At the end of this worksheet, you should be able to

$\vec{F} = m\vec{a}$

same direction

- discuss the quantities of displacement, velocity, and acceleration.
  vector
- interpret the meaning of the sign of the above quantities.
- discuss the cause of the change in those quantities.
- take a given state of motion of an object and predict its future state of motion.
- identify key features of a graph of these quantities over time.
- create a qualitative plot of these quantities over time given an interesting physical problem.
- solve a range of problems under the conditions of constant acceleration.

Displacement and Velocity

1. Sam moves from $x = 2$ m to $x = 10$ m in 4 sec.
   a) What is Sam's total displacement? Include +/–
   b) What is Sam's average velocity over this interval? Include +/–

-15 -10 -5 0 5 10 15

$\Delta x = x_f - x_i = 10 - 2 = 8m$

$v_{avg} = \frac{\Delta x}{t} = \frac{+8m}{4s} = 2 m/s$

2. Ford moves from $x = -2$ m to $x = 10$ m in 4 sec.
   a) What is Ford's total displacement? Include +/–
   b) What is Ford's average velocity over this interval? Include +/–

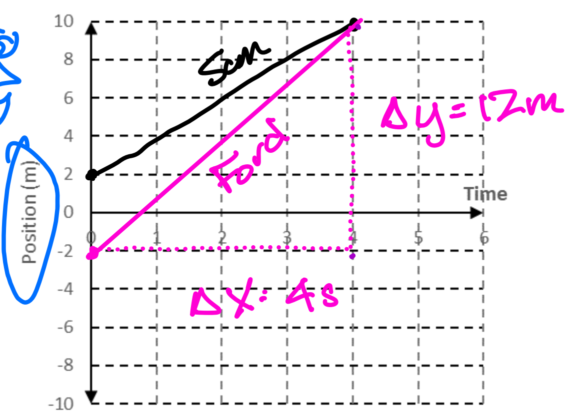
-15 -10 -5 0 5 10 15

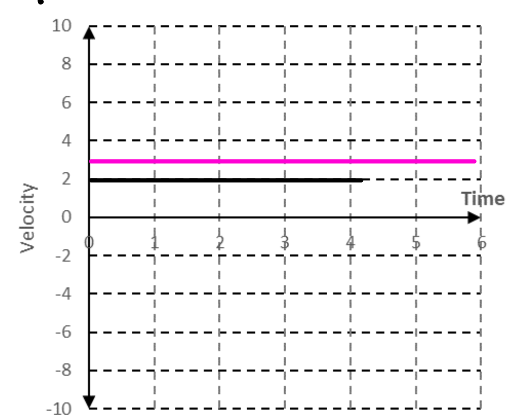
$\Delta x = x_f - x_i = 10 - (-2) = 12 m$

$v_{avg} = \frac{\Delta x}{t} = \frac{+12m}{4s} = 3 m/s$

3. Plot Sam and Ford's displacement and velocity on the grids. Compare the slopes.

constant velocity = linear





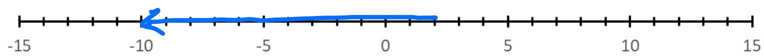
$\Delta x = x_f - x_i = v_i t + \frac{1}{2}\cancel{a}t^2$ constant velocity $a = 0$

$\Delta x = v_i t$ linear

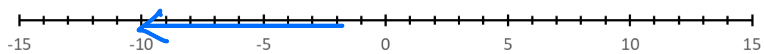
4. Sam moves from $x = 2$ m to $x = -10$ m in 4 sec. What is Sam's total displacement? Include +/– What is Sam's average velocity over this interval? Include +/–.

$\Delta x = -12m$

$V_{avg} = -3 m/s$



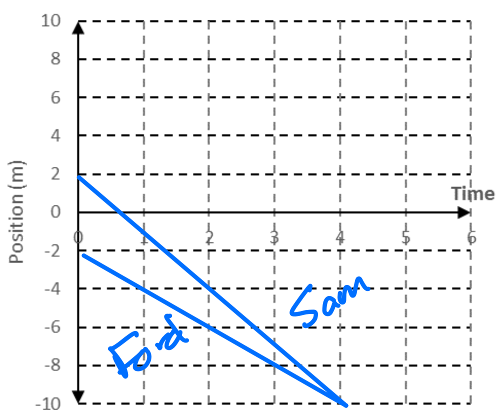
5. Ford moves from $x = -2$ m to $x = -10$ m in 4 sec. What is Ford's total displacement? Include +/–. What is Ford's average velocity over this interval? Include +/–.

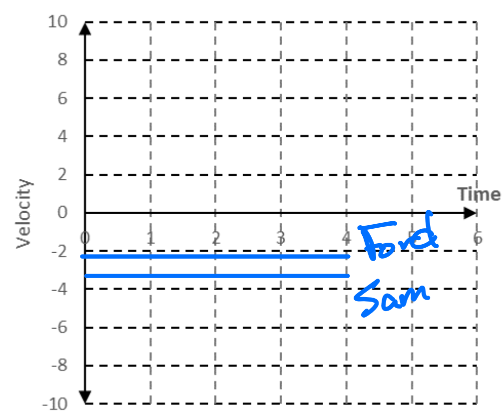


$\Delta x = -8 m$

$V_{avg} = -2 m/s$

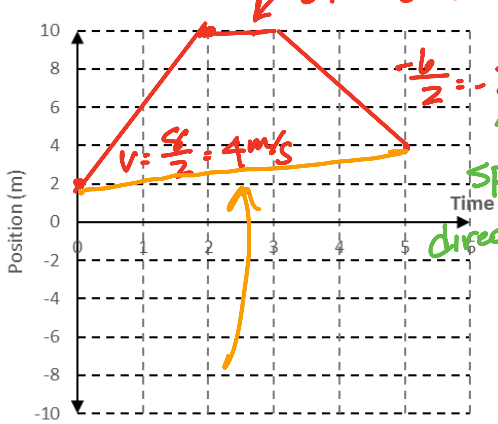
6. Plot Sam and Ford's displacement and velocity on the grids. Compare the slopes.





7. Sammie moves from $x = 2$ m to $x = 10$ m in 2 sec. Stands for 1 second and then moves back to $x = 4$ m in 2 seconds. Graph Sammie's displacement and velocity.

at rest (horizontal)

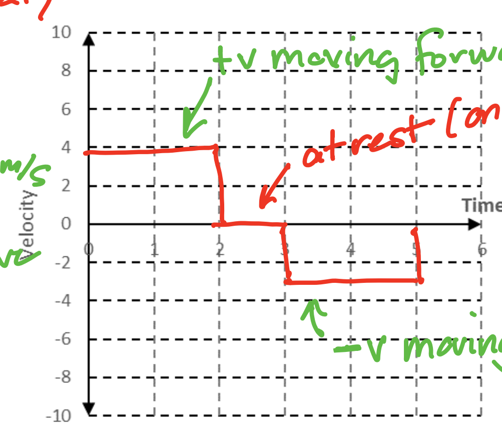


$\frac{-6}{2} = -3 m/s$

speed: 3 m/s
direction = negative

+v moving forward

at rest (on axis)



–v moving backward

avg velocity

$V_{avg} = \frac{+2m}{5 sec} = 0.4 m/s$

8. What is the difference between average velocity and instantaneous velocity? Give an example.

Slope of end points

Slope of position-time graph

$v_{avg} = \frac{\Delta x}{t} = \frac{+4-2m}{5s} = 0.4\ m/s$

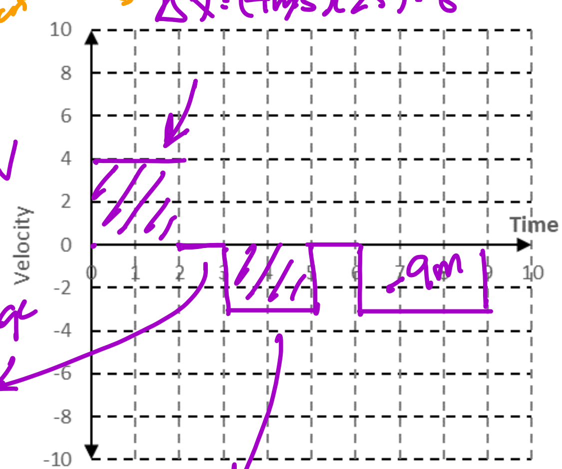
9. Can you generate a displacement graph if you know an object's velocity-time graph? Samantha moves with a constant velocity + 4 m/s for 2 seconds. She stands still for 1 second and then moves with a constant velocity –3 m/s for 2 seconds. She rests another second before continuing at – 3 m/s for 3 seconds. Do we need to know anything else to generate the position-time graph?

$\Delta x = vt$

Area = bh = t(v)

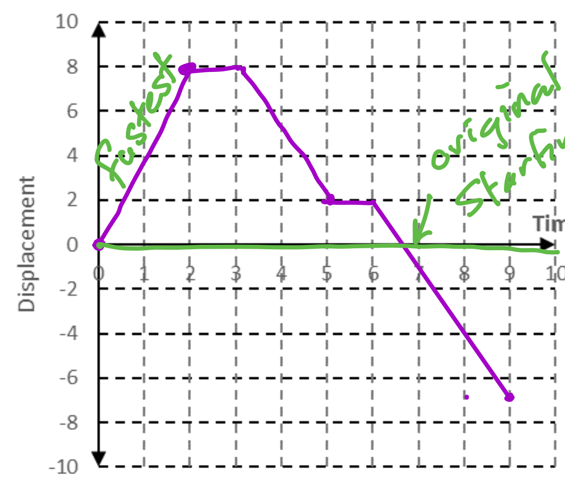
change in position

$\Delta x = (4m/s)(2s) = 8$



no change in Δx

Δx = area = $(-3m/s)(2s) = -6m$



displacement = Δx = change in position

position-time graph must have x_1 (starting position)

Kinematic equations

$x_2 = x_1 + v_1 t + \frac{1}{2}at^2$ (quadratic with time)

$v_2 = v_1 + at$ (linear with time) $\rightarrow a = \frac{\Delta v}{t}$

$\Delta x = v_{avg}\, t$

$v_2^2 = v_1^2 + 2a(\Delta x)$ "No time"

DISPLACEMENT, VELOCITY, AND ACCELERATION

$a \equiv \frac{\Delta v}{t}$

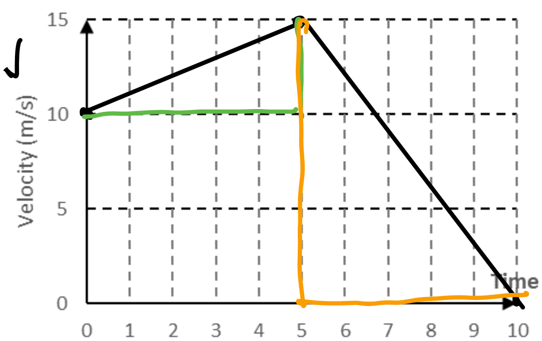
10. An object has an initial velocity of 10 m/s and speeds up to 15 m/s in 5 seconds. After another 5 seconds, it comes to a stop.

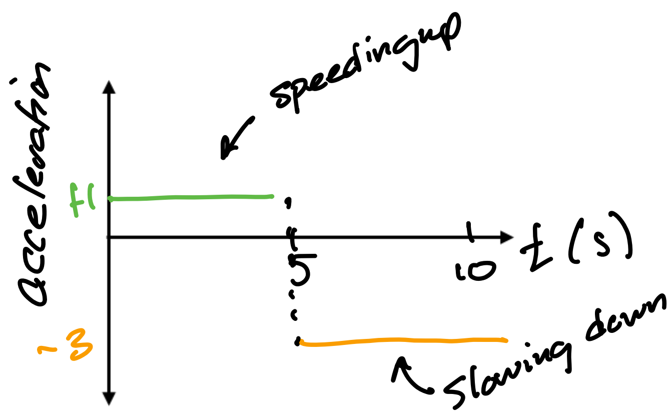
   a) Graph the object's velocity vs time.

   b) What is the object's acceleration for its first 5 seconds? $a = \frac{\Delta v}{t} = \frac{+15-10}{5} = +1\ m/s^2$

   c) What is the object's acceleration from *t* = 5s to *t* = 10 s. $a = \frac{0-15\,m}{5\,s} = -3\ m/s^2$

   d) Sketch the values. What is the significance of the signs of acceleration?





11. An object has an initial velocity of –10 m/s and speeds up to –15 m/s in 5 seconds. After another 5 seconds, it slows to a stop.

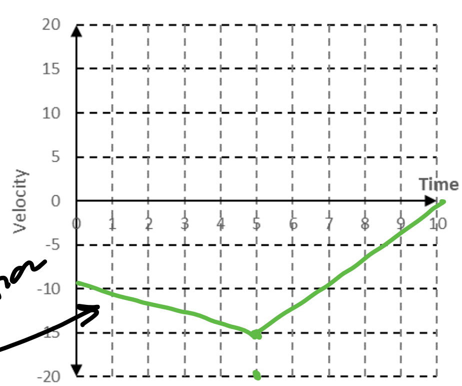
   a) Graph the object's velocity vs time.

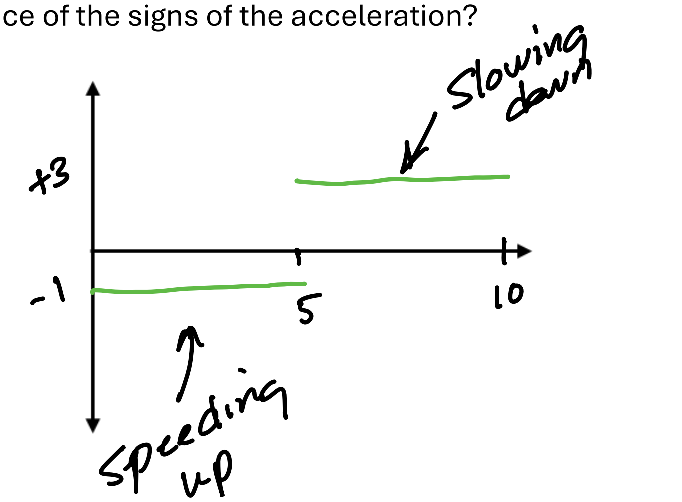
   b) What is the object's acceleration for its first 5 seconds? $a = \frac{-15-(-10)}{5} = -1\ m/s^2$

   c) What is the object's acceleration from *t* = 5s to *t* = 10 s. $a = \frac{0-(-15)}{5} = +3\ m/s^2$

   d) Sketch the values. What is the significance of the signs of the acceleration?

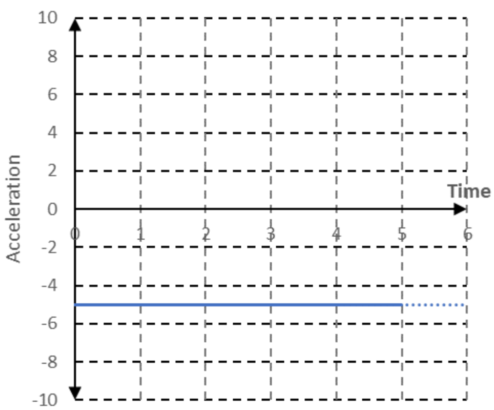
moving in negative direction

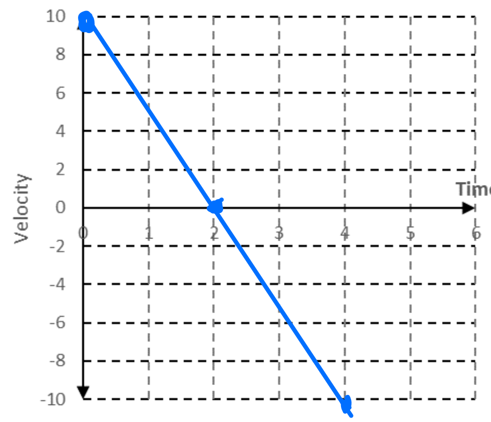




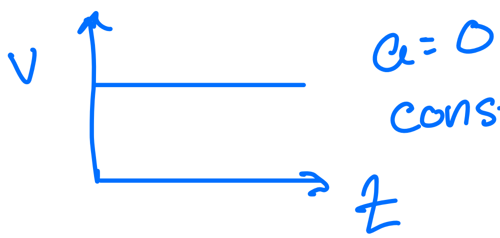
12. The following plot shows constant acceleration for 5 seconds. Sketch a graph of velocity vs. time when the initial velocity is $v_i = 10$ m/s.

$v_2 = v_1 + at = 10 - 5t$ ($v_1 = 10$ m/s)

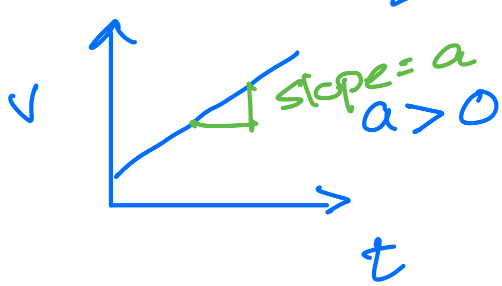


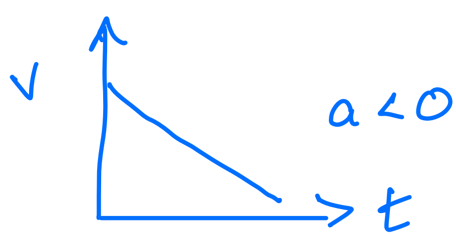


13. Sketch 3 plots of velocity vs time - one for a case of 0 acceleration, positive acceleration, and negative acceleration.

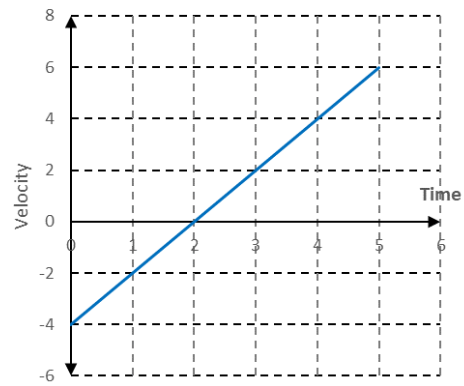


constant speed or at rest





14. An object moves according to the velocity-time graph shown.

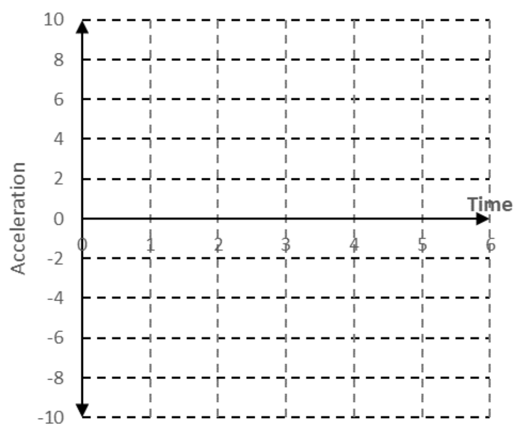


Skip for now

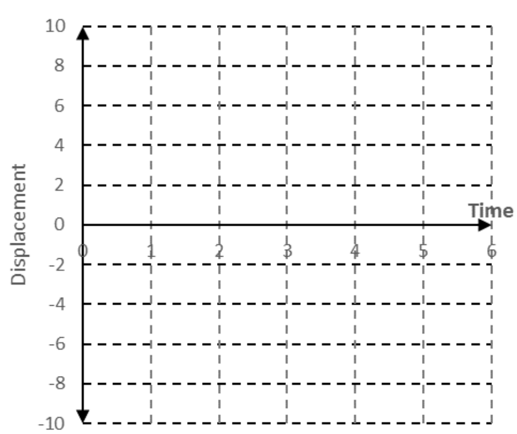
a) What is the initial velocity?

b) What is the acceleration of the motion?

c) Write the velocity kinematics equation for this motion. Draw the acceleration vs. time graph.

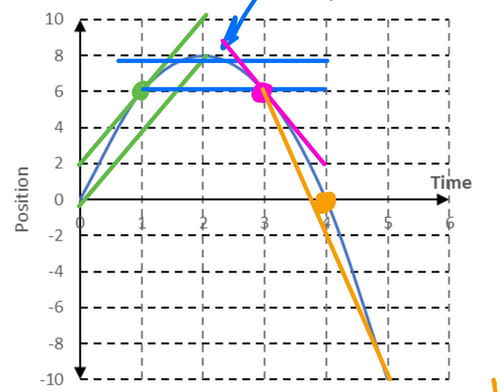


d) Write the position kinematics equation for this motion. Draw the displacement vs time graph. What do you not know about the position graph?



✓ 15. The graph shows the position of an object over time.

slope = 0



a) Estimate the velocity at 1, 2, 3, and 4 seconds.

Slope at a point (tangent line) = slope in a range if the point is the midpoint

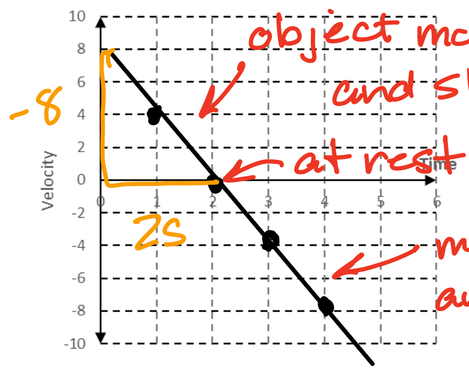
slope at 1s = slope from 0-2 sec

slope at 1s = $\frac{8-0\text{ m}}{2\text{ s}} = 4\text{ m/s}$

slope at 3s = −(slope at 1s)

Slope at 4s = $\frac{-10-6}{2}$ = −8 m/s

b) Roughly, graph the velocity-time data.



object moves forward/upward and slowing down

at rest and turning around

moves backward/falling and speeding up

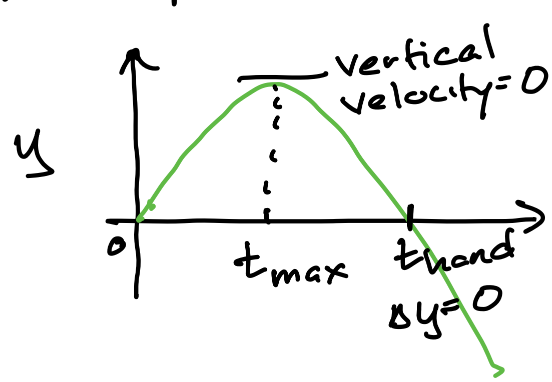
c) When does the object come to a stop? v = 2 at t = 2 seconds

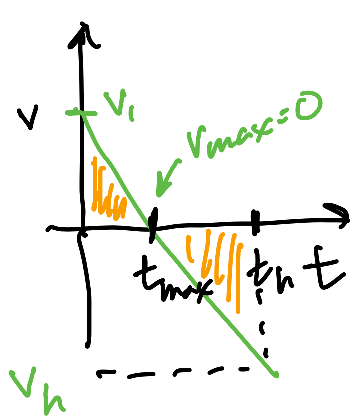
d) What is the acceleration of the object? Include +/ –

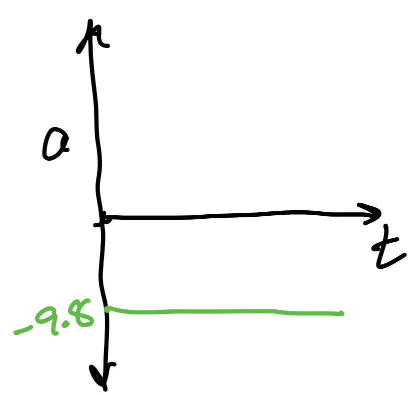
acceleration = $\frac{\Delta v}{t} = \frac{-8\text{ m/s}}{2\text{ s}} = 4\text{ m/s}^2$

Tossing Straight Up

displacements are equal





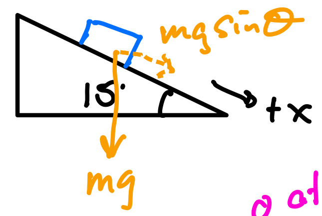


$t_{hand} = 2t_{max}$

$v_h = -v_i$

16. A 1-kg object initially at rest slides down a 15° ramp that has a frictionless surface. What is the acceleration of the object? How far does the object go down the plane in 3 seconds?

after free-fall



$F_x = ma_x$

$\cancel{m}g\sin\theta = \cancel{m}a_x$

a) $\boxed{a = g\sin\theta} = (9.8\,m/s^2)\sin 15°$

$\boxed{a = 2.54\,m/s^2}$

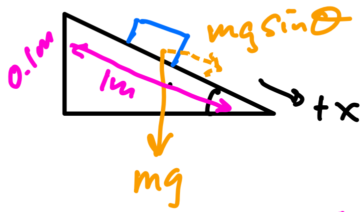
b) $\Delta x = \cancel{v_1}t + \frac{1}{2}at^2$ (0 at rest)

$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}(2.54\,m/s^2)(3s)^2 = 11.43\,m$

$\boxed{\Delta x = 11.43\,m}$

17. A 1-kg object initially at rest slides down a frictionless inclined plane. The *length of the plane* is 1 m, and the *height above the horizontal* is 10 cm. How long does it take the object to reach the bottom and how fast is it going when it gets there?

hyp = 1.0m
opp = 0.1m



a) time to reach bottom

$\Delta x = \cancel{v_1 t} + \frac{1}{2}at^2$ (1.0m)

$t = \sqrt{\frac{2(\Delta x)}{a}} = 1.4\,sec$

$a = 9.8\left(\frac{0.1}{1.0}\right)$

$a = 0.98\,m/s^2$

$\sin\theta = \frac{opp}{hyp} = \frac{0.1m}{1m} = 0.1$

$\theta = \sin^{-1}(0.1) = 5.7°$

b) $v_2^2 = \cancel{v_1^2} + 2a(\Delta x)$

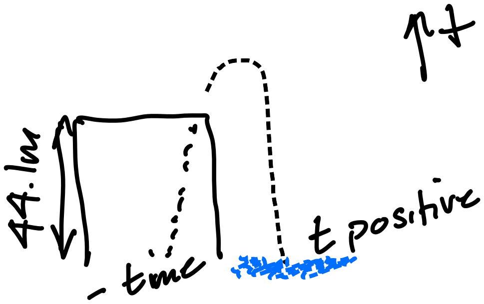
$v_2 = \sqrt{2a(\Delta x)} = \sqrt{2(0.98)(1.0)} = 1.4\,m/s$

~~18.~~ Examples 4.1, 4.2 and 4.3 from the textbook (pgs. 127-130) are excellent and you should use the space below to work those out for yourself.

Free-fall, 1-D only

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8.66 \pm \sqrt{(8.66)^2 - 4(-4.9)(44.1)}}{2(-4.9)}$$

19. Here is an example of turning a problem inside out. In the lecture video, I worked an example of a person throwing an object straight up and over the edge of a bridge. The object goes up and then comes down and splashes in the water below, 44.1 m below the place where it was released. In the example, we knew the time and found the initial velocity to be +8.66 m/s. In this problem, use an initial velocity of +8.66 m/s and determine the time to splash in the water (time-of-flight). (You should get 4 seconds)



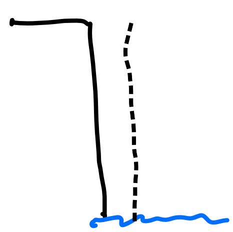
$$\Delta y = v_{y_1} t + \tfrac{1}{2} a t^2$$

$$-44.1 = (8.66\,\text{m/s})t + \tfrac{1}{2}(-9.8)t^2$$

$$-4.9t^2 + 8.66t + 44.1 = 0$$

← quadratic formula

20. I stand at the edge of a cliff, and I am curious about the cliff's height above the water below. To do this, I drop a rock off the edge and time how long it takes to hit the water. What is a general expression for the height of the cliff based on the amount of time it takes to hit the ground? If it takes 4 seconds to do this, how high is the cliff?



$$\Delta y = \cancel{v_{y_1}} t + \tfrac{1}{2} a t^2 \quad v = 0$$

$$\Delta y = \tfrac{1}{2}(-9.8\,\text{m/s}^2)(4\,\text{s})^2$$

$$\Delta y = -78.4\,\text{m} \text{ down}$$

21. A ball is tossed straight up into the air with an initial velocity of +19.6 m/s. Acceleration due to gravity on planet earth = 9.8 m/s^2.

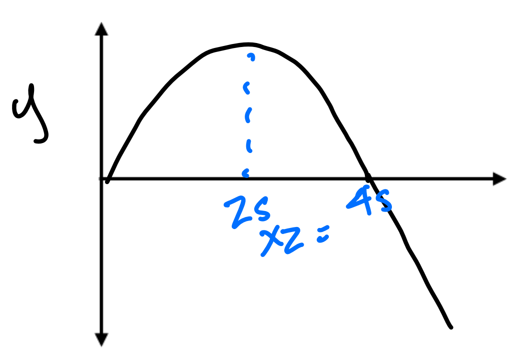
Method 1

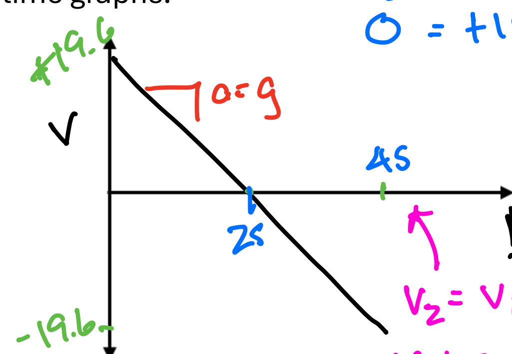
$$v_2 = v_1 + at$$

$$0 = +19.6 - 9.8t$$

$$t = 2\,\text{s}$$

a) Sketch the position-time and velocity-time graphs.





Method 2

$$v_2 = v_1 + at$$

$$-19.6 = +19.6 - 9.8t$$

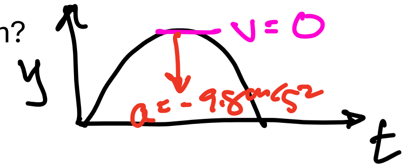
$$t = 4\,\text{s}$$

b) What is the velocity of a ball when it reaches its highest position?

max. height

$$V_{vertical} = 0$$

c) What is the acceleration of a ball when it reaches its maximum?



$a = g = -9.8\,m/s^2$

d) What is an expression for the amount of time it takes to reach its highest height? If the initial velocity is 15 m/s, then how much time does it take?

$$v_2 = v_1 + at$$
$$0 = 15 - 9.8t$$

e) How much time does it take for the ball thrown straight up to come back down?

See above

Method 1: solve t_{max}, $t_h = 2t_{max}$

Method 2: $v_{hand} = -v_1$

Method 3: $\Delta y = 0$

$0 = \Delta y = v_{y1}t + \frac{1}{2}at^2$

$0 = t\,(v_{y1} + \frac{1}{2}at)$

$0 = t$ or $(v_{y1} + \frac{1}{2}at) = 0$

$t = \frac{2v_{y1}}{-a} = \frac{2(15)}{-(-9.8)}$

f) So, if I throw a ball straight up and its total travel time from when it leaves my hand to when it comes back down is 4 seconds, then what was the original speed with which I threw it?

method 2 $v_1 = +19.6\,m/s$

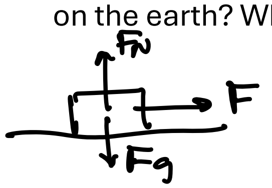
g) What is the final speed when the ball in the previous problem comes back down? (Assume final position is the same height from which I threw it.)

$v_2 = -19.6\,m/s$

go to #16

Forces and Motion

22. The moon has a gravitational field strength g = 1.6 N/kg, so objects feel lighter on the moon in terms of lifting them vertically, but what about pushing *horizontally*? If you wanted to accelerate a 10-kg object horizontally (on a frictionless surface) on the moon with an acceleration of 2 m/s², what force would you need to provide? Would this force be different on the earth? Why?



$$\Sigma F_x = ma_x$$
$$F = (10\,kg)(2\,m/s^2) = 20\,N$$

horizontal motion and vertical motion are independent

x, y motion

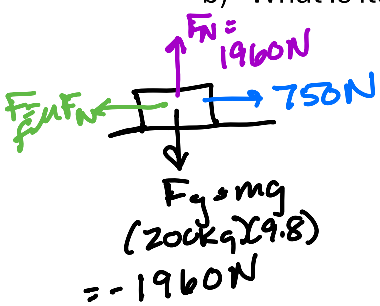
If question includes friction, $F_f = \mu F_N$ then yes!

Find normal force

23. A 750-N force pushes a 200-kg box with a velocity of + 3.0 m/s when it encounters a rough surface (coefficient of friction between the box and the floor, μ = 0.5).

a) What is the net force on the box?

b) What is its acceleration? What does the sign mean here?



$F_f = \mu F_N = (0.5)(1960) = 980N$

now: apply $\Sigma F_x = ma_x$

$750 - 980 = (200)a$

$\boxed{a = -1.15\ m/s^2}$

	x	y
F_g	0	-1960
F_N	0	+1960
F	750	
F_f	-980	
	-230	

c) How long does it take to come to a stop?

time

$v_2 = 0$ $v_1 = +3\ m/s$

$v_f = v_1 + at$

$0 = +3 + (-1.15)t$

$t = 2.6$ sec

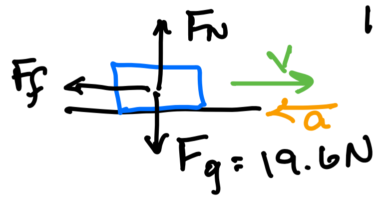
24. I give a 2-kg book a quick push to start it sliding across a table at 1.5 m/s. It slides 1.0 m across the table before it comes to a stop. What is the coefficient of friction between the book and table?

mass = 2kg

$v_1 = 1.5\ m/s$

$\Delta x = 1.0m$

$v_2 = 0$



1) Find acceleration

$v_2^2 = v_1^2 + 2a(\Delta x)$

$a = -1.13\ m/s^2$

2) F_f only force after book leaves hand

$\Sigma F_x = ma_x$

$-F_f = (2kg)(-1.13\ m/s^2) = -2.26N$

but $F_f = \mu F_N$

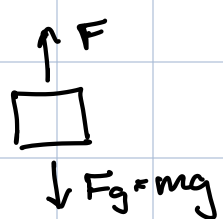
$-2.26N = \mu(+19.6N)$

direction

$\mu = 0.12$ (absolute value)

Last quiz

cup



$+\uparrow$

$\Sigma F_y = ma_y$

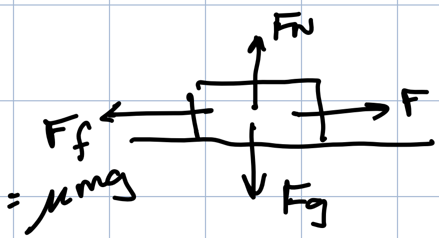
$F - mg = ma_y$

$F = mg + ma_y$

at rest $F = mg$

a up $F > mg$

a down $F < mg$



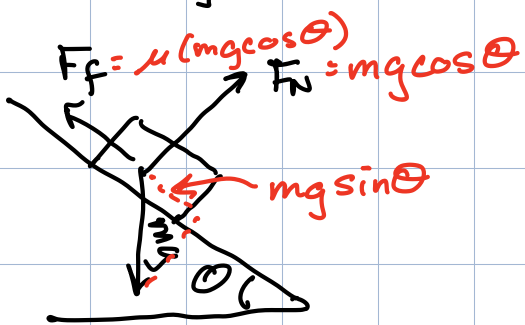
y

$F_N = F_g = mg$

Weight

x

$F - \mu mg = ma$



y

$F_N = mg\cos\theta$

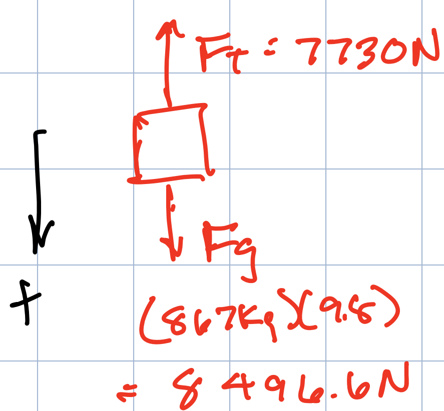
x

$mg\sin\theta - \mu mg\cos\theta = ma$

$a = 0$ (at rest, constant velocity) $\mu = \tan\theta$

Homework #9

$\boxed{\Delta t = 4\,sec \quad \Delta y = 5.0\,m}$



$\Sigma F_y = ma_y$

$(8496.6 - 7730) = (867\,kg)\,a$

$a = 0.884\ m/s^2$ down

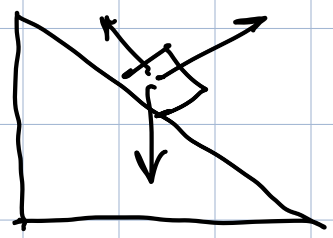
if starts from rest

$\Delta x = \cancel{v_i t} + \frac{1}{2}at^2$

$\Delta x = \frac{1}{2}(0.884)(4s)^2 = 7.1m$

that means +v initially

see if $v_2^2 = v_1^2 + 2a(\Delta y)$ works!



	x	y
F_g	$-mg\sin\theta$	$-mg\cos\theta$
F_N	0	$+mg\cos\theta$
F_f	$\mu mg\cos\theta$	

x direction

$$\Sigma F = mg\sin\theta - \mu mg\cos\theta$$