STRICT AND WEAK CHOICE IN RANDOM EXPECTED UTILITY MODELS

Evan Piermont & Roee Teper

Feb, 2019; Linz

Economic choice data is usually aggregated across

- many subjects, or,
- many different points in time, or both

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The analyst observes: $\rho_D(E)$ for $E \subseteq D$, representing the frequencies of choice.

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Can we still falsify individual rationality / expected utility?

Random Utility Models are a way of dealing with aggregated choice data:

Let $\mathcal{U}$ denote a set of utility functions. Then $\xi \in \Delta(\mathcal{U})$ is a Random Expected Utility Model (REUM) representing ρ if

$$\rho_D(E) = \xi\{u \in \mathcal{U} \mid \arg\max_D u \in E\}$$

Random utility models do not deal well with ties:

- If with positive ξ-probability $u(a) = u(b)$, what is $\rho_D(b)$?
- Depends on how indifferences are broken.
- Gul and Pesendorfer (2006) (and the following literature) assume ties occur with probability 0

We present a model of **random choice capacities** (RCCs), such that

- Represents upper-bounds of choice probability (sub-additive).
- Set of RCCs satisfying our axioms homeomorphic to the set of REUMs.

- The theory of RCCs can help us understand tie breaking from (observable) additive choice rules.
- Each additive choice rule corresponds to a set of RCCs (thus a set of REUMs)
    - When the choice rule is extreme (a la GP) then ties happen with probability 0.
    - There exists an RCC which minimizes tie breaking.
    - Choosing the model with the least tie-breaking is normatively appealing; errors are not costly.

Example

- $D = \{a, b, c = \frac{1}{2}a + \frac{1}{2}b\}$ is a decision problem.
- Let ξ be the uniform measure over the expected utility indices

$$u_1 = [1, 0], u_2 = [-1, 0], \text{ and } u_3 = [0, 0].$$

- if u_1 is realized then a is definitely chosen,
  - a is chosen $\geq \frac{1}{3}$;
- if u_2, b is definitely chosen,
  - a is chosen $\leq \frac{2}{3}$;
- if u_3, depends on tie breaking
  - Conclude: a is chosen $\in [\frac{1}{3}, \frac{2}{3}]$

Example

Our primitive, ρ_D, reflects the upper bounds of choice frequency.

- $\rho_D(c) = \frac{1}{3}$
- $\rho_D(a) = \rho_D(b) = \rho_D(\{a, c\}) = \rho_D(\{b, c\}) = \frac{2}{3}$
- $\rho_D(\{a, b\}) = \rho_D(\{a, b, c\}) = 1$

Set Up

$\mathcal{D}$ is the set of **decision problems**: all finite non-empty subsets of R^n.

Our primitive is a **random choice capacity** (RCC),

$$\rho = \{\rho_D : 2^D \to [0,1]\}_{D \in \mathcal{D}}.$$

that is

- **grounded**: $\rho_D(\emptyset) = 0$.
- **normalized**: $\rho_D(D) = 1$.
- **monotone**: $\rho_D(A \cup B) \geq \rho_D(A)$.
- not necessarily additive!

Random Linear Representations

Call ξ, a (finitely additive) probability measure over $\mathbb{R}^n$, a **random linear representation** (RLR). Say that ρ **maximizes** ξ if

$$\rho_D(A) = \xi(\{u \in \mathbb{R}^n \mid A \cap (\arg\max_{y \in D} u \cdot y) \neq \emptyset\})$$

for all (D, A).

Theorem

Every RLR has a unique maximizer and every ρ maximizes at most one RLR.

GP axioms

If ρ is additive then GP provide conditions for the existence of a RLR:

1. **Monotonicity**: $D \subseteq D' \implies \rho_D(a) \geq \rho_{D'}(a)$.
2. **Extremeness**: $\rho_D(\text{ext}(D)) = 1$
3. **Linearity**: $\rho_D(a) = \rho_{\lambda D + b}(\lambda a + b)$ for $\lambda > 0$.
4. **Mixture Cont**: $\rho_{\lambda D + \lambda' D'}$ is continuous in λ, λ' for $\lambda, \lambda' \geq 0$.

- We keep **Linearity** and **Mixture Continuity** and **Monotonicity**.
- Relax **Extremeness** to **Convex Modularity**.

Let $D = \{a, b, \frac{1}{2}a + \frac{1}{2}b\}$. Notice:

$$\rho_D(\{a\}) = \xi\big(\{u \mid u(a) > u(b)\}\big) + \xi\big(\{u \mid u(a) = u(b)\}\big), \text{ and}$$
$$\rho_D(\{b\}) = \xi\big(\{u \mid u(b) > u(a)\}\big) + \xi\big(\{u \mid u(a) = u(b)\}\big).$$

So,

$$\rho_D(\{a, b\}) = \rho_D(\{a\}) + \rho_D(\{b\}) - \xi\big(\{u \mid u(a) = u(b)\}\big)$$

Also,

$$u(a) = u(b) \iff \frac{1}{2}a + \frac{1}{2}b \in \arg\max_{z \in D} u(z)$$

Hence

$$\rho_D(\{a, b\}) = \rho_D(\{a\}) + \rho_D(\{b\}) - \rho_D(\{\frac{1}{2}a + \frac{1}{2}b\})$$

Convex-Modularity

Let $A, B \subseteq D$ be such that $\alpha A + (1-\alpha)B \subseteq D$ for $\alpha \in (0,1)$. Then

$$\rho_D(\alpha A + (1-\alpha)B) = \rho_D(A) + \rho_D(B) - \rho_D(A \cup B).$$

Theorem

The following are equivalent:

1. ρ satisfies Monotonicity, Convex-Modularity, Linearity, and Mixture-Continuity.
2. ρ maximizes a finitely additive RLR, ξ.

On additive choice rules

An additive choice rule μ is consistent with ξ + “some tie breaking”

$$\iff$$

μ is point-wise dominated by ρ, where ρ is the unique RCC that maximizes ξ

- To ensure consistency with an RLR, we only need to find a dominating ρ
- The (point-wise) minimal dominating RCC corresponds to the RLR that minimizes tie breaking.
- This can be done constructively.

$$\Gamma(\xi, D) = \Big\{ \int_{R^n} \tau_u(\cdot)\xi(du) \mid \tau_u \in \Delta\Big(\arg\max_{y \in D} u(y)\Big)\Big\}.$$

- the set of all possible choice rules constructed by first choosing a utility u according to ξ, and subsequently choosing among the maximizers in D according to some tie breaking procedure.

Theorem

Let ρ maximize ξ. Then $\rho_D(A) = \sup_{m \in \Gamma(\xi, D)} m(A)$ for all D.

More generally, for any linear, mixture continuous, monotone capacity ρ° and RLR ξ, TFAE

1. There exists a set of tie breaking rules $G(D) \subseteq \Gamma(\xi, D)$ such that $\rho^\circ_D(A) = \sup_{m \in G(D)} m(A)$.
2. $\rho^\circ_D(A) \leq \rho_D(A)$ for all $D \in \mathcal{D}$ and $A \subseteq D$, where ρ is the unique RCC that maximizes ξ.

Constructing ρ from μ

Let μ be an additive, linear, mx-cont, monotone, but **not** necessarily extreme choice rule.

- This is what would be observed.
- May admit ties.
- Always trivially dominated by $\rho \equiv 1$.

Let $\text{pi}(D, A) =$

$$\{x \in \text{conv}(D) \mid x = \alpha a + (1-\alpha)y, a \in A, y \in \text{conv}(D), \alpha \in (0, 1]\}$$

denote the projective interior of A in D.

- $\text{pi}(D, A)$ is the union of the relative interiors of all faces intersecting A.
- If $x \in \text{pi}(D, A)$ is chosen, then something in A is maximal.

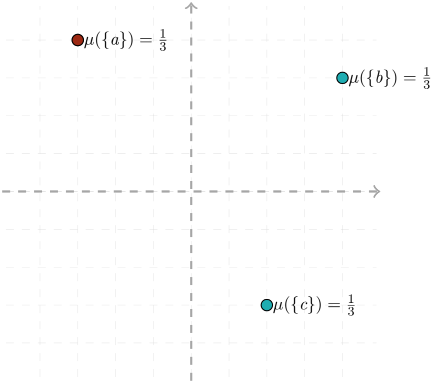
Let $CV(D)$ denote the set of decision problems with the same convex hull as D. Then set:

$$\rho_D(A) = \sup_{D' \in CV(D)} \mu_{D'}(\mathsf{pi}(D, A)).$$

- The resulting capacity is linear, mx-cont, monotone, and convex-**sub**-modular:

$$\rho_D(\alpha A + (1-\alpha)B) \leq \rho_D(A) + \rho_D(B) - \rho_D(A \cup B)$$

- Just need to increase LHS to make an equality.



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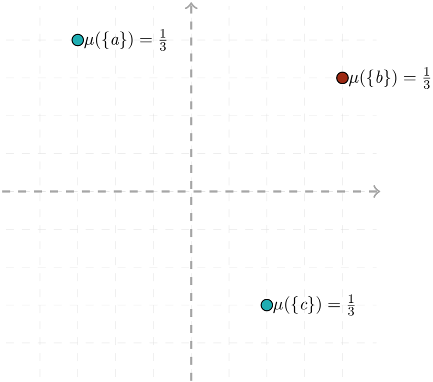

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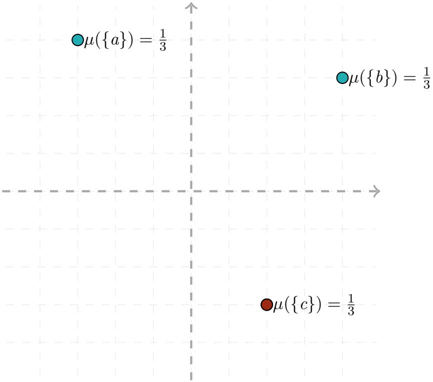

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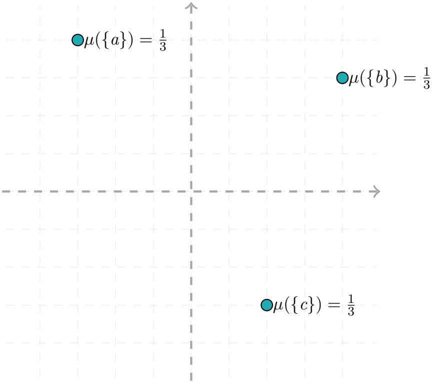

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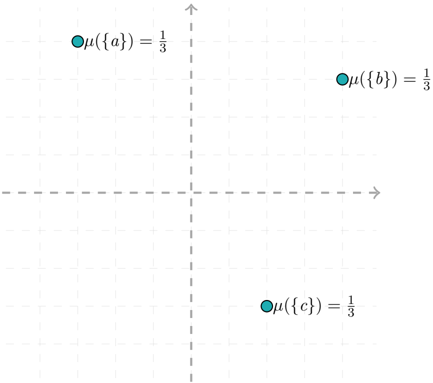

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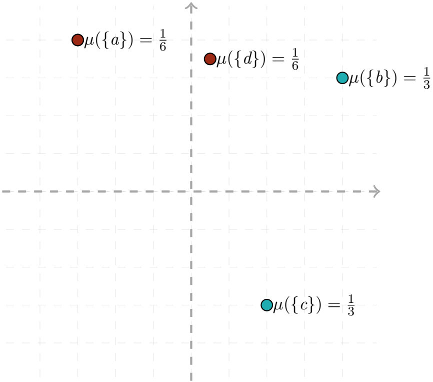

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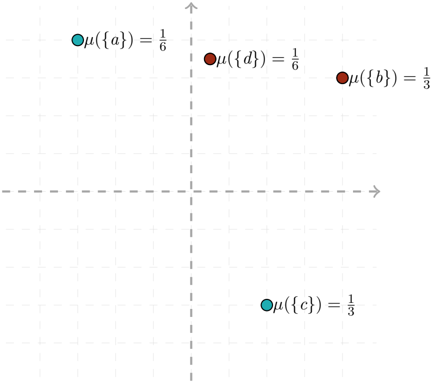

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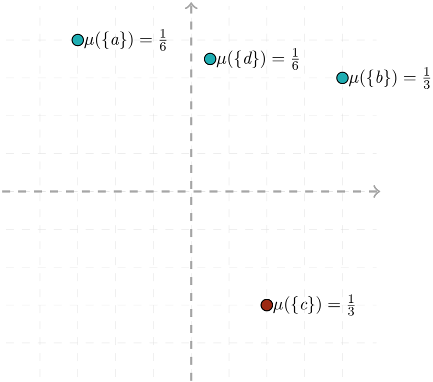

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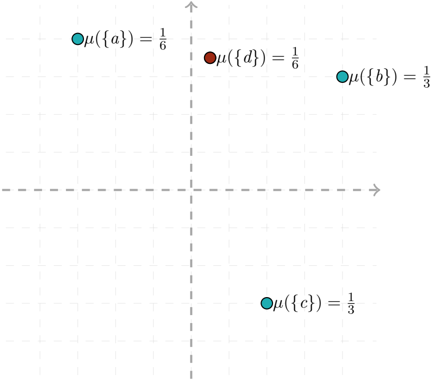

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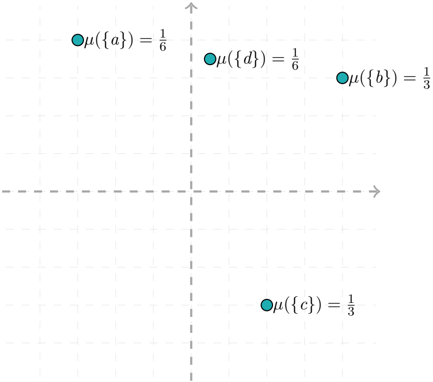

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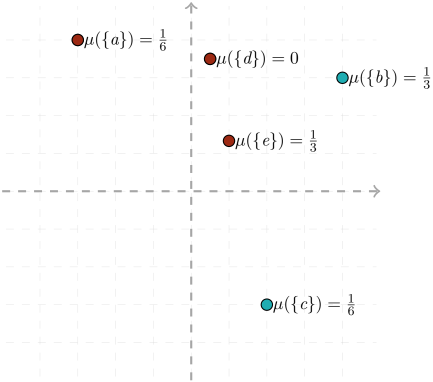

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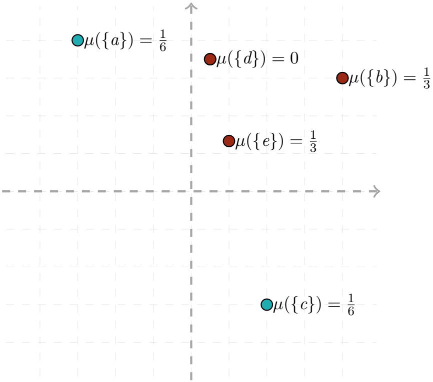

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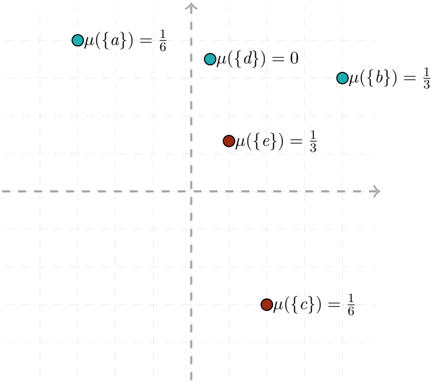

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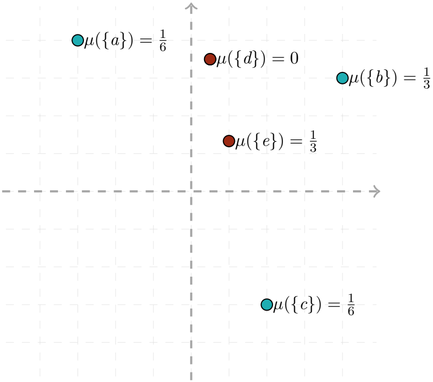

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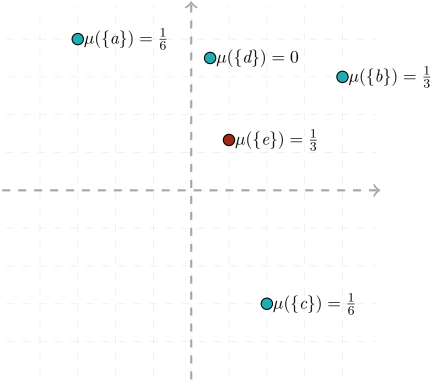

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