

Chapter 4ish - 5ish

wavelength - distance of an oscillation
- one electric field max to another max
- λ , meters, cm, nm

period - time for one oscillation to pass a point as the wave goes by
- T , seconds
 κ ← cursor key

wavenumber - $\frac{1}{\lambda} \kappa, k$

propagation constant $\frac{2\pi}{\lambda} = k$

frequency - $\nu = \frac{1}{T} = \text{Hz}$
 ω

angular frequency $\frac{2\pi}{T} = 2\pi\nu$

speed $\rightarrow v = \frac{\lambda}{T} = \lambda \cdot f = \lambda \nu$

in materials, light slows down

$C \rightarrow$ speed of light in vacuum
 $\rightarrow v < C$

$$\frac{C}{v} = n \leftarrow \text{index of refraction}$$

conventionally $n \geq 1$

but can be negative (metamaterials)

and can be complex (absorptive materials)

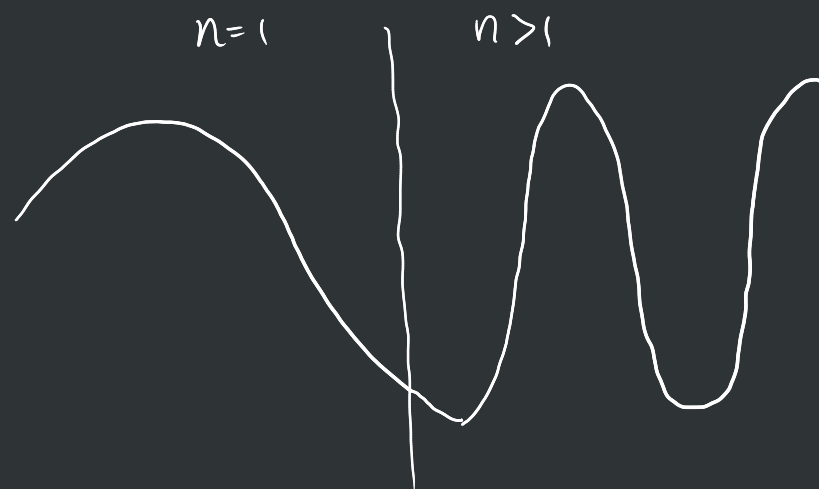
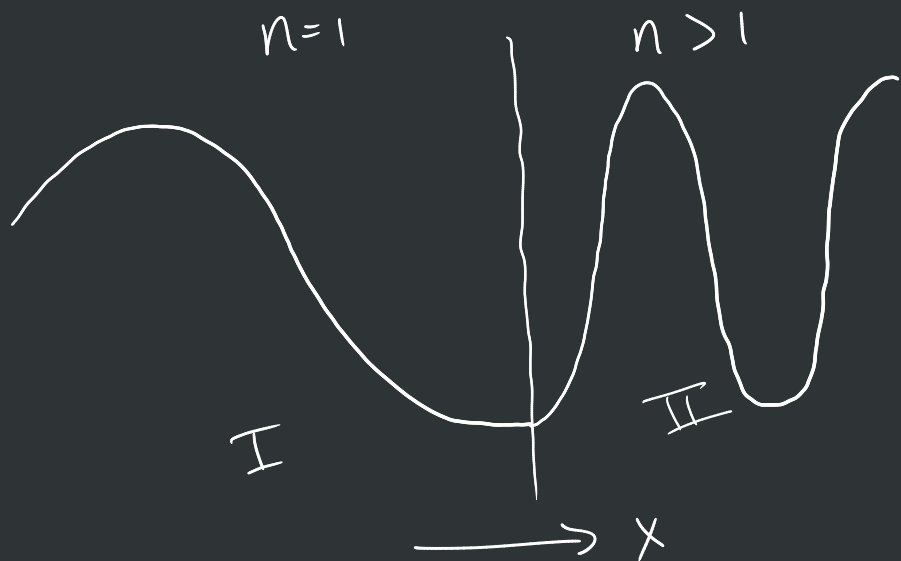
$$v = \frac{C}{n} = \lambda \cdot \nu$$

\rightarrow if n increases at some boundary
 λ decreases, but not ν .

ν is constant

$$E = h \cdot \nu = \frac{hc}{n\lambda}$$

\hookrightarrow Planck's constant



$$v = \frac{c}{n} = \lambda \nu$$

$$\frac{c}{n_1 \lambda_1} = \nu = \frac{c}{n_2 \lambda_2}$$

$$\frac{c}{n_1 \lambda_1} = \frac{c}{n_2 \lambda_2}$$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{n_2}{n_1} = \left(\frac{\lambda_2}{\lambda_1} \right)^{-1}$$

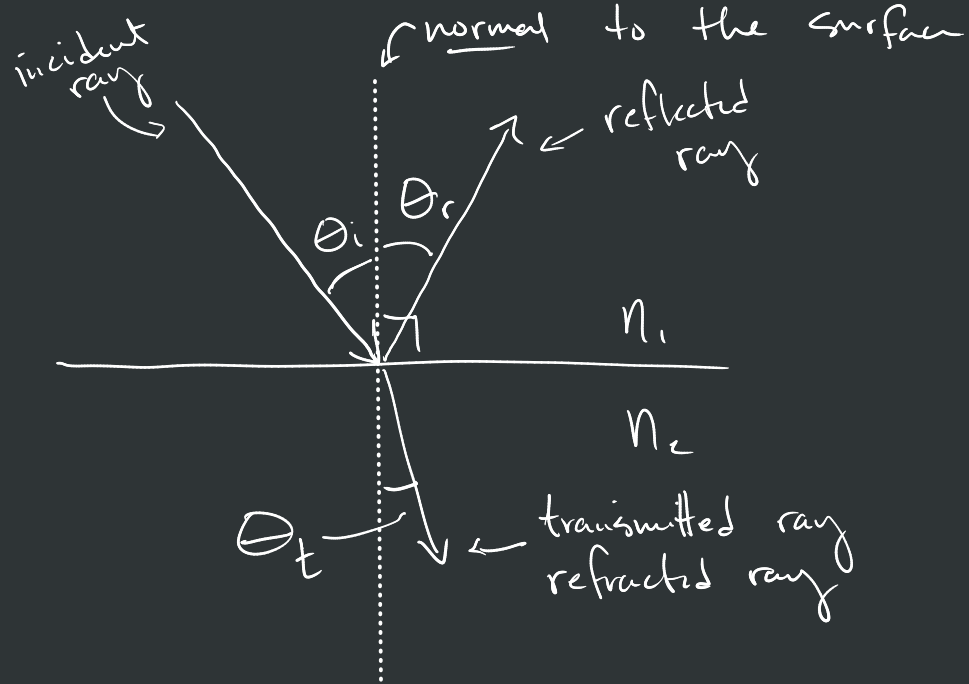
→ I is vacuum

$$\frac{n}{1} = \left(\frac{\lambda_2}{\lambda_0} \right)^{-1}$$

\uparrow \uparrow
 $n=1$ λ_0
 in vacuum in vacuum

$$\lambda_2 = \frac{\lambda_0}{n} \leftarrow n > 1$$

→ so this goes down w/ n



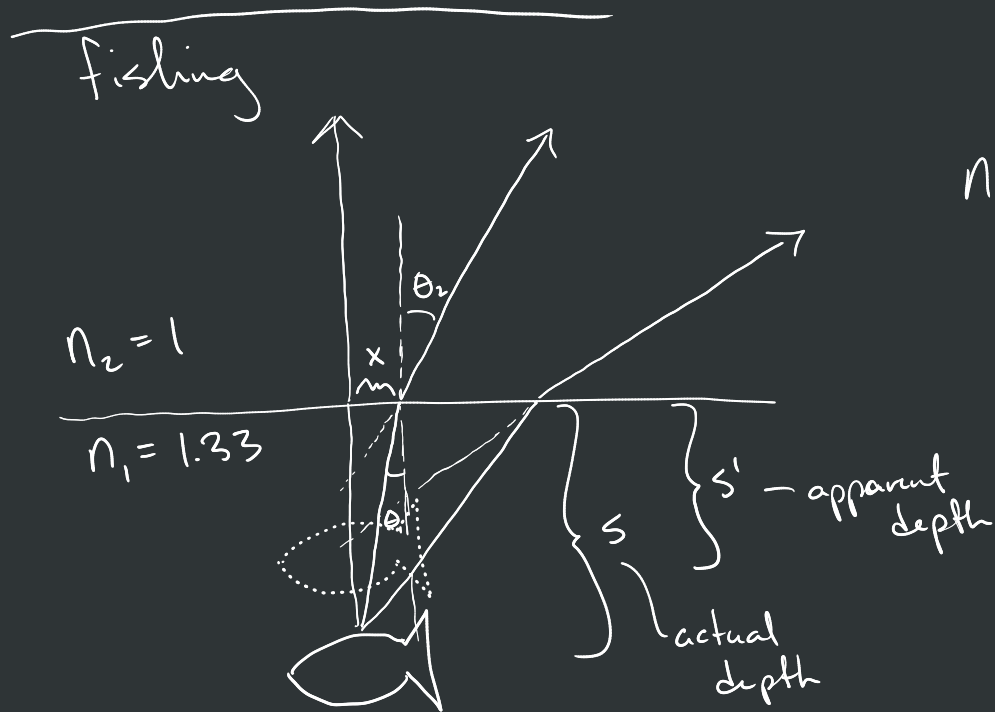
Law of Reflection

$$\rightarrow \theta_i = \theta_r$$

Law of Refraction (Snell's Law)

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\hookrightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2$$



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

for small θ , $\sin \theta \approx \tan \theta \approx \theta$

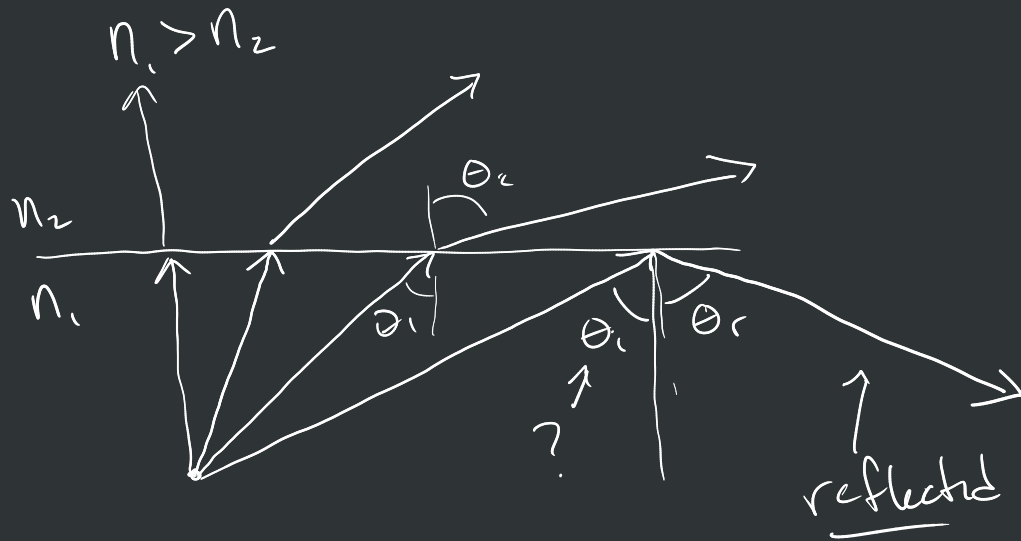
$$n_1 \tan \theta_1 = n_2 \tan \theta_2$$

$$\frac{n_1 \cancel{X}}{s} = \frac{n_2 \cancel{X}}{s'}$$

$$\frac{s'}{s} = \frac{n_2}{n_1} = \frac{1}{1.33} \approx 0.752$$

$$s' = 0.752 \cdot s$$

critical angle



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

$$\theta_2 = \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right)$$

What θ_1 causes θ_2 to be 90°

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

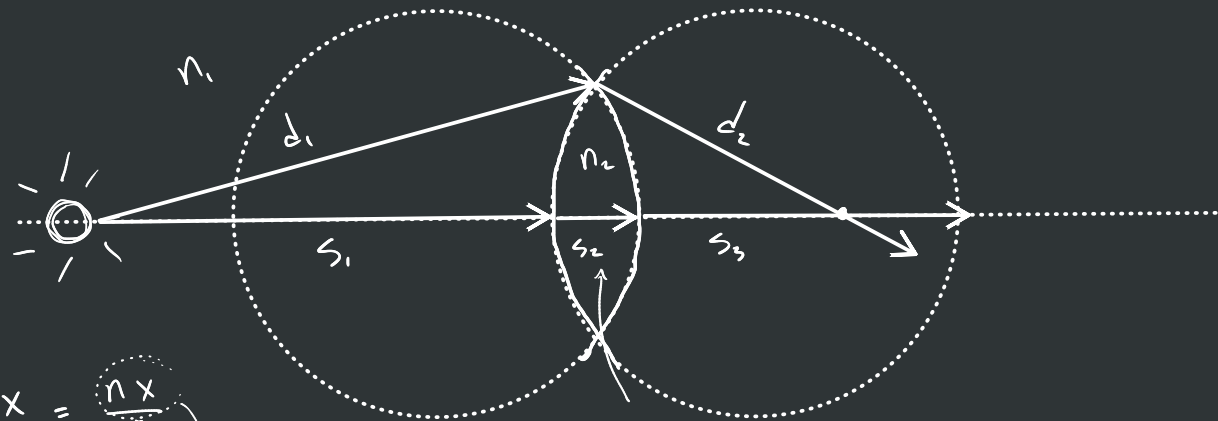
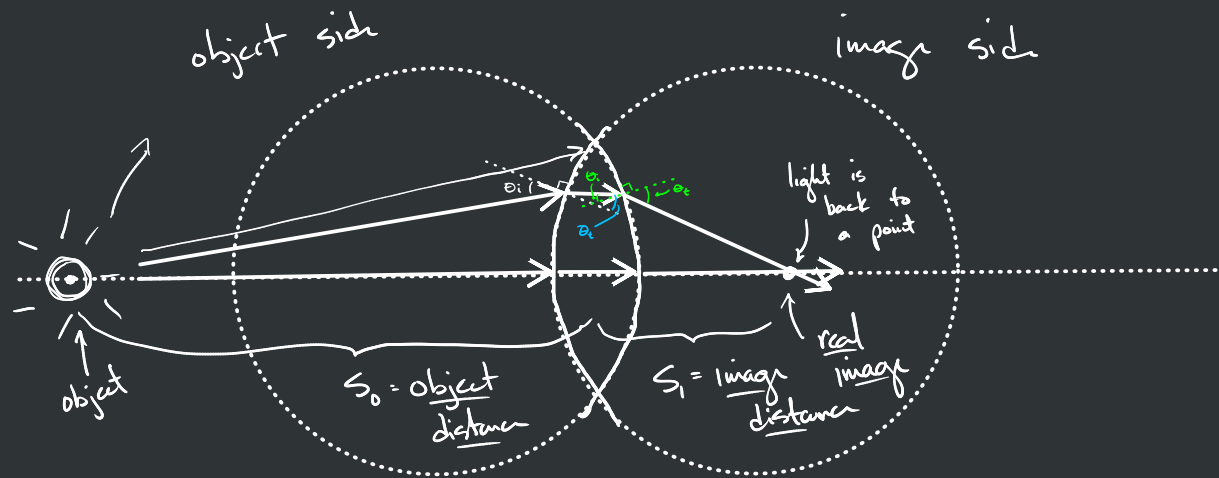
when this is ≥ 1

$$1 = \frac{n_1}{n_2} \sin \theta_1$$

critical angle $\rightarrow \theta_1 = \theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$

$\leftarrow < 1$

HW: Chapter 4: 6, 7, 8, 21, 24



$$t = \frac{x}{v} = \frac{n x}{c}$$

$v = \frac{c}{n}$

optical path length

$$\frac{n_1 d_1}{c} + \frac{n_1 d_2}{c} = \frac{n_1 s_1}{c} + \frac{n_2 s_2}{c} + \frac{n_1 s_3}{c}$$

$$n_1 d_1 + n_1 d_2 = n_1 s_1 + n_2 s_2 + n_1 s_3$$

} optical path lengths are the same!

perfect images

reflection

- ellipse
- hyperboloid
- parabola

refraction

- ellipse
- hyperboloid
- cartesian oval

Refraction



Thin lens

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

↑ object distance

↑ image distance

thin lens (Lens Makers Equation)

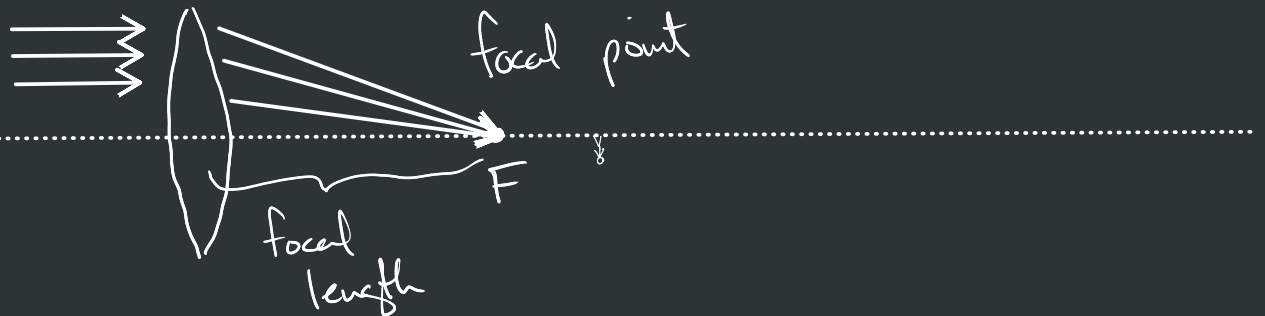
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

↑ focal length

(Thin Lens Equation)

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$



	+	-
s_o	real object	virtual object
s_i	real image	virtual image
f	converging lens	diverging lens
y_o	upright object	inverted object
y_i	upright image	inverted image

Optics Lab:

Conjugate Points: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

Focal length Exp: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$
 ~~$\frac{1}{s_o} \rightarrow \infty$~~
 $\frac{1}{s_i} = \frac{1}{f} \Rightarrow s_i = f$ for $s_o \rightarrow \infty$

Diverging Lens: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{-f}$

$$\frac{1}{s_i} = \frac{1}{-f} - \frac{1}{s_o}$$

$$s_i = \frac{1}{\frac{1}{-f} - \frac{1}{s_o}}$$

Finding a virtual image:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\left(\frac{1}{s_i} = \frac{1s_o}{fs_o} - \frac{1f}{s_o f} = \frac{s_o - f}{fs_o} \right)$$

$$s_i = \frac{s_o \cdot f}{s_o - f}$$

NINE: $s_o = 27 \text{ cm}$

$$f = -15 \text{ cm}$$

$$s_i = \frac{27(-15)}{27 + 15} = -9.60 \text{ cm}$$

Now for the check:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$s_i = 16.2 \text{ cm}$$

$$f = 10 \text{ cm}$$

$$s_o = 26.13$$

$$s_o = \frac{s_i \cdot f}{s_i - f}$$

$$s_o = \frac{16.2 \cdot 10}{16.2 - 10}$$

$$s_o = 26.13$$

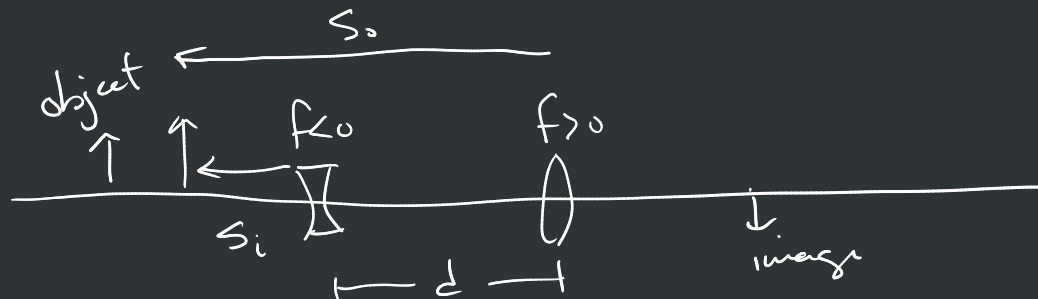
s_o now we need to compare

$$s_o = d - s_i$$

$$26.13 = d - (-9.6)$$

$$16.53 = d$$

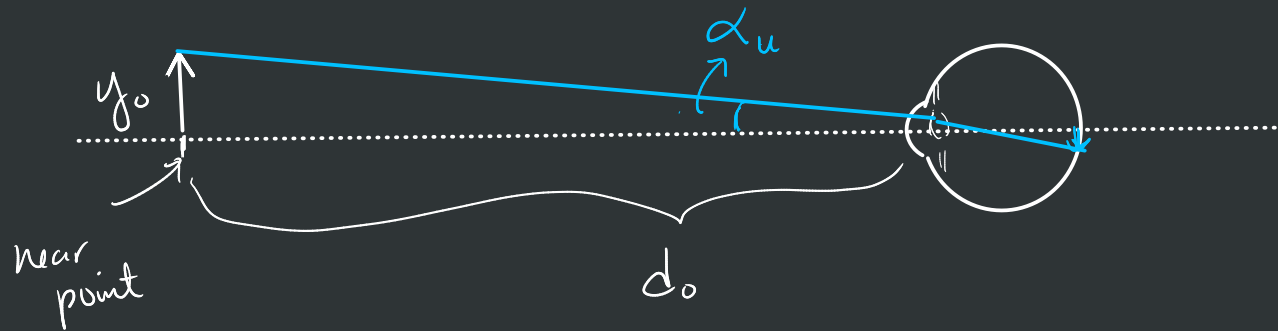
↖ 17.3 measured



Optical Devices

Magnifying Glass

Unaided Eye



magnifying power
angular magnification

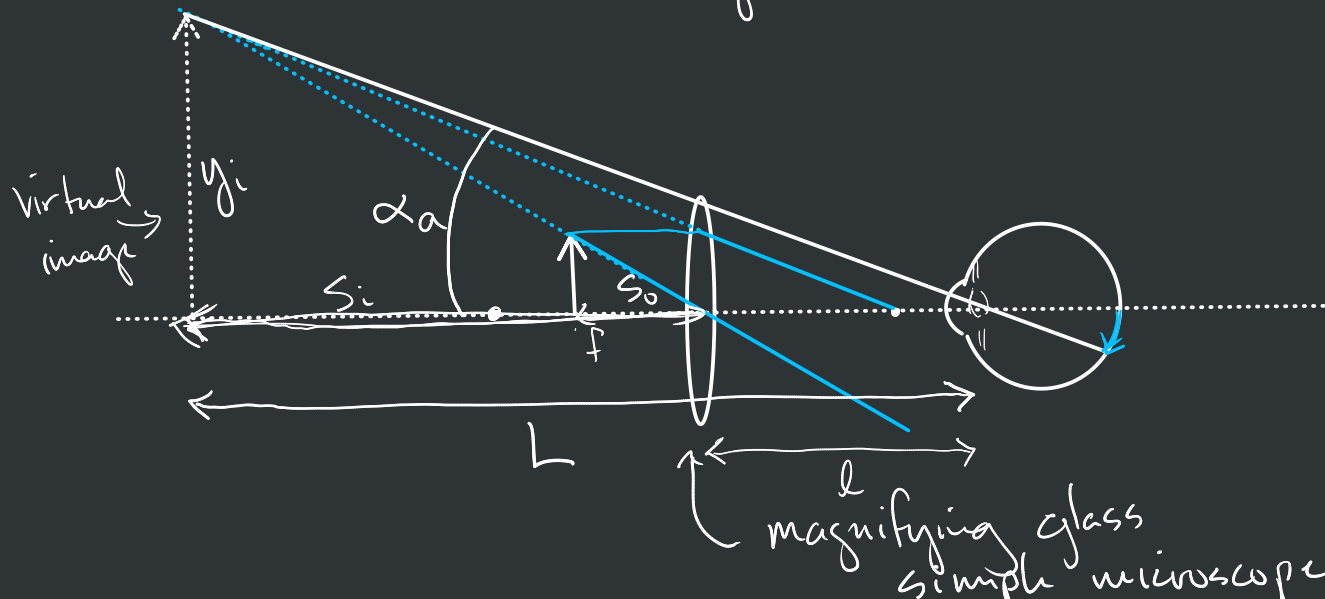
$$MP = M_A = \frac{\alpha_a}{\alpha_u}$$

radius
↓

$$\tan \alpha_u = \frac{y_o}{d_o} \approx \sin \alpha_u \approx \alpha_u$$

$$\tan \alpha_a = \frac{y_i}{L} \approx \alpha_a$$

Aided Eye



$$MP = \frac{y_i}{L} \cdot \frac{d_o}{y_o}$$

↑ this is positive

$$M_t = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

$$\rightarrow 57.3^\circ \leftarrow 1 \text{ rad} \cdot \frac{180}{\pi} = 57.3^\circ$$

$$\tan 1 \text{ rad} = 1.557$$

$$\tan 0.1 \text{ rad} = 0.10033$$

$$\hookrightarrow 5.73^\circ$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$s_i \left[\frac{1}{s_o} = \frac{1}{f} - \frac{1}{s_i} \right] s_i$$

$$\frac{s_i}{s_o} = \frac{s_i}{f} - 1$$

$$-\frac{s_i}{s_o} = \left[1 - \frac{s_i}{f} \right]$$

$$MP = - \frac{s_i}{s_o} \cdot \frac{d_o}{L}$$

$$MP = \left(1 - \frac{s_i}{f} \right) \frac{d_o}{L}$$

$$s_i = -(L - l)$$

$$MP = \left(1 + \frac{L - l}{f} \right) \cdot \frac{d_o}{L}$$

$$\frac{1}{f} = \mathcal{D} \leftarrow \text{dioptric power}$$

$$\boxed{MP = (1 + (L - l)\mathcal{D}) \cdot \frac{d_o}{L}} \text{ eq. 5.16}$$

\mathcal{D}

\mathcal{D}

Case 1: $l = f$

$$\begin{aligned} [MP]_{l=f} &= \left(1 + (L - l) \frac{1}{f} \right) \cdot \frac{d_o}{L} \\ &= \left(\cancel{1} + \frac{\cancel{L}}{f} - \frac{\cancel{L}}{\cancel{f}} \right) \cdot \frac{d_o}{\cancel{L}} \end{aligned}$$

$$[MP]_{l=f} = \frac{d_o}{f} = d_o \mathcal{D}$$

Case 2: $l = 0$

$$\begin{aligned} [MP]_{l=0} &= \left(1 + (L - \vec{l}) \mathcal{D} \right) \cdot \frac{d_o}{L} \\ &= \left(1 + L \mathcal{D} \right) \frac{d_o}{L} \end{aligned}$$

$$[MP]_{l=0} = \left(\frac{1}{L} + \mathcal{D} \right) \cdot d_o$$

if we shrink L to d_o ,

$$[MP]_{\substack{l=0 \\ L=d_o}} = \left(\frac{1}{d_o} + \mathcal{D} \right) \cdot d_o$$

$$[MP]_{\substack{l=0 \\ L=d_o}} = 1 + \mathcal{D} \cdot d_o$$

Case 3: We put the object at the focal point, $s_o = f$.
The image is formed

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{f} + \frac{1}{s_i} = \frac{1}{f}$$

$\xrightarrow{\quad}$

$$\frac{1}{s_i} = 0$$

$$s_i \rightarrow \infty$$

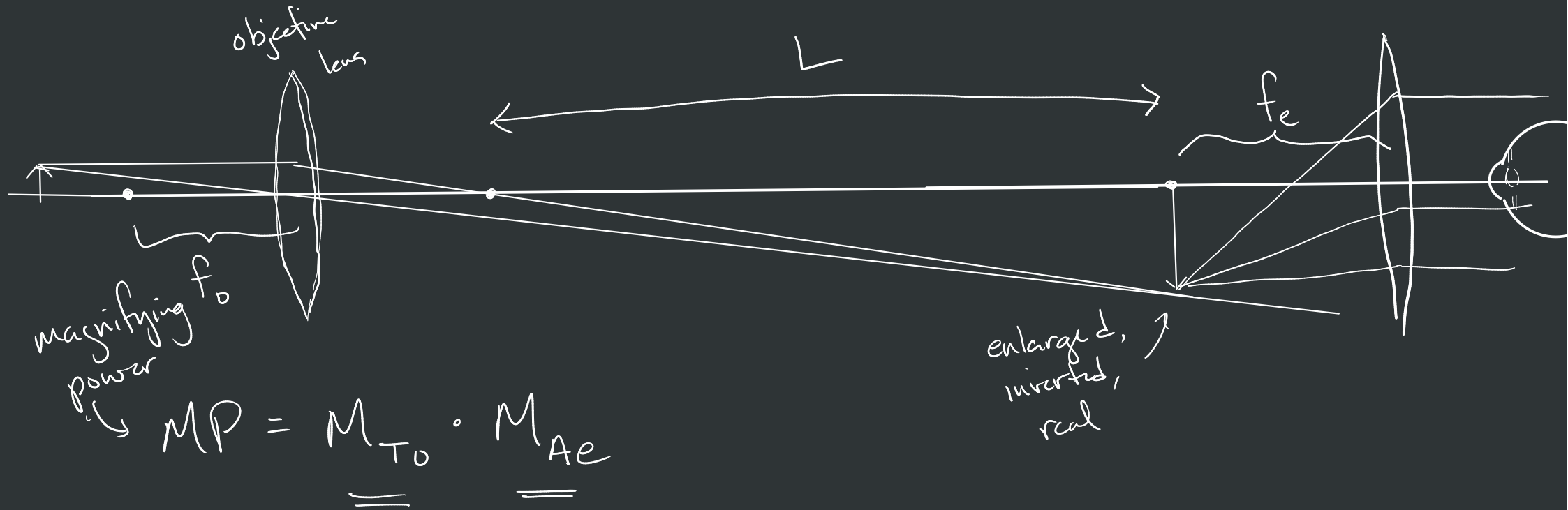
$$MP = (1 + (L - l)D) \cdot \frac{d_o}{L}$$

$$[MP]_{L \rightarrow \infty} = \cancel{\frac{d_o}{L}} + \frac{K \cdot D \cdot d_o}{\cancel{L}} - \frac{l D \cdot d_o}{\cancel{L}}$$

$$[MP]_{L \rightarrow \infty} = D \cdot d_o = \frac{d_o}{f}$$

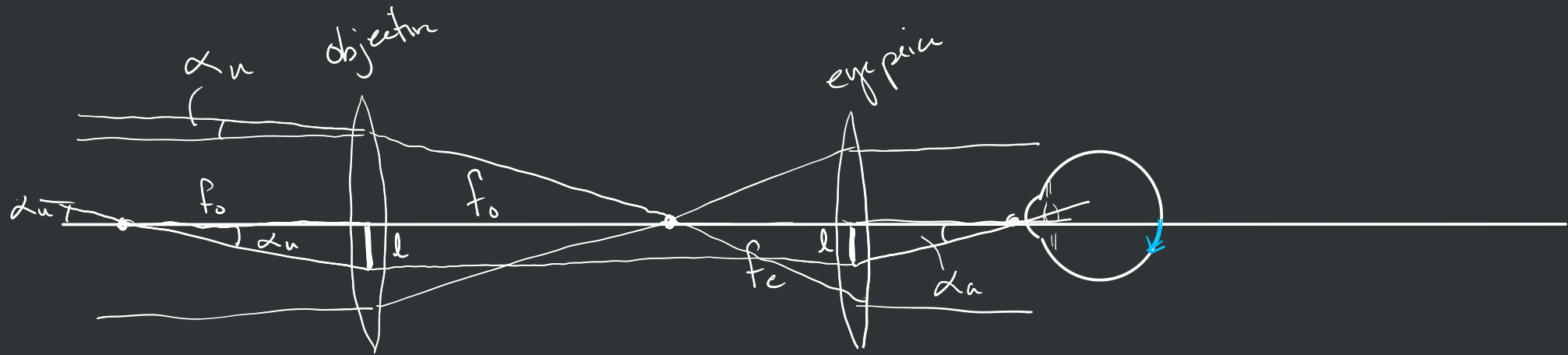
HW: ch 5. 25, 34, 42

Microscope



Objective lens usually has a very small focal length.
Eyepiece has a larger focal length

Telescope



$$MP = \frac{\alpha_a}{\alpha_u}$$

$$\alpha_u \approx \tan \alpha_u = \frac{l}{f_o}$$

$$\alpha_a \approx \tan \alpha_a = \frac{l}{f_e}$$

$$MP = \frac{\frac{l}{f_e}}{\frac{l}{f_o}}$$

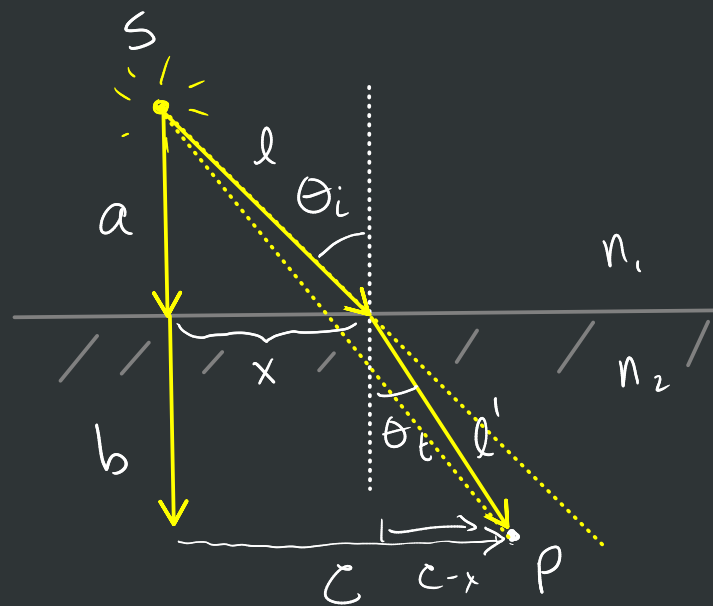
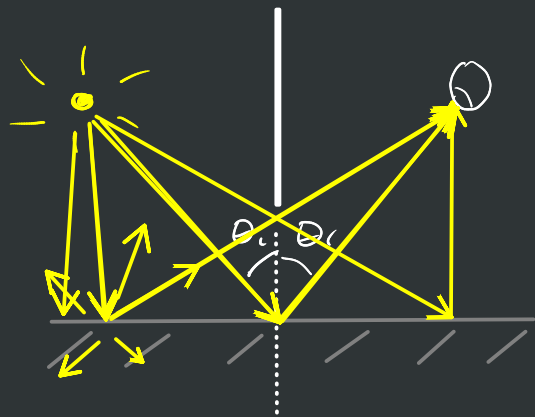
\longrightarrow so we want long focal length objectives

$$MP = \frac{f_o}{f_e}$$

\longrightarrow we want small focal length eyepieces

Temporarily back in Chapter 4

Law of Refraction: $n_1 \sin \theta_i = n_2 \sin \theta_t$



write an expression for the time:

$$t = \frac{l}{v_i} + \frac{l'}{v_t}$$

$$t = \frac{(a^2 + x^2)^{1/2}}{v_i} + \frac{(b^2 + (c-x)^2)^{1/2}}{v_t}$$

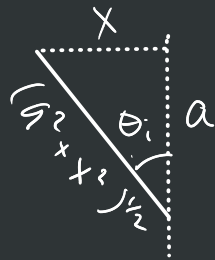
Fermat's principle

→ minimize t wrt. x

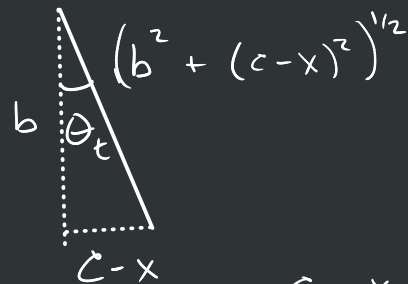
$$\frac{dt}{dx} = \frac{1}{2} \frac{(a^2 + x^2)^{-1/2}}{v_i} \cdot 2x + \frac{1}{2} \frac{(b^2 + (c-x)^2)^{-1/2}}{v_t} \cdot 2(c-x) \cdot (-1)$$

$$\frac{dt}{dx} = \frac{x}{v_i (a^2 + x^2)^{1/2}} - \frac{(c-x)}{v_t (b^2 + (c-x)^2)^{1/2}} = 0 \quad \leftarrow \text{minimize}$$

$$\frac{x}{v_i (a^2 + x^2)^{1/2}} = \frac{(c-x)}{v_t (b^2 + (c-x)^2)^{1/2}}$$



$$\sin \theta_i = \frac{x}{(a^2 + x^2)^{1/2}}$$



$$\sin \theta_t = \frac{c-x}{(b^2 + (c-x)^2)^{1/2}}$$

$$\frac{\sin \theta_i}{v_i} = \frac{\sin \theta_t}{v_t}$$

$$\frac{c}{v_i} = n_1$$

$$\frac{c}{v_t} = n_2$$

$$\frac{1}{v_i} = \frac{n_1}{c}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$PV = N k_B T \quad \leftarrow \text{Ideal Gas Law}$$

$$\frac{N}{V} = \frac{P}{k_B T} \quad \leftarrow \text{inversely proportional to } T$$

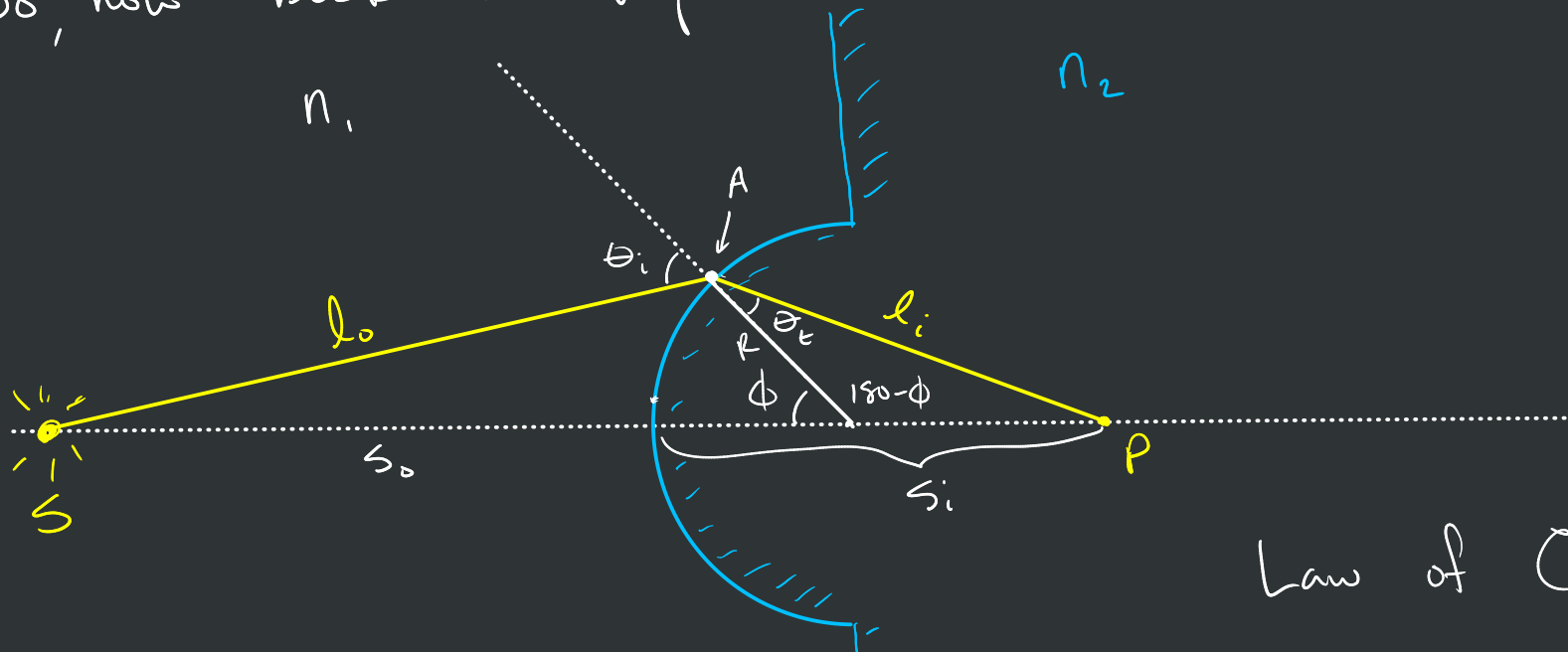
density

$$(n-1) \propto \frac{N}{V}$$

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{\text{distance}}{\frac{c}{n}} = \frac{(n \cdot \text{distance})}{c} \rightarrow \text{optical path length}$$

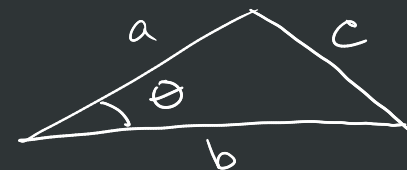
$\hookrightarrow v = \frac{c}{n}$

So, now back to chapter 5.



$$\text{OPL} = n_1 \cdot l_o + n_2 \cdot l_i$$

Law of Cosines

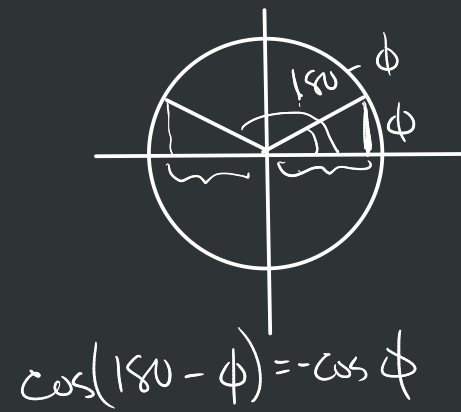


$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

$$l_o = \left(R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\phi \right)^{1/2}$$

$$l_i = \left(R^2 + (s_i - R)^2 - 2R(s_i - R)\cos(180 - \phi) \right)^{1/2}$$

$$= \left(R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\phi \right)^{1/2}$$



$$OPL = n_1 \left(R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\phi \right)^{1/2}$$

$$+ n_2 \left(R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\phi \right)^{1/2}$$

$$\frac{d(OPL)}{d\phi} = 0 \quad \leftarrow \text{Invoke Fermat's Principle}$$

$$0 = \frac{1}{2} \cdot n_1 \left(\underbrace{\quad}_{\frac{1}{l_o}} \right)^{-1/2} \left(-\cancel{2R}(s_o + R)(-\cancel{\sin\phi}) \right) + n_2 \frac{1}{2} \left(\underbrace{\quad}_{\frac{1}{l_i}} \right)^{-1/2} \left(\cancel{2R}(s_i - R)(-\cancel{\sin\phi}) \right)$$

$$\frac{n_1(s_0 + R)}{l_0} - \frac{n_2(s_i - R)}{l_i} = 0$$

$$\frac{n_1 s_0}{l_0} + \frac{n_1 R}{l_0} - \frac{n_2 s_i}{l_i} + \frac{n_2 R}{l_i} = 0$$

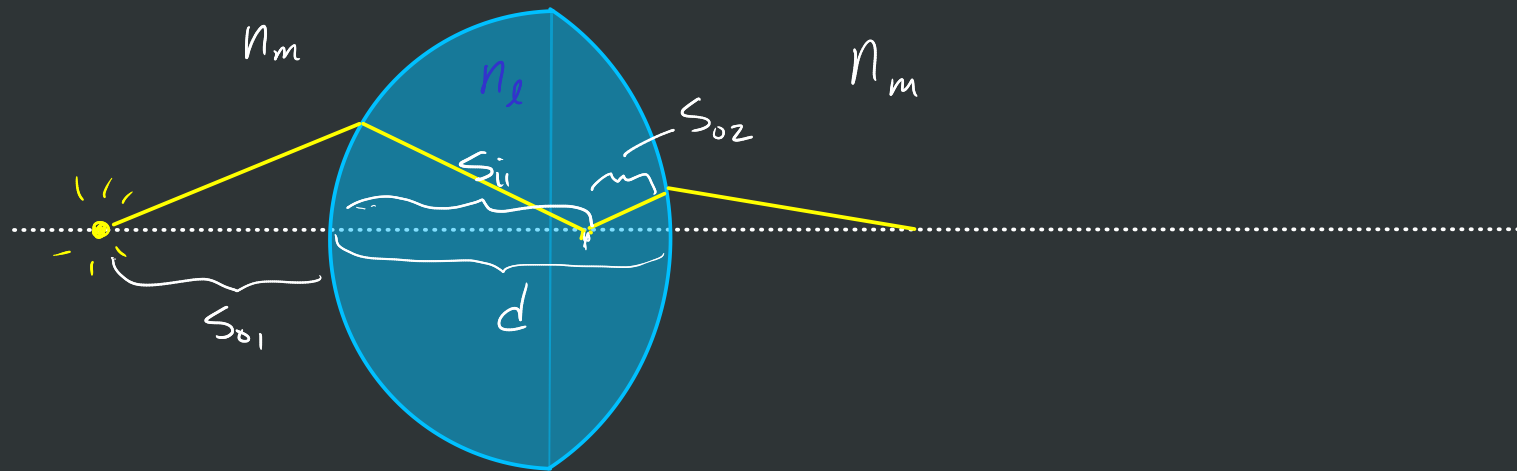
$$\frac{n_1 R}{l_0} + \frac{n_2 R}{l_i} = \frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0}$$

$$\frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R} \left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0} \right)$$

for small angles $\phi \approx 0$, $l_i \approx s_i$, $l_0 \approx s_0$, $\cos \phi \approx 1$

$$\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{1}{R} (n_2 - n_1)$$

This approximation is known first-order, paraxial, and Gaussian.



$$\bullet \quad \frac{n_m}{s_{o1}} + \frac{n_e}{s_{i1}} = \frac{n_e - n_m}{R_1} \quad \left. \vphantom{\frac{n_m}{s_{o1}} + \frac{n_e}{s_{i1}}} \right\}$$

$$\bullet \quad s_{o2} = \underbrace{d - s_{i1}}$$

$$\bullet \quad \frac{n_e}{s_{o2}} + \frac{n_m}{s_{i2}} = \frac{n_m - n_e}{R_2} \quad \left. \vphantom{\frac{n_e}{s_{o2}} + \frac{n_m}{s_{i2}}} \right\}$$

$$\frac{n_e}{d - s_{i1}} + \frac{n_m}{s_{i2}} = \frac{n_m - n_e}{R_2} \quad \left. \vphantom{\frac{n_e}{d - s_{i1}} + \frac{n_m}{s_{i2}}} \right\}$$

→ ADD TOGETHER



$$\frac{n_m}{s_{o1}} + \frac{n_e}{s_{i1}} + \frac{n_e}{d - s_{i1}} + \frac{n_m}{s_{i2}} = \frac{n_e - n_m}{R_1} + \frac{n_m - n_e}{R_2}$$

↓ some algebra

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_e - n_m) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{n_e \cdot d}{(s_{i1} - d) s_{i1}}$$

for a "thin lens" $d \rightarrow 0$

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_e - n_m) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

for

for $n_m = 1$

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_e - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

} Thin Lens Equation

$$\frac{1}{f} = (n_e - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

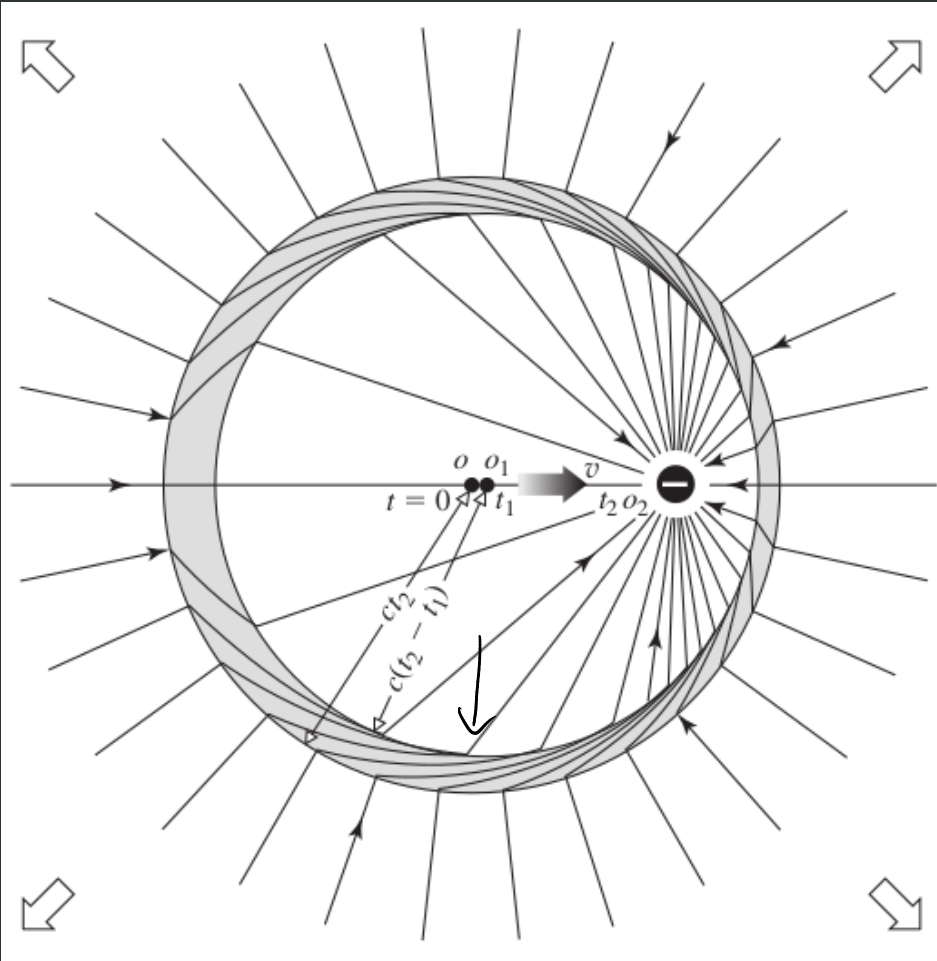
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$M_T = \frac{h'}{h} = - \frac{s_i}{s_o}$$

HW: watch those videos AND do chapter 5: 40, 43,

What is light? A wave of the electric field.

↳ Light originates from accelerating charges.



$$\vec{E} = \frac{-q}{4\pi\epsilon_0} \left[\frac{\hat{e}_r}{r^2} + \frac{r'}{c} \frac{d}{dt} \left(\frac{\hat{e}_r}{r^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \hat{e}_r \right]$$

$$\vec{E} = \frac{-q}{4\pi\epsilon_0 c^2} \frac{d^2 \hat{e}_r}{dt^2}$$

$$E_z(t) = \frac{-q}{4\pi\epsilon_0 c^2 r} a_z \left(t - \frac{r}{c} \right)$$

$$= E_0 \cos(\omega t)$$