Momentum - collisions!

Newton's 3rd $F_{AB} = -F_{BA}$ equal and opposite forces

$F = ma = m\frac{\Delta v}{\Delta t}$ ← change in speed.

time of contact between 2 objects during collision IS THE SAME

$\boxed{F\Delta t = (m\Delta v)}$ so change in momentum during a collision is equal and opposite

magnitude same same

$\boxed{p = mv}$

units: $kg\,m/s$ or Ns $(F\Delta t)$

Notes after Q2 before Q3

$F\Delta t = m\Delta v$

Two identical cars both moving 35m/s and come to rest

$(m\Delta v)_1 = m(0-35) = -35m$

$(m\Delta v)_2 = -35m$ same

Impulse $= F\Delta t$

$F\Delta t \rightarrow$ same for both!

Car 1 - 50's car

no seatbelts

big metal

Car 2 2023 car

seatbelts, air bags

plastic crunches

increases time

Force on human decreases

Types of Collisions

① explosion $P_i = 0$ P_f adds to zero
$P_{1f} = -P_{2f}$

② Perfectly Inelastic

- stick together — objects deform
- $P_i = (m_{total}) V_f$
- maximum energy is lost
- Δmv for each object is minimal
  force on each object is min.
- clay on door #2 Worksheet

③ Elastic = Energy is conserved
Rare = Billiard Balls

- $m_A V_{A1} + m_B V_{B1} = m_A V_{A2} + m_B V_{B2}$
- $\frac{1}{2} m_A V_{A1}^2 + \frac{1}{2} m_B V_{B1}^2 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{B2}^2$
- Δmv on each object is maximum
  Big force
- Bouncy Ball on door #2

Short cut

usually zero

$$V_{A2} = \frac{m_A - m_B}{m_A + m_B} V_{A1} + \frac{2m_B}{m_A + m_B} V_{B1}$$

$V_{A1} + V_{A2} = V_{B1} + V_{B2}$ +

$$V_{B2} = \frac{2m_A}{m_A + m_B} V_{A1} + \frac{m_B - m_A}{m_A + m_B} V_{B1}$$

$$V_{A1} + V_{A2} = V_{B1} + V_{B2}$$

At the end of this worksheet, you should be able to

- find the momentum of an object or collection of objects.
- find the change in momentum of an object or collections of objects due to an impulse.
- use the conservation of momentum to solve for an unknown quantity.
- use the principle of relative velocity to solve for an unknown in elastic collisions.

This week we are ignoring friction and air resistance. When using the conservation of momentum, we restrict ourselves to talking about the motion of the objects *immediately before* they collide and *immediately after* they collide. Friction and energy losses are insignificant compared to the force of the impact.

1. In this problem, a bug and a car collide.

   a) A 1 g bug flies east along a road at 5 m/s. What is the bug's momentum? Momentum is a vector, and the east is the positive direction in this class.

vector

$$\vec{p} = mv = (0.001\,Kg)(+5\,m/s) = +0.005\,Kgm/s$$

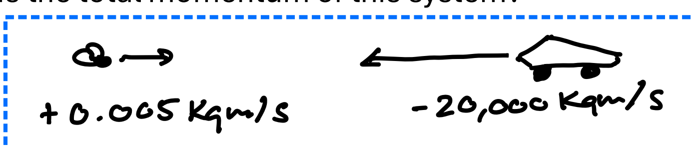
(east)

   b) A 1000 kg car travels west along this same road at a speed of 20 m/s. What is the car's momentum?

$$mv = -20{,}000\,Kgm/s$$

(west)

   c) What is the total momentum of this system?



system

$(mv)_{total} \sim 20{,}000\,\frac{Kgm}{s}$

$= 20{,}000 - 0.005$

   d) When the bug collides with the windshield of the car, what is the momentum of the bug-car system?

momentum is conserved if no external force acts on system

collision between bug/car is internal if system defined correctly. $F_{BC} = -F_{CB}$

Total momentum is conserved

$(\Delta mv)_{bug}$ $\quad \overrightarrow{5\,m/s} \quad \overleftarrow{-20\,m/s}$

$m(v_f - v_i) = (0.001\,Kg)(-20-5)$

$= -0.025\,Kgm/s$

$(\Delta mv)_{car} = +0.025\,Kgm/s$ equal and opposite

2. You want to close a door, but you do not want to get up. You look around and see that there is a bouncy ball that looks like you could throw it fast. For some reason there is also a wad of clay that you know would stick to the door if you threw it. The ball and the clay have the same mass. Which one should you throw against the door to close it most effectively?

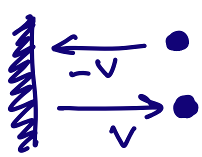
Some starters:

• Choose a mass for your ball and clay and choose a velocity. Or leave it as m and v and work it *in general*.

a) The ball hits the door and bounces back perfectly (its speed doesn't change after it bounces off the door).

i. How has its velocity changed?

vector



$$\Delta v = v_f - v_i = v - (-v) = 2v$$

ii. What is the ball's change in momentum?

$$m\Delta v = m(2v) = 2mv$$

elastic collision

$F\Delta t = m\Delta v$

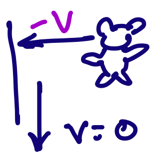
iii. What is the door's change momentum?

$$(m\Delta v)_1 = -2mv$$

equal + opposite

b) The clay hits the door and sticks. Its speed is zero after the collision.

i. How has its velocity changed?



$$\Delta v = 0 - (-v) = v$$

ii. What is the ball's change in momentum?

$$m\Delta v = mv$$

inelastic collision

iii. What is the door's change momentum?

$$(m\Delta v)_2 = -mv$$

c) Assume that the time interval of the collision is about the same for both cases. From Newton's 3rd Law, the force on the door by the thrown object is equal and opposite to the force on the thrown object by the door.

i. Which one of these scenarios imparts a larger force on the door?

$F\Delta t = m\Delta v$ ← The bounce ball because its $m\Delta v$ is twice the teddy bear's.

ii. What is the ratio of the force from the bouncy ball to the force of the clay wad?

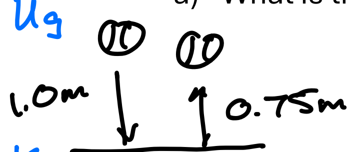
$$\frac{(m\Delta v)_1}{(m\Delta v)_2} = \frac{2mv}{mv} = 2$$

elastic has greater impulse

3. I drop a 0.60-kg basketball from a height of 1.0 m, and it bounces off the floor and rises to a height of 0.75 m.

a) What is the basketball's velocity right before it hits the ground? (energy)

$E_1 = U_g$



$E_2 = K$

$$mgh_1 = \tfrac{1}{2}mv^2$$

$$v = \sqrt{2gh_1} = -4.4\,m/s$$ ← energy does not tell direction

b) What is the basketball's velocity right after it hits the ground? Think about what it must be for it to rise to 0.75 m.

$$\tfrac{1}{2}mv^2 = mgh_2$$

$$v = \sqrt{2gh_2} = 3.8\,m/s$$

c) What is the change in momentum of the ball?

$$\Delta v = 3.8 - (-4.4) = 8.3\,m/s \quad \text{(sig figs)}$$

$$m\Delta v = (0.6\,kg)(8.3) = 4.96\,kg\,m/s \sim 5\,kg\,m/s$$

d) Is momentum conserved here? Why or why not?

BB collided with planet earth. Technically it is conserved if planet is included in system. But let's not do that (big numbers)

$\boxed{\text{System = ball only, } mv \text{ is not conserved}}$

e) What is the ratio of the change in kinetic energy?

$$\frac{KE_2}{KE_1} = \frac{U_{g2}}{U_{g1}} = \frac{mgh_2}{mgh_1} = \frac{0.75}{1.0} = 0.75$$

f) What is the change in kinetic energy?

$$mgh_2 - mgh_1 = mg(h_2 - h_1)$$

$$= (0.6\,kg)(9.8)(0.75 - 1.0)$$

$$= -1.47 \text{ Joules}$$

4. The kinetic energy of a 10 kg object is 100 J, then what is the momentum of the object? Now do this inside out. Now, do it *in general*. In other words, derive $K = \frac{p^2}{2m}$.

$p = mv$ $K = \frac{1}{2}mv^2$ $K = \frac{p^2}{2m}$

$\frac{1}{2m}\left(p^2 = m^2v^2\right)\frac{1}{2m}$

$\frac{p^2}{2m} = \frac{m^2v^2}{2m} = \frac{1}{2}mv^2$

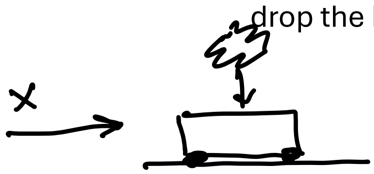
$p = \sqrt{2(10kg)(100J)}$ $p = \sqrt{2mK}$

$p = 44$ kgm/s

#7 homework

5. An empty 10-kg wagon is rolling past me at a speed of 5 m/s. I drop a 30-kg bag of concrete into the wagon right as it passes by. What is its speed immediately after I drop the bag?

Inelastic collision = momentum is conserved



$p_1 = p_2$ ← total

$(10kg)(5\,m/s) + (30kg)(0) = (10 + 30kg)(v_f)$

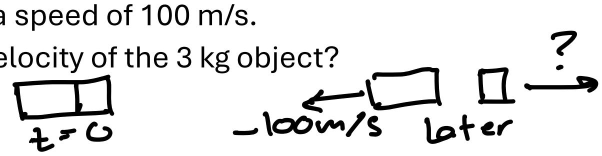
$v_f = 1.25$ m/s

6. Explosions are actually a kind of collision, but in reverse. Two indestructible objects are tied together with a stick of dynamite between them, and everything is at rest. When it explodes, the two objects fly away from each other in opposite directions. One of the objects has a mass of 10 kg and the other a mass of 3 kg. The 10 kg object flies away at a speed of 100 m/s.

Recoil

a) What is the velocity of the 3 kg object?

more room



$V = 333$ m/s to right

$p_1 = 0 = (10kg)(-100\,m/s) + (3kg)(v_f)$

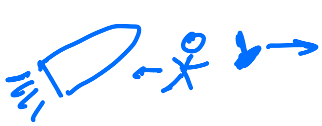
b) How much energy did the explosion give to the two objects? (There was more put into internal energy and sound but let's not consider that.)

$K_1 = 0$ but potential due chemical

$K_f = \frac{1}{2}(10kg)(100)^2 + \frac{1}{2}(3kg)(333)^2 = 2.2 \times 10^5$ J

c) What other problems in this worksheet is this similar to?

#6 homework



to right

$m_1 = 78$ kg
$m_w = 0.720$ kg $v_w = 5.0$ m/s $p_w = 3.6$ kgm/s
$m_s = 0.8$ kg $v_s = 8.0$ m/s $p_s = 6.4$ kgm/s
$m_m = 1.2$ kg $v_m = 6.0$ m/s $p_m = 7.2$ kgm/s

Total p right = 17.2 kgm/s

$p_1 = p_2$

$0 = 17.2 + (78kg)v_f$

$v_f = -0.22$ m/s speed is magnitude (abs. value)

7. A 5 kg gun fires a 10 g bullet ✓ at a velocity of 500 m/s ✓. The explosion that causes this shot takes place in 1 millisecond.

a) What force is exerted on the gun?

$F\Delta t = m\Delta v$

$F(0.001\,s) = (0.010\,kg)(500\,m/s)$

$F = 5000\,N$

b) What is the gun's recoil velocity?

$P_1 = P_2$

$0 = (0.01\,kg)(500\,m/s) + (5\,kg)v_f$

$v_f = -1.0\,m/s$ ← direction

Recoil/Explosion
- mv conserved
- mv inversely prop.
- less massive has greater K

c) Is Kinetic energy conserved? No

$K_1 = 0$

$K_b = \frac{1}{2}(0.01\,kg)(500)^2 = 1250\,J$

$K_g = \frac{1}{2}(5\,kg)(1)^2 = 2.5\,J$

8. Two objects collide [redacted]y, one has a mass of 1 kg and the second has a mass of 3 kg. The first object is traveling to the right at a speed of 3 m/s and the second is traveling in front of it initially at a speed of 1 m/s. What is the velocity of the objects after the collision? Think of and work as many variations of this problem as you can.

Take out elastic homework

a) What is the final velocity of the two objects if they collide perfectly inelastically and stick together?

* lab

1 kg → 3 m/s   3 kg → 1 m/s   same ↓

$P_1 = P_2$

$(1\,kg)(3\,m/s) + (3\,kg)(1\,m/s) = 6\,kg\,m/s = (1+3\,kg)(v_f)$

$v_f = +1.5\,m/s$ →

i. What is each object's change in momentum due to the collision?

$\Delta mv_{1kg} = 1\,kg(1.5 - 3\,m/s) = -1.5\,kg\,m/s$

$\Delta mv_{3kg} = (3\,kg)(1.5 - 1\,m/s) = +1.5\,kg\,m/s$

Sum = 0 always Newton 3rd

b) What is the final velocity of the objects if they are moving in the same direction, and they collide elastically?

energy conserved

$$v_{A2} = \frac{(m_A - m_B)}{(m_A + m_B)} v_{A1} + \frac{2m_B}{(m_A + m_B)} v_{B1} = \frac{(1-3)}{(1+3)}(3) + \frac{2(3)}{(1+3)}(1)$$

$v_{A2} = -1.5 + 1.5 = 0$ at rest $v_{B2} = v_{A1} + v_{A2} - v_{B1} = 2\,m/s$

i. What is each object's change in momentum due to the collision?

$\Delta mv_1 = 1\,kg(0 - 3) = -3\,kg\,m/s$

$\Delta mv_3 = 3\,kg(2 - 1) = +3\,kg\,m/s$

Sum = 0

Contrast a + b   perfectly inelastic $\Delta mv = 1.5\,kg\,m/s$

elastic $\Delta mv = 3.0\,kg\,m/s$

elastic always has greater impulse

c) What is the final velocity of the objects if they are moving in the opposite directions, and they collide elastically?

+/- important

$$V_{A2} = \frac{(1-3)}{(1+3)}(3) + \frac{2(3)}{(1+3)}(-1) = -1.5 - 1.5 = -3.0 \text{ m/s}$$

$$V_{B2} = V_{A1} + V_{A2} - V_{B1} = (3-3) - (-1) = +1 \text{ m/s}$$

i. What is each object's change in momentum due to the collision?

Velocities reversed

$$\Delta m v_1 = (1\,\text{kg})(-3-3) = -6 \text{ kg m/s}$$

$$\Delta m v_2 = (3\,\text{kg})(1-(-1)) = +6 \text{ kg m/s}$$

sum = 0

9. Reverse problem: Two objects collide elastically, one has a mass of 1 kg and the second has a mass of 3 kg. The first object is traveling to the right at a speed of 3 m/s. With what velocity would the second object need to be travelling so that after the collision, the first object was motionless?

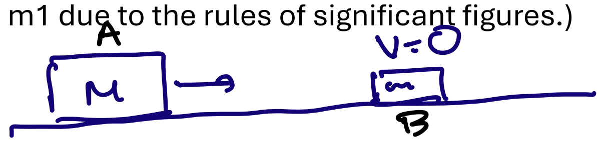
out of class

$$0 = V_{A2} = \frac{(m_A - m_B)}{(m_A + m_B)} V_{A1} + \frac{2m_B}{(m_A + m_B)} V_{B1}$$

$$0 = \frac{(1-3)}{(1+3)}(3 \text{ m/s}) + \frac{2(3)}{(1+3)} V_{B1}$$

10. A massive object moving with a velocity *v* collides elastically with a very small object that is initially at rest. What is the velocity of these two objects after they collide? (By massive and very small I mean when you add or subtract m1 and m2 they are indistinguishable from m1 due to the rules of significant figures.)

Rules of thumb #1



$m_A >> m_B$

$$V_{A2} = \frac{(m_A - \cancel{m_B})}{(m_A + \cancel{m_B})} V_{A1} + \frac{2\cancel{m_B}}{(m_A + \cancel{m_B})} \cancel{V_{B1}}^{\,0}$$

$V_{A2} = V_{A1}$ essentially unchanged

$$V_{B2} = \frac{2m_A}{(m_A + \cancel{m_B})} V_{A1} + \frac{(m_B - m_A)}{(m_A + m_B)} \cancel{V_{B1}}^{\,0}$$

$V_{B2} = 2 V_{A1}$

Rules of thumb #2

11. Reverse problem: A tiny object is colliding with velocity *v* with a huge object that is at rest. What is the final velocity of the two objects? elastic

$m_A << m_B$

$$V_{A2} = \frac{(\cancel{m_A} - m_B)}{(\cancel{m_A} + m_B)} V_{A1} + \frac{2m_B}{(m_A + m_B)} \cancel{V_{B1}}^{\,0} = -V_{A1}$$

reverse at approximately same speed

$$V_{B2} = \frac{2m_A}{(m_A + m_B)} V_{A1} + \frac{(m_B - m_A)}{(m_A + m_B)} \cancel{V_{B1}}^{\,0} = \text{small fraction of } V_{A1}$$

m like we do in the lab and do it *in general*. So, we have a bullet of initial velocity v_{bi} and mass *m*b and a target initially at rest with mass m_t. The collision is perfectly inelastic, and the pendulum (with the bullet inside) rises to a height *h* after the collision takes place. What is the initial velocity of the bullet?

13. Now in question #2 of the lab write-up it asks could this experiment be done with an elastic collision, given that we only measure the same things as we did in the inelastic collision case. So same parameters as before, but this time when the ball bounces off the target it has a speed v_{bf} and the pendulum rises to a height *h* but this time without the bullet embedded in it.

Equal mass - Pool Balls m_B at rest

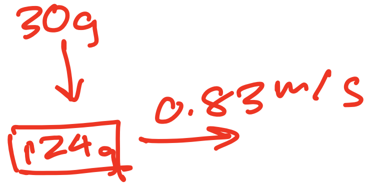
Rules of thumb #3

$$V_{A2} = \frac{\overbrace{(m_A - m_B)}^{=0}}{(m_A + m_B)} V_{A1} + \frac{2m_B}{(m_A + m_B)} \cancel{V_{B1}}^{\,0} = \text{comes to rest}$$

$$V_{B2} = \frac{2m_A}{\underbrace{(m_A + m_B)}_{2m_A}} V_{A1} + \frac{(m_B - m_A)}{(m_A + m_B)} \cancel{V_{B1}}^{\,0} = V_{A1} \text{ moves with } m_A \text{ speed}$$

exchange velocities → happens even if m_B is moving

#7



$$P_1 = P_2$$

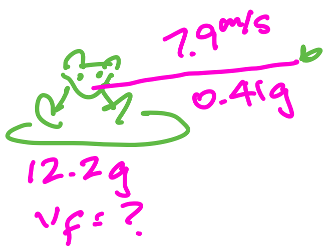
$$(124g)(0.83\,m/s) + (30g)(0) = (124+30)v_f$$

$$v_f = +0.668\,m/s$$

$$\Delta V = V_2 - V_1 = 0.668 - 0.83$$

$$\Delta V = -0.162\,m/s$$

#5



$P_1 = 0$ Recoil

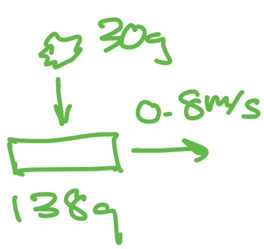
$P_2 = 0$ momentum conserved

$$0 = (12.2g)v_f + (0.41g)(7.9\,m/s)$$

$$v_f = -0.265\,m/s$$

Speed is absolute value of velocity

#7



↓ mv in not conserved

→ mv is conserved

$$P_1 = (138g)(0.8\,m/s) = 110.4\,g\,m/s$$

$$P_2 = 110.4\,g\,m/s \quad \text{(momentum conserved)}$$

$$P_2 = 110.4\,g\,m/s = (138+30)v_f$$

$$v_f = +0.657\,m/s$$

$$\Delta V = 0.657 - 0.8 = -0.143\,m/s$$