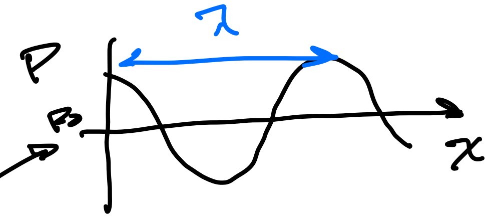
Sound

mechanical – requires molecules

longitudinal wave – $\leftrightarrow$ molecules } parallel
$\rightarrow$ wave

high P
compression

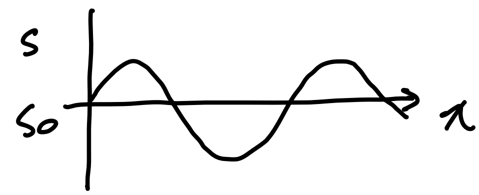
low P
rarefaction



Pressure : $P(x,t) = P_0 \cos(kx - \omega t)$

Displacement $s(x,t) = s_0 \sin(kx - \omega t)$

motion of
1 molecule



velocity of sound

$V_{solid} > V_{liquid} > V_{gas}$ does not travel in vacuum

$$V_1 = V_0 \sqrt{\frac{T_1}{T_0}}$$ must be in Kelvin

$$V = 331\,\text{m/s} \sqrt{\frac{T}{273K}}$$ in gas

Speed in gas is $331\,\text{m/s}$ at $273\,K$, $0°C$

At the end of this worksheet, you should be able to

- calculate the speed of waves in air at varying temperatures.
- operate the Kelvin temperature scale as an absolute temperature scale.
- calculate the intensity of a sound wave at any distance from the source.
- calculate the loudness of any sound source including the listener's distance from the source.
- apply the principles of standing wave to pipes open on one end or open on both ends.
- use the principle of beat frequency to calculate for an unknown frequency.

Speed of Sound in Air

1. The speed of sound in a gas is proportional to the square root of the *absolute* temperature. In the SI system, the Kelvin temperature scale is an absolute measure of temperature, because 0 K is the lowest conceivable temperature.

   a) What is the conversion between °C and Kelvin?

   $K = {}^\circ C + 273$

   b) What is 0°C in Kelvin?

   273 K

   c) What is room temperature (21°C) Kelvin?

   294 K

   d) What is 300 K in Celsius?

   27°C

   e) What is 100 K in Celsius?

   -173K

2. The size increments of the Kelvin and Celsius temperature scales are identical. If the outside temperature gets 5°C hotter, how much does it change in Kelvin?

   Same increment

   so + 5K

3. The velocity of sound in air is directly proportional to the square root of the absolute temperature of the air. The reference speed of sound in air of $v_0 = 331$ m/s at a temperature of $T = 0$°C or 273.15 K.

   a) What is the speed of sound at 20°C?

   $$v = 331\,\text{m/s}\sqrt{\frac{293}{273\,\text{K}}} = 343\,\text{m/s}$$

   b) At what temperature is the speed of sound equal to 300 m/s?

   $$300 = 331\sqrt{\frac{T}{273}}$$

   $$T = 224\,\text{K}$$

   c) At what temperature is it equal to 400 m/s?

4. The temperature of air decreases by 10%.

   a) By what factor does temperature decrease?

   $$\frac{T_2 - T_1}{T_1} * 100\% = 10\%$$

   factor $\frac{T_2}{T_1}$ $\rightarrow$ $\left(\frac{T_2}{T_1} - 1\right) = -0.1 \rightarrow \frac{T_2}{T_1} = 0.9$

   b) By what factor does the speed of sound decrease?

   $$\frac{v_2}{v_1} = \sqrt{0.9} = 0.95$$

   c) By what percent does the speed of sound decrease?

   $$(\text{factor} - 1) * 100\% = ?$$

   $$-5\%$$

Amplitude, Intensity, and Loudness
(I) decibels

$$I = \frac{\text{power}}{\text{Area}} = \frac{\text{power}}{4\pi r^2} \propto \text{pressure}^2 \qquad \text{units: } \frac{\text{Watts}}{m^2}$$

Area is the surface area of sphere

loudness

$$\beta = 10 \log \frac{I}{10^{-12} W/m^2}$$

Threshold of hearing
10^{-12} W/m²

$10^{\frac{\beta}{10}} \propto$

reminder

$\log(4632) = 3.67$

$10^{3.67} = 4632$

$$10^{\beta} = 10^{1}\left(\frac{I}{I_0}\right)$$

$$10^{\beta - 1} = \frac{I}{I_0}$$

Energy, Power, and Intensity of Sound and Sound Level

5. A firework explodes releasing 100 kJ of energy in 0.001 s.

$k = 10^3$

a) What power is this?

$$\mathbb{P} = \frac{\Delta E}{t} = \frac{100 \times 10^3\,J}{0.001\,s} = \frac{10^5\,J}{10^{-3}\,s} = 10^8\,W$$

$1 \times 10^8\,W$

b) Assume 10% of the fireworks' power goes into sound energy. What is the power of the sound?

$$\mathbb{P}_S = 10^8 (0.1) = 10^7\,W \checkmark$$

$1 \times 10^7\,W$

c) What is the intensity of the sound at 1.0 meter from the firework?

$$I = \frac{10^7\,W}{4\pi (1m)^2} = 795774 = 7.96 \times 10^5 \frac{W}{m^2}$$

d) What is the sound level of the sound at 1.0 meter? ($I_0 = 10^{-12}$ W/m^2)?

$$\beta = 10 \log \frac{I}{I_0} = 10 \log \frac{7.96 \times 10^5}{10^{-12}}$$

$$\beta = 10 \log (7.96 \times 10^{17}) = 179 \text{ dB}$$

converts Bels to dB

Double Power

6. A 2nd firework explodes releasing 200 kJ of energy in 0.001 seconds.

a) What is the intensity of the sound at 1.0 meter?

$$I_2 = 2 I_1 = 2(7.96 \times 10^5\,w/m^2)$$

8

$$I_2 = 16 \times 10^5\,w/m^2$$

b) What is the sound level of the sound at 1.0 meter?

$$\beta_2 = 10 \log (16 \times 10^{17})$$

$$\beta_2 = 182 \text{ dB}$$

Rule of Thumb: Double Intensity → add +3dB
Half Intensity → subtract 3dB

Effect of Distance on Intensity and Sound Level

8. A firework explodes releasing 100 kJ of energy in 0.001 s. $P_s = 10^7 \text{ W/m}^2$

a) At 2.0 meters (double distance)

- What is the intensity of the sound of the firework at 2.0 meters? By what *factor* does intensity change?

$$I_3 = \frac{10^7 \text{ W/m}^2}{4\pi (2\text{m})^2} = \frac{1}{4}\left(8\times10^5 \text{ W/m}^2\right)$$

$$I_3 = 2\times10^5 \text{ W/m}^2$$

- What is the sound level at 2.0 meters? What is the *change* in sound loudness level? Hint: subtract the sound level of I_2 and I_1 and then use the properties of logarithms that $\log A - \log B = \log (A/B)$

$$\beta = 10 \log\left(\frac{2\times10^5}{10^{-12}}\right) = 173 \text{ dB}$$

Rule of Thumb: Double distance → decrease −6dB
Half distance → increase +6dB

b) At 10.0 meters (10*distance)

- What is the intensity of the sound of the firework at 10.0 meters? By what *factor* does intensity change?

$$P = 10^7 \text{ W}$$

$$I = \frac{10^7 \text{ W}}{4\pi (10)^2} = \frac{1}{100}\left(8\times10^5 \text{ W/m}^2\right) = 8\times10^3 \text{ W/m}$$

- What is the sound level at 10.0 meters? What is the *change* in sound loudness level? Hint: subtract the sound level of I_2 and I_1 and then use the properties of logarithms that $\log A - \log B = \log (A/B)$

$$\beta = 10 \log\left(\frac{8\times10^4}{10^{-12}}\right) = 159 \text{ dB}$$

Rule of Thumb: Distance by a factor of 10 → ±20dB

9. An observer moves so that the intensity of the sound from a speaker doubles.

a) By what factor does the distance between the speaker and the observer change?

Closer

same, 4π

$$I = \frac{P}{4\pi r^2}$$

$$I \propto \frac{1}{r^2}$$

flipped/inverted

$$\frac{I_2}{I_1} = \frac{2I_1}{I_1} = \left(\frac{r_1}{r_2}\right)^2 \rightarrow \frac{r_1}{r_2} = \sqrt{\frac{2\cancel{I_1}}{\cancel{I_1}}}$$

$$r_2 = \frac{1}{\sqrt{2}} r_1 = 0.71\, r_1$$

b) What is the *change* in sound loudness level?

+ 3dB (see above)

7. The sound level of a normal conversation at 1.0 meter is 60 dB (at your ear).

a) What is the intensity of the sound?

$\beta = 10 \log \frac{I}{I_0}$ ← 10^{-12} W/m²

$60 = 10 \log \frac{I}{I_0}$

$6 = \log \frac{I}{I_0}$

$10^6 = \frac{I}{10^{-12}} \rightarrow I = 10^{-6}$ W/m²

b) If an eardrum (also called the tympanic membrane) has a diameter of 0.5 cm, what power is delivered to the eardrum?

$I = \frac{P}{4\pi r^2} \rightarrow P = I(4\pi r^2) = 10^{-6}(4\pi)(0.0025)^2$

$P = 7.9 \times 10^{-11}$ Watts

$r = \frac{0.5\text{cm}}{2} = 0.25\text{cm} = 0.0025\text{m}$

Sound in Tubes/Musical Instruments

10. What is the fundamental frequency and wavelength of a 1.0 m long pipe open at both ends at 24°C? What is the next highest frequency that supports a standing wave in the pipe?

273

$v = 331 \text{ m/s} \sqrt{\frac{297\text{K}}{273\text{K}}} = 345.24 \text{ m/s}$

$\lambda = \frac{2L}{n} = \frac{2(1.0\text{m})}{1} = 2.0\text{m}$

$f = \frac{v}{\lambda} = \frac{345 \text{ m/s}}{2.0\text{s}} = 173$ Hz

$f_2 = 2(173) = 345$ m/s

11. What is the fundamental frequency and wavelength of the same pipe when it is closed at one end? What is the next highest frequency that supports a standing wave in the pipe?

$\lambda = \frac{4L}{n} = \frac{4(1.0)}{1} = 4.0\text{m}$

$f = \frac{345}{4} = 87$ Hz

$f_3 = 3(87) = 261$ Hz

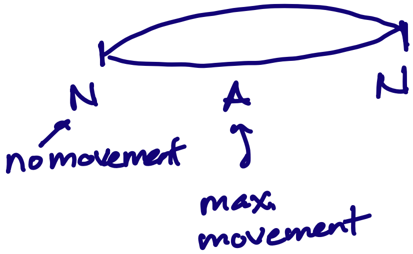
Tubes

String

$$v = \sqrt{\frac{F_T}{\mu}}$$

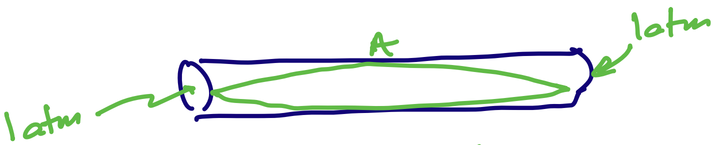
Tube

$$v = v_0\sqrt{\frac{T}{T_0}} = 331\sqrt{\frac{T}{273K}}$$



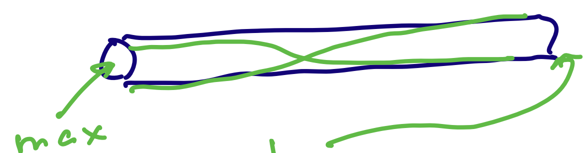
Open at both ends

pressure



first harmonic

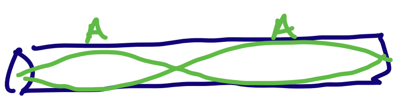
displacement of air



$\lambda = \frac{2L}{n}$ n = antinodes

v = set by temperature

$$f = \frac{v}{\lambda}$$



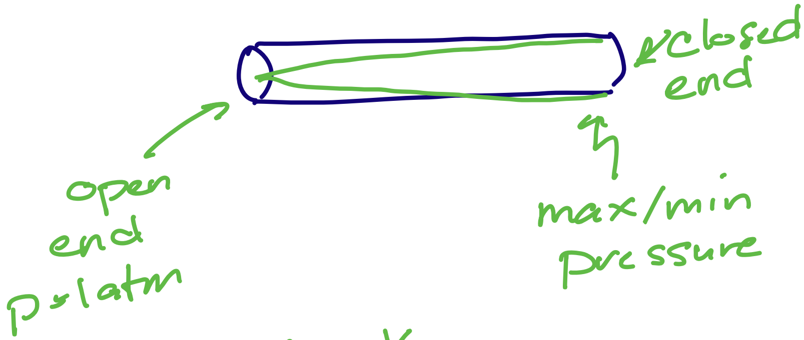
+ add $\frac{1}{2}\lambda$, then shrink to fit

$\lambda = \frac{2L}{n}$ $n = 2$

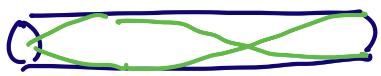
$$f_2 = 2f_1 = nf_1$$

all frequencies are integer multiples

Closed at one end

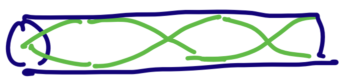


$\lambda = \frac{1}{4} L$



add $+\frac{1}{2}\lambda$

$\lambda = \frac{3}{4} L$



add $\frac{1}{2}\lambda$

$\lambda = \frac{5}{4} L$

$$\lambda = \frac{4L}{n}$$

Now $n = 1, 3, 5, 7 \ldots$

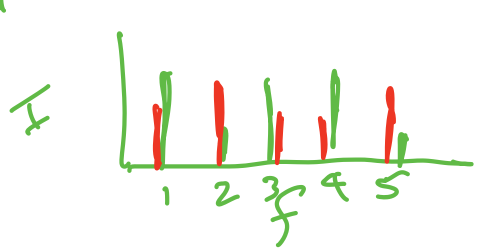
odd integers only

$$f = n f_1$$

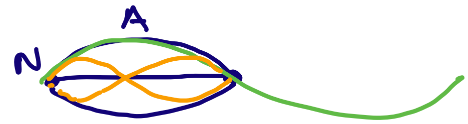
if $f_1 = 30\,Hz$ then $f_3 = 3(30) = 90\,Hz$

no $f_2 = 60\,Hz$ will resonate

Sound Profile



12. The speed of a wave on a 1.0 m long string is 100 m/s.



$\lambda = 2L$

n = # Antinodes loops

a) What is the fundamental wavelength and frequency?

$\lambda = \frac{2L}{n} = 2.0\,m$

$f = \frac{v}{\lambda} = \frac{100\,m/s}{2\,m/s} = 50\,Hz$

b) The air temperature around the string is 25ºC. What is the frequency and wavelength of the air pressure wave? Remember: v_0 = 331 m/s at T = 273 K.

$v = 331\sqrt{\frac{298}{273}} = 346\,m/s$

$f = 50\,Hz$

$\lambda = \frac{346}{50} = 6.9\,m$

c) The temperature drops to 10ºC. By what factor does the fundamental frequency change?

$v = v_1\sqrt{\frac{T}{T_1}}$

$v \propto \sqrt{T}$

$v = f\lambda$

combine

$\lambda \propto \sqrt{T}$

f stays the same

v, λ decrease

$\frac{\lambda_2}{\lambda_1} \propto \sqrt{\frac{T_2}{T_1}} \rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{283}{298}} = 0.97$

so $\lambda_2 = (0.97)(6.9) = 6.7\,m$ Shorter

13. An open pipe, 2.0 m long, creates resonant sounds. Assume the speed of sound is 340 m/s.

a) What are the three lowest standing wave frequencies?

$\lambda_1 = \frac{2L}{n} = \frac{2(2.0)}{1} = 4.0\,m$

$f_1 = \frac{340\,m/s}{4\,m} = 85\,Hz$

$f_2 = 2(85) = 170\,Hz$

$f_3 = 3(85) = 255\,Hz$

b) What are the three lowest standing wave frequencies when one end is closed?

$\lambda_1 = \frac{4L}{n} = 8\,m$

$f_1 = \frac{340}{8} = 42.5\,Hz$

$f_3 = 3(42.5) = 127.5\,Hz$

$f_5 = 5(42.5) = 212.5\,Hz$

14. Two strings of a guitar are being played at the same time. One string has a frequency of 400Hz. A beat frequency of 5.0 Hz can be heard.

a) What are the possibilities for the frequency of the second string?

$f_b = |f_2 - f_1|$

405 Hz
395 Hz

b) The tension in the second string is increased, and the beat frequency decreases to 2.0 Hz. What can you conclude about the frequency of the second string before it was tightened?

$\uparrow F_T$, $\uparrow v$, $\uparrow f$

must be 395Hz
$\uparrow f$ and got closer to 400Hz

c) By what factor and percentage was the tension in the string increased?

$v \propto f \propto \sqrt{F_T}$

$$\left(\frac{f_2}{f_1}\right)^2 = \frac{F_{T2}}{F_{T1}}$$

$$\text{factor} = \left(\frac{398}{395}\right)^2 = 1.015$$

% change = (factor − 1) * 100
% change = 1.5%

d) Reimagine the last problem and tightening the string had *increased* the beat frequency. What would you conclude about the original frequency now?

$f = 405$ Hz

Beats

loudness changes because $f_1 \neq f_2$ but close to each other

$f_b = |f_2 - f_1|$

$$\% \text{ change} = \left(\frac{x_2 - x_1}{x_1}\right) * 100\%$$

$$= \left(\frac{x_2}{x_1} - 1\right) * 100\%$$

factor

$$\boxed{\% \text{ change} = (\text{factor} - 1) * 100\%}$$

Table 12.2 Pressure Amplitudes, Intensities, and Intensity Levels of a Wide Range of Sounds in Air at 20°C (Room Temperature)

Sound	Pressure Amplitude (atm)	Pressure Amplitude (Pa)	Intensity (W/m^2)	Intensity Level (dB)
Threshold of hearing	3×10^{-10}	3×10^{-5}	10^{-12}	0
Leaves rustling	1×10^{-9}	1×10^{-4}	10^{-11}	10
Whisper (1 m away)	3×10^{-9}	3×10^{-4}	10^{-10}	20
Library background noise	1×10^{-8}	0.001	10^{-9}	30
Living room background noise	3×10^{-8}	0.003	10^{-8}	40
Office or classroom	1×10^{-7}	0.01	10^{-7}	50
Normal conversation at 1 m	3×10^{-7}	0.03	10^{-6}	60
Inside a moving car, light traffic	1×10^{-6}	0.1	10^{-5}	70
City street (heavy traffic)	3×10^{-6}	0.3	10^{-4}	80
Shout (at 1 m); or inside a subway train; risk of hearing damage if exposure lasts several hours	1×10^{-5}	1	10^{-3}	90
Car without muffler at 1 m	3×10^{-5}	3	10^{-2}	100
Construction site	1×10^{-4}	10	10^{-1}	110
Indoor rock concert; threshold of pain; hearing damage occurs rapidly	3×10^{-4}	30	1	120
Jet engine at 30 m	1×10^{-3}	100	10	130

<u>More Practice</u>

15. The intensity of music from a loudspeaker at a concert is 1.0 W/m^2 at a distance of 1.0 m away. What is the intensity 10 m away? 100 m away?