Week 11 covers sections of chapter 22 in the textbook. Topics include:

- electromagnetic waves
- index of refraction
- doppler shift

violet red
380 nm – 700 nm

1. What is the range of wavelengths of human vision? What is the range of frequencies? Approximately what wavelength is red? Green? Violet? A wave that has a wavelength of 1 nm could be used for what purpose? What is its frequency? Wifi usually works around a frequency of 2.4 GHz which is the same as a consumer microwave oven. What wavelength is this? WBHM is the Public Radio station in Birmingham and you can find it on the radio dial at 90.3 MHz. What wavelength is this?

$$f = \frac{3\cdot 10^8\ m/s}{1\cdot 10^{-9}\ m} = 3\cdot 10^{17}\ Hz$$ X-RAY

$$v = \lambda \cdot f$$

$$\lambda = \frac{v}{f} = \frac{3\cdot 10^8\ m/s}{2.4\cdot 10^9\ Hz} = 0.125\ m$$

12.5 cm

$$\lambda = \frac{3\cdot 10^8\ m/s}{90.3\cdot 10^6\ Hz} = 3.32\ m$$

kilo = 10^3
Mega = 10^6
Giga = 10^9
Tera = 10^{12}
Peta = 10^{15}
Exa = 10^{18}
Zetta = 10^{21}

$$f = \frac{1}{T}$$

$$\underset{\text{speed}}{v} = \frac{\overset{\text{wavelength}}{\lambda}}{\underset{\text{period}}{T}} = \overset{\text{wavelength}}{\lambda} \cdot \overset{\text{frequency}}{f}$$

$$v_{vac} = c = 3\cdot 10^8\ m/s$$

$$v = \lambda \cdot f$$

$$f_{violet} = \frac{v}{\lambda} = \frac{3\cdot 10^8\ m/s}{380\cdot 10^{-9}\ m} = 7.8\cdot 10^{14}\ Hz$$

$$f_{red} = \frac{3\cdot 10^8\ m/s}{700\cdot 10^{-9}\ m} = 4.3\cdot 10^{14}\ Hz$$

2. What frequency has the same magnitude as its wavelength? What region is this in?

$$v = \underset{x}{\lambda} \cdot \underset{x}{f}$$

$$v = x^2 \Rightarrow x = \sqrt{v} \Rightarrow x = \sqrt{3\cdot 10^8} = \underbrace{1.7\cdot 10^4\ m \text{ or } 1.7\cdot 10^4\ Hz}_{\text{radio waves}}$$

3. The sun is 93 000 000 miles away from earth. If the sun suddenly burned out, how long would it take for us to know?

$$V = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{V} \qquad \underline{t = 8.3 \text{ min}}$$

4. One light-year is the distance that light travels in a year. How many meters is this?

$$\text{distance} = v \cdot \text{time} \qquad \text{time of 1 year in seconds?}$$

$$d = 9.46 \cdot 10^{15} \text{ m}$$

$5.87 \cdot 10^{12}$ miles

5.87 Teramiles

5. What is the speed of light in water where $n = 1.33$? What about the speed of light in diamond where $n = 2.42$?

$$n = \frac{c}{v} \Rightarrow V = \frac{c}{n}$$

$$V_{H_2O} = \frac{3 \cdot 10^8 \text{ m/s}}{1.33} = 2.26 \cdot 10^8 \text{ m/s} \qquad V_c = \frac{3 \cdot 10^8 \text{ m/s}}{2.42} = 1.24 \cdot 10^8 \text{ m/s}$$

6. The speed of light in some unknown material is measured to be 1.3×10^6 m/s. What is the index of refraction?

$$n = \frac{c}{v} = \frac{3 \cdot 10^8 \text{ m/s}}{1.3 \cdot 10^6 \text{ m/s}} = 231$$

$\to 1 \cdot 10^{14}\,Hz$

7. If the frequency of light in water is 1×10^{14} Hz ~~in water~~ then what is it in diamond? What is the wavelength in water? What is the wavelength in diamond?

$V_{H_2O} = \lambda \cdot f$

$\lambda = \frac{2.26 \cdot 10^8\,m/s}{1 \cdot 10^{14}\,Hz} = 2.26 \cdot 10^{-6}\,m$

$2.26\,\mu m$

$V_c = \lambda_c \cdot f$

$\lambda_c = \frac{V_c}{f} = \frac{1.24 \cdot 10^8\,m/s}{1 \cdot 10^{14}\,Hz}$

$\lambda_c = 1.24 \cdot 10^{-6}\,m$

$\frac{c}{n} = V = \lambda \cdot f$

$\frac{c}{n} = \lambda \cdot f$

$\lambda = \frac{c}{nf} = \frac{c}{n} \cdot \frac{1}{f}$

$\lambda \propto n^{-1} \rightarrow \boxed{\frac{\lambda_c}{\lambda_{H_2O}} = \left(\frac{n_c}{n_{H_2O}}\right)^{-1}}$

8. Show that the quantity and units work out for the relationship between the speed of light in vacuum and the permittivity and permeability of free space:

$$c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$$

$\downarrow$ $3 \cdot 10^8\,m/s$

$\varepsilon_0 = 8.85 \cdot 10^{-12}\,\frac{C^2}{Nm^2}$

$\mu_0 = 4\pi \cdot 10^{-7}\,\frac{Tm}{A}$

9. The simplest model for an electromagnetic wave to take is a sine wave. The general form for using the sine wave and having the parameters match those of wavelength and period that we have discussed can be written like this:

$E_0 = 1\,\frac{N}{C}$

$$E(x,t) = E_0 \sin\left(\cancel{\frac{2\pi}{\lambda}}x - \frac{2\pi}{T}t\right)$$

This is just a function that gives you the value of the electric field at a particular place (x) at a particular time (t). For an electromagnetic wave of wavelength 100 nm, what is the period? Sketch two wavelengths of this wave on a graph when $t = 0$. Then sketch another plot 1/3 of a period later.

$V = \lambda \cdot f \qquad f = \frac{1}{T} \qquad V = \frac{\lambda}{T}$

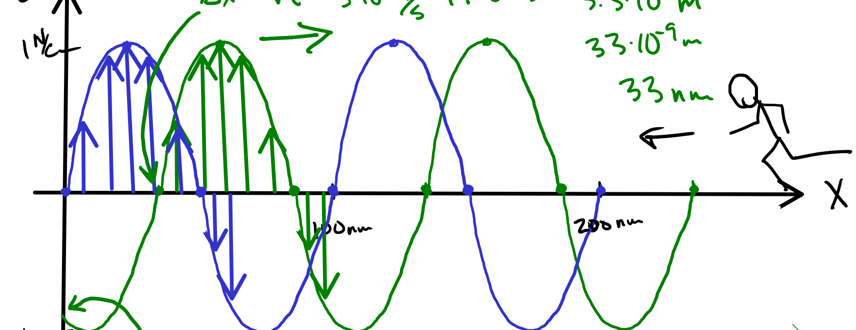
$T = \frac{\lambda}{V} = \frac{100 \cdot 10^{-9}\,m}{3 \cdot 10^8\,m/s} = 3.3 \cdot 10^{-16}\,s$

$\frac{T}{3} = 1.1 \cdot 10^{-16}\,s$

$\Delta x = vt = 3 \cdot 10^8\,m/s \cdot 1.1 \cdot 10^{-16}\,s = 3.3 \cdot 10^{-8}\,m$

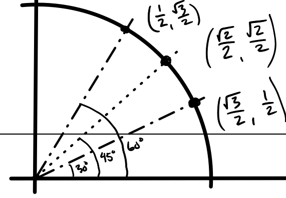
$33 \cdot 10^{-9}\,m$

$33\,nm$



$E(x=0\,m, t=1.1 \cdot 10^{-16}\,s) = 1\,\frac{N}{C} \cdot \sin\left(0 - \frac{2\pi}{3.3 \cdot 10^{-16}\,s} \cdot 1.1 \cdot 10^{-16}\,s\right)$

$= 1 \sin\left(-\frac{2\pi}{3}\right) = -0.866\,N/C$



10. *Doppler Effect.* Imagine yourself approaching a source of light and think of the light as waves. You are traveling in the opposite direction as the light propagation. You can see from some of the above graphs of waves traveling that if you are doing this, then the period of time between when crests of the light are detected by you will be shorter than for a stationary observer. This means the frequency you observe will be higher than the frequency of the source. The faster you go the higher this frequency shift will be. The opposite effect will happen if you are traveling *away* from the source of light; you will observe a lower frequency. The equation relating the frequency you observe, f_o, the frequency of the source f_s and the *relative velocity* between source and observer v_{rel} is

observed frequency

$$f_o = f_s\sqrt{\frac{1+\frac{v_{rel}}{c}}{1-\frac{v_{rel}}{c}}}$$

source frequency

In this equation v_{rel} is negative when the source and observer are moving away from each other and positive when they are approaching each other.

Using this equation, calculate how fast you would have to be going in your car in order for a red light to appear green.

$\lambda_{red} = 700\text{ nm}$ $\quad f_{red} = 4.3\cdot10^{14}\text{ Hz} = f_s$

$\lambda_{green} = 550\text{ nm}$ $\quad f_{green} = 5.4\cdot10^{14}\text{ Hz} = f_o$

$$\left(\frac{f_o}{f_s}\right)^2 = \left(\frac{5.4\cdot\cancel{10^{14}}\,\cancel{Hz}}{4.3\cdot\cancel{10^{14}}\,\cancel{Hz}}\right)^2 = 1.577$$

$$\left(1-\frac{v}{c}\right)\left(\frac{f_o}{f_s}\right)^2 = \frac{1+\frac{v}{c}}{\cancel{1-\frac{v}{c}}}\cancel{\left(1-\frac{v}{c}\right)}$$

$$\left(\frac{f_o}{f_s}\right)^2 - \left(\frac{f_o}{f_s}\right)^2\frac{v}{c} = 1+\frac{v}{c}$$

$$\left(\frac{f_o}{f_s}\right)^2 - 1 = \frac{v}{c} + \left(\frac{f_o}{f_s}\right)^2\cdot\frac{v}{c}$$

$$= \left[\left(\frac{f_o}{f_s}\right)^2 + 1\right]\cdot\frac{v}{c}$$

$$\frac{v}{c} = \frac{\left(\frac{f_o}{f_s}\right)^2 - 1}{\left(\frac{f_o}{f_s}\right)^2 + 1}$$

$$\frac{v}{c} = \frac{0.577}{2.577} = 0.22$$

$$\frac{v}{c} = 0.22$$

$$v = 0.22\cdot c$$

$$\boxed{v = 7\cdot10^7\text{ m/s}}$$

How long would you need to accelerate to go this fast?

$v_f = v_i + at$

$a = \frac{\Delta v}{\Delta t} = \frac{8000\text{ m/s}}{60\text{ s}} = 133.3\text{ m/s}^2$

13.5 g

$t = \frac{\Delta v}{a} = \frac{7\cdot10^7\text{ m/s}}{133.3\text{ m/s}^2} = 5\cdot10^5\text{ s}$

11. The speed of light is very very fast, and many times the relative velocities between source and observer are much smaller than that. The equation above can be simplified in cases

$5\cdot10^5\text{ s}\cdot\frac{1\text{ hr}}{3600\text{ s}}\cdot\frac{1\text{ day}}{24\text{ hr}} = 5.8\text{ days}$

$\frac{v_{rel}}{c} << 1$

when $v_{rel} << c$ to a much easier equation:

$$f_o = f_s\left(1 + \frac{v_{rel}}{c}\right)$$

The radar of a police officer's radar gun emits microwave radiation at about 3×10^{10} Hz. The officer is at driving at 35 mph some very reckless person is driving toward him at 55 mph in a 35 mph zone.

(a) What is the relative velocity between the officer and the driver?

35 mph + 55 mph = 90 mph

$$\frac{90\text{ miles}}{\text{hour}} \cdot \frac{1609\text{ m}}{1\text{ mile}} \cdot \frac{1\text{ hr}}{3600\text{ s}} = 40.2\ \frac{\text{m}}{\text{s}}$$

(b) What is the frequency of the radar that the speeding car observes?

$$f_o = f_s\left(1 + \frac{v}{c}\right)$$

speeder officer

$$= 3 \cdot 10^{10}\text{ Hz}\left(1 + \frac{40.2\text{ m/s}}{3 \cdot 10^8\text{ m/s}}\right)$$

$$= 3.0000004023 \cdot 10^{10}\text{ Hz}$$

$$= 30{,}000{,}004{,}023\text{ Hz}$$

(c) When this radiation hits the speeding car, it reflects back to the officer, but the reflected light is essentially re-emitted from the speeding car at the doppler shifted frequency it "observed". So now the speeding car is emitting radar (it is a now source) and the officer is observing this radiation, but since they are moving toward each other, it is doppler shifted again. What frequency of radar does the officer observe now?

$$f_o = f_s\left(1 + \frac{v_{rel}}{c}\right)$$

officer speeder

$$= 30{,}000{,}004{,}023\text{ Hz}\left(1 + \frac{40.2}{3 \cdot 10^8}\right)$$

$$= 3.0000008046 \cdot 10^{10}\text{ Hz}$$

$$= 30{,}000{,}008{,}046$$

(d) Actually measuring this frequency is hard, since they are so close together, but what can be done is measuring the *beat frequency* of the officer's emitted radar, and the observed beam reflected from the speeder. What is the beat frequency of these two waves?

$$f_b = |f_1 - f_2|$$

$$f_b = \underline{8046\,\text{Hz}} \leftarrow$$

12. At what relative speed does the approximate form of the doppler shift equation give an error of 1%. You should make a table (or even better an Excel sheet) and try out several values for v_{rel} to find out. Choose any f_s or just use the ratio of f_o/f_s.

$$f_o = f_s\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \quad \xrightarrow{\text{approx}} \quad f_o = f_s\left(1+\frac{v}{c}\right)$$

A	B	C	D	E	F	G
relative velocity	the exact doppler shift	the approximate doppler shift	% error		c	f_s
1	30000000100	30000000100	0		3E+08	3E+10
10	30000001000	30000001000	3.8147E-14			
100	30000010000	30000010000	5.55674E-12			
1000	30000100000	30000100000	5.55558E-10			
10000	30001000017	30001000000	5.55556E-08			
100000	30010001667	30010000000	5.55556E-06			
1000000	30100167224	30100000000	0.000555557			
10000000	31017236587	31000000000	0.055570996			
20000000	32071349029	32000000000	0.222469686			
30000000	33166247904	33000000000	0.501256289			
40000000	34306312494	34000000000	0.892875018	←	1%	error
50000000	35496478699	35000000000	1.398670282			
60000000	36742346142	36000000000	2.020410289			
70000000	38050309946	37000000000	2.760319028			
80000000	39427724440	38000000000	3.621118035			
100000000	42426406871	40000000000	5.719095842			

$\lambda = 479\,nm$ air $\to$ glass $n = 1.52$ $f_{glass} = ?$

$$f_{air} = \frac{c}{\lambda_{air}} = \frac{3 \cdot 10^{8}}{479 \cdot 10^{-9}} = 6.28 \cdot 10^{14}\,Hz = f_{glass}$$

$$V = \sqrt{\frac{3 k_B T}{m}}$$

Boltzmann's

[K]

[kg] $\to$ of 1 molecule

N_2

$$28\,\frac{g}{mol} \cdot \frac{1\,mol}{6.022 \cdot 10^{23}\,\text{molecules}} \cdot \frac{1\,kg}{1000\,g} = \quad \frac{kg}{\text{molecule}}$$

$$U = \frac{3}{2} nRT$$

[K]