IMAGE CONSCIOUS PREFERENCES

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When a DM takes an action:

- It results in a physical outcome:
  - Food, car, education, entertainment, ...
- It releases information about the motivation of the DM:
  - This DM is charitable, fashionable, erudite, ...

An image conscious DM cares not only about the physical consequences of his actions, but also about his image—how his actions are perceived by observers.

An image is the set of preferences consistent with the observed choice.

- Depends on the choice constraints.
    - Is a ‘revealed’ preference exercise. The best we can do with the observables.
- DM believes observer is naive; does not anticipate manipulation.
    - For tractability, but in line with psychology evidence.

Example

Tyrone Slothrop is deciding where to take Katje on a date. There are three restaurants, equal in all ways excepting their wine lists:

- D^l offers only low quality (l)
- D^m offers l and also a mid-tier (m)
- D^h offers l and m and also a high-tier (h)

Example

Despite the fact that Slothrop is an absolute cheapskate, he wishes to appear generous and refined:

- at D^m, figuring it worth the small expense to impress Katje, he would publicly choose m.
- at D^h, Slothrop would choose l: the cost of sending a signal of refinement—choosing h—is now too high.

Example

Anticipating this, Slothrop chooses to patronize D^m.

- At D^m or D^h he can manipulate his image by making a discriminating choice.
- At D^m the cost of effecting a positive image is lower.

Example

There is a connection between choice $\begin{cases} \textit{from} \text{ menus} \\ \textit{between} \text{ menus} \end{cases}$

- At D^m, Slothrop could have chosen l
- This would result in the same consumption and same image as choosing l from D^h.
- D^m is revealed preferred to D^h

Dual Self

The ‘observer’ need not be an external entity.

- DM cares about his self image.
- Oddly, this seems manipulable.
  - Gino et al. (2016); Grossman and Van Der Weele (2017).

The Model

Two stage choice:

1. (Private) choice over menus.
   - Choosing a restaurant.

2. (Public) choice from the menu chosen in stage 1.
   - Choosing the bottle of wine.

Another Model

If first stage choices are *private* how does the modeler assess them? Later, I consider only second stage choice.

- Consumption takes place in $\mathbb{R}^n$.
  - $x \in \mathbb{R}^n$ referred to an action or consumption object.
- $\mathcal{D}$ = finite non-empty subsets of $\mathbb{R}^n$,
  - $D \in \mathcal{D}$ referred to as stage 2 choice problem (2CP).
- $\mathscr{M}$ = finite non-empty subsets of $\mathcal{D}$,
  - $\mathcal{M} \in \mathscr{M}$ referred to as stage 1 choice problems (1CPs).

Our primitive is a pair $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$,

- $\mathscr{C}_1$ is a choice function over $\mathscr{M}$
- $\mathcal{C}_2$ is a choice function over $\mathcal{D}$

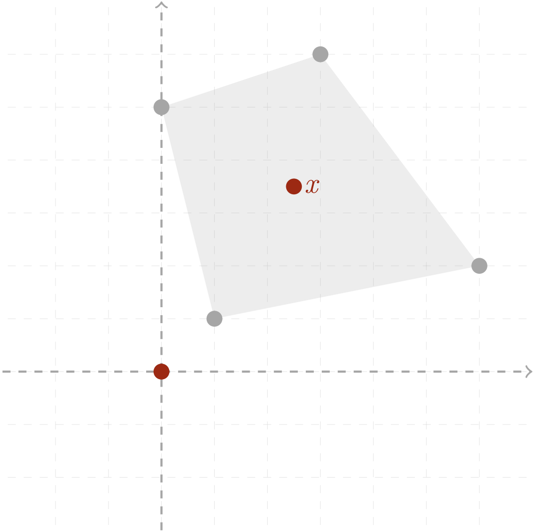
- $u \in \mathbb{R}^n$ defines a linear representation over $\mathbb{R}^n$:

$$u(x) = \sum_{i=1}^{n} u^i x^i.$$

- Call $I \subset \mathbb{R}^n$ an *image* if it is convex and $\lambda I \subseteq I$ for all $\lambda > 0$.
    - Let $\mathbb{I}$ denote the set of all images.

For $D \in \mathcal{D}$ and $x \in D$ let

$$I_D^x = \{u \in \mathbb{R}^n \mid u(x) \geq u(y), \text{ for all } y \in D\}.$$



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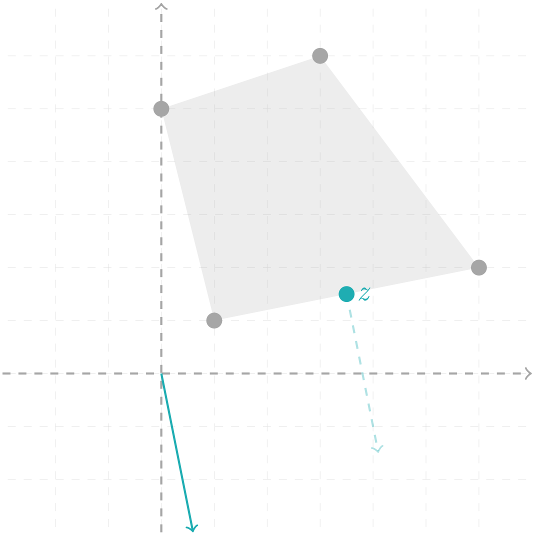

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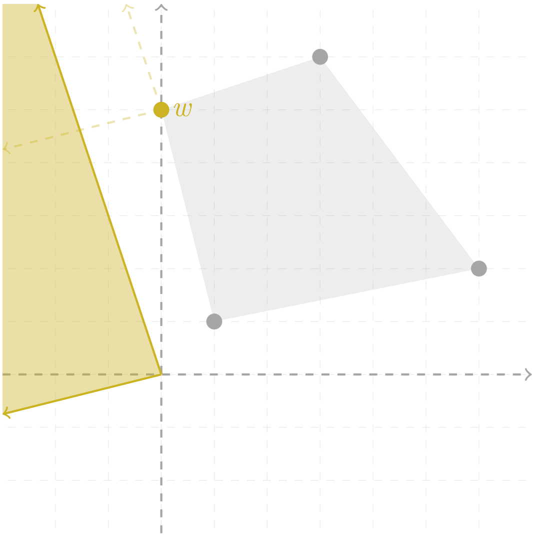

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An image conscious representation of $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is a pair $\langle u, \Gamma \rangle$ where $u \in \mathbb{R}^n$ and $\Gamma : \mathbb{I} \to \mathbb{R} \cup \{-\infty\}$ such that

$$\mathcal{C}_2(D) = \arg\max_{x \in D} \big(u(x) + \Gamma(I_D^x) \big) \quad \text{and} \tag{C2}$$

$$\mathscr{C}_1(\mathcal{M}) = \arg\max_{D \in \mathcal{M}} \Big(\max_{x \in D} \big(u(x) + \Gamma(I_D^x) \big) \Big), \tag{C1}$$

for all $D \in \mathcal{D}$ and $\mathcal{M} \in \mathscr{M}$.

Axiom 1—Stage 1 WARP

If $D, D' \in \mathcal{M} \cap \mathcal{M}'$, $D \in \mathscr{C}_1(\mathcal{M})$ and $D' \in \mathscr{C}_1(\mathcal{M}')$ then $D \in \mathscr{C}_1(\mathcal{M}')$.

- There is no image effect in the first stage.
- Consistency between choices.

Axiom 2—Weak Continuity

For all $D \in \mathcal{D}$, $UC(D) \cap \mathcal{D}^1$ and $LC(D) \cap \mathcal{D}^1$ are closed and non-empty.

- $UC(D)$ is the upper contour set of D.
- $\mathcal{D}^1$ are single element menus.

Axiom 3—Translation Invariance

For all $x \in \mathbb{R}^n$, $\mathcal{M} \in \mathscr{M}$ and $D \in \mathcal{D}$,

$$\mathscr{C}_1(\mathcal{M} + \{x\}) = \mathscr{C}_1(\mathcal{M}) + \{x\} \text{ and}$$
$$\mathcal{C}_2(D + x) = \mathcal{C}_2(D) + x$$

Axiom 4—Singleton Homogeneity

For all $\lambda \in \mathbb{R}_{++}$, $\mathcal{M} \subset \mathcal{D}^1$,

$$\mathscr{C}_1(\lambda \mathcal{M}) = \lambda \mathscr{C}_1(\mathcal{M}).$$

- No image concerns in singleton menus.

Lemma

If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfies A1-4 then there exists a value function, $V : \mathcal{D} \to \mathbb{R}$ such that

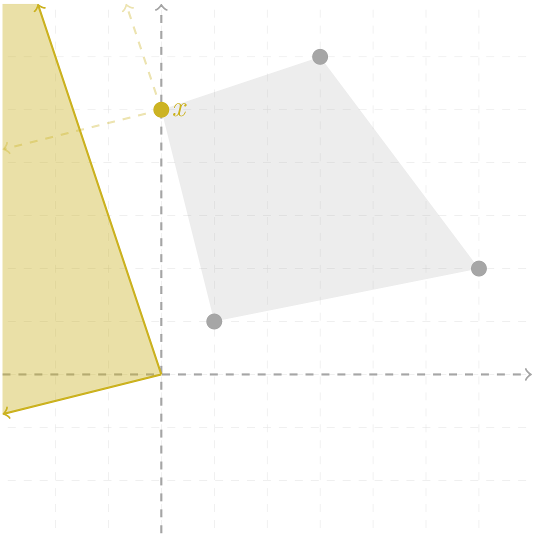
$$\mathscr{C}_1(\mathcal{M}) = \left\{ D \in \mathcal{M} \mid D \in \arg\max_{\mathcal{M}} V(D) \right\},$$

and, $u : x \mapsto V(\{x\})$ is a linear function over $\mathbb{R}^n$.

- Look at the restriction of $\mathscr{C}_1$ to D^1: satisfies EU axioms.
- Show that $UC(D) \cap LC(D) \cap D^1$ is non-empty for all $D \in \mathcal{D}$.

x is **more revealing** for D than D', written $D \overset{x}{\triangleright} D'$, if

1. we have $x \in D \cap D'$
2. $\mathrm{conv}(\alpha D' + (1-\alpha)x) \subseteq \mathrm{conv}(D)$.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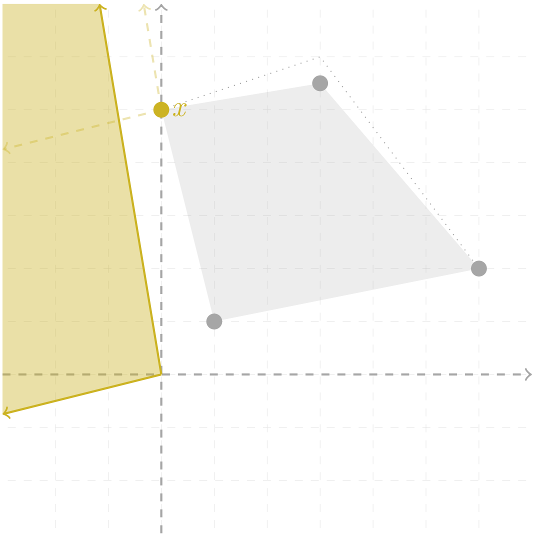
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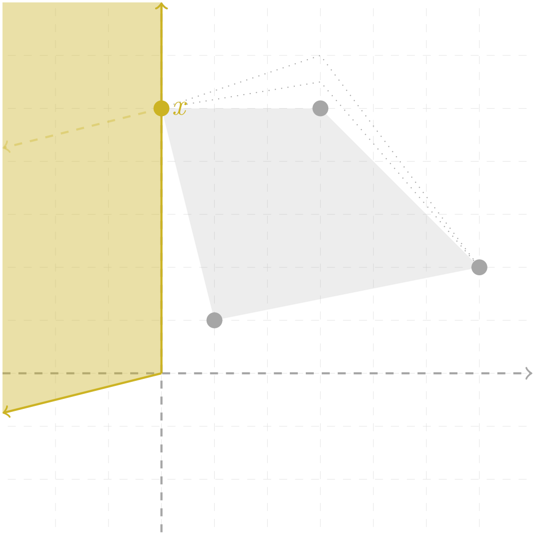

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x is equally revealing for D than D', written $D \overset{x}{\bowtie} D'$ if

$$D \overset{x}{\triangleright} D' \text{ and } D' \overset{x}{\triangleright} D$$

Axiom 5—Image Consistency

Let $D, D^{'} \in \mathcal{D}$ with $D \overset{x}{\bowtie} D^{'}$ be such that $x \in \mathcal{C}_2(D)$. Then $x \in \mathcal{C}_2(D^{'})$ if and only if $D \in \mathscr{C}_1(\{D, D^{'}\})$.

- Revealed preference.
- Recall Slothrop and his wine selection.

Theorem

The following are equivalent:

1. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfies A1-5,
2. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ has an image conscious representation $\langle u, \Gamma \rangle$.

Moreover, u is unique up to positive linear translations, and Γ is unique up to an additive constant on its effective domain.

Proof

For each $I \in \mathbb{I}$, define $\Gamma(I)$ to be

$$\Gamma(I) = \begin{cases} V(D) - u(x) & \text{if there exists } (D, x) \text{ with } \begin{cases} x \in \mathcal{C}_2(D) \\ I_D^x = I \end{cases} \\ -\infty & \text{otherwise.} \end{cases}$$

Proof: Γ is well defined

Let (D, x) and (D', x') be such that $I_D^x = I_{D'}^{x'}$ and $x \in \mathcal{C}_2(D)$ and $x' \in \mathcal{C}_2(D')$.

- WLOG $x = x'$ by Translation Invariance.
- $I_D^x = I_{D'}^x$ implies $D \overset{x}{\bowtie} D'$.
- By Image consistency and the prior Lemma, $V(D) = V(D')$:

$$V(D) - u(x) = V(D') - u(x).$$

Proof: $\langle u, \Gamma \rangle$ represents $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$

Let $x \in \mathcal{C}_2(D)$, and $y \in D$.

- Since x is chosen, $\Gamma(I_D^x) \neq -\infty$.
- If $\Gamma(I_D^y) = -\infty$, then $u(x) + \Gamma(I_D^x) > u(y) + \Gamma(I_D^y)$.
- If not, there exists a D' such that $y \in \mathcal{C}_2(D')$ and $I_{D'}^y = I_D^y$
  - By Image consistency, $V(D) \geq V(D')$.
- $u(x) + \Gamma(I_D^x) = V(D) \geq V(D') = u(y) + \Gamma(I_D^y)$

On Privacy

Call a DM, $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, privacy seeking if for all $D, D' \in \mathcal{D}$ with $D' \overset{x}{\rhd} D$: If $x \in \mathcal{C}_2(D')$ then $D \in \mathscr{C}_1(\{D, D'\})$.

- Choosing x from D' induces the same consumption and reveals more.
- If DM prefers privacy, this is worse than choosing x from D.

On Privacy

Call a DM, $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, privacy averse if for all $D, D' \in \mathcal{D}$ with $D \overset{x}{\triangleright} D'$: If $x \in \mathcal{C}_2(D')$ then $D \in \mathscr{C}_1(\{D, D'\})$.

- Choosing x from D' induces the same consumption and reveals less.
- If DM wants to transfer information, this is worse than choosing x from D.

On Privacy

Theorem

1. If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is privacy seeking then $D \in \mathscr{C}_1\{D, D'\}$ implies

$$D \in \mathscr{C}_1\{D, D \cup D'\}$$

2. If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is privacy averse then

$$D \cup D^{'} \in \mathscr{C}_1\{D, D \cup D'\}$$

What if $\mathscr{C}_1$ is unobservable. Because either

- The DM cannot choose constraints; it is degenerate.
- The DM makes choices privately, so the modeler cannot observe.

Idea: Recall in the example, WARP violations in stage 2 indicated image concerns.

Problem: Geometric dependence between menu and images—highly incomplete preferences.

Say $I, J \in \mathbb{I}$ are **directly comparable** if there is a D such that $I = I_D^x$, $J = I_D^y$ and $x \in \mathcal{C}_2(D)$

- We know:
$$\Gamma(I) - \Gamma(J) \geq u(y) - u(x).$$
- We can bound $\Gamma(I) - \Gamma(J)$ for all I, J in the transitive closure of the 'directly comparability' relation.
- This is an equivalence relation. It will turn out we can independently normalize over each class.

Axiom 1°—Scale Acyclicity

Let $0 < \lambda^1 < \lambda^2 < \lambda^3$ and $D \in \mathcal{D}$. If $x \in \frac{1}{\lambda^1}\mathcal{C}_2(\lambda^1 D) \cap \frac{1}{\lambda^3}\mathcal{C}_2(\lambda^3 D)$ then $x \in \frac{1}{\lambda^2}\mathcal{C}_2(\lambda^2 D)$.

- As λ gets bigger, consumption utility matters more.
- This is monotone.

In the limit (well defined by Scale Acyclicity) only consumption matter.

$$\mathcal{C}_2^\infty : D \mapsto \lim_{\lambda \to \infty} \frac{1}{\lambda} \mathcal{C}_2(\lambda D).$$

Axiom 2°—Limit Continuity

The upper and lower contour sets of $\mathcal{C}_2^\infty$ are closed, kinda.

Axiom 3°—Translation Invariance

We know what this looks like!

Lemma

If $\mathcal{C}_2$ satisfies A1°-3° then there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ such that

$$\mathcal{C}_2^\infty(D) \subseteq \arg\max_D u.$$

Moreover, u is unique up to positive linear transformations.

Define $\succcurlyeq$ over $(\mathbb{R}^n \times \mathbb{I})$:

- $(x, I) \succcurlyeq (y, J)$ if and only if
- exists $D \supseteq \{x, y\}$ with $I_D^x = I$ and $I_D^y = J$, and $x \in \mathcal{C}_2(D)$

Axiom 4°—$\{x \mid x \neq$ 'KITCHEN SINK'$\}$

$\succcurlyeq$ is acyclic and well behaved with respect to u.

Well behaved:

MONO $v(z) \geq 0$ and $(x, I) \succcurlyeq (y, J)$ then not $(y, J) \succcurlyeq (x + z, I)$

BND For all I, J: $\inf\{v(z) \mid (z, I) \succcurlyeq (\mathbf{0}, J)\} > -\infty$

CONT $v(x_n) \to 0$ and $(x_k, I) \succcurlyeq (y, J)$ for all k, then for any x with $v(x) = 0$, if $(y, J) \succcurlyeq (x, I)$ then $(x, I) \succcurlyeq (y, J)$

Theorem

The following are equivalent:

1. $\mathcal{C}_2$ satisfies A1°-4°
2. $\mathcal{C}_2$ has an image conscious representation $\langle u, \Gamma \rangle$.

Moreover, u is unique up to positive linear translations.

Proof Idea

Imagine some complete $\succcurlyeq^\star$ over $\mathbb{R}^n \times \mathbb{I}$ was magically identified and preserved the relevant structure and extended $\succcurlyeq$.

- Fix $I^* \in \mathbb{I}$ and set $\Gamma(I^*) = 0$
- For each I let $x^I \in \mathbb{R}^n$ such that $(x^I, I) \sim (\mathbf{0}, I^*)$
  - Exists by BND and CONT
- We can recover Γ:

$$\Gamma : I \mapsto -u(x^I)$$

Proof Idea

We need to complete $\succcurlyeq$ to $\succcurlyeq^\star$.

1. $\succcurlyeq$ by adding comparisons that must hold because of transitivity, monotonicity, or continuity.
2. If there are still images I and J such that no x satisfies $(x, I) \sim (\mathbf{0}, J)$: Just pick some x and extend the relation by adding $(x, I) \sim (\mathbf{0}, J)$.
   - If we added anything, go back to step 1.

Repeating the process for different I's and J's creates a partial order of extensions of $\succcurlyeq$, which, by Zorn's Lemma, has maximal element that must be complete.