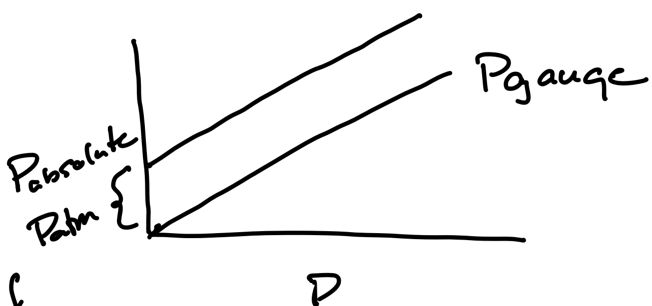
Pressure $= P = \frac{F}{A}$

$P = \frac{F}{\pi r^2}$ units: $\frac{N}{m^2}$ = Pascal

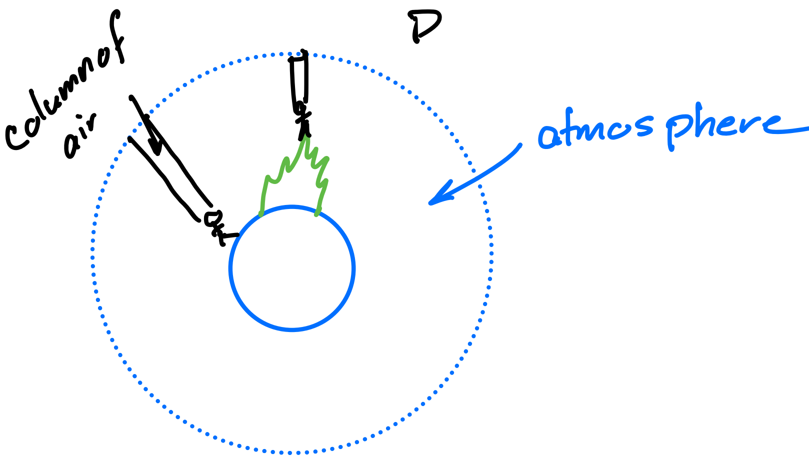
1 atm = 101 kPa = 14.7 psi

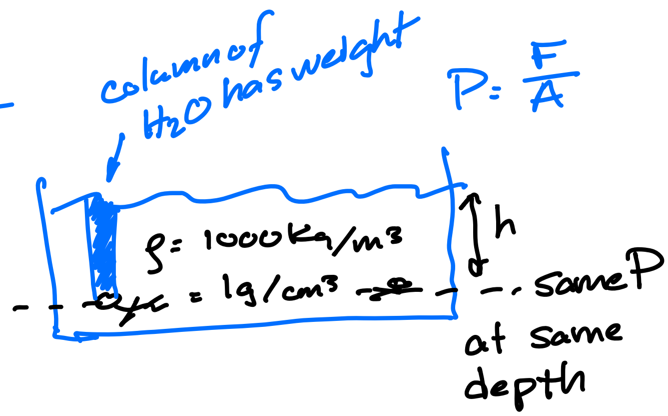
Q 1 + 2

Gauge Pressure measures pressure relative to atmospheric pressure



$P_{absolute} = P_{atm} + P_{gauge}$



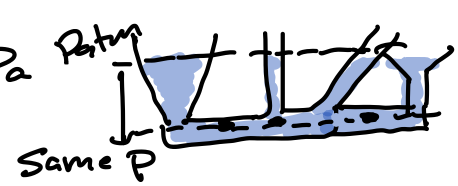


$P_{gauge} = \rho g h$ ← height of column depth in a pool (m)

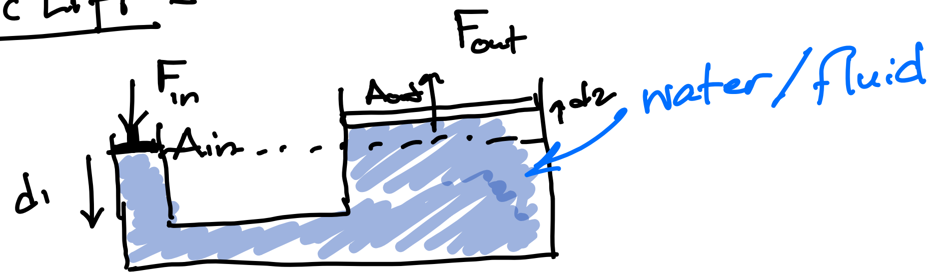
rho ρ = density of fluid (kg/m^3)

$P_{abs} = P_{atm} + P_{gauge}$

Pascals 1 atm = 101.3 kPa



Hydraulic Lift -



$$P = \frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \rightarrow F_{out} = F_{in}\left(\frac{A_{out}}{A_{in}}\right)$$

↑ lift increases force

Because mass + energy conserved → $F_1 d_1 = F_2 d_2$

smaller piston moves alot

Buoyancy - Static, still

When do objects float? $\rho_{object} < \rho_{fluid}$

$$\rho = \frac{\text{mass}}{\text{Volume}}$$

float → float on top, so mass is important

$\uparrow F_B$, $\downarrow F_g = mg$

$F_B = mg$

sink → $F_B \equiv$ weight of fluid displaced (defined as)

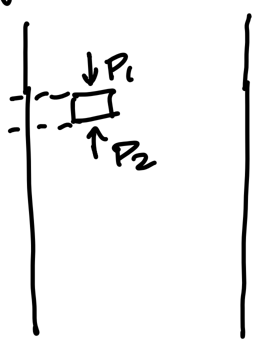
$$m_f g = (\rho_f V_f) g = \rho_f g V_{object}$$

So volume is important

$\rho = \frac{m}{V}$

$m = \rho V$

What causes Buoyancy? hydrostatic pressure



$$P_2 - P_1 = \Delta P = \rho g h$$

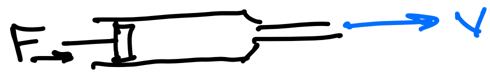
Volume of object doesn't change as it sinks so ΔP is constant

So as an object sinks

- P_{abs} increases
- ΔP stays the same so
- F_B stays the same

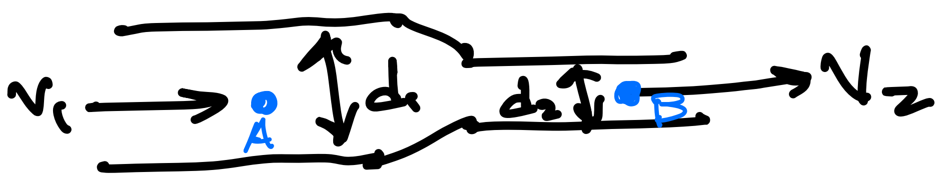
Fluid Dynamics - moving fluid (liquids)

Syringe



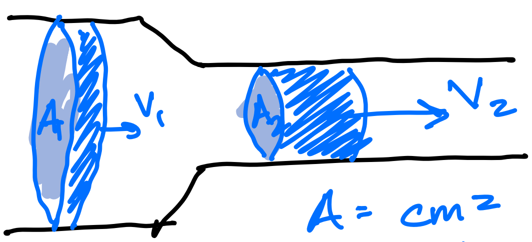
ΔP causes fluid to flow

Continuity Equation - conservation of mass



$$\frac{mass_A}{t} = \frac{mass_B}{t} \qquad \rho = \frac{m}{V}$$

volumetric flowrate $\rightarrow Q = \frac{Vol_A}{t} = \frac{Vol_B}{t}$



$A = cm^2$

$v = cm/s$

$$Q = A_1 v_1 = A_2 V_2$$

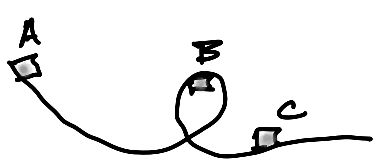
$$\frac{\cancel{\pi} d_1^2}{\cancel{4}} v_1 = \frac{\cancel{\pi} d_2^2}{\cancel{4}} V_2$$

$$d_1^2 v_1 = d_2^2 V_2$$

Bernoulli's Equation - conservation of energy

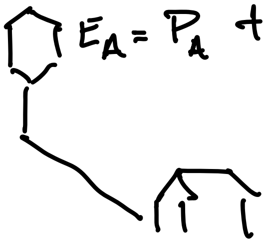
$E_A = E_B = E_C$ (no friction)

Rollercoaster



$E_A = W_A + mgh_A + \frac{1}{2} m v_A^2$

$E_B =$



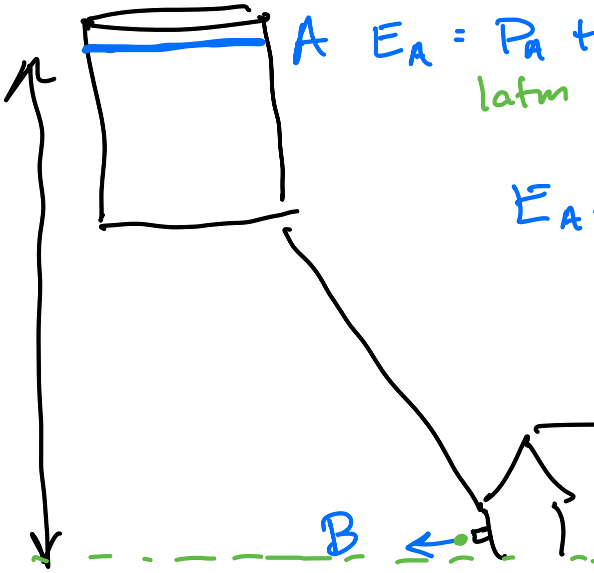
$E_A = P_A + \rho g h_A + \frac{1}{2}\rho v_A^2$

$E_B = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$

① What is P at A & B?
many times P_A : P_B cancel

② Ask velocity?
$v_A << v_B$
can be ignored

<u>example</u>



$E_A = P_A + \rho g h_A + \frac{1}{2}\rho v_A^2$

1atm $\quad h_A = h \quad$ essentially zero

$v_A << v_B$

$A_1 v_1 = A_2 v_2$

$E_A = \rho g h$

1atm $\quad$ zero

$E_B = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$

$E_B = \frac{1}{2}\rho v_B^2$

$E_A = E_B$

$\rho g h = \frac{1}{2}\rho v_B^2$

$v = \sqrt{2gh}$

At the end of this worksheet, you should be able to

- calculate the new quantities of pressure, gauge pressure, and density.
- apply the new principles of Pascal, Archimedes, and Bernoulli.
- use the continuity principle to solve for an unknown quantity.
- use the hydrostatic pressure principle to solve for an unknown quantity.

You will want to keep the following table of densities from your textbook handy.

Table 9.1 Densities of Common Substances (at 0°C and 1 atm unless otherwise indicated)

Gases	Density (kg/m^3)	Liquids	Density (kg/m^3)	Solids	Density (kg/m^3)
Hydrogen	0.090	Gasoline	680	Polystyrene	100
Helium	0.18	Ethanol	790	Cork	240
Steam (100°C)	0.60	Oil	800–900	Wood (pine)	350–550
Methane	0.72	Water (20°C)	998.21	Wood (oak)	600–900
Air (20°C)	1.20	Water (0°C)	999.84	Ice	917
Nitrogen	1.25	Water (3.98°C)	999.98	Wood (ebony)	1000–1300
Carbon monoxide	1.25	Seawater	1025	Bone	1500–2000
Air (0°C)	1.29	Blood (37°C)	1060	Concrete	2000
Oxygen	1.43	Mercury	13 600	Quartz, granite	2700
Carbon dioxide	1.98			Aluminum	2702
Argon	1.66			Iron, steel	7860
Xenon	5.86			Copper	8920
Radon	9.73			Lead	11 300
				Gold	19 300
				Platinum	21 500

1. A 500 N person stands on one foot that has an area 50.0 cm^2.
   a) What is the pressure on the floor?

   $A = 50 cm^2 \left(\frac{m}{100 cm}\right)^2 = 0.005 m^2$
   $5 \times 10^{3} m^2$

   $P = \frac{F}{A} = \frac{mg}{A} = \frac{500 N}{0.005 m^2} = 100,000 Pa$

   b) Now, this person stands on their heel in a high heeled shoe that has an area of 1.00 cm^2, what pressure is there?

   $A = 1 cm^2 \left(\frac{m}{100 cm}\right)^2$

   50x more pressure on bones

   $P = 5 \times 10^6 Pa$

c) What is the pressure on the floor if this person stands on a diamond that is cut to have a bottom facet that is 10,000 μm^2 (1 μm = 10^{-6} m)? *(Careful with unit conversions!)*.

$1\,\mu m = 10^{-6}\,m$

$A = 10{,}000\,\mu m^2 \left(\frac{10^{-6}\,m}{\mu m}\right)^2$

$A = 1\times10^{-8}\,m^2$

Really Big

$P = \frac{500N}{1\times10^{-8}\,m^2}$

2. A patient's blood systolic blood pressure when resting is 160 mmHg. What is this pressure in pascals, psi, and atm? (1 atm = 760 mm Hg = 14.7 psi = 101.3 kPa)

$160\,mmHg \left(\frac{101.3\,kPa}{760\,mmHg}\right) = 21.3\,kPa$

$160\,mmHg \left(\frac{14.7\,psi}{760\,mmHg}\right) = 3.09\,psi$

$160\,mmHg \left(\frac{1\,atm}{760\,mmHg}\right) = 0.21\,atm$

3. Throughout this worksheet and the homework, *gauge pressure* is a way of expressing the pressure measured by an instrument relative to atmospheric pressure, like your tire gauge. Therefore, when a gauge reads 0 pressure, the absolute pressure is 1 atm.

$$P_{abs} = P_{atm} + P_{gauge}$$

With a perfect vacuum, P_{abs} =0 so P_{gauge} = −1 atm.

$P_{atm} = 1\,atm = 101.3\,kPa$

a) What is the absolute pressure when the gauge reads 2 atm?

$P_{abs} = 1 + 2 = 3\,atm$

b) What is the gauge pressure when the absolute pressure is 30 kPa?

$P_{abs} = P_{atm} + P_g$

$30\,kPa = 101.3\,kPa + P_g$

$P_g = -71.3\,kPa$

c) What is the absolute pressure when gauge pressure is 1000 kPa?

$P_{abs} = 101.3 + 1000\,kPa$

$P_{abs} = 1101.3\,kPa$

101.3kPa

4. At the surface of a freshwater lake, the air pressure is 1 atm. At what depth under the water is the absolute pressure ~~4 atm~~? 405 kPa

$$405\ kPa = 101.3\ kPa + P_g$$

$$P_g = 303.7\ kPa$$

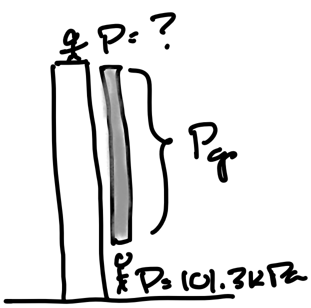
$$P_g = \rho g h$$

$$303700\ Pa = 303.7\ kPa = (1000\ kg/m^3)(9.8)\,h$$

$$h = 31\ m$$

101.3kPa

5. At sea level, the average atmospheric pressure is 1 atm. The density of air at this level is about 1.29 kg/m^3. Assuming the density of air is constant (it's not but just go with it), what is the air pressure at the Empire State Building that has a height of 381 m at the top deck? Assume that the pressure at the base of the building is at sea level.



$$P_{abs} = P_{atm} + P_g \leftarrow P_g = \rho g h$$

$$P_g = (1.29\ kg/m^3)(9.8)(381)$$

$$P_g = 4817\ Pa$$

$$P_{abs} = 101300 - 4817$$

$$P_{abs} = 96483\ Pa$$

6. A diver swims to a depth of 10 meters in a lake.
   a) What is the pressure on the diver's body?

(1000 kg/m^3)(9.8)(10)

$$P_{abs} = 101300\ Pa + P_g$$

$$= 101300 + 98000$$

$$= 199300\ Pa$$

b) What is the force on the diver's eardrums from the water if the area of the eardrum is 0.60 cm^2? (Ignore the fact that there is atmospheric pressure inside the eardrum, but also think about this problem if you didn't ignore that fact.)

$$A = 0.60\,cm^2 \left(\frac{1m}{100cm}\right)^2 = 6\times10^{-5}\,m^2$$

$$P = \frac{F}{A} \longrightarrow F = PA = (199300\,Pa)(6\times10^{-5}\,m^2)$$

$$F = 12\,N$$

7. The following cylinders are filled with two different fluids. The density of each fluid is given. Rank the cylinders according to the pressure at the bottom of each cylinder, smallest to largest.

(a) R = 40 cm, h = 80 cm, ρ = 1000 kg/m^3 80,000

(b) R = 40 cm, h = 100 cm, ρ = 1000 kg/m^3 100,000

(c) R = 50 cm, h = 100 cm, ρ = 800 kg/m^3 80,000

(d) R = 50 cm, h = 80 cm, ρ = 800 kg/m^3 64,000

(e) R = 50 cm, h = 125 cm, ρ = 800 kg/m^3 100,000

$d < a = c < b = e$

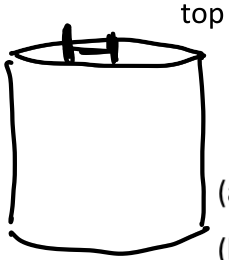
$$P_{abs} = P_{atm} + P_g$$

same

g same

so look at (ρh) only

(f) What happens to the pressure in each of these cylinders if a 10-kg dumbbell is placed on top of the piston?



$$\Delta P = \frac{10kg(9.8)}{\pi R^2}$$

Now look at surface area

(a) R = 40 cm, h = 80 cm, ρ = 1000 kg/m^3 $= \frac{98N}{\pi(0.4)^2} + 80{,}000 = \frac{612.5}{\pi} + 80{,}000$

(b) R = 40 cm, h = 100 cm, ρ = 1000 kg/m $\frac{98N}{\pi(0.4)^2} + 100{,}000 = \frac{612.5}{\pi} + 100{,}000$

(c) R = 50 cm, h = 100 cm, ρ = 800 kg/m^3 $\frac{98N}{\pi(0.5)^2} + 80{,}000 = \frac{392}{\pi} + 80{,}000$

(d) R = 50 cm, h = 80 cm, ρ = 800 kg/m^3 $\frac{392}{\pi} + 64{,}000$

(e) R = 50 cm, h = 125 cm, ρ = 800 kg/m^3 $\frac{392}{\pi} + 100{,}000$

New Rank: $d < c < a < e < b$

8. In a *hydraulic lift*, the radius of a small piston is 2 cm and the radius of the larger piston is 20 cm.

   a) What weight can the larger piston support when a force of 250 N is applied to the smaller piston?

$$P = \frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \qquad A_1 \propto 2^2 \qquad A_2 \propto 20^2$$

$$\frac{25000\,N}{9.8\,kg} = 2551\,kg$$

$$\frac{(250N)}{(2)^2} = \frac{F_{out}}{(20)^2}$$

$$F_{out} = (250N)\left(\frac{20}{2}\right)^2 = 25000\,N$$

Both cm → don't have to convert

   b) If the larger piston moves 5 cm, how far does the small piston move?

energy is conserved

$$F_1 d_1 = F_2 d_2$$

$$(250N)(5cm) = (25000)\, d_2$$

$$0.05\,cm = d_2$$

Buoyancy

9. Archimedes was charged with determining if King Heiron II's crown was not purely gold but a mixture of silver and gold. Archimedes knew that you can measure the volume of an object by submerging in water, $V_{object} = V_{displaced}$. Archimedes took the same mass of gold and one of silver, both equal in weight to the crown. He filled a vessel to the brim with water, put the silver in, and found how much water the silver displaced. He refilled the vessel and put the gold in. The gold displaced less water than the silver. He then put the crown in and found that it displaced more water than the gold and so was mixed with silver. Archimedes measured the weight of the crown; it was 24.5 N (Yes, he knew SI units!)

   a) What is the volume of silver that has a weight equal to 24.5 N? Answer in cm^3. (ρ = 10503 kg/m^3=10.503 g/cm^3)

$$V_{obj} = V_{displaced} \qquad \frac{24.5}{9.8} = 2.5\,kg$$

$$\rho = \frac{m}{V} \rightarrow V = \frac{m}{\rho} = \frac{2.5\,kg}{10503\,kg/m^3}$$

$$V = 2.38 \times 10^{-4}\,m^3 = 238\,cm^3$$

$$\left(\frac{100\,cm}{m}\right)^3$$

b) What is the volume of gold that has a weight equal to 24.5 N? Answer in cm^3. (ρ = 19300 kg/m^3=19.300 g/cm^3)

$2.5\,kg = 2500\,g$

$$V = \frac{2500\,g}{19.3\,g/cm^3} = 130\,cm^3$$

c) The crown displaced 157 cm^3. Was it entirely gold? How can you determine the fraction of gold in the crown using this information?

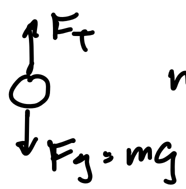
Not all gold

lab

Let's determine if the royal crown-maker cheated the king using buoyancy and SI units. Using a spring scale, you determine that the weight of the crown in air is 24.1 N. When the crown is lowered into a vat of water, the scale reads 22.85 N.

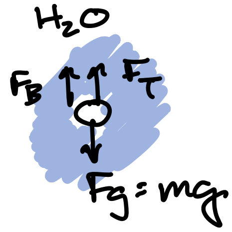
a) What is the buoyant force upward on the crown?

Air



$mg = F_T$

H_2O



$\Sigma F = 0$ H_2O $m_a g$ air $m_o g$

$F_B + F_T - mg = 0$

weight displaced fluid $m_f g$

$m_f \cancel{g} + m_a \cancel{g} - m_o \cancel{g} = 0$

$m_f = \rho_f V_f = \rho_f V_o$

$V_f = V_o = \frac{m_o}{\rho_o}$

b) What volume of water has been displaced?

$$\rho_f \left(\frac{m_o}{\rho_o}\right) + m_a - m_o = 0$$

$$\boxed{\rho_o = \rho_f \left(\frac{m_o}{m_o - m_a}\right)}$$

LAB

c) What is the volume of the crown?

d) What is the density of the crown?

10. A 5000-N object (rectangular prism) is *floating* in fresh water.

a) What is the net force on the object?

b) What is the magnitude of the buoyant force?

c) The bottom of the object is 1 meter below the surface of the water. What pressure is on the bottom of the object?

d) What is the area of the bottom surface of the object?

e) If the object has another 1 m sticking out of the water, then what is the volume of the object?

f) What is its density?

11. "ICEBERG DEAD AHEAD!" It is sometimes said that only 10% of an iceberg's volume actually sticks out above the surface of the water which is what made it so deadly to poor Jack and Rose on the *Titanic*. The density of ice is 917 kg/m^3 and the density of seawater is 1025 kg/m^3.

   a) What *fraction* of the iceberg is under the surface of the salt water? Work these *in general* for any density of object floating in any density of fluid.

   b) What is the fraction of the iceberg is above the surface of the water?

12. The time to fill a 5-gallon bucket with an ordinary garden hose is 1 minute when the valve is fully open. The diameter of the hose is 1 inch. (1 inch=0.0254 m, 1 gallon=0.00378 m^3, 1 min=60 s)

    a) What is the volumetric flowrate of the water?

    b) What is the speed of the water exiting the hose?

    c) How fast does it travel if you hold your thumb over the end of the hose and cover half of the area of the hose?

13. An artery has an inner diameter of 1.5 mm that narrows downstream to an inner diameter of 1.0 mm due to a buildup of plaque.

    d) By what factor does the speed of the blood flow change when it enters the narrow section of the artery?

    e) By what percentage does the speed of blood flow change?

14. The aorta is the artery from your heart that feeds 32 other major arteries. This is analogous to a single pipe splitting into many pipes. Assume that the aorta has an inner radius of 1 cm and each of the other arteries has an inner radius of 0.21 cm. If blood in the aorta has an average speed of 28 cm/s, then what is the average speed of blood in the major arteries? *Hint: you can just treat the major arteries as one big pipe that has an area 32 times bigger than the area of a single artery.*

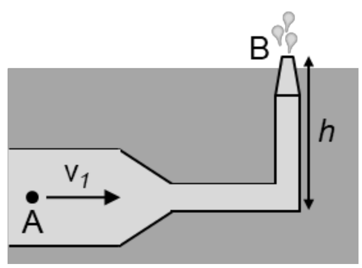
15. Show that *Bernoulli's Principle* really just reduces to the equation for pressure as a result of gravity when the fluid is not flowing (so $v_1 = v_2 = 0$). This equation is now the same as the *hydrostatic pressure* equation.

16. A cylindrical container of water is full to the brim when a hole is punctured 0.5 m from the top. What is the initial speed of the water as it comes out of the hole?

17. Following up on the previous problem, if you redirected this water straight up with the same speed, how high would it rise?

18. Water flows horizontally through a hose that has a radius of 1 cm at a speed of 2 m/s. If the nozzle of the hose narrows to 0.25 cm as the water sprays out, then what is the absolute pressure inside the hose? What is it as a gauge pressure?

19. A sprinkler system carries water 0.3 m below ground as shown. At point A, the pipe has a diameter of 2 cm. At Point B (ground level), the pipe has a diameter of 1 cm and the speed of the water is 8 m/s.



a) Calculate the speed of the water in the pipe at Point A.

b) Calculate the absolute water pressure in the pipe at point A.

c) How high does the water rise when it exits the sprinkler?