

Thermal Equilibrium - when objects have been in contact
and macroscopic coordinates have stopped changing

↳ involves an exchange of energy between two object, or an object + its surrounding

↳ volume (constant pressure) \rightarrow mercury/alcohol

- pressure

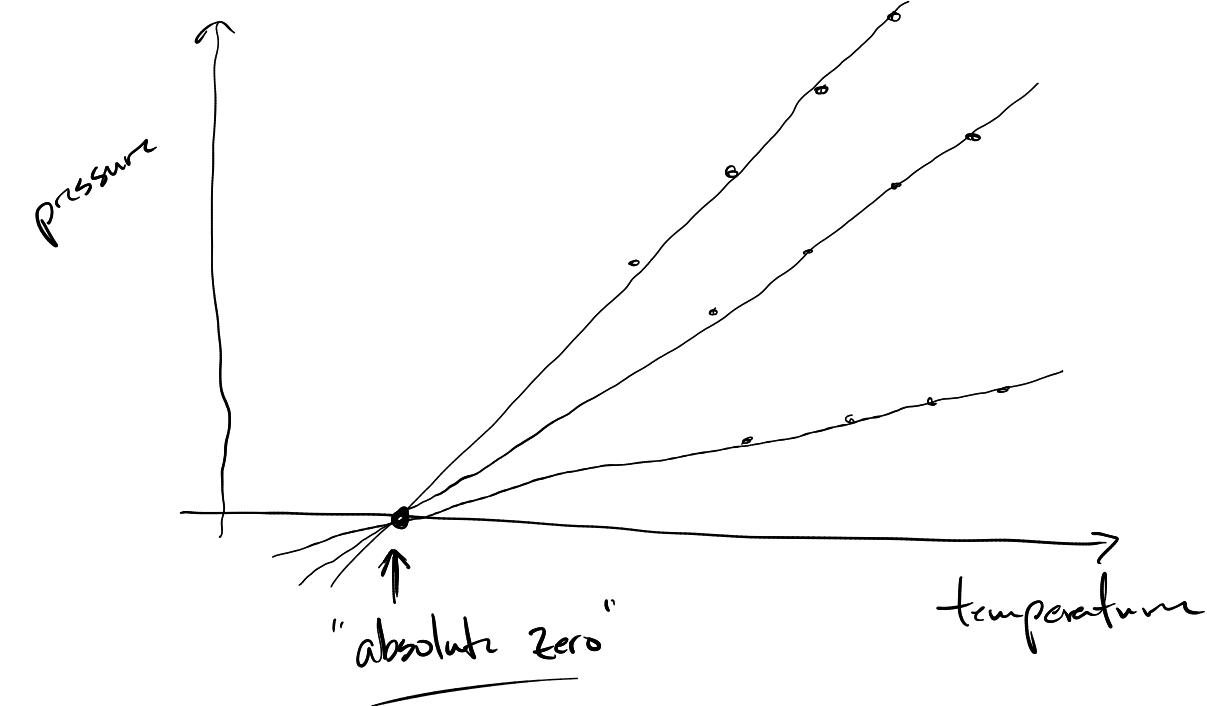
- electrical resistance

- radiation/thermal emf

$$\boxed{C_1 = 0^\circ C \quad C_2 = 100^\circ C}$$

$$F_1 = 32^\circ F \quad F_2 = 212^\circ F$$

$$C = mF + b$$



$$0K = -273.15^\circ C$$

1.2 Ideal Gas Law \rightarrow Equation of State \rightarrow relates all of the state variables together

$$PV = N k_B T$$

$P = \frac{F}{\text{Area}}$

$[P_a] = \left[\frac{N}{m^2} \right]$

number of particles

Boltzmann's constant

$$k_B = 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

$$PV = nRT$$

universal gas constant

number of moles

$$R = 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$N k_B = nR$$

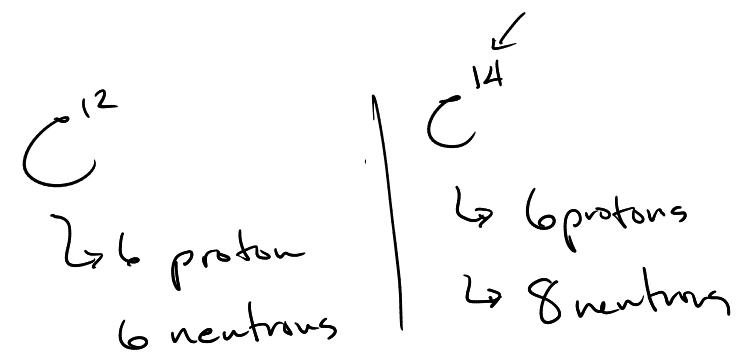
$$6.022 \cdot 10^{23} \cdot 1.38 \cdot 10^{-23} = R$$

$$8.31 \frac{\text{J}}{\text{mol K}} = R$$

Number, moles, molar, density

$$1 \text{ mole of things} = 6.022 \cdot 10^{23} \text{ things}$$

particle
atom
molecule



1 mole is 1 gram of protons + neutrons

Ex: mass of one proton in kg?

mass of one proton \times number of proton = mass of the collection

$$m \times N = M$$

$$M = \frac{1 \text{ g}}{N_A} = \frac{1}{6.022 \cdot 10^{23}} = 1.7 \cdot 10^{-24} \text{ grams} = 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2 ?

$$1 \text{ mole of } N_2 = 2 \left(\frac{14 \text{ g}}{\text{mol}} \right) = 28 \text{ g/mol}$$

Volume of 1 mole of air at room temp and atmospheric pressure?

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \text{ J/K} \cdot \text{mol} \cdot 300 \text{ K}}{10^5 \text{ Pa}} = 0.0249 \text{ m}^3$$

$$\hookrightarrow 1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$$

$$V_{\text{cube}} = D^3$$

$$D = 0.292 \text{ m} \sim 30 \text{ cm} \sim 1 \text{ ft}$$

1.17] $PV = nRT \left(1 + \frac{B(T)}{V_n} \right)$

a) $PV = nRT \left(1 + \frac{n}{V} \cdot B(T) \right)$

at atmospheric pressure $P = 10^5 \text{ Pa}$

Solve for $\frac{n}{V}$

$$P = \frac{n}{V} RT \left(1 + \frac{n}{V} B(T) \right)$$

$$0 = \frac{n}{V} RT + \left(\frac{n}{V} \right)^2 \cdot RT \cdot B(T) - P$$

T	B(T)	$\frac{n}{V}$	$\frac{n}{V} \cdot B(T)$
100	-160		
200	-35		
300	-4.2		
400	9.0		
500	16.9		
600	21.3		

b) $PV = nRT \left(1 + \frac{B(T)}{\sqrt{V_n}} \right)$ if

c) $\rightarrow PV = nRT \left(1 + \frac{B(T)}{\sqrt{V_n}} + \frac{C(T)}{(\sqrt{V_n})^2} \right)$

want: $B + C$ in terms of $a + b$

$$\left(P + \frac{an^2}{V^2} \right) \left(V - nb\frac{V}{V} \right) = nRT$$

$$\left(P + \frac{an^2}{V^2} \right) V \left(1 - \frac{nb}{V} \right) = nRT$$

$$\left(P + \frac{an^2}{V^2} \right) V = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV + \frac{an^2}{V} = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV = nRT \left(1 - \frac{nb}{V}\right)^{-1} - \frac{an^2}{V}$$

use our
 approx.

$$(1 + x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2 \quad px \ll 1$$

$$\begin{aligned} \left(1 + \left(-\frac{nb}{V}\right)\right)^{-1} &\approx 1 + (-1) \left(-\frac{nb}{V}\right) + \frac{1}{2}(-1)(-1-1) \left(-\frac{nb}{V}\right)^2 \\ &\approx 1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2 \end{aligned}$$

$$PV = nRT \left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2\right) - \frac{an^2}{V} \frac{(nRT)}{(nRT)} \longleftrightarrow PV = nRT \left(1 + \frac{n}{V} \cdot B(T) + \left(\frac{n}{V}\right)^2 \cdot C(T)\right)$$

$$PV = nRT \left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2 - \frac{an^2}{V \cdot nRT}\right) = nRT \left[1 + \frac{n}{V} \left(b - \frac{a}{RT}\right) + \left(\frac{n}{V}\right)^2 \cdot b^2\right]$$

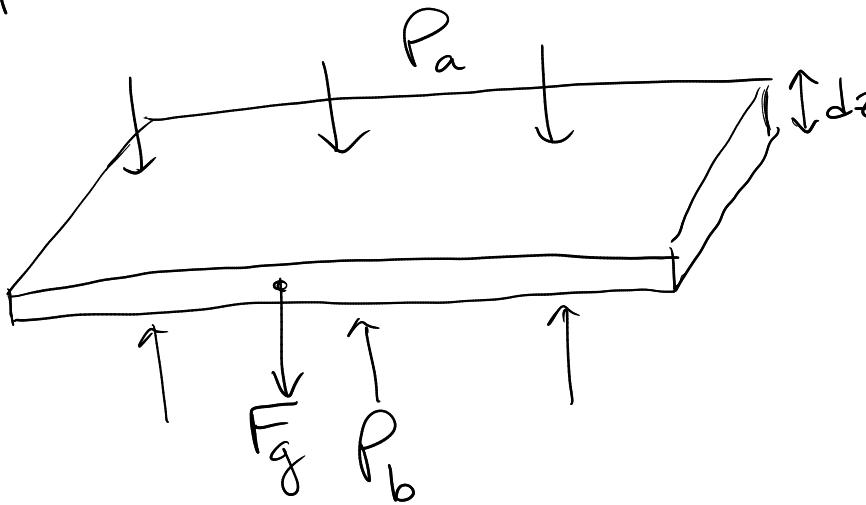
B(T) C(T)

d) \Rightarrow plot data from that table

$$\rightarrow \text{plot } B(T) = b - \frac{a}{RT} \quad \text{choose } b, a$$

1.1b
a) $\rho = \frac{M}{V}$ \leftarrow total mass

"rho"
volumetric
mass
density



$$V = A \cdot dz$$

$$\rho = \frac{m}{A \cdot dz}$$

$$m = \rho A dz$$

$$P_i = \frac{F}{A}$$

$$+ P_b \cdot A - P_a \cdot A - M \cdot g = 0$$

$$P_b \cdot A - P_a \cdot A - \rho A dz \cdot g = 0$$

$$-(P_b + P_a) = \rho A dz \cdot g$$

$$-(P_a - P_b) = \rho A dz \Rightarrow \frac{dP}{dz} = -\rho g$$

b) $PV = N k_B T$

$$m P = \frac{N k_B T}{V}$$

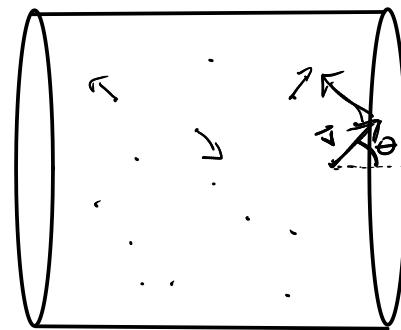
$$m P = \rho k_B T$$

$$\boxed{\frac{dP}{dz} = -\frac{mgP}{k_B T}}$$

1.2 b] Kinetic Theory and Equipartition of Energy

pressure \longleftrightarrow kinetic energy \longleftrightarrow temperature

$$\underset{\text{collision}}{\text{pressure}} = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \quad \overbrace{2mv \cos \theta}$$



$$d(\text{pressure}) = \frac{2mv \cos \theta}{dA \cdot dt} \cdot \text{number of particles hitting area } dA \quad \left. \begin{array}{l} \text{w/ velocity } v \text{ in } dt \text{ amount of time} \\ \text{integrate over all velocities} \\ \text{and over } \theta \end{array} \right\}$$

number of atoms
traveling in a particular
direction w/ a particular
speed fraction of them
 that are within
 striking distance
 of the surface dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

$$N = \sum_{x=0}^{\infty} N(x)$$

total outcomes

$$\text{rearrange } P(x) = \frac{N(x)}{N}$$

$$N(x) = P(x) \cdot N$$

normalized probabilities

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

• average value
• expectation value

→ weighted average by probability

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

Example (Blundell 3.3)

a. $P(x) = \frac{e^{-m} m^x}{x!}$; show $\sum_{x=0}^{\infty} P(x) = 1$

$$\sum_{x=0}^{\infty} \frac{e^{-m} m^x}{x!} ?= 1$$

$$e^{-m} \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!} ?= 1$$

$$\sum_{x=0}^{\infty} \frac{m^x}{x!} ?= e^m \checkmark$$

b. Show $\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x) ?= m$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-m} m^x}{x!} ?= m$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} ?= m$$

Looked up Taylor Series:
 $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} ?= m e^m$$

$$[x \cdot (x-1) \cdot (x-2) \cdot (x-3) \cdots]$$

$$\cancel{\frac{0 \cdot m^0}{0!}} + \frac{1 \cdot m^1}{1!} - + \frac{2 m^2}{2!} +$$

$$\Rightarrow \sum_{x=1}^{\infty} \frac{x m^x}{x!} = \sum_{x=1}^{\infty} \frac{m^x}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m^{x+1-1}}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m \cdot m^{x-1}}{(x-1)!}$$

$$x \cdot (x-1) \cdot (x-2) \cdots$$

$$\begin{aligned} x' &= x-1 \\ \hookrightarrow x &= x'+1 \end{aligned}$$

$$m \cdot \sum_{x'=0}^{\infty} \frac{m^{x'}}{x'!}$$

drop the primes!

$$C. \langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= 0 \cdot \frac{109}{200} + 1 \cdot \frac{65}{200} + 2 \cdot \frac{22}{200} + 3 \cdot \frac{3}{200} + 4 \cdot \frac{1}{100} + 5 \cdot \frac{0}{200}$$

$$= 0.61 = \frac{122}{200}$$

$$P(x) = \frac{e^{-0.61} \cdot 0.61^x}{x!}$$

```

import numpy as np
import matplotlib.pyplot as plt
from scipy.special import factorial

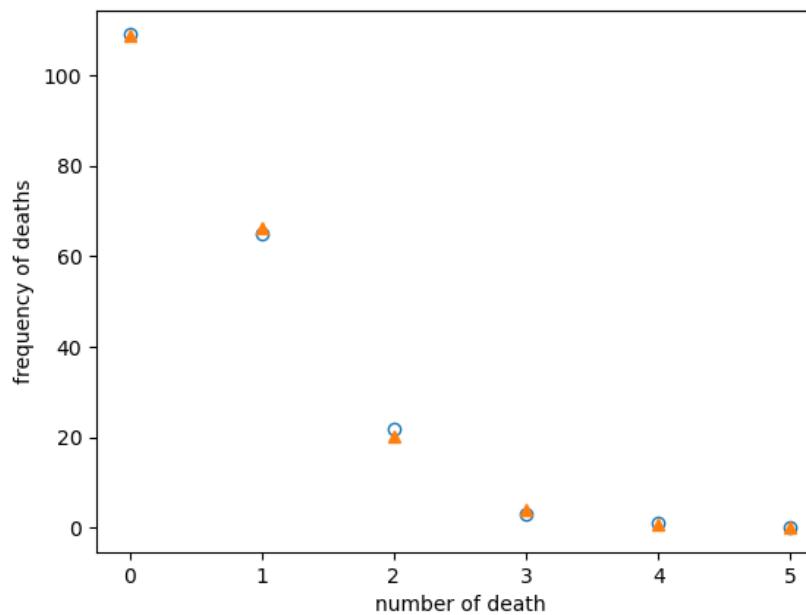
x = [0, 1, 2, 3, 4, 5]
y = [109, 65, 22, 3, 1, 0]

fig, ax = plt.subplots()
ax.plot(x, y, 'o', mfc='none')
ax.set_xlabel('number of death')
ax.set_ylabel('frequency of deaths')

def deaths(number):
    return(np.exp(-0.61)*np.power(0.61, number)/factorial(number))

ax.plot(x, deaths(x)*200, '^')

```



$$m \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!}$$

$$\underbrace{e^m}_{= e^m}$$

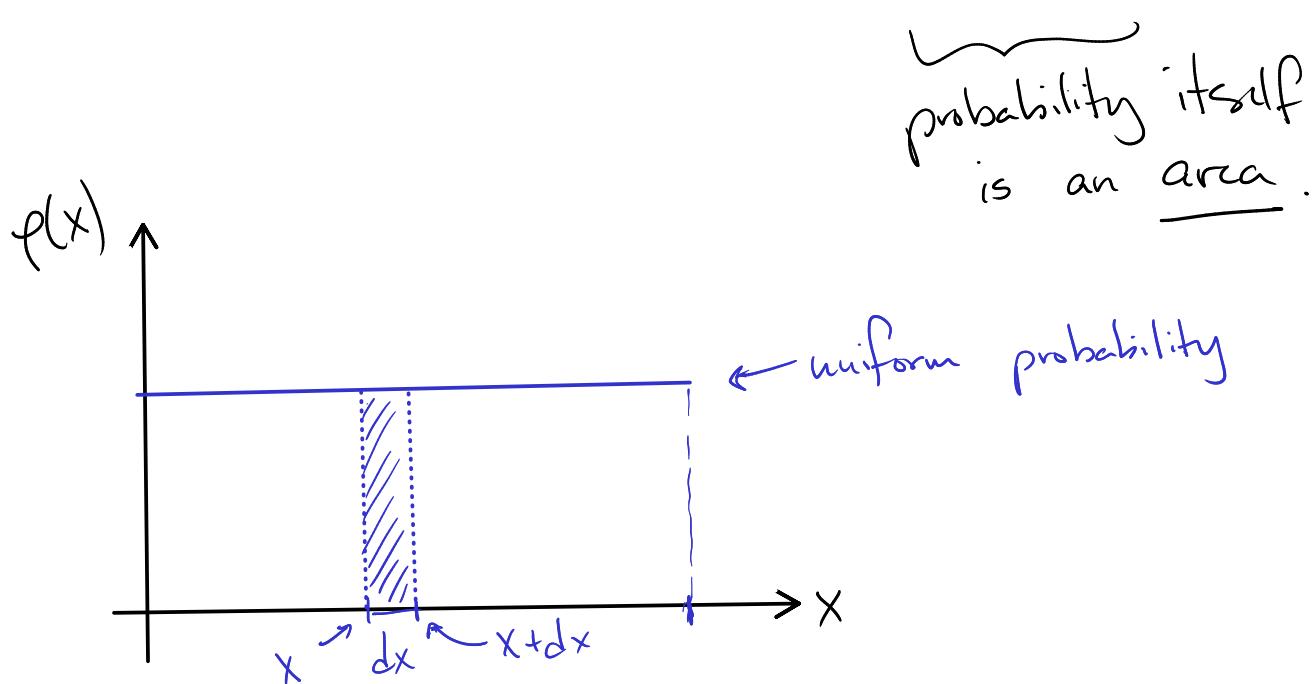
$$me^m ? \checkmark = me^m$$

$$\langle x \rangle = m$$

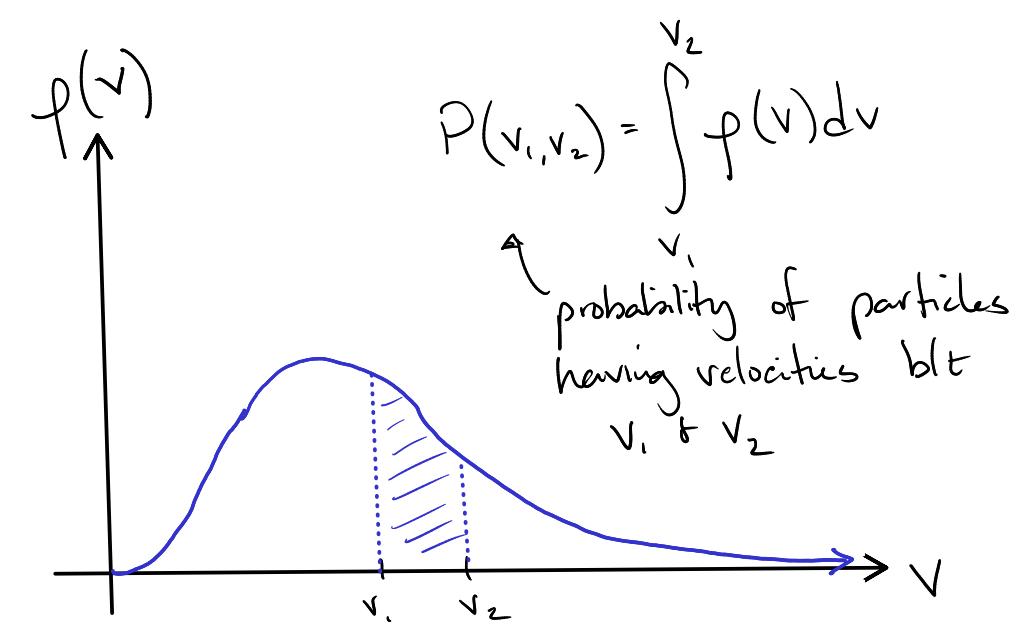
The continuous variable case follows by analogy but w/ some clarifications

$$\left\{ \begin{array}{l} \text{probability of choosing} \\ \text{a value at random} \\ \text{between } x \text{ and } x+dx \end{array} \right\} = f(x) \cdot dx$$

\hookrightarrow distribution function



probability itself
is an area.



Normalized

$$\int_{-\infty}^{\infty} f(v) dv = 1$$

Since we will be dealing w/ such large numbers

$$\left\{ \begin{array}{l} \text{probability that a} \\ \text{particle has a} \\ \text{velocity between} \\ v \text{ and } v+dv \end{array} \right\} = \left\{ \begin{array}{l} \text{fraction of particles} \\ \text{having a velocity} \\ \text{between } v + dv \end{array} \right\}$$

Example: (Blundell 3.4)

$$f(x)dx = A e^{-x/\lambda} dx$$

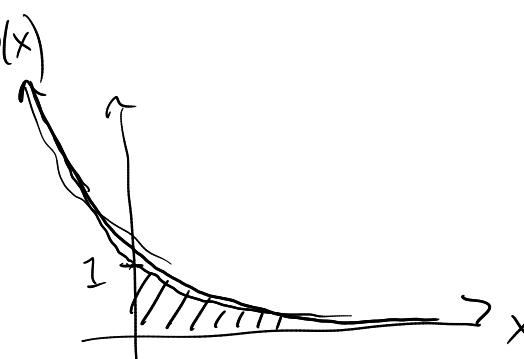
a) Find A so that

$$\int_0^\infty f(x)dx = 1$$

$$\int_0^\infty A e^{-x/\lambda} dx = 1$$

$$A(-\lambda) e^{-x/\lambda} \Big|_0^\infty = 1$$

$$A(-\lambda)(0 - -1) = 1$$



$$A\lambda = 1 \Rightarrow \boxed{A = \frac{1}{\lambda}}$$

Also mean

$$\langle v \rangle = \int_{-\infty}^{\infty} v \cdot f(v) dv$$

$$\text{or} \quad \langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 \cdot f(v) dv$$

$$\text{or} \quad \langle f(v) \rangle = \int_{-\infty}^{\infty} f(v) \cdot f(v) dv$$

$$\text{uv} \int_0^\infty - \int_0^\infty v du \quad u = x \quad dv = e^{-x/\lambda} dx \\ du = dx \quad v = -\lambda e^{-x/\lambda}$$

$$\text{b) } \langle x \rangle = \int_0^\infty x \cdot f(x) dx = \lambda$$

$$\int_0^\infty x \cdot \frac{1}{\lambda} e^{-x/\lambda} dx = \lambda$$

$$\frac{1}{\lambda} \left(-\lambda x e^{-x/\lambda} \right)_0^\infty - (-\lambda) \int_0^\infty e^{-x/\lambda} dx$$

$$c) \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$\sigma \rightarrow$ standard deviation

$\sigma^2 \rightarrow$ variance

$$\langle x \rangle = \lambda$$

$$\langle x \rangle^2 = \lambda^2$$

$$\langle x^2 \rangle = \int_0^\infty x^2 \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx$$

↪ LOOK IT UP!

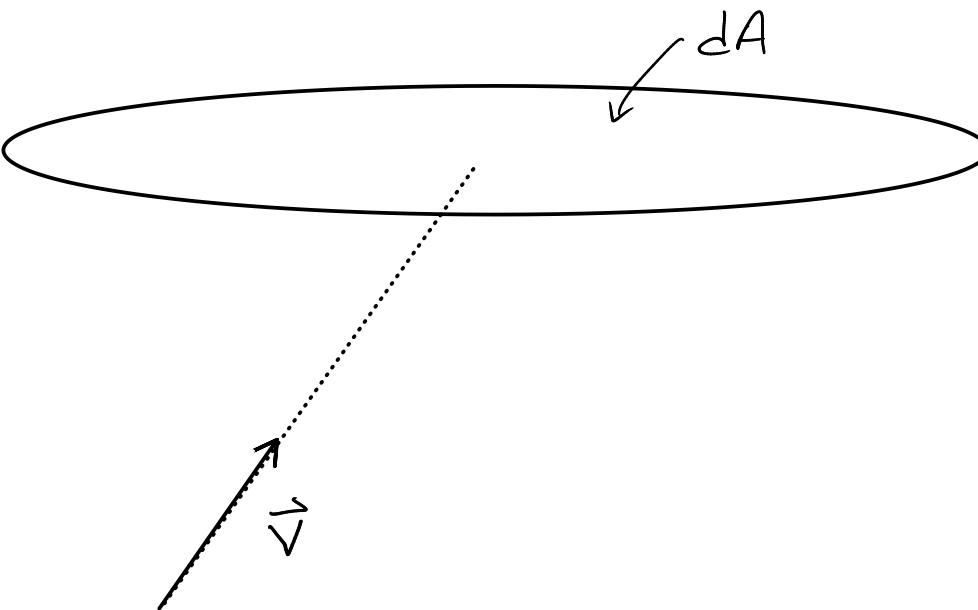
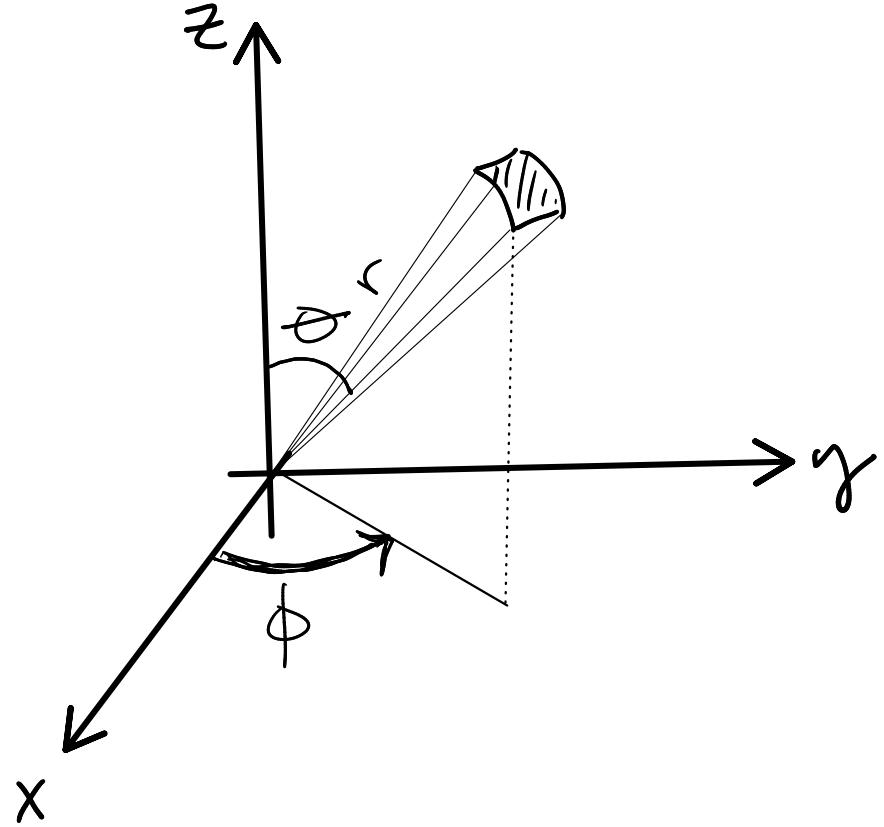
$$\langle x^2 \rangle = 2\lambda^2$$

$$\sigma^2 = 2\lambda^2 - \lambda^2 = \lambda^2 \leftarrow \text{variance}$$

$\sigma = \lambda \leftarrow$ standard deviation

$$\lambda = \langle x \rangle \checkmark$$

number of particles
traveling in a particular
direction w/ a
particular speed



$$U = \frac{3}{2} N k_B T$$

internal
energy

$$U = N \langle K \rangle$$

↑
avg of kinetic energy

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$= \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

$$= \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right)$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

↑ ↑ ↓

$$\frac{3}{2} k_B T = \frac{3}{2} m \langle v_x^2 \rangle$$

$$\frac{1}{2} k_B T = \frac{1}{2} m \langle v_x^2 \rangle$$

average of a
sum is the
sum of
averages

Equipartition Theorem

$$U_{\text{thermal}} = N \cdot f \cdot \frac{1}{2} k_B T$$

↗ degree of freedom (quadratic)

$$\frac{1}{2} m v_x^2, \frac{1}{2} m v_y^2, \frac{1}{2} m v_z^2, \frac{1}{2} I \omega_x^2, \frac{1}{2} k_s x^2$$

$f = 3 \leftarrow$ monoatomic gas

$f = 5 \leftarrow$ diatomic gas near room temp

$f = 6 \leftarrow$ solid

1.23

$$\nabla = 1 \text{ liter} = 0.001 \text{ m}^3 = 10^{-3} \text{ m}^3$$

$$p = 1.01 \cdot 10^5 \text{ Pa} = 10^5 \text{ Pa}$$

monoatomic $T = 293 \text{ K}$

$$\rightarrow U = \frac{3}{2} N k_B T \quad PV = N k_B T$$

$$U = \frac{3}{2} p \nabla = \frac{3}{2} (10^5 \text{ Pa}) (10^{-3} \text{ m}^3)$$

$$= 1.5 \cdot 10^2 \text{ J} = \underline{\underline{150 \text{ J}}}$$

diatomic

$$U = \frac{5}{2} N k_B T = \frac{5}{2} p \nabla$$

$$= 2.5 \cdot 10^2 = \underline{\underline{250 \text{ J}}}$$

$$PV = nRT$$

$$\frac{10^5 \text{ Pa} \cdot 10^{-3} \text{ m}^3}{8.31 \cdot 293} = 0.04 \text{ mol}$$

 \equiv

1.24

$$f = 6$$

$$U = \frac{6}{2} N k_B T$$

$$U = 3 N k_B T$$

$$U = 3 n_m \cdot R T = 3 \cdot 0.0048 \text{ mol} \cdot 8.31 \text{ J} \cdot \sum_{K=1}^{m=1} 293 = \underline{\underline{35 \text{ J}}}$$

1 g of Pb

207 g
mol

$$\frac{1 \text{ gr}}{207 \text{ gr/mol}} = \frac{0.0048 \text{ mol}}{1 \text{ mol}} \cdot \frac{6.022 \cdot 10^{23} \text{ atoms}}{1 \text{ mol}}$$

1.4 Heat and Work

spontaneous

flow of energy
due to a difference
in temperature

- force applied over distance
- energy through resistor

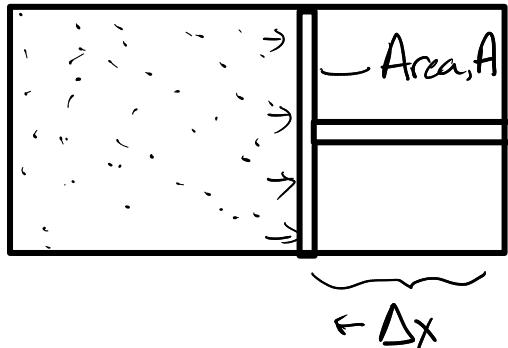
Equation: $dU = \delta Q + \delta W$

↑
small amount

NOT a small change
in amount

$$\delta Q + \delta W \mid Q + W$$

Compressive Work



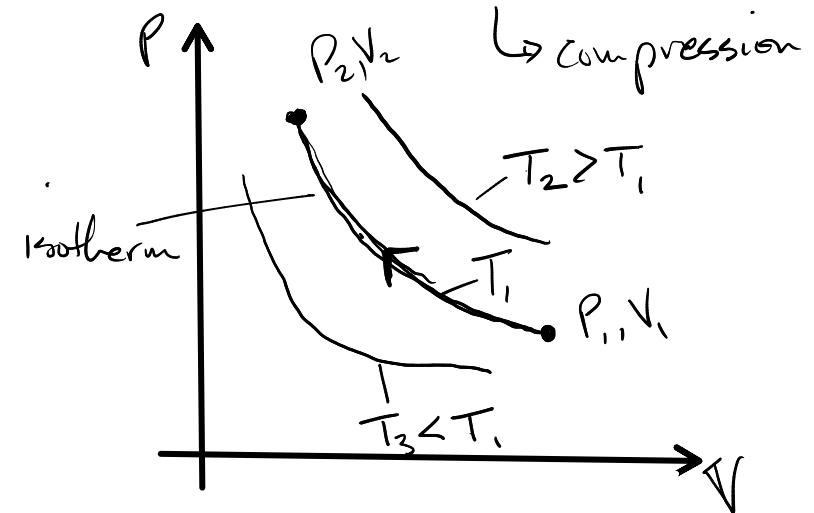
$$\delta W = \vec{F} \cdot \vec{dr} = \vec{F} \cdot \vec{\Delta x} = P \cdot A \cdot \Delta x$$

$\underbrace{-\Delta V}$ quasistatic, static-ish

$$\delta W = -P \Delta V = -P \Delta V$$

$$\delta W = -P(V) \Delta V$$

Isothermal process (constant temperature)



$$PV = Nk_B T$$

all constant

$$\left| P = \frac{Nk_B T}{V} \right| \leftarrow \text{hyperbola}$$

$$W = - \int p dV = - Nk_B T \int \frac{1}{V} dV$$

Constants
in this
case

$$\ln \left[\left(\frac{V_f}{V_i} \right)^{-1} \right]$$

$$W = Nk_B T \ln \left(\frac{V_i}{V_f} \right)$$

$$dU = dQ + dW$$

$$dQ = -dW \quad \xrightarrow{\text{isothermal}}$$

$$Q = -Nk_B T \ln \left(\frac{V_i}{V_f} \right)$$

$$Q = Nk_B T \ln \left(\frac{V_f}{V_i} \right)$$

$$U = \frac{f}{2} Nk_B T$$

$$\Delta U = \frac{f}{2} Nk_B \Delta T$$

$$dU = \frac{f}{2} Nk_B dT$$

$$dT = 0 \therefore dU = 0$$

Adiabatic process (Isentropic process) (no heat)

↳ compression

first law $\rightarrow dU = \cancel{dQ} + dW$

$$dU = dW \quad \text{for an ideal gas: } U = \frac{f}{2} N k_B T$$

$$\frac{f}{2} N k_B dT = dW$$

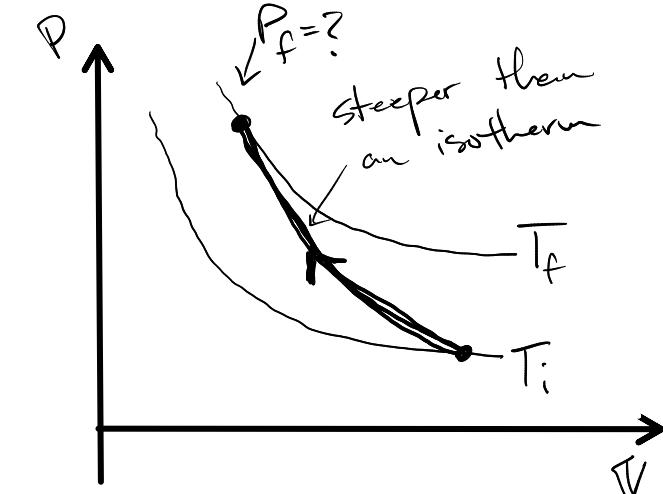
$$\frac{f}{2} N k_B dT = - P dV$$

$$\frac{f}{2} N k_B dT = - \frac{N k_B T}{V} dV$$

$$\frac{f}{2} dT = - \frac{T}{V} dV$$

$$\frac{f}{2} \frac{dT}{T} = - \frac{dV}{V}$$

$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = - \ln\left(\frac{V_f}{V_i}\right)$$



$$dU = \frac{f}{2} N k_B dT$$

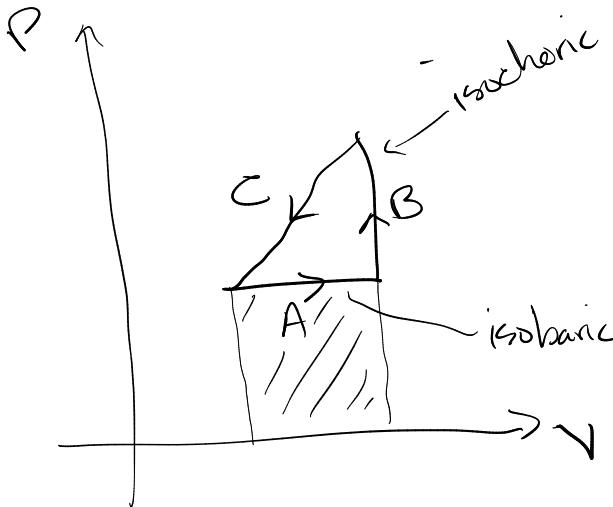
$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\rightarrow \left(\frac{T_f}{T_i}\right)^{\frac{f}{2}} = \frac{V_i}{V_f} \rightarrow V \propto \frac{1}{T^{\frac{f}{2}}} \\ V \propto T^{-\frac{f}{2}}$$

$$T_f^{\frac{f}{2}} \cdot V_f = T_i^{\frac{f}{2}} \cdot V_i \rightarrow V \cdot T^{\frac{f}{2}} = \text{constant of proportionality}$$

HW: 32, 33, 35 ←

1.33



A: $W < 0$

$Q = ?$

$$\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} P \Delta V$$

$$\Delta U > 0$$

$Q > 0$

$$N k_B \Delta T = P \Delta V$$

$$\Delta U = Q + W$$

↑

↑

↓

1.35

$$\sqrt{V T^{f/2}} = \text{a constant}$$

$$PV = \underline{N k_B T}$$

↑
constant

$$P V T^{-1} = \text{constant}$$

$$\sqrt{\gamma P} = \text{another constant}$$

$$\gamma = \frac{f+2}{f}$$

34, 36, 38 → see the end of this document for solutions

1.4 Heat Capacity

$$dU = \cancel{dQ} + \underbrace{\cancel{dW}}_{-PdV}$$

$$\cancel{dQ} = dU + PdV$$

$$\downarrow \underline{U(T, V)} \text{ or } U(T, P)$$

$$dU(T, V) = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad \leftarrow$$

$$\cancel{dQ} = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

$$\frac{\cancel{dQ}}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] \frac{dV}{dT}$$

Two cases: ① Constant volume
for a gas ② Constant pressure

① constant volume

$$dV = 0$$

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT \quad \text{or} \quad \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

\uparrow
heat capacity
at constant volume

For an ideal gas

$$U = \frac{f}{2} N k_B T$$

→ macroscopic form

$$U = \frac{f}{2} n_m R T$$

$$\left(\frac{\partial U}{\partial T}\right)_V = \frac{dU}{dT} = \frac{f}{2} n_m \cdot R = C_V$$

$$\frac{C_V}{n_m} = \frac{f}{2} \cdot R$$

heat capacity per mole
(molar specific heat)

monatomic
 $f=3$

$$\frac{C_V}{n_m} = \frac{3}{2} R$$

$$= 12.5 \text{ J/mol K}$$

diatomic
 $f=5$

$$\frac{C_V}{n_m} = \frac{5}{2} R$$

$$= 20.8 \text{ J/mol K}$$

② constant pressure

$$C_p \equiv \left(\frac{dQ}{dT} \right)_p = \left(\frac{\partial U}{\partial T} \right)_V + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

}

C_V

$$\rightarrow C_p = C_V + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

$$\rightarrow C_p - C_V = \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

for an ideal gas:

$$C_p = \frac{f}{2} n_m R + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

$$U = \frac{f}{2} n_m R \cdot T$$

$$\left(\frac{\partial U}{\partial V} \right)_T = 0$$

for an ideal gas

$$V = \frac{n_m R T}{P}$$

$$\left(\frac{\partial V}{\partial T} \right)_P = \frac{n_m R}{P}$$

$$C_p = \frac{f}{2} n_m R + n_m R$$

monatomic gas

$$f=3$$

$$C_p \approx 20.7$$

$$\frac{C_p}{n_m} = \frac{f}{2} R + R$$

$$\frac{C_p}{n_m} = \frac{C_V}{R} + R$$

diatom gas

$$C_p \approx 29$$

specific heat \rightarrow heat capacity per unit of mass

$$\hookrightarrow C_p = \frac{C_p}{M} \leftarrow \text{heat capacity}$$

\nwarrow total mass

$$C_v = \frac{C_v}{M} \leftarrow \begin{array}{l} \text{big size} \\ \downarrow \\ \text{little size} \end{array}$$

Latent Heat

$$\frac{dQ}{dT} = \infty$$

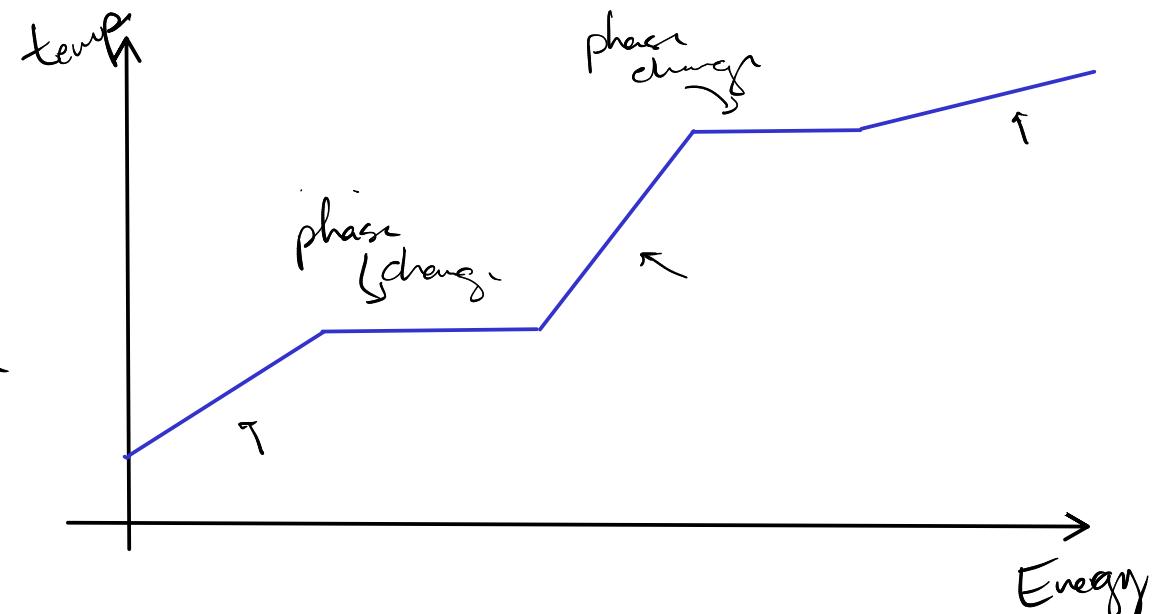
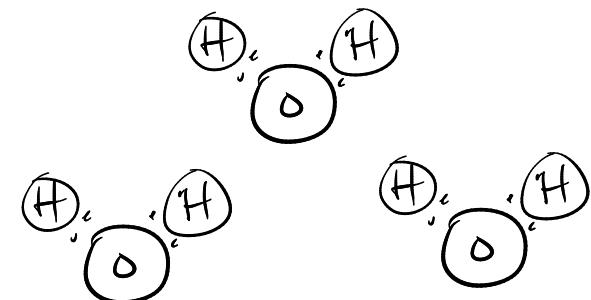
\nwarrow O

$Q \leftarrow$ total heat to completely change the phase
(depends on size)

$$\frac{Q}{m} = L \leftarrow \begin{array}{l} \text{Latent heat} \\ (\text{fusion, vaporization}) \end{array}$$

volumetric specific heat

$$= \frac{C_{v/p}}{V}$$



41, 45, 47

$$C_p = 75.29 \frac{J}{K \cdot mol}$$

41] $\sum \Delta U = 0$

$$\bar{C} = \frac{\Delta U}{\Delta T}$$

a) $\Delta U_{H_2O} = (250g) \cdot (4.186 \frac{J}{g \cdot K}) \cdot (24^\circ C - 20^\circ C)$

$$\Delta U = c \cdot m \cdot \Delta T$$

$$\Delta U = m \cdot C \cdot \Delta T$$

b) $\Delta U_{metal} = -\Delta U_{H_2O}$

c) $C = \frac{\Delta \bar{U}}{\Delta T} = \frac{\Delta U}{29-100}$

d) $\frac{C}{m} = c$

47] $\sum \Delta U = 0$

$$m_f C_f (65^\circ C - 100^\circ C) + \underbrace{ice \text{ warms}}_{m_i \cdot C_{ice}(0 - (-15))} + \underbrace{ice \text{ melts}}_{m_i \cdot L_f} + \underbrace{water \text{ warms}}_{m_i \cdot C_w (65^\circ C - 0^\circ C)} = 0$$

$$m_i \cdot C_{ice}(0 - (-15)) + m_i \cdot L_f + m_i \cdot C_w (65^\circ C - 0^\circ C) = 0$$

$\underbrace{333 J/g}$

1.45] $w = xy$ $x = yz$ ← given

a) $w = x \left(\frac{y}{z} \right)$ $\boxed{w = y^2 z}$ in terms of $y + z$

$\boxed{w = \frac{x^2}{z}}$ in terms of $x + y$

b) $\left(\frac{\partial w}{\partial x} \right)_y = y$ $\left(\frac{\partial w}{\partial x} \right)_z = \frac{2x}{z}$

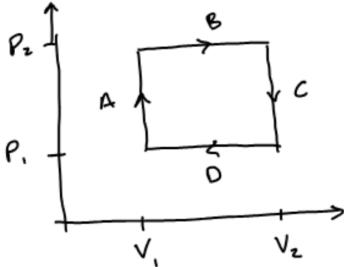
Since $y = \frac{x}{z}$

$$\left(\frac{\partial w}{\partial x} \right)_y = 2 \left(\frac{\partial w}{\partial x} \right)_z$$

c) $\left(\frac{\partial w}{\partial y} \right)_x = x$ $\left(\frac{\partial w}{\partial y} \right)_z = 2yz$

$$\left(\frac{\partial w}{\partial z} \right)_x = -\frac{x^2}{z^2} \quad \left(\frac{\partial w}{\partial z} \right)_y = y^2$$

1.34]



diatomic gas
f = 5

(a) work in each stage

A: 0

B: $-P_2(V_2 - V_1) = P_2(V_1 - V_2)$

C: 0

D: $-P_1(V_1 - V_2) = P_1(V_2 - V_1)$

 ΔU in each stage

A: $\Delta U = \frac{5}{2}Nk_B T - \frac{5}{2}NPV = \frac{5}{2}V_1(P_2 - P_1)$

B: $\Delta U = \frac{5}{2}PV = \frac{5}{2}P_2(V_2 - V_1)$

C: $\Delta U = \frac{5}{2}V_2(P_1 - P_2)$

D: $\Delta U = \frac{5}{2}P_1(V_1 - V_2)$

Q in each stage

A: $Q = \Delta U - W = \frac{5}{2}V_1(P_2 - P_1) \leftarrow \text{add heat}$

B: $\frac{5}{2}P_2(V_2 - V_1) - (-P_2(V_2 - V_1)) = \frac{5}{2}P_2(V_2 - V_1) + P_2(V_2 - V_1) = \frac{7}{2}P_2(V_2 - V_1) \leftarrow \text{add heat}$

C: $\frac{5}{2}V_2(P_1 - P_2) \leftarrow \text{remove heat}$

D: $\frac{7}{2}P_1(V_1 - V_2) \leftarrow \text{remove heat}$

(b) A \rightarrow adding heat at constant volume
to increase pressureB \rightarrow expanding gas works out to surroundings
add heat to maintain pressureC \rightarrow cool at constant volumeD \rightarrow do work to compress but cool
to keep constant pressure

c) Net Work: $W_{B\rightarrow D} = P_2(V_1 - V_2) + (-P_1(V_1 - V_2))$
 $= (V_1 - V_2)(P_2 - P_1)$
 $\underbrace{\quad}_{\geq 0} \quad \underbrace{\quad}_{\leq 0}$
 \uparrow
 \Rightarrow net work onto
surroundings

Net Change in Energy

$$\frac{5}{2}V_1(P_2 - P_1) + \frac{5}{2}P_2(V_2 - V_1) + \frac{5}{2}V_2(P_1 - P_2) + \frac{5}{2}P_1(V_1 - V_2)$$

$$- \frac{5}{2}V_2(P_2 - P_1) - \frac{7}{2}P_1(V_2 - V_1)$$

$$\begin{aligned} & \frac{5}{2}(V_2 - V_1)(P_2 - P_1) + (P_2 - P_1)(V_1 - V_2) \\ & \frac{5}{2}(V_2 - V_1)(P_2 - P_1) - (V_2 - V_1)(P_2 - P_1) \\ & \underbrace{\quad}_{=0} \end{aligned}$$

Net Heat Added

$$Q_{NET} = \frac{5}{2}V_1(P_2 - P_1) + \frac{7}{2}P_2(V_2 - V_1) + \frac{5}{2}V_2(P_1 - P_2) + \frac{7}{2}P_1(V_1 - V_2)$$

$$- \frac{5}{2}V_2(P_2 - P_1) - \frac{7}{2}P_1(V_2 - V_1)$$

$$= \frac{5}{2}(P_2 - P_1)(V_1 - V_2) + \frac{7}{2}(P_2 - P_1)(V_2 - V_1)$$

$$= -\frac{5}{2}(P_2 - P_1)(V_2 - V_1) + \frac{7}{2}(P_2 - P_1)(V_2 - V_1)$$

$$- (P_2 - P_1)(V_2 - V_1) \text{ exactly equal to work}$$

$\underbrace{\quad}_{\text{both } \geq 0 \text{ so net heat}}$
 is added and work
 is extracted.

$$(1.36) \quad (a) V_1 = 1 \text{ liter} = 0.001 \text{ m}^3 \quad P_1 = 1 \text{ atm} = 1 \cdot 10^5 \text{ Pa}$$

$$V_2 = ? \quad P_2 = 7 \text{ atm} = 7 \cdot 10^5 \text{ Pa}$$

$$\gamma = \frac{7}{5} \text{ for air}$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{\gamma}} = 1 \text{ liter} \left(\frac{1 \text{ atm}}{7 \text{ atm}} \right)^{\frac{1}{\gamma}}$$

$$V_2 = 0.249 \text{ liters}$$

(b) work done

$$dU = dW$$

$$\Delta U = W$$

$$\frac{5}{2} N k_B \Delta T - W$$

↑
so what is
final T and N?

It is easier to solve
for part c. first and
then do b, but the
alternative way is
possible w/ an
integral

(c)

$$VT^{\frac{1}{\gamma}} = \text{const}$$

$$\left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma}} = \frac{V_1}{V_2}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\frac{1}{\gamma}} = 300 \text{ K} \left(\frac{1 \text{ L}}{0.249 \text{ L}} \right)^{\frac{1}{\gamma}}$$

$$= 300 \text{ K} (1.74) = 523 \text{ K}$$

250°C

$$\Delta T = T_2 - T_1 = 223 \text{ K}$$

$$N = \frac{PV_1}{k_B T_1} = \frac{10^5 \cdot 0.001 \text{ m}^3}{1.38 \cdot 10^{-23} \cdot 300 \text{ K}}$$

$$N = 2.41 \cdot 10^{22} \approx 0.040 \text{ mol} = n$$

$$N k_B T_1 = PV = 10^5$$

$$\frac{5}{2} N k_B \Delta T$$

$$\frac{5}{2} N k_B (T_2 - T_1)$$

$$\frac{5}{2} N k_B T_1 \left(\frac{T_2}{T_1} - 1 \right)$$

$$10^5 \left(\left(\frac{V_1}{V_2} \right)^{\frac{1}{\gamma}} - 1 \right)$$

$$\frac{5}{2} \cdot 10^2 (0.74) = 185 \text{ J}$$

Alternative: $dW = -PdV$

$$PV^\gamma = \text{constant}$$

$$\frac{P_2}{P_1} = \frac{V_1^\gamma}{V_2^\gamma} \Rightarrow P(V) = \frac{P_1 V_1^\gamma}{V^\gamma}$$

$$dW = -P V_1^\gamma dV$$

$$W = - \int_{V_1}^{V_2} \frac{P_1 V_1^\gamma}{V^\gamma} dV = -P_1 V_1^\gamma \int_{V_1}^{V_2} \frac{dV}{V^\gamma}$$

$$= -P_1 V_1^\gamma \left[\frac{V_1^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$= \frac{P_1 V_1^\gamma}{\gamma-1} \left[V_1^{-\gamma+1} - V_2^{-\gamma+1} \right]$$

$$W = \frac{P_1 V_1^\gamma}{\gamma-1} \left(V_1^{-\gamma+1} \right)_{V_1}$$

$$W = \frac{P_1 V_1^\gamma}{\gamma-1} \left(V_2^{-\gamma+1} - V_1^{-\gamma+1} \right)$$

$$\begin{aligned} &= \frac{P_1}{\gamma-1} \left(\frac{V_1^\gamma}{V_2^{\gamma-1}} - V_1 V_2^{\gamma-1} \right) \\ &= \frac{P_1}{\gamma-1} \left(\frac{V_1^\gamma}{V_2^{\gamma-1}} - V_1 \right) \\ &= \frac{P_1 V_1}{\gamma-1} \left(\frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} - 1 \right) \\ &= \frac{P_1 V_1}{\gamma-1} \left(\left(\frac{V_1}{V_2} \right)^{\gamma-1} - 1 \right) \end{aligned}$$

$$= \frac{5 \cdot 10^5 \text{ Pa} \cdot 0.001 \text{ m}^3}{2} \left(\left(\frac{1}{0.249} \right)^{\frac{1}{\gamma}} - 1 \right) = 185 \text{ J}$$

$$\gamma^{-1} = \frac{7}{5} - \frac{5}{5} = \frac{2}{5}$$

1.38

