

Chapter 4ish - 5ish

wavelength - distance of an oscillation
- one electric field max to another max
- λ , meters, cm, nm

period - time for one oscillation to pass a point as the wave goes by

- T , seconds
κappa ↓ curvus kāps

wavenumber - $\frac{1}{\lambda}$ K, k

propagation constant $\frac{2\pi}{\lambda} = k$

frequency - ν = $\frac{1}{T}$ = Hz
nu

angular frequency $\frac{2\pi}{T} = 2\pi\nu$

speed $\rightarrow v = \frac{\lambda}{T} = \lambda \cdot f = \lambda \nu$

in materials, light slows down

$$\left. \begin{array}{l} c \rightarrow \text{speed of light in vacuum} \\ \Rightarrow v < c \end{array} \right\}$$

$$\underbrace{\frac{c}{v}}_{n} = n \leftarrow \text{index of refraction}$$

conventionally $n \geq 1$

but can be negative (metamaterials)

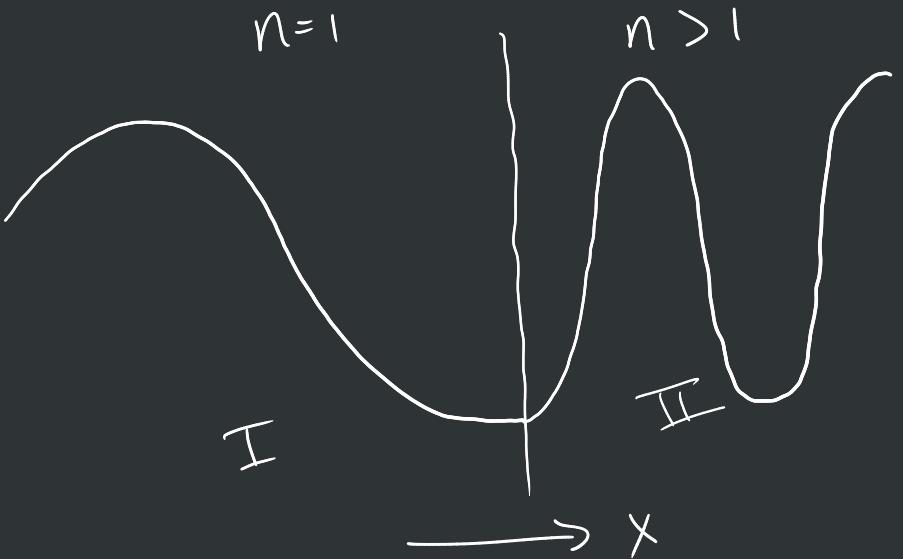
and can be complex (absorptive materials)

$$v = \frac{c}{n} = \underbrace{\lambda \cdot \gamma}_{\substack{\text{if } n \text{ increases at some boundary} \\ \lambda \text{ decreases, but not } \gamma.}}$$

γ is constant

$$\cdot E = h \cdot \gamma = \frac{hc}{\lambda}$$

\hookrightarrow Planck's constant



$$V = \frac{C}{n} = \lambda \nu$$

$$\frac{C}{n_1 \lambda_1} = \nu = \frac{C}{n_2 \lambda_2}$$

$$\frac{C}{n_1 \lambda_1} = \frac{C}{n_2 \lambda_2}$$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{n_2}{n_1} = \left(\frac{\lambda_2}{\lambda_1}\right)^{-1}$$

$\Rightarrow I$ is vacuum

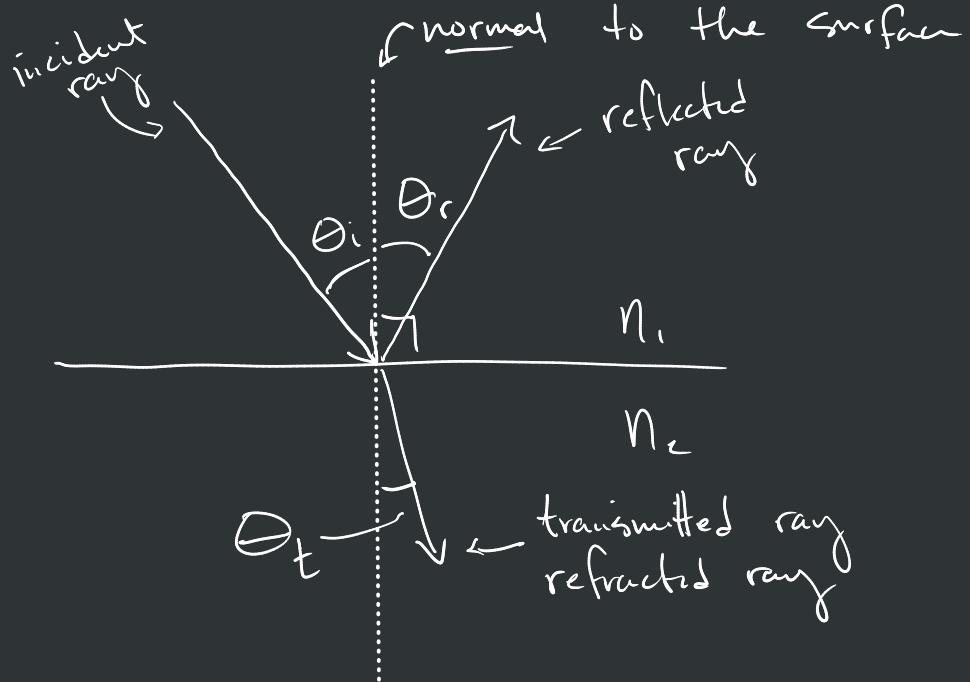
$$\frac{n}{1} = \left(\frac{\lambda_2}{\lambda_0}\right)^{-1}$$

\uparrow \uparrow

$n=1$
in vacuum in vacuum

$$\lambda_2 = \frac{\lambda_0}{n} \quad n > 1$$

\Rightarrow so this goes down w/n

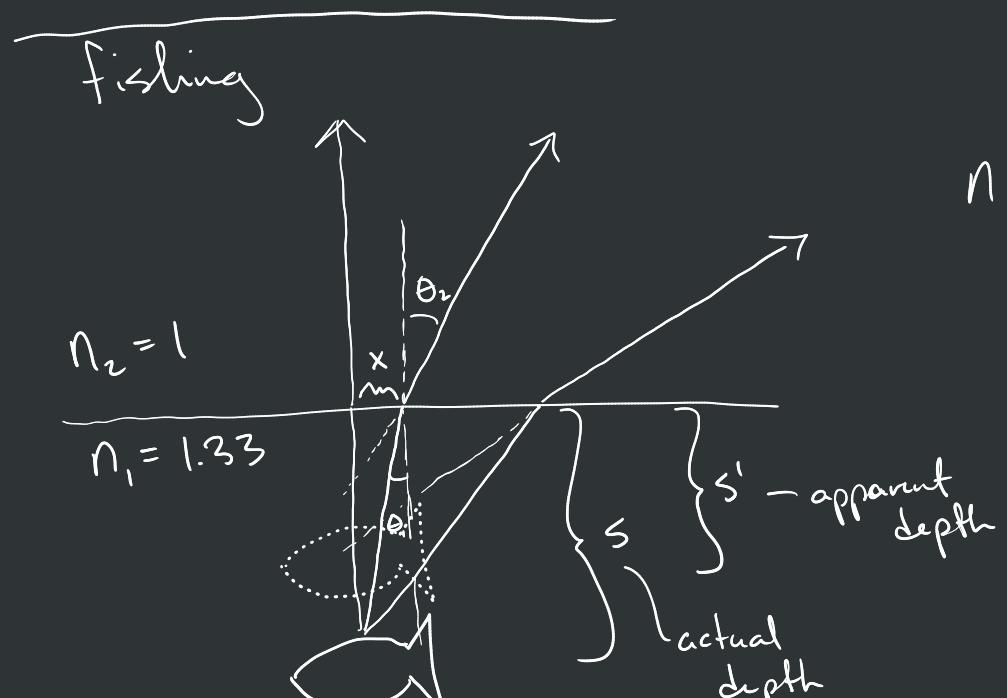


Law of Reflection
 $\rightarrow \theta_i = \theta_r$

Law of Refraction (Snell's Law)

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$(n_1 \sin \theta_i = n_2 \sin \theta_t)$$



$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

for small θ , $\sin \theta \approx \tan \theta \approx \theta$

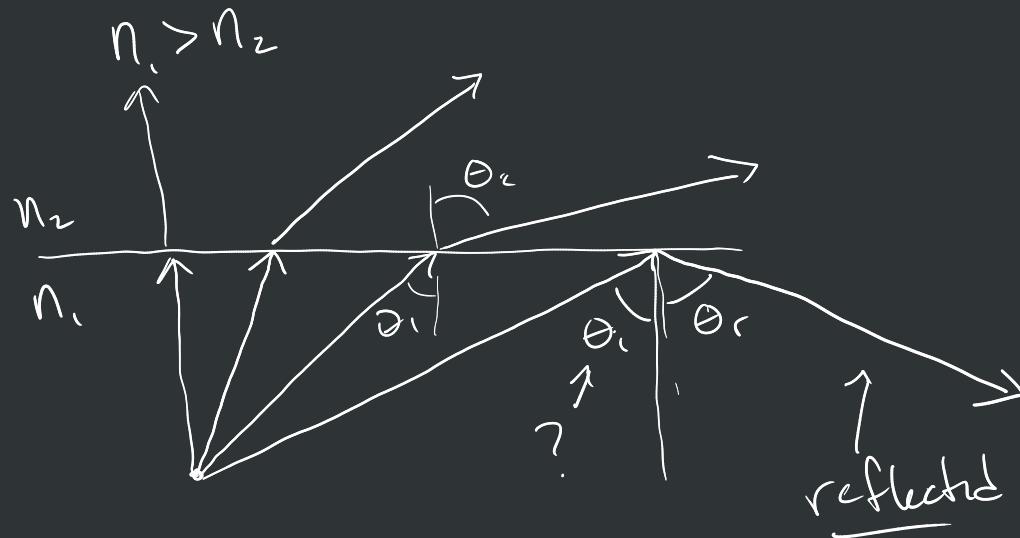
$$n_1 \tan \theta_i = n_2 \tan \theta_t$$

$$\frac{n_1}{s} = \frac{n_2}{s'}$$

$$\frac{s'}{s} = \frac{n_2}{n_1} = \frac{1}{1.33} \approx 0.752$$

$$s' = 0.752 \cdot s$$

Critical angle



$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

$$\sin \theta_r = \frac{n_1}{n_2} \sin \theta_i$$

$$\theta_r = \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_i \right)$$

What θ_i causes θ_r to be 90°

$$\underbrace{\sin \theta_r}_{= 1} = \frac{n_1}{n_2} \sin \theta_i$$

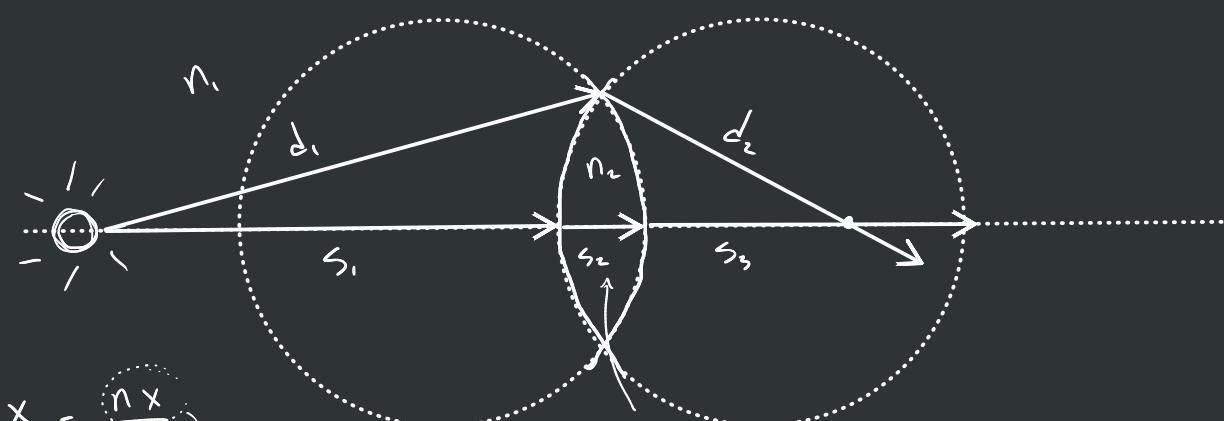
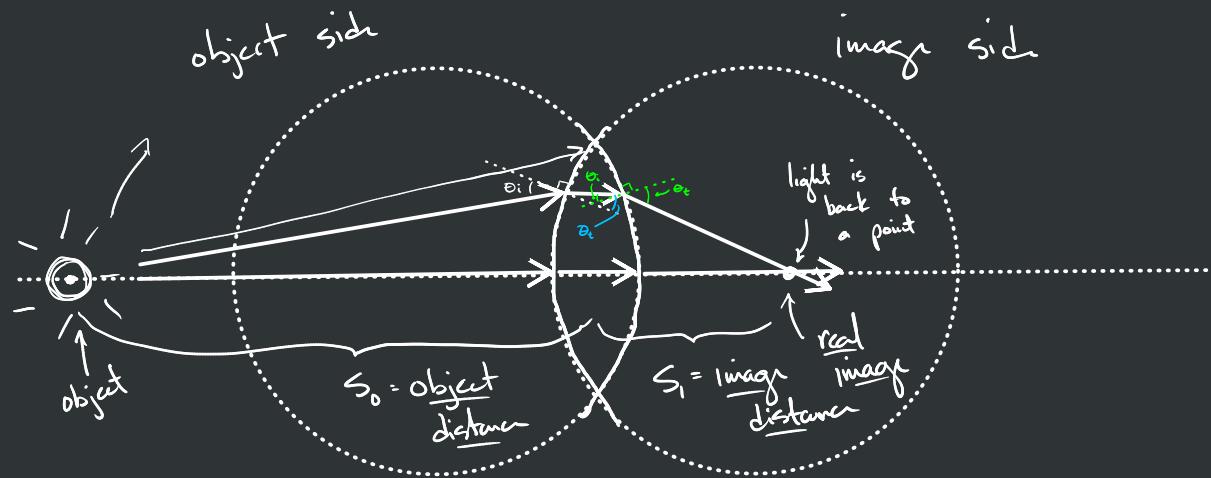
$\underbrace{\text{when this is } \geq 1}$

$$1 = \frac{n_1}{n_2} \sin \theta_i$$

Critical angle $\rightarrow \theta_i = \theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$

$\frac{n_2}{n_1} < 1$

HW: Chapter 4: 6, 7, 8, 21, 24



$$t = \frac{x}{v} = \frac{nx}{c}$$

$v = \frac{c}{n}$

optical path length

$$\frac{n_1 d_1}{c} + \frac{n_2 d_2}{c} = \frac{n_1 s_1}{c} + \frac{n_2 s_2}{c} + \frac{n_3 s_3}{c}$$

$$n_1 d_1 + n_2 d_2 = n_1 s_1 + n_2 s_2 + n_3 s_3$$

} optical path lengths
are the same!

perfect images

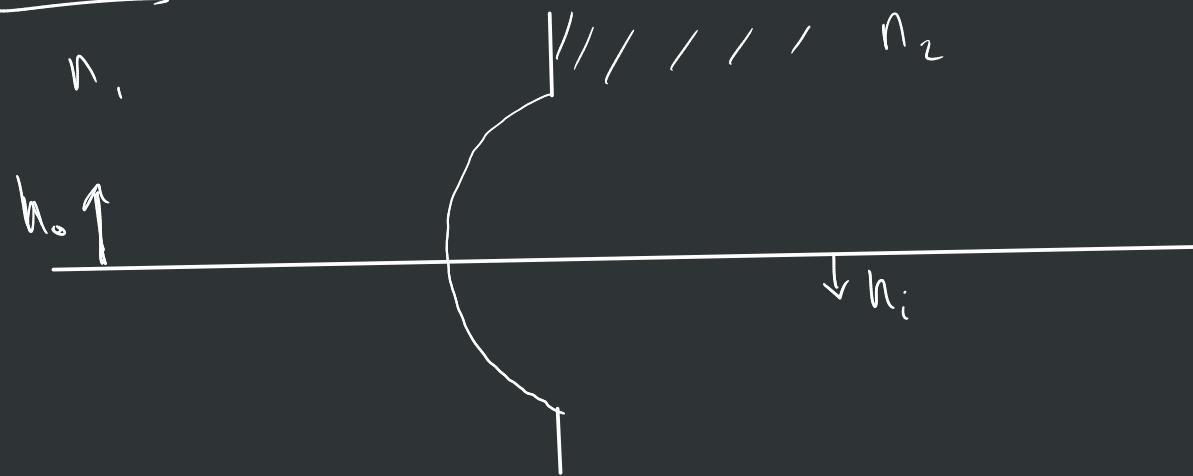
reflection

- ellips
- hyperboloid
- parabola

refraction

- ellips
- hyperboloid
- cartesian oval

Refraction



Thin lens

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

↑
object
distance ↓
image
distance

thin lens (Lens Makers Equation)

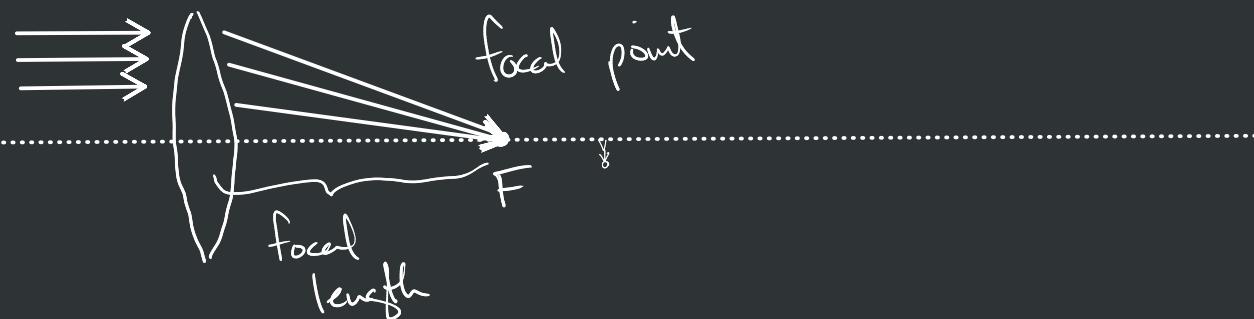
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

focal
length →
 (Thin Lens Equation)

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

infinity far away



	+	-
s_o	real object	virtual object
s_i	real image	virtual image
f	converging lens	diverging lens
y_o	upright object	inverted object
y_i	upright image	inverted image

Optics Lab:

Conjugate Points: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

Focal length Exp: ~~$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$~~ $\frac{1}{s_i} = \frac{1}{f} \Rightarrow s_i = f \quad \text{for } s_o \rightarrow \infty$

Diverging Lens: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{-f}$

$$\frac{1}{s_i} = \frac{1}{-f} - \frac{1}{s_o}$$

$$s_i = \frac{1}{\frac{1}{-f} - \frac{1}{s_o}}$$

Finding a virtual image:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\left(\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} \right) = \frac{s_o - f}{fs_o}$$

$$s_i = \frac{s_o \cdot f}{s_o - f}$$

NINE: $s_o = 27 \text{ cm}$

$f = -15 \text{ cm}$

$$s_i = \frac{27(-15)}{27 + 15} = -9.60 \text{ cm}$$

Now for the check:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$s_i = 16.2 \text{ cm}$$

$$f = 10 \text{ cm}$$

$$s_o = 26.13$$

$$s_o = \frac{s_i \cdot f}{s_i - f}$$

$$s_o = \frac{16.2 \cdot 10}{16.2 - 10}$$

$$s_o = 26.13$$

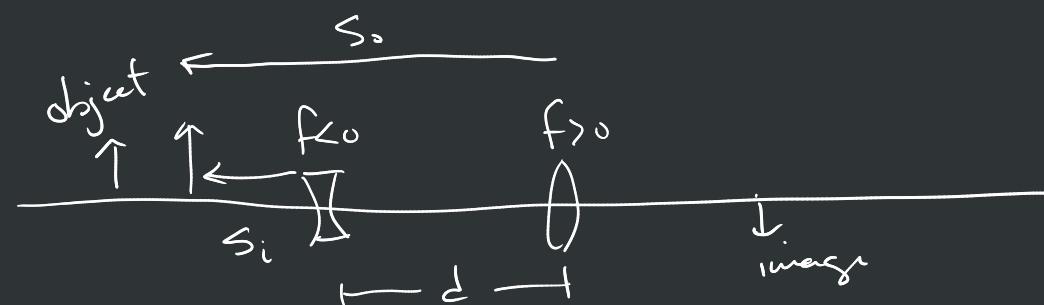
s_o now we need to compare

$$s_o = d - s_i$$

$$26.13 = d - (-9.6)$$

$$16.53 = d$$

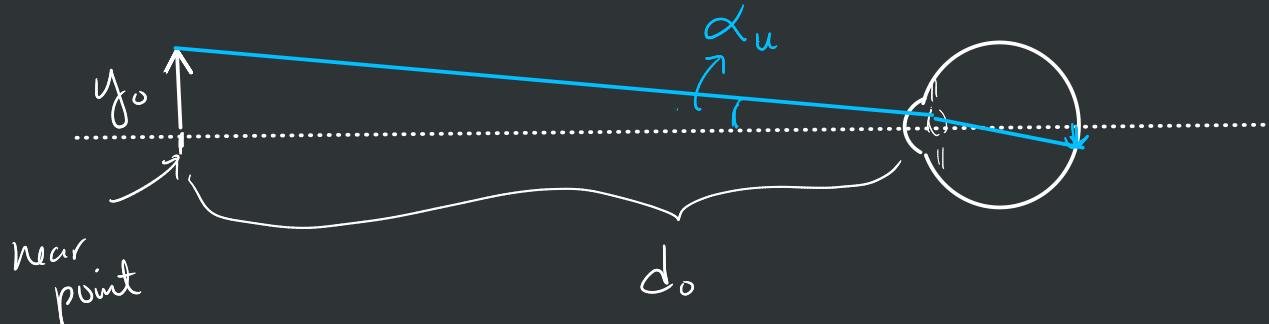
$\nwarrow 17.3 \underline{\text{measured}}$



Optical Devices

Magnifying Glass

Unaided Eye

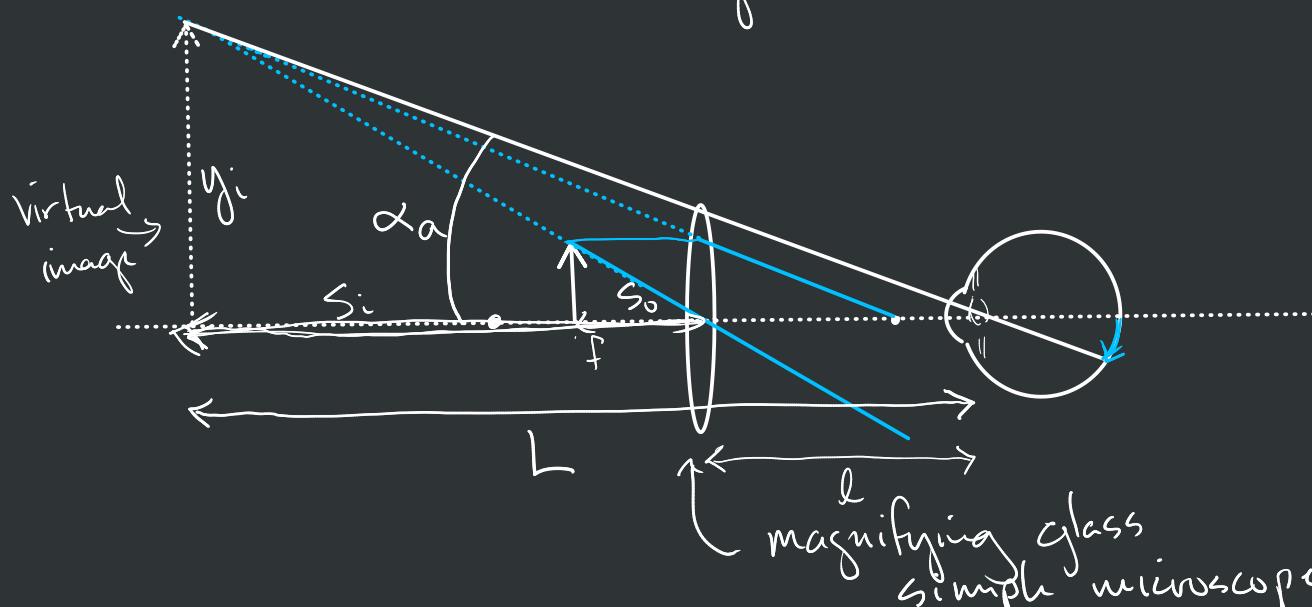


angular magnification
↓
magnifying power ↓
 $M_P = M_A = \frac{\alpha_a}{\alpha_u}$

radians ↓

$$\tan \alpha_u = \frac{y_o}{d_o} \approx \sin \alpha_u \approx \alpha_u$$

Aided Eye



$$\tan \alpha_a = \frac{y_i}{L} \approx \alpha_a$$

$$MP = \frac{y_i}{L} \cdot \frac{d_o}{y_o}$$

↑ this is positive

$$M_t = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

$$57.3^\circ \leftarrow 1 \text{ rad} \cdot \frac{180}{\pi} = 57.3^\circ$$

$$\tan 1 \text{ rad} = 1.557$$

$$\tan 0.1 \text{ rad} = 0.10033$$

$$\hookrightarrow 5.73^\circ$$

$$MP = -\frac{s_i}{s_o} \cdot \frac{d_o}{L}$$

$$MP = \left(1 - \frac{s_i}{f}\right) \frac{d_o}{L}$$

$$s_i = -(L - l)$$

$$MP = \left(1 + \frac{L - l}{f}\right) \cdot \frac{d_o}{L}$$

D

D

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$s_i \left[\frac{1}{s_o} = \frac{1}{f} - \frac{1}{s_i} \right] s_i$$

$$\frac{s_i}{s_o} = \frac{s_i}{f} - 1$$

$$-\frac{s_i}{s_o} = \boxed{\left|1 - \frac{s_i}{f}\right|}$$

$$\frac{1}{f} = D \leftarrow \text{dioptric power}$$

$$\boxed{MP = \left(1 + (L - l)D\right) \cdot \frac{d_o}{L}} \text{ eq. 5.76}$$

Case 1: $l = f$

$$\begin{aligned} [MP]_{l=f} &= \left(1 + (L - l) \frac{l}{f}\right) \cdot \frac{d_0}{L} \\ &= \left(\cancel{X} + \cancel{\frac{X}{f}} - \cancel{\frac{l}{f}}\right) \cdot \frac{d_0}{\cancel{X}} \end{aligned}$$

$$[MP]_{l=f} = \frac{d_0}{f} = d_0 D$$

Case 2: $l = 0$

$$\begin{aligned} [MP]_{l=0} &= \left(1 + (L - \cancel{l}) D\right) \cdot \frac{d_0}{L} \\ &= \left(1 + L D\right) \frac{d_0}{L} \end{aligned}$$

$$[MP]_{l=0} = \left(\frac{1}{L} + D\right) \cdot d_0$$

if we shrink L to d_0 ,

$$[MP]_{l=0} = \left(\frac{1}{d_0} + D\right) \cdot d_0$$

$$\begin{array}{c} [MP]_{l=0} \\ L=d_0 \end{array} = 1 + D \cdot d_0$$

Case 3: We put the object at the focal point, $s_o = f$.
 The image is formed

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{f} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{s_i} = 0$$

$$s_i \rightarrow \infty$$

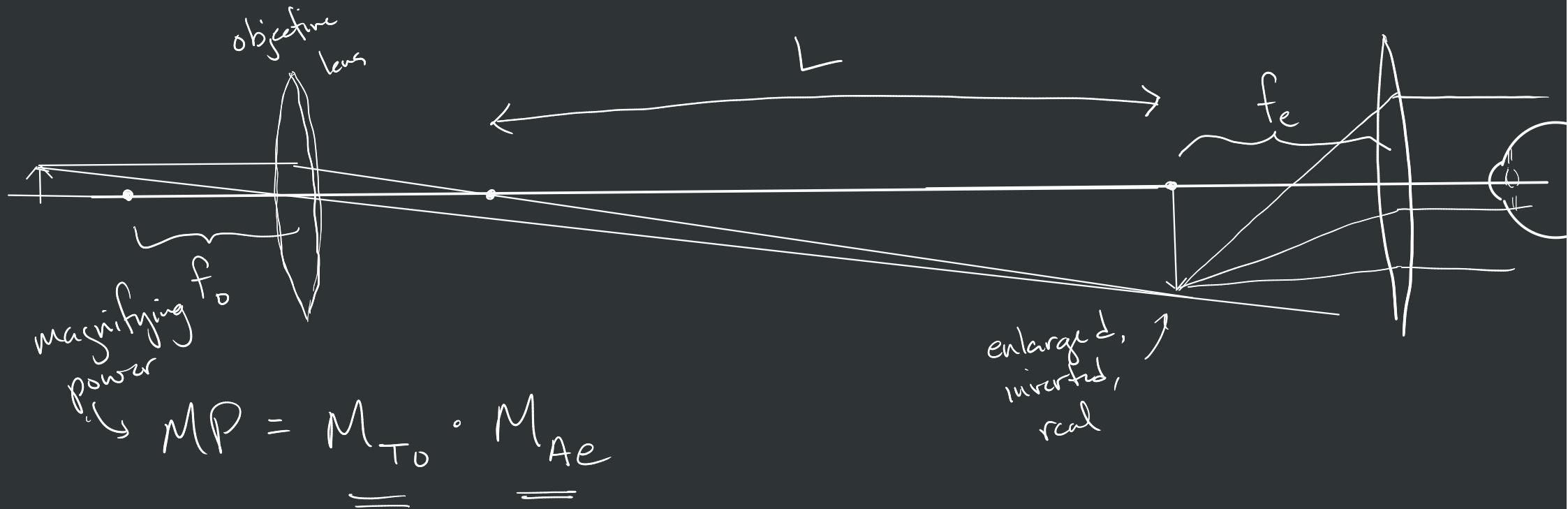
$$MP = \left(1 + (L - l)\mathcal{D}\right) \cdot \frac{d_o}{L}$$

$$[MP]_{L \rightarrow \infty} = \frac{d_o^0}{L} + \cancel{\frac{L \cdot \mathcal{D} \cdot d_o}{L}} - \cancel{\frac{l \cdot \mathcal{D} \cdot d_o}{L}}$$

$$[MP]_{L \rightarrow \infty} = \mathcal{D} \cdot d_o = \frac{d_o}{f}$$

HW: ch 5. 25, 34, 42

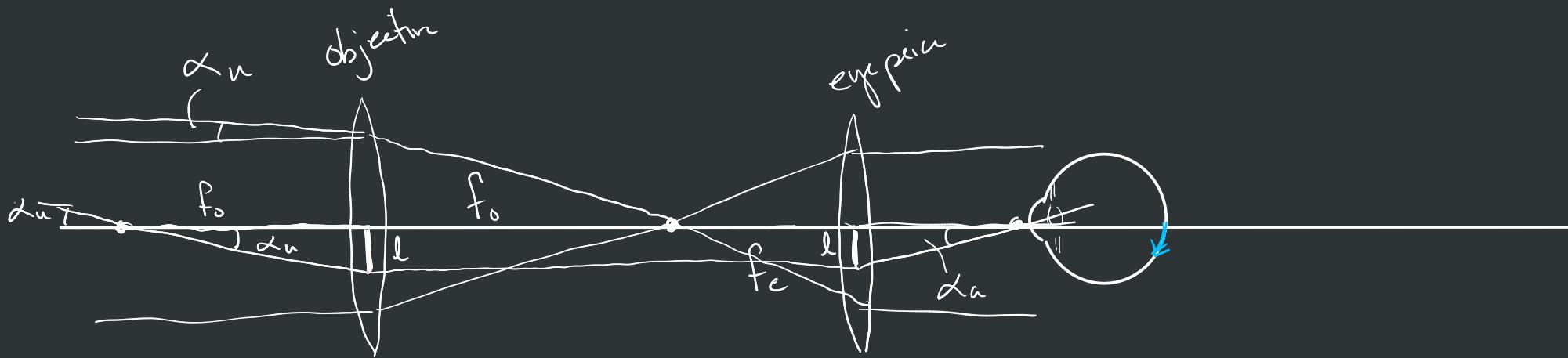
Microscope



Objective lens usually has a very small focal length.

Eyepiece has a larger focal length

Telescope



$$MP = \frac{\alpha_a}{\alpha_o} \quad \alpha_o \approx \tan \alpha_o = \frac{l}{f_o}$$

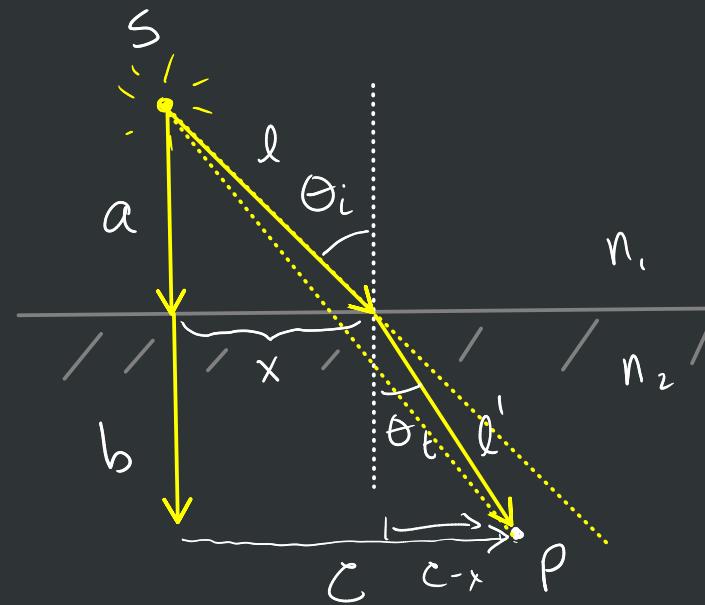
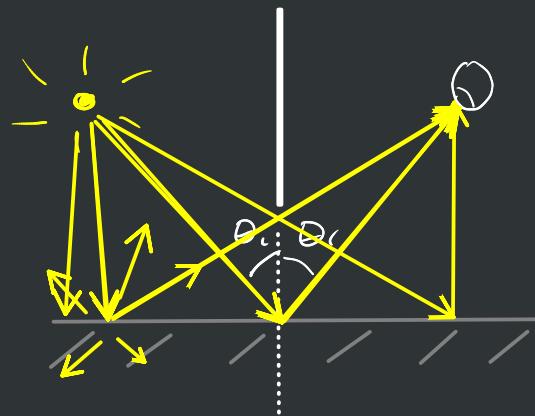
$$\alpha_a \approx \tan \alpha_a = \frac{l}{f_e}$$

$$MP = \frac{\frac{l}{f_e}}{\frac{l}{f_o}}$$

\rightarrow so we want long focal length objectives
 \rightarrow we want small focal length eyepieces

Temporarily back in Chapter 4

Law of Refraction : $n_1 \sin \theta_i = n_2 \sin \theta_t$



write an expression for the time:

$$t = \frac{l}{v_i} + \frac{l'}{v_t}$$

$$t = \frac{(a^2 + x^2)^{1/2}}{v_i} + \frac{(b^2 + (c-x)^2)^{1/2}}{v_t}$$

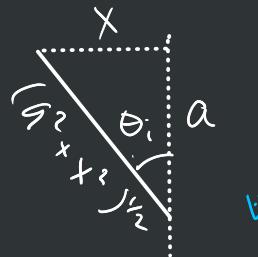
Fermat's principle

→ minimize t wrt. x

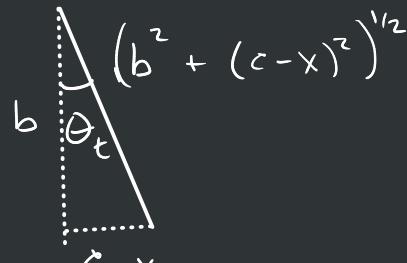
$$\frac{dt}{dx} = \frac{1}{2} \frac{(a^2 + x^2)^{-1/2}}{\sqrt{i}} \cdot 2x + \frac{1}{2} \frac{(b^2 + (c-x)^2)^{-1/2}}{\sqrt{t}} \cdot 2(c-x) \cdot (-1)$$

$$\frac{dt}{dx} = \frac{x}{\sqrt{i} (a^2 + x^2)^{1/2}} - \frac{(c-x)}{\sqrt{t} (b^2 + (c-x)^2)^{1/2}} = 0 \quad \text{minimize}$$

$$\frac{x}{\sqrt{i} (a^2 + x^2)^{1/2}} = \frac{(c-x)}{\sqrt{t} (b^2 + (c-x)^2)^{1/2}}$$



$$\sin \theta_i = \frac{x}{(a^2 + x^2)^{1/2}}$$



$$\sin \theta_t = \frac{c-x}{(b^2 + (c-x)^2)^{1/2}}$$

$$PV = N k_B T \quad \leftarrow \text{Ideal Gas Law}$$

$\frac{N}{V} = \frac{P}{k_B T} \leftarrow$ inversely proportional to T

$$(n-1) \propto \frac{N}{V}$$

$$\frac{\sin \theta_i}{\sqrt{i}} = \frac{\sin \theta_t}{\sqrt{t}}$$

$$\frac{c}{\sqrt{i}} = n_1$$

$$\frac{c}{\sqrt{t}} = n_2$$

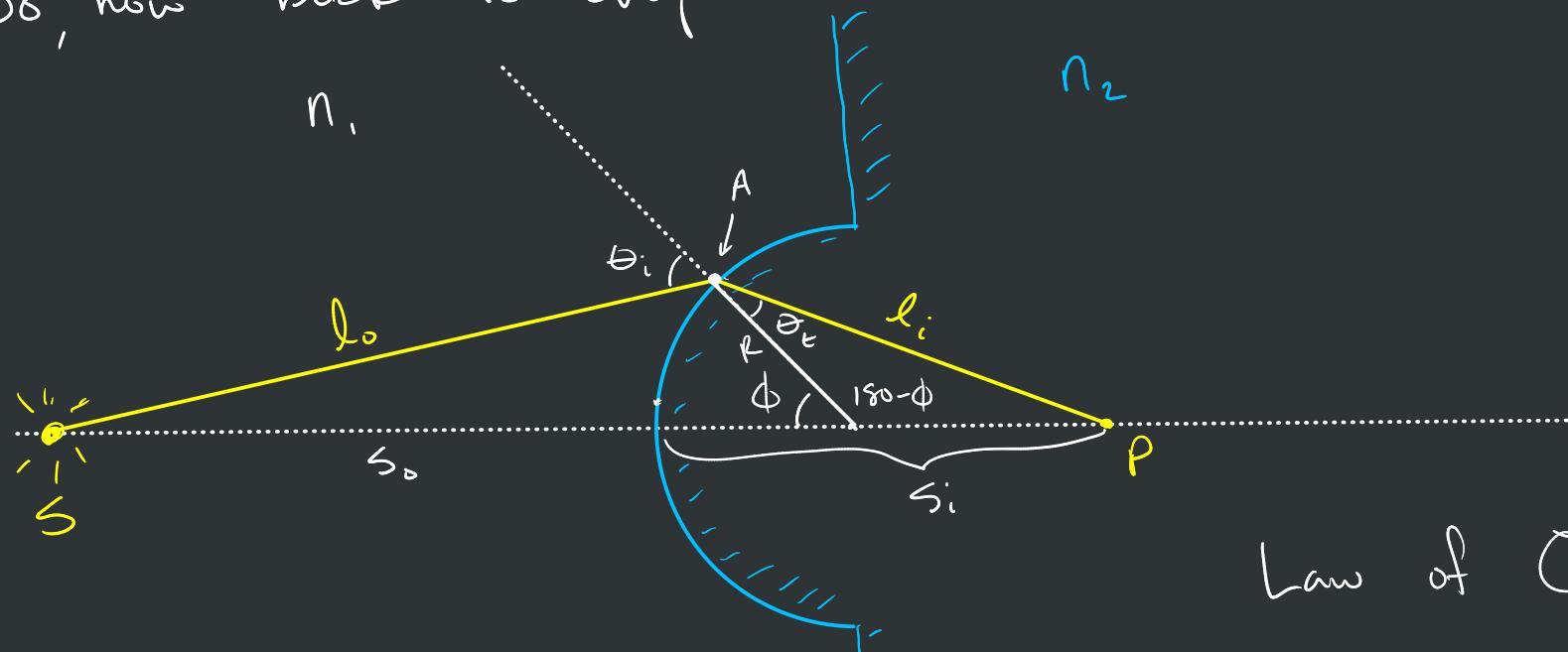
$$\frac{1}{\sqrt{i}} = \frac{n_1}{c}$$

$$\boxed{n_1 \sin \theta_i = n_2 \sin \theta_t}$$

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{\text{distance}}{\frac{c}{n}} = (\text{in. distance}) \rightarrow \frac{\text{optical path length}}{c}$$

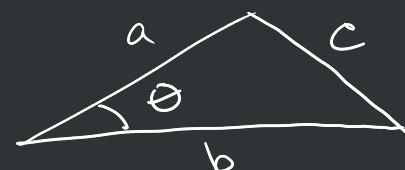
$\hookrightarrow v = \frac{c}{n}$

So, now back to chapter 5.



$$\text{OPL} = n_1 \cdot l_0 + n_2 l_1$$

Law of Cosines

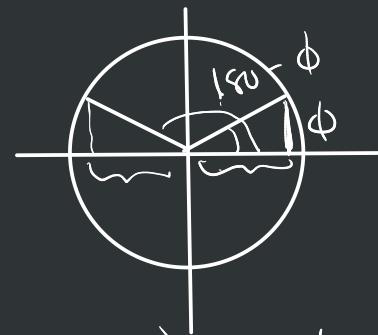


$$c^2 = a^2 + b^2 - 2ab \cos\theta$$

$$l_o = \left(R^2 + (S_o + R)^2 - 2R(S_o + R)\cos\phi \right)^{1/2}$$

$$l_i = \left(R^2 + (S_i - R)^2 - 2R(S_i - R)\cos(180 - \phi) \right)^{1/2}$$

$$= \left(R^2 + (S_i - R)^2 + 2R(S_i - R)\cos\phi \right)^{1/2}$$



$$\cos(180 - \phi) = -\cos\phi$$

$$OPL = n_1 \left(R^2 + (S_o + R)^2 - 2R(S_o + R)\cos\phi \right)^{1/2}$$

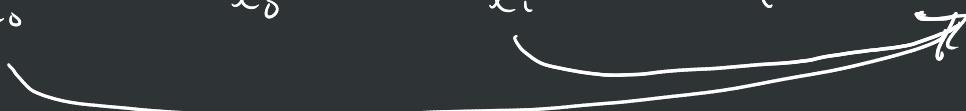
$$+ n_2 \left(R^2 + (S_i - R)^2 + 2R(S_i - R)\cos\phi \right)^{1/2}$$

$$\frac{d(OPL)}{d\phi} = 0 \quad \leftarrow \text{Invoke Fermat's Principle}$$

$$D = \frac{1}{2} \cdot n_1 \underbrace{\left(\dots \right)^{1/2}}_{\frac{1}{l_o}} \left(-2R(S_o + R)(-\sin\phi) \right) + n_2 \frac{1}{2} \underbrace{\left(\dots \right)^{1/2}}_{\frac{1}{l_i}} \left(2R(S_i - R)(-\sin\phi) \right)$$

$$\frac{n_1(s_o + R)}{l_o} - \frac{n_2(s_i - R)}{l_i} = 0$$

$$\frac{n_1 s_o}{l_o} + \frac{n_1 R}{l_o} - \frac{n_2 s_i}{l_i} + \frac{n_2 R}{l_i} = 0$$



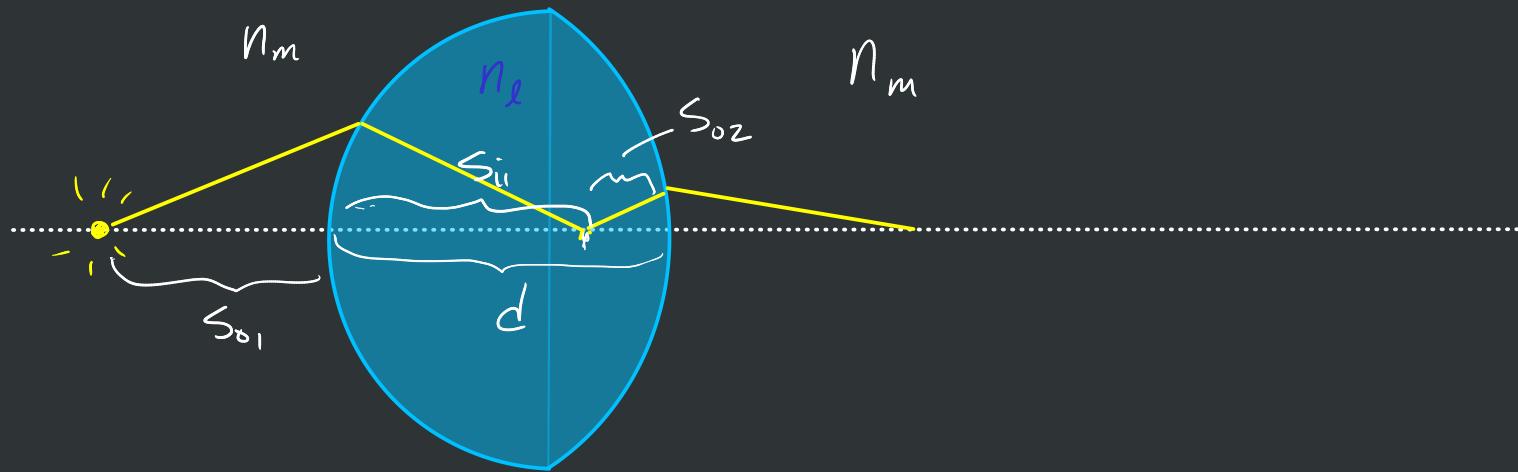
$$\frac{n_1 R}{l_o} + \frac{n_2 R}{l_i} = \frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}$$

$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R} \left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o} \right)$$

for small angles $\phi \approx 0$, $l_i \approx s_i$, $l_o \approx s_o$, $\cos\phi \approx 1$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{1}{R} (n_2 - n_1)$$

This approximation is known first-order, paraxial, and Gaussian.



$$\frac{n_m}{S_{o1}} + \frac{n_e}{S_{i1}} = \frac{n_e - n_m}{R_1} \quad \left. \right\}$$

- $S_{o2} = d - S_{i1}$

ADD TOGETHER

$$\frac{n_e}{S_{o2}} + \frac{n_m}{S_{i2}} = \frac{n_m - n_e}{R_2}$$

$$\frac{n_e}{d - S_{i1}} + \frac{n_m}{S_{i2}} = \frac{n_m - n_e}{R_2} \quad \left. \right\}$$

$$\frac{n_m}{s_{o1}} + \frac{n_e}{s_{i1}} + \frac{n_e}{d-s_{i1}} + \frac{n_m}{s_{i2}} = \frac{n_e - n_m}{R_1} + \frac{n_m - n_e}{R_2}$$

↓ some algebra

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_e - n_m) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{n_e \cdot d}{(s_{i1} - d)} s_{i1}$$

for a "thin lens" $d \rightarrow 0$

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_e - n_m) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

for

for $n_m = 1$

$$\frac{1}{S_o} + \frac{1}{S_i} = (n_e - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

} Thin Lens Equation

$$\frac{1}{f} = (n_e - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

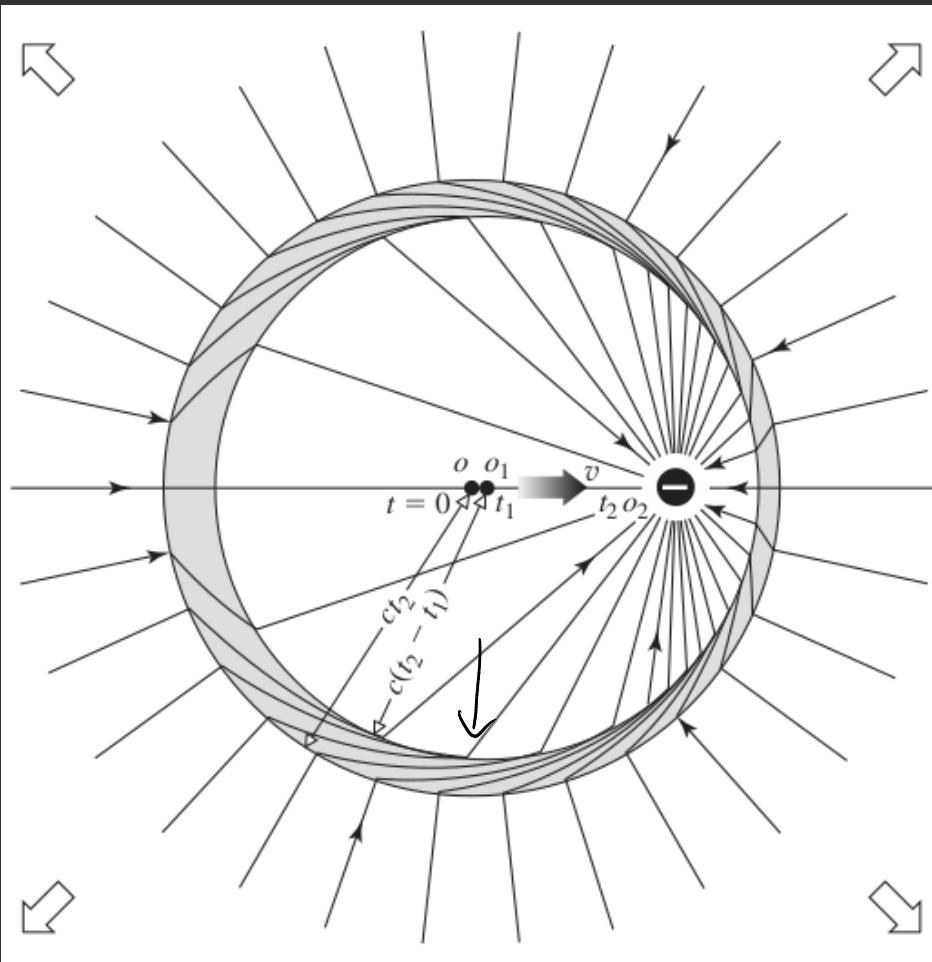
$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$M_t = \frac{h'}{h} = -\frac{S_i}{S_o}$$

HW: watch those videos AND do chapter 5: 40, 43,

What is light? A wave of the electric field.

↳ Light originates from accelerating charges.

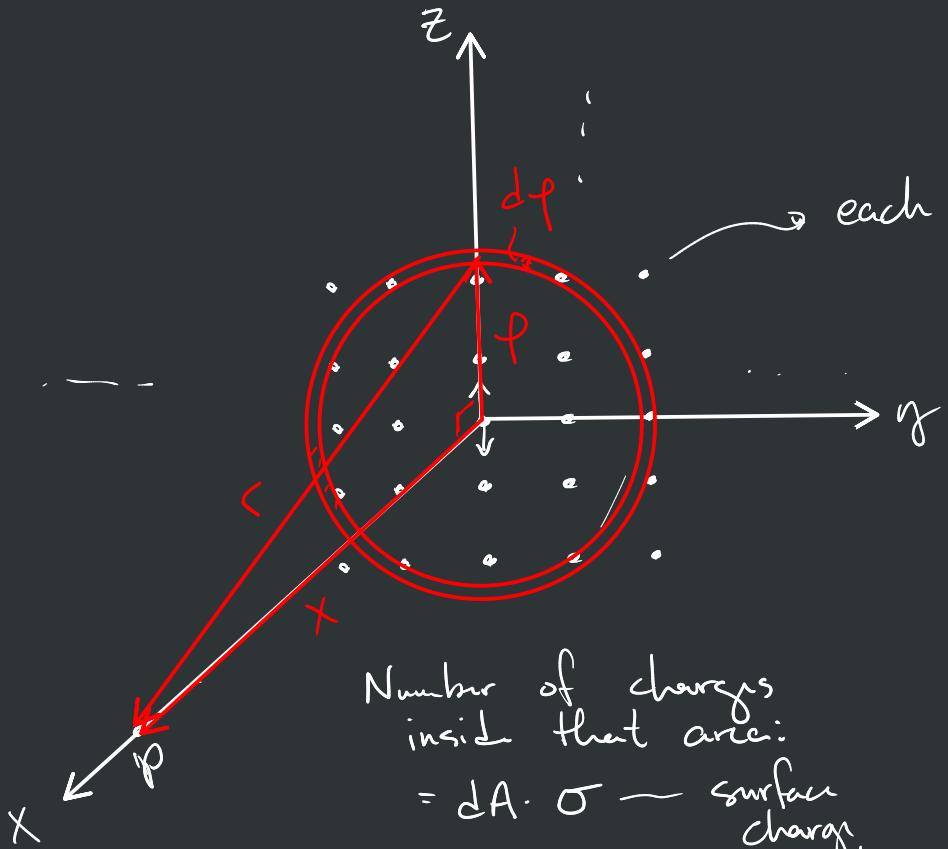


$$\vec{E} = \frac{-q}{4\pi G_0} \left[\frac{\hat{e}_r}{r'^2} + \frac{r'}{c} \frac{d}{dt} \left(\frac{\hat{e}_{r'}}{r'^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \hat{e}_{r'} \right]$$

$$\vec{E} = \frac{-q}{4\pi G_0 c^2} \frac{d^2 \hat{e}_{r'}}{dt^2}$$

$$E_z(t) = \frac{-q}{4\pi G_0 c^2 r} a_z \left(t - \frac{r}{c} \right)$$

HW: read about spherical mirrors, then work 5.71 + 5.74



Number of charges
inside that area:

$$= dA \cdot \sigma - \text{surface charge density}$$

$$= 2\pi f df \cdot \sigma$$

$$\sigma = \frac{Q}{A}$$

radiation from each charge will
be proportional to $-w^2 z_0 \cos wt$
 $-w^2 z_0 e^{iwt}$

Field at point P will be proportional
to $-w^2 z_0 e^{i\omega(t-\frac{r}{c})}$

$$\frac{q}{4\pi\epsilon_0 c^2} \cdot \frac{\omega^2 z_0 e^{i\omega(t-\frac{r}{c})}}{r}$$

$$\text{Total Field at pt. P} = \int_0^\infty \frac{q}{4\pi\epsilon_0 c^2} \frac{\omega^2 z_0 e^{i\omega(t-\frac{r}{c})}}{r} \cdot 2\pi f \cdot \sigma \cdot df$$

$$= \frac{q}{4\pi\epsilon_0 c^2} \frac{\omega^2 z_0 2\pi \cdot \sigma e^{iwt}}{2} \underbrace{\int_0^\infty \frac{e^{-i\frac{\omega}{c}r}}{r} \cdot f df}_{\rightarrow r^2 = f^2 + x^2}$$

When $f \rightarrow \infty$

$r \rightarrow \infty$

$$\rightarrow r^2 = f^2 + x^2$$

$$2rdr = 2f df$$

When $f \rightarrow 0$
 $f \rightarrow x$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\int_{r=x}^{r=\infty} \frac{e^{-i\omega/c} r}{r} \cdot r dr$$

$$\int_{r=x}^{r=\infty} e^{-i\omega/c} r dr$$

$$-\frac{c}{i\omega} e^{-i\omega/c} r \Big|_x^\infty$$

$$-\frac{c}{i\omega} \left[e^{-i\infty} - e^{-i\omega/c x} \right]$$

- $\underbrace{\text{not } 0}$, oscillates around 0
so make it 0.
- if σ fades as $r \rightarrow \infty$
then, it is also 0.
- if we keep track of all components of E , then it is 0

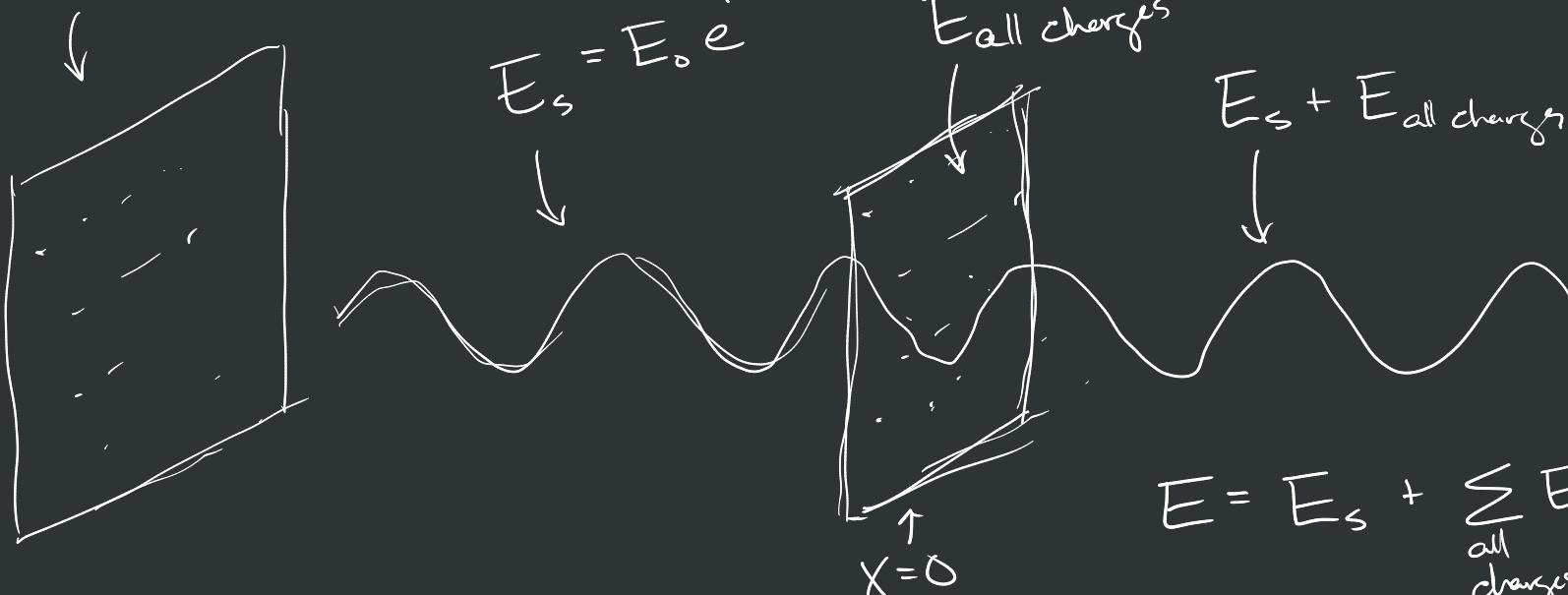
$$\text{Total field at } P = \frac{q}{2\epsilon_0 c^2} \omega^2 z_0 2\pi \cdot \sigma e^{i\omega t} \cdot \left(\frac{c}{i\omega} e^{-\frac{i\omega}{c} x} \right)$$

$$\frac{1}{i} \cdot \frac{i}{i} = -i$$

$$E_s = -\frac{q \omega z_0 \sigma}{2\epsilon_0 c} i \cdot e^{i\omega(t-\frac{x}{c})}$$

$$\cos \omega t - \frac{x}{c} + i \sin$$

Source



$$E = E_s + \sum_{\text{all charges}} E_{\text{each charge}}$$

assume these do not affect each other

If the plate has a thickness of Δx

$$t_{\text{through}} = \frac{\Delta x}{v} = \frac{\Delta x}{c_n} = \frac{n \cdot \Delta x}{c}$$

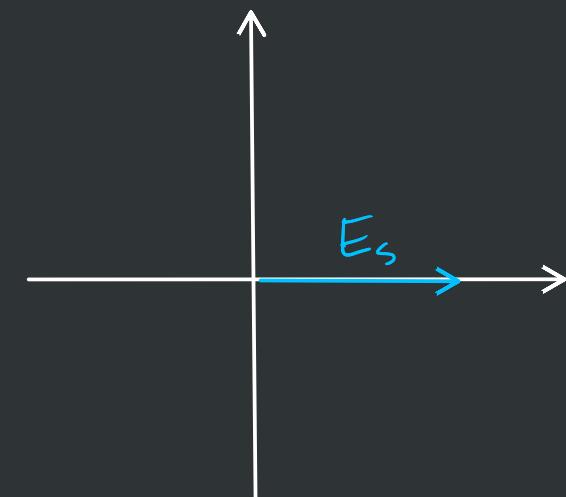
w/o the plate $t = \frac{\Delta x}{c}$

So the difference $\Delta t = \frac{n \cdot \Delta x}{c} - \frac{\Delta x}{c}$

$$\Delta t = (n-1) \frac{\Delta x}{c}$$

So the field after the plate is:

$$\begin{aligned} E_{\text{after plate}} &= E_0 e^{i\omega \left[t - (n-1) \frac{\Delta x}{c} - \frac{x}{c} \right]} \\ &= e^{-i\omega(n-1)\frac{\Delta x}{c}} \cdot E_0 e^{i\omega(t - \frac{x}{c})} \\ &\quad \underbrace{\qquad}_{\substack{\text{change of} \\ \text{phase of}}} \cdot \underbrace{E_0 e^{i\omega(t - \frac{x}{c})}}_{\substack{\text{original wave} = E_S}} \end{aligned}$$



This is what the delay means

Now let's make an approximation

$$e^x \approx 1 + x \quad \text{for small } x$$

$$\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ &\approx 1 + i\theta \end{aligned}$$

$$e^{-i\omega(n-1)\frac{\Delta x}{c}} \approx 1 - i\omega(n-1)\frac{\Delta x}{c}$$

$\brace{ \text{use this instead}}$

$$E_{\text{after plate}} = E_0 e^{i\omega(t-\frac{x}{c})} - i\omega(n-1)\frac{\Delta x}{c} E_0 e^{i\omega(t-\frac{x}{c})}$$

$\brace{E_s} \quad \brace{E_a \leftarrow \text{all charges}}$

In the material (which is very thin)

$$E_s = E_0 e^{i\omega(t-\frac{x}{c})}$$

at the plate $x=0$

$$E_s = E_0 e^{i\omega t}$$

Each electron will experience a force of $q_e E_s$, but from inside we will model the force from a nucleus as a SPRING

$$m\ddot{z} = -kz + \underbrace{(F_{\text{ext}})}_{\rightarrow qE} = q_e E_0 e^{i\omega t}$$

$$\rightarrow \ddot{z} = -\frac{k}{m}z + \underbrace{\frac{q_e E_0}{m} \cdot e^{i\omega t}}_{\omega^2}$$

$$\ddot{z} = -\omega^2 z + c\dot{z}$$

$$\dot{z} = f_y = v \leftarrow \text{in code}$$

Solution to this : $z(t) = \underbrace{\frac{q_e E_0}{m(\omega_0^2 - \omega^2)} \cdot e^{i\omega t}}_{Z_0} + \dots$

$$E = - \frac{g_f \omega z_0 \sigma_i}{2 \epsilon_0 C} e^{i\omega(t-\frac{x}{c})}$$

$$= - \frac{\sigma_f g_c}{2 \epsilon_0 C} \cdot i\omega z_0 e^{i\omega(t-\frac{x}{c})}$$

$$E = - \frac{\sigma_f g_c}{2 \epsilon_0 C} \cdot i\omega \frac{g_c E_0}{m(\omega_0^2 - \omega^2)} \cdot e^{i\omega(t-\frac{x}{c})}$$

Compare this to our hypothesized function above.

a wave that travels to the right

$$-\cancel{i\omega(n-1)\frac{\Delta x}{\epsilon}E_0} = -\frac{\sigma g_{fe}}{2\epsilon_0 c} \cdot i\omega \frac{g_{fe}E_0}{m(\omega_0^2 - \omega^2)}.$$

Hecht (Feynman)

$$(n-1)\frac{\Delta x}{\epsilon} = \frac{\sigma g_{fe}^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)}$$

$$\frac{\sigma}{\Delta x} = ? \quad \sigma = \frac{Q}{A} \Rightarrow \frac{\sigma}{\Delta x} = \frac{Q}{A \cdot \Delta x} = \frac{Q}{V} = N$$

volume charge density

$$n = 1 + \frac{N g_{fe}^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)}$$

index of refraction for thin
materials \hookrightarrow low density

$$n = \frac{v}{c}$$

~~C = v~~

$$C = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Ns^2}{C^2}$$

- vacuum permittivity
- permittivity of free space

what about v ?

$$C = K \cdot \epsilon_0 \cdot A \\ \downarrow \frac{d}{d}$$

$K = K_E \rightarrow$ relative permittivity

$$\epsilon = K_E \cdot \epsilon_0$$

\hookrightarrow permittivity
of a material

$$\frac{\epsilon}{\epsilon_0} = K_E$$

$K_m \leftarrow$ Hecht

$$\mu = K_B \cdot \mu_0$$

$$\frac{\mu}{\mu_0} = K_B \leftarrow$$

$$v = \sqrt{\frac{1}{\epsilon \mu}}$$

$$n = \frac{C}{V} = \frac{\sqrt{\frac{1}{\epsilon_0 \mu_0}}}{\sqrt{\frac{1}{\epsilon \mu}}} = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} = \sqrt{K_E K_B} = n$$

\vec{P} → polarization → way of talking about the material response to an external electric field

$$\vec{P} = \underbrace{(\epsilon - \epsilon_0)}_{\text{approximately true for most materials}} \vec{E}$$

$$K_E \approx 80$$

until $\nu = 10^{10} \text{ Hz}$

$$K_E \ll 80$$

$$\epsilon = K_E \epsilon_0$$

$$\vec{P} = (K_E - 1) \epsilon_0 \vec{E}$$

The dipole moment, $\vec{p} = q_e \Delta z$ \rightarrow distance that q is separated from "home".

So the polarization $\vec{P} = \vec{p} \cdot N$

$$\vec{P} = q_e \Delta z \cdot N$$

$\rightarrow \frac{q_e E_0}{m(\omega_0^2 - \omega^2)}$

$$P = (\kappa_E - 1) \epsilon_0 E_0 = \frac{q_e^2 E_0}{m(\omega_0^2 - \omega^2)} N$$

$$\kappa_E = 1 + \frac{q_e^2 N}{m \epsilon_0 (\omega_0^2 - \omega^2)}$$

$$N = \sqrt{\kappa_E K_B}$$

$\rightarrow K_B$ frequently 1

$$n^2 = k_E = 1 + \frac{g e^2 N}{m \epsilon_0 (\omega_0^2 - \omega^2)}$$

$$n = \left(1 + \frac{g e^2 N}{m \epsilon_0 (\omega_0^2 - \omega^2)} \right)^{1/2}$$

How different is this from the previous expression?

$$(1+x)^n \approx 1 + nx$$

Also for HW: 3: 62, 67

What about damping?

$$\ddot{z} = -\omega_0^2 z + \gamma \dot{z} + \frac{q_e E_0}{m} e^{i\omega t}$$

$$z(t) = \frac{q_e E_0}{m(\omega_0^2 - \omega^2 + \gamma i\omega)} e^{i\omega t}$$

$$\eta^2 = 1 + \frac{N q_e}{E_0 M} \cdot \frac{1}{\omega_0^2 - \omega^2 + \gamma i\omega}$$

$$\eta = \eta_R + i \eta_I$$

\downarrow absorption

A further improvement can be made w/ the following

$$\rightarrow \frac{n^2 - 1}{n^2 + 2} = \frac{N q_e^2}{\epsilon_0 M} \cdot \frac{1}{\omega_0^2 - \omega^2 + i\gamma\omega}$$

Suppose you had
the things for this:

C

$$\frac{n^2 - 1}{n^2 + 2} = C$$

$$n^2 - 1 = C n^2 + 2C$$

$$n^2(1 - C) = 2C + 1$$

$$n = \sqrt{\frac{2C + 1}{1 - C}}$$

The last extension:

$$n^2 = 1 + \frac{N q_e^2}{\epsilon_0 M_e} \sum_j \frac{f_j}{\omega_{ej}^2 - \omega^2 + i\gamma_j\omega}$$

$$\sum_i f_j = 1$$

\hookrightarrow transition probabilities

\hookrightarrow oscillator strengths

HW: 3.55, 5b, 65

What about metals? What is the index of refraction in metals?

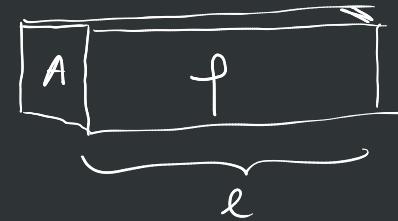
↳ What is a metal?

↳ conducting electrons which are "free" → not bound to any particular ion

$\sigma \leftarrow$ conductivity

$\rho \leftarrow$ resistivity

$$R = \rho \frac{l}{A}$$



material property $\rho = \Omega \text{ m}$

$$\sigma = \frac{1}{\rho} = \left[\frac{1}{\Omega \text{ m}} \right] = \left[\frac{\text{S}}{\text{m}} \right] \rightarrow \text{Sieman}$$

$$\vec{J} = \sigma \vec{E}$$

↳ current density

insulator

$$\sigma = 0$$

semiconductor

$$10^3 \frac{\text{S}}{\text{m}}$$

metal

$$5.8 \cdot 10^7 \frac{\text{S}}{\text{m}}$$

superconductor

$$\sigma \rightarrow \infty$$

If electrons are free to move around, then there is no restoring, no natural frequency.

$$z(t) = \frac{q_e}{m_e(\omega_0^2 - \omega^2 + i\gamma\omega)} E(t)$$

In reality, there will be a couple of terms in the index of refraction:

$$n^2 = 1 + \frac{N q_e^2}{\epsilon_0 m_e} \left[\frac{f_c}{\omega^2 + i\gamma\omega} + \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma_j\omega} \right]$$

\Rightarrow number of conduction electrons

Let's make a few simplification

$$n^2(\omega) = 1 - \left(\frac{N q_e^2}{\epsilon_0 m_e} \right) \frac{1}{\omega^2}$$

$$n^2(\omega) = 1 - \left(\frac{\omega_p}{\omega} \right)^2$$

↓ plasma frequency

$$\omega_p = \sqrt{\frac{N q_e^2}{\epsilon_0 m_e}}$$