1. We have gone through several kinds of equations now and lets sum up some of these *as proportions*:

   - acceleration is $\propto$ net force. $a = \frac{F_{net}}{m_{total}}$

     $\Sigma F = ma$

   - assuming constant acceleration and beginning at rest, an object's velocity is $\propto$ to the square root the displacement.

     $v_2^2 = \cancel{v_1^2} + 2ad$

     $v = \sqrt{2ad}$

   - assuming constant acceleration and beginning at rest, an object's displacement is quadratic the elapsed time.

     $\Delta x = \frac{1}{2}at^2$

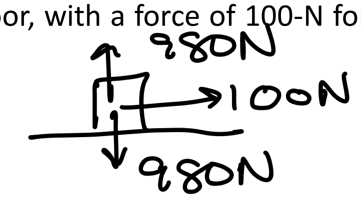
   - for an object that has been dropped, the distance it has fallen is quadratic its velocity at that distance.

     $v^2 = \cancel{v_1^2} + 2a(\Delta x)$

     $v^2 = 2a(\Delta x)$

2. I push a 100-kg box starting at rest along a *frictionless* floor, with a force of 100-N for 10 s. How fast is the box going at this point?

   - What is the net force on the box? 100N
   - What is the acceleration of the box?



$F = ma$

$100N = (100kg)a$

$a = 1\,m/s^2$

$v = v_1 + at = 0 + (1\,m/s^2)(10s) = 10\,m/s$

3. How fast is a 200-kg box moving if it starts from rest and I push it with a 100-N force for 10 s?

$a = \frac{100N}{200kg} = 0.5\,m/s^2$

$v = 0 + (0.5\,m/s^2)(10s) = 5\,m/s$

4. Next, I push a 100-kg box starting at rest along a *frictionless* floor, with a force of 100 N ~~over~~ **10 m**s for a displacement of 10 meters
How fast is the box going at this point? Some starters:

- What is the net force on the box? $F_{net} = 100N$
- What is the acceleration of the box? $a = 1.0 m/s^2$
- What is the final velocity after 10 m?

$$v^2 = v_i^2 + 2ad$$
$$v^2 = 0^2 + 2(1.0 m/s^2)(10 m)$$
$$v = \sqrt{20} = 4.5 m/s$$

5. If I did the same thing to a 200-kg box, then how fast is it going after 10 m?

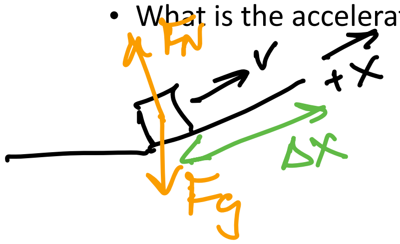
$$v^2 = 0^2 + 2(0.5 m/s^2)(10 m)$$
$$v = \sqrt{10} = 3.2 m/s$$

6. Back to the 100-kg box, I stop pushing after 10 m. What happens to the speed of the box?

frictionless, $v = 4.5 m/s$
constant velocity

7. The 100-kg moving at the speed above starts sliding up a *frictionless* 20° ramp. How far along the length of the ramp does the box rise? What height is this above the horizontal? Do the 100-kg and the 200kg box rise to the same height? Some starters:

- What is the net force on the box as it goes up the inclined plane? $\boxed{-mg\sin\theta = -335N}$
- What is the acceleration of the box as it goes up the inclined plane?

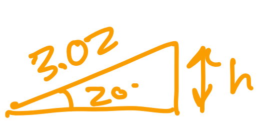


$$-\cancel{m}g\sin\theta = \cancel{m}a$$
$$\boxed{a = -g\sin\theta = -3.35 m/s^2}$$

	x	y
Fg	$-mg\sin\theta$	$-mg\cos\theta$
FN	0	$+mg\cos\theta$

at max. height

$$v^2 = v_i^2 + 2ad$$
$$0 = (4.5)^2 + 2(-3.35)(\Delta x)$$
$$\boxed{\Delta x = 3.02m}$$

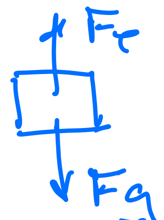


$\sin 20 = \frac{h}{3.02}$ so $h = 1.03m$

if they start at the same

Speed at the bottom of the ramp, yes!

8. A 50-kg crate of apples is lowered by a rope straight down and has an acceleration of 1.0 m/s² in the downward direction. What is the tension in the rope? State its magnitude and direction.



$mg = (50kg)(9.8)$

$490N$

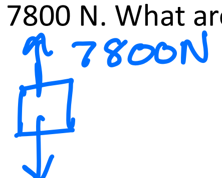
$\sum F_y = ma_y$

$F_T - F_g = ma$

$F_T = ma + F_g = (50kg)(-1.0\,m/s^2) + 490$

$F_T = 440N$

9. A 900-kg elevator moves downward 11.0 meters in 4 seconds. The tension in the supporting cable is 7800 N. What are the initial and final velocities of the elevator?



$mg = 8820N$

$\sum F = ma$

$8820 - 7800 = (900kg)a$

$a = \frac{1020N}{900kg} = 1.13\,m/s^2$ down

$\Delta y = v_1 t + \frac{1}{2}at^2$

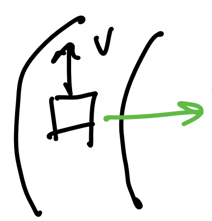
$11.0m = v_1(4s) + \frac{1}{2}(1.13\,m/s^2)(4s)^2$

$\boxed{v_1 = 0.49\,m/s \downarrow}$

$v_f = v_1 + at$

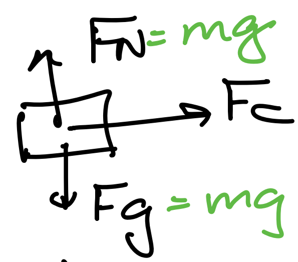
$\boxed{v_f = 5\,m/s \downarrow}$

10. A car travels at 17 m/s without skidding around a level curved road with a radius of 35m. What is the coefficient of static friction between the tires and the road if this speed is the fastest it can go around this curve without skidding?



$F_c = \mu F_N = \mu(mg)$

Top



Side

$\sum F_c = ma_c$

$\mu \cancel{m} g = \cancel{m}\frac{v^2}{r}$

$\mu = \frac{v^2}{rg} = 0.84$

11. If a soccer ball with a radius of 10 cm is rolls along the ground without slipping at 5 m/s, then how many revolutions does it roll through in 10 s and what distance has a point on the edge of the ball traveled? Some starters:
    - a) How fast is it *spinning*? By that we mean *angular speed*. $V = r\omega$
    - How many radians does the ball rotate through in this time? What is that in revolutions?
    - How far does it roll in this time? Is this the same distance as the distance of a point on the edge of the ball? Why or why not? yes
    - How many seconds does it take for the ball to complete one revolution? This amount of time is referred to as the *period* of its rotation, T.

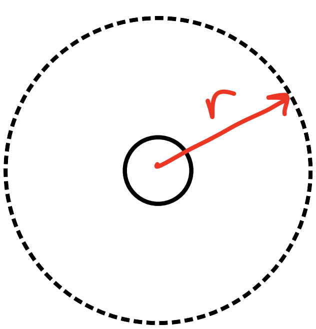
a) $\omega = \frac{v}{r} = \frac{5\,m/s}{0.1\,m} = 50\,rad/s$

b) $\theta = \omega t = (50\,rad/s)(10\,s) = 500\,rad\left(\frac{1\,rev}{2\pi}\right) = 79.6\,rev$

c) $d = vt = (5\,m/s)(10\,s) = 50\,m$

d) $\omega = \frac{2\pi}{T} \longrightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{50\,rad/s} = 0.125\,sec$

12. A satellite is in orbit around a distant planet. You observe the satellite is 5000 km from the center of the planet and revolving the planet once every 2 days. What is the period of the satellite's motion? How fast is the satellite moving around the planet? What is the angular speed What is the mass of the planet you have discovered?



$T = 2\,days\,(24\,h/d)(3600\,s/h) = 172{,}800\,s$

$\omega = \frac{2\pi}{T} = 3.6\times10^{-5}\,rad/s$

$V = r\omega = (5000\times10^{3}\,m)(3.6\times10^{-5}\,rad/s)$

$V = 182\,m/s$

$F_g = G\frac{m_p \cancel{m_s}}{r^2} = \cancel{m_s}\frac{V^2}{r}$

$m_p = \frac{V^2 r}{G} = \frac{(182)^2(5\times10^6)}{6.67\times10^{-11}}$

$m_p = 2.4 \times 10^{21}$ kg

Study

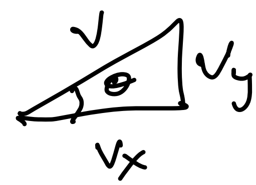
Unit conversion

Math relationships: proportionalities

Forces

Vectors $\quad v_x^2 + v_y^2 = v^2$

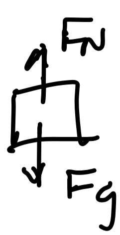
$$\theta = \tan^{-1} \frac{v_y}{v_x}$$



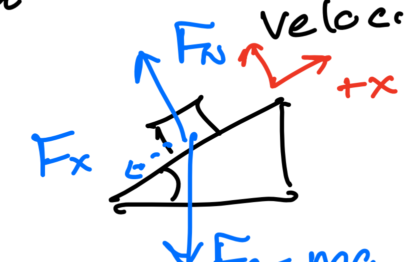
$F_f = \mu F_N$

Newton's Laws

① $\Sigma F = m\cancel{a} = 0$ (at rest, constant velocity)



$F_N = F_g$



$F_g = mg$

$F_N = -mg\cos\theta$

$F_x = -mg\sin\theta$

$F_f = \mu F_N = +\mu mg\cos\theta$

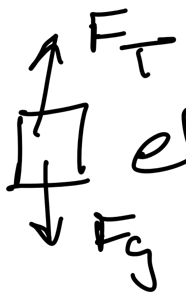
② $\Sigma F = ma, \; a \neq 0$

$$a = \frac{\Delta v}{\Delta t}$$

A speed changing

B direction changing

A



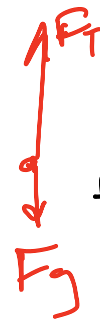
elevator

$$\Sigma F = ma$$

a is down,

$$\Sigma F = F_g - F_T$$

and a is +



a is up

$$\Sigma F = F_T - F_g$$

+y and a is +

apply our acceleration to Kinematics

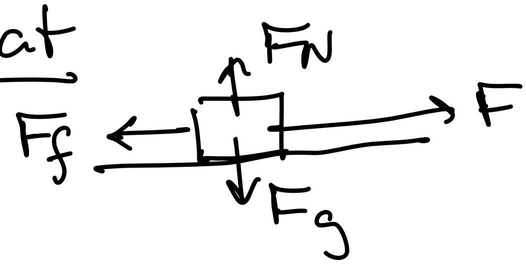
$$d \rightarrow \Delta x = v_1 t + \tfrac{1}{2} a t^2$$

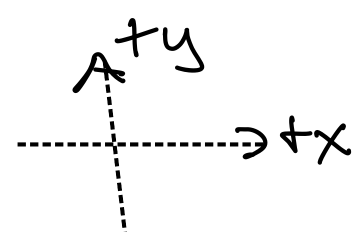
$$v_2 = v_1 + at$$

$$v_2^2 = v_1^2 + 2a(\Delta x)$$

d

Flat

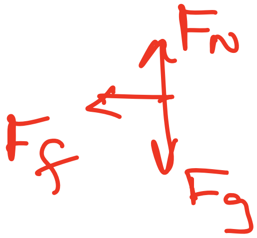




$$\Sigma F_x = ma_x$$

then Kinematics

$$F_f = \mu mg$$

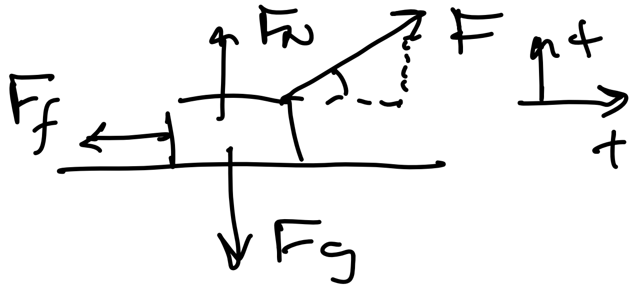


Slides to rest

$$\Sigma F_x = ma_x$$

$$\mu \cancel{m} g = \cancel{m} a$$

$$a = \mu g$$



	x	y
F_g	0	$-mg$
F	$F\cos\theta$	$+F\sin\theta$
F_N	0	$mg - F\sin\theta$
F_f	μF_N	
	a	0

y

$$\Sigma F_y = ma_y$$

$$F_N + F\sin\theta - mg = m\cancel{a}^{\,0}$$

$$F_N = mg - F\sin\theta$$

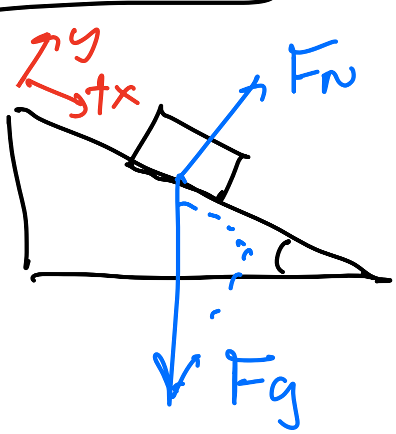
x

$$F\cos\theta - F_f = ma$$

$$F\cos\theta - \mu(mg - F\sin\theta) = ma$$

Nothing cancels

Incline



	X	Y
F_g	$mg\sin\theta$	$-mg\cos\theta$
F_N	0	$+mg\cos\theta$
F_f	μF_N	0

$$\Sigma F_x = ma_x$$

$$\cancel{m}g\sin\theta - \mu \cancel{m}g\cos\theta = \cancel{m}a$$

B Change direction $a_c = \frac{v^2}{r} = r\omega^2$

$360° = 2\pi$ radians $= 1$ revolution

T = period, time for 1 cycle

$$\omega = \frac{2\pi}{T} \qquad v = \frac{2\pi r}{T}$$

$$v = r\omega$$

$$\Sigma F = F_c = m\frac{v^2}{r} = mr\omega^2$$

r = separation of centers of masses

planets $G\frac{m_1 m_2}{r^2} = m_2\frac{v^2}{r}$

example: moon takes 28 days around earth

$$T = 28\,\cancel{day}\left(24\frac{\cancel{h}}{\cancel{d}}\right)\left(3600\frac{s}{\cancel{h}}\right)$$

$$T = 2.42 \times 10^6\ s$$

$$\omega = \frac{2\pi}{2.42\times10^6\ s} =$$

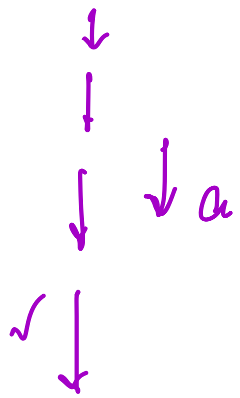
$$v = r\omega$$

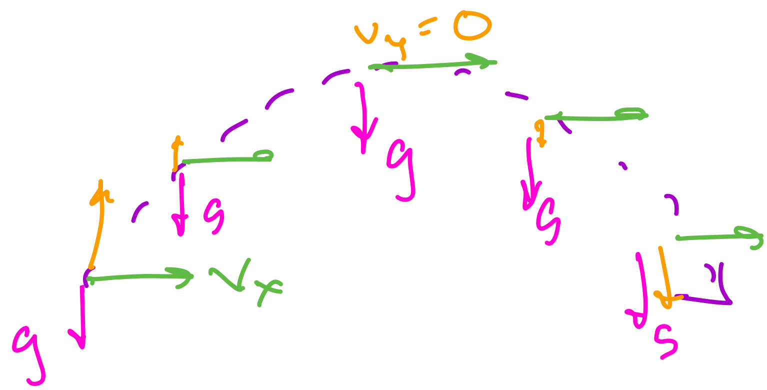
must have

$$d = vt = \underbrace{r\omega}_{v} t$$

Projectiles

$a = g = 9.8\ m/s^2$





V_x doesn't change

$V_y = V_1 + (-9.8)t$

$V_{land} = -V_{launch}$

$t_{peak} = \frac{1}{2} t_{total}$

all projectiles with same initial vertical velocity go to same peak height

No mention of time? Use

$$V_2^2 = V_1^2 + 2ad$$