Day 06 - Snow Day!
Day 07 - 260128 W
↳ Lots of demos

Day 08 - 260130 F

Chapter 1 - waves + review of E+M

function → $\dfrac{3}{10x^2+1} \xrightarrow{\text{wavify}} \dfrac{3}{10(x-vt)^2+1}$

So any function that can be written

$$\psi(x,t) = f(x \mp vt)$$

ψ ↑ wave function

$f(x \mp vt)$ ← specific function that has $x \mp vt$ in it,

$$f(\underbrace{x \mp vt}_{x'}) = f(x')$$

first derivative: $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial x'} \cdot \underbrace{\dfrac{\partial x'}{\partial x}}_{1} = \dfrac{\partial f}{\partial x'}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}$$

second derivative: $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial x}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial x'}\right)$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \frac{\partial x'}{\partial t} = \frac{\partial f}{\partial x'} \cdot (\mp v)$$

$$\frac{\partial f}{\partial t} = \mp v \frac{\partial f}{\partial x'}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$\dfrac{\partial f}{\partial t} = \mp v \dfrac{\partial f}{\partial x'}$

$$\frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1\text{, again!}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$= \mp v \frac{\partial^2 f}{\partial x'^2} \cdot (\mp v)$$

$$\frac{\partial^2 f}{\partial t^2} = \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial^2 f}{\partial x'^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}}$$

<u>wave</u> <u>equation</u>

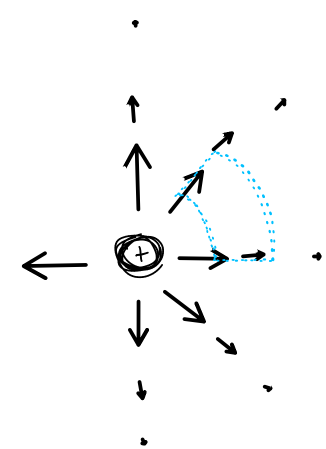
compare to

→ heat equation

→ schrodinger's

→ Laplace

$\nabla^2 \phi = 0$



Let's go back to Maxwell:

$\rho \leftarrow$ charge density

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$ (Gauss's Law) $\leftarrow$ Electric field spreads out if there is charge

$$\vec{\nabla}\cdot\vec{B} = 0$$ (Gauss's Law for magnetism) $\leftarrow$ Magnetic fields do not spread out (no magnetic monopole — yet!?)

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ (Maxwell-Faraday's Law) $\leftarrow$ There is electric field if a magnetic field is changing

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \epsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}$$ (Ampere's Law) $\leftarrow$ There is magnetic field if there is current or if the electric field changes.

For the case of light: no charge
in vacuum no current

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \epsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}$$

$\rightarrow$ take the curl of Faraday's

$$\underbrace{\vec{\nabla}\times(\vec{\nabla}\times\vec{E})} = -\frac{\partial}{\partial t}\underbrace{(\vec{\nabla}\times\vec{B})}_{\epsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}\,!}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}\underbrace{(\vec{\nabla}\cdot\vec{E})}_{=0!} - \vec{\nabla}^2\vec{E}$$

$$\boxed{\vec{\nabla}^2\vec{E} = \epsilon_0\mu_0\frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\frac{1}{v^2}$$

$$\epsilon_0 \mu_0 = \frac{1}{v^2}$$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{m\,kg}{C}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{s^2 C^2}{m^2 kg}$$

$$\boxed{v = 2.998 \cdot 10^8 \frac{m}{s}}$$

Maxwell's Equations again

$$\vec{\nabla}\cdot\vec{E} = \frac{\vec{\nabla}\cdot\vec{P}}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\ \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

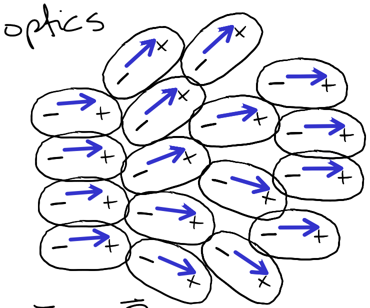
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}_{free} + \mu_0\frac{\partial\vec{P}}{\partial t} + \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

polarization has units of $\frac{\text{charge} \times \text{length}}{\text{volume}}$

$\vec{J}$ has units of $\frac{\text{charge} \times \text{velocity}}{\text{volume}}$

Charge density:

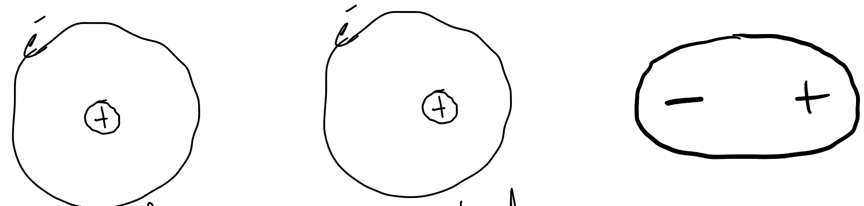
$$\rho = \underbrace{\cancel{\rho_{free}}}_{0\text{, for us in optics}} + \rho_P$$



$$\vec{\nabla}\cdot\vec{J}_P = \frac{\partial\rho_P}{\partial t} \longrightarrow \vec{J}_P = \frac{\partial\vec{P}}{\partial t}$$

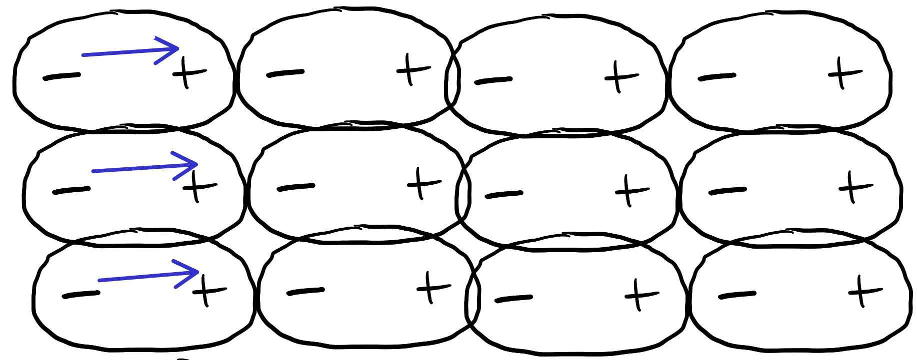
current density:

$$\vec{J} = \vec{J}_{free} + \underbrace{\cancel{\vec{J}_M}}_{0\text{, for us in optics}} + \vec{J}_P$$



neutral material; no polarization

neutral material polarization



$$\vec{J}_P = \frac{\partial\vec{P}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{J}_p = \frac{\partial \rho_p}{\partial t} \longrightarrow \vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

$$\frac{\partial(\vec{\nabla}\cdot\vec{P})}{\partial t} = \frac{\partial \rho_p}{\partial t}$$

$$\vec{\nabla}\cdot\vec{P} = \rho_p$$

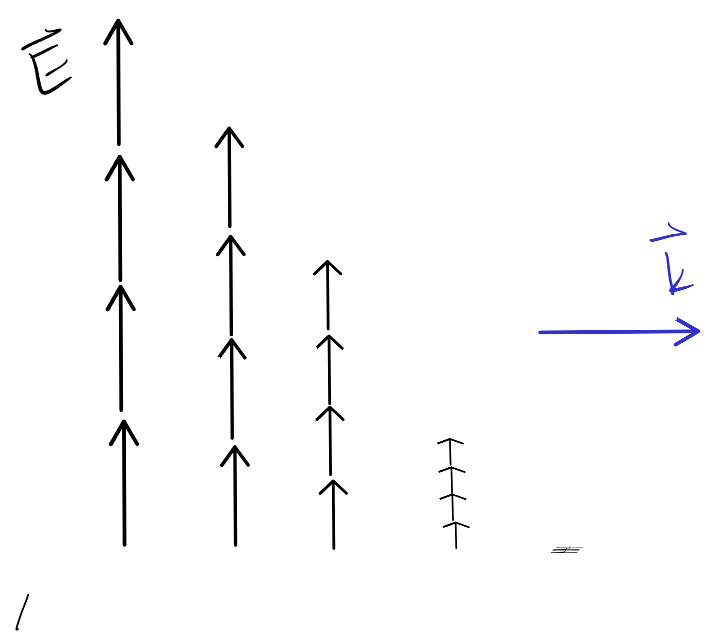
$$\underbrace{\vec{\nabla}^2 E - \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}_{\text{wave equation that we derived before.}} = \underbrace{\mu_0\frac{\partial J_{free}}{\partial t}}_{\text{present when current of free charge are flowing.}} + \underbrace{\mu_0\frac{\partial^2 P}{\partial t^2}}_{\text{dipole currents}} - \overbrace{\frac{1}{\epsilon_0}\nabla(\vec{\nabla}\cdot\vec{P})}^{\text{crystals}}$$

wave equation that we derived before.

present when current of free charge are flowing. reflection from mirror. also light through plasma.

dipole currents
dipole oscillations

crystals
(anisotropic materials)
not isotropic



Homework: 1.2, 4, 5, 7, 9

P1.2 Suppose that an electric field is given by $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$, where $\mathbf{k} \perp \mathbf{E}_0$ and ϕ is a constant phase. Show that

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{k}\times\mathbf{E}_0}{\omega}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$

is consistent with (1.3).

↳ assume: $\vec{k} = k_x\hat{x} + 0\hat{y} + 0\hat{z}$

↳ assume: $\vec{r} = x\hat{x} + 0\hat{y} + 0\hat{z}$

↳ $\vec{E}_0 = 0\hat{x} + E_{0y}\hat{y} + 0\hat{z}$

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}} \qquad \text{(Faraday's Law)} \qquad (1.3)$$

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \qquad \text{(Ampere's Law revised by Maxwell)} \qquad (1.4)$$

$$\vec{\nabla}\times\vec{E} = \vec{\nabla}\times\left(\vec{E}_0\cos(\vec{k}\cdot\vec{r} - \omega t + \phi)\right)$$

can: $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{E} = \hat{x}\underbrace{\left(-\frac{\partial E_y}{\partial z}\right)}_{0} - \hat{y}(0) + \hat{z}\underbrace{\left(\frac{\partial E_y}{\partial x}\right)}$$

$$-\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}\left(\frac{\vec{k}\times\vec{E}_0}{\omega}\cos(\vec{k}\cdot\vec{r} - \omega t + \phi)\right)$$

$$= -\frac{\vec{k}\times\vec{E}_0}{\omega}\left(-\sin(\vec{k}\cdot\vec{r} - \omega t + \phi)\cdot(-\omega)\right)$$

$$= \underbrace{-\vec{k}\times\vec{E}_0}\sin(\vec{k}\cdot\vec{r} - \omega t + \phi)$$

$$= \qquad \sin(k_x \cdot x - \omega t + \phi)$$

$$\vec\nabla \times \vec E = \hat z \left(\frac{\partial}{\partial x}\left(E_{oy}\cos(k_x x - \omega t + \phi)\right)\right)$$

$$= -\hat z\, E_{oy}\sin(k_x x - \omega t + \phi)\cdot k_x$$

$$= -E_{oy}\cdot k_x \cdot \sin(k_x x - \omega t + \phi)\,\hat z$$

$$\vec k \times \vec E = \begin{vmatrix} \hat x & \hat y & \hat z \\ k_x & 0 & 0 \\ 0 & E_{oy} & 0 \end{vmatrix}$$

$$= k_x E_{oy}\,\hat z$$

$$= -E_{oy}\,k_x \sin(k_x x - \omega t + \phi)\,\hat z$$

P1.4 Check that the **E** and **B** fields in P1.2 satisfy the rest of Maxwell's equations:

(a) (1.1). What must ρ be?

(b) (1.2).

(c) (1.4). What must **J** be?

$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	(Gauss' Law)	(1.1)
$\nabla \cdot \mathbf{B} = 0$	(Gauss' Law for magnetism)	(1.2)
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(Faraday's Law)	(1.3)
$\nabla \times \frac{\mathbf{B}}{\mu_0} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}$	(Ampere's Law revised by Maxwell)	(1.4)

P1.7 Show that the magnetic field in P1.2 is consistent with the wave equation derived in P1.6. What is the requirement on k and ω?