

ELICITING AWARENESS

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- ◇ A **decision maker** must choose a 'plan-of-action;' what action to take provided the future resolution of uncertainty
- ◇ He is unaware of some relevant contingencies and *knows this is possible*
 - ◇ Purchasing a home
 - ◇ Hiring a new AP
 - ◇ Writing legislation

What do we often do in such situations?

What do we often do in such situations? seek the council of an **expert** who is more aware

- ◇ Purchasing a home: **surveyor**
- ◇ Hiring a new AP: **letter writers, search committee**
- ◇ Writing legislation: **technical advisors**

Why is the interesting?

- ◇ When preferences are not aligned, the **expert** might strategically conceal her awareness
- ◇ Can the **dm** do anything to incentivize revelation?
- ◇ Importantly, even with full/complete contracting, the **dm** cannot articulate what he wants
- ◇ A(n unaware) designer may not be able to solve the problem, if mechanisms depend on the unknowns

Literature

- ◇ Contracting under unawareness

- ◇ Tirole (2009); Filiz-Ozbay (2012); Von Thadden and Zhao (2012); Auster (2013) Auster and Pavoni (2021); Piermont (2017); Lei and Zhao (2021); Francetich and Schipper (2021)

- ◇ Modeling unawareness and awareness of unawareness

- ◇ Halpern and Rêgo (2006, 2013); Heifetz et al. (2013); Karni and Vierø (2013, 2017), Halpern and Piermont (2020); Piermont, (2021)

- ◇ Robust Mechanism Design

- ◇ Bergemann and Morris (2005); Jehiel et al., (2006); Carroll (2015, 2019).

Example

- ◇ A politician (the decision maker) is trying to write environmental legislation that
 - ◇ can be contingent on the future realized environmental state-of-affairs, but
 - ◇ can depend only on those contingencies he is aware of.
- ◇ He can enlist the help of an environmental scientist (the expert) who may reveal what she is aware of

Example

- ◇ The true state-space is $\Omega = \{\omega, \nu\}$; equally likely
- ◇ Set of actions $\mathcal{A} = \{a, b, c\}$
- ◇ The politician must choose legislation $\mathfrak{c} : \Omega \rightarrow \mathcal{A}$

Example

The **expert** can tell distinguish the states, but the **politician** cannot.

$$\mathcal{P}_e = \{\{\omega\}, \{\nu\}\}$$

$$\mathcal{P}_d = \{\{\omega, \nu\}\}$$

$$u_{\textcolor{blue}{d}} = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$



$$u_{\textcolor{red}{e}} = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right.$$

How does the politician view payoffs in coarse states?

- ◇ Assume it is aggregated via expectations
- ◇ As if he correctly assesses randomness, but cannot explain what causes it

$$u_d = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$

$$u_d = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \{\omega, \nu\} & 2 & 3 & 1 \end{array} \right.$$

$$u_e = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right.$$

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What would the politician implement:

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What would the politician implement:

- ◇ Without revelation $c = b$
- ◇ $\mathbb{E}[u_d] = 3, \mathbb{E}[u_e] = 2$

$$u_d = \left\{ \begin{array}{c|ccc} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right. \quad \bigg| \quad u_e = \left\{ \begin{array}{c|ccc} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right.$$

What would the politician implement:

- ◇ With revelation: $c' : \begin{cases} \omega \mapsto a \\ \nu \mapsto b \end{cases}$
- ◇ $\mathbb{E}[u_d] = 5, \mathbb{E}[u_e] = 1$; So the expert won't reveal.

$$u_d = \left\{ \begin{array}{c|ccc} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right. \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|ccc} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right.$$

◇ But, $c^* : \begin{cases} \omega \mapsto c \\ \nu \mapsto b \end{cases}$ is a Pareto improvement over no revelation

◇ $\mathbb{E}[u_d] = 4, \mathbb{E}[u_e] = 3$

Example

- ◇ The Pareto improvement c^* , requires revelation
- ◇ But revealing allows the politician to exploit the expert
- ◇ What is the politician could commit:
 - ◇ Propose $c = b$ (his outside option)
 - ◇ After the expert reveals, propose some other contract c^\dagger
 - ◇ c^\dagger only get implemented if the expert agrees; else $c = b$

Example

Internalizing this, the politician solves:

$$\max_{\mathbf{c}^\dagger: \Omega \rightarrow \mathcal{A}} \mathbb{E}[u_{\mathbf{d}}(\mathbf{c}^\dagger)] \quad \text{subject to} \quad \mathbb{E}[u_{\mathbf{e}}(\mathbf{c}^\dagger)] \geq \mathbb{E}[u_{\mathbf{e}}(\mathbf{c})] \quad (\text{IC})$$

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- ◇ The solution is $\mathbf{c}^\star : \begin{cases} \omega \mapsto c \\ \nu \mapsto b \end{cases}$

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Does this always work? No

Example

What if the politician is initially unaware of action b ?

$$u_d = \left\{ \begin{array}{c|cc|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right.$$

$$u_d = \left\{ \begin{array}{c|cc|c} & a & & c \\ \hline \{\omega, \nu\} & 2 & & 1 \end{array} \right.$$

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What would the politician implement:

- ◇ Without revelation $c^* = a$
- ◇ $\mathbb{E}[u_d] = 2, \mathbb{E}[u_e] = 0$

$$u_d = \left\{ \begin{array}{c|ccc} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right\} \quad \bigg| \quad u_e = \left\{ \begin{array}{c|ccc} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right\}$$

What would the politician implement:

- ◇ Under full revelation (same as before): $c' : \begin{cases} \omega \mapsto a \\ \nu \mapsto b \end{cases}$
- ◇ $\mathbb{E}[u_d] = 5, \mathbb{E}[u_e] = 1$; this satisfies the incentive constraint.

$$u_d = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right\} \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right\}$$

- ◇ But, revealing only b leads to $c = b$
- ◇ $\mathbb{E}[u_d] = 3, \mathbb{E}[u_e] = 2$, partial revelation is preferred

$$u_d = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 4 & 0 & 2 \\ \hline \nu & 0 & 6 & 0 \end{array} \right\} \quad \Bigg| \quad u_e = \left\{ \begin{array}{c|c|c|c} & a & b & c \\ \hline \omega & 0 & 2 & 4 \\ \hline \nu & 0 & 2 & 4 \end{array} \right\}$$

◇ As before, $\mathbf{c}^* : \begin{cases} \omega \mapsto c \\ \nu \mapsto b \end{cases}$ is a Pareto improvement over $\mathbf{c} = b$

What if the procedure was repeated?

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- (4) The **expert** reveals the rest of her awareness (the partition $\{\{\omega\}, \{\nu\}\}$)
- (5) The **decision maker** solves

$$\max_{\mathbf{c}^\dagger: \Omega \rightarrow \mathcal{A}} \mathbb{E}[u_d(\mathbf{c}^\dagger)] \quad \text{subject to} \quad \mathbb{E}[u_e(\mathbf{c}^\dagger)] \geq \mathbb{E}[u_e(\mathbf{c})] \quad (\text{IC})$$

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- ◇ \mathbf{c}^* is the solution

Does this always work?

Does this always work? Yes, but what is 'always'?

Model

Hypothetical State-Space

Call $h = (W, A, (v_d, v_e), p)$ a **hypothetical states-space**,

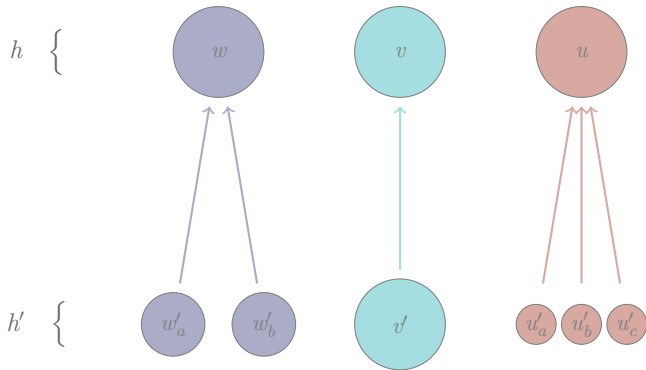
- ◇ W is a finite set of states
- ◇ A is a set of actions
- ◇ $v_i : W \times A \rightarrow \mathbb{R}$, for $i \in \{d, e\}$, determines payoffs, and,
- ◇ $p \in \Delta(W)$ is a probability over states

Let \mathcal{H} collect all hss; \mathcal{H} are the possible types of **expert**

Say that $h' = (W', A', (v'_d, v'_e), p')$ **refines** $h = (W, A, (v_d, v_e), p)$:

- ◇ $A \subseteq A'$
- ◇ Each state in h corresponds to an event in h' such that
 - ◇ probabilities aggregate
 - ◇ expected utilities of h 'measurable' acts are invariant

Then write: $h \preccurlyeq h'$.



Outcome Profiles

An **outcome profile** is a function from types to contracts:

$$\begin{array}{ccc} f: h & \mapsto & \mathfrak{c} \\ \cap & & \cap \\ \mathcal{H} & & \mathbb{R}^W \end{array}$$

Call an outcome profile **fully revealing** if the experts payoff is monotone in her type

$$\mathbb{E}_{p'}[v'_e(w, f(h'))] \geq \mathbb{E}_p[v_e(w, f(h))]$$

for any h' such that $h \preccurlyeq h'$

Iterated Revision Mechanism

An *iterated revelation mechanism* (IRM), is parameterized by a function from sequences of types to contracts:

$$\alpha : (h_0 \dots h_n) \mapsto (\mathfrak{c} : W_n \rightarrow \mathbb{R})$$

- (1) Set $n = 0$. The decision maker proposes $\mathfrak{c}_0 = \alpha(h_0)$.
- (2) The expert reveals h_{n+1} .
 - ◇ If $h_n \prec h_{n+1}$, the decision maker proposes $\mathfrak{c}_{n+1} = \alpha(h_{n+1})$. Set $n = n + 1$ and repeat step 2.
 - ◇ Otherwise, continue to step 3.
- (3) The mechanism is over and the expert selects from $\{\mathfrak{c}_0, \dots \mathfrak{c}_n\}$.

Full Revelation

Theorem

The following are equivalent for an outcome profile f

- (1) f can be implemented by an IRM
- (2) f is a fully revealing

The can be seen as an impossibility result:

- ◇ Without commitment to leave proposed contracts on the table, full revelation cannot be obtained.

Each proposed contract in an IRM specifies:

- (1) The outcome should the game end
 - ◇ **dm** wants to maximize his own payoff
- (2) The implicit incentive constraint should the game continue
 - ◇ **dm** wants to minimize the **expert's** payoff

In the examples, contracts solved (1) ignoring (2)

- ◇ The **dm** cannot conceive of what the **expert** is aware of
- ◇ It seems prudent, therefore, to consider *robust* strategies: those that maximize the worst case outcome
- ◇ Robust strategies turn out to be exactly those that follow the principle of myopic optimization

Robustness

Call an IRM, α , **robust** if at every sequence of (partial) revelations $\sigma = (h_0, \dots h_n)$:

- ◇ α maximizes the worst case payoff over all best responses that extend σ .
- ◇ relative to any other α' that coincides with α over σ

Robustness

Theorem

The following are equivalent:

- (1) α is robust
- (2) α is myopically optimal: at each sequence (h_0, \dots, h_n) ,

$$\begin{aligned} \alpha(h_0, \dots, h_n) \in \operatorname{argmax}_{\mathbf{c}: W \rightarrow \mathbb{R}} V_d(h, \mathbf{c}) & \quad \text{subject to} \\ V_e(h_n, \mathbf{c}) \geq V_e(h_n, \alpha(h_0 \dots h_m)) & \quad \text{for } 0 \leq m < n \end{aligned}$$

The Designers Problem

More generally, often awareness is decentralized:

- ◇ A **designer** wants the **decision maker** to take some action
- ◇ The **designer** does not know the **dm**'s or the **expert**'s awareness
- ◇ A **mechanism** elicits awareness and returns an action recommendation

Mechanism

A **mechanism** is a mapping from pairs of types into contracts:

$$\mathcal{M} : (h^d, h^e) \mapsto \mathcal{M}(h^d, h^e)$$

where $\mathcal{M}(h^d, h^e) : W(h^d \wedge h^e) \rightarrow \mathcal{A}$

Desiderata:

INDIVIDUAL RATIONALITY: the **dm** can not do better alone (there is no constraint for the **expert**)

INCENTIVE COMPATIBILITY: i prefers to report h^i than any $h \prec h^i$

PARETO OPTIMALITY: there is no feasible contract that dominates the outcome of the mechanism

These are all **ex-post** restrictions — they must hold for all type realizations

Consider the mechanism, \mathcal{M}^{IRM} , that implements an myopically optimal IRM.

Theorem

The mechanism \mathcal{M}^{IRM}

- ◇ is individually rational, incentive compatible, and Pareto optimal, and,
- ◇ V_d -dominates any other such mechanism (point-wise over the type-space)

Thank You!