HYPOTHETICAL EXPECTED UTILITY

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Good decision making requires thinking hypothetically

- Auctions
  - Thaler (JEP, 1988); Eyster and Rabin, (ETCA, 2005); Li (AER, 2017)
- Disclosure
  - Jin, Luca and Martin (WP, 2015), Enke (QJE, 2020)
- Voting
  - Feddersen (JEP, 2004); Esponda and Vespa (AEJ Micro, 2014)
- Construction of subjective likelihoods
  - Tversky and Kahneman (PsycR., 1983), Tversky and Koehler (PsycR., 1994)
- Strategic uncertainty
  - Eyster and Rabin, (2005), Esponda, (2008)

What is hypothetical thinking? How can it be flawed?

- Focusing on a subset, H, of the space of all possibilities and understanding
- what is true given this restriction: what H implies
- what might be true for the restriction to hold: what implies H

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general — can accommodate all of the examples above (and more); helps us understand *what* hypothetical thinking is

identifiable — is falsifiable and the parameters identifiable from standard economic data

The standard model of uncertainty: (Ω, μ).

- Ω is a state space, $H \subseteq \Omega$ is a **hypothesis**.
- μ is a measure over Ω; DM's uncertainty is captured by $\mu(H)$

The DM does properly interpret the hypothesis H. Instead she interprets is as some other event:

$$\pi : \quad 2^{\Omega} \to 2^{\Omega} \qquad \text{(Interpretation Map)}$$

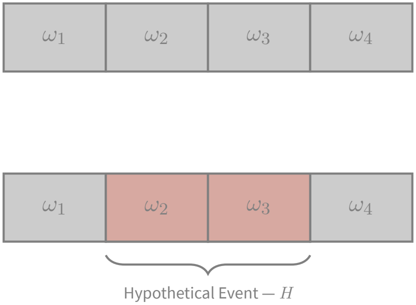
$$\pi : \quad H \mapsto \pi(H) \qquad \text{(Interpretation of } H\text{)}$$

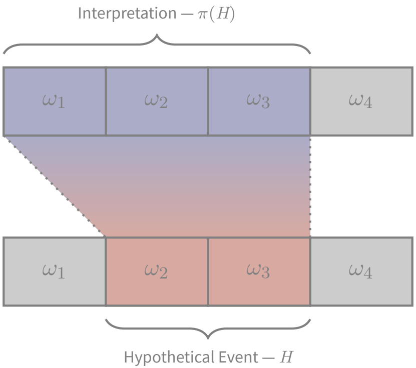
The *interpretational* model of uncertainty: (Ω, π, μ).

- DM's uncertainty is captured by $\mu(\pi(H))$
- This is a model of *misinterpretation*

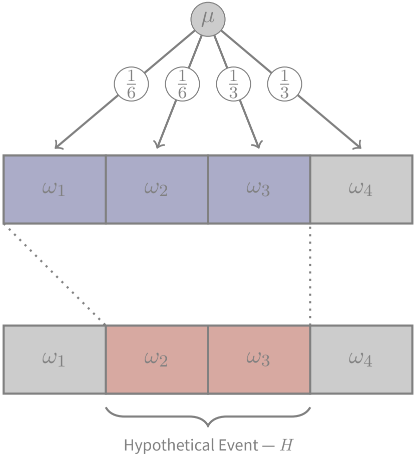
ω_1	ω_2	ω_3	ω_4

ω_1	ω_2	ω_3	ω_4

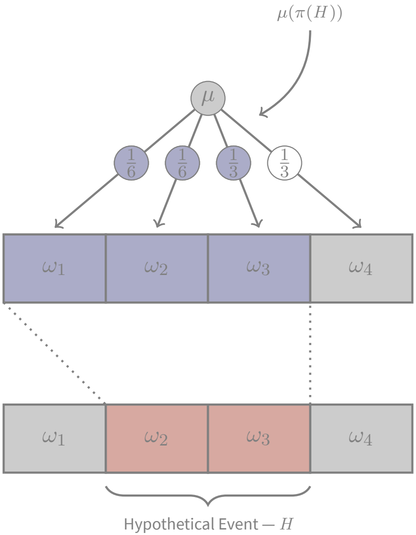

ω_1 ω_2 ω_3 ω_4
ω_1 ω_2 ω_3 ω_4
Hypothetical Event — H



Interpretation — $\pi(H)$
ω_1
ω_2
ω_3
ω_4
ω_1
ω_2
ω_3
ω_4
Hypothetical Event — H



μ
$\frac{1}{6}$
$\frac{1}{6}$
$\frac{1}{3}$
$\frac{1}{3}$
ω_1
ω_2
ω_3
ω_4
ω_1
ω_2
ω_3
ω_4
Hypothetical Event — H



$\mu(\pi(H))$
μ
$\frac{1}{6}$
$\frac{1}{6}$
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$\frac{1}{3}$
ω_1
ω_2
ω_3
ω_4
ω_1
ω_2
ω_3
ω_4
Hypothetical Event — H

The DM is ‘almost’ rational, restrict π:

TRUTH (T) $H \subseteq \pi(H)$

- Never rule out the true state of affairs.

Introspection (I) $\pi(\pi(H)) = \pi(H)$

- Cannot distinguish between an event and its interpretation

Monotonicity (M) $H \subseteq G$ implies $\pi(H) \subseteq \pi(G)$

- Weaker hypotheses remain weaker.

Call π **coherent** if it satisfies ($\mathtt{T}$), ($\mathtt{I}$) and ($\mathtt{M}$).

- Even under these rationality conditions, misinterpretation can explain many behavioral patterns

Example: the Monty Hall Problem

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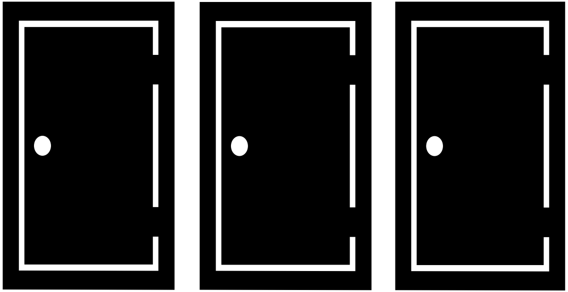
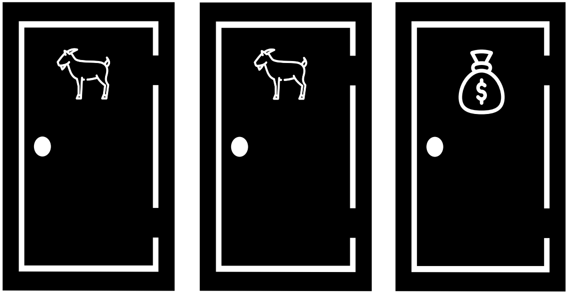
The winner of the game show *Let’s Make a Deal* is presented with 3 doors...



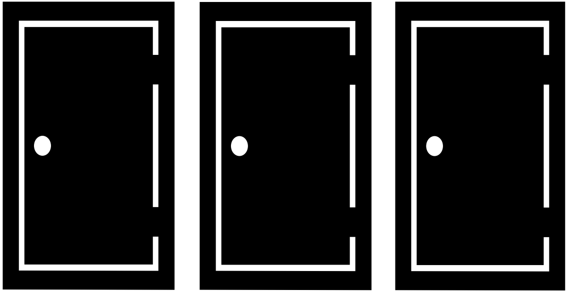
Example: the Monty Hall Problem

behind two of them stands a goat and the third a prize.



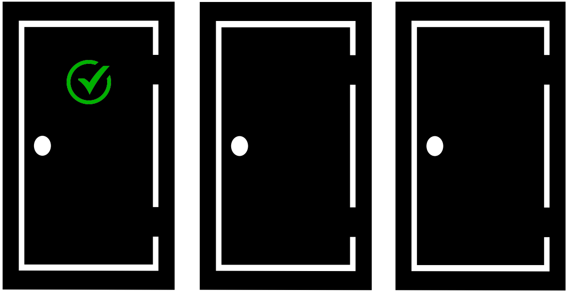
Example: the Monty Hall Problem

The Host, Monty, Knows the contents but the contestants do not.



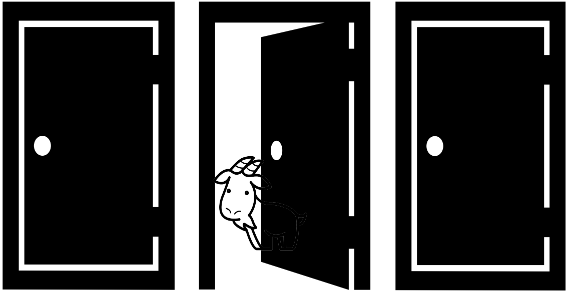
Example: the Monty Hall Problem

The contestant gets to choose a door.



Example: the Monty Hall Problem

Then Monty opens an unchosen door. **Critically: he always reveals a goat.**

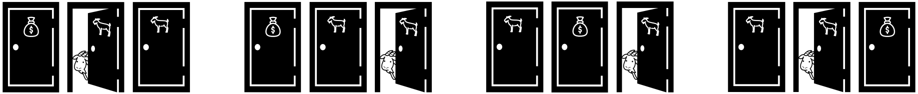


Example: the Monty Hall Problem

The contestant is afforded a final choice: keep his chosen door or switch to the other unopened door.

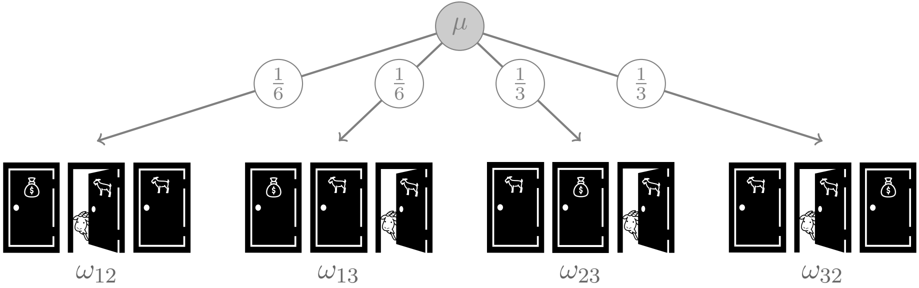
What should the contestant do?

We can analyze this will a simple 4 state model.



ω_{12} ω_{13} ω_{23} ω_{32}

- ω_{ij} — prize behind i, Monty opens j.



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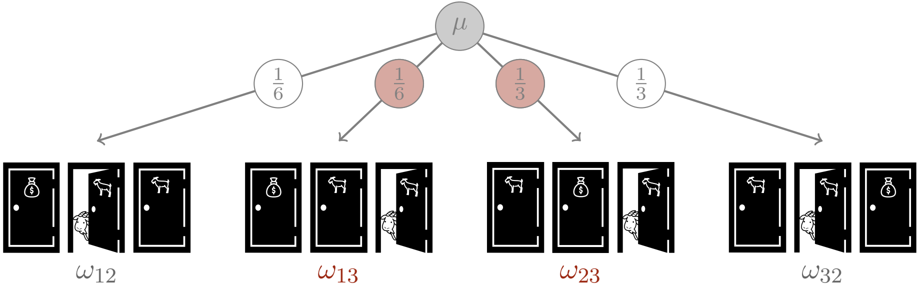
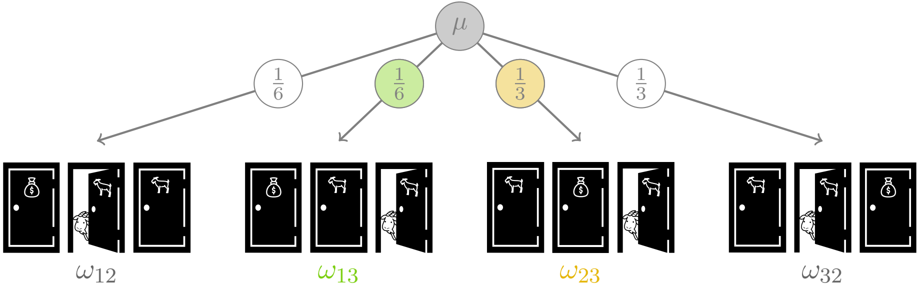


- ω_{ij} — prize behind i, Monty opens j.
- The event monty opens door 3 is $O_3 = \{\omega_{13}, \omega_{23}\}$.



- ω_{ij} — prize behind i, Monty opens j.
- The event monty opens door 3 is $O_3 = \{\omega_{13}, \omega_{23}\}$.
- The conditional probability of winning from sticking:

$$\mu(\{\omega_{12}, \omega_{13}\} \mid O_3) = \frac{\mu(\{\omega_{13}\})}{\mu(\{\omega_{13}, \omega_{23}\})} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3},$$

- And of winning by switching to door 2:

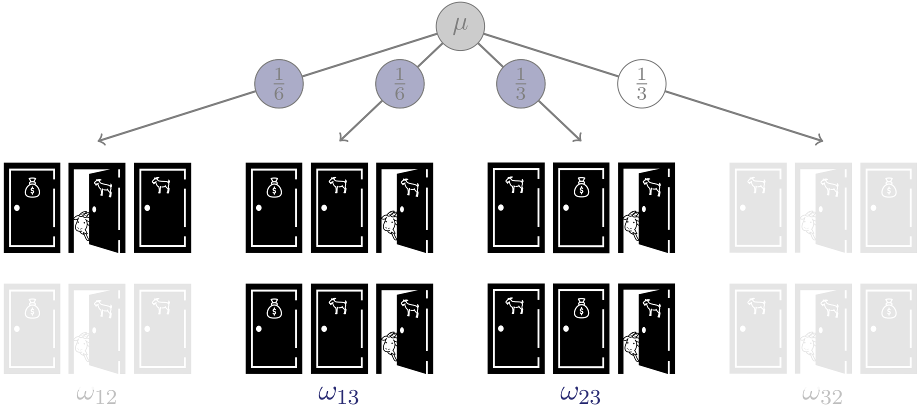
$$\mu(\{\omega_{23}\} \mid O_3) = \frac{\mu(\{\omega_{23}\})}{\mu(\{\omega_{13}, \omega_{23}\})} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}.$$

What happens if the contestant interprets

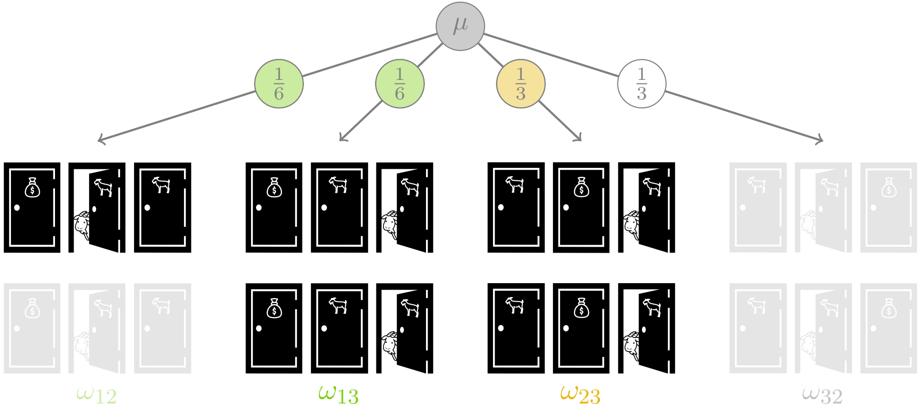
$$O_3 \text{ (door 3 is opened)}$$

as

$$\text{as } \textsc{not}(P_3) \text{ (the prize is not behind door 3)?}$$

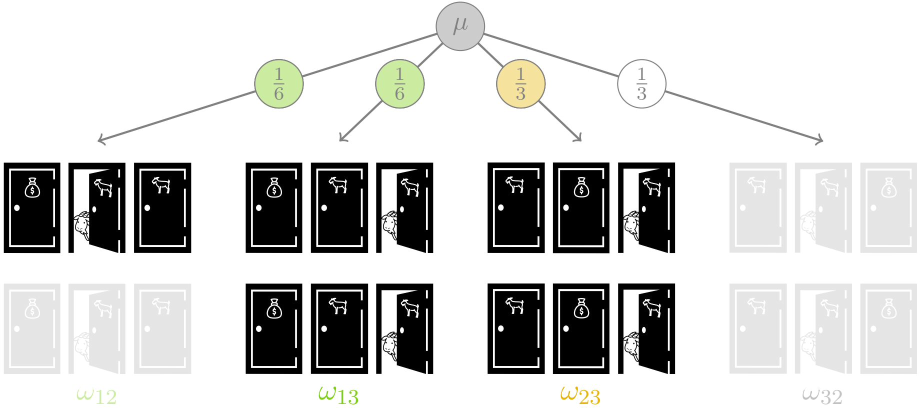


- $\pi(O_3) = \{\omega_{12}, \omega_{13}, \omega_{23}\}$.



- $\pi(O_3) = \{\omega_{12}, \omega_{13}, \omega_{23}\}$.
- The conditional probability of winning from sticking:

$$\mu(\{\omega_{12}, \omega_{13}\} \mid \pi(O_2)) = \frac{\mu(\{\omega_{12}, \omega_{13}\})}{\mu(\{\omega_{12}, \omega_{13}, \omega_{23}\})} = \frac{\frac{2}{6}}{\frac{2}{3}} = \frac{1}{2}$$



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Example: Esponda and Vespa (2014)

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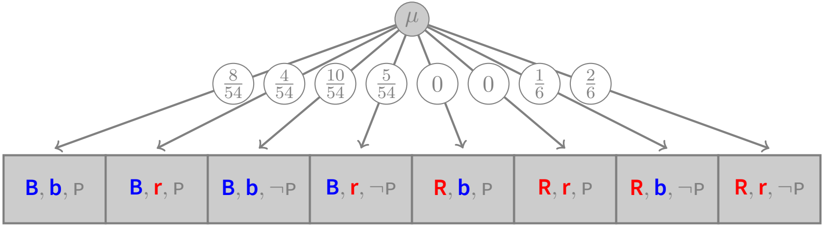
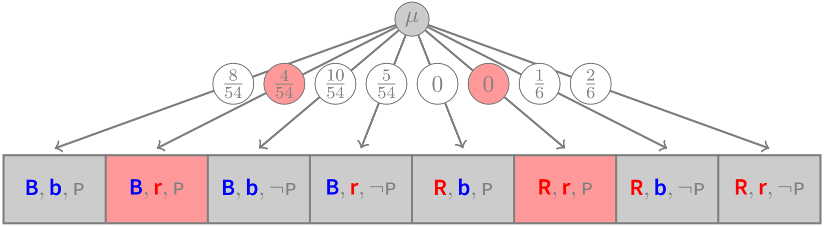
Subjects with the following decision problem:

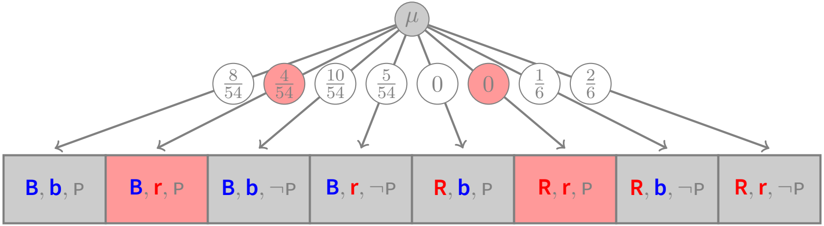
- State is **RED** or **BLUE** with equal prob
- Receive signal **r** or **b** with accuracy $\frac{2}{3}$.
- Must cast a vote for either **RED** or **BLUE**. In addition, two computers observe the state and also vote according to specific rule:
  - If **RED**: vote red
  - If **BLUE** : vote blue with probability $\frac{2}{3}$ and red with prob $\frac{1}{3}$
- Win if the color chosen by a simple majority matches the color of the state



◇ The objective state-space



- The objective state-space
- Conditioning event {r, P}



- The objective state-space
- Conditioning event $\{$**r**, P$\}$
- Conditional probability of **B** is $\mu(\mathbf{B} \mid \{\mathbf{r}, \mathrm{P}\}) = 1$.

Example

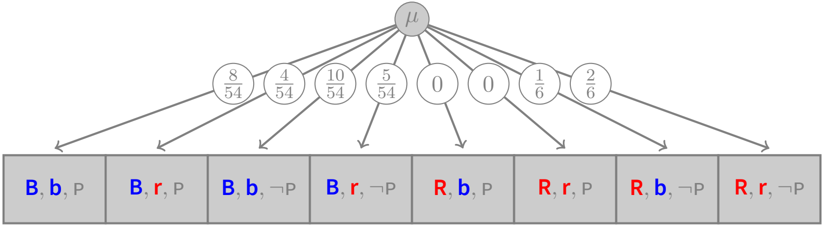
Subjects fail to reason contingently: *What must the world be like so that I got the information I did?*

- The subject interprets “The signal is **r** and I am pivotal” exactly as “The signal is **r**”
    - The former implies the latter but not the other way around.

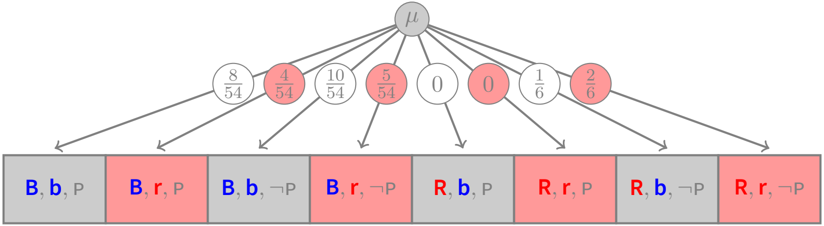
Example

Take the interpretation map which ignores pivotally:

$$
\begin{aligned}
\{(\mathbf{B}, \mathbf{b}, \mathrm{P})\} &\mapsto \{(\mathbf{B}, \mathbf{b}, \mathrm{P}), (\mathbf{B}, \mathbf{b}, \neg\mathrm{P})\} \\
\{(\mathbf{B}, \mathbf{r}, \mathrm{P})\} &\mapsto \{(\mathbf{B}, \mathbf{r}, \mathrm{P}), (\mathbf{B}, \mathbf{r}, \neg\mathrm{P})\} \\
\{(\mathbf{B}, \mathbf{b}, \neg\mathrm{P})\} &\mapsto \{(\mathbf{B}, \mathbf{b}, \mathrm{P}), (\mathbf{B}, \mathbf{b}, \neg\mathrm{P})\} \\
\{(\mathbf{B}, \mathbf{r}, \neg\mathrm{P})\} &\mapsto \{(\mathbf{B}, \mathbf{r}, \mathrm{P}), (\mathbf{B}, \mathbf{r}, \neg\mathrm{P})\} \\
\vdots \qquad & \qquad \vdots
\end{aligned}
$$

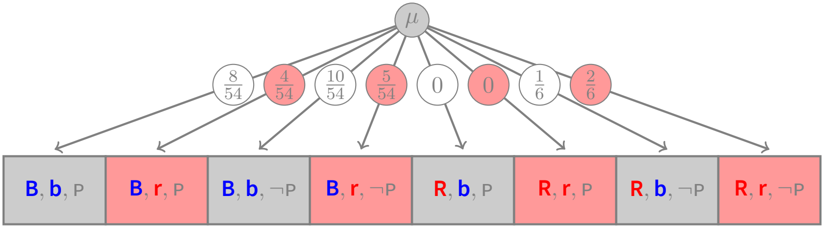


μ
$\frac{8}{54}$
$\frac{4}{54}$
$\frac{10}{54}$
$\frac{5}{54}$
0
0
$\frac{1}{6}$
$\frac{2}{6}$
B, b, P
B, r, P
B, b, ¬P
B, r, ¬P
R, b, P
R, r, P
R, b, ¬P
R, r, ¬P



- Conditioning event $\{r, P\}$ is interpreted as

$$\pi(\{r, P\}) = \{(B, r, P), (B, r, \neg P), (R, r, P), (R, r, \neg P)\}$$



- Conditioning event $\{\mathbf{r}, \mathsf{P}\}$ is interpreted as

$$\pi(\{\mathbf{r}, \mathsf{P}\}) = \{(\mathbf{B}, \mathbf{r}, \mathsf{P}), (\mathbf{B}, \mathbf{r}, \neg\mathsf{P}), (\mathbf{R}, \mathbf{r}, \mathsf{P}), (\mathbf{R}, \mathbf{r}, \neg\mathsf{P})\}$$

- Conditional probability of **B** is $\mu(\mathbf{B} \mid \pi(\{\mathbf{r}, \mathsf{P}\})) = \frac{\frac{4}{54}+\frac{5}{54}}{\frac{4}{54}+\frac{5}{54}+\frac{2}{6}} = \frac{1}{3}$.

Implication

The bridge between a decision maker's **choices** and her **interpretation of hypotheses** is implication.

- what is true given a hypothesis: what H implies
- what must be true for the hypothesis to hold: what implies H

Implication

H_S = "It is snowing" *implies* H_C = "It is cold out"

- Whenever the first hypothesis is true, so to the second.
- All the contingencies in H_S are also in H_C.
- $H_S \subseteq H_C$.

Implication

A DM with flawed hypothetical reasoning perceives implications subjectively

Implication

A DM with flawed hypothetical reasoning perceives implications subjectively

The DM π **perceives that H implies G** iff

$$\pi(H) \subseteq \pi(G)$$

The contestant interprets O_3 (door 3 is opened) as $\textsf{NOT}(P_3)$ (the prize is not behind door 3)

- he correctly perceives $O_3 \Rightarrow \textsf{NOT}(P_3)$
  - since $\pi(O_3) \subseteq \pi(\textsf{NOT}(P_3))$
- incorrectly perceives $\textsf{NOT}(P_3) \Rightarrow O_3$
  - since $\pi(\textsf{NOT}(P_3)) \subseteq \pi(O_3)$

Betting Behavior

A decision maker's perception of implication is revealed through her preferences.

Betting Behavior

- b_H is a bet on the hypothesis H
  - Pays 1 on H and 0 otherwise
- Assume we can observe $\succcurlyeq$, the DM's ranking over bets

When the DM perceives that H implies G, how does she value b_G and $b_{G \cup H}$?

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- $b_{G \cup H}$ pays if either H is true *or* G is true.
  - If G is true, both $b_{G \cup H}$ and b_G pay.
  - If G is false, then the DM perceives that H must be false too, neither $b_{G \cup H}$ nor b_G pays.

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    - If G is false, then the DM perceives that H must be false too, neither $b_{G \cup H}$ nor b_G pays.
- So: $b_G \sim b_{G \cup H}$

A DM, $\succcurlyeq$, **reveals she perceives that H implies G**, written $H \Rightarrow_{\succcurlyeq} G$, if

$$b_G \sim b_{G \cup H}$$

(π, μ) is an **HEU representation** of $\succcurlyeq$ if

$$b_H \succcurlyeq b_G \text{ if and only if } \mu(\pi(H)) \geq \mu(\pi(G))$$

Theorem

Let $\succcurlyeq$ be an HEU preference with representation (π, μ), then[1]

$$H \Rightarrow_{\succcurlyeq} G \text{ if and only if } \pi(H) \subseteq \pi(G)$$

[1] well, like, almost.

What if we begin with $=\!\succcurlyeq$ rather than π?

When does an interpretation exist from which it arises?

Properties of Implication

TRANSITIVITY (`trv`) If $H \mathrel{=\!\succcurlyeq} G$ and $G \mathrel{=\!\succcurlyeq} F$ then $H \mathrel{=\!\succcurlyeq} F$.

- DM perceives the transitive nature of cause and effect

Properties of Implication

MONOTONICITY (`mon`) If $H \subseteq G$ then $H \Rightarrow\!\succcurlyeq G$.

- The DM never inverts the order of objective implication

Properties of Implication

DEDUCTION (ded) If $H \Rightarrow\!\!\succcurlyeq G$ and $H' \Rightarrow\!\!\succcurlyeq G$ then $H \cup H' \Rightarrow\!\!\succcurlyeq G$.

- If the DM perceives that both H and H' would imply G she does not need to know which one holds in order to draw the conclusion

Theorem

The following are equivalent :

(1) $\Rrightarrow$ satisfies (`trv`), (`mon`), and (`ded`),

(2) There exists a coherent π such that $H \Rrightarrow G$ if and only if $\pi(H) \subseteq \pi(G)$.

Moreover, such a π is unique.

Proof

$$\pi : H \mapsto \bigcup \{ G \mid G \mathrel{=\!\succcurlyeq} H \}$$

- In the paper are other characterizations (diff properties of π)
- Uniqueness relies of properties π (i.e., unique *coherent* interpretation)
- If π preserves $\varnothing$, the image of π is a topology on Ω

What about μ, can we identify that too?

Richer Betting Behavior

$f : \Omega \to \mathbb{R}$ is an **act** — a contingent claim depending on the resolution of uncertainty.

We take $\succcurlyeq$ as the DM's preference over all acts.

How do we extend to general acts?

While $\mu \circ \pi$ is in general not a measure, it is a well-defined set function: we can use **Choquet** Integration.

- Choquet Integration generalized Lebesgue Integration, for non-additive capacities.
- It is additive over *co-monotone* acts
- Studied extensively in non-expected utility theory.

Call $\succcurlyeq$ a **Hypothetical Expected Utility** preference if there exists a (π, μ) such that

$$V(f) = \int^{\mathscr{C}} f \, \mathsf{d}(\mu \circ \pi), \tag{HEU}$$

represents $\succcurlyeq$.

Theorem

Let $\succcurlyeq$ satisfies some reasonable axioms then there exists a unique HEU representation.

We can strengthen `M` to:

CONSISTENCY (`C`) $\pi(H \cup G) = \pi(H) \cup \pi(G)$

- If a contingency can be ruled out by both H and G, it can be ruled out without knowing which H or G holds.

Consistency

Decomposition (`dcmp`) $F \mathrel{=\!\succcurlyeq} H \cup H'$ implies there exists G and G', such that $G \mathrel{=\!\succcurlyeq} H$, $G' \mathrel{=\!\succcurlyeq} H'$ and $F = G \cup G'$

- The DM can *decompose* complex implications into simpler ones.
- Ex. F implies either H = 'the student will do very well on the exam' or H' = 'the student will do very poorly on the exam' but does not determine which is true.
- Perhaps F = 'the student left the 2-hour final exam after 20 minutes.'
- Then F can be itself decomposed into 'the student is very bright' which implies H and 'the student is very apathetic' which implies H'.

Uncertainty Attitude

If π is coherent and consistent:

- $\pi(H \cap G) \subseteq \pi(H) \cap \pi(G)$
- $\pi(H \cup G) \subseteq \pi(H) \cup \pi(G)$

So $\mu \circ \pi$ is *concave*: $\mu \circ \pi(G \cap H) + \mu \circ \pi(H \cup G) \leq \mu \circ \pi(H) + \mu \circ \pi(G)$.

Theorem

Let $\succcurlyeq$ is an HEU preference than it is ambiguity loving: If $f \succcurlyeq g$, then $f \succcurlyeq \frac{1}{2}f + \frac{1}{2}g$.

Other Stuff

- Dual Models
- Syntax and subjective states spaces
- Relationship to Topology