

**ELICITING AWARENESS:  
ITERATED REVELATION MECHANISMS**

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## Cast of Characters

### Decision Maker (dm)

- ◇ Will take an action in the future
- ◇ Would like to condition the action on the resolution of uncertainty
- ◇ He is unaware/unable to express of some relevant aspects of the decision problem

### Expert (ex)

- ◇ Has no right to the decision, herself
- ◇ But can reveal aspects of the environment

(we will also add a **mechanism designer**, later)

- ◇ Writing legislation: politician; technical advisor
- ◇ Buying a house: buyer; surveyor
- ◇ Hiring: search committee; letter writer
- ◇ Investing: investor; financial advisor

## Contracts

- ◇  $\mathcal{A}$  is a set of actions
- ◇  $\Omega$  is a state-space
- ◇  $\mathbf{dm}$  must choose a **contract**:

$$\mathbf{c} : \Omega \rightarrow \mathcal{A}$$

## Contracts

- ◇ Not all contracts are feasible. **dm** may be
  - ◇ unable to express
  - ◇ unaware of
  - ◇ technologically unable to implement/condition on

some actions or events in the state-space

- ◇ **ex's** revelations are
  - ◇ verifiable
  - ◇ ex-ante uncontactable

effectively expand the set of feasible contracts

## Why is the interesting?

- ◇ When preferences are not aligned, **ex** might strategically conceal her awareness
- ◇ Can **dm** do anything to incentivize revelation?
- ◇ A(n unaware) designer may not be able to solve the problem, if mechanisms depend on the unknowns

# Literature

- ◇ Incomplete Contracting

- ◇ Grossman and Hart (1986); Maskin and Tirole (1999); Aghion and Holden (2011); Hart (2017)

- ◇ Contracting under unawareness

- ◇ Tirole (2009); Filiz-Ozbay (2012); Von Thadden and Zhao (2012); Auster (2013) Auster and Pavoni (2021); Piermont (2017); Lei and Zhao (2021); Francetich and Schipper (2021)

- ◇ Cheap talk

- ◇ Crawford and Sobel (1982); Seidmann and Winter (1997); Aumann and Hart (2003);

- ◇ Robust Mechanism Design

- ◇ Bergemann and Morris (2005); Jehiel et al., (2006); Carroll (2015, 2019).

## Example

- ◇ An **investor** (the **decision maker**) is trying to invest his wealth:
  - ◇ the composition of the portfolio can be contingent on the future realized state-of-affairs, but
  - ◇ can depend only on those contingencies he is aware of
  - ◇ can only invest in assets he is aware of
- ◇ He can enlist the help of a **financial advisor** (the **expert**) who may reveal novel contingencies/assets



## Example

- ◇ The true state-space is  $\Omega = \{\omega, \nu\}$ ; equally likely
- ◇ Set of actions  $\mathcal{A} = \{\alpha, \beta, \gamma\}$
- ◇ **dm** must choose an contract  $\mathfrak{c} : \Omega \rightarrow \mathcal{A}$
- ◇ Let  $V_i(\mathfrak{c})$  denote the expected utility to player  $i$

## Example

ex can distinguish the states, but dm cannot.

$$\mathcal{P}_e = \{\{\omega\}, \{\nu\}\}$$

$$\mathcal{P}_d = \{\{\omega, \nu\}\}$$

$$u_{\textcolor{blue}{d}} = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$



$$u_{\textcolor{red}{e}} = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right.$$

How does **dm** view payoffs in coarse states?

- ◇ Assume it is aggregated via expectations
- ◇ As if he correctly assesses randomness, but condition a contract on the source of this randomness because he
  - ◇ is unaware of what causes it, or
  - ◇ does not possess language describe it in a contract, or
  - ◇ does not have the technology to condition on it

$$u_d = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$

$$u_d = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \{\omega, \nu\} & 2 & 3 & 1 \end{array} \right.$$

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What would  $\text{dm}$  implement:

- ◇ Without revelation  $\mathfrak{c} = \beta$
- ◇  $V_d(\mathfrak{c}) = 3, V_e(\mathfrak{c}) = 2$

$$u_d = \left\{ \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \nu & 0 & 6 & 0 \end{array} \right\} \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline \omega & 0 & 2 & 4 \\ \nu & 0 & 2 & 4 \end{array} \right\}$$

What would **dm** implement:

- ◇ With revelation:  $\mathbf{c}' : \begin{cases} \omega \mapsto \alpha \\ \nu \mapsto \beta \end{cases}$
- ◇  $V_d(\mathbf{c}') = 5$ ,  $V_e(\mathbf{c}') = 1$ ; So **ex** won't reveal.



$$u_d = \left\{ \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \nu & 0 & 6 & 0 \end{array} \right\} \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline \omega & 0 & 2 & 4 \\ \nu & 0 & 2 & 4 \end{array} \right\}$$

- ◇ But,  $\mathbf{c}^* : \begin{cases} \omega \mapsto \gamma \\ \nu \mapsto \beta \end{cases}$  is a Pareto improvement over no revelation
- ◇  $V_d(\mathbf{c}^*) = 4, V_e(\mathbf{c}^*) = 3$

## Example

- ◇ The Pareto improvement  $c^*$ , requires revelation
- ◇ But revealing allows  $dm$  to exploit  $ex$
- ◇ What if  $dm$  could commit:
  - ◇ Propose  $c = \beta$  (his outside option)
  - ◇ After  $ex$  reveals, propose some other contract  $c^\dagger$
  - ◇  $c^\dagger$  only get implemented if  $ex$  agrees; else  $c = \beta$

## Example

Internalizing this,  $\text{dm}$  solves:

$$\max_{\mathfrak{c}^\dagger: \Omega \rightarrow \mathcal{A}} V_d(\mathfrak{c}^\dagger) \quad \text{subject to} \quad V_{\textcolor{red}{e}}(\mathfrak{c}^\dagger) \geq V_{\textcolor{red}{e}}(\mathfrak{c}) \quad (\text{IC})$$

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- ◇ The solution is  $\mathbf{c}^\star : \begin{cases} \omega \mapsto \gamma \\ \nu \mapsto \beta \end{cases}$

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So a two stage game with commitment to not revoke the prior proposal resulted in

- ◇ full revelation
- ◇ an efficient contract

Does this always work? No

## Example

What if  $dm$  is initially unaware of action  $\beta$ ?

$$u_d = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$

$$u_d = \left\{ \begin{array}{c|c|c|c} & \alpha & & \gamma \\ \hline \{\omega, \nu\} & 2 & & 1 \end{array} \right.$$

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What would  $\mathbf{dm}$  implement:

- ◇ Without revelation  $\mathbf{c}^* = \alpha$
- ◇  $V_d(\mathbf{c}^*) = 2, V_e(\mathbf{c}^*) = 0$

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What would  $\text{dm}$  implement:

- ◇ Under full revelation (same as before):  $\mathbf{c}' : \begin{cases} \omega \mapsto a \\ \nu \mapsto \beta \end{cases}$
- ◇  $V_d(\mathbf{c}') = 5$ ,  $V_e(\mathbf{c}') = 1$ ; this satisfies the incentive constraint.

$$u_d = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right\} \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|c|c|c} & \alpha & \beta & \gamma \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right\}$$

- ◇ But, revealing only  $\beta$  leads to  $\mathfrak{c} = \beta$
- ◇  $V_d(\mathfrak{c}) = 3$ ,  $V_e(\mathfrak{c}) = 2$ ; partial revelation is preferred

$$u_d = \left\{ \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right\} \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right\}$$

◇ As before,  $\mathbf{c}^* : \begin{cases} \omega \mapsto \gamma \\ \nu \mapsto \beta \end{cases}$  is a Pareto improvement over  $\mathbf{c} = \beta$

What if the procedure was repeated?

(1) Initially,  $\text{dm}$  commits (to  $\mathfrak{c}^* = \alpha$ )

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- (5) **dm** solves

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- ◇  $\mathbf{c}^*$  is the solution

Does this always work?

Does this always work? Yes, but what is 'always'?



# Model

The environment is described by

$\mathcal{A}$  — a set of actions

$\Omega$  — a state-space

$\pi$  — a probability over  $\Omega$

$(u_d, u_e)$  — state-dependent utility functions

Let  $V_i$  denote expectation operator w.r.t  $u_i$  and  $\pi$

# Types

- ◇ Let  $\mathbf{T}$  be a subset of finite subsets of  $\mathcal{A}^\Omega$
- ◇ A **type**  $t \in \mathbf{T}$  determines the set of expressible contracts (finite set of contracts)
- ◇ Say that type  $t$  is **more expressive** than type  $t'$ , written  $t' \subseteq t$ , if  $t' \subseteq t$
- ◇  $\mathbf{T}(t) \subseteq \mathbf{T}$  are those types expressive than  $t$ .
- ◇ Fix types  $t_d$  and  $t_e$ , and assume  $t_e \in \mathbf{T}(t_d)$ .

## Example

Each  $t$  is given by

$\mathcal{P}^t$  — a partition of  $\Omega$

$A^t$  — a subset of  $\mathcal{A}$

Then

$$t = \{ \mathfrak{c} : \Omega \rightarrow \mathcal{A}^t \mid \mathfrak{c} \text{ is } \mathcal{P}^t \text{ measurable} \}$$

◇  $t' \subseteq t$  if and only if  $\mathcal{P}^t$  refines  $\mathcal{P}^{t'}$  and  $A^{t'} \subseteq A^t$

## Outcome Profiles

An **outcome profile** is a function from types to contracts:

$$\begin{array}{ccc} f: & t & \mapsto \mathfrak{c} \\ & \cap & \cap \\ & \mathbf{T}(t_d) & \rightarrow \mathcal{A}^\Omega \end{array}$$

such that  $f(t) \in t$

Call  $f$  **incentive compatible** if **ex's** payoff is monotone in her type

$$V_e(f(h)) \leq V_e(f(h'))$$

whenever  $h \subseteq h'$

- ◇ There need not be any ‘strategic’ way of enacting an outcome profile.
- ◇ But if there is, it *must* be incentive compatible.

## Iterated Revision Mechanisms

An **iterated revelation mechanism** (IRM), is parameterized by a function from *sequences of types* to contracts:

$$m : (t_0 \dots t_n) \mapsto c \in t_n$$

**STEP 1** — Set  $n = 0$ . **dm** announces  $t_0 = t_d$ , and proposes  $m(t_0)$ .

**STEP 2** — **ex** reveals  $t_{n+1}$ .

- ◇ If  $t_n \subsetneq t_{n+1}$ , **dm** proposes  $m(t_{n+1})$ , goto **STEP 3**.
- ◇ Otherwise, the mechanism is over and  $m(t_n)$  get implemented.

**STEP 3** — **ex** can accept or reject the proposal,  $m(t_{n+1})$ :

- ◇ If she accepts, set  $n = n + 1$  and goto **STEP 2**.
- ◇ If she rejects, the mechanism is over and  $m(t_n)$  get implemented.

Importantly: the contracts proposed by an IRM are *jointly* expressible at the time of proposal

## Full Revelation

### Theorem

The following are equivalent for an outcome profile  $f$

- (1)  $f$  can be induced by an IRM
- (2)  $f$  is incentive compatible

◇ Induced:  $f(t) = m(\sigma)$  where  $\sigma$  is a *best response* over all expressible sequences for type  $t$ .



The can be seen as an impossibility result:

- ◇ Without commitment to leave proposed contracts on the table, full revelation cannot be guaranteed.

Each proposed contract in an IRM specifies:

- (1) The outcome should the game end
  - ◇ **dm** wants to maximize his own payoff
- (2) The implicit incentive constraint should the game continue
  - ◇ **dm** wants to minimize **ex's** payoff

In the examples, contracts solved (1) ignoring (2)

If **dm** cannot conceive of what **ex** is aware of it seems prudent to consider *robust* strategies:

- ◇ those that maximize the worst case outcome
- ◇ this is belief free: does not require conjecturing about probability of types
- ◇ Robust strategies turn out to be exactly those that follow the principle of myopic optimization

## Robustness

Call an IRM,  $m$ , **robust** if at every sequence of (possibly partial) revelations  $\sigma$ ,  $m$  maximizes the worst case payoff over

- ◇ all best responses that extend  $\sigma$ .
- ◇ for types for which  $\sigma$  would have been rational
- ◇ compared to any other  $m'$  that coincides with  $m$  over  $\sigma$

# Robustness

## Theorem

The following are equivalent (up to the implemented outcome profile)

- (1)  $m$  is robust
- (2)  $m$  is myopically optimal: at each sequence  $(t_0, \dots, t_n)$ ,

$$\begin{aligned} m(t_0, \dots, t_n) \in \operatorname{argmax}_{c \in t_n} V_d(c) & \quad \text{subject to} \\ V_e(c) \geq V_e(m(t_0 \dots t_{n-1})) \end{aligned}$$

## The Designers Problem

- ◇ A **designer** wants the **decision maker** to take some action
- ◇ The **designer** knows *neither* **dm**'s nor **ex**'s type
- ◇ A **mechanism** elicits types and returns an contract

## Mechanism

A **mechanism** is a mapping from pairs of types into contracts:

$$\mathcal{M} : (t_d, t_e) \mapsto \mathcal{M}(t_d, t_e)$$

where  $\mathcal{M}(t_d, t_e) \in t_e$

- ◇ It common knowledge that  $t_d \subseteq t_e$

## Desiderata:

INDIVIDUAL RATIONALITY: **dm** can not do better alone (there is no constraint for **ex**)

INCENTIVE COMPATIBILITY:  $i$  prefers to report  $t_i$  than any  $t \subsetneq t_i$

PARETO OPTIMALITY: there is no feasible contract that dominates the outcome of the mechanism

These are all **ex-post** restrictions — they must hold for all type realizations



Fixing  $t_d$ , a mechanism determines an outcome profile:

$$f^{t_d} : t \mapsto \mathcal{M}(t_d, t)$$

By incentive compatibility,  $f^{t_d}$  can be induced by an appropriate IRM.

Consider the mechanism,  $\mathcal{M}^{\text{MO}}$ , that implements a myopically optimal IRM:

- (1) first, the decision maker reveals  $t \in \mathbf{T}$
- (2) then we run a myopically optimal IRM,  $\mathfrak{m}^t$ :
  - ◇ starting from  $t$
  - ◇ multiple m.o. contracts  $\Rightarrow$  break ties in favor of the expert

## Theorem

The mechanism  $\mathcal{M}^{\text{MO}}$

- ◇ is individually rational, incentive compatible, and Pareto optimal, and,
- ◇ for any other such mechanism  $\mathcal{M}$ ,

$$V_d(\mathcal{M}^{\text{MO}}(t, t')) \geq V_d(\mathcal{M}(t, t'))$$

for all  $t, t' \in \mathbf{T}$  with  $t \subseteq t'$ .

- ◇ There is a ‘dual’ IRM that implements the  $V_e$ -dominant mechanism

## Distributed Awareness

What if we relax the assumption that  $t_d \subseteq t_e$ ?

### Theorem

Allowing for distributed awareness, there exists no incentive compatible and Pareto optimal mechanism.

Let  $\Omega = \{\omega\}$  and everything else defined by

	$\alpha$	$\beta$	$\gamma$
$u_d$	0	1	2
$u_e$	0	2	1

- ◇ each type is associated with a subset of  $\{\alpha, \beta, \gamma\}$
- ◇ Let  $\mathcal{M}$  be any Pareto optimal mechanism. This requires

$$\mathcal{M}(\{\alpha\}, \{\alpha, \gamma\}) = \gamma \quad \mathcal{M}(\{\alpha, \beta\}, \{\alpha\}) = \beta \quad \mathcal{M}(\{\alpha, \beta\}, \{\alpha, \gamma\}) \in \{\beta, \gamma\}$$

- ◇ if  $\mathcal{M}(\{\alpha, \beta\}, \{\alpha, \gamma\}) = \beta$ , then **dm** of type  $\{\alpha, \beta\}$  misreports as  $\{\alpha\}$ ,
- ◇ if  $\mathcal{M}(\{\alpha, \beta\}, \{\alpha, \gamma\}) = \gamma$ , then **ex** of type  $\{\alpha, \gamma\}$  misreports as  $\{\alpha\}$ ,

**Thank You!**