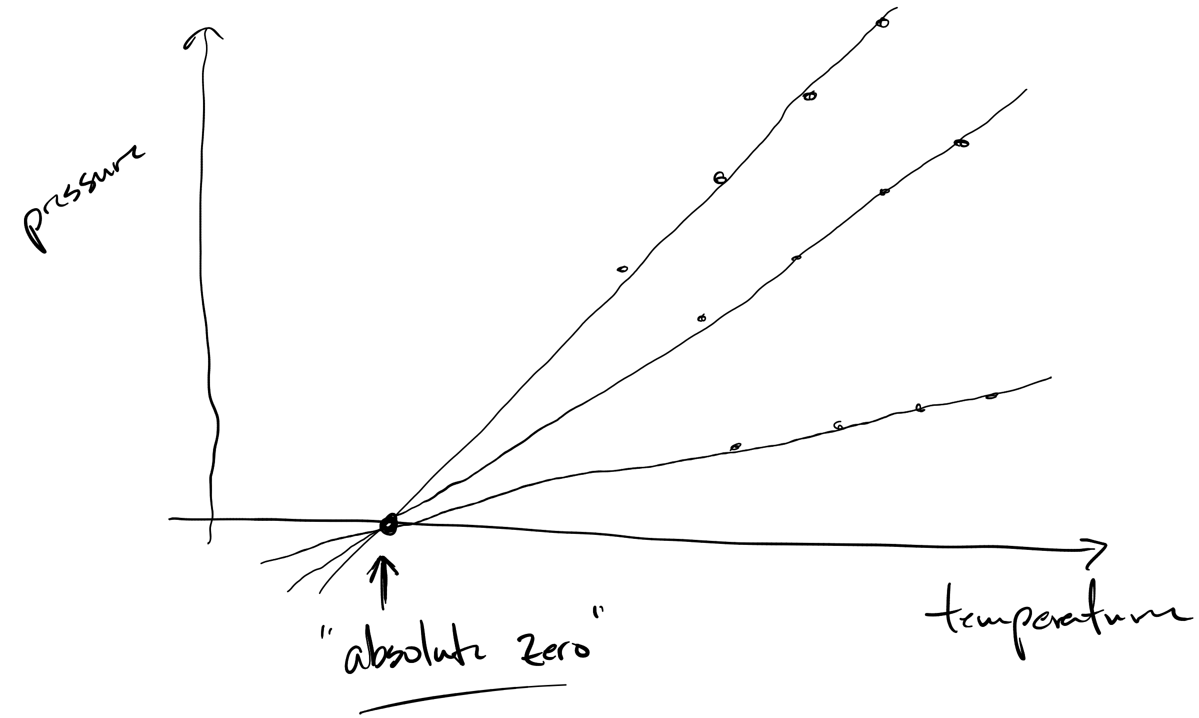
Thermal Equilibrium - when objects have been in contact and macroscopic coordinates have stopped changing

→ involves an exchange of energy between two object, or an object + its surrounding

→ volume (constant pressure) → mercury/alcohol

- pressure
- electrical resistance
- radiation/thermal emf



$0K = -273.15°C$

1) $C_1 = 0°C$ $C_2 = 100°C$

$F_1 = 32°F$ $F_2 = 212°F$

$$C = mF + b$$

1.2 Ideal Gas Law → Equation of State → relates all of the state variables together

$$pV = N k_B T$$

$p = \frac{F}{\text{Area}}$

$[Pa] = \left[\frac{N}{m^2}\right]$

N → number of particles

k_B → Boltzmann's constant

$k_B = 1.38 \cdot 10^{-23}\ J/K$

T → K

$$pV = nRT$$

n → number of moles

R → universal gas constant

$R = 8.31\ J/\text{mol}\cdot K$?

$$N k_B = nR$$

$$6.022 \cdot \cancel{10^{23}} \cdot 1.38 \cdot \cancel{10^{-23}} = R$$

$$8.31\ J/\text{mol}\,K = R$$

Number, moles, molar, density

1 mole of things = $6.022 \cdot 10^{23}$ things

particle
atom
molecule

C^{12}	C^{14}
→ 6 proton 6 neutrons	→ 6 protons → 8 neutrons

1 mole is 1 gram of protons + neutrons

Ex: mass of one proton in kg?

mass of one proton × number of proton = mass of the collection

$$m \times N = M$$

$$m = \frac{1g}{N_A} = \frac{1}{6.022 \cdot 10^{23}} = 1.7 \cdot 10^{-24} \text{ grams} = 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2?

$$1 \text{ mole of } N_2 = 2\left(14 \frac{g}{mol}\right) = 28 \, g/mol$$

Volume of 1 mole of air at room temp and atmospheric pressure?

↳ $1 atm = 1.013 \cdot 10^5 Pa$

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \, J/K \cdot mol \cdot 300K}{10^5 Pa} = 0.0249 \, m^3$$

$$V_{cube} = s^3$$

$$s = 0.292\ m \sim 30\ cm \sim 1\ ft$$

$\boxed{1.17}$

$$PV = nRT\left(1 + \frac{B(T)}{V/n}\right)$$

a) $$PV = nRT\left(1 + \frac{n}{V}\cdot B(T)\right)$$

at atmospheric pressure $P = 10^5\ Pa$

solve for $\frac{n}{V}$

$$P = \frac{n}{V}RT\left(1 + \frac{n}{V}B(T)\right)$$

$$0 = \frac{n}{V}RT + \left(\frac{n}{V}\right)^2\cdot RT\cdot B(T) - P$$

T	$B(T)$	$\frac{n}{V}$	$\frac{n}{V}\cdot B(T)$
100	-160		
200	-35		
300	-4.2		
400	9.0		
500	16.9		
600	21.3		

b) $$PV = nRT\left(1 + \frac{B(T)}{V/n}\right)$$ if

c) $\rightarrow$ $$PV = nRT\left(1 + \frac{B(T)}{V/n} + \frac{C(T)}{(V/n)^2}\right)$$

want: B + C in terms of a + b

$$\left(P + \frac{an^2}{V^2}\right)\left(V - nb\frac{V}{V}\right) = nRT$$

$$\left(P + \frac{an^2}{V^2}\right)V\left(1 - \frac{nb}{V}\right) = nRT$$

$$\left(P + \frac{an^2}{V^2}\right)V = nRT\left(1 - \frac{nb}{V}\right)^{-1}$$

$$PV + \frac{an^2}{V} = nRT\left(1 - \frac{nb}{V}\right)^{-1}$$

$$PV = nRT\underbrace{\left(1-\frac{nb}{V}\right)^{-1}}_{\text{use our approx.}} - \frac{an^2}{V}$$

$$(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2 \qquad px \ll 1$$

$$\left(1+\left(-\frac{nb}{V}\right)\right)^{-1} \approx 1 + (-1)\left(-\frac{nb}{V}\right) + \frac{1}{2}(-1)\overset{-2}{(-1-1)}\left(-\frac{nb}{V}\right)^2$$

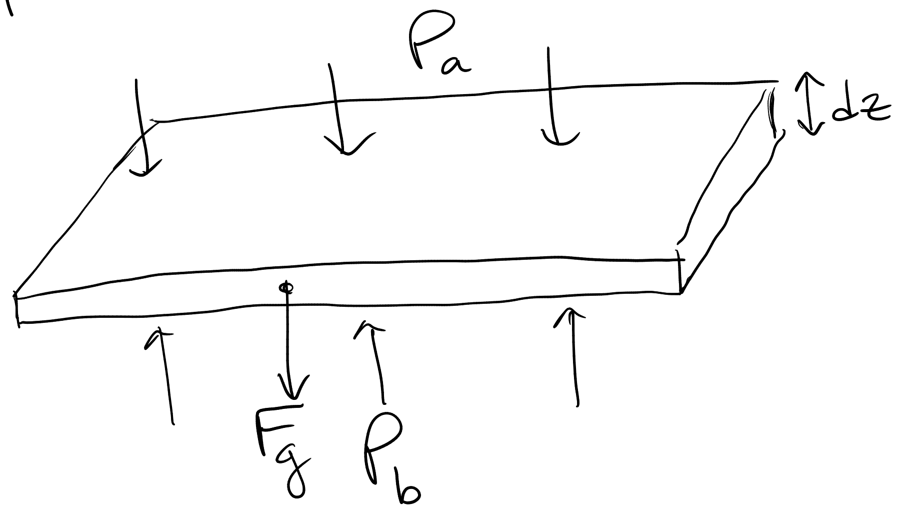
$$\approx \underbrace{1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2}$$

$$PV = nRT\left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2\right) - \frac{an^2}{V}\left(\frac{nRT}{nRT}\right) \longleftrightarrow PV = nRT\left(1 + \frac{n}{V}\cdot B(T) + \left(\frac{n}{V}\right)^2\cdot C(T)\right)$$

$$PV = nRT\left(1 + \frac{nb}{V} + \left(\frac{nb}{V}\right)^2 - \frac{a\cancel{n^2}}{V\cdot\cancel{n}RT}\right) = nRT\left[1 + \frac{n}{V}\underbrace{\left(b - \frac{a}{RT}\right)}_{B(T)} + \left(\frac{n}{V}\right)^2\cdot\underbrace{b^2}_{C(T)}\right]$$

d) → plot data from that table

→ plot $B(T) = b - \frac{a}{RT}$ choose b, a



1.16 a) $\rho = \frac{M}{V}$ ← total mass

"rho" volumetric mass density

$P = \frac{F}{A}$

$V = A \cdot dz$

$\rho = \frac{M}{A \cdot dz}$

$M = \rho A dz$

$+P_b \cdot A - P_a \cdot A - M \cdot g = 0$

$P_b \cdot \cancel{A} - P_a \cdot \cancel{A} - \rho \cancel{A} dz \cdot g = 0$

$-(-P_b + P_a) = \rho dz \cdot g$

$-\underbrace{(P_a - P_b)}_{dP} = \rho dz \Rightarrow \frac{dP}{dz} = -\rho g$

$$\boxed{\frac{dP}{dz} = -\frac{mg}{k_B T} P}$$

b) $PV = N k_B T$

$m P = \frac{m N k_B}{V} T$

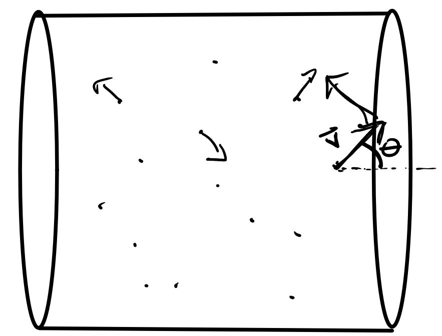
$m P = \rho k_B T$

1.2b Kinetic Theory and Equipartition of Energy

pressure $\longleftrightarrow$ kinetic energy $\longleftrightarrow$ temperature

$$\underset{\text{collision}}{\text{pressure}} = \frac{F}{A} = \frac{\Delta p}{A\Delta t}$$

Δp: $2mv\cos\theta$



$$d(\text{pressure}) = \frac{2mv\cos\theta}{dA \cdot dt} \cdot$$ number of particles hitting area dA w/ velocity v in dt amount of time

} integrate over all velocities and over θ

number of particles hitting area dA w/ velocity v in dt amount of time =

number of atoms traveling in a particular direction w/ a particular speed $\cdot$ fraction of them that are within striking distance of the surface dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

$$N = \sum_{x=0}^{\infty} N(x)$$

total outcomes

$$P(x) = \frac{N(x)}{N}$$

rearrange

$$N(x) = P(x) \cdot N$$

normalized probabilities

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \underbrace{\sum_{x=0}^{\infty} x \cdot P(x)}_{\text{weighted average by probability}}$$

- average value
- expectation value

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

Example (Blundell 3.3)

a. $P(x) = \dfrac{e^{-m} m^x}{x!}$; show $\sum_{x=0}^{\infty} P(x) = 1$

Looked up Taylor Series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sum_{x=0}^{\infty} \frac{e^{-m} m^x}{x!} \stackrel{?}{=} 1$$

$$e^{-m} \cdot \sum_{x=0}^{\infty} \frac{m^x}{x!} \stackrel{?}{=} 1$$

$$\sum_{x=0}^{\infty} \frac{m^x}{x!} \stackrel{?}{=} e^m \checkmark$$

b. Show $\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x) \stackrel{?}{=} m$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-m} m^x}{x!} \stackrel{?}{=} m$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{x \cdot m^x}{x!} \stackrel{?}{=} m$$

$$\sum_{x=0}^{\infty} \underbrace{\frac{x \cdot m^x}{x!}}_{} \stackrel{?}{=} m e^m$$

$x \cdot (x-1) \cdot (x-2)(x-3) \cdots 1$

$$\cancel{\frac{0 \cdot m^0}{0!}} + \frac{1 \cdot m^1}{1!} + \frac{2 m^2}{2!} + \cdots$$

$$\sum_{x=1}^{\infty} \frac{x m^x}{x!} = \sum_{x=1}^{\infty} \frac{m^x}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m^{x+1-1}}{(x-1)!} = \sum_{x=1}^{\infty} \frac{m \cdot m^{x-1}}{(x-1)!}$$

$x \cdot (x-1) \cdot (x-2) \cdots$

$x' = x - 1$

$x = x' + 1$

$$m \cdot \sum_{x'=0}^{\infty} \frac{m^{x'}}{x'!}$$

drop the primes!

c. $\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$

$$= 0 \cdot \frac{109}{200} + 1 \cdot \frac{65}{200} + 2 \cdot \frac{22}{200} + 3 \cdot \frac{3}{200} + 4 \cdot \frac{1}{100} + 5 \cdot \frac{0}{200}$$

$$= \underline{\underline{0.61}} = \frac{122}{200}$$

$$P(x) = \frac{e^{-0.61} \, 0.61^{x}}{x!}$$

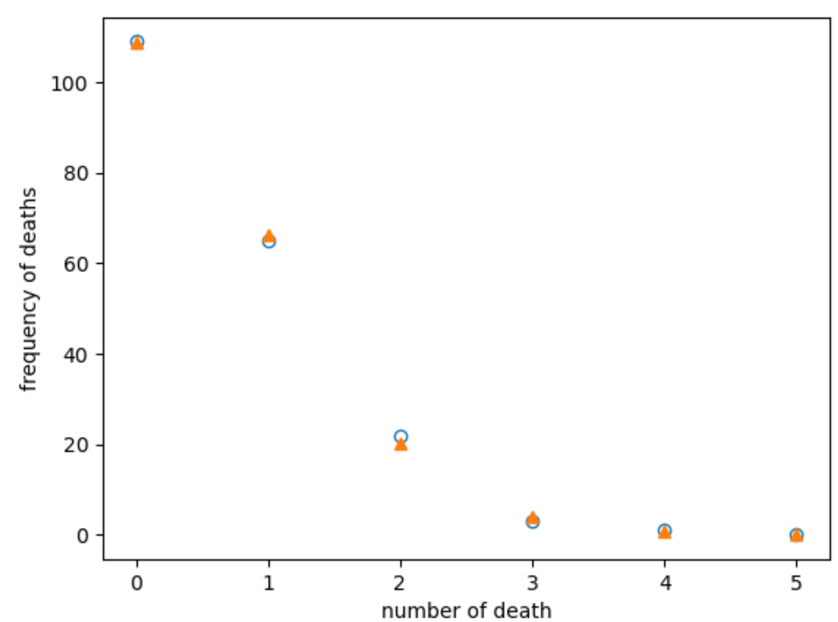
$$m \cdot \underbrace{\sum_{x=0}^{\infty} \frac{m^x}{x!}}_{= e^m}$$

$$me^m \overset{?}{=} me^m$$

$$\langle x \rangle = m$$

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import factorial

x = [0,1,2,3,4,5]
y = [109, 65, 22, 3, 1, 0]

fig, ax = plt.subplots()
ax.plot(x, y, 'o', mfc='none')
ax.set_xlabel('number of death')
ax.set_ylabel('frequency of deaths')

def deaths(number):
    return(np.exp(-0.61)*np.power(0.61, number)/factorial(number))

ax.plot(x, deaths(x)*200, '^')
```

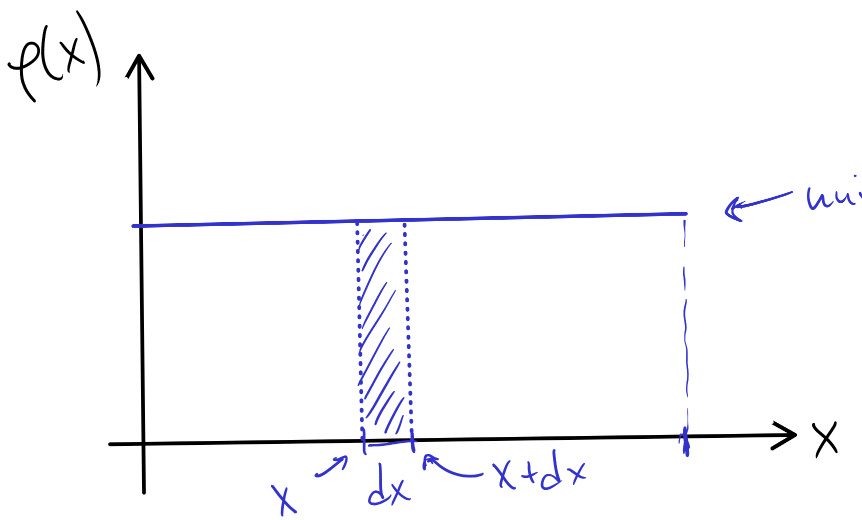
$\rightarrow$

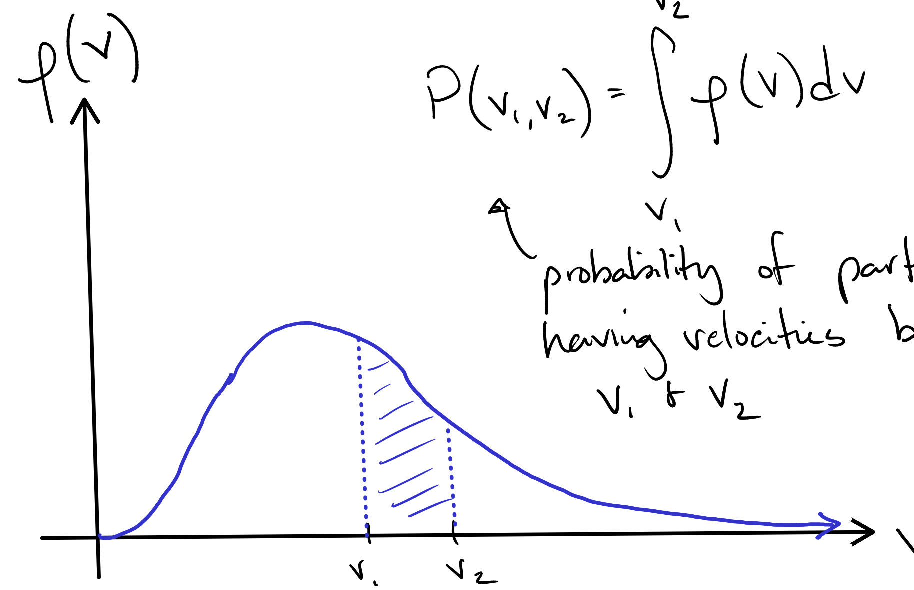


The continuous variable case follows by analogy but w/ some clarifications

$$\left\{\begin{array}{l}\text{probability of choosing}\\ \text{a value of random}\\ \text{between } x \text{ and } x+dx\end{array}\right\} = \underbrace{\rho(x)}_{\text{distribution function}} \cdot dx$$

probability itself is an area.





$$P(v_1,v_2) = \int_{v_1}^{v_2} \rho(v)\,dv$$

probability of particles having velocities b/t v_1 & v_2

Normalized

$$\int_{-\infty}^{\infty} \rho(v)\,dv = 1$$

Since we will be dealing w/ such large numbers

$$\left\{\begin{matrix}\text{probability that a} \\ \text{particle has a} \\ \text{velocity between} \\ v \text{ and } v+dv\end{matrix}\right\} = \left\{\begin{matrix}\text{fraction of particles} \\ \text{having a velocity} \\ \text{between } v + v+dv\end{matrix}\right\}$$

Also mean

$$\langle v \rangle = \int_{-\infty}^{\infty} v \cdot \rho(v)\,dv$$

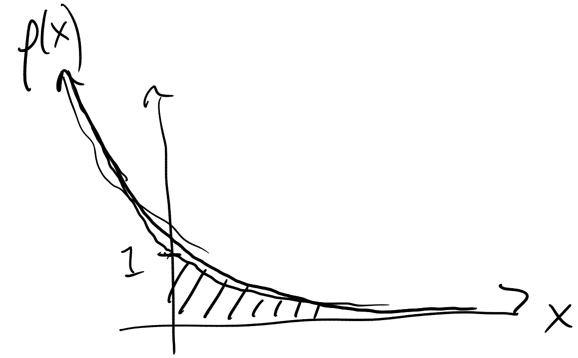
or

$$\langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 \cdot \rho(v)\,dv$$

or

$$\langle f(v) \rangle = \int_{-\infty}^{\infty} f(v) \cdot \rho(v)\,dv$$

<u>Example:</u> (Blundell 3.4)



$$\rho(x)\,dx = Ae^{-x/\lambda}\,dx$$

a) Find A so that

$$\int_0^{\infty} \rho(x)\,dx = 1$$

$$\int_0^{\infty} Ae^{-x/\lambda}\,dx = 1$$

$$A(-\lambda)e^{-x/\lambda}\Big|_0^{\infty} = 1$$

$$A(-\lambda)(0 - -1) = 1 \qquad A\lambda = 1 \Rightarrow \boxed{A = \frac{1}{\lambda}}$$

b) $$\langle x \rangle = \int_0^{\infty} x \,\rho(x)\,dx = \lambda$$

$$uv\Big|_0^{\infty} - \int_0^{\infty} v\,du$$

$$u = x \qquad dv = e^{-x/\lambda}dx$$

$$du = dx \qquad v = -\lambda e^{-x/\lambda}$$

$$\int_0^{\infty} x \cdot \frac{1}{\lambda} e^{-x/\lambda}\,dx \stackrel{?}{=} \lambda$$

$$\frac{1}{\lambda}\left(-\lambda x e^{-x/\lambda}\Big|_0^{\infty} - (-\lambda)\int_0^{\infty} e^{-x/\lambda}\,dx\right)$$

$\lambda = \langle x \rangle$ ✓

c) $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

$\sigma \to$ standard deviation

$\sigma^2 \to$ variance

$$\langle x \rangle = \lambda$$

$$\langle x \rangle^2 = \lambda^2$$

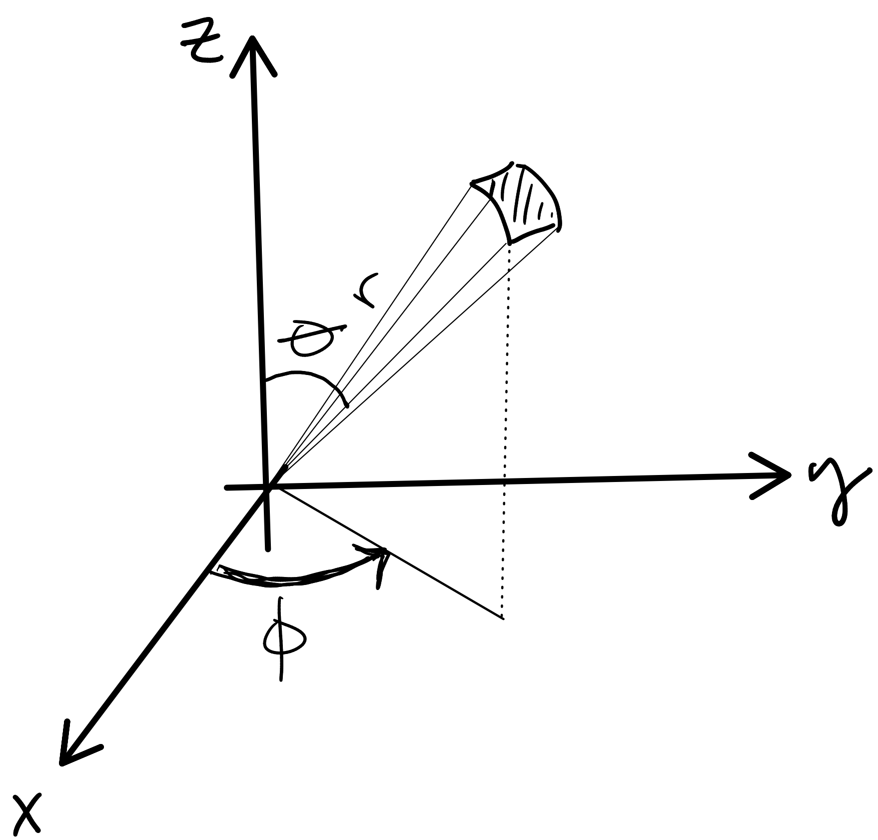
$$\langle x^2 \rangle = \int_0^\infty x^2 \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda}\, dx$$

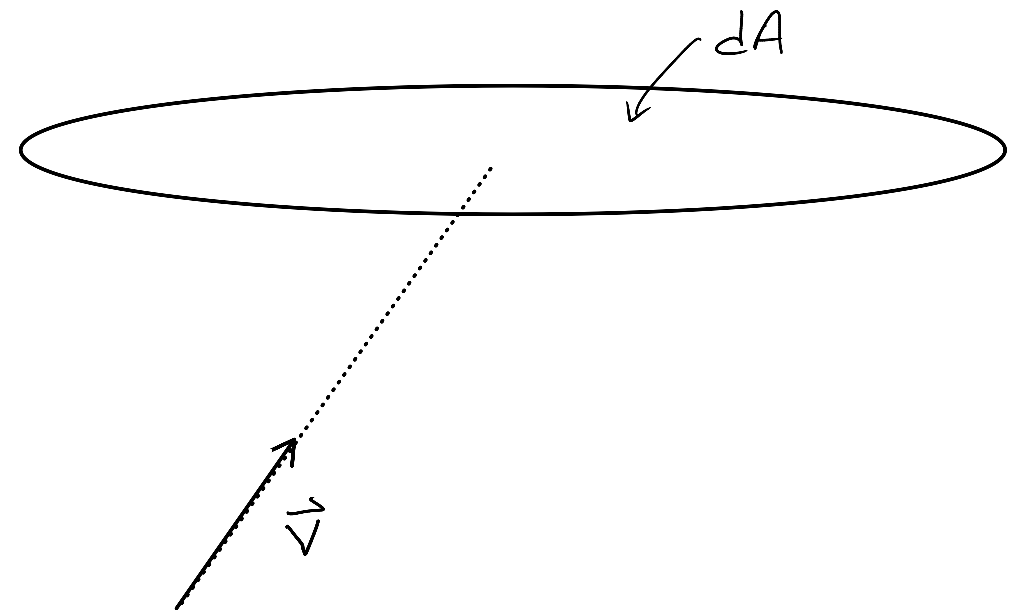
$\hookrightarrow$ LOOK IT UP!

$$\langle x^2 \rangle = 2\lambda^2$$

$\sigma^2 = 2\lambda^2 - \lambda^2 = \lambda^2 \leftarrow$ variance

$\sigma = \lambda \leftarrow$ standard deviation number of particles traveling in a particular direction w/ a particular speed





$$\boxed{U = \frac{3}{2} N k_B T}$$

internal energy

$$U = N \langle K \rangle$$

avg of kinetic energy

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$= \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

average of a sum is the sum of averages

$$= \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right)$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

$$\frac{3}{2} k_B T = \frac{3}{2} m \langle v_x^2 \rangle$$

$$\frac{1}{2} k_B T = \frac{1}{2} m \langle v_x^2 \rangle$$

Equipartition Theorem

$$U_{thermal} = N \cdot f \cdot \frac{1}{2} k_B T$$

f → degree of freedom (quadratic)

$$\frac{1}{2}mv_x^2, \ \frac{1}{2}mv_y^2, \ \frac{1}{2}mv_z^2, \ \frac{1}{2}I\omega_x^2, \ \frac{1}{2}k_s x^2$$

$f = 3 \leftarrow$ monatomic gas

$f = 5 \leftarrow$ diatomic gas near room temp

$f = 6 \leftarrow$ solid

1.23

$V = 1 \text{ liter} = 0.001 \, m^3 = 10^{-3} \, m^3$

$p = 1.01 \cdot 10^5 \, Pa = 10^5 \, Pa$

$T = 293 \, K$

monoatomic

$\rightarrow U = \frac{3}{2} N k_B T$ (f)

$pV = N k_B T$

$U = \frac{3}{2} pV = \frac{3}{2} (10^5 Pa)(10^{-3} m^3)$

$= 1.5 \cdot 10^2 \, J = \underline{150 \, J}$

diatomic

$U = \frac{5}{2} N k_B T = \frac{5}{2} pV$

$= 2.5 \cdot 10^2 = \underline{250 \, J}$

$pV = nRT$

$\frac{10^5 Pa \cdot 10^{-3} m^3}{8.31 \cdot 293} = \underline{\underline{0.04 \, mol}}$

1.24

$f = 6$

$U = \frac{6}{2} N k_B T$

$U = 3 N k_B T$

1g of Pb

$207 \frac{g}{mol}$

$\frac{1 \, gr}{207 \, gr/mol} = \underline{\underline{0.0048 \, \cancel{mol}}} \cdot \frac{6.022 \cdot 10^{23} \text{ atoms}}{1 \, \cancel{mol}}$

$U = 3 n_m R T = 3 \cdot 0.0048 \, mol \cdot 8.31 \, J/Kmol \cdot 293 = \underline{35 \, J}$

1.4 Heat and Work

spontaneous

flow of energy due to a difference in temperature

- force applied over distance
- energy through resistor

Equation: $dU = đQ + đW$

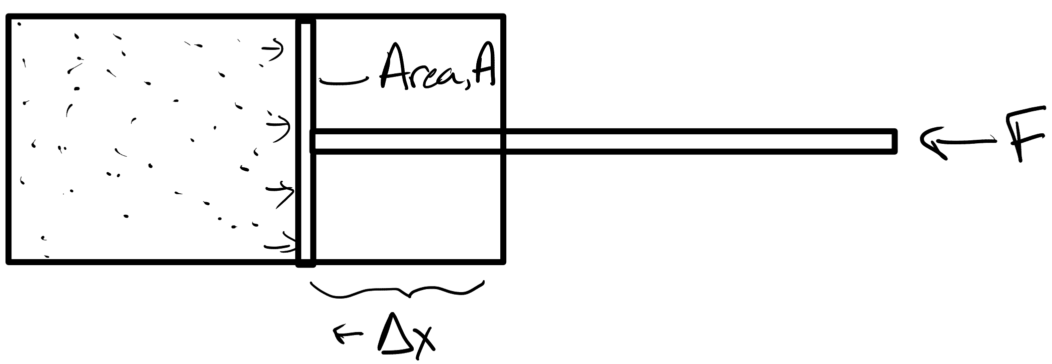
small amount

NOT a small change in amount

$\delta Q + \delta W \mid Q + W$

Compressive Work

$$đW = \vec{F}\cdot d\vec{r} = \vec{F}\cdot \Delta\vec{x} = P\cdot \underbrace{A\cdot \Delta x}_{-\Delta V}$$

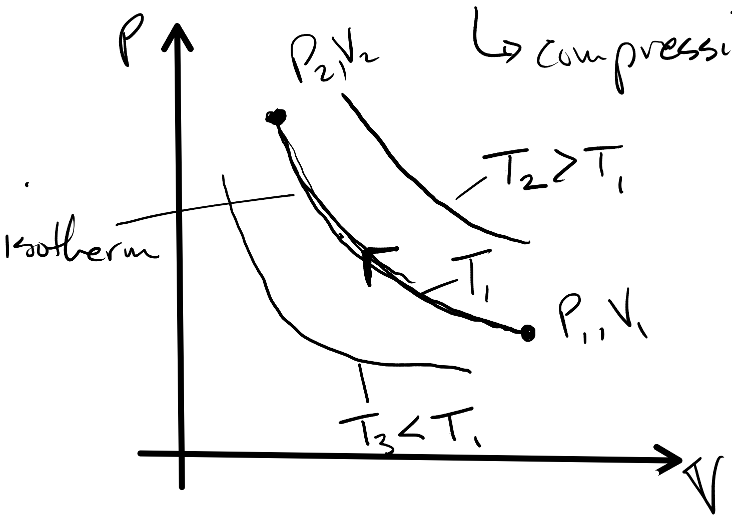


quasistatic, static-ish

$$đW = -P\,dV = -P\,\Delta V$$

$$đW = -P(V)\,dV$$

Isothermal process (constant temperature)



$$PV = \underline{\underline{Nk_BT}}$$

all constant

$$\boxed{P = \frac{Nk_BT}{V}} \leftarrow \text{hyperbola}$$

$$W = -\int p\,dV = -\underbrace{Nk_BT}_{\text{constants in this case}} \int_{V_i}^{V_f} \frac{1}{V}\,dV$$

$$W = -Nk_BT \ln\left(\frac{V_f}{V_i}\right) \qquad \ln\left[\left(\frac{V_f}{V_i}\right)^{-1}\right]$$

$$W = Nk_BT \ln\left(\frac{V_i}{V_f}\right)$$

$$\cancel{dU}^{\,0} = \text{đ}Q + \text{đ}W$$

$$0 = \text{đ}Q + \text{đ}W$$

$$\text{đ}Q = -\text{đ}W \leftarrow \text{isothermal}$$

$$Q = -Nk_BT \ln\left(\frac{V_i}{V_f}\right)$$

$$Q = Nk_BT \ln\left(\frac{V_f}{V_i}\right)$$

$$U = \frac{f}{2} Nk_BT$$

$$\Delta U = \frac{f}{2} Nk_B \Delta T$$

$$dU = \frac{f}{2} Nk_B dT$$

$$dT = 0 \therefore dU = 0$$

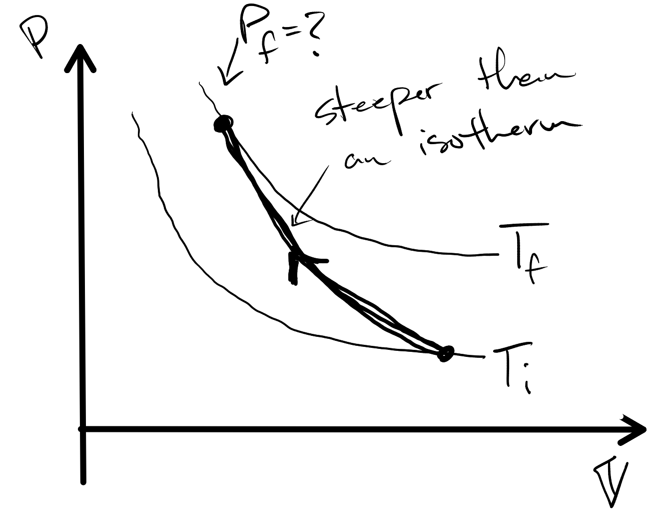
Adiabatic process (Isentropic process) (no heat)

↳ compression

first law → $dU = \cancel{đQ}^{\nearrow 0} + đW$

$dU = đW$ for an ideal gas: $U = \frac{f}{2} N k_B T$

$dU = \frac{f}{2} N k_B dT$



$$\frac{f}{2} N k_B dT = đW$$

$$\frac{f}{2} N k_B dT = -p\,dV$$

$$\frac{f}{2} \cancel{N k_B} dT = -\frac{\cancel{N k_B} T}{V} dV$$

$$\frac{f}{2} dT = -\frac{T}{V} dV$$

$$\frac{f}{2} \frac{dT}{T} = -\frac{dV}{V}$$

$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = -\ln\left(\frac{V_f}{V_i}\right)$$

$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\rightarrow \left(\frac{T_f}{T_i}\right)^{\frac{f}{2}} = \frac{V_i}{V_f} \rightarrow V \propto \frac{1}{T^{f/2}}$$

$$V \propto T^{-f/2}$$

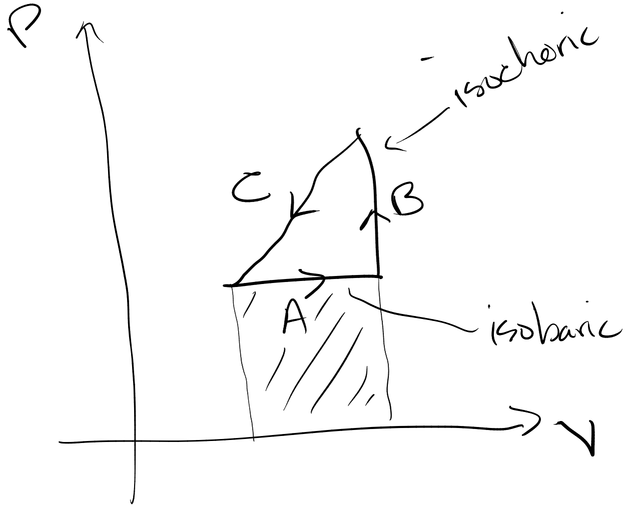
$$T_f^{f/2} \cdot V_f = T_i^{f/2} \cdot V_i \rightarrow V \cdot T^{f/2} = \text{constant of proportionality}$$

HW: 32, 33, 35 ←

$PΔV$

$Nk_B ΔT = ΔPV$

1.33



A: $W < 0$

$Q = ?$

$Q > 0$

$ΔU = \frac{3}{2} N k_B ΔT = \frac{3}{2} PΔV$

$ΔU > 0$

$ΔU = Q + W$

1.35

$V T^{f/2} = $ a constant

$PV = \underbrace{Nk_B}_{\text{constant}} T$

$V^{\gamma} P$ = another constant

PVT^{-1} = constant

$\gamma = \frac{f+2}{f}$

34, 36, 38 → see the end of this document for solutions

1.6 Heat Capacity

$$dU = đQ + \underbrace{đW}_{-p\,dV}$$

$$đQ = dU + p\,dV$$

$\downarrow$ $\underline{U(T,V)}$ or $U(T,P)$

$$dU(T,V) = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \leftarrow$$

$$đQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$$

$$\frac{đQ}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\frac{dV}{dT}$$

Two cases: ① Constant volume
for a gas ② Constant pressure

1. constant volume

$$dV = 0$$

$$đQ = \left(\frac{\partial U}{\partial T}\right)_V dT \quad \text{or} \quad \left(\frac{đQ}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_V \equiv \left(\frac{đQ}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

heat capacity at constant volume

For an ideal gas

$$U = \frac{f}{2} N k_B T$$

macroscopic form

$$U = \frac{f}{2} n_m R T$$

$$\left(\frac{\partial U}{\partial T}\right)_V = \frac{dU}{dT} = \frac{f}{2} n_m \cdot R = C_V$$

$$\frac{C_V}{n_m} = \frac{f}{2} \cdot R$$

← heat capacity per mole (molar specific heat)

monatomic

$f = 3$

$$\frac{C_V}{n_m} = \frac{3}{2} R = 12.5 \,\mathrm{J/mol\,K}$$

diatomic

$f = 5$

$$\frac{C_V}{n_m} = \frac{5}{2} R = 20.8 \,\mathrm{J/mol\,K}$$

(2) constant pressure

$$C_p \equiv \left(\frac{dQ}{dT}\right)_p = \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V} + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{dV}{dT}\right)_p$$

$$\rightarrow C_p = C_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{dV}{dT}\right)_p$$

$$\rightarrow C_p - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{dV}{dT}\right)_p$$

for an ideal gas:

$$C_p = \frac{f}{2} n_m \cdot R + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{dV}{dT}\right)_p$$

$$U = \frac{f}{2} n_m R \cdot T$$

$$\left(\frac{\partial U}{\partial V}\right)_T = 0 \leftarrow \text{for an ideal gas}$$

$$V = \frac{n_m R T}{p}$$

$$\left(\frac{dV}{dT}\right)_p = \frac{n_m R}{p}$$

$$C_p = \frac{f}{2} n_m R + n_m R$$

$$\frac{C_p}{n_m} = \frac{f}{2} R + R$$

$$\frac{C_p}{n_m} = \frac{C_V}{R} + R$$

monatomic gas

$f = 3$

$C_p \approx 20.7$

diatomic gas

$C_p \approx 29$

specific heat → heat capacity per unit of mass

$$c_p = \frac{C_p}{M}$$

C_p ← heat capacity

M ← total mass

volumetric specific heat

$$= \frac{C_{v/p}}{V}$$

$$c_v = \frac{C_v}{M}$$

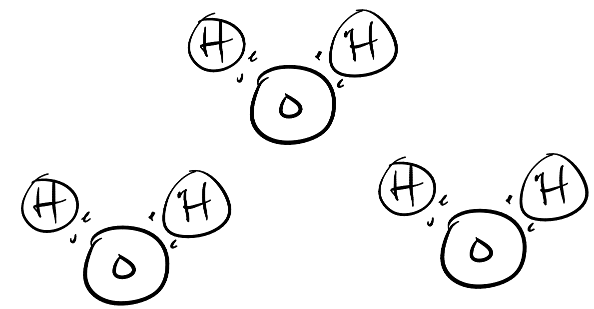
C_v ← big cee

c_v ← little cee

Latent Heat

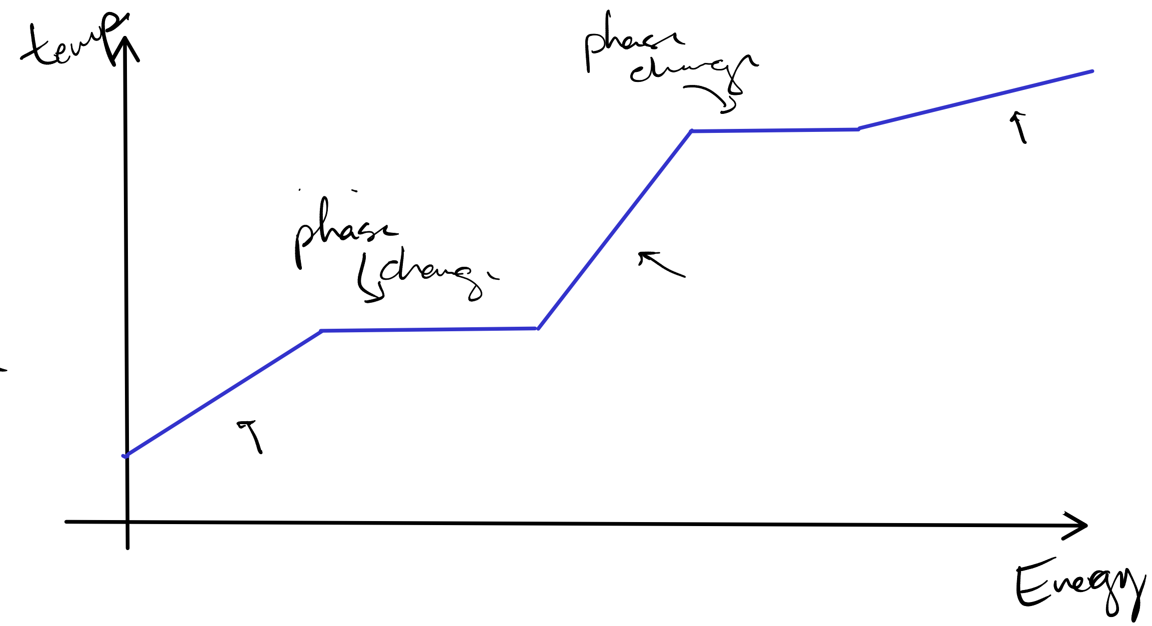
$$\frac{dQ}{dT} = \infty$$

dT ← 0



Q ← total heat to completely change the phase (depends on size)

$$\frac{Q}{m} = L$$ ← Latent heat (fusion, vaporization)



41, 45, 47

$C_p = 75.29 \, J/K\,mol$

41) $\underline{\Sigma \Delta U = 0}$

$\mathcal{C} = \frac{dU}{dT}$

a) $\Delta U_{H_2O} = (250g) \cdot (4.186 J/gK) \cdot (24°C - 20°C)$

$dU = c \cdot m \cdot dT$

$\Delta U = mc \cdot \Delta T$

b) $\Delta U_{metal} = -\Delta U_{H_2O}$

c) $\mathcal{C} = \frac{\Delta U}{\Delta T} = \frac{\Delta U}{29-100}$

d) $\frac{\mathcal{C}}{m} = c$

47) $\Sigma \Delta U = 0$

$m_t C_t (65° - 100°C) + \underline{\text{ice warms}} + \underline{\text{ice melts}} + \underline{\text{water warms}} = 0$

$m_i \cdot C_{ice}(0 - (-15)) + m_i \cdot L_f + m_i \cdot C_w \cdot (65°C - 0°C) = 0$

333 J/g

$\boxed{1.45}$ $w = xy \quad x = yz$ ← given

a) $w = x\left(\frac{x}{z}\right)$ $\quad \boxed{w = y^2 z}$ ← in terms of y + z

$\boxed{w = \frac{x^2}{z}}$ ← in terms of x + y

b) $\left(\frac{\partial w}{\partial x}\right)_y = y \qquad \left(\frac{\partial w}{\partial x}\right)_z = \frac{2x}{z}$

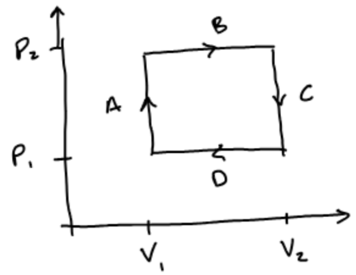
Since $y = \frac{x}{z}$

$$\left(\frac{\partial w}{\partial x}\right)_y = 2\left(\frac{\partial w}{\partial x}\right)_z$$

c) $\left(\frac{\partial w}{\partial y}\right)_x = x \qquad \left(\frac{\partial w}{\partial y}\right)_z = 2yz$

$\left(\frac{\partial w}{\partial z}\right)_x = -\frac{x^2}{z^2} \qquad \left(\frac{\partial w}{\partial z}\right)_y = y^2$

1.34]



diatomic gas

$f = 5$

(a) work in each stage

A: 0

B: $-P_2(V_2-V_1) = P_2(V_1-V_2)$

C: 0

D: $-P_1(V_1-V_2) = P_1(V_2-V_1)$

ΔU in each stage

A: $\Delta U = \frac{5}{2}Nk_BT - \frac{5}{2}\Delta PV = \frac{5}{2}V_1(P_2-P_1)$

B: $\Delta U = \frac{5}{2}P\Delta V = \frac{5}{2}P_2(V_2-V_1)$

C: $\Delta U = \frac{5}{2}V_2(P_1-P_2)$

D: $\Delta U = \frac{5}{2}P_1(V_1-V_2)$

Q in each stage

A: $Q = \Delta U - W = \frac{5}{2}V_1(P_2-P_1)$ ← add heat

B: $\frac{5}{2}P_2(V_2-V_1) - (-P_2(V_2-V_1)) = \frac{5}{2}P_2(V_2-V_1) + P_2(V_2-V_1) = \frac{7}{2}P_2(V_2-V_1)$ ← add heat

C: $\frac{5}{2}V_2(P_1-P_2)$ ← remove heat

D: $\frac{7}{2}P_1(V_1-V_2)$ ← remove heat

(b) A → adding heat at constant volume to increase pressure

B → expanding gas works out to surroundings add heat to maintain pressure

C → cool at constant volume

D → do work to compress but cool to keep constant pressure

c) Net Work: $W_B + W_D = P_2(V_1-V_2) + (-P_1(V_1-V_2))$

$$= \underbrace{(V_1-V_2)}_{<0}\underbrace{(P_2-P_1)}_{>0}$$

so net work onto surroundings

Net Change in Energy

$$\frac{5}{2}V_1(P_2-P_1) + \frac{5}{2}P_2(V_2-V_1) + \frac{5}{2}V_2(P_1-P_2) + \frac{5}{2}P_1(V_1-V_2)$$

$$-\frac{5}{2}V_2(P_2-P_1) \quad -\frac{5}{2}P_1(V_2-V_1)$$

$$\frac{5}{2}\left((V_2-V_1)(P_2-P_1) + (P_2-P_1)(V_1-V_2)\right)$$

$$\frac{5}{2}\underbrace{\left((V_2-V_1)(P_2-P_1) - (V_2-V_1)(P_2-P_1)\right)}_{=0}$$

Net Heat Added

$$Q_{NET} = \frac{5}{2}V_1(P_2-P_1) + \frac{7}{2}P_2(V_2-V_1) + \frac{5}{2}V_2(P_1-P_2) + \frac{7}{2}P_1(V_1-V_2)$$

$$-\frac{5}{2}V_2(P_2-P_1) \quad -\frac{7}{2}P_1(V_2-V_1)$$

$$= \frac{5}{2}(P_2-P_1)(V_1-V_2) + \frac{7}{2}(P_2-P_1)(V_2-V_1)$$

$$= -\frac{5}{2}(P_2-P_1)(V_2-V_1) + \frac{7}{2}(P_2-P_1)(V_2-V_1)$$

$$= \underbrace{(P_2-P_1)(V_2-V_1)}$$ exactly equal to work

both >0 so net heat is added and work is extracted.

1.36 (a) $V_1 = 1\text{ liter} = .001\ m^3$ $\quad P_1 = 1\text{ atm} = 1\cdot 10^5\ Pa$

$V_2 = ?$ $\quad P_2 = 7\text{ atm} = 7\cdot 10^5\ Pa$

$\gamma = 7/5$ for air

$$\frac{V_2^\gamma}{V_1^\gamma} = \frac{P_1}{P_2}$$

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/\gamma} = 1\text{ liter}\left(\frac{1\text{ atm}}{7\text{ atm}}\right)^{5/7}$$

$$V_2 = .249\text{ liters}$$

(b) work done

$$dU = dW$$

$$\Delta U = W$$

$$\frac{5}{2}Nk_B\Delta T = W$$

so what is final T and N?

It is easier to solve for part c. first and then to b., but the alternative way is possible w/ an integral

Alternative: $dW = -p\,dV$

$$pV^\gamma = \text{constant}$$

$$\frac{p_2}{p_1} = \frac{V_1^\gamma}{V_2^\gamma} \Rightarrow p(V) = \frac{p_1V_1^\gamma}{V^\gamma}$$

$$dW = -\frac{p_1V_1^\gamma}{V^\gamma}dV$$

$$W = -\int_{V_1}^{V_2}\frac{p_1V_1^\gamma}{V^\gamma}dV = -p_1V_1^\gamma\underbrace{\int\frac{dV}{V^\gamma}}$$

$$\int_{V_1}^{V_2} V^{-\gamma}dV$$

$$\frac{1}{-\gamma+1}V^{-\gamma+1}\Big|_{V_1}^{V_2}$$

$$-\frac{1}{\gamma-1}V^{-\gamma+1}\Big|_{V_1}^{V_2}$$

$$= \frac{p_1V_1}{\gamma-1}V^{-\gamma+1}\Big|_{V_1}^{V_2}$$

$$W = +\frac{p_1V_1^\gamma}{\gamma-1}\left(V^{-\gamma+1}\right)\Big|_{V_1}^{V_2}$$

$$W = \frac{p_1V_1^\gamma}{\gamma-1}\left(V_2^{-\gamma+1} - V_1^{-\gamma+1}\right)$$

just algebra to save keystrokes

$$= \frac{p_1}{\gamma-1}\left(\frac{V_1^\gamma}{V_2^{\gamma-1}} - V_1^\gamma V_1^{-\gamma+1}\right)$$

$$= \frac{p_1}{\gamma-1}\left(\frac{V_1^\gamma}{V_2^{\gamma-1}} - V_1\right)$$

$$= \frac{p_1V_1}{\gamma-1}\left(\frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} - 1\right)$$

$$= \frac{p_1V_1}{\gamma-1}\left(\left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1\right)$$

$$= \frac{5\cdot\overbrace{10^5\ Pa\cdot 0.001\ m^3}^{10^2}}{2}\left(\left(\frac{1}{0.25}\right)^{2/5} - 1\right) = 185\ J$$

$$\gamma - 1 = \frac{7}{5} - \frac{5}{5} = \frac{2}{5}$$

(c)

$$VT^{f/2} = \text{const}$$

$$\left(\frac{T_2}{T_1}\right)^{5/2} = \frac{V_1}{V_2}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{2/5} = 300K\left(\frac{1\text{ lt}}{.249\text{ lt}}\right)^{2/5}$$

$$= 300K\,(1.74) = 523\ K$$

$$250^\circ C$$

$$\Delta T = T_2 - T_1 = 223\ K$$

$$N = \frac{P_1V_1}{k_BT_1} = \frac{10^5\cdot .001\ m^3}{1.38\cdot 10^{-23}\cdot 300k}$$

$$N = 2.41\cdot 10^{22} \Rightarrow 0.040\text{ mol} = n$$

$$Nk_BT_1 = PV = 10^2$$

$$\frac{5}{2}Nk_B\Delta T$$

$$\frac{5}{2}Nk_B(T_2 - T_1)$$

$$\frac{5}{2}\underbrace{Nk_BT_1}_{10^2}\left(\frac{T_2}{T_1} - 1\right)$$

$$10^2\left(\left(\frac{V_1}{V_2}\right)^{2/5} - 1\right)$$

$$\frac{5}{2}\cdot 10^2(0.74) = 185\ J$$

1.38