1. We have gone through several kinds of equations now and lets sum up some of these *as proportions*:

$$\Sigma F = ma \rightarrow a \propto \Sigma F$$

$$a = \frac{\Sigma F}{m}$$

- acceleration is directly prop to the net force.
  assuming constant mass

- assuming constant acceleration and beginning at rest, an object's velocity is prop. to the square root of the displacement.
  twice as far

$$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta x \qquad v_f = \sqrt{2a\Delta x} \rightarrow v_f = \underbrace{(2a)^{1/2}}_{\text{constant of proportionality}} \Delta x^{1/2}$$

$$\frac{v_2}{v_1} = \left(\frac{\Delta x_2}{\Delta x_1}\right)^{1/2} \Rightarrow \frac{v_2}{v_1} = (2)^{1/2} = 1.41$$

- assuming constant acceleration and beginning at rest, an object's displacement is prop to the square of the elapsed time.

$$\Delta x = \cancel{v_i t}^{\,0} + \frac{1}{2}at^2$$

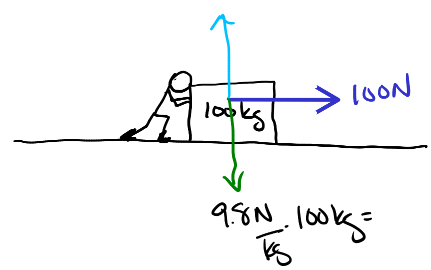
$$\Delta x = \frac{1}{2}at^2 \rightarrow \frac{\Delta x_2}{\Delta x_1} = \left(\frac{t_2}{t_1}\right)^2$$

- for an object that has been dropped, the distance it has fallen is prop to the square of its velocity at that distance.

$$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta y$$

$$\Delta y = \frac{v_f^2}{2a} \qquad \Delta y \propto v_f^2$$

2. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N over a distance of 10 m. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 m?
Some starters:



- What is the net force on the box? 100N +x
- What is the acceleration of the box? 1 m/s²
- What is the final velocity after 10 m

$$\Sigma F_x = ma_x$$

$$100\,N = 100\,kg \cdot a_x$$

$$a_x = 1\,m/s^2$$

$$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta x$$

$$v_f = \sqrt{2 \cdot 1\,m/s^2 \cdot 10\,m}$$

$$v_f = 4.5\,m/s$$

$$a = \frac{\Sigma F}{m} = \underbrace{\Sigma F}_{\text{constant}} \cdot m^{-1}$$

$$a \propto m^{-1}$$

$$\frac{a_2}{a_1} = \left(\frac{m_2}{m_1}\right)^{-1} = (2)^{-1} = \frac{1}{2}$$

$$a_2 = \frac{1}{2}a_1 = \frac{1}{2}(1\,m/s^2) = 0.5\,m/s^2$$

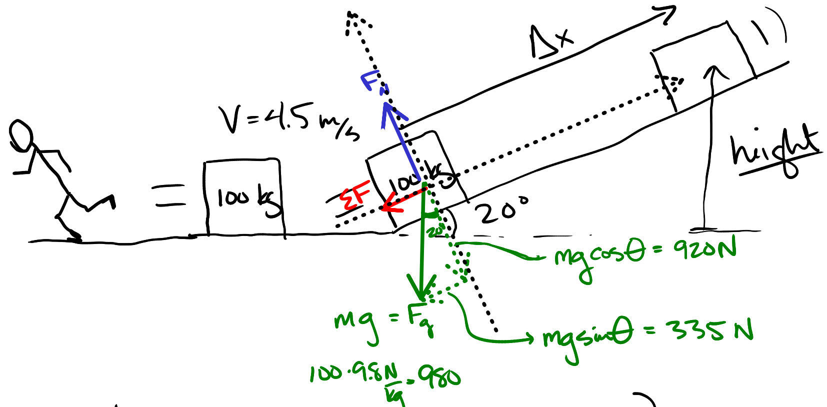
$$v_f = \sqrt{2a\Delta x}$$

$$v_f \propto a^{1/2} \Rightarrow \frac{v_2}{v_1} = \left(\frac{a_2}{a_1}\right)^{1/2} = \left(\frac{1}{2}\right)^{1/2} = 0.707$$

$$v_2 = 3.18\,m/s$$

3. Following up on the previous problem, if I stopped pushing after 10 m and the box continued with its speed, and then at then started sliding up a 20° ramp, then how far along the length of the ramp would the box rise? What height is this above the horizontal? Do the 100 kg and the 200 kg box rise to the same height?
   Some starters:

   - What is the net force on the box as it goes up the inclined plane?
   - What is the acceleration of the box as it goes up the inclined plane?
   - What is the sign of the displacement of the box going up the plane?
   - Is the sign of acceleration the same or different than displacement?



$\cancel{v_f^2}^{=0} = v_i^2 + 2a\Delta x$

$0 = (4.5)^2 + 2(-3.35)\Delta x$

-4.5^2

$-v_i^2 = 2a\Delta x$

$\frac{-v_i^2}{2a} = \Delta x = \frac{-(4.5)^2}{2(-3.35)} = 3.02\text{ m}$

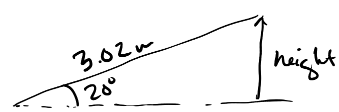
part 2

	x	y
F_N	0	
F_G	-335	
ΣF	-335	0

$a_x = \frac{\Sigma F_x}{m}$

$a_x = \frac{-335\text{N}}{100\text{ kg}} = -3.35\text{ m/s}^2$

part 1



$\sin\theta = \frac{\text{height}}{\text{hypotenuse}}$

$\sin 20° = \frac{h}{3.02}$

$\boxed{h = 1.03\text{ m}}$

4. We want to examine the idea of *mechanical advantage*.

   - How much force would it take to lift a 100 kg box straight up at constant speed a distance of 5 meters.

   - How much force would it take to push a box up an inclined plane that was 20 m long up to the same height? (again at constant speed)

   - What is the ratio of the two forces in these two cases? Which would you rather do? This ratio is known as *mechanical advantage*

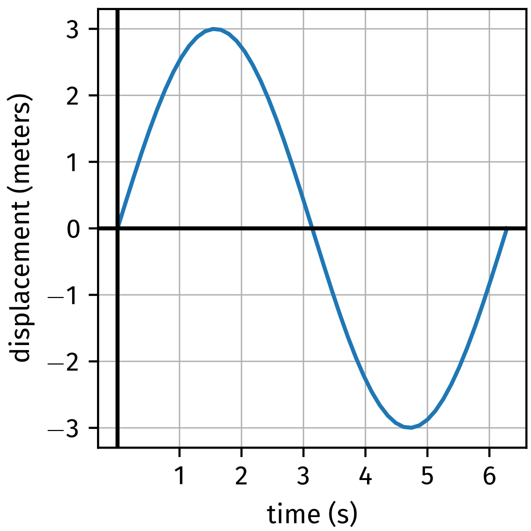
- What is the ratio of the displacement in these two cases?

- How can you use this idea to quickly figure out the force it would take you to push the box up a 100 m ramp that goes up the same height?

5. Let's talk about springs. *Hooke's Law* is the relationship between the force exerted by a spring and amount the spring has been stretched or compressed. The amount a spring has been stretched or compressed is the displacement of the end of the spring. Hooke's law says the magnitude of the force exerted is directly proportional to the displacement of the end of the spring. Write this in terms of a proportionality statement and again as an equation with a *constant of proportionality*. What are the units of the constant of proportionality?

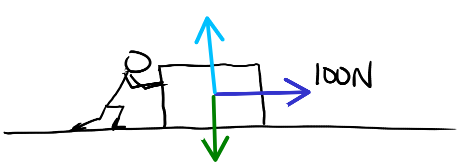
   Draw a qualitative plot of the magnitude of this force vs displacement. What is the slope of this graph?

6. Consider the graph below of displacement under condition of *not constant* acceleration. What would a graph of velocity vs. time look like for this case? What about acceleration vs time.



$v_i = 0$

7. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N for 10 s. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 s?



$\Sigma F_x = 100N = m \cdot a_x$

↑ 100kg

$a_x = 1 \, m/s$

$V_f = \cancel{V_i}^{0} + a t$

$= 1 \, m/s \cdot 10 s$

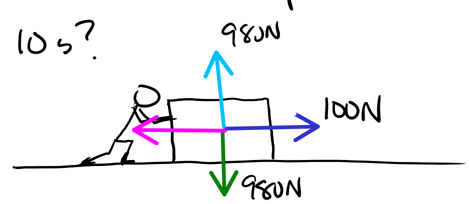
$= 10 \, m/s$

Bonus Q: How far did you push the box?

$\Delta x = \cancel{V_i t}^{0} + \frac{1}{2} a t^2$

$\Delta x = \frac{1}{2}(1)(10s)^2 = 50m$ ✓

What if $\mu = 0.1$, then what is the speed after 10 s?



$F_f = \mu \cdot F_N$

$= 0.1 \cdot 980N = 98N$

$\Sigma F_x = 2N = 100kg \cdot a$

$a = 0.02 \, m/s^2$

$V_f = 0.02 \, m/s^2 \cdot 10s = 0.2 \, m/s$

8. A follow up to the previous question. The *momentum* of an object is defined as the product of an object's mass and its velocity. ~~In this case~~ the momentum of this object changes because a force external to the object was exerted on it. What is the momentum

of the object initially? What is the momentum of the object at the end? It is said that the change in momentum of an object is equal to the *impulse*, where *impulse* is defined as the product of constant force and the amount of time the force is acting. Is that the case here? How could you use this to quickly find the final velocity of an object if the same force were pushing it for 100 s?

$y_f = y_i + v_{iy} t + \frac{1}{2} a_y t^2$

$0 = 0 + 1000\,m/s \cdot t + \frac{1}{2}(-9.8)t^2$

$0 = 1000\,m/s \cdot t + \frac{1}{2}(-9.8)t^2$

$0 = t\left(1000\,m/s + \frac{1}{2}(-9.8)t\right)$

$0 = 1000 + \frac{1}{2}(-9.8)t$

$-1000 = \frac{1}{2}(-9.8)t$

$2\left(\frac{-1000}{-9.8}\right) = t = 204.1\,s$

total time

$v_{yf} = v_{yi} + a_y t$

$0 = 1000 + (-9.8)t$

$-1000 = (-9.8)t$

$\left(\frac{-1000}{-9.8}\right) - t = 102\,s$

time to max height

$y_f = y_i + v_{iy} t + \frac{1}{2} a_y t^2$

$y_f = 0 + 1000\,m/s\,(102) + \frac{1}{2}(-9.8)(102)^2$

$y_f = 51{,}000\,m$

$\cancel{v_{yf}^2} = v_{yi}^2 + 2a\Delta y$

max height

9. If I launch a cannonball from ground level with a speed of 1000 m/s, then there are a variety of angles to choose from. If I fire it at 90°, then how long will it take to come down? How high will it go? How far will it go horizontally? What about 80°? What about 70°? Let's just make a table...

Angle	Time	Max Vertical Height	Horizontal Distance
90°	204.1 s	51,000 m	0
80°	201 s	49,500 m	34,900 m
70°	191.7	45,050	65,600
60°	176.7	38,300	88,400
50°	156.3	29,900	100,500
40°	131.2	21,100	100,500
30°	102	12,700	88,400
20°	69.8	5,970	65,600
10°	35.4	1,540	34,900
0°	0	0	0

$V_{yi} = 1000 \cdot \sin 80° = 984.8\,m/s$

$V_{xi} = 1000 \cos 80°$

$V_{xi} = 173.6\,m/s$

$v_{yf} = v_{yi} + a_y t$

$0 = 984.4 + (-9.8)t$

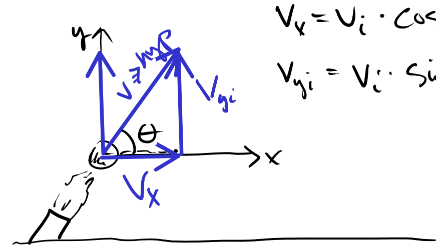
$t = \frac{984.4}{9.8} = 100.5$

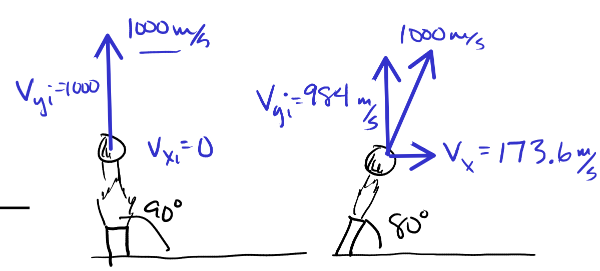
time to max height

total time $2(100.5)$

$V_x = V_i \cdot \cos\theta$

$V_{yi} = V_i \cdot \sin\theta$





$y_f = 984 \cdot (100.5) + \frac{1}{2}(-9.8)(100.5)^2$

$y_f = 49{,}500\,m$ max height

$\Delta x = v_x \cdot t$

$= 173.6 \cdot 201\,s$

$= 34{,}900\,s$ horizontal distance

10. If a soccer ball with a radius of 10 cm is rolls along the ground without slipping at 5 m/s, then how many revolutions does it roll through in 10 s and what distance has a point on the edge of the ball traveled? Some starters:

$r = 0.10\,m$

- How fast is it *spinning*? By that we mean *angular speed*.
- How many radians does the ball rotate through in this time? What is that in revolutions?
- How far does it roll in this time? Is this the same distance as the distance of a point on the edge of the ball? Why or why not?

11. Following up on the previous problem, how many seconds does it take for the ball to complete one revolution? This amount of time is referred to as the *period* of its rotation, and this is a similar characteristic time for the motion of the ball as the *period of a pendulum* was in the first lab.

12. Another follow up. How many revolutions does the ball travel through *per second*? You could convert this from angular speed ω that you would have calculated in the first instance of this problem, but if all you new was the period of the ball's rotation, how could you calculate it from there? *(Hint: what is the difference between revolutions per second and seconds per revoution?)* This quantity of revolutions per unit of time is sometimes called *frequency*.

13. Suppose a satellite is in orbit around a distant planet. You observe the the satellite to be 5000 km from the center of the planet, and rotating the planet once every 2 days. What is the mass of the planet you have discovered? ~~What is the period of the satellites motion~~?

$$2\text{ day} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{3600\text{ s}}{1\text{ hr}} = 172{,}800\text{ s}$$

$r = 5000\,km = 5\cdot10^6\,m$

$\omega = 3.6\cdot10^{-5}\,rad/s$

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{1.73\cdot10^5} = 3.6\cdot10^{-5}\,\frac{rad}{s}$ ✓

$v = \omega\cdot r = 180\,m/s$ ✓

~~What is its frequency?~~ How fast is the satellite moving around the planet? What is the angular speed?

$$F_g = \frac{G m_p m_s}{r^2} = \Sigma F_R = \frac{m_s v^2}{r}$$

$$\frac{G m_p}{r} = v^2$$

$$m_p = \frac{r\cdot v^2}{G} = \frac{5\cdot10^6\,m\cdot(180\,m/s)^2}{6.67\cdot10^{-11}} = 2.4\cdot10^{21}\,kg$$

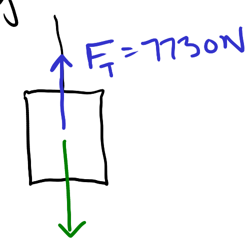
While an elevator of mass 927 kg moves downward, the tension in the supporting cable is a constant 7730 N. Between $t = 0$ and $t = 4.00$ s, the elevator's displacement is 5.00 m downward. What is the elevator's speed at $t = 4.00$ s?

v_f

know:
$m = 927\,kg$
$F_T = 7730\,N$
$t = 4\,s$
$\Delta y = -5\,m$

want:
v_f

- $v_f = v_i + at$
- $\Delta y = v_i\cdot t + \frac{1}{2}at^2$
- $v_f^2 = v_i^2 + 2a\Delta y$



$mg = 927\,kg\cdot9.8\,N/kg = 9084\,N$

$$\Sigma F_y = -9084\,N + 7730\,N = -1354\,N$$

$$\Sigma F_y = m\cdot a$$

$$\frac{-1354}{927} = a = -1.46\,m/s^2$$

$$\Delta y = v_i\cdot t + \frac{1}{2}at^2$$

$$\frac{\Delta y - \frac{1}{2}at^2}{t} = v_i$$

→ $v_i = 1.67\,m/s$

→ $v_f = v_i + at$

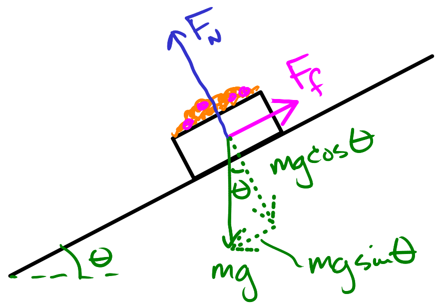
$$v_f = 1.67\,m/s - 1.46\,m/s^2\cdot4\,s$$

$$\boxed{v_f = -4.17\,m/s}$$

A crate of potatoes of mass 13.4 kg is on a ramp with angle of incline 30.0° to the horizontal. The coefficients of friction are μ_s = 0.750 and μ_k = 0.400.

$\mu_s \leq 0.750$

Find the frictional force on the crate, if the crate is at rest. If the direction of the frictional force is up the ramp, enter a positive value and if the direction is down the ramp, enter a negative value.



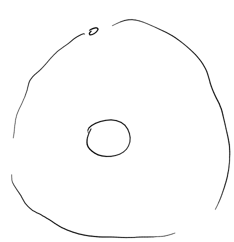
$$F_f = mg\sin\theta$$
$$= 13.4\,kg \cdot 9.8\,N/kg \cdot \sin 30°$$
$$= \underline{65.7\,N}$$

$$F_{fs} = \mu F_n \qquad F_n = mg$$

$$F_f = \mu \cdot m \cdot a_c = m a_R$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

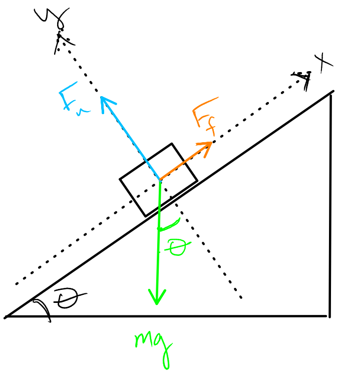
$$\mu = \frac{v^2}{gr}$$



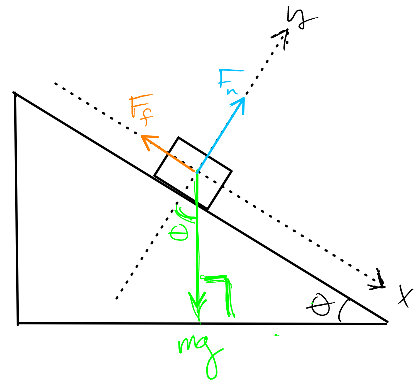
$$\omega = 1.5 \cdot 10^{-4}\, rad/s = \frac{\Delta\theta}{\Delta t}$$

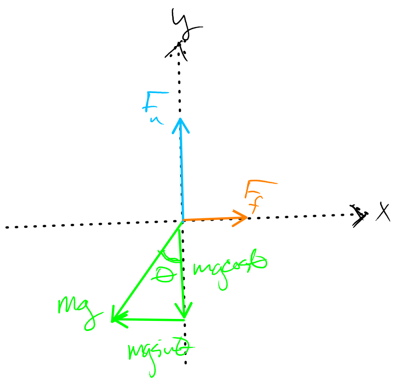
$$t = \frac{2\pi}{\omega} = \underline{\qquad s}$$

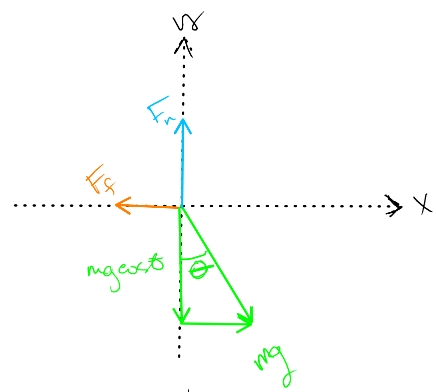
12 hr.



$F_{st} \leq \mu_s F_n$







x	y
$+F_f$	0
0	$+F_n$
$-mg\sin\theta$	$-mg\cos\theta$
0	0

$F_{NET,x}$ $F_{NET,y}$

$$F_f - mg\sin\theta = 0$$
$$F_f = +mg\sin\theta$$

$$0 = F_n - mg\cos\theta$$
$$F_n = mg\cos\theta$$

x	y
$-F_f$	0
0	$+F_n$
$+mg\sin\theta$	$-mg\cos\theta$
0	0

$$-F_f + mg\sin\theta = 0$$
$$-F_f = -mg\sin\theta$$
$$F_f = mg\sin\theta$$
$$\underbrace{F_f = 79.4\,\text{N}}$$

A certain satellite has an angular speed of $\underset{\omega}{1.45 \times 10^{-4}}$ rad/sec as it orbits the earth. How long would it take this satellite to do one complete orbit? (answer in *hours*, 3600 sec in 1 hr) t

$\theta = 1\text{rev} = 2\pi$ rad

$$\omega = \frac{\Delta\theta}{\Delta t} \quad \overset{?}{\longleftrightarrow} \quad \omega = \frac{v}{r}$$

$$(\Delta t)\, 1.45 \cdot 10^{-4} = \frac{2\pi}{\cancel{\Delta t}} \cancel{(\Delta t)} \Rightarrow \frac{\Delta t \cdot 1.45 \cdot 10^{-4}}{1.45 \cdot 10^{-4}} = \frac{2\pi}{1.45 \cdot 10^{-4}}$$

$$\Delta t = \frac{2\pi}{1.45 \cdot 10^{-4}} = 43{,}300\text{ s} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} = \underline{\underline{12 \text{ hr}}}$$

If a soccer ball with a radius of 15 cm rolls along the ground at constant angular speed of 10 radians per second, then how far along the ground does the ball roll in 10 seconds? Δx

$\omega = 10 \frac{rad}{sec}$ $\omega = \frac{\Delta\theta}{\Delta t}$ $\omega = \frac{V}{r}$ $\Delta x = ?$

$r = 0.15m$ θ

$\Delta t = 10s$

$V = \omega \cdot r = 10\frac{rad}{sec} \cdot 0.15m = 1.5 m/s$

$V = \frac{\Delta x}{\Delta t}$

$s = \Delta x$

$\Delta x = V \cdot \Delta t = 1.5 m/s \cdot 10s = 15m$

$V = 17 m/s$

A car travels at 17 m/s without skidding around a level curved road with a radius of 35m. What is the static friction coefficient between the tires and the road if this speed is as fast as the car can go without sliding? r $\mu = ?$

$a_R = \frac{V^2}{r}$ or $a_R = \omega^2 r$ $\omega = \frac{V}{r}$

$\omega = \frac{17}{35} = 0.49$

$F_f \leq \mu F_N$ $\boxed{F_f = \Sigma F_R = m \cdot a_R}$

$F_{fmax} = \mu F_N$

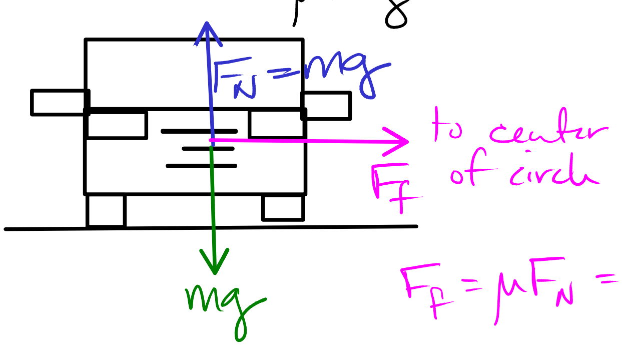
$= \mu m g$

$\mu \cancel{m} g = \frac{\cancel{m} V^2}{r}$ $\mu \cancel{m} g = \cancel{m}\omega^2 r$

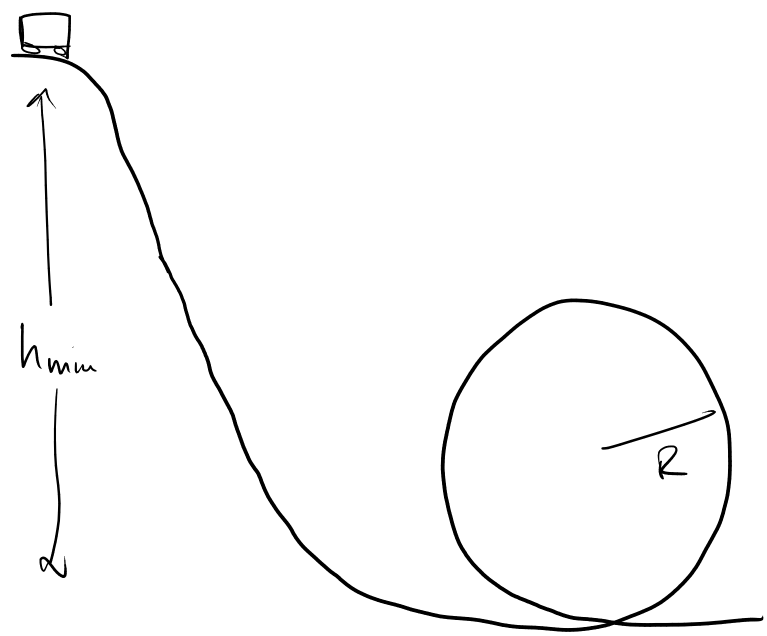
$\mu = \frac{V^2}{rg} = 0.84$ $\mu = \frac{\omega^2 r}{g}$

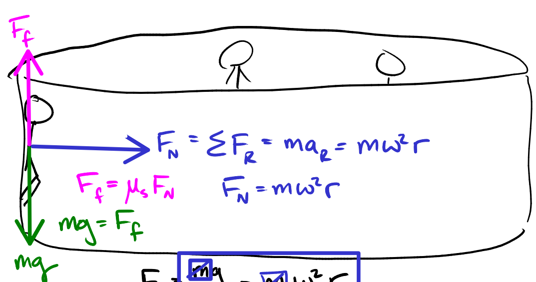
$= \frac{(0.49)^2 \cdot (35)}{9.8}$

$= 0.85$



$F_f = \mu F_N = \mu m g$





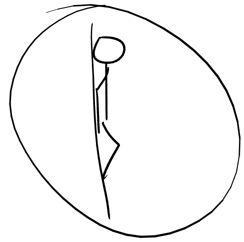
$$\boxed{F_N = \frac{\cancel{m}g}{\mu} = \cancel{m}\,\omega^2 r}$$

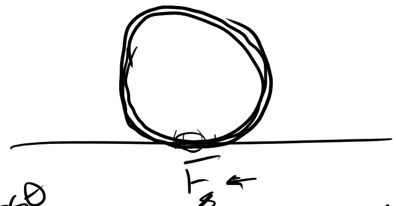
$$\frac{g}{\mu} = \omega^2 r$$

$$\boxed{\omega = \sqrt{\frac{g}{\mu r}}}$$

$$F_G = \frac{G m_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

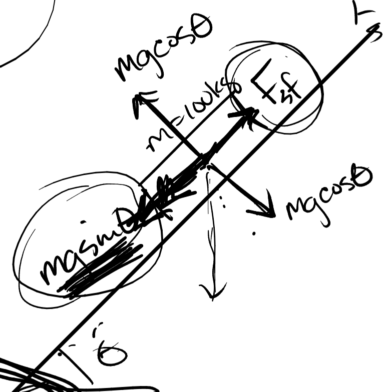
$\mu = \frac{F_f}{F_N}$





$F_{sf} \le \mu_s F_N$

$F_{kf} = \mu_k F_N$



$-mg\sin\theta + F_{sf} = 0$

$-mg\sin\theta + \mu mg\cos\theta = 0$

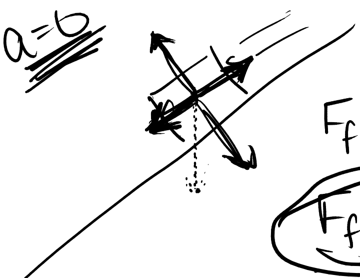
$\boxed{F_{sf_{max}} = \mu_s mg\cos\theta}$

μ_k

$\Sigma F_x = ma_x$

$F_f - mg\sin\theta = 0$

$a = 0$



$F_f = mg\sin\theta$

$F_f = \mu F_N$

$F_f = \mu mg\cos\theta$

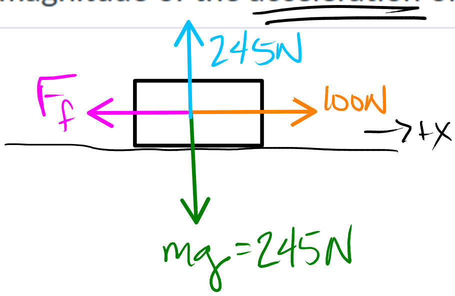
$\mu \cancel{mg}\cos\theta = \cancel{mg}\sin\theta$

$\mu = \frac{\sin\theta}{\cos\theta}$

$\mu = \tan\theta$

$\tan^{-1}(\mu) = \theta$

A 25 kg crate of apples is sliding across a level floor to the right with an applied force of 100 N, and a coefficient of friction between the crate and the floor being $\mu = 0.1$. What is the magnitude of the acceleration of the box?



$\Sigma F = ma_x$ → $+100N + (-24.5N) = 75.5N$

$\Sigma F_y = \cancel{ma_y}\ 0$

$-245N + F_N = 0$

$F_N = 245N$

$a = \frac{\Sigma F}{m}$

$a = \frac{75.5N}{25kg} = 3ish\ m/s^2$

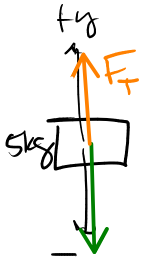
$F_f = \mu F_N$

$= 0.1(245) = 24.5N$

m

A 50 kg crate of apples is being lowered by a rope straight downwards and has an acceleration of 1 m/s^2 in the downward direction. What is the force the rope is applying ot the box? (magnitude and direction)

F_T



$F_g = -490N$

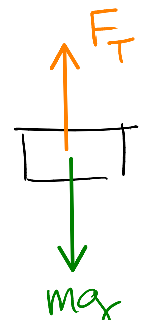
$a = -1\ m/s^2$

$\Sigma F = m \cdot a$

$\Sigma F = -50N$

$F_T - 490N = -50N$

$F_T = +440N$



$F_g = +490N$

+y

$a = +1\ m/s^2$

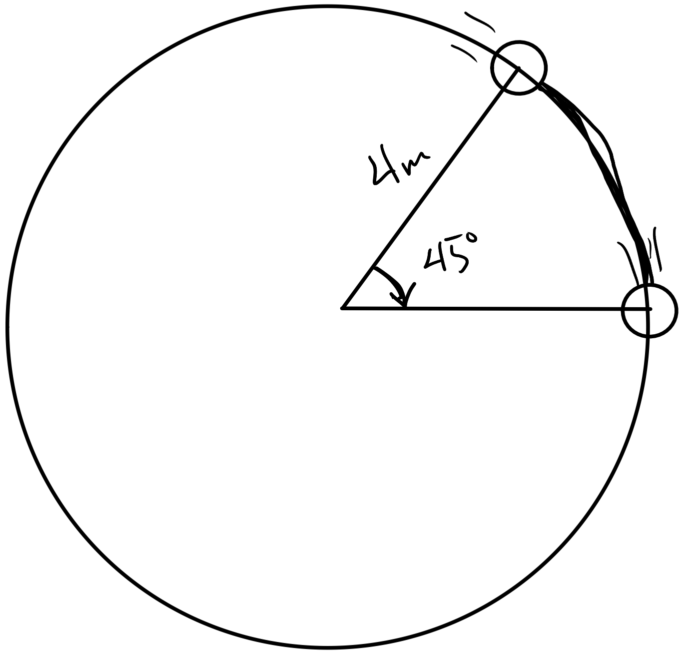
$\Sigma F = +50N$

$F_T + (490N) = +50N$

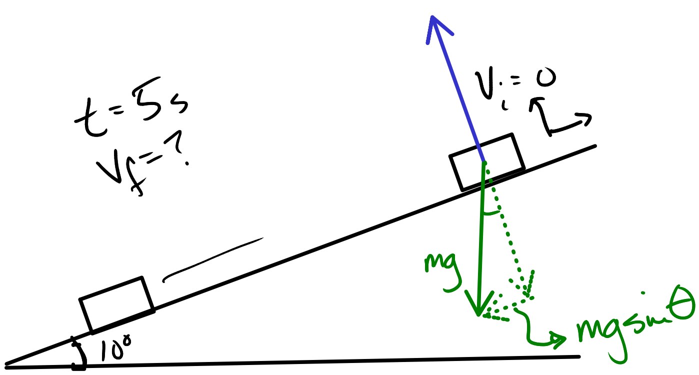
$F_T = -440N$

440N upward

In order to lift a certain box straight up at constant speed, I would need to exert a force of 1000 N on it. If I were to put the same box on an ramp inclined at 30 degrees to the horizontal, then what force would I need to exert to push it up the ramp at constant speed?



$$45° \cdot \frac{\pi \text{ rad}}{180°} = \underline{0.79 \text{ rad}} = \theta$$



$\rightarrow a = -g\sin\theta$

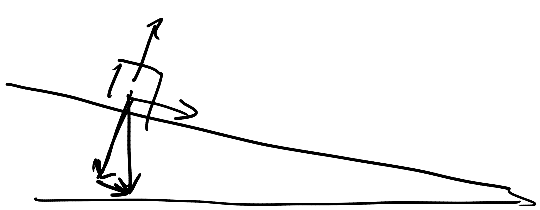
$F_{NET,x} = -\cancel{m}g\sin\theta = \cancel{m}a_x$

$-9.8\sin 10° = -1.7 \frac{m}{s^2} = a_x$

$\rightarrow V_f = V_i + at$

$V_f = 0 + -1.7 \cdot 5s$

$V_f = -$ ____



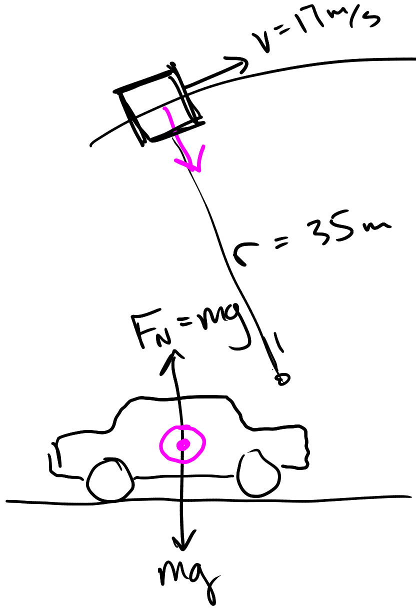
$V_f = +$ ____

$V = 17$

$r = 35$

$$\mu_s = \frac{F_f}{F_N}$$

$$F_f = \mu F_N$$



$$= \mu \cdot mg = F_{NET,R} = ma_R \qquad a_R = \frac{v^2}{r}$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$\mu = \frac{v^2}{gr}$$

$m = 1500\text{ kg}$

$V = 25.4\text{ m/s}$

$r = 35\text{ m}$

$\mu_s = ?$

$$F_f = \Sigma F_r = \frac{mv^2}{r} = \frac{1500\text{ kg} \cdot (25.4\text{ m/s})^2}{35\text{ m}}$$

$$= \frac{27{,}000\text{ N}}{mg}$$

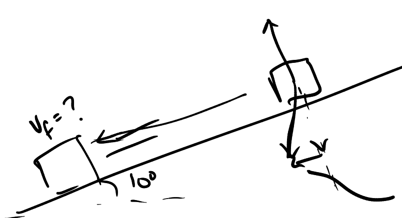
$$\mu mg = \sim\sim$$

$$\mu = \frac{\sim\sim}{mg} = 1.88 \leftarrow$$

?

oops ....

$\Delta t = 5\text{ s}$



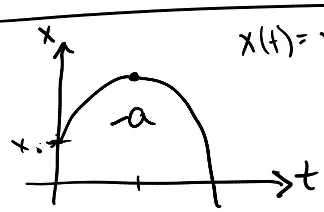
$$v_f = v_i + at$$

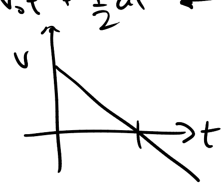
$$v_f = a_x \cdot t$$

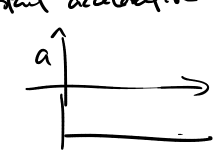
$$mg\sin\theta = \Sigma F_x = ma_x$$

$$a_x = g\sin\theta$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \leftarrow \text{constant acceleration}$$







$6.67 \cdot 10^{-11}$

$$G \text{ vs. } g = \frac{GM}{r^2}$$

8

$$\frac{GM_e \cancel{m_s}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}} \leftarrow \text{vs.} \rightarrow \left(\frac{1}{r}\right)\frac{GM_e \cancel{m_s}}{r^2} = \cancel{m_s}\omega^2 \cancel{r}\left(\frac{1}{\cancel{r}}\right)$$

$alt = 3.5 \cdot 10^6\,m$

$r_e = 6.37 \cdot 10^6\,m$ } r

$M_e = 6 \cdot 10^{24}\,kg$

$$\frac{GM_e m_s}{r^2} = \sum F_r = \frac{mv^2}{r} = m_s\omega^2 r$$

$$\frac{GM_e \cancel{m_s}}{r^2} = \cancel{m_s}\omega^2 r$$

$$\omega = \sqrt{\frac{GM_e}{r^3}}$$

$\omega = \frac{v}{r}$

$\Delta t_{rev} =$

$$\omega = \frac{\Delta\theta}{\Delta t} \leftarrow 2\pi$$

$$\Delta t = \frac{2\pi}{\omega} ? = 9700\,s = 2.7\,hr$$

$\omega =$

$m = 1225\,kg$

$v = 30\,m/s$

$r = 50\,m$

$$\sum F_r = ? = \frac{mv^2}{r} = \frac{1225\,kg \cdot (30)^2}{50}$$

$=$