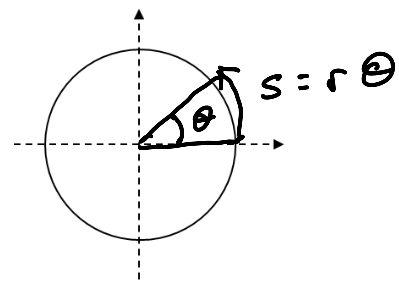
At the end of this worksheet you should be able to

- apply the relationships between angle and motion at the edge of a circle to describe the motion of an object in circular motion.
- apply Newton's 2nd law in the radial direction to solve interesting problems involving motion of objects in a circular path.
- apply the principles of radial net force and circular motion to planetary orbits and satellites as well horizontal and vertical paths near earth's surface.

<u>Rotational and Translational Motion</u>

1. How many degrees are in 1 radian?



$C = 2\pi r \qquad \frac{C}{r} = 2\pi$

$\boxed{s = \theta r}$ farther from axis

s + r are directly proportional
s + θ are directly proportional

Then $v = r\omega$
directly proportional

2. Salt and pepper shakers rest on a lazy-Susan with a radius of 10 cm. The pepper shaker rests 8 cm from the center and the salt shaker rests 6 cm from the center. A child spins the lazy-Susan <u>one-quarter a revolution</u>.

   1 revolution $= 360° = 2\pi$

   SP

   a) How many radians has each shaker moved through?
   b) What distance has each shaker travelled?
   c) If the motion took 2 seconds, what is the angular and tangential speeds of the shakers?

a) $\left(\frac{1}{4}\text{revolution}\right)\left(\frac{2\pi}{1\text{ rev.}}\right) = \frac{\pi}{2}$

radians are unitless
ratio $\frac{C}{r} = 2\pi$

b) Salt $s = r\theta = 6\left(\frac{\pi}{2}\right) = 3\pi = 9.4\text{ cm}$

$\frac{1}{4}(2\pi r) = 9.4\text{cm}$

Pepper $s = 8\left(\frac{\pi}{2}\right) = 4\pi = 12.5\text{cm}$

c) $\omega = \frac{2\pi}{\text{time}}$ Same angular speed $\frac{\pi/2}{2\text{ s}} = \pi/4 \text{ rad/s}$ 0.78 rad/s

$V_s = \frac{s}{t} = \frac{9.4\text{cm}}{2\text{s}} = 4.7\text{ cm/s} = r\omega = (6\text{ cm})(0.78\frac{\text{rad}}{\text{s}})$

$V_p = \frac{12.5\text{cm}}{t} = 6.25\text{ cm/s} = (8\text{ cm})(0.78\frac{\text{rad}}{\text{s}})$

3. A soccer ball with a radius of 10 cm spins through an angle of 20°.

   a) How many radians has it moved through? What distance has a point on the equator of the ball travelled? 1 rev = 2π = 360° always, conversion

$$20^\circ\left(\frac{2\pi}{360^\circ}\right) = 0.35\,\text{rad}$$

$$s = r\theta = 10\,\text{cm}\,(0.35\,\text{rad}) = 3.5\,\text{cm/s}$$

   b) What angle and distance does the point on the edge of the ball travel if it spins through 750°?

$$750^\circ = 13.1\,\text{rad}$$

same method

$$s = r\theta = 131\,\text{cm}$$

4. When you roll something along the ground, it is spinning/rotating, of course, but it is also moving linearly/translationally (its center of mass is moving). ***It turns out that the distance the edge of a soccer ball moves as it rolls is equal to the linear distance the ball moves, as long as it does not slip.*** $d = 2\pi r$ (circumference)

   A soccer ball of radius 10 cm rolls at constant angular speed through an angle of 500 rad

   a) How many revolutions is 500 radians?
   b) How far has the ball rolled?
   c) If it takes 10 seconds to do this, what was its angular speed and what was its linear speed?

θ a) $500\,\text{rad}\left(\frac{1\,\text{rev}}{2\pi}\right) = 79.6$ revolutions

s b) Point on surface has moved $r\theta = (10\,\text{cm})(500\,\text{rad})$

$s = 5000\,\text{cm}$

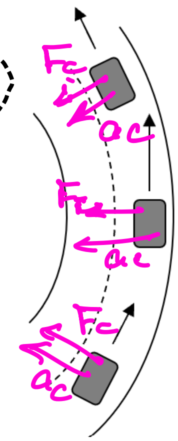
$$s = (79.6\,\text{rev})\left(\frac{2\pi r}{1\,\text{rev}}\right) = 50\,\text{m} = 5000\,\text{cm}$$

ω c) $\omega = \frac{\Delta\theta}{t} = \frac{500\,\text{rad}}{10\,\text{s}} = 50\,\text{rad/s}$

$$v = r\omega = 500\,\text{cm/s}$$

Circular Motion

5. When a car turns at constant speed, it travels along an approximately circular path. In which direction does the net force act ~~and~~ what provides this net force?



$a = \frac{\Delta v}{t}$ ← vector so a: change in speed or change in direction

$\vec{F}_c = m\vec{a}_c$ point in same direction

c = centripetal/radial

changes direction, not speed.

Forces that cause centripetal

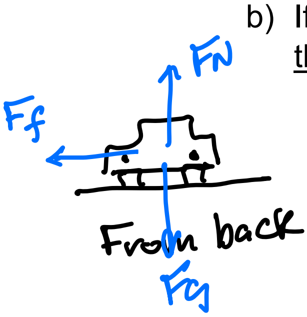
friction Normal

tension gravity

6. A 1000-kg car turns through a curve with a radius = 50 meters. The coefficient of friction between the tires and the road is $\mu = 0.5$.

   a) what is the maximum static force of friction that the road could provide to the car?

   b) If the car is going around a bend of radius 50 m, how how fast could it go around the bend without sliding?



$F_N = F_g$

a)

$F_f = \mu F_N = (0.5)(1000\,kg)(9.8)$

$F_f = 4900\,N$

b) x-direction

$\Sigma F = m a_c$ ← $\frac{v^2}{r}$

$F_f = m\frac{v^2}{r}$

$4900 = (1000\,kg)\frac{v^2}{50\,m}$

$v = \sqrt{\frac{4900}{1000} \cdot 50}$

$v = 15.7\ m/s$

Algebraically

$F_f = m\frac{v^2}{r}$

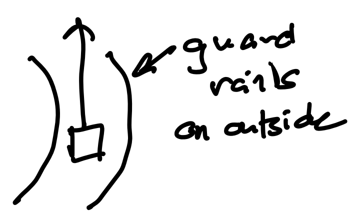
$\mu \cancel{m} g = \cancel{m}\frac{v^2}{r}$

max. speed → $v = \sqrt{r\mu g}$ Car going around a curve

   c) Now, the same 1000-kg car is attempting to go around a bend of radius 20 m, at 20 m/s. Can it do this safely without sliding? (μ = 0.5 still)

$v_{max} = \sqrt{(20\,m)(0.5)(9.8)}$

$v_{max} = 9.9\ m/s$



will collide with outer curve

7. A box of mass M rides on the flatbed of a truck which is traveling at constant speed *v* around a curve of radius *R*. The box is held in place by friction.

   a) Identify all the force vectors acting on the box.

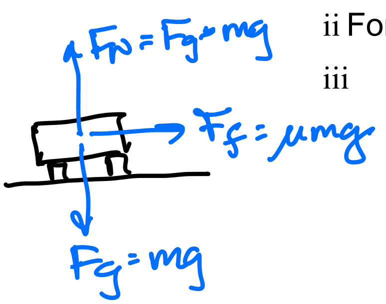
   b) Derive and expression for the coefficient of static friction μ between the box and the truck bed.

   c) If the truck makes a second trip with the box fully loaded with an additional mass of M, will the following increase, decrease, or remain the same?

      i Coefficient of friction, μ Same μ depends on 2 surfaces only

      ii Force of friction, F_f $F_f = \mu mg$ Yes Increase

      iii Maximum speed before the box slips same $v = \sqrt{r\mu g}$



x-direction $\Sigma F = ma_c$

$$F_f = m\frac{v^2}{r}$$

$$\mu \cancel{m} g = \cancel{m}\frac{v^2}{r}$$

$$\mu = \frac{v^2}{rg}$$

$$v = \sqrt{r\mu g}$$

Satellite Motion and Circular Motion

8. The earth orbits the sun, and while its path around the sun is not exactly circular, its close enough to treat that way here.

   a) What is the <u>angular velocity</u> of the earth around the sun? To do this, think about how long it takes to go one full revolution around the sun. How many radians is a revolution? So, how many radians per second does the earth travel around the sun?

T = period = time to complete 1 cycle

$$\omega = \frac{2\pi}{T}$$

$$T = (365.25\,\cancel{d})(24\,\tfrac{\cancel{h}}{\cancel{d}})(3600\,\tfrac{s}{\cancel{h}}) = 3.2\times10^{7}\,s$$

$$\omega = \frac{2(3.14)}{3.2\times10^{7}\,s} = 2\times10^{-7}\,rad/s$$

   b) What is the radius between the earth and the sun? (look this up in your book or google) Using the answer from the previous problem, what does this mean for the <u>tangential speed</u> of the earth around the sun?

$r = 1.5\times10^{11}\,m$

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$V = r\omega = (1.5\times10^{11}\,m)(2\times10^{-7}\,rad/s)$$

$$V = 30{,}000\,m/s$$

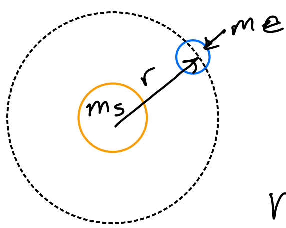
9. Now without looking it up, use this information to determine the <u>mass of the sun</u>? The formula for the force of gravity between to masses can be written as, $F_g = F_g = G\frac{m_1 m_2}{r^2}$ (G = 6.67 × 10-^{11} Nm^2/kg^2). Note: this not the form of the force gravity that we have been using. Why is that? Now look up the mass of the sun and see how close you get..

$F_g = G\frac{m_1 m_2}{r^2}$

$F_e = -F_s$ equal and opposite (Newton's 3rd)

object moving

$\Sigma F_c = m_e a_c$



$G\frac{m_s \cancel{m_e}}{r^2} = \cancel{m_e}\frac{v^2}{r}$ (same r)

$m_s = \frac{v^2}{r}\left(\frac{r^2}{G}\right) = \frac{rv^2}{G} = \frac{(1.5\times10^{11}\,m)(3\times10^4\,m/s)^2}{(6.67\times10^{-11}\,Nm^2/kg^2)} = \boxed{2\times10^{30}\,Kg}$

10. How can we use free-fall acceleration to get a measure of the mass of the earth? Suppose we go to the lab and measure an acceleration of a 1-kg mass to be 9.82 m/s^2. How can we calculate the <u>mass of the earth</u> with this information?

Good method for determining mass of central object/body

$F_g = G\frac{m_e \cancel{m_{obj}}}{r^2} = \cancel{m_{obj}}\, g$

$r_e = 6.4\times10^6 m$

radius of earth

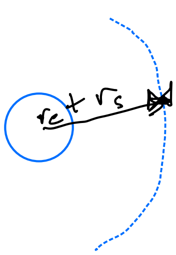
$m_e = \frac{gr^2}{G} = \frac{(9.8\,m/s^2)(6.4\times10^6\,m)^2}{6.67\times10^{-11}\,Nm^2/Kg^2}$

$m_e = 5.98\times10^{24}\,Kg$

6×10^{24}

11. In order to put a satellite into orbit around the earth, it needs to be traveling at a specific distance with a specific velocity, otherwise the force of gravity from the earth may be too large, and it will crash, or too small and it will fly away into space.

Suppose you wanted to put a 1000-kg satellite in orbit around the earth at a distance of 1000 km above the *surface of the earth*. How <u>fast</u> would this satellite need to be going in order to have this orbit?



$r = r_e + 1\times10^6 m$ ($r_e = 6.4\times10^6 m$)

$r = 7.4\times10^6 m$

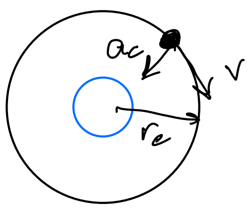
$G\frac{m_e \cancel{m_s}}{r^2} = \cancel{m_s}\frac{v^2}{r}$

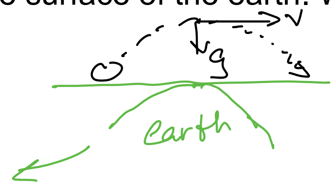
$v^2 = G\frac{m_e r}{r^2} = G\frac{m_e}{r}$

$v = \sqrt{G\frac{m_e}{r}} = \sqrt{\frac{(6.67\times10^{-11})(6\times10^{24}\,Kg)}{7.4\times10^6\,m}}$

$v = 7368\,m/s \sim 7400\,m/s$

12. Suppose you wanted to kick a soccer ball horizontally off a cliff and have it go into orbit near the surface of the earth. What velocity must the soccer ball have?





$$G\frac{m_e \cancel{m_{obj}}}{r^2} = \cancel{m_{obj}}\frac{v^2}{r} = \cancel{m}g$$

$$v = \sqrt{r_e g}$$

$$v = \sqrt{(6.4\times10^6 m)(9.8)}$$

$$v = \sim 7900 m/s$$

Notice:

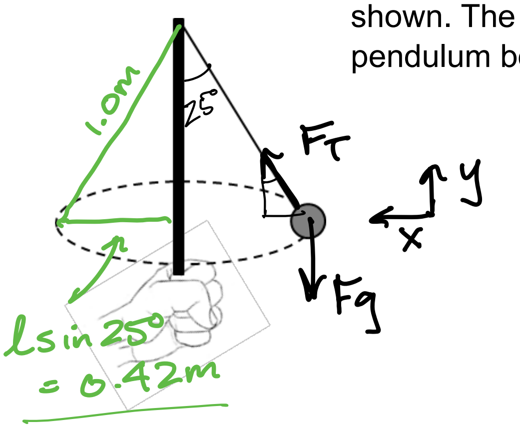
$$G\frac{m_e \cancel{m_{obj}}}{r^2} = \cancel{m_{obj}}g$$

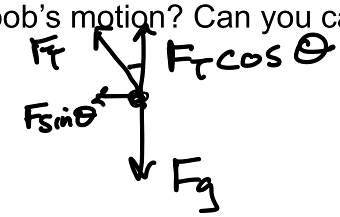
$$\boxed{G\frac{m_e}{r_e^2} = g}$$

Circular Motion

13. A 1-kg pendulum bob attached to a 1.0-meter string swings in a conical pendulum as shown. The string is angled 25° to the vertical. What is the radius of the pendulum bob's motion? Can you calculate the tension in the string?

$$F_T = \frac{(kg)(9.8)}{\cos 25°}$$





$F_T\cos\theta = F_g = mg = (1kg)(9.8 m/s^2)$

$F_T\sin\theta$ causes centripetal motion

$\boxed{F_T = 10.8N}$

$$F_T\sin\theta = m\frac{v^2}{r} \leftarrow 0.42m$$

$$(10.8N)\sin 25° = (1kg)\frac{v^2}{(0.42m)}$$

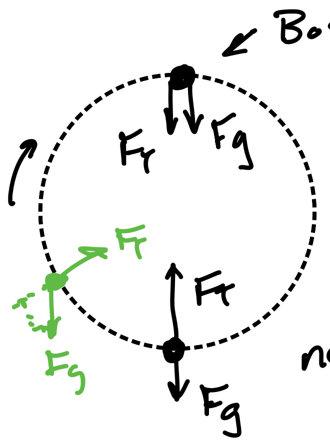
$v = 1.4 m/s$ tangential speed

14. You are swinging a 0.5-kg ball at the end of a string in a *vertical* circle with a radius equal to 0.6m.

a) Explain why the tension in the string is higher when the ball is at the bottom of its path, than when it is at the top of its path.

b) What is the maximum speed the ball can have without the string breaking if the maximum tension the string can have is 35 N.

Both are centripetal forces



$\Sigma F = ma_c$

$F_T + F_g = ma_c$

$F_T = ma_c - F_g$

$F_T \rightarrow 0$ at top

F_g can be sole force at top object behaves like a projectile for that moment

net force must point towards center so $F_T > F_g$ at this location

$\Sigma F = ma_c$

$F_T - F_g = ma_c$ ✓

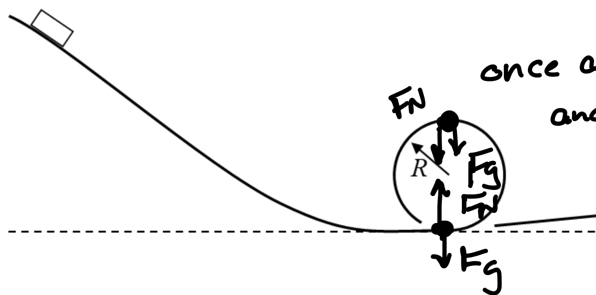
$F_T = ma_c + F_g$ max. tension

b) $F_T - F_g = m\frac{v^2}{r}$

$$35N - (0.5kg)(9.8m/s^2) = (0.5kg)\frac{v^2}{(0.6m)} \rightarrow v = 6.0 m/s$$

steps LHS, then ÷0.6/0.5, then √

15. A roller coaster cart is doing a loop-the-loop.



once again F_g can be only force and you won't die

a) Compare the normal forces on the cart when the cart is at the top and bottom of the loop. See question above. Exactly same reasoning but F_N, not F_T

b) What is the minimum force necessary for the cart to successfully navigate the loop? Is the cart still technically in contact with the track?

c) For a 30 m radius loop, what is the minimum speed the cart must be going to make the loop without losing contact with the track?

Top $\Sigma F = ma_c$

$$\cancel{F_N} + F_g = m\frac{v^2}{r}$$

Can go to zero (come out of seat)

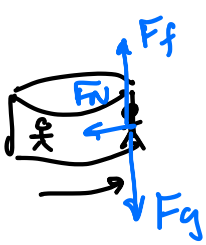
$$\cancel{m}g = \cancel{m}\frac{v^2}{r}$$

$$v = \sqrt{rg} = \sqrt{(30\,m)(9.8)}$$

$$v = 17\ m/s$$

Two equations linked (possibly 3: $F_f = \mu F_N$)

16.



y

$$F_f - F_g = m\cancel{a}^{\,0}$$

$$F_f = F_g = mg$$

x

$$F_N = m\frac{v^2}{r}$$

$$F_f = \mu F_N = \mu\left(m\frac{v^2}{r}\right)$$

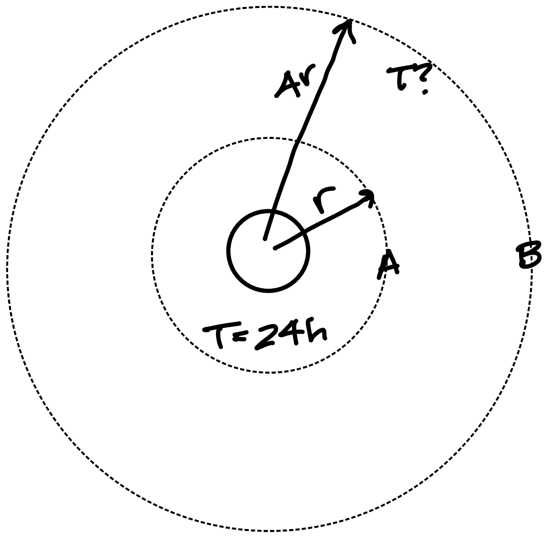
$$\mu\left(\cancel{m}\frac{v^2}{r}\right) = \cancel{m}g$$

$$v = \sqrt{\frac{rg}{\mu}}$$

$$\omega = \frac{v}{r} = \frac{1}{r}\sqrt{\frac{rg}{\mu}} = \sqrt{\frac{g}{r\mu}}$$

9/25 Homework #11

2 satellites



A

$$G\frac{m_p \cancel{m_A}}{r_A^2} = \cancel{m_A}\frac{v_A^2}{r_A}$$

B

$$G\frac{m_p \cancel{m_B}}{r_B^2} = \cancel{m_B}\frac{v_B^2}{r_B}$$

Same central body Gm_p = constant

$$Gm_p = v_A^2 r_A = v_B^2 r_B$$

also $v_A = r_A\omega = r_A\frac{2\pi}{T_A}$

$v_B = r_B\omega = r_B\frac{2\pi}{T_B}$

$$\left(r_A\frac{\cancel{2\pi}}{T_A}\right)^2 r_A = \left(r_B\frac{\cancel{2\pi}}{T_B}\right)^2 r_B$$

$$\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$$

$r \to r_A^3$; $24h \to T_A^2$; $r_B^3 \leftarrow 4r$

$$T_B^2 = T_A^2\left(\frac{r_B}{r_A}\right)^3$$

$$T_B = \sqrt{24^2\left(\frac{4\cancel{r}}{\cancel{r}}\right)^3}$$

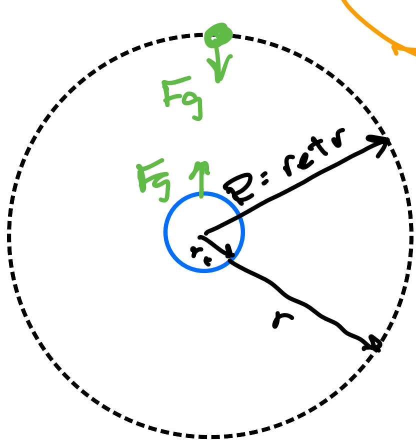
answer in hours

$6.9h\left(\frac{3600s}{h}\right) =$

$24840s$

$T = 6.9h$

#9



$v = \frac{2\pi R}{T}$

$m_e = 5.974\times10^{24}$ kg

$r_e = 6.371\times10^6$ m

$F_g = ma_c$ moving object

$$G\frac{m_e \cancel{m_s}}{R^2} = \cancel{m_s}\frac{v^2}{R} \quad R^2$$

$$R = \frac{Gm_e}{\cancel{v^2}} = \frac{Gm_e}{\left(\frac{2\pi R}{T}\right)^2}$$

$$R = \frac{Gm_eT^2}{4\pi^2R^2}$$

$$\sqrt{(6.67\times10^{-11})(5.97\times10^{24})(24840s)^2}$$

$$R^3 = \frac{G m_e T^2}{4\pi^2} \cdot \sqrt[3]{\frac{(6.67\times10 \ldots}{4\pi^2}}$$

6.22×10^{21}

$R = 1.8\times10^7 m = r_e + r$

$1.8\times10^7 - 6.37\times10^6$ $\quad$ r_e: 6.37×10^6

$r = 1.2\times10^7 m = 12024 Km$

$1.2E4$

$a_c = \frac{v^2}{r}$

$$\boxed{G\frac{m_e \cancel{m_s}}{R^2}} = \cancel{m}\left(\frac{v^2}{R}\right) = a_c$$

a_c

Big R in meters, not Km

$$\frac{(6.67\times10^{-11})(5.97\times10^{24} Kg)}{(1.8\times10^7)^2}$$