Do security prices rise or fall when margins are raised?

Jean-Marc Bottazzi [1] Mário Páscoa [2] Guillermo Ramírez [3]

[1] Capula and Paris School of Economics

[2] University of Surrey

[3] Nova School of Business and Economics

SUMMARY

- When assets cannot be shorted (as houses in mortgages), a collateral margins hike makes the asset price go down. How is it when the supply is endogenous, due to short sales?
- Securities serve as collateral in the repo market. When repo margins are raised by a central clearing counterparty (CCP), what is the impact on security prices?

- As both the long and the short must post margins, the impact depends on which side is more leveraged: traders long in the security or those short-selling it.
- If a raised margin forces more position unwind from the long than from the short, the price will go down to clear the market. However, if short positions are more hit then the long ones, a raised haircut leads to a higher security price!

- The way central clearing counterparties (CCPs) raise repo margins when security prices fall has been well documented and elaborated but we are more interested in the opposite direction.
- We notice that there were several episodes during the European sovereign debt crisis where margins went up but yields fell, contrary to a long standing view that haircuts are pro-cyclical. Let us look at 10 year bonds.

- For the *Irish bond*, margins rose on the 12th November 2010 to 15% and yields fell by 10% in two days.
- On the 1st of April 2011, margins went up to 45% and yields fell by 10% in the next three market dates.

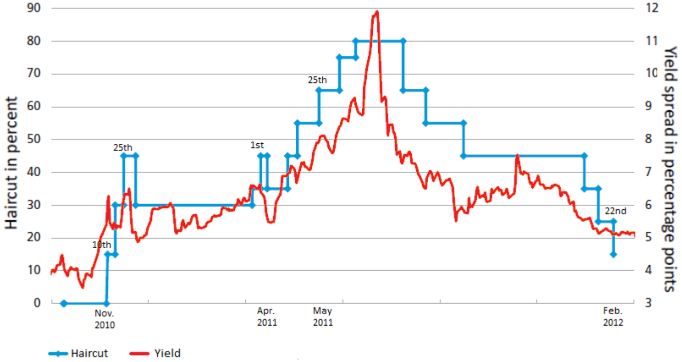
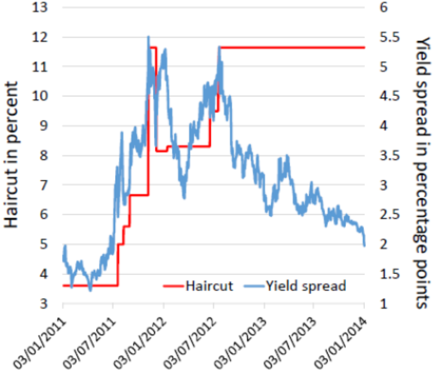


Figure: Haircuts and yield spread for 10 year **Irish** bond on 10 year German bond.

- For the 10 year *Italian bond*, margins rose (from 6.65% to 11.65%) on the 9th of November 2011 and yields fell by 9% within two days.
- Margins rose again at the same high level on the 23rd of July 2012 and yields fell by 10% within three days.
- For the 10 year *Spanish bond*, the margins update (to 11.8%) on the 19th of June 2012 led to yield reductions of 2%, 5%, 2% and 3% in the announcement date and the next three dates, respectively.



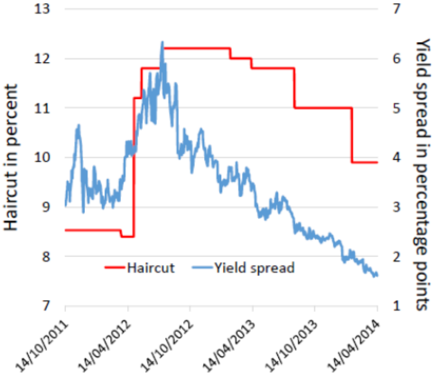


Figure: Haircuts and yield spread for 10 year **Italian** (left) and **Spanish** (right) bonds on 10 year German Government bond

Source: www.investing.com and www.lch.com

ITALY	Haircut	Yield	Change
Nov. 8, 2011	6.65%	6.707	-
Nov. 9, 2011	6.65%	7.286	8.63%
Nov. 10, 2011	11.65%*	6.957	-4.52%
Nov. 11, 2011	11.65%	6.512	-6.40%
Nov. 14, 2011	11.65%	6.702	2.92%

* Announced on the 9th, with impact on margin call in the morning of the 10th.

ITALY	Haircut	Yield	Change
July 23, 2012	9.5%	6.317	-
July 24, 2012	11.65%*	6.565	3.93%
July 25, 2012	11.65%	6.446	-1.81%
July 26, 2012	11.65%	6.047	-6.19%
July 27, 2012	11.65%	6.951	-1.59%
July 30, 2012	11.65%	6.014	1.06%

* Announced on the 23rd.

IRELAND	Haircut	Yield	Change
Nov. 9, 2010	≈0%	8.064	-
Nov. 10, 2010	≈0%	8.802	9.15%
Nov. 11, 2010	≈0%	9.108	3.48%
Nov. 12, 2010	15%*	8.391	-7.87%
Nov. 15, 2010	15%	8.162	-2.73%
Nov. 16, 2010	15%	8.368	2.52%

* Announced on the 10th.

IRELAND	Haircut	Yield	Change
Mar. 31, 2011	35%	10.349	-
Apr. 1, 2011	45%*	10.078	-2.62%
Apr. 4, 2011	45%	9.844	-2.32%
Apr. 5, 2011	45%	10.123	2.83%
Apr. 6, 2011	45%	9.530	-5.86%
Apr. 7, 2011	45%	9.462	-0.71%
Apr. 8, 2011	45%	9.337	-1.32%
Apr. 11, 2011	45%	9.304	-0.35%
Apr. 12, 2011	45%	9.274	-0.32%

* Announced on Apr. 1.

SPAIN	Haircut	Yield	Change
June 15, 2012	11.8%	6.870	-
June 18, 2012	11.8%	7.210	4.95%
June 19, 2012	11.8%	7.080	-1.80%
June 20, 2012	11.8%	6.730	-4.94%
June 21, 2012	11.8%	6.570	-2.38%
June 22, 2012	12.2%*	6.400	-2.59%
June 25, 2012	12.2%	6.640	3.75%
June 26, 2012	12.2%	6.690	0.75%

* Announced on June 20.

PORTUGAL	Haircut	Yield	Change
May 9, 2011	35%	9.849	-
May 10, 2011	35%	9.864	0.15%
May 11, 2011	45%*	9.464	-4.06%
May 12, 2011	45%	9.301	-1.72%
May 13, 2011	45%	9.259	-0.45%
May 16, 2011	45%	9.050	-2.26%
May 17, 2011	45%	9.157	1.18%

* Announced on May 10.

PORTUGAL	Haircut	Yield	Change
June 24, 2011	65%	11.551	-
June 27, 2011	65%	12.086	4.63%
June 28, 2011	65%	11.652	-3.59%
June 29, 2011	80%*	11.404	-2.13%
June 30, 2011	80%	11.243	-1.41%

* Announced on June 28.

The presumed procyclical nature of margins set by CCPs (more systematic than in over-the-counter (OTC) market) has even led to the view that CCPs might be factor of instability.

This view was nevertheless met with caution by European policy-makers that value CCPs' role as a firewall against the propagation of default shocks and find an increasingly centrally cleared market easier to monitor (speech by Constancio 2012).

In the case of the European sovereign debt crisis, a lot of positioning was done by banks and hedge funds. Banks tend to use CCPs and hedge funds use prime brokers who act as clearers for them (also charging margin regardless of whether the trader is short or long).

Other factors may have impacted on yields. Margin increases took place at times when agents' (and credit rating agencies') perceptions of default risk may have fluctuated, affecting also the yields.

It is not our purpose to try to adhere to those illustrative empirical examples closely.

However, we see the above mentioned counter-cyclical episodes as *suggesting* that the speculative leveraged positions on these sovereign bonds can be a factor explaining why prices may rise in response to higher margins.

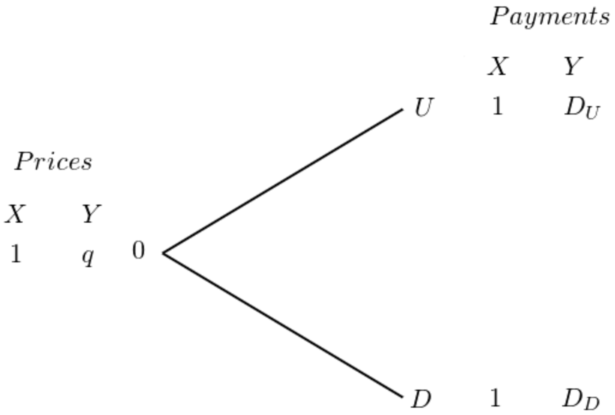
With this empirical motivation in mind we proceed to a theoretical analysis that challenges the current view on the procyclical role of repo haircuts and the resulting propagating down-spiral, as in Brunnermeider and Pedersen (2009), Adrian and Shin (2010), Gorton and Metrick (2012) or Fostel and Geanakoplos(2012).

Adrian and Shin (2010) observe a strong positive correlation between leverage and balance sheet size of US financial intermediaries, inferring from it a procyclical role of leverage.

Brunnermeider and Pedersen (2009) notice that short selling also requires capital in the form of margin but focus on margin spirals occurring when the deviation of prices from expected dividends makes margins go up, which tightens the finding constraints and, therefore, enlarges that deviation.

Gorton and Metrick (2012) argued that the spiral interaction of repo haircuts and prices of structured securities led to a run on repo.

As in Fostel and Geanakoplos (2012), we consider a binomial economy with two assets, cash X and security Y:



where $D_U < D_D$.

- Agent i position in asset x is denoted by $x^i \geq 0$.
- Agent i position in security y, is denoted by y^i.

Security y can be sold short.

- Security y trades are financed in a repo market.
- Agent i long and short positions on security y, in the repo market, are denoted by $\theta^i \geq 0$ and $\psi^i \geq 0$, respectively.
- The security can be pledged as collateral to borrow cash in the repo market (taking a short repo position).
- In order to short sell a security, a trader must have accepted the security as collateral (taking a long repo position).

Both operations are captured by the *box constraint*:

$$y^i + \theta^i - \psi^i \geq 0$$

When the collateral cannot be reused (as in the case of mortgages) the constraint would be:

$$y^i - \psi^i \geq 0$$

Our model is similar to that in Fostel and Geanakoplos (2012) with some important differences:

- Crucially, collateral can be reused (and short sold).
- Loans are recourse (as is typically the case in repo).
- Agents have endowments of the riskless asset also at the two nodes at the second date. This departure allows us to choose second date endowments high enough so that shorts can leverage up to full haircut potential without going bankrupt at the second date.

For unitary endowments of both assets at date 0, and state contingent endowments ω_s, agent i budget constraints are:

Date 0:

$$(x^i - 1) + q(y^i - 1) + q(\theta^i - \psi^i) + (1-h)q(\theta^i + \psi^i) \leq 0$$
$$y^i + \theta^i - \psi^i \geq 0, \qquad x^i, \theta^i, \psi^i \geq 0$$

Date 1:

$$C_s^i = \omega_s + x^i \cdot 1 + (1+\rho)q\Big[\theta^i - \psi^i + (1-h)(\theta^i + \psi^i)\Big] + y^i D_s \geq 0$$
$$s = U, D$$

For a given price q, and haircut $1-h$, budget and box constraints bound agents' portfolios. To see how, start with date 0 budget constraint:

$$\begin{aligned}
(x^i - 1) + q(y^i - 1) + q(\theta^i - \psi^i) + (1-h)q(\theta^i + \psi^i) \leq 0 \;&\Leftrightarrow \\
\underbrace{x^i}_{\geq 0} - 1 + q\underbrace{(y^i + \theta^i - \psi^i)}_{\geq 0 \text{ from Box}} - q + (1-h)q(\theta^i + \psi^i) \leq 0 \;&\Leftrightarrow \\
(1-h)q(\theta^i + \psi^i) \leq 1 + q \;&\Leftrightarrow \\
\theta^i + \psi^i \leq \frac{1+q}{q} \cdot \frac{1}{1-h} \equiv L(h,q)&
\end{aligned}$$

And, given the non-negativity of long and short repo positions we have that:

$$0 \leq \theta^i \leq L(h,q) \tag{1}$$

$$0 \leq \psi^i \leq L(h,q)$$

The bound in (1) and the box constraint together imply that:

$$L(h,q) \geq \theta^i \underbrace{\geq}_{\text{from box}} \psi^i - y^i \ \Leftrightarrow$$
$$-L(h,q) \leq y^i$$

Again, from date 0 budget constraint, we have that:

$$qy^i + q\left[(\theta^i - \psi^i) + (1-h)(\theta^i + \psi^i)\right] \leq 1 + q \ \Leftrightarrow$$
$$qy^i \leq 1 + q + q\psi^i \leq 1 + q + qL(h,q)$$

So we have the following bounds for y^i:

$$-L(h,q) \leq y^i \leq L(h,q)\left[(1-h) + q\right]$$

There is a continuum of agents indexed $i \in (0, 1)$ with payoff:

$$U^i(C_U^i, C_D^i) = \gamma^i C_U^i + (1 - \gamma^i) C_D^i$$

where $\gamma^i = i$.

An **Equilibrium** consists in a security price q, a repo rate ρ, and portfolios $(x^i, y^i, \theta^i, \psi^i)$ such that:

- Each agent's portfolio is a constrained payoff maximizer.
- $\int_0^1 x^i di = 1, \quad \int_0^1 y^i di = 1, \quad \int_0^1 \theta^i di + \Theta^{EX} = \int_0^1 \psi^i di$

where the CCP is investing in repo the margins it collects from both sides of the market:

$$\Theta^{EX} = (1 - h) \cdot \left(\int_0^1 \theta^i di + \int_0^1 \psi^i di \right)$$

Leverage is commonly defined as the ratio of a position over the down payment.

If an agent pledges his entire long position in the security, his leverage is $\frac{1}{1-h}$.

If an agent short sells the whole collateral that he has accepted, his leverage is, again, $\frac{1}{1-h}$.

Leverage is inversely related to the repo haircut. **Both parties cannot be fully leveraged.**

Suppose we had both short and long agents fully leveraged. This would imply:

$$\theta^H = -y^H = \psi^L = y^L = \frac{1+q}{q}\frac{1}{1-h}$$

Market clearing in the securities market would imply:

$$(1-m)y^H + my^L = -(1-m)\frac{1+q}{q}\frac{1}{1-h} + m\frac{1+q}{q}\frac{1}{1-h} = 1$$

This is:

$$(2m-1)\frac{1+q}{q}\frac{1}{1-h} = 1 \tag{2}$$

On the other hand, market clearing in the repo market would imply:

$$
\begin{aligned}
(2-h)(1-m)\theta^H - hm\psi^L = 0 &\Leftrightarrow (2-h)(1-m) - hm = 0 \\
&\Leftrightarrow m = \frac{2-h}{2}
\end{aligned}
\tag{3}
$$

Combining equations (2) and (3) we get:

$$
(2m-1)\frac{1+q}{q}\frac{1}{1-h} = (1-h)\frac{1+q}{q}\frac{1}{1-h} = 1
$$

This equation holds only if $1+q=q$, which is impossible.

Let us focus on equilibria where all shorts take the same short position and all longs take the same long position. We refer to such equilibria as *typical equilibria*.

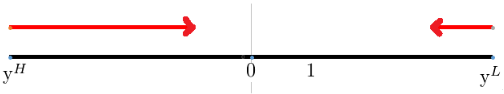
There are two types of typical equilibria for this economy: short and long market. In a short (long) market the shorts (the longs) are the traders that are more leveraged. More precisely,

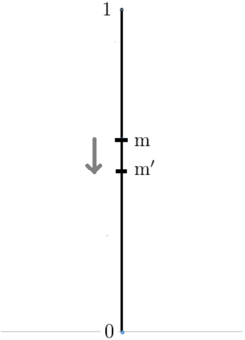
Definition
We say that a typical equilibrium $(q, \rho, x, y, \theta, \psi)$ is a short (long) market equilibrium if the net trade of a short is more (less, respectively) elastic with respect to the loan-to-value ratio h than the net trade of a long.

In both cases, there will be a *marginal agent* indexed $m \in (0, 1)$ that is indifferent between taking a long or a short position in security y. A generic agent in $(m, 1)$ is denoted by H and will be a short. A typical agent in $(0, m)$ is denoted by L and will be a long.

Intuition for a short market:

Market clearing requires $my^L + (1-m)y^H = 1$





$q = mD_U + (1-m)D_D$

Proposition

The security price q increases with the loan-to-value ratio h in a long market but decreases with h in a short market.

Proof

Market clearing requires $my^L + (1-m)y^H = 1$. As the loan-to-value ratio h changes, m has to change so that 1 remains a convex combination of the new positions y^L and y^H. More formally,

$$m = \frac{1-y^H}{y^L-y^H} = \frac{1}{1+\frac{y^L-1}{1-y^H}}$$

Short market equilibrium:

- When ω_D is large enough what ends up constraining the short position of the agents $i > m$ is their binding box ($y^H = -\theta^H$). They sell their endowments of X and Y to build up the largest possible short position in y:

$$y^H = -\theta^H = -\frac{1+q}{q} \cdot \frac{1}{1-h} \qquad \psi^H = x^H = 0 \quad C_U^H, C_D^H > 0$$

- If ω_D were not large enough what would constraint the short sale would be $C_D \geq 0$ instead.

By market clearing, $y^L = \frac{1}{m}\left[1 + (1-m)\frac{1+q}{q} \cdot \frac{1}{1-h}\right]$ and

$$\psi^L = (\frac{2-h}{h})(\frac{1-m}{m})\frac{1+q}{q(1-h)}$$

Also $x^L = \frac{1}{m}$. And $C_U^L = 0$, $C_D^L > 0$,

The box constraint $y^L - \psi^L \geq 0$ holds if and only if $h/2 \geq (1-m)\frac{1+q}{q}$

First order conditions for agent i problem:

$$y^i: \qquad q = E^i D + \sum_s \lambda_s^i (D_s - q) - q\mu_x^i + \mu^i$$

$$\theta^i: \qquad \rho(1 + \sum_s \lambda_s^i) = \mu_x^i - (\mu^i + \nu_\theta^i)/(q(2-h))$$

$$\psi^i: \qquad \rho(1 + \sum_s \lambda_s^i) = \mu_x^i - (\mu^i - \nu_\psi^i)/(qh) \ ,$$

For $i = m$, we have $E^m D = q$ with all shadow values zero. When $i > m$ FOC are satisfied for

$$\begin{aligned} \mu_x^H &= \frac{q - E^H D}{q(1-h)} > 0 \\ \mu^H &= q(2-h)\mu_x^H > 0 \\ \nu_\theta^H &= \lambda_U^H = \lambda_D^H = 0 \\ \nu_\psi^H &= 2q(1-h)\mu_x^H \end{aligned}$$

and when $i < m$ FOC are satisfied for:

$$\begin{aligned} \mu_x^L &= \mu^L = 0 \\ \nu_\theta^L &= \nu_\psi^L = \lambda_D^L = 0 \\ \lambda_U^L &= \frac{E^L D - q}{q - D_U} > 0 \end{aligned}$$

Assumption

We assume that $\omega_U = 0$ but ω_D is high enough so that $C_D^H \geq 0$, which holds if $\omega_D \geq (1 + D_D)\left(\frac{D_D - (2-h)D_U}{D_U(1-h)}\right)$.

Proposition

Under the previous assumption, markets are short and the security price decreases as leverage goes up.

Proof

$$C_U^L = 0 \Leftrightarrow H \equiv \frac{1}{1-h} = \left[\frac{m(1+q)}{q-D_U} - 1\right]\frac{q}{(1+q)(1-m)}$$

where $q = E^m D$. Now, $\frac{\partial H}{\partial m} > 0$ while $\frac{\partial H}{\partial q} < 0$.

Along any isoleverage curve we have: $\frac{dq}{dm} = -\frac{\frac{\partial H}{\partial m}}{\frac{\partial H}{\partial q}} > 0$

From $\frac{\partial H}{\partial m} > 0$ we get that, for a fixed value of q, increasing m implies moving to a isoleverage curve corresponding to a higher value of H. But the box constraint must be satisfied for the non-levered longs: the grey region in the next figure.

Equilibrium pairs (m, q) are found within this region where each isoleverage (for some h) crosses the dashed line given by $q = D_D + m(D_u - D_D)$.

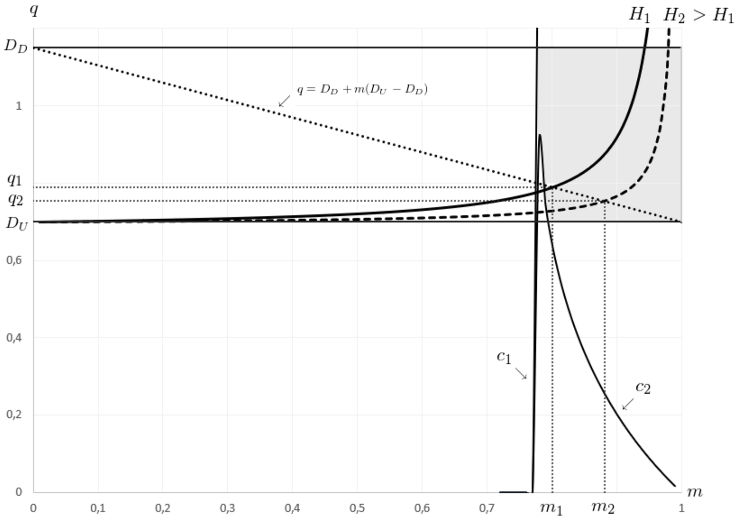
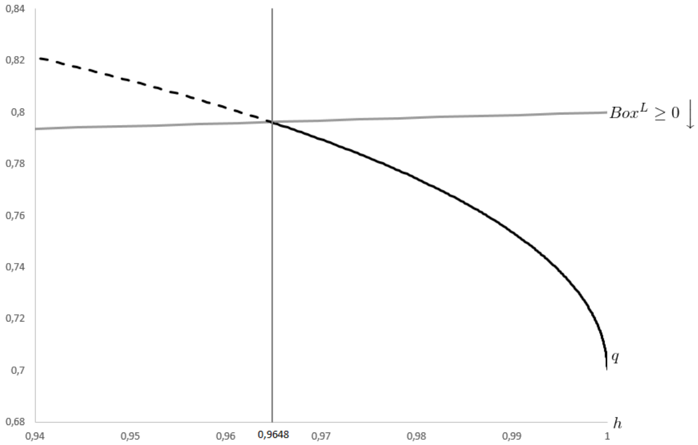
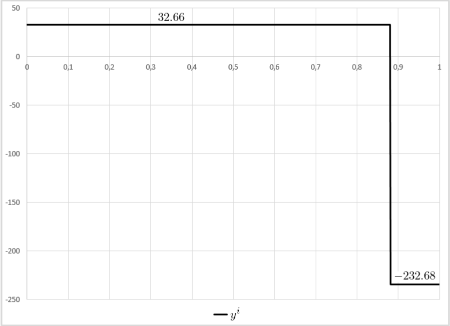


Figure: Impact of leverage on equilibrium when the market is short

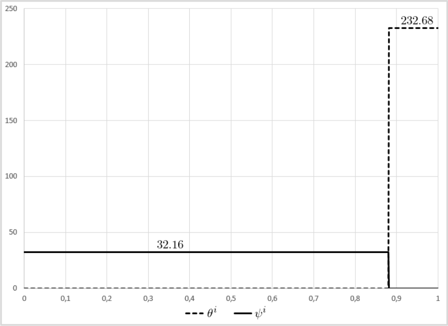
The isoleverages portrayed in the figure are for the haircuts of 1% and 3%.

Example: $D_U = 0.70$, $D_D = 1.15$, $\omega_D \geq (1 + D_D)\left(\frac{D_D-(2-h)D_U}{D_U(1-h)}\right)$, $\omega_U = 0$:



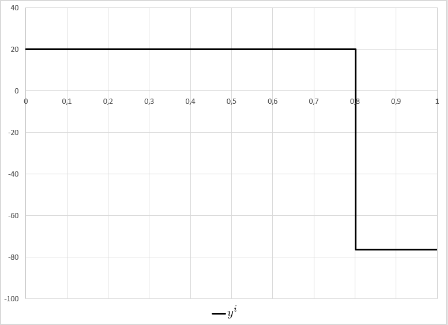


(a) y^i

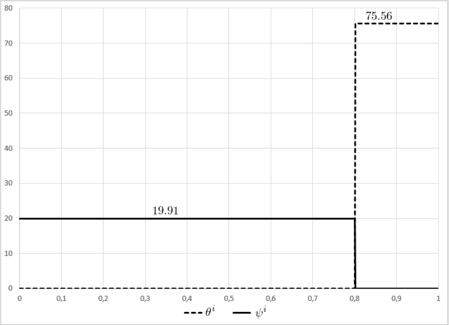


(b) θ^i and ψ^i

Figure: Equilibrium corresponding to haircut of 1% ($h = 0.99$): $m = 0.881$, $q = 0.754$, $x^i = 1/m$ for $i \leq m$ and $x^i = 0$ for $i > m$ (assuming $\omega_D \geq 107.6$).



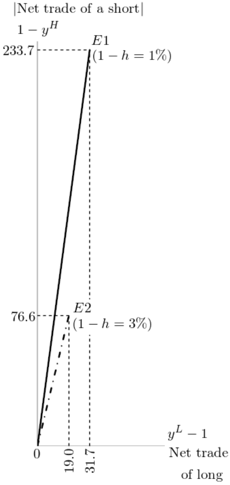
(a) y^i



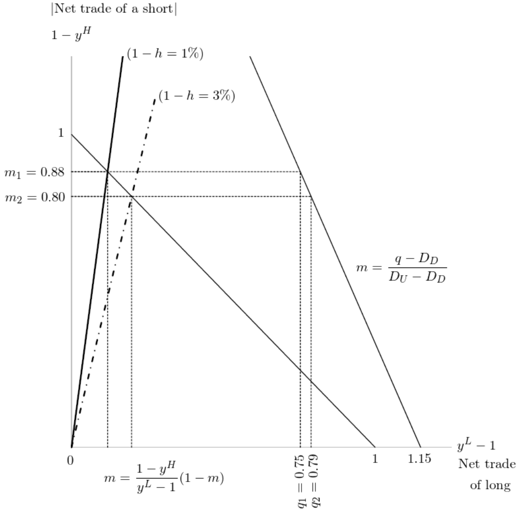
(b) θ^i and ψ^i

Figure: Equilibrium corresponding to haircut of 3% ($h = 0.97$): $m = 0.801$, $q = 0.789$, $x^i = 1/m$ for $i \leq m$ and $x^i = 0$ for $i > m$ (assuming $\omega_D \geq 34.7$).

We can compare how the increase in haircuts from 1% to 3% affects the shorts and the longs:



(a) Net trades of longs and shorts



(b) m and q determined from net trades ratio

Figure

Long market equilibrium.
We find an equilibrium with the following positions:

$$y^L = \psi^L = \frac{1+q}{q} \cdot \frac{1}{1-h}$$
$$\theta^L = x^L = 0$$
$$C_U^L, C_D^L > 0$$

$$y^H = \frac{1}{1-m}\left[1 - m \cdot \frac{1+q}{q} \cdot \frac{1}{1-h}\right]$$
$$\theta^H = \frac{h}{2-h}\frac{m}{1-m}\frac{1+q}{q(1-h)}$$
$$\psi^H = 0$$
$$x^H = \frac{1}{1-m}$$
$$C_U^H > 0 \quad C_D^H = 0$$

Assumption 2: *$\omega_D = 0$ and $\omega_U \geq (1 + D_D)\left(\frac{D_D-(2-h)D_U}{D_U(1-h)}\right)$ (to ensure that $C_U^L \geq 0$).*

Now, $C_D^H = 0$ implies that leverage should be such that

$$\frac{1}{1-h} = \left[\frac{(1-m)(1+q)}{D_D - q} + 1\right]\frac{q}{(1+q)m} \tag{4}$$

where $m = \frac{D_D-q}{D_D-D_U}$.

Proposition

If commodity endowments satisfy Assumption 2, then the market is long and the security price increases as leverage goes up (and therefore as margins decrease).

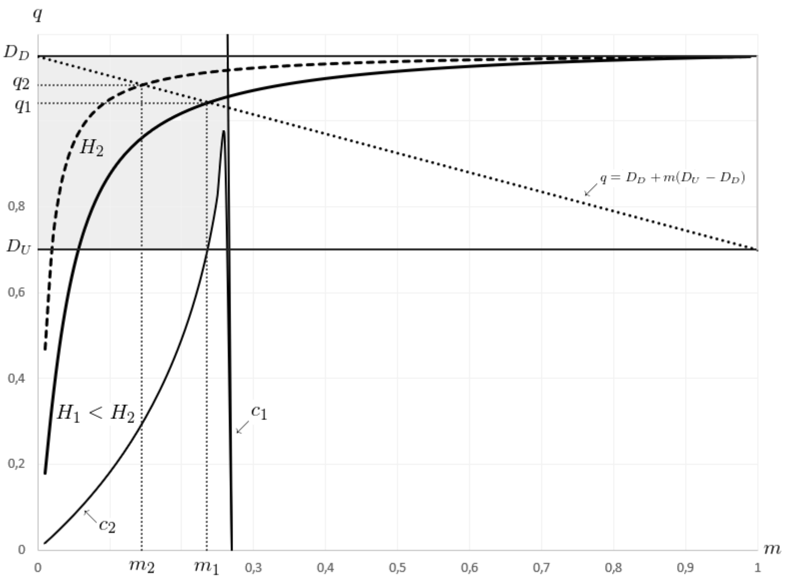


Figure: Impact of leverage on equilibrium when the market is long

With $D_U = 0.70$, $D_D = 1.15$, $\omega_U \geq (1 + D_D)\left(\frac{D_D-(2-h)D_U}{D_U(1-h)}\right)$ and $\omega_D = 0$ we have:

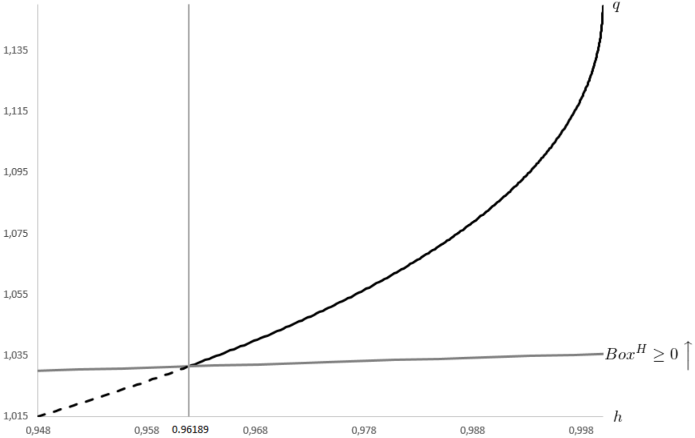


Figure: q vs. h - Long market

To summarize

The adjustment mechanism goes as follows. Looking at the impact of raised margin requirements separately for agents with a long or a short position in the asset, leverage reduction will mean reduction of position for both.

The price of the asset is the balancing factor that restores market clearing. If the position of a short is reduced by more than the position of a long - in what we call a 'short' market - the price of the security will go up.

The reverse happens when the market is 'long' and it has been the case studied by Fostel and Geanakoplos (2012), as this is the only possible outcome for the price of a house securing a mortgage, or for the price of other collateral used to asset back securities.

Other related work

In *“Determinants of Repo Haircuts and Bankruptcy” (discussion paper 2017)* we use a binomial model of over-the-counter (OTC) repo to study how traders should anticipate counterparty bankruptcy risk and choose repo haircuts accordingly.

We start with a one-security, two-agent case and construct bankruptcy equilibria where haircuts move together with the counterparty’s repayment rate. Charging a higher haircut means smaller loans, which in turn increases the proportion of the loan that is repaid in case of bankruptcy.

However, in a multi-agent and multi-security context, the haircut choice has a small impact on counterparties’s solvency, and the goal is now to reduce the counterparty’s net debt. For example, for an agent that is repo long,

- If the cash repayment is above the value of the collateral, the agent chooses to increase haircuts (lend less cash)
- Otherwise, he chooses to decrease haircuts (keep more of the collateral in case of counterparty bankruptcy)

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REPO PONZI SCHEMES AND LEVERAGED BUBBLES (Bottazzi, Luque and Pascoa)

In OTC repo the haircut benefit allows for Ponzi schemes: increasing by ϵ both the collateral accepted and the short sale generates a cash benefit equal to the haircut; this can be accommodated at the next nodes by choosing a new ϵ appropriately.

In centrally cleared repo there are no Ponzi schemes. For time and state additively separable preferences, equilibrium exists without having to assume uniform impatience.

Uniform impatience ruled out rational bubbles in models with naked short sales. Now, we get incomplete market bubbles when agents are not uniformly impatient (say when endowments are unbounded).

Agents hold on to a bubble to sell it later on at a more adverse node where a negative endowment shock occurs. When an agent wants to sell, another one wants to buy. This is possible as the shocks are idyosincratic and agents have different discounting patterns.