

Failures of Contingent Thinking

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Bounded Rationality in Choice, June 2022

We are motivated by two facts:

(1) There is a disconnect between the way uncertainty is modeled and how it is perceived by agents:

- ◇ Modeled by semantic state spaces; each state represents a complete description
 - ◇ State-space Ω , and probability μ over Ω .
- ◇ Real world uncertainty often doled out as a set of interconnected statements
 - ◇ “It is raining” or “The S&P500 went up today.”

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 - ◇ Real world uncertainty often doled out as a set of interconnected statements
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- (2) While rationality assumptions allow translating back and forth, humans are, unfortunately, not perfect reasoners.

Tversky and Kahneman (1983) provided subjects with the following vignette:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

When asked to rank the following statements in order of likelihood:

F = "Linda is active in the feminist movement."

T = "Linda is a bank teller."

$T \wedge F$ = "Linda is a bank teller and is active in the feminist movement."

- ◇ 85% of subjects ranked $F > T \wedge F > T$
- ◇ $T \wedge F$ implies T , any (classical) state space must rank the later weakly more likely.

1. Contingent thinking *is* the ability to recognize implications,
'if ... then ...'

- ◇ In the example, subjects do not recognize that $T \wedge F$ implies T
- ◇ The opposite is also possible — perceive an implication that does not exist
- ◇ We provide a precise behavioral criterion for perceiving implication

2. A state-space implicitly determines implication relations:

- ◇ we must start with a more primitive *syntactic* objects
- ◇ representation connects a subjective state-space to perceived implication

Failures of contingent thinking abound:

- ◇ Voting (pivotality)
 - ◇ Feddersen (JEP, 2004); Esponda and Vespa (AEJ Micro, 2014)
- ◇ Auctions (winning)
 - ◇ Thaler (JEP, 1988); Eyster and Rabin, (ETCA, 2005); Li (AER, 2017)
- ◇ Disclosure (no news)
 - ◇ Jin, Luca and Martin (WP, 2015), Enke (QJE, 2020)
- ◇ Information acquisition (generation process)
 - ◇ Enke and Zimmermann (ReStud, 2019), Enke (QJE, 2020)

Our goal is to provide:

- ◇ an abstract *definition* of contingent thinking that is context independent
- ◇ a methodology for *identifying* what the decision maker (DM) understands
- ◇ the limits of the *choice-data* necessary for such identification

Related (decision theoretic) Literature:

- ◇ Syntactic Decision Theory:
 - ◇ Tversky and Kahneman (1983); Mukerji (1997); Blume, Easley, Halpern (2021) and Bjorndahl and Halpern (2021)
- ◇ Incomplete or Inconsistent State-Spaces:
 - ◇ Inconsistent: Lipman (1999); Sadler (2020)
 - ◇ Incomplete: Fagin and Halpern (1988); Modica and Rustichini (1999); Heifetz, Meier, Schipper (2008)
- ◇ Misspecified Models:
 - ◇ Acemoglu et al. (2016); Mailath and Samuelson (2020); Frick et al. (2020); Eliaz et al. (2020); Ellis and Thysen (2021),

Decision environment

\mathbb{P} is a set of **primitive statements**

- ◇ In the example, “Linda is bank teller,” etc.

\mathcal{L} is the **language** induced by \mathbb{P} via negation, conjunction and disjunction

- ◇ If φ is in \mathbb{P} then it is in \mathcal{L} too
- ◇ If φ is in \mathcal{L} then $\neg\varphi$ (“not φ ”) is in \mathcal{L}
- ◇ If φ and ψ are in \mathcal{L} then $\varphi \wedge \psi$ (“ φ and ψ ”) is in \mathcal{L}
- ◇ If φ and ψ are in \mathcal{L} then $\varphi \vee \psi$ (“ φ or ψ ”) is in \mathcal{L}

Let ' $\varphi \rightarrow \psi$ ' mean that ϕ implies ψ according to the rules of classical logic.

- ◇ For example $\varphi \rightarrow \varphi \vee \psi$

Decision environment

An **act** is a function $f: \Phi \rightarrow [0, \infty)$ with finite domain $\Phi \subseteq \mathcal{L}$

- ◇ f yields (vNM) utility $f(\varphi)$ when $\varphi \in \Phi$ is *true*, and is *called off* if no $\varphi \in \Phi$ is true
- ◇ x_Φ denotes the constant act $\Phi \mapsto x$

The DM has a **strict** preference \succ over the set of acts

- ◇ Let $f \approx g$ if f and g satisfy the same \succ relations.

Interpretations of uncertainty

An **interpretation (of uncertainty)** is a list (Ω, t, μ) consisting of:

1. A set of *states* Ω
2. A *truth-valuation map* $t: \mathcal{L} \rightarrow 2^\Omega$
3. A *likelihood assessment* $\mu: \mathcal{A} \rightarrow [0, \infty)$, where \mathcal{A} is the algebra generated by the image of t

Interpretations of uncertainty

- ◇ $t(\varphi) \subseteq \Omega$ is the set of states in which φ is considered to true
 - ◇ t may not obey the usual logical dictates; e.g., $t(\varphi \wedge \psi) \not\subseteq t(\varphi)$

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 - ◇ t may not obey the usual logical dictates; e.g., $t(\varphi \wedge \psi) \not\subseteq t(\varphi)$
- ◇ $\mu(t(\varphi))$ quantifies the likelihood that statement φ is true
 - ◇ In the paper: μ may not be additive—not even monotone (e.g., $\mu(E) > \mu(F)$ for $E \subseteq F$)
 - ◇ For now: μ is a probability

Properties of t

- ◇ **Exact:** $\varphi \leftrightarrow \psi$ implies $t(\varphi) = t(\psi)$
- ◇ **Monotone:** $\varphi \rightarrow \psi$ implies $t(\varphi) \subseteq t(\psi)$
- ◇ **Distributive:** $t(\varphi \wedge \psi) = t(\varphi) \cap t(\psi)$
- ◇ **Symmetric:** $t(\neg\varphi) = \Omega \setminus t(\varphi)$
- ◇ **Sound:** all of the above

Interpretations of uncertainty allow for evaluating an act in two steps:

interpretation and **aggregation**

Evaluation of an act - Interpretation

- ◇ An act $f: \Phi \rightarrow [0, \infty)$ is defined over linguistic statements $\varphi \in \Phi$
- ◇ Thus, the first step will consist in *interpreting* f as a map $\mathbf{f}: \Omega \rightarrow [0, \infty)$
- ◇ This interpretation is subjective and hinges on (Ω, t)
- ◇ There is an obvious ambiguity involved: Where to map states $\omega \in t(\varphi) \cap t(\psi)$ (if any) for $\varphi, \psi \in \Phi$?

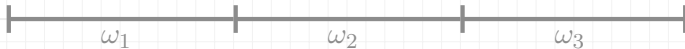
Evaluation of an act - Interpretation

◇ $\Phi = \{\varphi, \neg\varphi\}$ and $f: \varphi \mapsto 2, f: \neg\varphi \mapsto 3$



Evaluation of an act - Interpretation

◇ $\Omega = \{\omega_1, \omega_2, \omega_3\}$



Evaluation of an act - Interpretation

◇ $t: \varphi \mapsto \{\omega_1, \omega_2\}$



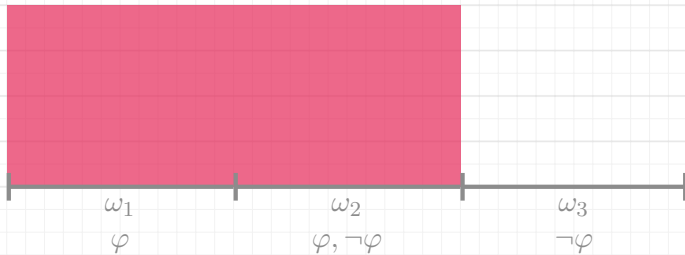
Evaluation of an act - Interpretation

$$\diamond \quad t: \neg\varphi \mapsto \{\omega_2, \omega_3\}$$



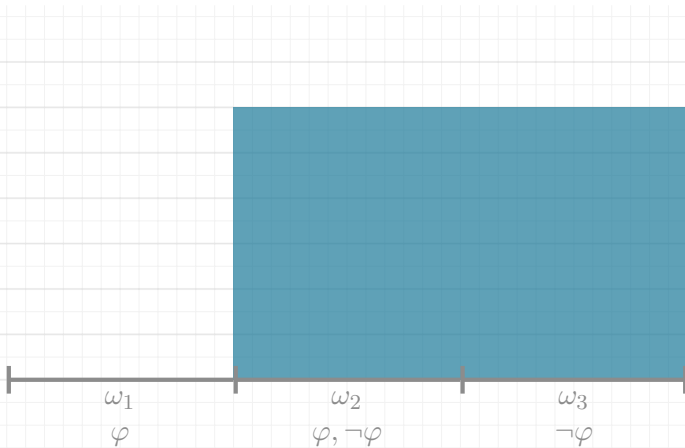
Evaluation of an act - Interpretation

◇ $f: \varphi \mapsto 2$



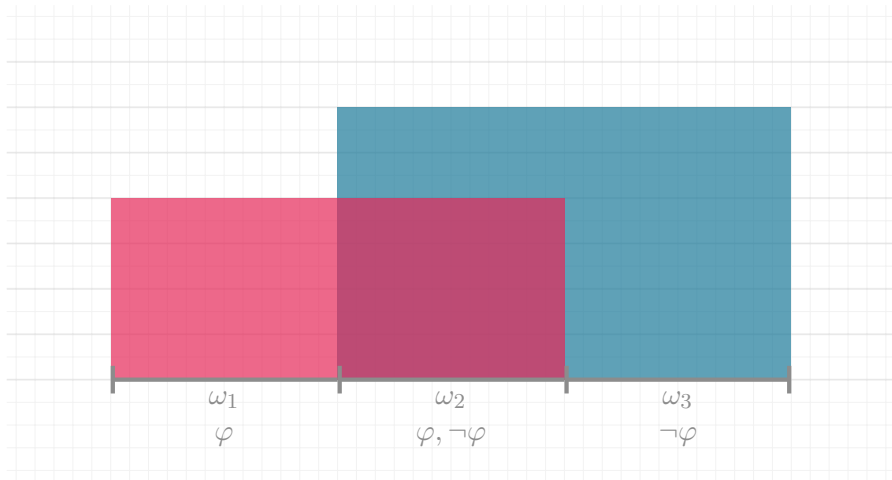
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◇ $f: \neg\varphi \mapsto 3$



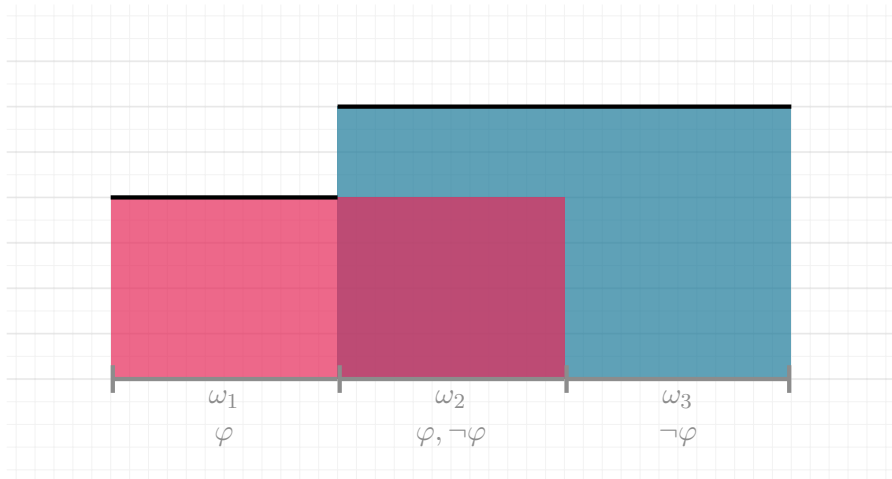
Evaluation of an act - Interpretation

- ◇ Payoff in state ω_2 is undefined



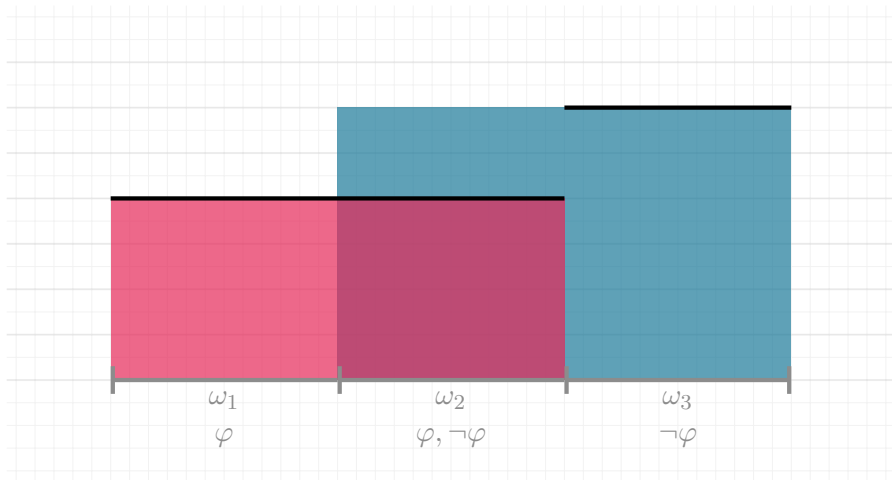
Evaluation of an act - Interpretation

- ◇ Maximal consistent act $\tilde{f}(\omega) = \sup\{f(\varphi) \mid \omega \in t(\varphi)\}$



Evaluation of an act - Interpretation

- Minimal consistent act $\underline{f}(\omega) = \inf\{f(\varphi) \mid \omega \in t(\varphi)\}$



Evaluation of an act - Interpretation

Fix act f with domain Φ

- ◇ $\mathbf{f}: t(\Phi) \rightarrow [0, \infty)$ is **consistent** with f if:

$$\mathbf{f}(\omega) \in \{f(\varphi) \mid \omega \in t(\varphi)\}$$

for every $\omega \in t(\Phi)$

- ◇ Let $\llbracket f \rrbracket$ collect all the maps $\mathbf{f}: t(\Phi) \rightarrow [0, \infty)$ consistent with f
 - ◇ The multiplicity of $\llbracket f \rrbracket$ represents the ambiguity arising in the interpretation of f

Evaluation of an act - Aggregation

- ◇ Fix act f with domain Φ
- ◇ For each $\mathbf{f} \in \llbracket f \rrbracket$ we have a well-defined **Expected Utility**: $\int \mathbf{f} d\mu$
- ◇ Thus, each syntactic act is associated to a subset of $[0, \infty)$:

$$\left\{ \int \mathbf{f} d\mu \mid \mathbf{f} \in \llbracket f \rrbracket \right\}$$

Interpretation-Dependent Expected Utility

\succ is an **interpretation-dependent expected utility (IDEU) preference** if there exists some interpretation (Ω, t, μ) that represents \succ ; i.e., such that, for every pair of acts f and g ,

$$f \succ g \iff \inf \left\{ \int \mathbf{f} d\mu \mid \mathbf{f} \in \llbracket f \rrbracket \right\} > \sup \left\{ \int \mathbf{g} d\mu \mid \mathbf{g} \in \llbracket g \rrbracket \right\}$$

- ◇ The ambiguity that arises when interpreting an act manifest as incompleteness of the preference
- ◇ We provide an axiomatization of these preferences in the paper.

The interpretation t formally captures the DM's contingent thinking:

- ◇ Benchmarks of rationality
- ◇ Understanding of contingencies via implication
- ◇ Relation to objective knowledge / models
- ◇ Belief updating

- ◇ Exact: $\varphi \leftrightarrow \psi$ implies $t(\varphi) = t(\psi)$
 - ◇ If $\varphi \leftrightarrow \psi$ and $\varphi \in \Phi$ then $x_\Phi \approx x_{\Phi \cup \psi}$

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- ◇ Distributive: $t(\varphi \wedge \psi) = t(\varphi) \cap t(\psi)$
 - ◇
$$\begin{cases} \varphi \mapsto x \\ \psi \mapsto 0 \end{cases} \approx \begin{cases} \varphi \mapsto x \\ \psi \wedge \varphi \mapsto 0 \end{cases}$$

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$$\begin{cases} \varphi \mapsto x \\ \psi \mapsto 0 \end{cases} \approx \begin{cases} \varphi \mapsto x \\ \psi \wedge \varphi \mapsto 0 \end{cases}$$
- ◇ Symmetric: $t(\neg\varphi) = \Omega \setminus t(\varphi)$
 - ◇ $x_{\{\varphi, \neg\varphi\}} \approx x_{\{\varphi \vee \neg\varphi\}}$

Perceived implications

For any $\varphi, \psi \in \mathcal{L}$, a DM with preference \succ **perceives** that φ implies ψ , denoted by $\varphi =\succ \psi$ if,

$$1_{\{\psi\}} \approx 1_{\{\varphi, \psi\}}.$$

- ◇ There is never any benefit to betting on φ given a bet on ψ
- ◇ We show $=\succ$ is a partial order over propositions
 - ◇ Not true if μ is not a measure

- ◇ $\varphi \not\Rightarrow \psi$ necessarily implies that $t(\varphi) \not\subseteq t(\psi)$ (otherwise $t(\{\psi\}) = t(\{\varphi\}) \cup t(\{\psi\})$)
- ◇ Thus, there exists some $\omega \in t(\varphi) \setminus t(\psi)$, i.e., it is conceivable for the DM that φ holds and ψ does not

Faithful Representation

An interpretation (Ω, t, μ) is **faithful** (for \succ) if it represents \succ and for every $\varphi, \psi \in \mathcal{L}$,

$$\varphi =_{\succ} \psi \text{ if and only if } t(\varphi) \subseteq t(\psi)$$

- ◇ Subjective implication exactly corresponds to set containment

Theorem

If \succ has an IDEU representation it has a faithful IDEU representation.

- ◇ Moreover this is unique in some meaningful sense.

Identification of theories

Typically, elicitation experiments rely on the analyst assuming some (non-logical) relation between statements.

- ◇ 'It is raining' implies 'The ground is wet'
- ◇ This implications is true but not logically necessary—it requires a **theory**

Question

- Could seemingly erratic behavior be explained as not understanding that some statement is true?
- Can we disentangle irrationality from differing beliefs?

Identification of theories

Theories

A set $\mathcal{T} \subseteq \mathcal{L}$ is a **theory** if, for any $\varphi \in \mathcal{T}$ and any $\psi \in \mathcal{L}$ the following two hold:

1. If $\varphi \rightarrow \psi$ then $\psi \in \mathcal{T}$
2. If $\varphi \rightarrow \neg\psi$ then $\psi \notin \mathcal{T}$

A theory \mathcal{T} allows for additional implications, denoted by $\xrightarrow{\mathcal{T}}$ (\mathcal{T} -implications)

E.g., if $\neg\varphi \vee \psi \in \mathcal{T}$ then $\varphi \xrightarrow{\mathcal{T}} \psi$ regardless of whether $\varphi \rightarrow \psi$ or not

An IDEU preference perceives \mathcal{T} -implications if $\varphi \succ \psi$ for every $\varphi, \psi \in \mathcal{L}$ such that $\varphi \xrightarrow{\mathcal{T}} \psi$

Identification of theories

Theorem

Let \succ be an IDEU preference that perceives implications, and let \mathcal{T} be a theory. Then, there exists a unique theory $\mathcal{T}^\succ \subseteq \mathcal{T}$ such that:

1. \succ perceives \mathcal{T}^\succ -implications
2. For any theory \mathcal{T}' such that $\mathcal{T}^\succ \subseteq \mathcal{T}' \subseteq \mathcal{T}$, \succ does not perceive all \mathcal{T}' -implications

We can identify the largest theory that rationalizes a rational DM's choices—even if inconsistent with the analyst's theory

What if we only observed preferences over simple bets of the form:

$$x_\varphi \mapsto \begin{cases} x & \text{if } \varphi \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

- ◇ In many environments (for example SEU) this suffices for identification.
- ◇ $x_\varphi \succ x_\psi$ implies φ is assessed as more likely than ψ .

- ◇ Given simple bets, we cannot distinguish failures of *logical* thinking from failures of *probabilistic* thinking.
- ◇ Non-rational t imparts the same preferences (over simple bets!) as weakening the conditions on μ

μ additive and	t sound and:
any t	N/A
t exact	any μ
t monotone	μ monotone
$t \wedge$ -distributive	μ totally monotone
t sound	μ additive

Thank You!