Waves

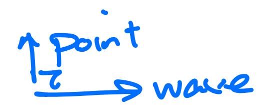
- energy disturbance, energy is transferred

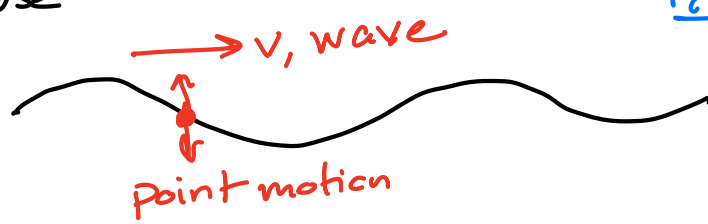
Two types

- mechanical - energy transferred by molecules
  string waves, sound waves
- electromagnetic - does not need matter to propagate

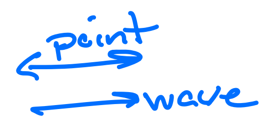
Two methods of wave transfer

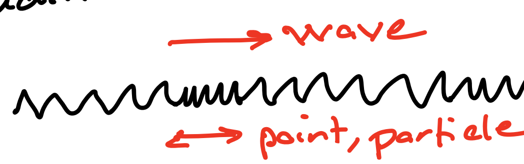
- Transverse wave





- Longitudinal (sound)

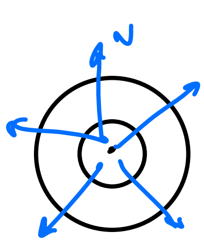




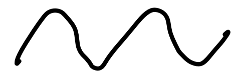
- Both point/particle stays relatively in place
  does not move with wave (energy).

Energy and Intensity

$$I \propto \frac{\text{Power}}{\text{Area}} \propto \frac{\text{Power}}{r^2} \qquad \text{units: } \frac{\text{Watts}}{m^2}$$



<u>Wave</u> Speed $v = \frac{d}{t} = \lambda f = \frac{\lambda}{T}$



- depends on material (water, air, string)
- does not depend on Amplitude, f, λ
- String $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T L}{m}}$

linear mass density $\mu = \frac{m}{L}$

F_T: tension, restoring force

-

At the end of this worksheet, you should be able to

- use the properties of waves to solve for an unknown quantity.
- use the mathematical description of a wave to plot waves motion over time.
- use the conditions of a standing wave on a string to solve for an unknown quantity.

$$F \propto F_T^{1/2} L^{1/2} m^{-1/2} \propto \sqrt{\frac{F_T L}{m}}$$

1. The speed of a wave on a string is proportional to the square root of the tension F_T in the string and the length L, and inversely proportional to the square root the mass m of the string.

   a) By what factor does the velocity of the wave change if the *tension doubles*?

   L, m don't change so $v \propto \sqrt{F_T}$

   $$\frac{v_2}{v_1} = \frac{\sqrt{F_2}}{\sqrt{F_1}} = \frac{\sqrt{2\cancel{F_1}}}{\sqrt{\cancel{F_1}}} = \sqrt{2} \text{ so } v_2 = \sqrt{2}\, v_1$$

   b) By what factor does the velocity of the wave change if the *mass triples*?

   $$v \propto \sqrt{\frac{1}{m}}$$

   $$\frac{v_2}{v_1} = \sqrt{\frac{1/m_2}{1/m_1}} = \sqrt{\frac{1/3m_1}{1/m_1}} = \frac{1}{\sqrt{3}} = 0.58$$

   c) By what factor does the velocity of the wave change if the *length halves (same mass)*?

   $$\frac{v_2}{v_1} = \sqrt{\frac{\frac{1}{2}L_1}{L_1}} = \frac{1}{\sqrt{2}} = 0.71$$

   d) By what factor does the velocity of the wave change if the *mass density doubles*? ✓

   $$v \propto \sqrt{\frac{1}{\mu}}$$

   $$\frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{1}{2}} = 0.71$$

   slows

   e) If the force doubles and the length triples, by what factor does the velocity change? ✓

   $$v \propto \sqrt{\frac{2F_T(3L)}{m}} \qquad \frac{v_2}{v_1} = \sqrt{6}$$

f) By what factor does the force need to change to double the velocity?

$$v \propto \sqrt{F}$$

$$\left(\frac{2\cancel{v_1}}{\cancel{v_1}}\right)^2 = \left(\sqrt{\frac{F_2}{F_1}}\right)^2 \rightarrow 4 = \frac{F_2}{F_1}$$

$$F_2 = 4F_1$$

g) By what factor does the mass density need to change to double the velocity?

$$\frac{2\cancel{v_1}}{\cancel{v_1}} = \sqrt{\frac{1/\mu_2}{1/\mu_1}} = \sqrt{\frac{\mu_1}{\mu_2}} \rightarrow \mu_2 = \frac{1}{4}\mu_1$$

h) If the velocity doubles and the mass density halves, by what factor does the force need to change?

$$v \propto \sqrt{\frac{F_T}{\mu}} \qquad \frac{F_{T2}}{F_{T1}} = \frac{(2v_1)^2(\frac{1}{2}\mu_1)}{v_1^2\mu_1}$$

$$F_T \propto v^2\mu \qquad F_{T2} = 2F_{T1}$$

2. A string is 2 m long and has a mass of 10 g. ← 0.010 kg

a) What is its mass density?

$$\mu = \frac{m}{L} = \frac{0.010\,\text{kg}}{2\,\text{m}} = 0.005\ \text{kg/m}$$

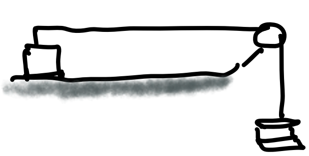
b) The string is cut in half, what is its new mass density?

Same μ, ratio $\frac{m}{L}$

c) A force of 10 N is given to the string, what will be the velocity of waves on the string?

$$v = \sqrt{\frac{F_T}{\mu}} \rightarrow v = \sqrt{\frac{10\,\text{N}}{0.005\,\text{kg/m}}} = 44.7\ \text{m/s}$$

d) What would the force need to be to make the velocity 100 m/s?



$$v = \sqrt{\frac{F_T}{\mu}}$$

$$v^2\mu = F_T \rightarrow F_T = (0.005\,\text{kg/m})(100\,\text{m/s})^2$$

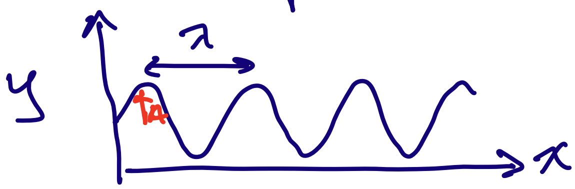
$$F_T = 50\,\text{N}$$

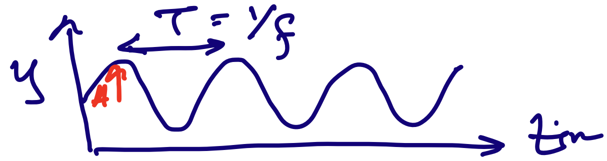
Periodic Waves

heartbeat   harmonic: sine or cosine wave
sinusoidal

characterized $v = \lambda f = \frac{\lambda}{T}$ ← depends on materials

Snapshot   History





motion of segment of string-like
mass-spring SHM

Same amplitude

$v_{max} = A\omega$

$a_{max} = A\omega^2$

together

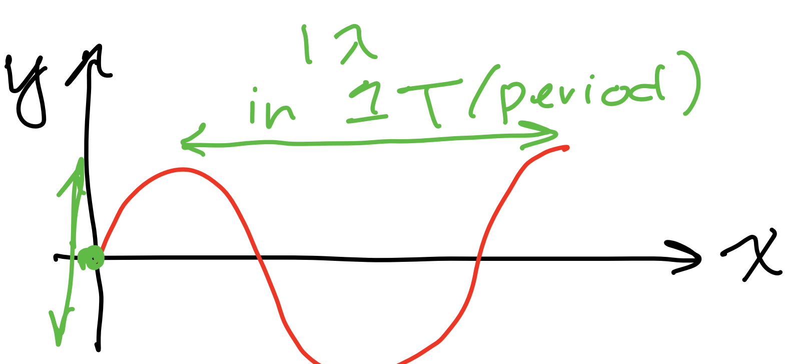
$$y = A\cos(\omega t - kx)$$

wavenumber

$\omega = \frac{2\pi}{T} = 2\pi f$ $k = \frac{2\pi}{\lambda}$

history graph   snapshot graph

not spring constant
reusing K, k



green dot
moves 4A
in 1 T

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda}$$

3. A string with a mass density of 10g/m has a tension of 100 N. A wave with a period of 0.1 s is created on the string.

$\mu = 0.010\ \text{kg/m}$

a) What is the frequency?

$$f = \frac{1}{T} = \frac{1}{0.1\text{s}} = 10\ \text{Hz}$$

b) What is the wavelength of this wave?

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{100\text{N}}{0.01}}$$

$$v = 100\ \text{m/s}$$

sub.

$v = \lambda f$??

$$100\ \text{m/s} = \lambda\ (10\ \text{Hz})$$

$$\lambda = 10\ \text{m}$$

c) What is the angular frequency and what is the wavenumber?

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{0.1} = 62.8\ \text{rad/s}$$

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2\pi}{10\text{m}} = 0.628\ \text{rad/m}$$

k T

4. The wavenumber of a wave is 20 rad/meter, and the period is 0.1seconds.

a) What is the frequency, angular frequency, wavelength, and velocity?

$$f = \frac{1}{T} = \frac{1}{0.1\text{s}} = 10\ \text{Hz}$$

$$\omega = \frac{2\pi}{T} = 62.8\ \text{rad/s}$$

$$k = \frac{2\pi}{\lambda}$$

$$20 = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{20} = 0.314\ \text{m}$$

$$v = \lambda f = \frac{\lambda}{T} = \frac{0.314}{0.1} = 3.14\ \text{m/s}$$

~~b)~~ The force on the string is ~~100N~~ 1.0N. What is the linear mass density of the string?

c) If the string is 10 grams, then what is the length of the string?

5. A 1-meter string with a linear mass density of 10g/m is attached to a wave generator, strung over a pulley, and attached to a 1-kg mass that hangs over the edge of a table (like the lab setup). What is the speed of waves on this string?

$$F_T = (1\,kg)(9.8) = 9.8\,N$$

$$v = \sqrt{\frac{9.8N}{0.01\,kg/m}} = 31.3\,m/s$$

6. An identical string (1 meter, 10 g/m) hangs vertically from a support with the 1kg mass tied on the end. What is the speed of a wave *near the mass*? What is the speed of a wave *near the top*?

more tension, greater velocity

$$F_T = (0.01\,kg/m)(1m)(9.8) = \mu L g$$

$$v = \sqrt{\frac{\cancel{\mu} L g}{\cancel{\mu}}} = \sqrt{Lg} = \sqrt{1m(9.8)}$$

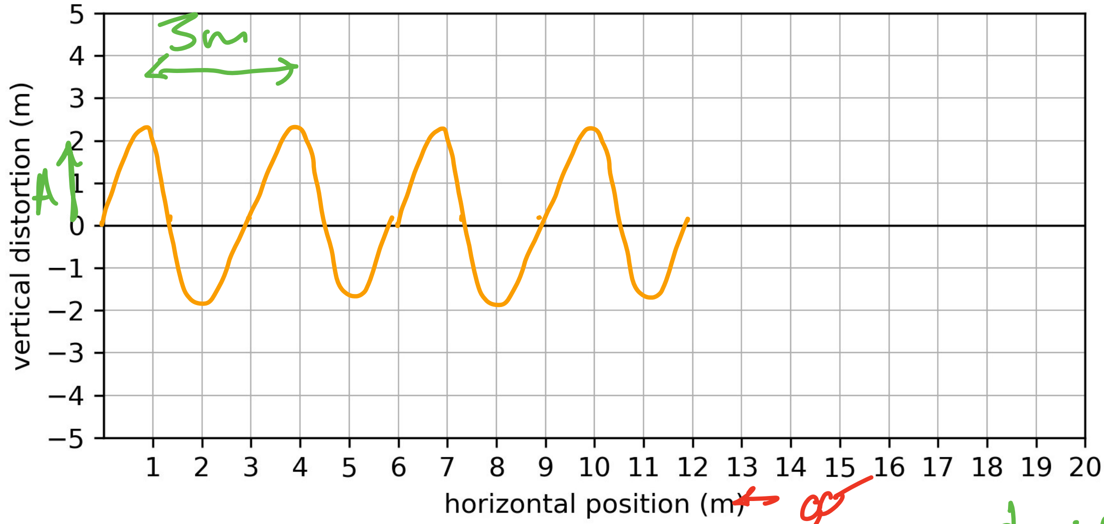
$$v = 3.13\,m/s$$

less tension, say 10 cm above end

$$v = \sqrt{(0.1m)(9.8)} = 1.0\,m/s$$

7. A wave has a wavelength of 3 m and an amplitude of 2 m. It travels at a speed of 5 m/s. If the wave has its maximum at the horizontal position $\boxed{\text{of 0 m when } t = 0\text{s,}}$ then sketch a plot (snapshot) of the wave at this time below:

sine, not cosine

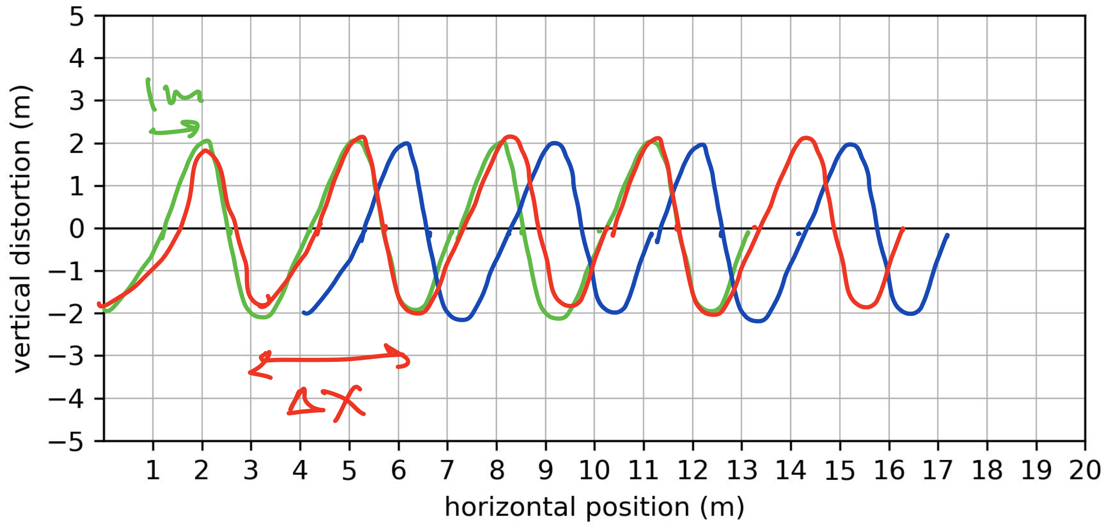


8. Next, sketch the wave after 0.2 seconds have gone by.

$d = vt$

$d = (5 m/s)(0.2s)$

1.0m



9. Now sketch the wave at t = 1.0 second on the same axis as above.

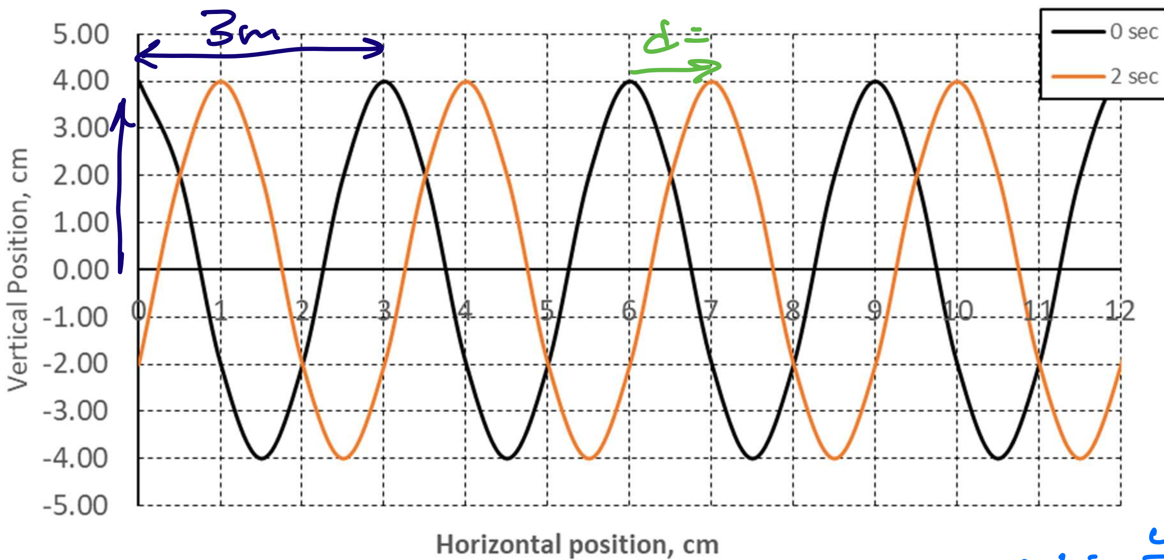
$d = (5 m/s)(1s) = 5m$

in phase - $\Delta x = n\lambda$ any integer

waves have same frequency
overlap entirely
crests at same location

Phase Shift = $1\lambda, 2\pi, 360^\circ$

10. The following plot shows a wave at $t = 0.0$ s and then later when $t = 2.0$ s. What is the amplitude, period, angular frequency, wavelength, wavenumber, and velocity?



$\omega = \frac{2\pi}{T}$

$T = 6.0s$

$A = 4cm$ $\lambda = 3m$

$k = \frac{2\pi}{\lambda} = \frac{2}{3}\pi$

$v = \frac{d}{t}$

$v = \frac{1 cm}{2}$

$v = 0.5 cm/s$

$v = \frac{\omega}{k}$

$0.5cm/s = \frac{\omega}{(2\pi/3)}$

$\omega = \frac{\pi}{3} = 1.05 \frac{rad}{s}$

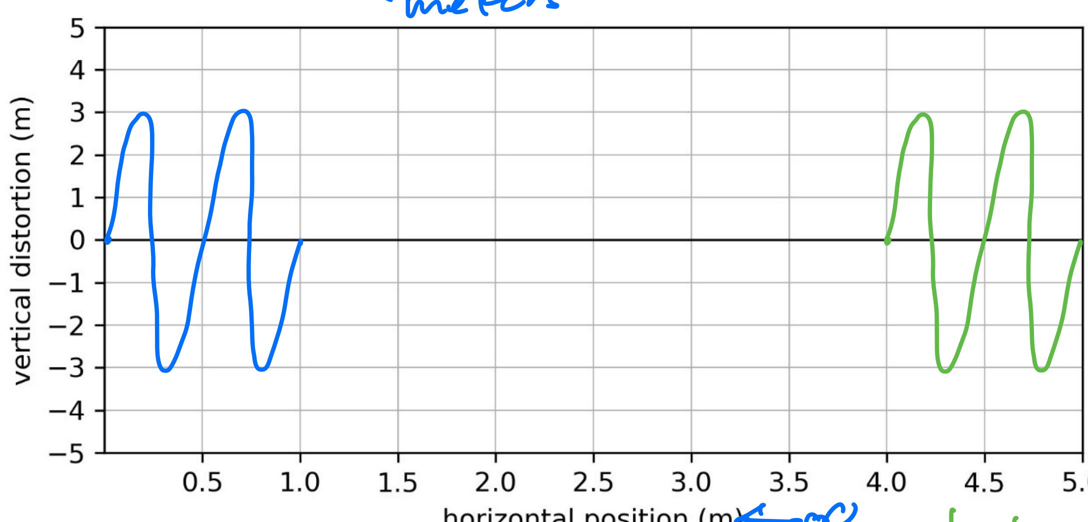
$T = 6.0sec$

11. A wave on a string is described by the equation below. Plot this function for $t = 0$s and again for $t = 2.0$s.

A $k = \frac{2\pi}{\lambda}$ meters

$y(x,t) = (3.0\text{m})\sin((4\pi \text{ rad/m})x-(8\pi \text{ rad/s})t)$

meters



$A = 3m$

$k = \frac{2\pi}{\lambda}$

$4\pi = \frac{2\pi}{\lambda}$

$v = \frac{\omega}{k}$

$= \frac{8\pi}{\ }$ m/s

at $t = 2.0sec$

$d = vt = (2)(2)$

$d = 4m$

Properties of waves

① Reflect

② Refract – move across a border between 2 media

③ Diffract (later – sound, light)

④ Interference with Superposition

Amplitudes add when overlappi

and waves move unaffected

$\lambda = \frac{1}{2}m$ 4π

12. Two strings with different linear densities are tied together forming a longer string. One string (String A) has a linear mass density of 10g/m; the other (String B) has a linear density of 40 g/m. The tension in the string is 100 N.

    a) What is the velocity of the wave on the first side of the knot?

    $$V = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{100N}{0.01 kg/m}} = 100 m/s$$

    b) What is the velocity on the second side?

    $$V = \sqrt{\frac{100N}{0.04 kg/m}} = 50 m/s$$

    ① Refraction wave passes from 1 material to another

    c) A wave with a wavelength of 0.2 meters travels along the string. It originates with String A. What is the frequency of the wave?

    $$V = \lambda f$$

    $$100 m/s = (0.2m) f$$

    $$f = 500 Hz \quad T = \frac{1}{f} = 0.002 sec$$

    d) A crest of the wave passes through the knot. The velocity of the wave changes when it travels in String B. What else about the wave changes? Frequency or wavelength?

    - In other words, how much time goes by before the next crest passes through the knot? What is the frequency of wave crests coming into the knot? What is the frequency of crests leaving the knot?

      frequency unaffected

      same frequency

    - How far has the first crest traveled in String B? What does this mean about the wavelength on the second side?

      wave travels 1λ in 1 period

      $T = 0.002 sec$

      $d = vT = (50 m/s)(0.002 s) = 0.1 m$

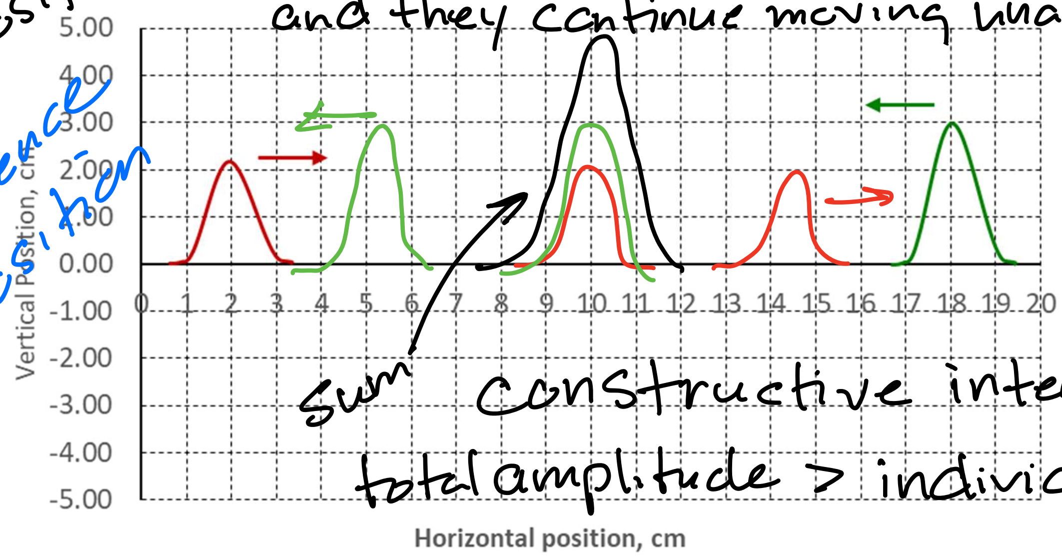
      half the distance in String A

- What can you conclude about waves passing through an interface of 2 different materials? When the speed of a wave changes suddenly at an interface does the wavelength change? Does the frequency change? Or do both change?

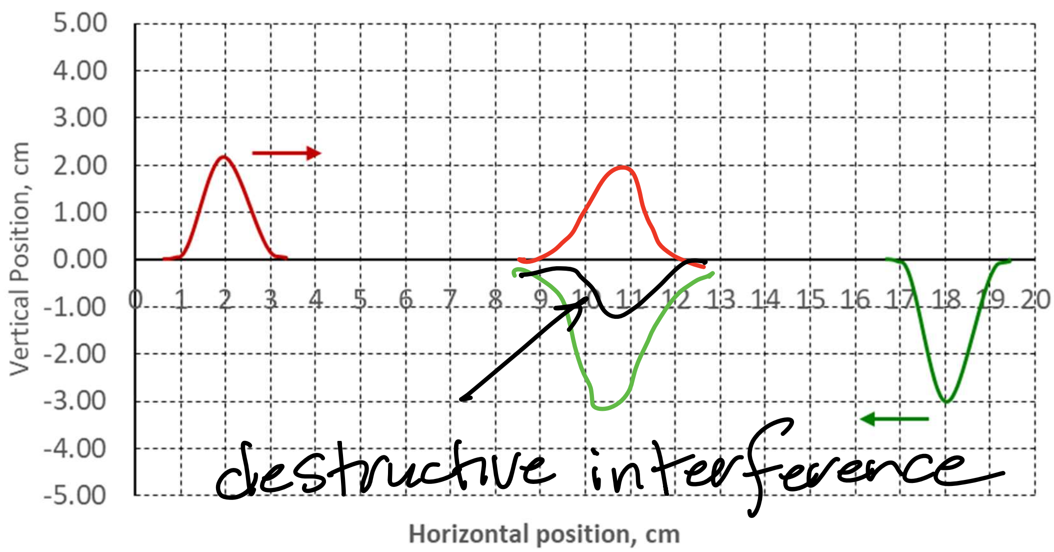
13. Two waves with the same wavelength approach each other with speed $v = 4.0$ m/s. At what time and where will these waves fully overlap (their max peaks line up) and what will the wave look like at that time?

Superposition – when 2 waves meet, Amplitudes add and they continue moving unaffected

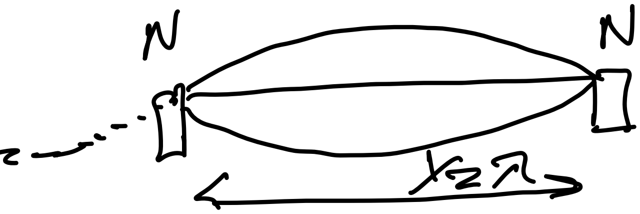
2 Interference by superposition



14. Two waves approach each other with speed $v = 6$m/s. At what time and where will these waves fully overlap and what will the wave look like at that time?



Standing Waves on a String



fixed ends so displacement nodes

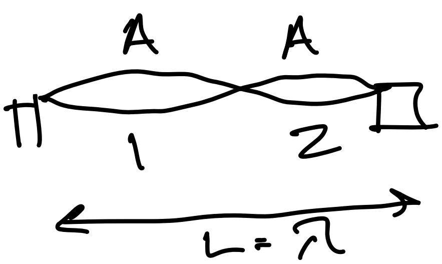
center is antinode
maximum movement

incident wave & reflected add constructively

$$v=\sqrt{\frac{F_T}{\mu}}=\lambda f$$

$$f=\frac{v}{\lambda}$$

$$f_1=\frac{v}{\lambda_1} \leftarrow 2L$$

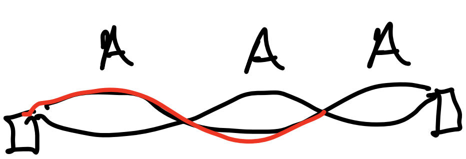


2^{nd} harmonic $n=2$

2 antinodes, 2 loops

$$\lambda=\frac{2L}{n}=\frac{2L}{2}=L$$

$$f=\frac{v}{\lambda}=2f_1$$



3^{rd} harmonic, $n=3$

$$\lambda=\frac{2L}{n}=\frac{2}{3}L$$

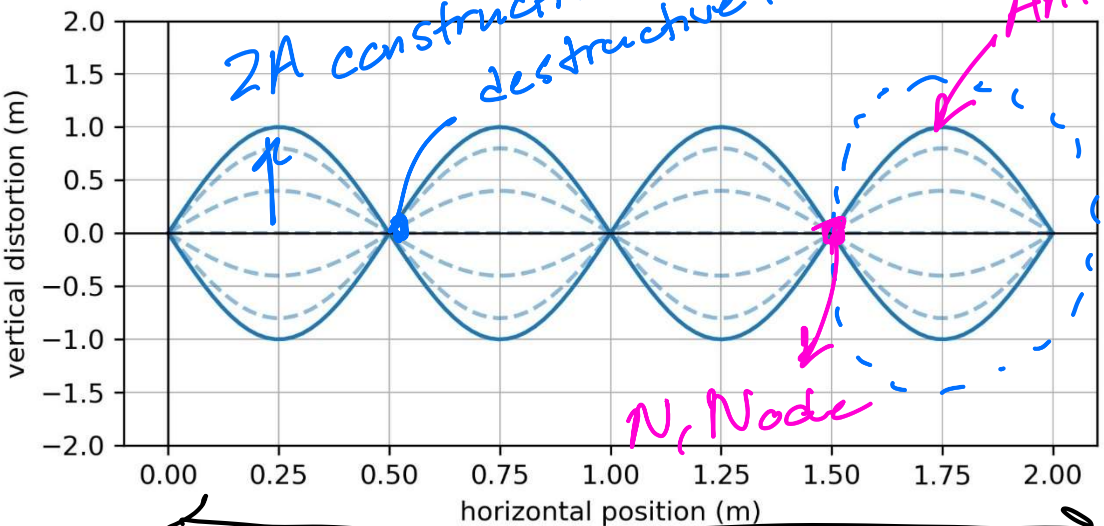
$$f=3f_1$$

15. The plot below represents a standing wave in a string.

2A constructive interference

destructive interference

Antinode, A



1 loop
1 antinode
2 nodes
$= \frac{1}{2}\lambda$

N, Node

L

a) How many antinodes are there?

4

b) How many nodes are there?

5

#Antinodes
#loops

c) What is the wavelength of the standing wave? $\lambda = 1\,m$ $\lambda = \frac{2L}{n}$

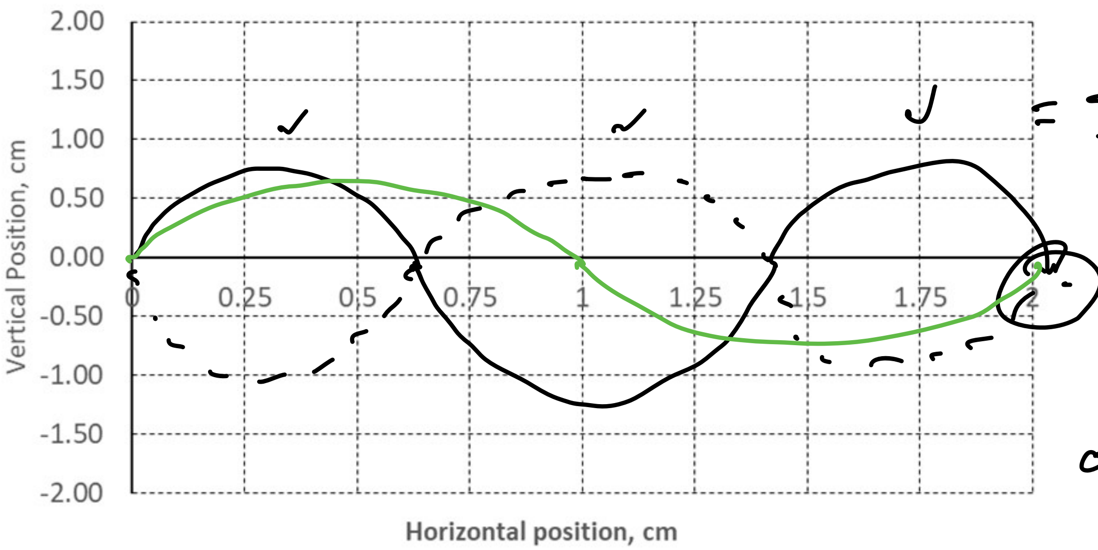
d) The speed of the wave on the string is v = 40 m/s. Calculate the frequency and the period of oscillation of the standing wave?

$$v = 40\,m/s = \lambda f$$

$$40\,m/s = (1.0\,m)\,f$$

$$f = 40\,Hz$$

16. Redraw the string from the above problem with three antinodes.



= 3

= L

length of string

a) What is the wavelength and frequency?

$$\lambda = \frac{2L}{n} = \frac{2(2m)}{3} = \frac{4}{3} = 1.3\,m$$

$$v = \lambda f \rightarrow f = \frac{40\,m/s}{1.3\,m}$$

$$f = 30.8\,Hz$$

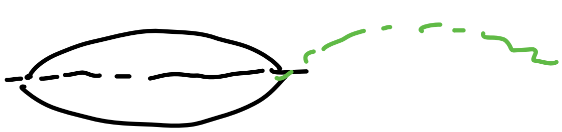
b) Redraw it with 2 antinodes.

$$\lambda = \frac{2L}{n} = \frac{2(2m)}{2} = 2m$$

c) What is the wavelength and frequency?

$$f = \frac{40\,m/s}{2m} = 20\,Hz$$

* d) Next, one antinode. What is the wavelength and frequency?



$$\lambda = \frac{2L}{n} = \frac{2(2m)}{1} = 4m$$

$$f = \frac{40\,m/s}{4m} = 10\,Hz$$

e) What is the 10th harmonic frequency? What is the 10th harmonic wavelength?

harmonic = n

$$f_2 = n f_1$$

$$f_1 = v\left(\frac{2L}{n}\right)$$

1

17. Instead of changing the frequency to make fewer anti-nodes, we could change the velocity instead, and the easiest way to do that is to adjust the tension in the string. So, if the standing wave goes from 4 antinodes to 3 antinodes, by what ratio (factor) does the *speed* of the wave change? If the speed of the wave changes by this ratio, then by what ratio does the tension on the string need to change?

$v = \sqrt{\frac{F_T}{\mu}} = \lambda f$ constant μ, f, L

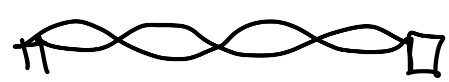
& see below in blue

$\sqrt{F_T} \propto \lambda = \frac{2L}{n} \propto \frac{1}{n}$

$\frac{\sqrt{F_{T3}}}{\sqrt{F_{T4}}} = \frac{n_4}{n_3} \rightarrow F_{T3} = \left(\frac{n_4}{n_3}\right)^2 F_{T4}$

18. A wave has a 4th harmonic frequency of 200Hz. What is the fundamental frequency?

$n = 4$



$f_4 = 4 f_1$

$200\text{Hz} = 4 f_1$

$f_1 = 50\text{Hz}$

19. A standing wave is produced with a frequency of 98Hz; the next standing wave frequency is 112Hz. What is the fundamental frequency? How many antinodes were there for these two standing waves?

$(n+1) f_1 - n f_1$

$n f_1 = 98\text{Hz}$

$(n+1) f_1 = 112\text{Hz}$

$\Delta f = [(\cancel{n}+1) - \cancel{n}] f_1 = 112 - 98$

$f_1 = 14\text{Hz}$

20. The tension in a string is increased by a factor of 1.3. By what factor is the fundamental frequency of a wave on the string changed?

$$v = \sqrt{\frac{F_T}{\mu}} = \lambda f$$

constant μ, λ

$$\sqrt{F_T} \propto f$$

$$\frac{\sqrt{F_{T2}}}{\sqrt{F_{T1}}} = \frac{f_2}{f_1}$$

$$F_{T2} = 1.3 F_{T1}$$

$$\sqrt{\frac{1.3}{1}} = \frac{f_2}{f_1}$$

$$f_2 = \sqrt{1.3}\, f_1$$

21. The tension in a string is increased by 10%. By what percent is the fundamental frequency changed?

factor = 1.1

$$\frac{f_2}{f_1} = \sqrt{1.1} = 1.049$$

$$\begin{array}{r} 1.049 \\ -1.0 \\ \hline 0.049 \end{array}$$

$$\%\Delta = 0.049 * 100 = 4.9\%$$

#17

$$v = \lambda f$$

constant f, L

λ changes

$$v \propto \lambda = \frac{2L}{n}$$

$v \propto \frac{1}{n}$ inversely proportional!

$$\frac{v_3}{v_4} = \frac{n_4}{n_3} = \frac{4}{3}$$

#13 homework

$L = 2.0\,m$

$F_T = 299\,N$

$f_1 = 27.5\,Hz$

$\frac{2L}{n}$

$v = \lambda f = \frac{2(L)}{1} f_1 = \frac{4}{1}(27.5\,Hz)$

$v = 110\,m/s$

$$\boxed{v = \sqrt{\frac{F_T}{\mu}} \quad \frac{m}{L}}$$

$$\mu = \frac{F_T}{v^2} = \frac{299\,N}{(110\,m/s)^2} = 0.0247$$

$$\mu = \frac{m}{L} \rightarrow m = \mu L = (0.0247)(2m)$$

$m = 0.0494\,m$

#10

$L = 3.1\,m \quad t = 0.0300\,s$

$$v = \frac{d}{t} = \frac{3.1m}{0.03} = 103.3\,m/s$$

$$\boxed{\lambda = \frac{2L}{n}} = \frac{2(3.1m)}{1} = 6.2\,m$$

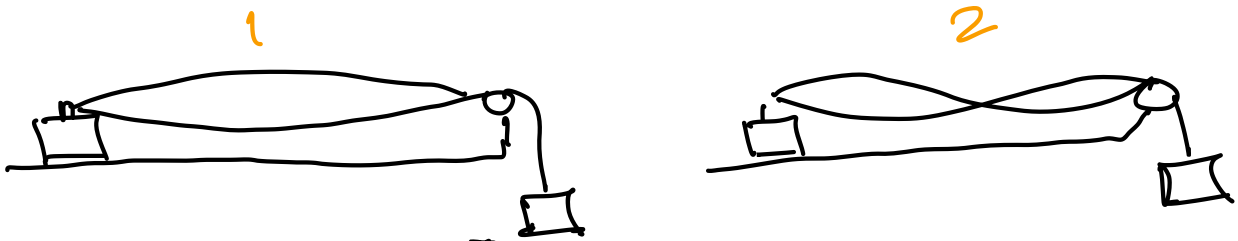
$\boxed{v = \lambda_1 f_1}$

$103.3\,m/s = (6.2m) f_1$

$f_1 = 16.7\,Hz$

$f_2 = 2f_1$

$f_3 = 3f_1$



changed the mass → change tension

1) By what factor does speed change?

constant μ L f

$$v = \sqrt{\frac{F_T}{\mu}} = \lambda f$$

$$v \propto \lambda = \frac{2L}{n} = \frac{1}{n}$$ inversely proportion

$$\frac{v_2}{v_1} = \frac{n_1}{n_2} \rightarrow \frac{1}{2}$$

2) Same tension. How does wavelength change?

$$\lambda = \frac{2L}{n}$$ inversely proportion

$$\frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2} = \frac{1}{2}$$

3) How does frequency change

$$v = \sqrt{\frac{F_T}{\mu}} = \text{constant} = \lambda f$$

$$f_2 = 2 f_1$$

Wave

$$y(x,t) = (5.0\,\text{cm})\cos(2\pi x - \underline{0.2\pi t})$$
period

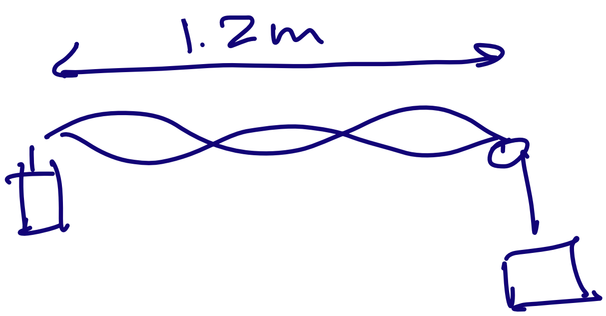
wavelength?

$$\boxed{k = \frac{2\pi}{\lambda}} = 2\pi \quad \text{so} \quad \lambda = 1.0\,\text{m}$$

Period

$$\omega = \frac{2\pi}{T} = 0.2\pi \quad \text{so} \quad T = 10\,\text{s}$$

Speed of wave $= \frac{\lambda}{T} = \frac{1.0\,\text{m}}{10\,\text{s}} = 0.1\,\text{m/s}$



what is the wavelength?

$$\lambda = \frac{2L}{n} = \frac{2(1.2\,\text{m})}{3} = \underline{0.8\,\text{m}}$$

$f = 60\,\text{Hz}$

what is speed of wave?

$$v = \lambda f = (0.8\,\text{m})(60\,\text{Hz}) = 48\,\text{m/s}$$

what is fundamental frequency?

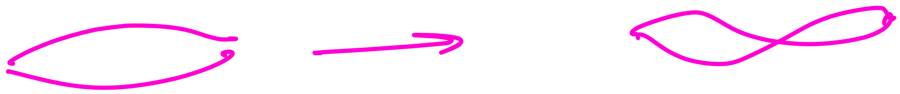
$$\boxed{f_n = n f_1}$$

$$f_3 = n f_1 = 3 f_1$$

$$60\,Hz = 3 f_1$$

$$f_1 = \frac{60\,Hz}{3} = 20\,Hz$$

No change in tension



λ_1 $\frac{1}{2}\lambda_1$

v_1 v_1 remains same