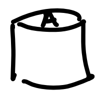
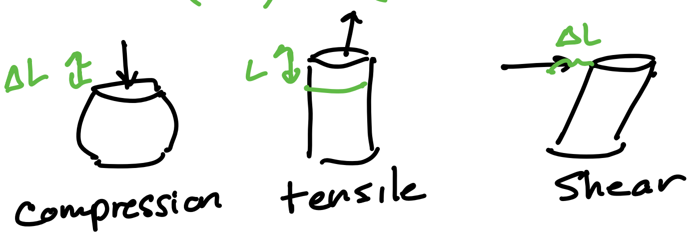
At the end of this worksheet, you should be able to

- calculate stress and strain and use Young's Modulus to solve for an unknown.
- plot all relevant quantities of simple harmonic motion over time.
- use the quantities of simple harmonic motion and the mathematical description to solve for an unknown.

stress strain

$$\left(\frac{F}{A}\right) = Y\left(\frac{\Delta L}{L}\right)$$





stress causes strain

Y - young's modulus

Table 10.1	Approximate Values of Young's Modulus for Various Substances
Substance	**Young's Modulus (GPa)**
Rubber	0.002–0.008
Human cartilage	0.024
Human vertebra	0.088 (compression); 0.17 (tension)
Collagen, in bone	0.6
Human tendon	0.6
Wood, across the grain	1
Nylon	2–6
Spider silk	4
Human femur	9.4 (compression); 16 (tension)
Wood, along the grain	10–15
Brick	14–20
Concrete	20–30 (compression)
Marble	50–60
Aluminum	70
Cast iron	100–120
Copper	120
Wrought iron	190
Steel	200
Diamond	1200

<u>Hooke's Law for Stress and Strain and Young's Modulus</u>

1. Consider a wire that is 0.1 mm in diameter and 2 m long and a Young's Modulus of 120 GPa (1 GPa = 10^9 Pa). A 100-N force is applied to the wire.

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.1\times 10^{-3} m)^2}{4}$$

   a) What is the stress on the wire?

$$\frac{F}{A} = \frac{100N}{7.85\times 10^{-9} m^2} = 1.27\times 10^{10} N/m^2$$

   b) What is the strain?

$$\frac{F}{A} = Y\frac{\Delta L}{L} \rightarrow \frac{\Delta L}{L} = \frac{F/A}{Y} = \frac{1.27\times 10^{10} N/m^2}{120\times 10^9 N/m^2}$$

$$\frac{\Delta L}{L} = 0.106$$

   c) By how much does its length change?

$$\Delta L = L(0.106) = 2m(0.106) = 0.212m$$

d) What is its new length?

$$L + \Delta L = 2.212\,m$$

$$L_2 = L + \Delta L = L\left(1 + \frac{\Delta L}{L}\right)$$

e) What percent change is this? Strain * 100%

$$\frac{L_2 - L_1}{L_1} = \frac{(\cancel{L_1} + \Delta L) - \cancel{L_1}}{L_1} = \frac{\Delta L}{L_1}$$

$0.106 \times 100\%$
$= 10.6\%$

2. Next, 1000-N force is applied to an identical wire. What is stress, strain, length change and percent change in the length from its original length?

$$10F_1 \rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}$$

Stress $= 10\,(1.27 \times 10^{10}\,N/m)$
strain, $\frac{\Delta L}{L} = 10\,(0.106) = 1.06$
% change $= 106\%$

3. If, instead, the wire was 1.0 m long with a 100-N applied force. Now, how much does it stretch? What percent change is this?

$$100N \rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}$$

now $L_2 = \frac{1}{2} L_1$ b/c $1m = \frac{1}{2}(2m)$

$\frac{F}{A}$ and $\frac{\Delta L}{L}$ will be the same 10.6% stretch

$$L = L + \Delta L = \underset{1m}{L}\left(1 + \frac{\Delta L}{L}\right) = 1.106\,m$$

4. Now, wire of same material and 2.0 m length but the cross-sectional diameter of the wire is half its original diameter ($d_2 = \frac{1}{2}(0.1mm)$) and a 100-N force is applied. How much does it stretch now? And what percent change is this?

$d_2 = \frac{1}{2} d_1$
$A_2 = \frac{1}{4} A_1$

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

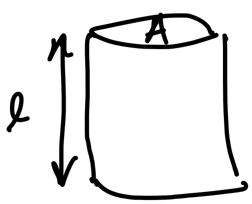
$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$4\left(\frac{F}{A}\right)_{original}$ so $4\left(\frac{\Delta L}{L}\right)_{original}$

% change $= 4\left(\frac{\Delta L}{L}\right)_o = 0.424 \times 100$

% change $= 42.4\%$

5. A wire of length ℓ_1 and volume V and cross-sectional area A_1 is stretched out to length ℓ_2, what is its new cross-sectional area? Think about this in terms of proportionality.



$$V_1 = V_2$$
$$A_1 \ell_1 = A_2 \ell_2$$
$$A_2 = A_1 \left(\frac{\ell_1}{\ell_2}\right) = A_1 \left(\frac{\ell_1}{\ell_1 + \Delta\ell}\right)$$
$$A_2 = A_1 \frac{\ell_1}{\ell_1\left(1 + \frac{\Delta\ell}{\ell_1}\right)} = \frac{A_1}{\left(1 + \frac{\Delta\ell}{\ell_1}\right)}$$

6. A 60 kg person stands upright. By how much does the femur shorten if each femur carries half the weight of the person? The cross-sectional area of a femur is about 4 cm^2 and the length is about 30 cm Also find the percent change in length. ($Y = 9.4 \times 10^9$ Pa) ← from list

L

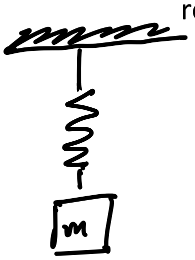
Stress: $\frac{F}{A} = \frac{30\,kg\,(9.8)}{(4)\left(\frac{m}{100\,cm}\right)^2} = 735000\ N/m^2$

Strain: $\frac{\Delta L}{L} = \frac{735000\ N/m^2}{9.4 \times 10^9\ N/m^2} = 7.82 \times 10^5$

$\Delta L = 30\,cm\,(7.82 \times 10^5) = 0.00235$

Hooke's Law for Springs

7. A 1-kg mass is attached to a spring of spring constant 1000 N/m. The mass is pulled back, so the spring is stretched 10 cm from its relaxed length. After you finish the following questions, make sure you can write down expressions for all of them in general as well as working them in reverse. ignore gravity for vertical



a) What is the initial total *energy* of the mass-spring system? What is it due to?

$E_1 = \frac{1}{2}kx^2 + \cancel{\frac{1}{2}mv^2}$ ← E_1 does not depend on mass

$E_1 = \frac{1}{2}(1000\,N/m)(0.10)^2 = 5$ Joules

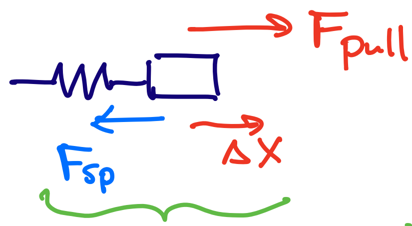
b) When the mass is released, it heads back toward equilibrium gaining kinetic energy as it goes. What is the maximum kinetic energy it can achieve? What is the position of the mass when kinetic energy is a maximum?

at $x = 0$, equilibrium

$K_{max} = 5$ Joules

Springs

Hooke's Law $F = -K(\Delta x)$



$F_{sp} = -F_{pull}$

negative, Δx is opposite Fspring

- Force is not constant
  - Fstring is constant, does not depend on length
- F depends on length → you cannot use Kinematic equations
  - $F = -K(\Delta x)$
  - $\Delta x = v_i t + \frac{1}{2} a t^2$
  - $F = ma = Kx$
  - $a \neq$ constant

– must energy to solve problems with moving/oscillating springs

Periodic Motion

examples: planetary motion, bouncing ball with elastic collision

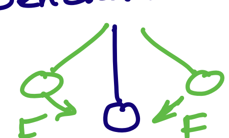
not oscillations

Oscillation – Simple Harmonic Motion (SHM)

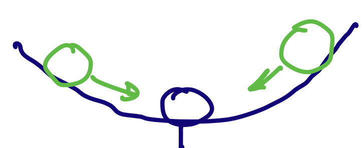
2 criteria

1. Force causing motion must be a restoring force, a force that pushes the object back to equilibrium

ex: pendulum

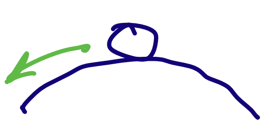


equilibrium



equilibrium

stable

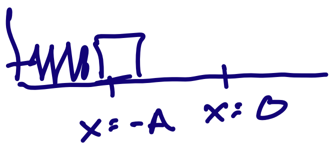


unstable equilibrium

② Restoring Force is proportional to displacement

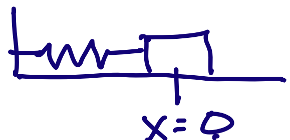
$$F = -K(\Delta x)$$

Forces during SHM of mass-spring



$a = \frac{-KA}{m}$

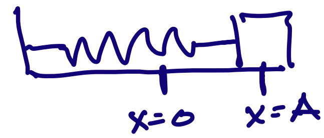
Same magnitude



equilibrium

$F = K\cancel{x}^{0} = 0$

no net Force



$F = Kx = KA$

$F = ma$

$KA = ma$

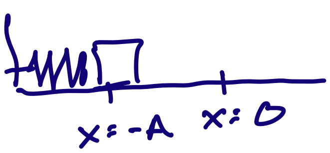
$a = \frac{KA}{m}$

$\therefore$ $F_{net} = 0$ at equilibrium

$F_{net} = max$ at max displacement A

Energy during oscillation

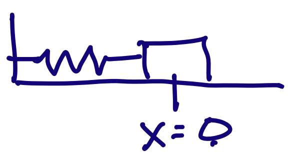
$E_{total} = \frac{1}{2}Kx^2 + \frac{1}{2}mv^2$ at every location



$V = 0$

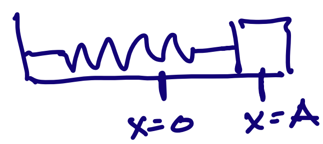
all $U_s = \frac{1}{2}Kx^2$

$U_s = \frac{1}{2}KA^2$



$V = V_{max}$

all $K = \frac{1}{2}mV_{max}^2$



$V = 0$

all $U_s = \frac{1}{2}KA^2$

c) What is the mass's velocity when it has maximum kinetic energy?

E_1

$\cancel{\tfrac{1}{2}} kA^2 = \cancel{\tfrac{1}{2}} m v_{max}^2$

$E_B = 5 \text{ Joules} = \tfrac{1}{2} m v^2$

$v_{max} = A\sqrt{k/m} = A\omega$

$5 = \tfrac{1}{2}(1\,kg)\,v^2 \rightarrow v = 3.16\ m/s$

d) What is the extension of the spring when the mass's potential energy and kinetic energy at that position are equal?

solve for x

$\tfrac{1}{2} U_s = \tfrac{1}{2}(5J) = 2.5J = \tfrac{1}{2}kx^2$

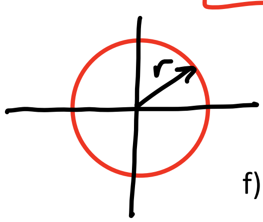
$2.5J = \tfrac{1}{2}(1000\,N/m)\,x^2$

$x = 0.071\,m \sim 7.1\,cm$

memorize

e) What is the *period* of this mass's motion? What is its frequency? What is its angular frequency?

$\omega\,(rad/s) = \sqrt{k/m} = 2\pi f = \frac{2\pi}{T}$ $\quad T = 1/f$



$1 = \cos^2\theta + \sin^2\theta$

$\theta = \omega t$

$y = A\cos(\omega t)$

initial pull, x

$\omega = \frac{2\pi}{T}$

$\sqrt{k/m} = \frac{2\pi}{T}$

$\sqrt{\frac{1000\,N/m}{1\,kg}} = \frac{2\pi}{T}$

angular frequency

$\omega = 31.6\ rad/s$ $\quad T = 0.2\,s$

$f = \frac{1}{0.2} = 5\,Hz$

f) What is the *force* on the mass when it is at its maximum displacement?

$F = -kx$

$F = (1000\,N/m)(0.1\,m)$

$F = 100\,N$

g) What is the force on the mass when it is at the equilibrium position?

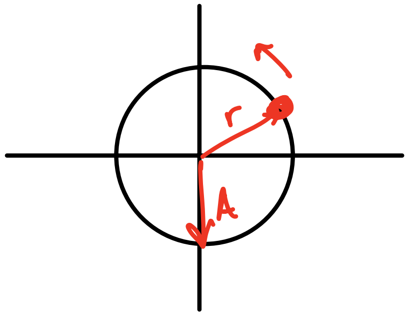
$F_{net} = -k(0) = 0$

h) What is the maximum acceleration of the mass? Where does this occur?

$a_{max} = \frac{F}{m} = \frac{kx}{m} =$ $\quad A$

$\omega = \sqrt{k/m}$

$a_{max} = \left(\frac{k}{m}\right)A = \omega^2 A$



Circular Motion

$v = r\omega$

$a = r\omega^2$

changes direction

SHM

$x = A\cos(\omega t)$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

$\omega = \sqrt{K/m}$

K: spring constant

m = mass moving

A mass oscillates with an amplitude of 0.2m and frequency of 12.0 Hz.

a) Period? $T = \frac{1}{12} = 0.083$ sec

b) angular frequency? $\omega = 2\pi f = 24\pi = 75.4$ rad/s

c) Spring constant, $K = 8$ N/m. What's the mass? $T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k}$

$\omega = \sqrt{K/m}$

$m = \frac{K}{\omega^2} = \frac{8}{(75.4)^2} = 0.0014$ kg $= 1.4$g

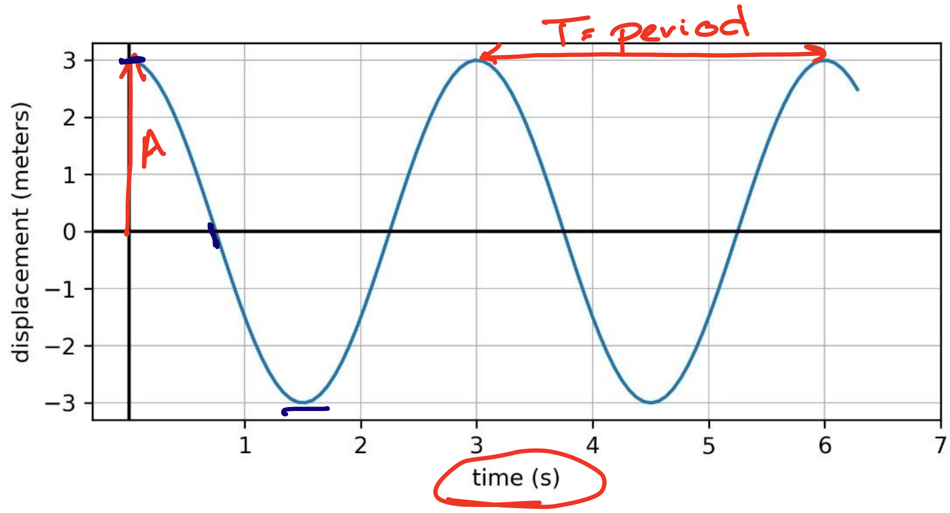
d) max velocity?

max acceleration?

To double the period $T = 2\pi\sqrt{m/k}$

- quadruple mass, 4m,
- quarter spring constant, ¼K,

8. A 4-kg mass attached to a spring oscillates according to the displacement graph that follows.



a) Determine the amplitude, period, natural frequency, angular frequency.

ω

$A = 3m$

$T = 3sec$

$f = \frac{1}{T} = \frac{1}{3}\ Hz = \frac{1}{3}\ /s$

$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{3} = 2.1\ rad/s$

b) Determine the spring constant of the spring.

$x = A\cos\omega t = (3m)\cos(2.1t)$

$\omega = \sqrt{K/m} \longrightarrow 2.1 = \sqrt{K/(4kg)}$ $\quad K = 17.6\ N/m$

c) Determine the mass's maximum velocity and maximum acceleration.

energy

forces $\longrightarrow$ max Force $= Kx = KA$

at $x = A$ at $x = 0$

$\frac{1}{2}KA^2 = \frac{1}{2}mv^2$

$v = A\sqrt{K/m} = A\omega = 3(2.1) = 6.3\ m/s$

$KA = ma$ (Newton 2nd)

$a = A\,K/m = A\omega^2 = 13.2\ m/s^2$

d) Determine the system's maximum kinetic energy, maximum potential energy, and total energy?

$KE_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}KA^2 = 79.2$ Joules

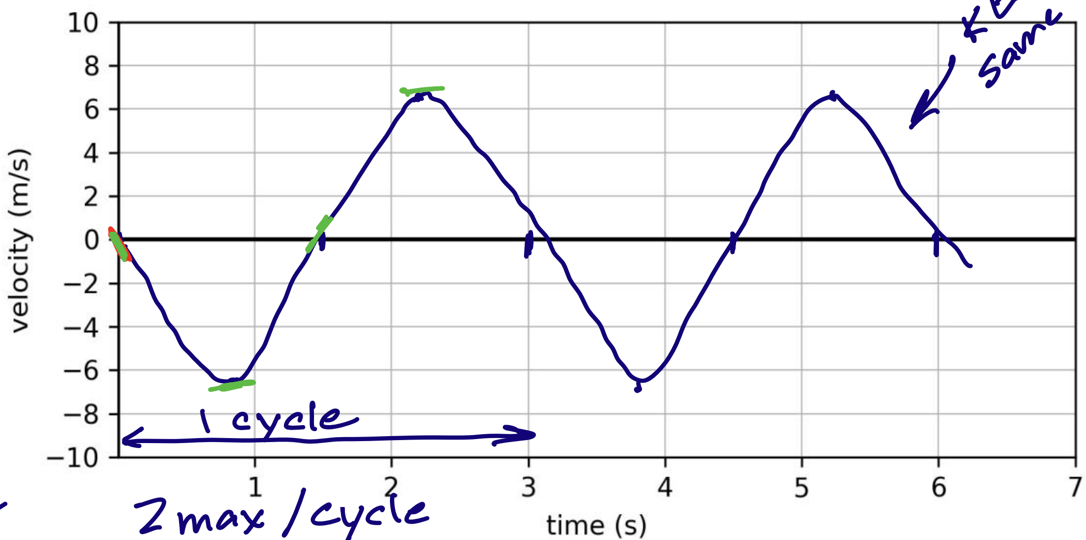
occurs at equilibrium

$U_{s,max} = 79.2\ J$ occurs at $+A, -A$

Total $E = 79.2\ J$

e) Also fill out the rest of the graphs.

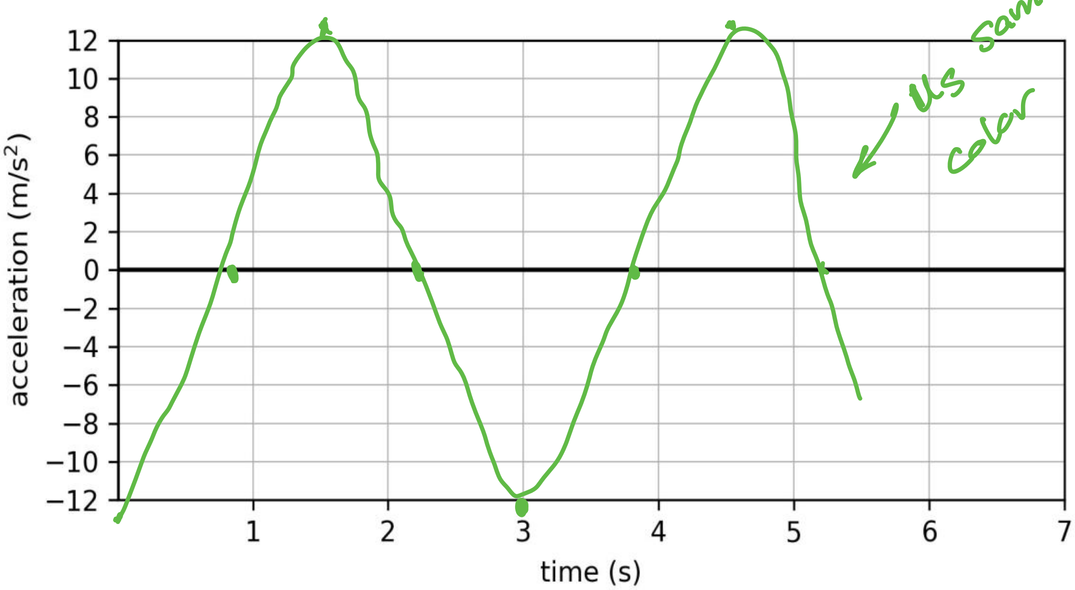
KE same color

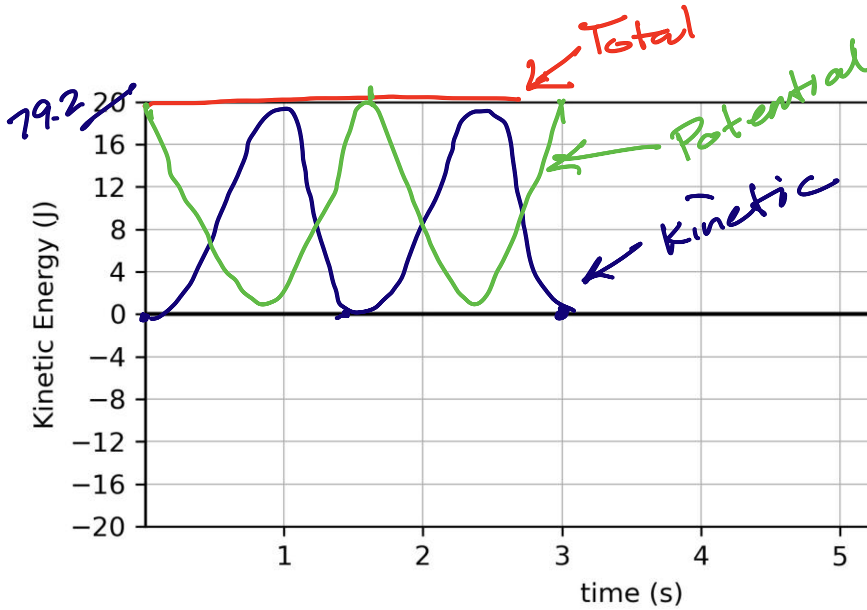


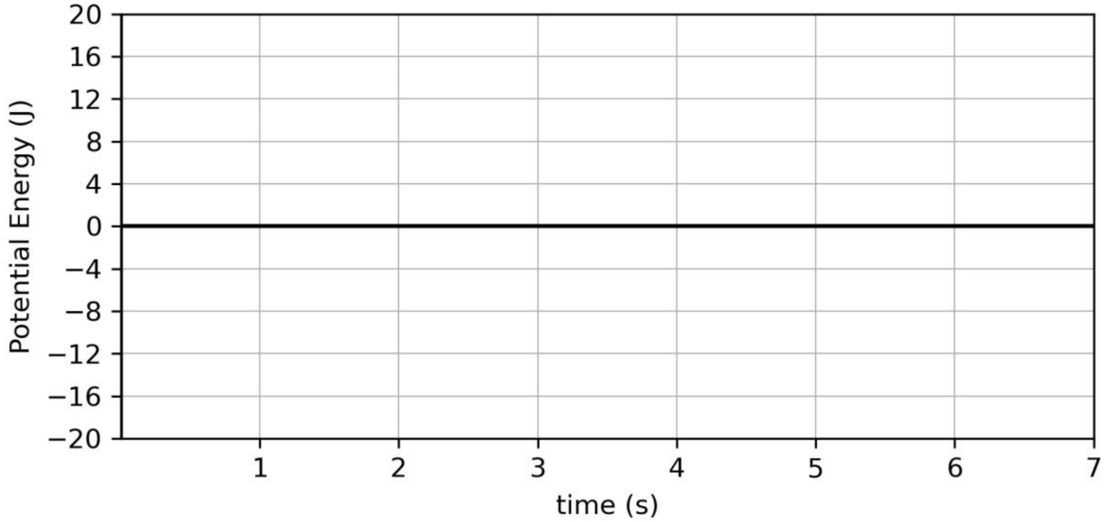
$\frac{1}{2}mv^2$

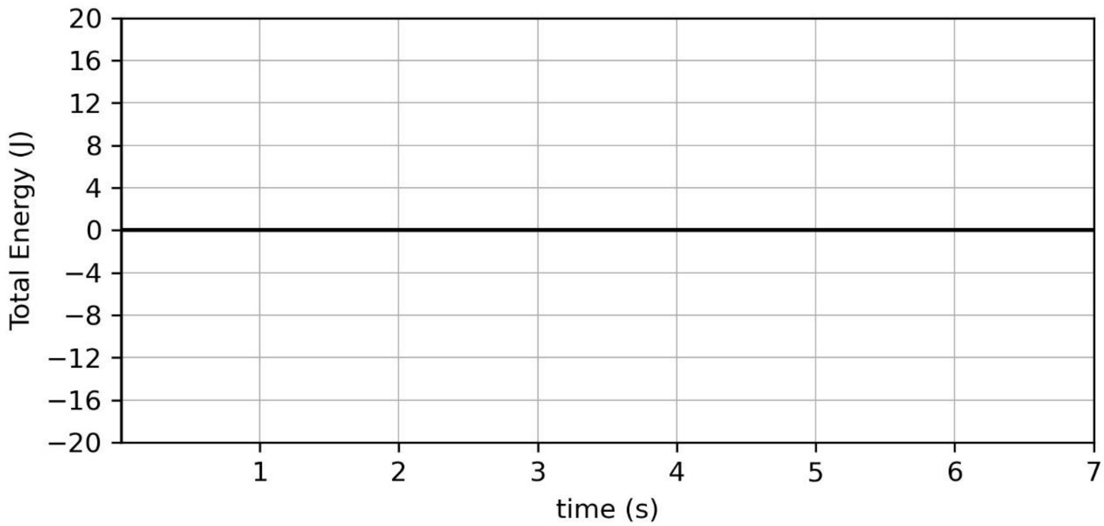
2 max / cycle
always positive

Us same color




Total
Potential
Kinetic
79.2
Kinetic Energy (J)
time (s)


Potential Energy (J)
time (s)


Total Energy (J)
time (s)

Stress and Strain

9. According to the table, which material stretches more, 2 m of steel or 1 m of copper of the same width?

Same Stress

2m Steel $Y_{st} = 200$ GPa

1m Copper $Y_{cu} = 120$ GPa

$$\frac{F}{A} = Y\frac{\Delta L}{L} \rightarrow \Delta L = \left(\frac{F/A}{Y}\right)L \propto \frac{L}{Y} \quad \text{b/c } F/A \text{ same}$$

$$\Delta L_{st} = \frac{2\text{m}}{200\text{GPa}} \qquad \Delta L_{cu} = \frac{1\text{m}}{120\text{GPa}}$$

10. Four ~~brass~~ Copper wires are subjected to the same ~~tensile stress~~ Force. The wires have the following unstretched lengths and widths. Rank them in order from least to most change in length.

Copper $Y = 120$ GPa

ΔL

(a) length *L*, diameter *d* $\frac{L}{d^2}$

(b) length 2*L*, diameter *d* $\rightarrow \frac{2L}{d^2} = 2(a)$

(c) length 4*L*, diameter *d*/2 $\frac{4L}{(d/2)^2} = \frac{4L}{(d^2/4)} = \frac{16L}{d^2} = 16(a)$

(d) length *L*/4, diameter *d*/2 $\rightarrow \frac{L/4}{(d/2)^2} = \frac{4(\frac{1}{4}L)}{d^2} = (a)$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$\Delta L = \left(\frac{F/A}{Y}\right)L = \frac{F}{AY}L \propto \frac{L}{d^2}$$

11. A 0.5-m long guitar string of cross-sectional area of 1.0×10^{-6} m^2 and Young's modulus $Y = 2.0$ GPa. By how much must you stretch the string to obtain a tension of 20 N.?

$$\frac{20\text{N}}{1\times10^{-6}\text{m}^2} = (2.0\times10^9\text{Pa})\frac{\Delta L}{0.5\text{m}}$$

$$\Delta L = 0.005\text{m} = 5\text{mm}$$

Nov 13

12. Compare Hooke's Law for springs ($F = kx$) to Hooke's Law for stress and strain($\frac{F}{A} = Y\frac{\Delta L}{L}$). Write an equation relating the spring constant, Young's modulus, length, and cross-sectional area. Solve for k. What does this tell you about what would happen to the spring constant of a spring if you cut the spring in half?

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$\underset{F}{F} = \underset{K}{\left(\frac{AY}{L}\right)}\underset{\Delta x}{\Delta L}$$

$$K = \frac{AY}{L}$$

geometry $\frac{A}{L}$

materials Y

13. If you attach two springs to an object side by side, then we say the springs are attached *in parallel*. This will result in two spring forces on the object that has been displaced some distance x_1. If you were to model this arrangement of springs in parallel as a single spring with a single spring constant that would have the same effect, then what would this single effective spring constant k_e be in terms of the original two spring constants k_1 and k_2?

14. If instead of a *parallel* arrangement, we attach one spring to another spring and then attach the two springs to the object. We say that these springs are connected *in series*. Derive an equation for an effective spring constant for this arrangement. Each spring will stretch a different amount based on its spring constant, but the object will experience one force and *both of the springs is exerting the same force.*

Homework #5

$A = 2.10 \times 10^{-8}$ m

$f = 20$ Hz $\longrightarrow$ $\omega = 2\pi f = 40\pi$

$v_{max}: A\omega = (2.10 \times 10^{-8}\text{ m})(40\pi)$

$a_{max}: A\omega^2 = (2.10 \times 10^{-8}\text{ m})(40\pi)^2$

2.1 E-8

#9

$\omega = \sqrt{k/m}$

$k = \omega^2 m$

$\omega = 16.5$ rad/s for m_1

ω_2 for $4m_1$

$\omega_1^2 m_1 = \omega_2^2 m_2$

$(16.5)^2 \cancel{m} = \omega_2^2 (4\cancel{m_1})$

$$\omega_1^2 = \omega_2^2 (4)$$

$$\sqrt{\omega_2^2} = \sqrt{\frac{\omega_1^2}{4}}$$

$$\omega_2 = \frac{\omega_1}{2}$$

#12

$$y(t) = \underbrace{(8\,cm)}_{A} \sin(\underbrace{2.76}_{\omega} t)$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{K/m}$$

$$V_{max} = A\omega \qquad V = A\omega \cos(2.76t)$$

$$a_{max} = A\omega^2 \qquad a = -A\omega^2 \sin(2.76t)$$