ITERATED REVELATION:

HOW TO INCENTIVIZE EXPERTS TO COMPLETE INCOMPLETE CONTRACTS

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Cast of Characters

Decision Maker (dm)

- Will take an action in the future
- Would like to condition the action on the resolution of uncertainty
- He is unaware/unable to express of some relevant aspects of the decision problem

Expert (ex)

- Has no right to the decision, herself
- But can reveal aspects of the environment

(we will also add a mechanism designer, later)

Writing legislation: politician; technical advisor

Research: Ph.D. student; supervisor

⋄ Investing: investor; financial expert

- An investor (the decision maker) is trying to invest his wealth:
 - the composition of the portfolio can be contingent on the future realized state-of-affairs, but
 - can depend only on those contingencies he is aware of
 - can only invest in assets he is aware of
- He can enlist the help of a financial advisor (the expert) who may reveal novel contingencies/assets

Contracts

- \diamond \mathcal{A} is a set of actions (assets)
- $\diamond \Omega$ is a state-space (state of economy)
- ♦ dm must choose a **contract**:

$$\mathfrak{c}:\Omega o\mathcal{A}$$

Contracts

- ⋄ Not all contracts are feasible. dm may be
 - unable to express
 - unaware of
 - technologically unable to implement/condition on

some actions or events in the state-space

- ex's revelations are
 - verifiable and voluntary
 - ex-ante uncontactable

Why is the interesting?

- When preferences are not aligned, ex might strategically conceal some facets of the problem
- Can dm do anything to incentivize revelation?
- A(n unaware) designer may not be able to solve the problem, if mechanisms depend on the unknowns

Literature

- Incomplete Contracting / Unawareness in Contracting
 - Grossman and Hart (1986); Maskin and Tirole (1999); Tirole (2009); Hart (2017); Piermont (2017); Lei and Zhao (2021); Francetich and Schipper (2021)
- Evidentiary disclosure
 - Dye, 1985; Green and Laffont, 1986; Grossman and Hart, 1986; Bull and Watson, 2007;
 Ben-Porath et al., 2019
- ♦ Strategic Information Transmission
 - Milgrom (1981), Crawford and Sobel (1982); Seidmann and Winter (1997); Aumann and Hart (2003); Chakraborty and Harbaugh (2010)
- Robust Mechanism Design
 - ♦ Bergemann and Morris (2005); Jehiel et al., (2006); Carroll (2015, 2019).

- \diamond The true state-space is $\Omega = \{\omega, \nu\}$; equally likely
- \diamond Set of actions $\mathcal{A} = \{\alpha, \beta, \gamma\}$
- \diamond dm must choose an contract $\mathfrak{c}:\Omega\to\mathcal{A}$
- \diamond Let $V_i(\mathfrak{c})$ denote the expected utility to player i

ex can distinguish the states, but dm cannot.

$$\mathscr{P}_{\mathbf{e}} = \big\{ \{\omega\}, \{\nu\} \big\} \qquad \qquad \mathscr{P}_{d} = \big\{ \{\omega, \nu\} \big\}$$

How does dm view payoffs in coarse states?

- Assume it is aggregated via expectations
- As if he correctly assesses randomness, but cannot contract it away because he
 - is unaware of what causes it, or
 - does not possess language describe it in a contract, or
 - does not have the technology to condition on it

$$u_{d} = \begin{cases} \begin{array}{c|c|c|c} \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} & u_{e} = \begin{cases} \begin{array}{c|c|c} \alpha & \beta & \gamma \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \\ \\ u_{d} = \begin{cases} \begin{array}{c|c|c} \alpha & \beta & \gamma \\ \hline \omega, \nu \} & 2 & 3 & 1 \end{array} & u_{e} = \begin{cases} \begin{array}{c|c|c} \alpha & \beta & \gamma \\ \hline \omega, \nu \} & 0 & 2 & 4 \end{array} \end{cases}$$

$$u_d = \begin{cases} \frac{\alpha}{\{\omega, \nu\}} & \frac{\beta}{2} & \frac{\gamma}{3} & 1 \end{cases} \qquad u_e = \begin{cases} \frac{\alpha}{\{\omega, \nu\}} & \frac{\beta}{2} & \frac{\gamma}{4} \end{cases}$$

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- Without revelation $\mathfrak{c} = \beta$
- $\diamond V_d(\mathbf{c}) = 3, V_e(\mathbf{c}) = 2$

$$\left\{ \begin{array}{c|ccccc}
 & \alpha & \beta & \gamma \\
\hline
 & \omega & 0 & 2 & 4 \\
\hline
 & \nu & 0 & 2 & 4
\end{array} \right.$$

$$\Rightarrow \text{ With revelation: } \mathfrak{c}' : \left\{ \begin{array}{l} \omega \mapsto \alpha \\ \nu \mapsto \beta \end{array} \right.$$

$$V_d(\mathbf{c'}) = 5$$
, $V_e(\mathbf{c'}) = 1$; So ex won't reveal.

$$u_d = \begin{cases} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \end{cases}$$

$$\diamond \; \mathsf{But}, \mathfrak{c}^\star : \left\{ \begin{matrix} \omega \mapsto \gamma \\ \nu \mapsto \beta \end{matrix} \text{ is a Pareto improvement over no revelation} \right.$$

$$\diamond V_d(\mathfrak{c}^{\star}) = 4, V_e(\mathfrak{c}^{\star}) = 3$$

- ⋄ The Pareto improvement c*, requires revelation
- ⋄ But revealing allows dm to exploit ex
- ♦ What if dm could commit:
 - ♦ Propose $\mathfrak{c} = \beta$ (his outside option)
 - \diamond After ex reveals, propose some other contract c^{\dagger}
 - $\diamond c^{\dagger}$ only get implemented if ex agrees; else $c = \beta$

Internalizing this, dm solves:

$$\max_{\mathfrak{c}^\dagger:\Omega\to\mathcal{A}}V_d(\mathfrak{c}^\dagger) \qquad \qquad \text{subject to} \qquad V_{\boldsymbol{e}}(\mathfrak{c}^\dagger) \geq V_{\boldsymbol{e}}(\mathfrak{c}) \tag{IC}$$

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(IC)

$$\diamond \text{ The solution is } \mathfrak{c}^* : \left\{ \begin{array}{l} \omega \mapsto \gamma \\ \nu \mapsto \beta \end{array} \right.$$



♦ full revelation

- ♦ an efficient contract

So a two stage game with commitment to not revoke the prior proposal resulted in

- ♦ full revelation
- ⋄ an efficient contract

Does this always work?

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- ♦ full revelation
- ♦ an efficient contract

Does this always work? No

What if dm is initially unaware of action β ?

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- Without revelation $c^* = \alpha$
- $\diamond V_d(\mathfrak{c}^*) = 2, V_e(\mathfrak{c}^*) = 0$

$$\left\{ \begin{array}{c|cccc}
 & \alpha & \beta & \gamma \\
\hline
 & \omega & 0 & 2 & 4 \\
\hline
 & \nu & 0 & 2 & 4
\end{array} \right.$$

- \diamond Under full revelation (same as before): $\mathbf{c}': \left\{ \begin{array}{l} \omega \mapsto a \\ \psi \mapsto \beta \end{array} \right.$
- $V_d(\mathbf{c}') = 5$, $V_e(\mathbf{c}') = 1$; this satisfies the incentive constraint.

$$u_d = \begin{cases} \begin{array}{c|cccc} \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \\ \end{array} \end{cases}$$

- But, revealing only β leads to $\mathfrak{c} = \beta$
- $V_d(\mathbf{c}) = 3$, $V_e(\mathbf{c}) = 2$; partial revelation is preferred

$$u_d = \begin{cases} & \alpha & \beta & \gamma \\ \hline \omega & 4 & 0 & 2 \end{cases}$$

$$\diamond$$
 As before, $\mathfrak{c}^{\star}: \left\{ egin{array}{l} \omega \mapsto \gamma \\ \nu \mapsto \beta \end{array} \right.$ is a Pareto improvement over $\mathfrak{c} = \beta$



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- (4) ex reveals more (the partition $\{\{\omega\}, \{\nu\}\}\)$

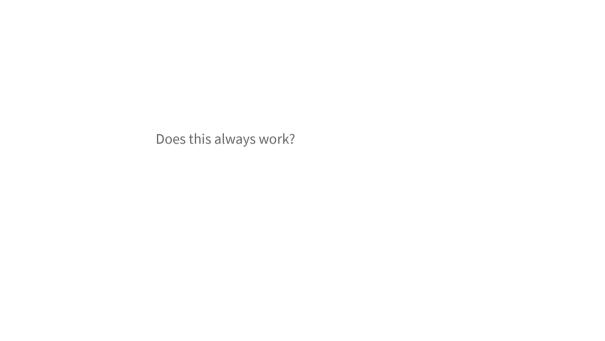
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- (5) dm solves

$$\max_{\mathfrak{c}^\dagger:\Omega o\mathcal{A}}V_d(\mathfrak{c}^\dagger)$$
 subject to $V_{m{e}}(\mathfrak{c}^\dagger)\geq V_{m{e}}(\mathfrak{c})$ (IC)

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$$\max_{\mathfrak{c}^{\dagger}:\Omega\to\mathcal{A}}V_d(\mathfrak{c}^{\dagger}) \qquad \text{subject to} \qquad V_e(\mathfrak{c}^{\dagger})\geq V_e(\mathfrak{c}) \qquad \text{(IC)}$$

⋄ c* is the solution



Does this always work? Yes, but what is 'always'?

Model

The environment is described by

A — a set of actions

 Ω — a state-space

 π — a probability over Ω

 (u_d, u_e) — state-dependent utility functions

Let V_i denote expectation operator w.r.t u_i and π

Types

- \diamond Let **T** be a collection of (compact) subsets of \mathcal{A}^{Ω}
- \diamond A **type** $t \in \mathbf{T}$ determines the set of expressible contracts
- \diamond Say that type t is **more expressive** than type t', if $t' \subseteq t$
- \diamond **T**(t) \subseteq **T** are those types expressive than t.
- \diamond Fix types t_d and t_e , and assume $t_e \in \mathbf{T}(t_d)$.

Example

Each t is given by

$$\mathscr{P}^t$$
 — a partition of Ω

$$A^t$$
 — a subset of \mathcal{A}

Then

$$t = \{\mathfrak{c} : \Omega \to \mathcal{A}^t \mid \mathfrak{c} \text{ is } \mathscr{P}^t \text{ measurable } \}$$

 $\diamond t' \subseteq t$ if and only if \mathscr{P}^t refines $\mathscr{P}^{t'}$ and $A^{t'} \subseteq A^t$

Outcome Profiles

An **outcome profile** is a function from types to contracts:

$$\begin{array}{cccc} f \colon & t & \mapsto & \mathfrak{c} \\ & & & & \cap \\ \mathbf{T}(t_d) & \to & t \end{array}$$

Call f monotone if ex's payoff is monotone in her type

$$V_{\boldsymbol{e}}(f(t)) \le V_{\boldsymbol{e}}(f(t')) \tag{1}$$

whenever $t \subseteq t'$, and **strongly monotone** if in addition (1) holds strictly whenever $f(t) \neq f(t')$.

- ♦ There need not be any 'strategic' way of enacting an outcome profile.
- ♦ But if there is, it *must* be monotone.

Iterated Revelation Mechanisms

An **iterated revelation mechanism** (IRM), is parameterized by a function from *sequences of types* to contracts:

$$\mathfrak{m}:(t_0\ldots t_n)\mapsto\mathfrak{c}\in t_n$$

STEP 1 — Set n = 0. dm announces $t_0 = t_d$, and proposes $\mathfrak{m}(t_0)$.

STEP 2 — ex reveals t_{n+1} .

- \diamond If $t_n \subseteq t_{n+1}$, dm proposes $\mathfrak{m}(t_{n+1})$, goto STEP 3.
- \diamond Otherwise, the mechanism is over and $\mathfrak{m}(t_n)$ get implemented.
- STEP 3 ex can accept or reject the proposal, $\mathfrak{m}(t_{n+1})$:
 - \diamond If she accepts, set n = n + 1 and goto STEP 2.
 - \diamond If she rejects, the mechanism is over and $\mathfrak{m}(t_n)$ get implemented.

	The only commitment is to the current outside option
	the contracts proposed by an IRM are jointly expressible at the time of

Importantly:

proposal

Full Revelation

Theorem

The following are equivalent for an outcome profile f

- (1) f can be implemented by an IRM
- (2) fis monotone

 \diamond Implemented: $f(t) = \mathfrak{m}(\sigma)$ where σ is a *best response* over all expressible sequences for type t.

The can be seen as an impossibility result:	
 Without commitment to leave proposed contracts on the table, full revelation cannot be guaranteed. 	

Full Revelation

Theorem

The following are equivalent for an outcome profile f

- (1) f can be fully implemented by an IRM (i.e., is the unique outcome)
- (2) f is a strongly monotone

Each proposed contract in an IRM specifies:

- (1) The outcome should the game end
 - dm wants to maximize his own payoff
- (2) The implicit incentive constraint should the game continue
 - dm wants to minimize ex's payoff

In the examples, contracts solved (1) ignoring (2)

If dm cannot conceive of what ex is aware of it seems prudent to consider *robust* strategies:

- those that maximize the worst case outcome
- this is belief free: does not require conjecturing about probability of types
- Robust strategies turn out to be exactly those that follow the principle of myopic optimization

Robustness

Call an IRM, \mathfrak{m} , **robust** if at every sequence of (possibly partial) revelations σ , \mathfrak{m} maximizes the worst case payoff over

- \diamond all best responses that extend σ .
- \diamond for types for which σ would have been rational
- \diamond compared to any other \mathfrak{m}' that coincides with \mathfrak{m} over σ

Robustness

Theorem

The following are equivalent (up to the implemented outcome profile)

- (1) m is robust
- (2) \mathfrak{m} is myopically optimal: at each sequence (t_0, \ldots, t_n) ,

$$\mathfrak{m}(t_0,\ldots,t_n)\in \mathop{\sf argmax}\limits_{\mathfrak{c}\in t_n}V_d(\mathfrak{c})$$
 subject to $V_e(\mathfrak{c})\geq V_e(\mathfrak{m}(t_0\ldots t_{n-1}))$

The Designers Problem

- A designer wants the decision maker to take some action
- ♦ The designer knows *neither* dm's nor ex's type
- ♦ A **mechanism** elicits types and returns a contract

Mechanism

A **mechanism** is a mapping from pairs of types into contracts:

$$\mathcal{M}:(t_d,t_{\mathbf{e}})\mapsto \mathcal{M}(t_d,t_{\mathbf{e}})$$

where $\mathcal{M}(t_d, t_e) \in t_e$

 $\diamond~$ It common knowledge that $t_d \subseteq t_{e}$

Desiderata:

INDIVIDUAL RATIONALITY: dm can not do better alone (there is no constraint for ex)

INCENTIVE COMPATIBILITY: i prefers to report t_i than any $t \subsetneq t_i$

PARETO OPTIMALITY: there is no feasible contract that dominates the outcome of the mechanism

These are all **ex-post** restrictions — they must hold for all type realizations

Fixing t_d , a mechanism determines an outcome profile:

$$f^{t_d}: t \mapsto \mathcal{M}(t_d, t)$$

By incentive compatibility, f^{t_d} is monotone, thus can be implemented by an appropriate IRM.

Consider the mechanism, \mathcal{M}^{MO} , that implements a myopically optimal IRM:

(1) first, the decision maker reveals $t \in \mathbf{T}$

- (2) then we run a myopically optimal IRM, \mathfrak{m}^t :
 - \diamond starting from t
 - \diamond multiple m.o. contracts \Rightarrow break ties in favor of the expert

Theorem

The mechanism \mathcal{M}^{MO}

- is individually rational, incentive compatible, and Pareto optimal, and,
- \diamond for any other such mechanism \mathcal{M} ,

$$V_d(\mathcal{M}^{MO}(t,t')) \ge V_d(\mathcal{M}(t,t'))$$

for all $t, t' \in \mathbf{T}$ with $t \subseteq t'$.

 \diamond There is a 'dual' IRM that implements the V_e -dominant mechanism

Distributed Awareness

What if we relax the assumption that $t_d \subseteq t_e$?

Theorem

Allowing for distributed awareness, there exists no incentive compatible and Pareto optimal mechanism.

Let $\Omega = \{\omega\}$ and everything else defined by

$$egin{array}{c|c} u_d & lpha & eta & lpha \ \hline u_d & 0 & 1 & 2 \ u_e & 0 & 2 & 1 \ \hline \end{array}$$

 \diamond each type is associated with a subset of $\{\alpha, \beta, \gamma\}$

 \diamond Let \mathcal{M} be any Pareto optimal mechanism. This requires

 $\mathcal{M}(\{\alpha\}, \{\alpha, \gamma\}) = \gamma$ $\mathcal{M}(\{\alpha, \beta\}, \{\alpha\}) = \beta$ $\mathcal{M}(\{\alpha, \beta\}, \{\alpha, \gamma\}) \in \{\beta, \gamma\}$

 \diamond if $\mathcal{M}(\{\alpha,\beta\},\{\alpha,\gamma\}) = \gamma$, then ex of type $\{\alpha,\gamma\}$ misreports as $\{\alpha\}$,

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