Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being 0 °C when water freezes and 100 °C when water boils at 1 atm of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. 105 °F, 98.6 °F, 72 °F, 32 °F, 0 °F. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

$$T_F = \frac{9}{5}\cdot T_c + 32$$

$-32 \qquad -32$

$$(T_F - 32) = \frac{9}{5} T_c$$

$$T_c = \frac{5}{9}(T_F - 32)$$

T_F	T_c
105°F	40.6
→ 98.6°F	37
72°F	22
32°F	0
0°F	-18
-40	-40

$$T = \frac{9}{5}\cdot T + 32$$

$$T - \frac{9}{5}T = 32$$

$$-\frac{4}{5}T = 32$$

$$\boxed{T = -40}$$

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by 30 °F, then by how much does the temperature change in Celsius? What does this mean about the "size" of a Celsius degree vs. the "size" of a Fahrenheit degree? Which one represents a larger change in temperature?

$$\Delta T_F = \frac{9}{5}\Delta T_c$$

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

$T_K = T_c + 273.15$

$T_c = T_K - 273.15$

T_c	T_K
-273.15	0
0	+273.15 K
20°C	293.15 K
-203.15°C	70 K
37°C	310 K

$\Delta T_c = \Delta T_K$

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

$T_{F,\text{abs zero}} = \frac{9}{5}(-273.15) + 32$

$T_{F,\text{abs zero}} = -459°F$

$T_F = \frac{9}{5}T_c + 32 \qquad T_c = \underbrace{T_K - 273.15}$

$T_F = \frac{9}{5}(T_K - 273.15) + 32$

$T_F = \frac{9}{5}T_K - 491 + 32$

$T_F = \frac{9}{5}T_K - 459$

5. What is the ~~molecular~~ atomic ~~weight~~ mass of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

12 g/mol

6p

6n

6p

8n

atomic mass − # of protons = # of neutrons

12 − 6 = 6

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

1 mole of things = $6.022 \cdot 10^{23}$ things

4g

12g

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

$$12g + 2(16g) = 44\,g/mol$$

mass of one thing × number of things = total mass of the collection

$$M_{CO_2} = \frac{44g}{6.022\cdot 10^{23}\text{ molecules}} = 7.1\cdot 10^{-23}\,g/\text{molecule} = 7.1\cdot 10^{-26}\,kg/\text{molecule}$$

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

$28g \cdot 0.78 =$ ____

$32g \cdot 0.21 =$ ____

$40g \cdot 0.01 =$ ____

29 g/mol

9. A balloon is filled with 0.4 mol of helium so that its volume is 0.010 m^3.

- Find the number of atoms.

$N = n \cdot N_A$ ← Avagadros

# of particles

number of moles

$N = 0.4\text{ mol} \cdot 6.022\cdot 10^{23}$ particles/mol

$N = 2.4\cdot 10^{23}$ He atoms

- Find the number density.

$$\text{number density} = \frac{N}{V} = \frac{2.4\cdot 10^{23}\text{ atoms}}{0.010\text{ m}^3} = 2.4\cdot 10^{25}\text{ He atoms}/m^3$$

10^{-2}

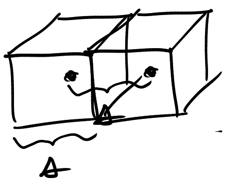
- Find the mass density.

$4\,g/mol \cdot 0.4\text{ mol} = 1.6g = 0.0016\,kg = M$

$$\text{mass density} = \rho = \frac{M}{V} = \frac{0.0016\,kg}{0.010\,m^3} = 0.16\,kg/m^3$$

- Estimate the average distance between atoms. To do this, fine the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

$$\text{number density} = \frac{\text{particles}}{\text{volume}} \quad \frac{\text{volume}}{\text{particles}} = 4.16 \cdot 10^{-26} \frac{m^3}{\text{particle}}$$



$$V = s^3 \Rightarrow s = \sqrt[3]{4.16 \cdot 10^{-26}\, m^3}$$

$$s = 3.5 \cdot \underbrace{10^{-9}\, m}_{\text{nanometer}}$$

$s = 3.5$ nanometers

$s = 35$ Å

10. You have a pound of feathers and a pound of lead.

    - Which one weighs more? same
    - Which one has more mass? same
    - Which one has the greater volume? feathers
    - Which one contains a larger number of moles? trick
    - Which one contains a larger number of atoms? feathers
    - Which one contains a larger number of protons and neutrons? same

$$F_g = mg$$

11. You check your car tire pressure and see that the pressure is 25 lb/in^2. What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

$$25\,\text{psi} \cdot \frac{1\,\text{atm}}{14.7\,\text{psi}} \cdot \frac{1.013 \cdot 10^5\,\text{Pa}}{1\,\text{atm}} = 1.7 \cdot 10^5\,\text{Pa}$$

$$P_{abs} = P_{gauge} + P_{atm}$$

$$= 1.7 \cdot 10^5\,\text{Pa} + 1.013 \cdot 10^5\,\text{Pa}$$

$$P_{abs} = 2.7 \cdot 10^5\,\text{Pa}$$

$39.7\,lb/in^2$

$+14.7\,lb/in^2$

12. You check you car tire pressure when it is 15 °C and it is 25 lb/in^2. By what factor do you increase the number of particles in the tire so that the pressure becomes that 30 lb/in^2? (*Hint: The volume and temperature do not change.*)

$+14.7\,lb/in^2$

$44.7\,lb/in^2$

$$\frac{P_2 \cancel{V_2} = N_2 \cancel{k_B} \cancel{T_2}}{P_1 \cancel{V_1} = N_1 \cancel{k_B} \cancel{T_1}}$$

$$\%\,change = \left(\frac{N_2 - N_1}{N_1}\right) \times 100$$

$$\%\,change = \left(\frac{N_2}{N_1} - 1\right) \times 100$$

$$\frac{P_2}{P_1} = \frac{N_2}{N_1} = \frac{44.7\,lb/in^2}{39.7\,lb/in^2} = 1.13$$

% change = 13% increase

absolute → $10^5\,Pa$

293K

13. The gas pressure inside of a 1 liter sealed container at room temperature is 1 atm. How many molecules are inside? How many moles of molecules?

$$1\,L \cdot \frac{1000\,mL}{1\,L} \cdot \frac{1\,cm^3}{1\,mL} \cdot \frac{(1\,m)^3}{(100\,cm)^3} = 0.001\,m^3$$

$$1000\,L = 1\,m^3$$

Boltzmann's Constant $1.38 \cdot 10^{-23}\,J/K$ (microscopic)

Ideal Gas Law

$PV = Nk_BT$ — Pa, m^3, number, Kelvin

$PV = nRT$ (macroscopic) — atm, Liters, moles, $0.08206\,\frac{atm \cdot L}{mol\,K}$, $\boxed{8.314\,\frac{J}{K\,mol}}$

$$PV = Nk_BT$$

$$N = \frac{PV}{k_BT} = \frac{10^5\,Pa \cdot 0.001\,m^3}{1.38 \cdot 10^{-23}\,J/K \cdot 293\,K} = 2.5 \cdot 10^{22}\,molecules$$

$$2.5 \cdot 10^{22}\,molecules \cdot \frac{1\,mol}{6.022 \cdot 10^{23}\,molecules} = 0.041\,mol$$

14. If the pressure inside a tank is 1 atm when the temperature is 100 K, then what is the pressure when the temperature rises to 200 K?

$$P = \frac{Nk_B}{V}T$$

$$\rightarrow P \propto T$$

$$\rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{200\,K}{100\,K} = 2$$

$$\frac{P_2}{P_1} = 2$$

$$P_2 = 2\,atm$$

$$P_2 = 2 \cdot 10^5\,Pa$$

15. If the pressure inside a tank is 1 atm when the temperature is 100 °C, then what is the pressure when the temperature rises to 200 °C? *CAREFUL!*

→ 373K

→ 473K

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{473}{373} = 1.26$$

$$\frac{P_2}{P_1} = 1.26 \qquad P_2 = 1.26\,atm$$

$$P_2 = 1.26 \cdot 10^5\,Pa$$

16. A gas is in a sealed container. By what factor does the pressure change if

$$P = \frac{Nk_BT}{V} = Nk_BT \cdot V^{-1}$$

- the volume is doubled?

$$P \propto V^{-1}$$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = (2)^{-1} = \frac{1}{2}$$

- the temperature is tripled?

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = 3$$

- the volume is double and the temperature is tripled?

$$P \propto T \cdot V^{-1}$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \cdot \left(\frac{V_2}{V_1}\right)^{-1} = 3 \cdot (2)^{-1} = 3/2$$

- the volume is halved?

$$P \propto V^{-1}$$

$$\frac{P_2}{P_1} = \left(\frac{1}{2}\right)^{-1} = 2$$

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

$$10\,\text{m} \cdot 15\,\text{m} \cdot 2\,\text{m} = 300\,\text{m}^3$$

$$PV = nRT$$

$$n = \frac{PV}{RT} = 12{,}300\ \text{mol}$$