ECON 1200: Midterm

Thursday July 9, 2015

DUE: Tuesday July 14, 2015

Q1 [25PTS]

Consider the following strategic-form game $G_1(x, y)$ where $x, y \in \mathbb{R}$ are parameters of the game.

Figure 1: Strategic-form game $G_1(x, y)$

Row: \ Column:	L	R
T	$4x, 3$	$4, 2$
B	$2, y$	$x, 7$

1. For what values of x does the Row player have a strictly dominated strategy? Provide both the set of possible values of x and the strategy that is dominated.
2. For what values of y does the Column player have a weakly dominated strategy? Provide both the values of y and the strategy that is dominated.
3. For values of (x, y) is (B, R) a pure strategy Nash Equilibrium. For what values of (x, y) is (T, L) a pure strategy Nash Equilibrium?
4. Now assume that $x = 5$ and $y = 2$. Find a totally mixed Nash Equilibrium, (i.e., $(p, 1-p)$ for Row, $(q, 1-q)$ for Column, such that $p, q \in (0, 1)$.)

Q2 [25PTS]

Two firms are price-competing as in the standard Bertrand model. Each faces the market demand function $D(p) = 100 - p$. Firm 1 has constant marginal cost $c_1 = 40$, while firm 2 has the constant marginal cost $c_2 = 50$. If either of the firms has the lower price, it captures the entire market, and when they charge exactly the same price, they split the market evenly, as in the standard model.

1. Draw the profit function $\pi_1(p_1, p_2)$ when $p_2 = 60$. (Mark sure to label all points of interest on all graphs: maximum values, axis crossings, points of discontinuity, axes labels)
2. Draw the profit function $\pi_2(p_1, p_2)$ when $p_1 = 60$.
3. Write out the best-response correspondences $B_1(p_2)$ and $B_2(p_1)$ as functions of p_2 and p_1 respectively.
4. Is there an equilibrium for this game as defined?
5. Suppose $S_1 = S_2 = \{0.00, 0.01, 0.02, \ldots, 1000.00\}$. That is, instead of any real number we force prices in whole cents.
   (a) Why must there now be an equilibrium, regardless of the exact profit functions?
   (b) Provide a Nash Equilibria in pure strategies for this game.
   (c) Calculate the profits for each firm under any one of these equilibria, accurate to the nearest cent.

Q3 [25PTS]

Recall in class, that we showed that the general solution for the N firm Cournot model was for each firm to produce $\frac{(a-c)}{N}$. Now we will investigate more closely the three firm case. Each firm $i \in \{1, 2, 3\}$ simultaneously chooses a quantity to produce q_i. The market price is determined by an inverse demand which depends on the total quantity produced by all three firms $Q = q_1 + q_2 + q_3$, where the price per unit is given by $P(Q) = \max\{a - Q, 0\}$. Each firm has the same marginal cost c of producing every unit of the good, such that $a > c > 0$. So the total profit is given by:

$$\pi_i(q_1, q_2, q_3) = \begin{cases} (a - q_1 - q_2 - q_3) \cdot q_i - c \cdot q_i & \text{if } q_1 + q_2 + q_3 \leqslant a \\ -c \cdot q_i & \text{if } q_1 + q_2 + q_3 > a \end{cases}$$

1. Write out the best-response correspondence $B_1(q_2, q_3)$ for every possible combination of q_2 and q_3
2. **Show** that the symmetric Nash Equilibrium quantity for each firm is given by $q_1^\star = q_2^\star = q_3^\star = \frac{(a-c)}{4}$. (I.e., show that this is a mutual best response)
3. Give an expression for each firm's equilibrium profits $\pi_i(q_1^\star, q_2^\star, q_3^\star)$ as a function of a and c at this equilibrium.
4. Provide any symmetric point $\hat{q}_1 = \hat{q}_2 = \hat{q}_3$ such that each firm's profit $\pi_i(\hat{q}_1, \hat{q}_2, \hat{q}_3) > \pi_i(q_1^\star, q_2^\star, q_3^\star)$.

Q4 [25PTS]

Consider the following game. There are two players. Player 1 has 10 dollars, and proposes any split of the ten dollars between himself and player 2 (i.e., $A_1 = [0, 10]$). Then, **after** the proposal, player 2 can accept of reject the split. The payoffs given by (a = accept, r = reject)

$$u_1(x, a) = 10 - x \quad u_1(x, r) = 0$$
$$u_2(x, a) = x \quad u_2(x, r) = 0$$

1. How many subgames are there in this game? Describe the set of all subgames.
2. Verify that the following strategies constitute a Nash Equilibrium: $(6; a \text{ if } x \geqslant 6, r \text{ if } x < 6)$.
3. Consider another NE strategy profile that achieves the same outcome (i.e., $x = 6$ and *accept*).
4. In a subgame perfect NE, what must P2's action be for any $x > 0$?
5. Show that in a subgame perfect NE, P2 cannot reject (with any positive probability) even after $x = 0$. (Hint: assume this was the case and find a contradiction).
6. What is the unique subgame perfect NE of this game?