EC5110: MICROECONOMICS

LECTURE 3: COMPARATIVE STATICS & DEMAND THEORY

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Comparative Statics is the study of how predictions of the model depend on parameters.

- How does a consumers welfare change with her income?
  - $\frac{\partial v}{\partial I}$.
- How does consumer demand change with income?
  - $\frac{\partial x_i^\star}{\partial I}$.
- How does consumer demand change with price changes?
  - $\frac{\partial x_i^\star}{\partial p_j}$.

This is easy with the envelope theorem. Recall:

Theorem. (The Constrained Envelope Theorem)

Let $f, g : \mathbb{R}^{n+k} \to \mathbb{R}$. We want to solve $\max_{\boldsymbol{x}} f(\boldsymbol{x}, \boldsymbol{a})$ subject to $g(\boldsymbol{x}, \boldsymbol{a}) = 0$. Let $f^\star$ be the optimized value. Then so long as both partial derivatives exist, we have

$$\frac{\partial f^\star}{\partial \boldsymbol{a}} = \frac{\partial f}{\partial \boldsymbol{a}} - \lambda \frac{\partial g}{\partial \boldsymbol{a}}$$

- $f^\star = v$
- $f = U$
- $g = \boldsymbol{p} \cdot \boldsymbol{x} - I$

$$\frac{\partial v}{\partial I} = \frac{\partial U}{\partial I} - \lambda \frac{\partial(\boldsymbol{p} \cdot \boldsymbol{x} - I)}{\partial I} = \lambda$$

- $\lambda \geq 0$ by definition
- The consumer is always better of with more money.

Theorem.

The indirect utility function $v(\boldsymbol{p}, I)$ is non-increasing in $\boldsymbol{p}$ and non-decreasing in I, homogeneous of degree 0, and quasi-convex in $\boldsymbol{p}$.

- Could have done whatever she was doing before; can't be worse off.

So when income goes up, you must be better off, but how does it change demand?

What is $\frac{\partial \boldsymbol{x}^\star}{\partial I}$

It is tempting to assume that $\frac{\partial x_i^\star}{\partial I} \geq 0$

- If I get a raise, I get more houseplants, tee-shirts, etc...
- This is plausible, but does *not* follow from any of our assumptions.
- I might want to change my consumption patterns to more expensive things.

If consumption of x_i increases when when income increase ($\frac{\partial x_i^\star}{\partial I} > 0$) we call x_i normal, if it decreases ($\frac{\partial x_i^\star}{\partial I} < 0$) we call it inferior.

Inferior goods happen because the consumer wishes to substitute towards more expensive goods.

- Inexpensive foods like instant noodles
- Financial services such as payday lending
- Long distance bus trips

FIGURE 5.1 Effect of an Increase in Income on the Quantities of x and y Chosen

As income increases from I_1 to I_2 to I_3, the optimal (utility-maximizing) choices of x and y are shown by the successively higher points of tangency. Observe that the budget constraint shifts in a parallel way because its slope (given by $-p_x/p_y$) does not change.

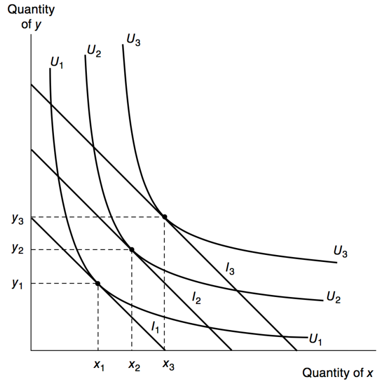
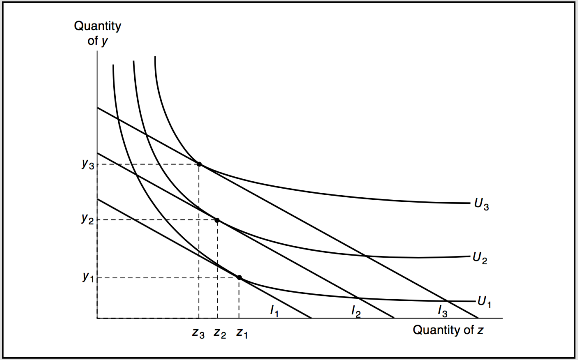


FIGURE 5.2 An Indifference Curve Map Exhibiting Inferiority

In this diagram, good z is inferior because the quantity purchased actually declines as income increases. Here, y is a normal good (as it must be if there are only two goods available), and purchases of y increase as total expenditures increase.



What about the demand for a good in response to a change in price: $\frac{\partial x_i^\star}{\partial p_j}$?

This is more involved then a change in income.

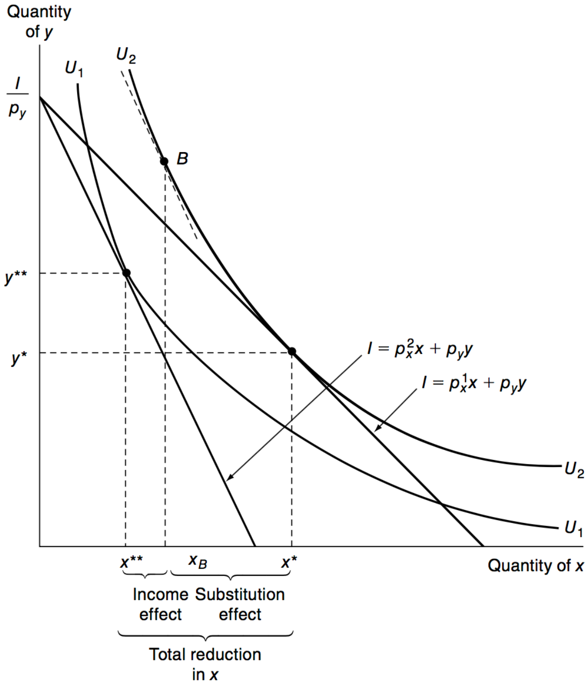
- A change in income shifted the budget curve up or down
- A change in prices rotates the budget curve.

There are two effects:

1. The substitution effect: change in the ratio of prices changes the tangency condition.
   - Might want to substitute one good for another.

2. The income effect: changes the real wealth of the consumer.
   - If a price increases, I can afford smaller bundles.

The total effect is the composition of these effects.


Quantity of y
$\frac{I}{p_y}$
U_1
U_2
B
y^{**}
y^*
$I = p_x^2 x + p_y y$
$I = p_x^1 x + p_y y$
U_2
U_1
x^{**}
x_B
x^*
Quantity of x
Income effect
Substitution effect
Total reduction in x

- The substitution effect is always negative:
  - increasing the price of x_i leads the consumer to shift consumption onto other goods, holding utility equal.
- The substitution effect can go either way:
  - Depends if the good is normal or inferior.

Theorem. (Slutsky Decomposition)

If u is continuous, locally non-satiated, and strictly quasi-concave, and h is differentiable, then:

$$\frac{\partial x_i^\star(\boldsymbol{p}, w)}{\partial p_j} = \frac{\partial h_i(\boldsymbol{p}, v(\boldsymbol{p}, I))}{\partial p_j} - \frac{\partial x_i^\star(\boldsymbol{p}, I)}{\partial I} x_j^\star(p, I)$$

- First term is the Substitution effect
  - Always negative by law of compensated demand.
- Second term is the Income effect.

Proof

- $h_i^\star(p,u) = x_i^\star(\boldsymbol{p}, e(\boldsymbol{p},u))$. Differentiating with respect to p_j (at the point $u = v(\boldsymbol{p}, I)$),

$$\frac{\partial h_i^\star(\boldsymbol{p},u)}{\partial p_j} = \frac{\partial x_i^\star(\boldsymbol{p},I)}{\partial p_j} + \frac{\partial x_i^\star(\boldsymbol{p},I)}{\partial I}\frac{\partial e(\boldsymbol{p},u)}{\partial p_j}.$$

- From properties of Hicksian demand

$$\frac{\partial e(\boldsymbol{p},u)}{\partial p_j} = h_j^\star(\boldsymbol{p},u)$$

- Daulity:
$h_j^\star(\boldsymbol{p},u) = x_j^\star(\boldsymbol{p}, e(\boldsymbol{p},u)) = x_j^\star(\boldsymbol{p}, e(\boldsymbol{p}, v(\boldsymbol{p},I))) = x_j^\star(\boldsymbol{p},I)$.

It might be possible that the income effect is so positive that it outweighs the substitution effect:

> "As Mr.Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it."
>
> —Alfred Marshall (Namesake of Marshallian Demand)

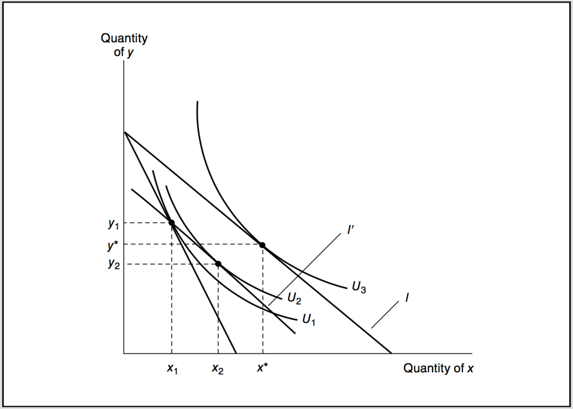
Such goods are called giffen goods, and there is no evidence they exist.

Is it better to raise taxes by taxing particular goods or income?

- Taxing income changes the affordable bundles.
  - Income effect.
- Taxing goods changes the affordable bundles *and* distorts tradeoffs.
  - Income and Substitution effects.

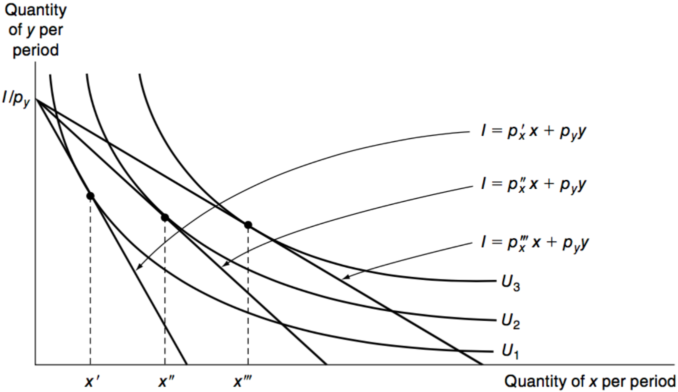
FIGURE 4.5 The Lump Sum Principle of Taxation

A tax on good x would shift the utility-maximizing choice from x^*, y^* to x_1, y_1. An income tax that collected the same amount would shift the budget constraint to I'. Utility would be higher (U_2) with the income tax than with the tax on x alone (U_1).

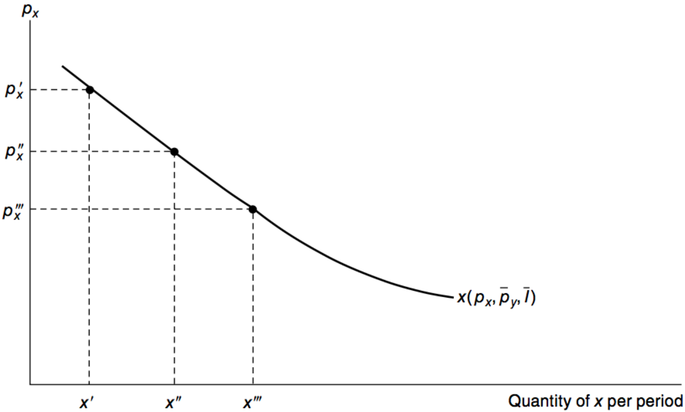


We can trace out the relationship between demand and price via a demand curve, which plots the demand against the price.

- This assumes everything else (income, other prices) stays the same.
- Changes of other parameters might shift the whole curve.



(a) Individual's indifference curve map



(b) Demand curve

- This is just a graph of $x_i^\star$ as a function of p_i
- The demand curve is downwards sloping unless the good is a Giffen good.

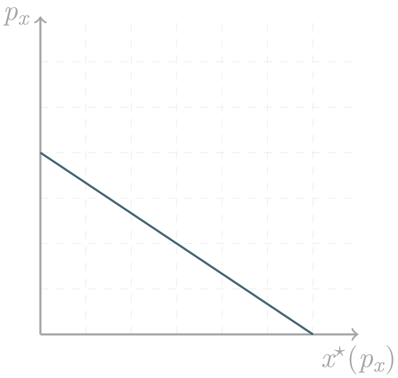
Consumer Surplus

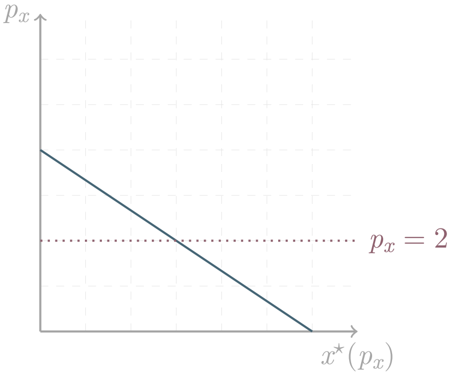
- If a consumer is buying bananas at 20p each, she will buy 10 bananas.
- When the price is 25p, she will only by 5.
- The first 5 bananas are worth more than 20p each:
  - if she only pays 20, she is making a surplus (gains from trade).

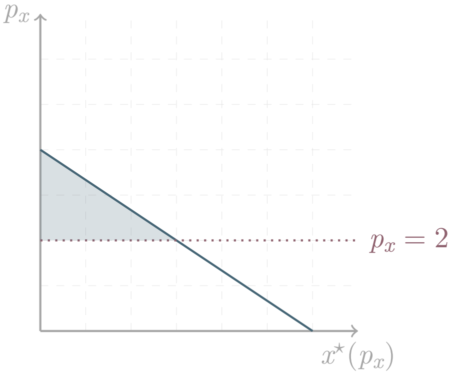
Consumer Surplus

How must is the total surplus from the purchase of a bundle?

- Its how much extra she would have paid for each unit.
- How much extra would she pay for the first unit, plus ....
- This is just the area under the demand curve, and above the price.


p_x
$x^\star(p_x)$


p_x
$p_x = 2$
$x^\star(p_x)$



p_x
$p_x = 2$
$x^\star(p_x)$

Example

A consumer has utility $U(x, y) = xy + y$. She has a wealth of I and faces prices p_x and p_y.

What is her consumer surplus from consuming x at price $\frac{I}{2}$?

Example

The Lagrangian from the example (taking strictly positive consumption as given) is:

$$\mathscr{L} = xy + y - \lambda(p_x x + p_y y - I)$$

We have the first order conditions:

$$\begin{aligned} \mathscr{L}_x : \qquad & y - \lambda p_x = 0 \\ \mathscr{L}_y : \qquad & x + 1 - \lambda p_y = 0 \\ \mathscr{L}_{\mu_1} : \qquad & p_x x + p_y y = I \end{aligned}$$

- We have $\frac{y}{x+1} = \frac{p_x}{p_y}$ or $y = \frac{p_x}{p_y}(x+1)$
- $p_x x + p_x(x+1) = I$ or $x^\star = \frac{I - p_x}{2p_x}$
- $y^\star = \frac{I + p_x}{2p_y}$
- (This requires that $p_x \leq I$, or we have a corner solution).

So the demand function is $x^{\star}(p_x, I) = \frac{I - p_x}{2p_x}$. The CS is:

$$\int_{\frac{I}{2}}^{I} \frac{I - p_x}{2p_x} \, d(p_x)$$

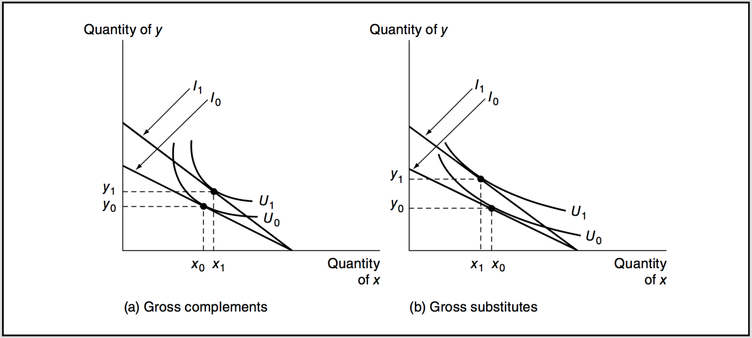
- Lower bound – actual price.
- Upper bound, where $x^{\star} = 0$.

We have analyzed how demand for x_i changes as a function of p_i, what about p_j?

- (Slutsky decomp still works)
- Changes in price of the same good are movements along the demand curve.
- Change in other prices (or income) are shifts of the curve.

FIGURE 6.1 Differing Directions of Cross-Price Effects

In both panels, the price of y has fallen. In (a), substitution effects are small so the quantity of x consumed increases along with y. Because $\partial x/\partial p_y < 0$, x and y are *gross complements*. In (b), substitution effects are large so the quantity of x chosen falls. Because $\partial x/\partial p_y > 0$, x and y would be termed *gross substitutes*.



Recall our prototypical utilities:

- Perfect complements: $U(x, y) = \min\{x, y\}$
- Perfect substitutes: $U(x, y) = x + y$

In general, a x, y are called gross complements if $\frac{\partial x^\star}{\partial p_y} < 0$ and gross substitutes if $\frac{\partial x^\star}{\partial p_y} > 0.$

In general, a x, y are called net complements if $\frac{\partial h_x^\star}{\partial p_y} < 0$ and net substitutes if $\frac{\partial h_y^\star}{\partial p_y} > 0.$

The idea of gross complements/substitutes is more intuitive

- If the price of apples increase do I buy more or less pears?

Example

A consumer has utility $U(x, y) = \ln(x) + y$. She has a wealth of I and faces prices p_x and p_y.

Are these goods gross complements or substitutes?

Example

The Lagrangian from the example (taking strictly positive consumption as given) is:

$$\mathscr{L} = \ln(x) + y - \lambda(p_x x + p_y y - I)$$

We have the first order conditions:

$$\begin{aligned}
\mathscr{L}_x : &\qquad \frac{1}{x} - \lambda p_x = 0 \\
\mathscr{L}_y : &\qquad 1 - \lambda p_y = 0 \\
\mathscr{L}_{\mu_1} : &\qquad p_x x + p_y y = I
\end{aligned}$$

- We have $\frac{1}{x} = \frac{p_x}{p_y}$ or $p_x x = p_y$
- $p_y + p_y y = I$ or $y^\star = \frac{I - p_y}{p_y}$
- Therefore $\frac{\partial y^\star}{\partial p_x} = 0$
- If p_y increases, then $y^\star$ decrease, so $x^\star$ increases.
  - I and p_x did not change.
  - Therefore $\frac{\partial x^\star}{\partial p_y} > 0$

Net substitutes on the other hand are symmetric:

$$\frac{\partial h_x^\star}{\partial p_y} = \frac{\partial h_y^\star}{\partial p_x}$$

- This follows from the fact that $\boldsymbol{h} = \nabla_p e$.