At the end of this worksheet you should be able to:

- convert units within the metric system.
- apply how to correctly convert area and volume units.
- apply scientific notation.
- apply the definition of percent change.
- convert between a ratio statement and percent change statement.
- apply the two forms of proportionality statements to make predictions from dependent variable change.

1. How many meters are in a decameter and how do you know?

   10m = 1 decameters

2. How many decameters are in a meter and why?

   $$\frac{10m}{10} = \frac{1 \text{ decameters}}{10}$$

   1m = 0.1 decameters

3. How many centimeters are in a meter? Draw on your paper approximately a centimeter.

   100 cm = 1m

4. How many millimeters are in a meter and how many meters are in a millimeter?

   1000 mm = 1 m

   1mm = 0.001m

5. The diameter of the earth is 6,380,000 m. What is this in kilometers?

   $$6{,}380{,}000 \text{ m} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = 6{,}380 \text{ km}$$

6. How many centimeters are in 3.2 km?

$$3.2\,\cancel{km} \cdot \frac{\overset{10^3}{1000\,\cancel{m}}}{1\,\cancel{km}} \cdot \frac{\overset{10^2}{100\,cm}}{1\,\cancel{m}} = 3.2 \cdot 10^5\ cm$$

7. How many inches are in 1 meter? Do this using the conversion of 1 in = 2.54 cm.

$$1\,\cancel{meter} \cdot \frac{100\,\cancel{cm}}{1\,\cancel{m}} \cdot \frac{1\ in}{2.54\,\cancel{cm}} = 39.4\ in$$

8. If I say your desk has an area of 0.5 meters, then what is wrong with that statement?

$$0.5\,m^2$$

9. Convert miles per hour to meters per second.

$$\frac{1\,\cancel{mile}}{\cancel{hour}} \cdot \frac{5280\,\cancel{ft}}{1\,\cancel{mile}} \cdot \frac{12\,\cancel{in}}{1\,\cancel{ft}} \cdot \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \cdot \frac{1\,m}{100\,\cancel{cm}} \cdot \frac{1\,\cancel{hour}}{3600\,s} = 0.45\ \frac{m}{s}$$

10. List as many formulas for the area of different shapes as you can remember (or look some up). What do these all have in common? What about volume formulas? What does this tell you about the units of these kinds of quantities?

Area: $A = l \cdot w$ $A_\square = s^2$

$A_\circ = \pi r^2$

Surface Area of a Sphere $= 4\pi r^2$

$A_\Delta = \frac{1}{2} b \cdot h$

Volume: $V = l \cdot w \cdot h$ $V = s^3$

$V_\circ = \frac{4}{3}\pi r^3$

$V_{cyl} = \pi r^2 \cdot h$

11. If your desk has an area of 0.5 meters^2, then what is its area in centimeters^2?

$$0.5\,m^2 \cdot \frac{(100\,cm)^2}{(1\ m)^2} = 0.5 \cdot 10^4\ cm^2 = 5 \cdot 10^3\ cm^2$$

$$= 5000\ cm^2$$

12. If a ball has a diameter of 18 cm, then what is the volume of the ball in meters^3?

$V = \frac{4}{3}\pi r^2$ radius not diameter!

18 cm → 0.18 m ← diameter

0.09 m ← radius

$V = \frac{4}{3}\pi (0.09\,m)^3$

$\boxed{V = 0.003\,m^3}$

13. Put the number 21,345,000,000 kg in scientific notation?

$2.1345 \cdot 10^{10}$ kg

14. Put the number 0.0000000234 km in scientific notation?

$2.34 \cdot 10^{-8}$ km

15. Correct the scientific notation of 140×10^{-3} seconds.

$1.40 \cdot 10^{-1}\,s = 0.140\,s$

16. Correct the scientific notation of 0.012×10^{-3} meters.

$1.2 \cdot 10^{-5}$ m

17. What is 0.000 345 meter in micrometers µm?

$1\,\mu m = 10^{-6}\,m$

$3.45 \cdot 10^{-4}\,m \cdot \frac{1\,\mu m}{10^{-6}\,m} = 3.45 \cdot 10^{2}\,\mu m = \boxed{345\,\mu m}$

18. What is 3.4×10^5 m in kilometers?

10^3

$3.4 \cdot 10^5\,km \cdot \frac{1000\,m}{1\,km} = 3.4 \cdot 10^8\,m$

19. If I have three times as many marbles as you do then what is the ratio of my marbles to yours? What is the ratio of your marbles to mine?

3 to 1

3:1

$\frac{3}{1} = \frac{\text{mine}}{\text{yours}}$ $\frac{1}{3} = \frac{\text{yours}}{\text{mine}}$

20. If I increase in speed from 25 mph to 40 mph, then by what percent has my speed changed? By what ratio has the speed changed?

Ratio:

$\frac{V_2}{V_1} = \frac{40\text{ mph}}{25\text{ mph}} = 1.6$

$V_2 = 1.6 \cdot V_1$

% Δ

$\%\Delta = \left(\frac{V_2 - V_1}{V_1}\right) \times 100\%$

$\%\Delta = \left(\frac{V_2}{V_1} - \frac{V_1}{V_1}\right) \times 100\%$

$\boxed{\%\Delta = \left(\frac{V_2}{V_1} - 1\right) \times 100\%}$

+60%

$\left(\frac{40-25}{25}\right) \times 100 = 60\%$

21. If my speed changes from 29 mph to 10 mph, then by what percent has my speed changed? What is the negative sign in the answer tell you? By what factor has your speed changed from initially to finally? Could a negative ratio make sense here?

$\%\Delta = \left(\frac{10}{29} - 1\right) \times 100 = -65.5\%$

65.5% decrease

$\frac{10}{29} = 0.34$ $< 1 \rightarrow$ decrease

22. If the generic variable *y* is inversely proportional to the a variable *x*, then write out this statement mathematically in two ways.

Proportion equation

$y \propto x^{-1}$

$\frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^{-1}$

equation w/ a constant of proportionality

$y = k \cdot x^{-1}$

23. Since the area of a triangle is directly proportional to both the base and height, then how would an *equation* for this look like and what is the constant of proportionality?

proportionality equation:

$\frac{A_2}{A_1} = \frac{b_2}{b_1} \cdot \frac{h_2}{h_1}$

equation w/ constant of proportionality

$A_\Delta = \frac{1}{2} \cdot b \cdot h$

$k = \frac{1}{2}$

24. The formula for the volume of a cylinder is $V = \pi r^2 h$. How would you write out a proportionality statement that was consistent with this formula? What is the constant of

$V \propto r^2 h$

$k = \pi$

proportionality?

As a proportion equation

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^2 \cdot \frac{h_2}{h_1}$$

$V = \pi r^2 h$

$$\frac{V_2}{V_1} = \frac{\cancel{\pi} r_2^2 \cdot h_2}{\cancel{\pi} r_1^2 \cdot h_1} = \frac{r_2^2}{r_1^2} \cdot \frac{h_2}{h_1} = \left(\frac{r_2}{r_1}\right)^2 \cdot \frac{h_2}{h_1}$$

"the new radius is 2.7 times bigger than the old one"

25. "If the radius of a sphere changes by a factor of 2.7," then by what factor does the volume change?

$\frac{r_2}{r_1} = 2.7$

$V \propto r^3$

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 = (2.7)^3 = 19.7$$

$$\frac{V_2}{V_1} = 19.7$$

26. How would you turn the previous problem "inside out"?

if the volume of a sphere increases by a factor of 19.7, then by what factor did the radius increase?

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 \Rightarrow [19.7]^{\frac{1}{3}} = \left[\left(\frac{r_2}{r_1}\right)^3\right]^{\frac{1}{3}} \Rightarrow 2.7 = \frac{r_2}{r_1} ✓$$

27. By what percent does the volume change in the previous two problems?

$$\%\Delta_V = \left(\frac{V_2}{V_1} - 1\right) \times 100\%$$

$$= (19.7 - 1) \times 100\% = 1870\%$$

"change" → by some %

28. If I increase the radius of a circle by 10%, then by what percent does the area change?

$$\%\Delta_r \longrightarrow \frac{r_2}{r_1} \longrightarrow \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 \longrightarrow \frac{A_2}{A_1} \to \%\Delta_A$$

$$10\% = \left(\frac{r_2}{r_1} - 1\right) \times 100\%$$

$$0.1 = \frac{r_2}{r_1} - 1 \Rightarrow \frac{r_2}{r_1} = 1.1$$

$+1 \quad +1$

$$\frac{A_2}{A_1} = (1.1)^2$$

$$\frac{A_2}{A_1} = 1.21$$

$$\%\Delta_A = \left(\frac{A_2}{A_1} - 1\right) \times 100\%$$

$$= (1.21 - 1) \times 100\%$$

$$= 21\%$$

negative

29. What if the radius decreases by 10%?

$$-10\% = \left(\frac{r_2}{r_1} - 1\right) \times 100$$

$$-0.1 = \frac{r_2}{r_1} - 1$$

$$\frac{r_2}{r_1} = 0.9$$

$$\frac{A_2}{A_1} = (0.9)^2$$

$$\frac{A_2}{A_1} = 0.81$$

$$\%\Delta_A = (0.81 - 1) \times 100\%$$

$$= -19\%$$

30. In order to "double" the volume of a sphere, by what factor must the radius change?

$$\frac{V_2}{V_1} = 2$$

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3$$

$$2 = \left(\frac{r_2}{r_1}\right)^3 \Rightarrow \frac{r_2}{r_1} = 1.26 \rightarrow \text{or } 26\%$$

"half" $\frac{V_2}{V_1} = \frac{1}{2}$ "third" $\frac{V_2}{V_1} = \frac{1}{3}$

The electric power P drawn from a generator by a lightbulb of resistance R is $P = V^2/R$, where V is the line voltage. The resistance of bulb B is 49% greater than the resistance of bulb A. What is the ratio P_B/P_A of the power drawn by bulb B to the power drawn by bulb A if the line voltages are the same?

$$P = \frac{V^2}{R} \Rightarrow P = V^2 \cdot R^{-1}$$

$$49\% = \left(\frac{R_B}{R_A} - 1\right) \times 100$$

$$\frac{P_B}{P_A} = \cancel{\left(\frac{V_B}{V_A}\right)^2}^{1} \cdot \left(\frac{R_B}{R_A}\right)^{-1}$$

$$\frac{R_B}{R_A} = 1.49$$

$$\frac{P_B}{P_A} = \left(\frac{R_B}{R_A}\right)^{-1}$$

$$\frac{P_B}{P_A} = (1.49)^{-1} = \underline{\underline{0.67}} \checkmark$$