DYNAMIC PARTIAL AWARENESS

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- Awareness is a pervasive form of ignorance:
- An agent is unaware of an object/event/statement/etc if it is *not on her radar screen*

Writing formal models that can capture the above is difficult because:

1. There are many subtleties and we want tractable models
2. We do not have a descriptive theory for how people behave

Why do we care?

- More realistic/nuanced representation of ignorance.
- Agents who reason about thier own limitations, may take actions to temper exposure to unawareness
  - Motivate learning, preference for the well understood
- Has important implications in economic markets:
  - Incomplete contracting, over-confidence, context-dependent preferences, &c.

This talk:

1. Provide a framework to analyze unawareness experimentally
   - Exposure to Unawareness increases cautious behavior
   - Risk preferences seem context-dependent

2. Develop a formal model of dynamic partial awareness
   - Modal logic extending Halpern Rego (2009, 2013)
   - This model is rigorous but **not** tractable
   - But, it has a simple (semantic) fragment that can used for applications!

Experiment

(with Felipe Augusto de Araujo)

We ran an experiment in which we induce unawareness.

Desiderata:

- Unawareness enters in a natural, endogenous way
- Subjects are aware of their unawareness
- ‘Level’ of awareness is varied

- Ran the exp via Amazon Mechanical Turk (online)
- Web app (python / JavaScript / sql)
- Data from 585 Subjects
  - Those who passed the comprehension quiz; started with 970
- Average payment $2.08; average time 8m40s.

The exp has two tasks:

1. Paid based on a real effort task: finding SETs
   - A constraint satisfaction / pattern finding task
   - Elicit beliefs about how well they did
   - We varied the subjects' understanding of how many possible solutions existed
     - Full: Knew ahead of time how many SETs
     - Surprise: Told after belief elicitation
     - Unawareness: never told
2. Risk elicitation:
   - How much to invest in a risky prospect (objective probability: 50/50)
   - We varied the context of the risk
     - In context: Risk based on SETs found in task 1
     - Out of context: Risk based on coin flip

Treatments and Sample Characteristics

	N		% Women		Age Group		$ Payoff	
	Out	In	Out	In	Out	In	Out	In
Full Awareness	86	98	56.6	42.7	1.98	1.92	2.10	1.99
Surprise Awareness	114	85	48.1	52.5	2.0	2.0	2.05	2.08
Unawareness	113	89	50.4	46.0	1.92	1.87	2.06	2.15

Age groups: 1 if between 18-30, 2 if 31-45, 3 if 46-65 and 4 if 65+.

We find that:

- The task does instill unawareness. Subjects' subjective beliefs are very wrong.
- Subjects more exposed to unawareness, or who has their beliefs 'shocked,' become more risk averse:
    - But, only when the risk elicitation is contextually related to the unawareness.

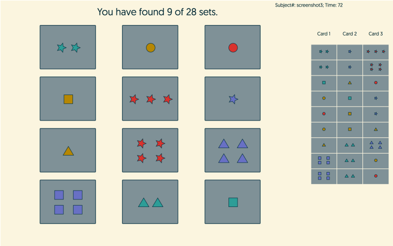
Prior work:

- Directly inducing unawareness:
  - Mengel, Tsakas and Vostroknutov (2016), Ma and Schipper (2017)
  - Both use surprising outcomes of lotteries.
- Context dependent risk preference:
  - Cohn, Engelmann, Fehr, Marechal (2015).
  - Framing, etc

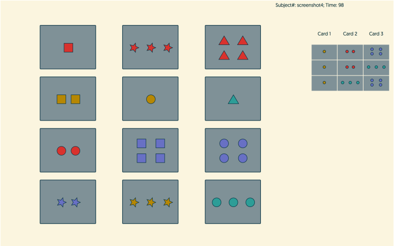
Task 1

https://faep2.herokuapp.com/

Task 1

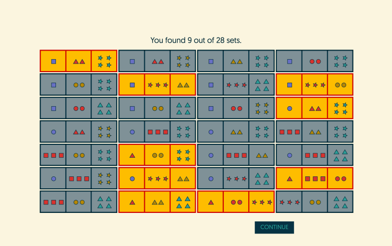


Full awareness treatments

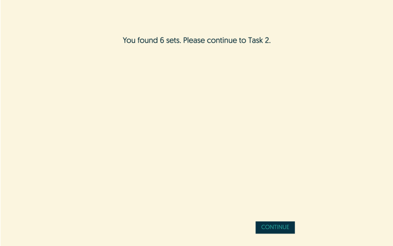


Surprise / No Info Treatments

Task 1

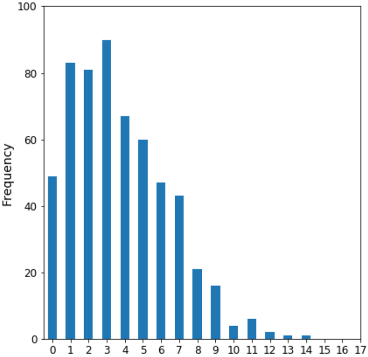


Full / Surprise treatments

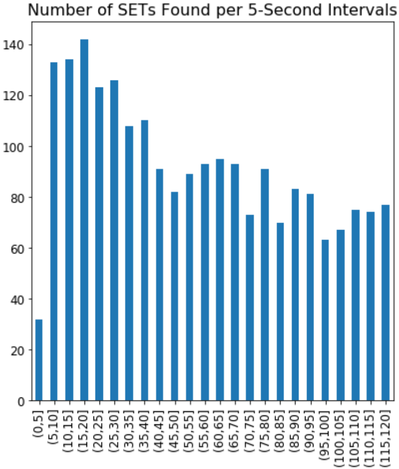


No Info Treatments

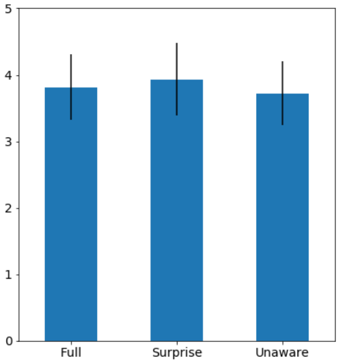
Number of SETs found:



Timing of SETs found:

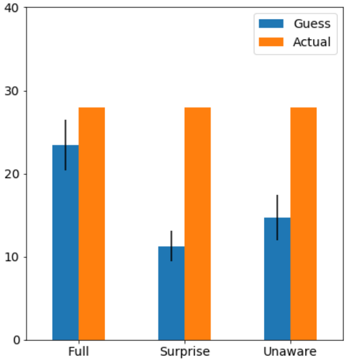


Number of SETs found by treatment:



Note: black lines are error bars.

Belief about the number of SETs found by treatment:



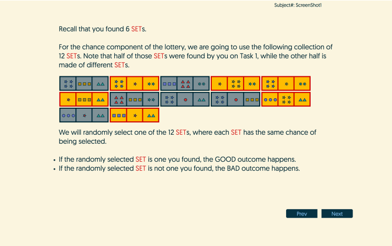
Note: black lines are error bars.

Task 2

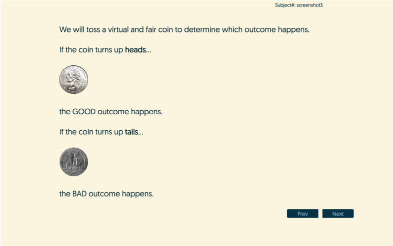
Subjects invest up to 100 cents in a risky asset:

- Asset pays investment*3 with probability $\frac{1}{2}$
- The subject keeps whatever is not invested.
- Adapted from Gneezy Potters (1997).

Task 2

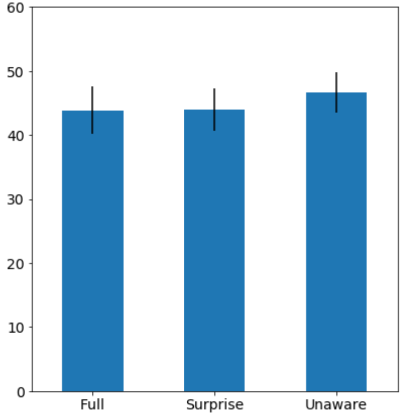


In Context treatments



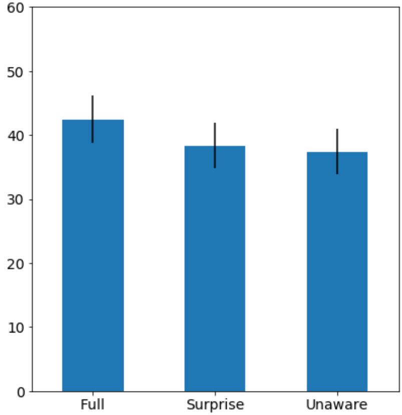
Out of Context Treatments

Average Amount Bet on Out-of-Context Lottery, by Treatment



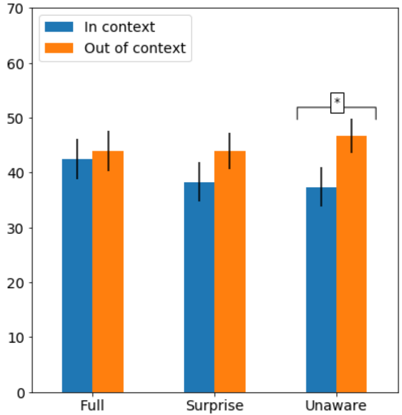
Note: black lines are error bars.

Average Amount Bet on In-Context Lottery, by Treatment



Note: black lines are error bars.

Average Amount Bet Lottery, by Treatment and Lottery Context

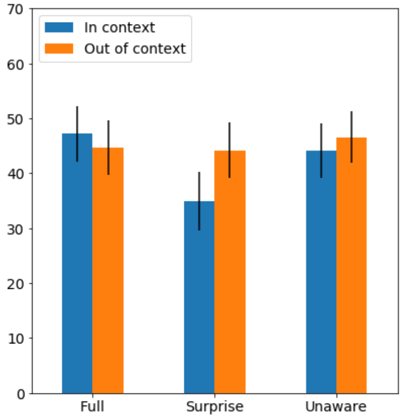


Note: black lines are error bars. * Difference significant at 5% level (Mann-Whitney, p=0.043).

Being unaware seems to make people more cautious, but only when the risk is related to the unawareness.

- Effect seems to be stronger when the unawareness is unresolved.
- Subjects were already aware they were unaware?
  - Look at the subjects who better understood the task (found more 4 or more SETs)

Average Amount Bet Lottery, by Treatment and Lottery Context (subjects with 4+ sets)



Note: black lines are error bars.

Being unaware seems to make people more cautious:

- Subjects who understood the task were ‘surprised’ by the number of sets
  - Surprise treatment had a high exposure to unawareness
- Subjects who did not understand the task were not surprised
  - Unawareness treatment had a high exposure to unawareness

Theory

(with Joseph Y. Halpern)

For use, we want a model that is primarily semantic. However, awareness is a syntactic problem.

- Awareness is description dependent: we are aware of things in our language.
- Aware of φ and not of ψ even though $\varphi \Leftrightarrow \psi$.

The language has the building blocks:

1. objects $d, d' \ldots$
2. properties: unary predicates: $P, Q, \ldots$
   - $P(d)$: d has property P.

The logic also has two modalities:

- $A\varphi$: agent i is **aware** of the formula φ
- $K\varphi$: agent i **explicitly knows** formula φ.

We capture introspection via quantification (over both objects and properties).

$$K(\exists P \neg A(P))$$

- The agent knows there exists a property she is not aware of.

A consistent truth assignment to each sentence tells us about the agent's knowledge and awareness.

- But this is very hard to work with.
- We need to know if every sentence is true or false.

Semantics

- Ω of possible *states*.
- Each state ω is associated with a language (a set of objets and properties)
- Each state determines the truth of all sentences in its associated language.

A model is a state space plus:

- An interpretation I: for each state ω, we have a function I_ω taking properties to subsets of objects.
  - Determines 'physical' properties of states.
  - Which objects satisfy which properties.
- An awareness set: for each state ω, a set of objects and properties $\mathcal{A}(\omega)$.
  - $A\varphi$ is true at ω if all the objects and properties referenced in φ are in $\mathcal{A}(\omega)$.
- A knowledge relation: for each state ω, a set of states $\mathcal{K}(\omega)$.
  - $K\varphi$ is true at ω if $A\varphi$ is true at ω and φ is true for all $\omega' \in \mathcal{K}(\omega)$.

Why different languages at different states? Otherwise:

$$\neg K \neg \forall P (AP) \Rightarrow K \forall P (AP)$$

The state space Ω is God's state-space. The 'objective' view of the world.

But the agent's view is limited by her own language.

An event $E \subseteq \Omega$ is ω-**conceivable** of there is a sentence φ such that

1. The agent is aware of φ at ω.
2. φ is true exactly on E.

Under basic regularity conditions

$$\Sigma_\omega = \{E \subseteq \Omega \mid \text{ is } \omega\text{-conceivable}\}$$

is a σ-algebra on Ω.

A modeler can obtain a subjective probability $\pi \in \Delta(\Omega, \Sigma_\omega)$

- Use bets on the truth of φ (usual decision theory).
- Only consider sentences the agent is already aware of.
- The support of π is $\mathcal{K}(\omega)$.

Now what happens when the agent becomes more aware, e.g., of a new property P.

- Her language gets richer
  - Implies: She might be able to differentiate new states
- She learns that she used to be unaware of whatever she discovered
  - Implies: she might condition her beliefs

Formally: becoming aware of φ changes the model from M to $M^{[\varphi]}$:

- The 'physical' properties of the states do not change: $\Omega = \Omega^{[\varphi]}$, and $P(d)$ is true at ω if and only if it is true at $\omega^{[\varphi]}$.
- The agent becomes aware of all the symbols in φ

$$\mathcal{A}^{[\varphi]}(\omega) = \mathcal{A}(\omega) \cup \mathsf{SYM}(\varphi)$$

- The agent learns she was unaware of φ:

$$\mathcal{K}^{[\varphi]}(\omega^{[\varphi]}) = \mathcal{K}(\omega) \cap \{\omega \mid \text{the agent could have been unaware of } \varphi\}$$

Say π_0 is ex-ante probability and π_1 is ex-post (the agent becomes aware of φ):

- $\Sigma_\omega \subseteq \Sigma_\omega^{[\varphi]}$ (can differentiate new states)
- $\mathsf{supp}(\pi_1) \subseteq \mathsf{supp}(\pi_0)$ ('condition' her beliefs).

Bayes' rule states $\pi_1(E) = \frac{\pi_0(E \cap \mathsf{supp}(\pi_1))}{\pi_0(\mathsf{supp}(\pi_1))}$, but what if $\mathsf{supp}(\pi_1) \notin \Sigma_\omega$?

Theorem.

If for all $E, E' \in \Sigma_\omega$ with $E, E' \subseteq \mathsf{supp}(\pi_1)$, we have

$$\frac{\pi_0(E)}{\pi_0(E')} = \frac{\pi_1(E)}{\pi_1(E')}$$

and $\pi_0(E) \leq \pi_1(E)$ then it is *as if* π_1 is a conditional distribution of π_0.

- There exists a $\pi^* \in \Delta(\Omega, \Sigma_\omega^{[\varphi]})$ such that π^* is an extension of π_0 and
$$\pi_1(E) = \frac{\pi^*(E \cap \mathsf{supp}(\pi_1))}{\pi^*(\mathsf{supp}(\pi_1))}$$

Hence the model:

- Allows for growing awareness to change beliefs about previously describable events
  - Has testable predictions (with more detailed data)
- This is in juxtaposition to Karni and Viero (2015) and Dominiak and Tserenjigmid (2019)
- Can explain why subjects risk preferences change