At the end of this worksheet you should be able to

- discuss Newton's laws and provide examples of the application of each.
- apply Newton's first law to solve interesting physical problems.
  An object stays at rest or constant velocity if $F_{net} = 0$ — Special Case of Newton's 2nd — no acceleration
- apply Newton's second law to solve interesting physical problems for objects that accelerate.
  $F = ma$ $F = m\cancel{a}^{0} = 0$
- apply Newton's third law to situations involving the motion of multiple objects.
  equal + opposite

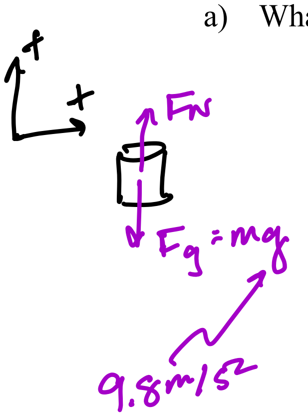
1. What base units are the composite force units of Newtons equal to?

$$a = \frac{F_{net}}{m_{total}} \qquad F_{net} = ma \qquad \Sigma F = ma$$

$$(N) = (kg)(m/s^2)$$

2. I hold a 5-kg cup motionless in my hand.

   a) What force do I provide to the cup?

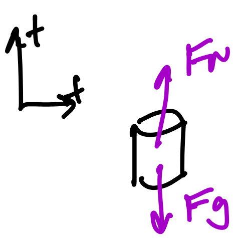


$$\Sigma F_y = m\cancel{a}^{0}$$

static equilibrium — rest — equal forces

$$F_N - F_g = 0$$
$$F_N = F_g = mg = (5kg)(9.8\,m/s^2)$$
$$F_N = 49N$$

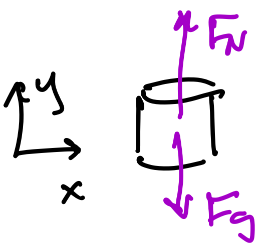
   b) What force do I need to provide from my hand to the cup if I raise the cup with my hand *at constant speed*?



$$\Sigma F_y = m\cancel{a}^{0}$$
$$F_N = 49N$$

elevator problems

c) Still holding the 5-kg cup. What force do I need to provide from my hand to the cup if I raise the cup with my hand *with an acceleration of + 1 m/s² (upward)*?



$\Sigma F_y = ma \quad a = +1\,m/s^2$

$F_N - F_g = ma$

$F_N = F_g + ma = (5kg)(9.8\,m/s^2) + (5kg)(+1.0)$

Bathroom scale reads normal force

$F_N = 5kg\,(9.8 + 1\,m/s^2) = 54N$

d) What force do I need to provide from my hand to the cup if I raise the cup with my hand *with an acceleration of – 1 m/s² (downward)*?

Same as

$F_N = F_g + ma$

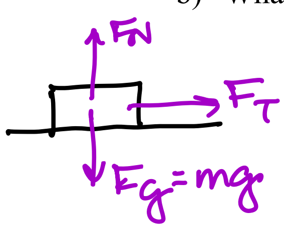
$F_N = (5kg)(9.8) + (5kg)(-1.0\,m/s^2)$

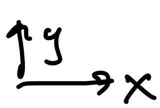
$F_N = 44N$

3. I push a 100-kg box with an applied force of 700 N along a *frictionless* surface.

a) Calculate the force of gravity on the box, the normal force, and the net force.

b) What is the acceleration of the box?





x, y motion are independent of each other

$\Sigma F_y = m\cancel{a_y}^{\,0}$

$F_N - F_g = 0$

$F_N = F_g = mg$

$F_N = (100kg)(9.8\,m/s^2)$

$F_N = 980N$

$\Sigma F_x = ma_x$

$F = ma$

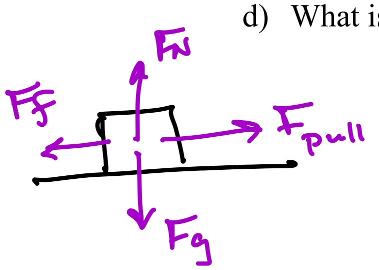
$700N = (100kg)\,a$

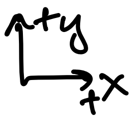
$a = \frac{700N}{100kg} = 7\,m/s^2$

c) What is the net force on the box if the surface is not frictionless (coefficient of friction is μ = 0.5)?

$F_f = \mu F_N$

d) What is the acceleration of the box with the friction force given in the above example?





	x (N)	y (N)
F_g	0	-980
F_N	0	+980
F_N	700	0
F_f	-490	
sum	210	0

$F_f = \mu F_N$
$= (0.5)(980)$

$F_y = m\cancel{a_y}^{0}$

$F_N - F_g = m\cancel{a}^{0}$

$F_N = F_g = mg$

must do y-direction first because $F_f = \mu F_N$ → $F_N = +980N$

$F_x = ma_x$

$F - F_f = ma$

$700N - 490N = m(a)$

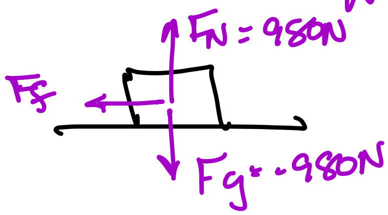
$210N = (100kg)a$

$a = \frac{210N}{100kg}$

$a = 2.1 m/s^2$ →

e) What happens when I stop pushing in each of the previous scenarios?

What is the acceleration of the box?



$F_f = -490N$

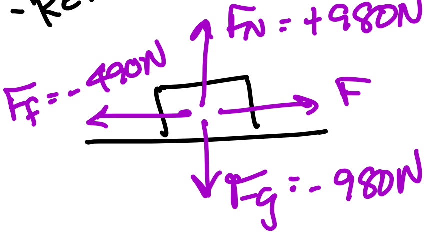
$\Sigma F_x = ma$

$-490N = (100kg)a$

$a = -4.9 m/s^2$

f) What must the pushing force be to move the box at constant speed if is μ = 0.5?

-Reverse'



$F_x = m\cancel{a}^{0}$

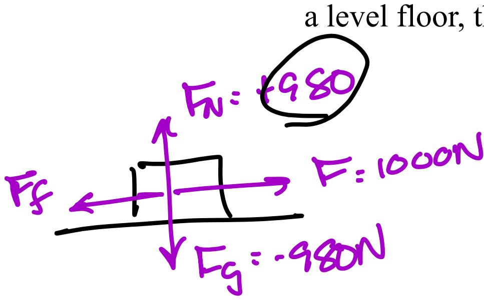
$F - 490N = 0$

$F = 490N = F_f$

all forces are balanced (cancel to zero)

g) If I need to provide a 1000 N force to keep a 100-kg box moving at constant speed along a level floor, then what is the coefficient of friction between the floor and the box?

μ?



$\Sigma F_x = ma_x$

$F - F_f = m\cancel{a}^{0}$

$F = F_f = \mu F_N$

by definition $F_f = \mu F_N$

$F = F_f = 1000N = \mu(980N)$

$\mu = 1.02$ no units

$F_g = mg = (1000\,kg)(9.8\,m/s^2) = 9800\,N$

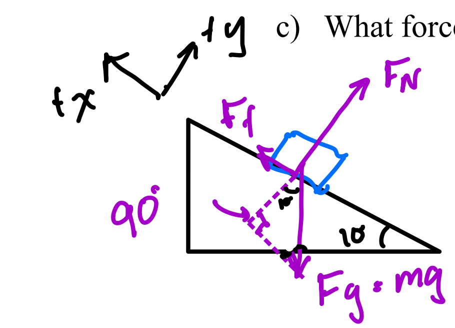
at rest

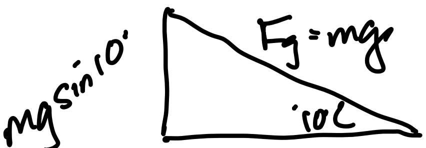
4. A 1000-kg car is parked on a hill that has an angle of 10° with respect to the horizontal.

   a) What is the weight of the car?

   b) What is the normal force on the car?

   c) What force is keeping the car from sliding down the hill? How large is that force?





$mg\cos 10°$
$= 9800\cos 10°$
$= 9651\,N$

Force	x (N)	y (N)
F_g	-1702	-9651
F_N	0	= 0-(-9651) = +9651
F_f	+1702	0
Sum (parked)	0	0

$\Sigma F_y = m\cancel{a_y}$ 0

$\Sigma F_x = m\cancel{a_x}$ 0

What is μ_s

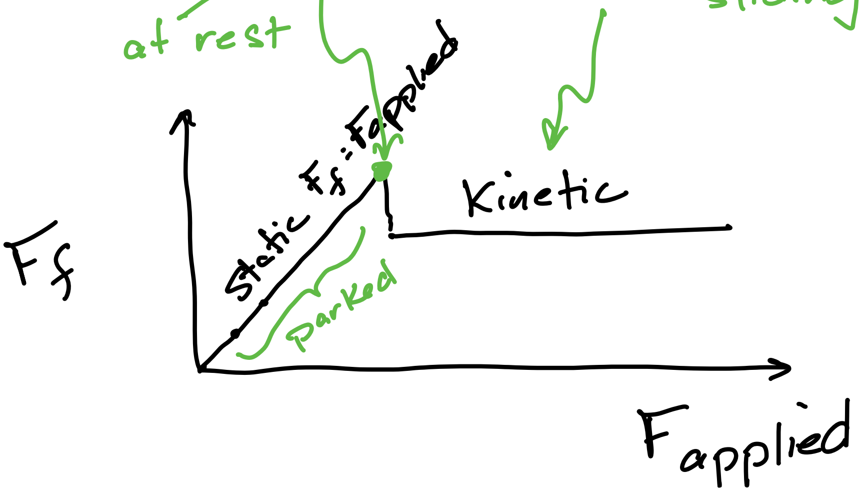
5. ~~The same car rests on a steeper 20° incline, what is the weight, normal force and the frictional force on the car now?~~

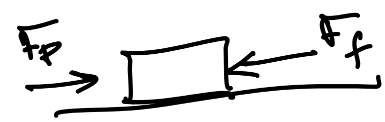
Skip #5

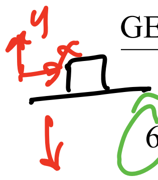
μ_s and μ_k

at rest

sliding



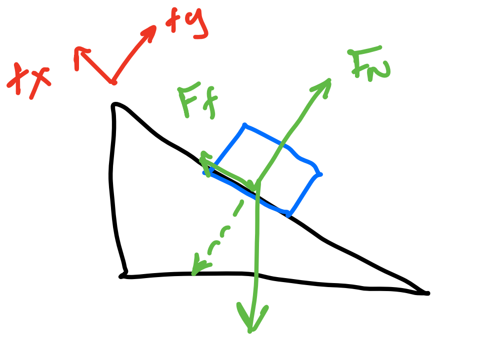




1000-kg

6. If the maximum incline that the car can be parked on without sliding is 25°, then what is the coefficient of friction between the tires and the road?

$F_f = \mu F_N \quad \mu = 0.47$



$F_g = mg$

$F_g = (1000kg)(9.8)$

$F_g = 9800N$

y motion

$\Sigma F_y = ma_y$

$F_N - F_g\cos\theta = m\cancel{a_y}^{0}$

$F_N = F_g\cos\theta$

$F_N = (9800N)\cos 25°$

$F_N = 8882N$

x motion

$\Sigma F_x = ma_x$

$F_f - F_g\sin\theta = m\cancel{a_x}^{0}$

$F_f = F_g\sin\theta$ ✓

$F_f = 4141 N$

Now Apply $F_f = \mu F_N$

$4141 N = \mu(8882N) \longrightarrow \mu = \frac{4141}{8882} = 0.47$

7. Now, work the above problem in reverse. You know the coefficient of friction and must calculate the maximum angle of an incline the car can successfully park on.

apply algebra → $\mu = \frac{F_f}{F_N} = \frac{\cancel{F_g}\sin\theta}{\cancel{F_g}\cos\theta} = \tan\theta$

Not numbers

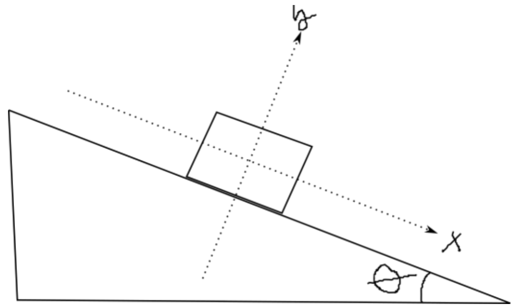
weight cancels

when mass is included in the forces that cause all motion, it cancels

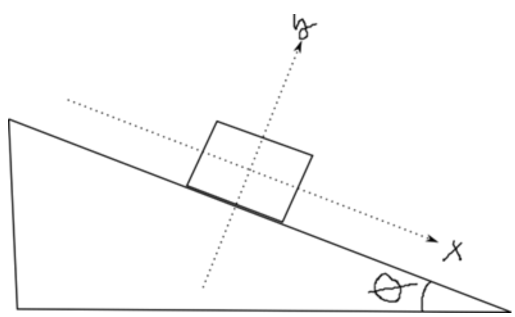
$\mu_k = \tan\theta$ sliding at constant speed

$\mu_s = \tan\theta$ parked on hill with max angle

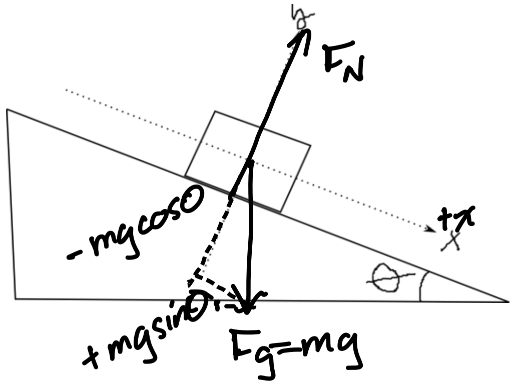
8. A 100-kg box slides down a *frictionless* inclined plane that has an angle of 30° to the horizontal. What is the net force on the box and what is the acceleration of the box?



9. Same box, same incline but now with friction. A 100-kg box slides down a ~~friction-full~~ rough inclined plane that has an angle of 30° to the horizontal and a coefficient of friction of $\mu = 0.1$. What is the net force on the box and what is the acceleration of the box?



10. Let's do the friction-less inclined plane problem *in general* for any mass and incline. Follow the same procedure as before but with the variable m for mass and θ for incline angle. Find an expression for the net force on the mass as a function of θ and for the acceleration as a function of θ.



	$x(N)$	$y(N)$
F_g	$mg\sin\theta$	$-mg\cos\theta$
F_N	0	$+mg\cos\theta$
a	$g\sin\theta$	0

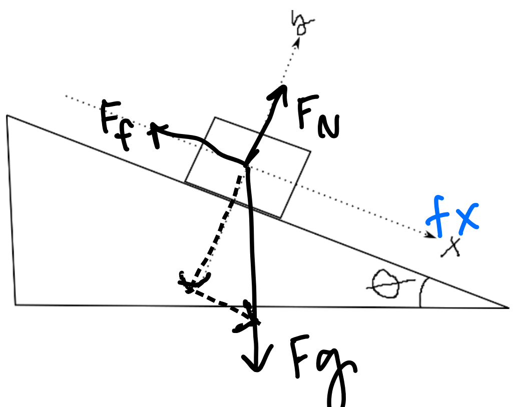
① FBD
② assign axes
③ components of Forces
④ Apply F=ma to x,y direction separately

$\Sigma F_x = ma_x$

$\cancel{m}g\sin\theta = \cancel{m}a_x$

so $a_x = g\sin\theta$

11. Now let's do the inclined plane with friction *in general*. Just like the previous problem, use m for mass, θ for angle, and now use μ as a variable for coefficient of friction. Find an expression for the acceleration of the mass as a function of θ, m, and μ. g



	$X(N)$	$Y(N)$
F_g	$+mg\sin\theta$	$-mg\cos\theta$
F_N	0	$mg\cos\theta$
F_f	$-F_f$	0
	?	0

X motion

$-F_f + mg\sin\theta = ma_x$

$-F_f + mg\sin\theta = ma_x$

but 2nd relationship $F_f = \mu F_N = \mu mg\cos\theta$

$-\mu\cancel{m}g\cos\theta + \cancel{m}g\sin\theta = \cancel{m}a_x$

$a_x = g\sin\theta - \mu g\cos\theta$

unitless unitless

m/s² m/s² unitless m/s²

units!

12. Now, work the above problem in reverse. You know the coefficient of friction and must calculate the maximum angle of an incline the car can successfully park on.

on incline parked $a_x = g\sin\theta - \mu g\cos\theta$

$$0 = \cancel{g}\sin\theta - \mu\cancel{g}\cos\theta = g(\sin\theta - \mu\cos\theta)$$

$$0 = \sin\theta - \mu\cos\theta$$

$$\mu\cos\theta = \sin\theta$$

$$\boxed{\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta}$$

$$\theta = \tan^{-1}\mu$$

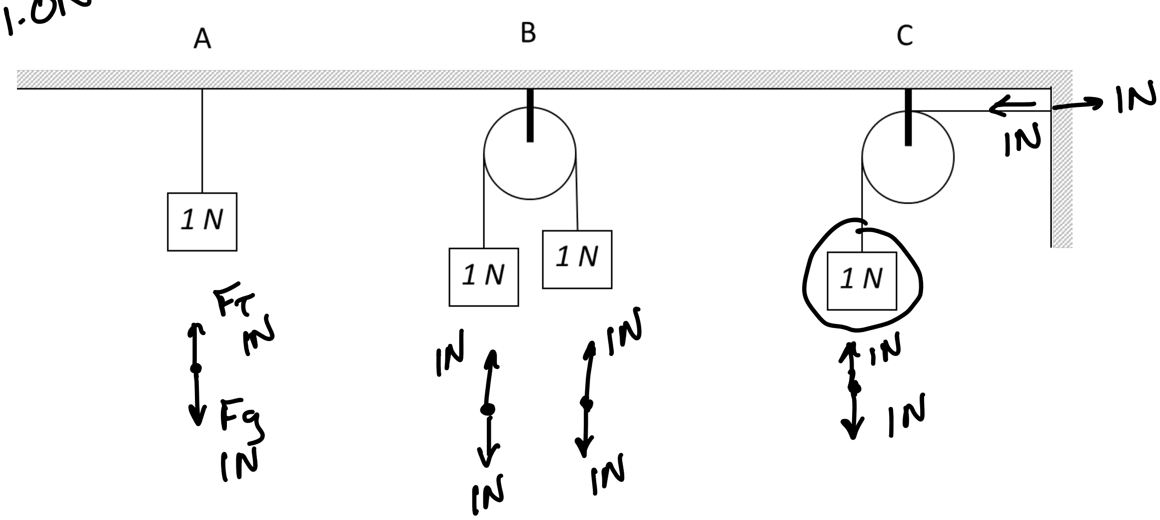
13. What force is necessary to keep a box *motionless on a frictionless inclined plane*? Is there a difference between the force to hold it motionless on the incline and a force to push it up the incline *at constant speed*?

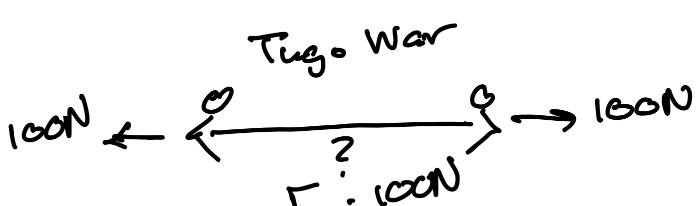
same friction force

only happens when $a = 0$ at rest, constant velocity

14. A ~~50~~-N weight is suspended 3 different ways. What is the tension in each rope?

1.0N



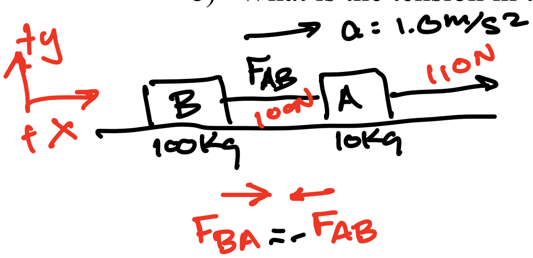


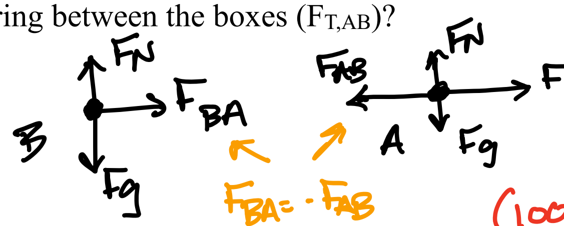
Newton's 3rd

15. Two boxes (A and B) rest on a frictionless table. Box A has a mass of 10 kg, Box B has a mass of 100 kg. The boxes are connected by a string. I pull Box A and the boxes accelerate at 1.0 m/s^2.

 a) What is my pulling force (tension in the string attached to my hand and Box A)?

 b) What is the tension in the string between the boxes ($F_{T,AB}$)?





+x

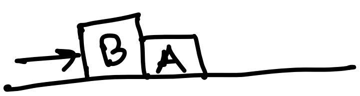
$(100kg)(1.0 m/s^2) = 100N$

$B_x: \cancel{F_{BA}} = m_B a_B$ $\quad a = a_B = a_A$

$A_x: F - \cancel{F_{AB}} = m_A a_A$

$F = m_B a + m_A a = a(m_B + m_A)$

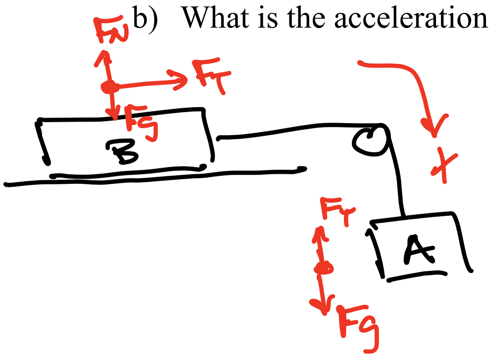
$F = (1.0 m/s^2)(100+10kg) = 110N$



16. Mass A (m_A) is connected to Mass B (m_B) that is hanging off the edge of a frictionless table. The string passes over a pulley at the edge of the table so there is no friction from the string. Mass B falls to the ground and pulls Mass A across the frictionless table. Answer the following in terms of m_A, m_B, and g.

 a) What is the force causing the acceleration?

 b) What is the acceleration of the two objects?



$B: \cancel{F_T} = m_B a_B$

$A: F_{gA} - \cancel{F_T} = m_A a_A$

$F_{gA} = a(m_B + m_A)$

$m_A g$

$m_A g = a(m_B + m_A)$

$a = \frac{m_A g}{(m_A + m_B)} = \frac{F_{net}}{m_A + m_B}$

 c) Repeat the experiment with a table with friction. The coefficient of friction is μ.

Newton's 3rd : equal and opposite
the 2 forces act on separate objects so
it involves 2 separate Newton's 2nd, $F = ma$

$\underline{F = ma}$

Ball Falling

$F_{Be} = F_g$ ← Force on ball due to earth

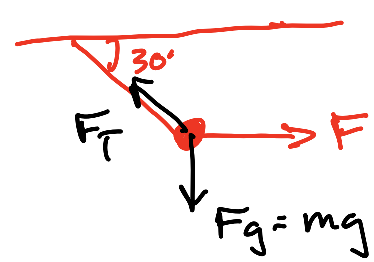
F_{eB} ← Force on earth due to ball

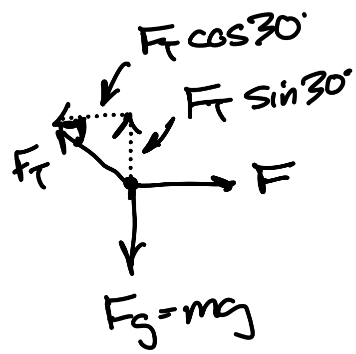
$F_{Be} = m_B a_B$

equal magnitude

$F_{eB} = m_e a_e$

Tension





$$\sum F_y = m\cancel{a_y}^{\,0}$$
$$F_T \sin\theta - mg = 0$$
$$F_T = \frac{mg}{\sin\theta}$$

$$\sum F_x = m\cancel{a_x}^{\,0}$$
$$F - F_T \cos\theta = 0$$
$$F = F_T \cos\theta$$
$$F = \left(\frac{mg}{\sin\theta}\right)\cos\theta$$
$$F = \frac{mg}{\tan\theta}$$