Image Conscious Preferences[*]

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Abstract

An *image conscious* decision maker (DM) cares not only about the physical consequences of his actions, but also how his actions are perceived by others. When a DM takes a choice, the resulting *image* is the set of preferences that are consistent with the observed choice. This paper axiomatizes the behavior of a DM who derives utility directly via consumption and also via the induced image. Because the image depends on what *could* have been chosen, the DM will display menu-dependent preferences. I consider two models: in the first, the modeler observes two stages of choice—over menus and then from the chosen menu; in the second, only the latter choices are observed. The two models share the same representation but uniqueness is obtained only in the first.

Key words: Image consciousness; menu-dependent preferences; reluctant giving; self-images.

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1 Introduction

When a decision maker (DM) takes an action, he reveals something about his motivation to anyone observing his choice. An *image conscious* decision maker (DM) cares not only about the physical consequences of his actions, but also how his actions are perceived by others—when taking an action, he considers both the direct effect of his choice and also the information that could be inferred by observing his behavior. This paper investigates the choices of an image conscious decision maker.[1]

Image consciousness nests two distinct phenomena that have been studied in isolation. The first is the DM's desire to conceal information about his motivation; for example, a DM who is not altruistic but is ashamed of his selfishness would ideally like to act selfishly without exposing this motivation.[2] The second is the DM's desire to unveil information about his motivation; for example, a DM who who chooses the most expensive wine from a wine list to signal gentility might prefer large wine lists as it make such a signal stronger. By examining these motivations in conjunction, we see that they are in fact captured by the same general framework, and that it is consistent that a DM might conceal information in some situations and divulge it in others (as evidenced by the below Example).

An *image* in this paper is a set of preferences; specifically, when a DM takes a choice, the resulting image is the set of preferences that are consistent with the observed choice. Therefore, a DM's image depends both on his choice and also on the set of actions that *could have been* chosen. Fixing the chosen action, adding unchosen alternatives (potentially) changes the associated image, since the observer can now rule out additional preferences. As such, an image conscious DM will not behave in accordance to the normative model of choice theory.

Example. Slothrop is deciding where to take Katje on a date. There are three restaurants, D^l, D^m, and D^h equal in all ways excepting their wine lists. The wine list at restaurant D^l offers only an inexpensive low quality bottle (l); D^m offers this and also a mid-tier bottle (m); D^h, in addition to l and m, offers a costly and high quality bottle (h).

Despite the fact that Slothrop is an absolute cheapskate, he wishes to appear generous and refined. That is, privately, Slothrop would prefer to consume l, but, all else equal, would prefer Katje to think that he prefers more expensive items to less. Hence, at D^m, figuring it worth the small expense to impress Katje, he would publicly choose m. When at D^h, however, Slothrop would revert to his private optimum, choosing l. This is because when m is chosen in favor of h, Katje rules out the possibility that Slothrop prefers grandeur,

[1] Image conscious behavior is so perversive, its existence is a truism. Image concerns manifest in decisions regarding credit cards (Rao et al., 2018), monetary allocations in experiments, (Andreoni and Bernheim, 2009), clothing brands (Han et al., 2010), voting in national elections (DellaVigna et al., 2016), the wearing or not wearing of flat brimmed 'trucker' hats (Barker, 2004), charitable donations (DellaVigna et al., 2012), home design (Wagner, 2018), music genres (Berger and Heath, 2008) etc.

[2] See Dana et al. (2006) for experimental evidence of this effect, and Dillenberger and Sadowski (2012) for a theoretical account.

believing instead that he has middling taste; the cost of sending a signal of refinement—choosing h—is now too high.

Anticipating his contrived wine selection, it is reasonable that Slothrop choose D^m of the three restaurants. The addition of m to D^l allows him to manipulate his image, appearing to have discriminating tastes, without sacrificing too much in terms of personal consumption value. Further adding h eliminates this interpretation of choosing m, causing him to revert his choice. ∎

As the above example shows, the preferences of an image conscious DM will be reflected in his preference over decision problems themselves. There is a clear connection between Sothrop's choice reversal when choosing *from* D^m and D^h and his choice *between* them. Notice that at both restaurants, he could choose l, effecting the same image (that he is a cheapskate) and the same physical consumption. The fact that he did not choose l from D^m signals that he must prefer his induced outcome, and so, from the ex-ante perspective, he must prefer D^m to D^h. Choice reversals across different menus reveal the DM's value of images.

If arbitrary consumption objects and images could be paired together—that is, for all consumption objects x and y and images I and J, the modeler knew whether the DM preferred (x, I) to (y, J) or vice-versa—then understanding the DM's image concerns would be a simple application of existing decision theoretic tools. However, there is a strict geometric dependence between images and choice problems results in an observability problem; in the example, Slothrop is never able to choose a cheap wine while looking refined or expensive wine while looking cheap.[3] In what follows, I show that this observability problem can be overcome: a modeler can identify the DM's consumption and image utility by examining only his observable actions.

The model. I take consumption objects to be elements of the n-dimensional Euclidian space. This generality permits the instantiation that consumption objects are lotteries, Anscombe-Aumann acts, social allocations regarding multiple agents, multi-attribute objects, etc. The DM entertains a linear utility function over consumption objects $u : \mathbb{R}^n \to \mathbb{R}$. In the absence of image concerns, the DM's preference over consumption is represented by u. A *choice problem* is a finite set of consumption objects: $D \subset \mathbb{R}^n$. When an observer sees the DM choose x from a second stage choice problem, D, she believes that the DM's utility function was maximized by x: the DM's image is therefore

$$I_D^x = \{u : \mathbb{R}^n \to \mathbb{R} \mid u \text{ is linear}, u(x) \geq u(y) \text{ for all } y \in D\}.$$

As a starting point, I take as the primitive a pair of choice functions: a first stage choice function, $\mathscr{C}_1$, defined on sets of choice problems and a second stage choice function, $\mathcal{C}_2$,

[3]That this dependence is geometric will follow from the assumption that images are collections of linear functionals maximized at the chosen point—see figure 1.

defined on choice problems themselves. To continue with the language of the example, $\mathscr{C}_1$ indicates the DM's choice of restaurant and $\mathcal{C}_2$ his choice of wine from the restaurant chosen at the first stage. The interpretation is that the first stage choice problem (choosing which restaurant to patronize) is not observed by the parties the DM cares about impressing. The second stage choice (choosing which wine, at the restaurant) is public, so the DM takes into account not only his consumption value but how his choice will be interpreted by other parties. Because of observational concerns, I later consider a variant of the model where *only* second stage choices are observed by the modeler; this is discussed shortly.

Letting $\mathbb{I}$ denote the set of all possible images (collections of linear utility functions), the DM's image concerns are represented by the function $\Gamma : \mathbb{I} \to \mathbb{R}$. Thus, the DM's choice from D, when observed by someone whose opinion he cares about, is given by

$$\mathcal{C}_2(D) = \operatorname*{arg\,max}_{x \in D} \big(u(x) + \Gamma(I_D^x)\big). \tag{C2}$$

In the first stage, when considering which restaurant to go to, the DM anticipates his image consciousness, and therefore seeks to maximize his eventual utility as given by (C2). Hence, his choice from a collection of choice problems, $\mathcal{M} = \{D_1, \dots D_k\}$ is given by

$$\mathscr{C}_1(\mathcal{M}) = \operatorname*{arg\,max}_{D \in \mathcal{M}} \Big(\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big)\Big), \tag{C1}$$

The pair $\langle u, \Gamma \rangle$ forms an *image conscious (IC) representation* if $\mathscr{C}_1$ and $\mathcal{C}_2$ jointly satisfy (C1) and (C2).

When both $\mathscr{C}_1$ and $\mathcal{C}_2$ are accessible to the modeler, the axiomatic characterization of image conscious choice is remarkably simple. Beyond the standard axioms to achieve a linear representation over consumption, only one additional restriction is needed. This key axiom, called *Image Consistency*, ties together the two stages of choice. Let D and D' be two choice problems containing x and such that choosing x induces the same image, I, from either choice problem. *Image Consistency* states that if x is chosen from D' in the second stage, then D must be weakly preferred to D' in the first stage. The intuition is a simple revealed preference argument, as exhibited by Slothrop's preferences in the example. Since x is chosen from D', the value of the menu is $u(x)+\Gamma(I)$, and since x *could have been* chosen from D the value must be at least $u(x) + \Gamma(I)$. Anticipating this, the DM must prefer D.

IC without First Stage Choices. If first stage choices are *completely* private, they are by definition unobservable to the modeler. Even when theoretically observable by the modeler, in many contexts first stage choice problems will pervasively be degenerate. For example, a businessman who cares about his professional image might routinely find himself responsible for choosing the wine at restaurants chosen by clients. Because of this observational hurdle, Section 5 considers a variant of the model where only $\mathcal{C}_2$ is observable.

I provide axioms equivalent to an IC representation, but uniqueness is no longer ensured (in fact, full identification is *never* possible). To understand how partial identification is still possible, recall how, in the example, choice reversals indicated a preference for one image over another. This inference relies only on second stage choices. Further, mixing a menu with a fixed alternative changes the dispersion of utility across the menu without changing the possible images. Then, by identifying the point at which choice reversals happen, the modeler can identify the relative difference (in utility terms) between images.

Not all images can be compared in this manner, even indirectly; the resulting representation is not unique. In the two stage model, the observational limitation imposed by the geometry of images is fully overcome. This is no longer true when only first stage choices are considered by the modeler.

Organization. The next section outlines the requisite notation. Then, Section 3 provides the axiomatization and representation theorem for image conscious choice under the assumption that both stages of choice are observable. This assumption is relaxed in Section 5, which provides a representation when only the second stage is observed. Literature is reviewed at the end, in Section 6.

2 Notation and Primitives

Notation. Consumption takes place in $\mathbb{R}^n$ for $n \geq 2$. Let $\mathcal{D}$ denote the set finite non-empty subsets of $\mathbb{R}^n$, referred to as stage 2 choice problems (2CPs). The topology on $\mathcal{D}$ is induced by the Hausdorff metric, $d_h^{\mathcal{D}}$. Let $\mathscr{M}$ denote the set of all finite non-empty subsets of $\mathcal{D}$, refereed to as stage 1 choice problems (1CPs). Endow $\mathscr{M}$ with the associated Hausdorff metric, $d_h^{\mathscr{M}}$. For any $k \in \mathbb{N}$ let $\mathcal{D}^k = \{D \in \mathcal{D} \mid \#[D] = n\}$ denote the set of 2CPs with n elements.

Our primitive is a pair $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, where $\mathscr{C}_1$ is a choice function over $\mathscr{M}$ (i.e., $\mathscr{C}_1 : \mathscr{M} \to \mathscr{M}$ such that $\mathscr{C}_1(\mathcal{M}) \subseteq \mathcal{M}$ for all $\mathcal{M} \in \mathscr{M}$) and $\mathcal{C}_2$ is a choice function over $\mathcal{D}$ (i.e., $\mathcal{C}_2 : \mathcal{D} \to \mathcal{D}$ such that $\mathscr{C}_1(D) \subseteq D$ for all $D \in \mathcal{D}$). The interpretation is that of two stage choice; in the first stage the DM faces a 1CP, $\mathcal{M}$, (a set of 2CPs) from which to choose the next periods constraint. Her first stage choice is given by $\mathscr{C}_1(\mathcal{M})$. In the next period she faces one of the acceptable 1CPs $D \in \mathscr{C}_1(\mathcal{M})$, from which she must make a choice of consumption object. Her second stage choice is given by $\mathcal{C}_2(D)$.[4]

For sets $D, D' \in \mathcal{D}$ and $\lambda \in \mathbb{R}$, define $\lambda D + \lambda' D' = \{\lambda x + \lambda' x' \mid x \in D, x' \in D'\}$. Likewise, for sets $\mathcal{M}, \mathcal{M}' \in \mathscr{M}$ and $\lambda \in [0,1]$, define $\lambda \mathcal{M} + \lambda' \mathcal{M}' = \{\lambda D + \lambda' D' \mid D \in \mathcal{M}, D' \in \mathcal{M}'\}$. For any $D \in \mathcal{D}$ denote by $UC(D)$ the upper contour set of D with respect to $\mathscr{C}_1$: $UC(D) = \{D' \in \mathcal{D} \mid D' \in \mathscr{C}_1(\{D, D'\})\}$, the set of all decision problems chosen over x. Define the lower contour set, $LC(D)$, in dual fashion $LC(D) = \{D' \mid D \in UC(D')\}$.

Utilities and Representation. For each $u \in \mathbb{R}^n$, u defines a linear representation (i.e., expected utility function) over $\mathbb{R}^n$. This is via the obvious duality which views u as the

[4] In general, flat font face is used for consumption objects (i.e., x, D), calligraphic lettering is used second stage choice objects (i.e, $\mathcal{M}, \mathcal{D}, \mathcal{C}_2$) and script lettering is used for first stage choice objects (i.e. $\mathscr{M}$, $\mathscr{C}_1$).

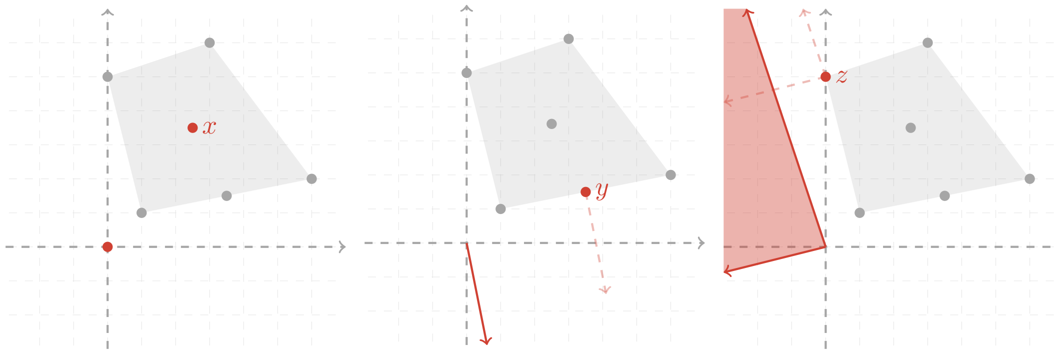

Figure 1: I_D^w for $w \in \{x, y, z\}$ and the same D. The image is shown in red. The dotted lines, representing the image shifted to the chosen object, highlight the geometric dependence between choice problems and images.

function taking x to its inner product with u. When a decision maker makes a public choice, she receives utility directly from consumption, but also, from her *image*—the set of utilities that observers believe she might have. Of course, choices over lotteries can only reveal utilities up to affine transformations, so we identify utilities which are rescalings of one another: call $I \subset \mathbb{R}^n$ an *image* if I is convex and $\lambda I \subseteq I$ for all $\lambda > 0$. Let $\mathbb{I}$ denote the set of all images.

Given any $D \in \mathcal{D}$ and $x \in D$ we denote by $I_D^x \in \mathbb{I}$ the set of utilities such that x would maximize u given D. We have

$$I_D^x = \{u \in \mathbb{R}^n \mid u(x) \geq u(y), \text{ for all } y \in D\}.$$

Notice that I_D^x depends only on the convex hull of D, so we can view it as an operation on convex sets, where it is referred to in the general literature as the *normal cone* of x in D. An *image valuation* is a function $\Gamma : \mathbb{I} \to \mathbb{R} \cup \{-\infty\}$ such that $\{\Gamma(I) \mid I = I_D^x, x \in D\} \neq \{-\infty\}$ for all $D \in \mathcal{D}$.

With these definitions in place, we can formally state the definition of an image conscious representation.

Definition. An *image conscious representation* of $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is a pair $\langle u, \Gamma \rangle$ where $u \in \mathbb{R}^n$ and Γ is an image valuation such that

$$\mathcal{C}_2(D) = \operatorname*{arg\,max}_{x \in D} \big(u(x) + \Gamma(I_D^x)\big) \qquad \text{and} \tag{C2}$$

$$\mathscr{C}_1(\mathcal{M}) = \operatorname*{arg\,max}_{D \in \mathcal{M}} \Big(\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big)\Big), \tag{C1}$$

for all $D \in \mathcal{D}$ and $\mathcal{M} \in \mathscr{M}$.

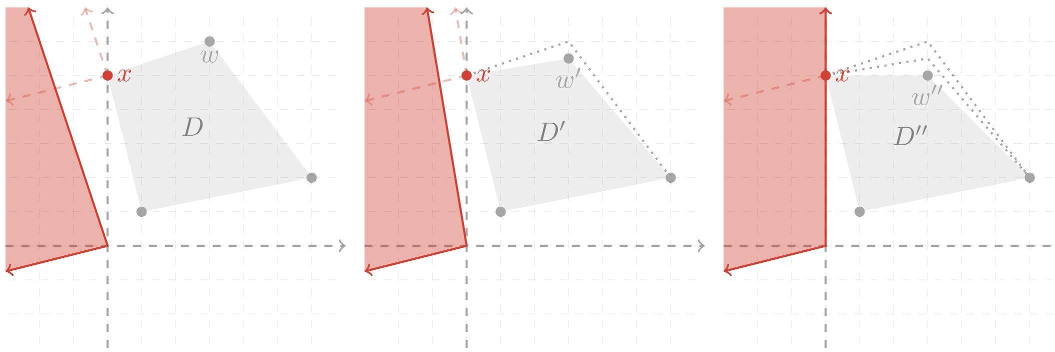

Figure 2: When the element w is replaced with w', then w'' the image associated with choosing x expands. Under each replacement, the new choice problem lies within the convex hull of the previous ones. Hence: $I^x_D \subset I^x_{D'} \subset I^x_{D''}$

3 Two Stage Image Conscious Choice

Because the DM makes the first stage privately, she has a more standard preference in the first stage. Specifically, there are no context effects, and so the DM will have a well defined value function over decision problems. When considering the class of degenerate (i.e., singleton) choice problems, every second stage choice, by nature of being degenerate, instills the same image. Hence the DM's consideration of such problems depends only on her consumption utility.

Axiom 1—Singleton Expected Utility. There exists a value function, $V : \mathcal{D} \to \mathbb{R}$ such that

$$\mathscr{C}_1(\mathcal{M}) = \big\{D \in \mathcal{M} \mid D \in \arg\max_{\mathcal{M}} V(D)\big\},$$

where $u : x \mapsto V(\{x\})$ is a linear function over $\mathbb{R}^n$.

A1 imposes a lot of structure and is admittedly somewhat divorced from the observable primitive. Because of this, Appendix A contains a (rather straightforward if uninspiring) set of axioms directly on the primitive, equivalent to A1.

The next axiom ensures that an image depends only on the difference between what was chosen and what could have been, and not the aggregate level of consumption. In other words, changing the baseline level of consumption by shifting all consumption alternatives by a constant amount does not distort the induced image set.

Axiom 2—Translation Invariance. For all $x \in \mathbb{R}^n$, $\mathcal{M} \in \mathscr{M}$ and $D \in \mathcal{D}$,

$$\mathscr{C}_1(\mathcal{M} + \{\{x\}\}) = \mathscr{C}_1(\mathcal{M}) + \{\{x\}\} \text{ and}$$
$$\mathcal{C}_2(D + \{x\}) = \mathcal{C}_2(D) + \{x\}$$

What remains is to ensure the value function V reflects image consciousness. In particular, we want to show that the gap between $V(D)$ and $u(\mathcal{C}_2(D))$ is dependent only on the induced image.

When $I^x_D = I^x_{D'}$, then choosing x from D or D' induces the same perception in an observer; hence, the DM should find choosing x from D exactly as appealing as choosing x from D'. Thus, if x is chosen from D but not chosen from D' it must mean that whatever is chosen from D' is even better than consuming x with image I^x_D. From a period 1 perspective, this indicates that D' is preferred to D. The following axiom embodies exactly this logic.

Axiom 3—Image Consistency. Assume $I^x_D = I^x_{D'}$ and $x \in \mathcal{C}_2(D)$. Then $x \in \mathcal{C}_2(D')$ if and only if $D \in \mathscr{C}_1(\{D, D'\})$

Image consistency, along with the linear structure imposed by A1 and A2, is enough to ensure the existence of an image conscious representation.

3.1 Representation Result

Theorem 3.1. *The following are equivalent:*

1. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *satisfies* A1–3*,*
2. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *satisfies* A2–5*,*
3. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *has an image conscious representation* $\langle u, \Gamma \rangle$*.*

Moreover, u *is unique up to positive linear translations, and* Γ *is unique up to an additive constant on its effective domain.*

Proof. The equilence between (1) and (2) is given by Lemma 2. That (3) implies (1) is standard. We will show that (1) implies (3). Let $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfy A1 and A3. Take u and V as defined in A1. For each $I \in \mathbb{I}$, define $\Gamma(I)$ to be

$$\Gamma(I) = \begin{cases} V(D) - u(x) & \text{if there exists } (D, x) \text{ with } x \in \mathcal{C}_2(D) \text{ and } I^x_D = I \\ -\infty & \text{otherwise.} \end{cases} \tag{3.1}$$

We here verify that Γ is well defined and an image valuation. Let (D, x) and (D', x') be such that $I^x_D = I^{x'}_{D'}$ and $x \in \mathcal{C}_2(D)$ and $x' \in \mathcal{C}_2(D')$. By A2 it is without loss of generality to assume $x = x'$. By A3 it must be that $V(D) = V(D')$. This of course implies that $V(D) - u(x) = V(D') - u(x)$, so that Γ is well defined. Since $\mathcal{C}_2$ is non-empty, for at least one $x \in D$, $\Gamma(I^x_D) \neq -\infty$; Γ is an image valuation.

Next, we claim that for all D we have $\mathcal{C}_2(D) = \arg\max_{x \in D} u(x) - \Gamma(I^x_D)$. First, assume that $x \in \mathcal{C}_2(D)$ and let $y \in D$. Since $x \in \mathcal{C}_2(D)$ it follows that $\Gamma(I^x_D) \neq -\infty$, so if $\Gamma(I^y_D) = -\infty$, we have $u(x) + \Gamma(I^x_D) \geq u(y) + \Gamma(I^y_D)$ immediately. So, to make the problem extra hard, assume $\Gamma(I^y_D) \neq -\infty$. It must be there exists a (D', y') such that $I^{y'}_{D'} = I^y_D$ with

$y' \in \mathcal{C}_2(D')$. By A2, we can choose $y' = y$, so that $I^y_{D'} = I^y_D$ and that $y \in \mathcal{C}_2(D')$. Thus, we can conclude that $V(D) \geq V(D')$. Appealing to (3.1) implies that

$$u(x) + \Gamma(I^x_D) = V(D) \geq V(D') = u(y) + \Gamma(I^y_{D'}) = u(y) + \Gamma(I^y_D).$$

Next assume that $x \in \arg\max_{x \in D} u(x) - \Gamma(I^x_D)$ but $x \notin \mathcal{C}_2(D)$. Let $y \in \mathcal{C}_2(D)$. This implies $\Gamma(I^y_D) \neq -\infty$, and so also that $\Gamma(I^x_D) \neq -\infty$. Thus, there exists a (D', x) such that $I^x_{D'} = I^x_D$ with $x \in \mathcal{C}_2(D')$; since $x \in \mathcal{C}_2(D')$ and $x \notin \mathcal{C}_2(D)$ we can conclude via A3 that $V(D) > V(D')$. Hence, by the definition of Γ,

$$u(y) + \Gamma(I^y_D) > u(x) + \Gamma(I^x_{D'}) = u(x) + \Gamma(I^x_D),$$

another clear contradiction. ∎

4 Attitudes Towards Information Revelation

A DM who has a *preference for revelation* wants to reveal the motivations behind his actions: instilling in the observer, the maximal information about his preference. For example, a generous, if vain, DM who wishes to make it known that he is generous. Conversely, a DM who has a *preference for concealment* prefers, all else equal, to limit the inference an observer can make regarding his underlying motivations. For example, the selfish DM who wishes to take selfish actions without tarnishing his reputation. Formally:

Definition. A DM has a *preference for revelation* if $I \subset J$ implies that $\Gamma(J) \leq \Gamma(I)$, and has a *preference for concealment* if $I \subset J$ implies that $\Gamma(I) \leq \Gamma(J)$.

There is an interesting link between attitudes towards the revelation of information and classical axioms placed on menu preferences. In particular, set betweenness, which plays a key role in Dillenberger and Sadowski (2012), arises from a preference for concealing information, whereas Krepsian flexibility from a preference for revealing information.

Theorem 4.1. *Assume that $\langle u, \Gamma \rangle$ represents $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, then if a DM has a preference for revelation then she displays a preference for flexibility: $D \cup D' \in \mathscr{C}_1\{D, D \cup D'\}$. Conversely, he has a preference concealment then she displays set betweenness: $D \in \mathscr{C}_1\{D, D'\}$ implies $D \in \mathscr{C}_1\{D, D \cup D'\}$.*

Proof. Assume that $\langle u, \Gamma \rangle$ represents $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$. Assume the DM has a preference for revelation. Notice that for all $D, D' \in \mathcal{D}$ and $x \in D$, $I^x_{D \cup D'} \subseteq I^x_D$, hence, by the definition of a preference for revelation,

$$V(D \cup D') \geq \max_{x \in D} u(x) + \Gamma(I^x_{D \cup D'}) \geq \max_{x \in D} u(x) + \Gamma(I^x_D) = V(D).$$

The proof for concealment is parallel and so omitted. ∎

If a DM (globally) displays both a preference for concealment and revelation, then every image is valued equally and the DM is a classical expected utility decision maker over 2CPs so that each menu is indifferent to its maximizer. However, at evidenced by the Example in the introduction, these attitudes towards information are generally not held globally; the DM sometimes (strictly) wants to reveal information about his motivation, and sometimes (strictly) wants to conceal it. In the example, Slothrop values the addition of a medium priced wine because it allows him to send a signal placing him in a favorable light. He dislikes the further addition of a high priced wine, though, since its existence places bounds on his urbanity should he continue to choose the medium quality wine. As such, he prefers $\{l, m\}$ to both $\{m\}$ and $\{l, m, h\}$—a violation of of both global preferences to information.

4.1 A Simple Model of Signaling and Altruism

To better understand how a preference for concealment/revelation arises in a natural economic environment, we will consider a simple model where a DM chooses between payoff vectors for himself and for another agent. Thus, the consumption objects are points in $\mathbb{R}^2$. The DM cares about both payoffs but cares weakly more about his own; furthermore, the DM, independently of how much he actually cares about his partner's payoff, prefers to appear altruistic.

We can normalize the utility functions to be for the form:

$$u(x) = x_1 + \beta x_2,$$

and we will further make the assumption that $\beta \in [0, 1]$ with the two extremes indicating purely selfish and purely utilitarian preferences.[5]

Because of this, we can think of each image I as an interval of possible β's. Now, how does the DM value I? First, if the interval of an image does not intersect $[0, 1]$, assume it confers irrationality, and is valued at $-\infty$. For the rest of the images, we consider two simple options: first, that the intervals are ordered according to their lower-bounds (intersect the unit interval), and second according to their upper-bounds (again, intersect the unit interval). The former corresponds to a cynical observer, who believes the DM is as selfish as possible within the set of preferences consistent with the observed behavior; the latter to a charitable observer who believes the DM is as altruistic as is consistent with the observed behavior.[6]

In the 'lower-bound' model, the DM has a preference for revelation, since the absence of information is interpreted as selfishness. This can be easily seen by the fact that if $I \subset J \subset [0, 1]$ then $\min\{\beta \in J\} \leq \min\{\beta \in I\}$. Of course, the converse logic applies to the 'upper-bound' model, where the DM displays a preference for concealment; the DM is only considered selfish when he explicitly acts selfishly. This latter model is a special case of Dillenberger and Sadowski (2012) where the DM's *norm* is to care equally about his own

[5]The obvious monotonicity axiom on singletons ensures that the coefficients are positive and the further restriction that $\{x_1 + z, x_2\} \in \mathscr{C}_1(\{\{x_1 + z, x_2\}, \{x_1, x_2 + z\}\})$ for any $z > 0$, ensures that $\beta \leq 1$.

[6]These restrictions also can be recast axiomatically over choices: the 'lower-bound' model requires that only the lower bound of the image matters, hence given $x \in \mathcal{C}_2(D)$ the value of D depends only on (and is increasing in) the anti-clockwise adjacent element of $\text{ext}(D)$ to x. In the 'upper-bound' model, the value depends only on the clock-wise adjacent element.

and the other agent's payoffs.

4.2 Comparative Image Consciousness

Consider two different IC DMs: $\langle \mathscr{C}_1^i, \mathcal{C}_2^i \rangle$ and $\langle \mathscr{C}_1^j, \mathcal{C}_2^j \rangle$. We are interested in understanding how the image concerns of the two decision makers relate to one another. In this section, we consider IC representations such that $\Gamma(I) > -\infty$ if and only there exists a (x, D) such that $x \in \mathcal{C}_2(D)$ and $I_D^x = I$. Call the set of such images the induced images.

Definition. Say that i and j have the *same image ranking* if (i) the set of induced images is the same for i and for j, and (ii) for all $D_i, D_i', D_j, D_j' \in \mathcal{D}$, such that $I_{D_i}^x = I_{D_j}^x$ and $I_{D_i'}^x = I_{D_j'}^x$ and x is chosen from all four 2CPs, then

$$D_i \in \mathscr{C}_1^i(\{D_i, D_i'\}) \iff D_j \in \mathscr{C}_1^j(\{D_j, D_j'\}).$$

When i chooses x from D_i, it imparts the same image as when j choses x from D_j, likewise, when the x is chosen from D_i' and D_j'. Since the consumption utility is invariant across the two choices, each DM's ranking of the primed and un-primed 2CPs depends only on his ranking of the induced images. Since the rankings coincide, the DMs rank images identically. The first part of the definition ensures that every pair of images that can be compared in this manner by i can also be compared by j.

Theorem 4.2. *$\langle u^i, \Gamma^i \rangle$ and $\langle u^j, \Gamma^j \rangle$ represent preferences which have the same image ranking if and only if Γ^i is an strictly increasing transformation of Γ^j.*

Proof. Assume i and j have the same image rankings and let $\Gamma^i(I) \geq \Gamma^i(I') > -\infty$. Then there exists some $(x_i, D_i), (x_i', D_i')$ such that $x_i \in \mathcal{C}_2^i(D_i)$, $x_i' \in \mathcal{C}_2^i(D_i')$ and $I_{D_i}^{x_i} = I$ and $I_{D_i'}^{x_i'} = I'$. Since i and j have the same induced images there also exists some analogous $(x_j, D_j), (x_j', D_j')$. By translation invariance, we can choose $x_i = x_i' = x_j = x_j'$. Since consumption utility is constant for each DM, we have

$$\begin{aligned} \Gamma^i(I) \geq \Gamma^j(I') &\iff V^i(D_i) \geq V^i(D_i') \\ &\iff V^j(D_j) \geq V^j(D_j') \iff \Gamma^j(I) \geq \Gamma^j(I'). \end{aligned}$$

The other direction is immediate. ∎

With this definition in place, we can now discuss when DM i is more or less sensitive to image effects than j. Of course, such a definition only has real bite when the DMs also entertain the same image ranking, and the same ranking of consumption alternatives.

Definition. Say that i is *more image conscious* than j if (i) i and for j have the same image ranking, and (ii) $\mathscr{C}_1^i$ and $\mathscr{C}_1^j$ coincide on $\mathcal{D}^1$ and (iii) if $\{y\} \in \mathscr{C}_1^i(\{\{x\}, \{y\}) = \mathscr{C}_1^j(\{\{x\}, \{y\})$ and $D_i, D_i', D_j, D_j' \in \mathcal{D}$, such that $I_{D_i}^x = I_{D_j}^x$ and $I_{D_i'}^y = I_{D_j'}^y$ and $x \in \mathcal{C}_2^i(D_i) \cap \mathcal{C}_2^j(D_j)$ and $y \in \mathcal{C}_2^i(D_i') \cap \mathcal{C}_2^j(D_j')$, then

$$D_j \in \mathscr{C}_1^j(\{D_j, D_j'\}) \implies D_i \in \mathscr{C}_1^i(\{D_i, D_i'\}).$$

If i is more image conscious than j, then he cares relatively more about changes in image utility than does j. In the definition, since y is preferred to x, the fact that D_j is chosen by j—so y is consumed—indicates that he finds the image utility more than makes up for gap in consumption utility. Since i must also make the same choices, it must be that he also finds the image utility sufficient to overcome the gap in consumption utility. As expected, this increases sensitivity can be captured by the relation that Γ^i is "more spread out" than Γ^j.

Theorem 4.3. *Let $\langle u^i, \Gamma^i \rangle$ and $\langle u^j, \Gamma^j \rangle$ represent preferences such that i is more image conscious than j then $u^i = u^j$ and*

$$|\Gamma^i(I) - \Gamma^i(I')| > |\Gamma^j(I) - \Gamma^j(I')|$$

for all $I, I' \in \mathbb{I}$.

Proof. Let i be more image conscious than j. That $u^i = u^j$ is immediate, so call the joint representation u. Choose some induced images $I, I' \in \mathbb{I}$ and without loss of generality, assume that $\Gamma^j(I) \geq \Gamma^j(I')$. Choose some $x, y \in \mathbb{R}^n$ such that $\Gamma^j(I) - \Gamma^j(I') = u(y) - u(x) \geq 0$. Then by translation invariance we can find some $D_i, D_i', D_j, D_j' \in \mathcal{D}$ such that $I^x_{D_i} = I^x_{D_j} = I$ and $I^y_{D_i'} = I^y_{D_j'} = I'$ and $x \in \mathcal{C}^i_2(D_i) \cap \mathcal{C}^j_2(D_j)$ and $y \in \mathcal{C}^i_2(D_i') \cap \mathcal{C}^j_2(D_j')$.

By our assumption we have $V^j(D_j) = u(x) + \Gamma^j(I) = u(y) + \Gamma^j(I') = V(D_j')$. Therefore, since i is more image conscious than j, we have also that $u(x) + \Gamma^i(I) = V^i(D_i) \geq V^i(D_i') = u(y) + \Gamma^i(I')$. Rearranging yields, $\Gamma^i(I) - \Gamma^i(I') \geq u(y) - u(x) = \Gamma^j(I) - \Gamma^j(I')$, as desired. The other implication follows from a similar line of reasoning, and is omitted. ∎

5 Single Stage Image Conscious Choice

The interpretation of two stage choice is that $\mathscr{C}_1$ represents a choice over 2CPs that is made in the absence of image concerns. Hence, in many scenarios, this choice function will not be observable. This section considers the image conscious model when only second stage choice is accessible to the modeler; it posits axioms only on $\mathcal{C}_2$ equivalent to (C2) of the IC representation.

Limited observability bears a cost. First, the effective uniqueness of Γ is no longer ensured. Second, the axiomatic structure and concomitant proof rely more directly on technical assumptions, and so, are correspondingly more involved. This latter point is self evident, but to understand the failure of uniqueness, consider the following.

Say $I, J \in \mathbb{I}$ are *directly* comparable if there is a D such that $I = I^x_D$, $J = I^y_D$ and either x or y is in $\mathcal{C}_2(D)$. When I and J are directly comparable (and, say x is chosen), we have an bound on the utility difference between I and J in terms of consumption utility:

$$\Gamma(I) - \Gamma(J) \geq u(y) - u(x).$$

Say that $I, J \in \mathbb{I}$ are *indirectly comparable* if they are contained in the transitive closure of the direct comparability relation. If two images are not indirectly comparable, then there is no restriction imposed by the observed choices on the relative values of the images. Indirect comparability is an equivalence relation; Γ in the resulting representation normalized independently across the classes of this equivalence relation.

5.1 Axioms

Scaling a choice problem may result in non-linear tradeoffs. As λ increases, choice from λD places more importance on consumption utility. The first axiom allows $\mathcal{C}_2(\lambda D)$ to vary non-linearly in λ, but ensures that deviations are consistent with increasing importance on consumption utility.

Axiom 1°—Scale Acyclicity. Let $0 < \lambda < \lambda' < \lambda''$ and $D \in \mathcal{D}$. If $x \in \frac{1}{\lambda}\mathcal{C}_2(\lambda D) \cap \frac{1}{\lambda''}\mathcal{C}_2(\lambda'' D)$ then $x \in \frac{1}{\lambda'}\mathcal{C}_2(\lambda' D)$.

In the limit, as $\lambda \to \infty$, only consumption utility matters. Indeed, this is the manner in which u might be identified. To get at this, we can define the following map, which is well defined given A1° and the finiteness of each D:

$$\mathcal{C}_2^\infty : D \mapsto \lim_{\lambda\to\infty} \frac{1}{\lambda}\mathcal{C}_2(\lambda D).$$

To identify u we need $\mathcal{C}_2^\infty$ to be well behaved; ideally this would just entail the imposition of WARP. Unfortunately, it is possible that $u(x) = u(y)$ but $y \neq \mathcal{C}_2^\infty(\{x, y\})$; this happens whenever $\Gamma(I^x_{\{x,y\}}) > \Gamma(I^y_{\{x,y\}})$. To deal with this, we impose WARP on perturbed choice problems.

Axiom 2°—Sequential Limit Consistency. Let $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converge to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all $k \in \mathbb{N}$. Then for any D' with $y \in \mathcal{C}_2^\infty(D')$ there exists a sequence $D'_k \to D'$ such that $x \in \mathcal{C}_2^\infty(D'_k \cup \{x\})$ for all k.

Translation invariance (A2) remains, but is transcribed here for completeness.

Axiom 3°—Translation Invariance. For all $x \in \mathbb{R}^n$ and $D \in \mathcal{D}$,

$$\mathcal{C}_2(D + x) = \mathcal{C}_2(D) + x$$

With these three axioms, u can be identified.

Lemma 1. *If $\mathcal{C}_2$ satisfies A1°—3° then there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ such that*

$$\mathcal{C}_2^\infty(D) \subseteq \arg\max_D u.$$

Moreover, u is unique up to positive linear transformations.

Proof. In section B.1. ∎

From $\mathcal{C}_2$ we can define $\succcurlyeq \subset (\mathbb{R}^n \times \mathbb{I}) \times (\mathbb{R}^n \times \mathbb{I})$ via $(x, I) \succcurlyeq (y, J)$ iff there exists a $D \supseteq \{x, y\}$ with $I^x_D = I$ and $I^y_D = J$, and such that $x \in \mathcal{C}_2(D)$. The next axioms place restrictions on $\succcurlyeq$

but these can be translated back into choice behavior in the obvious, but tedious, manner. Per normal let $\succ$ and $\sim$ denote the asymmetric and symmetric components.

The relation $\succcurlyeq$ will necessarily be highly incomplete; for example, images with overlapping relative interiors will never be comparable. Because of this, $\succcurlyeq$ will not be transitive; it should, however, be extendable to a transitive relation.

Axiom 4°—Acyclicity. $\succ$ is acyclic.

Finally, we impose three restrictions that relate the choice over $\succcurlyeq$ to the consumption utility as identified by Lemma 1: *monotonicity* states that if $(x, I) \succcurlyeq (y, J)$ and $u(x') > u(x)$ then not $(y, J) \succ (x', I)$—ceteris paribus, more consumption is better; *boundedness* states that $(x, I) \succ (y, J)$ cannot hold for all x—I cannot be 'infinitely' better than J; *continuity* states that if $u(x_n) \to u(x)$ and $(x_n, I) \succcurlyeq (y, J)$ for all n, then not $(y, J) \succ (x, I)$—preferences cannot be reversed in the limit.

In the proof of the representation theorem, we will extend $\succcurlyeq$ to a complete binary relation, showing these properties still hold; as such, it is helpful to define things for a general relation R defined over $(\mathbb{R}^n \times \mathbb{I})$.

Definition. Let $v : \mathbb{R}^n \to \mathbb{R}$. Call a relation R (with asymmetric component S) defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-monotone* if whenever

1. $v(z) > 0$ and $(x, I)R(y, J)$, or,
2. $v(z) \geq 0$ and $(x, I)S(y, J)$,

then not $(y, J)R(x + z, I)$.

Definition. Let $v : \mathbb{R}^n \to \mathbb{R}$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-bounded* if for all $I, J \in \mathbb{I}$, it is true that $\inf\{v(z) \mid (z, I)R(\mathbf{0}, J)\} > -\infty$.

Definition. Let $v : \mathbb{R}^n \to \mathbb{R}$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-continuous* if for all $I, J \in \mathbb{I}$, if whenever there exists a sequence $\{x_k\}_{k \in \mathbb{N}}$ such that $v(x_n) \to 0$ and $(x_k, I)R(y, J)$ for all k, then for any x with $v(x) = 0$, if $(y, J)R(x, I)$ then $(x, I)R(y, J)$.

Let $\succcurlyeq^{TC}$ denote the transitive closure of $\succcurlyeq$.

Axiom 5°—Consumption Regularity. The relation $\succcurlyeq^{TC}$ is u-monotone, u-continuous, and u-bounded.

These axioms are equivalent to the existence of a image conscious representation $\langle u, \Gamma \rangle$ which represents $\mathcal{C}_2$ as (C2).

Theorem 5.1. *The following are equivalent:*

1. *$\mathcal{C}_2$ satisfies* A1°—5°

2. $\mathcal{C}_2$ has an image conscious representation $\langle u, \Gamma \rangle$.

Moreover, u is unique up to positive linear translations and Γ is unique up-to an additive constant within each equivalence class generated by the indirect comparability relation.

Proof. In section B.2. ∎

In contrast to the proof of Theorem 3.1, the proof of Theorem 5.1 is rather involved. The main difficulty surrounds the intrinsic incompleteness of the induced preference relation on $\mathbb{R}^n \times \mathbb{I}$, owing to the geometric dependence between the set of consumption alternatives and the consequent images. Indeed, imagine that some complete $\succcurlyeq^\star$ over $\mathbb{R}^n \times \mathbb{I}$ was magically identified and preserved the relevant structure and extended $\succcurlyeq$. Then, fixing $I^* \in \mathbb{I}$ and setting $\Gamma(I^*) = 0$, we can recover the entirety of Γ is by simply setting:

$$\Gamma : I \mapsto -u(x^I)$$

where x^I is a consumption alternative such that $(x^I, I) \sim (\mathbf{0}, I^*)$. Such an alternative exists by the u-boundedness and u-continuity assumptions, and its utility is unique by u-monotonicity. Translation invariance and transitivity then ensure the resulting $\langle u, \Gamma \rangle$ actually represents $\succcurlyeq^\star$, and hence $\mathcal{C}_2$.

Guaranteeing that $\succcurlyeq$ can be extended to a complete $\succcurlyeq^\star$ (while preserving the axiomatic structure) turns out to be pain, but mostly for technical reasons. The relatively simple core idea is as follows: we can first extend $\succcurlyeq$ by adding comparisons that were not observed by $\mathcal{C}_2$ but must hold because of transitivity, monotonicity, or continuity. The resulting relation extends $\succcurlyeq$ because of A4° and A5°. Still, there will be images I and J such that no x satisfies $(x, I) \sim (\mathbf{0}, J)$. What can we do? Just pick some x and extend the relation by adding $(x, I) \sim (\mathbf{0}, J)$ (and then again adding all the consequences of transitivity, monotonicity, or continuity). Repeating the process for different I's and J's creates a partial order of extensions of $\succcurlyeq$, which, by Zorn's Lemma, has maximal element that must be complete.[7]

This process also elucidates the exact nature of non-uniqueness. If two images are initially comparable, that is there exists an x and y such that $(x, I) \sim (y, J)$ is implied by the initial choice function, then the difference between $\Gamma(I)$ and $\Gamma(J)$ is identified (up to a common normalization) by the difference between $u(x)$ and $u(y)$. Thus, identification is made over the equivalence classes of initially comparable images (that comparability is an equivalence relation is Lemma 5(i)), but, these equivalence classes can be independently normalized.

6 Discussion, Including a Brief Review of Related Literature

Image conscious behavior is ubiquitous and has long been studied within economics. In a work of classical importance, Veblen (1899) coined the term *conspicuous consumption*

[7]This is one of the many instantiations of Szpilrajn's extension theorem with additional structure being preserved by the extension.

referring to purchases in which the primary value is derived indirectly by signaling wealth or status. Such spending habits are alive and well in the modern era.

Recently, experimental economists and psychologists have exposed the importance of image concerns in sundry other contexts. A common theme is the discord between and individual's personal preference and his desire to be seen as acting in a normative manner: the DM faces a tradeoff between direct utility and image utility. This tradeoff is central to the present model as captured by the IC representation.

Dana et al. (2006) find that subjects in the dictator game are willing to pay a positive cost to ensure the receivers did not know the game was to be played (i.e., the dictator gets the full pie, less the cost, and the receiver is never informed there was a decision to be made). Because the decision to keep everything is always available, paying the cost serves only to effect a more desirable image. This result is echoed in Andreoni and Bernheim (2009), where subjects' choice of fair (i.e., 50-50) allocations in the dictator game depends very much on who can observer the dictators' choices. When there is a commonly known chance that unfair allocations get implemented irrespective of the dictator's choice, and these nature-chosen outcomes are indistinguishable from dictator-chosen outcomes to receivers, then the rate of fair allocations dramatically declines.

DellaVigna et al. (2012) find, in a door-to-door field experiment, that many donation decisions seem to be predicated on social pressure. When given the ability to avoid face to face contact with a solicitor, donations decrease. This effect is concentrated in small donations, an effect that is predicted by the present model. DellaVigna et al. (2016) find that social pressure plays a key role in the decision to vote; potential voters are more likely to vote when they expect that other will ask them about their voting record.

Bénabou and Tirole (2006) provide the canonical utility function for image concerned agents and explore how direct incentives to act pro-socially can have the opposite effect by skewing the images associated with certain actions. Their model is behavioral rather than decision theoretic, in the sense that they are less concerned with identification from observables, and the generality of the types of images that can be entertained. For instance, they make the assumption that actions can be linearly ordered and that everyone prefers a "higher" image to a "lower" one.

Closest to this model are the models of Dillenberger and Sadowski (2012) and Evren and Minardi (2017) who investigate the axiomatic characterizations of shame driven preferences and of *warm glow*, respectively. Although image consciousness, shame, and warm glow are all distinct phenomena, there are two major similarities between the models: (i) all can promote normative behavior and (ii) in these models, the *menu* from which an element is chosen changes the derived value from consumption. This later property was is also shared by the model of Gul and Pesendorfer (2001) who consider a DM who seeks to limit his options to curtail the effect of temptation.

The Dual-Self Interpretation of Images. It need not be that an IC DM cares about the opinion of any *third* party, but rather, the 'observer' might be him himself. We can interpret the utility of a given image as the psychological benefit/cost of adhering to or deviating from the DM's ideal preferences. For example, a DM might *want* to be a charitable person but also not want to give up on personal consumption. In situations where his hands are tied—when there is no opportunity to give—he circumvents the psychological cost of selfish behavior. But, when confronted with a choice, he must either forgo direct consumption or address his avarice.

While this story can clearly explain choice reversals at second stage choices, it may also make sense in the context of first stage choice. A completely rational and forward looking DM would understand that choosing to avoid the future option of donating money is effectively choosing not to donate, and would therefore be unable to skip out on the phycological bill. Of course, like all benchmarks of rationality, there is a growing body of evidence suggesting humans do not meet this standard; Gino et al. (2016); Grossman and Van Der Weele (2017) show that "meta-decisions"—which do not implement actions directly, but affect the decision making process; e.g., information collection—and action-implementing decisions have different effects on shaping self-images. Within the present context, the interpretation being that a phycological cost is levied only when consumption decisions are actually made, so DM might avoid situations where donating to a charity is possible, even if donating nothing is always an option.

A Axiomatization of Axiom A1

We need a value function:

Axiom 3—WARP. If $D, D' \in \mathcal{M} \cap \mathcal{M}'$, $D \in \mathscr{C}_1(\mathcal{M})$ and $D' \in \mathscr{C}_1(\mathcal{M}')$ then $D \in \mathscr{C}_1(\mathcal{M}')$.

Even if we do not insist that the preference over images is continuous, the fact that the DM's preference over consumption objects is continuous requires that when considering only degenerate 2CPs, the DM has a continuous preferences—the projection of contour sets onto $\mathcal{D}^1$ must be closed.

Axiom 4—Weak Continuity. For all $D \in \mathcal{D}$, $UC(D) \cap \mathcal{D}^1$ and $LC(D) \cap \mathcal{D}^1$ are closed and non-empty.

The non-emptiness restriction ensures that images are not lexicographically preferred to dis-preferred to one another. For example, if $UC(D) \cap \mathcal{D}^1$ was empty, then there no consumption object, no matter how good, that is preferred (along with the trivial image) to D. Because we are interested in linear utilities over consumption objects, this would indicate that the image associated with the choice from D is infinitely good.[8]

When all image concerns are obviated, we want preferences to be expected utility.

[8] If we want, in addition, the value function over images to be continuous, we can strengthen A4 as

Axiom 5—Singleton Independence. For all $\lambda \in \mathbb{R}_{++}$, $\mathcal{M}, \mathcal{M}' \subset \mathcal{D}^1$,

$$\mathscr{C}_1(\lambda\mathcal{M} + \lambda'\mathcal{M}') = \lambda\mathscr{C}_1(\mathcal{M}) + \lambda'\mathscr{C}_1(\mathcal{M}').$$

These axioms provide the scaffolding of the representation, a value function over choice problems that, when looking at degenerate problems, reflects a linear preference over consumption objects.

Lemma 2. *If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfies A3—5 if and only if it satisfies A1. (For the readers connivence: A1 imposes the existance a translation invariant value function, $V : \mathcal{D} \to \mathbb{R}$ representing $\mathscr{C}_1$ such that $u : x \mapsto V(\{x\})$ is a linear function over $\mathbb{R}^n$.)*

Proof. Necessity is standard. Towards sufficiency, consider the projection of $\mathscr{C}_1$ to $\mathcal{M} \subset \mathcal{D}^1$. Over this space, $\mathscr{C}_1$ satisfies the expected utility axioms. Therefore, there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ rationalizing the projection of $\mathscr{C}_1$.

Now consider any $D \in \mathcal{D}$. We claim that $UC(D) \cap LC(D) \cap \mathcal{D}^1$ is non-empty—the Lemma then follows directly by setting $V(D) = u(x)$ for an $\{x\}$ in the intersection. Assume the claim did not hold. Take $x \in \arg\max_{LC(D)\cap\mathcal{D}^1} u(x)$ and $y \in \arg\min_{UC(D)\cap\mathcal{D}^1} u(x)$ which exist and are distinct by A4 and our assumption. By A3 and the linearity of u, $u(x) < u(\frac{1}{2}x + \frac{1}{2}y) < u(y)$. Thus, $\{\frac{1}{2}x + \frac{1}{2}y\}$ is in neither the upper nor the lower contour set of D, a contradiction to the non-emptiness of $\mathscr{C}_1$. ∎

B Proofs Omitted From the Text

B.1 Proof of Lemma 1

Define the preference relation, $\dot{\succcurlyeq}$, on $\mathbb{R}^n$ as follows: $x \dot{\succcurlyeq} y$ if there exists a $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converging to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all k. We claim that $\dot{\succcurlyeq}$ is an expected utility preference; this would complete the lemma, for if $x \in \mathcal{C}_2^\infty(D)$ then $x \dot{\succcurlyeq} y$ by taking the constant sequence D, and hence $x \in \arg\max_D u$ for any representation of $\dot{\succcurlyeq}$.

Completeness. Fix $x, y \in \mathbb{R}^n$, and take a sequence $\{y_k\}_{k\in\mathbb{N}}$ converging to y. Since $\mathcal{C}_2$ is non-empty there exists a subsequence (w.l.o.g., indexed by the same k) such that for all k either $x \in \mathcal{C}_2^\infty(\{x, y_k\})$ or $x \notin \mathcal{C}_2^\infty(\{x, y_k\})$. If it is the former, we are done and $x \dot{\succcurlyeq} y$. If it is the latter, we can appeal to translation invariance, and for each k, shift by $y - y_k$ to obtain a sequence $\{x + y - y_k, y\}$ such that y is always chosen, so $y \dot{\succcurlyeq} x$.

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follows:

Axiom 4*—k-Weak Continuity. For all $D \in \mathcal{D}$ and $k \in \mathbb{N}$, $UC(D) \cap \mathcal{D}^k$ and $LC(D) \cap \mathcal{D}^k$ are closed.

Notice that even when the DM's utility over images is continuous, $\mathscr{C}_1$ is *not* continuous (in the relevant topology) because the map carrying a choice to its associated image is not itself continuous. To see this, notice that if $D_\lambda = \{x, y\}$ for $x \neq y$. For $\lambda \in (0,1)$, $I^{\lambda x + \lambda' x}_{\lambda D + \lambda' x} \neq \mathbb{R}^n$ whereas the limiting choice indices the image $I^x_{\{x\}} = \mathbb{R}^n$. Such complications arise whenever two elements collide in the limit, a problem which does not happen when restricting the domain to $\mathscr{M}^k$.

Transitivity. Let $x \dot{\succsim} y$ and $y \dot{\succsim} z$. Consider the choice problem $D = \{x, y, z\}$. If $x \in \mathcal{C}_2^\infty(D)$ then $x \dot{\succsim} z$ and we are done. If $y \in \mathcal{C}_2^\infty(D)$, then we can appeal to A2° to obtain a sequence $D_k \to D$ such that $x \in \mathcal{C}_2^\infty(D_k \cup \{x\})$ for all k, hence $x \dot{\succsim} z$ (notice, $x \dot{\succsim} y$ definitionally implies the antecedent for A2°). Finally, assume $z \in \mathcal{C}_2^\infty(D)$. Then by the above reasoning, we have a sequence $D_k \to D$ such that $y \in \mathcal{C}_2^\infty(D_k \cup \{y\})$. Now since $x \dot{\succsim} y$, we can, for each D_k find a further sequence $D^k_{k'} \to D_k \cup \{y\}$ such that $x \in \mathcal{C}_2^\infty(D^k_{k'} \cup \{x\})$ for all $k, k' \in \mathbb{N}$. For each $m \in \mathbb{N}$, set $\hat{D}_m$ to be the first element of $\{D^m_{k'}\}_{k' \in \mathbb{N}}$ such that $d_H(D^m_{k'} - D_m) \leq \frac{1}{m}$. This is a sequence converging to D and with x always chosen.

Continuity. Let $\{y_k\}_{k \in \mathbb{N}}$ converge to y and be such that $x \dot{\succsim} y_k$ for all k. Then by definition, we have a sequence of sequences $\{\{D^k_{k'}\}_{k' \in \mathbb{N}}\}_{k \in \mathbb{N}}$ such that $D^k_{k'} \to D_k$ for all k and $x \in \mathcal{C}_2^\infty(D^k_{k'} \cup \{x\})$. As above, we can find a sequence of sets converging to $\{x, y\}$ such that x is chosen from each. Closure of the lower contour sets is the analogous.

Independence. Let $x \dot{\succsim} y$; we have $\{D_k\}_{k \in \mathbb{N}} \subset \mathcal{D}$ converging to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all k. Set $\lambda \in (0,1)$ and $z \in \mathbb{R}^n$. We know $x \in \mathcal{C}_2^\infty(D_k)$ indicates by definition that $x \in \lim\limits_{\gamma \to \infty} \frac{1}{\gamma}\mathcal{C}_2(\gamma D_k) = \frac{1}{\lambda} \lim\limits_{\gamma \to \infty} \frac{1}{\gamma}\mathcal{C}_2(\gamma \lambda D_k)$ or, multiplying by λ, that $\lambda x \in \mathcal{C}_2^\infty(\lambda D_k)$. Then by A2 we have that $\lambda x + \lambda' z \in \mathcal{C}_2^\infty(\lambda D_k + \lambda' z)$. Since $\lambda D_k + \lambda' z$ converges to $\lambda D + \lambda' z$, we have that $\lambda x + \lambda' z \dot{\succsim} \lambda y + \lambda' z$, as desired. ∎

B.2 Proof of Theorem 5.1

Definition. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *translation invariant* if for all $x, y, z \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$ we have $(x, I)R(y, J)$ if and only if $(x + z, I)R(y + z, J)$.

Since $\mathcal{C}_2$ is translation invariant, $\succsim^{TC}$ is as well.

Definition. Let R_1 and R_2 denote two binary relations on a set X (with asymmetric components S_1 and S_2). We say that R_1 *extends* R_2 if $R_2 \subseteq R_1$ and if xS_2y then also xS_1y.

That is, R_1 includes all comparisons that R_2 includes, but does not break any asymmetric comparison into a symmetric one. Because $\succ$ is acyclic, $\succsim^{TC}$ extends $\succsim$.

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R (with asymmetric component S) defined over $(\mathbb{R}^n \times \mathbb{I})$ *strongly-v-monotone* if $(x+z, I)R(x, I)$ whenever $v(z) \geq 0$ and $(x+z, I)S(x, I)$ whenever $v(z) > 0$.

Notice that a transitive and strongly-v-monotone relation is also v-monotone. Let $\succsim^{\#}$ denote $\succsim^{TC} \cup \{((x + z, I), (x, I)) \mid x, z \in \mathbb{R}^n, u(z) \geq 0, I \in \mathbb{I}\}$ and $\succsim^*$ its transitive closure.

Lemma 3. *$\succsim^*$ is reflexive, transitive, translation invariant, strongly-u-monotone, u-bounded, u-continuous and extends $\succsim$.*

Proof. That $\succsim^*$ is reflexive follows from the addition of $((x+\mathbf{0}, I), (x, I))$; that it is transitive is immediate in that it is a transitive closure; that it is translation invariant follows from that translation invariance of $\succcurlyeq^{TC}$ and the fact that all added relations are added in a translation invariant way. Next, notice that $\succcurlyeq^{\#}$ is obviously u-monotone and u-bounded. Further, notice that, because of u-monotonicity, the addition comparisons added to $\succcurlyeq^{TC}$ cannot turn a strict preference into an indifference; hence $\succcurlyeq^{\#}$ extends $\succcurlyeq^{TC}$.

$\succcurlyeq^{*}$ EXTENDS $\succcurlyeq^{TC}$. Assume this was not the case so that we have a finite sequence

$$(x_1, I_1) \succcurlyeq^{\#} (x_2, I_2) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_m, I_m)$$

such that $(x_m, I_m) \succ^{TC} (x_1, I_i)$.

Notice that for at least one $j < m$ we have

$$(x_{j+1}, I_{j+1}) = (x_j - z'_j, I_j) \tag{B.1}$$

for some z'_j with $u(z'_j) \geq 0$. If this was not the case, then each relation holds also for $\succcurlyeq^{TC}$, indicating that $(x_1, I_1) \succcurlyeq^{TC} (x_m, I_m)$, a clear contradiction.

So, let $B \subseteq \{1 \ldots m\}$ denote the non-empty set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) \geq 0$. We have:

$$(x_1, I_1) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_j, I_j) \succcurlyeq^{\#} (x_{j+1}, I_{j+1}) = (x_j - z'_j, I_j) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_m, I_m)$$

By translation invariance, we can, for the lowest $j \in B$, add z'_j from the all terms after $j+1$ to obtain

$$(x_1, I_1) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_{j-1}, I_{j-1}) \succcurlyeq^{\#} (x_j, I_j) = (x_{j+1} + z'_j, I_j) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_m + z'_j, I_m)$$

Continuing to delete terms in this manner for all $i \in B$, we are left with a sequence, contained within $\succcurlyeq^{TC}$, asserting $(x_1, I_1) \succcurlyeq^{TC} (x_m + \sum_{i \in B} z'_i, I_m)$, contradicting u-monotonicity.

STRONG-u-MONOTONICITY. By way of contradiction, assume that by taking the transitive closure we generate a violation of strong-u-monotonicity. That $(x + z, I) \succcurlyeq^{*} (x, I)$ is immediate, so assume this holds only weakly: for some (x, I), $(x, I) \succcurlyeq^{*} (x + z, I)$ for $z \in \mathbb{R}^n$ with $u(z) > 0$.

This requires a sequence of comparisons

$$(x, I) = (x_1, I_1) \succcurlyeq^{\#} (x_2, I_2) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_m, I_m) = (x + z, I)$$

As above, let $B \subseteq \{1 \ldots m\}$ denote the set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) > 0$. If $B = \emptyset$, we could exhibit this sequence via $\succcurlyeq^{TC}$, violating u-monotonicity. Therefore, as above, we can appeal to translation invariance to delete terms for each $i \in B$: the resulting sequence is contained within $\succcurlyeq^{TC}$ and asserts $(x, I) \succcurlyeq^{TC} (x + z + \sum_{j \in B} z'_j, I)$, contradicting u-monotonicity.

u-BOUNDEDNESS. Fix $z \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$. Let $(z, I) \succcurlyeq^{*} (\mathbf{0}, J)$ so that there exists a finite sequence

$$(z, I) = (x_1, I_1) \succcurlyeq^{\#} (x_2, I_2) \succcurlyeq^{\#} \ldots \succcurlyeq^{\#} (x_m, I_m) = (\mathbf{0}, J)$$

Once again, let $B \subseteq \{1 \ldots m\}$ denote the set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) > 0$. If $B = \emptyset$, then this sequence would exist within $\succcurlyeq^{TC}$, indicating

$\inf\{u(y) \mid (y, I) \succcurlyeq^{TC} (\mathbf{0}, J)\} \leq u(z)$. If B is not empty, we can proceed by the usual trick to conclude $(z, I) \succcurlyeq^{TC} (\sum_{j \in B} z'_j, J)$, or by translation invariance, $(z - \sum_{j \in B} z'_j, I) \succcurlyeq^{TC} (\mathbf{0}, J)$. This indicates that $\inf\{u(y) \mid (y, I) \succcurlyeq^{TC} (\mathbf{0}, J)\} \leq u(z) - \sum_{j \in B} u(z'_j) \leq u(z)$. Since $u(z)$ was arbitrary, the infimum with respect to $\succcurlyeq^*$ can be no lower than with respect to $\succcurlyeq^{TC}$, which was bounded below.

u-CONTINUITY. Let x be such that $u(x) = 0$ and a sequence $\{x_k\}_{k \in \mathbb{N}}$ converging with $u(x_n) \to 0$. Let (y, J) be such that $(x_k, I) \succcurlyeq^* (y, J)$ for all k and $(y, J) \succcurlyeq^* (x, I)$. We can use the now standard trick to find following relations:

$$(x_k - z_k, I) \succcurlyeq^{TC} (y, J)$$

for each k, and

$$(y, J) \succcurlyeq^{TC} (x + z, I)$$

with $u(z_k) \geq 0$ for each k and $u(z) \geq 0$. Necessarily, $u(z) = 0$, or else, eventually $u(x_k - z_k) < u(x + z)$ creating a violation of u-monotonicity. For the same reason, it must be that for all $u(z_k) \leq u(x_k)$. Hence $u(x_k - z_k) \to 0$, and by u-continuity $(x + z, I) \succcurlyeq^{TC} (y, J)$. Now, since $u(-z) = 0$ we have that $(x, I) \succcurlyeq^{\#} (x + z, I) \succcurlyeq^{\#} (y, J)$, and hence, $(x, I) \succcurlyeq^* (y, J)$. ★

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *strongly-v-continuous* if for all $I, J \in \mathbb{I}$, if whenever there exists a sequence $\{x_k\}_{k \in \mathbb{N}}$ such that $v(x_n) \to 0$ and $(x_k, I) R (y, J)$ for all k, then for any x with $v(x) = 0$, $(x, I) R (y, J)$.

Lemma 4. *Let $\succcurlyeq^+$ be the transitive closure of*

$$\succcurlyeq^* \cup \Big\{((x, I), (y, J)) \mid \textit{Exists } \{z_k\}_{k \in \mathbb{N}}, u(z_n) \to 0, (x + z_k, I) \succcurlyeq^* (y, J) \textit{ for all } k\Big\}$$

Then $\succcurlyeq^+$ is reflexive, transitive, translation invariant, strongly-u-monotone, u-bounded, strongly-u-continuous and extends $\succcurlyeq^*$ (hence $\succcurlyeq$).

Proof. Reflexivity, transitivity, translation invariance, and strong-u-continuity are all immediate.

$\succcurlyeq^+$ EXTENDS $\succcurlyeq^*$. Let $(y, J) \succcurlyeq^+ (x, I)$. Then there must exist a sequence $\{(x_j, I_j)\}_{j=1}^m$, with $(x_1, I_1) = (y, J)$ and $(x_m, x_m) = (x, I)$, and such that for each $j < m$ there is a sequence $\{z^j_k\}_{k \in \mathbb{N}}, u(z^j_k) \to 0$ (possibly the constant sequence $\mathbf{0}$, if $(x_i, I_i) \succcurlyeq^* (x_{i+1}, I_{i+1})$) such that $(x_j + z^j_k, I_j) \succcurlyeq^* (x_{j+1}, I_j)$ for all k. It is without loss of generality to assume that $u(z^j_k) \geq 0$ for all j, k. But notice we have

$$(x_1 + \sum_{i=1}^{m} z^i_k, I_1) \succcurlyeq^* (x_2 + \sum_{i=2}^{m} z^i_k, I_2) \succcurlyeq^* \ldots (x_j + \sum_{i=j}^{m} z^i_k, I_j) \succcurlyeq^* \ldots \succcurlyeq^* (x_m, I_m)$$

for each k. This indicates that $(y + \sum_{i=1}^m z^i_k, J) \succcurlyeq^* (x, I)$ where $u(\sum_{i=1}^m z^i_k) \to 0$. So by the u-continuity of $\succcurlyeq^*$, we cannot have $(x, I) \succ^* (y, J)$: therefore $\succcurlyeq^+$ extends $\succcurlyeq^*$.

STRONG-u-MONOTONICITY. We have that $(x, I) \succcurlyeq^+ (x + z, I)$ immediately; since $\succcurlyeq^+$ extends $\succcurlyeq^*$ it cannot be that $(x + z, I) \succcurlyeq^+ (x, I)$.

u-BOUNDEDNESS. Fix $x \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$. Let $(x, I) \succcurlyeq^{+} (\mathbf{0}, J)$. Using the same trick as in the proof of extension, we can find a (finite) collection of sequences, $\{\{z_k^j\}_{k\in\mathbb{N}}\}_{j=1}^m$ such that $u(z_k^j) \to 0$ for each j and $(x + \sum_{i=1}^m z_k^i, I) \succcurlyeq^{*} (\mathbf{0}, J)$. Since $u(x + \sum_{i=1}^m z_k^i) \to u(x)$ we have that $u(x) \geq \inf\{v(z) \mid (z, I) \succcurlyeq^{*} (\mathbf{0}, J)\}$. ★

Lemma 5. *Let $v : \mathbb{R}^n \to \mathbb{R}$ be a linear function and R be a preorder on $(\mathbb{R}^n \times \mathbb{I})$ that is translation invariant, strongly-v-monotone, v-bounded and strongly-v-continuous. Call $I, J \in \mathbb{I}$ R-*comparable *if there exists an $x \in \mathbb{R}^n$ such that $(x, I)R(\mathbf{0}, J)$ and $(\mathbf{0}, J)R(x, I)$. Then*

1. *R-comparability is an equivalence relation.*
2. *If I, J are not comparable, then there exists $\bar{x} \in \mathbb{R}^n$ such that neither $(\bar{x}, I)R(\mathbf{0}, J)$ nor $(\mathbf{0}, J)R(\bar{x}, I)$*
3. *If $\bar{I}, \bar{J}$ are not comparable, and $\bar{x}$ is as in (2), then, R^* defined as the transitive closure of $R^{\#} = R \cup \{(\bar{x}+z, \bar{I})R(z, \bar{J}), (z, \bar{J})R(\bar{x}+z, \bar{I}) \mid z \in \mathbb{R}^n\}$ is also a translation invariant, strongly-v-monotone, v-bounded, and strongly-v-continuous preorder that extends R.*

Proof. (1) Reflexivity is immediate. Symmetry follows from translation invariance. Transitivity follows from the transitivity and translation invariance of R, in the obvious way.

(2) Consider the sets $\{v(x) \mid (x, I)R(\mathbf{0}, J)\} \subseteq \mathbb{R}$ and $\{v(x) \mid (\mathbf{0}, J)R(x, I)\} \subseteq \mathbb{R}$. By strong-$v$-monotonicity, these are (possibly empty) intervals, the former upward-closed and the later downward-closed. By v-boundedness neither is $\mathbb{R}$ itself. By strong-v-continuity they are closed. If these intervals overlap, then I and J are comparable, so assume they do not overlap. Since $\mathbb{R}$ is connected, so there must be a point not in either interval.

(3) Fix $\bar{I}, \bar{J}$ that are not comparable for some R. Let $R^{\#}$ and R^* be as in the statement of the Lemma, and let and S, $S^{\#}$ and S^* denote respective asymmetric components. Reflexivity, transitivity, and translation invariance are immediate.

R^* EXTENDS R. Assume it did not: there exists a (x, I) and (y, J) such that $(x, I)S(y, J)$ but $(y, J)R^*(x, I)$. This last relations indicates the existence of a sequence,

$$(y, J)R^{\#}(x_1, I_1)R^{\#} \ldots R^{\#}(x_m, I_m)R^{\#}(x, I).$$

As in the proof of Lemma 3, there must be some relation not contained in R, so that for some $j < m$, we have $(x_j, I_j) = (\bar{x} + z, \bar{I})$ and $(x_{j+1}, I_{j+1}) = (z, \bar{J})$ (or vice versa, with an analogous proof following). It is without loss of generality that there is a single index j such that $(x_j, I_j), (x_{j+1}, I_{j+1}) \notin R$.[9] Capitalizing on the fact that R is transitive, we can further

[9] To see why: consider the following sequence

$$(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(x_i, I_i)R \ldots (x_{i+j}, I_{i+j})R(\bar{x} + z', \bar{I})R^{\#}(z', \bar{J})$$

where the "…" contains only R relations. If $v(z') < v(z)$, then $(z, \bar{J})S(z', \bar{J})$ by strong-v-monotonicity, and delete all other relations, we have

$$(y, J)R(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(x, I).$$

We can split the above sequence and swapping the order, recall $(x, I)S(y, J)$, leaving us with:

$$(z, \bar{J})R(x, I)S(y, J)R(\bar{x} + z, \bar{I}).$$

By the translation invariance of R, this implies $(\mathbf{0}, \bar{J})R(\bar{x}, \bar{I})$, a contradiction to the definition of $\bar{I}, \bar{J}$ and $\bar{x}$.

Strong-v-monotonicity. We have that $(x, I)R^*(x + z, I)$ immediately; since R^* extends R it cannot be that $(x + z, I)R^*(x, I)$.

v-boundedness. Fix $I, J \in \mathbb{I}$. Define the following constants.

$$a_1 = \inf\{v(z) \mid (z, I)R(\mathbf{0}, J)\}$$
$$a_2 = \inf\{v(z) \mid (z, I)R(\bar{x}, \bar{I})\}$$
$$a_3 = \inf\{v(z) \mid (z, \bar{J})R(\mathbf{0}, J)\}$$

Let $(x, I)R^*(\mathbf{0}, J)$. If this relation can be exhibited by R, then $u(x) \leq a_1$. So, to make things interesting, assume it cannot be; by the above arguments we can find the following sequence of relations:

$$(x, I)R(\bar{x} + z', \bar{I})R^{\#}(z', \bar{J})R(\mathbf{0}, J)$$

By the definition of a_2, and translation invariance, the first relation indicates that $u(x) \geq a_2 + v(z')$. The last relation likewise indicates that $u(z') \geq a_3$; hence $u(x) \geq a_2 + a_3$. In either case, $u(x) \geq \min\{a_1, a_2 + a_3\}$ and is hence bounded from below.

u-continuity. Let x be such that $u(x) = 0$ and a sequence $\{x_k\}_{k \in \mathbb{N}}$ converging with $u(x_n) \to 0$. Let (y, J) be such that $(x_k, I)R^*(y, J)$ for all k. By taking a subsequence if necessary, it is without loss of generality to restrict attention to the case where either $(x_k, I)R(y, J)$ for all k or not $(x_k, I)R(y, J)$ for all k. The former is a direct application of the strong-u-continuity of R. Assume the latter: we have,

$$(x_k, I)R(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(y, J)$$

By the strong-u-continuity of R, $(x, I)R(\bar{x} + z, \bar{I})$ and hence $(x, I)R^*(y, J)$. ★

Lemma 6. *There exists a translation invariant, strongly-u-monotone, u-bounded, and strongly-u-continuous preorder on* $(\mathbb{R}^n \times \mathbb{I})$, $\succcurlyeq^\star$, *that extends* $\succcurlyeq^+$ *such that all* $I, J \in \mathbb{I}$ *are* $\succcurlyeq^\star$ *comparable.*

we can make the same inference deleting one $R^{\#}$ relation. If $v(z') \geq v(z)$ we have a contradiction: we have

$$(z', \bar{J})R(z, \bar{J})R(x_i, I_i)R \ldots (x_{i+j}, I_{i+j})R(\bar{x} + z', \bar{I})$$

where the first relation is from strong-v-monotonicity. This implies, however, via translation invariance, that $(\mathbf{0}, \bar{J})R(\bar{x}, \bar{I})$, a contradiction to the definition of $\bar{I}, \bar{J}$ and $\bar{x}$.

Proof. Consider the set of all translation invariant, strongly-u-monotone, u-bounded, and strongly-u-continuous preorders on $(\mathbb{R}^n \times \mathbb{I})$ that extend $\succcurlyeq^+$. Say that $R \leq R'$ if R' extends R. Clearly, $\leq$ is a partial order, and every chain (totally ordered subset) is bounded by its union. Hence, we can apply Zorn's lemma to conclude the existence of a maximal (with respect to the extension induced order) relation over $(\mathbb{R}^n \times \mathbb{I})$. Call this relation $\succcurlyeq^\star$. By Lemma 5 part (iii), all I, J are $\succcurlyeq^\star$-comparable, or else we could find a further extension, contradicting the maximality of $\succcurlyeq^\star$. ★

For each $I \in \mathbb{I}$, define let x^I denote an element such that $(x^I, I) \sim^\star (\mathbf{0}, \mathbf{0})$. Then define $\Gamma : \mathbb{I} \to \mathbb{R}$ by

$$\Gamma : I \mapsto -u(x^I) \tag{B.2}$$

We now claim that $\langle u, \Gamma \rangle$ forms a IC representation for $\mathcal{C}_2$. Take a menu $D \in \mathcal{D}$. Assume that $x \in \mathscr{C}_1(D)$. Then $(x, I_D^x) \succcurlyeq (y, I_D^y)$ for all $y \in D$. Since $\succcurlyeq^\star$ extends $\succcurlyeq^+$ (Lemma 6), hence $\succcurlyeq$ (Lemma 3), we have $(x, I_D^x) \succcurlyeq^\star (y, I_D^y)$ for all $y \in D$. Therefore, by definition, and translation invariance,

$$(x - x^{I_D^x}, \mathbf{0}) \sim^\star (x, I_D^x) \succcurlyeq^\star (y, I_D^y) \sim^\star (y - x^{I_D^y}, \mathbf{0}).$$

Moreover, by strong-u-monotonicity this indicates that

$$u(x - x^{I_D^x}) \geq u(y - x^{I_D^y}),$$

or, from the definition of Γ and and the linearity of u,

$$u(x) + \Gamma(I_D^x) \geq u(y) + \Gamma(I_D^y),$$

for all $y \in D$. So $\mathcal{C}_2(D) \subseteq \arg\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big)$.

Now assume that $x \notin \mathcal{C}_2(D)$. Then there exists a $y \in D$ such that $(y, I_D^y) \succ (x, I_D^x)$. Since $\succcurlyeq^\star$ extends $\succcurlyeq$, we have $(y, I_D^y) \succ^\star (x, I_D^x)$. From repetition of the above with strict preference/inequality we conclude that

$$u(y) + \Gamma(I_D^y) > u(x) + \Gamma(I_D^x),$$

So $\arg\max_{x \in D} \big(u(x) + \Gamma(I_D^x)\big) \subseteq \mathcal{C}_2(D)$ and we have established the existence of an image conscious representation. ∎

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