$$f(x) = \frac{3}{10x^2+1} \longleftarrow x \to x - vt \longrightarrow f(x,t) = \frac{3}{10(x-vt)^2+1}$$

$$\psi(x,t) = f(x,t)$$

psi

$$\psi = f(x \mp vt) = f(x')$$

1st

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1} = \frac{\partial f}{\partial x'}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{= \mp v} = \mp v \cdot \frac{\partial f}{\partial x'}$$

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} &= \mp\frac{1}{v} \frac{\partial f}{\partial t} \\ \frac{\partial f}{\partial x'} &= \mp \frac{1}{v} \frac{\partial f}{\partial t} \end{aligned}}$$

2nd

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'}\right) = \frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1 \text{ again}} = \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$$\frac{\partial f}{\partial t} = \mp v \frac{\partial f}{\partial x'}$$

$$= \mp v \frac{\partial^2 f}{\partial x'^2} \cdot \mp v$$

$$= \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

$$\underbrace{\frac{\partial^2 f}{\partial x^2}} = \underline{\frac{\partial^2 f}{\partial x'^2}}$$

$$\underbrace{\mp \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}} = \underline{\frac{\partial^2 f}{\partial x'^2}}$$

$\vec{F} = \dot{\vec{p}}$

- Schrodinger's
- Heat
- Laplace

$$\rightarrow \boxed{\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

The <u>wave equation</u>

HW 1,3 so far of Chapter 2 by Wed.

Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$ (Gauss's Law) ← Electric field spread out if there is charge.

$$\vec{\nabla} \cdot \vec{B} = 0$$ (Gauss's Law for Magnetism) ← Magnetic field do not spread out.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ (Maxwell-Faraday's Law) ← There is an electric field if a magnetic field changes over time

$$\vec{\nabla} \times \vec{B} = \mu_0\left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$ (Ampere's Law) ← There is a magnetic field if there is current <u>or</u> if an electric field is changing over time

In the case of no ρ and no $\vec{J}$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\left.\begin{aligned} \vec{\nabla} \times \vec{E} &= -\frac{\partial B}{\partial t} \\ \vec{\nabla} \times \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}\right\} \text{take the curl of each}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \underbrace{(\vec{\nabla} \times \vec{B})}_{\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$= -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \underbrace{(\vec{\nabla} \cdot \vec{E})}_{=0} - \underbrace{(\vec{\nabla} \cdot \vec{\nabla}) \vec{E}}_{\vec{\nabla}^2 \vec{E}}$$

$$\frac{\partial^2 E_x}{\partial x^2} \hat{x} + \frac{\partial^2 E_y}{\partial y^2} \hat{y} + \dots$$

$$-\vec{\nabla}^2\vec{E} = -\mu_0\epsilon_0 \frac{\partial^2\vec{E}}{\partial t^2}$$

$$\underline{\underline{\vec{\nabla}^2\vec{E} = \mu_0\epsilon_0 \frac{\partial^2\vec{E}}{\partial t^2}}} \longrightarrow \frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial x^2}\hat{x} + \dots = \mu_0\epsilon_0 \frac{\partial^2 E_x}{\partial t^2} + \dots$$

So is $\epsilon_0 \cdot \mu_0 = \dfrac{1}{v^2}$? or $v = \dfrac{1}{\sqrt{\epsilon_0\mu_0}}$

$$\left.\begin{aligned} \epsilon_0 &= 8.85\cdot 10^{-12}\ \frac{s^2 C^2}{m^3 kg} \\ \mu_0 &= 4\pi\cdot 10^{-7}\ \frac{m\,kg}{C^2} \end{aligned}\right\} \quad v = \frac{1}{\sqrt{8.85\cdot 10^{-12}\cdot 4\pi\cdot 10^{-7}}} = \underline{2.99\cdot 10^{8}\ m/s}$$

Harmonic waves   HW: 2.9, 18, 22

$$\psi(x,t) = A \cdot \sin\left(k(x - vt)\right) \leftarrow \text{harmonic wave}$$

↳ repetition in space → wavelength, λ [m], [nm]

↳ repetition in time → period, T [s]

$$k\lambda = 2\pi$$

wavenumber → $k = \frac{2\pi}{\lambda} \left[\frac{rad}{m}\right]$

$$\omega \cdot T = 2\pi$$

angular frequency → $\omega = \frac{2\pi}{T}$ [rad/sec]

$$\nu = \frac{1}{T}$$

(natural) frequency, [Hz] = [s^{-1}]

$$\omega = 2\pi\nu$$

$K = \frac{1}{\lambda}$

[m^{-1}] [nm^{-1}]

kappa [cm^{-1}]

relationship b/t λ + T is v

$$v = \frac{\lambda}{T} = \nu \cdot \lambda$$

$$\rightarrow v = \frac{\omega}{k}$$

$$\psi(x,t) = A \cdot \sin\left(k(x \mp vt)\right)$$

$$\boxed{\psi(x,t) = A \cdot \sin\left(kx \mp \omega t\right)}$$

$$\psi(x,t) = A \sin\left(\frac{2\pi}{\lambda} x \mp \frac{2\pi}{T} t\right)$$

$$\psi(x,t) = A \sin\left(\frac{2\pi}{\lambda} x \mp 2\pi\nu \cdot t\right)$$

$$\psi(x,t) = A \sin 2\pi\left(\frac{x}{\lambda} \mp \nu t\right)$$

$$\psi(x,t) = A \sin 2\pi\left(k x \mp \nu t\right)$$

Superposition - multiple waves are present in the same place