

Thermal Equilibrium - when objects have been in contact
and macroscopic coordinates have stopped changing

↳ involves an exchange of energy between two objects, or an object + its surroundings

↳ volume (constant pressure) → mercury/alcohol

- pressure

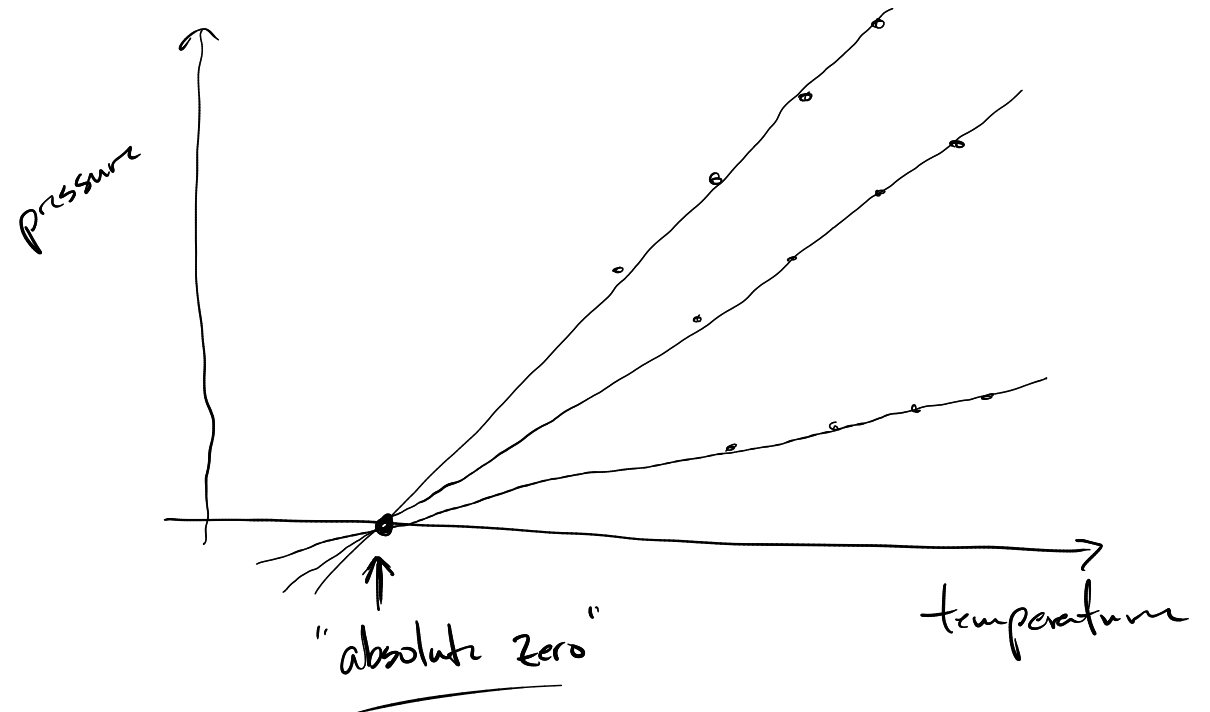
- electrical resistance

- radiation / thermal emf

$$1) \quad C_1 = 0^\circ\text{C} \quad C_2 = 100^\circ\text{C}$$

$$F_1 = 32^\circ\text{F} \quad F_2 = 212^\circ\text{F}$$

$$C = mF + b$$



$$0\text{K} = -273.15^\circ\text{C}$$

1.2 Ideal Gas Law \rightarrow Equation of State \rightarrow relates all of the state variables together

$$pV = N k_B T$$

$p = \frac{F}{\text{Area}}$
 $[Pa] = \left[\frac{N}{m^2} \right]$

N \rightarrow number of particles
 k_B Boltzmann's constant
 $k_B = 1.38 \cdot 10^{-23} \frac{J}{K}$

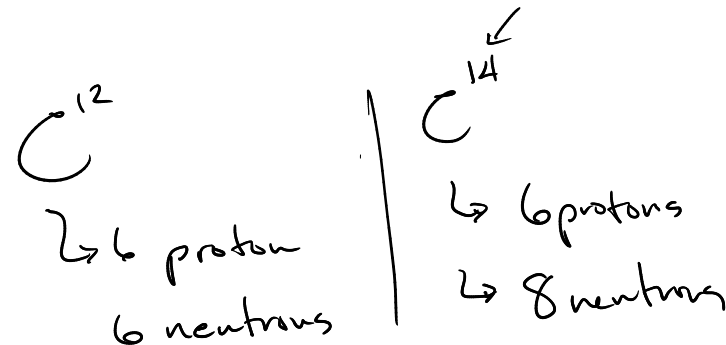
$$pV = nRT$$

n \rightarrow number of moles
universal gas constant
 $R = 8.31 \frac{J}{mol \cdot K} ?$

$$N k_B = nR$$
$$6.022 \cdot 10^{23} \cdot 1.38 \cdot 10^{-23} = R$$
$$8.31 \frac{J}{mol K} = R$$

Number, moles, molar, density

1 mole of things = $6.022 \cdot 10^{23}$ things
particle
atom
molecule



1 mole is 1 gram of protons + neutrons

Ex: mass of one proton in kg?

mass of one proton \times number of proton = mass of the collection

$$m \times N = M$$

$$m = \frac{1g}{N_A} = \frac{1}{6.022 \cdot 10^{23}} = 1.7 \cdot 10^{-24} \text{ grams} = \underline{1.7 \cdot 10^{-27} \text{ kg}}$$

What about N_2 ?

$$1 \text{ mole of } N_2 = 2 \left(14 \frac{g}{\text{mol}} \right) = 28 \frac{g}{\text{mol}}$$

Volume of 1 mole of air at room temp and atmospheric pressure?

$$\hookrightarrow 1 \text{ atm} = \underline{1.013 \cdot 10^5 \text{ Pa}}$$

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \frac{J}{K \cdot \text{mol}} \cdot 300K}{10^5 \text{ Pa}} = 0.0249 \text{ m}^3$$

$$V_{\text{cube}} = \Delta^3$$

$$\Delta = 0.292 \text{ m} \sim 30 \text{ cm} \sim 1 \text{ ft}$$

1.17] $PV = nRT \left(1 + \frac{B(T)}{V/n} \right)$

a) $PV = nRT \left(1 + \frac{n}{V} \cdot B(T) \right)$

at atmospheric pressure $P = 10^5 \text{ Pa}$

→ solve for $\frac{n}{V}$

$$P = \frac{n}{V} RT \left(1 + \frac{n}{V} B(T) \right)$$

$$0 = \frac{n}{V} RT + \left(\frac{n}{V} \right)^2 \cdot RT \cdot B(T) - P$$

T	B(T)	$\frac{n}{V}$	$\frac{n}{V} \cdot B(T)$
100	-16.0		
200	-3.5		
300	-4.2		
400	9.0		
500	16.9		
600	21.3		

$$b) \quad PV = nRT \left(1 + \frac{B(T)}{V/n} \right) \quad \text{if}$$

$$c) \rightarrow PV = nRT \left(1 + \frac{B(T)}{V/n} + \frac{C(T)}{(V/n)^2} \right) \quad \left. \vphantom{PV = nRT} \right\} \text{ want: } B + C \text{ in terms of } a + b$$

$$\left(P + \frac{an^2}{V^2} \right) \left(V - nb \frac{V}{V} \right) = nRT$$

$$\left(P + \frac{an^2}{V^2} \right) V \left(1 - \frac{nb}{V} \right) = nRT$$

$$\left(P + \frac{an^2}{V^2} \right) V = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV + \frac{an^2}{V} = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV = nRT \left(1 - \frac{nb}{V} \right)^{-1} - \frac{an^2}{V}$$

use our approx.

$$(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2 \quad px \ll 1$$

$$\left(1 + \left(-\frac{nb}{V} \right) \right)^{-1} \approx 1 + (-1) \left(-\frac{nb}{V} \right) + \frac{1}{2}(-1)(-1-1) \left(-\frac{nb}{V} \right)^2$$

$$\approx 1 + \frac{nb}{V} + \left(\frac{nb}{V} \right)^2$$

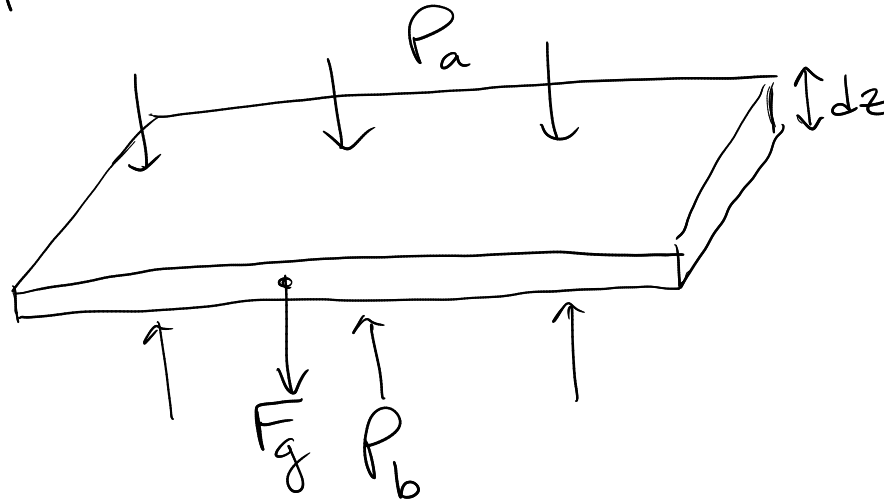
$$PV = nRT \left(1 + \frac{nb}{V} + \left(\frac{nb}{V} \right)^2 \right) - \frac{an^2}{V} \begin{matrix} nRT \\ nRT \end{matrix} \longleftrightarrow PV = nRT \left(1 + \frac{n}{V} \cdot B(T) + \left(\frac{n}{V} \right)^2 \cdot C(T) \right)$$

$$PV = nRT \left(1 + \frac{nb}{V} + \left(\frac{nb}{V} \right)^2 - \frac{an^2}{V \cdot nRT} \right) = nRT \left[1 + \underbrace{\frac{n}{V} \left(b - \frac{a}{RT} \right)}_{B(T)} + \underbrace{\left(\frac{n}{V} \right)^2 \cdot b^2}_{C(T)} \right]$$

d) → plot data from that table ←
 → plot $B(T) = b - \frac{a}{RT}$ choose b, a

1.16 | a) $\rho = \frac{M}{V}$ ← total mass

↑
 "rho"
 volumetric
 mass
 density



$$V = A \cdot dz$$

$$\rho = \frac{M}{A \cdot dz}$$

$$M = \rho A dz$$

$$P = \frac{F}{A}$$

$$+ P_b \cdot A - P_a \cdot A - M \cdot g = 0$$

$$P_b \cdot A - P_a \cdot A - \rho A dz \cdot g = 0$$

$$-(P_b - P_a) = \rho dz \cdot g$$

$$-(\underbrace{P_a - P_b}_{dP}) = \rho dz \Rightarrow \frac{dP}{dz} = -\rho g$$

$$\left| \frac{dP}{dz} = -\frac{mg}{k_B T} P \right|$$

b) $PV = Nk_B T$

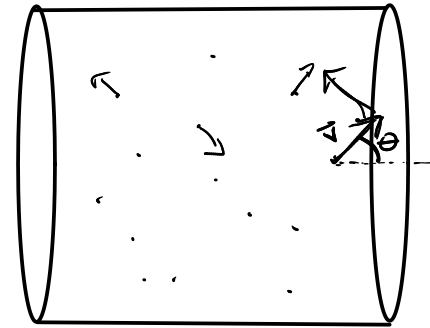
$$m P = \frac{m N k_B T}{V}$$

$$m P = \rho k_B T$$

1.2b | Kinetic Theory and Equipartition of Energy

pressure \longleftrightarrow kinetic energy \longleftrightarrow temperature

$$\text{pressure}_{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \quad \leftarrow \underbrace{2mv \cos \theta}$$



$$d(\text{pressure}) = \frac{2mv \cos \theta}{dA \cdot dt} \cdot \left. \begin{array}{l} \text{number of particles hitting area } dA \\ \text{w/ velocity } v \text{ in } dt \text{ amount of time} \end{array} \right\} \begin{array}{l} \text{integrate over all velocities} \\ \text{and over } \theta \end{array}$$

number of atoms
traveling in a particular
direction w/ a particular
speed

• fraction of them
that are within
striking distance
of the surface dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

total outcomes \nearrow

$$N = \sum_{x=0}^{\infty} N(x)$$

rearrange \nwarrow

$$P(x) = \frac{N(x)}{N}$$

$$N(x) = P(x) \cdot N$$

normalized probabilities

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

\uparrow
• average value
• expectation value

$\underbrace{\hspace{10em}}_{\rightarrow \text{weighted average by probability}}$

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

Example (Blundell 3.3)

a. $P(x) = \frac{e^{-m} m^x}{x!}$; show

$$\sum_{x=0}^{\infty} P(x) = 1$$

Looked up Taylor Series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sum_{x=0}^{\infty} \frac{e^{-m} m^x}{x!} \stackrel{?}{=} 1$$

$$e^{-m} \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!} \stackrel{?}{=} 1$$

$$\sum_{x=0}^{\infty} \frac{m^x}{x!} \stackrel{?}{=} e^m \checkmark \checkmark$$

b. Show $\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x) \stackrel{?}{=} m$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-m} m^x}{x!} \stackrel{?}{=} m$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} \stackrel{?}{=} m$$

$$\sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} \stackrel{?}{=} m e^m$$

$$\underbrace{\quad}_{x \cdot (x-1) \cdot (x-2) \cdot (x-3) \cdots 1}$$

$$\cancel{\frac{0 \cdot m}{0!}} + \frac{1 \cdot m^1}{1!} + \frac{2 m^2}{2!} + \dots$$

$$\sum_{x=1}^{\infty} \frac{x m^x}{x!} = \sum_{x=1}^{\infty} \frac{m^x}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m^{x+1-1}}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m \cdot m^{x-1}}{(x-1)!}$$

$$x \cdot (x-1) \cdot (x-2) \cdots$$

$$\begin{aligned} x' &= x-1 \\ \hookrightarrow x &= x'+1 \end{aligned}$$

$$m \cdot \sum_{x'=0}^{\infty} \frac{m^{x'}}{x'!}$$

drop the primes!

$$C. \langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= 0 \cdot \frac{109}{200} + 1 \cdot \frac{65}{200} + 2 \cdot \frac{22}{200} + 3 \cdot \frac{3}{200} + 4 \cdot \frac{1}{200} + 5 \cdot \frac{0}{200}$$

$$= \underline{\underline{0.61}} = \frac{122}{200}$$

$$P(x) = \frac{e^{-0.61} \cdot 0.61^x}{x!}$$

$$M \cdot \sum_{x=0}^{\infty} \frac{M^x}{x!}$$

$= e^M$

$$M e^M \stackrel{?}{=} M e^M$$

✓

$$\langle x \rangle = M$$

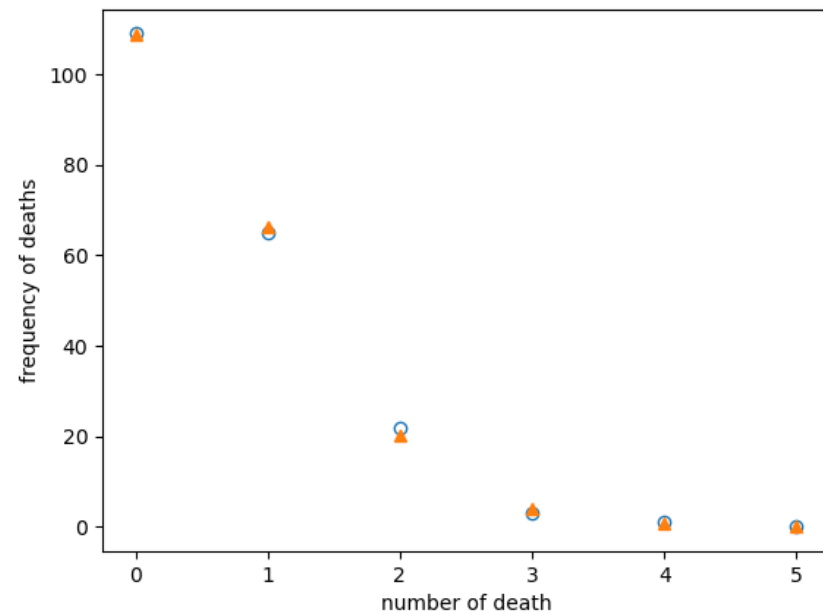
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import factorial

x = [0,1,2,3,4,5]
y = [109, 65, 22, 3, 1, 0]

fig, ax = plt.subplots()
ax.plot(x, y, 'o', mfc='none')
ax.set_xlabel('number of death')
ax.set_ylabel('frequency of deaths')

def deaths(number):
    return(np.exp(-0.61)*np.power(0.61, number)/factorial(number))

ax.plot(x, deaths(x)*200, '^')
```

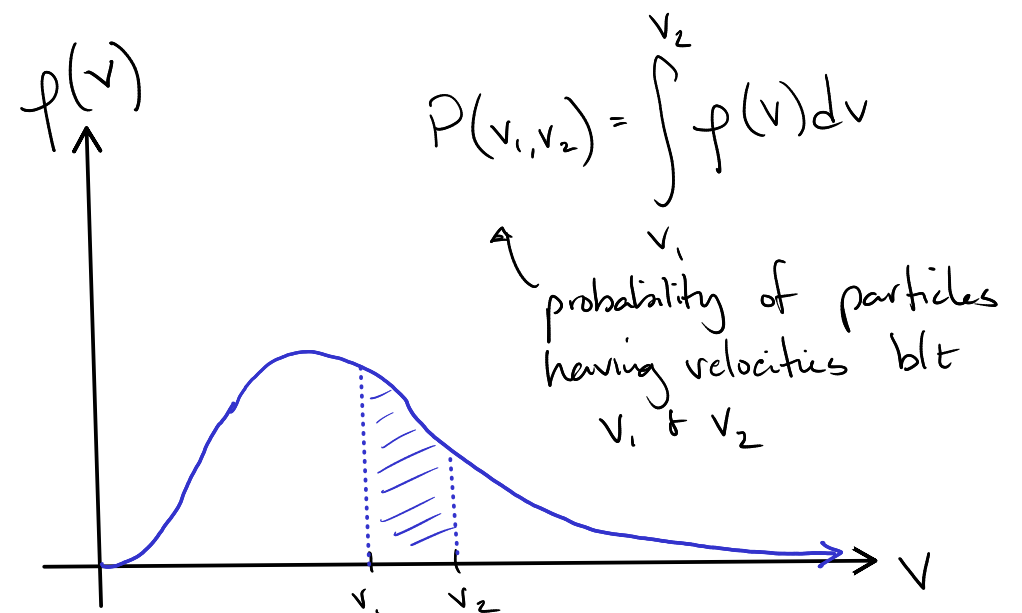
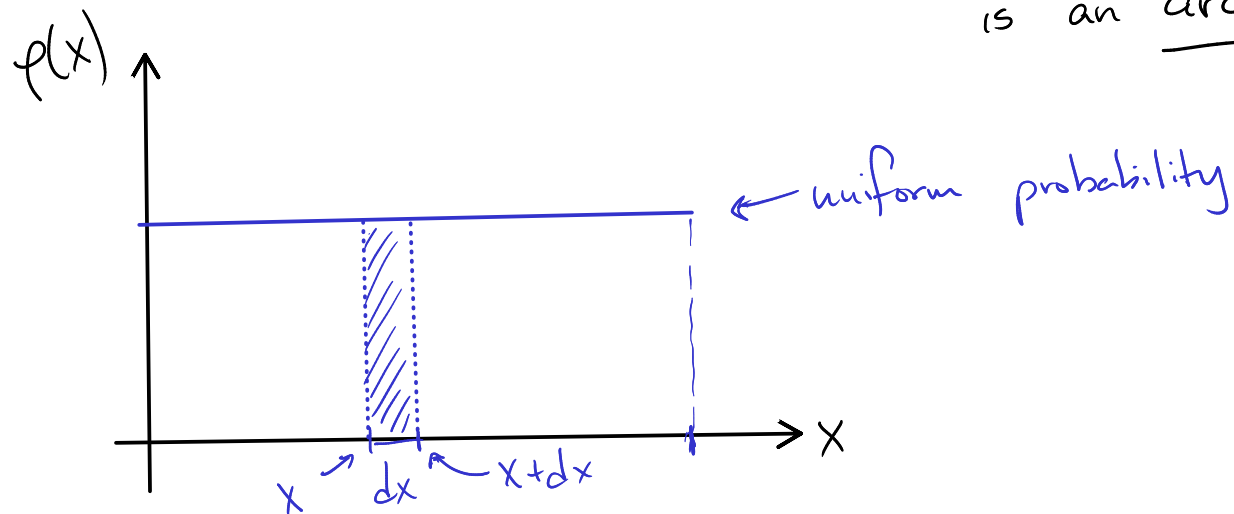


The continuous variable case follows by analogy but w/ some clarifications

$$\left\{ \begin{array}{l} \text{probability of choosing} \\ \text{a value at random} \\ \text{between } x \text{ and } x+dx \end{array} \right\} = p(x) \cdot dx$$

\rightarrow distribution function

probability itself
is an area.



Normalized

$$\int_{-\infty}^{\infty} p(v) dv = 1$$

Since we will be dealing w/ such large numbers

$$\left\{ \begin{array}{l} \text{probability that a} \\ \text{particle has a} \\ \text{velocity between} \\ v \text{ and } v+dv \end{array} \right\} = \left\{ \begin{array}{l} \text{fraction of particles} \\ \text{having a velocity} \\ \text{between } v \text{ + } v+dv \end{array} \right\}$$

Example: (Blundell 3.4)

$$p(x)dx = A e^{-x/\lambda} dx$$

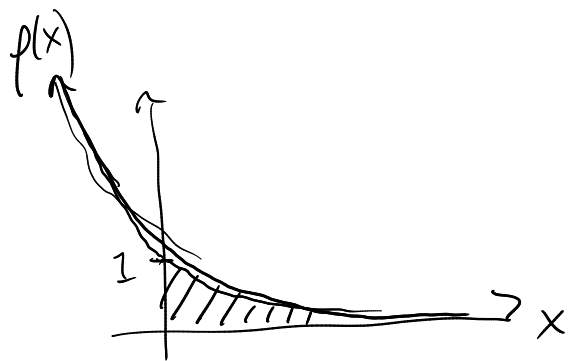
a) Find A so that

$$\int_0^{\infty} p(x) dx = 1$$

$$\int_0^{\infty} A e^{-x/\lambda} dx = 1$$

$$A(-\lambda) e^{-x/\lambda} \Big|_0^{\infty} = 1$$

$$A(-\lambda)(0 - -1) = 1 \quad A\lambda = 1 \Rightarrow \boxed{A = \frac{1}{\lambda}}$$



Also mean

$$\langle v \rangle = \int_{-\infty}^{\infty} v \cdot p(v) dv$$

$$\text{or} \quad \langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 \cdot p(v) dv$$

$$\text{or} \quad \langle f(v) \rangle = \int_{-\infty}^{\infty} f(v) \cdot p(v) dv$$

$$b) \langle x \rangle = \int_0^{\infty} x p(x) dx = \lambda$$

$$\int_0^{\infty} x \cdot \frac{1}{\lambda} e^{-x/\lambda} dx = \lambda$$

$$\frac{1}{\lambda} \left(-\lambda x e^{-x/\lambda} \Big|_0^{\infty} - (-\lambda) \int_0^{\infty} e^{-x/\lambda} dx \right)$$

$$uv \Big|_0^{\infty} - \int_0^{\infty} v du$$

$$u = x \quad dv = e^{-x/\lambda} dx$$

$$du = dx \quad v = -\lambda e^{-x/\lambda}$$

$$c) \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$\sigma \rightarrow$ standard deviation

$\sigma^2 \rightarrow$ variance

$$\langle x \rangle = \lambda$$

$$\langle x \rangle^2 = \lambda^2$$

$$\langle x^2 \rangle = \int_0^{\infty} x^2 \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx$$

\hookrightarrow LOOK IT UP!

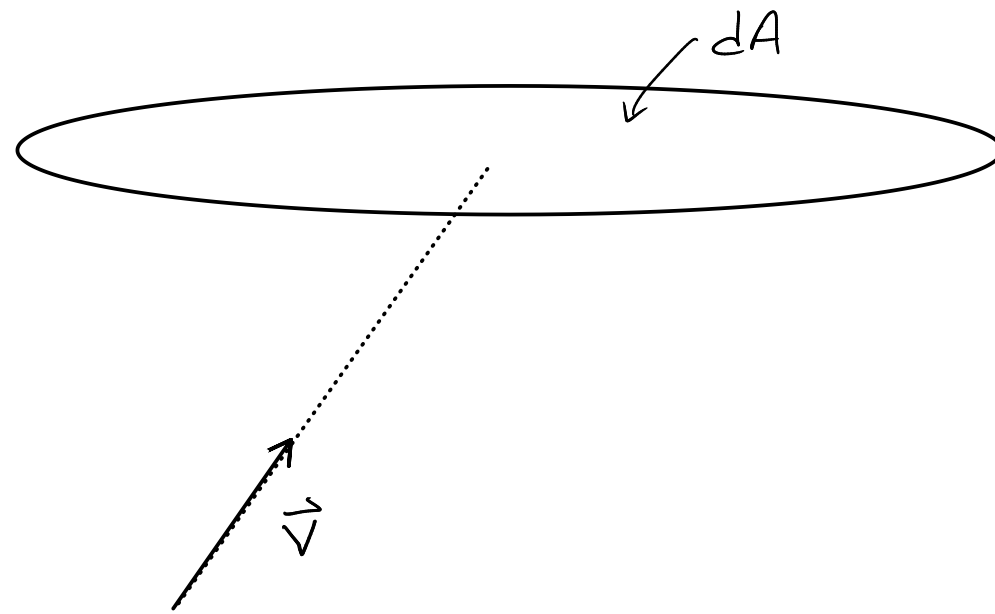
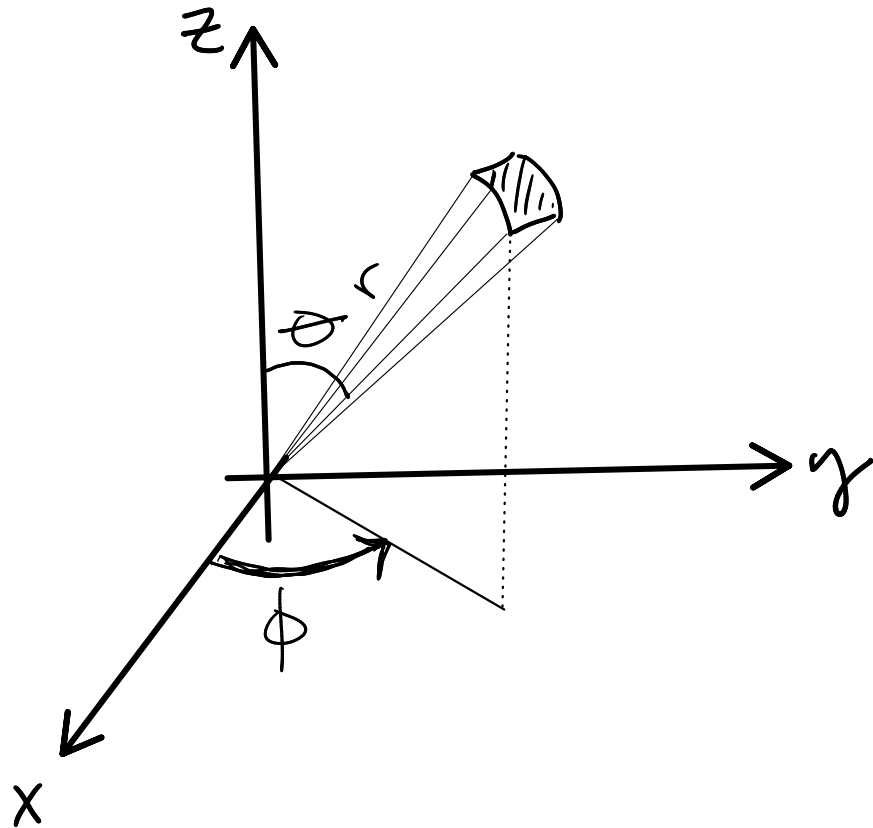
$$\langle x^2 \rangle = 2\lambda^2$$

$$\sigma^2 = 2\lambda^2 - \lambda^2 = \lambda^2 \leftarrow \text{variance}$$

$$\sigma = \lambda \leftarrow \text{standard deviation}$$

$$\lambda = \langle x \rangle \checkmark$$

number of particles
traveling in a particular
direction w/ a
particular speed



internal energy

$$U = \frac{3}{2} N k_B T$$

$$U = N \langle K \rangle$$

↑ avg of kinetic energy

$$\begin{aligned} \langle K \rangle &= \frac{1}{2} m \langle v^2 \rangle \\ &= \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle \quad \begin{array}{l} \text{average of a} \\ \text{sum is the} \\ \text{sum of} \\ \text{averages} \end{array} \\ &= \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right) \end{aligned}$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

$$\frac{3}{2} k_B T = \frac{3}{2} m \langle v_x^2 \rangle$$

$$\frac{1}{2} k_B T = \frac{1}{2} m \langle v_x^2 \rangle$$

Equipartition Theorem

$$U_{\text{thermal}} = N \cdot f \cdot \frac{1}{2} k_B T$$

↳ degree of freedom (quadratic)

$$\frac{1}{2} m v_x^2, \frac{1}{2} m v_y^2, \frac{1}{2} m v_z^2, \frac{1}{2} I \omega_x^2, \frac{1}{2} k_s x^2$$

$$f = 3 \leftarrow \text{monatomic gas}$$

$$f = 5 \leftarrow \text{diatomic gas near room temp}$$

$$f = 6 \leftarrow \text{solid}$$

1.23

$$V = 1 \text{ liter} = 0.001 \text{ m}^3 = 10^{-3} \text{ m}^3$$

$$p = 1.01 \cdot 10^5 \text{ Pa} = 10^5 \text{ Pa}$$

$$T = 293 \text{ K}$$

monatomic $\rightarrow U = \frac{3}{2} N k_B T$

$$pV = N k_B T$$

$$U = \frac{3}{2} pV = \frac{3}{2} (10^5 \text{ Pa}) (10^{-3} \text{ m}^3)$$

$$= 1.5 \cdot 10^2 \text{ J} = \underline{150 \text{ J}}$$

diatomic

$$U = \frac{5}{2} N k_B T = \frac{5}{2} pV$$

$$= 2.5 \cdot 10^2 = \underline{250 \text{ J}}$$

$$pV = nRT$$

$$\frac{10^5 \text{ Pa} \cdot 10^{-3} \text{ m}^3}{8.31 \cdot 293} = \underline{\underline{0.04 \text{ mol}}}$$

1.24

$$f = 6$$

$$U = \frac{6}{2} N k_B T$$

$$U = 3 N k_B T$$

1 g of Pb

$$207 \frac{\text{g}}{\text{mol}}$$

$$\frac{1 \text{ g}}{207 \frac{\text{g}}{\text{mol}}} = \underline{\underline{0.0048 \text{ mol}}} \cdot \frac{6.022 \cdot 10^{23} \text{ atoms}}{1 \text{ mol}}$$

$$U = 3 n_m \cdot R T = 3 \cdot 0.0048 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K mol}} \cdot 293 = \underline{35 \text{ J}}$$

1.4 Heat and Work

spontaneous
flow of energy
due to a difference
in temperature

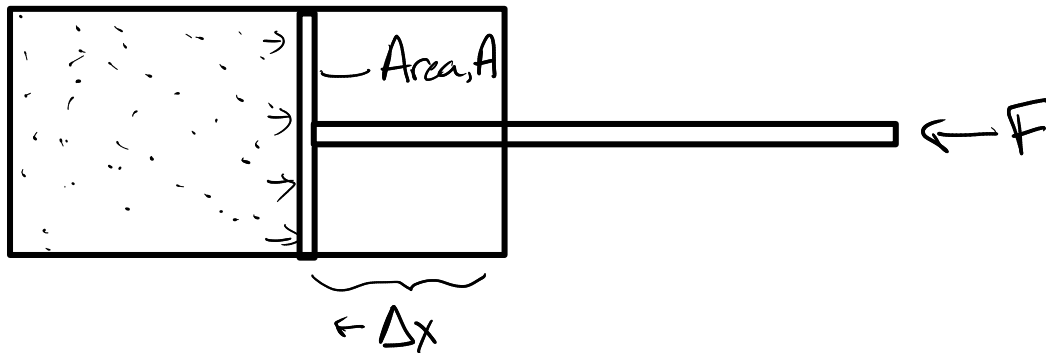
- force applied over distance
- energy through resistor

Equation: $dU = \delta Q + \delta W$

↑ small amount
NOT a small change
in amount

$$\delta Q + \delta W \quad | \quad Q + W$$

Compressive Work



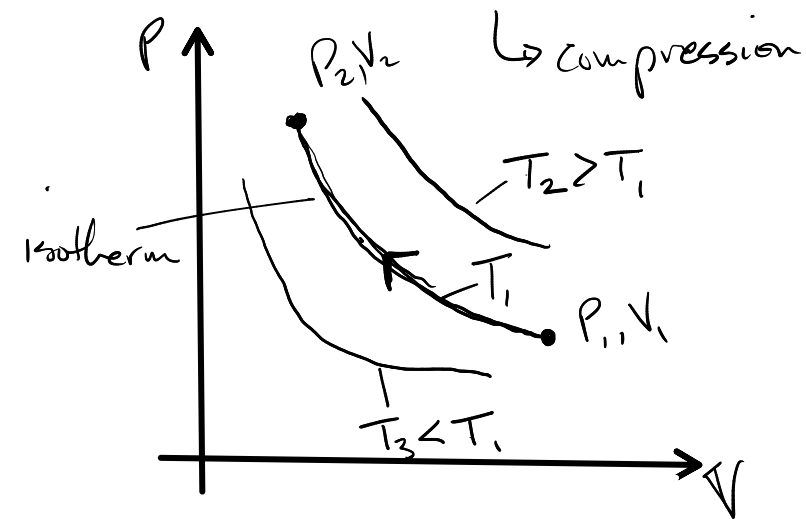
$$\delta W = \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta \vec{x} = P \cdot \underbrace{A \cdot \Delta x}_{-\Delta V}$$

quasistatic, static-ish

$$\delta W = -p \delta V = -p \Delta V$$

$$\delta W = -p(V) \delta V$$

Isothermal process (constant temperature)



$$PV = Nk_B T$$

all constant

$$P = \frac{Nk_B T}{V} \leftarrow \text{hyperbola}$$

$$W = - \int p dV = - Nk_B T \int_{V_i}^{V_f} \frac{1}{V} dV$$

Constants in this case

$$W = - Nk_B T \ln \left(\frac{V_f}{V_i} \right) \quad \ln \left[\left(\frac{V_f}{V_i} \right)^{-1} \right]$$

$$W = Nk_B T \ln \left(\frac{V_i}{V_f} \right)$$

$$dU = \delta Q + \delta W \quad \rightarrow \quad 0 = \delta Q + \delta W$$

$$\delta Q = -\delta W \leftarrow \text{isothermal}$$

$$Q = -Nk_B T \ln \left(\frac{V_i}{V_f} \right)$$

$$Q = Nk_B T \ln \left(\frac{V_f}{V_i} \right)$$

$$U = \frac{f}{2} Nk_B T$$

$$\Delta U = \frac{f}{2} Nk_B \Delta T$$

$$dU = \frac{f}{2} Nk_B dT$$

$$dT = 0 \therefore dU = 0$$

Adiabatic process (Isentropic process) (no heat)

↳ compression

first law $\rightarrow dU = \cancel{dQ} + dW$

$dU = dW$ for an ideal gas: $U = \frac{f}{2} N k_B T$
 $dU = \frac{f}{2} N k_B dT$

$$\frac{f}{2} N k_B dT = dW$$

$$\frac{f}{2} N k_B dT = -p dV$$

$$\frac{f}{2} \cancel{N k_B} dT = - \frac{\cancel{N k_B} T}{V} dV$$

$$\frac{f}{2} dT = - \frac{T}{V} dV$$

$$\frac{f}{2} \frac{dT}{T} = - \frac{dV}{V}$$

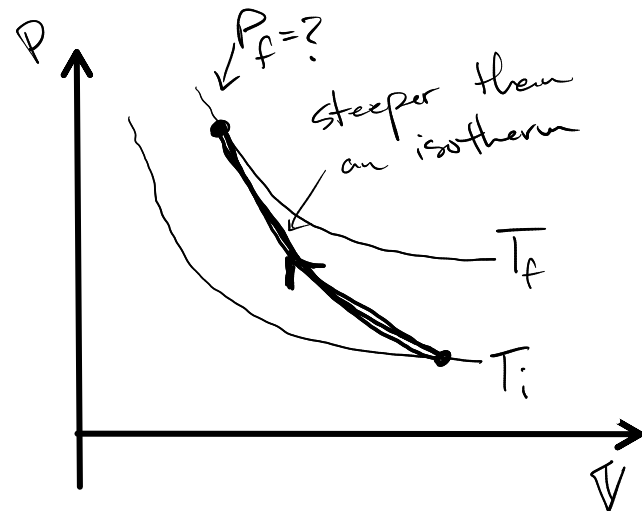
$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = - \ln\left(\frac{V_f}{V_i}\right)$$

$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\rightarrow \left(\frac{T_f}{T_i}\right)^{\frac{f}{2}} = \frac{V_i}{V_f} \rightarrow V \propto \frac{1}{T^{f/2}}$$

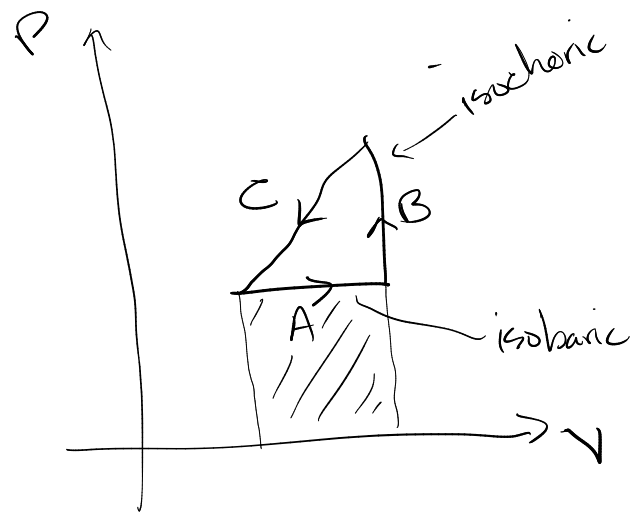
\downarrow
 $V \propto T^{-f/2}$

$$T_f^{f/2} \cdot V_f = T_i^{f/2} \cdot V_i \rightarrow V \cdot T^{f/2} = \text{constant of proportionality}$$



HW: 32, 33, 35 ←

1.33



A: $W < 0$
 $Q = ?$
 $\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} P \Delta V$
 $\Delta U > 0$

$$N k_B \Delta T = P \Delta V$$

$$\Delta U = Q + W$$

\uparrow \uparrow \downarrow

1.35

$$V T^{f/2} = \text{a constant}$$

$$V^\gamma P = \text{another constant}$$

$$\gamma = \frac{f+2}{f}$$

$$P V = N k_B T$$

↑
constant

$$P V T^{-1} = \text{constant}$$

↓

34, 36, 38

1.6 Heat Capacity

$$dU = \pm Q + \underbrace{\pm W}_{-p dV}$$

$$\pm Q = dU + p dV$$

$$\downarrow \underline{U(T, V)} \text{ or } U(T, P)$$

$$dU(T, V) = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV \quad \leftarrow$$

$$\pm Q = \left(\frac{\partial U}{\partial T} \right)_V dT + \left[\left(\frac{\partial U}{\partial V} \right)_T + p \right] dV$$

$$\frac{\pm Q}{dT} = \left(\frac{\partial U}{\partial T} \right)_V + \left[\left(\frac{\partial U}{\partial V} \right)_T + p \right] \frac{dV}{dT}$$

Two cases: ① Constant volume
② Constant pressure

① constant volume

$$dV = 0$$

$$\delta Q = \left(\frac{\partial U}{\partial T} \right)_V dT \quad \text{or} \quad \left(\frac{\delta Q}{dT} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V$$

$$C_V \equiv \left(\frac{\delta Q}{dT} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V$$

↑
heat capacity
at constant volume

For an ideal gas

$$U = \frac{f}{2} N k_B T$$

↪ macroscopic form ↪

$$U = \frac{f}{2} n_m R T$$

$$\left(\frac{\partial U}{\partial T} \right)_V = \frac{dU}{dT} = \frac{f}{2} n_m R = C_V$$

$$\frac{C_V}{n_m} = \frac{f}{2} R$$

↪ heat capacity per mole
(molar specific heat)

monatomic
 $f=3$

$$\frac{C_V}{n_m} = \frac{3}{2} R$$

$$= 12.5 \text{ J/mol K}$$

diatomic
 $f=5$

$$\frac{C_V}{n_m} = \frac{5}{2} R$$

$$= 20.8 \text{ J/mol K}$$

② constant pressure

$$C_p \equiv \left(\frac{dQ}{dT} \right)_p = \underbrace{\left(\frac{\partial U}{\partial T} \right)_V}_{C_v} + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{dV}{dT} \right)_p$$

$$C_p = C_v + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{dV}{dT} \right)_p$$

$$C_p - C_v = \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{dV}{dT} \right)_p$$

for an ideal gas:

$$C_p = \frac{f}{2} n_m R + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{dV}{dT} \right)_p$$

$$U = \frac{f}{2} n_m R \cdot T$$

$$\left(\frac{\partial U}{\partial V} \right)_T = 0 \quad \leftarrow \text{for an ideal gas}$$

$$V = \frac{n_m R T}{P}$$

$$\left(\frac{dV}{dT} \right)_p = \frac{n_m R}{P}$$

$$C_p = \frac{f}{2} n_m R + n_m R$$

$$\frac{C_p}{n_m} = \frac{f}{2} R + R$$

$$\frac{C_p}{n_m} = \frac{C_v}{R} + R$$

monatomic gas

$$f=3$$

$$C_p \approx 20.7$$

diatomic gas

$$C_p \approx 29$$