ELICITING AWARENESS

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Internal Seminar

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- A decision maker must choose a ‘plan-of-action;’ what action to take provided the future resolution of uncertainty

- He is unaware of some relevant contingencies and *knows this is possible*
  - Purchasing a home
  - Hiring a new AP
  - Writing legislation

What do we often do in such situations?

What do we often do in such situations? seek the council of an expert who is more aware

- Purchasing a home: **surveyor**
- Hiring a new AP: **letter writers, search committee**
- Writing legislation: **technical advisors**

Why is the interesting?

- When preferences are not aligned, the expert might strategically conceal her awareness
- Can the dm do anything to incentivize revelation?
- Importantly, even with otherwise unconstrained contracting, the dm cannot articulate what he wants
- A(n unaware) designer may not be able to solve the problem, if mechanisms depend on the unknowns

Literature

- Contracting under unawareness
  - Tirole (2009); Filiz-Ozbay (2012); Von Thadden and Zhao (2012); Auster (2013) Auster and Pavoni (2021); Piermont (2017); Lei and Zhao (2021); Francetich and Schipper (2021)
- Modeling unawareness and awareness of unawareness
  - Halpern and Rêgo (2006, 2013); Heifetz et al. (2013); Karni and Vierø (2013, 2017), Halpern and Piermont (2020); Piermont, (2021)
- Robust Mechanism Design
  - Bergemann and Morris (2005); Jehiel et al., (2006); Carroll (2015, 2019).

Example

- A politician (the decision maker) is trying to write environmental legislation that
  - can be contingent on the future realized environmental state-of-affairs, but
  - can depend only on those contingencies he is aware of.
- He can enlist the help of an environmental scientist (the expert) who may reveal what she is aware of

Example

- The true state-space is $\Omega = \{\omega, \nu\}$; equally likely
- Set of actions $\mathcal{A} = \{a, b, c\}$
- The politician must choose legislation $\mathfrak{c} : \Omega \to \mathcal{A}$

Example

The expert can distinguish the states, but the politician cannot.

$$\mathscr{P}_e = \{\{\omega\}, \{\nu\}\} \qquad \mathscr{P}_d = \{\{\omega, \nu\}\}$$

$$u_{d} = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$

$$u_{e} = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right.$$

How does the politician view payoffs in coarse states?

- Assume it is aggregated via expectations
- As if he correctly assesses randomness, but cannot explain what causes it

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$$u_d = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \{\omega, \nu\} & 2 & 3 & 1 \end{array} \right.$$

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What would the politician implement:

- Without revelation $\mathfrak{c} = b$
- $\mathbb{E}[u_d] = 3, \mathbb{E}[u_e] = 2$

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What would the politician implement:

- With revelation: $\mathfrak{c}' : \begin{cases} \omega \mapsto a \\ \nu \mapsto b \end{cases}$
- $\mathbb{E}[u_d] = 5, \mathbb{E}[u_e] = 1$; So the expert won't reveal.

$$u_d = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$

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- But, $\mathfrak{c}^\star : \begin{cases} \omega \mapsto c \\ \nu \mapsto b \end{cases}$ is a Pareto improvement over no revelation
- $\mathbb{E}[u_d] = 4, \mathbb{E}[u_e] = 3$

Example

- The Pareto improvement $\mathfrak{c}^{\star}$, requires revelation
- But revealing allows the politician to exploit the expert
- What if the politician could commit:
  - Propose $\mathfrak{c} = b$ (his outside option)
  - After the expert reveals, propose some other contract $\mathfrak{c}^{\dagger}$
  - $\mathfrak{c}^{\dagger}$ only get implemented if the expert agrees; else $\mathfrak{c} = b$

Example

Internalizing this, the politician solves:

$$\max_{\mathfrak{c}^\dagger : \Omega \to \mathcal{A}} \mathbb{E}[u_d(\mathfrak{c}^\dagger)] \qquad \text{subject to} \qquad \mathbb{E}[u_e(\mathfrak{c}^\dagger)] \geq \mathbb{E}[u_e(\mathfrak{c})] \qquad \text{(IC)}$$

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- The solution is $\mathfrak{c}^\star : \begin{cases} \omega \mapsto c \\ \nu \mapsto b \end{cases}$

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Does this always work? No

Example

What if the politician is initially unaware of action b?

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What would the politician implement:

- Without revelation $\mathfrak{c}^* = a$
- $\mathbb{E}[u_d] = 2, \mathbb{E}[u_e] = 0$

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What would the politician implement:

- Under full revelation (same as before): $\mathfrak{c}' : \begin{cases} \omega \mapsto a \\ \nu \mapsto b \end{cases}$
- $\mathbb{E}[u_d] = 5, \mathbb{E}[u_e] = 1$; this satisfies the incentive constraint.

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- But, revealing only b leads to $\mathfrak{c} = b$
- $\mathbb{E}[u_d] = 3, \mathbb{E}[u_e] = 2$, partial revelation is preferred

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◇ As before, $\mathfrak{c}^\star : \begin{cases} \omega \mapsto c \\ \nu \mapsto b \end{cases}$ is a Pareto improvement over $\mathfrak{c} = b$

What if the procedure was repeated?

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(5) The decision maker solves

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- $\mathfrak{c}^\star$ is the solution

Does this always work?

Does this always work? Yes, but what is ‘always’?

Model

Hypothetical State-Space

Call $h = (W, A, (v_d, v_e), p)$ a hypothetical states-space,

- W is a finite set of states
- A is a set of actions
- $v_i : W \times A \to \mathbb{R}$, for $i \in \{d, e\}$, determines payoffs, and,
- $p \in \Delta(W)$ is a probability over states

Let $\mathscr{H}$ collect all hss; $\mathscr{H}$ are the possible types of expert

Say that $h' = (W', A', (v'_d, v'_e), p')$ **refines** $h = (W, A, (v_d, v_e), p)$:

- $A \subseteq A'$
- Each state in h corresponds to an event in h' such that
  - probabilities aggregate
  - expected utilities of h 'measurable' acts are invariant

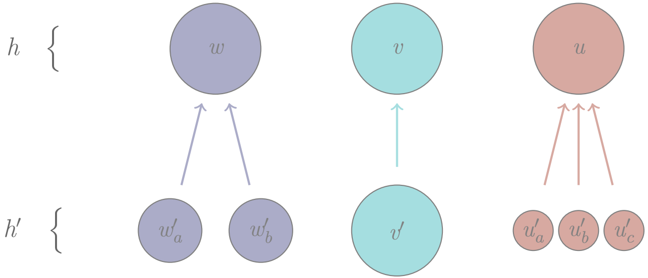
Then write: $h \preccurlyeq h'$.

Formally, $h \preccurlyeq h'$ if $A \subseteq A'$ and there exists a surjection $q : W' \to W$ such that

$$\sum_{w' \in q^{-1}(w)} p'(w') = p(w) \qquad \text{for all } w \in W \tag{H1}$$

and

$$\sum_{w' \in q^{-1}(s)} w_i'(w', \cdot) p'(w') = v_i(w, \cdot) \qquad \text{for all } w \in W \text{ and } i \in \{d, e\} \tag{H2}$$



h
h'
w
v
u
w'_a
w'_b
v'
u'_a
u'_b
u'_c

Outcome Profiles

An **outcome profile** is a function from types to contracts:

$$
\begin{array}{cccc}
f: & h & \mapsto & \mathfrak{c} \\
 & \cap & & \cap \\
 & \mathscr{H} & \to & A^W
\end{array}
$$

Call and outcome profile **incentive compatible** if the experts payoff is monotone in her type

$$\mathbb{E}_{p'}\left[v'_e(w, f(h'))\right] \geq \mathbb{E}_p\left[v_e(w, f(h))\right]$$

for any h' such that $h \preccurlyeq h'$

- There need not be any ‘strategic’ way of enacting an outcome profile.
- But if there is, it *must* be incentive compatible.

Iterated Revision Mechanism

An *iterated revelation mechanism* (IRM), is parameterized by a function from sequences of types to contracts:

$$\alpha : (h_0 \dots h_n) \mapsto (\mathfrak{c} : W_n \to A_n)$$

(1) Set $n = 0$. The decision maker proposes $\mathfrak{c}_0 = \alpha(h_0)$.

(2) The expert reveals h_{n+1}.
   - If $h_n \prec h_{n+1}$, the decision maker proposes $\mathfrak{c}_{n+1} = \alpha(h_{n+1})$. Set $n = n + 1$ and repeat step 2.
   - Otherwise, continue to step 3.

(3) The mechanism is over and the expert selects from $\{\mathfrak{c}_0, \dots \mathfrak{c}_n\}$.

Full Revelation

Theorem

The following are equivalent for an outcome profile f

(1) f can be implemented by an IRM
(2) f is a incentive compatible

The can be seen as an impossibility result:

- Without commitment to leave proposed contracts on the table, full revelation cannot be obtained.

Each proposed contract in an IRM specifies:

(1) The outcome should the game end
   - dm wants to maximize his own payoff

(2) The implicit incentive constraint should the game continue
   - dm wants to minimize the expert's payoff

In the examples, contracts solved (1) ignoring (2)

- The dm cannot conceive of what the expert is aware of
- It seems prudent, therefore, to consider *robust* strategies: those that maximize the worst case outcome
- Robust strategies turn out to be exactly those that follow the principle of myopic optimization

Robustness

Call an IRM, α, **robust** if at every sequence of (partial) revelations $\sigma = (h_0, \ldots h_n)$:

- α maximizes the worst case payoff over all best responses that extend σ.
- relative to any other α' that coincides with α over σ

Robustness

Theorem

The following are equivalent (up to the implemented outcome profile)

(1) α is robust

(2) α is myopically optimal: at each sequence $(h_0, \ldots, h_n)$,

$$\alpha(h_0, \ldots, h_n) \in \operatorname*{argmax}_{\mathfrak{c}\colon W \to \mathbb{R}} V_d(h, \mathfrak{c}) \qquad \text{subject to}$$

$$V_e(h_n, \mathfrak{c}) \geq V_e(h_n, \alpha(h_0 \ldots h_{n-1}))$$

The Designers Problem

More generally, often awareness is decentralized:

- A designer wants the decision maker to take some action
- The designer does not know the dm's or the expert's awareness
- A **mechanism** elicits awareness and returns an action recommendation

Mechanism

A mechanism is a mapping from pairs of types into contracts:

$$\mathcal{M} : (h^d, h^e) \mapsto \mathcal{M}(h^d, h^e)$$

where $\mathcal{M}(h^d, h^e) : W^e \to \mathcal{A}^e$

Desiderata:

INDIVIDUAL RATIONALITY: the dm can not do better alone (there is no constraint for the expert)

INCENTIVE COMPATIBILITY: i prefers to report h^i than any $h \prec h^i$

PARETO OPTIMALITY: there is no feasible contract that dominates the outcome of the mechanism

These are all **ex-post** restrictions — they must hold for all type realizations

Consider the mechanism, $\mathcal{M}^{IRM}$, that implements a myopically optimal IRM.

Theorem

The mechanism $\mathcal{M}^{IRM}$

- is individually rational, incentive compatible, and Pareto optimal, and,
- V_d-dominates any other such mechanism (point-wise over $\mathscr{H}$)

- There is a ‘dual’ IRM that implements the V_e-dominant mechanism

Thank You!