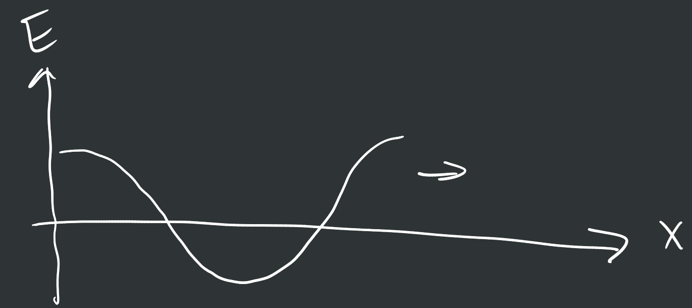
Chapter 7 - Superposition - when waves combine at the same place at the same, the displacements add together

$$\psi = \psi_1 + \psi_2$$



same frequency

$$E_1(x,t) = E_1 \cos(kx_1 - \omega t + \phi_1)$$

$$E_2(x,t) = E_2 \cos(k x_2 - \omega t + \phi_2)$$

different phase

$$\alpha_1 = kx_1 + \phi_1$$

$$\alpha_2 = kx_2 + \phi_2$$

phase difference $\rightarrow \alpha_2 - \alpha_1 = k(x_2 - x_1) + (\phi_2 - \phi_1)$

What if : $\alpha_2 - \alpha_1 = 2\pi \cdot m \} \rightarrow$ even integer of π

any integer

$$E_R = E_1 + E_2 = E_1\cos(\alpha_1 - wt) + E_2\cos(\alpha_2 - wt)$$

$$= E_1\cos(\alpha_1 - wt) + E_2\cos(2\pi\cdot m + \alpha_1 - wt)$$

$$\cos(x) = \cos(x + 2\pi\cdot m)$$

$$= \underbrace{(E_1 + E_2)}_{\text{constructive interference}}\cos(\alpha_1 - wt)$$

But, what if $\alpha_2 - \alpha_1 = (2m-1)\pi$ → odd integers of π.

any integer

$$E_t = E_1 + E_2 = E_1\cos(\alpha_1 - wt) + E_2\cos(\alpha_2 - wt)$$

$$= E_1\cos(\alpha_1 - wt) + E_2\cos(\alpha_1 + (2m+1)\pi - wt)$$

$$-\cos x = \cos(x + (2m-1)\pi)$$

destructive interference

$$= (E_1 - E_2)\cos(\alpha_1 - wt)$$

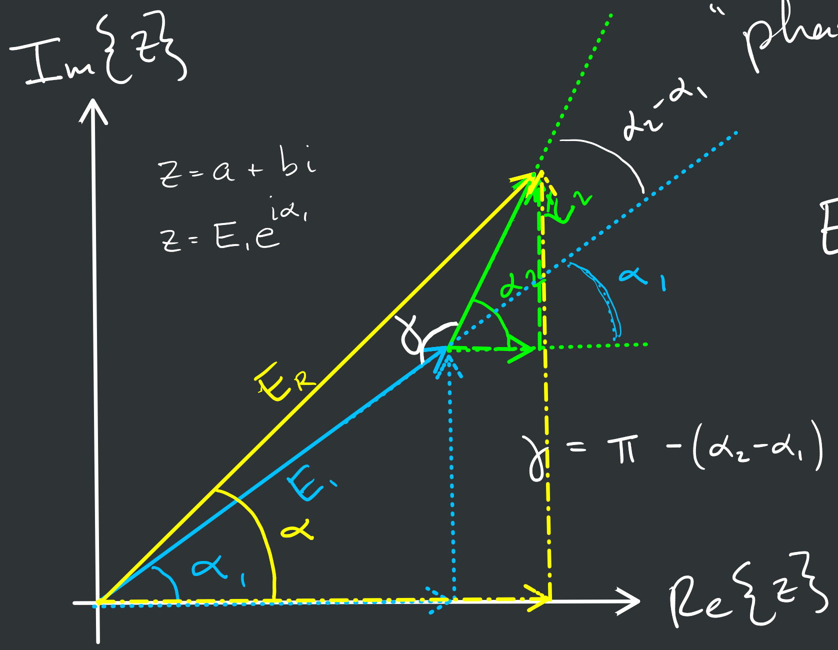
What if any other phase shift?

↳ switch from trig to complex notation

$$E_R = E_1 + E_2 = \mathrm{Re}\left\{ E_1 e^{i(\alpha_1 - \omega t)} + E_2 e^{i(\alpha_2 - \omega t)} \right\}$$

$$= \mathrm{Re}\left\{ e^{-i\omega t} \underbrace{\left(E_1 e^{i\alpha_1} + E_2 e^{i\alpha_2} \right)} \right\}$$

"phasor diagram" → treat complex number like vectors



$$E_R(x,t) = \mathrm{Re}\left\{ E_R e^{i(\alpha - \omega t)} \right\}$$

? ?

$$E_R^2 = E_1^2 + E_2^2 - 2E_1E_2 \underbrace{\cos\gamma}$$

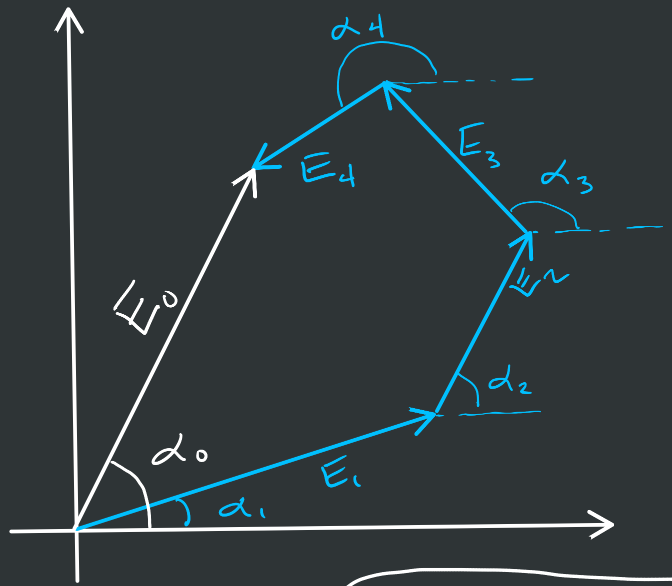
$\cos(\pi - \alpha_2 + \alpha_1)$

$-\cos(-\alpha_2 + \alpha_1)$

$-\cos(\alpha_1 - \alpha_2)$

$\rightarrow \quad E_R^2 = E_1^2 + E_2^2 + 2E_1E_2\cos(\alpha_1 - \alpha_2)$

$\rightarrow \quad \tan\alpha = \dfrac{E_1\sin\alpha_1 + E_2\sin\alpha_2}{E_1\cos\alpha_1 + E_2\cos\alpha_2}$



$$\tan\alpha_0 = \frac{\sum_{i=1}^{N} E_i\sin\alpha_i}{\sum_{i=1}^{N} E_i\cos\alpha_i}$$

$$E_0^2 = \underbrace{\left(\sum_{i=1}^{N} E_i\cos\alpha_i\right)^2}\ + \left(\sum_{i=1}^{N} E_i\sin\alpha_i\right)^2$$

$$\left(\sum_{i=1}^{N} E_i\cos\alpha_i\right)^2 = \sum_{i}^{N} E_i^2\cos^2\alpha_i + \underbrace{\sum_{i}^{N} 2E_i\cos\alpha_i \sum_{j>i}^{N} E_j\cos\alpha_j}_{2\sum_i \sum_{j>1} E_iE_j\cos\alpha_i\cos\alpha_j}$$

$(a + b + c + d)^2$

$$E_0^2 = \underbrace{\sum_i^N E_i^2 \cos^2\alpha_i + \sum_i^N E_i^2 \sin^2\alpha_i}_{\sum_i^N E_i^2 \left(\underbrace{\cos^2\alpha_i + \sin^2\alpha_i}_{1}\right)} + \underbrace{2\sum_i \sum_{j>i} E_i E_j \cos\alpha_i \cos\alpha_j + 2\sum_i \sum_{j>i} E_i E_j \sin\alpha_i \sin\alpha_j}_{2\sum_i^N \sum_{j>i}^N E_i E_j \left(\underbrace{\cos\alpha_i \cos\alpha_j + \sin\alpha_i \sin\alpha_j}_{\cos(\alpha_j - \alpha_i)}\right)}$$

$$E_0^2 = \sum_i^N E_i^2 + 2\sum_i \sum_{j>i} E_i E_j \cos(\alpha_j - \alpha_i)$$

if all sources are equal in magnitude $\rightarrow E_i = E_1$

if all sources are random phases and short durations ($< 10\,ns$)

$$\sum_i \sum_{j>i} E_i E_j \cos(\alpha_j - \cos\alpha_i) \rightarrow 0$$

$$\rightarrow E_0^2 = N E_1^2 \Rightarrow E_0 = \sqrt{N} \cdot E_1$$

$$E_0 \propto \sqrt{N}$$

irradiance $\rightarrow I = \frac{1}{2}\epsilon_0 c E_0^2$

$\boxed{I \propto N}$ experimentally verified ✓

if phases are not random, but coherent

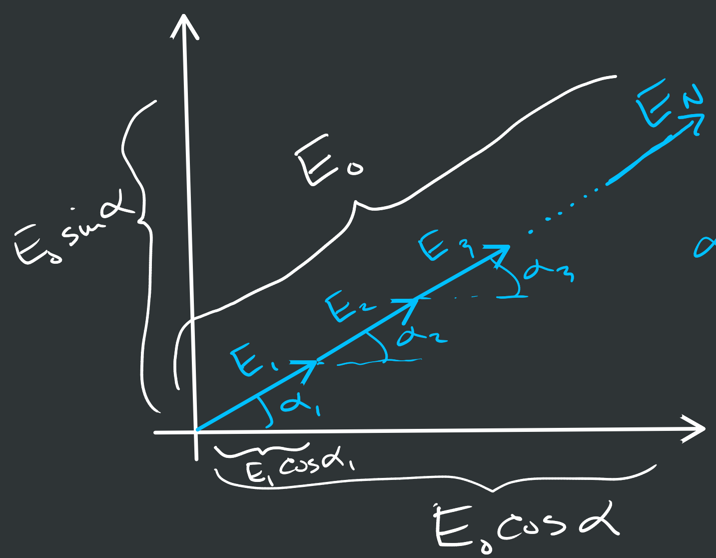
↳ same frequency + waveform

↳ same phase

$$E_0^2 = \sum_i^N E_i^2 + 2\sum_i \sum_{j>i} E_i E_j \underbrace{\cos(\alpha_j - \alpha_i)}_{\cos(0) = 1}$$

$$E_0^2 = \sum_i^N E_i^2 + 2\sum_i \sum_{j>i} E_i E_j$$

→ if sources are equal in magnitude



$\alpha_1 = \alpha_2 = \alpha_3 \cdots = \alpha$

$$E_0^2 = (NE_1 \cos\alpha)^2 + (NE_1 \sin\alpha)^2$$

$$E_0^2 = N^2 E_1^2 \underbrace{\left(\cos^2\alpha + \sin^2\alpha\right)}_{1}$$

$$E_0 = N E_1$$

$$I = \frac{1}{2}\epsilon_0 c N^2 E_1^2$$

$$I \propto N^2$$

$$\frac{I_{coherent}}{I_{random}} = \frac{N^2}{N} = N$$

Standing wave $\Rightarrow$ interference of a wave with is own reflection

as two waves: $E_1 = E_0 \sin(-kx + \omega t) \leftarrow$ to the right

$E_2 = E_0 \sin(kx + \omega t) \leftarrow$ to the left

put in a phase shift to account for the reflection at the boundary $\rightarrow -\phi_R$

$$E_R = E_0\left(\sin(\underbrace{-kx + \omega t}_{A}) + \sin(\underbrace{kx + \omega t - \phi_R}_{B})\right)$$

$$\sin A + \sin B = 2\sin\left(\tfrac{1}{2}(A+B)\right)\cos\left(\tfrac{1}{2}(A-B)\right)$$

$$E_R = E_0 \cdot 2\sin\left(\frac{1}{2}\left(-\cancel{kx}+\omega t + \cancel{kx} + \omega t - \phi_R\right)\right)\cos\left(\frac{1}{2}\left(-kx+\cancel{\omega t} - kx - \cancel{\omega t} + \phi_R\right)\right)$$

$$E_R = 2E_0 \sin\left(\omega t - \frac{\phi_R}{2}\right)\cos\left(-kx + \phi_R/2\right)$$

lets take the important case of $\phi_R = \pi$

$$E_R = 2E_0 \sin\left(\omega t - \frac{\pi}{2}\right)\cos\left(-kx + \pi/2\right)$$

$$= 2E_0\left(-\cos(\omega t)\cdot \sin(kx)\right)$$

$$E_R = \underbrace{2E_0 \cdot \sin(kx)}_{\text{spacial amplitude}}\underbrace{\cos(\omega t)}$$

variation of that amplitude as time goes by.

spacial amplitude

the places where this are equal to 0 will always be zero

positions are nodes

where do the nodes appear?

$$kx = m\pi \qquad m = 0, \pm 1, \pm 2, \ldots \rightarrow \text{is an integer}$$

$$k = \frac{2\pi}{\lambda} \qquad\qquad \omega = \frac{2\pi}{T}$$

$$\frac{2\cancel{\pi}}{\lambda} \cdot x = m\cancel{\pi}$$

$$x = m \cdot \frac{\lambda}{2}$$ ← positions where amplitude is always zero
positions of the nodes

$$x_2 - x_1 = \frac{2\lambda}{2} - \frac{1\lambda}{2}$$

$$\Delta x = \frac{\lambda}{2}$$ ← distance between adjacent nodes

$$\lambda = 2\Delta x$$

when do the maxima occure?

$$\cos(\omega t) = 1$$

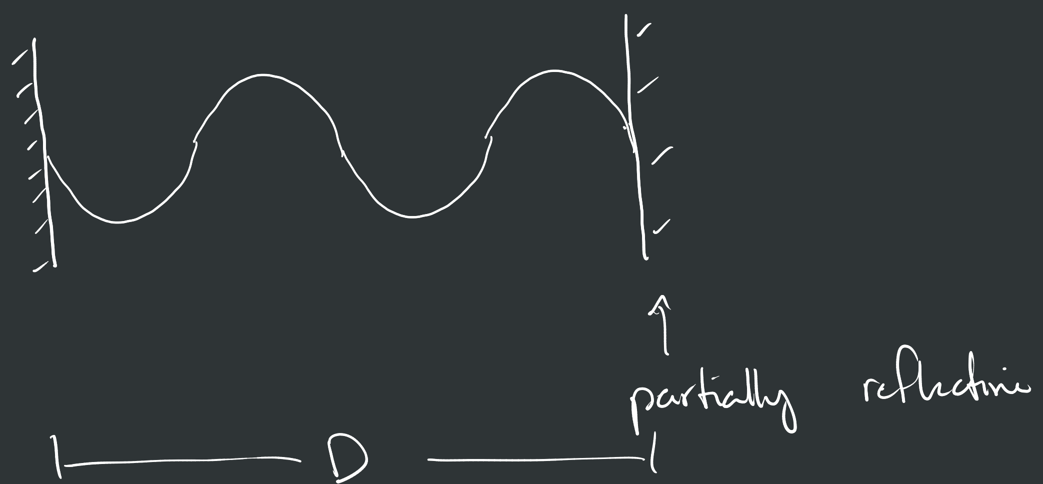
$$\omega t = 0, \pi, 2\pi, \ldots \qquad \omega t = m\pi \qquad m = 0, \pm 1, \pm 2 \rightarrow \text{integer}$$

$$\omega = \frac{2\pi}{T}$$

$$t_{max} = m\frac{T}{2}$$

$$\Delta t_{max} = \frac{T}{2}$$

How do we do this?



$$D = m\left(\frac{\lambda}{2}\right)$$

number of nodes

$m = 1, 2, 3, 4 \ldots$