Week 1

At the end of this worksheet you should be able to:

- convert units within the metric system.
- apply how to correctly convert area and volume units.
- apply scientific notation.
- apply the definition of percent change.
- convert between a ratio statement and percent change statement.
- apply the two forms of proportionality statements to make predictions from dependent variable change.

1. How many meters are in a decameter and how do you know?

2. How many decameters are in a meter and why?

3. How many centimeters are in a meter? Draw on your paper approximately a centimeter.

4. How many millimeters are in a meter and how many meters are in a millimeter?

5. The diameter of the earth is 6,380,000 m. What is this in kilometers?

6. How many centimeters are in 3.2 km?

7. How many inches are in 1 meter? Do this using the conversion of 1 in = 2.54 cm.

8. If I say your desk has an area of 0.5 meters, then what is wrong with that statement?

9. Convert miles per hour to meters per second.

10. List as many formulas for the area of different shapes as you can remember (or look some up). What do these all have in common? What about volume formulas? What does this tell you about the units of these kinds of quantities?

11. If your desk has an area of 0.5 meters^2, then what is its area in centimeters^2?

12. If a ball has a diameter of 18 cm, then what is the volume of the ball in meters^3?

13. Calculate your age in seconds on your last birthday.

14. Make up your own unit conversion problem that involves at least two conversions chained together.

15. Put the number 21,345,000,000 kg in scientific notation?

16. Put the number 0.0000000234 km in scientific notation?

17. Correct the scientific notation of 140×10^{-3} seconds.

18. Correct the scientific notation of 0.012×10^{-3} meters.

19. What is $0.000\,345$ meter in micrometers µm?

20. What is 3.4×10^{5} m in kilometers?

21. If I have three times as many marbles as you do then what is the ratio of my marbles to yours? What is the ratio of your marbles to mine?

22. If I increase in speed from 25 mph to 40 mph, then by what percent has my speed changed? By what ratio has the speed changed?

23. If my speed changes from 29 mph to 10 mph, then by what percent has my speed changed? What is the negative sign in the answer tell you? By what factor has your speed changed from initially to finally? Could a negative ratio make sense here?

24. If the generic variable y is inversely proportional to the a variable x, then write out this statement mathematically in two ways.

25. Since the area of a triangle is directly proportional to both the base and height, then how would an *equation* for this look like and what is the constant of proportionality?

26. The formula for the volume of a cylinder is $V = \pi r^2 h$. How would you write out a proportionality statement that was consistent with this formula? What is the constant of proportionality?

27. If the radius of a sphere changes by a factor of 2.7, then by what factor does the volume change?

28. How would you turn the previous problem ”inside out”?

29. By what percent does the volume change in the previous two problems?

Week 2

At the end of this worksheet you should be able to

- define a force and discuss the types responsible for most observable interactions
- add vectors graphically
- decompose vectors and recombine components
- discuss Newton's first and third law.

1. What is a force?

2. How much force is 225 lbs in newtons?

3. How much mass in kg is 225 lbs.

4. Draw two vectors represented as arrows, with one of them being approximately twice as big as the other, and pointed in different directions.

5. Graphically add these vectors up.

6. Show your results to someone else and look at their results. Draw their vector problem in the space below.

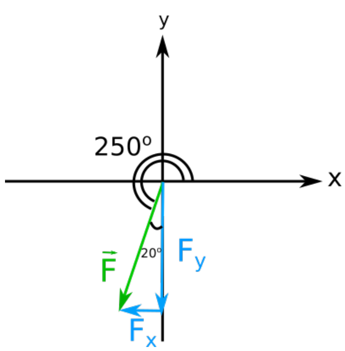
7. A vector is described as 20° *north of east*. Draw a sketch of this on the coordinate plane. Also sketch in its x- and y-components.

8. A vector is described as 20° *south of east*. Draw a sketch of this on the coordinate plane. Also sketch in its x- and y-components.

9. A vector is described as 20° *east of north*. Draw a sketch of this on the coordinate plane. Also sketch in its x- and y-components.

10. A vector is described as 20° *west of north*. Draw a sketch of this on the coordinate plane. Also sketch in its x- and y-components.

11. Draw out your own vector in any direction on the x-y coordinate plane, and describe its coordinates from both the nearest axis, and from the positive x-axis. I have done an example of this in the space below; you do another one.



12. A force is described as being 200° from the positive x direction. Express this in three other equivalent ways

13. A force is described as being 300° from the positive x direction. Express this in three other equivalent ways

14. A vector is described as being at $-30°$. Where is this and describe it in two other ways.

15. A book is resting on a table. What forces are acting on the book? What forces are acting on the table?

16. You are holding a glass in your hand. What forces are acting on the glass.

17. A book is sliding down a table that is inclined on one side. What forces are acting on the book?

18. What are the x and y components of a 100 N force that is acting on an object at an angle describes as 20° North of West?

19. Vector $\vec{A}$ is 10 N 30° *north of east* and vector $\vec{B}$ is 40 N *east*. What is the vector sum of $\vec{A} + \vec{B}$?

20. Use the simulation at `https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_en.html` to create a vector addition problem, and work throught it with our methods to check the results.

21. A phone is resting on an horizontal table. The phone has a weight (Force of gravity) of 10 N. What must the normal force from the table be for there to be 0 N *net force* on the phone?

22. One side of the table that the phone is resting on is raised so that the surface is at a 10° angle with respect to the horizontal. For this problem, make your coordinate system so that the x-axis is parallel to the surface of the table, and the y-axis is perpendicular to the table. With this coordinate system, what are the x and y components of the weight of the phone?

23. From the same problem as above, what would the normal force need to be so that the net force in the y-direction is 0 N? What would the force of friction need to be so that the net force in the x-direction is 0 N?

24. If a vector has an x-component of $+10$ N, and a y-component of $+20$ N, then what is the magnitude and direction of the vector?

25. If a vector has an x-component of -10 N, and a y-component of $+20$ N, then what is the magnitude and direction of the vector?

26. If a vector has an x-component of $+10$ N, and a y-component of -20 N, then what is the magnitude and direction of the vector?

27. If a vector has an x-component of $-10\,\mathrm{N}$, and a y-component of $-20\,\mathrm{N}$, then what is the magnitude and direction of the vector?

28. A person is pulling a sled with a rope across level snowy ground (frictionless). The sled has a weight of $1000\,\mathrm{N}$, and the person is pulling with a force of $500\,\mathrm{N}$ at an angle of $25°$ with respect to the horizontal. What are the x- and y- components of this tension force and what would the normal force need to be so that the net force is zero in the y-direction? What is the net force in the x-direction?

Week 3

At the end of this worksheet you should be able to

- discuss Newton's laws and provide examples of the application of each.
- discuss the quantities of displacement, velocity, and acceleration.
- apply Newton's first law to solve interesting physical problems.
- apply Newton's second law to solve interesting physical problems for objects that accelerate.
- apply Newton's third law to situations involving the motion of multiple objects.

1. What is the difference between mass and weight? Why can we use them interchangeably at the grocery store?

2. What base units are the composite force units of Newtons equal to?

3. What is your mass in kg, your weight in Newtons, and your weight in lbs? Start with which ever one you know and convert to the others. If you don't want to use your weight, then make up a weight a person could be and do the same thing.

4. What is the force of gravity between the moon and the earth? You will need to look up some values.

5. What is the force of gravity between two 75 kg people standing 1m apart?

6. Find the radius from the center of the earth where Earth's gravitational field strength would

    - $\frac{1}{2}$ of its value at the surface.
    - $\frac{1}{3}$ of its value at the surface.
    - What would be the *altitude* where the gravitational field is $\frac{1}{4}$ its value at the surface?
    - What is the gravitational field strength at the altitude that the space station orbits the earth?

7. What is the normal force on a 75 kg person standing on the floor? Where does Newton's first law come into play here?

8. If the person from the previous problem jumped, by exerting a 10 000 N force on the floor (in addition to their weight) then what is the normal force from the floor on the person?

9. If I hold a 5 kg cup motionless in my hand, what force do I provide to the cup?

10. If I raise a 5 kg cup with my hand, *at constant speed*, then what force do I need to provide from my hand to the cup?

11. If I accelerate the cup upwards with an acceleration of $+1\,\mathrm{m/s^2}$, then what force does my hand need to provide to the cup?

12. If I accelerate the cup downwards with an acceleration of $-1\,\mathrm{m/s^2}$, then what force does my hand need to provide to the cup?

13. If I push a 100 kg box with an applied force of 700 N along a friction-less surface. Find the force of gravity on the box, the normal force, and the net force.

14. Take the same problem from above and now add friction. The coefficient of friction is $\mu = 0.5$ then what is the net force?

15. In order to keep a box moving *at constant speed* along a friction-less level surface, what pushing force is required?

16. If I have to exert a force to keep a box moving along a level surface at constant speed, then what other force is probably present? What is the net force on the box in this case?

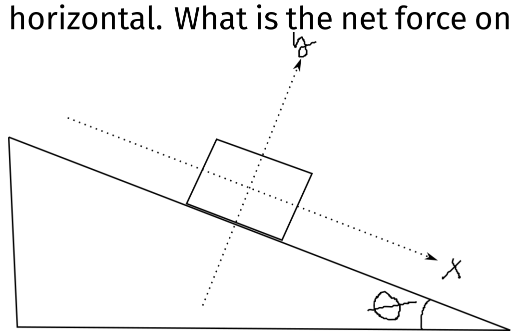
17. If I need to provide a 1000 N force to keep a 100 kg box moving at constant speed along a level floor, then what is the coefficient of friction between the floor and the box?

18. A 1000 kg car is parked on a hill that has an angle of 10° with respect to the horizontal. What is the weight of the car? What is the normal force on the car? What force is keeping the car from sliding down the hill? How large is that force?

19. If the hill that the car is parked on from the previous problem is somehow made steeper to a 20° incline, what is the normal force and the frictional force on the car now? It is still motionless.

20. If the maximum incline that the car can be parked on without sliding is 25°, then what is the coefficient of friction between the tires and the road?

21. A 100 kg box slides down a friction-less inclined plane that has an angle of 30° to the horizontal. What is the net force on the box and then what is the acceleration of the box?



22. A 100 kg box slides down a friction-full inclined plane that has an angle of 30° to the horizontal and a coefficient of friction of $\mu = 0.1$. What is the net force on the box and then what is the acceleration of the box?

23. Let's do the friction-less inclined plane problem *in general* for any mass and incline. Follow the same procedure as before but with the variable m for mass and θ for incline angle. Find an expression for the net force on the mass as a function of θ and for the acceleration as a function of θ.

24. Now let's do the inclined plane with friction *in general*. Just like the previous problem, use m for mass, θ for angle, and now use μ as a variable for coefficient of friction. Find an expression for the acceleration of the mass as a function of θ, m, and μ.

25. In order to *hold* a box on a friction-less inclined plane, that is keep it motionless, what force would be necessary to do that? Is there a difference between the force to hold it motionless on the incline and the force to push it up the incline *at constant speed*?

26. What is a displacement? What is velocity? What is acceleration? How do you know you have moved places or how do you know you are in motion? How do you know you are accelerating?

27. If I go from $x = 10\,\text{m}$ to $x = 28\,\text{m}$, then what is my displacement? If it takes 12 seconds to do that, then what is my average velocity over that interval. Does this mean that my velocity has this value at every instant along the path?

28. If I am initially going $+10\,\text{m/s}$ and it takes me 15 seconds to speed up to $+25\,\text{m/s}$, then what is my acceleration?

29. If my acceleration is $+10\,\text{m/s}^2$, then if I started at rest, how long would it take me to speed up to $+45\,\text{m/s}$? How fast would I be going after 10 seconds?

Week 4

At the end of this worksheet you should be able to

- discuss the quantities of displacement, velocity, and acceleration.
- interpret the meaning of the sign of the above quantities.
- discuss the cause of the change of those quantities.
- take a given state of motion of an object and predict its future state of motion.
- identify key features of a graph of these quantities over time.
- create a qualitative plot of these quantities over time given an interesting physical problem.
- solve a range of problems under the conditions of constant acceleration.

1. I move from a place marked $x = 2\,\text{m}$ to a place marked $x = 10\,\text{m}$ in $4\,\text{sec}$. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

2. I move from a place marked $x = -2\,\text{m}$ to a place marked $x = 10\,\text{m}$ in $4\,\text{sec}$. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

3. I move from a place marked $x = 2\,\text{m}$ to a place marked $x = -10\,\text{m}$ in $4\,\text{sec}$. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

4. I move from a place marked $x = -2\,\text{m}$ to a place marked $x = -10\,\text{m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

5. What is the difference between average velocity and instantaneous velocity? Give an example.

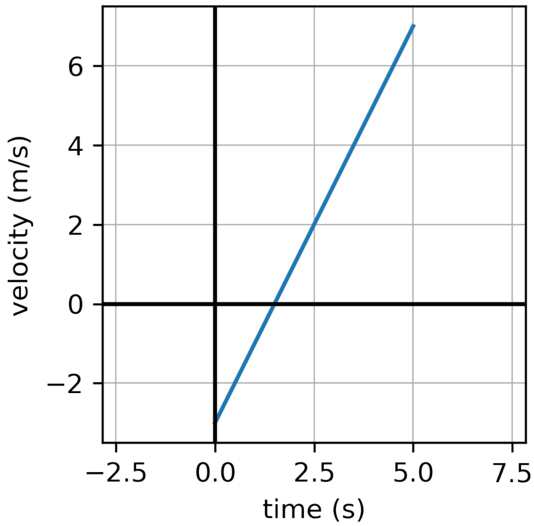
6. If the rate of change of position is called velocity, and the rate of change of velocity is called acceleration, then what is the rate of change of acceleration called? (this is not in your book, you'll have to search for it) Does the rate of change of *that* have a name?

7. An object has a velocity of $10\,\text{m/s}$ and then 5 seconds later has a velocity of $15\,\text{m/s}$. What is its average acceleration over this time interval? In what *direction* did the velocity change?

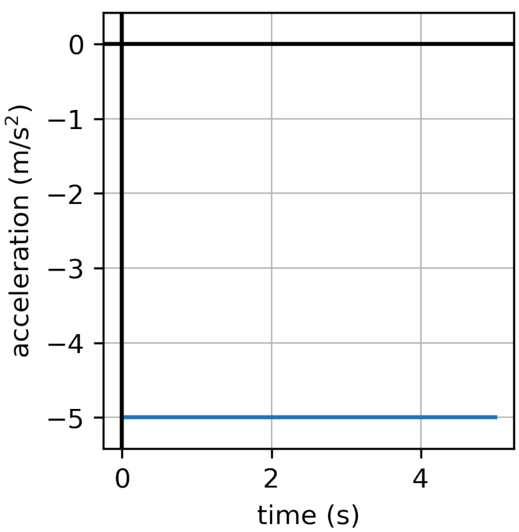
8. An object has a velocity of $15\,\text{m/s}$ and then 5 seconds later it came to a stop. What is its average acceleration over this time interval? What is the significance of the sign?

9. An object has a velocity of $-10\,\text{m/s}$ and then 5 seconds later it has a velocity of $-25\,\text{m/s}$. What is its average acceleration over this time interval? What is the significance of the sign?

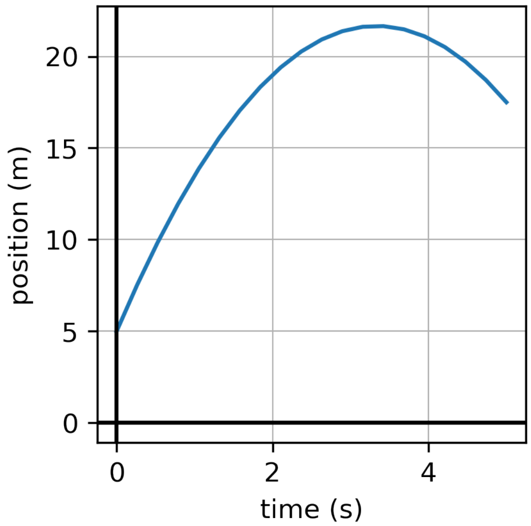
10. The following plot shows constant velocity vs. time. What is this acceleration? What is the initial velocity? Draw the acceleration vs. time graph. Draw the position vs time graph? What do you not know about the position graph?



11. The following plot shows constant acceleration over time. If the initial velocity is $v_i = 5\,\mathrm{m/s}$, then sketch a graph of velocity vs. time.



12. The following plot shows the position of an object over time. What is the sign of the acceleration? What is the sign of the initial velocity? Approximately when does the object come to a stop?



13. A turtle starts walking in a straight line at a steady speed from a position $x = 2.1\,\mathrm{m}$ to a position of $x = 5.4\,\mathrm{m}$ in 10 seconds. Draw a plot of position vs time for this motion. Draw a plot of velocity vs. time for this motion. Label all the important features on these graphs.

14. Draw a plot of velocity vs time for a case of 0 acceleration. Positive acceleration? Negative acceleration?

15. Draw a plot of velocity vs time for an object that is initially moving forward, but then stops and starts to move backward. Assume constant acceleration. What is the sign of the acceleration and what does that mean for the slope of this plot?

16. What does a plot of position vs time look like for the same case as before (initially moving forward, but then stopping and moving backward)?

17. Since $g = 9.8\,\mathrm{N/kg}$ also happens to correspond to acceleration due to gravity when no other forces are acting on the object, many people say $g = 9.8\,\mathrm{m/s^2}$. What is g in feet/second^2?

18. A 1 kg object slides down a friction-less inclined plane. The plane has an incline angle of 15°. What is the acceleration of the object? How far does the object gone down the plane in 3 seconds?

19. A 1 kg object slides down a friction-less inclined plane. The *length of the plane* is 1 m, and the *height above the horizontal* is 10 cm. How long does it take the object to reach the bottom and how fast it is going when it gets there?

20. Examples 4.1, 4.2 and 4.3 from the text book (pgs. 127-130) are excellent and you should use the space below to work those out for yourself.

21. Here is an example of turning a problem inside out. In the lecture video, I worked an example of a person throwing an object up and off the edge of a bridge. The object goes up and then comes down and splashes in the water below, 44.1 m below the place where it was released. In the example, we knew the time and found the initial velocity to be $+8.66\,\text{m/s}$. In this problem, use this information to find the time. (You should get 4 seconds)

22. I am standing at the very edge of a cliff and I want to know how far down it is to the ground or the water below me. To do this, I drop a rock off the edge, and count how long it takes to hit the ground. What is a general expression for the height of the cliff based on the amount of time it takes to hit the ground? If it takes 4 seconds to do this, how high is the cliff?

23. What is the velocity of a ball when it reaches its highest height after being thrown straight up?

24. What is the acceleration of a ball when it reaches its highest height after being thrown straight up?

25. If I throw a ball upward, what is an expression for the amount of time it takes to reach its highest height? If the initial velocity is $15\,\text{m/s}$, then how much time does it take?

26. How much time does it take for the ball thrown straight up to come back down?

27. So if I throw a ball straight up and its total travel time from when it leaves my hand to when it comes back down is 4 seconds, then what was the original speed with which I threw it?

28. What is the final speed when the ball in the previous problem comes back down? (Assume final position is the same height from which I threw it.)

29. The moon has a gravitational field strength $g = 1.6\,\mathrm{N/kg}$, so objects feel lighter on the moon in terms of lifting them vertically, but what about pushing horizontally? If you wanted to accelerate an 10 kg object horizontally (on a frictionless surface) on the moon with an acceleration of $2\,\mathrm{m/s^2}$, what force would you need to provide? Would this force be different on the earth? Why?

30. I am pushing a 200 kg box across a friction-full surface with 750 N force. The coefficient of friction between the box and the floor is $\mu = 0.5$. What is the net force on the box? What is the acceleration? What does the sign mean here? If the box was moving at 3 m/s when I began to encounter the friction patch on the floor, how long does it take to come to a stop?

31. I give a 2 kg book a quick push to start it sliding across a table at 10 m/s initially after my push it done, and it slides about 1 m across the table before it comes to a stop, then what is the coefficient of friction between the book and table?

The next two problems are here to set up the lab next week and are more in the vein of last week's problems, but still apply and are good to work through.

32. I am pulling two objects connected by a string. They are both 100 kg. There is no friction between the objects and the surface, and they are accelerating at $1\,\mathrm{m/s^2}$. What is my pulling force and what tension force is between the boxes?

33. One object m_1 connected by a string to another object m_2 that is hanging off the edge of the table. There is a pulley at the edge of the table so the string is not introducing any friction to the system. If this is a friction-less table, then what is an expression for the acceleration of the objects?

34. Now do the previous problem again, but with a coefficient of friction μ between the sliding mass (m_1) and the surface.

Week 5

At the end of this worksheet you should be able to

- apply the concepts of vector components to velocity.
- use Newton's 1st and 2nd laws to discuss the motion of objects in 2 dimensional projectile motion.
- apply the kinematic equations to be able to completely describe the motion of a projectile.
- apply the relationships between angle and motion at the edge of a circle to describe the motion of an object in circular motion.
- apply Newton's 2nd law in the radial direction to solve interesting problems involving motion of objects in a circular path.
- apply the principles of radial net force and circular motion to planetary orbits and satellites as well horizontal and vertical paths near earth's surface.

1. I initially throw a baseball at an angle of 10° with respect to horizontal. The initial speed of the ball is 60 miles per hour. What is the x- and y- component of the initial velocity?

2. If the x-component of a velocity vector is $+10$ m/s and the y-component is $+20$ m/s, then what is the speed of the ball and what is the angle of its initial trajectory with respect to the horizontal?

3. A baseball is thrown and then lands with a speed of 25 m/s at an angle of 30° degrees to the horizontal. If I described this as an angle with respect to the vertical what would I have said. What is the x- and y-components of the final velocity of this ball?

4. In the last problem, what is the initial horizontal velocity of the baseball? What information would you need in order to find what the initial vertical velocity component was?

5. I throw a baseball with a speed of 40 m/s at an angle of 60° degrees above the horizontal. The ball is released at a height of 1.6 meters. For each second that goes by, calculate the horizontal velocity, the vertical velocity, the horizontal position and the vertical position.

time	x-vel	y-vel	x-pos	y-pos
0 s			0 m	1.6 m
1				
2				
3				
4				
5				
6				
7				
8				

6. Plot your data for the last problem vs. time. (v_x vs t, v_y vs t, x vs t, y vs t)

7. I throw a baseball with a speed of 40 m/s at an angle of 60° degrees above the horizontal. The ball is released at a height of 1.6 meters. What is the vertical displacement when the baseball hits the ground? When will the ball reach its maximum height, and when will it hit the ground?

8. For a soccer ball that is kicked horizontally off the edge of a 10 m cliff with an initial speed of 20 m/s, what are the x- and y- components? When does the ball land?

9. If I just dropped the soccer ball off the 10 m cliff, how long would that take to land? Compare this answer to the previous problem. Is that surprising? What if you kicked the ball with 100 m/s initial speed horizontal velocity? Surely that would matter...

10. OK back to the soccer ball that is kicked horizontally off the edge of a 10 m cliff with an initial speed of 20 m/s. How far from the base of the cliff will it land?

11. For the previous problem, what is the *speed* of the ball when it lands and at what angle with respect to the horizontal does it hit the ground?

12. Let's do the last problem inside out. So take how far away from the base of the cliff the soccer ball lands, and work backwards to find its initial horizontal speed. The only assumption is that it is initially kicked exactly horizontally.

13. Now let's do the previous problem *in general*. If I kick a ball directly horizontally off a cliff of height h, and its lands a distance x away from the base of the cliff, then what was the initial velocity of the ball? (We will use this result in lab soon.)

14. How many degrees are in 1 rad?

15. A soccer ball of radius 10 cm spins through an angle of 20°, then how many radians is that? What distance has a point on the equator of the ball traveled? What if it spins through 750°, then what distance has a point on the edge traveled?

16. When you roll something along the ground, it is spinning of course, but it is also moving linearly (its center of mass is moving). It turns out that the distance the edge of a soccer ball moves as it spins is equal to the linear distance the ball moves, as long as it does not slip. So if a soccer ball of radius 10 cm rolls at constant angular speed through an angle of 500 rad, then how far has it rolled? If it takes 10 seconds to do this, what was its angular speed and what was its linear speed?

17. When a car turns at constant speed, it travels along an approximately circular path. In which direction does the net force act and what provides this net force?

18. For a 1000 kg car turning like in the previous problem, if the coefficient of friction between the tires and the road is $\mu = 0.5$, then what is the maximum static force of friction that

the road could provide to the car? If the car is going around a bend of radius 50 m, how how fast could it go around the bend without sliding?

19. If the same 1000 kg car is attempting to go around a bend of radius 20 m, at 20 m/s, then can it do this safely without sliding? ($\mu = 0.5$ still)

20. The earth orbits the sun, and while its path around the sun is not exactly circular, its close enough to treat that way here. What is the angular velocity of the earth around the sun? To do this, think about how long it takes to go one full revolution around the sun. How many radians is a revolution? So now how many radians per second does the earth travel around the sun?

21. What is the radius between the earth and the sun? (look this up in your book or google) Using the answer from the previous problem, what does this mean for the *tangential speed* of the earth around the sun?

22. Now without looking it up, how could we use this information to determine the mass of the sun? Remember that the formula for the force of gravity between to masses can be written as, $F_g = \frac{Gm_1m_2}{r^2}$ ($G = 6.67 \times 10^{-11}\,\mathrm{Nkg^2/m^2}$). Now look up the mass of the sun and see how how close we got?

23. By the way, how can we use free fall to get a measure of the mass of the earth? If we got to the lab and measure an acceleration of a 1 kg mass to be $9.82\,\mathrm{m/s^2}$, then how can we calculate the mass of the earth?

24. In order to put a satellite into orbit around the earth, it needs to be traveling at a specific distance with a specific velocity, otherwise the force of gravity from the earth may be too large, and it will crash, or too small and it will fly away into space. So suppose you wanted to put a 1000 kg satellite in orbit around the earth at a distance of 1000 km above the *surface of the earth*. How fast would this satellite need to be going in order to have this orbit?

25. If you wanted to kick a soccer ball horizontally off a cliff and have it go into orbit near the surface of the earth, then what velocity would you need to give it to achieve this?

26. (OMIT) A pendulum is swinging in a horizontal circle. The length of the pendulum is 1 m. If the angle of the pendulum string is 25°, then what is the radius of travel of the pendulum bob?

27. (OMIT) The mass of the pendulum bob from the previous problem is 1 kg. What upward force is necessary to keep the pendulum from moving up and down? What does this imply about the tension in the string? What does this mean for the radial tension force? How fast must this pendulum bob be moving?

28. (OMIT) When you are swinging a ball at the end of a string in a *vertical* circle, explain why the tension in the string is higher when the ball is at the bottom of its path, than when it is at the top of its path.

29. A roller coaster cart is doing a loop-the-loop. When the cart is at the top, what forces are acting on the cart to keep it in its circular path? What is the minimum force that would still technically mean that the cart is still in contact with the track? For a 30 m radius loop, what is the minimum speed that the cart must be going to make the loop without losing contact with the track?

Week 6

1. We have gone through several kinds of equations now and lets sum up some of these *as proportions*:

   - acceleration is ____________ net force.

   - assuming constant acceleration and beginning at rest, an object's velocity is ____________ the displacement.

   - assuming constant acceleration and beginning at rest, an object's displacement is ____________ the elapsed time.

   - for an object that has been dropped, the distance it has fallen is ____________ its velocity at that distance.

2. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N over a distance of 10 m. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 m?
   Some starters:

   - What is the net force on the box?
   - What is the acceleration of the box?
   - What is the final velocity after 10 m

3. Following up on the previous problem, if I stopped pushing after 10 m and the box continued with its speed, and then at then started sliding up a 20° ramp, then how far along the length of the ramp would the box rise? What height is this above the horizontal? Do the 100 kg and the 200 kg box rise to the same height?
   Some starters:
   - What is the net force on the box as it goes up the inclined plane?
   - What is the acceleration of the box as it goes up the inclined plane?
   - What is the sign of the displacement of the box going up the plane?
   - Is the sign of acceleration the same or different than displacement?

4. We want to examine the idea of *mechanical advantage*.
   - How much force would it take to lift a 100 kg box straight up at constant speed a distance of 5 meters.
   - How much force would it take to push a box up an inclined plane that was 20 m long up to the same height? (again at constant speed)
   - What is the ratio of the two forces in these two cases? Which would you rather do? This ratio is known as *mechanical advantage*

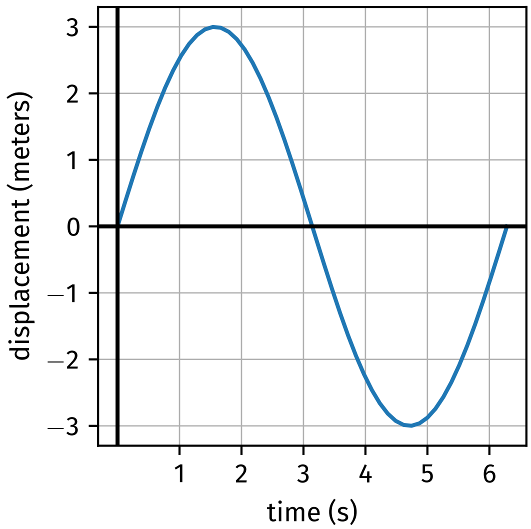
- What is the ratio of the displacement in these two cases?

- How can you use this idea to quickly figure out the force it would take you to push the box up a 100 m ramp that goes up the same height?

5. Let's talk about springs. *Hooke's Law* is the relationship between the force exerted by a spring and amount the spring has been stretched or compressed. The amount a spring has been stretched or compressed is the displacement of the end of the spring. Hooke's law says the magnitude of the force exerted is directly proportional to the displacement of the end of the spring. Write this in terms of a proportionality statement and again as an equation with a *constant of proportionality*. What are the units of the constant of proportionality?

   Draw a qualitative plot of the magnitude of this force vs displacement. What is the slope of this graph?

6. Consider the graph below of displacement under condition of *not constant* acceleration. What would a graph of velocity vs. time look like for this case? What about acceleration vs time.



7. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N for 10 s. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 s?

8. A follow up to the previous question. The *momentum* of an object is defined as the product of an object's mass and its velocity. In this case the momentum of this object changes because a force external to the object was exerted on it. What is the momentum

of the object initially? What is the momentum of the object at the end? It is said that the change in momentum of an object is equal to the *impulse*, where *impulse* is defined as the product of constant force and the amount of time the force is acting. Is that the case here? How could you use this to quickly find the final velocity of an object if the same force were pushing it for 100 s?

9. If I launch a cannonball from ground level with a speed of 1000 m/s, then there are a variety of angles to choose from. If I fire it at 90°, then how long will it take to come down? How high will it go? How far will it go horizontally? What about 80°? What about 70°? Let's just make a table...

Angle	Time	Max Vertical Height	Horizontal Distance
90°			
80°			
70°			
60°			
50°			
40°			
30°			
20°			
10°			
0°			

10. If a soccer ball with a radius of 10 cm is rolls along the ground without slipping at 5 m/s, then how many revolutions does it roll through in 10 s and what distance has a point on the edge of the ball traveled? Some starters:

- How fast is it *spinning*? By that we mean *angular speed*.
- How many radians does the ball rotate through in this time? What is that in revolutions?
- How far does it roll in this time? Is this the same distance as the distance of a point on the edge of the ball? Why or why not?

11. Following up on the previous problem, how many seconds does it take for the ball to complete one revolution? This amount of time is referred to as the *period* of its rotation, and this is a similar characteristic time for the motion of the ball as the *period of a pendulum* was in the first lab.

12. Another follow up. How many revolutions does the ball travel through *per second*? You could convert this from angular speed ω that you would have calculated in the first instance of this problem, but if all you new was the period of the ball's rotation, how could you calculate it from there? *(Hint: what is the difference between revolutions per second and seconds per revoution?)* This quantity of revolutions per unit of time is sometimes called *frequency*.

13. Suppose a satellite is in orbit around a distant planet. You observe the the satellite to be 5000 km from the center of the planet, and rotating the planet once every 2 days. What is the mass of the planet you have discovered? What is the period of the satellites motion?

What is its frequency? How fast is the satellite moving around the planet? What is the angular speed

Week 7

At the end of this worksheet you should be able to

- to discuss the relationships between the quantities of work, energy, displacement, velocity.
- differentiate between a conservative force and a non-conservative force.
- apply the work energy theorem to solve interesting problems that would be hard to use Newton's Laws.
- discuss the principle of conservation of energy and explain when it is useful.

1. *Work* is defined as a transfer of *energy*. This transfer occurs by one object exerting a force on another object *over some displacement*. But the relative directions of these two vector quantities (force and displacement) matters. Summarize the work done in 5 different cases that are represented below by drawing the object and the vectors representing force $\vec{F}$ and displacement $\Delta\vec{x}$. In each case I have provided a simple example to illustrate what I mean. You provide another one.
   - The force and displacement point in exactly the same direction. (I push a box across a level floor with 100 N a distance of 10 m).
   - The force and displacement point in different directions, but the angle between them is less than 90°. (I pull a box across a level floor with a string, directing 100 N at an angle of 20° with respect to the floor.)
   - The force and the displacement are exactly perpendicular to each other. (A 10 kg box is sliding across a friction-less surface, and the normal force is acting on the box.)

- The force and displacement point in different directions and the angle between them is greater than 90°. (I bring a sliding box to a stop by exerting a 100 N force on it at and angle of 30° with respect to the horizontal.)

- The force and displacement point in exactly opposite directions. (I bring a sliding box to a stop by exerting a 100 N force over a distance of 10 m.)

2. What amount of work is done by a person to lift a 100 kg object a distance of 1 meter high? What amount of work is done by the force of gravity? If the person dropped the box what amount of work would the force of gravity do on the box as it fell? What velocity would it achieve before it hit the ground?

3. What amount of work is required to push a 100 kg object up a friction-less inclined plane of that is 10 m long that's end it 1 meter high? How does this compare to the work done to lift it? Show that this can be used to derive the formula $\frac{F_{\text{push}}}{weight} = \frac{height}{l_{\text{plane}}}$.

4. An object has some initial velocity at the bottom of a friction-less ramp and it begins to slide up the ramp. The force of gravity does negative work here and the object slows down to stop. The ramp has an incline angle of 20° with respect to the horizontal. Calculate the work done by the force of gravity and see that it is a negative value in three ways:

   - What is the angle between the displacement and the force of gravity? Use this angle and the definition of work to calculate the work.

- What is another angle between the displacement and the force of gravity? Now use this angle and the definition of work to calculate the work.

- What is the *component* of the force of gravity that is in the direction of the displacement? Are these vectors in the same direction or opposite directions? What is the work done using component and the displacement?

5. *Kinetic energy* is the energy of an object that has velocity. In order to calculate it, use $K = \frac{1}{2}mv^2$. Calculate the kinetic energy of a 10 kg object that has a velocity of 10 m/s. If you do some work to *double* the velocity of the object, what is the new kinetic energy? What is the ratio of the kinetic energy final to the initial kinetic energy? What is the *change* in kinetic energy? How much *work* would be required to cause this change in kinetic energy? If this was done by a force pointed in the direction of the objects motion acting over a distance of 10 meters, what is the magnitude of the force?

6. I push an object at constant velocity of 1 m/s over a friction-full surface. I exert a force of 100 N and do this over a distance of 10 m. What work have I done? What work has the force of friction done? What is the net work done? What is the kinetic energy initially? Does the kinetic energy change? What power am I providing?

7. I pull a 10 kg object with a rope at an angle of 20° to a horizontal friction-full surface. I use a force of 100 N on the rope, and the coefficient of kinetic friction is $\mu = 0.1$. I start at rest and exert this force over a distance of 100 m.

   - What work do I do?
   - What work does the force of friction do?
   - What work does the normal force do?
   - What work does the force of gravity do?
   - Using these works, what is the net work?
   - What is the change in kinetic energy?
   - What is the final velocity?

- What is the net force?

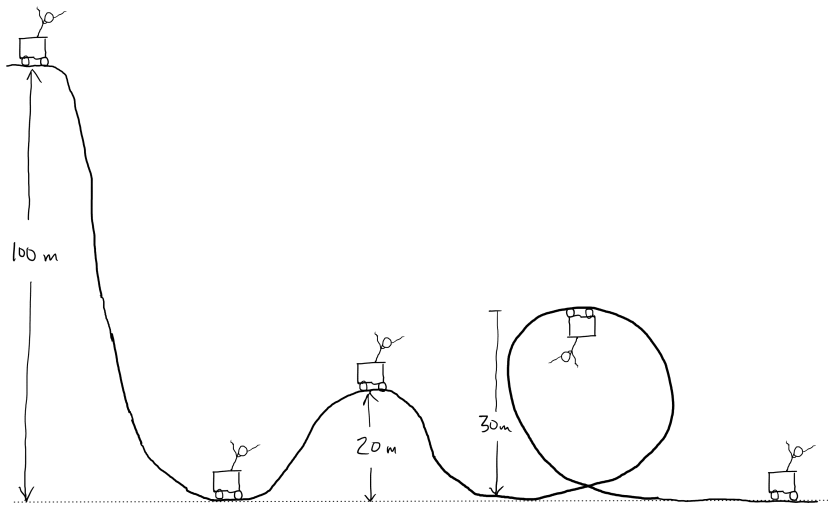
- Using the net force, what is the net work?

8. What forces are conservative forces and what are not conservative forces?

9. When a 10 kg object is 10 m high, what is its potential energy? If it begins it to fall, what is its potential energy after it falls 1 m? How much has its kinetic energy changed?

10. An object falls from a height of 100 m then how fast is it going when it hits the ground? Solve this using kinematics an then again using conservation of energy?

11. A roller coaster starts from rest at the top of a hill and rolls down its course. Find its kinetic and potential energy at each position marked.



12. A 10 kg box slides down a friction-full inclined plane ($\mu = 0.1, \theta = 30°$). The height of the plane is 1 m above the horizontal. What is the speed of the box at the bottom of the plane? How does this compare to if there were no friction? How much work has been done by the force of friction?

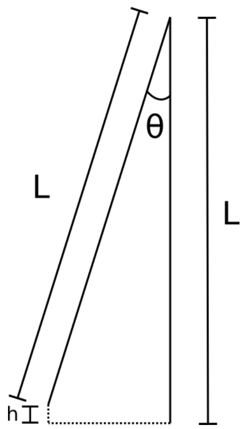
13. Here is an example of a problem that would be much more difficult to do with Newton's Laws. Take the same inclined plane as the problem above, but make *most* of the plane friction-less and only a 10 cm portion in the middle of the plane have friction $\mu = 0.1$. Now what is the speed at the bottom of the plane? Think about how you would have solved this using Newton's Laws and kinematics.

14. The maximum speed of a child on a swing is 5 m/s. At this point the child is 1 m above the ground. What is the maximum height of the child above the ground? Do this two ways, once with $U_g = 0$ at the ground, and once again with $U_g = 0$ at the bottom of the swing's path.

15. A 100 kg cyclist (and bike) has at constant speed of 10 m/s up an incline of 5% (vertical height/horizontal distance as a percent). What power output does the rider need to provide to do this?

16. A pendulum swings from some maximum height where its velocity is zero to a minimum where is velocity is a maximum and then back up to a maximum height. If the maximum height is 0.1 meters above the minimum height, then what is the speed of the pendulum at the bottom of its swing?

17. For a pendulum, it is hard to measure its maximum height, but it is easy to measure its length and to measure its angle from the vertical. If the maximum angle from the vertical of a 1 m long pendulum is 20° then how high is this above the horizontal? (*Hint: draw a line from the end of the pendulum when it is at its maximum height perpendicularly to the line when it is at is minimum height.*)

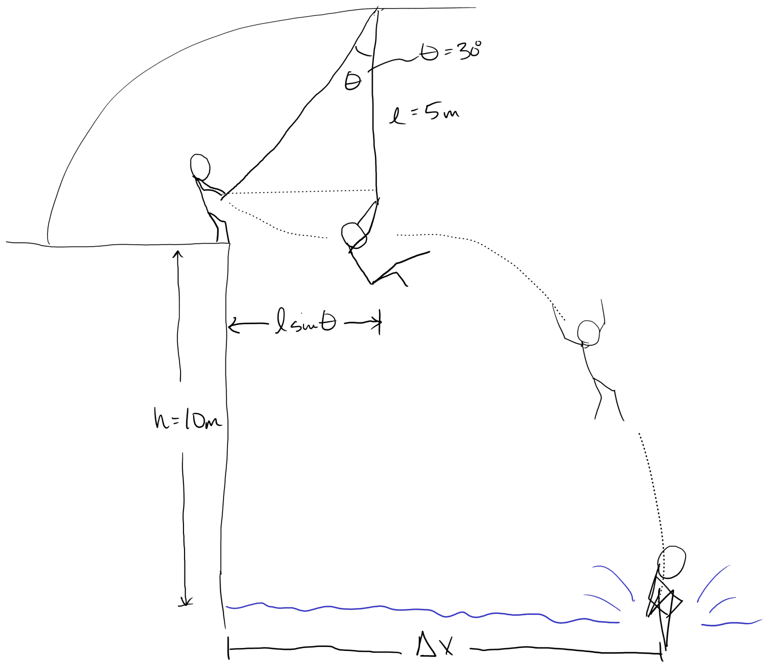


18. A horse pulls a 250 kg cart a distance of 1.5 km. The frictional force on the cart is a constant 250 N. The horse eats oats in the morning to prepare for this trip. Each gram of oats provides 10 kJ of energy, but only 10% of this energy can go into work pulling the cart. How many grams of oats must the horse eat?

19. I want to try a weight loss program that involves repeatedly lifting a 50 kg barbell from the ground to over my head at a height of 2 m. I can do this about 5 times per minute. How long will it take me to burn 0.5 kg of fat? "Burning" fat means I have used it to supply energy to do work. Each gram of fat has roughly 39 kJ of energy to the body, but the muscles can only use 10% of this to do work.

20. What is the minimum height h that a roller coaster needs to start at rest in order to do a loop-the-loop of radius r and not lose contact with the track?

21. If you are doing a rope swing and then at the lowest point in the swing you let go and drop into a lake below, how far from the edge of the cliff do you land in the water. See the diagram below for the relevant parameters.



Week 9

At the end of this worksheet you should be able to

- find the momentum of an object or collection of objects.
- find the change in momentum of an object or collections of objects due to an impulse.
- use the conservation of momentum to solve for an unknown quantity.
- use the principle of relative velocity to solve for an unknown in elastic collisions.

Note that at in every problem this week we will be ignoring friction and air resistance. Its not that momentum can't work with those quantities, but momentum does not really help tell us anything new about them, so our problems will not involve friction unless explicitly specified. When using the conservation of momentum in this way, we really restrict ourselves to talking about the motion of the objects *just before* they collide as well as *just after* they have stopped colliding.

1. In this problem a bug and car collide. Assume the car is coasting frictionlessly. Also assume you can measure quantities with perfect accuracy.
    - A 1 g bug is flying east through the air along a road at 5 m/s. What is the bug's momentum? Remember that momentum is a *vector* so if you are calling the momentum positive that means east.
    - A 1000 kg car is traveling west along this same road at a speed of 20 m/s. What is the car's momentum. Remember the sign!
    - What is the total momentum of this system?

- When the bug collides with the windshield of the car, what is the momentum of the bug-car system? What is the total mass of the car now? What is the speed of the car?

2. You want to close a door but you don't want to get up. You look around and see that there is a bouncy ball that looks like you could throw it pretty fast. For some reason there is also a wad of clay that you know would stick to the door if you threw it. The ball and the clay have the same mass. Which one should you throw against the door to close it most effectively?
   Some starters:

   - Choose a mass for your ball and clay, and choose a velocity. Or leave it as m and v and work it *in general*.
   - If the ball hits the door and bounces back perfectly, its speed should be basically the same after it bounces off the door. How has its velocity changed? How much has its momentum changed?
   - If the clay hits the door and sticks, then what is its change in momentum? Its speed is basically zero after the collision.
   - Which one of these cases would close the door more effectively? If the time interval of the collision is about the same for both of these cases, and we model the force of collision as constant, then what is the ratio of the force from the bouncy ball, to the force of the clay wad?

3. You know the kinetic energy of a 10 kg object is 100 J, then what is the momentum of the object? Now do this *inside out*. Now do it *in general*. In other words show where $K = \dfrac{p^2}{2m}$ comes from.

4.
   - If I am are standing motionless, what is my momentum? I have a mass of 75 kg.
   - What if I am wearing a backpack with 100 - 0.5 kg baseballs while standing on ice (with skates)?
   - I throw one of the baseballs with a velocity at $-15\,\text{m/s}$. What is its momentum?
   - What is my mass after I throw it (think about the mass of my backpack)? What is my momentum after I throw the baseball? What is my velocity?
   - If I throw another baseball with a velocity of $-15\,\text{m/s}$ *relative to my current velocity*, then what is the velocity of the ball relative to the ice?
   - What is my new velocity after throwing this second baseball? How much did my velocity change from before?
   - If I did this again, throwing a baseball with a velocity of $-15\,\text{m/s}$ *relative to my current velocity*, what is my velocity after I throw it?

- I keep doing this until I am out of baseballs. How fast am I going?

5. An empty 10 kg wagon is rolling past me at a speed of 5 m/s while I am holding a 30 kg bag of concrete. If I drop the concrete bag into the wagon right at it gets to me, what is its speed immediately after that?

6. I drop a 1 kg basketball from a height of 1 m and it bounces off the floor and rises to a height of 0.75 m. How much energy was converted to internal energy?
   Some starters:
   - What is the basketball's velocity right before it hits the ground?
   - What is the basketball's velocity right after it hits the ground? Think about what it must be for it to rise to 0.75 m.
   - What is the ratio of the change in kinetic energy?
   - What is the change in kinetic energy?
   - What is the change in momentum of the ball?
   - Is momentum conserved here? Why or why not?

7. Explosions are actually a kind of collision, but in reverse. Let's work a problem to see. Two indestructible objects are tied together with a stick of dynamite between them and everything is at rest. When it explodes, they fly away from each other in opposite directions. One of the objects has a mass of 10 kg and the other a mass of 3 kg. The 10 kg object flies away with a speed of 100 m/s. What is the velocity of the 3 kg object? At least how much energy was the explosion (some of it was probably also put into internal energy and sound)? What other problem in this worksheet is this similar to?

8. A 5 kg gun fires a 10 g bullet at a velocity of 500 m/s. The explosion that causes this shot takes place in 1 millisecond. What force is exerted on the gun and what is the gun's recoil velocity?

9. Two objects collide elastically, one has a mass of 1 kg and the second has a mass of 3 kg. The first object is traveling to the right at a speed of 3 m/s and the second is traveling in front of it initially at a speed of 1 m/s. What is the velocity of the objects after the collision. Think of and work as many variations of this problem as you can.

10. Two objects collide elastically. Two objects collide elastically, one has a mass of 1 kg and the second has a mass of 3 kg. The first object is traveling to the right at a speed of 3 m/s. With what velocity would the second object need to be travelling so that after the collision, the first object was motionless?

11. A massive object moving with a velocity v and is going to collide elastically with a very small object that is initially at rest. What is the velocity of these two objects after they collide? (*By massive and very small I mean when you add or subtract m_1 and m_2 they are indistinguishable from m_1 due to the rules of significant figures.*)

12. Reverse the previous problem. A tiny object is colliding with velocity v with a huge object that is at rest. What is the final velocity of the two objects?

13. Let's work the ballistic pendulum problem like we do in the lab and do it *in general*. So we have a bullet of initial velocity v_{bi} and mass m_b and a target initially at rest with mass m_t. The collision is perfectly inelastic, and the pendulum (with the bullet inside) rises to a height h after the collision takes place. What is the initial velocity of the bullet?

14. Now in question #2 of the lab write-up it asks could this experiment be done with an *elastic* collision, given that we only measure the same things as we did in the inelastic collision case. So same parameters as before, but this time when the ball bounces off the target it has a speed v_{bf} and the pendulum rises to a height h but this time without the bullet embedded in it.

Week 10

At the end of this worksheet you should be able to

- describe the quantities involved in discussing rotational motion and the connection of them to translational motion.
- calculate the quantities describing rotational motion.
- use the conditions of equilibrium to solve for an unknown quantity.
- use the principle of conservation of angular momentum to solve for an unknown quantity.

1. In the lecture video, I talked about the analogy between linear (translational) motion and rotational motion. All of the most important equations that we have used have a corresponding version in rotational motion. I wrote down in the notes many of these rotational version, but I skipped over the kinematic equations so let's do them here. I am not interested in you working with these equations since we have conservation of energy and conservation of angular momentum now, but do this as an exercise in understanding the analogy.

2. You exert a force of 100N to open a door. To get the door swinging you exert this force for 1 second and you apply this force perpendicularly to the door. The door is 1m wide and 2m tall and has a mass of 30kg. The door handle is 0.9m from the hinges.
    - What torque do you exert on the door?
    - What is the rotational inertia of the door? $I = \frac{1}{3}ML^2$
    - What is the angular acceleration of the door?

- How fast is the door moving when you stop pulling and let it swing?

- What is the initial angular momentum of the door? What angular impulse did you give the door?

- What is the door's final angular momentum? Use this to find the angular speed when you stop changing the door's momentum.

- What is the door's kinetic energy after you stop pulling?

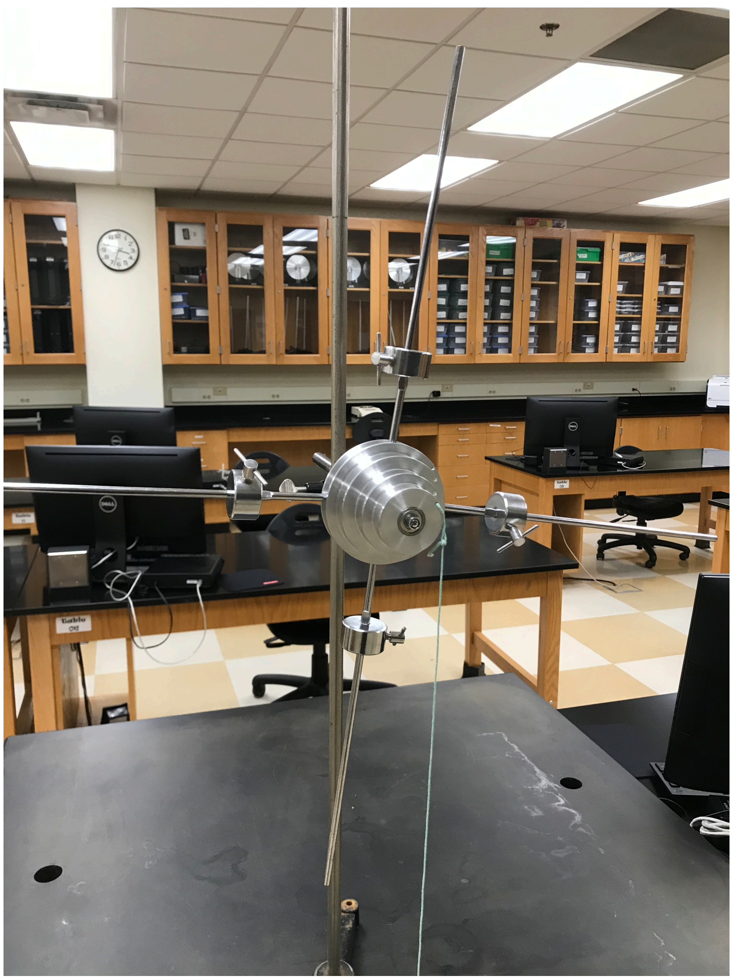
- What was its initial kinetic energy? How much work did you do to open the door?

- Over what angular displacement did you exert this 100N force?

- I can't think of anything else to calculate about this; can you?

3. Look up and estimate the rotational inertia of some objects around you. I see a basketball, a book, an iPad, a car tire, and a barbell. Pick some objects and look up their dimensions and calculate their rotational inertia.

4. Let's calculate the rotational inertia of the demonstration apparatus that I used in the video. This is a compound shape, but we only need to add together the individual rotational inertia of each. Also since the point masses on the rods and the rods themselves are repeated 4 times around the device, we just need to find that once and multiply by 4. The pulley in the center is 4 discs stacked together, so we will model each one and add them together.



5. In lab next week, we will measure the rotational inertia of a disk and also of a hollow cylinder. To do this we will apply a constant torque to a pulley below the spinning object and measure the angular acceleration of the spinning object. To apply a constant force to the pulley, we will use a mass that will hang down from a string and have a constant force of gravity applied to it. Lets do this problem *in general* and develop an equation to calculate the rotational inertia from knowing the hanging mass, the radius at which the string is applying a torque to the pulley. *Be careful here! The force applied to the pulley is not mg. The hanging mass is accelerating, but not with* $9.8\,\mathrm{m/s^2}$
Some starters:
   - Draw a free body diagram on the hanging mass. You know the weight but you do not know the tension.
   - In lab we will measure the angular acceleration, so how can we write the acceleration fo the hanging mass in terms of the angular acceleration of the spinning pulley its attached to?
   -

6. To generate torque with a wrench, you exert a force at the end of the wrench, 30cm away from the bolt. You push with 500N in the direction to loosen the bolt, but it will not budge. Why is it not rotating? In order to generate more torque, you go get a longer wrench, this one 55cm. You can still only push with 500N of force, so how much torque have you applied now? By what factor have you changed the length of the lever arm? By what factor have you changed the torque?

7. A person laying on the floor is doing a leg lift with an ankle weight on. The ankle weight is 50 N. Explain why the exercise is hard at first when the leg is horizontal but very easy when the leg gets to vertical. Find the torque on the persons leg when their leg is at 30° with respect to the horizontal. The ankle weight is essentially at the end of their leg which is 1m long, and the mass of their leg is 12.75kg. Take the center of mass of the leg to be the geometric center of the leg. Calculate this again when their leg is at an 80° with respect to the horizontal.

8. Two people are carrying a 1000N wooden beam that is 5m long. One person is positioned at one end of the beam, and the other person is positioned 1m away from the other end. What force is each person exerting on the beam?

9. Let's do the previous problem inside out. A question might read, "Where should the second person be positioned so that his force on the beam is ______________ ?" Provide the force from the previous question and solve for his position from one end.

10. Another followup, where would the second person need to be so that his force was 750 N?

11. Chris and Jamie are carrying Wayne on a horizontal stretcher. The uniform stretcher is 2.00 m long and weighs 100 N. Wayne weighs 800 N. Wayne's center of gravity is 75.0 cm from Chris. Chris and Jamie are at the ends of the stretcher. What force is Chris providing to support the stretcher? What force is Jamie providing to support the stretcher?

12. A pole-vaulter holds out a 4.75m pole horizontally in front of him. Assuming the pole is uniform in construction, and that he holds the pole with one hand at the very end, and one hand 0.75m from the end, what is the ratio of the force applied by the hand on the end of the pole to the weight of the pole?

13. Will wants to hit a PR in bench press. His target total weight is 147.5lbs. The bar is 45lbs, and he has two 45 lb plates, two 10lb plates, two 5lb plates, and two 2.5 lb plates to choose from. This collection of plates cannot be made into 147.5lbs shared symmetrically on both sides. Determined to hit his PR, he puts 55lbs on one side of the bar, and 57.5 lbs on the other side. He thinks that he can grip the bar off center, and apply the same force to the bar as he would have if he had a combination of plates that would allow the weight to be evenly distributed. His spotter John is skeptical that this will work and it sounds dangerous, so they come to you to solve the problem. The weights are 5 ft apart and can be treated like point masses. He likes to grip the bar with his hands two feet apart. Can he lift this weight with the same force on both hands and if so how far should he position his hands off center? *Don't worry about any conversions to metric here. Pounds are already a force and multiplying them by feet gives foot-pounds, which is the US customary system units of torque.*

Week 11

At the end of this worksheet you should be able to

- calculate the new quantities of pressure, gauge pressure, and density.
- apply the new principles of Pascal, Archimedes, and Bernoulli.
- use the continuity principle to solve for an unknown quantity.
- use the hydrostatic pressure principle to solve for an unknown quantity.

You will want to keep the following table of densities from your textbook handy.

Table 9.1 Densities of Common Substances (at 0°C and 1 atm unless otherwise indicated)

Gases	Density (kg/m^3)	Liquids	Density (kg/m^3)	Solids	Density (kg/m^3)
Hydrogen	0.090	Gasoline	680	Polystyrene	100
Helium	0.18	Ethanol	790	Cork	240
Steam (100°C)	0.60	Oil	800–900	Wood (pine)	350–550
Methane	0.72	Water (20°C)	998.21	Wood (oak)	600–900
Air (20°C)	1.20	Water (0°C)	999.84	Ice	917
Nitrogen	1.25	Water (3.98°C)	999.98	Wood (ebony)	1000–1300
Carbon monoxide	1.25	Seawater	1025	Bone	1500–2000
Air (0°C)	1.29	Blood (37°C)	1060	Concrete	2000
Oxygen	1.43	Mercury	13 600	Quartz, granite	2700
Carbon dioxide	1.98			Aluminum	2702
Argon	1.66			Iron, steel	7860
Xenon	5.86			Copper	8920
Radon	9.73			Lead	11 300
				Gold	19 300
				Platinum	21 500

1. If 500 N person stands on one foot that has an area 50.0 cm^2, what is the pressure on the floor? If this person stands on their heel in a high heeled shoe that has an area of 1.00 cm^2, what pressure is there? What if this person stands on a diamond that is cut to have a bottom facet that is 10 000 μm^2 ($1\,\mu m = 10^{-6}$ m)? *(Careful with unit conversions!).*

2. A patient's blood systolic blood pressure when resting is 160 mmHg. What is this pressure in pascals, psi, and atm?

3. Throughout this worksheet and the homework *gauge pressure* is a way of expressing the pressure measured by an instrument relative to the atmospheric pressure. So 1 atm of absolute pressure is set to 0 atm of gauge pressure. Perfect vacuum is absolute zero pressure, so that would be -1 atm of gauge pressure. 2 atm gauge pressure is now what? 30 kPa of absolute pressure is what gauge pressure? What about a gauge pressure of 1000 kPa as absolute pressure?

4. In a hydraulic lift, the radius of a small piston is 2 cm and the radius of the larger piston is 20 cm, what weight can the larger piston support when a force of 250 N is applied to the smaller piston? What is the increase in pressure caused by the 250 N force on the small piston? If the larger piston moves 5 cm, how far does the small piston move?

5. At the surface of a freshwater lake, the air pressure is 1 atm. At what depth under the water is the water pressure 4 atm?

6. At sea level, the average atmospheric pressure is 1 atm. The density of air at this level is about 1.2 kg/m^3. Assuming the density of air is constant (its not but just go with it), what is the air pressure at the Empire State Building that has a height of 381 m at the top deck, and we will just assume that its base is at sea level?

7. A diver swims to a depth of 10 meters in a lake. What is the pressure on the diver's body? What is the force on the diver's eardrums if they have an ear of 0.60 cm^2? What would this be if the diver was swimming in sea water?

8. A 5000 N object is floating in fresh water.

   - What is the net force on the object?

   - What is the magnitude of the buoyant force?

   - If the the bottom of the object is 1 meter below the surface of the water, then what pressure is on the bottom of the object?

- What is the area of the bottom surface of the object?

- If the object has another 1 m sticking out of the water, then what is the volume of the object?

- What is its density?

9. The following cylindrical barrels are filled to the brim with fluids of the given density. Put these in order from smallest to largest pressure at the bottom of the barrel.

   (a) $R = 40\,\text{cm}, h = 80\,\text{cm}, \rho = 1000\,\text{kg/m}^3$

   (b) $R = 40\,\text{cm}, h = 100\,\text{cm}, \rho = 1000\,\text{kg/m}^3$

   (c) $R = 50\,\text{cm}, h = 100\,\text{cm}, \rho = 800\,\text{kg/m}^3$

   (d) $R = 50\,\text{cm}, h = 80\,\text{cm}, \rho = 800\,\text{kg/m}^3$

   (e) $R = 50\,\text{cm}, h = 125\,\text{cm}, \rho = 800\,\text{kg/m}^3$

10. Let's do a problem based on the famous Archimedes legend. The story goes that a King Hiero II of Syracuse commissioned an ornate golden crown to be made, but when he got it, he was suspicious that silver had been mixed in with the gold. He charged Archimedes to figure out how to determine the density of the crown without damaging it. Archimedes' "Eureka" moment came when he realized he could determine the volume of a complex shape by submerging it in a tub of water that had been filled to the brim. The amount of water that spilled over the edge would be equal to the volume of the crown as long as you could catch all of the spilled water and measure its volume. So with all of that said, suppose the amount of water that spilled over the edge weighed 1.0 N but there was still some water stuck to the sides of the bucket that didn't get weighed so maybe call it 1.1 N I don't know this is pretty messy. The crown itself weighed 24.1 N. So what is the density of the crown and how does it compare to gold? *Hint: what is the volume of water that spilled over the edge?*

11. The above story is ridiculous. What if the water that spilled over the edge had weighed 1.2 N or 1.0 N? This is far too imprecise for something as serious as an allegation of cheating the king out of a pure gold crown. So instead we want to measure the buoyant force on the crown. So, if you weight the crown to be 24.1 N and then you lower it into the water fully submerged and weight it then (by tying a string to it and lowering it into the water but the string is attached to a balance) and then its "weight" is 22.85 N. What is the buoyant force upward on the crown? What volume of water has been displaced? What is the volume of the crown? What is the density of the crown? What is better about this method?

12. "ICEBERG DEAD AHEAD!" It is sometimes said that only 10% of an iceberg's volume actually sticks out above the surface of the water which is what made them so deadly to poor Jack

and Rose on the *Titanic*. If the density of ice is $917\,\mathrm{kg/m^3}$ and the density of seawater is $1025\,\mathrm{kg/m^3}$, then what is the ratio of the volume of ice under the water to the entire volume of ice? What is the ratio of the volume of ice above the water to the entire volume of ice? Next work these *in general* for any density of object floating in any density of fluid.

13. An artery has an inner diameter of 1.5 mm, but narrows to an inner diameter of 1.0 mm due to a build up of plaque. By what percent does the speed of blood flow change when it enters the narrow section?

14. What if we have a case where a single pipe splits into many pipes. Following an example from the textbook, the aorta is the artery from your heart that feeds 32 other major arteries. The aorta has an inner radius of 1 cm and assume each of the other arteries has an inner radius of 0.21 cm. If blood in the aorta has an average speed of 28 cm/s, then what is the average speed of blood in the major arteries? *Hint: you can just treat the major arteries as one big pipe that has an area 32 times bigger than the area of a single artery.*

15. Suppose it takes one minute to fill a 5 gallon bucket with water from your garden hose that is open all the way. The diameter of your hose is 1 inch. How fast is water traveling out of the end of the hose? How fast does it travel if you hold your thumb over the end of the hose and cover half of the area of the hose?

16. Show that Bernoulli's Principle really just reduces to the equation for pressure as a result of gravity when the fluid is not flowing (so $v_1 = v_2 0$). This equation is known as the *hydrostatic pressure* equation.

17. A cylindrical container of water is full to the brim when a hole is punctured 0.5 m from the top. What is the speed of the water as it comes out of the hole?

18. Following up on the previous problem, if you redirected this water straight up with the same speed, how high would it rise?

19. If water flows horizontally through a hose that has a radius of 1 cm at a speed of 2 m/s. If the nozzle of the hose narrows to 0.25 cm as the water sprays out, then what is the pressure inside the hose? What is it as a gauge pressure?

Week 12

At the end of this worksheet you should be able to

- calculate stress and strain and use Young's Modulus to solve for an unknown.
- plot all relevant quantities of simple harmonic motion over time.
- use the quantities of simple harmonic motion and the mathematical description to solve for an unknown.

1. Consider a wire that is 0.1 mm in diameter and 2 m long and a Young's Modulus of 120 GPa ($1\,\text{GPa} = 10^9\,\text{Pa}$). If you applied 100 N to this wire, then what is the stress on the wire? What is the strain? By how much does its length change? What is its new length? What percent change is this?

2. If instead a 200 N force or a 1000 N force is applied then what is stress, strain, length change and percent change in the length from its original length?

3. If instead the wire was 1 m long with 100 N applied, then how much does it stretch? What percent change is this?

4. What if the wire had half of its cross sectional diameter with 100 N applied?

5. Now comparing the form of Hooke's Law for springs $F = kx$ to Hooke's Law for stress and strain $\frac{F}{A} = Y\frac{\Delta l}{L}$, how could you write an expression for spring constant in terms of Young's modulus, length, and cross sectional area? What does this tell you about what would happen to the spring constant of a spring if you cut the spring in half?

6. Speaking of springs, if you attach two springs to an object side by side, then we say the springs are attached *in parallel*. This will result in two spring forces on the object that has been displaced some distance x_1. If you were to model this arrangement of springs in parallel as a single spring with a single spring constant that would have the same effect, then what would this single effective spring constant k_e be in terms of of the original two spring constants k_1 and k_2?

7. If instead of a *parallel* arrangement, we attach one spring to another spring and then to the object, we say that these springs are connected *in series*. Let's find an expressions for an effective spring constant for this arrangement. Each spring will stretch a different

amount based on its spring constant, but the object will experience one force and *both of the springs is exerting the same force.*

8. A wire of length l_1 and volume V and cross sectional area A_1 is stretched out to length l_2, what is its new cross sectional area? Think about this in terms of proportionality.

9. A 60 kg person upright. By how much does the femur shorten if each femur carries half the weight of the person? The cross sectional area of a femur is about 4 cm^2 and the length is about 30 cm Also find the percent change in length.

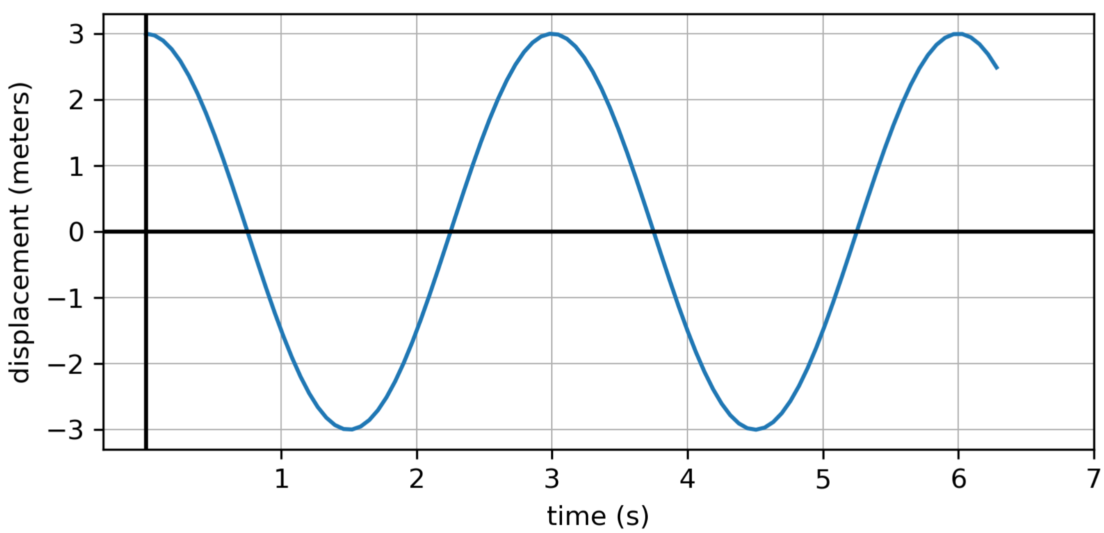
10. A 1 kg mass attached to a spring of spring constant 1000 N/m is positioned so that the spring is stretched 10 cm from its relaxed length. After you finish the following questions, make sure you can write down expressions for all of them in general as well as working them inside out.

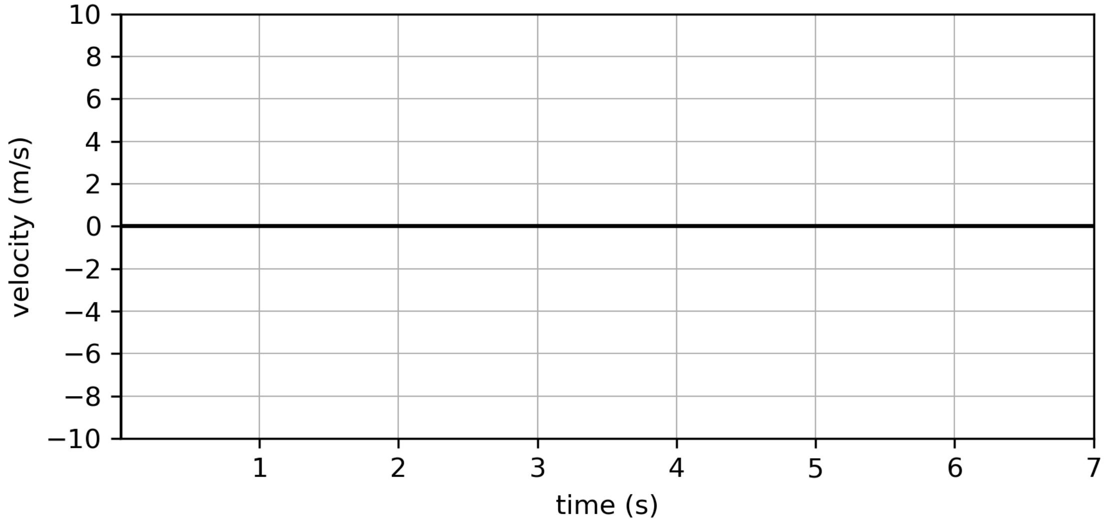
    - How much spring potential energy does it have at this position?
    - When it is released, and it heads back toward equilibrium gaining kinetic energy as it does, then what is the maximum kinetic energy it can achieve? How stretched out is the spring when it has this much kinetic energy?
    - What is the mass's velocity when it has maximum kinetic energy?
    - What is the period of this mass's motion? What is its frequency? What is its angular frequency?
    - What is the force on the mass when it is at its maximum displacement?
    - What is the force on the mass when it is at the equilibrium position?

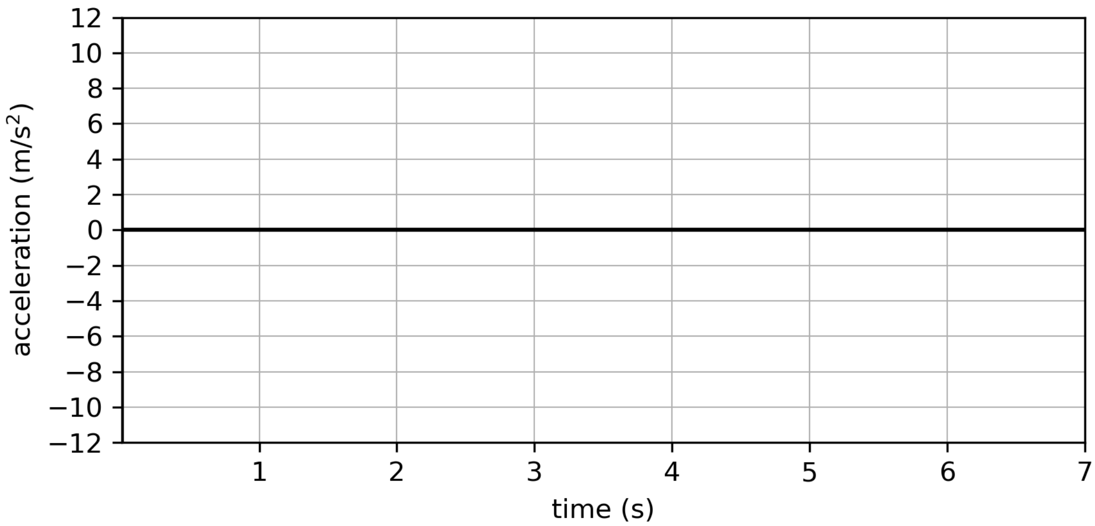
- What is the maximum acceleration of the mass? Where does this occur?

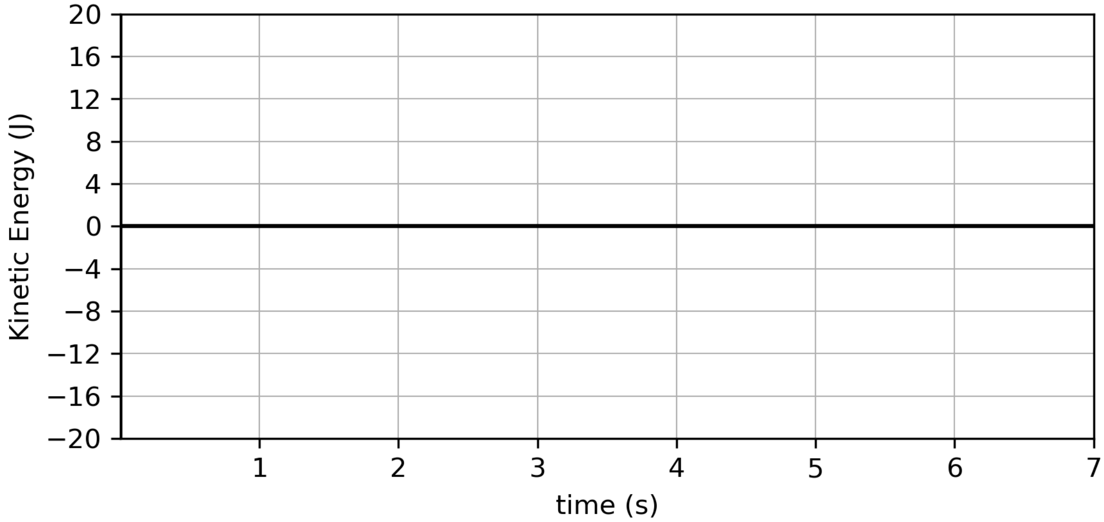
- Where is the mass when its potential energy and kinetic energy at that position are equal?

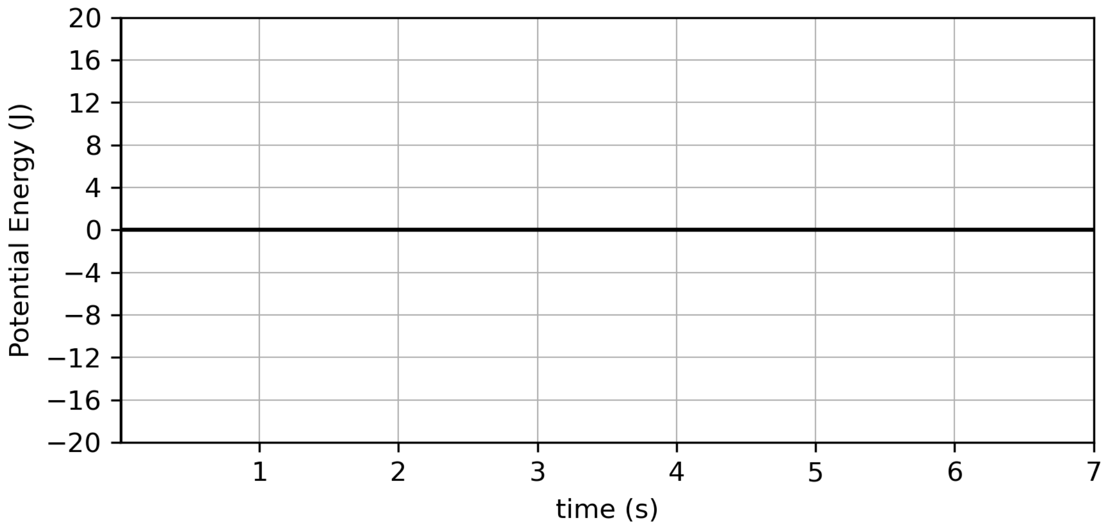
11. A 1 kg mass on a spring has the displacement graph that follows. What is the angular frequency, natural frequency, period, spring constant, amplitude, maximum velocity, maximum acceleration, maximum kinetic energy, maximum potential energy, and total energy? Also fill out the rest of the graphs.

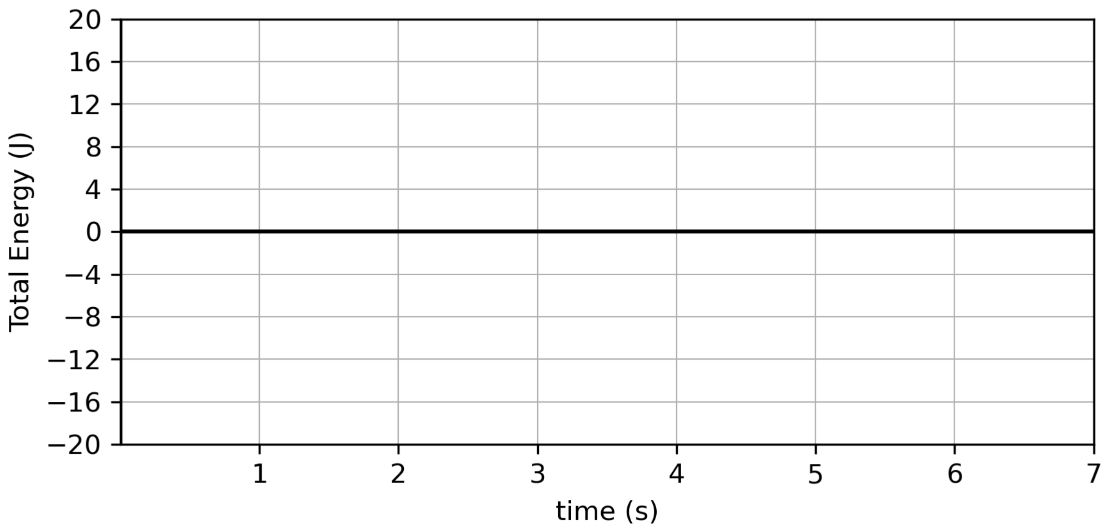



velocity (m/s)
10
8
6
4
2
0
−2
−4
−6
−8
−10
1
2
3
4
5
6
7
time (s)


acceleration (m/s²)
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
1
2
3
4
5
6
7
time (s)


Kinetic Energy (J)
20
16
12
8
4
0
−4
−8
−12
−16
−20
1
2
3
4
5
6
7
time (s)


Potential Energy (J)
20
16
12
8
4
0
−4
−8
−12
−16
−20
1
2
3
4
5
6
7
time (s)



12. According to the table below, which material stretches more, 2 m of steel or 1 m of copper of the same width?

13. Four brass wires are subjected to the same tensile stress. The wires have the following unstretched lengths and widths. Rank them in order from least to most change in length.

    (a) length L, diameter d
    (b) length $2L$, diameter d
    (c) length $4L$, diameter $d/2$
    (d) length $L/4$, diameter $d/2$

14. A 0.5 m long guitar string of cross sectional area of 1.0×10^{-6} m and Young's modulus $Y = 2.0$ GPa. By how much must you stretch the string to obtain a tension of 20 N.

15. Young's modulus is like the spring constant for materials, but if you stretch a material beyond a limit then the material will not exactly return to its original length. This point of stress is called the *elastic limit* of a material. After this point, *plastic deformation* begins to occur which means the material will be permanently deformed. More stress can be applied beyond this but the material will fracture and break when the stress reaches the *breaking point*. A hair breaks under a tension of 1.2 N and the tensile stress of the breaking point is 200 MPa. What is the diameter of the hair?

16. A copper wire of length 3.0 m is observed to stretch by 2.1 mm when a weight of 120 N is hung from the end. What is the diameter of the wire and what is the stress in the wire? If the breaking point of copper is 400 MPa, what is the maximum weight that may be hung from this wire?

Table 10.1 Approximate Values of Young's Modulus for Various Substances

Substance	Young's Modulus (GPa)
Rubber	0.002–0.008
Human cartilage	0.024
Human vertebra	0.088 (compression); 0.17 (tension)
Collagen, in bone	0.6
Human tendon	0.6
Wood, across the grain	1
Nylon	2–6
Spider silk	4
Human femur	9.4 (compression); 16 (tension)
Wood, along the grain	10–15
Brick	14–20
Concrete	20–30 (compression)
Marble	50–60
Aluminum	70
Cast iron	100–120
Copper	120
Wrought iron	190
Steel	200
Diamond	1200

Week 13

At the end of this worksheet you should be able to

- use the properties of waves to solve for an unknown quantity.
- use the mathematical description of a wave to plot waves motion over time.
- use the conditions of a standing wave on a string to solve for an unknown quantity.

1. The speed of a wave on a string is proportional to the square root of the tension F in the string and the length L, and inversely proportional to the square root the mass m of the string.
    - By what factor does the velocity of the wave change if the tension doubles?
    - What about if the length halves?
    - What if the mass triples?
    - What if the mass density doubles?
    - If the force doubles and the length triples, by what factor does the velocity change?

- What if the mass density triples and the force doubles?

- By what factor does the force need to change to double the velocity?

- By what factor does the length need to change to double the velocity?

- By what factor does the length need to change to quarter the velocity?

- If the velocity doubles and the length halves, by what factor does the force need to change?

2. A string is 2 m long and has a mass of 10 g. What is its mass density? If you cut the string in half, what is its mass density then? If you exerted a force of 10 N, then what is the velocity of waves on the string? What would the force need to be to make the velocity 100 m/s?

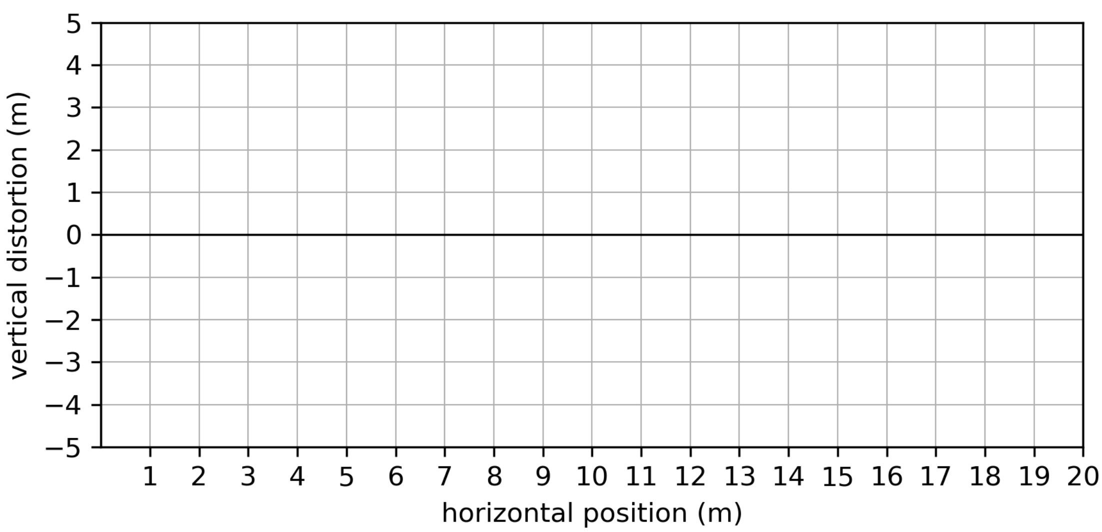
3. A string with a mass density of 10 g/m has a tension of 100 N. A periodic waveform that has a period of 0.1 s is put into this string. What is the frequency? What is the wavelength of this wave? What is the angular frequency and what is the wavenumber?

4. The wavenumber of a wave is 20 rad/meter, and the period is 0.1 seconds.
   - What is the frequency, angular frequency, wavelength, and velocity?
   - If the force on the string that is carrying this wave is 100 N, then what is the linear mass density?
   - If the string is 10 grams then what is the length of the string?

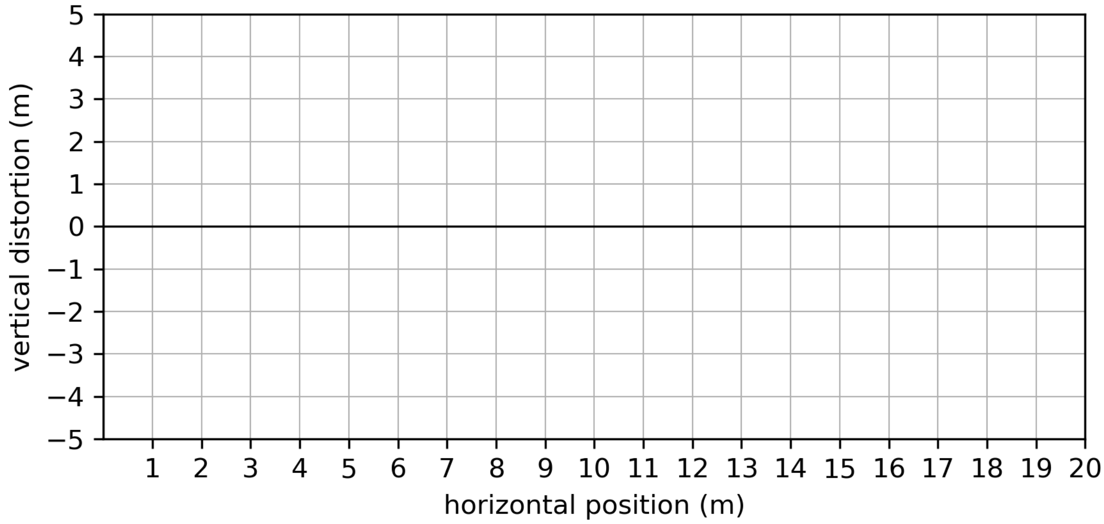
5. A 1 meter long string with a linear mass density of 10 g/m is oriented horizontally and a pulley at one end allows a 1 kg to hang down and put tension in the string. What is the speed of waves on this string.

6. The same string from above is hanging vertically from a support with the 1 kg mass tied on the end. What is the speed of a wave *near the mass*? What is the speed of a wave *near the top*?

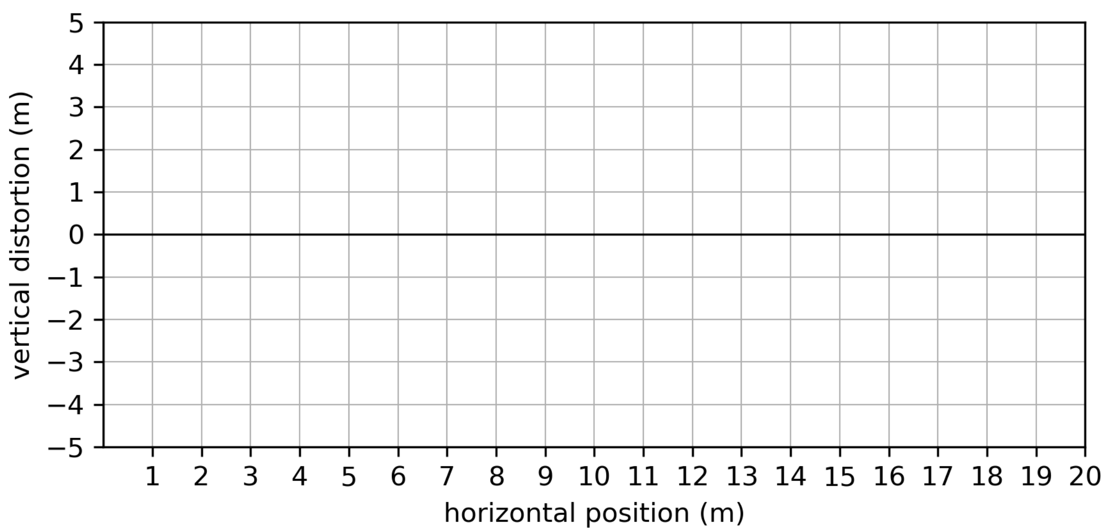
7. A wave has a wavelength of 3 m and an amplitude of 2 m. It travels with a speed of 5 m/s. If the wave has its maximum at the horizontal position of 0 m when $t = 0$ s, then sketch a plot of the wave at this time below:



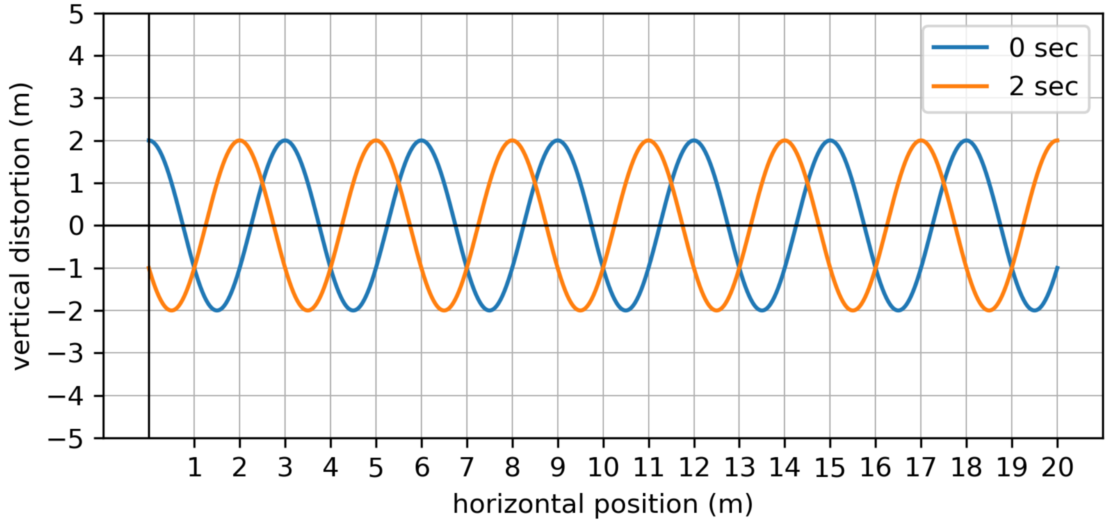
8. Now sketch the wave after 0.2 seconds have gone by.



9. Now sketch the wave 1 second after the begining.

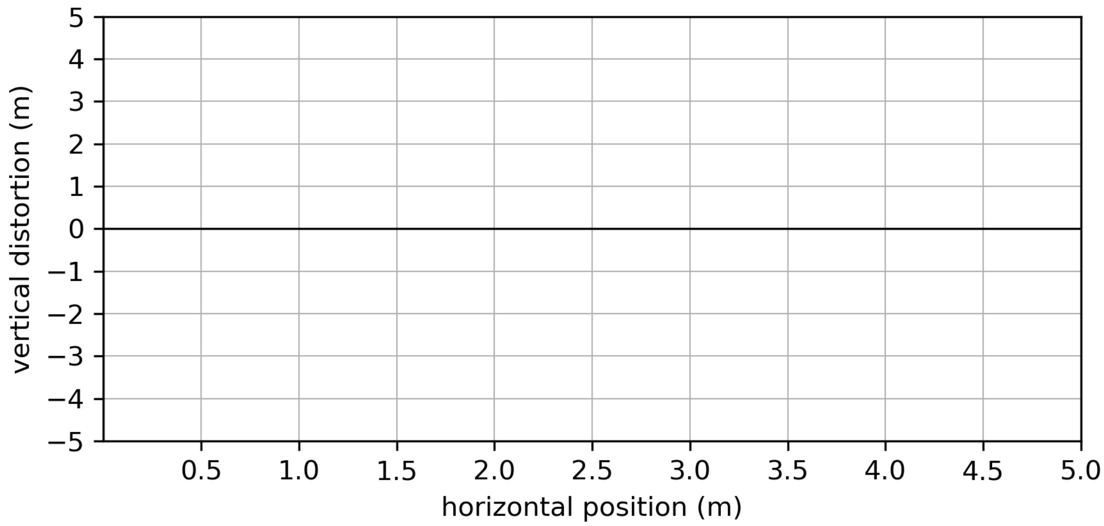


10. The following plot shows a wave at $t = 0\,\text{s}$ and then later when $t = 2\,\text{s}$. What is the wavelength, amplitude, period, angular frequency, wavenumber, and velocity?



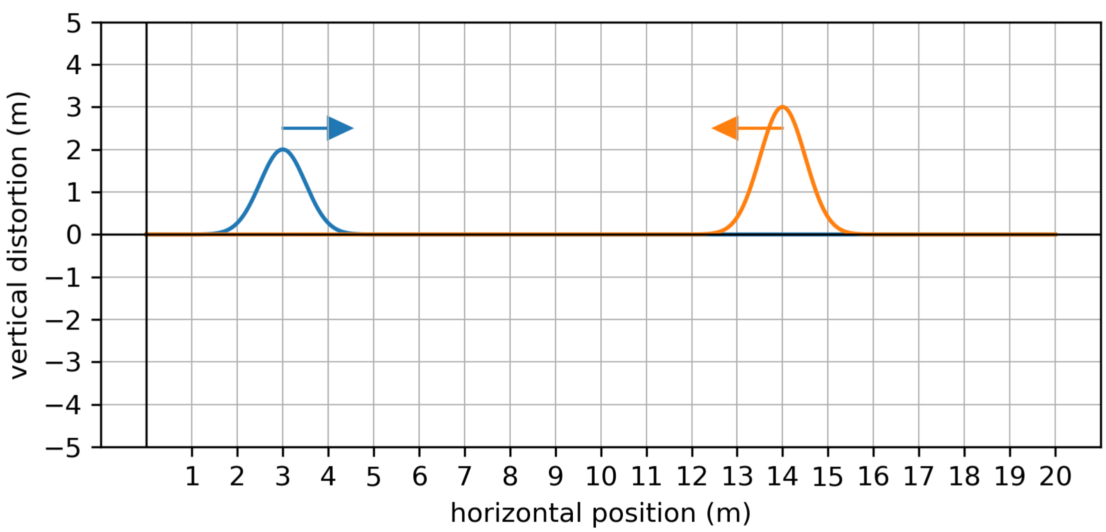
11. A wave on a string is described by the equation below. Plot this function for $t = 0\,\text{s}$ and again for $t = 2\,\text{s}$.

$$y(x,t) = 3.5\,\text{m}\sin\left((6\pi\,\text{rad/m})x - (8\pi\,\text{rad/s})t\right)$$

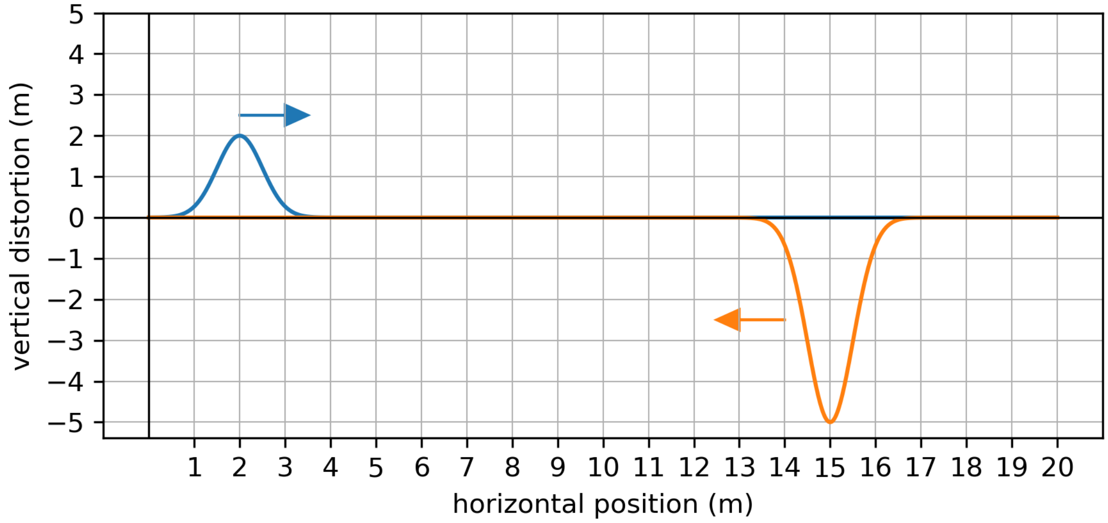


12. A string has a tension of 100 N. But this string has two sections that are knotted together. One side has a linear mass density of 10 g/m and the other side has a linear mass density of 35 g/m. What is the velocity of the wave on the first side of the knot? What is the velocity on the second side? If a wave with a wavelength of 0.2 meters is traveling on the first side, what is its frequency? When the crest of this wave from the first side reaches the knot, this crest simply passes through the knot. But the velocity has changed. So how much time goes by to when the next crest passes the knot? How far has the first crest traveled in the second side of the string? What does this mean about the wavelength on the second side? What is the frequency of wave crests coming into the knot? What is the frequency of crests leaving the knot? On the basis of this analysis, when the speed of a wave changes suddenly at an interface does the wavelength change or does the frequency or do both change?

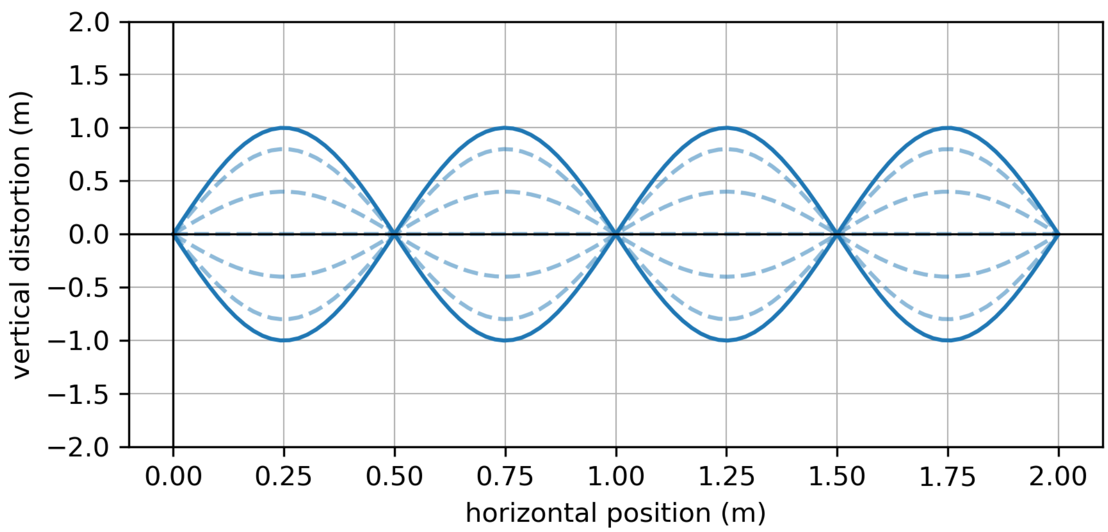
13. Two waves are approaching each other with speed $v = 4\,\text{m/s}$. At what time and where will these waves fully overlap (their max peaks line up) and what will the wave look like at that time?



14. Again, two waves approach each other with speed $v = 6\,\mathrm{m/s}$. At what time and where will these waves fully overlap and what will the wave look like at that time?



15. The plot below represents a standing wave in a string. How many antinodes are there? How many nodes are there? What is the wavelength of the string. If the speed of the wave on the string is $v = 40\,\mathrm{m/s}$, then what input frequency produces this standing wave? What period of oscillation of the input frequency?



16. If the string from the above problem has three antinodes, then what is the wavelength? If the velocity of the wave is still $v = 40\,\mathrm{m/s}$, then what is the frequency? What is the wavelength and frequency when there are two antinodes? What about when there is one antinode? What is the fundamental frequency? What is the 10th harmonic frequency? What is the 10th harmonic wavelength?

17. Instead of changing the frequency to make a fewer anti-nodes, we could change the velocity instead, and the easiest way to do that is to adjust the force in the string. So if the standing wave goes from 4 antinodes to 3 antinodes, by what ratio does the *speed* of the wave change? If the speed of the wave changes by this ratio, then by what ratio does the force on the string need to change?

18. Suppose a wave has a 4th harmonic frequency of 200 Hz then what is the fundamental frequency?

19. If a standing wave is produced a frequency of 98 Hz and a the next standing wave frequency is 112 Hz. What is the fundamental frequency and how many antinodes were there for these two standing waves?

20. If you increase the tension in a string by a factor of 1.3, then by what factor do you change the fundamental frequency of a waves in that string? If you increase the tension in string by 10%, then by what percent do you change the fundamental frequency?

Week 14

At the end of this worksheet you should be able to

- calculate the speed of waves in air at varying temperatures.
- operate the Kelvin temperature scale as an absolute temperature scale.
- calculate the intensity of a sound wave at any distance from the source.
- calculate the loudness of any sound source including the listener's distance from the source.
- apply the principles of standing wave to pipes open on one end or open on both ends.
- use the principle of beat frequency to calculate for an unknown frequency.

1. According to the notes, the speed of sound in a gas like air is proportional to the square root of the temperature. But much like when we talked about pressure, we need to use an *absolute* temperature scale. In the SI system the Kelvin temperature scale is an absolute measure of temperature, because 0 K is the lowest conceivable temperature. The relationship of the Kelvin scale to the Celsius scale is that 0 K is -273.15 °C. So what is 0 °C in Kelvin? What is 20 °C in Kelvin? What is 100 °C in Kelvin? What is 300 K in Celsius? What is 100 K in Celsius?

2. The size of the degrees are the same in both the Kelvin and Celsius temperature scales. So if the air temperature changes from 10 °C to 36 °C, by how much does the temperature change in Kelvin? If the air temperature changes by 20 °C, then how much does the temperature change in Kelvin?

3. The Celsius scale has its zero point set at the freezing point of water. While this puts the temperatures that humans most commonly deal with at a useful size, it is not acceptable when using proportionality reasoning. For example, does it make sense to refer to doubling the temperature in the Celsius scale? Is $20\,°\mathrm{C}$ twice the temperature of $10\,°\mathrm{C}$? If not, what is the proper ratio of these two temperatures? What is the percent change going from $10\,°\mathrm{C}$ to $20\,°\mathrm{C}$?

4. So in these problems in order to find the speed of sound at any temperature, we need to use a known reference speed at a known temperature. We will use the reference speed of sound in air of $v_0 = 331\,\mathrm{m/s}$ at the temperature of $T = 0\,°\mathrm{C}$ or $273.15\,\mathrm{K}$. So what is the speed of sound at $20\,°\mathrm{C}$? At what temperature would the speed of sound be $300\,\mathrm{m/s}$? At what temperature would it be $400\,\mathrm{m/s}$?

5. If the temperature of air decreased by 10%, then by what percent would the speed of sound decrease?

6. $40\,°\mathrm{C}$ is just about as hot as it ever gets in Alabama in the summer and $-15\,°\mathrm{C}$ is just about as cold as it ever gets in the winter. What is this ratio of temperatures and what is the ratio of speeds of sound in the hottest day of the summer to speeds of sound in the coldest night in the winter? What is the speed of sound at these temperatures?

7. A firework explodes releasing $100\,\mathrm{kJ}$ of energy in $0.001\,\mathrm{s}$. What power is this? If 10% of this power goes into sound energy what power is that? What is the intensity of this sound $1\,\mathrm{m}$ away from the explosion? What is the intensity $10\,\mathrm{m}$ away? $100\,\mathrm{m}$ away?

8. If the intensity of music from a loudspeaker at a concert is $1\,\mathrm{W/m^2}$ at a distance of $1\,\mathrm{m}$ away, then what is the intensity $10\,\mathrm{m}$ away? $100\,\mathrm{m}$ away?

9. For all of the intensities in the previous problems, calculate their sound level (i.e loudness) relative to the threshold of hearing, $I_0 = 10^{-12}\,\mathrm{W/m^2}$.

10. If the sound level at the location of your ear is 60 dB, what is the intensity of sound? If your eardrum (also apparently called the tympanic membrane)has a diameter of 0.5 cm, then what power is delivered to your eardrum?

11. The intensity of a sound is doubled. What is the difference in the sound level *change*? Hint: subtract the sound level of $2I$ and I and then use the properties of logarithms that $\log A - \log B = \log (A/B)$

12. The smallest change in sound level that humans can detect is about 1 dB. What is the ratio of intensity does this represent?

13. What is the sound level 1 m away from a 100 W sound source? What is it 2 m away? Plot this change in sound level as a function of distance away from the source. If you double your distance away from a source of sound how much does the sound level change?

14. What is the fundamental frequency and wavelength of a 1 m long pipe open at both ends at 24 °C? What is the fundamental frequency and wavelength of the same pipe if it is closed at one end? For each pipe, what is the next highest frequency that supports a standing wave in the pipe?

15. On a guitar each string has a different mass density and tension, but also the fret board of the guitar allows you to shorten the length of each string. The frequency of the top E string at is full length of about a meter is about 164 Hz.

    - If I press the string on a fret to half the length of the string, then how much does the fundamental frequency change? State it as a difference, and a ratio, and a percent change.

    - On a guitar, the frets are spaced so that there are 12 divisions. So if the divisions are equally spaced (they aren't for reasons that are outside the scope of this problem) then how much does the length change with each fret?

- So how much does each fret shorten the string by? If I press on the first fret by how much did the frequency change? As a difference and a percent change?

- What about the next fret as a difference and percent change?

16. The speed of a wave on a 1 m long string is 100 m/s. What is the fundamental wavelength and frequency? If this string is in air at 25 °C what is the frequency and wavelength of the pressure wave? If the temperature drops to 10 °C, by how much does the fundamental frequency change?

17. What are the three lowest standing wave frequencies of a 2 m long pipe that is open on both ends when the speed of sound is 340 m/s? What if the pipe were closed on one end?

18. Two strings of a guitar are being played at the same time. One string has a frequency of 400Hz. A beat frequency of 5 Hz can be heard. What are the possibilities for the frequency of the second string? If the tension in the second string is increased, and the beat frequency decreases to 2Hz, then what can you conclude about the frequency of the second string before it was tightened? By what percent was the tension in the string increased? By what percent would the tension need to be increased in order to eliminate the beating?

19. Imagine the last problem if tightening the string had *increased* the beat frequency. What would you conclude about the original frequency then?

Table 12.2 Pressure Amplitudes, Intensities, and Intensity Levels of a Wide Range of Sounds in Air at 20°C (Room Temperature)

Sound	Pressure Amplitude (atm)	Pressure Amplitude (Pa)	Intensity (W/m^2)	Intensity Level (dB)
Threshold of hearing	3×10^{-10}	3×10^{-5}	10^{-12}	0
Leaves rustling	1×10^{-9}	1×10^{-4}	10^{-11}	10
Whisper (1 m away)	3×10^{-9}	3×10^{-4}	10^{-10}	20
Library background noise	1×10^{-8}	0.001	10^{-9}	30
Living room background noise	3×10^{-8}	0.003	10^{-8}	40
Office or classroom	1×10^{-7}	0.01	10^{-7}	50
Normal conversation at 1 m	3×10^{-7}	0.03	10^{-6}	60
Inside a moving car, light traffic	1×10^{-6}	0.1	10^{-5}	70
City street (heavy traffic)	3×10^{-6}	0.3	10^{-4}	80
Shout (at 1 m); or inside a subway train; risk of hearing damage if exposure lasts several hours	1×10^{-5}	1	10^{-3}	90
Car without muffler at 1 m	3×10^{-5}	3	10^{-2}	100
Construction site	1×10^{-4}	10	10^{-1}	110
Indoor rock concert; threshold of pain; hearing damage occurs rapidly	3×10^{-4}	30	1	120
Jet engine at 30 m	1×10^{-3}	100	10	130