Chapter 4ish - 5ish

wavelength - distance of an oscillation
- one electric field max to another max
- λ, meters, cm, nm

period - time for one oscillation to pass a point as the wave goes by
- T, seconds

kappa

cursive kay

wavenumber - $\frac{1}{\lambda}$ K, k

propogation constant $\frac{2\pi}{\lambda} = k$

nu

frequency - $\nu = \frac{1}{T} = Hz$

angular frequency $\frac{2\pi}{T} = 2\pi\nu$

speed $\rightarrow$ $v = \frac{\lambda}{T} = \lambda \cdot f = \lambda\nu$

in materials, light slows down

$c \to$ speed of light in vacuum

$\to v < c$

$$\underbrace{\frac{c}{v}}= n \leftarrow \text{index of refraction}$$

conventionally $n \geq 1$

but can be negative (metamaterials)

and can be complex (absorptive materials)

$$v = \frac{c}{n} = \lambda \cdot \nu$$

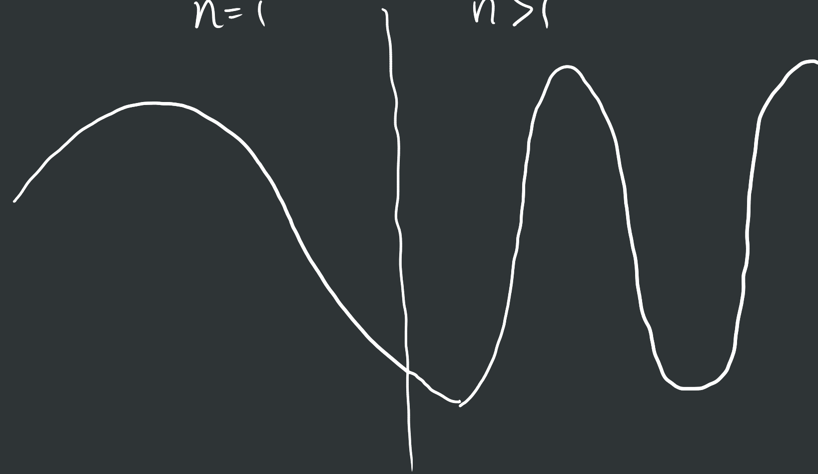
$\to$ if n increases at some boundary

λ decreases, but not ν.

ν is constant

- $E = h \cdot \nu = \frac{hc}{n\lambda}$

$\to$ Planck's constant



$$V = \frac{c}{n} = \lambda \nu$$

$$\frac{c}{n_1 \lambda_1} = \nu = \frac{c}{n_2 \lambda_2}$$

$$\frac{\cancel{c}}{n_1 \lambda_1} = \frac{\cancel{c}}{n_2 \lambda_2}$$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{n_2}{n_1} = \left(\frac{\lambda_2}{\lambda_1}\right)^{-1}$$

I is vacuum

$$\frac{n}{1} = \left(\frac{\lambda_2}{\lambda_0}\right)^{-1}$$

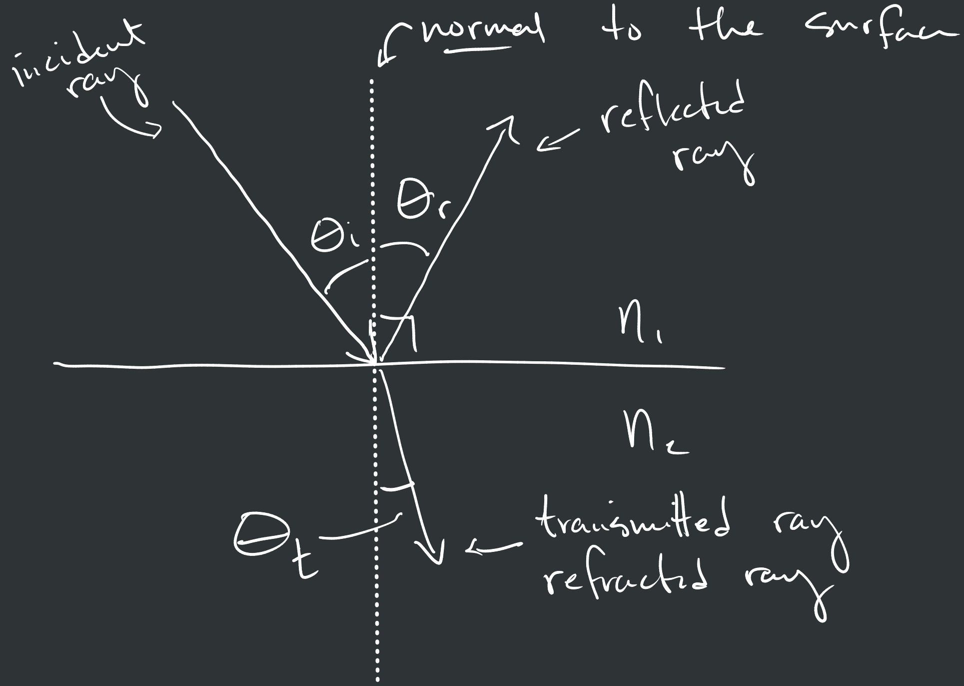
n=1 in vacuum

in vacuum

$$\lambda_2 = \frac{\lambda_0}{n}$$

n > 1

so this goes down w/ n



Law of Reflection

$\rightarrow \theta_i = \theta_r$

Law of Refraction (Snell's Law)

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\hookrightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

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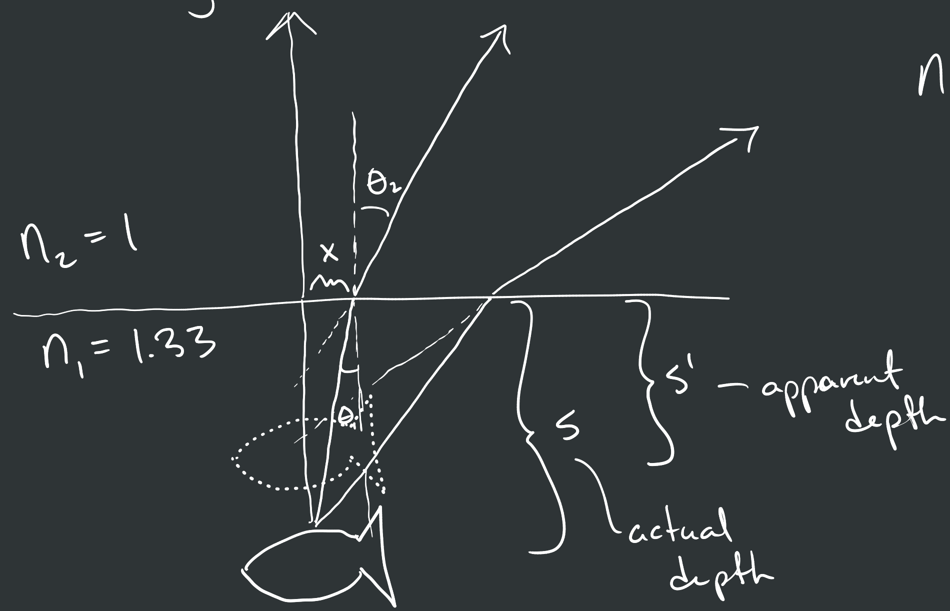
fishing



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

for small θ, $\sin\theta \approx \tan\theta \approx \theta$

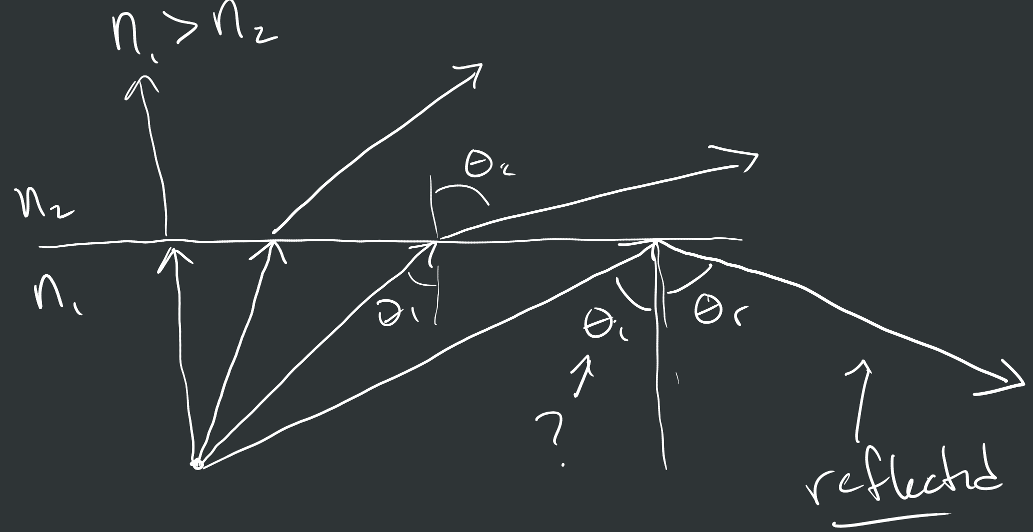
$$n_1 \tan\theta_1 = n_2 \tan\theta_2$$

$$\frac{n_1 \cancel{x}}{s} = \frac{n_2 \cancel{x}}{s'}$$

$$\frac{s'}{s} = \frac{n_2}{n_1} = \frac{1}{1.33} \approx 0.752$$

$s' = 0.752 \cdot s$

critical angle



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

what θ_1 causes θ_2 to be 90°

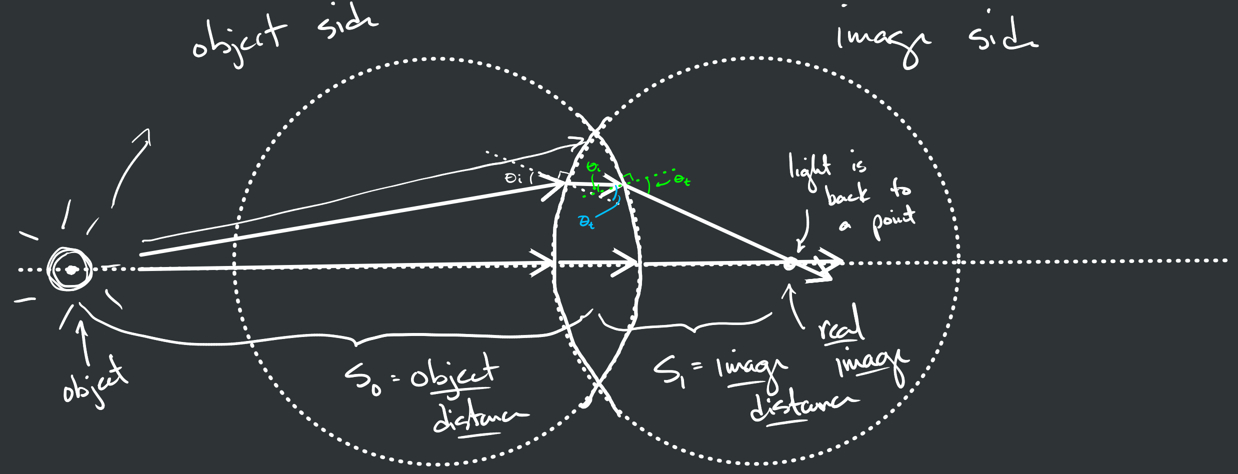
$$\underbrace{\sin\theta_2}_{} = \underbrace{\frac{n_1}{n_2}\sin\theta_1}_{\text{when this is } \geq 1}$$

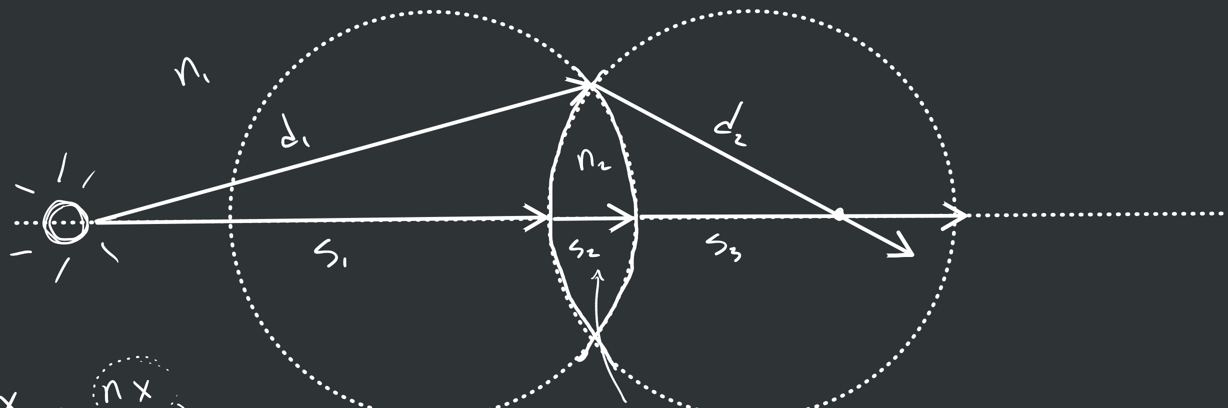
$$1 = \frac{n_1}{n_2}\sin\theta_1$$

critical angle $\longrightarrow$ $\theta_1 = \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

< 1

HW: Chapter 4: 6, 7, 8, 21, 24





$$t = \frac{x}{v} = \frac{n x}{c}$$

$\frac{nx}{c}$ → optical path length

$$v = \frac{c}{n}$$

$$\frac{n_1 d_1}{\cancel{c}} + \frac{n_1 d_2}{\cancel{c}} = \frac{n_1 S_1}{\cancel{c}} + \frac{n_2 S_2}{\cancel{c}} + \frac{n_1 S_3}{\cancel{c}}$$

$$n_1 d_1 + n_1 d_2 = n_1 S_1 + n_2 S_2 + n_1 S_3$$ } optical path lengths are the same!

perfect images

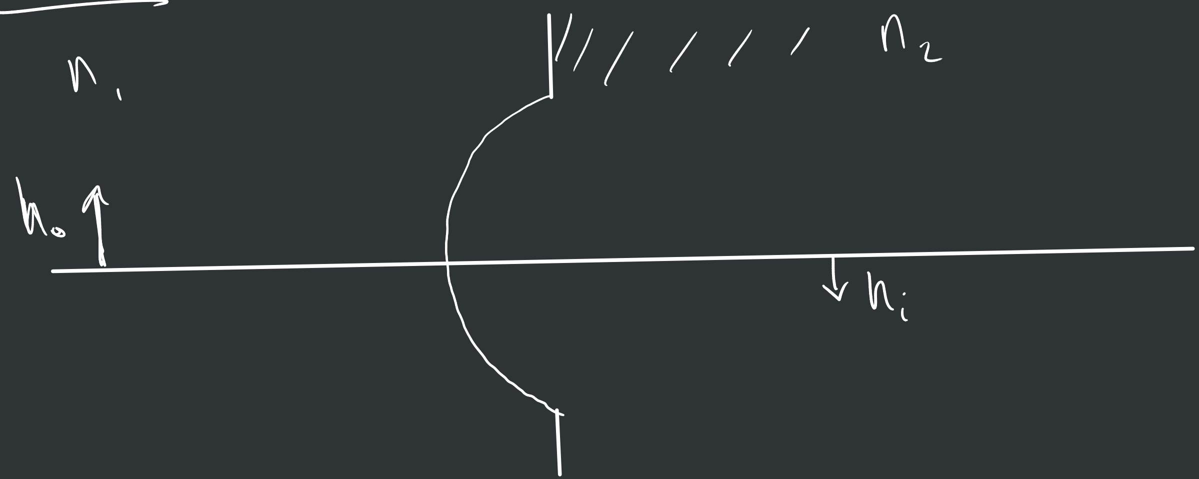
reflection

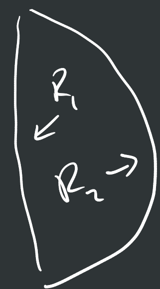
- ellipson
- hyperboloid
- parabola

refraction

- ellipson
- hyperboloid
- cartesian oval

Refraction





Thin lens

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R}$$

S_o: object distance

S_i: image distance

thin lens (Lens Makers Equation)

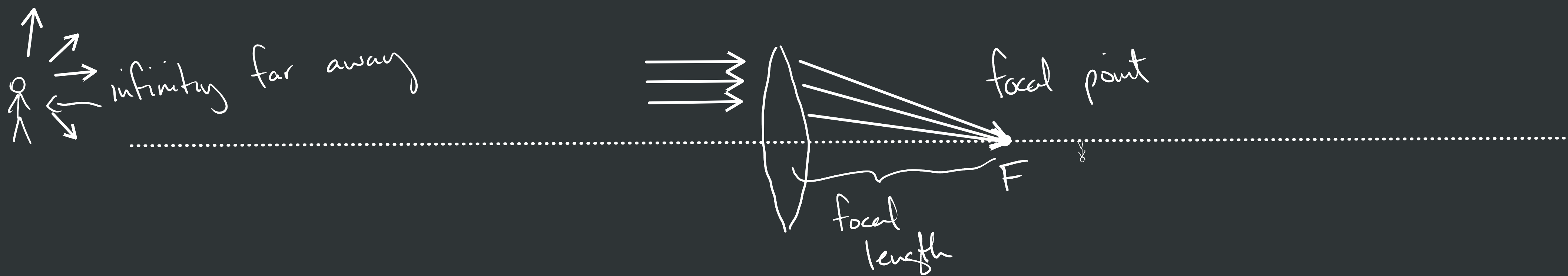
$$\frac{1}{S_o} + \frac{1}{S_i} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

f: focal length

(Thin Lens Equation)

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$



	+	−
s_o	real object	virtual object
s_i	real image	virtual image
f	converging lens	diverging lens
y_o	upright object	inverted object
y_i	upright image	inverted image

Optics Lab:

Conjugate Points: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

Focal Length Exp: $\cancel{\frac{1}{s_o \to \infty}}^{\to 0} + \frac{1}{s_i} = \frac{1}{f}$

$\frac{1}{s_i} = \frac{1}{f} \Rightarrow s_i = f$ for $s_o \to \infty$

Diverging Lens: $$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{-f}$$

$$\frac{1}{S_i} = \frac{1}{-f} - \frac{1}{S_o}$$

$$S_i = \frac{1}{\frac{1}{-f} - \frac{1}{S_o}}$$

Finding a virtual image:

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$\frac{1}{S_i} = \frac{1\,S_o}{f\,S_o} - \frac{1\,f}{S_o\,f} = \frac{S_o - f}{f\,S_o}$$

$$S_i = \frac{S_o \cdot f}{S_o - f}$$

NINE: $S_o = 27\text{ cm}$

$f = -15\text{ cm}$

$$S_i = \frac{27(-15)}{27+15} = -9.60\text{ cm}$$

Now for the check:

$s_i = 16.2$ cm

$f = 10$ cm

$s_o = 26.13$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

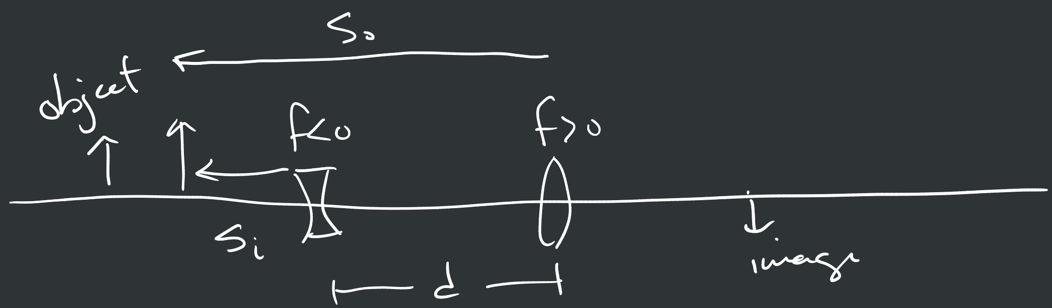
$$s_o = \frac{s_i \cdot f}{s_i - f}$$

$$s_o = \frac{16.2 \cdot 10}{16.2 - 10}$$

$$s_o = 26.13$$

So now we need to compare

$$s_o = d - s_i$$



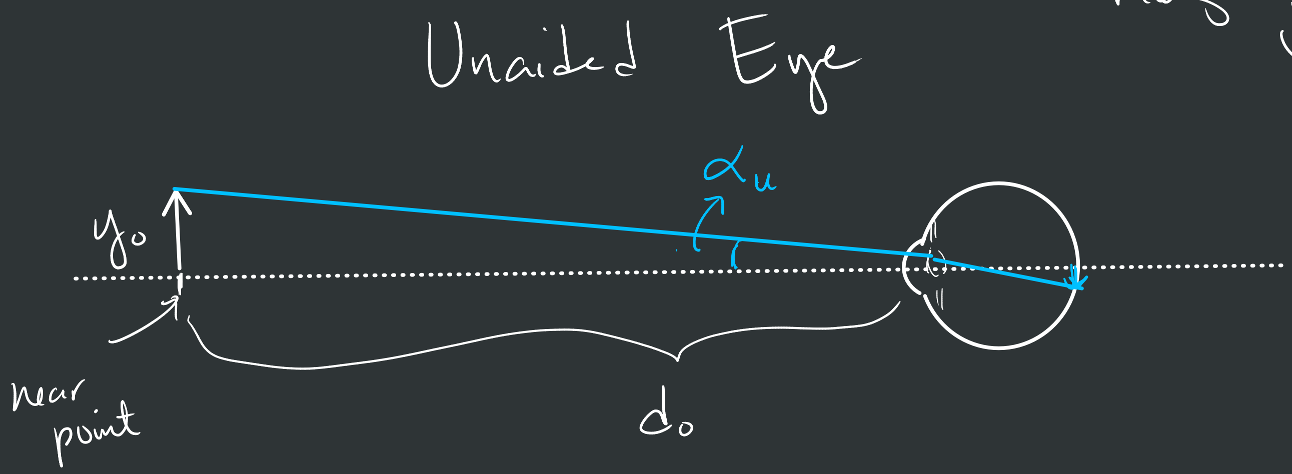
$26.13 = d - (-9.6)$

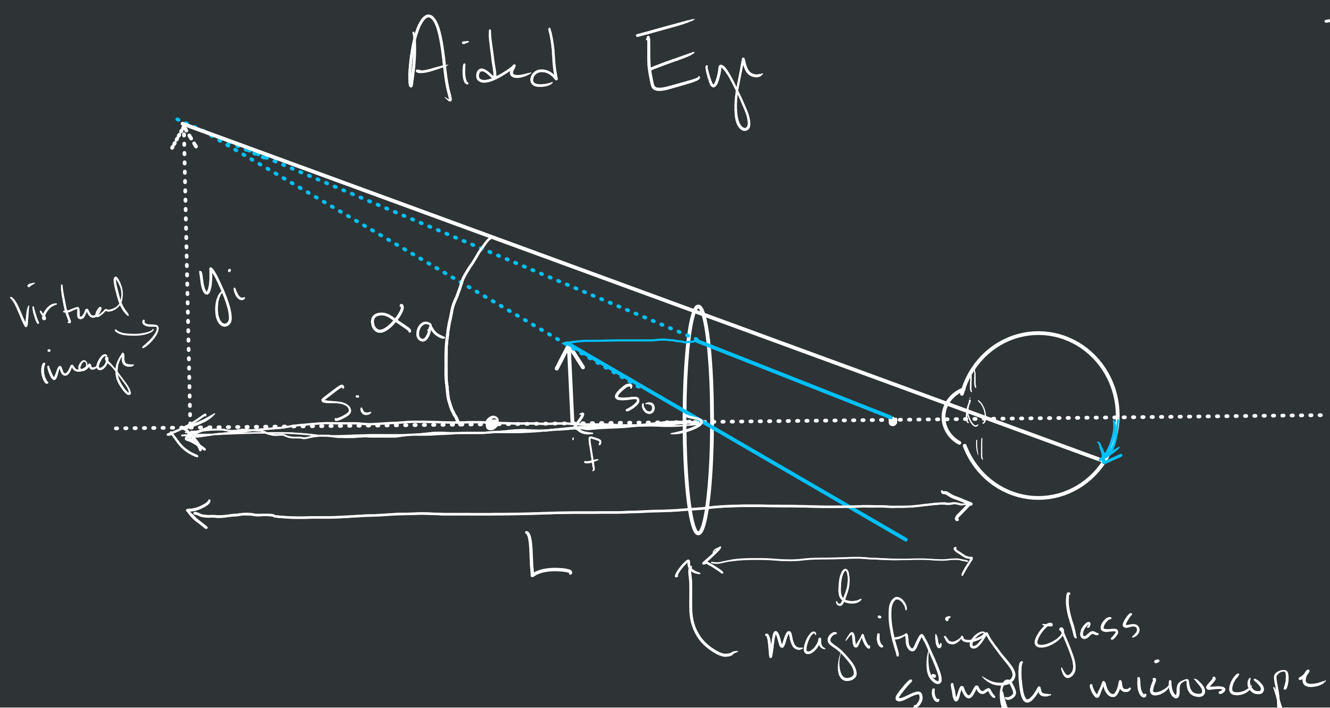
$16.53 = d$

17.3 <u>measured</u>

Optical Devices

Magnifying Glass





magnifying power

angular magnification

$$MP = M_A = \frac{\alpha_a}{\alpha_u}$$

radians

$$\tan \alpha_u = \frac{y_o}{d_o} \approx \sin \alpha_u \approx \alpha_u$$

$$\tan \alpha_a = \frac{y_i}{L} \approx \alpha_a$$

$$MP = \frac{y_i}{L} \cdot \frac{d_o}{y_o}$$

this is positive

$$M_t = \frac{y_i}{y_o} = -\frac{S_i}{S_o}$$

$\nearrow 57.3^\circ \leftarrow 1\,\text{rad} \cdot \frac{180}{\pi} = 57.3^\circ$

$\tan 1\,\text{rad} = 1.557$

$\tan 0.1\,\text{rad} = 0.10033$

$\hookrightarrow 5.73^\circ$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$S_i \left[\frac{1}{S_o} = \frac{1}{f} - \frac{1}{S_i} \right] S_i$$

$$\frac{S_i}{S_o} = \frac{S_i}{f} - 1$$

$$-\frac{S_i}{S_o} = \boxed{1 - \frac{S_i}{f}}$$

$$MP = -\frac{S_i}{S_o} \cdot \frac{d_o}{L}$$

$$MP = \left(1 - \frac{S_i}{f}\right) \frac{d_o}{L}$$

$$S_i = -(L - \ell)$$

$$MP = \left(1 + \frac{L - \ell}{f}\right) \cdot \frac{d_o}{L}$$

$$\frac{1}{f} = \mathcal{D} \leftarrow \text{dioptric power}$$

$$\boxed{MP = (1 + (L - \ell)\mathcal{D}) \cdot \frac{d_o}{L}} \quad \text{eq. } 5.76$$

$\mathcal{D}$

$\mathcal{D}$

Case 1: $l = f$

$$[MP]_{l=f} = \left(1 + (L - l)\frac{1}{f}\right) \cdot \frac{d_o}{L}$$

$$= \left(\cancel{1} + \frac{\cancel{L}}{f} - \cancel{\frac{l}{f}}^{\,1}\right) \cdot \frac{d_o}{\cancel{L}}$$

$$[MP]_{l=f} = \frac{d_o}{f} = d_o \mathcal{D}$$

Case 2: $l = 0$

$$[MP]_{l=0} = \left(1 + (L - \cancel{l}^{\,0})\mathcal{D}\right) \cdot \frac{d_o}{L}$$

$$= (1 + L\mathcal{D})\frac{d_o}{L}$$

$$[MP]_{l=0} = \left(\frac{1}{L} + \mathcal{D}\right) \cdot d_o$$

if we change L to d_o,

$$[MP]_{\substack{l=0 \\ L=d_o}} = \left(\frac{1}{d_o} + \mathcal{D}\right) \cdot d_o$$

$$[MP]_{\substack{l=0 \\ L=d_o}} = 1 + \mathcal{D} \cdot d_o$$

Case 3: We put the object at the focal point, $s_o = f$.
The image is formed

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{f} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{s_i} = 0$$

$$s_i \to \infty$$

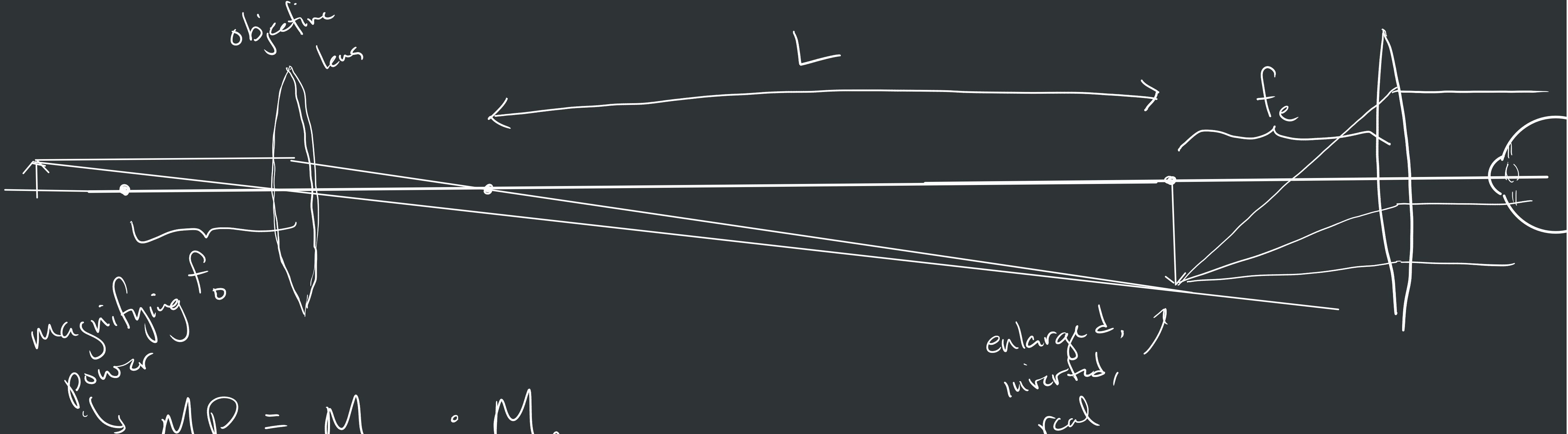
$$MP = \left(1 + (L - l)\mathcal{D}\right) \cdot \frac{d_o}{L}$$

$$\left[MP\right]_{L\to\infty} = \cancelto{0}{\frac{d_o}{L}} + \frac{\cancel{L} \cdot \mathcal{D} \cdot d_o}{\cancel{L}} - \cancelto{0}{\frac{l\,\mathcal{D} \cdot d_o}{L}}$$

$$\left[MP\right]_{L\to\infty} = \mathcal{D} \cdot d_o = \frac{d_o}{f}$$

HW: ch5. 25, 34, 42

Microscope



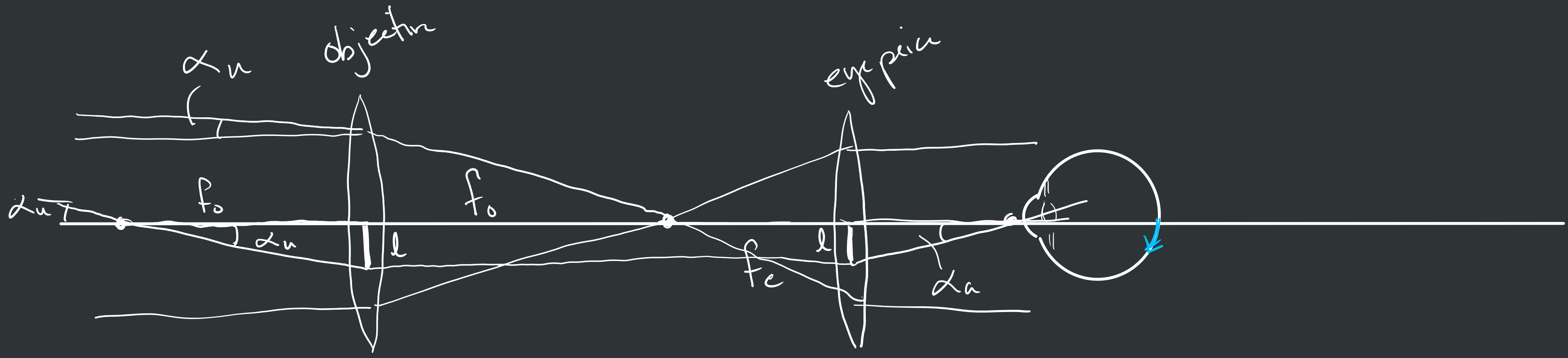
magnifying power

$$MP = M_{To} \cdot M_{Ae}$$

Objective lens usually has a very small focal length.

Eyepiece has a larger focal length

Telescope



$$MP = \frac{\alpha_a}{\alpha_u}$$

$$\alpha_u \approx \tan \alpha_u = \frac{l}{f_o}$$

$$\alpha_a \approx \tan \alpha_a = \frac{l}{f_e}$$

$$MP = \frac{l/f_e}{l/f_o}$$

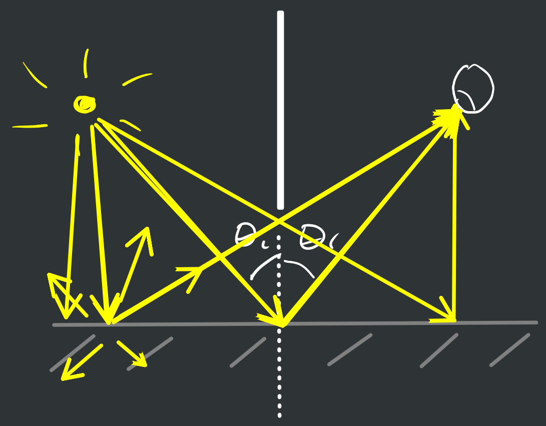
$$MP = \frac{f_o}{f_e}$$

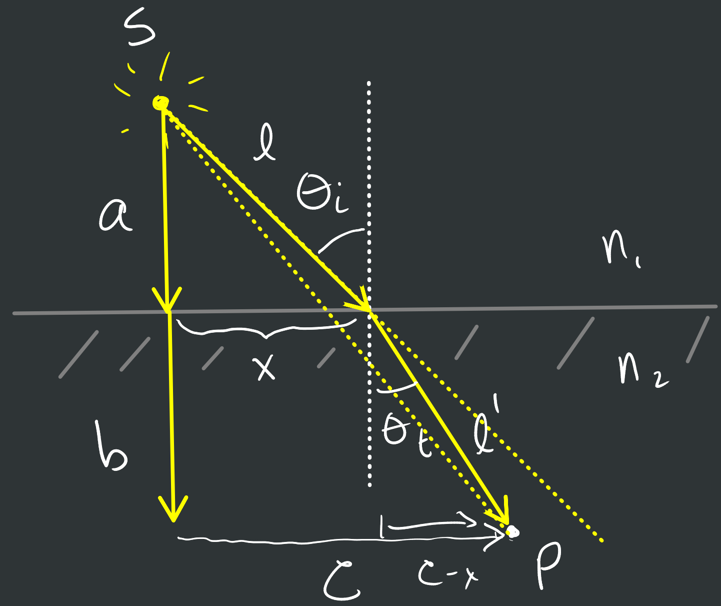
→ so we want long focal length objectives

→ we want small focal length eyepieces

Temporarily back in Chapter 4

Law of Refraction: $n_1 \sin\theta_i = n_2 \sin\theta_t$





write an expression for the time:

$$t = \frac{\ell}{v_i} + \frac{\ell'}{v_t}$$

$$t = \frac{(a^2 + x^2)^{1/2}}{v_i} + \frac{(b^2 + (c-x)^2)^{1/2}}{v_t}$$

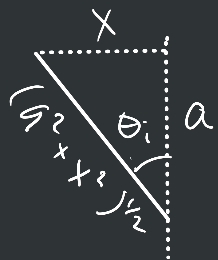
Fermat's principle

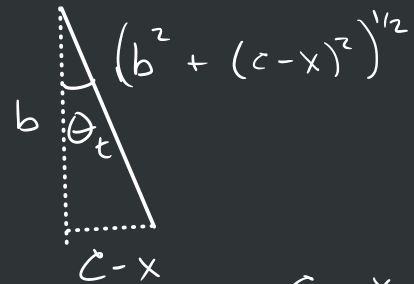
$\hookrightarrow$ minimize t w.r.t. x

$$\frac{dt}{dx} = \frac{1}{\cancel{2}} \frac{(a^2+x^2)^{-1/2}}{V_i} \cdot \cancel{2}x + \frac{1}{\cancel{2}} \frac{\left(b^2+(c-x)^2\right)^{-1/2}}{V_t} \cdot \cancel{2}(c-x)\cdot(-1)$$

$$\frac{dt}{dx} = \frac{x}{V_i (a^2+x^2)^{1/2}} - \frac{(c-x)}{V_t\left(b^2+(c-x)^2\right)^{1/2}} = 0$$ ← minimize

$$\frac{x}{V_i (a^2+x^2)^{1/2}} = \frac{(c-x)}{V_t\left(b^2+(c-x)^2\right)^{1/2}}$$





$$\sin\theta_i = \frac{x}{(a^2+x^2)^{1/2}}$$

$$\sin\theta_t = \frac{c-x}{\left(b^2+(c-x)^2\right)^{1/2}}$$

$PV = N k_B T$ ← Ideal Gas Law

$\frac{N}{V} = \frac{P}{k_B T}$ ← inversely proportional to T

$\frac{N}{V}$ ~ density

$(n-1) \propto \frac{N}{V}$

$$\frac{\sin\theta_i}{V_i} = \frac{\sin\theta_t}{V_t}$$

$$\frac{c}{V_i} = n_1 \qquad \frac{c}{V_t} = n_2$$

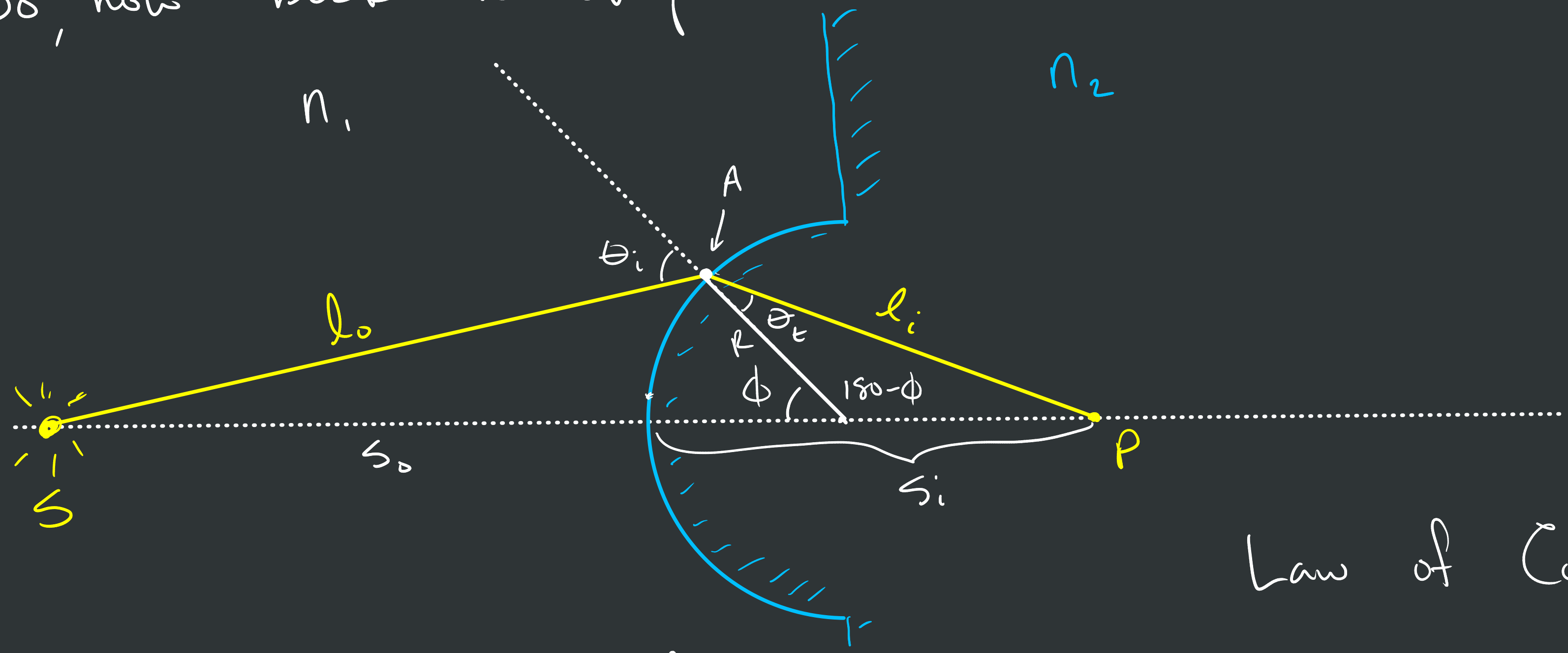
$$\frac{1}{V_i} = \frac{n_1}{c}$$

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{\text{distance}}{c/n} = \frac{n \cdot \text{distance}}{c}$$ → optical path length

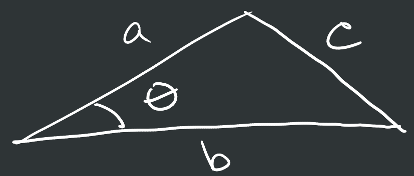
↳ $v = \frac{c}{n}$

So, now back to chapter 5.



$$OPL = n_1 \cdot l_o + n_2 l_i$$

Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$l_o = \left(R^2 + (s_o+R)^2 - 2R(s_o+R)\cos\phi\right)^{1/2}$$

$$l_i = \left(R^2 + (s_i-R)^2 - 2R(s_i-R)\cos(180-\phi)\right)^{1/2}$$

$$= \left(R^2 + (s_i-R)^2 + 2R(s_i-R)\cos\phi\right)^{1/2}$$

$$\cos(180-\phi) = -\cos\phi$$

$$OPL = n_1\left(R^2 + (s_o+R)^2 - 2R(s_o+R)\cos\phi\right)^{1/2}$$

$$+ n_2\left(R^2 + (s_i-R)^2 + 2R(s_i-R)\cos\phi\right)^{1/2}$$

$$\frac{d(OPL)}{d\phi} = 0 \quad \leftarrow \text{Invoke Fermat's Principle}$$

$$0 = \frac{1}{\cancel{2}} \cdot n_1 \underbrace{(\qquad)^{-1/2}}_{\frac{1}{l_o}} \left(-\cancel{2}\cancel{R}(s_o+R)(-\cancel{\sin\phi})\right) + n_2 \frac{1}{\cancel{2}} \underbrace{(\qquad)^{-1/2}}_{\frac{1}{l_i}} \left(\cancel{2}\cancel{R}(s_i-R)(-\cancel{\sin\phi})\right)$$

$$\frac{n_1(s_o+R)}{l_o} - \frac{n_2(s_i-R)}{l_i} = 0$$

$$\frac{n_1 s_o}{l_o} + \frac{n_1 R}{l_o} - \frac{n_2 s_i}{l_i} + \frac{n_2 R}{l_i} = 0$$

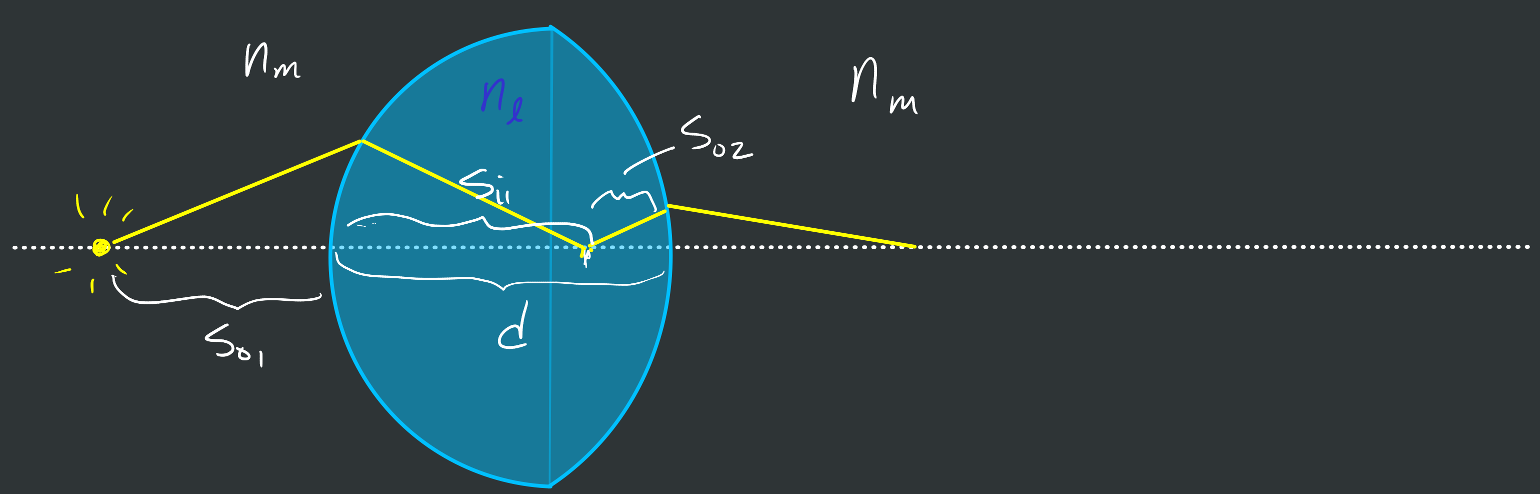
$$\frac{n_1 R}{l_o} + \frac{n_2 R}{l_i} = \frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}$$

$$\frac{n_1}{l_o} + \frac{n_2}{l_o} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right)$$

for small angles $\phi \approx 0$, $l_i \approx s_i$, $l_o \approx s_o$, $\cos\phi \approx 1$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{1}{R}(n_2 - n_1)$$

This approximation is known first-order, paraxial, and Gaussian.



- $$\frac{n_m}{S_{o1}} + \frac{n_\ell}{S_{i1}} = \frac{n_e - n_m}{R_1}$$

- $$S_{o2} = \underbrace{d - S_{i1}}$$

- $$\frac{n_\ell}{S_{o2}} + \frac{n_m}{S_{i2}} = \frac{n_m - n_e}{R_2}$$

$$\frac{n_e}{d - S_{i1}} + \frac{n_m}{S_{i2}} = \frac{n_m - n_e}{R_2}$$

ADD TOGETHER

$$\frac{n_m}{s_{o1}} + \frac{n_e}{s_{i1}} + \frac{n_e}{d - s_{i1}} + \frac{n_m}{s_{i2}} = \frac{n_e - n_m}{R_1} + \frac{n_m - n_e}{R_2}$$

$$\Big\downarrow \text{ some alegebra}$$

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_e - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_e \cdot d}{(s_{i1} - d)\, s_{i1}}$$

for a "thin lens" $d \to 0$

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_e - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

for

for $n_m = 1$

$$\boxed{\frac{1}{S_o} + \frac{1}{S_i} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

} Thin Lens Equation

$$\boxed{\frac{1}{f} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

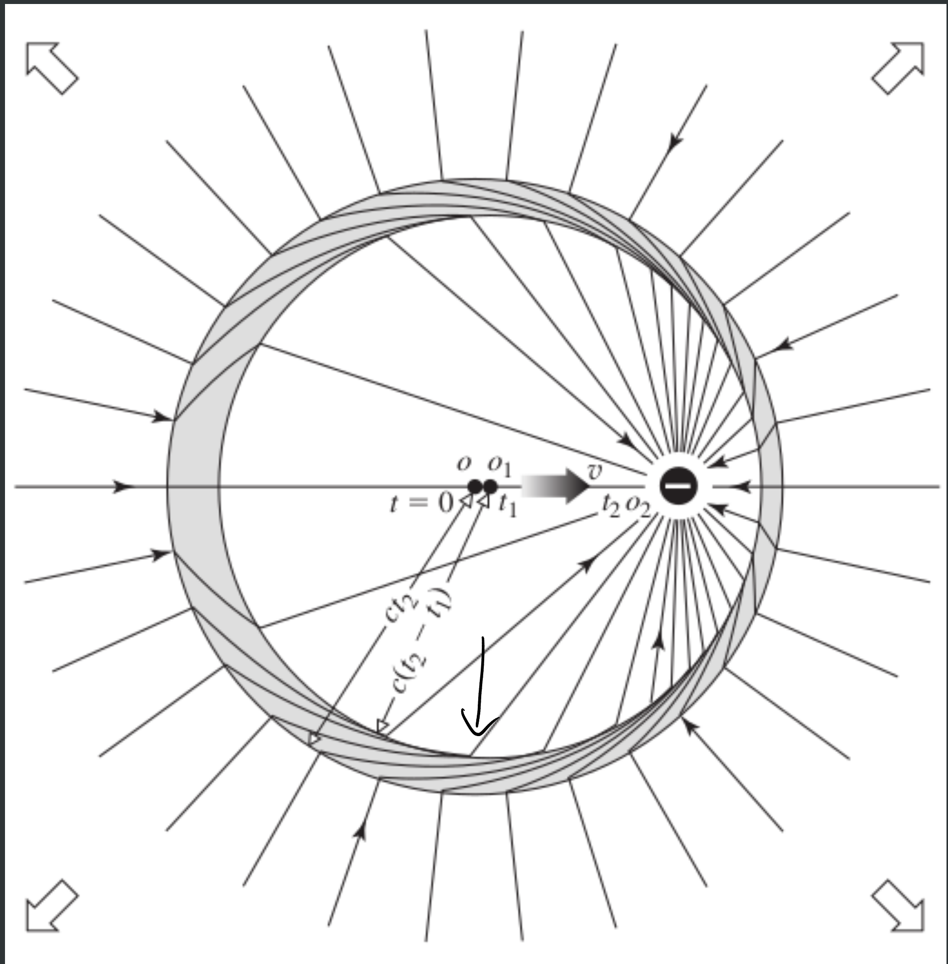
$$\boxed{\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}}$$

$$M_T = \frac{h'}{h} = -\frac{S_i}{S_o}$$

<u>HW:</u> watch those videos AND do chapter 5: 40, 43,

What is light? A wave of the electric field.

↳ Light originates from accelerating charges.

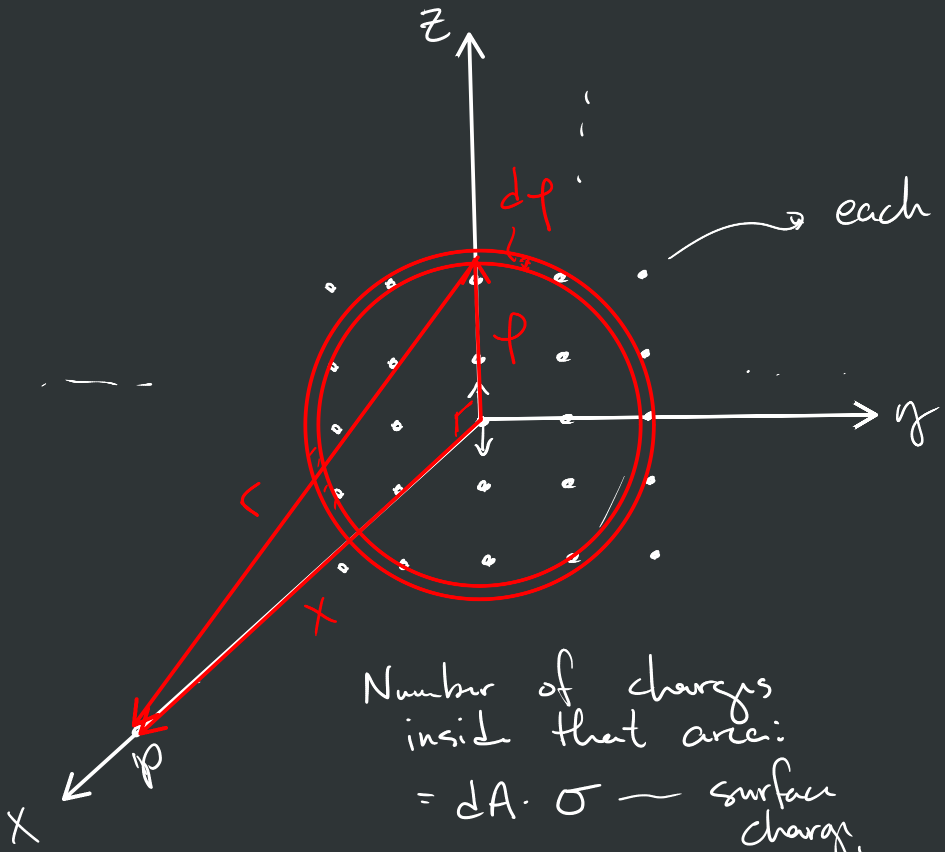


$$\vec{E} = \frac{-q}{4\pi\epsilon_0}\left[\cancelto{0}{\frac{\hat{e}_{r'}}{r'^2}} + \cancelto{0}{\frac{r'}{c}\frac{d}{dt}\left(\frac{\hat{e}_{r'}}{r'^2}\right)} + \frac{1}{c^2}\frac{d^2\hat{e}_{r'}}{dt^2}\right]$$

$$\vec{E} = \frac{-q}{4\pi\epsilon_0 c^2}\frac{d^2\hat{e}_{r'}}{dt^2}$$

$$\underline{E_z}(t) = \frac{-q}{4\pi\epsilon_0 c^2 r}\, a_z\!\left(t - \frac{r}{c}\right)$$

<u>HW:</u> read about spherical mirrors, then work 5.71 & 5.74



each charge is oscillating

$z_0 \cos \omega t$

radiation from each charge will be proportional to $-\omega^2 z_0 \cos \omega t$

$-\omega^2 z_0 e^{i\omega t}$

Number of charges inside that area:

$= dA \cdot \sigma$ — surface charge density

$= 2\pi \rho \, d\rho \cdot \sigma \qquad \sigma = \dfrac{Q}{A}$

field at point P will be proportional to $-\omega^2 z_0 e^{i\omega(t - \frac{r}{c})}$

$$\frac{q}{4\pi\epsilon_0 c^2} \cdot \frac{\omega^2 z_0 e^{i\omega(t-\frac{r}{c})}}{r}$$

Total Field at pt. P $= \displaystyle\int_0^\infty \frac{q}{4\pi\epsilon_0 c^2} \frac{\omega^2 z_0 e^{i\omega(t-\frac{r}{c})}}{r} \cdot 2\pi\rho \cdot \sigma \cdot d\rho$

$$= \frac{q}{\cancel{4\pi}_{2}\,\epsilon_0 c^2} \omega^2 z_0 \cancel{2\pi} \cdot \sigma \, e^{i\omega t} \underbrace{\int_0^\infty \frac{e^{-i\frac{\omega}{c} \cdot r}}{r} \cdot \rho \, d\rho}$$

$\rightarrow r^2 = \rho^2 + x^2$

$\cancel{2} r dr = \cancel{2} \rho d\rho$

when $\rho \rightarrow \infty$

$r \rightarrow \infty$

when $\rho \rightarrow 0$

$r \rightarrow x$

$$\int_{r=x}^{r=\infty} \frac{e^{-i\frac{\omega}{c}r}}{\cancel{r}} \cdot \cancel{r}\, dr$$

$$\int_{r=x}^{r=\infty} e^{-i\frac{\omega}{c}r}\, dr$$

$$-\frac{c}{i\omega} e^{-i\frac{\omega}{c}r} \Big|_{x}^{\infty}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$-\frac{c}{i\omega}\left[\underbrace{e^{-i\infty}} - e^{-i\frac{\omega}{c}x}\right]$$

- not 0, oscillates around 0 so make it 0.
- if σ fades as $r \to \infty$ then, it is also 0.
- if we keep track of all components of E, then it is 0

$$\text{Total field at } P = \frac{q}{\cancel{4}_{2}\pi \epsilon_0 c^{\cancel{2}}} \, \omega^{\cancel{2}} z_0 \cancel{2\pi} \cdot \sigma \, e^{i\omega t} \cdot \left(\frac{\cancel{c}}{i\cancel{\omega}} e^{-\frac{i\omega}{c} x} \right)$$

$$\frac{1}{i} \cdot \frac{i}{i} = -i$$

$$= -\frac{q \omega z_0 \sigma i}{2 \epsilon_0 c} \underbrace{e^{i\omega\left(t - \frac{x}{c}\right)}}_{\cos \omega\left(t - \frac{x}{c}\right) + i \sin}$$