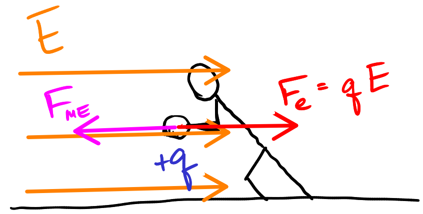
Week 5 covers sections of chapter 17 in the textbook. Topics include:

- work and electric potential energy
- electric potential or voltage
- conservation of energy
- electric field lines and equipotential surfaces
- energy storage in capacitors

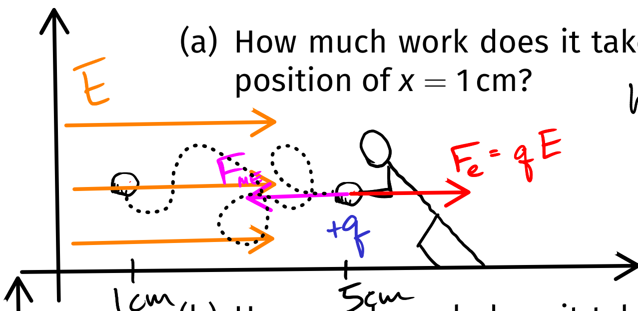
$\Delta U = Q + W$

$-p\Delta V$



$\text{Work} = |\text{Force}| \times \cos\theta \times |\text{displacement}|$

1. An uniform electric field has a strength of 1000 N/C and points in the positive x-direction. It doesn't really matter what is causing the electric field but you can imagine we are inside of a capacitor. A 1 μC charge is moved around within this field by an external force.



   (a) How much work does it take to move the charge from the position $x = 5\,\text{cm}$ to a position of $x = 1\,\text{cm}$?

   $W_{ME} = |F_{ME}| \cos\theta |\Delta x|$

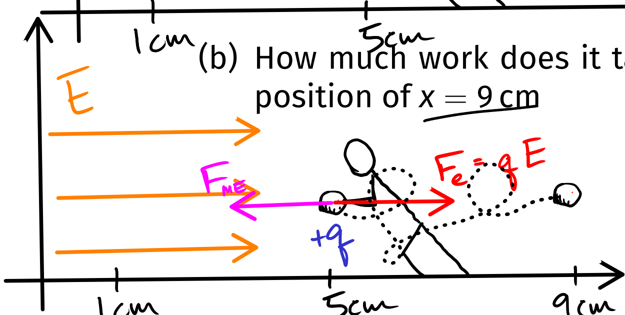
   $= |qE| \cos 0° \, |0.04\,\text{m}|$ $\quad 10^3 \quad 4\cdot10^{-2}$

   $= 1\cdot10^{-6} \cdot 1000 \cdot \cos 0° \cdot 0.04$

   $W_{ME} = 4\cdot10^{-5}\,\text{J}$

   $W_E = |F_E| \cos\theta \cdot \Delta x$ $\quad \cos 180°$

   $W_E = -4\cdot10^{-5}\,\text{J}$



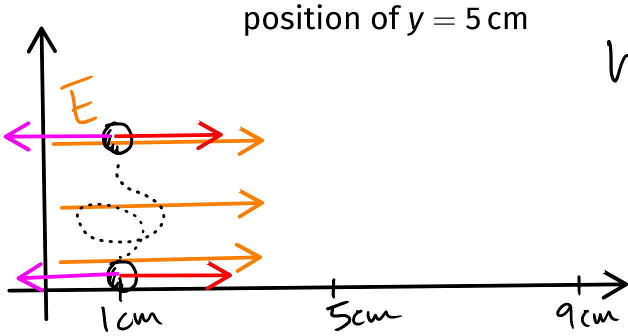
   (b) How much work does it take to move the charge from the position $x = 5\,\text{cm}$ to a position of $x = 9\,\text{cm}$

   $W_{ME} = -4\cdot10^{-5}\,\text{J}$

   $W_E = +4\cdot10^{-5}\,\text{J}$

   (c) How much work does it take to move the charge from the position $x = -5\,\text{cm}$ to a position of $x = 1\,\text{cm}$

   (d) How much work does it take to move the charge from the position $y = 0\,\text{cm}$ to a position of $y = 5\,\text{cm}$



   $W = F \cdot \cos\theta \cdot \Delta y$

   $\cos 90° = 0$

   $W = 0$

$$\cancel{K_i}^{0} + U_i + W_{NC} = \cancel{K_f}^{0} + U_f$$

$$W = \Delta U$$

volume

$\downarrow$

V

2. For the above problem, what is the amount of work *per unit of charge* that was done in each step?

change in voltage

$$\Delta V = \frac{W}{q} = \frac{4 \cdot 10^{-5}\,J}{1 \cdot 10^{-6}\,C} = 4 \cdot 10^{1} = 40\,\frac{J}{C} \qquad 10^{+6}$$

all for a constant electric field

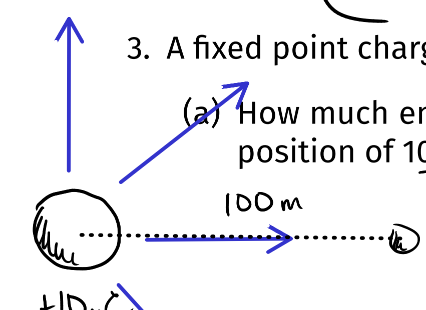
$$\Delta U = W = F_{ME} \cdot \cos\theta \cdot \Delta x \quad \} \text{ constant force}$$

$$\Delta U = -q\vec{E} \cdot \Delta \vec{x}$$

change in electric potential (voltage) (potential difference)

$$\frac{\Delta U}{q} = -\vec{E} \cdot \Delta\vec{x} = \Delta V \quad \rightarrow \quad E = -\frac{\Delta V}{\Delta x}$$

3. A fixed point charge of 10 µC creates an electric field throughout space around it.



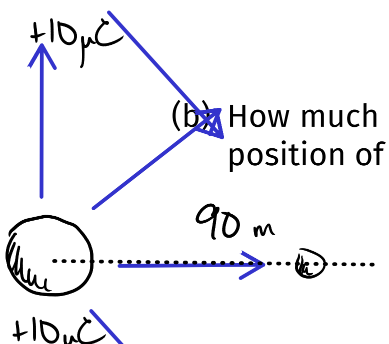
(a) How much energy would it take to move a 1 µC charge from infinitely far away to a position of 100 m away?

$$W_{ME} = \Delta U = \frac{k q_0 q_1}{r} \qquad \frac{\Delta U}{q_1} = \Delta V = \frac{k q_0}{r}$$

relative to ∞ far away

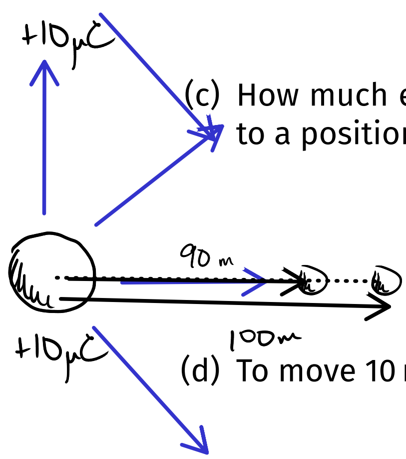
$$\Delta U = \frac{9 \cdot 10^{9} \cdot 10^{1} \cdot 10^{-6} \cdot 1 \cdot 10^{-6}}{\underbrace{100}_{10^{2}}} \quad 10^{-2}$$

$$= 9 \cdot 10^{-4}\,J$$



(b) How much energy would it take to move a 1 µC charge from infinitely far away to a position of 90 m away?

$$\Delta U = \frac{\cancel{9} \cdot 10^{9} \cdot 10 \cdot 10^{-6} \cdot 1 \cdot 10^{-6}}{\underset{\cancel{9} \cdot 10^{1}}{90}} = 1 \cdot 10^{-3}\,J$$



(c) How much energy would it take to move a 1 µC charge from a position 100 m away to a position of 90 m away?

$$\Delta U = 1 \cdot 10^{-3} - 9 \cdot 10^{-4} = 1 \cdot 10^{-4}\,J$$

$$= 10 \cdot 10^{-4} - 9 \cdot 10^{-4} = 1 \cdot 10^{-4}\,J$$

(d) To move 10 m closer, how much more energy would it take?

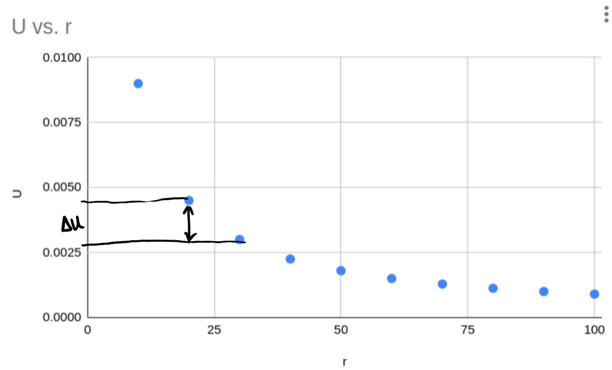
$$\Delta U = U(80\,m) - U(90\,m)$$

$$\frac{k q_0 q_1}{80\,m} - 1 \cdot 10^{-3}\,J$$

$$= 1.125 \cdot 10^{-3} - 1 \cdot 10^{-3}\,J = 1.25 \cdot 10^{-4}\,J$$

(e) How much energy would it take for each 10 m displacement closer to the fixed charge?

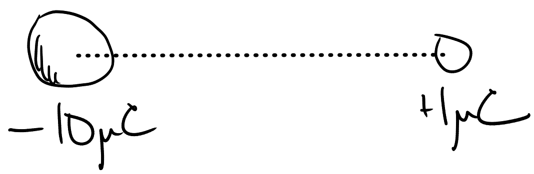
r	U(r)	delta U	V(r)	delta V
100	0.0009		900	
90	0.001	0.0001	1000	100
80	0.001125	0.000125	1125	125
70	0.001285714286	0.000160714285	1285.714286	160.7142857
60	0.0015	0.000214285714	1500	214.2857143
50	0.0018	0.0003	1800	300
40	0.00225	0.00045	2250	450
30	0.003	0.00075	3000	750
20	0.0045	0.0015	4500	1500
10	0.009	0.0045	9000	4500



4. Work the steps in the above problem for the voltage change rather than the energy change. Also plot a graph of voltage vs. radius and illustrate on the graph what it looks like to move from say 50 m to 40 m in terms of the voltage difference.

$$V = \frac{U}{q_1} \longrightarrow \frac{kq_0\cancel{q_1}}{r\cancel{q_1}} \Rightarrow V(r) = \frac{kq_0}{r} \Big\} \text{ single point charge}$$

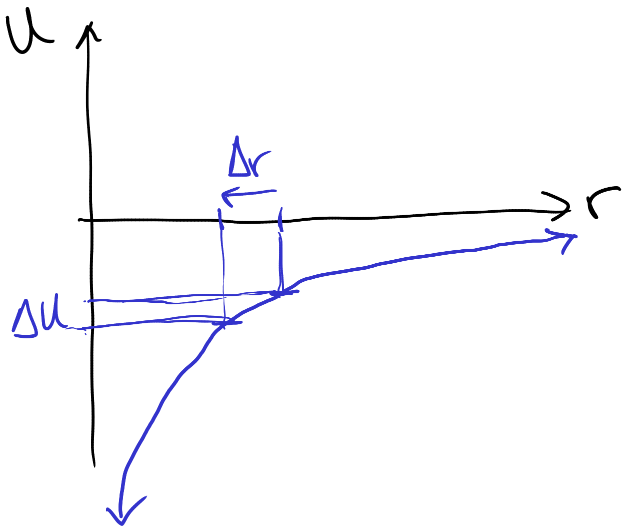
5. Now imagine that the source charge is negative. What does this change about all of the answers to the previous problem?



−10μC

$$U(r) = \frac{k q_0 q_1}{r}$$

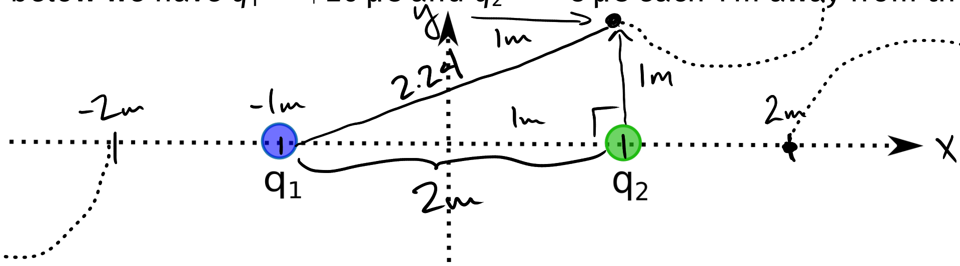
$$\Delta U = \frac{k q_0 q_1}{r_2} - \frac{k q_0 q_1}{r_1}$$



6. What are some inside out versions of the previous problems. (Write out the wording and I'll feature some of them in the space below.)

7. In order to find the voltage at a place around multiple charges, you simply find the voltage at a location due to each charge individually and then add them up. Same goes for finding the potential energy of putting a charge at a place around other charges. So in the figure

below we have $q_1 = +20\,\mu C$ and $q_2 = -5\,\mu C$ each 1 m away from the origin.



pt. charge

$V(r) = \frac{kq}{r}$

(a) What is the voltage at the location of $x = +2$ m on the x-axis relative to infinitely far away?

$$V_t = V_{q_1} + V_{q_2}$$

$$= \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \frac{9\cdot 10^9 \cdot 20\cdot 10^{-6}}{(3m)} + \frac{9\cdot 10^9 \cdot (-5)\cdot 10^{-6}}{(1m)} = 15{,}000\,V$$

15kV

(b) What is the voltage at the location of $x = -2$ m on the x-axis relative to infinitely far away?

$$V_t = V_{q_1} + V_{q_2}$$

$$V_t = \frac{9\cdot 10^9 \cdot 20\cdot 10^{-6}}{1m} + \frac{9\cdot 10^9 \cdot (-5)\cdot 10^{-6}}{3m} = 165{,}000\,V$$

165 kV

(c) What is the voltage at the location of $x = -2$ m on the x-axis relative to the location of $x = +2$ m on the x-axis?

$$\Delta V = V_2 - V_1 = 165{,}000V - 15{,}000V = 150{,}000V$$

(d) What is the voltage at the location of $x = +1$ m, $y = +1$ m (again relative to infinitely far away; if I ever forget to say this then this is what I mean)?

$$V_t = \frac{9\cdot 10^9 \cdot 20\cdot 10^{-6}}{2.24m} + \frac{9\cdot 10^9 \cdot (-5)\cdot 10^{-6}}{1m} = 35{,}500\,V$$

(e) Where along the x-axis would the voltage be equal to zero? What does this mean for the amount of work it would take to move a charge to that location from infinitely far away?

$X = 1.67m$

8. Now we need to put together the change in potential energy with the kinetic energy so that we can use the conservation of energy. To review some equations that we used last semester, lets start with a statement of the conservation of energy

$U_g = mgh$

$K = \frac{1}{2}mv^2$ $\rightarrow$ $$K_f + U_f = K_i + U_i + W_{non\text{-}conservative}$$

$U_s = \frac{1}{2}k\Delta x^2$

The only non-conservative forces we will have are the ones from outside that increase the potential energy of a charge, but there will be no friction. So if a charge starts from rest at some place you move it to another place and it arrives there at rest then the equation becomes:

$$\rightarrow 0 + U_f = 0 + U_i + W_{nc}$$
$$\Delta U = W_{nc}$$

which is what we have been using this entire time.

But, if instead of doing work to move it, what if we just let go? The charge would begin to move in response to the electric field that it was in and the potential energy would go down, and in its place the kinetic energy would increase since the charge is speeding up. There are several ways to represent this:

no W_{nc}

$$\rightarrow K_f + U_f = K_i + U_i + 0$$
$$\Delta K = -\Delta U$$

$K_f + U_f = K_i + U_i$

$K_f - K_i = -U_f + U_i$

Ok now for an actual question. Take this last expression and do a substitution to express the change in kinetic energy as a change in voltage.

$\Delta K = -(U_f - U_i)$

$\Delta K = -\Delta U$

$\Delta V = \frac{\Delta U}{q_1}$

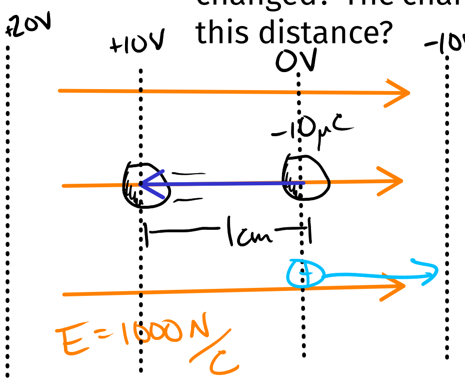
$\Delta U = q_1 \Delta V$

$\boxed{\Delta K = -q_1 \Delta V}$

$V_f = \frac{kq_0}{r_f}$

$V_i = \frac{kq_0}{r_i}$

9. A $-10\,\mu C$ charge is in a uniform electric field of 1000 N/C that points to the right. If the charge travels a distance of 1 cm, then how much has the voltage changed within the field? How much has the potential energy changed? How much has the kinetic energy changed? The charge has a mass of 1 mg. How fast is the charge going after it has traveled this distance?



$\Delta V = -\vec{E} \cdot \Delta \vec{x}$

$\Delta V = -(+1000\, N/C) \cdot (-0.01\, m)$

$\Delta V = +10\, V$

$\Delta U = q_1 \Delta V$

$\Delta U = (-10 \cdot 10^{-6}) \cdot (10\, V) = -1 \cdot 10^{-4}\, J$

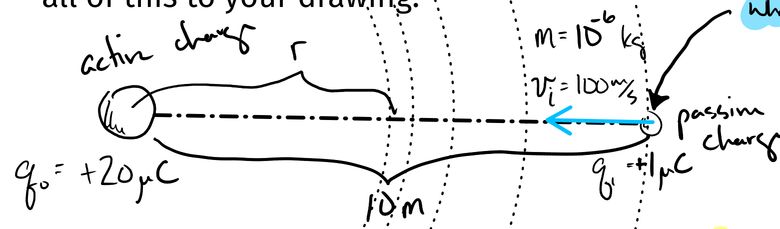
$\Delta K = -\Delta U$

$\Delta K = 1 \cdot 10^{-4}\, J$

$\Delta K = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}^{\,0}$

$1 \cdot 10^{-4}\, J = \frac{1}{2}(10^{-6}\, kg)v_f^2 \Rightarrow \boxed{v_f = 14.1\, m/s}$

10. Draw a capacitor and an electric field within it pointing to the right. What do the equipotential surfaces look like within this capacitor? If the electric field within the capacitor is 500 N/C and the capacitor plates are 1 cm apart, then what is the voltage between the two plates? Divide this voltage difference into a sensible number of "steps" of voltage. Make these the equipotential surfaces and then find the distance between surfaces. Add all of this to your drawing.



what is the voltage at this point?

$$V(r) = \frac{kq_0}{r} = \frac{9\cdot 10^9 \cdot 20\cdot 10^{-6}}{10} = 18000\,V = 1.8\cdot 10^4\,V$$

V_i

where does the q_1 stop?

$K_f = 0\,J$

$\Delta K = -q\Delta V$

18000V

$$\cancel{\tfrac{1}{2}mv_f^2}^{\,0} - \tfrac{1}{2}mv_i^2 = -q\left(\frac{kq_0}{r_f} - \boxed{\frac{kq_0}{r_i}}\right)$$

$$-\tfrac{1}{2}(10^{-6}\,kg)(100\,m/s)^2 = -10^{-6}\left(\frac{9\cdot 10^9 \cdot 20\cdot 10^{-6}}{r_f} - 18000\right)$$

$-0.5\cdot 10^{-2} =$

$-5\cdot 10^{-3}\,J =$ $\boxed{r_f = 7.8\,m}$

How fast is it going when it is 20 m away?

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -10^{-6}\left(\frac{9\cdot 10^9\cdot 20\cdot 10^{-6}}{20} - \frac{9\cdot 10^9\cdot 20\cdot 10^{-6}}{7.8}\right)$$

$v_i = 0$ when $r = 7.8\,m$

$\tfrac{1}{2}(10^{-6}\,kg)v_f^2 = 1.41\cdot 10^{-2}\,J$

$v_f = 168\,m/s$

11. Refer to your drawing in the above problem. If you put a positive charge in between the plates of this capacitor which way will it accelerate? Is this in the direction of increasing or decreasing voltage or neither? What about a negative charge? The equation for relating the potential energy and the voltage does need a sign on the charge.

How fast is it going when it is "infinity" far away?

$v = 0$ when $r_i = 7.8\,m$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -q_1\left(\frac{kq_0}{r_f} - \frac{kq_0}{r_i}\right)$$

$r_f \to \infty$ $\quad \frac{kq_0}{r_f} \to 0$

So... $\quad \tfrac{1}{2}mv_f^2 = -q_1\left(0 - \frac{9\cdot 10^9 \cdot 20\cdot 10^{-6}}{7.8}\right)$ $\quad (q_1 = 10^{-6})$

$v_f = 214\,m/s$

12. Suppose the capacitor plates in the problems above has an area of $0.1\,m^2$. What is the capacitance? How much charge is stored on the capacitor? How much energy is stored in the capacitor?

13. Start with the equation for the energy stored in a capacitor and the definition of capacitance, and using substitution derive two other expressions for the energy stored on a capacitor.

14. Derive an expression for the energy density of a capacitor. Start with one of the expressions above for the energy of the capacitor and divide it by the volume of the capacitor. Use some clever substitution to get an expression for the energy density of the capacitor in terms of the electric field inside the capacitor. Design you're own capacitor and then determine its energy density. Compare the energy density of your capacitor to that of an Li-ion battery as well as gasoline (use google).

15. IN PROGRESS...

$$\underline{m_i} L_f + \underline{m_i} \cdot c_w \cdot (\underset{5^\circ}{T_f} - \underset{0^\circ}{T_i}) + m_w c_w \cdot (\underset{5^\circ}{T_f} - \underset{20}{T_i})$$

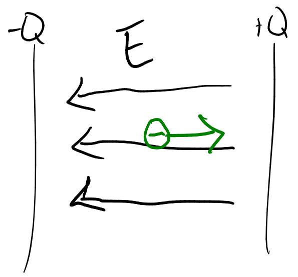
$$mc\Delta T = Q$$

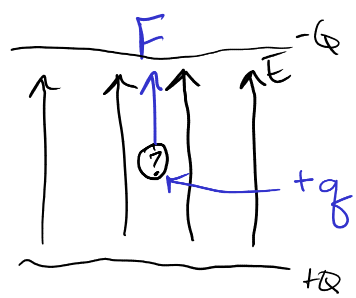
$$(1\,\text{kg})\left(4.184\,\frac{\text{kJ}}{\text{kg K}}\right)(\Delta T) = Q$$

$$P = \frac{Q}{t} \Rightarrow t = \frac{Q \leftarrow \text{J}}{P \leftarrow 600\,\text{W}}$$

$$Q = \_\_\_\ \text{kJ} \rightarrow \text{J}$$

$$W = -P\Delta V$$





$$F = q \cdot E$$

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