

At the end of this worksheet, you should be able to

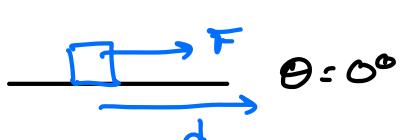
Scalars

- to discuss the relationships between the quantities of work, energy, displacement, velocity.
- differentiate between a conservative force and a non-conservative force.
- apply the work energy theorem to solve interesting problems that would be hard to use Newton's Laws.
- discuss the principle of conservation of energy and explain when it is useful.

1. Work is defined as a transfer of energy. This transfer occurs by one object exerting a force on another object over some displacement. But the relative directions of these two vector quantities (force and displacement) matters. Summarize the work done in 5 different cases that are represented below by drawing the object and the vectors representing force \vec{F} and displacement $\Delta\vec{x}$. In each case I have provided a simple example to illustrate what I mean. You provide another one.

$$W = F \parallel d = (F \cos\theta)d : Fd \cos\theta$$

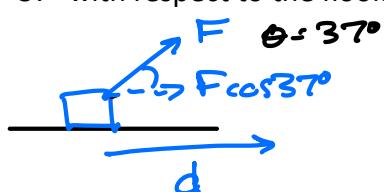
- The force and displacement point in the same direction. (I push a box across a level floor with 100-N force over a displacement of 10 m).



$$W = Fd \cos 0^\circ : (100N)(10m) = 1000 \text{ Joules}$$

- The force and displacement point in different directions, but the angle between them is less than 90° . (I pull a box 10 m across a level floor with a 100-N force, directed at an angle of 37° with respect to the floor.)

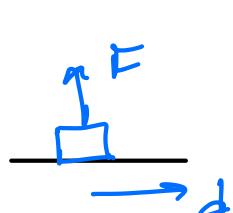
luggage on rollers



$$W = (F \cos 37^\circ)d : (100N) \cos 37^\circ \times 10m$$

$$W \approx 800J$$

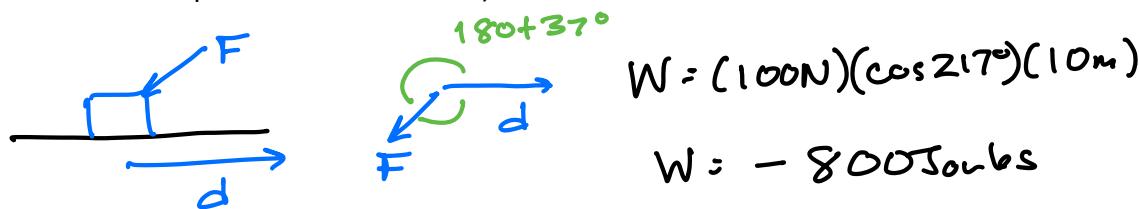
- The force and the displacement are perpendicular to each other. (A 10 kg box is sliding across a rough surface with friction, and the normal force is acting on the box.)



$$W = (100N) \cos 90^\circ \times 10m = 0$$

Centripetal never do 'Work'
change speed of an object
they may change velocity, not speed

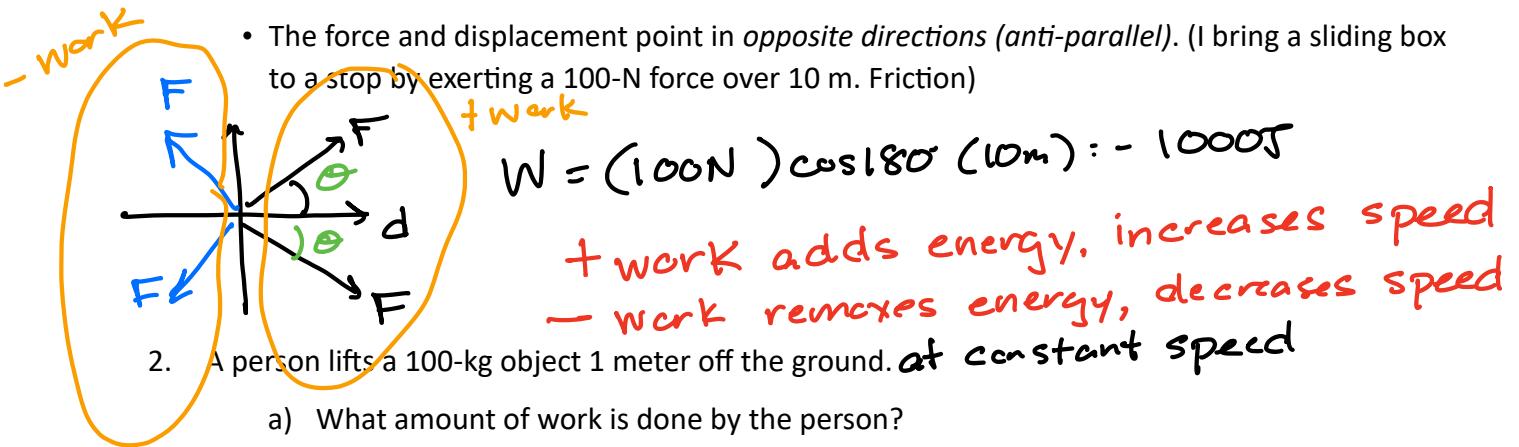
- The force and displacement point in *different directions and the angle between them is greater than 90°.* (I bring a sliding box to a stop by exerting a 100-N force on it at an angle of 37° with respect to the horizontal.)



$$W = (100\text{N})(\cos 217^\circ)(10\text{m})$$

$$W = -800 \text{ Joules}$$

- The force and displacement point in *opposite directions (anti-parallel).* (I bring a sliding box to a stop by exerting a 100-N force over 10 m. Friction)

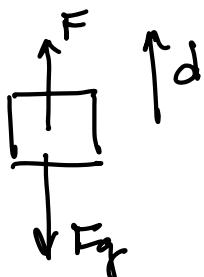


$$W = (100\text{N}) \cos 180^\circ (10\text{m}) = -1000\text{J}$$

+ work adds energy, increases speed
- work removes energy, decreases speed
at constant speed

2. A person lifts a 100-kg object 1 meter off the ground.

- What amount of work is done by the person?
- What amount of work is done by the force of gravity?
- If the person drops the box, what amount of work would the force of gravity do on the box as it fell?



a) $F = F_g = mg \quad (\text{constant speed})$

$$W = (mg)d \cos 0^\circ = (100\text{kg})(9.8)(1\text{m}) = 980\text{J}$$

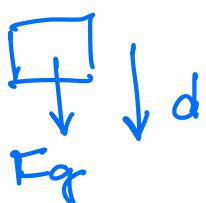
b) $W_g = (mg)d \cos 180^\circ = -980\text{J}$

$$W_{\text{net}} = +980 - 980 = 0 \quad \text{constant speed}$$

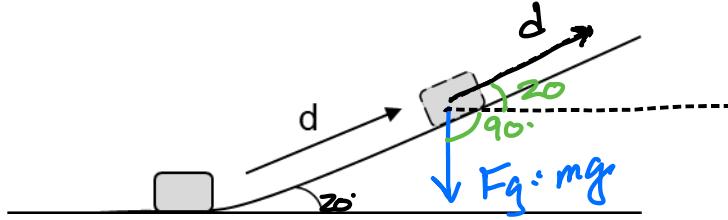
c) $W = Fd \cos 0^\circ = +980\text{J}$

box will speed up as it falls

$$W = Fd \cos \theta = +980 = \frac{1}{2}mv^2$$



1. An object sliding across a *frictionless* surface approaches a *frictionless* ramp and begins to slide up it. The force of gravity does negative work (removes energy), and the object slows down to a maximum height, d , before sliding back down. The ramp has an incline angle of 20° with respect to the horizontal.



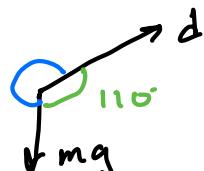
Calculate the work done by the force of gravity and see that it is a negative value in three ways:

- a) What is the angle between the displacement and the force of gravity? Use this angle and the definition of work to calculate the work.

$$\theta = 110^\circ \text{ tail-tail analysis for } \theta$$

$$W = (mg)d \cos 110^\circ = -0.342 \text{ slows down}$$

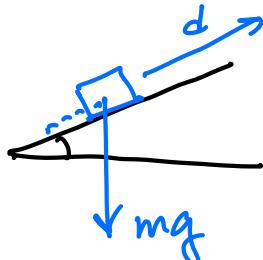
- b) What is another angle between the displacement and the force of gravity? Now use this angle and the definition of work to calculate the work.



$$\theta = 360^\circ - 110^\circ = 250^\circ$$

$$W = (mg)d \cos 250^\circ = -0.342$$

- c) What is the *component* of the force of gravity that is in the direction of the displacement? Now use this angle and the definition of work to calculate the work.



$$F = -mg \sin 20^\circ = -0.342 \text{ same value}$$

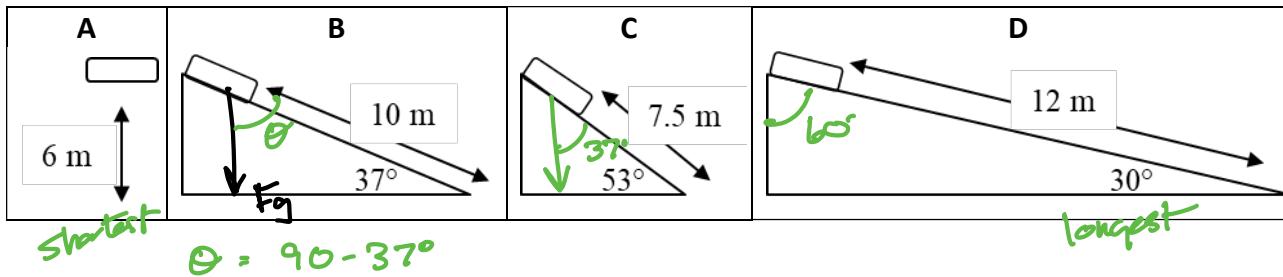


$$\sin \theta = \frac{A}{\text{hyp}}$$



$$\cos \theta = \frac{A}{\text{hyp}}$$

2. Identical boxes are either dropped or slide down different frictionless inclines. The boxes are all identical, and the distance down the incline, and angle of the incline are given.



- a. Rank the work done by gravity in each case from greatest to least.

$$W_A = (mg)(6) \cos 0^\circ = mg(6)$$

$$W_B = (mg)(10m) \cos 53^\circ = mg(6)$$

$$W_C = (mg)(7.5m) \cos 37^\circ = mg(6)$$

$$W_D = (mg)(12m) \cos 60^\circ = mg(6)$$

$$W = F \parallel d$$

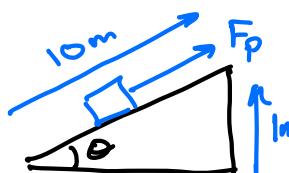
\uparrow
y-displacement
vertical height
Path (A,B,C,D) has no effect
on work

- b. Rank the time to strike the ground in each case from shortest time to longest time.

Energy cannot give information on 'when' time
Forces tell accelerations and time $\Delta x = \frac{1}{2}at^2$ when $v_i=0$

$$t_A < t_C < t_B < t_D$$

3. What amount of work is required to push a 100-kg object up a *frictionless* inclined plane that is 10 m long, and its end is 1 meter high at constant velocity? How does this compare to the work done to lift it? Show that this can be used to derive the formula $\frac{F_{push}}{\text{weight}} = \frac{\text{height}}{d_{plane}}$



$$W_{\text{push}} = mg \underbrace{\cos(90-\theta)}_{\sin\theta} d_{\text{plane}}$$

$$\sin\theta = \frac{h}{d}$$



$$W_{\text{lif}} = mg d \cos 0^\circ = mg(h)$$

$$\sin\theta = \frac{F_{\text{push}} = mgs \sin\theta}{\text{weight} = mg} = \frac{h}{d}$$

for same Work, F and d inversely proportional ⁴

\therefore Double v, Quadruple K, Quadrupled for same F

4. Kinetic energy is the energy of an object that has velocity. $K = \frac{1}{2}mv^2$

- a) Calculate the kinetic energy of a 10 kg object that has a velocity of 10 m/s.

$$K_i = \frac{1}{2}mv^2 = \frac{1}{2}(10\text{kg})(10\text{m/s})^2 = 500\text{J}$$

$$F = \frac{500\text{J}}{100\text{N}}$$

- b) If you do some work to double the velocity of the object, what is the new kinetic energy?

$$K_f = \frac{1}{2}(10\text{kg})(20\text{m/s})^2 = 2000\text{J}$$

$$F = 5\text{m}$$

- c) What is the ratio of the kinetic energy final to the initial kinetic energy?

$$\frac{K_f}{K_i} = \frac{2000}{500} : 4:1$$

$$F_2 = \frac{2000\text{J}}{100\text{N}}$$

$$F_2 = 20\text{m}$$

- d) What is the change in kinetic energy?

$$\Delta K = 2000 - 500 = 1500\text{J}$$

- e) How much work would be required to cause this change in kinetic energy?

$$\text{Same } W = \Delta K = 1500\text{J}$$

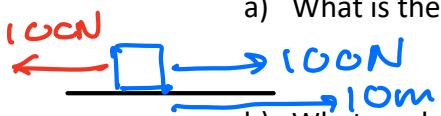
- f) What is the magnitude of a force that acts parallel to displacement and moves the object 10 meters?

What if the force is the same? 100N $W = F \parallel d$ $(500\text{J} = F(10\text{m}))$

$$F = 150\text{N}$$

5. A 5-kg object, initially at rest, is pushed by a 100-N force over a rough surface with friction for 10 meters. The object moves with constant velocity of 1 m/s.

- a) What is the work done by the force?



$$W_p = F \parallel d = (100\text{N})(10\text{m}) = 1000\text{J}$$

- b) What work has the force of friction done?

constant velocity $F_{net} = 0$ $F_f = -100\text{N}$ $W = (-100\text{N})(0\text{ m}) = -1000\text{J}$

- c) What is the net work done?

$$W_{net} = 1000 - 1000 = 0$$

- d) What is the kinetic energy initially?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5\text{kg})(1\text{m/s})^2 = 2.5 \text{ Joules}$$

- e) Does the kinetic energy change?

$$\Delta K = 0$$

- f) What power am I providing?

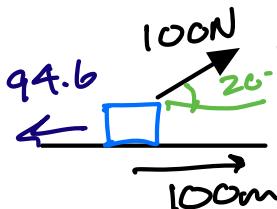
$$P = \frac{W}{t} = \frac{+1000\text{J}}{10\text{s}} = 100 \text{ Watts}$$

$$d = vt$$

$$10\text{m} = (1.0\text{m/s})t$$

$$t = 10\text{s}$$

6. A 10-kg object, starting from rest, is pulled by a rope at an angle of 20° to a horizontal along a rough surface for 100 meters. The tension in the rope is 100 N, and the coefficient of kinetic friction between the floor and the object is $\mu = 0.1$.



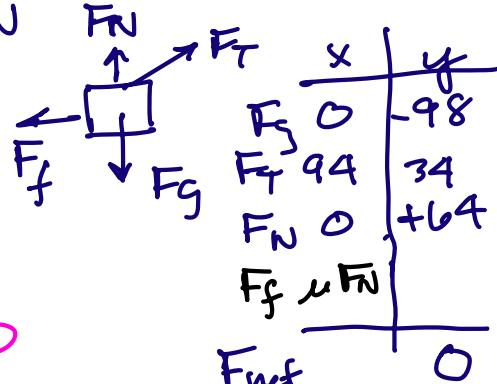
a) What work is done by the rope?

$$W_T = Fd \cos\theta = (100N)(100m) \cos 20^\circ$$

$$W_T = 9397 \text{ Joules}$$

b) What work is done by the force of friction?

$$F_f = \mu F_N = (0.1)(64 \text{ N}) = 6.4 \text{ N}$$



c) What work is done by the normal force?

$$W_N = 64(100m) \cos 90^\circ = 0$$

d) What work is done by the force of gravity?

$$W_g = 98(100m) \cos(-90^\circ) = 0$$

e) What is the net work done on the object?

$$W_{\text{net}} = 9397 - 640 + 0 + 0 = 8757 \text{ J}$$

f) What is the change in the object's kinetic energy?

$$\Delta K = 8757 \text{ J same}$$

g) What is the object's final velocity?

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \rightarrow \text{Square velocity before subtracting}$$

h) What is the net force on the object?

$$F_{\text{net}} = 94 - 6.4 = 87.6 \text{ N}$$

$$V = 41.8 \text{ m/s}$$

$$F = \frac{8757 \text{ J}}{100m} = 87.6 \text{ N}$$

i) Using the net force, what is the net work done on the object?

$$W = F_{\text{net}} d = (87.6 \text{ N})(100 \text{ m})$$

$$W_{\text{net}} = 8760 \text{ J}$$

$$\begin{aligned} F_N &= mg \\ F_f &= \mu F_N = (0.1)(64) \\ F_g &= mg = (10 \text{ kg})(9.8) \\ F_g &= 98 \text{ N} \end{aligned}$$

$$\textcircled{2} \quad \text{Solve } F_f + F_f = \mu F_N$$

$$\begin{array}{|c|c|} \hline & x & y \\ \hline F_T & 94 & 34 \\ F_f & ? & 0 \\ F_N & 0 & ? \\ F_g & 0 & mg = 98 \text{ N (down)} \\ \hline F_{\text{net}} & 0 & 0 \\ \hline \end{array}$$

$$\textcircled{2} \quad F_f = \mu F_N = (0.1)(64)$$

$$\textcircled{1} \quad \text{Solve } y$$

$$\begin{aligned} \sum F &= ma^y \\ F_N + 34 - 98 &= 0 \\ F_N &= 98 - 34 = 64 \text{ N} \end{aligned}$$

7. What forces are conservative forces and what are not conservative forces?

can store energy
energy fully recovered, not dependent on path (inclines)

examples

gravity - $mg\Delta y$
springs - $\frac{1}{2}k(\Delta x)^2$
electric.

Nonconservative

friction \rightarrow heat
applied

8. A 10-kg object is 10 m above the ground.

- a) What is its potential energy?

No friction $U_g = mg(\Delta y) = mgh$ \leftarrow vertical height

$$E_1 = E_2 \quad U_g = (10\text{kg})(9.8)(10\text{m}) = 980\text{ J} = E_1$$

mgh

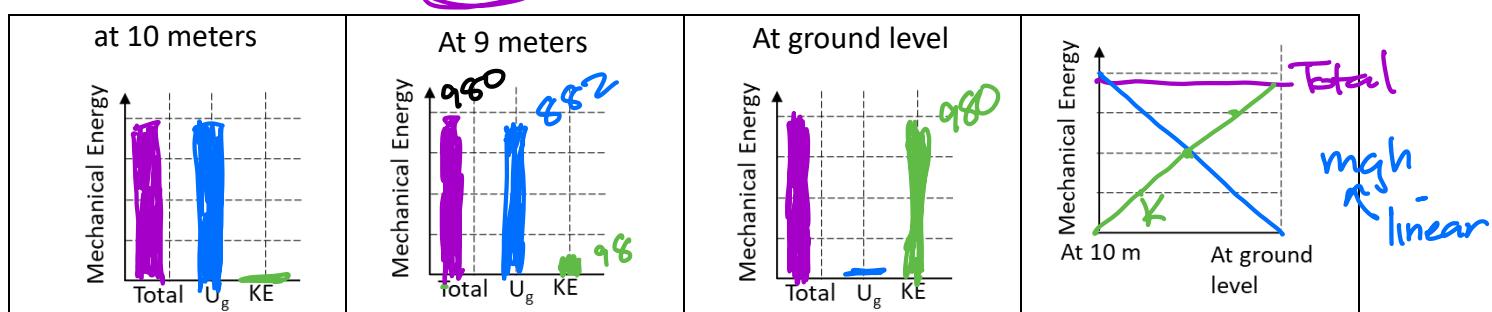
- b) If it begins to fall, what is its potential energy after it falls 1 m? What is its kinetic energy at this point?

$$U_g = (10\text{kg})(9.8)(9\text{m}) = 882\text{ J}$$

$$980 = E_2 = U_g + K = 882 + K = 982$$

- c) What is its kinetic energy when it hits the ground? Calculate its velocity at this point.

- d) Demonstrate this in the bar graphs below.



9. An object falls from a height of 100 m then how fast is it going when it hits the ground?

Solve this using kinematics and then again using conservation of energy?

Forces/Kinematics

No time $v_2^2 = v_1^2 + 2ad$ $\downarrow a=g$

$0^2 = 0^2 + 2gd$

$v_2 = \sqrt{2gd}$

exact same result!

Energy

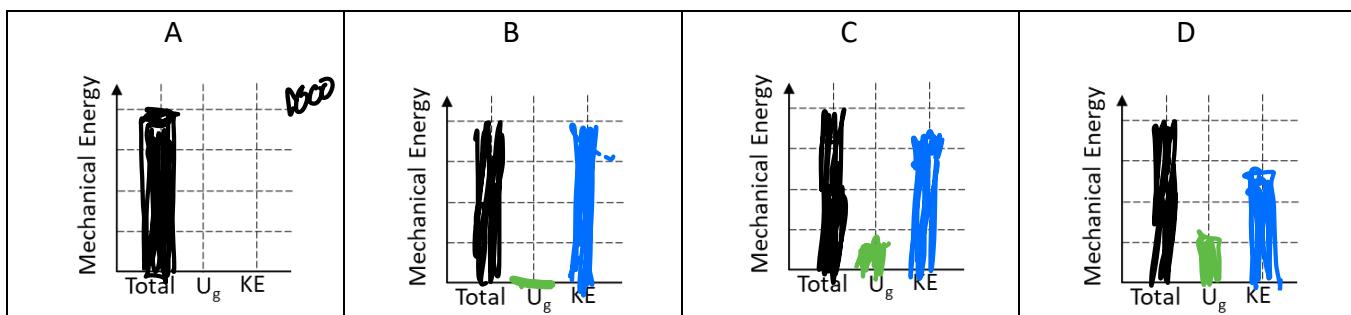
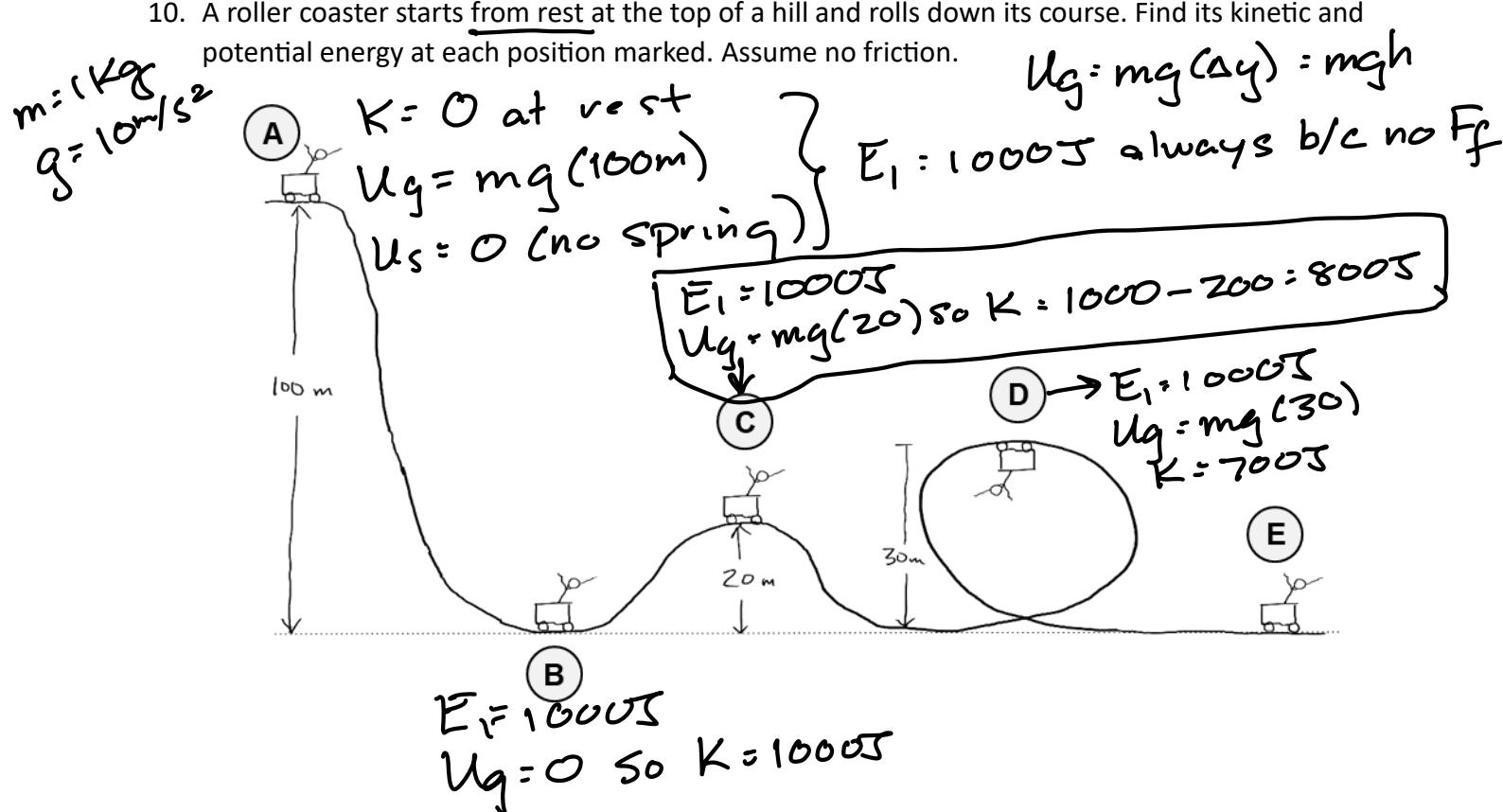
$v_i = 0$ so $K = 0$ $\Rightarrow E_1 = mgh$

$U_g = mgh$

$\Delta y = h$ No losses

$U_g = 0 \Rightarrow E_2 = \frac{1}{2}mv^2$

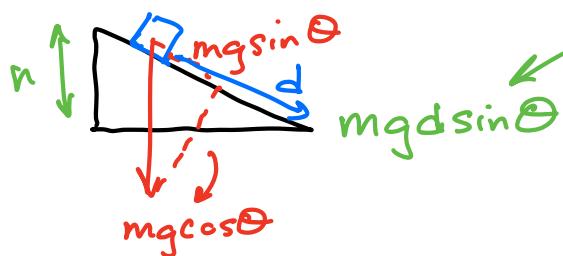
10. A roller coaster starts from rest at the top of a hill and rolls down its course. Find its kinetic and potential energy at each position marked. Assume no friction.



11. What is the minimum height h that a roller coaster needs to start at rest to successfully complete a loop-the-loop of radius r and not lose contact with the track?

12. A 10-kg box slides down a rough inclined plane ($\mu = 0.1$, $\theta = 30^\circ$). The height of the plane is 1 m above the horizontal. Release from rest

a) What is the speed of the box at the bottom of the plane?



$$E_1 + W_{nc} = E_2 \rightarrow y_2 mv^2$$

$$F_f = \mu F_N = \mu (mg \cos \theta) \leftarrow \text{Caution: this is a force}$$

$$W_f = -\mu (mg \cos \theta) d$$

$$\cancel{mgdsin\theta - (\mu mgcos\theta)d} = y_2 mv^2$$

$$\sqrt{2gd(\sin\theta - \mu\cos\theta)} = v$$

b) How does this compare to if there were no friction? How much work has been done by the force of friction?

$$U_g = mgh, \sin\theta = \frac{h}{d} \text{ so } h = d \sin\theta, U_g = mgdsin\theta$$

$$K_1 = 0$$

$$U_2 = 0, K_2 = y_2 mv^2$$

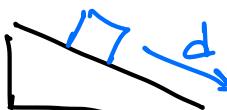
$$E_1 = E_2$$

$$mgdsin\theta = y_2 mv^2$$

$$v = \sqrt{2gdsin\theta}$$

13. Here is an example of a problem that would be much more difficult to do with Newton's Laws. Take the same inclined plane as the problem above but make most of the plane frictionless and only a 10 cm portion in the middle of the plane have friction $\mu = 0.1$. Now what is the speed at the bottom of the plane? Think about how you would have solved this using Newton's Laws and kinematics.

Now



$$mgdsin\theta - \mu mgcos\theta(0.1m) = \frac{1}{2}mv^2$$

$$10\text{cm} = 0.1\text{m}$$

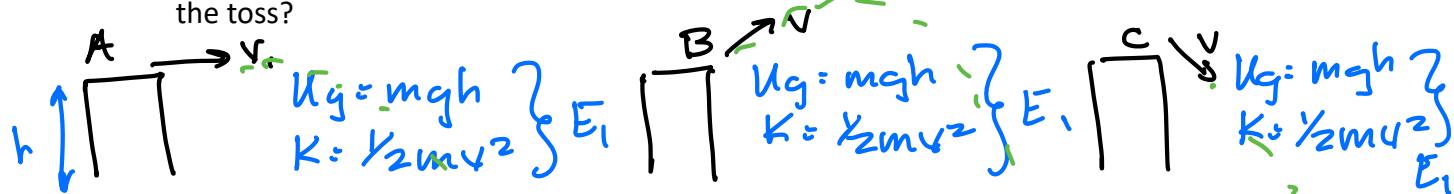
only portion with friction

drop or slide w/o friction. same Path doesn't matter!

$$E_i = K + U_g + U_s$$

14. Three identical balls are tossed from the top of a building at the same speed. Ball A is tossed horizontally, Ball B is tossed upward at a 45° angle, and Ball C is tossed downward.

- a) What is each ball's total mechanical energy at the top of the building immediately after the toss?



- b) What is the speed of each ball as it strikes the ground?

Top: $E_i = mgh + \frac{1}{2}mv_i^2$ all same

Bottom: $E_f = \frac{1}{2}mv_f^2$ all same

$$v_f = \sqrt{2gh + v_i^2}$$

- c) Will energy tell us when the ball lands?

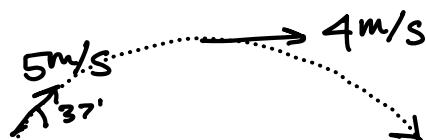
No - time is not in energy eqns
(in power)

Path didn't matter

- d) Will energy tell us how far each ball lands from the base of the building?

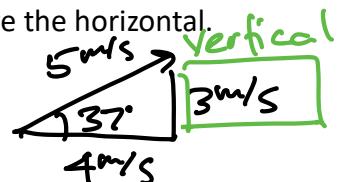
No - Kinematics tell that

15. A soccer ball is kicked across a field with a velocity of 5 m/s at a 37° angle above the horizontal. How can you determine the maximum height of the ball using energy?



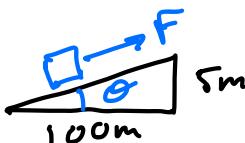
$\downarrow mg$ so only use
K in vertical direction

$$K = \frac{1}{2}m(3\text{ m/s})^2 \quad \text{vertical}$$



16. A 100 kg cyclist (and bike) has at constant speed of 10 m/s up an incline of 5% (vertical height/horizontal distance as a percent). What power output does the rider need to provide to do this?

$$P = \frac{W}{t} = \frac{F_{\text{up}} \cos \theta}{t} = (F \cos \theta) \left(\frac{d}{t}\right)$$



$$\tan \theta = \frac{5}{100}$$

$$\theta = 3^\circ$$

$$F_{\text{up}} = m g \sin \theta$$

$$F = (100 \text{ kg})(9.8) \sin 3^\circ$$

$$F = 51 \text{ N}$$

$$P = Fv = (51 \text{ N})(10 \text{ m/s}) = 510 \text{ Watts}$$

17. The maximum speed of a child on a swing is 5 m/s. At this point the child is 1 m above the ground. What is the maximum height of the child above the ground? Do this two ways, once with $U_g = 0$ at the ground, and once again with $U_g = 0$ at the bottom of the swing's path.

Method 1

$$E_1 = \frac{1}{2}mv^2 + mgh_1$$

$$E_2 = mgh_2$$

$$U_g = 0$$

$$\frac{1}{2}mv^2 + mg(1m) = mgh_2$$

$$h_2 = \frac{v^2}{2g} + 1m$$

Method 2

$$E_1 = \frac{1}{2}mv^2$$

$$E_2 = mgh$$

$$\frac{1}{2}mv^2 = mgh$$

$$h = \frac{v^2}{2g} = \frac{(5m/s)^2}{2(9.8)} = 1.25m$$

$$h = 1.3m \text{ but } 2.3 \text{ above ground}$$

18. A pendulum swings from some maximum height where its velocity is zero to a minimum where its velocity is a maximum and then back up to a maximum height. If the maximum height is 0.1 meters above the minimum height, then what is the speed of the pendulum at the bottom of its swing?

$$E_1 = mgh$$

$$U_g = 0 \rightarrow \frac{1}{2}mv^2$$

$$E_1 = E_2$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

down incline
down rollercoaster
pendulum

19. For a pendulum, it is hard to measure its maximum height, but it is easy to measure its length and to measure its angle from the vertical. If the maximum angle from the vertical of a 1 m long pendulum is 20° then how high is this above the horizontal? (Hint: draw a line from the end of the pendulum when it is at its maximum height perpendicularly to the line when it is at its minimum height.)

$$h = L - L\cos\theta = L(1 - \cos\theta)$$

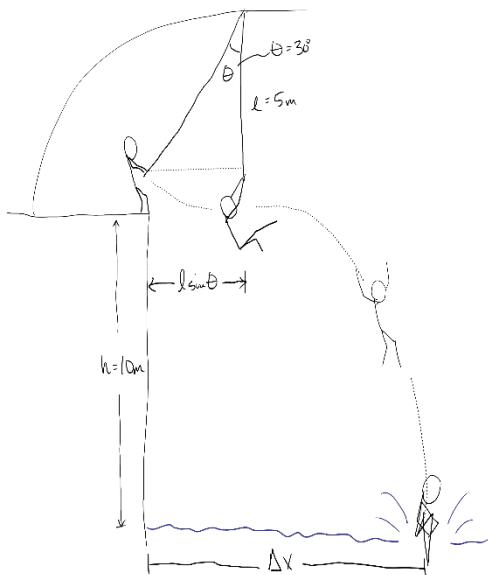
$$mgh = \frac{1}{2}mv^2$$

$$mgL(1 - \cos\theta) = \frac{1}{2}mv^2$$

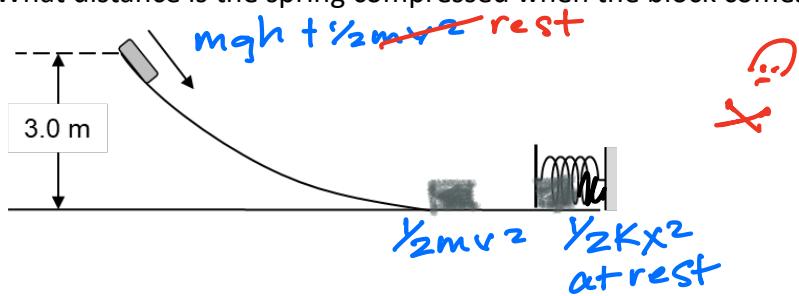
$$v = \sqrt{2gL(1 - \cos\theta)}$$

$$v = \sqrt{2(9.8)(1m)(1 - \cos20^\circ)}$$

- ~~X~~ 20. If you are doing a rope swing and then at the lowest point in the swing you let go and drop into a lake below, how far from the edge of the cliff do you land in the water. See the diagram below for the relevant parameters.



21. A 2-kg mass rests at the top of a frictionless curved ramp 3 meters above its base. The mass slides down a frictionless curved ramp and strikes a spring with a spring constant $k = 20 \text{ N/m}$. What distance is the spring compressed when the block comes to rest?



$$E_1 = E_2 = E_3 \quad (\text{no friction})$$

$$E_1 = E_3$$

$$mgh = Y_2 K x^2 \quad \text{X} \quad \text{mass stays in equation}$$

$$x = \sqrt{\frac{2mgh}{K}}$$

$$x = \sqrt{\frac{2(2\text{kg})(9.8)(3\text{m})}{20\text{N/m}}} = 2.4 \text{ m/s}^{12}$$

Conservative Forces

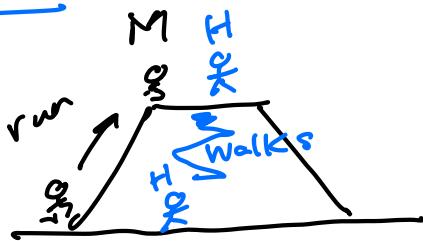
- work/energy put into object can
 - be fully recovered
 - does not depend on path

$$E_{\text{initial}} + W_{nc} = W_{\text{final}}$$

\nwarrow nonconservative (friction)

$$K_1 + U_{g1} + U_{s1} + W_{nc} = K_2 + U_{g2} + U_{s2}$$

Power



M and H have same mass, M

$$\Delta U_g = Mgh \text{ same}$$

M and H power

$$P_M > P_H$$

$$\frac{Mgh}{t}$$

New Spring Problems



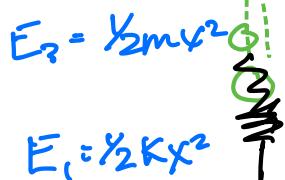
$$E_1 = \frac{1}{2} Kx^2$$

$$E_2 = mgh$$

$$\frac{1}{2} Kx^2 = mgh$$

$$\frac{Kx^2}{2mg} = h$$

$$E_2 = mgh$$

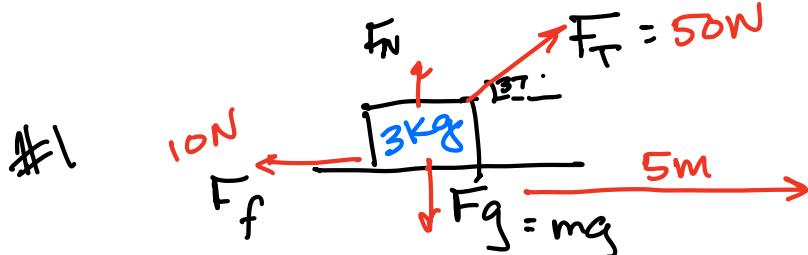
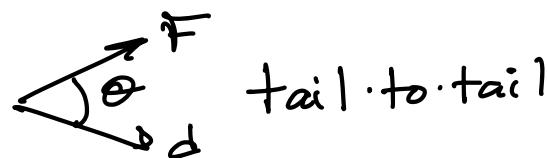


$$E_1 = \frac{1}{2} Kx^2$$

$$\frac{1}{2} Kx^2 = mgh$$

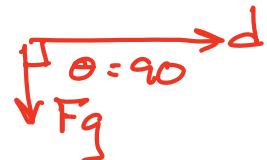
$$\frac{Kx^2}{2mg} = h$$

$$\text{Work} = Fd \cos \theta$$



a) Work done by each force

$$\begin{cases} W_g = mgd \cos 90^\circ \\ W_N = F_N d \cos 90^\circ \end{cases}$$



$$W_T = (50\text{N})(5\text{m}) \cos 37^\circ = 200 \text{ Joules}$$

$$W_f = (10\text{N})(5\text{m}) \cos 180^\circ = -50 \text{ Joules}$$

b) what is the object's speed at 5m if starts from rest?

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\frac{200 - 50}{3\text{Kg}} = \frac{1}{2}(3\text{Kg})v^2$$

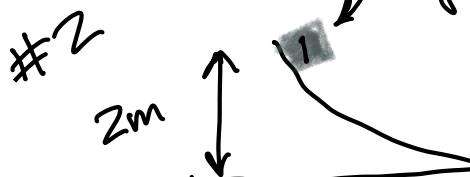
$$10 \text{ m/s} = v$$

c) What is the final speed if $v_0 = 6 \text{ m/s}$?

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}m(6)^2 \quad \text{square before subtracting}$$

$$\underbrace{150 + \frac{1}{2}(3)(6)^2}_{204} = \frac{1}{2}(3\text{Kg})v^2$$

$$11.7 \text{ m/s} = v$$



What is the block's final speed?

a) No friction

$$E_1 = mgh$$

$$E_2 = \gamma_2 m v^2$$

$$E_1 = E_2$$

$$\cancel{mgh} = \cancel{\gamma_2 m v^2}$$

$$\sqrt{1} = \sqrt{\gamma_2 g h} = \sqrt{2(9.8)(2m)} = 2\sqrt{9.8}$$

Ask at each location

1. Above lowest level? U_g

2. Moving? $K = \gamma_2 m v^2$

3. Spring? $U_s = \gamma_2 k x^2$

Set Equal,

b) final velocity if $m = 6\text{Kg}$?

$$V = 1.3 \text{ m/s}$$

but E_1 and E_2 will be double (a)

c) Work done by friction if $v_f = 3.0 \text{ m/s}$?



$$E_1 = mgh$$

$$E_2 = \gamma_2 m v^2 = \gamma_2 m (3.0 \text{ m/s})^2$$

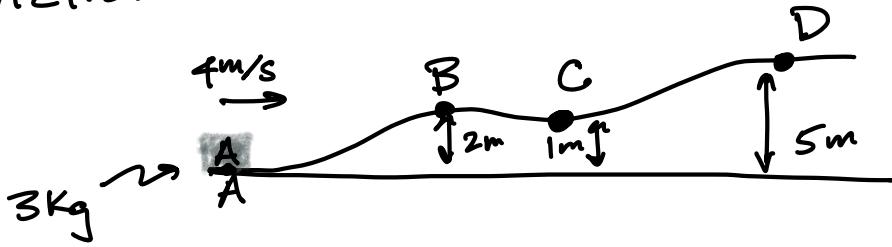
$$E_1 + W = E_2$$

$$W = E_2 - E_1$$

$$W = \frac{1}{2} (3\text{kg})(3.0 \text{ m/s})^2 - (3\text{kg})(9.8)(2\text{m})$$

$$W = -45 \text{ Joules}$$

#3 Frictionless



a) Speed at C

$$E_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(3\text{kg})(4\text{m/s})^2 = 24\text{J}$$

what additional energy does block need to make to Point D?

$$E_D = mgh = (3\text{kg})(9.8)(5)$$

$$E_D = 147 \text{ Joules}$$

$$E_A = 24 \text{ Joules}$$

$$W = \Delta E = 123 \text{ Joules}$$

$$E_C = \frac{1}{2}mv_C^2 + mgh$$

$$E_A = E_C$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_C^2 + mgh$$

$$24\text{J} = \frac{1}{2}(3\text{kg})v^2 + \underbrace{(3\text{kg})(9.8)(1\text{m})}_{29.4\text{J}}$$

won't get to C

a) Speed at A $v_A = 10\text{m/s}$

$$E_A = E_C$$

$$\frac{1}{2}(3\text{kg})(10\text{m/s})^2 = \frac{1}{2}mv_C^2 + (3\text{kg})(9.8)(1\text{m})$$

$$120.6 = \frac{1}{2}(3\text{kg})v_C^2$$

$$9\text{m/s} = v_C$$

b) will block make it to Point D when $v_A = 10\text{m/s}$

$$E_A = \frac{1}{2}mv_A^2$$

$$E_D = mgh$$

$$\frac{1}{2}(3\text{kg})(10\text{m/s})^2 = (3\text{kg})(9.8)h_{\max}^?$$

$$h_{\max} = 5.10\text{m} \quad \text{Yes!}$$