

Chapter 2 - Plane Waves + Index of Refraction

260208 M
Day 12

plane wave:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \cdot \cos(\vec{k} \cdot \vec{r} - \omega t + \phi)$$

"wave vector" \rightarrow $\vec{k} = k \hat{u} = \frac{2\pi}{\lambda_{vac}} \hat{u}$
"k"
direction of propagation

angular frequency $\rightarrow \omega = \frac{2\pi \cdot c}{\lambda_{vac}} = 2\pi \cdot \nu$

$k + \omega$ are related just like $\lambda + \nu$

$$c = \frac{\lambda_{vac}}{T} = \lambda_{vac} \cdot \nu = \frac{\omega}{k}$$

this relationship
is known as the
"dispersion relation"

"kappa"
wave number
 $\frac{1}{\lambda_{vac}} = K = [cm^{-1}]$

$c = v_{vac} \leftarrow$ velocity of light in vacuum

$\nu \rightarrow$ "nu"
frequency

similarly for magnetic field:

$$\vec{B}(\vec{r}, t) = \vec{B}_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \phi)$$

$\vec{B}_0 = \frac{\vec{k} \times \vec{E}_0}{\omega}$, so \vec{B}_0 is not independent, but is determined by other parameters.

$$\vec{B}_0 \perp \vec{k} \perp \vec{E}$$

also think about magnitude

$$B_0 = \frac{k E_0}{\omega} = \frac{E_0}{c} \leftarrow \text{since this is so large we will focus on the } \vec{E} \text{ field.}$$

Complex plane waves:

$$\vec{E}(\vec{r}, t) = \text{Re} \left\{ \vec{\tilde{E}}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right\}$$

$$\vec{\tilde{E}}_0 = \vec{E}_0 e^{i\phi} \leftarrow \text{phase shift}$$

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

Summarize some facts that we know now:

$\lambda \rightarrow$ wavelength

$T \rightarrow$ period

$\nu \rightarrow$ frequency ($\frac{1}{T}$)

$k \rightarrow$ wave vector ($\frac{2\pi}{\lambda}$)

$\omega \rightarrow$ angular frequency ($\frac{2\pi}{T}$)

$$V = \frac{\omega}{k}$$

$$V = \frac{\lambda}{T} = \lambda \cdot \nu \leftarrow \text{any wave}$$

$$c = \lambda_{\text{vac}} \cdot \nu$$

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \Rightarrow c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

permittivity of free space $\rightarrow \epsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$

permeability of free space $\rightarrow \mu_0 = 4\pi \cdot 10^{-7} \frac{T}{Am^2}$

Speed of Light in matter

\hookrightarrow light slows down in materials

$$\frac{c}{v} = n \leftarrow \text{index of refraction}$$

$$n=1 \leftarrow \text{vacuum}$$

$$n=1.0003 \leftarrow \text{air}$$

$$n=1.33 \leftarrow \text{water}$$

$$n=1.5 \leftarrow \text{glass}$$

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Day 13

$$\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \frac{k^2}{\omega^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

Q1: what is light? ✓

Q2: where does light come from? ✓

Q3: can the speed of light change? see Q6

Q4: what affects the brightness of light?

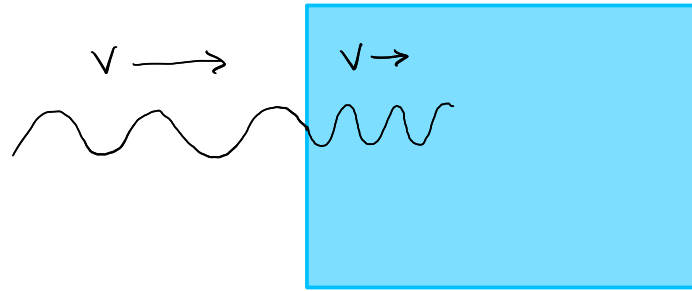
Q5: how do human perceive light amount + color

Q6: why does light slow down in materials

Q3

Q7: what is reflection

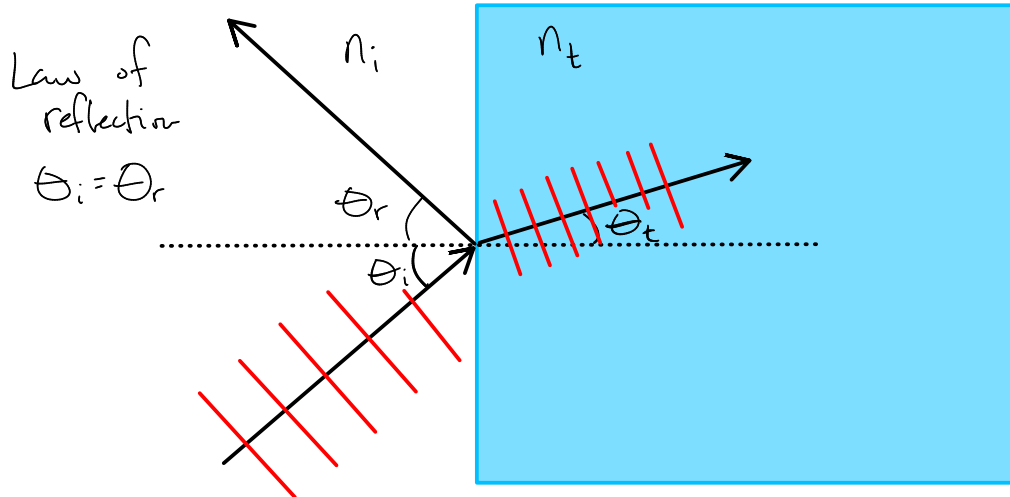
$\lambda \cdot \nu = v = \frac{c}{n}$
 \downarrow
 gets smaller as n increases



$\lambda_{vac} \nu = c \Leftrightarrow c = n \cdot \nu$
 \uparrow
 $\lambda \cdot \nu$

$\lambda_{vac} \nu = n \cdot \lambda \cdot \nu$
 \leftarrow wavelength in material

$\lambda = \frac{\lambda_{vac}}{n}$ } λ will be smaller than λ_0 since $n > 1$



Snell's Law
 $n_i \sin \theta_i = n_t \sin \theta_t$

Table 23.1 Indices of Refraction for $\lambda = 589.3 \text{ nm}$ in Vacuum (at 20°C Unless Otherwise Noted)

Material	Index
Solids	
Ice (at 0°C)	1.309
Fluorite	1.434
Fused quartz	1.458
Polystyrene	1.49
Lucite	1.5
Plexiglas	1.51
Crown glass	1.517
Plate glass	1.523
Sodium chloride	1.544
Light flint glass	1.58
Dense flint glass	1.655
Sapphire	1.77
Zircon	1.923
Diamond	2.419
Titanium dioxide	2.9
Gallium phosphide	3.5
Liquids	
Water	1.333
Acetone	1.36
Ethyl alcohol	1.361
Carbon tetrachloride	1.461
Glycerin	1.473
Sugar solution (80%)	1.49
Benzene	1.501
Carbon disulfide	1.628
Methylene iodide	1.74

static case

$$E(\vec{r}) = \frac{kq_0}{r^2}$$

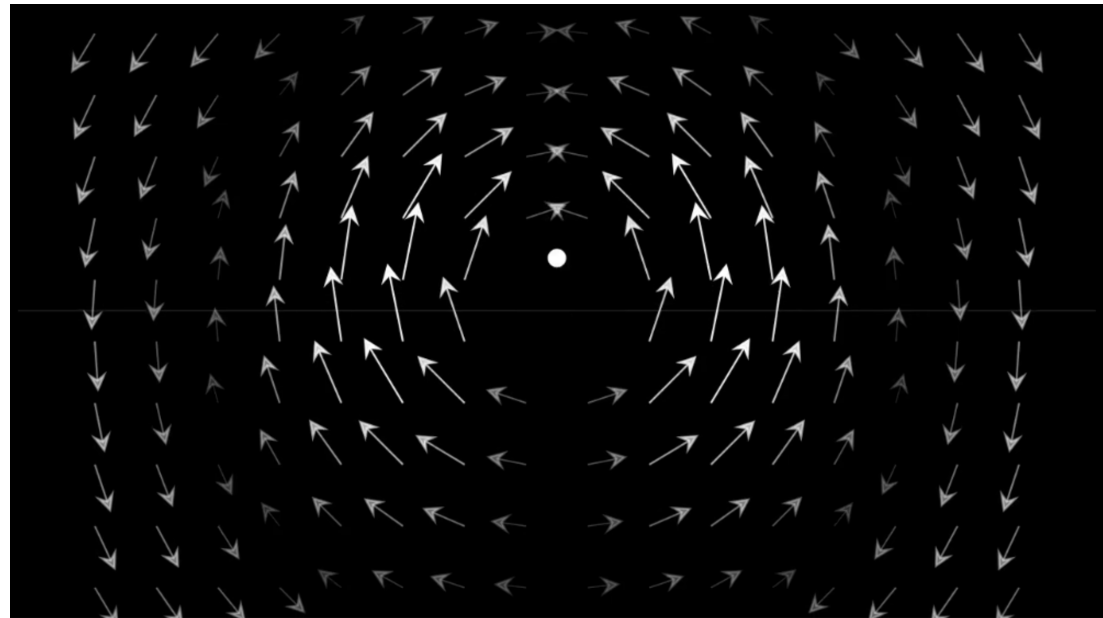
oscillating charge

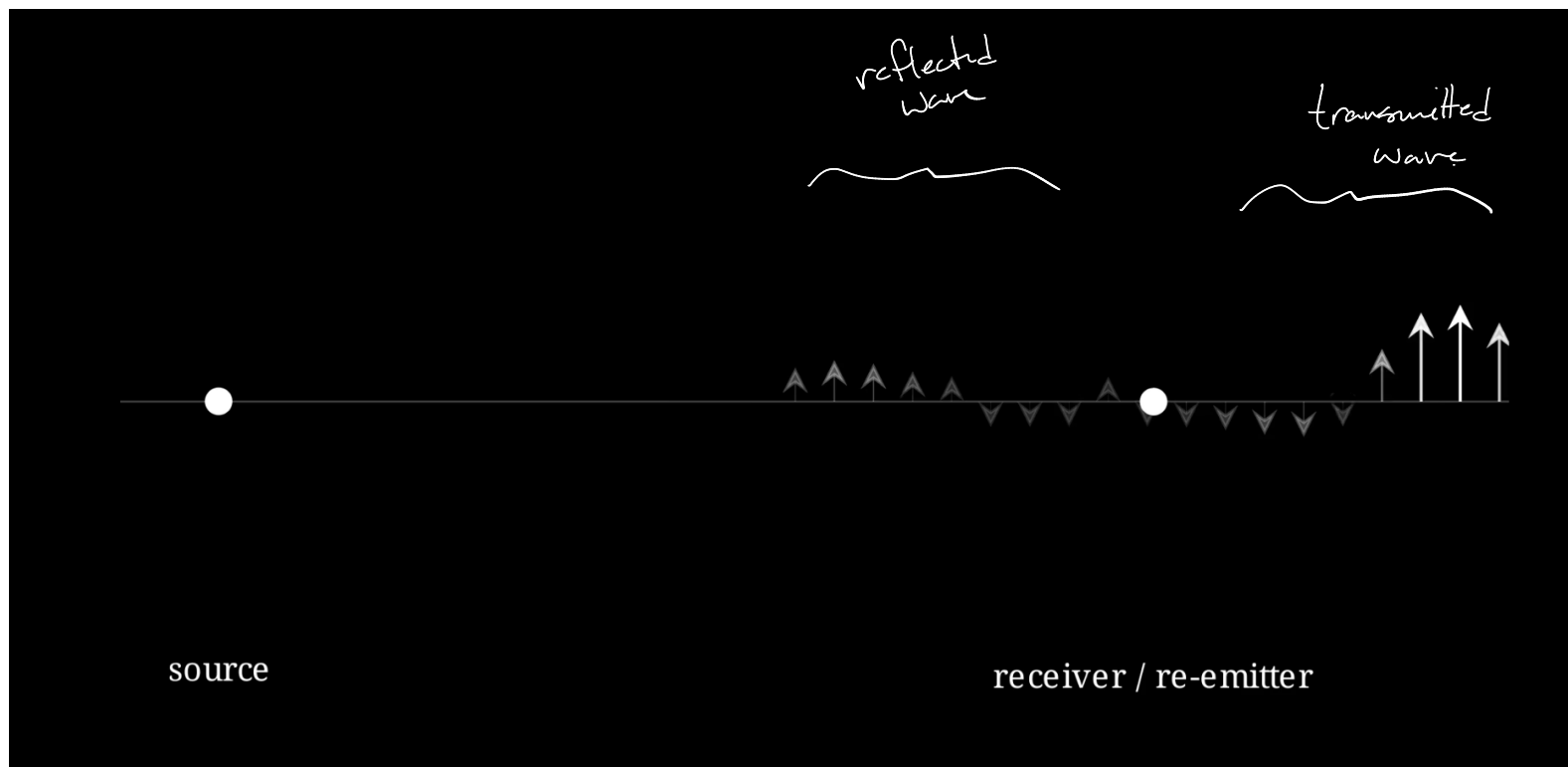
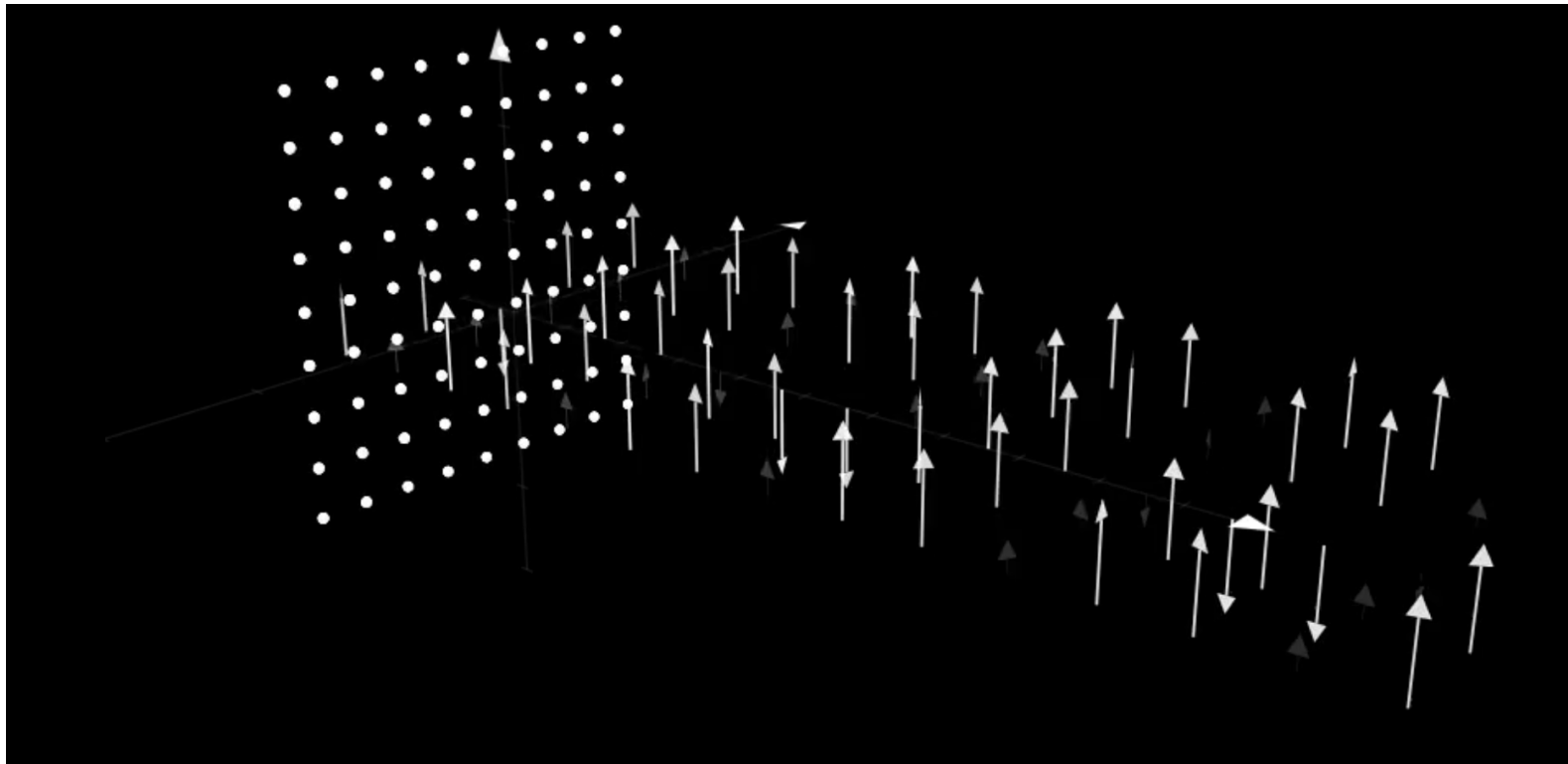
$$\vec{E}(\vec{r}, t) = \vec{E}_{\text{static}} + \vec{E}_{\text{rad}}$$

wave behavior

$$\vec{E}_{\text{rad}} \propto \frac{\vec{a}_{\perp}(t - r/c)}{r}$$

compare to $\vec{r} - vt$





Dielectric Media

260216 M
Day 15

↳ insulator (like glass, water, air)

Assumptions: (1) isotropic → same in all directions

(2) homogeneous → same over distance

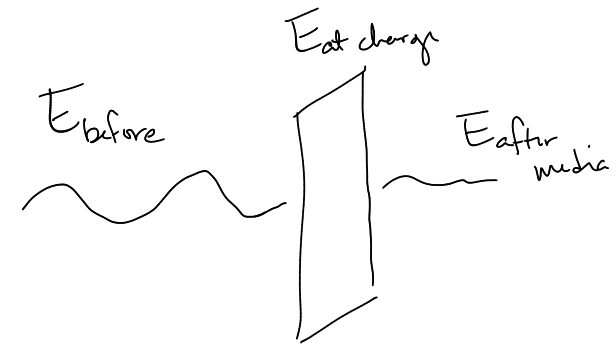
(3) non-conducting (insulator/dielectric) $\vec{J}_{\text{free}} = 0$

(4) very thin material
(air or some other thin gas)

} we will relax this later on

Light in a thin gas → Feynman, vol 1, Ch 31

Light in matter = incident field + field reradiated by driven charges



$$\vec{E}_{\text{after plate}} = \vec{E}_s + \sum_{\text{all other charges}} \vec{E}_{\text{each charge}}$$

$$\vec{E}_s = \vec{E}_0 \cos(kx - \omega t)$$

$$\vec{E}_s = \vec{E}_0 \cos\left(k\left(x - \frac{\omega}{k}t\right)\right)$$

$$\vec{E} = \vec{E}_0 \cos\left(\omega\left(\frac{k}{\omega}x - t\right)\right)$$

$$\vec{E} = \vec{E}_0 \cos\left(-\omega\left(t - \frac{k}{\omega}x\right)\right)$$

$$\vec{E} = \vec{E}_0 \cos\left(\omega\left(t - \frac{x}{c}\right)\right)$$

$$\vec{E} = \vec{E}_0 e^{i\omega\left(t - \frac{x}{c}\right)}$$

So now what about the field from other charges?

plate thickness = Δx

$$t_{\text{vac}} = \frac{\Delta x}{c}$$

$$t_{\text{plate}} = \frac{n \Delta x}{c}$$

$$\Delta t = t_{\text{plate}} - t_{\text{vac}}$$

$$= (n-1) \frac{\Delta x}{c}$$

$$E_{\text{after plate}} = E_0 e^{i\omega\left(t - \Delta t - \frac{x}{c}\right)}$$

$$= E_0 e^{i\omega\left(t - (n-1) \frac{\Delta x}{c} - \frac{x}{c}\right)}$$

$$= e^{-i\omega(n-1) \frac{\Delta x}{c}} E_0 e^{i\omega\left(t - \frac{x}{c}\right)}$$

$\underbrace{\hspace{1.5cm}}$ w/ a phase shift
 $\underbrace{\hspace{1.5cm}}$ just like the original wave
 But w/ a

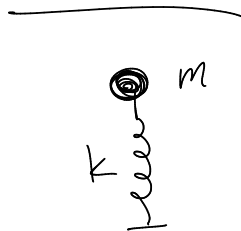
$$e^{-i\omega(n-1)\frac{\Delta x}{c}}, \Delta x \text{ is very small}$$

$$\underbrace{e^s \approx 1 + s, \text{ for small } s}$$

$$e^{-i\omega(n-1)\frac{\Delta x}{c}} \approx 1 - i\omega(n-1)\frac{\Delta x}{c}$$

$$E_{\text{after plate}} = \underbrace{E_0 e^{i\omega(t - \frac{x}{c})}}_{E_s} - \underbrace{\frac{i\omega(n-1)\Delta x}{c} \cdot E_0 e^{i\omega(t - \frac{x}{c})}}_{E_a}$$

$E_a \rightarrow$ new electric field that is the result of all moving charges



$$ma = \sum F = -kz$$

$$m \frac{d^2 z}{dt^2} = -kz$$

$$\boxed{\frac{d^2 z}{dt^2} = -\frac{k}{m} z}$$

$$\cancel{A \omega^2 \cos(\omega t + \phi)} = \cancel{-\frac{k}{m} \cdot A \cos(\omega t + \phi)}$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

\swarrow one of our constants

one way

$$\rightarrow z = A \cos(\omega t + \phi)$$

$$\frac{dz}{dt} = -A\omega \sin(\omega t + \phi)$$

$$\rightarrow \frac{d^2 z}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$$

another way

$$z = A e^{i\omega t + \phi}$$

$$\frac{dz}{dt} = A i \omega e^{i\omega t + \phi}$$

$$\frac{d^2 z}{dt^2} = A i^2 \omega^2 e^{i\omega t + \phi}$$

$$A \omega^2 e^{i\omega t + \phi} = -\frac{k}{m} A e^{i\omega t + \phi}$$

$$\omega^2 = -\frac{k}{m}$$

$$z = A \cos(\sqrt{\frac{k}{m}} t + \phi) \leftarrow \text{general solution to that DE}$$

\hookrightarrow given by initial conditions

$$\text{at } t=0, z=1 \Rightarrow A=1 \quad \phi=0$$

(A + ϕ are determined by initial conditions)

Now we have a charge connected
by a spring being driven by $\vec{E}(t)$

$$z = 1 \cdot \cos\left(\sqrt{\frac{k}{m}} \cdot t\right)$$

260218 W
Day 16

$$\frac{d^2 z}{dt^2} = -\frac{k}{m} z + \frac{q_e E_0}{m} e^{i\omega t + \phi}$$

the frequency of the light

ω_0^2

↳ natural frequency of the electrons

Guess: $z = z_0 e^{i\omega t + \phi}$

$$\dot{z} = z_0 i\omega e^{i\omega t + \phi}$$

$$\ddot{z} = z_0 i^2 \omega^2 e^{i\omega t + \phi}$$

ω
-1

So now we plug in:

$$-z_0 \omega^2 e^{i\omega t + \phi} = -\omega_0^2 z_0 e^{i\omega t + \phi} + \frac{q_e E_0}{m} e^{i\omega t + \phi}$$

↳ solve for z_0

$$z_0(\omega_0^2 - \omega^2) = \frac{q_e E_0}{m}$$

$$z_0 = \frac{q_e E_0}{m(\omega_0^2 - \omega^2)}$$

so... →

$$z(t) = \frac{q_e E_0}{m(\omega_0^2 - \omega^2)} e^{i\omega t + \phi}$$

motion of charges → Electric field that results

$$E_a = \frac{-\eta q_e}{2\epsilon_0 c} i\omega z_0 e^{i\omega(t - \frac{x}{c})}$$

↓

← from Griffiths
or Feynman

$$E_a = \frac{-\eta q_e}{2\epsilon_0 c} \cdot \frac{i\omega \cdot q_e E_0}{m(\omega_0^2 - \omega^2)} \cdot e^{i\omega(t - \frac{x}{c})} \quad \text{vs.} \quad E_a = \frac{-i\omega(n-1)\Delta x}{c} \cdot E_0 e^{i\omega(t - \frac{x}{c})}$$

↙ from before

so putting what survives together

$$\frac{\eta q_e^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)} = (n-1)\Delta x$$

$\eta \rightarrow$ number of charges
per unit of area

$N \rightarrow$ number of charges
per unit of volume

$$\eta = N \cdot \Delta x$$

$$\frac{N \cdot \Delta x q_e^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)} = (n-1)\Delta x$$

$$n = 1 + \frac{N \cdot q_e^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)}$$

ω_0 is often much
larger than ω
for visible light

as ω rises the denominator
gets smaller, so the fraction
gets larger!

\vec{E} from a static
sheet of charge

$$E = \frac{Q}{2\epsilon_0 A}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

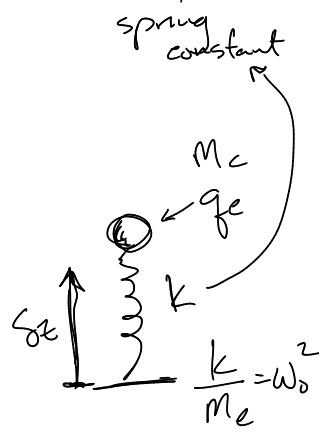
$$E = \frac{\eta q_e}{2\epsilon_0}$$

Let's do this again, but better. \rightarrow index of refraction for heavier dielectrics

\rightarrow Go back to the wave equation from Maxwell's:

$$\nabla^2 \vec{E} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

\rightarrow what is \vec{P} ?
polarization of an atom
(not the polarization of light)



$$\vec{P} = N \cdot q \cdot S_z$$

\uparrow
number
per
volume

plug in!

$$\begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \\ \vec{P} = \vec{P}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \end{cases}$$

$$i^2 k^2 E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} - \epsilon_0 \mu_0 (+i^2 \omega^2) \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = \mu_0 (+i^2 \omega^2) \vec{P}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$\underbrace{-1}_{\text{wave vector, } k}$

$$-k^2 \vec{E}_0 + \epsilon_0 \mu_0 \omega^2 \vec{E}_0 = -\mu_0 \omega^2 \vec{P}_0$$

$$\vec{P}_0 = \epsilon_0 \chi(\omega) \vec{E}_0 \quad \leftarrow \text{constitutive relation}$$

\hookrightarrow susceptibility

\hookrightarrow kicking the can down the road!

\hookrightarrow complex!

\hookrightarrow a function of ω (frequency)
just like $\vec{P}_0 + \vec{E}_0$

\hookrightarrow plug in and let \vec{E}_0 cancel out

$$-k^2 + \epsilon_0 \mu_0 \omega^2 = -\epsilon_0 \mu_0 \omega^2 \chi(\omega)$$

→ solve for k , $\epsilon_0 \mu_0 = \frac{1}{c^2}$

$$k^2 = \epsilon_0 \mu_0 \omega^2 (1 + \chi(\omega))$$

still wave vector $k \rightarrow$ $k = \frac{\omega}{c} \sqrt{1 + \chi(\omega)}$

recall: $k = \frac{\omega}{v}$ and $v = \frac{c}{n}$

$$k = \frac{\omega \cdot n}{c}$$

$$\tilde{k} = \frac{\tilde{n} \omega}{c}$$

$$\tilde{k} = (n + i\kappa) \frac{\omega}{c}$$

$$\epsilon_0 (1 + \chi(\omega)) = \epsilon(\omega) \leftarrow \text{permittivity of a material}$$

$$\mathcal{N}(\omega) \equiv n(\omega) + i\kappa(\omega) = \sqrt{1 + \chi(\omega)}$$

compare!

$$\tilde{n} = \sqrt{1 + \chi(\omega)}$$

$$\tilde{n} = n(\omega) + i\kappa(\omega)$$

→ light "slows down"

→ absorption coefficient

$$\tilde{n} = n' + i n''$$

Let's plug this back in to \vec{E}

$$\begin{aligned} \vec{E}(\vec{r}, t) &= \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = \vec{E}_0 e^{i(\tilde{n} + i\tilde{\kappa}) \frac{\omega}{c} \hat{u} \cdot \vec{r} - i\omega t} \\ &= \vec{E}_0 e^{-\kappa \frac{\omega}{c} \hat{u} \cdot \vec{r} + i(n \frac{\omega}{c} \hat{u} \cdot \vec{r} - \omega t)} \end{aligned}$$

$$= \vec{E}_0 e^{-\frac{k\omega}{c} \hat{a} \cdot \vec{r}} e^{i(\frac{n\omega}{c} \hat{a} \cdot \vec{r} - \omega t)}$$

$$\vec{E}_0 = \vec{E}_0 e^{i\phi} \leftarrow \text{bad notation so watch out!}$$

$$= \vec{E}_0 e^{-\frac{k\omega}{c} \hat{a} \cdot \vec{r}} \cdot \cos\left(\frac{n\omega}{c} \hat{a} \cdot \vec{r} - \omega t + \phi\right) + \text{some imaginary stuff that we ignore}$$

this wave
decays as it
goes into this
material

$$V_{\text{phase}}(\omega) = \frac{c}{n(\omega)} \leftarrow \text{real part of } \tilde{n}$$

