

At the end of this worksheet, you should be able to

- calculate stress and strain and use Young's Modulus to solve for an unknown.
- plot all relevant quantities of simple harmonic motion over time.
- use the quantities of simple harmonic motion and the mathematical description to solve for an unknown.

**Table 10.1**

Approximate Values of  
Young's Modulus for Various  
Substances

Substance	Young's Modulus (GPa)
Rubber	0.002–0.008
Human cartilage	0.024
Human vertebra	0.088 (compression); 0.17 (tension)
Collagen, in bone	0.6
Human tendon	0.6
Wood, across the grain	1
Nylon	2–6
Spider silk	4
Human femur	9.4 (compression); 16 (tension)
Wood, along the grain	10–15
Brick	14–20
Concrete	20–30 (compression)
Marble	50–60
Aluminum	70
Cast iron	100–120
Copper	120
Wrought iron	190
Steel	200
Diamond	1200

### Hooke's Law for Stress and Strain and Young's Modulus

1. Consider a wire that is 0.1 mm in diameter and 2 m long and a Young's Modulus of 120 GPa ( $1 \text{ GPa} = 10^9 \text{ Pa}$ ). A 100-N force is applied to the wire.
  - a) What is the stress on the wire?
  - b) What is the strain?
  - c) By how much does its length change?

- d) What is its new length?
- e) What percent change is this?
2. Next, 1000-N force is applied to an identical wire. What is stress, strain, length change and percent change in the length from its original length?
3. If, instead, the wire was 1.0 m long with a 100-N applied force. Now, how much does it stretch? What percent change is this?
4. Now, wire of same material and 2.0 m length but the cross-sectional diameter of the wire is half its original diameter ( $d_2 = \frac{1}{2} (0.1\text{mm})$ ) and a 100-N force is applied. How much does it stretch now? And what percent change is this?

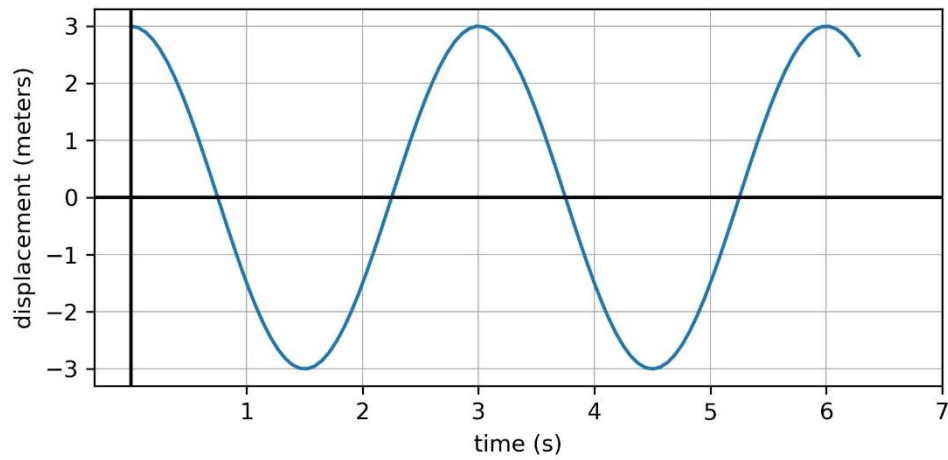
5. A wire of length  $\ell_1$  and volume  $V$  and cross-sectional area  $A_1$  is stretched out to length  $\ell_2$ , what is its new cross-sectional area? Think about this in terms of proportionality.
6. A 60 kg person stands upright. By how much does the femur shorten if each femur carries half the weight of the person? The cross-sectional area of a femur is about  $4 \text{ cm}^2$  and the length is about 30 cm Also find the percent change in length. ( $Y = 9.4 \times 10^9 \text{ Pa}$ )

### **Hooke's Law for Springs**

7. A 1-kg mass is attached to a spring of spring constant 1000 N/m. The mass is pulled back, so the spring is stretched 10 cm from its relaxed length. After you finish the following questions, make sure you can write down expressions for all of them in general as well as working them in reverse.
- a) What is the initial total *energy* of the mass-spring system? What is it due to?
  - b) When the mass is released, it heads back toward equilibrium gaining kinetic energy as it goes. What is the maximum kinetic energy it can achieve? What is the position of the mass when kinetic energy is a maximum?

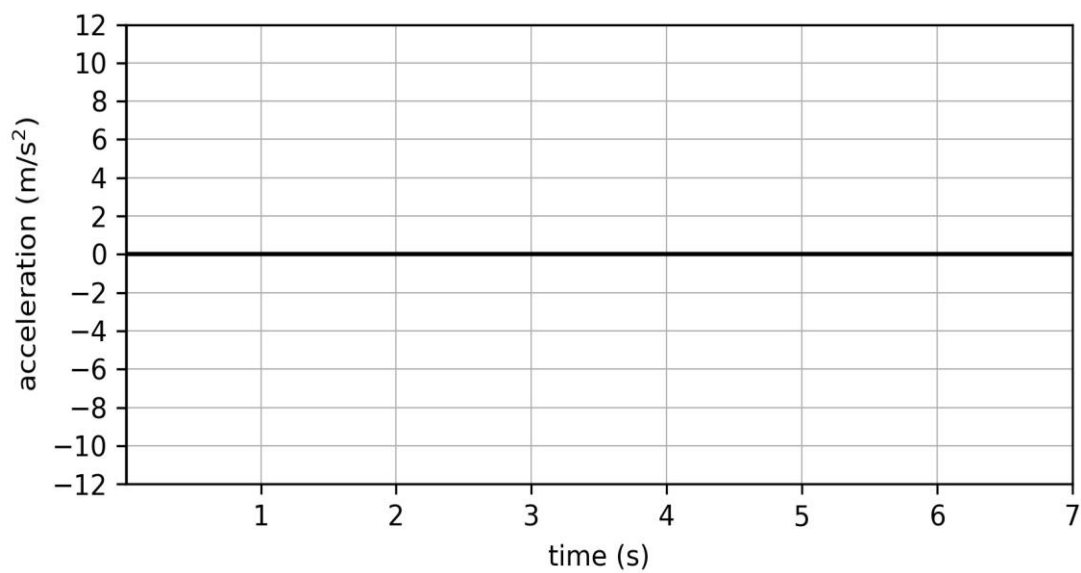
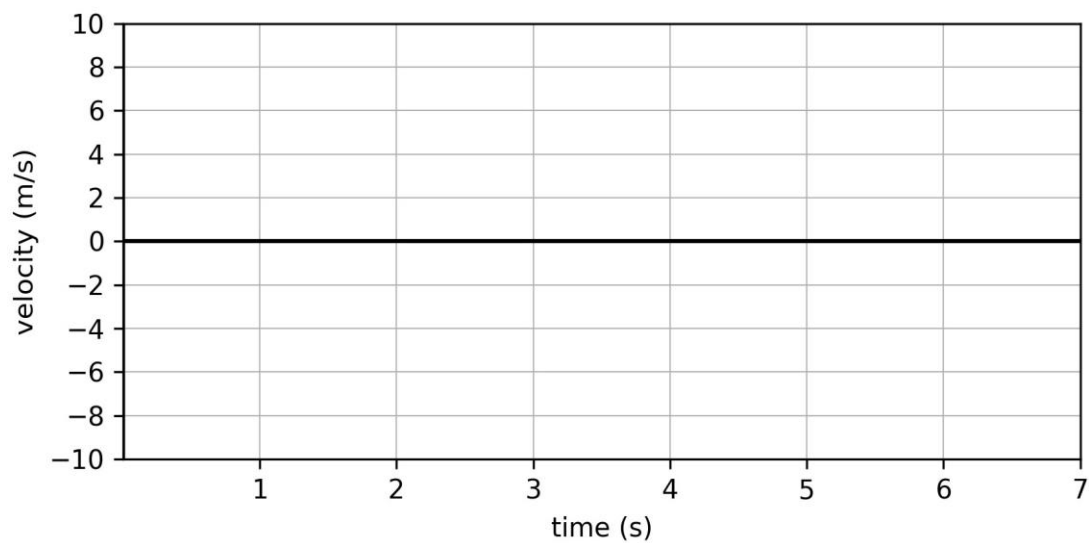
- c) What is the mass's velocity when it has maximum kinetic energy?
  
  
  
  
  
  
  
  
  
  
- d) What is the extension of the spring when the mass's potential energy and kinetic energy at that position are equal?
  
  
  
  
  
  
  
  
  
  
- e) What is the *period* of this mass's motion? What is its frequency? What is its angular frequency?
  
  
  
  
  
  
  
  
  
  
- f) What is the *force* on the mass when it is at its maximum displacement?
  
  
  
  
  
  
  
  
  
  
- g) What is the force on the mass when it is at the equilibrium position?
  
  
  
  
  
  
  
  
  
  
- h) What is the maximum acceleration of the mass? Where does this occur?

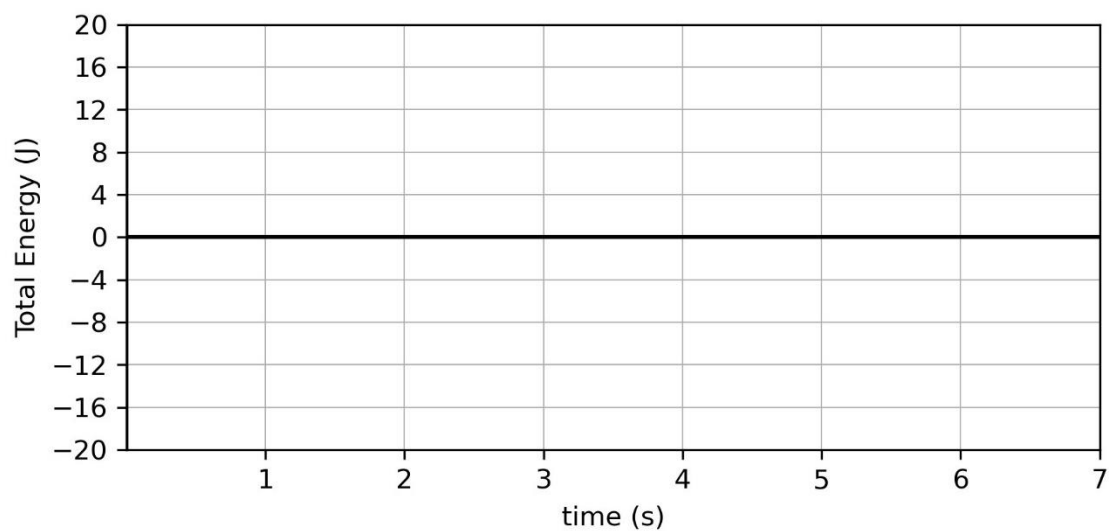
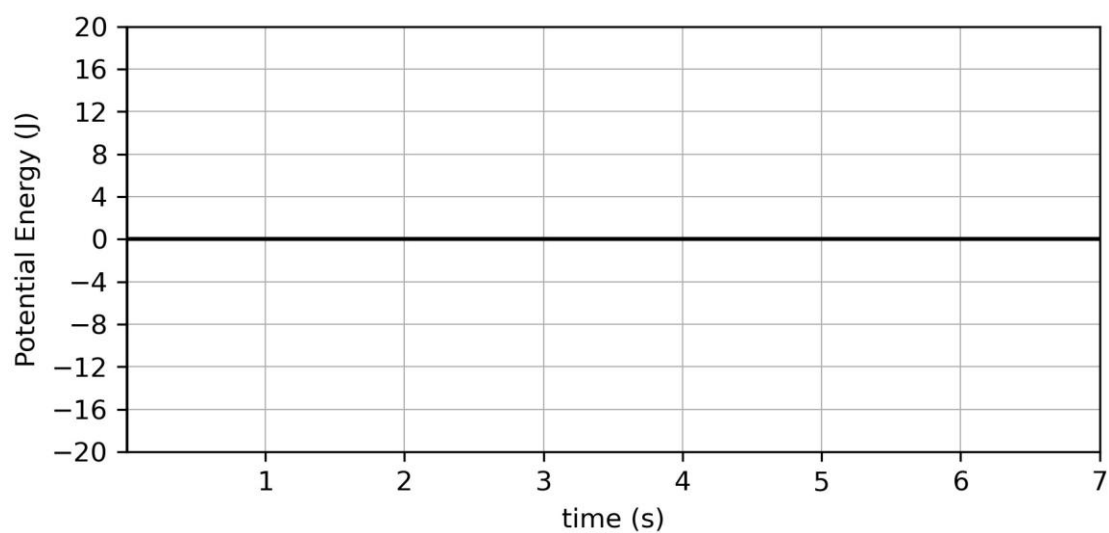
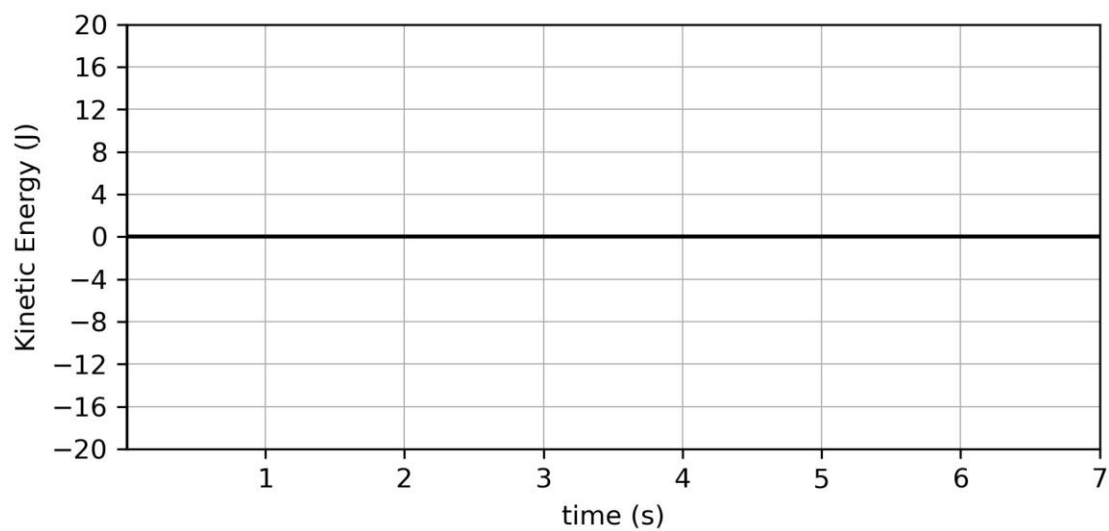
8. A 4-kg mass attached to a spring oscillates according to the displacement graph that follows.



- Determine the amplitude, period, natural frequency, angular frequency.
- Determine the spring constant of the spring.
- Determine the mass's maximum velocity and maximum acceleration.
- Determine the system's maximum kinetic energy, maximum potential energy, and total energy?

e) Also fill out the rest of the graphs.





**Stress and Strain**

9. According to the table, which material stretches more, 2 m of steel or 1 m of copper of the same width?
10. Four brass wires are subjected to the same tensile stress. The wires have the following unstretched lengths and widths. Rank them in order from least to most change in length.
- (a) length  $L$ , diameter  $d$
  - (b) length  $2L$ , diameter  $d$
  - (c) length  $4L$ , diameter  $d/2$
  - (d) length  $L/4$ , diameter  $d/2$
11. A 0.5-m long guitar string of cross-sectional area of  $1.0 \times 10^{-6} \text{ m}^2$  and Young's modulus  $Y = 2.0 \text{ GPa}$ . By how much must you stretch the string to obtain a tension of 20 N.
12. Compare Hooke's Law for springs ( $F = kx$ ) to Hooke's Law for stress and strain ( $\frac{F}{A} = Y \frac{\Delta L}{L}$ ). Write an equation relating the spring constant, Young's modulus, length, and cross-sectional area. Solve for  $k$ . What does this tell you about what would happen to the spring constant of a spring if you cut the spring in half?



13. If you attach two springs to an object side by side, then we say the springs are attached *in parallel*. This will result in two spring forces on the object that has been displaced some distance  $x_1$ . If you were to model this arrangement of springs in parallel as a single spring with a single spring constant that would have the same effect, then what would this single effective spring constant  $k_e$  be in terms of the original two spring constants  $k_1$  and  $k_2$ ?
14. If instead of a *parallel* arrangement, we attach one spring to another spring and then attach the two springs to the object. We say that these springs are connected *in series*. Derive an equation for an effective spring constant for this arrangement. Each spring will stretch a different amount based on its spring constant, but the object will experience one force and *both of the springs is exerting the same force*.