Menus as Frames: The Informational Content of Decision Problems *

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Abstract

This paper examines a model where the menu from which a decision maker must choose acts as a frame that changes his perception of uncertainty. Specifically, this paper proposes a set of axioms such that each menu induces a subjective belief over the type-space concerning the type of goods offered in the menu. The decision maker's preferences are dependent on the realization of the type-space. The resulting representation is analogous to state-dependent expected utility in each menu, where the beliefs are menu-dependent and the utility index is not. The DM's utility index and beliefs are uniquely identified. This paper also proposes a framework wherein the type-space can be endogenous and identified from behavior. In addition to identification, the representation accounts for two behaviors that are not readily explainable by previous models. First, preference reversals as a response to the inclusion or exclusion of (seemingly) unrelated alternatives. Second, preference of a menu over its superset.

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1 Introduction

1.1 Menus as Frames

In standard decision theory, it is assumed that the decision maker (DM) has a preference over the set of all alternatives, and that this preference remains unchanged when restricted to any subset. This corresponds directly to the Weak Axiom of Revealed Preference (henceforth, WARP) in a choice theoretic setting. There is, nonetheless, strong psychological and experimental evidence that real life decisions are affected by their environment [Kahneman and Tversky, 1984]. The process where seemingly unrelated information alters a DM's choice is referred to as framing, and the information itself as a frame [Tversky and Kahneman, 1981]. While there are many external factors that potentially exert influence on the decision making process, this paper examines a model in which the set of alternatives that are *currently* available (a menu) acts as a frame.

Framing is often associated with notions of bounded rationality. This paper, however, interprets menu-induced framing (and consequent WARP violations) as being purely rational. If the decision maker believes that there is information contained in the menu, conditioning his preference on such information is a rational action. Specifically, the model assumes there are many (ex-ante indistinguishable) types of each alternative. The DM's utility from consumption depends not only on the chosen outcome, but also on which 'type' of outcome is realized.[1] The DM, before consumption, is uncertain about the type of each alternative in the choice set. He does, however, hold a belief (a probability distribution) over the type space; in a given decision problem, the DM maximizes his expected utility according to his belief.

This is related to Sen's notion of the epistemic content of decision problems [Sen, 1993, 1997]. In the current context, the two considerations, epistemic content and framing, coincide. The epistemic content of a menu is the frame it induces –the information that alters the decision maker's belief. In this paper, each frame corresponds to a perception of the alternatives –a belief over the 'type' of each outcome.

Before expounding the finer points of the model, it is worth considering an example to better illuminate why more complex menu-dependent preferences are indeed necessary to explain many common decision making scenarios. The following is a variant of an example posited by Luce and Raiffa [1957]. A man is looking to purchase a painting to

[1]An alternative's type is any initially unobservable feature. For practical purposes we can think of a type as being the alternative's quality, longevity, integration with other goods, suitability for a particular task, etc.

hang in his home. There are four possible (classes of) paintings from which to choose: a conservative painting by a known artist, and conservative painting by an unknown artist, an avant-garde painting by a known artist, or an avant-garde painting by an unknown artist. When in a gallery that sells only unknown artists, the DM (strictly) prefers the a conservative painting. Nonetheless, if the store *also* offered an avant-garde painting by a known artist, the DM would (strictly) prefer the avant-garde painting by an unknown artist to a conservative painting by an unknown artist. As stated, this is a clear violation of the WARP. The DM, however, has a simple and intuitive justification of his preference. He is unsure of current trends concerning up-and-coming art and does not want to purchase a work that will mark him as uncouth. Without any additional information, he would prefer the safe conservative option. When the gallery represents a known artist, the DM infers that the gallery is aware of current trends, and puts more faith in the store's curation. The DM would believe with a higher probability that the unknown artist's painting will be well received, and therefore prefers it to the conservative painting.

1.2 The Confused Decision Maker

The thoughtful reader might object here: this line of reasoning could lead the DM to hold conflicting beliefs. If, say, there were two galleries next to one another, each offering one of the two menus described above, then doesn't the DM simultaneously believe that a unknown artist's avant-garde painting has a high and a low probability of being well received? This conflict arises from the classical WARP-induced assumption that outcomes with the same ex-ante label are in-fact the same across different decision problems. However, the DM could easily consider a wide range of artists to all be unknown, all of which are ex-ante indistinguishable. There is no reason to think that the two menus offer the same paintings, only that each painting falls within its designated ex-ante label.

Indeed, this model characterizes framing as a local phenomenon. The alternatives that are available in the menu influence the DM's belief, but only over the alternatives that are contained in that particular menu. This is to say, a menu's contents is revealing about itself, but indicates nothing in regards to other menus. Thus, Gallery A's collection can influence the decision maker's belief about Gallery A's unknown artist's avant-garde painting but contains no information about Gallery B's.

1.3 Choice Between Menus

This paper also considers the effect of epistemic value on preferences over menus themselves. If we consider a menu to simply be a restriction of the available choice, then the obvious extension of preference over outcomes to preference over menus is to value each menu according to the value of its most preferred element. This assignment assumes that the DMs preference is the same when he chooses the menu and when he choose *from* the menu.[2]

If preferences are the same across menus (i.e., WARP), then it is clear that a menu is preferred to any of its subsets. When the epistemic content of menus is taken into account, the property fails (and with good reason). If the smaller menu induces a more favorable frame, the DM may strictly prefer the the smaller menu. Such inter-menu comparisons are possible only because epistemic content is local in its reach; different frames can co-exist simultaneously.

Consider another example. A diner is in an unknown town with two restaurants, both of which post their menus outside their doors. The first has a small, contemporary selection of good cuts of beef; the second a sprawling menu containing many different types of food. Further, we can assume that the second restaurant offers everything the first does, and much more. The diner, a meat lover, would choose a steak in either restaurant. The canonical theory of choices between menus dictates that the diner will at least weakly prefer the second restaurant with the larger menu. Nonetheless, our diner prefers the first restaurant. He believes that the first restaurant has likely mastered preparing its small menu, whereas the second restaurant prepares many things adequately, but none exceptionally. The diner strictly prefers the first restaurant because his belief about the realization of types (i.e. the quality of the restaurant) is more favorable when conditioned on the first menu.

1.4 Endogenous Type Space

In the model described above we made the assumption that the type-space was observable to the modeler. Nonetheless, it is possible to relax the assumption that the modeler is aware of the exact uncertainty which the DM faces regarding the types of alternatives. Indeed, we show that (at the cost of added complexity) the mechanics of the principle model are largely retained and a subjective type-space can be identified from behavior. The is

[2] There is a large class of models, starting with Kreps [1979], in which some or all uncertainty is resolved before the choice *from* the menu. This framework is utilized in 5. These models, in general, also satisfy monotonicity with respect to set inclusion, and are discussed in the Literature Review.

accomplished by examining the DM's preference over menus, relying on the framework of Dekel et al. [2001].

Kochov [2010] proposes a model that also considers the epistemic value of menus in a subjective state space framework. In his model, the DM's state space, rather than his beliefs, change in response to epistemic concerns. This may result in a DM who is confused in the aforementioned sense. For a lengthier discussion on the differences between these models, see Section 6.

The rest of the paper is as follows. Section 2 is an informal discussion about the mechanics of the model. It also provides a numerical formulation of the above examples to motivate and elucidate the formalities of the next section. The formal model and main results are contained in Section 3; Sections 3.2 and 3.3 consider choices of acts from within a menu, Section 3.4 considers the menus themselves as the object of choice. Section 4 examines an extension to the basic model: the model with realization-independent preferences. A discussion and basic results regarding the the model with an subjective type space is presented in Section 5. Section 6 contains a survey of the relevant literature. Section 7 concludes. All proofs are contained in the Appendix.

2 Discussion and Examples

As in the standard model of subjective expected utility, the DM has preference over acts: functions that assign a consumption outcome to each realization of the type-space, T. Unlike the standard model, each act is conditioned on a menu. Denote f_A as an act, $f : T \to A$, when the DM faces menu A. In the interim stage, when facing a menu A, the DM can choose only between acts that assign lotteries in A. Intuitively, an act is a contingent plan for the DM; a commitment to consume a particular object from the menu contingent on the realization of the type space. The primitive of the model is the DM's ex-ante preference over all feasible act-menu pairs.

The canonical subjective expected utility (SEU) axioms are imposed on each menu-induced interim preference relation (the DM's preference over elements in the menu). To capture menu-induced framing, the belief in the interim stage is a function of the menu that is realized. Conversely, the DM's tastes over ex-post outcomes are fixed across menus. The resulting representation is a family of subjective expected utility representations, indexed by menus. Theorem 3.1 states the behavioral restrictions such that for all acts that assign lotteries in the menu A,

$$f_A \succcurlyeq g_A \Leftrightarrow \sum_t \mu_A(t) \sum_x u(t,x) f(t,x) \succcurlyeq \sum_t \mu_A(t) \sum_x u(t,x) g(t,x) \tag{R1}$$

where $f(t,x)$ is the probability assigned by lottery $f(t)$ to outcome $x \in X$; $u(t,x)$ is a menu-independent utility index; $\mu_A(t)$ is the DM's menu-induced belief, taking into account the epistemic content of menu A. A consistency axiom ensures that the utility index $u(t,x)$ is the same in every menu –the DM's ex-post tastes are fixed. The belief, μ_A, is expressly dependent on the menu. The epistemic value of a menu in entirely subjective. The menu-induced beliefs are identified from preferences; different DMs need not have the same menu-induced beliefs.

In general, the DM's induced preference for ex-post outcomes is type-dependent. The ranking over acts depends not only on the subjective probability with which an outcome is consumed, but also qualitatively on which type is realized. Type-dependence is the driving force behind explaining epistemically motivated preference reversals. To see this, consider the following formalization of the example given in the introduction.

Example 1. *There are three possible painting: unknown artist avant-garde (UA), unknown artist conservative (UC), and known artist avant-garde (KA). There are two galleries, A and B. The menus are as follows: $A = \{UA, UC\}$ and $B = \{UA, UC, KA\}$[3]. There are two types of each painting: good (g) or bad (b). Of course, the type of each painting is unobservable ex-ante; perhaps, the DM will only consider the painting to be 'good' if it is still in favorable standing in 10 years. The DM has the following menu-dependent preferences*

$$UC \succ_A UA$$

$$UA \succ_B UC \succ_B KA$$

were UC is the constant act that assigns a conservative painting by an unknown artist in every realization, etc. A representation that rationalizes this preference reversal and retains the SEU structure is given by

$$u(g,UA) = 1 \quad u(g,UC) = \tfrac{2}{3} \quad u(g,KA) = 0 \qquad \mu_A(g) = \tfrac{1}{3} \quad \mu_B(g) = \tfrac{2}{3}$$

$$u(b,UA) = 0 \quad u(b,UC) = \tfrac{1}{3} \quad u(b,KA) = 0 \qquad \mu_A(b) = \tfrac{2}{3} \quad \mu_B(b) = \tfrac{1}{3}$$

[3] In fact, to be consistent with the formal model, they offer all possible lotteries over these paintings.

Calculating the utility according to (R1)*, the value of* UC*,* $\frac{4}{9}$*, exceeds that of* UA*,* $\frac{3}{9}$*, at gallery A. However, at B, the value of* UA*,* $\frac{6}{9}$*, exceeds that of* UC*,* $\frac{4}{9}$*.*

Example 1, although simple, illustrates the importance of type-dependent preferences. If preferences were type-independent, ensuring that in any representation $u(g,\cdot) = u(b,\cdot)$, it is clear that no change in beliefs could reverse preferences over constant acts. Thus, type-dependence is a necessary condition for such behavior. In other words, if the realization of types is only used as a betting device, and has no intrinsic effect on the utility of the DM, than epistemic content cannot reverse preference over constant acts.

Recall that in the basic SEU model, the representation assigns a value to each act, but cannot specify the correct decomposition of that value into tastes and beliefs. Ordinarily, the separation of tastes from beliefs is guaranteed by imposing state-independence as in Anscombe and Aumann [1963]. Here, however, the identification of tastes and beliefs must come from elsewhere, namely the consistency of ex-post preference across menus. Any change in the decomposition of the value of an act into its constituent parts –tastes (a type dependent utility index) and beliefs (a probability distribution over the realization of types)– must be admissible simultaneously for every menu-induced representation. Given sufficient richness of the menu space, only one decomposition will jointly satisfy all menu-induced representations.

Ex-ante preferences also specify the DM's ordering over acts from different menus. The DM has preferences over f_A and g_B (in terms of example one, preference between an unknown artist's painting from from the first gallery or known artists painting from from the second). The obvious candidate to represent inter-menu preferences is the comparison of the interim value of each act, given the menu-specific utility function. Unfortunately, this representation requires a bit more structure. The vNM axioms, richness, and consistency, do not rule out the possibility that different menus are evaluated according to different scales. Including a universal best and worst outcome assures that the preferences in each menu are evaluated using a common scale. The representation of ex-ante preferences, as given by Corollary 3.2,

$$f_A \succcurlyeq g_B \Leftrightarrow \sum_t \mu_A(t) \sum_x u(t,x) f(t,x) \geqslant \sum_t \mu_B(t) \sum_x u(t,x) g(t,x) \tag{R2}$$

This representation is a generalization of the interim preference representation: when the two acts come from the same menu, i.e. $A = B$, (R2) collapses into the interim preference representation (R1). This representation also makes clear that the change in the value of an act across menus is solely due to changing beliefs. If f_A is preferred to f_B, it is because the DMs beliefs puts more weight on favorable type realizations when facing A than when facing B.

The ability to compare acts across menus is principally important when the DM does not yet face a menu. Thus, the obvious implication is the DM's ability, in the ex-ante stage, to choose between menus themselves. If a DM evaluates a menu as the value of its most preferred act, the resulting representation is immediate:

$$A \mathrel{\hat{\succcurlyeq}} B \Leftrightarrow \max_{f_A \in F_A} \sum_t \mu_A(t) \sum_x u(t,x) f(t,x) \geqslant \max_{f_B \in F_B} \sum_t \mu_B(t) \sum_x u(t,x) f(t,x)$$

An important implication of this representation is that it allows for non-monotonicity of set inclusion. It is admissible for a menu A to be preferred to its strict superset B. This is a direct emanation of framing effects; a smaller menu may impart a favorable outlook to the DM. Previous models either could not explain such behavior or relied on dynamic inconsistencies to do so. This model allows for the preference of smaller menus to larger ones but does not exclude the opposite. The value of a menu is characterized not only by the outcomes it contains, but also by the frame it induces. If removing an outcome, x, from a menu increases the subjective likelihood of a favorable realization of types (regarding other outcomes in the menu), then it is possible to prefer the menu without x.

This directly accounts for the anecdotal behavior of our diner in the introduction; as well as more general behavior where small specialized menus are preferred because they impart a favorable belief over the type space. As with preference reversals, this non-monotonicity over set inclusion requires type-dependence. A second example will help to illuminate the mechanism behind this result.

Example 2. *A diner can conceive of N possible entres, e, indexed by the set $\{1, 2 \dots N\}$. Assume the entres are ordered by preference, so that within any menu the diner prefers a higher numbered entre to lower numbered entre. Given a menu, there are two types of each entre: good, g, or bad, b. Given a particular entre, the diner would prefer to consume it in the restaurant where it is more likely to be good. To make the example succinct, assume that this belief, as specified by the ex-ante preferences, is a function only of how many dishes the restaurant serves (with less entres indicating the restaurant is more likely to be good). This is indicated by the ex-ante preference that given an entre, the diner prefers to consume it in the restaurant with a smaller selection. A representation that respects this ordering is*

$$u(g, e_i) = N + i \qquad \mu_A(g) = \tfrac{N+1-|A|}{N}$$

$$u(b, e_i) = i \qquad \mu_A(b) = 1 - \tfrac{N+1-|A|}{N}$$

Where $|A|$ is the number of entres offered in menu A. The belief that the restaurant is good is decreasing in the number dishes offered, while the utility from a dish is increase in its placement in the index. Now consider two menus, A and B

$$\begin{aligned} A &= \Delta(\{e_{N-1}, e_N\}) \\ B &= \Delta(\{e_{N-2,} e_{N-1}, e_N\}) \end{aligned}$$

By looking at the utility index, it is immediately clear that the diner will choose the constant act e_N from both restaurants. Calculating the value of e_N in each menu, it becomes clear that $U_A(e_N) = 2(N-1) + 1 > 2(N-1) = U_A(e_N)$. Therefore, $(e_N)_A \succ (e_N)_B$.

Again, as in Example 1, realization-dependence in the mechanism by which epistemic effects explain the anomalous behavior. If outcomes are evaluated identically across different states, then the idea of a positive and negative perception regarding the menu is meaningless. If $u(g, \cdot) = u(b, \cdot)$, than no belief system can make the smaller menu better. Theorem 4.3 formally shows that realization-dependence is a necessary condition for non-monotonicity of set inclusion.

3 Model

3.1 Structure and Primitives

There is a finite set of alternative consumption bundles represented by the set X. Let x, y denote arbitrary elements of X. Let $\Delta(X)$ be the set of all lotteries over X, endowed with the topology of weak convergence; $\Delta(X)$ is therefore metrizable by a metric d_X. Define $\mathcal{K}(\Delta(X))$ to be the set of non empty, convex, compact subsets of $\Delta(X)$. $A, B \in \mathcal{K}(\Delta(X))$ are menus –subsets of the possible lotteries over consumption bundles. Let $\mathcal{K}(\Delta(X))$ be endowed with the Hausdorff metric, d_H, defined using d_X. Let T be some finite type-space, with $s, t \in T$.

For each menu, we can define the set of feasible acts, $F_A = \{f_A : T \to A\}$; functions from the type space and whose images are contained in A. Acts are commitments to a particular consumption conditional on the realization of the type-space. Ex-ante a DM has preferences over all feasible acts $F = \bigcup_{A \in \mathcal{K}(\Delta(X))} F_A$. However, in the interim, after a menu is known, but before the realization of the type, the DM has preferences over only the restricted acts F_A. Intuitively, when the DM reaches a menu, he can only consider acts that assign available outcomes –elements in the menu he faces. The implied timeline of the decision making process is shown in figure 1.

With regards to the notation, each feasible act that is denoted by the same letter will assign the same outcome to each realization. For example f_A and f_B assign the same outcome to each realization; the difference being that f_A is the act when the DM faces menu A, and f_B when he faces menu B. Of course, f_A and f_B are only well defined if the image of f is in the intersection of A and B. For each act, $f(t)$ is the lottery obtained in realization t, and $f(t, x)$ is the probability assigned by lottery $f(t)$ to outcome x.

The primitive of the DM is an ex-ante preference over all available act-menu pairs. Formally, this is a binary relation $\succcurlyeq$ over F. $f_A \succcurlyeq g_B$ is interpreted in the usual manor: facing menu A and obtaining the act that assigns f is at least as good as facing menu B and obtaining the act g. Defining preferences over act-menu pairs captures behavior where the DM's preferences are dependent on the available outcomes. What would generally be consider a WARP violation is here perfectly reasonable; a DM can prefer f_A to g_A but g_B to f_B. Additional structure on the preferences will allow the representation to retain the desirable functional properties of SEU models, and a simple behavioral rational in the form of framing effects.

From the primitive $\succcurlyeq$ we can define the interim preferences given a menu A, $\succcurlyeq_A$ –DMs preferences after facing a menu A but before the uncertainty about the true realization is resolved. $\succcurlyeq_A$ is a binary relation over the set of restricted acts F_A defined such that $f_A \succcurlyeq_A g_A$ if and only if $f_A \succcurlyeq g_A$. Because, $\succcurlyeq_A$ is defined only over acts from menu A we can drop the subscript notation from the acts: $f_A \succcurlyeq_A g_A$ becomes$f \succcurlyeq_A g$.

3.2 Interim Representation

Although we are principally interested in the more general ex-ante preferences, we will first examine preferences at the interim stage, once a DM is facing a menu. This somewhat backwards seeming approach is motivated by two concerns. Practically, we can then use the structure imposed on the interim stage to back-out a more tractable representation of ex-ante preferences. Also, philosophically, ex-ante preferences are never directly

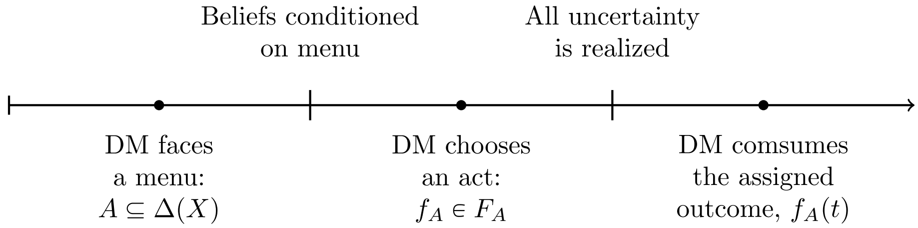

Figure 1: Implied timeline of the decision making process

observable, since every real-world choice is made from a menu. It therefore makes sense to begin by imposing structure on the observable preferences, and then identifying the implications on the unobservable ex-ante preferences.

First, to obtain a continuous representation, basic restrictions on ex-ante preference are needed: weak order and continuity.

Definition. *$\succcurlyeq$ is a **weak order** if it a complete and transitive binary relation. That is for all $f_A, g_B \in F \times \mathcal{K}(\Delta(X))$ either $f_A \succcurlyeq g_B$ or $g_B \succcurlyeq f_A$ or both (completeness). For all $f_A, g_B, h_C \in F \times \mathcal{K}(\Delta(X))$ if $f_A \succcurlyeq g_B$ and $g_B \succcurlyeq h_C$ then $f_A \succcurlyeq h_C$ (transitivity). From this we can define $\succ$ and $\sim$ in the usual manor.*

Definition. *$\succcurlyeq$ is **conditionally-continuous** (c-continuous) if for all f_A, the conditional upper and lower contour sets –$\{g_A \in F_A : g_A \succcurlyeq f_A\}$ and $\{g_A \in F_A : f_A \succcurlyeq g_A\}$, respectively– are closed.*

We can also consider a stronger notion of continuity that is not conditional on a menu.

Definition. *$\succcurlyeq$ is **continuous** if for all f_A, the upper and lower contour sets –$\{g_B \in F : g_B \succcurlyeq f_A\}$ and $\{g_B \in F : f_A \succcurlyeq g_B\}$, respectively– are closed.*

As stated previously, we will constrain the interim preferences to be subjective-expected-utility. Note that weak order and c-continuity are inherited by the conditional preference from the ex-ante preference. Thus, to obtain SEU we need only a notion of independence.

Definition. *$\succcurlyeq$ is **conditionally-independent** (c-independent) if for all $A \in \mathcal{K}(\Delta(X))$, $f_A, g_A, h_A \in F_A$ and $\alpha \in [0,1]$ if $f_A \succcurlyeq g_A$ then $(\alpha f + (1-\alpha)h)_A \succcurlyeq (\alpha g + (1-\alpha)h)_A$.*

Where $\alpha f + (1-\alpha)h$ is defined as the the act that assigns lottery $\alpha f(t) + (1-\alpha)h(t)$ if t realized; given that f, h are in A so is $\alpha f + (1-\alpha)h$ by the convexity of menus. Independence is defined over interim preferences instead of ex-ante preferences for two reasons. The first is purely technical: F is not a mixture space. The second reason is much more substantive. Independence (if we consider a different setup in which F is a mixture space) would be too strong. Independence restricts the DM from changing his preferences between acts because of the inclusion of new acts: the exact behavior we wish to capture.[4] This, of course, raises the well deserved question: why then impose independence over interim preferences? We assume that the DM's non-standard behavior is the result of framing. So, once the menu is realized, and the DM's beliefs accordingly conditioned, his preference can no longer change. Thus, interim preferences should behave according to the standard model.

When considering only menu-conditional (interim) preferences, the above restrictions are exactly the Von Neumann–Morgenstern axioms that provide the following representation.

Theorem (von Neumann Morgenstern)**.** *$\succcurlyeq$ is a* ***c-continuous****,* ***c-independent weak order*** *if and only if there exists some $w : T \times X \times \mathcal{K}(\Delta(X)) \to \mathbb{R}$ such that $\forall A \in \mathcal{K}(\Delta(X))$ and all $f, g \in A$*

$$f \succcurlyeq_A g \Leftrightarrow \sum_t \sum_x w(t,x,A) f(t,x) \geqslant \sum_t \sum_x w(t,x,A) g(t,x)$$

Moreover, if $w(t,x,A)$ and $\hat{w}(t,x,A)$ both represent $\succcurlyeq_A$ then, $w(t,x,A) = a(t)\hat{w}(t,x,A) + b(t)$ where $\forall t \in T$ $a(t) \in \mathbb{R}_{++}$ and $b(t) \in \mathbb{R}$

Recall that $f(t,x)$ and $g(t,x)$ are given, objective probabilities. Specifically, they denote the probabilities of obtaining x in the lottery assigned by the act in under realization t. The index $w(\cdot)$ can be decomposed into tastes and beliefs by choosing some probability distribution $p \in \Delta(T)$ such that $p(t) > 0$ and utility index $u(t,x,A) = \frac{w(t,x,A)}{p(t)}$. Of course, this creates the classic problem of multiple rationalizing beliefs: if we consider some other $p' \in \Delta(T)$ such that $p'(t) > 0$, then it is clear that p' and $u'(t,x,a) = \frac{w(t,x,A)}{p'(t)}$ also represent the same preferences. We still can not identify the DM's tastes for ex-post outcomes or his beliefs; the two are jointly determined.

In order to deal with the multiplicity of possible beliefs, Anscombe and Aumann restricted preferences to be state independent. State dependency is a very restrictive assumption, since it interprets states as abstract probabilistic events that have no mean-

[4] More specifically, it imposes that the DM's beliefs vary linearly with the menu. I.e., that if $C = \alpha A + (1-\alpha)B$ then $\mu_C = \alpha\mu_A + (1-\alpha)\mu_B$. We view this restriction as too strong, as it rules out the possibility that the inclusion of an element (even with very small probability) drastically alters the DM's perception of the menu. It is easy to see how, in Sen's example with the heroin addict (see the Literature Review), the offer of heroin, even with very small probability, could sway the acquaintance's preference from having tea to going home.

ing outside of their use as betting devices. As discussed in the previous section, state-dependence (type-dependence) is a necessary requirement for the behavior we wish to capture. For these reasons, this model retains type-dependence. However, this model has a somewhat similar restriction to state-independence; preferences need be menu consistent. However, first we need to consider the idea of a null realization. Unlike the standard model, here a realization can be null in one menu and not in another.

Definition. *A realization t is* ***null for menu A*** *(denoted null-A) if $\forall f \in F_A$, $\rho \in A$*

$$(f_{-t}\rho)_A \sim f_A$$

Null realizations, in the general subjective states literature, have two indistinguishable interpretations. First, that the DM is indifferent between all available options conditional on the realization of the null state, t; second, that the DM places zero probability on t. However, assuming that the DM is consistent with his tastes across different menus (an assumption that will be formalized shortly), it is possible to differentiate these two interpretations of null events. If a realization t is null-A, but there exists a different menu, B, where the DM displays a strict preference over elements of A (given realization t), it must be that t was assigned zero probability when facing A. This is because the DM cannot be indifferent to all elements of A (contingent on t) since he displays strict preference in the menu B. This is formalized by evidently-null states, first considered in Karni et al. [1983].

Definition. *A realization t is* ***evidently null for menu A*** *(denoted e-null-A) if t is null-A and there exists some menu B such that*

$$(f_{-t}\rho)_B \succ f_B$$

for some f, ρ in $A \cap B$.

Let N_A denote the union of all e-null-A events, and N the union all events which are null for all menus. With this definition in mind we can now define menu consistency.

Definition. $\succcurlyeq$ *is* ***menu-consistent*** *if for all $A, B \in \mathcal{K}(\Delta(X))$, $\rho \in \Delta(X)$, all $t \in T$ such that $t \notin N_A \cup N_B$, and all f, g such that $f(t) = g(t)$,*

$$(f_{-t}\rho)_A \succcurlyeq f_A \Leftrightarrow (g_{-t}\rho)_B \succcurlyeq g_B$$

If $\succcurlyeq$ is menu-consistent, then the DM's tastes for outcomes are identical across menus. It is important to note that this does not rule out preference reversals, even over constant acts. Each menu carries with it a frame that changes beliefs, and can therefore change the DM's preferences for acts. However, if $\succcurlyeq$ is menu-consistent then any preference reversal is due entirely to the change in beliefs, and not because of changes in ex-post tastes. Setting $f = g$ in the definition, consistency guarantees that the ordering between $f_{-t}\rho$ and $f_{-t}\pi$ hold regardless of the menu.

Menu-consistency also implies a form of realization-dependent monotonicity. To see this, set $A = B$ in the definition; menu consistency now states that if $h(t)$ is preferred to $f(t)$ *for realization* t when part of act f, then $h(t)$ is preferred to $f(t)$ *for realization* t regardless of the act. If h and f are such that $(f_{-t}h)_A \succ f_A$ for all t, then menu-consistency (and transitivity) implies $h \succ_A f$. This differs from standard monotonicity as implied by state-independence because the comparison of f and h must be made on a realization-by-realization basis.

Definition. *$\succcurlyeq$ is **rich** if for every event $E \subset T$ there exists a menu, A, such that $E = N_A$.*

Richness is an admittedly strong assumption; although it buys a strong result: the uniqueness of tastes. If $\succcurlyeq$ is rich and consistent than the value of an act can be uniquely decomposed into tastes and beliefs.

Theorem 3.1. *(a) $\succcurlyeq$ is a **c-continuous**, **c-independent**, **menu-consistent, weak order** if and only if there exists type-dependent utility index $u : T \times X \to \mathbb{R}$ and family of beliefs $\{\mu_A \in \Delta(T)\}_{A \in \mathcal{K}(\Delta(X))}$ such that for all $A \in \mathcal{K}(\Delta(X))$ and all $f_A, g_A \in F_A$*

$$f_A \succcurlyeq g_A \Leftrightarrow \sum_t \mu_A(t) \sum_x u(t,x) f(t,x) \geqslant \sum_t \mu_A(t) \sum_x u(t,x) g(t,x)$$

For each menu A, $\mu_A(t) = 0$ if and only if $t \in N_A \cup N$.
*(b) Moreover, if $|T| > 2$ and $\succcurlyeq$ is **rich** then $\{\mu_A\}_{A \in \mathcal{K}(\Delta(X))}$ is unique, and the utility index, $u(\cdot)$, is unique up to realization-independent affine transformations.*

The representation provided by Theorem 3.1 achieves two seemingly unrelated objectives. First, it allows for preference reversals –which under the canonical structure would be considered WARP violations– resulting from the inclusion or exclusion of alternatives. Secondly, it guarantees a unique type-dependent utility index for each set of menu-induced preferences. The utility index is menu-independent –a direct consequence of the menu-consistency axiom. The subjective beliefs, however, depend directly on the menu; each menu is associated with a particular distribution over the type-space. It is through the menu-dependent beliefs that this structure allows for preference reversals in the face of new alternatives. It follows that the types of preference reversals that are allowable is very limited. Because the utility index is unchanging, it is only through the shifting of probabilities by which preferences can change. Thus, it is clear that if an act f is preferred to g on a realization-by-realization basis, then it is preferred to g in every menu.

3.3 Ex-ante Representation

We can now step back to our ex-ante preferences and utilize the interim representation to impose a tractable representation. It is immediate that the interim preferences must be represented as above; what remains to be shown is how the interim preferences are compared to one another. Without imposing any further restrictions on preferences, it is possible for the DM to have intrinsic affinity for a menu. That is, the DM receives utility simply from facing a menu –unrelated to any epistemic change in her beliefs. This is obviously an undesirable property of preference; a DM could prefer an act f_A to g_B, even though g_B strictly dominates f_A on a realization-by-realization ex-post comparison. Ideally, acts from different menus would be measured against a common scale.

A somewhat unsatisfying solution consists of including a hypothetical best and worst outcome to act as the common scale. Let $\overline{x}, \underline{x}$ be the hypothetical best and worst outcomes, respectively. For any act, menu, realization combination it must be true that $(f_{-t}\overline{x})_A \succcurlyeq f_A$ and $f_A \succcurlyeq (f_{-t}\underline{x})_A$. These elements do not need to actually exist but only be hypothetically conceivable by the DM. Potential candidates for best and worst outcome include winning the lottery and death by boiling oil, respectively. For each menu A, define its *hypothetical* extension $\overline{A}$ as $\Delta(A \cup \{\overline{x}, \underline{x}\})$. We can define preferences to apply over menu extensions as the following

Definition. *$\succcurlyeq$ is* ***extendable*** *if the following hold*

$$\begin{aligned} f_A &\sim f_{\overline{A}} \\ (\alpha\overline{x} + (1-\alpha)\underline{x})_{\overline{A}} &\sim (\alpha\overline{x} + (1-\alpha\underline{x})_{\overline{B}} \end{aligned}$$

for any $\alpha \in [0, 1]$.

The first condition states that the extended preferences coincide with the original preferences. This is the sense in which the extension is hypothetical; the DM can conceive of comparing available outcomes the universal best and worst outcome, but these elements do not change the epistemic value of the menu. The second property ensures that the universal best and worst outcomes are indeed universal (do not depend on the menu).

Corallary 3.2. *If there exists a hypothetical best and worst outcome, then $\succcurlyeq$ satisfies the conditions for Theorem 3.1 and is **extendable** if and only if there exists a type-dependent utility index $u : T \times X \to \mathbb{R}$ and family of beliefs $\{\mu_A \in \Delta(T)\}_{A \in \mathcal{K}(\Delta(X))}$ such that*

$$f_A \succcurlyeq g_B \Leftrightarrow \sum_t \mu_A(t) \sum_x u(t,x) f(t,x) \geqslant \sum_t \mu_B(t) \sum_x u(t,x) g(t,x)$$

Where, for each menu A, and its extension $\overline{A}$, $\mu_A(\cdot) = \mu_{\overline{A}}(\cdot)$ and $u(\cdot)$ is as in Theorem 3.1.

The intuition behind Corollary 3.2 is straight forward. If each menu is measured against some universal act, and the universal acts are measured against the same scale in each menu, then the menus themselves are measured against that same scale. In other words, the utility of an act is measured in terms of the certainty equivalent with regards to these universal outcomes: $\alpha\overline{x} + (1-\alpha)\underline{x}$. Introducing universal acts is a less restrictive form of type-independence. We do not need type-independence outright, only a over the universal extreme acts.

3.4 Preferences Over Menus

The analysis thus far has focused only on the DM's choice from within a menu, or from within a set of menus. The tacit implication being that the menu was exogenously given (or previously chosen). Under the assumption of time-consistency, the ex-ante preferences also define the DM's preferences over menus themselves. As in the previous section, let $Q(F_A)$ denote the maximal set of acts from the restricted set of acts F_A. Formally, $Q(F_A) = \{f_A \in F_A |\ f_A \succcurlyeq g_A, \forall g_A \in F_A\}$. Since all menus are non-empty and compact, the transitivity, completeness, and c-continuity of the ex-ante preference guarantee that $Q(\cdot)$ is non-empty for all menus. Define a binary relation, $\hat{\succcurlyeq}$, on the collection of menus, $\mathcal{K}(\Delta(X))$, with the following rule[5]

$$A \,\hat{\succcurlyeq}\, B \Leftrightarrow Q(F_A) \succcurlyeq Q(F_B)$$

[5]This is an abuse of notation. Technically, $\hat{\succcurlyeq}$ should be defined by $A \,\hat{\succcurlyeq}\, B \Leftrightarrow f_A \succcurlyeq g_B$ for all $f_A \in Q(F_A)$ and $g_B \in Q(F_B)$. In addition, all results hold true if we restrict $Q(\cdot)$ to be the maximal constant acts. Thus the interpretation that the DM chooses his most preferred element (rather that act) is perfectly valid and will not distort the analysis.

The DM anticipates that he will choose an act from the maximal set of any menu, and therefore compares menus by his ex-ante ordering of the maximal elements. It is immediate that $\hat{\succcurlyeq}$ inherits the properties of transitivity and completeness from the ex-ante preferences. Thus, $\succcurlyeq$ is a preference ordering and is represented by a value function. Further, since the ordering of menus is derived from the ex-ante ordering of particular acts, the representation is of the same form as the ex-ante representation.

Corallary 3.3. *(a) $\succcurlyeq$ satisfies the conditions for Corollary 3.2 if and only if*

$$A \hat{\succcurlyeq} B \Leftrightarrow \underset{f_A \in F_A}{max} \sum_t \mu_A(t) \sum_x u(t,x) f(t,x) \succcurlyeq \underset{f_B \in F_B}{max} \sum_t \mu_B(t) \sum_x u(t,x) f(t,x)$$

Each menu is evaluated as the maximum utility over all acts that assign outcomes available in the menu. We can call the value of a menu $V(A)$

$$V(A) = \max_{f_A \in F_A} \sum_t \mu_A(t) \sum_x u(t,x) f(t,x)$$

If all of the conditions except extendability are met then $V(A)$ contains affine coefficients that depend of A. If $\succcurlyeq$ is continuous rather than c-continuous, preferences are continuous over changes in the menu: $V(\cdot)$ is a continuous function. This gives way to the intuitive interpretation that a small change in the menu can have only a small effect on the DM's induced beliefs. An important consequence of incorporating epistemic effects is that, even with temporally static preferences, it is perfectly admissible for a DM to strictly prefer a menu to its superset: $V(A) > V(B)$ for some $A \subset B$.

4 Realization-Independence

In this section we will quickly examine what the above model would look like with the additional restriction that preferences are realization-independent. That is, the ex-post ordering of outcomes is the same regardless of realization of the type-space. This consideration should explicit the roll of the axioms, and relate this model to the standard Ascombe-Aumann (AA) set up. Note that the type-space is mathematically the same as the state-space in AA, and imposing realization-independence is the same as restricting interim preferences to be AA. In particular, the comparison between realization-independent and realization-dependent preferences makes salient the trade-off between the identification of preferences and the flexibility of the representation. We can retain very similar structure, while imposing that preferences for ex-post outcomes are realization-independent by strengthening the notion of menu-consistency.

Definition. *$\succcurlyeq$ is **strongly-menu-consistent** if for all $A, B \in \mathcal{K}(\Delta(X))$, $\rho \in \Delta(X)$, all $t \in T$ such that $t \notin N_A$ and $s \notin N_B$, and all f, g such that $f(s) = g(t)$,*

$$(f_{-t}\rho)_A \succcurlyeq f_A \Leftrightarrow (g_{-s}\rho)_B \succcurlyeq g_B$$

By setting $s = t$ it is clear that strong-menu-consistency implies menu-consistency. In addition, strong-menu-consistency dictates that if the ex-post realization-outcome pair (s, ρ) is preferred to $(s, f(s))$ than it must also be that (t, ρ) is preferred to $(t, f(s))$ for all $t \in T$. Effectively, states do not effect the ex-post preference orderings. The previous conditions along with strong-menu-consistency imply AA's axioms on each of the conditional preferences induced by each menu. Uniqueness of beliefs is guaranteed by AA's state-independent representation; richness is not needed.

Theorem 4.1. *(a) $(\succcurlyeq)$ is a **c-continuous**, **c-independent**, **strongly-menu-consistent, weak order** if and only if $\exists\ u : X \to \mathbb{R}$ and family of beliefs $\{\mu_A \in \Delta(T)\}_{A \in \mathcal{K}(\Delta(X))}$ such that $\forall A \in \mathcal{K}(\Delta(X))$ and all $f_A, g_A \in A$*

$$f_A \succcurlyeq g_A \Leftrightarrow \sum_t \mu_A(t) \sum_x u(x) f(t,x) \geqslant \sum_t \mu_A(t) \sum_x u(x) g(t,x) \tag{1}$$

For each menu A, $\mu_A(t) = 0$ if and only if $t \in N_A$.

(b) Moreover, the family of induced beliefs, $(\mu)_{\mathcal{K}(\Delta(X))}$, and the utility index, $u(\cdot)$, are unique up to realization independent affine transformations over all non-null realization-menu pairs.

The only visible difference between this representation and Theorem 3.1 is that the utility index no longer takes the realization, t, as an argument. The proof relies on the fact that the axioms imply both AA's representation on conditional preferences and Theorem 3.1. Also, note that the uniqueness result applies to all non-null realizations since the concepts of evidently-null and null coincide when preferences are realization-independent.

Corallary 4.2. *(a) $(\succcurlyeq)$ is a **c-continuous**, **c-independent**, **strongly-menu-consistent, weak order** and $\rho_A \sim \rho_B$ for any constant act ρ and menus A, B if and only if $\exists\ U : X \to \mathbb{R}$ such that for all non-null $A \in \mathcal{K}(\Delta(X))$ and all f, g*

$$f_A \succcurlyeq g_B \Leftrightarrow \sum_t \mu_A(t) \sum_x u(x) f(t,x) \geqslant \sum_t \mu_B(t) \sum_x u(x) g(t,x)$$

Where, for each menu A, $\mu_A(\cdot)$ and $u(\cdot)$ are as in Theorem 4.1.

It is straightforward to show that extendability and strong menu consistency together imply that $\rho_A \sim \rho_B$ for any constant act ρ and menus A, B. Hence, realization-independence removes the need for an explicit universal best and worst outcomes. Instead, restricting that the DM does not care from which menu a constant acts originates, ensures that preferences over different menus are measured against the same scale. This is an intuitive restriction, since a change in the distribution over the type-space should have no affect on the valuation of a constant act.

While realization-independence removes some of the uglier axioms of the model, it comes at a cost. Realization-independent preferences cannot explain preference reversals over constant acts. Strong menu consistency posits that regardless of the realized type, ex-post consumption bundles are evaluated by the same ordering. Therefore, the value of an act is dictated only by the total probability assigned to each consumption bundle, and not by the realization to which the bundle is allocated. Changing the distribution has no effect on the value of the act. Therefore, constant acts must be ordered in the same way across menus.

Monotonicity (with respect to subsets) is preserved with realization independent preferences.

Theorem 4.3. *If $\succcurlyeq$ is strongly-menu-consistent then $A \subset B \Rightarrow V(A) \leqslant V(B)$.*

A formal proof is contained in the appendix, but the intuition is straightforward. Although within a particular menu the set of ex-post consumption bundles is fixed, any allocation out of the fixed set of outcomes can be considered (i.e. any act with an image in the menu can be considered). Therefore, as the belief over the type-space changes the value of the maximal act remains the same (even as the act that provides the maximal utility changes). Thus if A and is a strict subset of B then any utility attainable in A with belief μ_A is still attainable with belief μ_B: the utility cannot decrease. Without allowing preferences to be conditioned on the realization, epistemic value has little bite. In the absence of realization-dependence, changes in the beliefs over the type-space do nothing but change on which realizations the DM bets.

5 Endogenous Type Space

The type space, which is an exogenous part of the model presented in Section 3, is supposed to encompass the uncertainty the DM faces with respect to his preference over the (ex-ante indistinguishable) outcomes. Because the DM has type-dependent preference, each realization corresponds to an expected utility function over the outcomes. This raises the question, from where did the type-space arise? Surely, the set of preference orderings which the DM deems possible is subjective, and not readily identifiable from the modelers perspective. From this point of view, the exogeneity of the type-space is a short coming of the model.

This section proposes an environment where the type-space is identified from behavior rather than imposed by a modeler. While this allows the relaxation of assumptions regarding the type-space it comes at the cost of added complexity and weaker identification. The environment is adapted from the Dekel et al. [2001] (henceforth DLR) framework; the object of choice is menus rather than acts. However, as in the above model, preference over the alternatives will be conditioned decision problems (here menus of menus).

As before let X be a finite set of consumption outcomes. Subsets of X are referred to as global menus. For each global menu, denoted $A \subseteq X$, we can also define $\mathcal{K}(\Delta(A))$ be the set of non empty, convex, compact subsets of $\Delta(A)$. Elements of global menus, denoted by bold letters, $\boldsymbol{a}_A, \boldsymbol{b}_A \in \mathcal{K}(\Delta(A))$, are referred to as local menus. One can think of a global menu as the collection of all the locally available items. For example, a city comprised of many neighborhoods: the city is the amalgamation of the the neighborhoods, each of which offers its own consumption alternatives (stores, apartments, restaurant, etc).

The DM has preferences over local menus, given a global menu. Like in models of unforeseen contingencies, the DM prefers local flexibility under the assumption that the uncertainty regarding type-space will be realized before a choice of a final consumption outcome. In terms of the example, we could think of a DM ranking the neighborhoods in each city; neighborhoods with more amenities can maximize the DMs utility for a larger number of type-space realizations.

The DM, as in the model presented in Section 3, is effected by the epistemic information of the menu, but acts as an expected utility maximizer within a menu. Here, epistemic information is contained in the global menu but not in the local menu. That is, the DM after seeing the global menu, acts as a consistent DLR maximizer –maximizing the final period utility according to some belief over a type-space. In analogy to the main framework, the belief over the type-space is conditioned the global menu, however, the subjective type space is the same for all global menus. Epistemic concerns can influence beliefs but not tastes. Although the preferences are static, there is an implied timing to the decision making process as shown in figure 2.

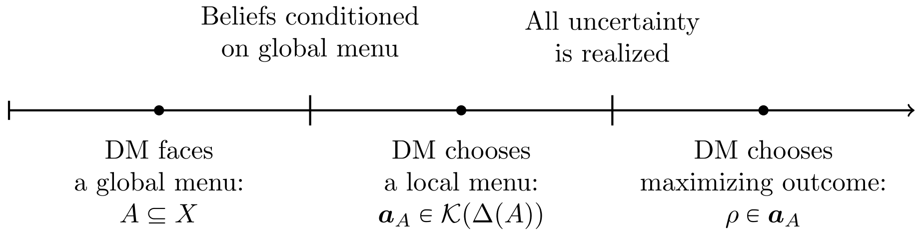

Figure 2: Implied timeline of the decision making process with a subjective type-space

5.1 Axioms and Representation

The DM has preference over pairs of local and global menus (where the local menu must contain only lotteries over the global menu) –in direct comparison to the main framework where the DM has preference over pairs of acts and menus. This is a non-trivial binary relation $\hat{\succcurlyeq}$, over $\bigcup_{A \subseteq X} \mathcal{K}(\Delta(A))$.

We will impose largely the same restrictions on $\hat{\succcurlyeq}$ as were imposed in the main model. There is a natural translation of the many of the restrictions imposed in section 3 to this environment. Indeed, with the sole exception that the object of choice is a menu rather than an act, weak order, c-continuity and c-independence are identical their counterparts in the main framework.

With regards to extendability, we require a different construction of universal best and worst outcomes. The hypothetical best and worst outcomes in this domain satisfy the following relations for any $\boldsymbol{a}$ and for any global menus A.

$$\{\overline{x}\}_A \mathrel{\hat{\succcurlyeq}} \boldsymbol{a}_A$$
$$\boldsymbol{a}_A \mathrel{\hat{\sim}} (\boldsymbol{a} \cup \{\underline{x}\})_A$$

With this definition of hypothetical best and worst outcomes, extendability can be written in an analogous way as before, albeit with menus rather than acts.

We also need two novel restrictions that were not present the above model. The first imposes a preference for flexibility –a standard axiom regarding choice over menus.

Definition. $\hat{\succcurlyeq}$ *is* ***c-monotone*** *if and only if for all* $A \subseteq X$*, and all* $\boldsymbol{a} \subseteq \boldsymbol{b} \subseteq \Delta(A)$*,*

$$\boldsymbol{b}_A \mathrel{\hat{\succcurlyeq}} \boldsymbol{a}_A$$

Monotonicity dictates that flexibility is never disadvantageous *conditional on a particular global menu.* That is, after any framing effects, larger menus are preferred to their subsets. Because this type of monotonicity applies only to local menus it does not rule out the possibility of a global menu being preferred to its superset in the sense discussed in Section 3.4.

Lastly, we need a notion of consistency that parallels menu-consistency. Recall that menu-consistency dictated that the DM's tastes did not change when considering different menus –his type-dependent preference ordering was the same for each realization of types.

With an endogenous, and hence unobservable, type-space we cannot overtly condition on the type realization. Rather, we can impose that framing effects do not change the DM's tastes by requiring that the type-space is the same across different global menus. That is, the DM has the same preference for flexibility when choosing from different global menus.

Definition. *$\hat{\succcurlyeq}$ is **globally menu-consistent** if and only if for all $A, B \subseteq X$, $\boldsymbol{a} \subseteq \Delta(A) \cap \Delta(B)$ and $\rho \in \Delta(A) \cap \Delta(B)$,*

$$(\boldsymbol{a} \cup \rho)_A \mathrel{\hat{\succ}} \boldsymbol{a}_A \Longleftrightarrow (\boldsymbol{a} \cup \rho)_B \mathrel{\hat{\succ}} \boldsymbol{a}_B$$

Global menu consistency states that if, conditioning on the epistemic information obtained from A, the DM conceives of a particular realization of uncertainty such that ρ will be preferred to all elements of $\boldsymbol{a}$, then the same realization must be conceivable after B. In terms of the representation, this implies that the DMs type-space, after and framing effect, must be the same regardless of the conditioning event. Of course, global menu consistency says nothing about the magnitude of the improvement when ρ is added to the menu. Thus, the DMs preference over local menus need not be the same across global menus. The discrepancy between interim preferences, as in the main model, arises because epistemic concerns can alter the DMs beliefs over the type-space.

These restrictions generate a representation that is analogous to the representation of Theorem 3.1.

Theorem 5.1 (DLR)**.** *If $\hat{\succcurlyeq}$ satisfies **weak order**, **c-continuity**, **c-independence**, **extendability**, **monotonicity**, and **global menu consistency** then there exists of type space T, a set of type-dependent expected utility functionals, $U : \Delta(X) \times T \to \mathbb{R}$ and a positive finitely additive measure over T, μ_A, for each $A \subseteq X$, such that*

$$\boldsymbol{b}_B \mathrel{\hat{\succcurlyeq}} \boldsymbol{a}_A \Leftrightarrow \int_T \max_{\rho \in \boldsymbol{a}} U(\rho, t)\mu_A(dt) \geqslant \int_T \max_{\rho \in \boldsymbol{b}} U(\rho, t)\mu_B(dt)$$

This result relies heavily on the DLR framework and results. The interim preferences are characterized by the DLR axioms and therefore imply such a representation. Global menu consistency then implies that the corresponding type-spaces are all the same. Finally, extendability guarantees that a common normalization is possible.

As in Theorem 3.1, the DMs tastes (here represented by the type-space) are unaffected by epistemic concerns. The variation in preference is entirely characterized by a change in the belief over the type-space.

6 Related Literature

The formulation of revealed preference, a collection of decision problems and their chosen alternatives, dates back to Samuelson [1938]. The conditions under which a revealed preference can be rationalized by a preference relation, attributed to Houthakker [1950], is now referred to as the Weak Axiom of Revealed Preference, or WARP. Loosely, it states that if x is preferred to y in some decision problem, then there can be no other decision problem in which y is preferred to x. While WARP has a strong mathematical and intuitive appeal, it is not hard to construct instances where it will fail.

WARP is violated, by definition, if decision environment changes the preference of the DM. Tversky and Kahneman [1981] developed the notion of framing –the idea that a decisions are influenced by their surrounding context. Framing has a large literature, both in the theoretical, experimental, and psychological settings [Simonson and Tversky, 1993, Salant and Rubinstein, 2008, Tversky and Shafir, 1992]. A particular type of framing, is the consideration of a menu's epistemic content. That a menu may contain information that is relevant to the DM's choice therein was first articulated by Luce and Raiffa [1957] and expounded upon by Sen [1993, 1997]. Sen describes the notion of the epistemic value of a menu with more tact than I could hope to achieve:

> What is offered for choice can give us information about the underlying situation, and can thus influence our preference over the alternatives, as we see them. For example, the chooser may learn something about the person offering the choice on the basis of what he or she is offering. To illustrate, given the choice between having tea at a distant acquaintance's home (x), and not going there (y), a person who chooses to have tea (x), may nevertheless choose to go away (y), if offered –by that acquaintance– a choice over having tea (x), going away (y), and having some cocaine (z). The menu offered may provide information about the situation –in this case say something about the

distant acquaintance, and this can quite reasonably affect the ranking of the alternatives x and y.

The epistemic aspect of decision problems has been studied by Kochov [2010] in a model that shares many philosophical motivations with this one. Kochov seeks to explain the discrepancy in the DM's preference across decision problems by associating each decision problem its own rationalizing preference relation. The difference in these preference relations is attributed to the epistemic content of associated decision problem. Kochov's model, similarly to the one presented in 5, defines a decision problems as collections of menus, and imposes the DLR structure on each preference relation to back out a subjective state space.

There are two major differences between Kochov's model and this one. The first is that the primary mechanism by which epistemic content alters the decision makers preference in Kochov's model is by changing the composition of the subjective state-space. That is to say that each decision problem induces its own set of "unforeseen contingencies." This is problematic from the modelers point of view, as it is impossible to observe a decision maker who is both aware and unaware of a particular contingency. This paper, on the other hand, explains the same behavior while avoiding problem of a confused decision maker, by confining the framing effect to be a local one.

Secondly, Kochov's model does not posses the language to describe preferences across menus or between them. This is a consequence of the global nature of the unforeseen contingencies interpretation –the decision maker cannot simultaneously consider the different epistemic effects of different menus. The local interpretation of menus as frames, on the other hand, is conducive to explaining a larger class of behavior including non-monotone preferences over menus.

The appeal to a DM who holds multiple beliefs in the identification of tastes has been explored in the literature on state dependent preferences. Karni et al. [1983] propose a DM who ranks alternatives after some hypothetical event with a known probability. Like this paper, the identification relies on the DM's consistency in ex-post tastes. This strategy is also common in the decision theoretic literature regarding "moral hazard", or a DM who has the ability to influence the realization of the state of the world fist attributed to Drèze [1987]. Identification of tastes is possible over states that can be manipulated by he DM, a condition similar in spirit to the richness axiom introduced here. Drèze and Rustichini [2004] provides a survey on state dependent preferences and discusses both hypothetical choice and choice with moral hazard.

There are several models that account for preference reversals and WARP violations by appealing to a DM who optimizes his choice relative to multiple (different) preferences [Kalai et al., 2002, Manzini and Mariotti, 2007]. These models allow for preference reversals that arise from the inclusion (or exclusion) of elements in the choice set by applying the preference rules in a sequential order. These models, however, fail to account for epistemic value. The underling (multiple) preferences are the same for every menu, and thus the DM's preference is *not* dependent on the menu. Although the maximal element of a choice set can be influenced by (seemingly) unrelated elements in the set, the preferences over the menu does not reflect any information contained therein.

There are a variety of other models where the menu acts as a frame, affecting the decision makers preference. One prevalent brach of this literature is models of endogenous reference dependence. In these models the decision problem is associated with a reference level of utility by which the DM evaluates each outcome [Kőzsegi and Rabin, 2006, Ortoleva et al., 2014]. As such, adding outcomes that will effect the reference point will thereby change the DM's preferences. These models can be thought of as a specific case of epistemic concerns; the reference point is information about some underlying state variable. A decision problem associated with reference point, r, is an indicator that the state-of-the-world is s_r.

This paper is also derived from the literature on choices *over* menus. Here the DM is choosing a menu, under the (often unmodeled) assumption that he will later choose from it. The largest strand of this literature is that of unforeseen contingencies, beginning with Kreps [1979]. Models of unforeseen contingencies, like this model, assume that the that the DM faces uncertainty about his preference (or equivalently, that he has state-dependent preference, and faces uncertainty about the state-of-the-world). This uncertainty is resolved in the interim –after choosing the menu, but before choosing from the menu– and creates a preference for flexibility as a larger menu can facilitate the optimal choice for a larger number of possible preference realizations. This is the intuition behind Kreps [1979], Kreps [1992], Ahn and Sarver [2013] and to a large extent Dekel et al. [2001].[6] The model presented here does not assume that uncertainty is resolved fully after the choice of menu. More importantly, by allowing for epistemic concerns this model assumes that the residual uncertainty is a function of which menu is chosen; the

[6] The model presented in Dekel et al. [2001] allows for smaller menus to be preferred to larger ones, however, only because some outcomes are aggregated as negatives (i.e. will cause disutilty). Thus, outcomes are treated the same (i.e. as good or bad) in every menu, and as such, all menus are comonotonic with respect to the inclusion or exclusion of particular element. This is reflected in the identification strategy that underlies the paper.

posterior belief is a function of the decision problem. This allows for the discrepancy between preference in different decision problems.

Previous models have explained menu-dependent preferences by appealing to dynamic inconsistencies. For example, a DM who anticipates his future temptation to choose a immediately gratifying –but overall suboptimal– option, may choose the smaller menu as a commitment device was consider in Gul and Pesendorfer [2001, 2004]. These 'time-inconsistency' models capture a different effect, as they change the 'tastes' of the DM rather than his 'beliefs'. That is to say, the diner who prefers the smaller menu is not inconsistent, he just wants the better steak and believes it will be served at the restaurant with the smaller menu.

7 Conclusion

This paper develops a framework that allows preferences to change in response to a change in the menu, or set of possible consumption outcomes. In particular, this paper identifies axioms over preferences that incorporate the epistemic value of the decision problem. That is, preferences such that the DM will subjectively condition his beliefs on an type-space. The change in his beliefs subsequently effects his preferences over realization-contingent allocations. Therefore, preferences are endogenously defined by the outcome space.

The result is a tractable representation of preferences that allows menu-induced framing effects (of a particular kind). In addition, this paper identifies the conditions such that DM's beliefs over the type-space, and utility index over ex-post outcomes, are unique. Representing preferences in this framework explains two behavioral traits that are unexplainable by previous work. First, these preferences allow for preference reversals as a response to the inclusion or exclusion of (seemingly) unrelated outcomes. Second, these preferences can explain why a smaller menu would be (strictly) preferred to its superset, without appealing to time-inconsistencies.

A Proofs

Proof. **Proof of Theorem 3.1(a)** (Sufficiency). We need to show that $u(t, x)$ does not depend on the menu. Assume the opposite: that there exists two non-null menu-realization pairs such that the menu-induced utility indices are not affine transformations of one another. By assumption then there exists some menus A, D such that for some $s \notin N_A \cup N_D$ there exists some $x, y, z \in A \cap D$ such that $\frac{u(s,x,A)}{u(s,z,A)} \neq \frac{u(s,x,D)}{u(s,z,D)}$. WLOG assume $x \prec_A y \prec_A z$. This is without loss since it is merely a relabeling of x and z, since we can always find a y in between by the continuity axiom and c-independence. Utilizing two degrees of freedom (one from each representation), we can normalize $u(s, y, A) = u(s, y, D) = 0$. Let f be an act such that $f(s, y) = 1$ and h^α be a sequence of acts such that $h^\alpha(s, x) = \alpha$ and $h^\alpha(s, z) = (1 - \alpha)$ for $\alpha \in (0, 1)$. These acts are in $A \cap D$ by the convexity of menus. We know from consistency and the vNM theorem that

$$\sum_t \sum_x u(t, x, A)\mu_A(t)[f_{-s}h^\alpha(t, x) - f(t, x)] \geqslant 0$$

$$\Leftrightarrow$$

$$\sum_t \sum_x u(t, x, D)\mu_B(t)[f_{-s}h^\alpha(t, x) - f(t, x)] \geqslant 0$$

Removing like terms from both sides and utilizing our normalization this implies that for all α

$$\mu_A(s)\left[u(s, x, A)\alpha + (1 - \alpha)u(s, z, A)\right] \geqslant 0 \Leftrightarrow \mu_B(s)\left[u(s, x, D)\alpha + (1 - \alpha)u(s, z, D)\right] \geqslant 0 \tag{2}$$

Since $u(s, x, A) < u(s, y, A) = 0 < u(s, z, A)$ we know there exists an α^* such that the left side of (1) is equal to zero. However, by assumption the righthand side can not be equal to zero at α^*. By the continuity of the above equations there must also exist an α^{**} such that (1) is violated. This contradicts menu consistency. Thus, for every pair of non-null menus the utility indices are affine transformations of one another. Thus, by the vNM theorem if $u(s, x)$ represents $\succcurlyeq_A$, it represents all interim preferences.

It only remains to show that $s \in N_A \cup N$ iff $\mu_A(s) = 0$. For all $t \in N$, and by menu-consistency, we have $(f_{-t}\rho)_A \sim (f_{-t}\pi)_A$ for all A and acts f and ρ, π in A. It follows that $\mu_A(t) > 0$ for any A implies that $u(t, \cdot)$ is constant over X. But it is therefore without loss that we set $\mu_A(t) = 0$, and rescale the probabilities on $T \backslash t$ accordingly. Now assume that $t \in N_A$. By definition of evidently-null, $u(s, x)$ is not constant over A (since if it was constant, than there would be no other menu with strict preference over elements in A). Let $x, y \in A$ such that $u(s,x) > u(s,y)$. Consider two acts, f and g, that coincide on all allocations off of state s and in all lotteries that allocated other that x, y. That is, f allocates more probability to (s,x) by taking away for (s,y), when compared to g. Therefore,

$$\mu_A(s)[u(s,x)\,(f(s,x) - g(s,x)) + u(s,y)\,(f(s,y) - g(s,y))] = 0$$

where $f(s,x) > f(s,y)$ and $g(s,y) > g(s,x)$. This can only hold if $\mu_A(s) = 0$. The other direction is immediate, since if $\mu_A(t) = 0$ then by the representation is is clear that $(f_{-t}\rho)_A \sim (f_{-t}\pi)_A$ for all acts f and ρ, π in A. Hence, $t \in N_A \cup N$.

(Necessity). The necessity of continuity, independence, and weak order follow from vNM theorem. Let $\succcurlyeq$ be represented as in Theorem 3.1. Fix A and B and $f, g \in F, \rho \in A \cap B$ such that $f(s) = g(s)$ and $s \notin N_A \cup N_B$. Assume $(f_{-s}\rho) \succ f_A$. This implies that

$$\mu_A(s)u(s,x)[f(s,x) - \rho(x)] > 0$$

Since $\mu_A(s)$ cannot be negative, it must be that $u(s,x)[f(s,x) - \rho(x)] = u(s,x)[g(s,x) - \rho(x)] > 0$. Therefore, $u(s,x)$ cannot be constant over B (since $g, \rho \in F_B$). Since s is not evidently-B-null by assumption, $\mu_B(s)$ is strictly positive. This directly implies that

$$\begin{aligned}
&\mu_B(s)u(s,x)[g(s,x) - \rho(x)] \\
&= \sum_{t \in S/s} \mu_B(t) \sum_x u(t,x)[g(t,x) - g(t,x)] + \mu_B(s)u(s,x)[g(s,x) - \rho(x)] > 0
\end{aligned}$$

By the representation, $(g_{-s}\rho)_B \succ g_B$. Hence, $\succcurlyeq$ is menu-consistent.

Proof of Theorem 3.1(b). First, note that richness implies that $N = \varnothing$ since for any $t \in T$ there is a menu, A such that $t \in N_A$ which implies there is some other menu, B such that t is non-null-B.

Assume that, for some menu, D, (μ_D, u) and $(\hat{\mu}_D, \hat{u})$ both represent $\succcurlyeq_D$. We will first show that the number of states on which $\mu_D(t) \neq \hat{\mu}_D(t)$ cannot be more than 2. From the vNM theorem we know that $\mu_D(t)u(t,x) = \hat{\mu}_D(t)\hat{u}(t,x) = w(t,x,D)$ for all $t \in T, x \in X$. Thus, $\hat{u}(t,x) = \frac{\mu_D(t)}{\hat{\mu}_D(t)}u(t,x)$ for all t in the support of μ_D. Since $\hat{\mu}_D$ and μ_D must have the same support, this is admissible. Since we are assuming that $\mu_D \neq \hat{\mu}_D$ there must be some realization, $t \in T$, such that $\mu_D(t) \neq \hat{\mu}_D(t)$. Reindex the types such that $\mu_D(t) \neq \hat{\mu}_D(t)$ on the range t_1 to t^*.

If $t^* > 2$ then there are two types t_l and t_k such that $\mu_D(t_l) > \hat{\mu}_D(t_l)$ and $\mu_D(t_k) > \hat{\mu}_D(t_k)$. By our richness assumption there exists a menu A such that $N_A = S \backslash \{t_l, t_k\}$. By part (a) we have that $\mu_B(t)u(t,x) = \hat{\mu}_B(t)\hat{u}(t,x) = w(t,x,B)$ for all $t \in T, x \in X$ and so $\mu_B(t)u(t,x) = \hat{\mu}_B(t)\frac{\mu_D(t)}{\hat{\mu}_D(t)}u(t,x)$. Therefore, $\frac{\mu_A(t)}{\hat{\mu}_A(t)} = \frac{\mu_D(t)}{\hat{\mu}_D(t)}$ for $t = t_l, t_k$. Clearly, this implies that $\sum_{t=t_l,t_k} \hat{\mu}(t) < 1$, a contradiction to $N_A = T/\{t_l, t_k\}$. Therefore, $t^* = 2$.

So assume for menu D that $t^* = 2$. First assume that there exists some $t_i > t^*$ such that $t_i \notin N_D$. So there exists a menu, B, such that $N_B = S \backslash \{t_1, t_i\}$. By the same argument as above, we know that $\frac{\mu_B(t)}{\hat{\mu}_B(t)} = \frac{\mu_D(t)}{\hat{\mu}_D(t)}$ for $t = t_1, t_i$, and so $\mu_B(t_i) = \alpha\mu_D(t_i)$ and $\hat{\mu}_B(t_i) = \alpha\hat{\mu}_D(t_i)$ for some $\alpha > 0$. Also, $\mu_B(t_1) + \mu_B(t_i) = \hat{\mu}_B(t_1) + \hat{\mu}_B(t_i) = 1$ which implies

$$\frac{1 - \alpha\mu_D(t_1)}{1 - \alpha\hat{\mu}_D(t_1)} = \frac{\mu_D(t_i)}{\hat{\mu}_D(t_i)}$$

a contradiction to the definition of t^* (if $\alpha \neq 1$, then $\mu_D(t_i) \neq \hat{\mu}_D(t_i$, and if $\alpha = 1$ then $\mu_D(t_2) = \hat{\mu}_D(t_2) = 0$). □

Lastly, assume that there for all $t_j > t^*$, $t_j \in N_D$. Since $|T| > 2$, there exists a menu, C, such that $N_C = S \backslash \{t_1, t_j\}$ for some $t_j > t^*$. We know from the above arguments that $\mu_C(t_1) \neq \hat{\mu}_C(t_1)$. Also, $\mu_C(t_1) + \mu_C(t_j) = \hat{\mu}_C(t_1) + \hat{\mu}_C(t_j) = 1$ and so $\mu_C(t_j) \neq \hat{\mu}_C(t_j)$. But then, by richness, there is some menu, E, such that $N_C = S \backslash \{t_1, t_2, t_j\}$. Further, by the above arguments $\frac{\mu_E(t)}{\hat{\mu}_E(t)} = \frac{\mu_D(t)}{\hat{\mu}_D(t)}$ for $t = t_1, t_2$ and $\frac{\mu_E(t)}{\hat{\mu}_E(t)} = \frac{\mu_C(t)}{\hat{\mu}_C(t)}$ for $t = t_1, t_j$. So, μ_E and $\hat{\mu}_E$ disagree on more than 2 states, a contradiction.

Proof. **Proof of Corollary 3.2** (Sufficiency). From Debreu's theorem we know that there exists a continuous (in $f(s,x)$) utility function that represents $\succcurlyeq$. Denote this utility function $V(f(s,x), A)$. Since $V(f(s,x), A)$ also represents $\succcurlyeq_A$ for each $A \in \mathcal{K}(\Delta(X))$, then from vNM it must be that

$$V(f(s,x), A) = a(A)\left[\sum_s \sum_x w(s,x,A)f(s,x)\right] + b(A) \tag{3}$$

From Theorem 3.1 we can rewrite (2) utilizing the decomposition of $w(\cdot)$.

$$V(f(s,x), A) = a(A)\left[\sum_s \mu_A(s) \sum_x u(s,x)f(s,x)\right] + b(A) \tag{4}$$

The properties of extendability, along with the ability to construct menus or arbitrary support, allows us to normalize $u(s, \overline{x}) = 1$ and $u(s, \underline{x}) = 0$. By evaluating the constant acts $\underline{x}, \overline{x}$ we get

$$\begin{aligned} a(A)\,[0] + b(A) &= a(B)\,[0] + b(B) \\ a(A) + b(A) &= a(B) + b(B) \end{aligned}$$

The only solution to this system is obviously, $a(\cdot)$ and $b(\cdot)$ are constant. Normalizing $a(\cdot) = 1$ and $b(\cdot) = 0$ gives the result. It follows from $f_A \sim f_{\overline{A}}$ for all $f \in F_A$ and $A \in \mathcal{K}(\Delta(X))$ that $w(s, x, A) = w(s, x, \overline{A})$ for all menus. Therefore, we get that $\mu_A = \mu_{\overline{A}}$ since $w(s, x, \cdot)$ is uniquely decomposed by Theorem 3.1.

(Necessity). Satisfying the conditions for Theorem 3.1 follows from the proof thereof. Assume preferences are represented by as given in 3.2. Since $\mu_A = \mu_{\overline{A}}$ and $u(s, x)$ is constant it must be that $f_A \sim f_{\overline{A}}$. Further, since the utility index of $\underline{x}, \overline{x}$ are constant across realizations, it is clear that any convex combination is equally preferred across menus. □

Proof. **Proof of Theorem 4.1**. Strong-menu-consistency (along with the other restrictions) imply the Anscombe and Aumann [1963] axioms hold for each $\succcurlyeq_A$. Assume $(f_{-s}\rho)_A \succ f_A \Leftrightarrow (g_{-t}\rho)_B \succ g_B$. By letting $A = B$ we have state-independence exactly. With the addition of the vNM restrictions we know that each menu is represented by AA's representation, along with a unique belief. Since strong-menu-consistency implies menu-consistency, the proof of Theorem 3.1 guarantees that the utility index is menu-independent. It remains only to show the necessity of strong-menu-consistency. The proof is remarkably similar to the necessity proof of Theorem 3.1, and is therefore omitted. □

Proof. **Proof of Corollary 4.2**. The conditions of Theorem 4.1 are satisfied, therefore, it suffices to show that $a(A) = a(B)$ and $b(A) = b(B)$ for all menus A and B. Fix an arbitrary A, B and two constant acts ρ, π such that $\rho_A \succ \pi_A$. This is ensured to exist by non-triviality. We know from Theorem 4.1 that

$$\begin{aligned} a(A)\left[\sum_s \mu_A(s) \sum_x u(x)\rho(x)\right] + b(A) &= a(B)\left[\sum_s \mu_B(s) \sum_x u(x)\rho(x)\right] + b(B) \\ a(A)\left[\sum_s \mu_A(s) \sum_x u(x)\pi(x)\right] + b(A) &= a(B)\left[\sum_s \mu_B(s) \sum_x u(x)\pi(x)\right] + b(B) \end{aligned}$$

The interior of the square brackets in identical within each equation, but different across equations; the only solution to this system is $a(A) = a(B)$ and $b(A) = b(B)$. We can therefore normalize $a(\cdot) = 1$ and $b(\cdot) = 0$ to obtain the desired representation. $\square$

Proof. **Proof of Theorem 4.3**. Assume that the conditions for Theorem 4.3 hold, namely strong-menu-consistency, but there exists an A, B such that $A \subset B$ and $V(A) > V(B)$. By definition this means that there exists $f \in F_A$ such that

$$\sum_s \mu_A(s) \sum_x u(x) f(s,x) \geqslant \sum_s \mu_B(s) \sum_x u(x) g(s,x)$$

for all $g \in F_B$. Define $\nu \in \Delta(X)$, such that $\nu(x) = \sum_s \mu_A(s) f(s,x)$. $\nu(x)$ is a probability distribution that represents the subjective probability of obtaining outcome x given the lottery $f(s)$ and the subjective belief over the type space. Further, since ν is constructed as the convex combination of elements of A, and because A is convex, it must be that $\nu \in A$. Therefore, the constant act that assigns ν is in F_A and therefore also in F_B. But by construction

$$\sum_s \mu_A(s) \sum_x u(x) f(s,x) = \sum_s \mu_B(s) \sum_x u(x) \nu(x)$$

Therefore the constant act ν_B is weakly preferred to f_A contradicting the assumption. $\square$

Proof. **Proof of Theorem 5.1**. (Sufficiency) That $\hat{\succcurlyeq}$ satisfies weak order, c-continuity and c-independence, and monotonicity allows us to apply Theorem 4 in Dekel et al. [2001]. The consequence of which is that, for each $A \subseteq X$, there exists a unique set of type-dependent expected utility functionals, $U_A(\cdot, t)$ for $t \in T_A$ and a positive finitely additive measure μ_A, such that

$$\boldsymbol{b}_A \mathrel{\hat{\succcurlyeq}} \boldsymbol{a}_A \Leftrightarrow \int_{T_A} \max_{\rho \in \boldsymbol{a}} U_A(\rho, t) \mu_A(dt) \geqslant \int_{T_A} \max_{\rho \in \boldsymbol{b}} U_A(\rho, t) \mu_A(dt) \tag{5}$$

for each $\boldsymbol{a}, \boldsymbol{b} \subset A$.

Note that the uniqueness claim is made modulo some normalization (i.e., common affine transformations). Because the ex-ante preference specifies preference between local menus from different global menus, it is not in general possible to keep a consistent normalization *across* global menus. Thus, if $t \in T_A \cap T_B$ then $U_A(\cdot, t) = U_B(\cdot, t)$ up to an affine transformation.

We now claim that $T_A = T_X|_{\Delta(A)}$ for all global menus $A \subseteq X$. That is, there is a minimal, unique, type-space such that each conditional preference is a projection thereof.

Choose some $A \subseteq X$ and consider the corresponding type space T_A. Let $\hat{T}_A$ denote the set of linear functionals characterized by the projection of $U_{\Delta(X)}(\cdot, t)$ onto $\Delta(A)$. The claim is proven if $T_A = \hat{T}_A$, since it is without loss of generality to extend T_A to elements not in A. Choose some common normalization for $T_A, \hat{T}_A$. Under a common normalization, there is no reason to believe that these type spaces represent the ex-ante preference –however, they still represent the interim projections (choices from the same global menu).

Assume to the contrary that $T_A \neq \hat{T}_A$. In particular, that there exists some $t_0 \in T_A \backslash \hat{T}_A$ (the other direction follows from a symmetric argument). For each $t \in T_A \cup \hat{T}_A$ let $\mathcal{L}_t$ denote a lower contour set, intersect $\Delta(A)$, of the associated preference, $\hat{\succcurlyeq}_t$. Let $\boldsymbol{c}$ be the intersection of all these lower contour sets, and let $\hat{\boldsymbol{c}}$ be the intersection of all lower contour sets except $\mathcal{L}_{t_0}$. Since both $\boldsymbol{c}$ and $\hat{\boldsymbol{c}}$ are the intersection of compact sets contained in $\Delta(A)$ they are elements of $\mathcal{K}(\Delta(A))$.

In an argument identical to the one contained in the proof of Theorem 1.B in Dekel et al. [2001] the lower contour sets can be chosen so as to ensure:

$$\max_{\rho \in \boldsymbol{c}} U_X(\rho, t) = \max_{\rho \in \hat{\boldsymbol{c}}} U_X(\rho, t) \qquad \forall t \in \hat{T}_A \tag{6}$$

$$\max_{\rho \in \boldsymbol{c}} U_A(\rho, t) = \max_{\rho \in \hat{\boldsymbol{c}}} U_A(\rho, t) \qquad \forall t \in T_A, t \neq t_0 \tag{7}$$

$$\max_{\rho \in \boldsymbol{c}} U_X(\rho, t_0) < \max_{\rho \in \hat{\boldsymbol{c}}} U_X(\rho, t_0) \tag{8}$$

Since $(U_X, \hat{T}_A, \mu_X)$ represent the interim preferences (conditional on X) over local menus contained in $\Delta(A)$, equation (6) implies (by equation (5)) that $\hat{\boldsymbol{c}}_X \mathrel{\hat{\sim}} \boldsymbol{c}_X$. Likewise, since (U_A, T_A, μ_A) represent the interim preferences (conditional on A) over local menus contained in $\Delta(A)$, equations (7) and (8) imply that $\hat{\boldsymbol{c}}_A \mathrel{\hat{\succ}} \boldsymbol{c}_A$. But this is a contradiction to the comontonicity property implied by global menu consistency.

So up to set-wise affine transformations, each interim type-spaces is a projection of a common larger type-space. It remains to show that these type spaces can accommodate a common normalization. This is accomplished in much the same manner as in Theorem 3.1 utilizing the properties of extendability. □

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