

Chapter 4

Heat Engine \rightarrow absorbs heat, produces work

can not convert
all of it to work.

Heat comes in
increase the entropy
of the engine



to start the cycle
over, entropy must
be taken out of
the engine

heat exhausted

$$\Delta U_{\text{cycle}} = 0 = Q + W_{\text{gas}}$$

$$0 = Q_h - Q_c + W_{\text{gas}}$$

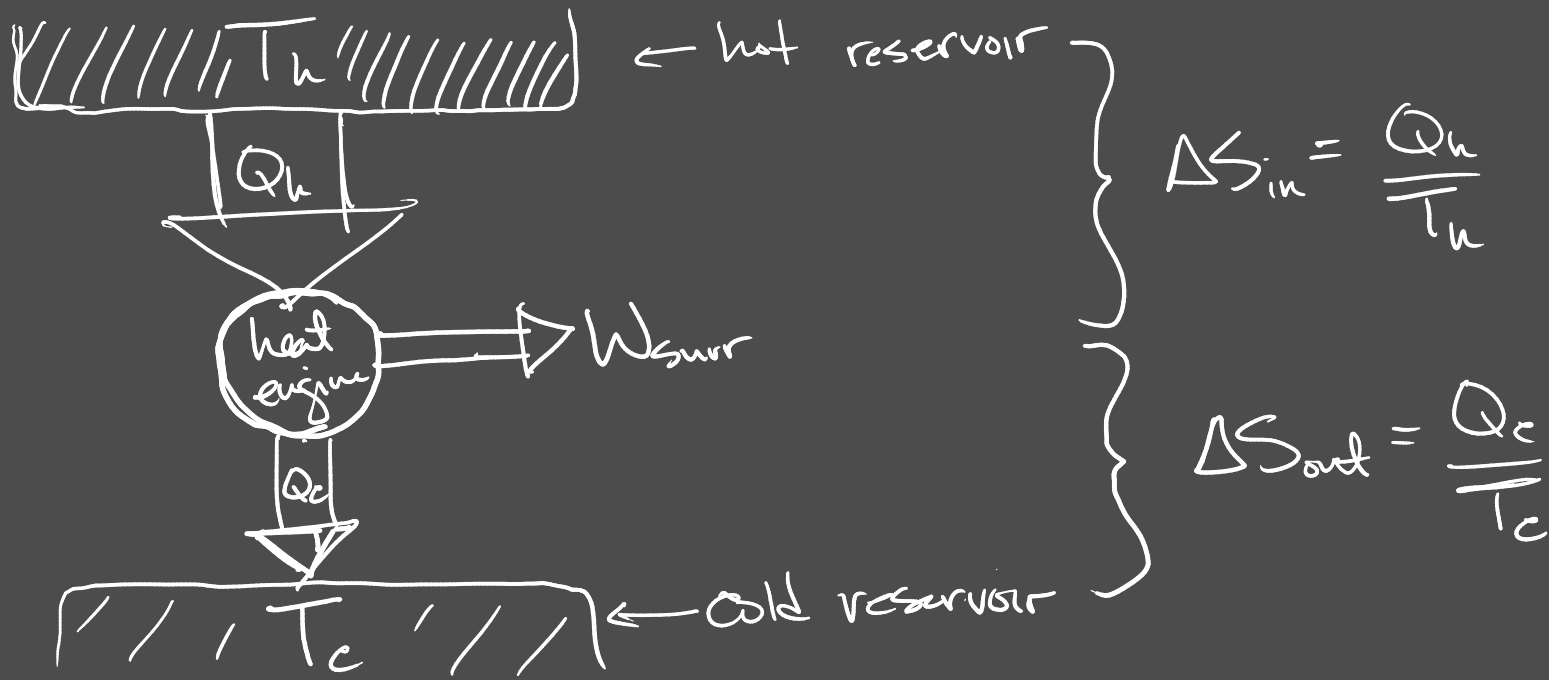
heat in \nearrow \nwarrow heat out

$$-W_{\text{gas}} = Q_h - Q_c$$

$$\boxed{W_{\text{surr}} = Q_h - Q_c} \quad \text{1st Law of Thermo}$$

$$\text{efficiency} \equiv \frac{\text{what you get}}{\text{what you pay for}} = \frac{\text{benefit}}{\text{cost}}$$

$$e = \frac{W_{\text{surr}}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$



at best, w/ no new entropy created

$$\Delta S_{in} = \Delta S_{out}$$

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

more realistically

$$\frac{Q_h}{T_h} \leq \frac{Q_c}{T_c}$$

$$\frac{T_c}{T_h} \leq \frac{Q_c}{Q_h} \Rightarrow e \leq 1 - \frac{T_c}{T_h}$$

how can we achieve max efficiency

$$\frac{Q_h}{T_h} = \frac{Q_h}{T_{gas}}$$

$\underbrace{\hspace{1cm}}$
 \hookrightarrow removed
from res

$\underbrace{\hspace{1cm}}$
 \hookrightarrow gained
by working
substances

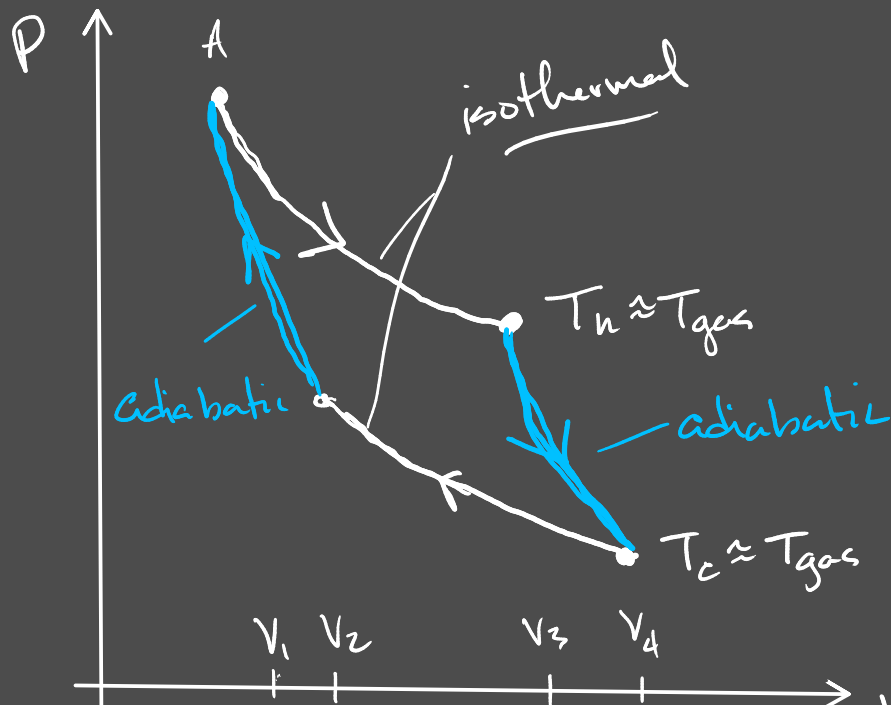
$$\rightarrow T_h = T_{gas}$$

no heat will transfer
 $\hookrightarrow T_{gas} + dT = T_h$

keep this T_{gas} constant
isothermal expansion

for exhaust,

$$\frac{Q_c}{T_c} = \frac{Q_c}{T_{gas}}$$



- Carnot cycle $\rightarrow e = 1 - \frac{T_c}{T_h}$

4.5 isothermal expansion

$$dU^0 = \delta Q + \delta W$$

$$\delta Q = -\delta W = p dV$$

$$p = \frac{Nk_B T}{V}$$

$$Q = Nk_B T \int_{V_i}^{V_f} \frac{dV}{V} = Nk_B T \ln\left(\frac{V_f}{V_i}\right)$$

$$Q_h = Nk_B T_h \ln\left(\frac{V_3}{V_1}\right)$$

$$Q_c = Nk_B T_c \ln\left(\frac{V_2}{V_4}\right)$$

$$e = 1 - \frac{Q_c}{Q_h} \stackrel{?}{=} 1 - \frac{T_c}{T_h}$$

$$1 - \frac{T_c \ln\left(\frac{V_2}{V_4}\right)}{T_h \ln\left(\frac{V_3}{V_1}\right)} = 1 - \frac{T_c}{T_h} \quad \text{only if} \quad \underbrace{\frac{V_2}{V_4} = \frac{V_3}{V_1}}_{\text{prove this}} \quad \checkmark$$

adiabatic expansion ✓

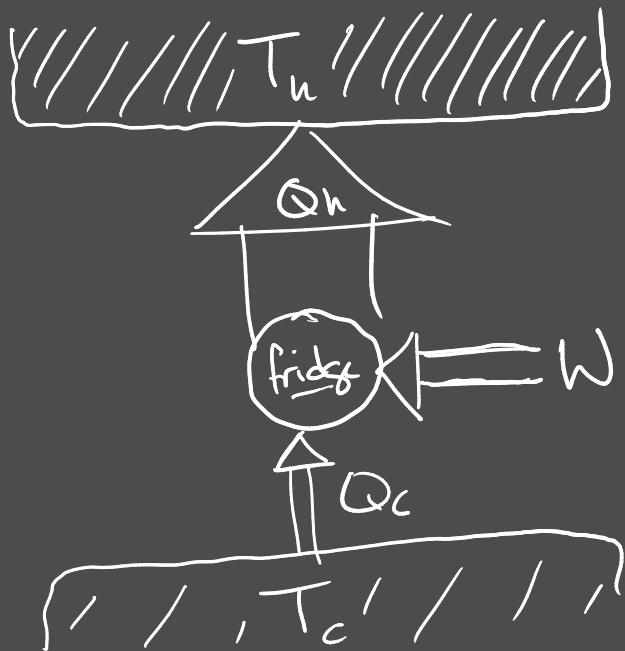
$$V_f T_f^{f/2} = V_i T_i^{f/2}$$

Refrigerator → operates oppositely to the heat engine

↳ W in from surroundings

Q_c out of cold res.

Q_h into hot res.



fridge | air conditioner | heat pump

operate in the same way, difference is what you want

efficiency \Rightarrow coefficient of performance

$$\text{COP} = \frac{\text{what you get}}{\text{what you want}} = \frac{Q_c}{W} \quad \left. \vphantom{\frac{Q_c}{W}} \right\} \text{fridge}$$

heat pump

$$\text{COP} = \frac{Q_h}{W}$$

$$\text{COP} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{1}{\frac{Q_h}{Q_c} - 1}$$

\swarrow 1st law of thermo

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c} \Rightarrow \frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$$

$$\text{COP} \leq \frac{1}{\frac{T_h}{T_c} - 1}$$

$$\text{COP} \leq \frac{T_c}{T_h - T_c}$$

Ex. kitchen freezer

$$T_c = 255 \text{ K}$$

$$T_h = 295 \text{ K}$$

$$\text{COP} \leq \frac{255}{40} \leq 6.3 \geq \frac{Q_c}{W}$$

\downarrow

each joule of work moves
6.3 J of heat out of the
freezer

What heat is dumped into
the kitchen?

$$Q_h = Q_c + W = \underline{7.3 \text{ J}}$$

4.10 Heat leaks into fridge at a rate of 300 Watts.
What is the power drawn from the wall?

$$\text{COP} = \frac{Q_c}{W} = \frac{Q_c/\Delta t}{W/\Delta t} = \frac{300 \text{ Watts}}{P_{\text{wall}}} = \text{COP} \leftarrow$$

↑ Power

$$P_{\text{wall}} = \frac{300 \text{ W}}{7.3} = 41 \text{ Watts} \leftarrow \text{electrical power from outlet.}$$