Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being 0 °C when water freezes and 100 °C when water boils at 1 atm of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. 105 °F, 98.6 °F, 72 °F, 32 °F, 0 °F. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

$$T_F = \frac{9}{5}T_c + 32$$

$$\frac{9}{5}T_c = T_F - 32$$

$$T_c = \frac{5}{9}(T_F - 32)$$

T_F	T_c
105°F	40.6°C
98.6°F	37°C
72°F	22
32	0
0°F	-18
-40°F	-40°C

$$T = \frac{9}{5}T + 32$$

$$T = -40$$

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by 30 °F, then by how much does the temperature change in Celsius? What does this mean about the "size" of a Celsius degree vs. the "size" of a Fahrenheit degree? Which one represents a larger change in temperature?

$$\Delta T_F = \frac{9}{5}\Delta T_c$$

$$\frac{5}{9}\Delta T_F = \Delta T_c$$

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

$T_K = T_C + \underline{273.15°C}$

$T_C = T_K - 273.15°C$

T_C	T_K
0°C	273.15 K
20°C	293.15 K
-203°C	70 K
37°C	310 K

$\underline{\Delta T_C = \Delta T_K}$

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

$T_F = \frac{9}{5} T_C + 32$

$T_F = \frac{9}{5}(-273.15) + 32$

$= -459°F$

$T_F = \frac{9}{5} T_C + 32 \qquad T_C = T_K - 273.15°C$

$T_F = \frac{9}{5}(T_K - 273.15) + 32$

$T_F = \frac{9}{5} T_K - 491 + 32$

$T_F = \frac{9}{5} T_K - 459.67 \leftarrow$

5. What is the ~~molecular~~ atomic ~~weight~~ mass of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

6p

6n

6p

8n

atomic mass − # of protons = # of neutrons

12 − 6p = 6n

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

1 mole of things = $6.022 \cdot 10^{23}$ things

4g

12g

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

(kg)

$$12g + 2(16g) = 44\,g/mol$$

→ mass of one thing × number of things = total mass of the collection

$$m_{CO_2} = \frac{44\,g}{6.022\cdot 10^{23}\text{ molecules}} = 7.1\cdot 10^{-23}\,\frac{g}{\text{molecule}} = 7.1\cdot 10^{-26}\,kg$$

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

$28\,g/mol \cdot 0.78 =$ ____

$32\,g/mol \cdot 0.21 =$ ____

$40\,g/mol \cdot 0.01 =$ ____

$29\,g/mol$

9. A balloon is filled with 0.4 mol of helium so that its volume is 0.010 m^3.

- Find the number of atoms.

$$\rightarrow N = n \cdot N_A \leftarrow \text{Avogadro's}$$

N: number of particles; n: number of moles

$$N = 0.4\,mol \cdot 6.022\cdot 10^{23}\,\text{He atoms}/mol$$

$$N = 2.4\cdot 10^{23}\text{ He atoms}$$

- Find the number density.

$$\text{number density} = \frac{N}{V} = \frac{2.4\cdot 10^{23}\text{ atoms}}{0.010\,m^3} = 2.4\cdot 10^{25}\text{ He atoms}/m^3$$

- Find the mass density.

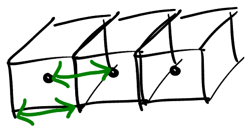
$$\text{mass density} = \rho = \frac{M}{V}$$

$$M = 4\,\frac{g}{mol}\cdot 0.4\,mol = 1.6\,g = 0.0016\,kg$$

$$\rho = \frac{0.0016\,kg}{0.010\,m^3} = 0.16\,kg/m^3$$

- Estimate the average distance between atoms. To do this, fine the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

$$\frac{N}{V} \to \frac{V}{N} = \frac{1}{2.4 \cdot 10^{25}\ \text{He atoms}/m^3} = 4.16 \cdot 10^{-26}\ m^3/\text{particle}$$ ← Volume of cube



$$V_{cube} = s^3$$

$$s = \sqrt[3]{V}$$

$$s = 3.46 \cdot 10^{-9}\ m$$

$$s = 3.5\ nm = 35\ Å$$

10. You have a pound of feathers and a pound of lead.

   - Which one weighs more? same
   - Which one has more mass? same
   - Which one has the greater volume? feathers
   - Which one contains a larger number of moles? bad question
   - Which one contains a larger number of atoms? feathers
   - Which one contains a larger number of protons and neutrons? same

11. You check your car tire pressure and see that the pressure is 25 lb/in^2 (psi). What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

$$25\ psi \cdot \frac{1\ atm}{14.7\ psi} \cdot \frac{1.013 \cdot 10^5\ Pa}{1\ atm} = 1.7 \cdot 10^5\ Pa$$

$$\to P_{abs} = P_{gauge} + P_{atm}$$

$$= 1.7 \cdot 10^5\ Pa + 1.013 \cdot 10^5\ Pa$$

$$P_{abs} = 2.7 \cdot 10^5\ Pa$$

12. You check you car tire pressure when it is 15 °C and it is 25 lb/in². By what factor do you increase the number of particles in the tire so that the pressure becomes that 30 lb/in²? (*Hint: The volume and temperature do not change.*)

44.7 psi

$$\frac{P_2 V_2 = N_2 k_B T_2}{P_1 V_1 = N_1 k_B T_1} \Rightarrow \frac{P_2}{P_1} = \frac{N_2}{N_1}$$

$$\frac{N_2}{N_1} = \frac{44.7\,psi}{39.7\,psi} = 1.13$$

$$\%\Delta = \left(\frac{N_2 - N_1}{N_1}\right) \times 100$$

$$\%\Delta = \left(\frac{N_2}{N_1} - 1\right) \times 100$$

$$\%\Delta = +13\%$$

13% increase

13. The gas pressure inside of a 1 liter sealed container at room temperature is 1 atm. How many molecules are inside? How many moles of molecules? Ideal Gas Law

$$1\,L \cdot \frac{1000\,mL}{1\,L} \cdot \frac{1\,cm^3}{1\,mL} \cdot \frac{(1\,m)^3}{(100\,cm)^3} = 0.001\,m^3$$

$$20^\circ C + 273.15 = 293.15\,K$$

$$1\,atm = 1.013 \cdot 10^5\,Pa$$

$$N = \frac{PV}{k_B T} = \frac{1.013 \cdot 10^5\,Pa \cdot 0.001\,m^3}{1.38 \cdot 10^{-23}\,\frac{J}{K} \cdot 293.15\,K} = 2.5 \cdot 10^{22}\ part.$$

$$2.5 \cdot 10^{22}\ part \cdot \frac{1\ mol}{6.022 \cdot 10^{23}\ part} = 0.041\ mol$$

(microscopic)

$PV = N k_B T$

P: Pa, V: m^3, k_B: Boltzmann, T: K

$k_B = 1.38 \cdot 10^{-23}\ J/K$

(macroscopic)

$PV = nRT$ → K → K

P: Pa, V: m^3, n: mol, R: 8.31 J/K mol

P: atm, V: L, n: mol, R: 0.0821 atm·L/K mol

$$n = \frac{PV}{RT} = \frac{1}{0.0821 \cdot 293} = 0.041\ mol$$

14. If the pressure inside a tank is 1 atm when the temperature is 100 K, then what is the pressure when the temperature rises to 200 K?

P_{abs}

$$PV = Nk_BT$$

$$P = \left(\frac{Nk_B}{V}\right) T$$

constant

$$P \propto T$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{200}{100} = 2$$

$$P_2 = 2 \cdot P_1$$

$$P_2 = 2 \cdot 1\,atm = 2\,atm = 2 \cdot 10^5\,Pa$$

15. If the pressure inside a tank is 1 atm when the temperature is 100 °C, then what is the pressure when the temperature rises to 200 °C? *CAREFUL!*

P_1, 473 K, 373 K

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{473}{373} = 1.26$$

$$\frac{P_2}{P_1} = 1.26$$

$$P_2 = 1.26\,atm$$

$$P_2 = 1.26 \cdot 10^5\,Pa$$

16. A gas is in a sealed container. By what factor does the pressure change if

- the volume is doubled?

$$\frac{P_2V_2 = \cancel{N_2}\cancel{k_B}\cancel{T_2}}{P_1V_1 = \cancel{N_1}\cancel{k_B}\cancel{T_1}}$$

$$\frac{P_2V_2}{P_1V_1} = 1 \qquad \frac{V_2}{V_1} = 2 \qquad \frac{V_1}{V_2} = \frac{1}{2}$$

$$\frac{P_2}{P_1} = \frac{V_1}{V_2}$$

$$PV = Nk_BT$$

$$P = \frac{Nk_BT}{V}$$

$$P = Nk_BT\,V^{-1}$$

$$P \propto V^{-1}$$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = (2)^{-1} = \underline{\frac{1}{2}}$$

- the temperature is tripled?

$$P \propto T$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = 3$$

- the volume is double and the temperature is tripled?

$$P \propto T \cdot V^{-1}$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \cdot \left(\frac{V_2}{V_1}\right)^{-1} = 3 \cdot (2)^{-1} = 3 \cdot \frac{1}{2} = \frac{3}{2}$$

- the volume is halved?

$$P \propto V^{-1}$$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$$

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

$$V = l \cdot w \cdot h = 10\,m \cdot 15\,m \cdot 2\,m = 300\,m^3$$

$$PV = nRT$$

$$R = 8.31$$

$$n = \frac{PV}{RT} = \frac{1 \cdot 10^5\,Pa \cdot 300\,m^3}{8.31\,J/K\cdot mol \cdot 293\,K} = 12{,}300\ mol$$

18. RT, 1atm, 1mol, how big? show me

$$V = \frac{nRT}{P} = \frac{1\,mol \cdot 8.31 \cdot 293}{1 \cdot 10^5\,Pa} = 0.024\,m^3$$

$$s = \sqrt[3]{V}$$

$$s = 0.289\,m \rightarrow 30\,cm$$

11.8 in

$$V_{rms} = \sqrt{\frac{3k_BT}{m}}$$

$\hookrightarrow$ mass of an particle in kg

O_2 at RT, how fast is each molecule moving?

$\hookrightarrow 2 \cdot 16 g/mol = 32 g/mol$ $\quad 0.032 \frac{kg}{mol} \cdot \frac{1 \text{ mol}}{6.022 \cdot 10^{23} \text{ part}} = 5.31 \cdot 10^{-26}$ kg

$$V_{rms} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 293K}{5.31 \cdot 10^{-26} kg}}$$

$$= 478 \text{ m/s}$$

$$V_{rms} = \left(\frac{3k_BT}{m}\right)^{1/2} = (3k_B)^{1/2} T^{1/2} m^{-1/2}$$

$V_{rms} \propto T^{1/2}$ for same m

$$\frac{V_2}{V_1} = \left(\frac{T_2}{T_1}\right)^{1/2}$$

$V_{rms} \propto m^{-1/2}$ for constant T

$$\frac{V_{O_2}}{V_{N_2}} = \left(\frac{m_{O_2}}{m_{N_2}}\right)^{-1/2}$$

For your HW problem:

2 mol, N_2, $(31 cm)^3$, 2.9 atm

$PV = nRT$

$$T = \frac{PV}{nR} = \frac{2.9 \cdot 10^5 \cdot (0.31 m)^3}{2 \text{ mol} \cdot 8.31}$$

$$= 519.8 K$$

$m = .028 \frac{kg}{mol} \cdot \frac{1 \text{ mol}}{6.02 \cdot 10^{23} \text{ parts}} = 4.65 \cdot 10^{-26}$ kg

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

$$V_{rms} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 519.8}{4.65 \cdot 10^{-26}}}$$

$V_{rms} = 680$ m/s