Chapter 0 – Mathematical Review

$$\boxed{\frac{\vec{r}}{|\vec{r}|} = \hat{r}}$$

$C \equiv$ Coulomb

Vectors

unit vectors

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r} = \frac{kq\vec{r}}{r^3}$$ } fine, but incomplete → it assumes the charge is located at the origin

$\left[\frac{N}{C}\right]$

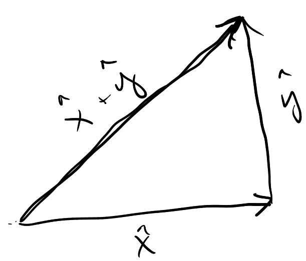
$9 \cdot 10^9 \frac{Nm^2}{C^2}$

$$k = \frac{1}{4\pi\epsilon_0}$$

$8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$

more general

$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(\vec{r} - \vec{r}_0)}{|\vec{r} - \vec{r}_0|^3}}$$



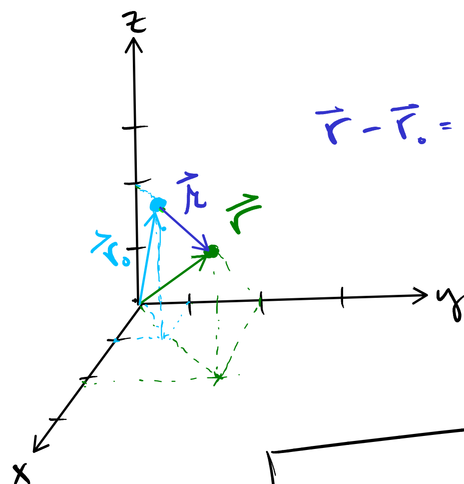
$\vec{r} \Rightarrow$ location where we want to know the electric field

$\vec{r}_0 = \vec{r}$ naught → location of the charge

Example 0.1

$$\vec{r} = (2\hat{x} + 2\hat{y} + 2\hat{z})$$

$$\vec{r}_0 = (1\hat{x} + 1\hat{y} + 2\hat{z})$$

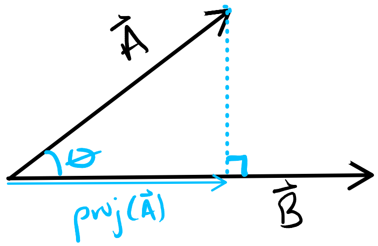


$\vec{r} - \vec{r}_0 = \vec{\mathscr{r}} \Rightarrow \vec{r}_0 + \vec{\mathscr{r}} = \vec{r}$

$$\vec{r} - \vec{r}_0 = 1\hat{x} + 1\hat{y} + 0\hat{z} = \hat{x} + \hat{y}$$

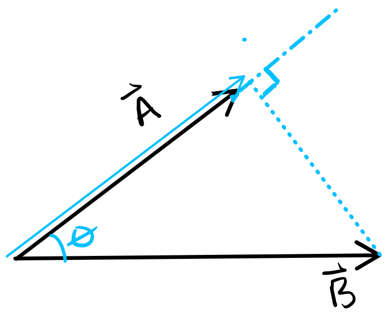
$$|\vec{r} - \vec{r}_0|^3 = |\hat{x} + \hat{y}|^3 = (\sqrt{2})^3 = 2^{3/2} = 2 \cdot \sqrt{2} \longrightarrow \boxed{\vec{E}(\vec{r}) = \frac{q(\hat{x} + \hat{y})}{8\pi\epsilon_0\sqrt{2}}}$$

Dot Product (inner product)



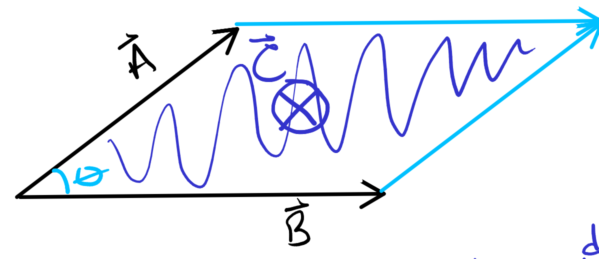
$$\vec{A}\cdot\vec{B} = \overbrace{A_xB_x + A_yB_y + A_zB_z}^{\text{scalar}}$$

$$= |A|\cdot|B|\cdot\cos\theta$$



Cross Product (vector product)

(perp. to both $\vec{A}$ + $\vec{B}$)



determinant

$$\underbrace{\vec{C}}_{\text{vector}} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

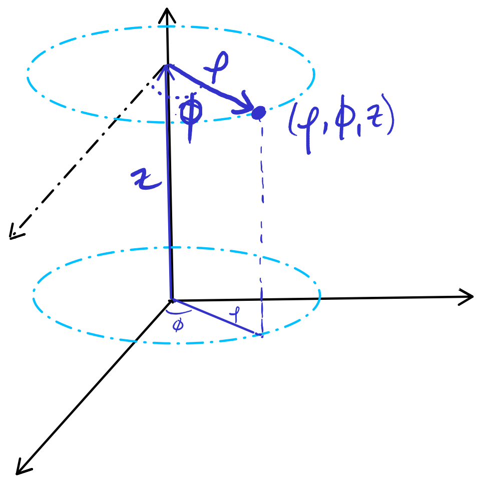
$$= \hat{x}(A_yB_z - A_zB_y)$$

$$-\hat{y}(A_xB_z - A_zB_x)$$

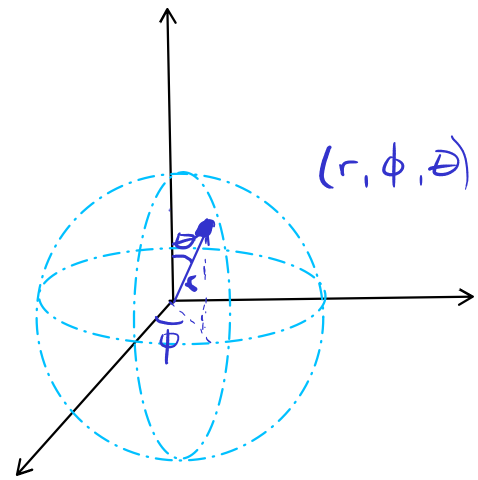
$$+\hat{z}(A_xB_y - A_yB_x)$$

$$|\vec{C}| = |\vec{A}\times\vec{B}| = |A|\cdot|B|\cdot\sin\theta$$

Cylindrical Coords.



Spherical Coords.



Gradient: of a scalar function

(directional derivative)

$$\vec{\nabla} f(x,y,z) = \underbrace{\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}}_{\text{vector result}}$$

scalar function

vector result points in the direction of steepest decline

Divergence: of a vector function

$$\vec{\nabla} \cdot \vec{E} = \underbrace{\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}}_{\text{scalar}}$$

Curl: of a vector function

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\underbrace{= \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{z}\left(\qquad \right)}_{\text{vector}}$$