

1. Express the wave function for one-dimensional harmonic waves in terms of the following parameters:

- Frequency and wavelength
- Period and wavelength
- Angular frequency and wave vector

$$1) \eta(x) = A \cos\left(\frac{2\pi x}{\lambda} - 2\pi v t + \phi\right)$$

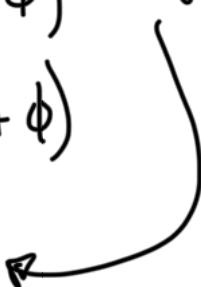
$$2) \eta(x) = A \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi}{T} t + \phi\right)$$

3)

starting place \rightarrow

$$\eta(x) = A \cos(k(x - vt) + \phi)$$

$$\text{or} \\ \eta(x) = A \cos(kx - \omega t + \phi)$$



2. Consider the plane electromagnetic wave in vacuum (in SI units) given by the expressions:

$$E_x = 0$$

$$E_y = 4 \cos[2\pi * 10^{14}(t - x/c) + p/2]$$

$$E_z = 0$$

$$\omega\left(t - \frac{x}{c}\right) = \omega t - \underbrace{\frac{\omega x}{c}}_k$$

not sure about

$$\text{but } k \cdot v = \omega$$

$$\text{and } v - \underbrace{\frac{\omega}{k}}_{\lambda} = \lambda \cdot v$$

$$\text{so since } \omega = 2\pi \cdot 10^{14} \quad \omega = 2\pi v$$

$$\text{then } v = 10^{14} \text{ Hz}$$

$$\lambda = \frac{2\pi}{\omega} = \frac{2\pi c}{\omega}$$

$$\text{but also } \lambda = \frac{v}{f}$$

$$\text{so } \lambda = \frac{c}{10^{14}} = \frac{3 \cdot 10^8}{10^{14}} = 3 \cdot 10^{-6} = 3 \mu\text{m}$$

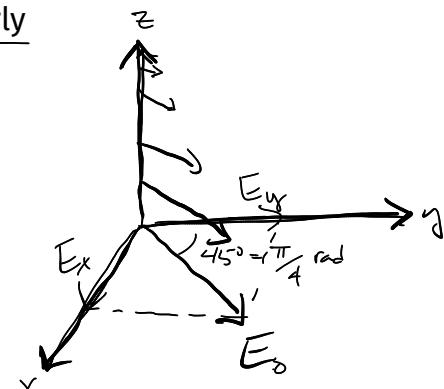
3. Write an expression for the \vec{E} and \vec{B} fields that constitute a plane harmonic wave traveling in the $+z$ -direction. The wave is linearly polarized with its plane of vibration at 45° to the yz -plane.

$$\vec{k} = k_z \hat{z}$$

$$E_x = \frac{E_0 \sqrt{2}}{2} \cos(k_z z - \omega t)$$

$$E_y = \frac{E_0 \sqrt{2}}{2} \cos(k_z z - \omega t)$$

$$E_z = 0$$



$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & k_z \\ E_x & E_y & 0 \end{vmatrix}$$

4. A 500-nm harmonic EM wave, whose electric field is in the z -direction, is traveling in the y -direction in vacuum.

$$\vec{k} = 1.26 \cdot 10^7 \frac{\text{rad}}{\text{m}} \hat{z}$$

- What is the frequency of the wave?
- Determine both v and k for this wave. $\rightarrow v = c$, $k = \frac{2\pi}{\lambda} = 1.26 \cdot 10^7 \frac{\text{rad}}{\text{m}}$
- If the electric field amplitude is 700 V/m, what is the amplitude of the magnetic field?
- Write an expression for both $E(t)$ and $B(t)$ given that each is zero at $x = 0$ and $t = 0$. Put in all the appropriate units.

$$|B| = \frac{|E|}{c} = \frac{700 \text{ V/m}}{3 \cdot 10^8 \text{ m/s}} = 2.3 \cdot 10^{-6} \text{ T}$$

$$\lambda = 500 \text{ nm}, \text{ so } v = \frac{c}{\lambda} = \frac{3 \cdot 10^8 \text{ m/s}}{500 \cdot 10^{-9} \text{ m}} = 6 \cdot 10^{14} \text{ Hz}$$

$$\omega = 2\pi \cdot f = 3.77 \cdot 10^{15} \frac{\text{rad}}{\text{s}}$$

$$E_x(t) = 700 \frac{\text{V}}{\text{m}} \cos\left(1.26 \cdot 10^7 \frac{\text{rad}}{\text{m}} \cdot z - 3.77 \cdot 10^{15} \frac{\text{rad}}{\text{s}} t\right)$$

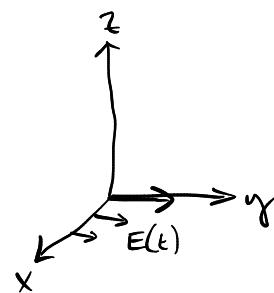
$$B_y(t) = \frac{700 \text{ V}}{3 \cdot 10^8 \text{ m/s}} \cos\left(1.26 \cdot 10^7 \frac{\text{rad}}{\text{m}} \cdot z - 3.77 \cdot 10^{15} \frac{\text{rad}}{\text{s}} t\right)$$

$$B_y(t) = 2.33 \text{ T} \cdot \cos\left(1.26 \cdot 10^7 \frac{\text{rad}}{\text{m}} \cdot z - 3.77 \cdot 10^{15} \frac{\text{rad}}{\text{s}} t\right)$$

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega} = \frac{k_z \hat{z} \times E_x \hat{x}}{\omega} = \frac{E_x (+i)}{c}$$

5. Consider a linearly polarized plane electromagnetic wave traveling in the $+x$ -direction in free space having as its plane of vibration the xy -plane. Given that its frequency is 5 MHz and its amplitude is $E_0 = 0.05 \text{ V/m}$

- Find the period and wavelength of the wave.
- Write an expression for $E(t)$ and $B(t)$.
- ~~Find the flux density, $\langle S \rangle$, of the wave.~~



$$\nu = 5 \cdot 10^6 \text{ Hz} \quad \frac{1}{\nu} = T = 2 \cdot 10^{-7} \text{ s} \quad \checkmark$$

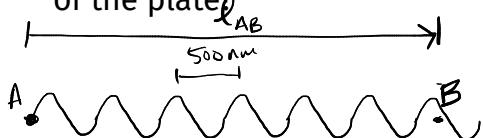
$$\lambda \cdot \nu = c$$

$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \text{ m/s}}{5 \cdot 10^6 \text{ Hz}} = 60 \text{ m}$$

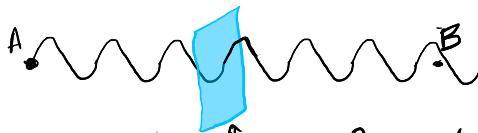
$$E_y = 0.05 \frac{\text{V}}{\text{m}} \cos\left(\frac{2\pi}{60 \text{ m}} x - \frac{2\pi}{2 \cdot 10^7 \text{ s}} t\right)$$

$$E_z = 0$$

6. A light wave travels from point A to point B in vacuum. Suppose we introduce into its path a flat glass plate ($n = 1.50$) of thickness $L = 1.00 \text{ mm}$. If the vacuum wavelength is 500 nm, how many waves span the space from A to B with and without the glass in place? What phase shift is introduced with the insertion of the plate?



$$\frac{l_{AB}}{\lambda_{vac}} = \# \text{ of wavelengths from A to B in vacuum}$$



$$\lambda_{glass} = \frac{\lambda_{vac}}{n} = \frac{500 \text{ nm}}{1.50} = 333 \text{ nm}$$

$$\frac{l_{AB} - 1.00 \text{ mm}}{\lambda_{vac}} + \frac{1.00 \text{ mm}}{\lambda_{glass}}$$

$$\frac{l_{AB} - 1.00 \text{ mm}}{\lambda_{vac}} + \frac{1.00 \text{ mm} \cdot n}{\lambda_{vac}}$$

$$\frac{l_{AB} - 1.00 \text{ mm} + 100 \text{ mm} \cdot n}{\lambda_{vac}} = \left[\frac{l_{AB} + 100 \text{ mm} \cdot (n-1)}{\lambda_{vac}} \right] / 3$$

wavelengths from
A to B w/
glass in the
way

