

Thermal Equilibrium - when objects have been in contact
and macroscopic coordinates have stopped changing

↳ involves an exchange of energy between two object, or an object + its surrounding

↳ volume (constant pressure) \rightarrow mercury/alcohol

- pressure

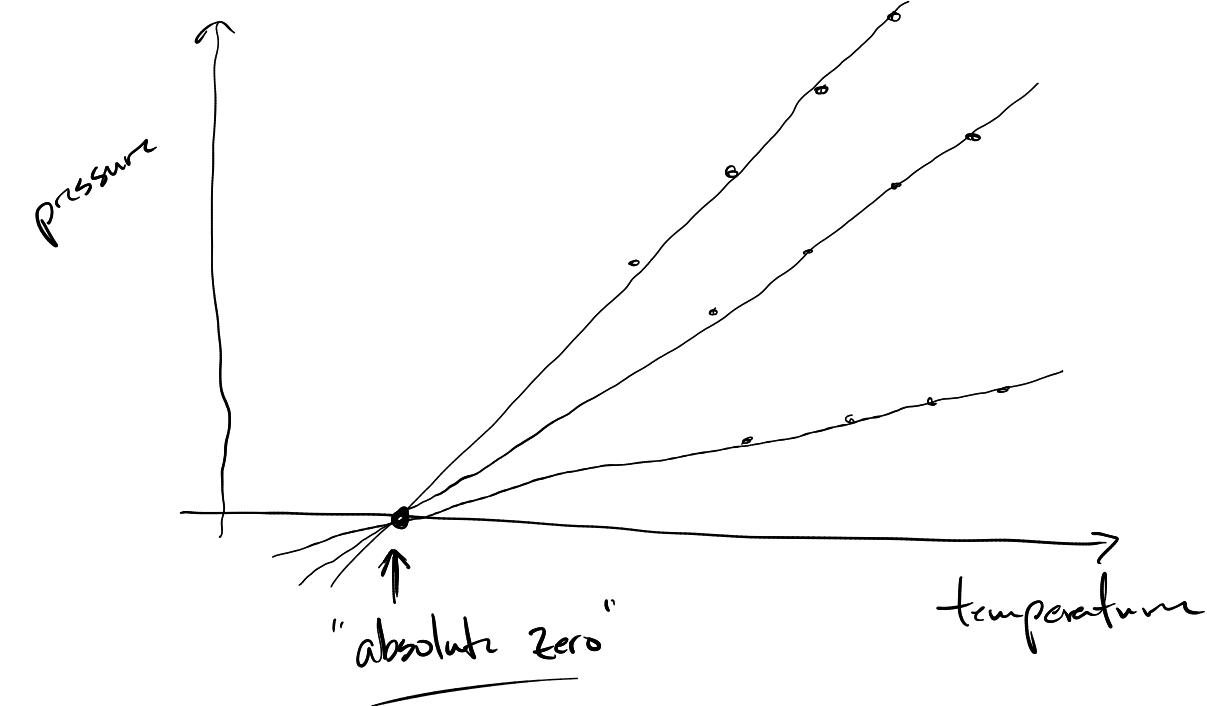
- electrical resistance

- radiation/thermal emf

$$\boxed{C_1 = 0^\circ C \quad C_2 = 100^\circ C}$$

$$F_1 = 32^\circ F \quad F_2 = 212^\circ F$$

$$C = mF + b$$



$$0K = -273.15^\circ C$$

1.2 Ideal Gas Law \rightarrow Equation of State \rightarrow relates all of the state variables together

$$PV = N k_B T$$

$P = \frac{F}{\text{Area}}$

$[P_a] = \left[\frac{N}{m^2} \right]$

number of particles

Boltzmann's constant

$$k_B = 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

$$PV = nRT$$

universal gas constant

number of moles

$$R = 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$N k_B = nR$$

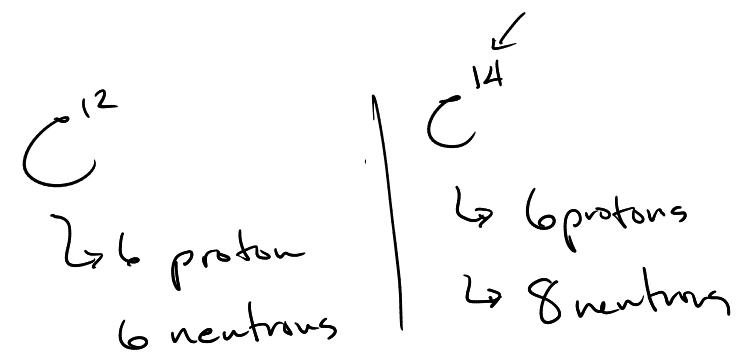
$$6.022 \cdot 10^{23} \cdot 1.38 \cdot 10^{-23} = R$$

$$8.31 \frac{\text{J}}{\text{mol K}} = R$$

Number, moles, molar, density

$$1 \text{ mole of things} = 6.022 \cdot 10^{23} \text{ things}$$

particle
atom
molecule



1 mole is 1 gram of protons + neutrons

Ex: mass of one proton in kg?

mass of one proton \times number of proton = mass of the collection

$$m \times N = M$$

$$M = \frac{1 \text{ g}}{N_A} = \frac{1}{6.022 \cdot 10^{23}} = 1.7 \cdot 10^{-24} \text{ grams} = 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2 ?

$$1 \text{ mole of } N_2 = 2 \left(\frac{14 \text{ g}}{\text{mol}} \right) = 28 \text{ g/mol}$$

Volume of 1 mole of air at room temp and atmospheric pressure?

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \text{ J/K} \cdot \text{mol} \cdot 300 \text{ K}}{10^5 \text{ Pa}} = 0.0249 \text{ m}^3$$

$$\hookrightarrow 1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$$

$$V_{\text{cube}} = D^3$$

$$D = 0.292 \text{ m} \sim 30 \text{ cm} \sim 1 \text{ ft}$$

1.17] $PV = nRT \left(1 + \frac{B(T)}{V_n} \right)$

a) $PV = nRT \left(1 + \frac{n}{V} \cdot B(T) \right)$

at atmospheric pressure $P = 10^5 \text{ Pa}$

Solve for $\frac{n}{V}$

$$P = \frac{n}{V} RT \left(1 + \frac{n}{V} B(T) \right)$$

$$0 = \frac{n}{V} RT + \left(\frac{n}{V} \right)^2 \cdot RT \cdot B(T) - P$$

T	B(T)	$\frac{n}{V}$	$\frac{n}{V} \cdot B(T)$
100	-160		
200	-35		
300	-4.2		
400	9.0		
500	16.9		
600	21.3		

b) $PV = nRT \left(1 + \frac{B(T)}{\sqrt{V_n}} \right)$ if

c) $\rightarrow PV = nRT \left(1 + \frac{B(T)}{\sqrt{V_n}} + \frac{C(T)}{(\sqrt{V_n})^2} \right)$

want: $B + C$ in terms of $a + b$

$$\left(P + \frac{an^2}{V^2} \right) \left(V - nb\frac{V}{V} \right) = nRT$$

$$\left(P + \frac{an^2}{V^2} \right) V \left(1 - \frac{nb}{V} \right) = nRT$$

$$\left(P + \frac{an^2}{V^2} \right) V = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV + \frac{an^2}{V} = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV = nRT \left(1 - \frac{nb}{V}\right)^{-1} - \frac{an^2}{V}$$

use our approx.

$$(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2 \quad px \ll 1$$

$$\left(1 + \left(-\frac{nb}{V}\right)\right)^{-1} \approx 1 + (-1) \left(-\frac{nb}{V}\right) + \frac{1}{2}(-1)(-1-1) \left(-\frac{nb}{V}\right)^2$$

$$\approx 1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2$$

$$PV = nRT \left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2\right) - \frac{an^2}{V} \frac{(nRT)}{(nRT)} \longleftrightarrow PV = nRT \left(1 + \frac{n}{V} \cdot B(T) + \left(\frac{n}{V}\right)^2 \cdot C(T)\right)$$

$$PV = nRT \left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2 - \frac{an^2}{V \cdot nRT}\right) = nRT \left[1 + \frac{n}{V} \left(b - \frac{a}{RT}\right) + \left(\frac{n}{V}\right)^2 \cdot b^2\right]$$

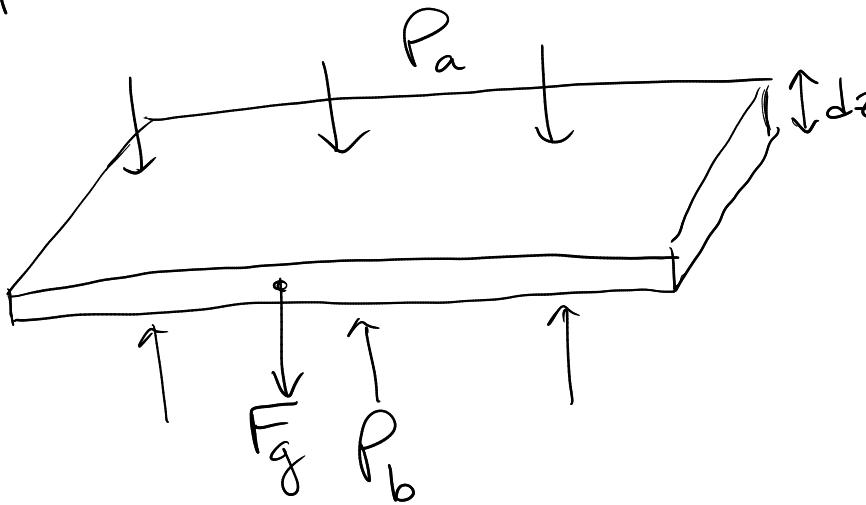
$B(T)$ $C(T)$

d) \Rightarrow plot data from that table

$$\rightarrow \text{plot } B(T) = b - \frac{a}{RT} \quad \text{choose } b, a$$

1.1b
a) $\rho = \frac{M}{V}$ \leftarrow total mass

"rho"
volumetric
mass
density



$$V = A \cdot dz$$

$$\rho = \frac{m}{A \cdot dz}$$

$$m = \rho A dz$$

$$P_i = \frac{F}{A}$$

$$+ P_b \cdot A - P_a \cdot A - M \cdot g = 0$$

$$P_b \cdot A - P_a \cdot A - \rho A dz \cdot g = 0$$

$$-(P_b + P_a) = \rho A dz \cdot g$$

$$-(P_a - P_b) = \rho A dz \Rightarrow \frac{dP}{dz} = -\rho g$$

$$\boxed{\frac{dP}{dz} = -\frac{\rho g P}{K_B T}}$$

b) $PV = N k_B T$

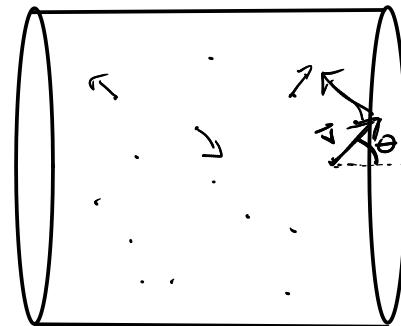
$$m P = \frac{N k_B T}{V}$$

$$m P = \rho k_B T$$

1.2 b] Kinetic Theory and Equipartition of Energy

pressure \longleftrightarrow kinetic energy \longleftrightarrow temperature

$$\frac{\text{pressure}}{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \quad \overbrace{2mv \cos \theta}$$



$$d(\text{pressure}) = \frac{2mv \cos \theta}{dA \cdot dt} \cdot \text{number of particles hitting area } dA \quad \left. \begin{array}{l} \text{w/ velocity } v \text{ in } dt \text{ amount of time} \\ \text{integrate over all velocities} \\ \text{and over } \theta \end{array} \right\}$$

number of atoms
traveling in a particular
direction w/ a particular
speed fraction of them
 that are within
 striking distance
 of the surface dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

$$N = \sum_{x=0}^{\infty} N(x)$$

↑
total outcomes

$$P(x) = \frac{N(x)}{N}$$

rearrange

$$N(x) = P(x) \cdot N$$

normalized probabilities

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

• average value
• expectation value

→ weighted average by probability

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

Example (Blundell 3.3)

a. $P(x) = \frac{e^{-m} m^x}{x!}$; show $\sum_{x=0}^{\infty} P(x) = 1$

$$\sum_{x=0}^{\infty} \frac{e^{-m} m^x}{x!} ?= 1$$

$$e^{-m} \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!} ?= 1$$

$$\sum_{x=0}^{\infty} \frac{m^x}{x!} ?= e^m \checkmark$$

b. Show $\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x) ?= m$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-m} m^x}{x!} ?= m$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} ?= m$$

Looked up Taylor Series:
 $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} ?= m e^m$$

$\underbrace{x \cdot (x-1) \cdot (x-2) \cdot (x-3) \cdots 1}_{\cancel{0 \cdot m}} + \frac{1 \cdot m}{1!} - + \frac{2 m^2}{2!} + \dots$

$$\sum_{x=1}^{\infty} \frac{x m^x}{x!} = \sum_{x=1}^{\infty} \frac{m^x}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m^{x+1-1}}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m \cdot m^{x-1}}{(x-1)!}$$

$x \cdot (x-1) \cdot (x-2) \cdots$

$x' = x - 1$
 $\hookrightarrow x = x' + 1$

$$m \cdot \sum_{x'=0}^{\infty} \frac{m^{x'}}{x'!}$$

drop the primes!

$$C. \langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= 0 \cdot \frac{109}{200} + 1 \cdot \frac{65}{200} + 2 \cdot \frac{22}{200} + 3 \cdot \frac{3}{200} + 4 \cdot \frac{1}{100} + 5 \cdot \frac{0}{200}$$

$$= 0.61 = \frac{122}{200}$$

$$P(x) = \frac{e^{-0.61} \cdot 0.61^x}{x!}$$

```

import numpy as np
import matplotlib.pyplot as plt
from scipy.special import factorial

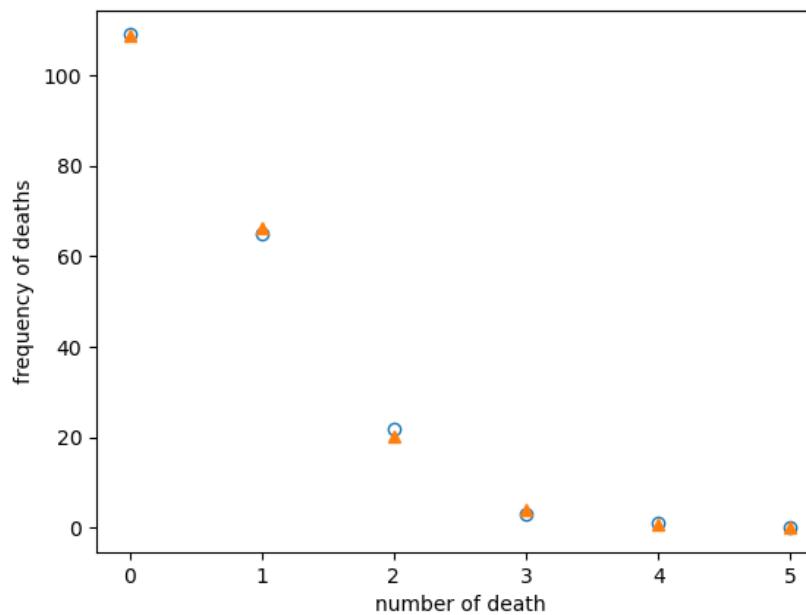
x = [0, 1, 2, 3, 4, 5]
y = [109, 65, 22, 3, 1, 0]

fig, ax = plt.subplots()
ax.plot(x, y, 'o', mfc='none')
ax.set_xlabel('number of death')
ax.set_ylabel('frequency of deaths')

def deaths(number):
    return(np.exp(-0.61)*np.power(0.61, number)/factorial(number))

ax.plot(x, deaths(x)*200, '^')

```



$$m \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!}$$

$$\underbrace{e^m}_{= e^m}$$

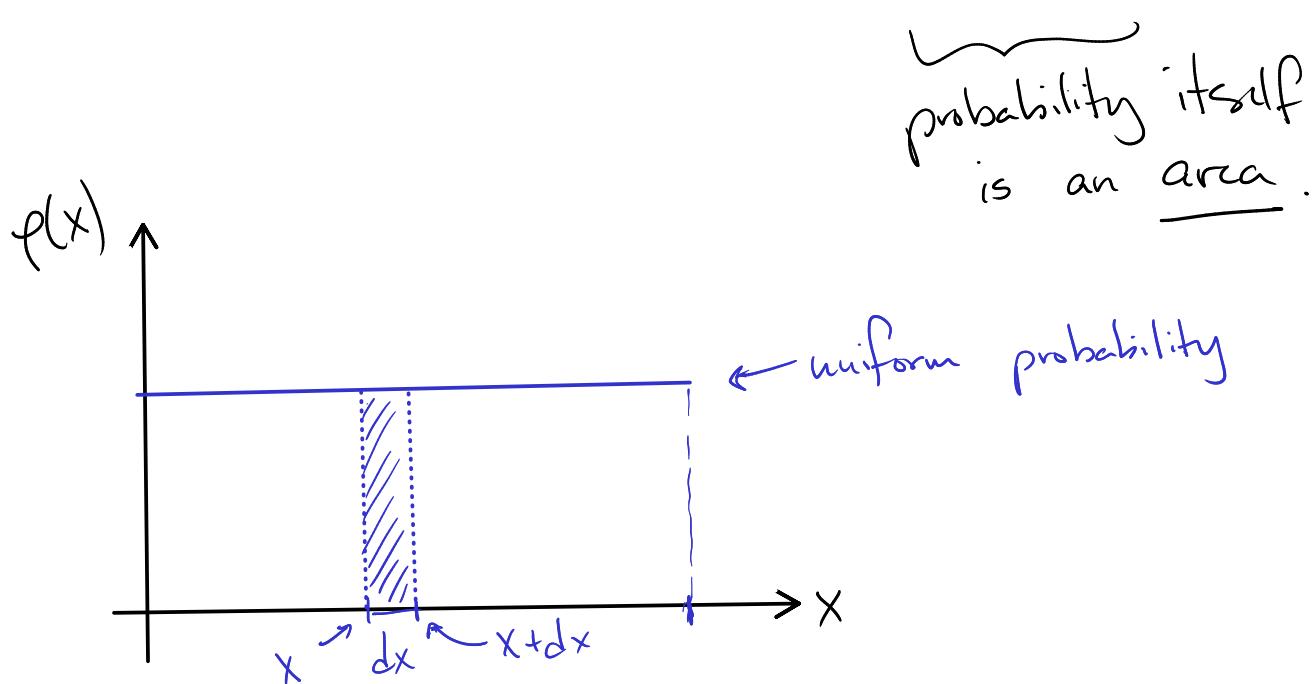
$$me^m ? \checkmark = me^m$$

$$\langle x \rangle = m$$

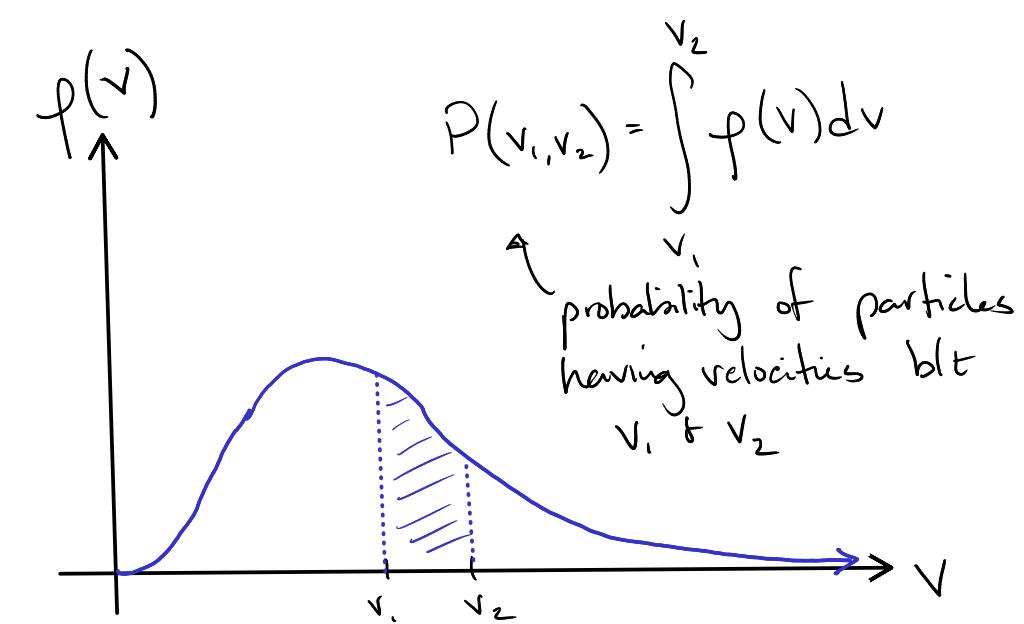
The continuous variable case follows by analogy but w/ some clarifications

$$\left\{ \begin{array}{l} \text{probability of choosing} \\ \text{a value at random} \\ \text{between } x \text{ and } x+dx \end{array} \right\} = f(x) \cdot dx$$

\hookrightarrow distribution function



probability itself
is an area.



Normalized

$$\int_{-\infty}^{\infty} f(v) dv = 1$$

Since we will be dealing w/ such large numbers

$$\left\{ \begin{array}{l} \text{probability that a} \\ \text{particle has a} \\ \text{velocity between} \\ v \text{ and } v+dv \end{array} \right\} = \left\{ \begin{array}{l} \text{fraction of particles} \\ \text{having a velocity} \\ \text{between } v + dv \end{array} \right\}$$

Example: (Blundell 3.4)

$$f(x)dx = A e^{-x/\lambda} dx$$

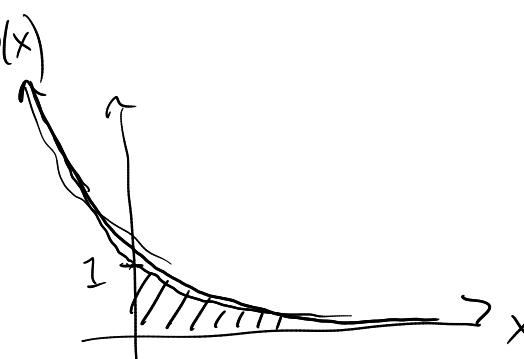
a) Find A so that

$$\int_0^\infty f(x)dx = 1$$

$$\int_0^\infty A e^{-x/\lambda} dx = 1$$

$$A(-\lambda) e^{-x/\lambda} \Big|_0^\infty = 1$$

$$A(-\lambda)(0 - -1) = 1$$



$$A\lambda = 1 \Rightarrow \boxed{A = \frac{1}{\lambda}}$$

Also mean

$$\langle v \rangle = \int_{-\infty}^{\infty} v \cdot f(v) dv$$

$$\text{or} \quad \langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 \cdot f(v) dv$$

$$\text{or} \quad \langle f(v) \rangle = \int_{-\infty}^{\infty} f(v) \cdot f(v) dv$$

$$\text{uv} \int_0^\infty - \int_0^\infty v du \quad u = x \quad dv = e^{-x/\lambda} dx \\ du = dx \quad v = -\lambda e^{-x/\lambda}$$

$$\text{b) } \langle x \rangle = \int_0^\infty x \cdot f(x) dx = \lambda$$

$$\int_0^\infty x \cdot \frac{1}{\lambda} e^{-x/\lambda} dx = \lambda$$

$$\frac{1}{\lambda} \left(-\lambda x e^{-x/\lambda} \Big|_0^\infty - (-\lambda) \int_0^\infty e^{-x/\lambda} dx \right)$$

$$c) \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$\sigma \rightarrow$ standard deviation

$\sigma^2 \rightarrow$ variance

$$\langle x \rangle = \lambda$$

$$\langle x \rangle^2 = \lambda^2$$

$$\langle x^2 \rangle = \int_0^\infty x^2 \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx$$

↪ LOOK IT UP!

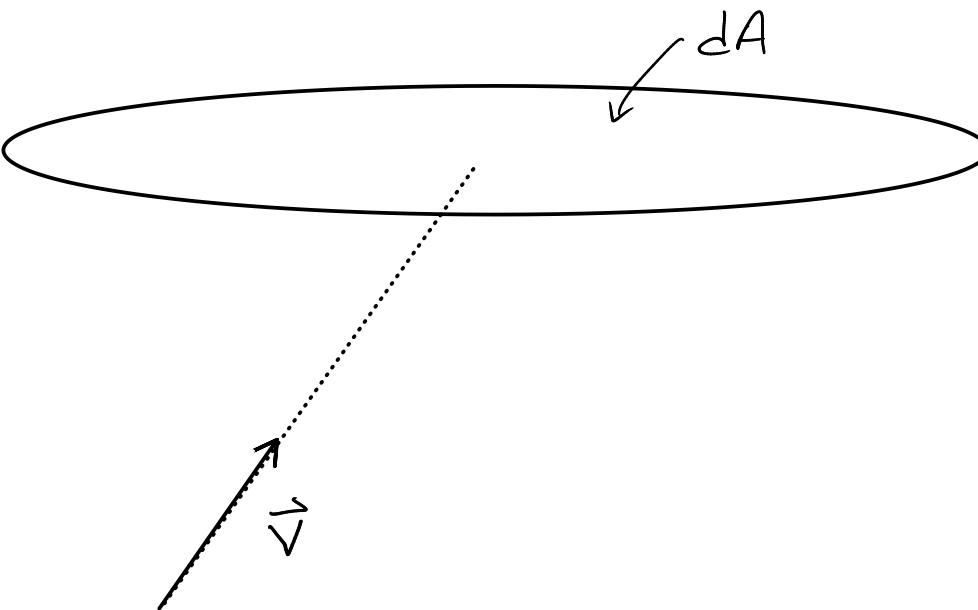
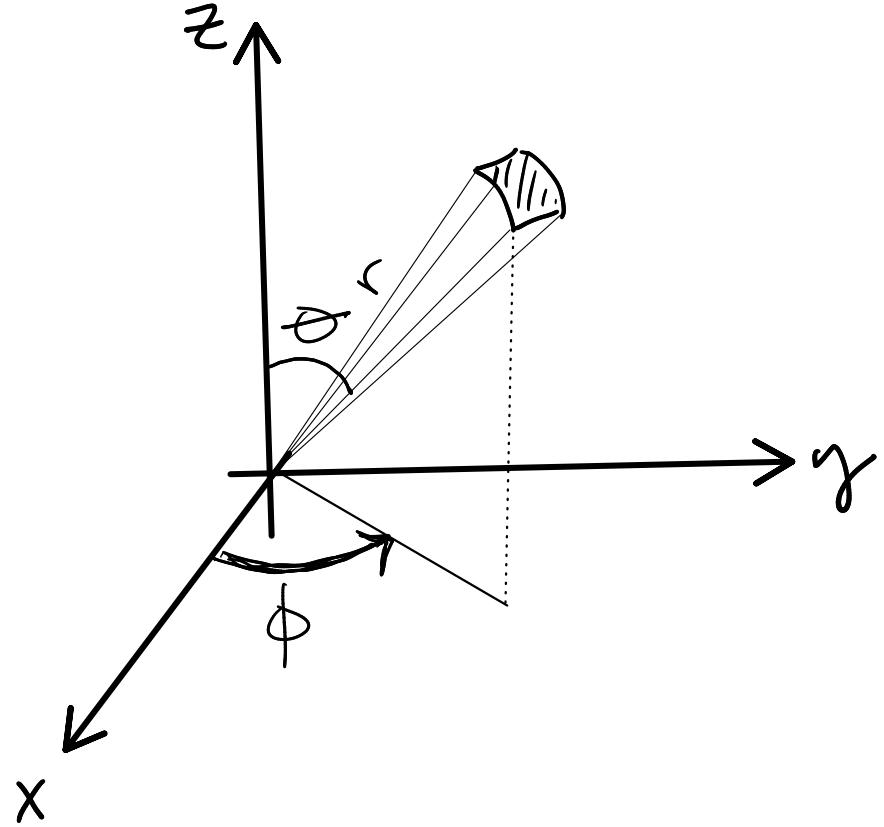
$$\langle x^2 \rangle = 2\lambda^2$$

$$\sigma^2 = 2\lambda^2 - \lambda^2 = \lambda^2 \leftarrow \text{variance}$$

$\sigma = \lambda \leftarrow$ standard deviation

$$\lambda = \langle x \rangle \checkmark$$

number of particles
traveling in a particular
direction w/ a
particular speed



$$U = \frac{3}{2} N k_B T$$

internal
energy

$$U = N \langle K \rangle$$

↑
avg of kinetic energy

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$= \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

$$= \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right)$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

↑ ↑ ↓

$$\frac{3}{2} k_B T = \frac{3}{2} m \langle v_x^2 \rangle$$

$$\frac{1}{2} k_B T = \frac{1}{2} m \langle v_x^2 \rangle$$

average of a
sum is the
sum of
averages

Equipartition Theorem

$$U_{\text{thermal}} = N \cdot f \cdot \frac{1}{2} k_B T$$

↗ degree of freedom (quadratic)

$$\frac{1}{2} m v_x^2, \frac{1}{2} m v_y^2, \frac{1}{2} m v_z^2, \frac{1}{2} I \omega_x^2, \frac{1}{2} k_s x^2$$

$f = 3 \leftarrow$ monoatomic gas

$f = 5 \leftarrow$ diatomic gas near room temp

$f = 6 \leftarrow$ solid

1.23

$$\nabla = 1 \text{ liter} = 0.001 \text{ m}^3 = 10^{-3} \text{ m}^3$$

$$p = 1.01 \cdot 10^5 \text{ Pa} = 10^5 \text{ Pa}$$

monoatomic $T = 293 \text{ K}$

$$\rightarrow U = \frac{3}{2} N k_B T \quad PV = N k_B T$$

$$U = \frac{3}{2} p \nabla = \frac{3}{2} (10^5 \text{ Pa}) (10^{-3} \text{ m}^3)$$

$$= 1.5 \cdot 10^2 \text{ J} = \underline{\underline{150 \text{ J}}}$$

diatomic

$$U = \frac{5}{2} N k_B T = \frac{5}{2} p \nabla$$

$$= 2.5 \cdot 10^2 = \underline{\underline{250 \text{ J}}}$$

$$PV = nRT$$

$$\frac{10^5 \text{ Pa} \cdot 10^{-3} \text{ m}^3}{8.31 \cdot 293} = 0.04 \text{ mol}$$

1.24

$$f = 6$$

$$U = \frac{6}{2} N k_B T$$

$$U = 3 N k_B T$$

$$U = 3 n_m \cdot R T = 3 \cdot 0.0048 \text{ mol} \cdot 8.31 \text{ J} \cdot \sum_{Km=1}^{23} 293 = \underline{\underline{35 \text{ J}}}$$

1 g of Pb

207 g
mol

$$\frac{1 \text{ gr}}{207 \text{ gr/mol}} = \frac{0.0048 \text{ mol}}{1 \text{ mol}} \cdot \frac{6.022 \cdot 10^{23} \text{ atoms}}{1 \text{ mol}}$$

1.4 Heat and Work

spontaneous

flow of energy
due to a difference
in temperature

- force applied over distance
- energy through resistor

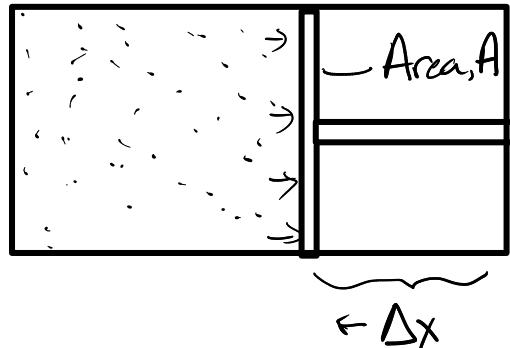
Equation: $dU = \oint Q + \oint W$

↑
small amount

NOT a small change
in amount

$$dQ + dW \mid Q + W$$

Compressive Work



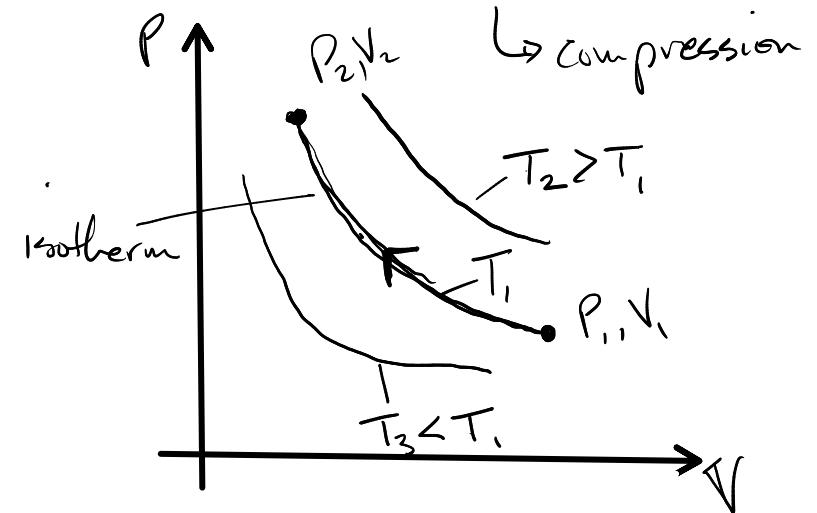
$$dW = \vec{F} \cdot d\vec{r} = \vec{F} \cdot \vec{\Delta x} = P \cdot A \cdot \Delta x$$

$\underbrace{-\Delta V}$ *quasistatic, static-ish*

$$dW = -P dV = -P \Delta V$$

$$dW = -P(V) dV$$

Isothermal process (constant temperature)



$$PV = Nk_B T$$

all constant

$$\left| P = \frac{Nk_B T}{V} \right| \leftarrow \text{hyperbola}$$

$$W = - \int p dV = - Nk_B T \int \frac{1}{V} dV$$

Constants
in this
case

$$\ln \left[\left(\frac{V_f}{V_i} \right)^{-1} \right]$$

$$W = Nk_B T \ln \left(\frac{V_i}{V_f} \right)$$

$$dU = \cancel{\delta Q} + \delta W$$

$$\delta Q = - \cancel{\delta W} \quad \xrightarrow{\text{isothermal}}$$

$$Q = - Nk_B T \ln \left(\frac{V_i}{V_f} \right)$$

$$Q = Nk_B T \ln \left(\frac{V_f}{V_i} \right)$$

$$U = \frac{f}{2} Nk_B T$$

$$\Delta U = \frac{f}{2} Nk_B \Delta T$$

$$dU = \frac{f}{2} Nk_B dT$$

$$dT = 0 \therefore dU = 0$$

Adiabatic process (Isentropic process) (no heat)

↳ compression

first law $\rightarrow dU = \cancel{dQ} + dW$

$$dU = dW \quad \text{for an ideal gas: } U = \frac{f}{2} N k_B T$$

$$\frac{f}{2} N k_B dT = dW$$

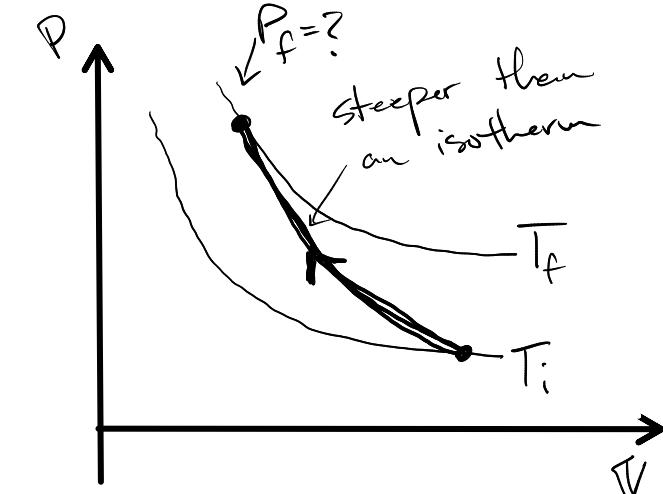
$$\frac{f}{2} N k_B dT = - P dV$$

$$\frac{f}{2} N k_B dT = - \frac{N k_B T}{V} dV$$

$$\frac{f}{2} dT = - \frac{T}{V} dV$$

$$\frac{f}{2} \frac{dT}{T} = - \frac{dV}{V}$$

$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = - \ln\left(\frac{V_f}{V_i}\right)$$



$$dU = \frac{f}{2} N k_B dT$$

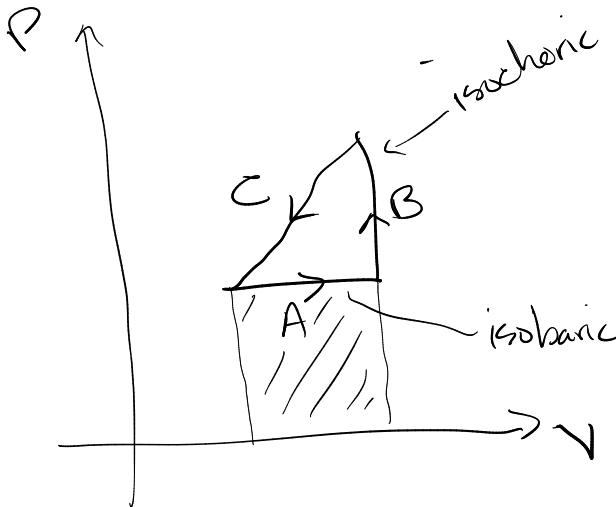
$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\rightarrow \left(\frac{T_f}{T_i}\right)^{\frac{f}{2}} = \frac{V_i}{V_f} \rightarrow V \propto \frac{1}{T^{\frac{f}{2}}} \\ V \propto T^{-\frac{f}{2}}$$

$$T_f^{\frac{f}{2}} \cdot V_f = T_i^{\frac{f}{2}} \cdot V_i \rightarrow V \cdot T^{\frac{f}{2}} = \text{constant of proportionality}$$

HW: 32, 33, 35 ←

1.33



A: $W < 0$

$Q = ?$

$$\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} P \Delta V$$

$$\Delta U > 0$$

$Q > 0$

$$N k_B \Delta T = P \Delta V$$

$$\Delta U = Q + W$$

↑

↑

↓

1.35

$$\sqrt{V T^{f/2}} = \text{a constant}$$

$$PV = \underline{N k_B T}$$

↑
constant

$$P V T^{-1} = \text{constant}$$

$$\gamma = \frac{f+2}{f}$$

↓

34, 36, 38

1.4 Heat Capacity

$$dU = \cancel{dQ} + \underbrace{\cancel{dW}}_{-PdV}$$

$$\cancel{dQ} = dU + PdV$$

$$\downarrow \underline{U(T, V)} \text{ or } U(T, P)$$

$$dU(T, V) = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad \leftarrow$$

$$\cancel{dQ} = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

$$\frac{\cancel{dQ}}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] \frac{dV}{dT}$$

Two cases: ① Constant volume
for a gas ② Constant pressure

① constant volume

$$dV = 0$$

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT \quad \text{or} \quad \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

\uparrow
heat capacity
at constant volume

For an ideal gas

$$U = \frac{f}{2} N k_B T$$

→ macroscopic form

$$U = \frac{f}{2} n_m R T$$

$$\left(\frac{\partial U}{\partial T}\right)_V = \frac{dU}{dT} = \frac{f}{2} n_m \cdot R = C_V$$

$$\frac{C_V}{n_m} = \frac{f}{2} \cdot R$$

heat capacity per mole
(molar specific heat)

monatomic
 $f=3$

$$\frac{C_V}{n_m} = \frac{3}{2} R$$

$$= 12.5 \text{ J/mol K}$$

diatomic
 $f=5$

$$\frac{C_V}{n_m} = \frac{5}{2} R$$

$$= 20.8 \text{ J/mol K}$$

② constant pressure

$$C_p \equiv \left(\frac{dQ}{dT} \right)_p = \left(\frac{\partial U}{\partial T} \right)_V + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

}

C_V

$$\rightarrow C_p = C_V + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

$$\rightarrow C_p - C_V = \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

for an ideal gas:

$$C_p = \frac{f}{2} n_m R + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

$$U = \frac{f}{2} n_m R \cdot T$$

$$\left(\frac{\partial U}{\partial V} \right)_T = 0$$

for an ideal gas

$$V = \frac{n_m R T}{P}$$

$$\left(\frac{\partial V}{\partial T} \right)_P = \frac{n_m R}{P}$$

$$C_p = \frac{f}{2} n_m R + n_m R$$

monatomic gas

$$f=3$$

$$C_p \approx 20.7$$

$$\frac{C_p}{n_m} = \frac{f}{2} R + R$$

$$\frac{C_p}{n_m} = \frac{C_V}{R} + R$$

diatom gas

$$C_p \approx 29$$

specific heat \rightarrow heat capacity per unit of mass

$$\hookrightarrow C_p = \frac{C_p}{M} \leftarrow \text{heat capacity}$$

\nwarrow total mass

$$C_v = \frac{C_v}{M} \leftarrow$$

big size

little size

Latent Heat

$$\frac{dQ}{dT} = \infty$$

\nwarrow O

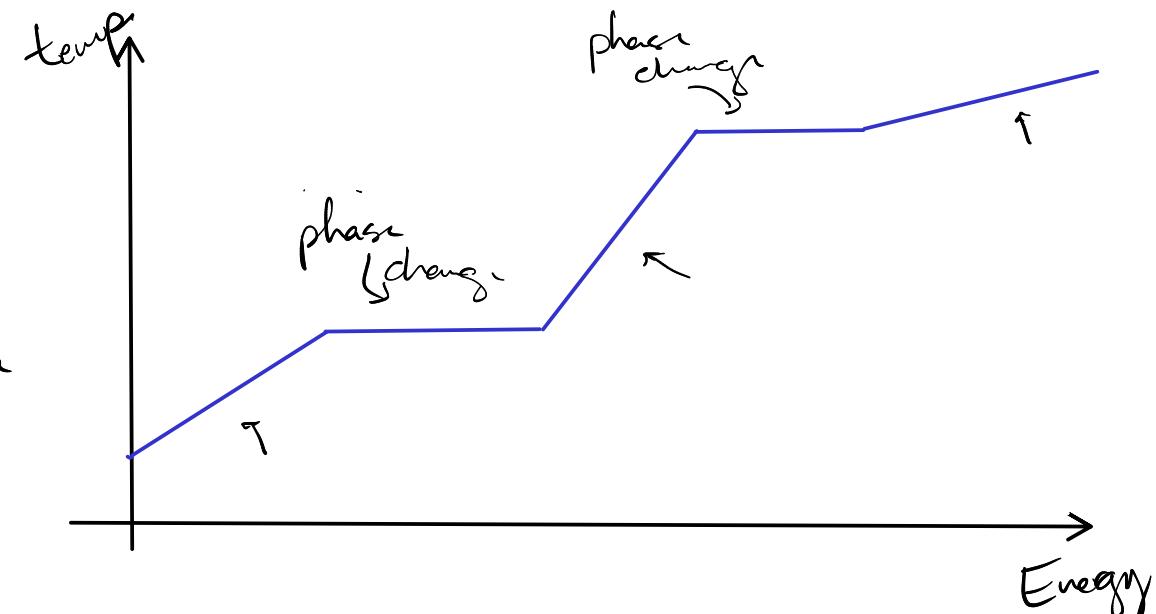
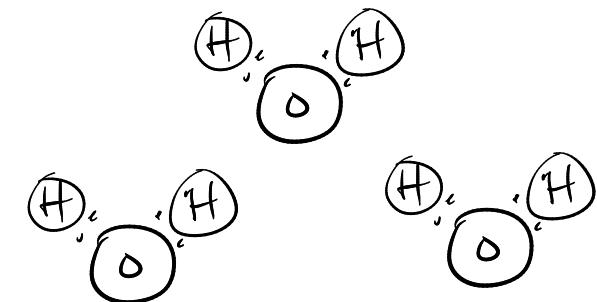
$Q \leftarrow$ total heat to completely change the phase
(depends on size)

$$\frac{Q}{m} = L \leftarrow \text{Latent heat}$$

(fusion, vaporization)

volumetric specific heat

$$= \frac{C_{v/p}}{\nabla}$$



41, 45, 47

$$C_p = 75.29 \frac{\text{J}}{\text{K} \cdot \text{mol}}$$

41] $\sum \Delta U = 0$

$$\bar{C} = \frac{\Delta U}{\Delta T}$$

a) $\Delta U_{H_2O} = (250 \text{ g}) \cdot (4.186 \frac{\text{J}}{\text{g} \cdot \text{K}}) \cdot (24^\circ\text{C} - 20^\circ\text{C})$

$$\Delta U = c \cdot m \cdot \Delta T$$

$$\Delta U = m \cdot C \cdot \Delta T$$

b) $\Delta U_{\text{metal}} = -\Delta U_{H_2O}$

c) $C = \frac{\Delta \bar{U}}{\Delta T} = \frac{\Delta U}{29 - 100}$

d) $\frac{C}{m} = c$

47] $\sum \Delta U = 0$

$$m_f C_f (65^\circ\text{C} - 100^\circ\text{C}) + \underbrace{ice \text{ warms}}_{+} + \underbrace{ice \text{ melts}}_{+} + \underbrace{water \text{ warms}}_{=} = 0$$

$$m_i \cdot C_{icw} (0 - (-15)) + m_i \cdot L_f + M_i \cdot C_w (65^\circ\text{C} - 0^\circ\text{C}) = 0$$

$\underbrace{333 \frac{\text{J}}{\text{g}}}_{}$

