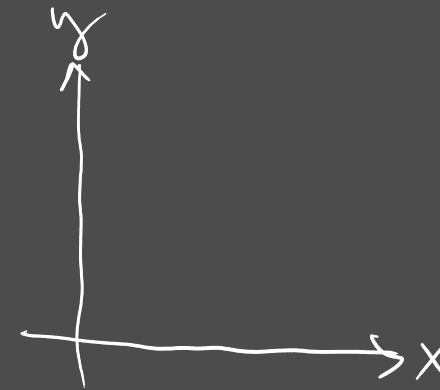
Chapter 3

Chapter 3 - Free-fall and Projectile Motion



Free-fall
* constant acceleration -> g = 9.8 m/s^2
  10 m/s^2 is close enough in most cases
* can be positive or negative depending on
  your perspective
* otherwise use the constant acceleration
  equations we have.

$$\rightarrow v_f = v_i + a \cdot t$$

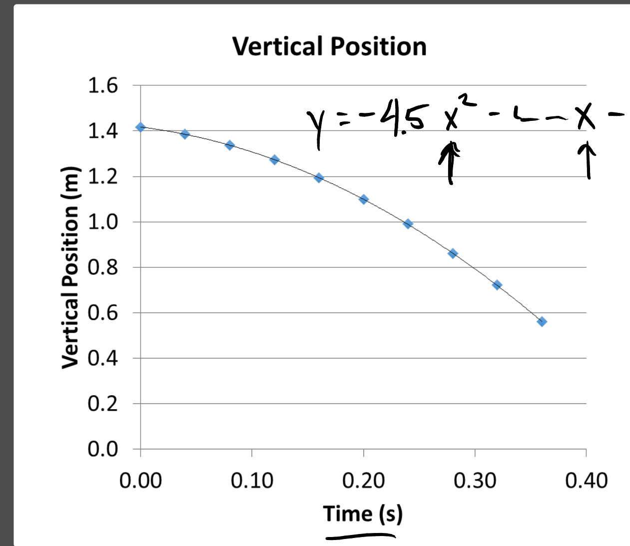
$$g = (\pm 9.8\ m/s^2)$$

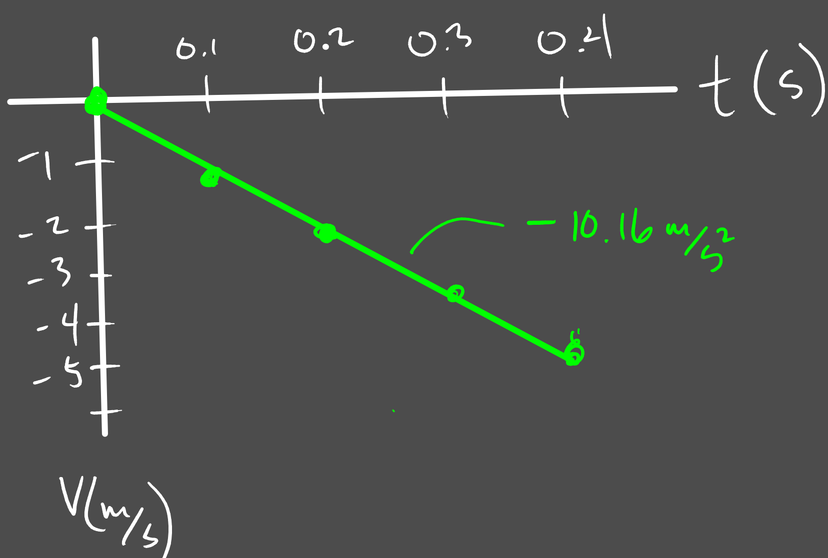
$$y_f = \underline{y_i + v_i \cdot t} + \frac{1}{2} a \cdot t^2$$

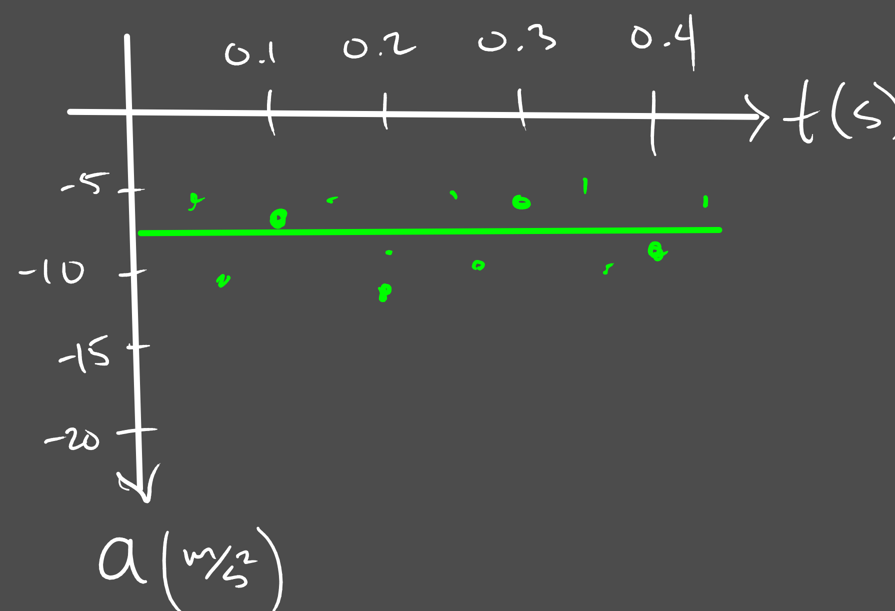
$$g$$

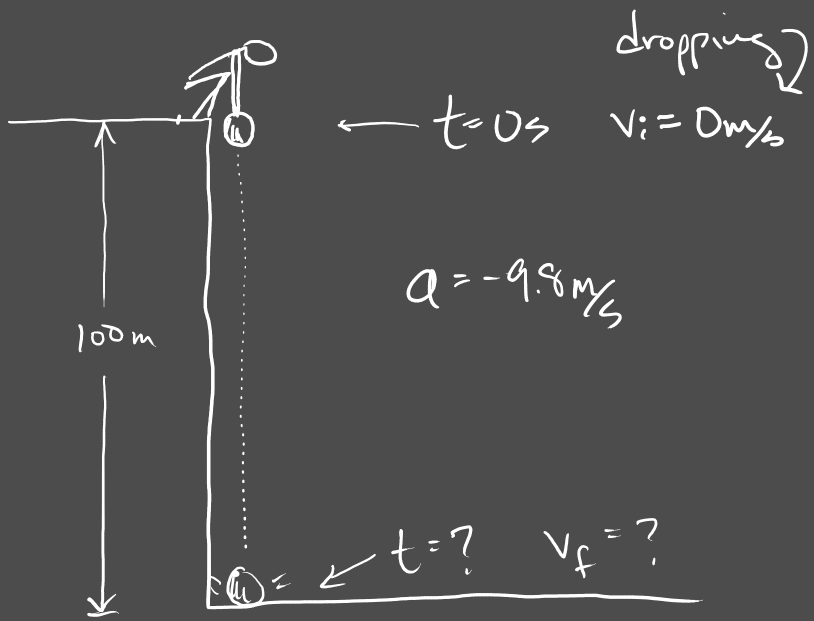
Vertical Motion of Free-Fall Object

Time (s)	Vertical Position, y (cm)	Vertical Position, y (m)	Delta y (m)	Vertical Velocity (m/s)	Vertical Acceleration (m/s²)
0.00	15.80	1.417			
0.04	15.45	1.386	-0.031	-0.78	
0.08	14.90	1.336	-0.049	-1.23	-11.21
0.12	14.20	1.274	-0.063	-1.57	-8.41
0.16	13.30	1.193	-0.081	-2.02	-11.21
0.20	12.25	1.099	-0.094	-2.35	-8.41
0.24	11.05	0.991	-0.108	-2.69	-8.41
0.28	9.60	0.861	-0.130	-3.25	-14.01
0.32	8.05	0.722	-0.139	-3.48	-5.61
0.36	6.25	0.561	-0.161	-4.04	-14.01
				Average Acceleration	-10.16









$$y_f = y_i + v_i \cdot t + \tfrac{1}{2} a \cdot t^2$$
$$0 = +100m + \cancel{0 \cdot t} + \tfrac{1}{2}(-9.8 m/s^2)t^2$$
$$0 = 100m + (-4.9 m/s^2)t^2$$
$$-100 \quad -100$$
$$\frac{-100m}{-4.9 m/s^2} = \frac{-4.9 m/s^2 \cdot t^2}{-4.9 m/s^2}$$
$$\sqrt{20.4 s^2} = \sqrt{t^2}$$
$$\boxed{4.52 s = t}$$

$$v_f = v_i + at$$
$$? \quad 0 \quad -9.8 m/s^2 \quad 4.52s$$
$$v_f = 0 - 9.8 m/s^2 \cdot 4.52 s$$
$$v_f = -44.296 m/s$$
$$\boxed{v_f = -44.3 m/s}$$

$$y_f = y_i + v_i \cdot t + \tfrac{1}{2} a t^2$$
$$100m \quad 0m \quad 0m/s \quad 10 m/s^2$$
$$100m = \cancel{0} + \cancel{0 \cdot t} + \tfrac{1}{2}(10 m/s^2)t^2$$
$$100m = 5 m/s^2 \, t^2$$
$$\sqrt{20 s^2} = \sqrt{t^2}$$
$$4.47s = t$$

$$v_f = \cancel{v_i} + a \cdot t$$
$$v_f = 10 m/s^2 \cdot (4.47s)$$
$$v_f = +44.7 m/s$$

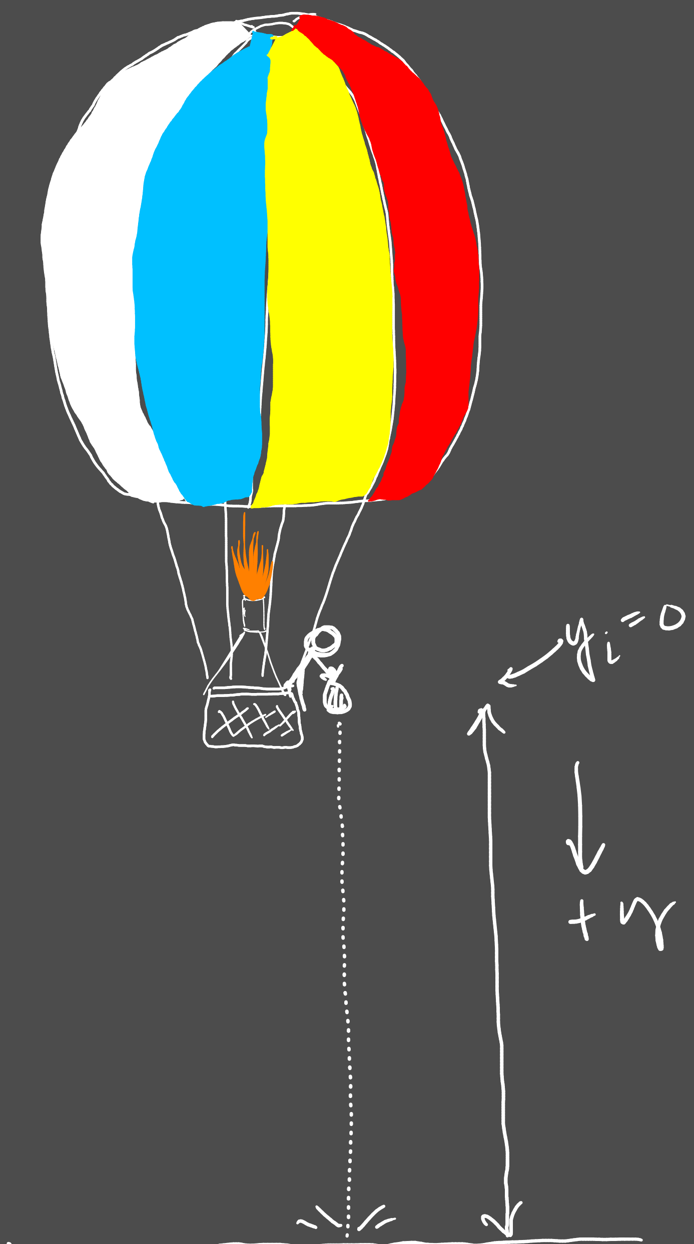
* A person drops a ball off a 100 m cliff, and it falls ~~with an acceleration of 9.8 m/s/s,~~ find the final time and final velocity of the ball when it hits the ground.

How could I word the description of the problem above?
* If I drop a ball from a 100 m high cliff, how long is it in the air? And how fast is it travelling when it hits the ground?
* How long would it take for a ball to drop 100 m with no resistance starting from stand still (rest) and how fast is it traveling when it hits the ground?

What if I threw the object down to start with at 10 m/s initially?

quadratic equation → more math → more bad

Recall Quiz: If it takes 3.5 seconds for an object you drop to hit the ground, then how high are you above the ground?



$$y_f = \underbrace{y_i}_{0} + \underbrace{v_i}_{0} \cdot t + \frac{1}{2}\underbrace{a}_{+9.8\,m/s^2} t^2$$

$$y_f = \frac{1}{2}(9.8\,m/s^2)(3.5\,s)^2$$

$$y_f = 60.0\,m$$

how fast is the object going?

$$v_f = \cancel{v_i}^{\,0} + a \cdot t$$

$$v_f = +9.8\,m/s^2\,(3.5\,s)$$

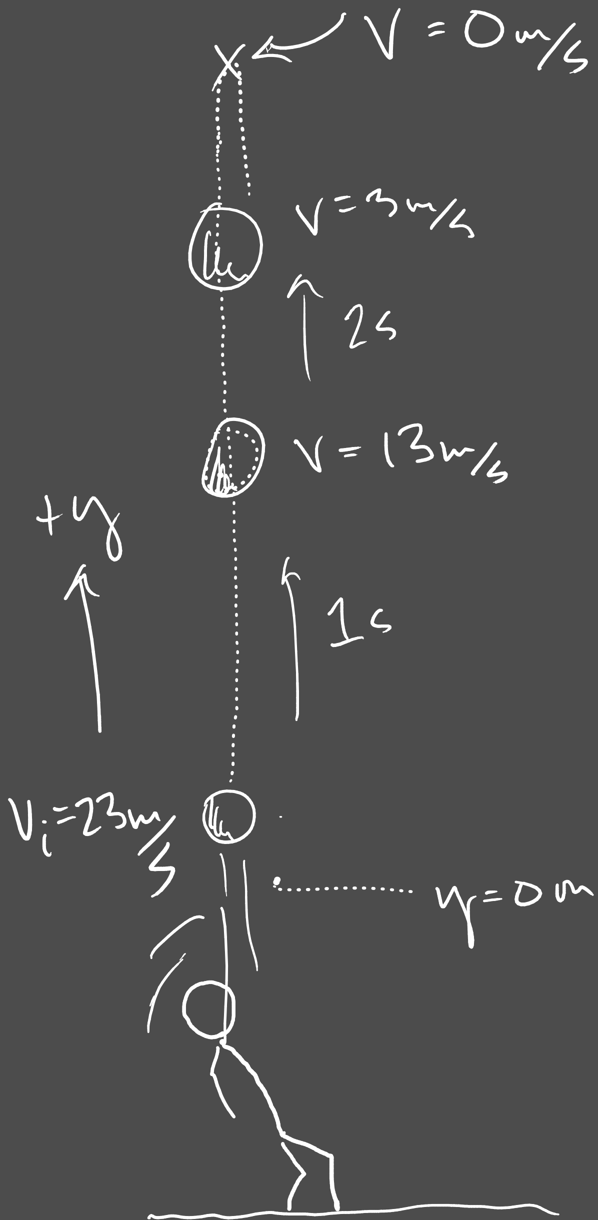
$$v_f = 35\,m/s$$

acc in free fall

$$a = g = 9.8\,\frac{m}{s^2}$$

$$\approx 10\,m/s^2$$

An object thrown straight up.



$$\rightarrow y_f = y_i + v_i \cdot t + \frac{1}{2}at^2$$

how far?

how long?

$$V_f = V_i + at$$

$0\,m/s \quad +23\,m/s \quad -9.8\,m/s^2$

$$0\,m/s = \cancel{+23\,m/s} - 9.8\,m/s^2 \cdot t$$

$-23 \qquad -23$

$$\frac{-23\,m/s}{-9.8\,m/s^2} = \frac{-9.8\,m/s^2 \cdot t}{-9.8\,m/s^2}$$

$$\boxed{2.35 = t}$$

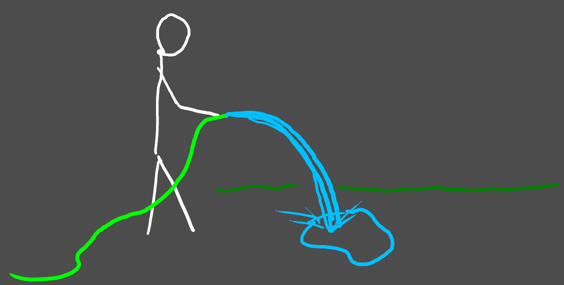
$$Y_f = 0\,m + \underbrace{23\,m/s \cdot (2.35)} + \underbrace{\frac{1}{2}(-9.8\,m/s^2)(2.35s)^2}$$

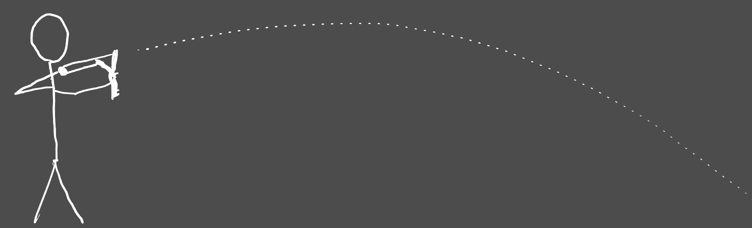
$$Y_f = 54.05\,m - 26.95\,m$$

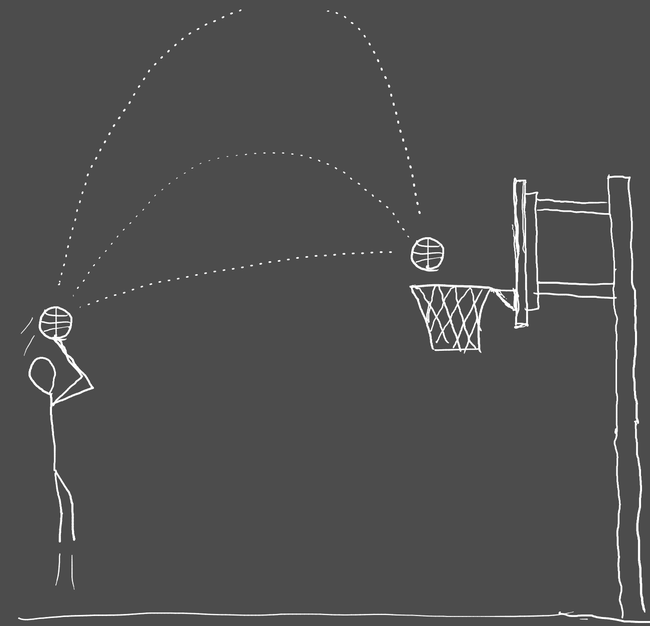
$$Y_f = 26.9\,m = \underline{27\,m}$$

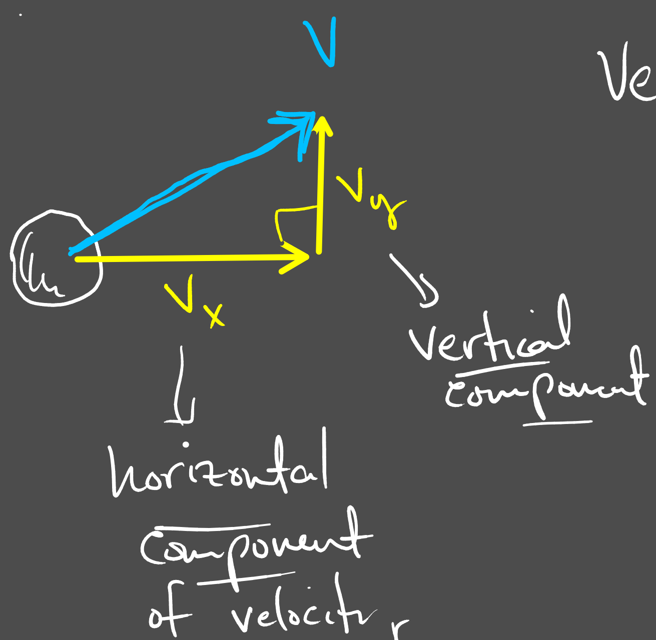
Two dimensional projectile motion

Paths are in the shape of parabolas



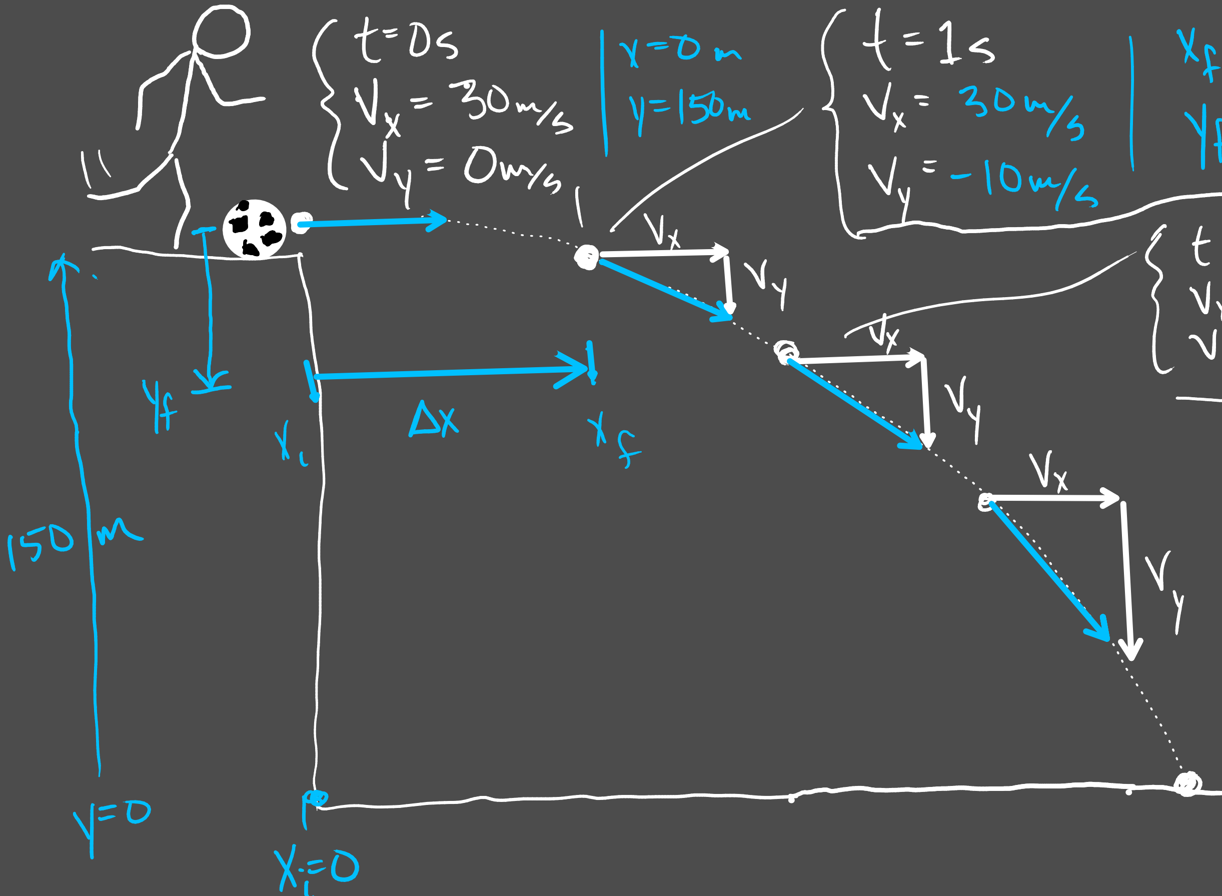






Velocity is a vector quantity

- length of the arrow is proportional to size of the quantity
- direction of the arrow is the direction of the quantity.



$t = 0\,s$, $v_x = 30\,m/s$, $v_y = 0\,m/s$ | $y = 0\,m$, $y = 150\,m$

$t = 1\,s$, $v_x = 30\,m/s$, $v_y = -10\,m/s$

$x_f = x_i + v_x \cdot t = 0\,m + 30\,\frac{m}{s} \cdot 1\,s = 30\,m$

$y_f = y_i + v_{yi} \cdot t + \frac{1}{2} a_y \cdot t^2 = 150\,m + 0\,\frac{m}{s} \cdot 1\,s + \frac{1}{2}\left(-10\,\frac{m}{s^2}\right) \cdot (1\,s)^2$

$= 150\,m - 5\,m$

$= 145\,m$

$t = 2\,s$, $v_x = 30\,m/s$, $v_y = -20\,m/s$

$x_f = 60\,m$

$y_f = y_i + v_{yi} \cdot t + \frac{1}{2} a_y t^2$

$= 150\,m + 0(2\,s) + \frac{1}{2}\left(-10\,\frac{m}{s^2}\right)(2\,s)^2$

$= 150\,m - 20\,m$

$= 130\,m$

$t = 3\,s$, $v_x = 30\,m/s$, $v_y = -30\,m/s$

$x_f = 90\,m$

$y_f = 105\,m$

So how long is the ball in the air?

$$y_f = y_i + \underline{v_i} \cdot t + \frac{1}{2} a_y t^2$$

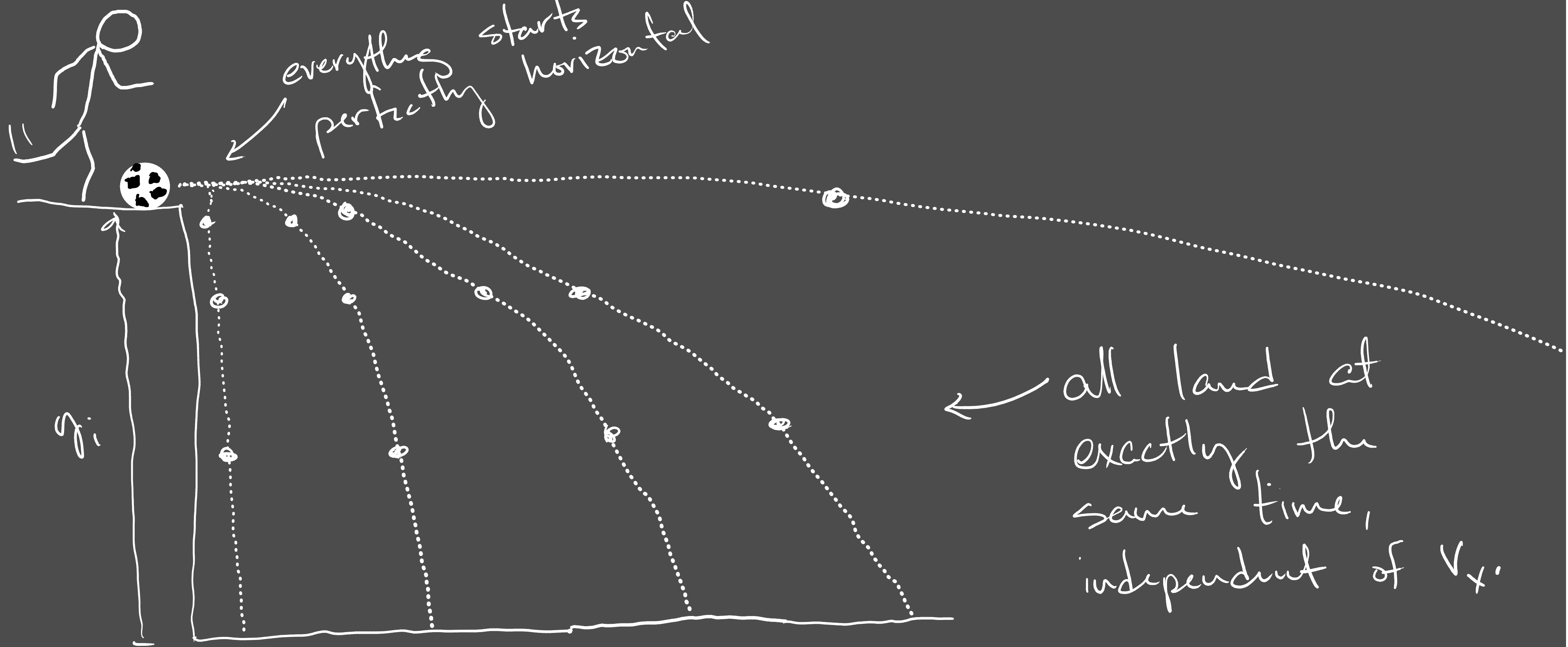
$$0\,m = 150\,m + \cancel{0 \cdot t}^{\,0} + \frac{1}{2}\left(-10\,\frac{m}{s^2}\right) \cdot t^2$$

$$0\,m = 150\,m - 5\,\frac{m}{s^2} t^2$$

$$-150\,m \quad -150\,m$$

$$-150\,m = -5\,\frac{m}{s^2} \cdot t^2$$

$$30\,s^2 = t^2 \implies \sqrt{30\,s^2} = t = \underline{5.5\,s}$$



if $v_x = 30\,\mathrm{m/s}$, how far from the base of cliff does the object land?

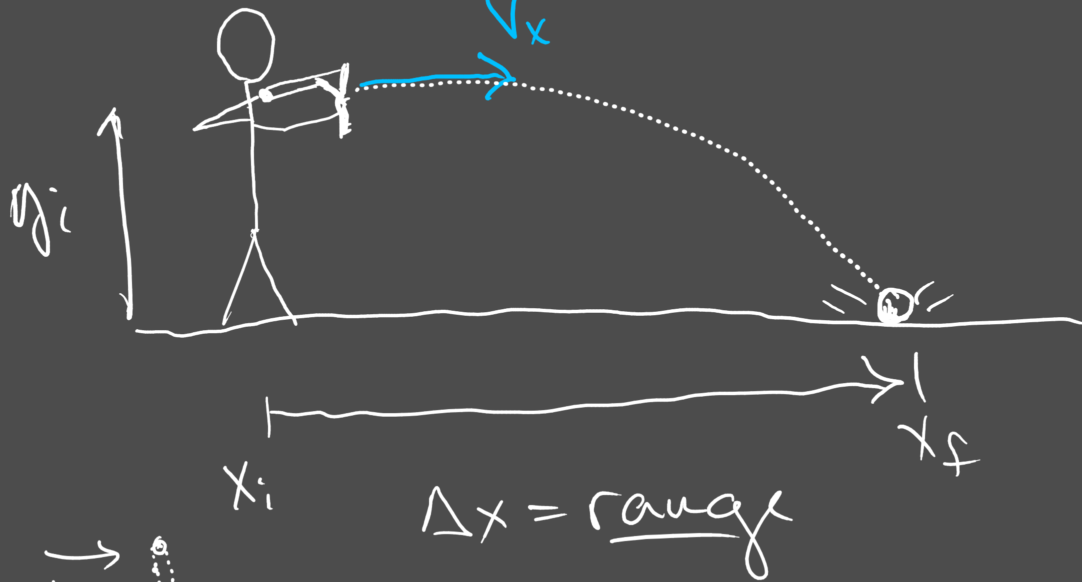
$$x_f = x_i + v_x \cdot t + \cancel{\tfrac{1}{2} a_x \cdot t^2}$$

no acceleration in x-direction

$a_x = 0$

x_i: 0

$$x_f = 30\,\mathrm{m/s} \cdot 5.5\,\mathrm{s} = \underline{\underline{165\,\mathrm{m}}}$$

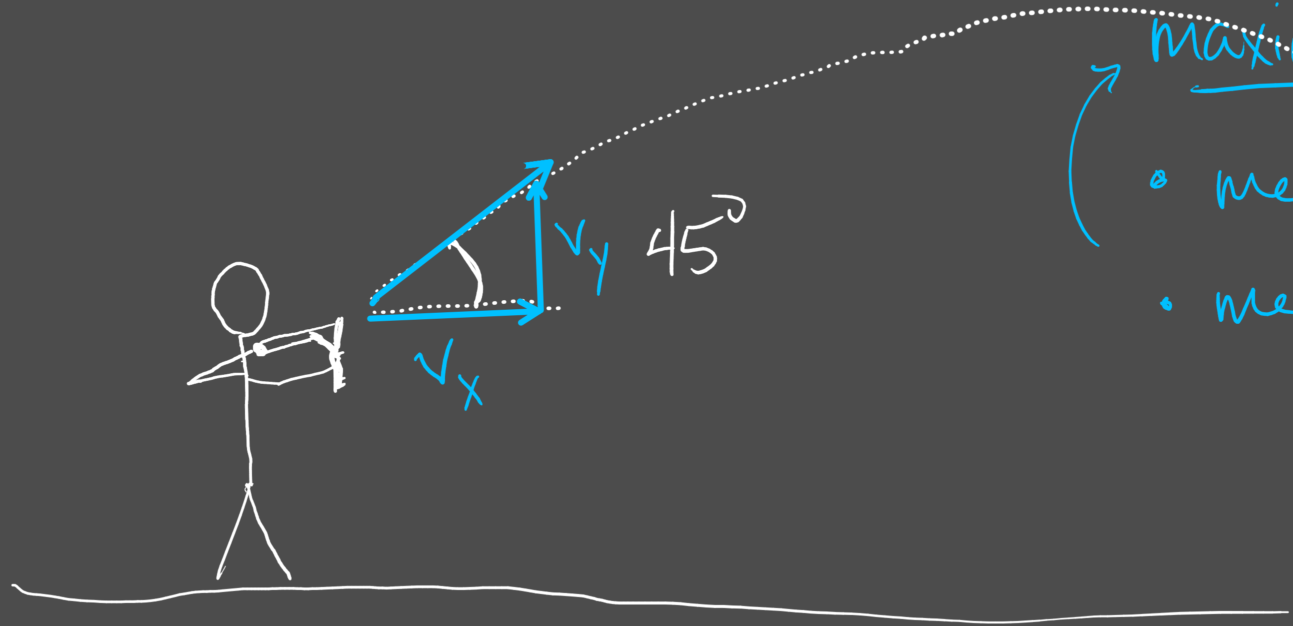


<u>Completely horizontal initial velocity</u>

- fast horizontal speed
- short travel time based on initial height

Completely vertical initial velocity

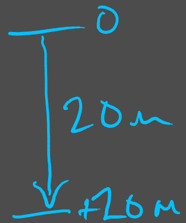
- zero horizontal velocity → no range
  no horizontal travel
- long travel time in the air



<u>maximum</u> <u>range</u>

- medium travel time
- medium horizontal speed

1. If I drop a ball from a height of 20m, how long will it take to land?



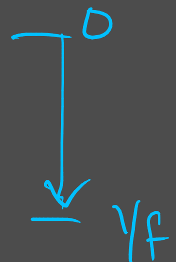
$$y_f = y_i + v_i \cdot t + \frac{1}{2} a t^2$$

y_f: +20m, y_i: 0m, v_i: 0m/s, a: +9.8 m/s²

$$20m = \frac{1}{2}(+9.8 m/s^2)\, t^2$$

$$4s^2 = t^2 \Rightarrow \boxed{t = 2s}$$

2. If I drop a ball and it takes 10 seconds to land, how high is the ledge?



know: $t = 10s$, $v_i = 0$, $a = +9.8 m/s^2$

want: height $= y_f$

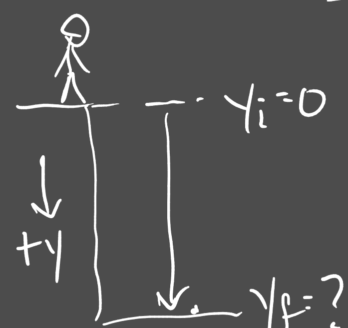
$$y_f = \frac{1}{2}(+9.8 m/s^2)(10s)^2$$

$$y_f = 500m \rightarrow \text{height}$$

3. If I throw the ball ball downwards with an initial velocity on 10 m/s and it takes 10 seconds to hit the ground, then how fast is it going when it gets there? How high is the ledge?

we know: $t = 10s$, $v_i = 10 m/s$

we want: $v_f = ?$, y_f or $\cancel{y_i}$?



$$v_f = v_i + at$$

$$v_f = +10 m/s + (+9.8 m/s^2)(10s)$$

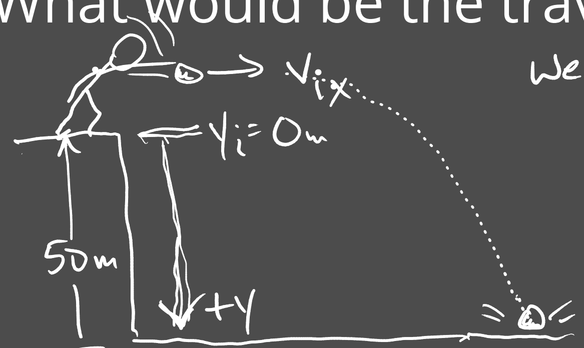
$$v_f = 110 m/s$$

$$y_f = 0 + 10 m/s \cdot (10s) + \frac{1}{2}(9.8 m/s^2)(10s)^2$$

$$y_f = +650m$$

downwards

4. If I throw a ball horizontally from a 50m cliff, how long will it take to land? What would be the travel time if I dropped it?



we know: $y_i = 0m$, $y_f = 50m$, $a_y = +9.8 m/s^2$, $v_{iy} = 0 m/s$

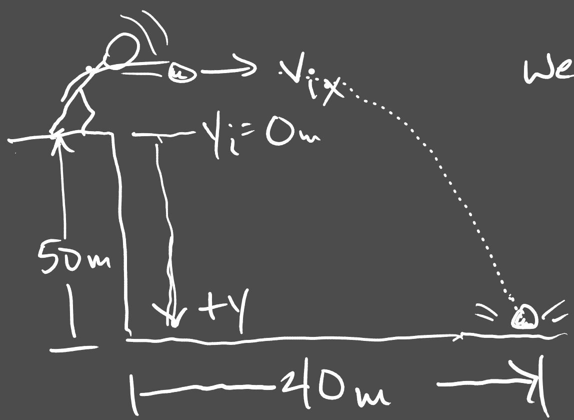
we want: t

$$y_f = y_i + \underbrace{v_i t}_{0} + \frac{1}{2} a t^2$$

$$+50m = \frac{1}{2}(+9.8 m/s^2) \cdot t^2$$

$$\boxed{t = 3.2s}$$

If I throw a ball horizontally from a 50 m cliff and a friend measures that ball landed 40 m from the base of the cliff, what was the initial velocity with which I threw the ball?



we know: $y_i = 0m$

$y_f = 50m$

$a_y = +9.8\,m/s^2$

$V_{iy} = 0\,m/s$

$x_f = 40m$

$x_i = 0\,m$

we found

$t = 3.2s$

we want:

V_{ix}

$$\cancel{V_{fx} = V_{ix} + a_x t}$$

$$\boxed{X_f = X_i + V_{ix} t} + \cancel{\tfrac{1}{2} a_x t^2}$$

40m, 0, ?, 3.2s

$$40m = V_{ix} \cdot 3.2s$$

$$V_{ix} = 12.5\,m/s$$