Regression Models Course Project

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Executive summary

This document represents the project in the Coursera/Johns Hopkins Regression Models course. The overall objective was to analyze the `mtcars` data in the R `datasets` package using regression models and exploratory analyses. Specifically, the following two questions were asked: "Is an automatic or manual transmission better for MPG?", and "Quantify the MPG difference between automatic and manual transmissions."

Manual transmission was associated with a 7.2 miles/gallon improvement compared to automatic (P = 0.001). However, in multivariable regression analysis holding weight, horse power and number of cylinders constant, manual transmission showed a non-significant positive association with MPG (1.8 MPG improvement, P = 0.21). In this model, weight and horse power seem to be the most important predictors.

Exploratory analysis and statistical inference

Due to space constraints, I can not show all code in this report. Several external packages were loaded for this analysis. Details on the dataset can be found through `?mtcars`. I transformed `mtcars` into `carsFactor`, changing the following variables from numeric to factor: `cyl, vs, am, gear, carb`.

I constructed a boxplot of MPG by transmission type (Appendix, Fig 1) that shows clearly increased MPG for cars with manual transmission. This relationship can be evaluated by `t.test(mpg ~ am, data = carsFactor)`), which demonstrates a highly significant difference in the means, P = 0.0014. Next, I examined the correlation between MPG and the other variables in `mtcars`:

```
sort(cor(mtcars)[1,])
```

```
##         wt        cyl       disp         hp       carb       qsec
## -0.8676594 -0.8521620 -0.8475514 -0.7761684 -0.5509251  0.4186840
##       gear         am         vs       drat        mpg
##  0.4802848  0.5998324  0.6640389  0.6811719  1.0000000
```

Several variables show a high correlation with MPG, as can also be visualised in a pairwise plot for `MPG`, variables with cor>0.7 (`cyl, disp, hp, wt`) and colour coded for `am` (Appendix, Fig 2a). This is further emphasized in Fig 2b, a scatter plot of MPG vs weight by transmission. From this exploratory analysis, it seems clear that it is oversimplistic to only study the relation between `mpg` and `am`.

Linear regression and model selection

Table 1: fitBase <- lm(mpg ~ am, data = carsFactor)

	Estimate	Std Err	t value	Pr(>|t|)
(Intercept)	17.147368	1.124602	15.247492	0.000000
amManual	7.244939	1.764422	4.106127	0.000285

This simple linear regression model shows a high correlation between transmission and MPG. The coefficent $\beta 0$ represents the mean MPG for automatic transmission (`am` = 0), and $\beta 1$ is the mean difference between automatic and manual. However, the adjusted R^2 is only 0.33, meaning that a large fraction of the variance remains unexplained by this model. Compare this to the full model (results suppressed due to space constraints):

```
fitAll <- lm(mpg ~ ., data = carsFactor)
```

In the full model, the adjusted R^2 is 0.78, so a larger fraction of the variance is explained. However, no variables are significant, and inclusion of too many varibles can lead to over-fitting. Going back to the correlation above, the variables with cor>0.7 (`cyl, disp, hp, wt`) all make sense to include in a model, since the weight and engine efficency should have an impact on MPG. Different models were compared:

```
fitAddWt <- update(fitBase, .~. + wt)
fitAddWtCyl <- update(fitAddWt, .~. + cyl)
fitAddWtCylDisp <- update(fitAddWtCyl, .~. + disp)
fitAddWtCylDispHp <- update(fitAddWtCylDisp, .~. + hp)
anova(fitBase, fitAddWt, fitAddWtCyl, fitAddWtCylDisp, fitAddWtCylDispHp)
```

```
## Analysis of Variance Table
##
## Model 1: mpg ~ am
## Model 2: mpg ~ am + wt
## Model 3: mpg ~ am + wt + cyl
## Model 4: mpg ~ am + wt + cyl + disp
## Model 5: mpg ~ am + wt + cyl + disp + hp
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1     30 720.90
## 2     29 278.32  1    442.58 73.5623 6.452e-09 ***
## 3     27 182.97  2     95.35  7.9244   0.00216 **
## 4     26 182.87  1      0.10  0.0165   0.89895
## 5     25 150.41  1     32.46  5.3954   0.02862 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

So, by the ANOVA we can see statistically significant differences from the base model until we add `disp`, but `hp` again seems to contribute. This leads us to the final model:

```
fitFinal <- update(fitAddWtCylDisp, .~. -disp + hp)
anova(fitBase, fitFinal)
```

```
## Analysis of Variance Table
##
## Model 1: mpg ~ am
## Model 2: mpg ~ am + wt + cyl + hp
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     30 720.90
## 2     26 151.03  4    569.87 24.527 1.688e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(fitFinal, fitAll)
```

```
## Analysis of Variance Table
##
## Model 1: mpg ~ am + wt + cyl + hp
## Model 2: mpg ~ cyl + disp + hp + drat + wt + qsec + vs + am + gear + carb
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     26 151.03
## 2     15 120.40 11    30.623 0.3468 0.9588
```

The ANOVA results demonstrate both a significant difference from the base model, and at the same time no significant difference from the full model. Let's take a look at the final model summary together with 95% CI estimates:

Table 2: fitFinal <- lm(mpg ~ am + wt + cyl + hp, data = carsFactor)

	Estimate	Std Err	t value	Pr(>|t|)	95% CI (lower)	95% CI (upper)
(Intercept)	33.7083239	2.6048862	12.940421	0.0000000	28.3539037	39.0627441
amManual	1.8092114	1.3963045	1.295714	0.2064597	-1.0609336	4.6793564
wt	-2.4968294	0.8855878	-2.819404	0.0090814	-4.3171812	-0.6764776
cyl6	-3.0313445	1.4072835	-2.154040	0.0406827	-5.9240572	-0.1386318
cyl8	-2.1636753	2.2842517	-0.947214	0.3522509	-6.8590220	2.5316713
hp	-0.0321094	0.0136926	-2.345025	0.0269346	-0.0602549	-0.0039639

In the final model, it is clear from the $\beta 1$ coefficient that manual transmission is associated with a 1.8 MPG increase, holding weight, horse power and number of cylinders constant. This result does not reach statistical significance at the $\alpha = 0.05$ level, however, as demonstrated both by the estimated p-value and the 95% CI. In this model, the other predictors seem to be more important. Finally, the adjusted R^2 of the final model is 0.84 (with multiple $R^2 = 0.87$), demonstrating a clear improvement in the fraction of the variance explained as compared to the base model.

Conclusion and diagnostics

From Table 2 above, we can

APPENDIX

Fig 1. Boxplot of MPG by transmission type

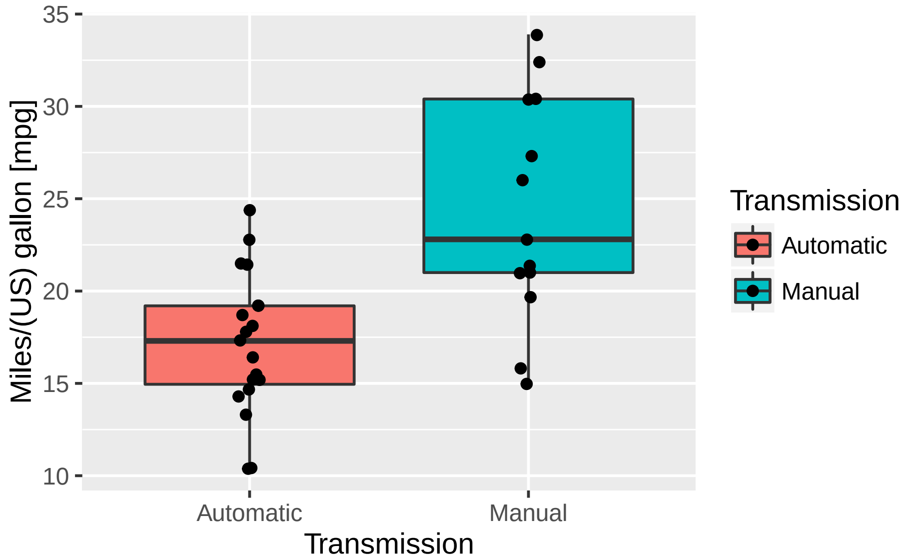
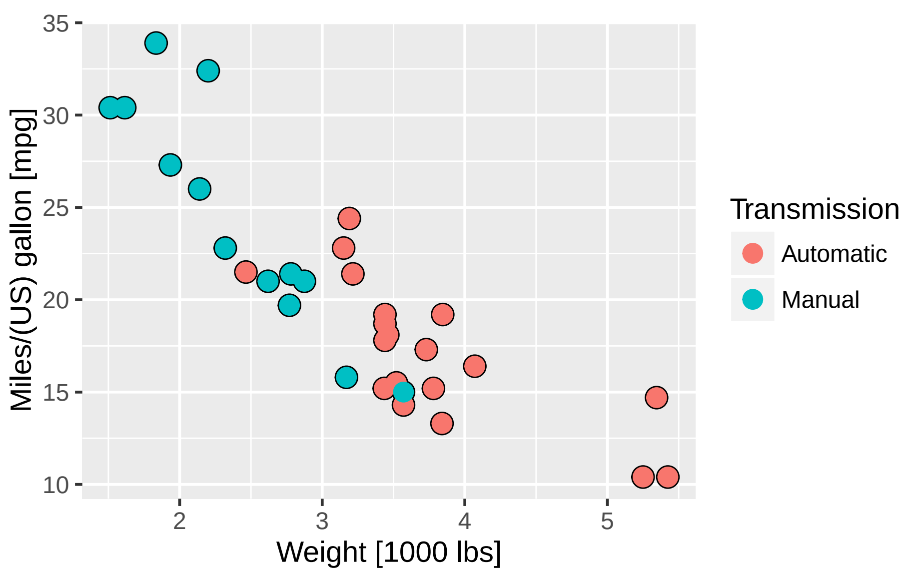


Fig 2a. Pairwise plot of selected variables from the `mtcars` dataset, colour coded by transmission

Fig 2b. Scatter plot of MPG vs weight by transmission



Diagnostic plot (FUNDERA PÅ NAMN)