PARTIAL AWARENESS

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- Awareness is a pervasive form of ignorance:
- An agent is unaware of an object/event/statement/etc if it is *not on her radar screen*.

Existing formal models treat awareness as binary:

- An agent is either aware of a statement or an event or she is not.
- This is something like knowledge: either you know something or you don't. But with probability, we can have gradations.

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- An agent is either aware of a statement or an event or she is not.
- This is something like knowledge: either you know something or you don't. But with probability, we can have gradations.
- Satoshi Nakamoto is *fully aware* of bitcoin.
- My grandmother is *fully unaware* of bitcoin.
- Most people, however, are in-between these extremes.
    - The concept is on their radar but not fully understood.

Why do we care?

- More realistic/nuanced representation of ignorance.
- As a form of introspection: allows the agent to reason about her own limitations.
    - Motivate learning, filling in missing data, etc.
- Has important implications in economic markets.

This paper:

1. Develops a formal model of partial awareness.
   - Modal logic extending Halpern Rego (2009, 2013).
2. Applies the model to an (economic) contracting model with partially aware agents.
   - Examines the effect of awareness on welfare and efficiency of contracts.

The language has three building blocks:

- Objects, $\mathscr{O}$ (and object variables, $\mathcal{V}^{\mathscr{O}}$)
  - Following Levesque (1990), $\mathscr{O}$ consists of *standard names* $d_1, d_2, \ldots$
- Properties, $\mathscr{P}$: unary predicates (and predicate variables, $\mathcal{V}^{\mathscr{P}}$)
- Concepts $\mathscr{C}$: Boolean combinations of properties.

In each state (possible world), each agent is aware of a subset of objects, properties, and concepts.

The logic also has two modalities:

- $A_i\varphi$: agent i is **aware** of the formula φ
- $K_i\varphi$: agent i **explicitly knows** formula φ.
    - Read: φ is true at all possible worlds and $A_i\varphi$

and quantification over both objects and properties.

$\mathcal{L}(\mathscr{O}, \mathscr{P}, \mathscr{C})$ is the resulting Language.

Suppose a quantum computer (Q) is defined as a computer (C) that possesses an additional “quantum property” QP.

$$Q := C \wedge QP$$

1. A *partially aware* agent is aware of Q but unaware of the specific Boolean combination of properties that characterizes it.
2. A *fully unaware* agent is unaware of even the concept of a quantum computer
3. A *fully aware* agent is unaware is aware of both the concept of a quantum computer and also what it means to be one, i.e., the properties C and QP.

Quantification allow for expressions of introspection.

- $K_i(\exists P \forall x(Q(x) \Leftrightarrow C(x) \wedge P(x)))$
- Although the agent is unaware of QP, she knows something about Q (quantum computers): she knows that they're computers that satisfy some extra property P.

Semantics

- Ω of possible *states*.
- we take the domain D to be the standard names in $\mathscr{O}$.
- a binary relation $\mathcal{K}_i$ on states; states i considers possible.
- an awareness set $\mathcal{A}_i(\omega) \subseteq \mathcal{L}$, the symbols i is aware of.

An interpretation I: for each state ω, we have a function I_ω:

- taking $\mathscr{O}$ to elements of the domain D, standard names are mapped to themselves, so that $I_w(d_i) = d_i$.
- $\mathscr{P}$ to subsets of D,
- $\mathscr{C}$ to $\mathcal{L}^{bc}$, Boolean combinations of properties (i.e., predicates).

The truth of a sentences at a state ω in

$$M = (\Omega, D, \Phi, \mathcal{A}_1 \ldots, \mathcal{A}_n, \mathcal{K}_1, \ldots, \mathcal{K}_n, I)$$

are defined as usual .

Of interest:

- $(M, \omega) \models C(d)$ iff $(M, \omega) \models C^I_\omega(d)$,
- $(M, \omega) \models \forall Y \varphi$ iff $(M, \omega) \models \varphi[Y/\psi]$, where $\psi \in \mathcal{L}^{bc}$
- $(M, \omega) \models A_i \varphi$ iff $\varphi \in \mathcal{L}(\mathcal{A}_i(\omega))$,
- $(M, \omega) \models K_i \varphi$ iff $(M, \omega) \models A_i \varphi$ and $(M, \omega') \models \varphi$ for all $\omega' \in \mathcal{K}_i(\omega)$.

We assume agents know what they are aware of:

- if $(\omega, \omega') \in \mathcal{K}_i$, then $\mathcal{A}_i(\omega) = \mathcal{A}_i(\omega')$

and $\mathcal{K}_i$ is an equivalence relation, and thus partitions the states in Ω.

An Application: Contracting

- Two agents, each has an endowment, $End_i \subset D$, and preferences over objects.
  - Utility of d depends only on the properties of d, and cannot vary (across states) unless the agent is aware of a difference between the states.
- Agents each consumes 1 object
- A contract is a mapping from a set of formulas to consumption for each agent.
  - Formulas must be mutually exclusive, collectively exhaustive, in the awareness set of the agents.

A contract is

- **Acceptable** for agent i if at least as good as consuming out of End_i in each possible state.
- **Efficient** if there is no contract both agents would prefer and one strictly.

Theorem.

Given M and a state $\omega^* \in \Omega$ such that $\mathcal{A}_1(\omega^*) = \mathcal{A}_2(\omega^*)$ then there exists a contract that is efficient and acceptable at ω^*.

- Limited by symmetric awareness does not impinge the efficacy of contracts.
- The symmetry of awareness cannot be dropped.

Example 1

A buyer (agent 1) is trying to purchase a computer from a firm (agent2).

- $End_1 = \{d^{\$}\}$: where $d^{\$}$ is a fixed amount of money
- $End_2 = \{d^{cmp}\}$, and d^{cmp} is the computer in question.
- There are two states: $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and information partitions the whole space.
- There are three predicates, $\mathscr{P} = \{P, Q, R\}$.
- One concept: $\mathscr{C} = \{QC\}$

	ω_1	ω_2	ω_3
P	$\{d^{cmp}\}$	$\{d^{cmp}\}$	$\emptyset$
Q	$\{d^{cmp}\}$	$\emptyset$	$\emptyset$
R	$\{d^{\$}\}$	$\{d^{\$}\}$	$\{d^{\$}\}$
QC	$P \wedge Q$	$P \wedge Q$	$\neg P \wedge \neg Q \wedge \neg R$
U_1	$d^{cmp} \succ d^{\$}$	$d^{\$} \succ d^{cmp}$	$d^{\$} \succ d^{cmp}$
U_2	$d^{\$} \succ d^{cmp}$	$d^{\$} \succ d^{cmp}$	$d^{\$} \succ d^{cmp}$

For now, assume both agents have full awareness.

- Because the buyer does not know the state, she is unwilling to make any unconditional trade (all constant contracts are unacceptable for the buyer).

However, this is easily remedied by
the use of a contract. The obvious contract,

$$\begin{aligned} Q(d^{cmp}) &\mapsto (d^{cmp}, d^{\$}) \\ \neg Q(d^{cmp}) &\mapsto (d^{\$}, d^{cmp}) \end{aligned}$$

is clearly efficient and acceptable to all parties.

Example 2

Same as Example 1 except both agents are unaware of Q.

- The previous contract is no longer articulable.

We can circumvent the agents' linguistic limitations by writing a contract in terms of the concept QC. Indeed, the contract

$$
\begin{aligned}
P(d^{cmp}) \wedge QC(d^{cmp}) &\mapsto (d^{cmp}, d^{\$}) \\
\neg\big(P(d^{cmp}) \wedge QC(d^{cmp})\big) &\mapsto (d^{\$}, d^{cmp})
\end{aligned}
$$

- With limited awareness, we can use concepts as a method of indirect reference.

Thank You!

Example 3

- $End_1 = \{d_1\}$ and $End_2 = \{d_2\}$
- Three states, ω_1, ω_2, and ω_3
- three predicate symbols P, Q, and R.
  - $P^I_{\omega_1} = \{d_1\}$,
  - $Q^I_{\omega_3} = \{d_2\}$,
  - $P^I_{\omega_2} = P^I_{\omega_3} = Q^I_{\omega_1} = Q^I_{\omega_2} = \emptyset$,
  - $R^I_{\omega} = \{d_1\}$ for all states ω,

- both agents consider all three worlds possible at all worlds
- $\mathcal{A}_1(\omega) = (\mathscr{O}, \{P, R\}, \emptyset)$, and $\mathcal{A}_2(\omega) = (\mathscr{O}, \{Q, R\}, \emptyset)$ for each state ω.

Consider what happens when

- agent 1 wants d_1 only when it has property P
  - wants to trade in states ω_2, ω_3
- agent 2 wants d_2 only when it has property Q
  - wants to trade in states ω_1, ω_2

- an efficient contract must induce trade in state ω_2
- acceptable contract cannot induce trade except in state ω_2.
- neither agent alone can propose such a contract
    - from agents 1's perspective, states ω_2 and ω_3 are indistinguishable (he is unaware of Q)
    - from 2's perspective, states ω_1 and ω_2 are indistinguishable (she is unaware of P)