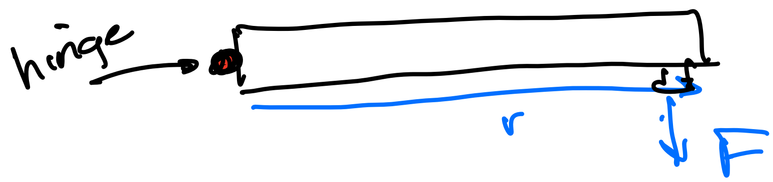
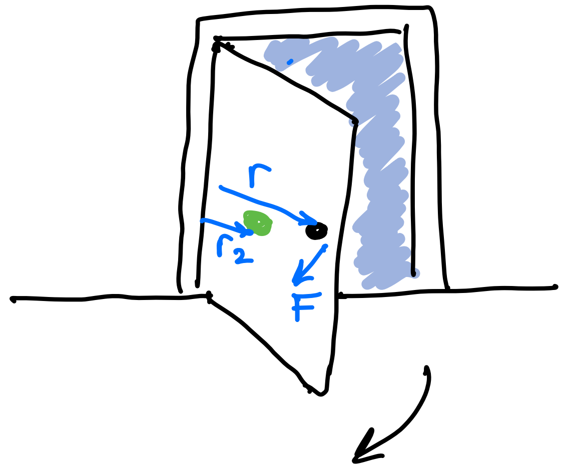
Top View



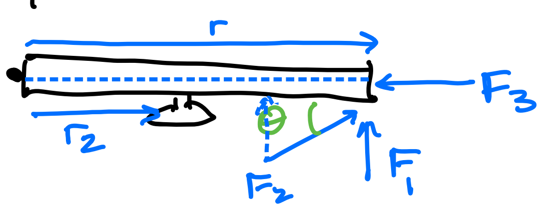


What force must I use to open door if the Knob is located at ½ width?

$$\tau = r F \cancel{\sin 90^\circ} \rightarrow 1.0$$

For same torque at ½r,
Force must be 2F (double)

Torque = $rF\sin\theta = r_\perp F$



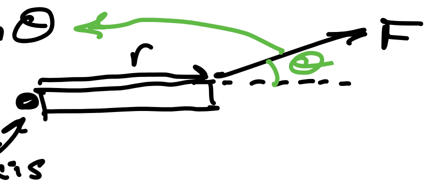
$\tau_1 = r F_1 \sin 90^\circ = r F_1$

$\tau_2 = r F_2 \sin\theta$ less than τ_1

$\tau_3 = (0) F_3 = 0$

Oct 30 - Rotation

Newton's 2nd $\qquad \tau = r_\perp F = r F \sin\theta$

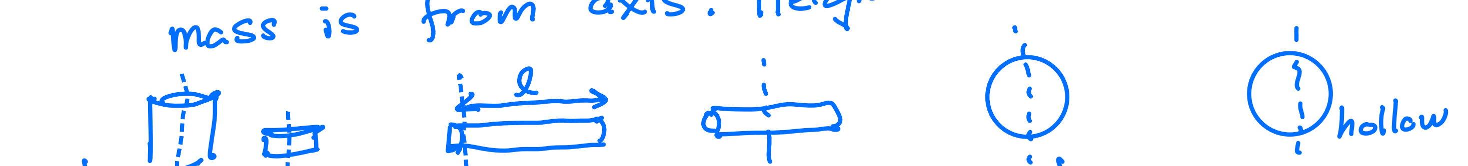


$\tau = I \alpha$

I — rotational inertia depends on mass & how mass is distributed

α — angular acceleration

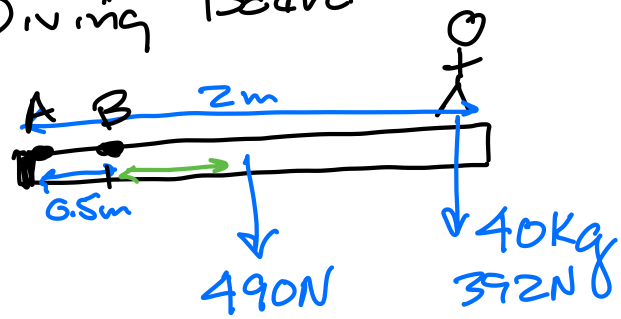
Rotational Inertia depends on axis. and how far mass is from axis. Height is not in equations



Solid $\quad I = \frac{1}{2}mr^2$ same $\qquad I = \frac{1}{3}ml^2 \qquad I = \frac{1}{12}ml^2 \qquad$ solid $I = \frac{2}{5}mr^2 \qquad I = \frac{2}{3}mr^2$

More on Static Equilibrium

Diving Board



Force at screw A (place axis at B)

All distances are relative to axis point

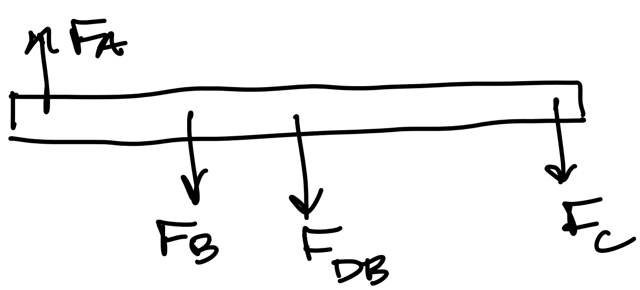
$$\Sigma \tau = 0 = (1.5\,m)(392\,N) + (0.5\,m)(490\,N) - (0.5\,m)F$$

$$F = \frac{(1.5)(392) + (0.5)(490)}{0.5} = 1666\,N$$

Force at B

$$\Sigma F = 0 = 1666 + F_B - 490 - 392$$

$$F_B = -784\,N$$



At the end of this worksheet, you should be able to

- describe the quantities involved in discussing rotational motion and the connection of them to translational motion.
- calculate quantities describing rotational motion.
- use the conditions of equilibrium to solve for an unknown quantity.
- use the principle of conservation of angular momentum to solve for an unknown quantity.

1. Write the translational kinematics equations and their rotational counterparts. Also, write the rotational equations for Newton's 2nd Law, kinetic energy, and angular momentum.

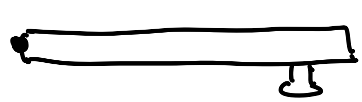
translational	rotational
$\Delta x = v_1 t + \frac{1}{2} a t^2$	$\Delta\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
$v_2 = v_1 + at$	$\omega_2 = \omega_1 + \alpha t$
$v_2^2 = v_1^2 + 2a(\Delta x)$	$\omega_2^2 = \omega_1^2 + 2\alpha(\Delta\theta)$
$\bar{F} = m\bar{a}$	$\bar{\tau} = rF\sin\theta = I\bar{\alpha}$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
$p = mv$	$L = I\omega$
$F\Delta t = m\Delta v$	$\tau\Delta t = I\Delta\omega$

Rotational kinematics

2. You exert a force of 100N to open a door. To get the door swinging you exert this force for 1 second and you apply this force perpendicularly to the door. The door is 1.0 m wide and 2.0 m tall and has a mass of 30 kg. The door handle is 0.9 m from the hinges.

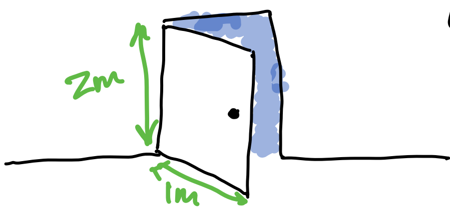
   a) What torque do you exert on the door? $\sin\theta = \sin 90 = 1.0$

$$\tau = rF\sin\theta = (0.9\,m)(100\,N)\sin 90^\circ = 90\ Nm$$



b) What is the rotational inertia of the door? $I = \frac{1}{3}ML^2$ $\frac{1}{3}ML^2$

L: distance from axis to the mass



↳ I depends on mass and how mass is distributed

resistance to rotation

$I = \frac{1}{3}ML^2 = \frac{1}{3}(30\,kg)(1.0\,m)^2 = 10\,kgm^2$

↖ short or tall will only affect mass.

c) What is the angular acceleration of the door?

Newton's 2nd law for rotation ($F = ma$)

$\tau = I\alpha$

$90\,Nm = (10\,kgm^2)\alpha \longrightarrow \alpha = 9\,rad/s^2$

d) How fast is the door moving when you stop pulling and let it swing?

0 at rest

$\omega_2 = \cancel{\omega_1} + \alpha t = (9\,rad/s^2)(1\,s) = 9\,rad/s$

e) What is the initial angular momentum of the door? What angular impulse did you give the door?

↳ L ↳ $\tau\Delta t$

f) What is the door's final angular momentum? Use this to find the angular speed when you stop changing the door's momentum. ↘ L

g) What is the door's kinetic energy after you stop pulling?

↳ K

h) What was its initial kinetic energy? How much work did you do to open the door?

i) Over what angular displacement did you exert this 100N force?

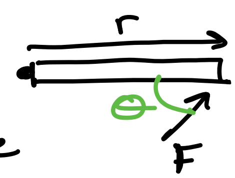
Torque

3. To generate torque with a wrench, you exert a force at the end of the wrench, 30cm away from the bolt. You push with 500N in the direction to loosen the bolt, but it will not budge. Why is it not rotating? In order to generate more torque, you go get a longer wrench, this one 55cm.

   a) Calculate the torque each wrench applies to the bolt.

   b) By what factor have you changed the length of the lever arm? $\frac{55}{30} = 1.8$

   c) By what factor have you changed the torque? $\frac{275}{150} = 1.8$

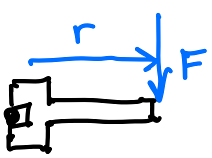


lever arm

Torque is product of $\vec{r} \times \vec{F} = rF\sin\theta$

r: distance from axis of rotation to force

θ = angle between r and F



units

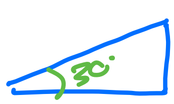
a) $\tau_1 = rF\sin\theta = (0.30m)(500N) = 150Nm$

$\tau_2 = (0.55m)(500N) = 275\ Nm$

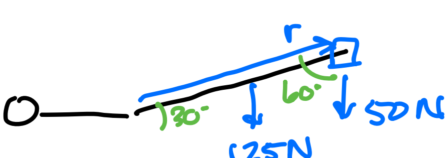
$\tau = rF$: torque is directly proportional to r, F

4. A person laying on the floor is doing a leg lift with an ankle weight on. The ankle weight is 50 N. The ankle weight is essentially at the end of their leg which is 1m long, and the mass of their leg is 12.75kg. Take the center of mass of the leg to be the geometric center of the leg. 125N

   a) Find the torque on the person's leg when their leg is at 30° with respect to the horizontal. Torque is a vector and sum like forces



$$\tau = (1.0m)(50N)\sin 60° + (0.5m)(125N)\sin 60°$$



$$\tau = 97.4\ Nm$$

   b) Calculate this again when their leg is at an 80° with respect to the horizontal.

$$\tau = [(1.0)(50N) + (0.5)(125)]\sin 10°$$

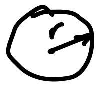
$$\tau = 19.5\ Nm$$

coefficient is important!!!
lower coefficient ⇒ easier to rotate, greater α

Rotational Inertia

5. List some equations to determine the rotational inertia of some common objects.

solid disk $I = \frac{1}{2}mr^2$ solid sphere $= \frac{2}{5}mr^2$

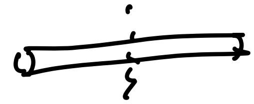

m

hoop/ring $I = mr^2$ hollow sphere $= \frac{2}{3}mr^2$

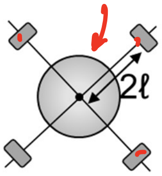
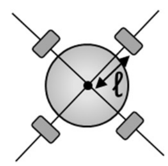
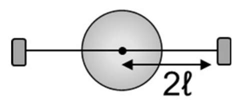
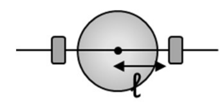
height is not in equation

rod on edge $I = \frac{1}{3}ml^2$

rod at center $= \frac{1}{12}ml^2$



6. Four identical light-weight pulleys have identical masses, *m*, attached at different locations (ℓ and 2ℓ) away from the central axis as shown. Pulleys A and B have 4 masses attached. Pulleys C and D have two masses attached. The central pulley has a rotational inertia equal to I_d. Calculate the rotational inertia for each pulley.

disk  $I_1 = m(2l)^2 = 4ml^2$ $4I_1 = 4(4ml^2)$ $I_{disk} = \frac{1}{2}MR^2$ $I_{total} = \frac{1}{2}MR^2 + 16ml^2$ A	 $I_1 = ml^2$ $4I_1 = 4ml^2$ $I_{disk} = \frac{1}{2}MR^2$ $I_{total} = \frac{1}{2}MR^2 + 4ml^2$ easier than A B	 $I_1 = m(2l)^2 = 4ml^2$ $2I = 8ml^2$ $I_{total} = \frac{1}{2}MR^2 + 8ml^2$ B is harder to translate (side to side) but easier to rotate than C C	 $I_1 = ml^2$ $2I_1 = 2ml^2$ $I_{total} = \frac{1}{2}MR^2 + 2ml^2$ D

$\tau = I\alpha$ Inversely proportional

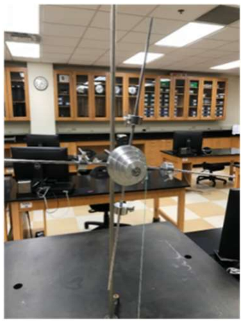
Summary

A and B have more total mass. (More than C, D)
So harder to translationally accelerate
$F = ma$
But for rotation I, $\tau = I\alpha$
$I_A > I_C > I_B > I_D$ easiest to rotate high α

hardest to rotate, low α

7. Determine the rotational inertia of the compound pulley shown. Compare it to its theoretical value.

9



r_1 = 0.0202 m, r_2 = 0.02865 m, r_3 = 0.03852 m

I_{disk} = 0.00058 kg m^2, $4I_{rods}$ = 0.0127 kg m^2, 1 mass = 0.185 kg

L_{rods} = 0.11m and 0.34m

I_{total} ~ 0.098 kg m^2 for fully extended, 0.013 kg m^2

$\tau = rF = I\alpha$

changed Torque LHS

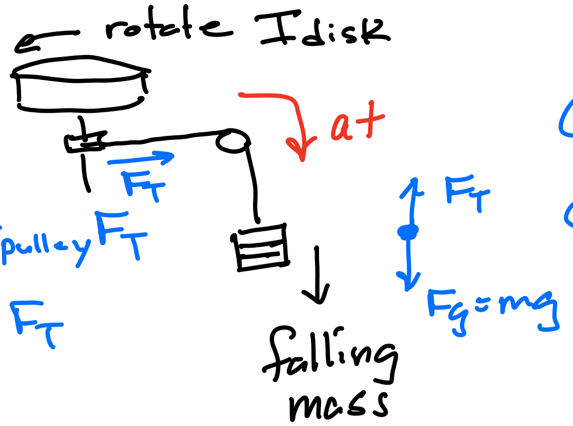
	Torque effect		Inertia effect			
Trial	Falling mass (kg)	Pulley radius (m)	L between masses and axis (m)	Time (s)	a (m/s^2)	Conclusions
1	0.200	0.02865	0.34	13	X	control
2	0.200	**0.0202**	same I 0.34	16.1	lower α	Smaller lever arm lower $\tau = rF$
3	**0.400**	0.02865	0.34	8.4	greater α	Increased F greater $\tau = rF_T$
4	0.200	0.02865	**0.11**	6.0	greater α	lowered I

This changes R.H.S of equation

Be careful here! The force applied to the pulley is not mg. The hanging mass is accelerating, but not with 9.8 m/s^2.

8. In lab this week, we will *determine the rotational inertia* of a disk and a hollow cylinder. To do this we will apply a constant torque to a pulley below the spinning object and measure the angular acceleration of the spinning object. To apply a constant force to the pulley, we will use a mass that will hang down from a string and have a constant force of gravity applied to it. Let's do this problem *in general* and develop an equation to calculate the rotational inertia from knowing the hanging mass, the radius at which the string is applying a torque to the pulley. *Be careful here! The force applied to the pulley is not mg. The hanging mass is accelerating, but not with* 9.8 m/s^2. Some starters:

   - Draw a free body diagram on the hanging mass. You know the weight but you do not know the tension.
   - In lab we will measure the angular acceleration (α) so solve for rotational inertia in terms of angular acceleration.



$\tau = r_{pulley} F_T$

$\tau = r_p F_T$

Newton's 2nd Law $\Sigma F = ma$

① $mg - F_T = m_h a$

② $r_p F_T = I_{disk} \alpha$

③ $a = r_p \alpha$

$$m_h g - \frac{I\alpha}{r_p} = m_h (r_p \alpha)$$

$$I = m_h \left(\frac{g r_p}{\alpha} - r_p^2 \right)$$

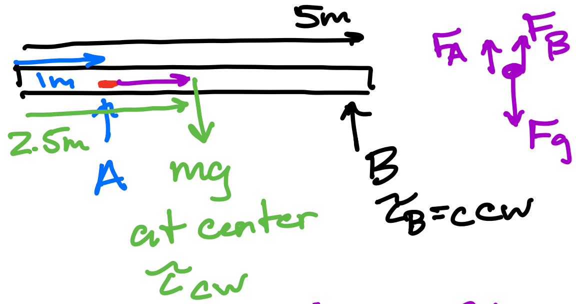
Axis can be placed anywhere

Static Equilibrium

$\Sigma F = 0$ not moving translationally

$\Sigma \tau = 0$ not rotating

9. Two people are carrying a 1000N wooden beam that is 5m long. One person is positioned at one end of the beam, and the other person is positioned 1m away from the other end.
What force is each person exerting on the beam?



$F_B + F_A - F_g = 0$

If you want to Find F_B, place axis at A because $\tau_A = rF_A$ but $r = 0$

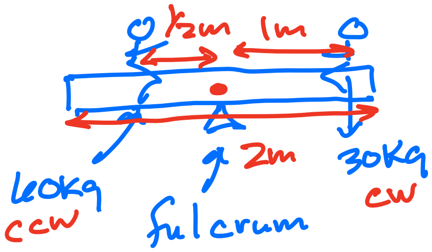
$\Sigma \tau = \tau_{mg} + \tau_B = (1.5m)(mg) - (4m)(F_B) = 0$

$\boxed{F_B = \frac{(1.5)(1000N)}{4} = 375N}$

② Use $\Sigma F = 0$ to Find $F_A = 1000N - 375 = 625N$

10. Let's do the previous problem inside out. A question might read, "Where should the second person be positioned so that his force on the beam is ______________ ?" Provide the force from the previous question and solve for his position from one end.

Person closer to center (natural axis) is carrying greater load (force)

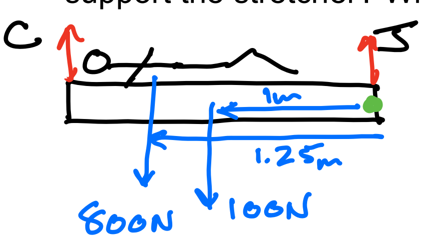


Where should parent (60kg) sit to balance seesaw with child at edge?

$\Sigma \tau = 0 = (1m)(30kg)(9.8) - r(60kg)(9.8)$

axis at fulcrum $r = \frac{1(30)(9.8)}{(60)(9.8)} = 0.5m$ from fulcrum

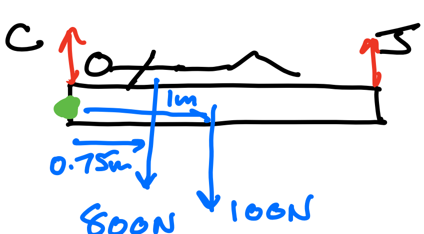
11. Chris and Jamie carry Wayne on a horizontal stretcher. The uniform stretcher is 2.00 m long and weighs 100 N. Wayne weighs 800 N. Wayne's center of gravity is 75.0 cm from Chris. Chris and Jamie are at the ends of the stretcher. What force is Chris providing to support the stretcher? What force is Jamie providing to support the stretcher?



a) Chris Force, place axis at Jamie

$\Sigma \tau = 0 = 2m(F_c) - (1.25m)(800N) - (1m)(100N)$

$F_c = \frac{1.25(800) + 1(100)}{2} = 550N$



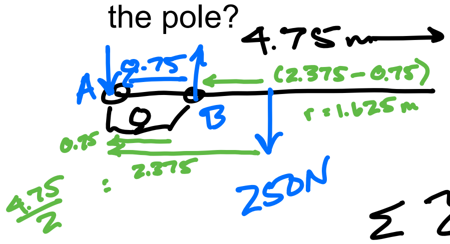
b) Force due to Jamie

$\Sigma \tau = 0 = 2m(F_J) - (0.75)(800) - (1)(100)$

$F_J = \frac{(0.75)(800) + 1(100)}{2} = 350N$

Check: $\Sigma F = 0 = 550 + 350 - 800 - 100 = 0$ ✓

12. A pole-vaulter holds out a 4.75m pole horizontally in front of him. He places one hand at the very end of the pole, and the other hand 0.75m from the end. Assuming the pole weighs 250 N and is uniform in construction. What is the force in his hand at the end of the pole?



Force we don't care about is at Point B

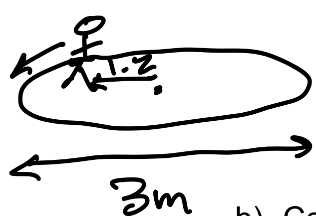
$$\Sigma\tau = 0 = (1.625\,m)(250) + (0.75\,m)F_R$$

$$F_R = \frac{1.625(250)}{-0.75} = -542\,N$$

Correct magnitude but assumed direction of force incorrectly. Merely change direction of force

<u>**Angular Momentum and its Conservation**</u> $L = I\omega$ $I_1\omega_1 = I_2\omega_2$

13. A 80-kg child stands on a merry-go-round that spins with angular velocity 0.6 rev/s. The platform of the merry-go-round is 3 meters in diameter and has a mass of 60 kg. The child initially stands 1.2 m from the axis of rotation. He moves closer to the center 0.3 m from the axis while the platform is spinning.
    a) Calculate the rotational inertia of the child-platform when the child is 1.2 m from the axis of rotation.



disk

$$I_1 = \tfrac{1}{2}mr^2 + m_c r_c^2$$

182.7 Kgm²

$$I_1 = \tfrac{1}{2}(60\,Kg)(1.5\,m)^2 + (80\,Kg)(1.2\,m)^2$$

    b) Calculate the rotational inertia of the child-platform when the child is 0.3 m from the axis of rotation.

$$L_1 = I_1\omega_1 = (182.7\,Kgm^2)(0.6\,rev/s) = 109.6$$

conserved

$$L_2 = L_1 = 109.6 = (74.7\,Kgm^2)(\omega_2)$$

$$\boxed{\omega_2 = 1.5\,rev/s}$$

$$I_2 = \tfrac{1}{2}(60\,Kg)(1.5\,m)^2 + (80\,Kg)(0.3\,m)^2$$

$$I_2 = 74.7\,Kgm^2$$

c) What is the final angular speed of the platform when the child is 0.3 m from the center? Answer in revolutions/s.

Rolling — Friction must be present. Use Energy!

14. Three objects roll down an incline. Determine the final speed of the object using energy.

a) A solid sphere

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

from formula

$v = r\omega$

Both translational ($\frac{1}{2}mv^2$)
rotational ($\frac{1}{2}I\omega^2$)

b) A solid cylinder

c) A hollow cylinder (ring)

In a race
1 Bowling Ball fastest
2 Solid disk
3 Hoop

d) How do these values compare to an object sliding down a frictionless incline?

Race with icecube (no friction)
Frictionless has greatest acceleration and final velocity