^{\mathcal{M}, g[x/d]}\} \\
= \quad \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \ \& \ i \in \{k \in c : \langle d, k_2 \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}}\}\} \\
= \quad \{d \cdot i : d \in \mathcal{D}_{\mathcal{M}} \ \& \ i \in c \ \& \ \langle d, i_2 \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}}\}
\end{array}$$

**Updating info. state  $c_2$**

$$\begin{aligned}
& c_2 \llbracket \exists x B^2 x \top_2 \rrbracket^{\mathcal{M}, g} \\
&= \{b \cdot i : i \in c_2 \ \& \ \langle b, i_2 \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}}\} \\
&= \{b \cdot i : i \in \{\langle j, a \rangle : a = \llbracket a \rrbracket_{\mathcal{M}} \ \& \ a \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \ \& \ a \in \llbracket P^1 \rrbracket_{\mathcal{M}} \ \& \ \langle j, a \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \ \& \ j \in \llbracket G^1 \rrbracket_{\mathcal{M}}\} \ \& \ \langle b, i_2 \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}}\} \\
&= \{\langle b, j, a \rangle : a = \llbracket a \rrbracket_{\mathcal{M}} \ \& \ a \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \ \& \ a \in \llbracket P^1 \rrbracket_{\mathcal{M}} \ \& \ \langle j, a \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \ \& \ j \in \llbracket G^1 \rrbracket_{\mathcal{M}} \ \& \ \langle b, a \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}}\}
\end{aligned}$$

**Truth in  $\mathcal{M}$**

$$\mathcal{M} \models \exists x (x = a \wedge Y^1 x \wedge P^1 x) \wedge \exists x (J^2 x \top_1 \wedge G^1 x) \wedge \exists x B^2 x \top_2$$

$$iff \exists b, j, a \in \mathcal{D}_{\mathcal{M}} (a = \llbracket a \rrbracket_{\mathcal{M}} \ \& \ a \in \llbracket Y^1 \rrbracket_{\mathcal{M}} \ \& \ a \in \llbracket P^1 \rrbracket_{\mathcal{M}} \ \& \ \langle j, a \rangle \in \llbracket J^2 \rrbracket_{\mathcal{M}} \ \& \ j \in \llbracket G^1 \rrbracket_{\mathcal{M}} \ \& \ \langle b, a \rangle \in \llbracket B^2 \rrbracket_{\mathcal{M}})$$