Chapter 2 - Plane Waves + Index of Refraction

plane wave:

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cdot \cos(\vec{k}\cdot\vec{r} - \omega \cdot t + \phi)$$

"wave vector" "k" $\rightarrow \vec{k} = k\hat{u} = \frac{2\pi}{\lambda_{vac}}\hat{u}$

$\hat{u}$: direction of propogation

angular frequency $\rightarrow \omega = \frac{2\pi \cdot c}{\lambda_{vac}} = 2\pi \cdot \nu$

k + ω are related just like λ + ν

$$c = \frac{\lambda_{vac}}{T} = \lambda_{vac} \cdot \nu = \frac{\omega}{k}$$

this relationship is known as the "dispersion relation"

$$\frac{1}{\lambda_{vac}} = \kappa = [cm^{-1}]$$

κ: "kappa", wavenumber

$c = v_{vac} \leftarrow$ velocity of light in vacuum

ν ν $\leftarrow$ "nu"

ν: frequency

similarly for magnetic field:

$$\vec{B}(\vec{r},t) = \vec{B}_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \phi)$$

$$\vec{B}_0 = \frac{\vec{k}\times\vec{E}_0}{\omega}$$, so $\vec{B}_0$ is not independent, but is determined by other parameters.

$$\vec{B}_0 \perp \vec{k} \perp \vec{E}$$

also think about magnitude

$$B_0 = \frac{kE_0}{\omega} = \frac{E_0}{c}$$ ← since this is so large we will focus on the $\vec{E}$ field.

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complex plane waves:

$$\vec{E}(r,t) = \mathrm{Re}\left\{\tilde{\vec{E}}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right\}$$

$$\tilde{\vec{E}}_0 = \vec{E}_0 e^{i\phi}$$ ← phase shift

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

Summarize some facts that we know now:

$\lambda \to$ wavelength

$T \to$ period

$\nu \to$ frequency $\left(\frac{1}{T}\right)$

$k \to$ wave vector $\left(\frac{2\pi}{\lambda}\right)$

$\omega \to$ angular frequency $\left(\frac{2\pi}{T}\right)$

$$V = \frac{\omega}{k}$$

$$V = \frac{\lambda}{T} = \lambda \cdot \nu \leftarrow \text{any wave}$$

$$c = \lambda_{vac} \cdot \nu$$

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \Rightarrow c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

permittivity of free space $\to \epsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$

permeability of free space $\to \mu_0 = 4\pi \cdot 10^{-7} \frac{T}{Am^2}$

- $\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$
- $\nabla^2 \vec{E} = \frac{1}{V^2} \frac{\partial^2 E}{\partial t^2}$
- $\nabla^2 \vec{E} = \frac{k^2}{\omega^2} \frac{\partial^2 E}{\partial t^2}$

Q1: what is light? ✓

Q2: where does light come from? ✓

Q3: can the speed of light change? see Q6

Q4: what affects the brightness of light?

Q5: how do human perceive light amount + color

Q6: why does light slow down in materials Q3

Q7: what is reflection

Speed of Light in matter

↳ light slows down in materials

$$\frac{c}{V} = n \leftarrow \text{index of refraction}$$

$n = 1 \leftarrow$ vacuum

$n = 1.0003 \leftarrow$ air

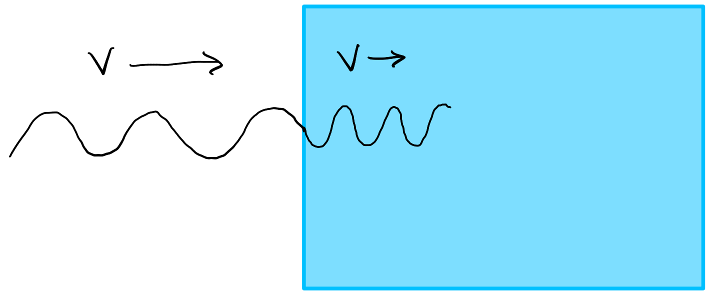
$n = 1.33 \leftarrow$ water

$n = 1.5 \leftarrow$ glass

constant as n increases

$$\lambda \cdot \nu = v = \frac{c}{n}$$

λ gets smaller as n increases



$$\lambda_{vac}\nu = c \iff c = n \cdot v$$

$\lambda \cdot \nu$

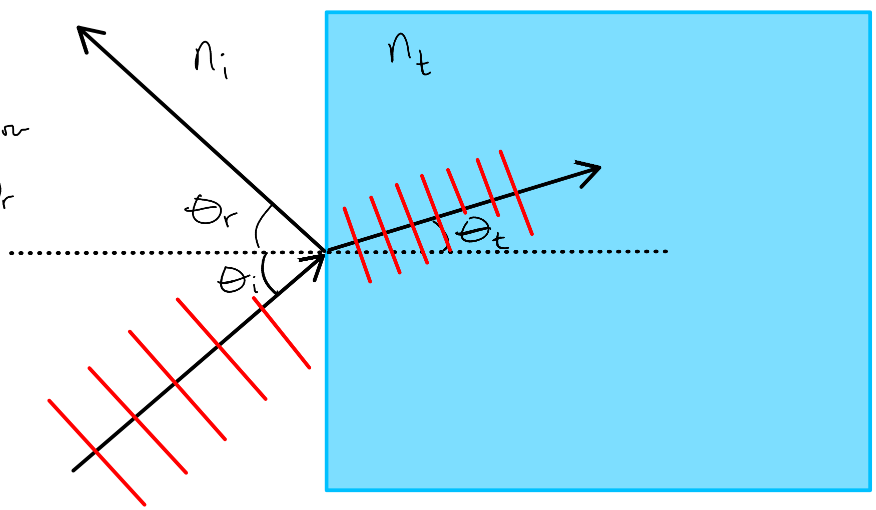
$$\lambda_{vac}\cancel{\nu} = n \cdot \lambda \cdot \cancel{\nu}$$

wavelength in material

$$\lambda = \frac{\lambda_{vac}}{n}$$ $\Big\}$ λ will be smaller than λ_0 since $n > 1$

Law of reflection

$\theta_i = \theta_r$



Snell's Law

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Table 23.1 Indices of Refraction for $\lambda = 589.3$ nm in Vacuum (at 20°C Unless Otherwise Noted)

Material	Index
Solids	
Ice (at 0°C)	1.309
Fluorite	1.434
Fused quartz	1.458
Polystyrene	1.49
Lucite	1.5
Plexiglas	1.51
Crown glass	1.517
Plate glass	1.523
Sodium chloride	1.544
Light flint glass	1.58
Dense flint glass	1.655
Sapphire	1.77
Zircon	1.923
Diamond	2.419
Titanium dioxide	2.9
Gallium phosphide	3.5
Liquids	
Water	1.333
Acetone	1.36
Ethyl alcohol	1.361
Carbon tetrachloride	1.461
Glycerin	1.473
Sugar solution (80%)	1.49
Benzene	1.501
Carbon disulfide	1.628
Methylene iodide	1.74

static case

$E(\vec{r}) = \frac{kq_0}{r^2}$ $\longrightarrow$

oscillating charge

$\vec{E}(\vec{r},t) = \vec{E}_{staticish} + \underbrace{\vec{E}_{rad}}$

$\vec{E}_{rad} \propto \frac{\vec{a}_\perp \overbrace{(t - r/c)}^{\text{wave behavior}}}{r}$

compare to $\vec{r} - vt$