Chapter 3 - Free-fall and Projectile Motion

Recall Problem:

How far to you go in the first second of accelerating from rest at 10 m/s^2?

How far do you go in the second second? Third second?

$$x_f = x_i + v_i \cdot t + \frac{1}{2} \cdot a \cdot t^2$$

$$x = 0 + 0\,m/s \cdot 1s + \underbrace{\frac{1}{2} \cdot 10\,m/s^2 \cdot (1s)^2}$$

$$x = 5\,m$$

$$x = 0 + 0\,m/s \cdot 2s + \frac{1}{2}(10)(2s)^2$$

$$x = 20\,m$$

$$\Delta x_{1 \to 2} = 15\,m$$

$$x_f = 5m + (10\,m/s)(1s) + \frac{1}{2}(10\,m/s^2)(1s)^2$$

$$= 5m + 10m + 5m$$

$$x_f = 20\,m \qquad \Delta x = 15\,m$$

$$x_f = 0m + 0\,m/s(3s) + \frac{1}{2}(10\,m/s^2)(3s)^2$$

$$x_f = 45\,m$$

$$\Delta x = 25\,m$$

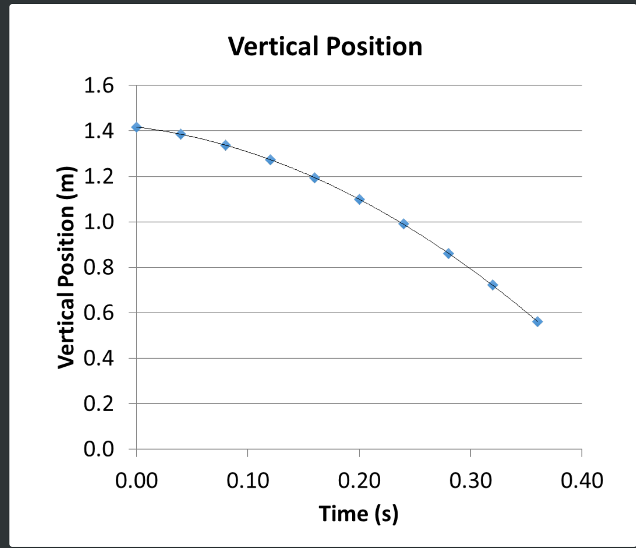
Free-fall

* constant acceleration -> g = 9.8 m/s^2
  10 m/s^2 is close enough in most cases
* can be positive or negative depending on your perspective
* otherwise use the constant acceleration equations we have.

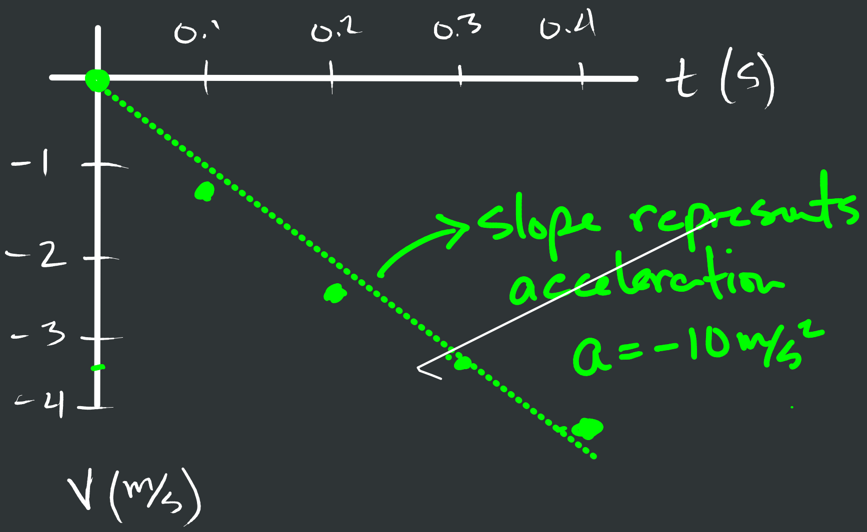
$$\longrightarrow \quad x_f = x_i + v_i \cdot t + \frac{1}{2} a t^2$$

$$v_f = v_i + a \cdot t$$

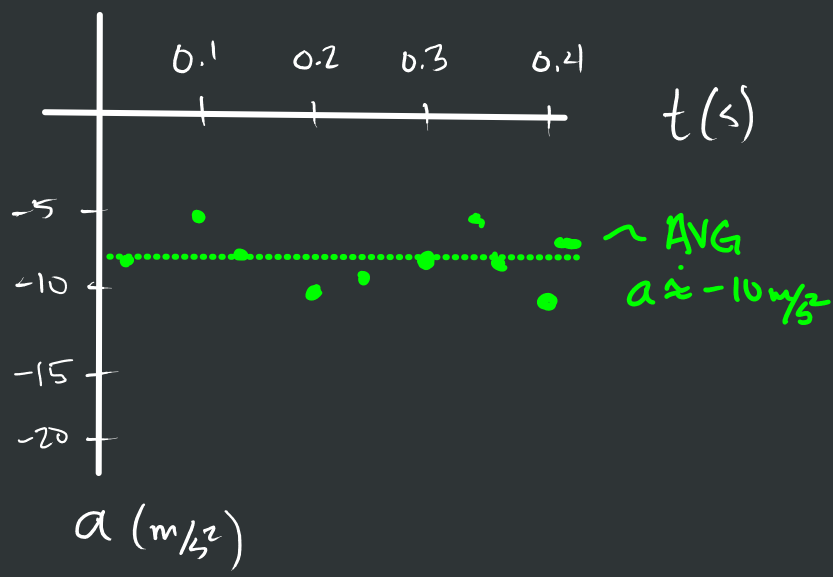
Vertical Motion of Free-Fall Object						
	Time	Vertical Position, y	Vertical Position, y	Delta y	Vertical Velocity	Vertical Acceleration
	(s)	(cm)	(m)	(m)	(m/s)	(m/s^2)
	0.00	15.80	1.417			
	0.04	15.45	1.386	-0.031	-0.78	
	0.08	14.90	1.336	-0.049	-1.23	-11.21
	0.12	14.20	1.274	-0.063	-1.57	-8.41
	0.16	13.30	1.193	-0.081	-2.02	-11.21
	0.20	12.25	1.099	-0.094	-2.35	-8.41
	0.24	11.05	0.991	-0.108	-2.69	-8.41
	0.28	9.60	0.861	-0.130	-3.25	-14.01
	0.32	8.05	0.722	-0.139	-3.48	-5.61
	0.36	6.25	0.561	-0.161	-4.04	-14.01
					Average Acceleration	-10.16

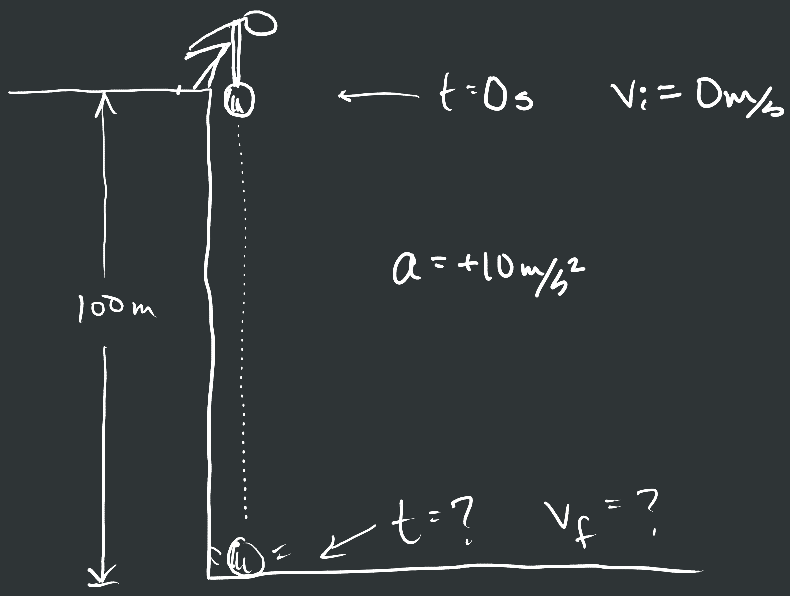


now plot velocity vs. time!



now acceleration vs. time





$$y_f = \overset{0}{y_i} + \cancel{v_i}^{\,0} t + \frac{1}{2}at^2$$

$$100\,m = \underbrace{\frac{1}{2}(10\,m/s^2)}_{5}\,t^2$$

$$\frac{100\,m}{5\,m/s^2} = \frac{5\,m/s^2 \cdot t^2}{5\,m/s^2}$$

$$\sqrt{20\,s^2} = \sqrt{t^2}$$

$$4.47\,s = t$$

$$v_f = \cancel{v_i}^{\,0} + a \cdot t$$

$$v_f = 10\,m/s^2 \cdot (4.47\,s) = 44.7\,m/s$$

How could I word the description of the problem above?
* If I drop a ball from a 100 m high cliff, how long is it in the air? And how fast is it travelling when it hits the ground?
* How long would it take for a ball to drop 100 m with no resistance starting from stand still (rest) and how fast is it traveling when it hits the ground?

What if I threw the object down to start with at 10 m/s initially?

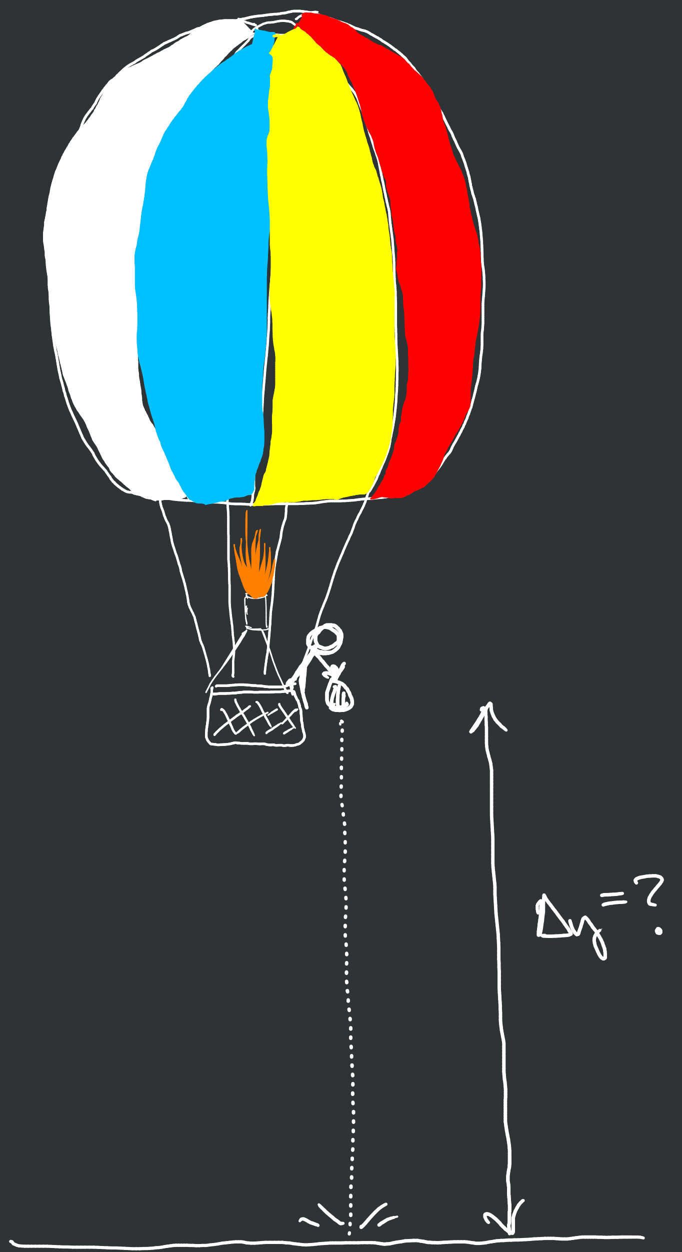
$$y_f = y_i + v_i \cdot t + \frac{1}{2}at^2 \rightsquigarrow 100\,m = 10\,m/s\, t + \frac{1}{2}(10\,m/s^2) \cdot t^2$$

$$0 = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Recall Quiz: If it takes 3.5 seconds for an object you drop to hit the ground, then how high are you above the ground?

$v_i = 0$



$$\Delta y = \underbrace{v_i \cdot t}_{0} + \frac{1}{2} a \cdot t^2$$

$$\Delta y = \frac{1}{2}(10\,m/s^2)(3.5\,s)^2$$

$$\Delta y = \underline{61\,m}$$

$$v_f = v_i + a \cdot t \iff a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t}$$

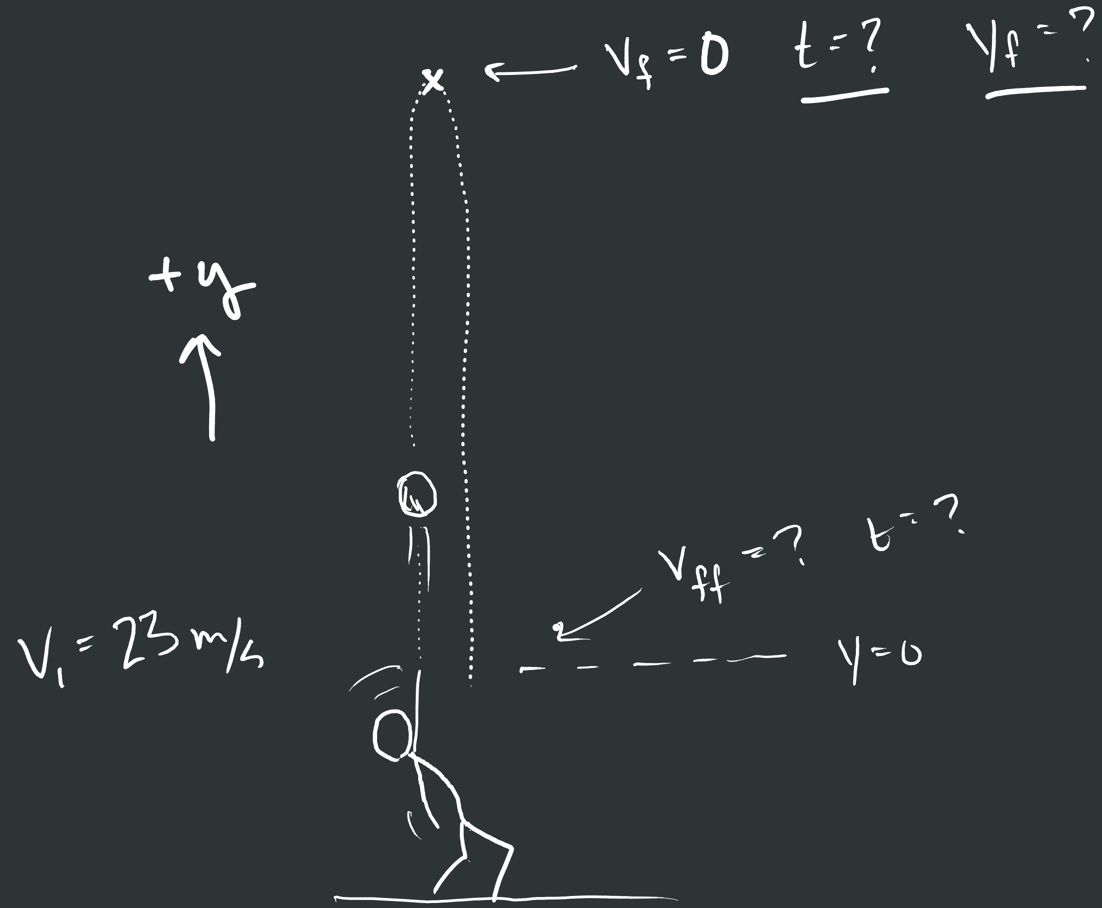
$$v_f = 0\,m/s + 10\,m/s^2 (3.5\,s)$$

$$v_f = 35\,m/s$$

$$a = g \equiv 9.8\,m/s^2$$
$$\approx 10\,m/s^2$$

~~Throwing an object up.~~

An object thrown straight up.



how long does it take to get to the highest position

$$a = \frac{\Delta v}{t}$$

$$\frac{a \cdot t}{a} = \frac{\Delta v}{a} \qquad v_f = v_i + a \cdot t$$

$$t = \frac{\Delta v}{a} = \frac{23\,m/s}{10\,m/s^2} = 2.3\,s$$

$$v_f = v_i + at$$

$$0\,m/s = +23\,m/s + (-10\,m/s^2) \cdot t$$

$$-23\,m/s \quad -23\,m/s$$

$$-23\,m/s = -10\,m/s^2 \cdot t$$

$$2.3\,s = t$$

$$\Delta y = \underset{+23\,m/s}{v_i} \cdot t + \frac{1}{2} \underset{-10\,m/s^2}{a} t^2$$

$$\Delta y = 23\,m/s \cdot (2.3\,s) + \frac{1}{2}(-10\,m/s^2)(2.3\,s)^2$$

$$\Delta y = 52.9\,m + (-26.45\,m)$$

$$\underline{\Delta y = +26.45\,m} \leftarrow \text{highest point}$$

where is the object at some other time between the ending?

is it going up or coming down?

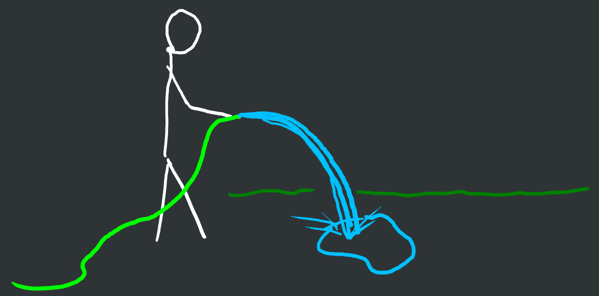
$$\Delta y = v_i \cdot t + \frac{1}{2} a t^2$$

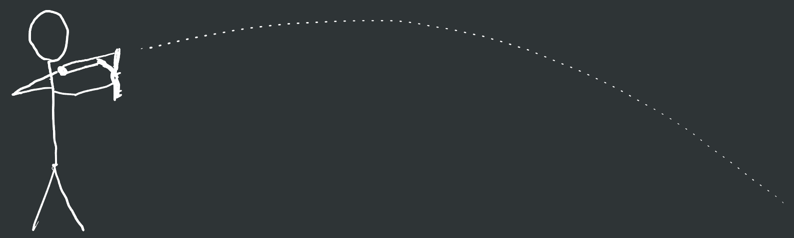
$$v_f = \overset{23\,m/s}{v_i} + a\overset{3\,s}{t}$$

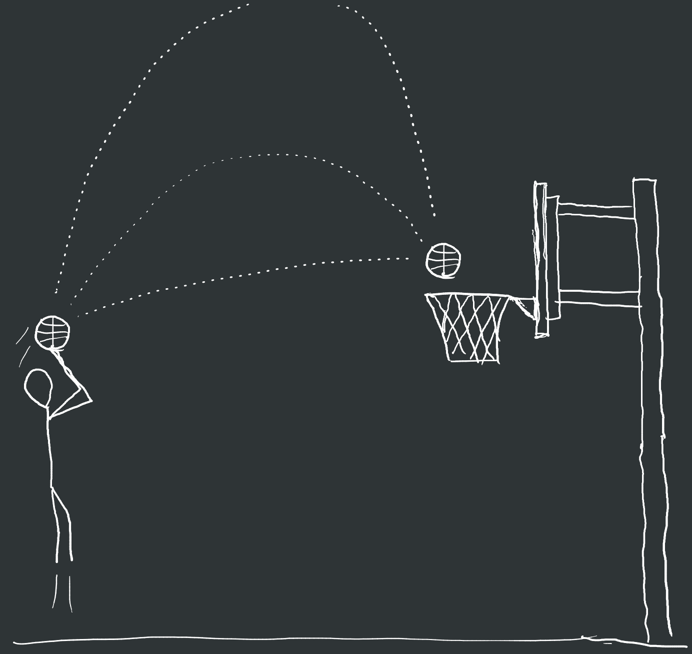
$$v_f = 23\,m/s + (-10\,m/s^2)(3\,s) = -7\,m/s$$

Two dimensional projectile motion

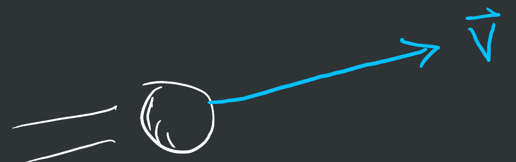
Paths are in the shape of parabolas







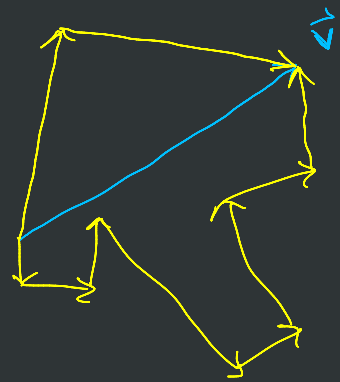
Velocity is a <u>vector</u>.



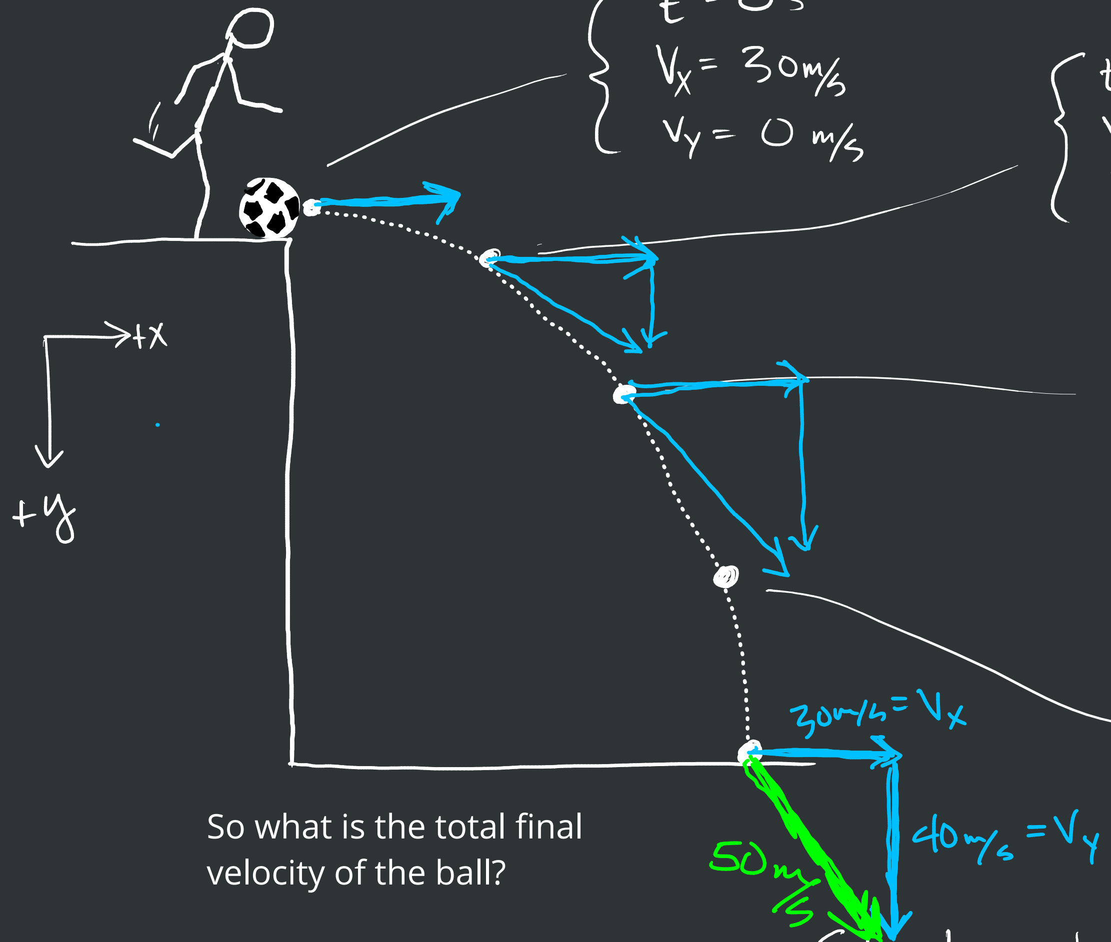
- length represent size of the quantity
- direction follows the arrow

Unusual properties of vectors:

- can be broken up into other vectors which add up to the original
- when broken up into perpendicular vectors these are called <u>components</u>.



In projectile motion, the acceration caused by gravity only changes the vertical component of the velocity, not the horizontal part.



So what is the total final velocity of the ball?

$t = 0\,s$, $V_x = 30\,m/s$, $V_y = 0\,m/s$

$t = 1\,s$, $V_x = 30\,m/s$, $V_y = 10\,m/s \downarrow$

$\Delta x = 30\,m$

$\Delta y = V_{iy} \cdot t + \frac{1}{2} a_y t^2 = 0\,m/s\,(1s) + \frac{1}{2}(10\,m/s^2)(1s)^2 = 5\,m$

$t = 2\,s$, $V_x = 30\,m/s$, $V_y = 20\,m/s$

$\Delta x = 60\,m$

$\Delta y = \underbrace{V_{iy} \cdot t}_{0} + \frac{1}{2} a_y \cdot t^2 = 0 + \frac{1}{2}(10\,m/s^2)(2s)^2 = 20\,m$

$t = 3\,s$, $V_x = 30\,m/s$, $V_y = 30\,m/s$

$\Delta x = 90\,m$

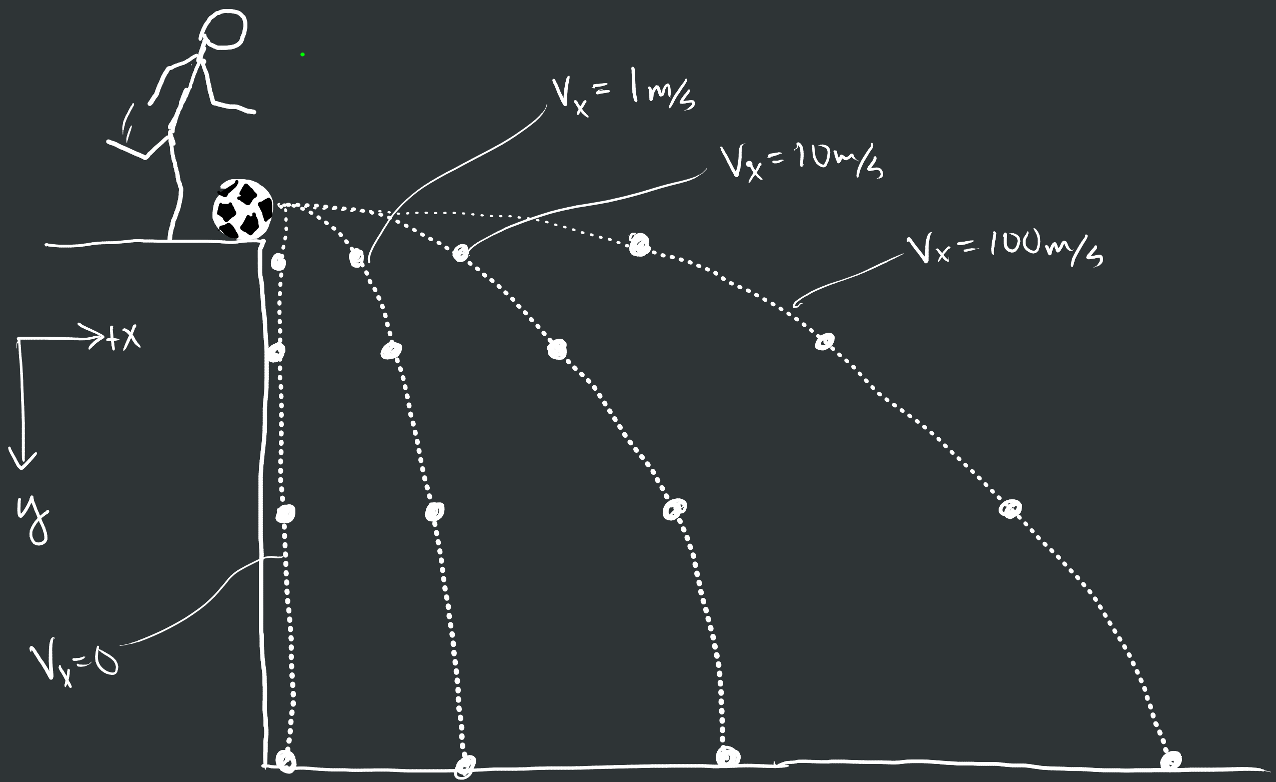
$\Delta y = \frac{1}{2}(10)(3s)^2$

$t = 4\,s$, $V_x = 30\,m/s$, $V_y = 40\,m/s$

$\Delta x = 30\,m/s \cdot 4s = 120\,m$

$\Delta y = 0\,m/s \cdot 4s + \frac{1}{2}(10\,m/s^2)(4s)^2 = 80\,m$

So what changes here with different initial horizontal velocities?
(in each case the initial vertical component is zero)



$\Delta y = \frac{1}{2} a t^2$ $\quad (v_{iy} \cdot t = 0)$

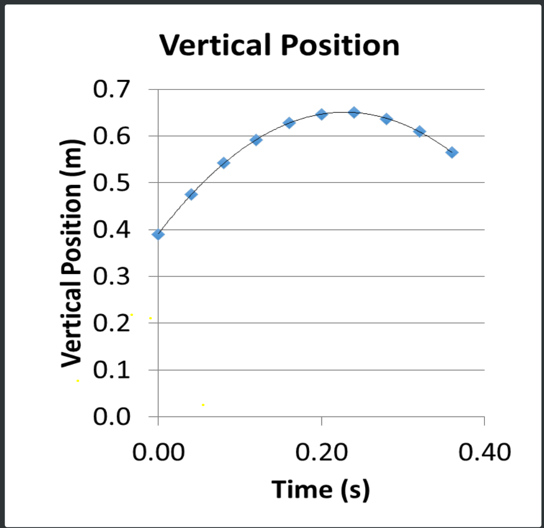
$\hookrightarrow t = \sqrt{\frac{2\Delta y}{a}}$

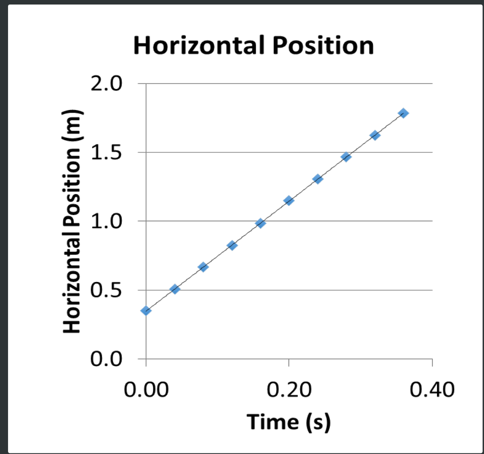
$\Delta x = V_x \cdot t$ $\quad (\frac{1}{2} a_x t^2 = 0)$

$$\boxed{\Delta x = V_x \sqrt{\frac{2\Delta y}{a}}}$$

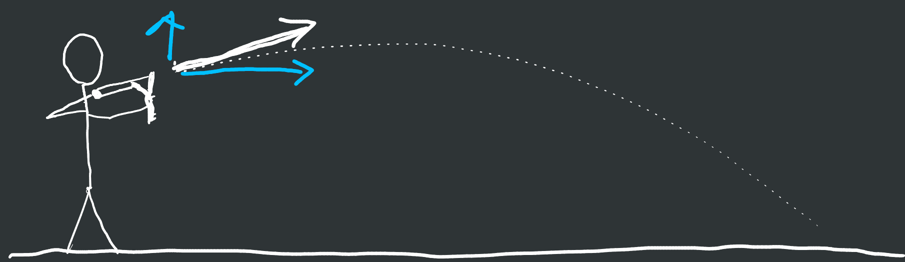
distance from the base of the cliff

We measured in lab that the acceleration only occured in the vertical positions, not in the horizontal ones.





Takeaway for projectile motion and vectors:
The initial velocity is broken into horizontal and vertical components.



thus one determines how long the object is in the air

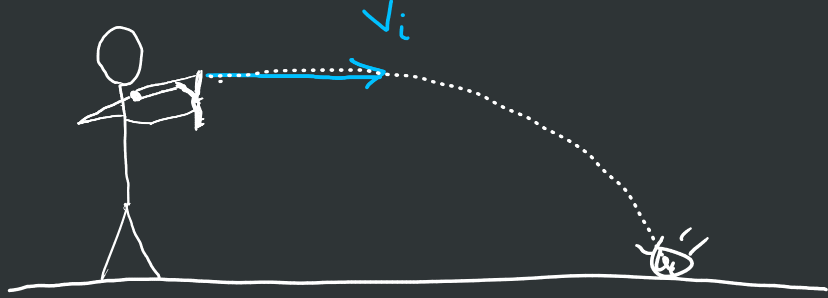
thus one is how fast it is travelling horizontally

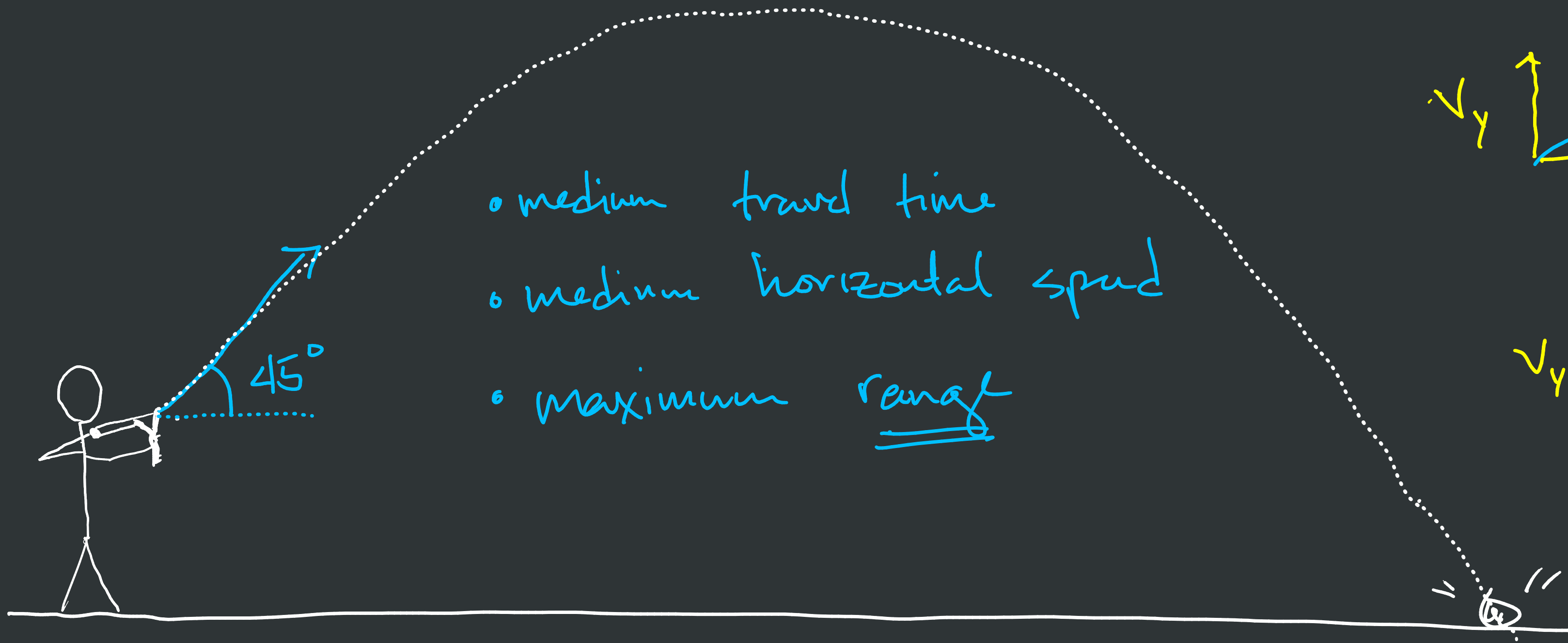
completely vertical velocity

- long time in the air
- no horizontal travel

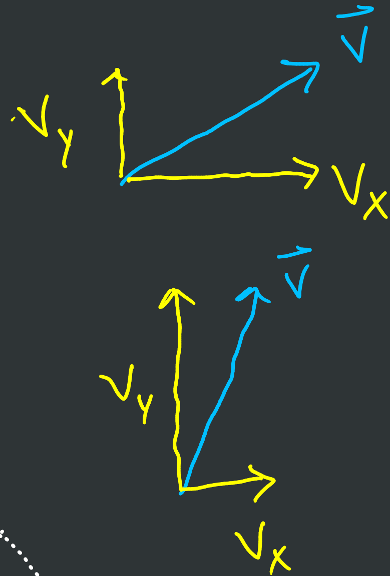
completely horizontal velocity

- fast horizontal speed
- short travel time





- medium travel time
- medium horizontal speed
- maximum range



1. If I drop a ball from a height of 20 m how long will it take to land?

20m

$\downarrow +y$

$\Delta y = 20m$

$v_i = 0 m/s$

$a = +10 m/s^2$

$\Delta y = \cancel{v_i \cdot t}^{0} + \frac{1}{2}at^2$

$20m = \frac{1}{2} \cdot 10 m/s^2 \cdot t^2$

$= 5t^2$

$\frac{20}{5} = t^2$

$\sqrt{4s^2} = \sqrt{t^2} \Rightarrow \underline{t = 2}$

2. If I drop a ball and it takes 10 seconds to land, how high is the ledge?

$\Delta y = \cancel{v_i t}^{0} + \frac{1}{2}at^2$

$\Delta y = \frac{1}{2}(10 m/s^2)(10s)^2 = 500 m$

3. If I throw the ball downwards with an initial velocity of 1$\underline{0m/s}$
and it takes 10 seconds to hit the ground,then how fast is it going
when it gets there? How high is the ledge?

$\downarrow +y$

$v_i = +10 m/s$

$a = +10 m/s^2$

$t = 10s$

$v_f = ?$

$v_f = v_i + at$

$= 10 m/s + 10 m/s^2 \cdot 10s$

$v_f = 110 m/s$

$\Delta y = 10 m/s \cdot 10s + \frac{1}{2} 10 \cdot 10^2$

$\underline{\Delta y = 600 m}$

4. If I throw a ball horizontally from a 50 m cliff, how long will it
take to land? What would be the time it took to land if I dropped it?

$\downarrow +y$

$\Delta y = 50m$

$v_i = 0 m/s$

$a = 10 m/s^2$

$t = ?$

$\Delta y = \cancel{v_i t}^{0} + \frac{1}{2}at^2$

$50m = \frac{1}{2}(10 m/s^2)t^2$

$10s^2 = t^2$

$t = \sqrt{10} = 3.16s$

5. If I throw a ball horizontally from a 50 m cliff and a friend measures that the ball landed 40 m from the base of the cliff what was the initial velocity with which I threw the ball?

$\Delta y = 50m$

$V_{iy} = 0 m/s$

$a_y = 10 m/s^2$

$t = ?$

$\Delta y = V_{iy} \cdot t + \frac{1}{2} a_y t^2$

$50m = \frac{1}{2}(10 m/s^2) t^2$

$\boxed{t = 3.16 s}$

$\Delta x = V_x \cdot t$

? ?

$40m = V_x \cdot 3.16s$

$V_x = \frac{40}{3.16} = \underline{12.7 m/s}$ ✓

6. If I throw a ball horizontally with an initial velocity of 30 m/s off a cliff and it takes 10 sec to hit the ground, then how high is the cliff and how far from the base did the ball land?

$\Delta x = V_x \cdot t = 30 m/s \cdot 10 s = 300 m$

$\Delta y = \cancel{V_{iy}}^{0} \cdot t + \frac{1}{2} a_y t^2 = \frac{1}{2}(10 m/s^2)(10s)^2 = \underline{500m}$