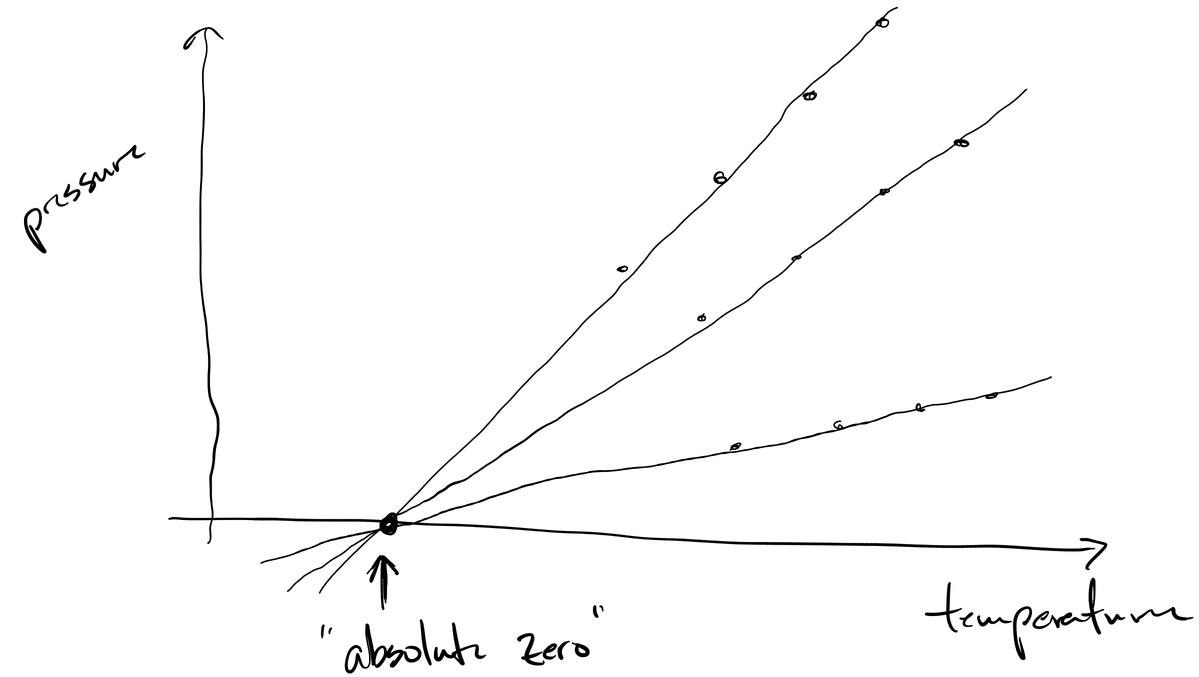
Thermal Equilibrium - when objects have been in contact and macroscopic coordinates have stopped changing

→ involves an exchange of energy between two object, or an object + its surrounding

→ volume (constant pressure) → mercury/alcohol

- pressure
- electrical resistance
- radiation/thermal emf



$0K = -273.15°C$

1) $C_1 = 0°C$ $C_2 = 100°C$

$F_1 = 32°F$ $F_2 = 212°F$

$C = mF + b$

1.2 Ideal Gas Law → Equation of State → relates all of the state variables together

$$pV = N k_B T$$

$p = \frac{F}{\text{Area}}$

$[Pa] = \left[\frac{N}{m^2}\right]$

N → number of particles

k_B → Boltzmann's constant

$k_B = 1.38 \cdot 10^{-23}\ J/K$

T → K

$$pV = nRT$$

n → number of moles

R → universal gas constant

$R = 8.31\ J/\text{mol}\cdot K$?

$$N k_B = nR$$

$$6.022 \cdot \cancel{10^{23}} \cdot 1.38 \cdot \cancel{10^{-23}} = R$$

$$8.31\ J/\text{mol}\,K = R$$

Number, moles, molar, density

1 mole of things = $6.022 \cdot 10^{23}$ things

particle
atom
molecule

C^{12}
→ 6 proton
6 neutrons

C^{14}
→ 6 protons
→ 8 neutrons

1 mole is 1 gram of protons + neutrons

Ex: mass of one proton in kg?

mass of one proton x number of proton = mass of the collection

$$m \times N = M$$

$$m = \frac{1g}{N_A} = \frac{1}{6.022 \cdot 10^{23}} = 1.7 \cdot 10^{-24} \text{ grams} = 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2?

$$1 \text{ mole of } N_2 = 2\left(14 \tfrac{g}{mol}\right) = 28 \text{ g/mol}$$

Volume of 1 mole of air at room temp and atmospheric pressure?

↳ $1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \text{ J/K·mol} \cdot 300\text{K}}{10^5 \text{ Pa}} = 0.0249 \text{ m}^3$$

$$V_{cube} = s^3$$

$$s = 0.292\ m \sim 30\ cm \sim 1\ ft$$

1.17

$$PV = nRT\left(1 + \frac{B(T)}{V/n}\right)$$

a)

$$PV = nRT\left(1 + \frac{n}{V} \cdot B(T)\right)$$

at atmospheric pressure $P = 10^5\ Pa$

solve for $\frac{n}{V}$

$$P = \frac{n}{V} RT\left(1 + \frac{n}{V} B(T)\right)$$

$$0 = \frac{n}{V} RT + \left(\frac{n}{V}\right)^2 \cdot RT \cdot B(T) - P$$

T	$B(T)$	$\frac{n}{V}$	$\frac{n}{V} \cdot B(T)$
100	-160		
200	-35		
300	-4.2		
400	9.0		
500	16.9		
600	21.3		

b) $$PV = nRT\left(1 + \frac{B(T)}{V/n}\right) \quad \text{if}$$

c) $\rightarrow$ $$PV = nRT\left(1 + \frac{B(T)}{V/n} + \frac{C(T)}{(V/n)^2}\right)$$

$$\left(P + \frac{an^2}{V^2}\right)\left(V - nb\frac{V}{V}\right) = nRT$$

<u>want</u>: B + C in terms of a + b

$$\left(P + \frac{an^2}{V^2}\right)V\left(1 - \frac{nb}{V}\right) = nRT$$

$$\left(P + \frac{an^2}{V^2}\right)V = nRT\left(1 - \frac{nb}{V}\right)^{-1}$$

$$PV + \frac{an^2}{V} = nRT\left(1 - \frac{nb}{V}\right)^{-1}$$

$$PV = nRT\underbrace{\left(1-\frac{nb}{V}\right)^{-1}}_{\text{use our approx.}} - \frac{an^2}{V}$$

$$(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2 \qquad px \ll 1$$

$$\left(1+\left(-\frac{nb}{V}\right)\right)^{-1} \approx 1 + (-1)\left(-\frac{nb}{V}\right) + \frac{1}{2}(-1)(\overset{-2}{-1-1})\left(-\frac{nb}{V}\right)^2$$

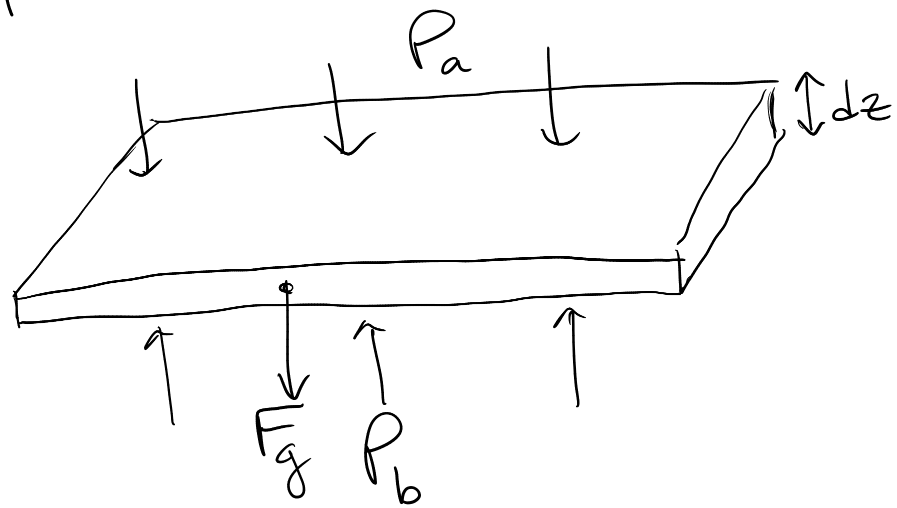
$$\approx \underbrace{1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2}$$

$$PV = nRT\left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2\right) - \frac{an^2}{V}\left(\frac{nRT}{nRT}\right) \longleftrightarrow PV = nRT\left(1 + \frac{n}{V}\cdot B(T) + \left(\frac{n}{V}\right)^2\cdot C(T)\right)$$

$$PV = nRT\left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2 - \frac{an^{\cancel{2}}}{V\cdot\cancel{n}RT}\right) = nRT\left[1 + \frac{n}{V}\underbrace{\left(b-\frac{a}{RT}\right)}_{B(T)} + \left(\frac{n}{V}\right)^2\cdot\underbrace{b^2}_{C(T)}\right]$$

d) → plot data from that table

→ plot $B(T) = b - \frac{a}{RT}$ choose b, a



1.16

$$\rho = \frac{M}{V}$$

"rho"
volumetric mass density

$$P = \frac{F}{A}$$

$$+P_b \cdot A - P_a \cdot A - M \cdot g = 0$$

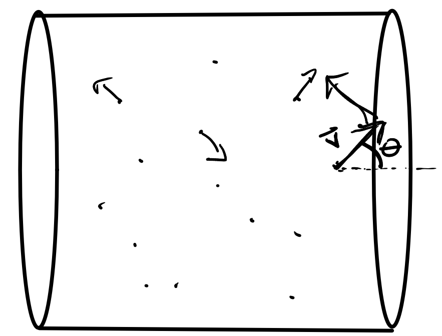
$$\frac{dP}{dz} = \underbrace{\qquad\qquad}_{\rho}$$

1.2b Kinetic Theory and Equipartition of Energy

pressure $\longleftrightarrow$ kinetic energy $\longleftrightarrow$ temperature

$$\underbrace{\text{pressure}}_{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A\,\Delta t}$$

Δp ← $\underbrace{2mv\cos\theta}$



$d(\text{pressure}) = \dfrac{2mv\cos\theta}{dA \cdot dt} \cdot$ $\underbrace{\text{number of particles hitting area } dA \text{ w/ velocity } v \text{ in } dt \text{ amount of time}}$ } integrate over all velocities and over θ

number of atoms traveling in a particular direction w/ a particular speed $\cdot$ fraction of them that are within striking distance of the surface dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

$$N = \sum_{x=0}^{\infty} N(x)$$

total outcomes

$$P(x) = \frac{N(x)}{N}$$

rearrange

$$N(x) = P(x) \cdot N$$

normalized probabilities

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \underbrace{\sum_{x=0}^{\infty} x \cdot P(x)}$$

- average value
- expectation value

weighted average by probability

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

Example (Blundell 3.3)

a. $P(x) = \frac{e^{-m} m^x}{x!}$; show $\sum_{x=0}^{\infty} P(x) = 1$

Looked up Taylor Series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sum_{x=0}^{\infty} \frac{e^{-m} m^x}{x!} \stackrel{?}{=} 1$$

$$e^{-m} \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!} \stackrel{?}{=} 1$$

$$\sum_{x=0}^{\infty} \frac{m^x}{x!} \stackrel{?}{=} e^m \checkmark$$

b. Show $\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x) \stackrel{?}{=} m$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-m} m^x}{x!} \stackrel{?}{=} m$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} \stackrel{?}{=} m$$

$$\sum_{x=0}^{\infty} \underbrace{\frac{x \cdot m^x}{x!}}_{} \stackrel{?}{=} m e^m$$

$$x \cdot (x-1) \cdot (x-2)(x-3) \cdots 1$$

$$\cancel{\frac{0 \cdot m^0}{0!}}^{\,0} + \frac{1 \cdot m^1}{1!} + \frac{2 m^2}{2!} + \ldots$$

$$\sum_{x=1}^{\infty} \frac{x m^x}{x!} = \sum_{x=1}^{\infty} \frac{m^x}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m^{x+1-1}}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m \cdot m^{x-1}}{(x-1)!}$$

$$x \cdot (x-1) \cdot (x-2) \ldots$$

$$x' = x - 1$$

$$x = x' + 1$$

$$m \cdot \sum_{x'=0}^{\infty} \frac{m^{x'}}{x'!}$$

drop the primes!

c.

$$m \cdot \underbrace{\sum_{x=0}^{\infty} \frac{m^x}{x!}}_{= e^m}$$

$$me^m \stackrel{?}{=} me^m$$

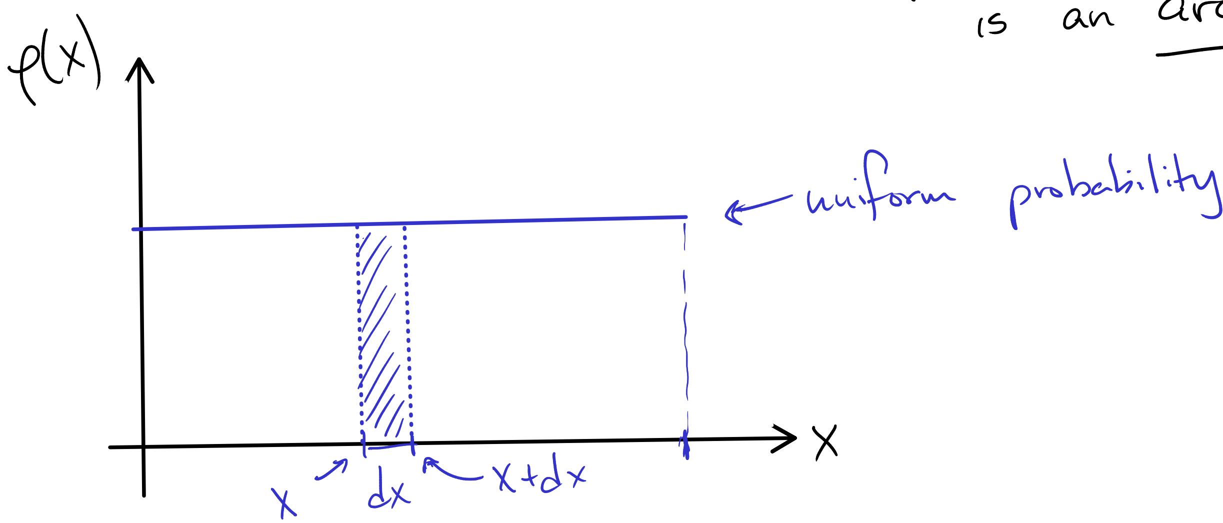
✓

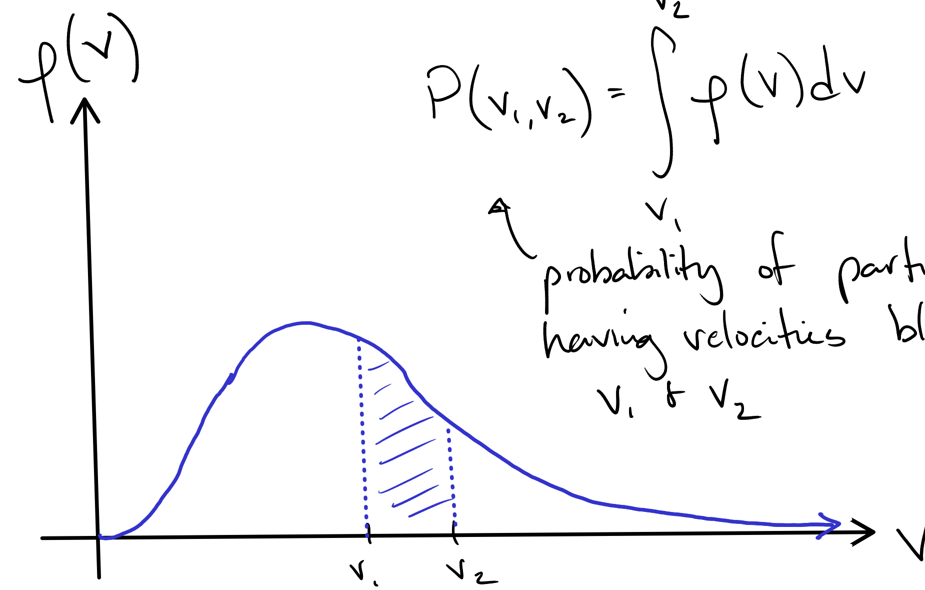
$$\langle x \rangle = m$$

The continuous variable case follows by analogy but w/ some clarifications

$$\left\{\begin{array}{l}\text{probability of choosing} \\ \text{a value at random} \\ \text{between } x \text{ and } x+dx\end{array}\right\} = \underbrace{\rho(x)\cdot dx}_{\text{probability itself is an area.}}$$

$\rho(x) \rightarrow$ distribution function





$$P(v_1,v_2) = \int_{v_1}^{v_2} \rho(v)\,dv$$

probability of particles having velocities b/t v_1 & v_2

Normalized

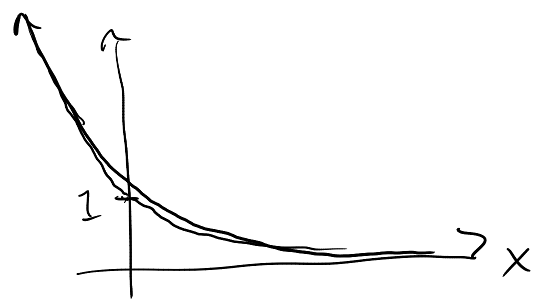
$$\int_{-\infty}^{\infty} \rho(v)\,dv = 1$$

Since we will be dealing w/ such large numbers

$$\left\{\begin{array}{c}\text{probability that a}\\ \text{particle has a}\\ \text{velocity between}\\ v \text{ and } v+dv\end{array}\right\} = \left\{\begin{array}{c}\text{fraction of particles}\\ \text{having a velocity}\\ \text{between } v \text{ \& } v+dv\end{array}\right\}$$

Example: (Blundell 3.4)

$$\rho(x)dx = Ae^{-x/\lambda}dx$$



a) Find A so that

$$\int_0^\infty \rho(x)dx = 1$$

Also mean

$$\langle v\rangle = \int_{-\infty}^{\infty} v\cdot\rho(v)dv$$

or

$$\langle v^2\rangle = \int_{-\infty}^{\infty} v^2\cdot\rho(v)dv$$

or

$$\langle f(v)\rangle = \int_{-\infty}^{\infty} f(v)\cdot\rho(v)dv$$