ITERATED REVELATION:

HOW TO INCENTIVIZE EXPERTS TO REVEAL NOVEL ACTIONS

Evan Piermont
Royal Holloway, University of London

Time, Uncertainties & Strategies X

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- Many economic models (cheap talk, persuasion, delegation, etc) find a decision maker seeking the council of an expert.

- Almost always: expert provides *statistical* info about the resolution of uncertainty

- But, there is another common motivation: to learn new actions.

Decision Maker	Expert	Information	Novel Actions
investor	analyst	economic forecast	assets, firms, strat.
politician	scientific advisor	climate forecast	technologies
Ph.D student	supervisor	pred. of success	research ideas
homeowner	architect	???	house design

Example

- An investor (the decision maker) is trying to invest his wealth:
  - harbors uncertainty about eventual state of the economy
  - can only invest in assets he is aware of
  - choice determines payoffs for both players
- He can enlist the help of a financial advisor (the expert) who
  - may have additional information about the state
  - may be aware of novel assets

Verifiability

Revelations (novel assets)

- verifiable
- “You can invest in `NVIDIA`”
- ex-ante uncontactable

Signals (about the state)

- unverifiable / cheap talk
- “There will be a recession next year”

Why is the interesting?

- When preferences are not aligned, ex might strategically conceal some facets of the problem
- Can dm do anything to incentivize revelation?
- Direct mechanisms do not exist!
  - "*If you reveal the existence of* `NVIDIA`*, I will invest in it*" is not allowed, and probably nonsensical

Literature

- Incomplete Contracting / Unawareness in Contracting
  - Grossman and Hart (1986); Maskin and Tirole (1999); Tirole (2009); Hart (2017); Piermont (2017); Lei and Zhao (2021); Francetich and Schipper (2021)
- Evidentiary disclosure
  - Dye, 1985; Green and Laffont, 1986; Grossman and Hart, 1986; Bull and Watson, 2007; Ben-Porath et al., 2019
- Strategic Information Transmission
  - Milgrom (1981), Crawford and Sobel (1982); Seidmann and Winter (1997); Aumann and Hart (2003); Chakraborty and Harbaugh (2010)
- Robust Mechanism Design
  - Bergemann and Morris (2005); Jehiel et al., (2006); Carroll (2015, 2019).

Example A

- No uncertainty about payoffs (i.e., the state)
  - each asset given by (x_d, x_e); x_d is dm's payoff, x_e is ex's
- The dm is initially aware of two assets:

$$x = (2, 2) \qquad y = (0, 0)$$

- The ex is also aware of:

$$a = (3, 3) \qquad b = (4, 1)$$

Example A

- If the ex had full control over what to reveal: simply reveal $a = (3, 3)$
- However, not all assets can be independently revealed:
  - Revealing one asset in a class reveals the existence of the whole class, etc
- What if a and b must be revealed together?

Example A

$$x = (2, 2)$$
$$y = (0, 0)$$

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$$y = (0, 0)$$

Example A

$$* \, x = (2,2) \, *$$
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$$\downarrow$$

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Example A

- The expert would choose not to reveal. This is Pareto Inefficient
- What if dm and ex can create the following contract (before revelation):
  - agree to an 'outside option' (that the dm is aware of)
  - this can be re-negotiated after revelation
  - the dm can propose a new action, but the ex can veto (therefore implement outside option)

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Does this always work? No

Example B

- What if the dm was also initially unaware of x
- $\{x\}$ and $\{a, b\}$ can be revealed independently

Example B

$$*y = (0, 0)*$$

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Model

The environment is described by

$\mathcal{A}$ — a (for talk, finite) set of actions

Ω — a state-space

π — a probability over Ω (for now, no private information)

(u_d, u_e) — state-dependent utility functions $\mathcal{A} \times \Omega \to \mathbb{R}$

Let V_i denote expectation operator w.r.t u_i and π

Revelation Types

- Let $\mathcal{R}$ be a collection of subsets of $\mathcal{A}$ such that
- A **revelation type** $r \in \mathcal{R}$ is a set of actions
- Say that r is **more expressive** than r', if $r' \subseteq r$
- Fix types r_d and r_e, and assume $r_d \subseteq r_e$.

Outcome Profiles

An **outcome profile** is a function from types to actions:

$$\begin{array}{rccc} f: & r & \mapsto & a \\ & \Cap & & \Cap \\ & \mathcal{R} & \to & r \end{array}$$

Call f **monotone** if ex's payoff is monotone in her type

$$V_e(f(r')) \leq V_e(f(r)) \tag{1}$$

whenever $r' \subseteq r$, and **strongly monotone** if in addition (1) holds strictly whenever $f(r) \neq f(r')$.

- If direct mechanisms existed, monotonicity is necessary
- Direct mechanisms don't exist: even with monotonicity, there need not be any 'strategic' way of enacting an outcome profile.

Iterated Revelation Protocol

INITIAL STEP — The decision maker announces $r_0 \in \mathcal{R}$, and proposes $p_0 \in r_0$.

ITERATIVE STEP — Given $(r_0, \ldots, r_n)$ distinct prior revelations, the expert reveals $r_{n+1} \in \mathcal{R}$.

- If $r_n \subsetneq r_{n+1}$, the decision maker proposes $p_{n+1} \in r_{n+1}$, and the ITERATIVE STEP is repeated.
- Otherwise, the protocol moves to the FINAL STEP

FINAL STEP — Given $(r_0, \ldots, r_n)$ distinct revelations, the expert chooses an action $a \in \bigcup_{m \leq n} p_m$.

Importantly:

- This protocol can be explained / contracted to without having to express any specific actions/outcomes
- Specifically, the only contractual obligations in an IRP are actions that *have already been* revealed.

Strategies

Given the IRP, a **strategy**

- for the dm is a function from *sequences of revelations* to actions:

$$\mathfrak{m} : (r_0 \dots r_n) \mapsto a \in r_n$$

- for the ex is a function from *sequences of proposals* to revelations:

$$\sigma : (p_0 \dots p_n) \mapsto r_{n+1} \in \mathcal{R}$$

(and a choice out of the final set of proposals)

Implementation

Let $\mathfrak{m}(\sigma)$ denote the action enacted by playing strategies $\mathfrak{m}$ and σ.

Say that $\mathfrak{m}$ **implements** the outcome profile f if for all $r \in \mathcal{R}$

$$f(r) = \mathfrak{m}(\sigma) \quad \text{for some best response for type } r$$

and **fully implements** f

$$f(r) = \mathfrak{m}(\sigma) \quad \text{for every best response for type } r$$

Full Revelation

Theorem

The following are equivalent for an outcome profile f

(1) f is monotone (resp. strongly monotone)

(2) there exists some $\mathfrak{m}$ that implements f, (resp. fully implements)

Each proposal in an IRP specifies:

(1) The outcome should the game end
   - dm wants to maximize his own payoff

(2) The implicit incentive constraint should the game continue
   - dm wants to minimize ex's payoff

In the examples, IRPs solved (1) ignoring (2)

Definition

Call a strategy $\mathfrak{m}$

- **locally rational** if for all $(r_0 \dots r_n)$, there is no $a \in r_n$ such that

$$V_e(\mathfrak{m}(r_0 \dots r_{n-1})) \leq V_e(a) < V_e(\mathfrak{m}(r_0 \dots r_n)) \quad \text{and} \quad V_d(\mathfrak{m}(r_0 \dots r_n)) < V_d(a)$$

- **locally optimal** if for all $(r_0 \dots r_n)$, there is no $a \in r_n$ such that

$$V_e(\mathfrak{m}(r_0 \dots r_{n-1})) \leq V_e(a) \quad \text{and} \quad V_d(\mathfrak{m}(r_0 \dots r_n)) < V_d(a)$$

Definition

Call a monotone outcome profile f

- **undominated** if for all $r \in \mathcal{R}$, there is no other monotone outcome profile f' such that

$$\begin{aligned} V_d(f(r')) &\leq V_d(f'(r')) && \text{for all } r' \supseteq r, \\ V_d(f(r')) &< V_d(f'(r')) && \text{for some } r' \supseteq r \end{aligned}$$

- **cautious** if for all $r \in \mathcal{R}$, there is no other monotone outcome profile f' such that

$$\inf_{r' \supseteq r} V_d(f(r')) < \inf_{r' \supseteq r} V_d(f'(r'))$$

Theorem

An outcome profile f is

(1) cautious if and only if it is implemented by a locally optimal $\mathfrak{m}$.

(2) undominated if and only if it is implemented by a locally rational $\mathfrak{m}$.

- ‘if’ direction requires a richness condition on $\mathcal{R}$

Payoff Uncertainty

- The implementation above presupposes dm can anticipate ex's acceptance / rejection
- What happens with private information:
  - assume ex knows the state, $\omega \in \Omega$
  - dm does not, entertains prob π

Example C

- $\Omega = \{\omega_L, \omega_R\}$, ex knows the state, dm believes equally likely
  - Each asset is therefore given by $(\langle x_{d,L}, x_{d,R}\rangle, \langle x_{e,L}, x_{e,R}\rangle)$.
- The dm is initially aware of one asset:

$$x = (\langle 1, 1\rangle, \langle 1, 1\rangle)$$

- The ex is also aware of:

$$a_L = (\langle 3, 0\rangle, \langle 3, 0\rangle) \qquad a_R = (\langle 0, 3\rangle, \langle 0, 3\rangle) \qquad b = (\langle 2, 2\rangle, \langle 2, 2\rangle)$$

- The only revelation type is $\{a_L, a_R, b\}$.

Example C

$$*x = \langle 1, 1 \rangle, \langle 1, 1 \rangle *$$

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Example C

- Preferences are completely aligned, but IRP does not allow delegation
- the protocol cannot use ex's private info.
  - this *creates* inefficiency
- Instead, dm chooses a **set of actions** $p_1 \subseteq r$. After revelation, propose

$$p_1 = \{a_L, a_R\}$$

and let the ex choose.

- A **generalized IRP** allows the dm to choose a set of actions at each step:
  - At each $(r_0 \dots r_n)$, $\mathfrak{m}(r_0 \dots r_n) \subseteq r_n$

- A **generalized outcome profile** is a function $f : \Omega \times \mathcal{R} \to \mathcal{A}$
  - For each $r \in \mathcal{R}$, $w \in \Omega$, we have $f(\omega, t) \in t$

Full Revelation

Theorem

The following are equivalent for a gen. outcome profile f

(1) f can be implemented by a gen. IRP

(2) f is monotone: for all $\omega, r \in \Omega \times \mathcal{R}$

$$V_e(f(\omega', r') \mid \omega) \leq V_e(f(\omega, t) \mid \omega)$$

for any other $\omega' \in \Omega$ and $r' \subseteq r$.

Also in the paper

- Examine a designer's problem
  - doesn't know either player's type
  - wants to be Pareto efficient
- Characterize all efficient mechanisms in terms of IRPs
- Relate payoff bounds to cautious outcome profiles

Conclusion

Thank You!