CENTERED CHOICE

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&
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Marge is taking Bart and Lisa to see a film. There are three films playing:

avengers, **b**atman, and **c**aptain america

The children's preferences:

Bart	Lisa
a	b
b	c
c	a

- Marge Chooses **b**atman.

The children's preferences:

Bart	Lisa
a	b
b	~~c~~
~~c~~	a

- When they arrive, **c**aptain america is sold out.

The children's preferences:

Bart	Lisa
a	b
b	~~c~~
~~c~~	a

- Now Marge is indifferent between a and b.

Attraction Effect

- Alternatives are multi-dimensional.
- Two alternatives, a and b, are superior in different dimensions.
  - Indifference (or close to)
- Add an element c—dominated by b but not a.
  - Choices tend to shift to choose b.
- Violates rational choice theory (WARP, IIA, etc).
- Well documented empirically: Tversky and Kahneman, 1981; Huber et al., 1982; Rabin 1998, etc.

What drives the attraction effect?

- Context matters: reference dependence
    - Elements are evaluated not in absolute terms but relative to a reference point
    - Reference point is determined by the choice set.
- Comparisons are “non-linear”: loss aversion
    - Losses are more costly than gains are beneficial.
    - Otherwise everything washes out.

Centered Choice

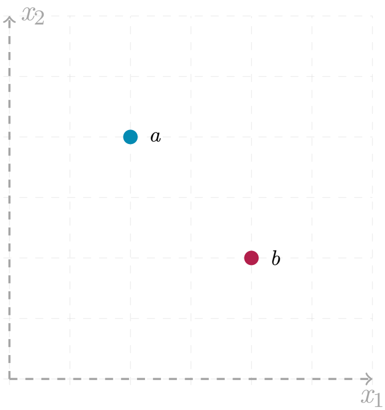
We want the simplest model of reference dependence accommodating loss aversion.

- Alternatives are vectors in $\mathbb{R}^k$.
- For a set $A \subset \mathbb{R}^k$, the reference point, $\mathbb{E}(A)$, is the average point.
- The DM entertains a loss function: $l : \mathbb{R}^k \to \mathbb{R}$.
- Choice minimizes relative loss

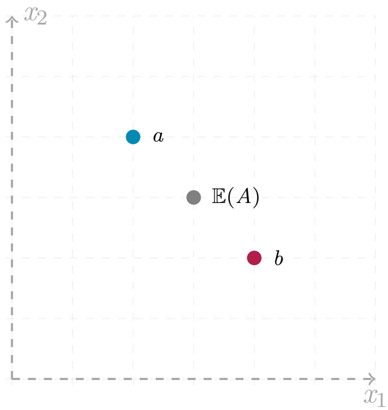
$$\mathcal{C}(A) = \arg\min_{a \in A} l(\mathbb{E}(A) - a)$$

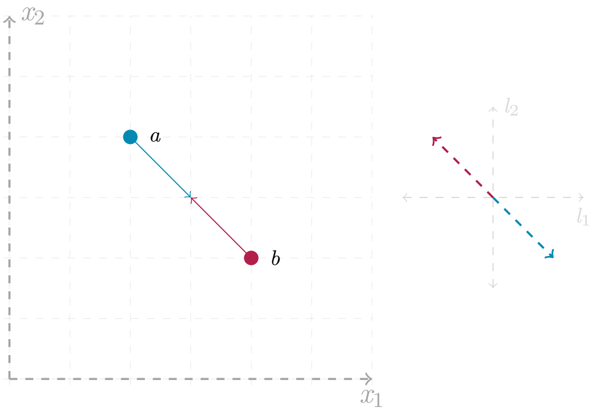
Example

- Points in $\mathbb{R}^2$ with $l((x_1, x_2)) = \frac{1}{2}e^{x_1} + e^{x_2}$.
- Consider the four objects: $a = (1, 2), b = (2, 1), c = (1.8, .9)$
- $A = \{a, b\}$, $A' = \{a, b, c\}$.

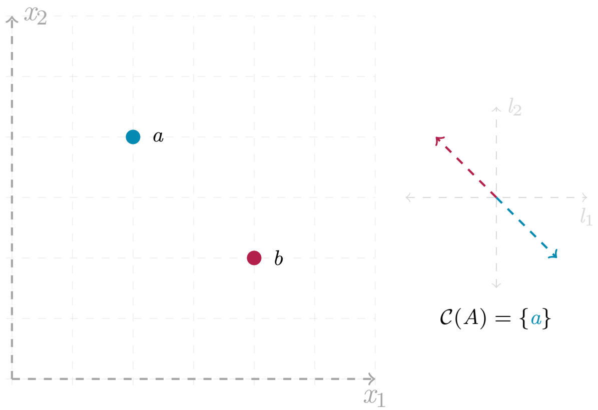


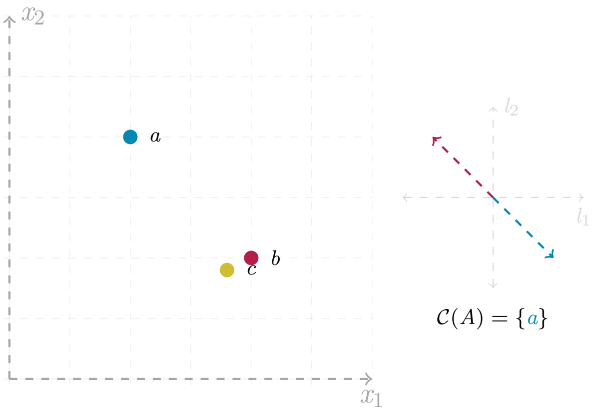
x_2
a
b
x_1


x_2
a
$\mathbb{E}(A)$
b
x_1

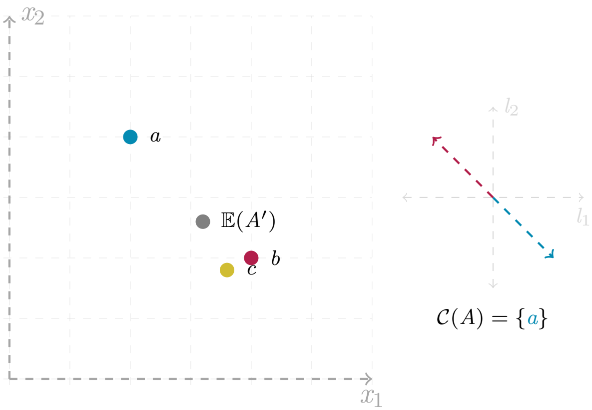


x_2
a
b
x_1
l_2
l_1

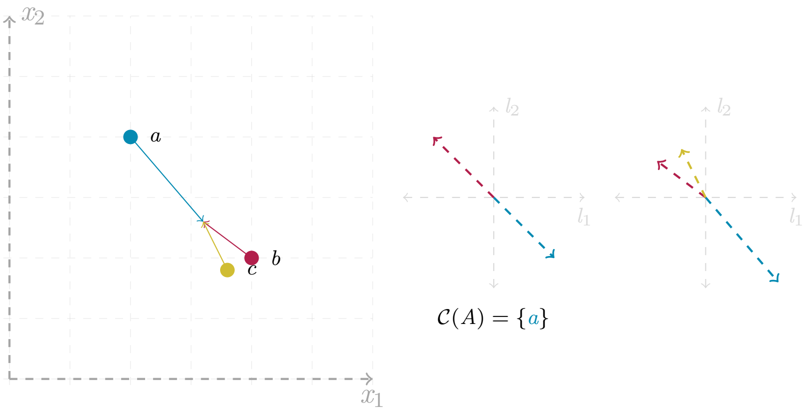

x_2
a
b
x_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$



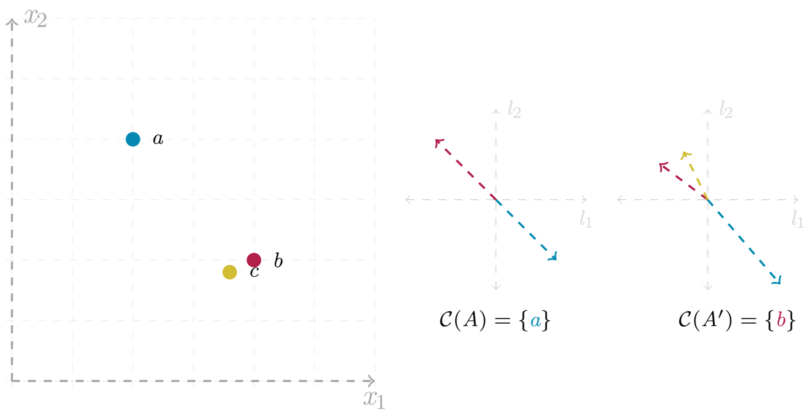
$$\mathcal{C}(A) = \{a\}$$



$\mathcal{C}(A) = \{a\}$



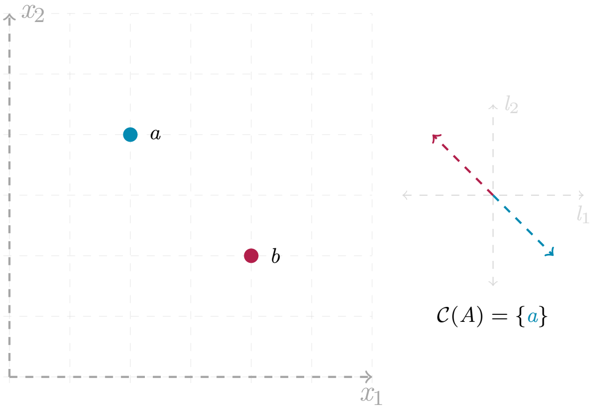
x_2
x_1
a
b
c
l_2
l_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$



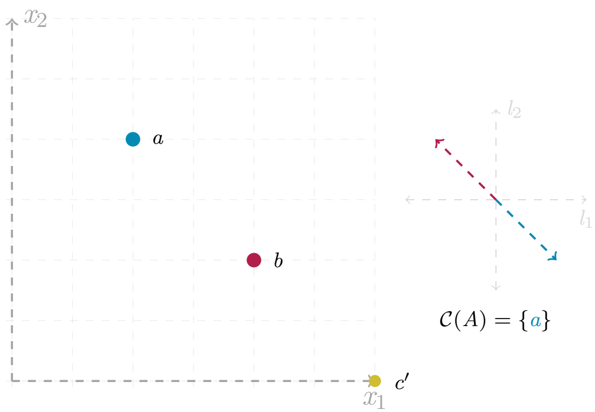
x_2
x_1
a
b
c
l_2
l_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$
$\mathcal{C}(A') = \{b\}$

Comprimise Effect

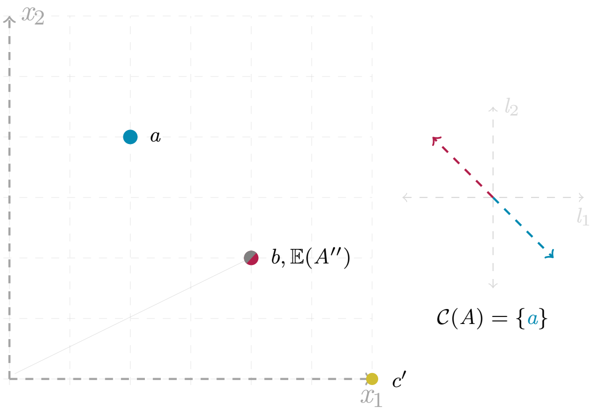
- Add an element c—so as to make b close to the center of $\{a, b, c\}$.
  - Choices tend to shift to choose b.
- Also well documented empirically.
- Can also be explained by centered choice.
  - Take the previous example with $c' = (3, 0)$.



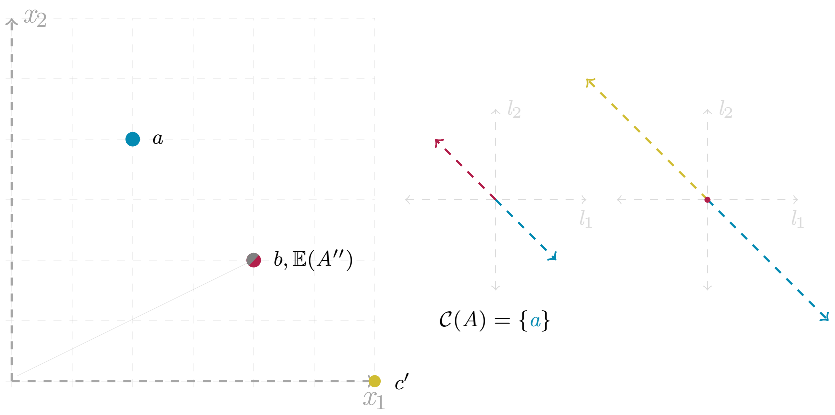
x_2
a
b
x_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$



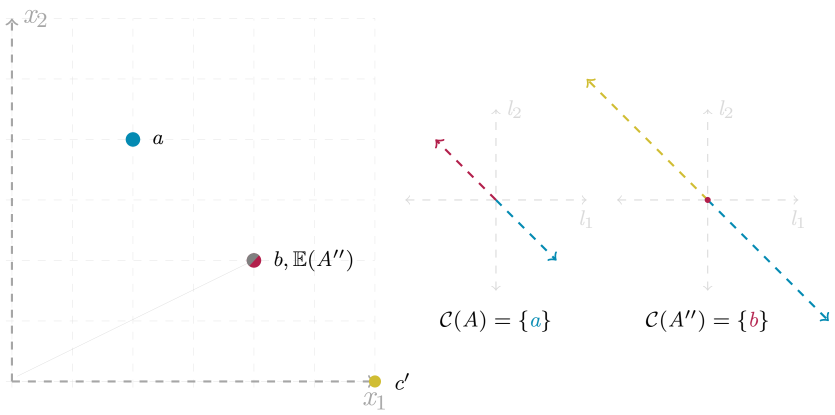
x_2
a
b
c'
x_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$



x_2
a
$b, \mathbb{E}(A'')$
c'
x_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$



x_2
a
$b, \mathbb{E}(A'')$
c'
x_1
l_2
l_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$



x_2
a
$b, \mathbb{E}(A'')$
c'
x_1
l_2
l_1
l_2
l_1
$\mathcal{C}(A) = \{a\}$
$\mathcal{C}(A'') = \{b\}$

Cumulative Reference Dependence

What if $l((x_1, x_2)) = \alpha e^{x_1} + e^{x_2}$ for small α?

- The addition of c does not change preference.
  - Loss in dimension 2 is much more costly than dimension 1.
- Adding an additional decoy near b might reverse choices.
- Conversely, a decoy near a will “balance” reference effects.

Comp > Attr.

CC with a convex loss function also predicts that the compromise effect should be more powerful than the attraction effect.

- Introducing a decoy trades gain in one dimension for loss in another.
- Convexity: effect increases as reference point gets close to b.
- This is found empirically, Huber et al., 1982

Talk Today

1. Axiomatic treatment of general CCR.
2. Identification of l.
3. Online experiment testing cumulative reference dependence.

Preliminaries

- Alternatives are points in $\mathbb{R}^k$, $k \geq 2$.
- $\mathcal{M} = \{A, B, C \ldots\}$ are **choice problems**, all non-empty finite subsets of $\mathbb{R}^k$.
- A **choice rule**, $\mathcal{C}$, is a function $\mathcal{M} \to \mathcal{M}$ such that $\mathcal{C}(A) \subseteq A$ for all $A \in \mathcal{M}$.
- $\mathbb{E}(A) \in \mathbb{R}^k$ is the **center** of A; $\mathbb{E}(A)^i = \sum_{a \in A} \frac{a^i}{\#[A]}$.

The CC model.

Sat that $\mathcal{C}$ has a centered choice representation (CCR) if

$$\mathcal{C}(A) = \arg\min_{a \in A} l(\mathbb{E}(A) - a)$$

with l strictly monotone and continuous.

- Call a CC loss averse if l is strictly quasi-convex.
- Call a CC addative if $l = \sum_{i \leq k} l^i(\mathbb{E}(A)^i - a^i)$ for continuous, monotone $l^i : \mathbb{R} \to \mathbb{R}$.

Preliminaries

Define the sets:

- $\textsf{CONTAIN}(a) \subset \mathcal{M}$—all choice problems that contain a.
- $\textsf{CENTER}(a) \subset \mathcal{M}$—all choice problems centered with $\mathbb{E}(A) = a$.
- $UC(a) = \{b \mid \exists A \in \textsf{CONTAIN}(a) \cap \textsf{CENTER}(\mathbf{0}), b \in \mathcal{C}(A)\}$.
- $LC(a) = \{b \mid a \in UC(b)\}$.

Monotonicity (M)

Let $a, b \in A$. If $a > b$ then $b \notin \mathcal{C}(A)$

- $a > b$ if $a_i \geq b_i$ for all i and at least one of the inequalities is strict.
- Implies that consumption is good (or, conversely that loss is bad).

Translation Invariance (TI)

$\mathcal{C}(A + x) = \mathcal{C}(A) + x$ for any $x \in \mathbb{R}^k$ (where $+$ is the Minkowski sum).

- Preferences only reflect relative comparisons.
- As we move the entire problem, the relative gains and losses remain fixed.

Continuity (C)

For all $a \in \mathbb{R}^k$, $UC(x)$ and $LC(x)$ are closed.

- Recall, $UC(a)$ only pertains to choice problems centered at $\mathbf{0}$.
- We impose continuity only on such problems.
- TI takes of the rest.

Barycentric WARP (B-WARP)

Fix some $A, B \in \mathcal{M}$ with $\mathbb{E}(A) = \mathbb{E}(B)$ then if $a, b \in A \cap B$, $a \in \mathcal{C}(A)$ and $b \in \mathcal{C}(B)$, then $a \in \mathcal{C}(B)$.

- When the center of the menu is fixed, reference effects are constant.
- Behavior is “rational.”

Theorem

A choice rule $\mathcal{C}$ satisfies M, TI, C, and B-WARP if and only if it admits and CCR.

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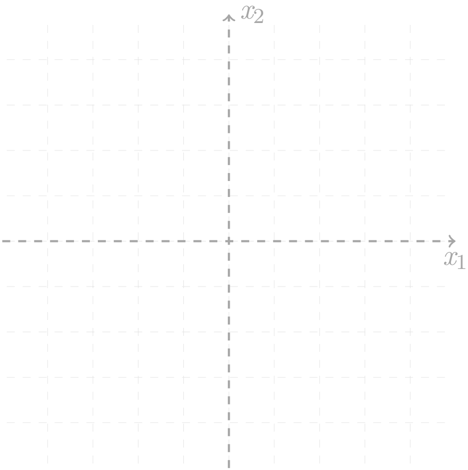
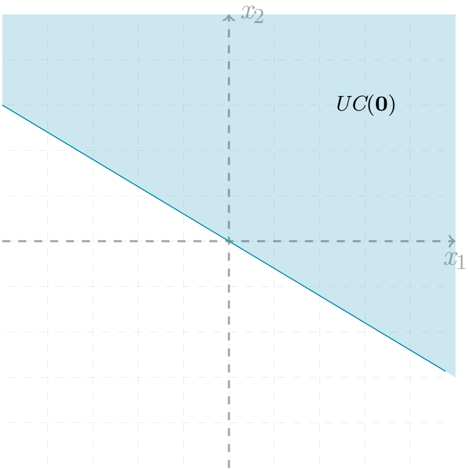
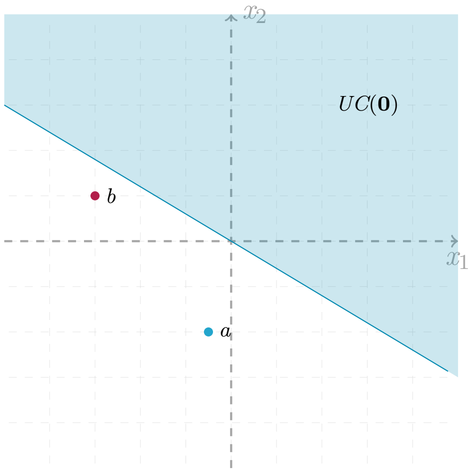
A choice rule $\mathcal{C}$ satisfies M, TI, C, and B-WARP if and only if it admits and CCR.

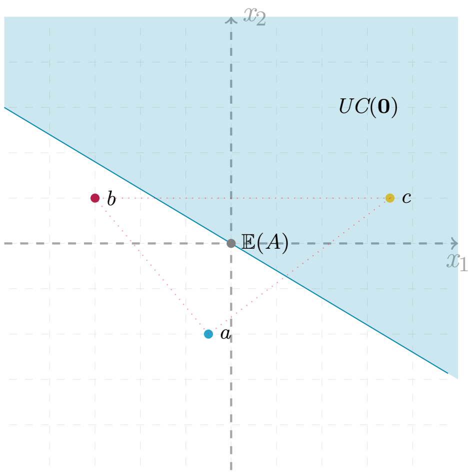
- Uniqueness? Not even up to monotone transforms.


x_2
x_1



x_2
$UC(\mathbf{0})$
x_1


x_2
$UC(\mathbf{0})$
b
x_1
a



x_2
$UC(\mathbf{0})$
b
c
$\mathbb{E}(A)$
x_1
a

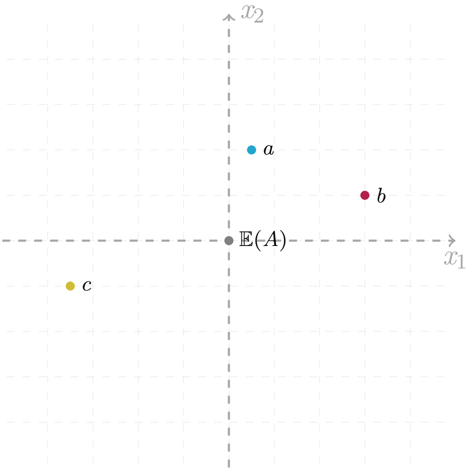
Sketch of Proof

- Find a CCR for CENTER($\mathbf{0}$), then appeal to TI.
- Define $\succcurlyeq$ over $\mathbb{R}^n$ as $a \succcurlyeq b$ if
  1. $a \in UC(b)$, or
  2. $a \geq b$
- $\succcurlyeq$ is reflexive and monotone by (2).
- Need to show transitivity.

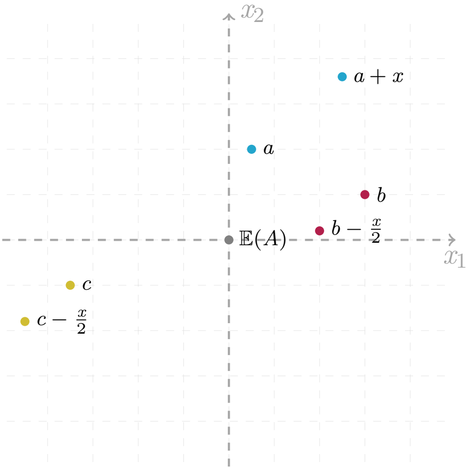
Sketch of Proof

- Say $a \in UC(b)$. We need $a + x \in UC(b)$ for $x \in \mathbb{R}^+$.
- There exists $A \supset \{a, b\}$, $\mathbb{E}(A) = \mathbf{0}$, $a \in C(a)$.
- Perturb this to find a menu where $a + x$ is chosen.

Sketch of Proof



Sketch of Proof



Sketch of Proof

- Now, say $a \in UC(b)$ and $b \in UC(c)$.
    - There exists $A \supset \{a, b\}$, $\mathbb{E}(A) = \mathbf{0}$, $a \in C(A)$.
    - There exists $B \supset \{b, c\}$, $\mathbb{E}(B) = \mathbf{0}$, $b \in C(B)$.
- Consider $A \cup B$ (but perturb everything to deal with overlap).
- B-WARP implies a is chosen.

Sketch of Proof

- $\succcurlyeq$ is reflexive, transitive, continuous, and monotone.
- It admits a partial utility representation $U : \mathbb{R}^n \to R$.
- Set $L(x) = -U(-x)$.
- $UC(a)$ is upward closed
  - If a is ever chosen, $UC(a) = \{b \mid b \succcurlyeq a\}$.
  - L rationalizes $\mathcal{C}$.

Strict Convexity (SCV)

For all a, $UC(a)$ is strictly convex.

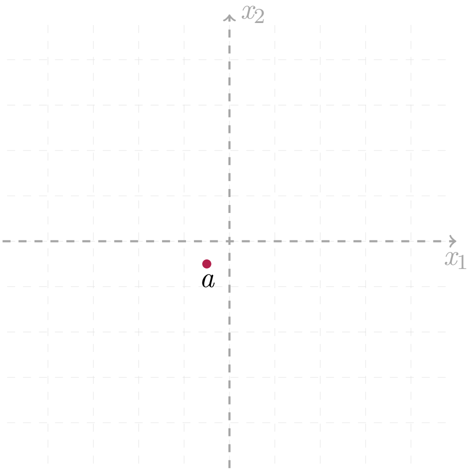
- If b, b' are both better than a (with reference $\mathbf{0}$ then $\alpha b + (1-\alpha)b'$ is strictly better.

Theorem

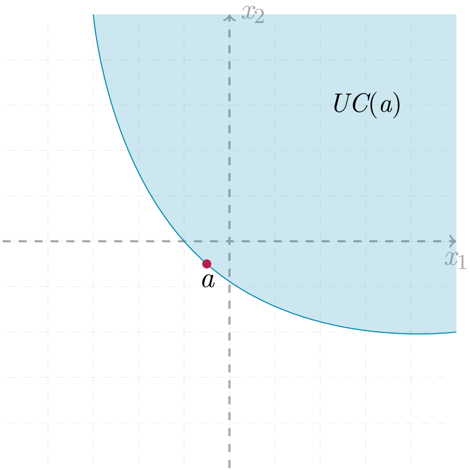
A choice rule $\mathcal{C}$ satisfies M, TI, C, B-WARP and SCV if and only if it admits and loss averse CCR. Moreover L is ordinally unique.

- Convexity implies the convexity of $\succcurlyeq$ implies the quasi-convexity of L.
- Uniqueness: we can now compare every a and b; either $a \in UC(b)$ or $b \in UC(a)$; $\succcurlyeq$ is complete.
  - Add elements that will never be chosen but move $\mathbb{E}(A)$.

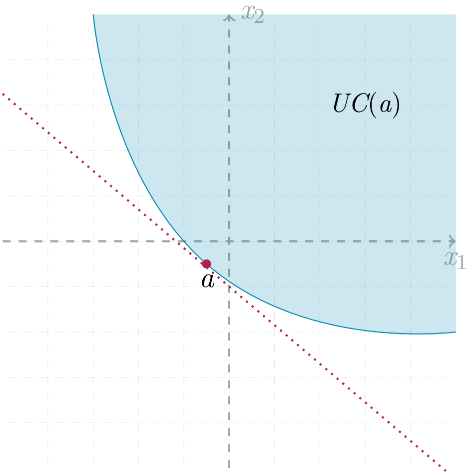
Sketch of Proof



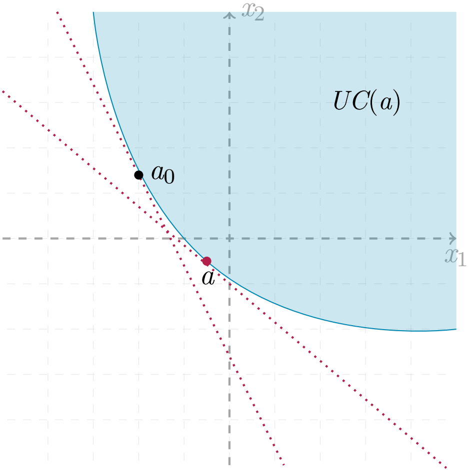
Sketch of Proof



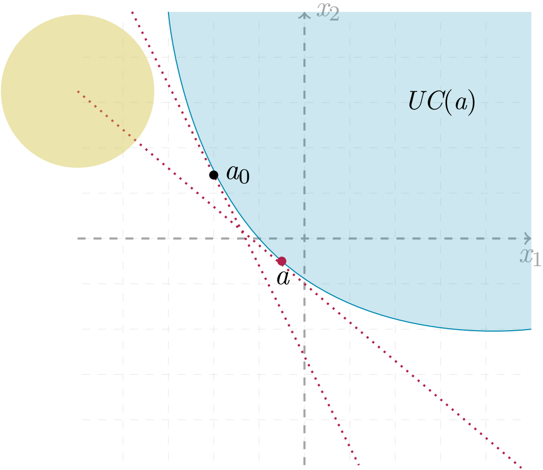
Sketch of Proof



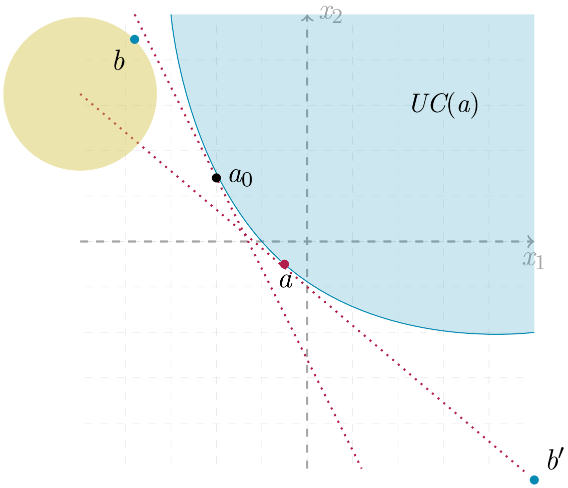
Sketch of Proof



Sketch of Proof



Sketch of Proof



Experiment (Super Preliminary)

- Ran a short experiment via Amazon Mechanical Turk.
- Purpose: understand the attraction effect in the presence of multiple decoy options
- 120 subjects; average payment \$3.08; average duration 126 seconds.

Design

- Within Subject design.
- Each subject evaluated 5 decision problems made up of lotteries.
  - A lottery is a magnitude of payment and a probability of winning.
  - Two specifications, with different lotteries (60 subjects each).
- One decision problem was randomly selected for payment.

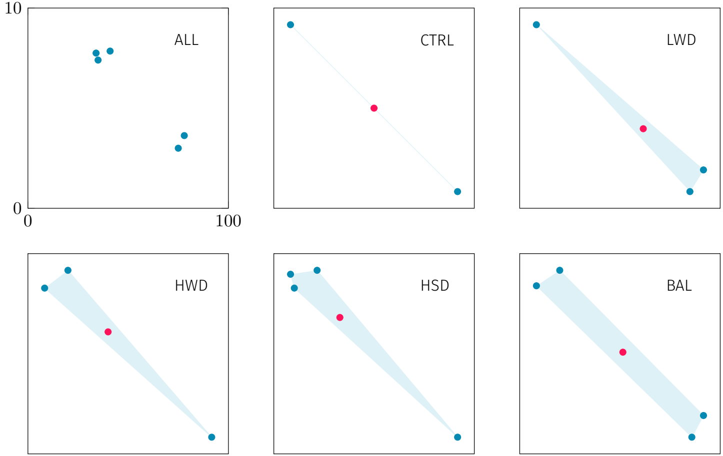
Lotteries

	S1	S2
h	$(\$7.85, .41)$	$(\$7.85, .41)$
l	$(\$3.63, .78)$	$(\$3.33, .78)$
ld	$(\$3.00, .75)$	$(\$2.70, .75)$
hd1	$(\$7.50, .35)$	$(\$7.50, .35)$
hd2	$(\$7.75, .34)$	$(\$7.75, .34)$

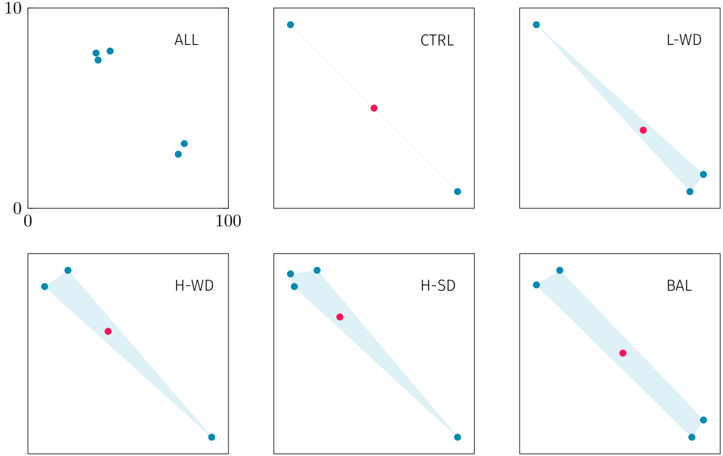
Decision problems

1. Control: CRTL $= \{h, l\}$
2. Low weak decoy: L-WD $= \{h, l, ld\}$
3. High weak decoy: H-WD $= \{h, l, hd1\}$
4. High strong decoy: H-SD $= \{h, l, hd1, hd2\}$
5. Balanced: BAL $= \{h, l, ld, hd\}$.

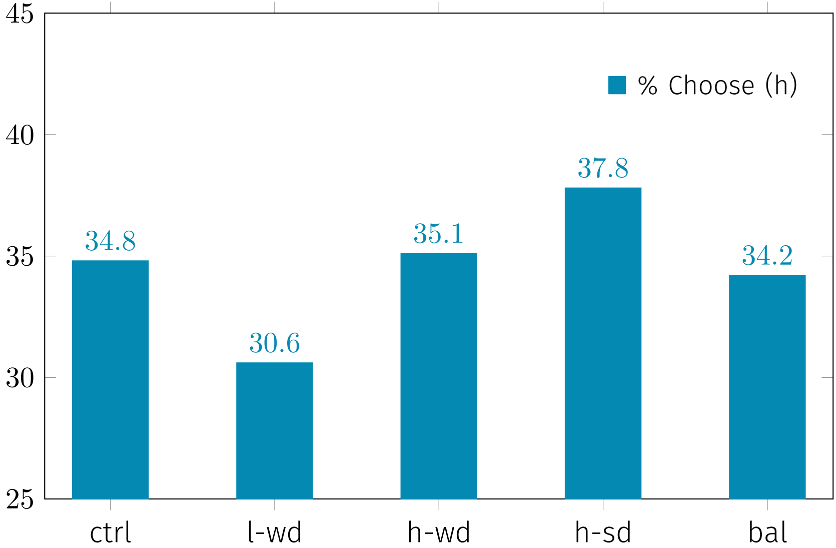
Decision problems: S1

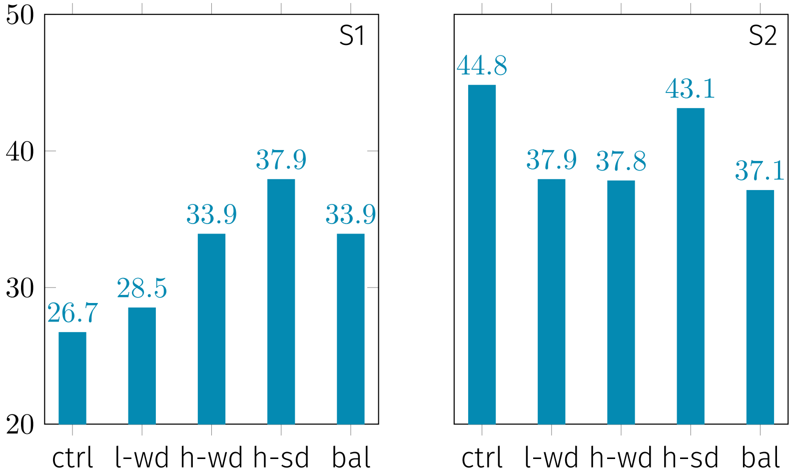


Decision problems: S2



Results (Aggregate)





50
40
30
20
S1
26.7
28.5
33.9
37.9
33.9
ctrl
l-wd
h-wd
h-sd
bal
S2
44.8
37.9
37.8
43.1
37.1
ctrl
l-wd
h-wd
h-sd
bal

Conclusions

- Centered Choice can explain the attraction and compromise effect.
- It has an perspicuous axiomatization.
- People may or may not behave according to CC.