Chapter 1 - waves + review of E+M

Day 06 - Snow Day! → 260126 M

Day 07 - 260128 W → Lots of demos

Day 08 - 260130 F

function → $\frac{3}{10x^2+1} \xrightarrow{\text{wavify}} \frac{3}{10(x-vt)^2+1}$

So any function that can be written

$$\psi(x,t) = f(x \mp vt)$$

wave function

specific function that has $x \mp vt$ in it,

$$f(\underbrace{x \mp vt}_{x'}) = f(x')$$

first derivative: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1} = \frac{\partial f}{\partial x'}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}$$

second derivative: $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'}\right)$

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \frac{\partial x'}{\partial t} = \frac{\partial f}{\partial x'} \cdot (\mp v)$

$$\frac{\partial f}{\partial t} = \mp v \frac{\partial f}{\partial x'}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$\frac{\partial f}{\partial t} = \mp v \frac{\partial f}{\partial x'}$

$$\frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1\text{, again!}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$= \mp v \frac{\partial^2 f}{\partial x'^2} \cdot (\mp v)$$

$$\frac{\partial^2 f}{\partial t^2} = \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial^2 f}{\partial x'^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}}$$

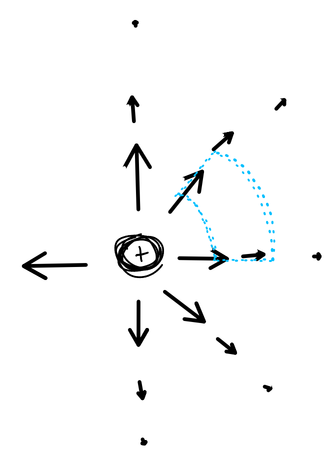
wave equation

compare to

→ heat equation

→ schrodinger's

→ Laplace $\nabla^2 \phi = 0$



Let's go back to Maxwell:

$\rho \leftarrow$ charge density

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$ (Gauss's Law) $\leftarrow$ Electric field spreads out if there is charge

$$\vec{\nabla} \cdot \vec{B} = 0$$ (Gauss's Law for magnetism) $\leftarrow$ Magnetic fields do not spread out (no magnetic monopole — yet!?)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ (Maxwell-Faraday's Law) $\leftarrow$ There is electric field if a magnetic field is changing

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$ (Ampere's Law) $\leftarrow$ There is magnetic field if there is current or if the electric field changes.

For the case of light: no charge
in vacuum no current

$$\vec{\nabla} \cdot \vec{E} = 0$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$\rightarrow$ take the curl of Faraday's

$$\underbrace{\vec{\nabla} \times (\vec{\nabla} \times \vec{E})} = -\frac{\partial}{\partial t} \underbrace{(\vec{\nabla} \times \vec{B})}_{\epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\,!}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}\underbrace{(\vec{\nabla} \cdot \vec{E})}_{=0!} - \vec{\nabla}^2 \vec{E}$$

$$\boxed{\vec{\nabla}^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\frac{1}{v^2}$$

$$\epsilon_0 \mu_0 = \frac{1}{v^2}$$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{m\,kg}{C}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{s^2 C^2}{m^2 kg}$$

$$\boxed{v = 2.998 \cdot 10^{8} \frac{m}{s}}$$

Maxwell's Equations again

$$\vec\nabla\cdot\vec E = \frac{\vec\nabla\cdot\vec P}{\epsilon_0}$$

$$\vec\nabla\cdot\vec B = 0$$

$$\vec\nabla\times\vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t}$$

$$\vec\nabla\times\vec B = \mu_0\vec J_{free} + \mu_0\frac{\partial\vec P}{\partial t} + \epsilon_0\mu_0\frac{\partial\vec E}{\partial t}$$

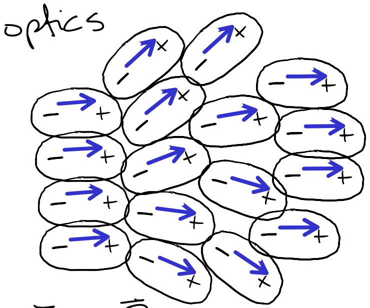
polarization has units of $\frac{\text{charge} \times \text{length}}{\text{volume}}$

$\vec J$ has units of $\frac{\text{charge} \times \text{velocity}}{\text{volume}}$

<u>charge density</u>:

0, for us in optics

$$\rho = \cancel{\rho_{free}} + \rho_p$$

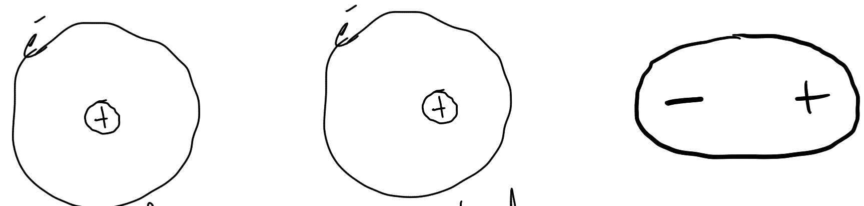


$$\vec\nabla\cdot\vec J_p = \frac{\partial\rho_p}{\partial t} \longrightarrow \vec J_p = \frac{\partial\vec P}{\partial t}$$

<u>current density</u>:

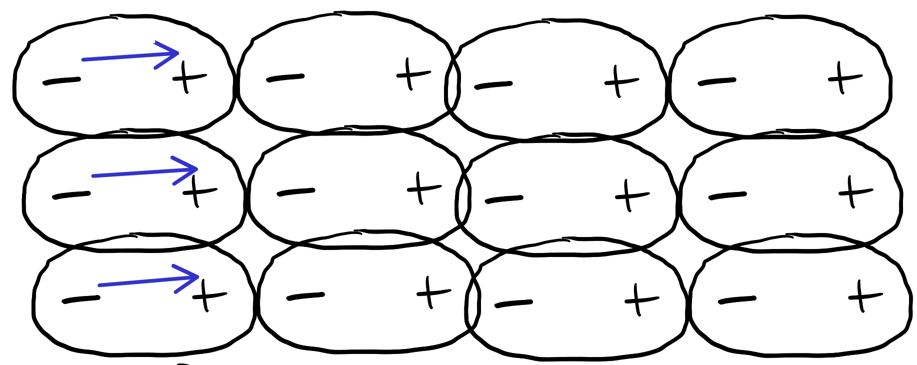
0, for us in optics

$$\vec J = \vec J_{free} + \cancel{\vec J_M} + \vec J_p$$



neutral material; no polarization

neutral material polarization



$$\vec J_p = \frac{\partial\vec P}{\partial t}$$

$$\vec{\nabla}\cdot\vec{J}_p = \frac{\partial \rho_p}{\partial t} \longrightarrow \vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

$$\frac{\partial(\vec{\nabla}\cdot\vec{P})}{\partial t} = \frac{\partial \rho_p}{\partial t}$$

$$\vec{\nabla}\cdot\vec{P} = \rho_p$$

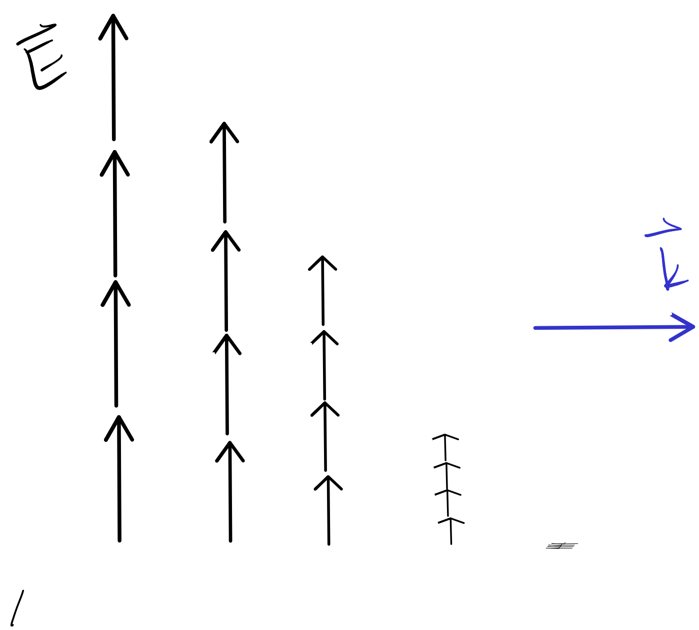
crystals
(anisotropic materials)
↳ not isotropic

$$\underbrace{\vec{\nabla}^2 E - \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}_{\text{wave equation that we derived before.}} = \underbrace{\mu_0\frac{\partial J_{free}}{\partial t}}_{\text{present when current of free charge are flowing.}} + \underbrace{\mu_0\frac{\partial^2 P}{\partial t^2}}_{\text{dipole currents}} - \frac{1}{\epsilon_0}\nabla(\vec{\nabla}\cdot\vec{P})$$

wave equation that we derived before.

present when current of free charge are flowing. reflection from mirror. also light through plasma.

dipole currents
dipole oscillations



Homework: 1, 2, 4, 5, 7, 9

260204 W
Day 10

P1.2 Suppose that an electric field is given by $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$, where $\mathbf{k} \perp \mathbf{E}_0$ and ϕ is a constant phase. Show that

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{k}\times\mathbf{E}_0}{\omega}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$

is consistent with (1.3).

↳ assume: $\vec{k} = k_x\hat{x} + 0\hat{y} + 0\hat{z}$

↳ assume: $\vec{r} = x\hat{x} + 0\hat{y} + 0\hat{z}$

↳ $\vec{E}_0 = 0\hat{x} + E_{0y}\hat{y} + 0\hat{z}$

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}} \qquad \text{(Faraday's Law)} \qquad (1.3)$$

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \qquad \text{(Ampere's Law revised by Maxwell)} \qquad (1.4)$$

$$\vec{\nabla}\times\vec{E} = \vec{\nabla}\times\left(\vec{E}_0\cos(\vec{k}\cdot\vec{r} - \omega t + \phi)\right)$$

can: $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{E} = \underbrace{\hat{x}\left(-\frac{\partial E_y}{\partial z}\right)}_{0} - \hat{y}(0) + \underbrace{\hat{z}\left(\frac{\partial E_y}{\partial x}\right)}$$

$$-\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}\left(\frac{\vec{k}\times\vec{E}_0}{\omega}\cos(\vec{k}\cdot\vec{r} - \omega t + \phi)\right)$$

$$= -\frac{\vec{k}\times E_0}{\omega}\left(-\sin(\vec{k}\cdot\vec{r} - \omega t + \phi)\cdot(-\omega)\right)$$

$$= \underbrace{-\vec{k}\times\vec{E}_0}\sin(\vec{k}\cdot\vec{r} - \omega t + \phi)$$

$$= \qquad \sin(k_x \cdot x - \omega t + \phi)$$

$$\vec{\nabla}\times\vec{E} = \hat{z}\left(\frac{\partial}{\partial x}\left(E_{oy}\cos(k_x x - \omega t + \phi)\right)\right)$$

$$= -\hat{z}\, E_{oy}\sin(k_x x - \omega t + \phi)\cdot k_x$$

$$= -E_{oy}\cdot k_x \cdot \sin(k_x x - \omega t + \phi)\,\hat{z}$$

$$\vec{k}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & 0 & \cancel{0} \\ 0 & E_{oy} & \cancel{0} \end{vmatrix}$$

$$= k_x E_{oy}\,\hat{z}$$

$$= -E_{oy}\,k_x \sin(k_x x - \omega t + \phi)\,\hat{z}$$

P1.4 Check that the **E** and **B** fields in P1.2 satisfy the rest of Maxwell's equations:

(a) (1.1). What must ρ be?

(b) (1.2).

(c) (1.4). What must **J** be?

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \text{(Gauss' Law)} \qquad (1.1)$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \text{(Gauss' Law for magnetism)} \qquad (1.2)$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \text{(Faraday's Law)} \qquad (1.3)\ \checkmark$$

$$\underbrace{\nabla\times\frac{\mathbf{B}}{\mu_0} = \epsilon_0\frac{\partial\mathbf{E}}{\partial t} + \mathbf{J}} \qquad \text{(Ampere's Law revised by Maxwell)} \qquad (1.4)$$

$$\mu_0\epsilon_0 = \frac{k^2}{\omega^2}$$

HINT!

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\cancel{\frac{\partial E_x}{\partial x}}^{0} + \frac{\partial E_y}{\partial y} + \cancel{\frac{\partial E_z}{\partial z}}^{0}$$

$$\frac{\partial E_y}{\partial y} = \frac{\partial}{\partial y}\left(\underbrace{E_{oy}\cos(k_x x - \omega t + \phi)}_{\text{does not depend on } y \text{ at all!}}\right) = 0$$

So $\rho = 0$

similarly for B since B_z is the only term and B_z does not depend on z.

$$B(\vec r,t) = \frac{\vec k \times \vec E}{\omega}\cos(\vec k\cdot\vec r - \omega t + \phi) = \frac{k_x E_{0y}}{\omega}\cos(k_x x - \omega t + \phi)$$

P1.7 Show that the magnetic field in P1.2 is consistent with the wave equation derived in P1.6. What is the requirement on k and ω?

$$\nabla^2 \vec E = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$ ← this we did in class! see day08!

$$\nabla^2 \vec B = \epsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

side note!

Following what we did in class last time:

$$\left.\begin{array}{l}\vec k = (k_x, 0, 0)\\ \vec E_0 = (0, E_{0y}, 0)\end{array}\right\} \text{ so } \vec k \times \vec E = \underbrace{k_x E_{0y}\,\hat z}$$

which means $\vec B$ is entirely in the $\hat z$ direction!

also $\vec k \cdot \vec r = k_x \cdot x$

$$\underbrace{\epsilon_0 \mu_0 = \frac{1}{v^2} \quad + \quad \frac{\omega}{k} = v}$$

$$\epsilon_0 \mu_0 = \frac{k^2}{\omega^2}$$

this could be useful!

$$\omega = \left[\frac{\text{rad}}{s}\right] \qquad k = \left[\frac{\text{rad}}{m}\right] \qquad \frac{\omega}{k} = \left[\frac{m}{s}\right] = v$$

from chapter 0 notes:

$$\nabla^2 \vec B = \left(\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2}\right)\hat x + \left(\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_y}{\partial z^2}\right)\hat y + \left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2}\right)\hat z$$

both zero!

also zero!

B_z only depends on x

$$\nabla^2 \vec B = \frac{\partial^2 B_z}{\partial x^2} = \frac{k_x E_{0y}}{\omega}\cdot\left(-\cos^2(k_x x - \omega t + \phi)\right)\cdot k_x^2$$

$$\frac{k_x^2}{\omega^2}\frac{\partial^2 B}{\partial t^2} = \frac{k_x^2}{\cancel{\omega^2}}\cdot\frac{k_x E_{0y}}{\omega}\left(-\cos^2(kx - \omega t + \phi)\cdot\cancel{\omega^2}\right)$$

$$\nabla^2 \vec{B} = -\frac{k_x^3 E_{oy}}{\omega}\cos^2(k_x x - \omega t + \phi) \quad \Bigg| \quad \frac{k^2}{\omega^2}\frac{\partial^2 B}{\partial t^2} = -\frac{k_x^3 E_{oy}}{\omega}\cos^2(kx - \omega t + \phi)$$

Same!

problem 1.9

P1.9 (a) Show that $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos\left(k(\hat{\mathbf{u}}\cdot\mathbf{r} - ct) + \phi\right)$ is a solution to the vacuum wave equation (1.41), where $\hat{\mathbf{u}}$ is an arbitrary unit vector, $c = 1/\sqrt{\epsilon_0\mu_0}$, and k is a constant with units of inverse length.

→ exactly the same as P1.7, just w/ Electric Field instead of Magnetic

(b) Show that each wavefront forms a plane, which is why such solutions are often called 'plane waves'. HINT: A wavefront is a surface in space where the argument of the cosine (i.e. the *phase* of the wave) has a constant value. Set the cosine argument to an arbitrary constant and see what positions are associated with that phase.

$k(\hat{u}\cdot\vec{r} - ct) + \phi$ must be constant at a moment in time. t is constant at a moment of time by definition. c is constant always. ϕ is constant. So what is $\hat{u}\cdot\vec{r}$

if $\hat{u}$ is arbitrary, then I can choose:

$$\hat{u} = \hat{x}$$

and then $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

so $\hat{u}\cdot\vec{r} = x$

and $k(\hat{u}\cdot\vec{r}) = k\cdot x$

this means that for any x-value (and only x value) the wave has a constant value in the y + z directions.

(c) Determine the speed $v = \Delta r/\Delta t$ that a wavefront moves in the $\hat{\mathbf{u}}$ direction. HINT: Set the cosine argument to a constant, and consider a change in position along $\hat{\mathbf{u}}$ with its associated change in time.

$$V = \frac{\Delta r}{\Delta t}$$

(d) By analysis of this wave, determine the wavelength λ in terms of k. HINT: Holding time constant, find the distance between identical wavefronts by changing the position along $\hat{\mathbf{u}}$ and allowing the cosine argument to evolve through 2π.

(e) Use (1.33) to show that $\mathbf{E}_0$ and $\hat{\mathbf{u}}$ must be perpendicular to each other in vacuum.

1.9 a – again

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos\left(k(\hat{u}\cdot\vec{r} - ct) + \phi\right)$$

$\hat{u}$ is arbitrary so I will choose $\hat{x}$

$$\vec{E}_0 = (0, E_{0y}, 0)$$

$$\vec{k} = k\hat{u} = (k, 0, 0)$$

x-direction

- $\vec{\nabla}^2\vec{E} = \epsilon_0\mu_0 \dfrac{\partial^2\vec{E}}{\partial t^2}$
- $\vec{\nabla}^2\vec{E} = \dfrac{1}{v^2}\dfrac{\partial^2\vec{E}}{\partial t^2}$

$$v = \frac{\omega}{k}$$

- $\vec{\nabla}^2\vec{E} = \dfrac{k^2}{\omega^2}\dfrac{\partial^2\vec{E}}{\partial t^2}$ ← use this!

$$\vec{\nabla}^2\vec{E} = \left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right)\hat{x}$$

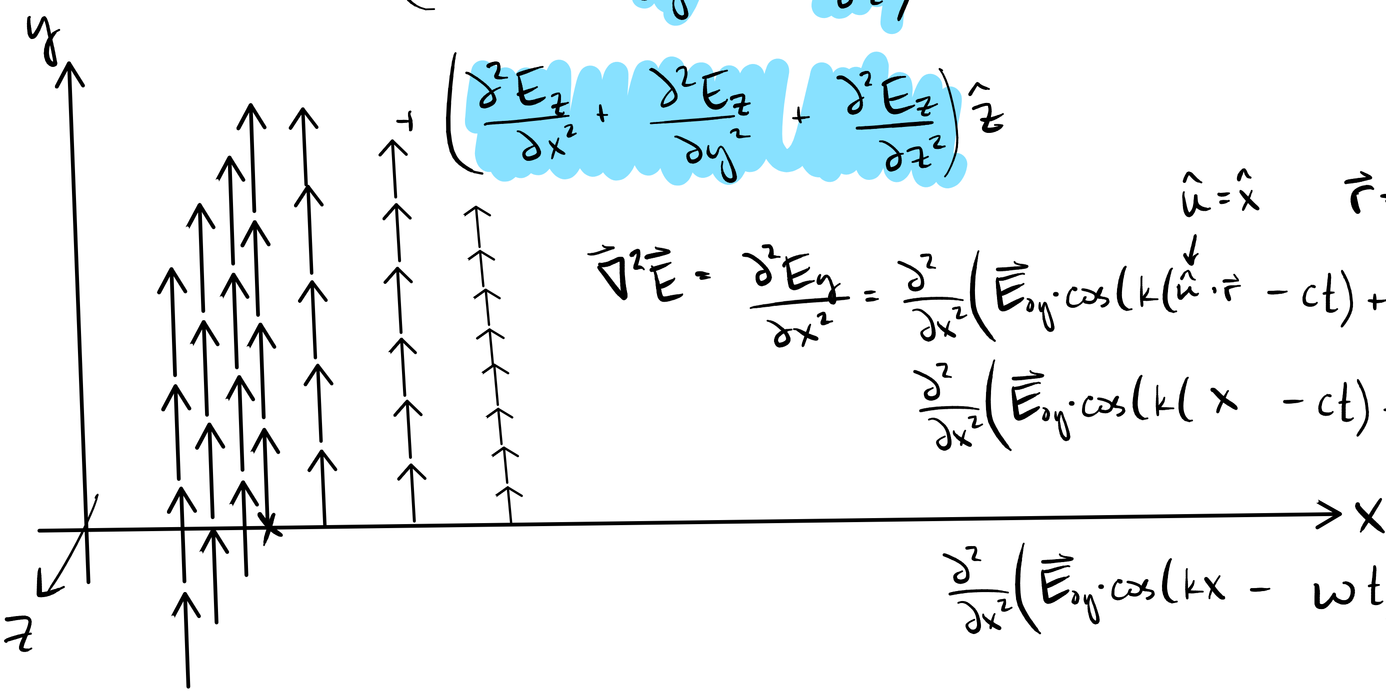
$$+ \left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right)\hat{y}$$

$$+ \left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right)\hat{z}$$

$\hat{u} = \hat{x}$ $\quad$ $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\vec{\nabla}^2\vec{E} = \frac{\partial^2 E_y}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left(\vec{E}_{0y}\cdot\cos(k(\hat{u}\cdot\vec{r} - ct) + \phi)\right)$$

$$\frac{\partial^2}{\partial x^2}\left(\vec{E}_{0y}\cdot\cos(k(x - ct) + \phi)\right)$$



$$\frac{\partial^2}{\partial x^2}\left(\vec{E}_{0y}\cdot\cos(kx - \omega t) + \phi\right)$$