Week 5 covers sections of chapter 17 in the textbook. Topics include:

E

F_{ME}

$F_e = qE$

$+q$

- work and electric potential energy

- electric potential or voltage

- conservation of energy

$\Delta U = Q + W$

$-p\Delta V$

- electric field lines and equipotential surfaces

- energy storage in capacitors
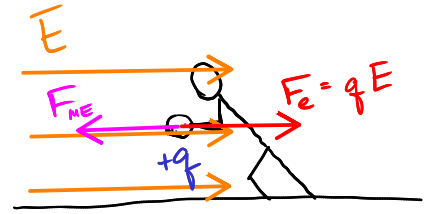
$$Work = \left|\underline{\underline{Force}}\right| \times \cos\theta \times \left|\underline{\underline{displacement}}\right|$$

1. An uniform electric field has a strength of 1000 N/C and points in the positive x-direction. It doesn't really matter what is causing the electric field but you can imagine we are inside of a capacitor. A 1 μC charge is moved around within this field by an external force.

(a) How much work does it take to move the charge from the position $x = 5$ cm to a position of $x = 1$ cm?

E

F_{ME}

$F_e = qE$

$+q$

1 cm   5 cm

$W_{ME} = \left|F_{ME}\right| \cos\theta \left|\Delta x\right|$

$= \left|qE\right| \cos\theta \left|\Delta x\right|$

$= \left|10^{-6} \cdot 10^{3}\right| \cos 0 \left|-4 cm\right|$

$= 4 \cdot 10^{-5} J$

$W_E = \left|F_E\right| \cos\theta \cdot \Delta x$

$\cos 180°$

$W_E = -4 \cdot 10^{-5} J$

(b) How much work does it take to move the charge from the position $x = 5$ cm to a position of $x = 9$ cm

E

F_{ME}

$F_e = qE$

$+q$

1 cm   5 cm   9 cm

$W_{ME} = -4 \cdot 10^{-5} J$
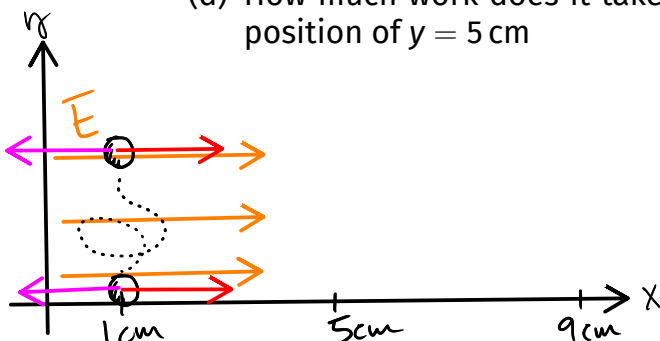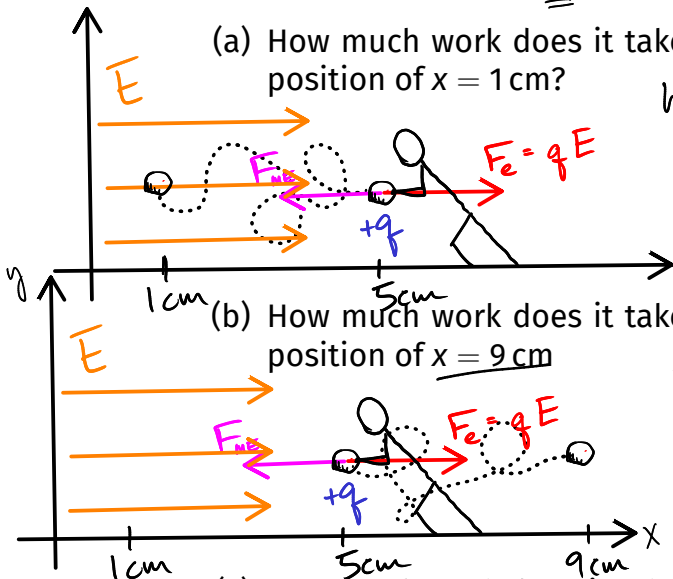
$W_E = +4 \cdot 10^{-5} J$

(c) How much work does it take to move the charge from the position $x = -5$ cm to a position of $x = 1$ cm

(d) How much work does it take to move the charge from the position $y = 0$ cm to a position of $y = 5$ cm

E

1 cm   5 cm   9 cm

$W = \left|qE\right| \cos 90° \, \Delta x$

0

$W = 0 J$

volume
↓
V

v
↑
velocity

2. For the above problem, what is the amount of work *per unit of charge* that was done in each step?

$$\Delta V = \frac{W}{q} = \frac{4 \cdot 10^{-5} \text{ J}}{10^{-6} \text{ C}} = 40 \frac{\text{J}}{\text{C}} = 40 \text{ Volts}$$
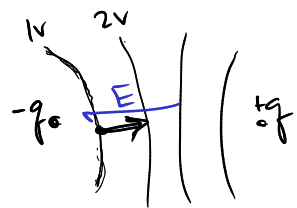
$V \leftarrow$ electric potential

all for constant $\vec{E}$ field

$$\Delta U = W_{NC} = F_{ME} \cdot \cos\Theta \cdot \Delta x \quad \Big\} \text{ constant force}$$

$$\Delta U = -q\vec{E} \cdot \vec{\Delta x}$$

$$\frac{\Delta U}{q} = -\vec{E} \cdot \vec{\Delta x} = \Delta V$$

change in voltage (voltage?) (potential difference)

$$E = -\frac{\Delta V}{\Delta x}$$

3. A fixed point charge of 10 μC creates an electric field throughout space around it.

(a) How much energy would it take to move a 1 μC charge from infinitely far away to a position of 100 m away?

$$W_{ME} = \Delta U = U(100m) - U(\infty)$$

$$\boxed{U(r) = \frac{k q_0 \cdot q_i}{r}}$$

$$= \frac{k q_0 q_i}{100m} - \frac{k q_0 q_i^{0}}{\infty}$$

$$= \frac{9 \cdot 10^9 \cdot 10 \cdot 10^{-6} \cdot 10^{-6}}{100m} = 9 \cdot 10^{-4} \text{ J}$$

100 m

(b) How much energy would it take to move a 1 μC charge from infinitely far away to a position of 90 m away?

$$\Delta U = \frac{9 \cdot 10^9 \cdot 10 \cdot 10^{-6} \cdot 10^{-6}}{90} = 1 \cdot 10^{-3} \text{ J}$$

90 m

(c) How much energy would it take to move a 1 μC charge from a position 100 m away to a position of 90 m away?

$$\Delta U = U(90) - U(100)$$

$$= 1 \cdot 10^{-3} \text{ J} - 9 \cdot 10^{-4} \text{ J}$$

$$= 10 \cdot 10^{-4} \text{ J} - 9 \cdot 10^{-4} \text{ J} = 1 \cdot 10^{-4} \text{ J}$$

90 m

100m

(d) To move 10 m closer, how much more energy would it take?

$$\Delta U = U(80m) - U(90m)$$

$$= \frac{9 \cdot 10^9 \cdot 10 \cdot 10^{-6} \cdot 10^{-6}}{80m} - 1 \cdot 10^{-3} \text{ J}$$

$$= 1.125 \cdot 10^{-3} \text{ J} - 1 \cdot 10^{-3} \text{ J} = \frac{0.125 \cdot 10^{-3}}{1.25 \cdot 10^{-4}} \quad \Delta U$$

2

$U(r) = \dfrac{kq_0 q_i}{r}$ (e) How much energy would it take for each 10 m displacement closer to the fixed charge? $V(r) = \dfrac{kq}{r}$

r	U(r)	delta U	V(r)	delta V
100	0.0009		900	
90	0.001	0.0001	1000	100
80	0.001125	0.000125	1125	125
70	0.001286	0.000161	1285.714	160.7143
60	0.0015	0.000214	1500	214.2857
50	0.0018	0.0003	1800	300
40	0.00225	0.00045	2250	450
30	0.003	0.00075	3000	750
20	0.0045	0.0015	4500	1500
10	0.009	0.0045	9000	4500

U(r)

$U(r) = \dfrac{kq_0 q_i}{r}$

r (meters)
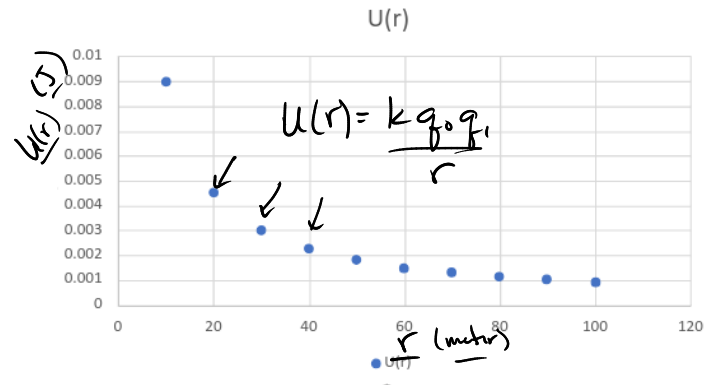
4. Work the steps in the above problem for the voltage change rather than the energy change. Also plot a graph of voltage vs. radius and illustrate on the graph what it looks like to move from say 50 m to 40 m in terms of the voltage difference.

$\Delta V(50m \rightarrow 40m) = 450 V$

V(r)

V_f V_i
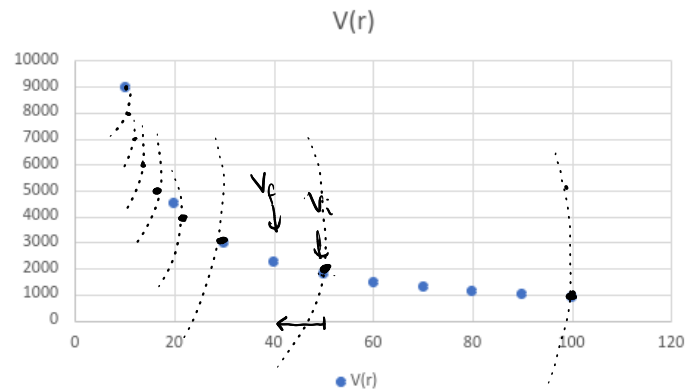
5. Now imagine that the source charge is negative. What does this change about all of the answers to the previous problem?

U

V

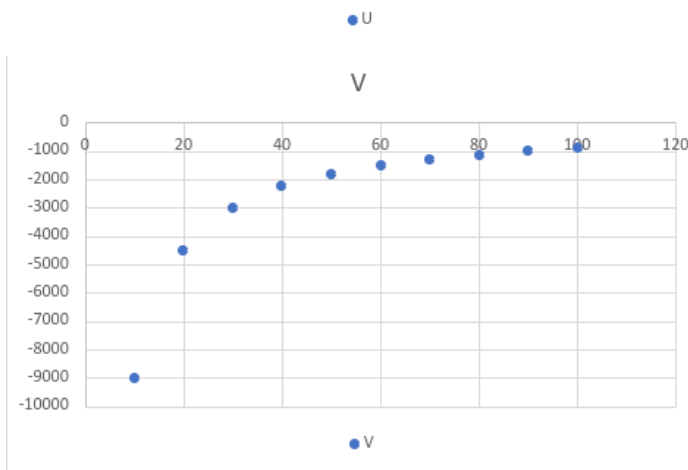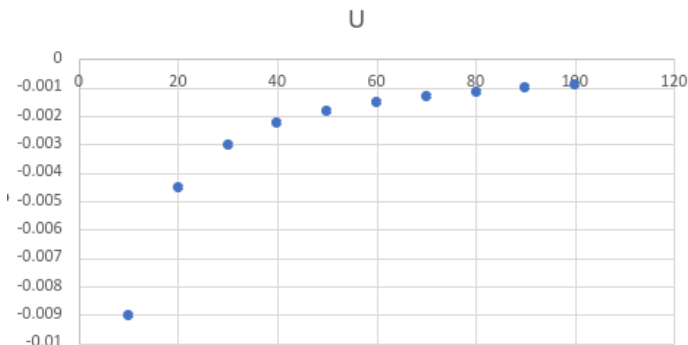$$U = 9 \cdot 10^9 \cdot \frac{(-10 \cdot 10^{-6} C)(1 \cdot 10^{-6})}{r}$$

$$\Delta U (50m \rightarrow 40m)$$

$$\Delta U = \frac{k q_0 q_i}{40m} - \frac{k q_0 q_i}{50m}$$

$$= k q_0 q_i \left(\frac{1}{40} - \frac{1}{50} \right)$$

$$\Delta U = -4.5 \cdot 10^{-4} J$$

6. What are some inside out versions of the previous problems. (Write out the wording and I'll feature some of them in the space below.)

7. In order to find the voltage at a place around multiple charges, you simply find the voltage at a location due to each charge individually and then add them up. Same goes for finding the potential energy of putting a charge at a place around other charges. So in the figure

$r_1 = \sqrt{2^2 + 1^2} = \sqrt{5} = 2.24\,m$

below we have $q_1 = +20\,\mu C$ and $q_2 = -5\,\mu C$ each 1 m away from the origin.
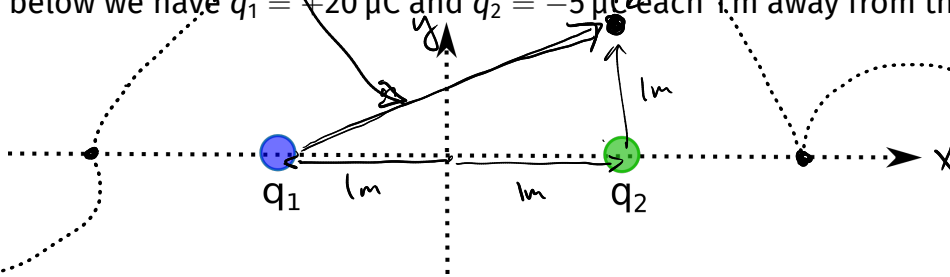
(a) What is the voltage at the location of $x = +2$ m on the x-axis relative to infinitely far away?

$\Delta V = \Delta V_{q_1} + \Delta V_{q_2}$

$= V_{q_1}(x=2m) - V_{q_1}(r=\infty)^0 + V_{q_2}(x=2m) - V_{q_2}(\infty)^0$

$= \dfrac{kq_1}{r_1} + \dfrac{kq_2}{r_2}$

$\Delta V = \dfrac{9\cdot 10^9 \cdot 20 \cdot 10^{-6}}{3m} + \dfrac{9\cdot 10^9 \cdot (-5\cdot 10^{-6})}{1m}$

$\Delta V = 15,000\,V$

$\Delta V = 15\,kV$

(b) What is the voltage at the location of $x = -2$ m on the x-axis relative to infinitely far away?

$\Delta V = \dfrac{9\cdot 10^9 \cdot 20 \cdot 10^{-6}}{1m} + \dfrac{9\cdot 10^9 \cdot (-5\cdot 10^{-6})}{3m} = 165,000\,V$

$= 165\,kV$

(c) What is the voltage at the location of $x = -2$ m on the x-axis relative to the location of $x = +2$ m on the x-axis?

$\Delta V = \Delta V_{(b)} - \Delta V_{(a)} = 165,000V - 15,000V = 150,000\,V$

(d) What is the voltage at the location of $x = +1$ m, $y = +1$ m (again relative to infinitely far away; if I ever forget to say this then this is what I mean)?

$\Delta V = \dfrac{kq_1}{2.24m} + \dfrac{kq_2}{1m} = 35,500\,V$

(e) Where along the x-axis would the voltage be equal to zero? What does this mean for the amount of work it would take to move a charge to that location from infinitely far away?

$0 = V_1 + V_2$

$0 = \dfrac{kq_1}{r_1} + \dfrac{kq_2}{r_2}$

$0 = \dfrac{kq_1}{x+1} + \dfrac{kq_2}{x-1}$

$-\dfrac{q_2}{x-1} = \dfrac{q_1}{x+1}$

$-q_2(x+1) = q_1(x-1)$

$-q_2 x - q_2 = q_1 x - q_1$

$q_1 - q_2 = q_1 x + q_2 x$

$= x(q_1 + q_2)$

$\dfrac{q_1 - q_2}{q_1 + q_2} = x = \dfrac{20\cdot 10^{-6} - (-5\cdot 10^{-6})}{20\cdot 10^{-6} + (-5\cdot 10^{-6})}$

$x = 1.67\,m$

8. Now we need to put together the change in potential energy with the kinetic energy so that we can use the conservation of energy. To review some equations that we used last semester, lets start with a statement of the conservation of energy

$$K_i + U_i + W_{nc} = K_f + U_f$$
$$K_f + U_f = K_i + U_i + W_{non\text{-}conservative}$$

The only non-conservative forces we will have are the ones from <u>outside</u> that increase the potential energy of a charge, but there will be no friction. So if a charge starts from rest at some place you move it to another place and it arrives there at rest then the equation becomes:

$$\overset{K_f}{0} + U_f = \overset{K_i}{0} + U_i + W_{nc}$$
$$\underline{\underline{\Delta U}} = \underline{\underline{W_{nc}}}$$

which is what we have been using this entire time.

But, if instead of doing work to move it, what if we just let go? The charge would begin to move in response to the electric field that it was in and the potential energy would go down, and in its place the kinetic energy would increase since the charge is speeding up. There are several ways to represent this:

$$W_{nc} \rightarrow 0 \qquad\qquad K_f - K_i = -U_f + U_i$$
$$\rightarrow \quad K_f + U_f = K_i + U_i + 0 \qquad\qquad = -(U_f - U_i)$$
$$\Delta K = -\Delta U \qquad\qquad \Delta K = -\Delta U$$

Ok now for an actual question. Take this last expression and do a <u>substitution</u> to express the change in kinetic energy as a change in voltage.

$$\Delta K = -\Delta U$$

$$\Delta V = \frac{\Delta U}{q}$$

$$\boxed{\Delta K = -q\,\Delta V}$$

$$\Delta U = q\,\Delta V$$

$$V_f = \frac{kq_o}{r_f}$$

$$V_i = \frac{kq_o}{r_i}$$

9. A $-10\,\mu C$ charge is in a uniform electric field of $1000\,N/C$ that points to the right. If the charge travels a distance of 1 cm, then how much has the <u>voltage changed</u> within the field? How much has the <u>potential energy changed</u>? How much has the <u>kinetic energy</u> changed? The charge has a mass of 1 mg. How fast is the charge going after it has traveled this distance?
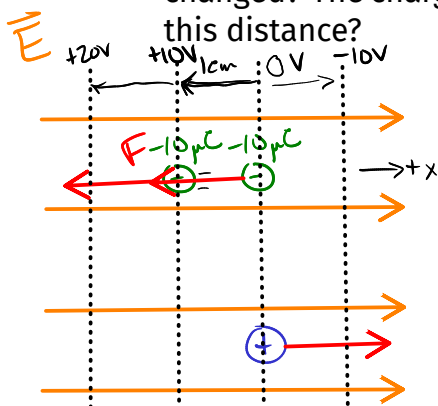
$$\Delta V = -\vec{E}\cdot\vec{\Delta x}$$
$$= -\left(+1000\tfrac{N}{C}\right)\cdot(-0.01\,m)$$
$$\underline{\Delta V = +10\,V}$$

$$\Delta U = q\cdot\Delta V$$
$$= (-10\cdot10^{-6}C)\cdot(+10V)$$
$$\Delta U = -1\cdot10^{-4}\,J$$

$$\Delta K = -\Delta U$$
$$\Delta K = +1\cdot10^{-4}\,J$$
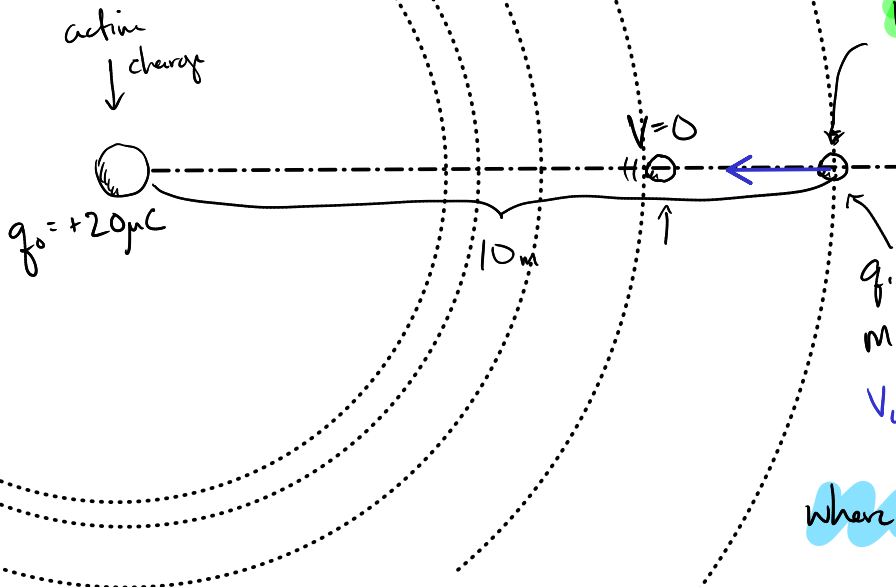
$$K = \tfrac{1}{2}mv^2 \quad\text{velocity}$$

$$\Delta K = K_f - \cancel{K_i}^{0} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}m\cancel{v_i^2}^{0}$$

$$\Delta K = \tfrac{1}{2}mv_f^2$$

$$V_f = \sqrt{\frac{2\Delta K}{m}} = 14.1 \text{ m/s}$$

action
↓ charge

$q_0 = +20\mu C$

$V = 0$

10 m

$q_i = +1\mu C$
$M = 10^{-6} kg$
$V_i = 100 \text{ m/s}$

What is the voltage at this point?

$$\Delta V = V(r_i) - V(\infty)$$

$$V(10m) = \frac{kq_0}{r} = \frac{9 \cdot 10^9 \cdot 20 \cdot 10^{-6}}{10m} \quad V = ?$$

$$V(10m) = 18000 V$$

Where does q_i stop?

$$\Delta K = -\Delta U$$

$$\Delta K = -q_i \Delta V$$

$$\frac{1}{2}mv_f^2{\nearrow}^0 - \frac{1}{2}mv_i^2 = -q_i\left(\frac{kq_0}{r_f} - \frac{kq_0}{r_i}\right)$$

$10^{-6} \quad 100 \quad 10^{-6} \quad\quad 20 \cdot 10^{-6} \quad 18000V$

$10m$

$$\boxed{r_f = 7.8 \text{ m}}$$

How fast is this charge going when it is 20m away?

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2{\nearrow}^0 = -q_i\left(\frac{kq_0}{20m} - \frac{kq_0}{7.8m}\right)$$

$$\boxed{V_f = 168 \text{ m/s}}$$

How fast is it going when it is infinitely far away?

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2{\nearrow}^0 = -q_i\left(\frac{kq_0}{r_f}{\nearrow}^0 - \frac{kq_0}{7.8m}\right)$$

$$r_f \to \infty$$

$$\boxed{V_f = 214 \text{ m/s}}$$

$$\Delta K = K_f - K_i = -q \, \Delta V$$

$$K_f = -q \, \Delta V + K_i$$

$$\underset{=}{m_i} L_f + \underset{=}{m_i} \cdot C_w \cdot \left(T_f - T_i \right) + \quad m_w C_w \cdot \left(T_f - T_i \right)$$

$$5° \qquad 0° \qquad\qquad 5° \qquad 20$$

$$mc\Delta T = Q$$

$$\left(1 kg \right) \left(4.184 \, kJ/_{kg\,k} \right) \left(\Delta T \right) = Q$$

$$P = \frac{Q}{t} \Rightarrow t = \frac{Q}{P} \xleftarrow{} J$$

$$\xleftarrow{} 600 W$$

$$Q = \underline{\quad} \, kJ \rightarrow J$$

$$W = - p \Delta V$$

$$-Q \qquad E \qquad +Q$$

$$F \qquad -Q$$
$$E$$
$$+q$$
$$+Q$$

$$F = q \cdot E$$
$$\frac{\pm}{?}$$