$$f(x) = \frac{3}{10x^2+1} \longleftarrow x \to x - vt \longrightarrow f(x,t) = \frac{3}{10(x-vt)^2+1}$$

$$\psi(x,t) = f(x,t)$$

psi

$$\psi = f(x \mp vt) = f(x')$$

1st

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1} = \frac{\partial f}{\partial x'}$$

2nd

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'}\right) = \frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1 \text{ again}} = \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{= \mp v} = \mp v \cdot \frac{\partial f}{\partial x'}$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} = \frac{\mp 1}{v}\frac{\partial f}{\partial t} \qquad \frac{\partial f}{\partial x'} = \mp\frac{1}{v}\frac{\partial f}{\partial t}}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$$\frac{\partial f}{\partial t} = \mp v \frac{\partial f}{\partial x'}$$

$$= \mp v \frac{\partial^2 f}{\partial x'^2} \cdot \mp v$$

$$= \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2} \qquad \Big| \qquad \mp\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$\vec{F} = \dot{\vec{p}}$$

$$\rightarrow \boxed{\frac{\partial^2 f}{\partial x^2} = \mp\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

The <u>wave</u> <u>equation</u>

- Schrodinger's
- Heat
- Laplace

HW 1,3 so far of Chapter 2 by Wed.

Maxwell's equations:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$ (Gauss's Law) ← Electric field spread out if there is charge.

$$\vec{\nabla}\cdot\vec{B} = 0$$ (Gauss's Law for Magnetism) ← Magnetic field do not spread out.

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$ (Maxwell-Faraday's Law) ← There is an electric field if a magnetic field changes over time

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$ (Ampere's Law) ← There is a magnetic field if there is current <u>or</u> if an electric field is changing over time

In the case of no ρ and no $\vec{J}$

$$\vec{\nabla}\cdot\vec{E}=0$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\left.\begin{aligned}\vec{\nabla}\times\vec{E}&=-\frac{\partial B}{\partial t}\\ \vec{\nabla}\times\vec{B}&=\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\}$$ take the curl of each

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=-\frac{\partial}{\partial t}\underbrace{(\vec{\nabla}\times\vec{B})}_{\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

$$=-\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{=0})-\underbrace{(\vec{\nabla}\cdot\vec{\nabla})\vec{E}}_{\vec{\nabla}^2\vec{E}}$$

$$\frac{\partial^2 E_x}{\partial x^2}\hat{x}+\frac{\partial^2 E_y}{\partial y^2}\hat{y}+\ldots$$

$$-\vec{\nabla}^2\vec{E} = -\mu_0\epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\underline{\underline{\vec{\nabla}^2\vec{E} = \mu_0\epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}} \longrightarrow \frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial x^2}\hat{x} + \ldots = \mu_0\epsilon_0 \frac{\partial^2 E_x}{\partial t^2} + \ldots$$

So is $\epsilon_0 \cdot \mu_0 = \frac{1}{v^2}$? or $v = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

$$\left.\begin{aligned} \epsilon_0 &= 8.85 \cdot 10^{-12}\ \frac{s^2C^2}{m^3kg} \\ \mu_0 &= 4\pi \cdot 10^{-7}\ \frac{m\,kg}{C^2} \end{aligned}\right\} \quad v = \frac{1}{\sqrt{8.85 \cdot 10^{-12} \cdot 4\pi \cdot 10^{-7}}} = \underline{2.99 \cdot 10^{8}\ m/s}$$

Harmonic waves   HW: 2.9, 18, 22

$$\psi(x,t) = A \cdot \sin\left(k(x - vt)\right)$$ ← harmonic wave

↳ repetition in space → wavelength, λ [m], [nm]

↳ repetition in time → period, T [s]

$k\lambda = 2\pi$

→ $k = \frac{2\pi}{\lambda}$ $\left[\frac{rad}{m}\right]$

propogation number

$\omega \cdot T = 2\pi$

$\omega = \frac{2\pi}{T}$

angular frequency [rad/sec]

$\nu = \frac{1}{T}$

(natural) frequency, [Hz] = [s^{-1}]

$\omega = 2\pi\nu$

wavenumber

$K = \frac{1}{\lambda}$

[m^{-1}][nm^{-1}]

kappa [cm^{-1}]

k

relationship b/t λ + T is v

$$v = \frac{\lambda}{T} = \nu \cdot \lambda$$

$$\rightarrow v = \frac{\omega}{k}$$

$$\psi(x,t) = A \cdot \sin\left(k\left(x \mp vt\right)\right)$$

$$\boxed{\psi(x,t) = A \cdot \sin\left(kx \mp \omega t\right)}$$

$$\psi(x,t) = A \sin\left(\frac{2\pi}{\lambda} x \mp \frac{2\pi}{T} t\right)$$

$$\psi(x,t) = A \sin\left(\frac{2\pi}{\lambda} x \mp 2\pi\nu \cdot t\right)$$

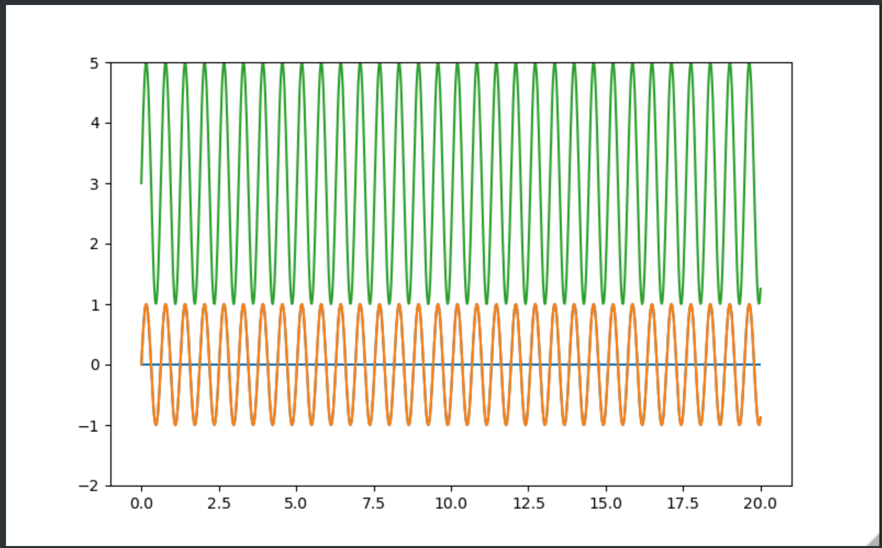
$$\psi(x,t) = A \sin 2\pi\left(\frac{x}{\lambda} \mp \nu t\right)$$

$$\psi(x,t) = A \sin 2\pi\left(\kappa x \mp \nu t\right)$$

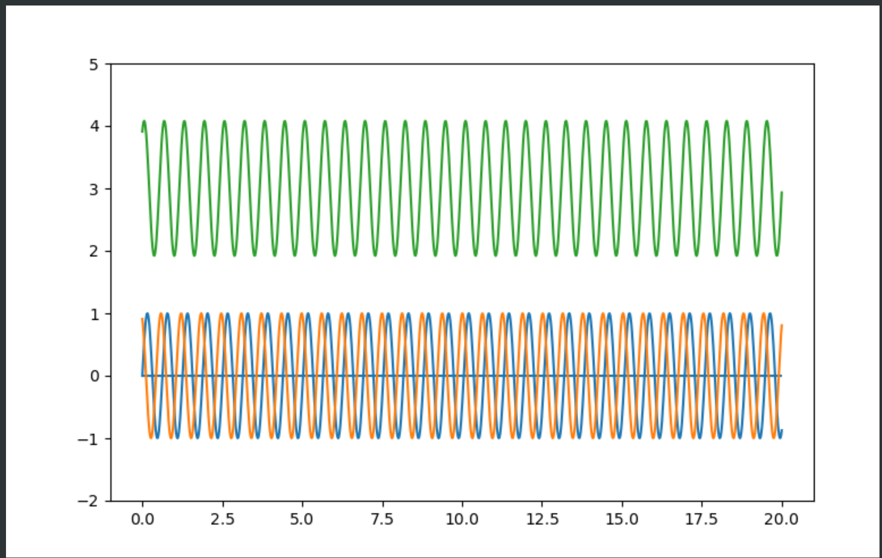
Superposition - multiple waves are present in the same place

Interference

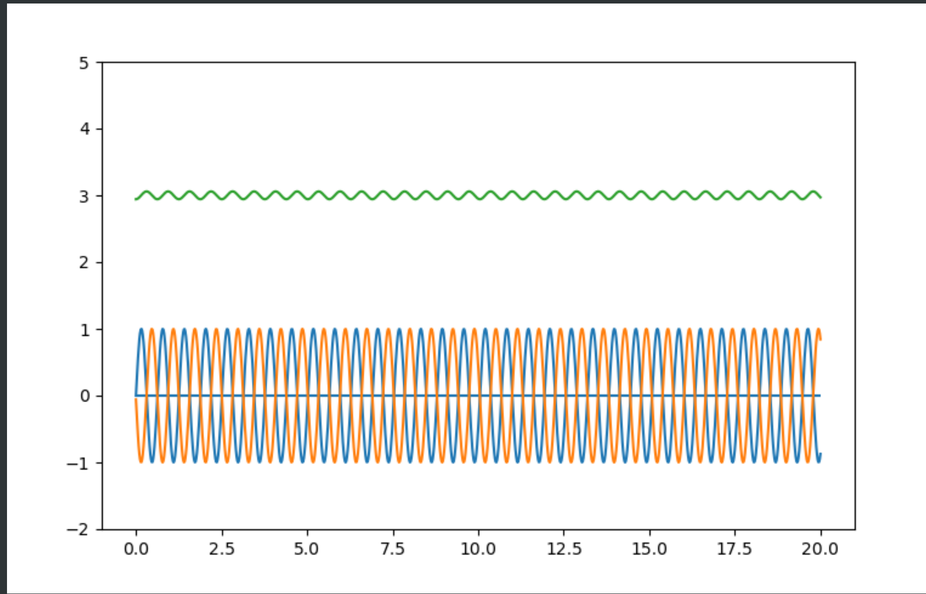
perfectly constructive interference



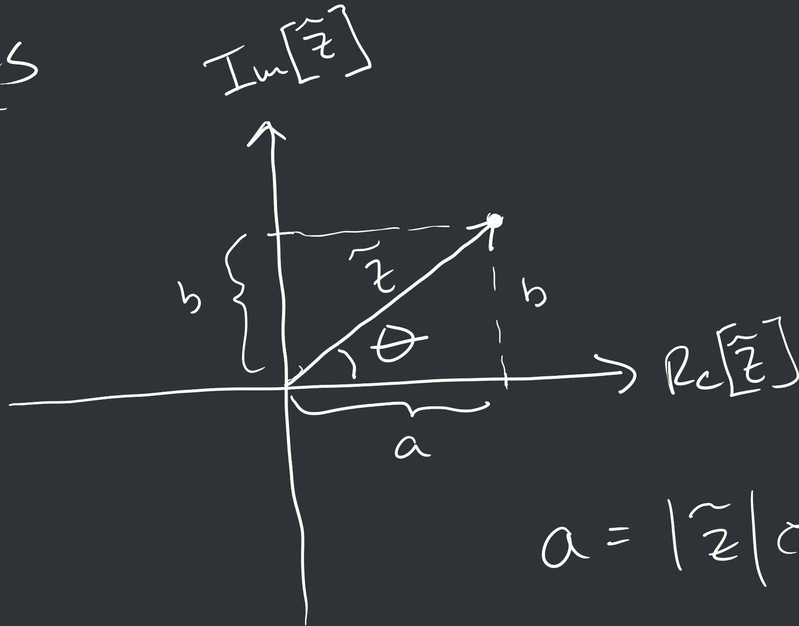
interference



destructive interference



Complex numbers + waves



$$\tilde{z} = a + ib$$

$$\tilde{z}^* = a - ib$$

$$[\tilde{z}]^2 = (a + ib)(a + ib)$$

$$a^2 + 2iba - b^2$$

$$\tilde{z} \cdot \tilde{z}^* = (a + ib)(a - ib)$$

$$a^2 - iab + iab - i^2 b = a^2 + b^2$$

$$a = |\tilde{z}| \cos\theta$$

$$b = |\tilde{z}| \sin\theta$$

$$\tilde{z} = |\tilde{z}| (\cos\theta + i \sin\theta)$$

$$\underbrace{e^{i\theta}}_{\text{abuse of notation}} = \cos\theta + i\sin\theta$$ Euler's identity

$$e^2$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \ldots$$

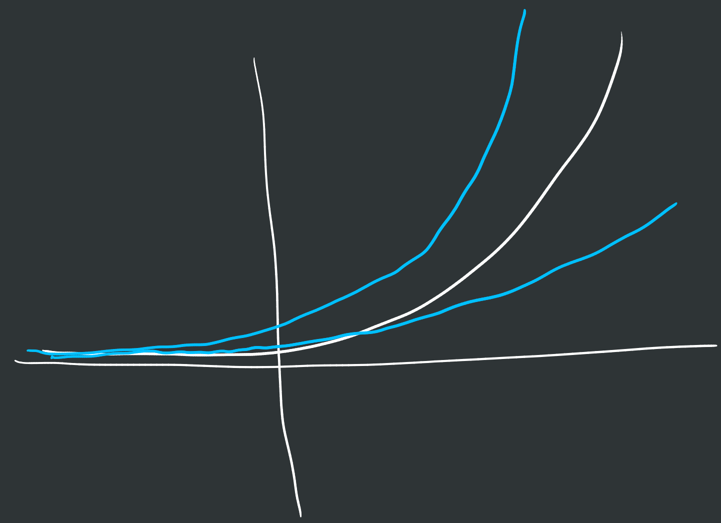
$$e^{i\theta} = \underbrace{1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots}_{\cos\theta} + i\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\ldots\right)}_{\sin\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\pi} = -1$$

$$e^{i\pi} + 1 = 0$$

$$\frac{da^x}{dx} = c \cdot a^x$$



$$\tilde{z} = a + ib \qquad \tilde{z} = |\tilde{z}| e^{i\theta}$$

$$\tilde{z}^* = a - ib \qquad \tilde{z}^* = |\tilde{z}| e^{-i\theta}$$

$$\tilde{z}\tilde{z}^* = |\tilde{z}| e^{i\theta} \cdot |\tilde{z}| e^{-i\theta}$$

$$= |\tilde{z}| \cdot |\tilde{z}|$$

$$\tilde{z}\tilde{z}^* = |\tilde{z}|^2$$

$$\mathrm{Re}\{\tilde{z}\} = a = \frac{\tilde{z} + \tilde{z}^*}{2}$$

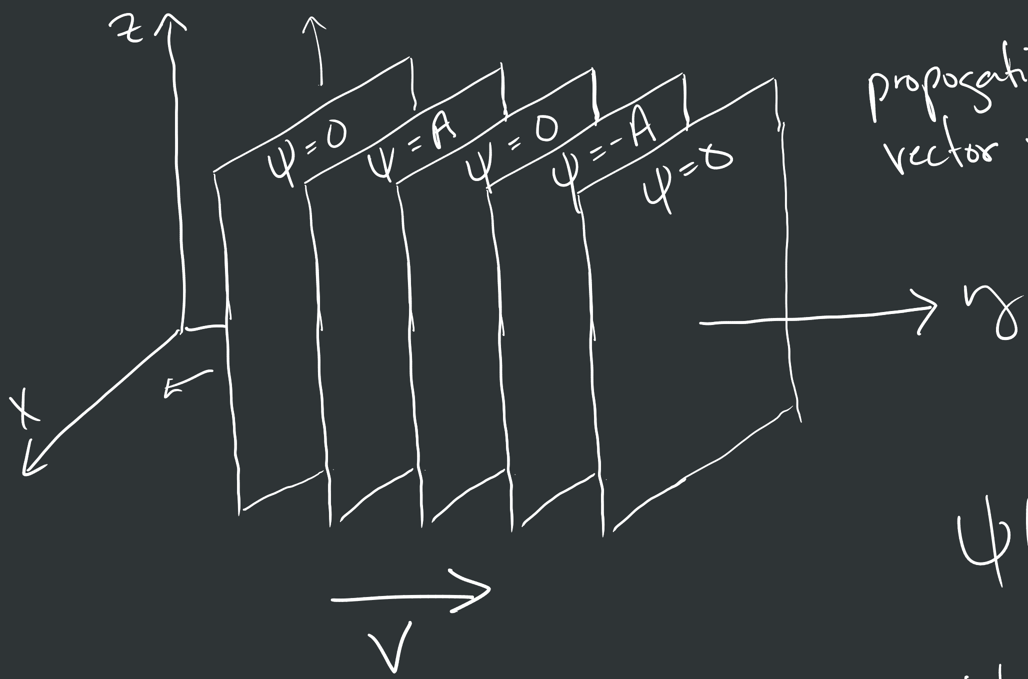
$$\mathrm{Im}\{\tilde{z}\} = b = \frac{\tilde{z} - \tilde{z}^*}{2i}$$

$$\psi(x,t) = A\cos(kx - \omega t + \phi)$$

$$\psi(x,t) = \mathrm{Re}\left\{ A e^{i(kx - \omega t + \phi)} \right\}$$

$$\psi(x,t) = A e^{i(kx - \omega t + \phi)} \longleftarrow$$

<u>3-D</u> <u>Waves</u> $\rightarrow$ <u>Plane</u> <u>wave</u>



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

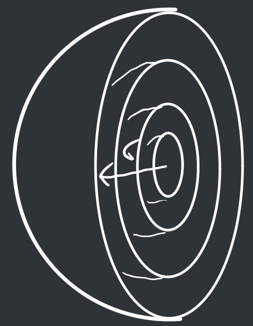
propogation vector $\rightarrow \vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$$\psi(\vec{r},t) = A\sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$\psi(\vec{r},t) = A e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

→ Spherical Wave



$$\psi(\vec{r},t) = \left(\frac{A}{r}\right) \sin\left(\vec{k}\cdot\vec{r} - \omega t\right)$$

$$\psi(\vec{r},t) = \left(\frac{A}{r}\right) e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$