Image Conscious Preferences

Evan Piermont[*]

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Abstract

An *image conscious* decision maker (DM) cares not only about the physical consequences of his actions, but also how his actions are perceived by others. When a DM takes a choice, the resulting *image* is the set of preferences that are consistent with the observed choice. This paper axiomatizes the behavior of a DM who derives utility directly via consumption and also via the induced image. Because the image depends on what *could* have been chosen, the DM will display menu-dependent preferences. I consider two models: in the first, the modeler observes two stages of choice—over menus and then from the chosen menu; in the second, only the latter choices are observed. The two models share the same representation but uniqueness is obtained only in the first.

Key words: Image consciousness; menu-dependent preferences; reluctant giving; self-images.

[*]Royal Holloway, University of London, Department of Economics, `evan.piermont@rhul.ac.uk`

1 Introduction

When a decision maker (DM) takes an action, he reveals something about his motivation to anyone observing his choice. An *image conscious* decision maker (DM) cares not only about the physical consequences of his actions, but also how his actions are perceived by others—when taking an action, he considers both the direct effect of his choice and also the information that could be inferred by observing his behavior. This paper investigates the choices of an image conscious decision maker.[1]

An *image* is a set of preferences; specifically, when a DM takes a choice, the resulting image is the set of preferences that are consistent with the observed choice. Therefore, a DM's image depends both on his choice and also on the set of actions that *could have been* chosen. When, for example, the DM faces a degenerate choice—with only a single action at his disposal—he reveals nothing about his motivation; any preference is consistent with the observation. Fixing the chosen action, adding unchosen alternatives (potentially) changes the associated image, since the observer can now rule out additional preferences. As such, an image conscious DM will not behave in accordance to the normative model of choice theory; adding unchosen elements to a choice set can change the total value to each action, and, for sufficiently costly changes in the image, cause choice reversals.

Example. Slothrop is deciding where to take Katje on a date. There are three restaurants, D^l, D^m, and D^h equal in all ways excepting their wine lists. The wine list at restaurant D^l offers only an inexpensive low quality bottle (l); D^m offers this and also a mid-tier bottle (m); D^h, in addition to l and m, offers a costly and high quality bottle (h).

Despite the fact that Slothrop is an absolute cheapskate, he wishes to appear generous and refined. That is, privately, Slothrop would prefer to consume l, but, all else equal, would prefer Katje to think that he prefers more expensive items to less. Hence, at D^m, figuring it worth the small expense to impress Katje, he would publicly choose m. When at D^h, however, Slothrop would revert to his private optimum, choosing l. This is because when m is chosen in favor of h, Katje rules out the possibility that Slothrop prefers grandeur, believing instead that he has middling taste; the cost of sending a signal of refinement—choosing h—is now too high.

Anticipating his contrived wine selection, it is reasonable that Slothrop choose D^m of the three restaurants. The addition of m to D^l allows him to manipulate his image, appearing to have discriminating tastes, without sacrificing too much in terms of personal consumption value. Further adding h eliminates this interpretation of choosing m, causing him to revert his choice.

[1] Image conscious behavior is so perversive, its existence is a truism. Image concerns manifest in decisions regarding credit cards (Rao et al., 2018), monetary allocations in experiments, (Andreoni and Bernheim, 2009), clothing brands (Han et al., 2010), voting in national elections (DellaVigna et al., 2016), the wearing or not wearing of flat brimmed 'trucker' hats (Barker, 2004), charitable donations (DellaVigna et al., 2012), home design (Wagner, 2018), music genres (Berger and Heath, 2008) etc. Despite the glut of evidence of image driven behavior, I am aware of no prior general theories outlining the identification of image effects.

As the above example shows, the preferences of an image conscious DM will be reflected in his preference over decision problems themselves. There is a clear connection between Sothrop's choice reversal when choosing *from* D^m and D^h and his choice *between* them. Notice that at both restaurants, he could choose l, effecting the same image (that he is a cheapskate) and the same physical consumption. The fact that he did not choose l from D^m signals that he must prefer his induced outcome, and so, from the ex-ante perspective, he must prefer D^m to D^h. Choice reversals across different menus reveal the DM's value of images.

I take consumption objects to be elements of the n-dimensional Euclidian space. This generality permits the instantiation that consumption objects are lotteries, Anscombe-Aumann, multi-attribute objects, etc. The DM entertains a linear utility function over consumption objects $u : \mathbb{R}^n \to \mathbb{R}$. In the absence of image concerns, the DM's preference over consumption is represented by u. A *choice problem* is a finite set of consumption objects: $D \subset \mathbb{R}^n$.

As a starting point, I take as the primitive a pair of choice functions: a first stage choice function, $\mathscr{C}_1$, defined on sets of choice problems and a second stage choice function, $\mathcal{C}_2$, defined on choice problems themselves. To continue with the language of the example, $\mathscr{C}_1$ indicates the DM's choice of restaurant and $\mathcal{C}_2$ his choice of wine from the restaurant chosen at the first stage. The interpretation is that the first stage choice problem (choosing which restaurant to patronize) is not observed by the parties the DM cares about impressing. The second stage choice (choosing which wine, at the restaurant) is public, so the DM takes into account not only his consumption value but how his choice will be interpreted by other parties. Because of observational concerns, I later consider a variant of the model where *only* second stage choices are observed by the modeler; this is discussed shortly.

When an observer sees the DM choose x from a second stage choice problem, D, she believes that the DM's utility function was maximized by x: the DM's image is therefore

$$I_D^x = \{u : \mathbb{R}^n \to \mathbb{R} \mid u \text{ is linear}, u(x) \geq u(y) \text{ for all } y \in D\}.$$

Letting $\mathbb{I}$ denote the set of all possible images (collections of linear utility functions), the DM's image concerns are represented by the function $\Gamma : \mathbb{I} \to \mathbb{R}$. Thus, the DM's choice from D, when observed by someone whose opinion he cares about, is given by

$$\mathcal{C}_2(D) = \arg\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big). \tag{C2}$$

In the first stage, when considering which restaurant to go to, the DM anticipates his image consciousness, and therefore seeks to maximize his eventual utility as given by (C2). Hence, his choice from a collection of choice problems, $\mathcal{M} = \{D_1, \dots D_k\}$ is given by

$$\mathscr{C}_1(\mathcal{M}) = \arg\max_{D \in \mathcal{M}} \Big(\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big)\Big), \tag{C1}$$

The pair $\langle u, \Gamma \rangle$ forms an *image conscious (IC) representation* if $\mathscr{C}_1$ and $\mathcal{C}_2$ jointly satisfy (C1) and (C2).

When both $\mathscr{C}_1$ and $\mathcal{C}_2$ are accessible to the modeler, the axiomatic characterization of image conscious choice is remarkably simple. Beyond the standard axioms to achieve a linear representation over consumption, only one additional restriction is needed. This key axiom, called *Image Consistency*, ties together the two stages of choice. Let D and D' be two choice problems containing x and such that choosing x induces the same image, I, from either choice problem. *Image Consistency* states that if x is chosen from D' in the second stage, then D must be weakly preferred to D' in the first stage. The intuition is a simple revealed preference argument, as exhibited by Slothrop's preferences in the example. Since x is chosen from D', the value of the menu is $u(x)+\Gamma(I)$, and since x *could have been* chosen from D the value must be at least $u(x) + \Gamma(I)$. Anticipating this, the DM must prefer D.

IC without First Stage Choices. If first stage choices are *completely* private, they are by definition unobservable to the modeler. Even when theoretically observable by the modeler, in many contexts first stage choice problems will pervasively be degenerate. For example, a businessman who cares about his professional image might routinely find himself responsible for choosing the wine at restaurants chosen by clients. Because of this observational hurdle, Section 4 considers a variant of the model where only $\mathcal{C}_2$ is observable. Axioms are provided equivalent to an IC representation, but uniqueness is no longer ensured (in fact, full identification is *never* possible). Recall how, in the example, choice reversals indicated a preference for one image over another—an inference that relies only on second stage choices. Further, mixing a menu with a fixed alternative changes the dispersion of utility across the menu without changing the possible images. Then, by identifying the point at which choice reversals happen, the modeler can identify the relative difference (in utility terms) between images. However, not all images can be compared in this way, so the resulting representation is not unique.

The Dual-Self Interpretation of Images. It need not be that an IC DM cares about the opinion of any *third* party, but rather, the 'observer' might be him himself. We can interpret the utility of a given image as the psychological benefit/cost of adhering to or deviating from the DM's ideal preferences. For example, a DM might *want* to be a charitable person but also not want to give up on personal consumption. In situations where his hands are tied—when there is no opportunity to give—he circumvents the psychological cost of selfish behavior. But, when confronted with a choice, he must either forgo direct consumption or address his avarice.

While this story can clearly explain choice reversals at second stage choices, it may also make sense in the context of first stage choice. A completely rational and forward looking DM would understand that choosing to avoid the future option of donating money is effectively choosing not to donate, and would therefore be unable to skip out on the phycological bill. Of course, like all benchmarks of rationality, there is a growing body of evidence suggesting humans do not meet this standard; it seems as if "meta-decisions"—which do not implement actions directly, but affect the decision making process; e.g., information collection—and action-implementing decisions have different effects on shaping self-images.[2] Within the present context, the interpretation being that a phycological cost is levied only when consumption decisions are actually made, so DM might avoid situations where donating to a charity is possible, even if donating nothing is always an option.

Organization. The next section outlines the requisite notation. Then, Section 3 provides the axiomatization and representation theorem for image conscious choice under the assumption that both stages of choice are observable. This assumption is relaxed in Section 4, which provides a representation when only the second stage is observed. Literature is reviewed at the end, in Section 5.

2 Notation and Primitives

Notation. Consumption takes place in $\mathbb{R}^n$ for $n \geq 2$. For a vector $x \in \mathbb{R}^n$, and $1 \leq i \leq n$, let x^i denote the i^{th} component of x. Let $\mathcal{D}$ denote the set finite non-empty subsets of $\mathbb{R}^n$, referred to as stage 2 choice problems (2CPs). The topology on $\mathcal{D}$ is induced by the Hausdorff metric, $d_h^{\mathcal{D}}$. Let $\mathscr{M}$ denote the set of all finite non-empty subsets of $\mathcal{D}$, refereed to as stage 1 choice problems (1CPs). Endow $\mathscr{M}$ with the associated Hausdorff metric, $d_h^{\mathscr{M}}$. For any $k \in \mathbb{N}$ let $\mathcal{D}^k = \{D \in \mathcal{D} \mid \#[D] = n\}$ denote the set of 2CPs with n elements.

Our primitive is a pair $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, where $\mathscr{C}_1$ is a choice function over $\mathscr{M}$ (i.e., $\mathscr{C}_1 : \mathscr{M} \to \mathscr{M}$ such that $\mathscr{C}_1(\mathcal{M}) \subseteq \mathcal{M}$ for all $\mathcal{M} \in \mathscr{M}$) and $\mathcal{C}_2$ is a choice function over $\mathcal{D}$ (i.e., $\mathcal{C}_2 : \mathcal{D} \to \mathcal{D}$ such that $\mathscr{C}_1(D) \subseteq D$ for all $D \in \mathcal{D}$). The interpretation is that of two stage choice; in the first stage the DM faces a 1CP, $\mathcal{M}$, (a set of 2CPs) from which to choose the next periods constraint. Her first stage choice is given by $\mathscr{C}_1(\mathcal{M})$. In the next period she faces one of the acceptable 1CPs $D \in \mathscr{C}_1(\mathcal{M})$, from which she must make a choice of consumption object. Her second stage choice is given by $\mathcal{C}_2(D)$.[3]

For sets $D, D' \in \mathcal{D}$ and $\lambda \in \mathbb{R}$, define $\lambda D + \lambda' D' = \{\lambda x + \lambda' x' \mid x \in D, x' \in D'\}$. Likewise, for sets $\mathcal{M}, \mathcal{M}' \in \mathscr{M}$ and $\lambda \in [0, 1]$, define $\lambda \mathcal{M} + \lambda' \mathcal{M}' = \{\lambda D + \lambda' D' \mid D \in \mathcal{M}, D' \in \mathcal{M}'\}$. For any $D \in \mathcal{D}$ denote by $UC(D)$ the upper contour set of D with respect to $\mathscr{C}_1$: $UC(D) = \{D' \in \mathcal{D} \mid D' \in \mathscr{C}_1(\{D, D'\})\}$, the set of all decision problems chosen over x. Define the lower contour set, $LC(D)$, in dual fashion $LC(D) = \{D' \mid D \in UC(D')\}$.

Utilities and Representation. For each $u \in \mathbb{R}^n$, u defines a linear representation (i.e.,

[2]For a general account of how the moral implications of meta-decisions, e.g., information acquisition, seem to be ignored, see Gino et al. (2016); Grossman and Van Der Weele (2017).

[3]In general, flat font face is used for consumption objects (i.e., x, D), calligraphic lettering is used second stage choice objects (i.e, $\mathcal{M}, \mathcal{D}, \mathcal{C}_2$) and script lettering is used for first stage choice objects (i.e. $\mathscr{M}$, $\mathscr{C}_1$).

expected utility function) over $\mathbb{R}^n$. This is via the obvious duality

$$u(x) = \sum_{i=1}^{n} u^i x^i,$$

which views u as the function taking x to its inner product with u. When a decision maker makes a public choice, she receives utility directly from consumption, but also, from her *image*—the set of utilities that observers believe she might have. Of course, choices over lotteries can only reveal utilities up to affine transformations, so we identify utilities which are rescalings of one another: call $I \subset \mathbb{R}^n$ an *image* if $\lambda I \subseteq I$ for all $\lambda > 0$. Let $\mathbb{I}$ denote the set of all images.

Given any $D \in \mathcal{D}$ and $x \in D$ we denote by $I_D^x \in \mathbb{I}$ the set of utilities such that x would maximize u given D. We have

$$I_D^x = \{u \in \mathbb{R}^n \mid u(x) \geq u(y), \text{ for all } y \in D\}.$$

Notice that I_D^x depends only on the convex hull of D, so we can view it as an operation on convex sets, where it is referred to in the general literature as the *normal cone* of x in D. For any finite subset of $\mathbb{R}^n$, $D \in \mathcal{D}$, denote the set of images associated to D as $\textsc{img}(D) = \{I \in \mathbb{I} \mid I = I_D^x, x \in D\}$

With these definitions in place, we can formally state the definition of an image conscious representation.

Definition. An *image conscious representation* of $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is a pair $\langle u, \Gamma \rangle$ where $u \in \mathbb{R}^n$ and $\Gamma : \mathbb{I} \to \mathbb{R}$ such that

$$\mathcal{C}_2(D) = \operatorname*{arg\,max}_{x \in D} \big(u(x) + \Gamma(I_D^x)\big) \quad \text{and} \tag{C2}$$

$$\mathscr{C}_1(\mathcal{M}) = \operatorname*{arg\,max}_{D \in \mathcal{M}} \Big(\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big)\Big), \tag{C1}$$

for all $D \in \mathcal{D}$ and $\mathcal{M} \in \mathscr{M}$.

3 Two Stage Image Conscious Choice

Because the DM makes the first stage privately, she has a more standard preference in the first stage. Specifically, there are no context effects, and so the DM will have a well defined value function over decision problems.

Axiom 1—WARP. If $D, D' \in \mathcal{M} \cap \mathcal{M}'$, $D \in \mathscr{C}_1(\mathcal{M})$ and $D' \in \mathscr{C}_1(\mathcal{M}')$ then $D \in \mathscr{C}_1(\mathcal{M}')$.

Even if we do not insist that the preference over images is continuous, the fact that the DM's preference over consumption objects is continuous requires that when considering only degenerate 2CPs, the DM has a continuous preferences—the projection of contour sets onto $\mathcal{D}^1$ must be closed.

Axiom 2—Weak Continuity. For all $D \in \mathcal{D}$, $UC(D) \cap \mathcal{D}^1$ and $LC(D) \cap \mathcal{D}^1$ are closed and non-empty.

The non-emptiness restriction ensures that images are not lexicographically preferred to dis-preferred to one another. For example, if $UC(D) \cap \mathcal{D}^1$ was empty, then there no consumption object, no matter how good, that is preferred (along with the trivial image) to D. Because we are interested in linear utilities over consumption objects, this would indicate that the image associated with the choice from D is infinitely good.[4]

We now want to impose a bit of structure that relates the value of D with the *elements* of D: when D is a single element we want the valuation to reflect only the change in consumption value, and hence behave linearly. That is, when all image concerns are obviated, we want preferences to be expected utility. The usual independence axiom is split into two: translation invariance and homogeneity. The reason is that while translation invariance applies to all problems, scaling a choice problem changes the relative importance of consumption utility and image concerns, and can therefore change the value in non-linear ways. Nonetheless, over singleton choice problems, the induced image also remains fixed, preserving the full independence structure.

Axiom 3—Translation Invariance. For all $x \in \mathbb{R}^n$, $\mathcal{M} \in \mathscr{M}$ and $D \in \mathcal{D}$,

$$\mathscr{C}_1(\mathcal{M} + \{x\}) = \mathscr{C}_1(\mathcal{M}) + \{x\} \text{ and}$$
$$\mathcal{C}_2(D + x) = \mathcal{C}_2(D) + x$$

Axiom 4—Singleton Homogeneity. For all $\lambda \in \mathbb{R}_{++}$, $\mathcal{M} \subset \mathcal{D}^1$,

$$\mathscr{C}_1(\lambda\mathcal{M}) = \lambda\mathscr{C}_1(\mathcal{M}).$$

These axioms provide the scaffolding of the representation, a value function over choice problems that, when looking at degenerate problems, reflects a linear preference over consumption objects.

Lemma 1. *If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfies A1—4 then there exists then there exists of a value function, $V : \mathcal{D} \to \mathbb{R}$ such that*

$$\mathscr{C}_1(\mathcal{M}) = \big\{ D \in \mathcal{M} \mid D \in \arg\max_{\mathcal{M}} V(D) \big\},$$

such that, $u : x \mapsto V(\{x\})$ is a linear function over $\mathbb{R}^n$.

Proof. Consider the projection of $\mathscr{C}_1$ to $\mathcal{M} \subset \mathcal{D}^1$. Over this space, $\mathscr{C}_1$ satisfies the expected utility axioms. Therefore, there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ rationalizing the projection of $\mathscr{C}_1$.

[4] If we want, in addition, the value function over images to be continuous, we can strengthen A2 as follows:

Axiom 2*—k-Weak Continuity. For all $D \in \mathcal{D}$ and $k \in \mathbb{N}$, $UC(D) \cap \mathcal{D}^k$ and $LC(D) \cap \mathcal{D}^k$ are closed.

Notice that even when the DM's utility over images is continuous, $\mathscr{C}_1$ is *not* continuous (in the relevant topology) because the map carrying a choice to its associated image is not itself continuous. To see this, notice that if $D_\lambda = \{x, y\}$ for $x \neq y$. For $\lambda \in (0,1)$, $I^{\lambda x + \lambda' x}_{\lambda D + \lambda' x} \neq \mathbb{R}^n$ whereas the limiting choice indices the image $I^x_{\{x\}} = \mathbb{R}^n$. Such complications arise whenever two elements collide in the limit, a problem which does not happen when restricting the domain to $\mathscr{M}^k$.

Now consider any $D \in \mathcal{D}$. We claim that $UC(D) \cap LC(D) \cap \mathcal{D}^1$ is non-empty—the Lemma then follows directly by setting $V(D) = u(x)$ for an $\{x\}$ in the intersection. Assume the claim did not hold. Take $x \in \arg\max_{LC(D)\cap\mathcal{D}^1} u(x)$ and $y \in \arg\min_{UC(D)\cap\mathcal{D}^1} u(x)$ which exist and are distinct by A2 and our assumption. By A1 and the linearity of u, $u(x) < u(\frac{1}{2}x + \frac{1}{2}y) < u(y)$. Thus, $\{\frac{1}{2}x + \frac{1}{2}y\}$ is in neither the upper nor the lower contour set of D, a contradiction to the non-emptiness of $\mathscr{C}_1$. ∎

What remains is to ensure the value function V reflects image consciousness. In particular, we want to show that the gap between $V(D)$ and $u(\mathcal{C}_2(D))$ is dependent only on the induced image. Towards this, we first define, from the properties of choice problems, when one choice reveals more than another about the preferences of the DM. In other words, when will the observing party make a more discriminating inference from the choice of the DM.

We say that x is *more revealing* for D than D', written $D \overset{x}{\rhd} D'$ if we have $x \in D \cap D'$ and for some $\alpha \in (0,1)$ we have $\text{conv}(\alpha D' + (1-\alpha)x) \subseteq \text{conv}(D)$. To understand the motivation for this definition, notice that when a menu is mixed with a singleton, it does not change the maximizer of any expected utility function. Hence, the same inference can be made by the observation that x is chosen out of $\alpha D' + (1-\alpha)x$ as from the observation that x was chosen from D': scaling does not change the induced image. Next, consider what happens when an observer can rule out the DM having preference $v \in \mathbb{R}^n$ after observing $x \in \mathcal{C}_2(D')$, or equivalently, that $x \in \mathcal{C}_2(\alpha D' + (1-\alpha)x)$. This indicates that there is some $y \in \alpha D' + (1-\alpha)x$ such that $v(y) > v(x)$, so x would not have been chosen. But $y \in \text{conv}(D)$, so there must also be some $z \in D$ such that $v(z) > v(x)$—the observer could make the same inference from observing $x \in \mathcal{C}_2(D)$. Hence, observing $x \in \mathcal{C}_2(D)$ reveals at least as much as observing $x \in \mathcal{C}_2(D')$.

Lemma 2. *$D \overset{x}{\rhd} D'$ if and only if $I^x_D \subseteq I^x_{D'}$*

Proof. In Section A.1. ∎

Say that x is *equally revealing* for D than D', written $D \overset{x}{\bowtie} D'$, if $D \overset{x}{\rhd} D'$ and $D' \overset{x}{\rhd} D$. When x is equally revealing for two choice problems, D and D', then choosing x from either menus induces the same perception in an observer; of course choosing x also imparts the same consumption utility, and hence, the DM should find choosing x from D exactly as appealing as choosing x from D'. Thus, if x is chosen from D but not D' it must mean the choice from D' imparts a better combination of consumption and image value: from a period 1 perspective, D' is preferred to D. The following axiom embodies exactly this logic.

Axiom 5—Image Consistency. Let $D, D' \in \mathcal{D}$ with $D \overset{x}{\bowtie} D'$ be such that $x \in \mathcal{C}_2(D)$. Then $x \in \mathcal{C}_2(D')$ if and only if $D \in \mathscr{C}_1(\{D, D'\})$.

3.1 Representation Result

Theorem 3.1. *The following are equivalent:*

1. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *satisfies* A1—5,
2. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *has an image conscious representation* $\langle u, \Gamma \rangle$.

Moreover, u is unique up to positive linear translations, and Γ is unique up to an additive constant on its effective domain.

Proof. That (2) implies (1) is standard. Now, let $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfy A1–5 . Take u and V as defined in Lemma 1. For each $I \in \mathbb{I}$, define $\Gamma(I)$ to be

$$\Gamma(I) = \begin{cases} V(D) - u(x) & \text{if there exists } (D, x) \text{ with } x \in \mathcal{C}_2(D) \text{ and } I_D^x = I \\ -\infty & \text{otherwise.} \end{cases} \tag{3.1}$$

We here verify that Γ is well defined. Let (D, x) and (D', x') be such that $I_D^x = I_{D'}^{x'}$ and $x \in \mathcal{C}_2(D)$ and $x' \in \mathcal{C}_2(D')$. By A3 it is without loss of generality to assume $x = x'$. By the assumption that the $I_D^x = I_{D'}^{x'}$, Lemma 2 provides that $D \overset{x}{\bowtie} D'$. Hence by A5 and Lemma 1, $V(D) = V(D')$. This of course implies that $V(D) - u(x) = V(D') - u(x)$, so that Γ is well defined.

Next, we claim that for all D we have $\mathcal{C}_2(D) = \arg\max_{x \in D} u(x) - \Gamma(I_D^x)$. First, assume that $x \in \mathcal{C}_2(D)$ and let $y \in D$. Since $x \in \mathcal{C}_2(D)$ it follows that $\Gamma(I_D^x) \neq -\infty$, so if $\Gamma(I_D^y) = -\infty$, we have $u(x) + \Gamma(I_D^x) \geq u(y) + \Gamma(I_D^y)$ immediately. So, to make the problem extra hard, assume $\Gamma(I_D^y) \neq -\infty$. It must be there exists a (D', y') such that $I_{D'}^{y'} = I_D^y$ with $y' \in \mathcal{C}_2(D')$. By A3, we can choose $y' = y$. Notice, we have $D \overset{y}{\bowtie} D'$ and that $y \in \mathcal{C}_2(D')$. Thus, we can conclude that $V(D) \geq V(D')$. Appealing to 3.1 implies that

$$u(x) + \Gamma(I_D^x) \geq u(y) + \Gamma(I_{D'}^y) = u(y) + \Gamma(I_D^y).$$

The other direction follows by an anbalagous argument and is therefore omitted.

Next assume that $x \in \arg\max_{x \in D} u(x) - \Gamma(I_D^x)$ but $x \notin \mathcal{C}_2(D)$. Let $y \in \mathcal{C}_2(D)$–so that $\Gamma(I_D^y) \neq -\infty$, which by our assumption implies that $\Gamma(I_D^x) \neq -\infty$. Thus, there exists a (D', x) such that $I_{D'}^x = I_D^x$ with $x \in \mathcal{C}_2(D')$. Again we have $D \overset{x}{\bowtie} D'$ and that $x \in \mathcal{C}_2(D')$; we can conclude that $V(D) \geq V(D')$. Further appeal to A5 shows that in fact $V(D) > V(D')$ (since otherwise, it would be that $x \in \mathcal{C}_2(D)$, which we assumed it was not). Hence, by the definition of Γ,

$$u(y) + \Gamma(I_D^y) > u(x) + \Gamma(N(D', x)) = u(x) + \Gamma(I_D^x),$$

another clear contradiction. ∎

3.2 Attitudes Towards Privacy

A DM who is *privacy seeking* prefers, all else equal, to limit the inference an observer can make regarding his underlying motivations and *privacy averse* if he prefers the opposite:

that an observer can extract the maximal information. Formally:

Definition. Call a DM, $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, *privacy seeking* (resp. *privacy averse*) if for all $D, D' \in \mathcal{D}$ with $D' \overset{x}{\rhd} D$ (resp. $D' \overset{x}{\rhd} D$): If $x \in \mathcal{C}_2(D')$ then $D \in \mathscr{C}_1(\{D, D'\})$.

Recall, $D' \overset{x}{\rhd} D$ indicates that choosing x from D' is more revealing that choosing x from D. Hence a privacy seeking DM would get more utility from consuming X from D rather than D'. Since whatever is chosen from D must be at least as good as x, then if x is chosen from D', where it reveals more than from D, it must be that D can provide as much utility as D'.

There is an interesting link between attitudes towards privacy and classical axioms placed on menu preferences.

Theorem 3.2. *If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is privacy seeking then $D \in \mathscr{C}_1\{D, D'\}$ implies $D \in \mathscr{C}_1\{D, D \cup D'\}$. If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is privacy averse then $D \cup D' \in \mathscr{C}_1\{D, D \cup D'\}$.*

Proof. Follows immediately from Lemma 2 and the observation that $I^x_{D \cup D'} \subseteq I^x_D$ for all $D, D' \in \mathcal{D}$ and $x \in D$. ∎

4 Single Stage Image Conscious Choice

The interpretation of two stage choice is that $\mathscr{C}_1$ represents a choice over 2CPs that is made in the absence of image concerns. Hence, in many scenarios, this choice function will not be observable. This section considers the image conscious model when only second stage choice is accessible to the modeler; it posits axioms only on $\mathcal{C}_2$ equivalent to (C2) of the IC representation.

Limited observability bears a cost. First, the effective uniqueness of Γ is no longer ensured. Second, the axiomatic structure and concomitant proof rely more directly on technical assumptions, and so, are correspondingly more involved. This latter point is self evident, to understand the failure of uniqueness, consider the following.

Say $I, J \in \mathbb{I}$ are *directly* comparable if there is a D such that $I = I^x_D$, $J = I^y_D$ and $x \in \mathcal{C}_2(D)$. When I and J are directly comparable, we have an bound on the utility difference between I and J in terms of consumption utility:

$$\Gamma(I) - \Gamma(J) \geq u(y) - u(x).$$

Say that $I, J \in \mathbb{I}$ are *indirectly comparable* if they are contained in the transitive closure of the direct comparability relation. If two images are not indirectly comparable, then there is no restriction imposed by the observed choices on the relative values of the images. Indirect comparability is an equivalence relation; Γ in the resulting representation normalized independently across the classes of this equivalence relation.

4.1 Axioms

As discussed when introducing A4, scaling a choice problem may result in non-linear tradeoffs. As λ increases, choice from λD places more importance on consumption utility. The first axiom allows $\mathcal{C}_2(\lambda D)$ to vary non-linearly in λ, but ensures that deviations are consistent with increasing importance on consumption utility.

Axiom 1°—Scale Acyclicity. Let $0 < \lambda < \lambda' < \lambda''$ and $D \in \mathcal{D}$. If $x \in \frac{1}{\lambda}\mathcal{C}_2(\lambda D) \cap \frac{1}{\lambda''}\mathcal{C}_2(\lambda'' D)$ then $x \in \frac{1}{\lambda'}\mathcal{C}_2(\lambda' D)$.

In the limit, as $\lambda \to \infty$, only consumption utility matters. Indeed, this is the manner in which u might be identified. To get at this, we can define the following map, which is well defined given A1° and the finiteness of each D:

$$\mathcal{C}_2^\infty : D \mapsto \lim_{\lambda \to \infty} \frac{1}{\lambda}\mathcal{C}_2(\lambda D).$$

To identify u we need $\mathcal{C}_2^\infty$ to be well behaved; ideally this would just entail the imposition of WARP. Unfortunately, it is possible that $u(x) = u(y)$ but $y \neq \mathcal{C}_2^\infty(\{x, y\})$; this happens whenever $\Gamma(I^x_{\{x,y\}}) > \Gamma(I^y_{\{x,y\}})$. To deal with this, we impose WARP on perturbed choice problems.

Axiom 2°—Sequential Limit Consistency. Let $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converge to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all $k \in \mathbb{N}$. Then for any D' with $y \in \mathcal{C}_2^\infty(D')$ there exists a sequence $D'_k \to D'$ such that $x \in \mathcal{C}_2^\infty(D'_k \cup \{x\})$ for all k.

Translation invariance (A3) remains, but is transcribed here for completeness.

Axiom 3°—Translation Invariance. For all $x \in \mathbb{R}^n$ and $D \in \mathcal{D}$,

$$\mathcal{C}_2(D + x) = \mathcal{C}_2(D) + x$$

With these three axioms, u can be identified.

Lemma 3. *If $\mathcal{C}_2$ satisfies A1°—3° then there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ such that*

$$\mathcal{C}_2^\infty(D) \subseteq \arg\max_D u.$$

Moreover, u is unique up to positive linear transformations.

Proof. In section A.2. ∎

From $\mathcal{C}_2$ we can define $\succcurlyeq \subset (\mathbb{R}^n \times \mathbb{I}) \times (\mathbb{R}^n \times \mathbb{I})$ via $(x, I) \succcurlyeq (y, J)$ iff there exists a $D \supseteq \{x, y\}$ with $I^x_D = I$ and $I^y_D = J$, and such that $x \in \mathcal{C}_2(D)$. The next axioms place restrictions on $\succcurlyeq$ but these can be translated back into choice behavior in the obvious, but tedious, manner. Per normal let $\succ$ and $\sim$ denote the asymmetric and symmetric components.

The relation $\succcurlyeq$ will necessarily be highly incomplete; for example, images with overlapping relative interiors will never be comparable. Because of this, $\succcurlyeq$ will not be transitive; it should, however, be extendable to a transitive relation.

Axiom 4°—Acyclicity. $\succ$ is acyclic.

Finally, we impose three restrictions that relate the choice over $\succcurlyeq$ to the consumption utility as identified by Lemma 3: *monotonicity* states that if $(x, I) \succcurlyeq (y, J)$ and $u(x') > u(x)$ then not $(y, J) \succ (x', I)$—ceteris paribus, more consumption is better; *boundedness* states that $(x, I) \succ (y, J)$ cannot hold for all x—I cannot be 'infinitely' better than J; *continuity* states that if $u(x_n) \to u(x)$ and $(x_n, I) \succcurlyeq (y, J)$ for all n, then not $(y, J) \succ (x, I)$—preferences cannot be reversed in the limit.

In the proof of the representation theorem, we will extend $\succcurlyeq$ to a complete binary relation, showing these properties still hold; as such, it is helpful to define things for a general relation R defined over $(\mathbb{R}^n \times \mathbb{I})$.

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R (with asymmetric component S) defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-monotone* if whenever

1. $v(z) > 0$ and $(x, I)R(y, J)$, or,
2. $v(z) \geq 0$ and $(x, I)S(y, J)$,

then not $(y, J)R(x + z, I)$.

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-bounded* if for all $I, J \in \mathbb{I}$, it is true that $\inf\{v(z) \mid (z, I)R(\mathbf{0}, J)\} > -\infty$.

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-continuous* if for all $I, J \in \mathbb{I}$, if whenever there exists a sequence $\{x_k\}_{k \in \mathbb{N}}$ such that $v(x_n) \to 0$ and $(x_k, I)R(y, J)$ for all k, then for any x with $v(x) = 0$, if $(y, J)R(x, I)$ then $(x, I)R(y, J)$.

Let $\succcurlyeq^{TC}$ denote the transitive closure of $\succcurlyeq$.

Axiom 5°—Consumption Regularity. The relation $\succcurlyeq^{TC}$ is u-monotone, u-continuous, and u-bounded.

These axioms are equivalent to the existence of a image conscious representation $\langle u, \Gamma \rangle$ which represents $\mathcal{C}_2$ as (C2).

Theorem 4.1. *The following are equivalent:*

1. *$\mathcal{C}_2$ satisfies A1°—5°*
2. *$\mathcal{C}_2$ has an image conscious representation $\langle u, \Gamma \rangle$.*

Moreover, u is unique up to positive linear translations.

Proof. In section B. ∎

In contrast to the proof of Theorem 3.1, the proof of Theorem 4.1 is rather involved. The main difficulty surrounds the intrinsic incompleteness of the induced preference relation on $\mathbb{R}^n \times \mathbb{I}$, owing to the geometric dependence between the set of consumption alternatives and the consequent images. Indeed, imagine that some complete $\succcurlyeq^\star$ over $\mathbb{R}^n \times \mathbb{I}$ was magically identified and preserved the relevant structure and extended $\succcurlyeq$. Then, fixing $I^* \in \mathbb{I}$ and setting $\Gamma(I^*) = 0$, we can recover the entirety of Γ is by simply setting:

$$\Gamma : I \mapsto -u(x^I)$$

where x^I is a consumption alternative such that $(x^I, I) \sim (\mathbf{0}, I^*)$. Such an alternative exists by the u-boundedness and u-continuity assumptions, and its utility is unique by u-monotonicity. Translation invariance and transitivity then ensure the resulting $\langle u, \Gamma\rangle$ actually represents $\succcurlyeq^\star$, and hence $\mathcal{C}_2$.

Guaranteeing that $\succcurlyeq$ can be extended to a complete $\succcurlyeq^\star$ (while preserving the axiomatic structure) turns out to be pain, but mostly for technical reasons. The relatively simple core idea is as follows: we can first extend $\succcurlyeq$ by adding comparisons that were not observed by $\mathcal{C}_2$ but must hold because of transitivity, monotonicity, or continuity. The resulting relation extends $\succcurlyeq$ because of A4° and A5°. Still, there will be images I and J such that no x satisfies $(x, I) \sim (\mathbf{0}, J)$. What can we do? Just pick some x and extend the relation by adding $(x, I) \sim (\mathbf{0}, J)$ (and then again adding all the consequences of transitivity, monotonicity, or continuity). Repeating the process for different I's and J's creates a partial order of extensions of $\succcurlyeq$, which, by Zorn's Lemma, has maximal element that must be complete.[5]

This process also elucidates the exact nature of non-uniqueness. If two images are initially comparable, that is there exists an x and y such that $(x, I) \sim (y, J)$ is implied by the initial choice function, then the difference between $\Gamma(I)$ and $\Gamma(J)$ is identified (up to a common normalization) by the difference between $u(x)$ and $u(y)$. Thus, identification is made over the equivalence classes of initially comparable images (that comparability is an equivalence relation is Lemma 6(i)), but, these equivalence classes can be independently normalized.

5 A Brief Review of Related Literature

Image conscious behavior is ubiquitous and has long been studied within economics. In a work of classical importance, Veblen (1899) coined the term *conspicuous consumption* referring to purchases in which the primary value is derived indirectly by signaling wealth or status. Such spending habits are alive and well in the modern era.

Recently, experimental economists and psychologists have exposed the importance of image concerns in sundry other contexts. A common theme is the discord between and individual's personal preference and his desire to be seen as acting in a normative manner:

[5]This is one of the many instantiations of Szpilrajn's extension theorem with additional structure being preserved by the extension.

the DM faces a tradeoff between direct utility and image utility. This tradeoff is central to the present model as captured by the IC representation.

In an experimental setting, Snyder et al. (1979) asked subjects to choose one of two films to watch, one being shown in a room with a disabled person. Subjects overwhelming stated a preference for the film in the room *without* the disabled person, regardless of which film this happened to be. The authors conclude that subjects want to avoid being near disabled people but do not want others to believe they feel this way. Recent studies suggest similar 'reluctant adherence' to many other social norms is motivated by image consciousness.

Dana et al. (2006) find that subjects in the dictator game are willing to pay a positive cost to ensure the receivers did not know the game was to be played (i.e., the dictator gets the full pie, less the cost, and the receiver is never informed there was a decision to be made). Because the decision to keep everything is always available, paying the cost serves only to effect a more desirable image. This result is echoed in Andreoni and Bernheim (2009), where subjects' choice of fair (i.e., 50-50) allocations in the dictator game depends very much on who can observer the dictators' choices. When there is a commonly known chance that unfair allocations get implemented irrespective of the dictator's choice, and these nature-chosen outcomes are indistinguishable from dictator-chosen outcomes to receivers, then the rate of fair allocations dramatically declines.

DellaVigna et al. (2012) find, in a door-to-door field experiment, that many donation decisions seem to be predicated on social pressure. When given the ability to avoid face to face contact with a solicitor, donations decrease. This effect is concentrated in small donations, an effect that is predicted by the present model. DellaVigna et al. (2016) find that social pressure plays a key role in the decision to vote; potential voters are more likely to vote when they expect that other will ask them about their voting record.

Bénabou and Tirole (2006) provide the canonical utility function for image concerned agents and explore how direct incentives to act pro-socially can have the opposite effect by skewing the images associated with certain actions. Their model is behavioral rather than decision theoretic, in the sense that they are less concerned with identification from observables, and the generality of the types of images that can be entertained. For instance, they make the assumption that actions can be linearly ordered and that everyone prefers a "higher" image to a "lower" one.

In fact, the decision theoretic exploration of image consciousness is practically non-existent. Closest to this model is the model of Evren and Minardi (2017) who investigate the axiomatic characterization of *warm glow*. Although warm glow is a distinct phenomena, there are two major similarities between the models: (i) both image consciousness and warm glow can promote normative behavior and (ii) in both models, the *menu* from which an element is chosen changes the derived value from consumption. This later property was is also shared by the model of Gul and Pesendorfer (2001) who consider a DM who seeks to limit his options to curtail the effect of temptation.

A Proofs Omitted From the Text

A.1 Proof of Lemma 2

The if direction is essentially Gul and Pesendorfer (2006), Lemma 1. For completeness: Assume that $I = I^x_D \subseteq I^x_{D'} = I'$. For some $D \subset \mathbb{R}^n$, let $pos(D) = \{x \in \mathbb{R}^n \mid x = \sum_i \lambda_i x_i, \lambda_i \geq 0, x_i \in D\}$. Since normal cones are contra-variant with respect to subset inclusion, $I^{\mathbf{0}}_{I'} \subseteq I^{\mathbf{0}}_{I}$. By Gul and Pesendorfer (2006), proposition 1(ii), this indicates $pos(D' - \{x\}) \subseteq pos(D - \{x\})$. Let $y \in D' - \{x\} \subseteq pos(D' - \{x\}) \subseteq pos(D - \{x\})$. It follows that $y = \sum_i \lambda_i y_i$ with $\lambda_i \geq 0$ and $y_i \in D' - \{x\}$. Therefore, $\alpha_y y \in \text{conv}(D - \{x\})$ for sufficiently small $\alpha_y \in (0,1)$; letting $\alpha = \min\{\alpha_y \mid y \in D' - \{x\}\}$. We have, $\alpha(y + x) + (1-\alpha)x \in \text{conv}(\alpha D + (1-\alpha)x) \subseteq \text{conv}(D)$.

Towards the only if assume that $D \overset{x}{\triangleright} D'$, witnessed by $\lambda \in (0,1)$, and let $v \in I^x_D$, so that $v(x) \geq v(y)$ for all $y \in D$. Since v is linear, this indicates that $v(x) \geq v(y)$ for all $y \in \text{conv}(D)$, and thus for all $y \in \lambda D' + \lambda' x$. So $v \in I^x_{\lambda D' + \lambda' x} = I^x_{D'}$.

A.2 Proof of Lemma 3

Define the preference relation, $\dot{\succcurlyeq}$, on $\mathbb{R}^n$ as follows: $x \dot{\succcurlyeq} y$ if there exists a $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converging to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}^\infty_2(D_k)$ for all k. We claim that $\dot{\succcurlyeq}$ is an expected utility preference; this would complete the lemma, for if $x \in \mathcal{C}^\infty_2(D)$ then $x \dot{\succcurlyeq} y$ by taking the constant sequence D, and hence $x \in \arg\max_D u$ for any representation of $\dot{\succcurlyeq}$.

Completeness. Fix $x, y \in \mathbb{R}^n$, and take a sequence $\{y_k\}_{k\in\mathbb{N}}$ converging to y. Since $\mathcal{C}_2$ is non-empty there exists a subsequence (w.l.o.g., indexed by the same k) such that for all k either $x \in \mathcal{C}^\infty_2(\{x, y_k\})$ or $x \notin \mathcal{C}^\infty_2(\{x, y_k\})$. If it is the former, we are done and $x \dot{\succcurlyeq} y$. If it is the latter, we can appeal to translation invariance, and for each k, shift by $y - y_k$ to obtain a sequence $\{x + y - y_k, y\}$ such that y is always chosen, so $y \dot{\succcurlyeq} x$.

Transitivity. Let $x \dot{\succcurlyeq} y$ and $y \dot{\succcurlyeq} z$. Consider the choice problem $D = \{x, y, z\}$. If $x \in \mathcal{C}^\infty_2(D)$ then $x \dot{\succcurlyeq} z$ and we are done. If $y \in \mathcal{C}^\infty_2(D)$, then we can appeal to A2° to obtain a sequence $D_k \to D$ such that $x \in \mathcal{C}^\infty_2(D_k \cup \{x\})$ for all k, hence $x \dot{\succcurlyeq} z$ (notice, $x \dot{\succcurlyeq} y$ definitionally implies the antecedent for A2°). Finally, assume $z \in \mathcal{C}^\infty_2(D)$. Then by the above reasoning, we have a sequence $D_k \to D$ such that $y \in \mathcal{C}^\infty_2(D_k \cup \{y\})$. Now since $x \dot{\succcurlyeq} y$, we can, for each D_k find a further sequence $D^k_{k'} \to D_k \cup \{y\}$ such that $x \in \mathcal{C}^\infty_2(D^k_{k'} \cup \{x\})$ for all $k, k' \in \mathbb{N}$. For each $m \in \mathbb{N}$, set $\hat{D}_m$ to be the first element of $\{D^m_{k'}\}_{k'\in\mathbb{N}}$ such that $d_H(D^m_{k'} - D_m) \leq \frac{1}{m}$. This is a sequence converging to D and with x always chosen.

Continuity. Let $\{y_k\}_{k\in\mathbb{N}}$ converge to y and be such that $x \dot{\succcurlyeq} y_k$ for all k. Then by definition, we have a sequence of sequences $\{\{D^k_{k'}\}_{k'\in\mathbb{N}}\}_{k\in\mathbb{N}}$ such that $D^k_{k'} \to D_k$ for all k and $x \in \mathcal{C}^\infty_2(D^k_{k'} \cup \{x\})$. As above, we can find a sequence of sets converging to $\{x, y\}$ such that x is chosen from each. Closure of the lower contour sets is the analogous.

Independence. Let $x \dot{\succcurlyeq} y$; we have $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converging to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all k. Set $\lambda \in (0,1)$ and $z \in \mathbb{R}^n$. We know $x \in \mathcal{C}_2^\infty(D_k)$ indicates by definition that $x \in \lim_{\gamma\to\infty} \frac{1}{\gamma}\mathcal{C}_2(\lambda D_k) = \frac{1}{\lambda}\lim_{\gamma\to\infty}\frac{1}{\gamma}\mathcal{C}_2(\gamma\lambda D_k)$ or, multiplying by λ, that $\lambda x \in \mathcal{C}_2^\infty(\lambda D_k)$. Then by A3 we have that $\lambda x + \lambda' z \in \mathcal{C}_2^\infty(\lambda D_k + \lambda' z)$. Since $\lambda D_k + \lambda' z$ converges to $\lambda D + \lambda' z$, we have that $\lambda x + \lambda' z \dot{\succcurlyeq} \lambda y + \lambda' z$, as desired. ∎

B Proof of Theorem 4.1

Definition. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *translation invariant* if for all $x, y, z \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$ we have $(x, I)R(y, J)$ if and only if $(x + z, I)R(y + z, J)$.

Since $\mathcal{C}_2$ is translation invariant, $\succcurlyeq^{TC}$ is as well.

Definition. Let R_1 and R_2 denote two binary relations on a set X (with asymmetric components S_1 and S_2). We say that R_1 *extends* R_2 if $R_2 \subseteq R_1$ and if xS_2y then also xS_1y.

That is, R_1 includes all comparisons that R_2 includes, but does not break any asymmetric comparison into a symmetric one. Because $\succ$ is acyclic, $\succcurlyeq^{TC}$ extends $\succcurlyeq$.

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R (with asymmetric component S) defined over $(\mathbb{R}^n \times \mathbb{I})$ *strongly-v-monotone* if $(x+z, I)R(z, I)$ whenever $u(z) \geq 0$ and $(x+z, I)S(z, I)$ whenever $u(z) > 0$.

Notice that a transitive and strongly-v-monotone relation is also v-monotone. Let $\succcurlyeq^{\#}$ denote $\succcurlyeq^{TC} \cup \{((x + z, I), (x, I)) \mid x, z \in \mathbb{R}^n, u(z) \geq 0, I \in \mathbb{I}\}$ and $\succcurlyeq^*$ its transitive closure.

Lemma 4. *$\succcurlyeq^*$ is reflexive, transitive, translation invariant, strongly-u-monotone, u-bounded, u-continuous and extends $\succcurlyeq$.*

Proof. That $\succcurlyeq^*$ is reflexive follows from the addition of $((x+\mathbf{0}, I), (x, I))$; that it is transitive is immediate in that it is a transitive closure; that it is translation invariant follows from that translation invariance of $\succcurlyeq^{TC}$ and the fact that all added relations are added in a translation invariant way. Next, notice that $\succcurlyeq^{\#}$ is obviously u-monotone and u-bounded. Further, notice that, because of u-monotonicity, the addition comparisons added to $\succcurlyeq^{TC}$ cannot turn a strict preference into an indifference; hence $\succcurlyeq^{\#}$ extends $\succcurlyeq^{TC}$.

$\succcurlyeq^*$ extends $\succcurlyeq^{TC}$. Assume this was not the case so that we have a finite sequence

$$(x_1, I_1) \succcurlyeq^{\#} (x_2, I_2) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_m, I_m)$$

such that $(x_m, I_m) \succ^{TC} (x_1, I_i)$.

Notice that for at least one $j < m$ we have

$$(x_{j+1}, I_{j+1}) = (x_j - z'_j, I_j) \tag{B.1}$$

for some z'_j with $u(z'_j) \geq 0$. If this was not the case, then each relation holds also for $\succcurlyeq^{TC}$, indicating that $(x_1, I_1) \succcurlyeq^{TC} (x_m, I_m)$, a clear contradiction.

So, let $B \subseteq \{1 \dots m\}$ denote the non-empty set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) \geq 0$. We have:

$$(x_1, I_1) \succsim^{\#} \dots \succsim^{\#} (x_j, I_j) \succsim^{\#} (x_{j+1}, I_{j+1}) = (x_j - z'_j, I_j) \succsim^{\#} \dots \succsim^{\#} (x_m, I_m)$$

By translation invariance, we can, for the lowest $j \in B$, add z'_j from the all terms after $j+1$ to obtain

$$(x_1, I_1) \succsim^{\#} \dots \succsim^{\#} (x_{j-1}, I_{j-1}) \succsim^{\#} (x_j, I_j) = (x_{j+1} + z'_j, I_j) \succsim^{\#} \dots \succsim^{\#} (x_m + z'_j, I_m)$$

Continuing to delete terms in this manner for all $i \in B$, we are left with a sequence, contained within $\succsim^{TC}$, asserting $(x_1, I_1) \succsim^{TC} (x_m + \sum_{i \in B} z'_i, I_m)$, contradicting u-monotonicity.

STRONG-u-MONOTONICITY. By way of contradiction, assume that by taking the transitive closure we generate a violation of strong-u-monotonicity. That $(x + z, I) \succsim^* (x, I)$ is immediate, so assume this holds only weakly: for some (x, I), $(x, I) \succsim^* (x + z, I)$ for $z \in \mathbb{R}^n$ with $u(z) > 0$.

This requires a sequence of comparisons

$$(x, I) = (x_1, I_1) \succsim^{\#} (x_2, I_2) \succsim^{\#} \dots \succsim^{\#} (x_m, I_m) = (x + z, I)$$

As above, let $B \subseteq \{1 \dots m\}$ denote the set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) > 0$. If $B = \emptyset$, we could exhibit this sequence via $\succsim^{TC}$, violating u-monotonicity. Therefore, as above, we can appeal to translation invariance to delete terms for each $i \in B$: the resulting sequence is contained within $\succsim^{TC}$ and asserts $(x, I) \succsim^{TC} (x + z + \sum_{j \in B} z'_j, I)$, contradicting u-monotonicity.

u-BOUNDEDNESS. Fix $z \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$. Let $(z, I) \succsim^* (\mathbf{0}, J)$ so that there exists a finite sequence

$$(z, I) = (x_1, I_1) \succsim^{\#} (x_2, I_2) \succsim^{\#} \dots \succsim^{\#} (x_m, I_m) = (\mathbf{0}, J)$$

Once again, let $B \subseteq \{1 \dots m\}$ denote the set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) > 0$. If $B = \emptyset$, then this sequence would exist within $\succsim^{TC}$, indicating $\inf\{u(y) \mid (y, I) \succsim^{TC} (\mathbf{0}, J)\} \leq u(z)$. If B is not empty, we can proceed by the usual trick to conclude $(z, I) \succsim^{TC} (\sum_{j \in B} z'_j, J)$, or by translation invariance, $(z - \sum_{j \in B} z'_j, I) \succsim^{TC} (\mathbf{0}, J)$. This indicates that $\inf\{u(y) \mid (y, I) \succsim^{TC} (\mathbf{0}, J)\} \leq u(z) - \sum_{j \in B} u(z'_j) \leq u(z)$. Since $u(z)$ was arbitrary, the infimum with respect to $\succsim^*$ can be no lower than with respect to $\succsim^{TC}$, which was bounded below.

u-CONTINUITY. Let x be such that $u(x) = 0$ and a sequence $\{x_k\}_{k \in \mathbb{N}}$ converging with $u(x_n) \to 0$. Let (y, J) be such that $(x_k, I) \succsim^* (y, J)$ for all k and $(y, J) \succsim^* (x, I)$. We can use the now standard trick to find following relations:

$$(x_k - z_k, I) \succsim^{TC} (y, J)$$

for each k, and

$$(y, J) \succsim^{TC} (x + z, I)$$

with $u(z_k) \geq 0$ for each k and $u(z) \geq 0$. Necessarily, $u(z) = 0$, or else, eventually $u(x_k - z_k) <$

$u(x+z)$ creating a violation of u-monotonicity. For the same reason, it must be that for all $u(z_k) \leq u(x_k)$. Hence $u(x_k - z_k) \to 0$, and by u-continuity $(x+z, I) \succsim^{TC} (y, J)$. Now, since $u(-z) = 0$ we have that $(x, I) \succsim^{\#} (x+z, I) \succsim^{\#} (y, J)$, and hence, $(x, I) \succsim^{*} (y, J)$. ★

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *strongly-v-continuous* if for all $I, J \in \mathbb{I}$, if whenever there exists a sequence $\{x_k\}_{k \in \mathbb{N}}$ such that $v(x_n) \to 0$ and $(x_k, I) R (y, J)$ for all k, then for any x with $v(x) = 0$, $(x, I) R (y, J)$.

Lemma 5. *Let $\succsim^+$ be the transitive closure of*

$$\succsim^* \cup \Big\{ ((x, I), (y, J)) \mid \textit{Exists } \{z_k\}_{k \in \mathbb{N}}, u(z_n) \to 0, (x + z_k, I) \succsim^* (y, J) \textit{ for all } k \Big\}$$

Then $\succsim^+$ is reflexive, transitive, translation invariant, strongly-u-monotone, u-bounded, strongly-u-continuous and extends $\succsim^*$ (hence $\succsim$).

Proof. Reflexivity, transitivity, translation invariance, and strong-u-continuity are all immediate.

$\succsim^+$ EXTENDS $\succsim^*$. Let $(y, J) \succsim^+ (x, I)$. Then there must exist a sequence $\{(x_j, I_j)\}_{j=1}^m$, with $(x_1, I_1) = (y, J)$ and $(x_m, x_m) = (x, I)$, and such that for each $j < m$ there is a sequence $\{z_k^j\}_{k \in \mathbb{N}}, u(z_k^j) \to 0$ (possibly the constant sequence $\mathbf{0}$, if $(x_i, I_i) \succsim^* (x_{i+1}, I_{i+1})$) such that $(x_j + z_k^j, I_j) \succsim^* (x_{j+1}, I_j)$ for all k. It is without loss of generality to assume that $u(z_k^j) \geq 0$ for all j, k. But notice we have

$$(x_1 + \sum_{i=1}^m z_k^i, I_1) \succsim^* (x_2 + \sum_{i=2}^m z_k^i, I_2) \succsim^* \ldots (x_j + \sum_{i=j}^m z_k^i, I_j) \succsim^* \ldots \succsim^* (x_m, I_m)$$

for each k. This indicates that $(y + \sum_{i=1}^m z_k^i, J) \succsim^* (x, I)$ where $u(\sum_{i=1}^m z_k^i) \to 0$. So by the u-continuity of $\succsim^*$, we cannot have $(x, I) \succ^* (y, J)$: therefore $\succsim^+$ extends $\succsim^*$.

STRONG-u-MONOTONICITY. We have that $(x, I) \succsim^+ (x+z, I)$ immediately; since $\succsim^+$ extends $\succsim^*$ it cannot be that $(x+z, I) \succsim^+ (x, I)$.

u-BOUNDEDNESS. Fix $x \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$. Let $(x, I) \succsim^+ (\mathbf{0}, J)$. Using the same trick as in the proof of extension, we can find a (finite) collection of sequences, $\{\{z_k^j\}_{k \in \mathbb{N}}\}_{j=1}^m$ such that $u(z_k^j) \to 0$ for each j and $(x + \sum_{i=1}^m z_k^i, I) \succsim^* (\mathbf{0}, J)$. Since $u(x + \sum_{i=1}^m z_k^i) \to u(x)$ we have that $u(x) \geq \inf\{v(z) \mid (z, I) \succsim^* (\mathbf{0}, J)\}$. ★

Lemma 6. *Let $v : \mathbb{R}^n \to \mathbb{R}$ be a linear function and R be a preorder on $(\mathbb{R}^n \times \mathbb{I})$ that is translation invariant, strongly-v-monotone, v-bounded and strongly-v-continuous. Call $I, J \in \mathbb{I}$ R-*comparable *if there exists an $x \in \mathbb{R}^n$ such that $(x, I) R (\mathbf{0}, J)$ and $(\mathbf{0}, J) R (x, I)$. Then*

1. *R-comparability is an equivalence relation.*

2. *If I, J are not comparable, then there exists $\bar{x} \in \mathbb{R}^n$ such that neither $(\bar{x}, I) R (\mathbf{0}, J)$ nor $(\mathbf{0}, J) R (\bar{x}, I)$*

3. If $\bar{I}, \bar{J}$ are not comparable, and $\bar{x}$ is as in (2), then, R^* defined as the transitive closure of $R^{\#} = R \cup \{(\bar{x}+z, \bar{I})R(z, \bar{J}), (z, \bar{J})R(\bar{x}+z, \bar{I}) \mid z \in \mathbb{R}^n\}$ is also a translation invariant, strongly-v-monotone, v-bounded, and strongly-v-continuous preorder that extends R.

Proof. (1) Reflexivity is immediate. Symmetry follows from translation invariance. Transitivity follows from the transitivity and translation invariance of R, in the obvious way.

(2) Consider the sets $\{v(x) \mid (x, I)R(\mathbf{0}, J)\} \subseteq \mathbb{R}$ and $\{v(x) \mid (\mathbf{0}, J)R(x, I)\} \subseteq \mathbb{R}$. By strong-$v$-monotonicity, these are (possibly empty) intervals, the former upward-closed and the later downward-closed. By v-boundedness neither is $\mathbb{R}$ itself. By strong-v-continuity they are closed. If these intervals overlap, then I and J are comparable, so assume they do not overlap. Since $\mathbb{R}$ is connected, so there must be a point not in either interval.

(3) Fix $\bar{I}, \bar{J}$ that are not comparable for some R. Let $R^{\#}$ and R^* be as in the statement of the Lemma, and let and S, $S^{\#}$ and S^* denote respective asymmetric components. Reflexivity, transitivity, and translation invariance are immediate.

R^* extends R. Assume it did not: there exists a (x, I) and (y, J) such that $(x, I)S(y, J)$ but $(y, J)R^*(x, I)$. This last relations indicates the existence of a sequence,

$$(y, J)R^{\#}(x_1, I_1)R^{\#} \dots R^{\#}(x_m, I_m)R^{\#}(x, I).$$

As in the proof of Lemma 4, there must be some relation not contained in R, so that for some $j < m$, we have $(x_j, I_j) = (\bar{x} + z, \bar{I})$ and $(x_{j+1}, I_{j+1}) = (z, \bar{J})$ (or vice versa, with an analogous proof following). It is without loss of generality that there is a single index j such that $(x_j, I_j), (x_{j+1}, I_{j+1}) \notin R$.[6] Capitalizing on the fact that R is transitive, we can further delete all other relations, we have

$$(y, J)R(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(x, I).$$

We can split the above sequence and swapping the order, recall $(x, I)S(y, J)$, leaving us with:

$$(z, \bar{J})R(x, I)S(y, J)R(\bar{x} + z, \bar{I}).$$

By the translation invariance of R, this implies $(\mathbf{0}, \bar{J})R(\bar{x}, \bar{I})$, a contradiction to the definition of $\bar{I}, \bar{J}$ and $\bar{x}$.

Strong-v-monotonicity. We have that $(x, I)R^*(x + z, I)$ immediately; since R^* extends R it cannot be that $(x + z, I)R^*(x, I)$.

[6] To see why: consider the following sequence

$$(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(x_i, I_i)R \dots (x_{i+j}, I_{i+j})R(\bar{x} + z', \bar{I})R^{\#}(z', \bar{J})$$

where the "..." contains only R relations. If $v(z') < v(z)$, then $(z, \bar{J})S(z', \bar{J})$ by strong-v-monotonicity, and we can make the same inference deleting one $R^{\#}$ relation. If $v(z') \geq v(z)$ we have a contradiction: we have

$$(z', \bar{J})R(z, \bar{J})R(x_i, I_i)R \dots (x_{i+j}, I_{i+j})R(\bar{x} + z', \bar{I})$$

where the first relation is from strong-v-monotonicity. This implies, however, via translation invariance, that $(\mathbf{0}, \bar{J})R(\bar{x}, \bar{I})$, a contradiction to the definition of $\bar{I}, \bar{J}$ and $\bar{x}$.

v-BOUNDEDNESS. Fix $I, J \in \mathbb{I}$. Define the following constants.

$$a_1 = \inf\{v(z) \mid (z, I)R(\mathbf{0}, J)\}$$
$$a_2 = \inf\{v(z) \mid (z, I)R(\bar{x}, \bar{I})\}$$
$$a_3 = \inf\{v(z) \mid (z, \bar{J})R(\mathbf{0}, J)\}$$

Let $(x, I)R^*(\mathbf{0}, J)$. If this relation can be exhibited by R, then $u(x) \le a_1$. So, to make things interesting, assume it cannot be; by the above arguments we can find the following sequence of relations:

$$(x, I)R(\bar{x} + z', \bar{I})R^{\#}(z', \bar{J})R(\mathbf{0}, J)$$

By the definition of a_2, and translation invariance, the first relation indicates that $u(x) \ge a_2 + v(z')$. The last relation likewise indicates that $u(z') \ge a_3$; hence $u(x) \ge a_2 + a_3$. In either case, $u(x) \ge \min\{a_1, a_2 + a_3\}$ and is hence bounded from below.

u-CONTINUITY. Let x be such that $u(x) = 0$ and a sequence $\{x_k\}_{k \in \mathbb{N}}$ converging with $u(x_n) \to 0$. Let (y, J) be such that $(x_k, I)R^*(y, J)$ for all k. By taking a subsequence if necessary, it is without loss of generality to restrict attention to the case where either $(x_k, I)R(y, J)$ for all k or not $(x_k, I)R(y, J)$ for all k. The former is a direct application of the strong-u-continuity of R. Assume the latter: we have,

$$(x_k, I)R(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(y, J)$$

By the strong-u-continuity of R, $(x, I)R(\bar{x} + z, \bar{I})$ and hence $(x, I)R^*(y, J)$. ★

Lemma 7. *There exists a translation invariant, strongly-u-monotone, u-bounded, and strongly-u-continuous preorder on $(\mathbb{R}^n \times \mathbb{I})$, $\succcurlyeq^\star$, that extends $\succcurlyeq^+$ such that all $I, J \in \mathbb{I}$ are $\succcurlyeq^\star$ comparable.*

Proof. Consider the set of all translation invariant, strongly-u-monotone, u-bounded, and strongly-u-continuous preorders on $(\mathbb{R}^n \times \mathbb{I})$ that extend $\succcurlyeq^+$. Say that $R \le R'$ if R' extends R. Clearly, $\le$ is a partial order, and every chain (totally ordered subset) is bounded by its union. Hence, we can apply Zorn's lemma to conclude the existence of a maximal (with respect to the extension induced order) relation over $(\mathbb{R}^n \times \mathbb{I})$. Call this relation $\succcurlyeq^\star$. By Lemma 6 part (iii), all I, J are $\succcurlyeq^\star$-comparable, or else we could find a further extension, contradicting the maximality of $\succcurlyeq^\star$. ★

For each $I \in \mathbb{I}$, define let x^I denote an element such that $(x^I, I) \sim^\star (\mathbf{0}, \mathbf{0})$. Then define $\Gamma : \mathbb{I} \to \mathbb{R}$ by

$$\Gamma : I \mapsto -u(x^I) \tag{B.2}$$

We now claim that $\langle u, \Gamma \rangle$ forms a IC representation for $\mathcal{C}_2$. Take a menu $D \in \mathcal{D}$. Assume that $x \in \mathscr{C}_1(D)$. Then $(x, I_D^x) \succcurlyeq (y, I_D^y)$ for all $y \in D$. Since $\succcurlyeq^\star$ extends $\succcurlyeq^+$ (Lemma 7), hence $\succcurlyeq$ (Lemma 4), we have $(x, I_D^x) \succcurlyeq^\star (y, I_D^y)$ for all $y \in D$. Therefore, by definition, and translation invariance,

$$(x - x^{I_D^x}, \mathbf{0}) \sim^\star (x, I_D^x) \succcurlyeq^\star (y, I_D^y) \sim^\star (y - x^{I_D^y}, \mathbf{0}).$$

Moreover, by strong-u-monotonicity this indicates that

$$u(x - x^{I_D^x}) \geq u(y - x^{I_D^y}),$$

or, from the definition of Γ and and the linearity of u,

$$u(x) + \Gamma(I_D^x) \geq u(y) + \Gamma(I_D^y),$$

for all $y \in D$. So $\mathcal{C}_2(D) \subseteq \arg\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big)$.

Now assume that $x \notin \mathcal{C}_2(D)$. Then there exists a $y \in D$ such that $(y, I_D^y) \succ (x, I_D^x)$. Since $\succcurlyeq^\star$ extends $\succcurlyeq$, we have $(y, I_D^y) \succ^\star (x, I_D^x)$. From repetition of the above with strict preference/inequality we conclude that

$$u(y) + \Gamma(I_D^y) > u(x) + \Gamma(I_D^x),$$

So $\arg\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big) \subseteq \mathcal{C}_2(D)$ and we have established the existence of an image conscious representation. ∎

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