Chapter 6 - Work, Energy, Oscillations

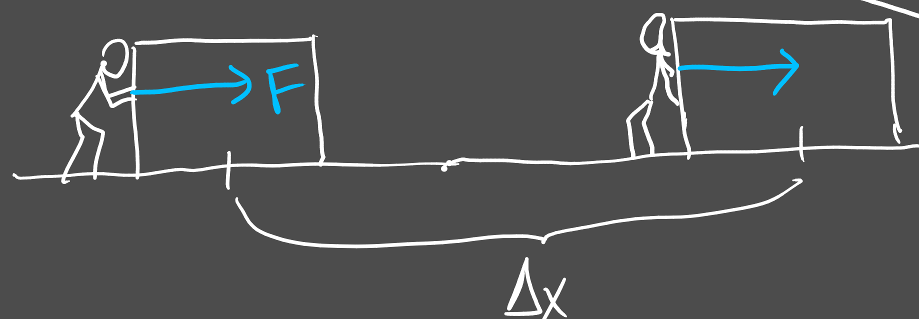
→ **Work** = Force × distance ⟶ transfer of energy from one system to another

$W = F \cdot \Delta x$

$[N][m]$

$\left[\frac{kg\,m}{s^2}\right][m] = \left[\frac{kg\,m^2}{s^2}\right]$

= Joule



Work depends on the amount of force parallel to displacement

→ Work is not done by ⊥ forces.

Energy - capacity to do work.

forms that energy can take:

* energy of motion - kinetic energy
* energy stored as the result of an interaction - potential energy
    * gravitational potential energy
    * spring potential energy
    * chemical potential energy - fuel, food
    * electric potential energy
* heat
* light
* nuclear energy

Conservation of Energy

slight amendment

Total Energy before = Total Energy after ← universally true

Total Energy before + Work in or out = Total Energy after

Kinetic Energy

Kinetic Energy = $\frac{1}{2}$ x mass x velocity^2

$$\boxed{K = \frac{1}{2} m v^2}$$

$$[kg] \cdot \left[\frac{m}{s}\right]^2 = \left[\frac{kg\, m^2}{s^2}\right] = [\text{Joule}]$$

another unit

[Calorie]

$\underbrace{\hspace{6em}}_{2m}$

$K_f = 0$

$$K_i = \frac{1}{2} m v^2$$

$$= \frac{1}{2}(10kg)(10\, m/s)^2$$

$$K_i = 500\,J$$

$$\boxed{E_i + W_{in/out} = E_f}$$ ← conservation of energy

$$500J + W = 0$$

$$W = -500J$$

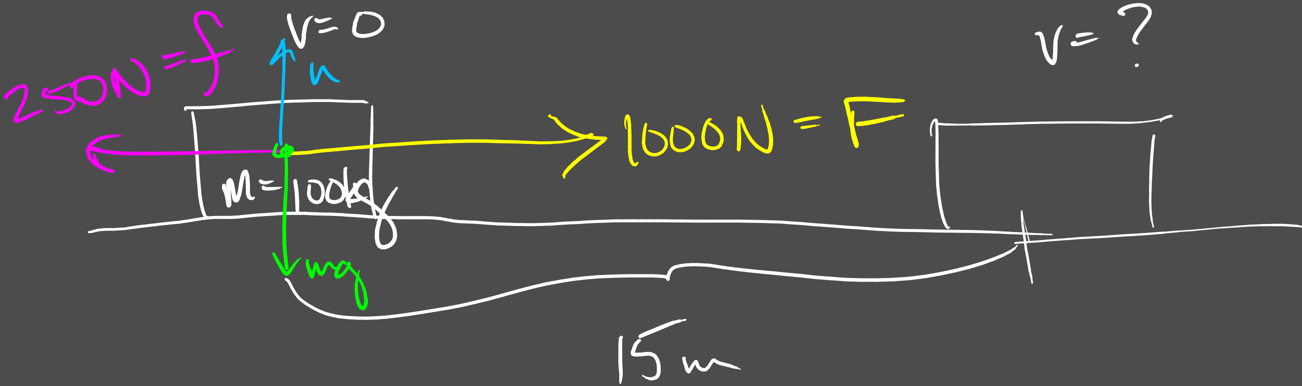
what frictional force does this?

$$W = F \cdot \Delta x = -500J = f \cdot 2m$$

$$f = -250N$$

↳ energy has been removed from the system

friction <u>always</u> removes <u>energy</u>



$$\cancel{E_i}^{\,0} + W_{in/out} = E_f$$

$$W_{in} = 1000\,N \cdot 15\,m = 15000\,J$$

$$W_{out} = -250\,N \cdot 15\,m = -3750\,J$$

$$0\,J + 15000\,J - 3750\,J = E_f$$

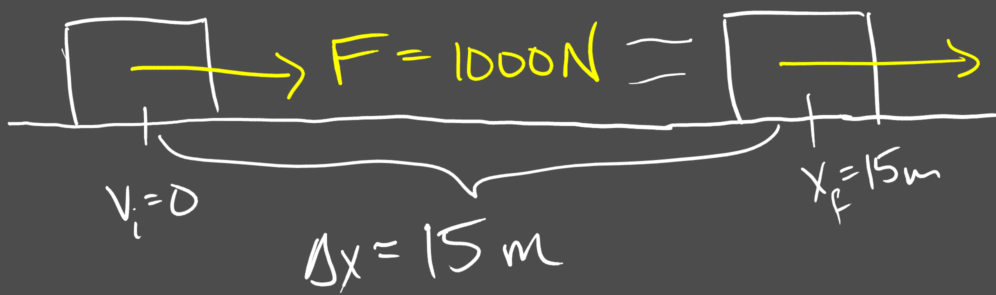
$$11{,}250\,J = E_f = K_f$$

$$11{,}250\,J = \frac{1}{2} m v_f^2$$

$$11{,}250\,J = \frac{1}{2}(100\,kg)\,v_f^2$$

$$\sqrt{225} = \sqrt{v_f^2} \qquad \boxed{v_f = 15\,m/s}$$

$m = 100\,kg$ $\qquad$ $v_f = ?$



$K = \frac{1}{2}mv^2$

$$\underbrace{E_i}_{=0} + W = E_f$$

$$W = E_f - \cancel{E_i}^{\to 0}$$

$$W = E_f$$

$$W = F \cdot \Delta x = 1000N \cdot 15m$$

$$W = 15{,}000\ J$$

$$15000J = E_f = K_f$$

$$15000\,J = \frac{1}{2}mv_f^2$$

$$15000J = \frac{1}{2}(100kg)v^2$$

$$\frac{1500\cancel{0}}{5\cancel{0}} = \frac{\cancel{50}\,v^2}{\cancel{50}}$$

$$\sqrt{300} = \sqrt{v^2}$$

$$\boxed{v = 17.3\ m/s}$$

→ $E_i + W_{in/out} = E_f$ → $K = \frac{1}{2}mv^2$

↓

expand to include potential energy

work done by the force of gravity – potential energy

→ conservative forces

- gravitational potential energy

$$U_g = \text{mass} \cdot g \cdot \text{height}$$

$$U_g = mgh$$

So $E_i + W_{i/o} = E_f$ becomes

$$\boxed{K_i + U_i + W_{i/o} = K_f + U_f}$$

external forces
- push / pull
- friction

$m = 10kg$

$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}(10)(0)^2 = 0$

$U_i = m \cdot g \cdot h = 10kg \cdot 9.8 \, m/s^2 \cdot 20m$

$U_i = 1960 \text{ J}$

$W_{i/o} = 0 \text{ J}$ no external forces/friction

$U_f = mgh_f = 10kg(9.8)(0m) = 0 \text{ J}$

$K_f = ? \rightarrow v_f = ?$

$\cancel{K_i}^{0} + U_i + W_{i/o} = K_f + \cancel{U_f}^{0}$ ← conservation of energy

$0J + 1960J + 0J = K_f + 0J$

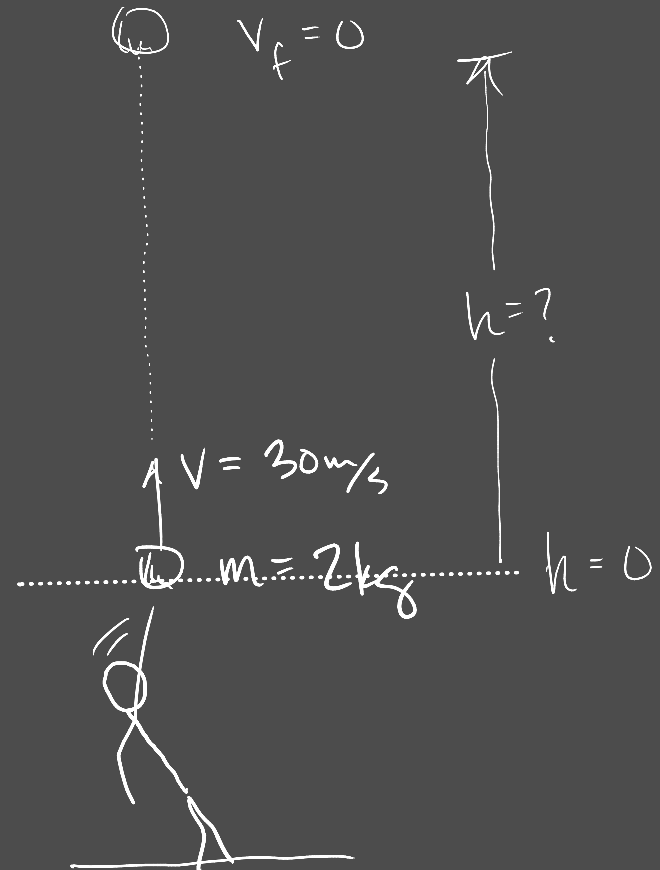
$1960J = K_f$

$1960J = \frac{1}{2} m v^2$

$1960J = \frac{1}{2}(10kg) v^2$

$\frac{1960J}{5} = v^2$ | $\sqrt{v^2} = \sqrt{392}$

$\boxed{v = 19.8 \, m/s}$



$$K_i + \cancel{U_i}^{0} + \cancel{W_{i/o}}^{0} = \cancel{K_f}^{0} + U_f$$

$$K_i = U_f$$

$$U_f = 900J = mgh$$

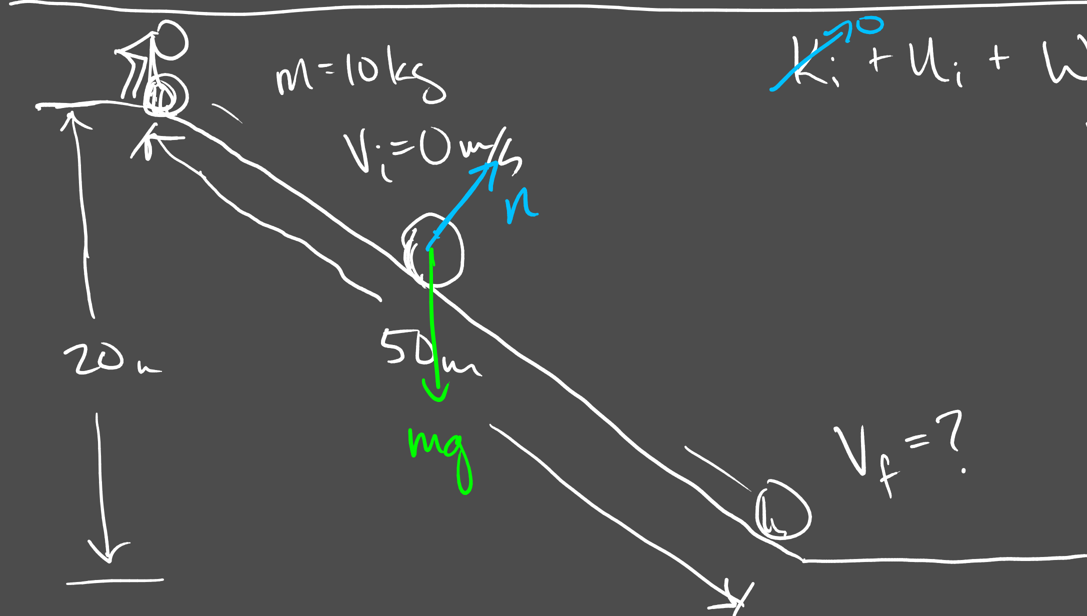
$$K_i = \frac{1}{2}mv^2$$

$$K_i = \frac{1}{2}(2kg)(30 m/s)^2$$

$$K_i = 900 J$$

$$900J = 2kg \cdot 9.8 m/s^2 \cdot h$$

$$\boxed{h = 45.9 m}$$



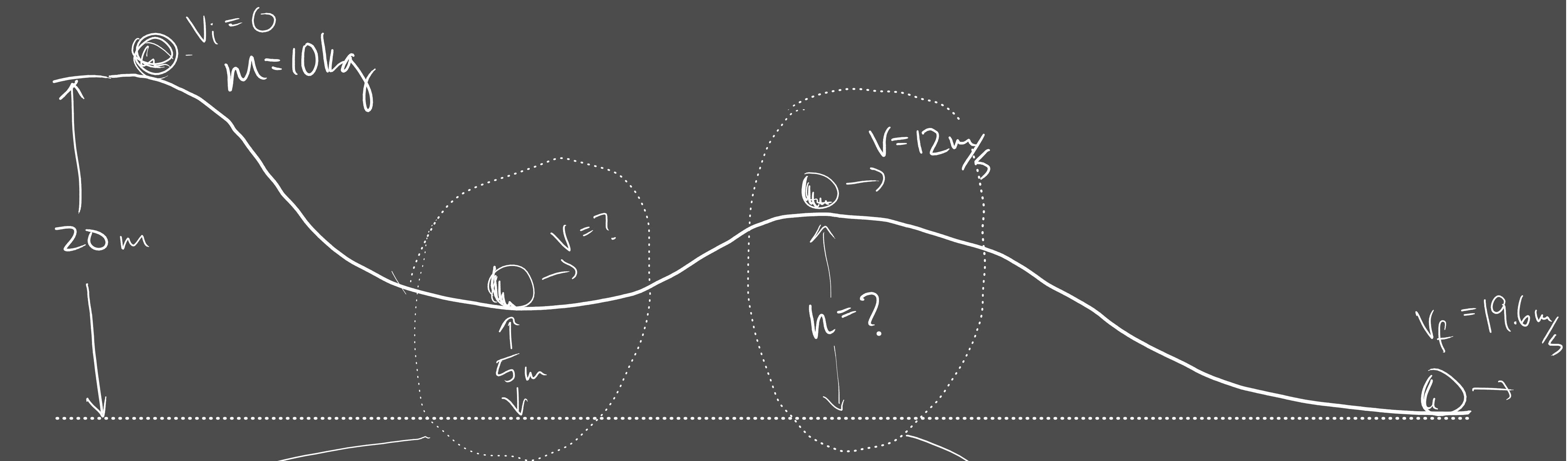
$$\cancel{K_i}^{0} + U_i + W_{i/o} = K_f + \cancel{U_f}^{0}$$

$W_{i/o} = ?$

No work is done by the normal force b/c normal force is $\perp$ to displacement

$$U_i = K_f$$

$$10kg \cdot 9.8 \cdot 20m = \frac{1}{2}(10kg) \cdot v^2 \rightarrow V = 19.8 m/s$$



mgh

$10kg \cdot 9.8 m/s^2 \cdot 5m$

$$\cancel{K_i}^{0} + U_i = K_f + U_f$$

$$1960J = K_f + 490J$$

$$1470J = K_f$$

$$1470J = \frac{1}{2}(10kg)v^2$$

$$v = 17.1\ m/s$$

$$\cancel{K_i}^{0} + U_i = K_f + U_f$$

$$1960J = K_f + U_f$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(10)(12)^2$$

$$K = 720J$$

$$1960J = 720J + U_f$$

$U_f = 1240\,J$

$1240\,J = mgh$

$1240\,J = 10kg \cdot 9.8 \cdot h$

$\boxed{h = 12.7m}$

Spring Potential Energy

$F_s = k \cdot x$ → NOT a constant force

BUT, it is conservative force

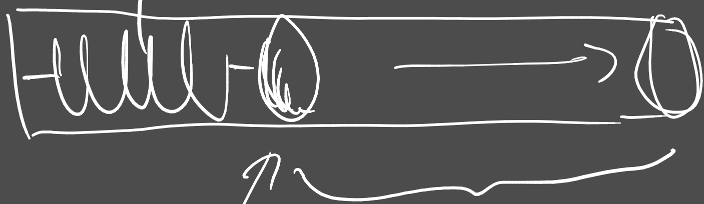
k → spring constant

x → stretch/compression distance

$$\boxed{U_s = \frac{1}{2} k x^2}$$

$k = 100\,N/m$

$\rightarrow v = ?$



$5\,cm = 0.05\,m$

$m = 0.1\,kg$

$$U_s = \frac{1}{2}(100\,N/m)(.05\,m)^2$$

$$U_s = 0.125\,J$$

$$\cancel{K_i}^{\,0} + U_i = K_f + \cancel{U_f}^{\,0}$$

$$0.125\,J = K_f$$

$$K_f = \frac{1}{2}mv^2$$

$$0.125\,J = \frac{1}{2}(0.1\,kg)\,v^2$$

$$2.5 = v^2$$

$$\boxed{v = 1.58\,m/s}$$

$v = 4\,m/s$

$m = 0.1\,kg$

$x = 0.05\,m$

$k = ?$

$$\rightarrow \cancel{K_i}^{\,0} + U_i = K_f + \cancel{U_f}^{\,0}$$

$$K_f = \frac{1}{2}mv^2$$

$$K_f = \frac{1}{2}(.1)(4)^2$$

$$K_f = 0.8\,J$$

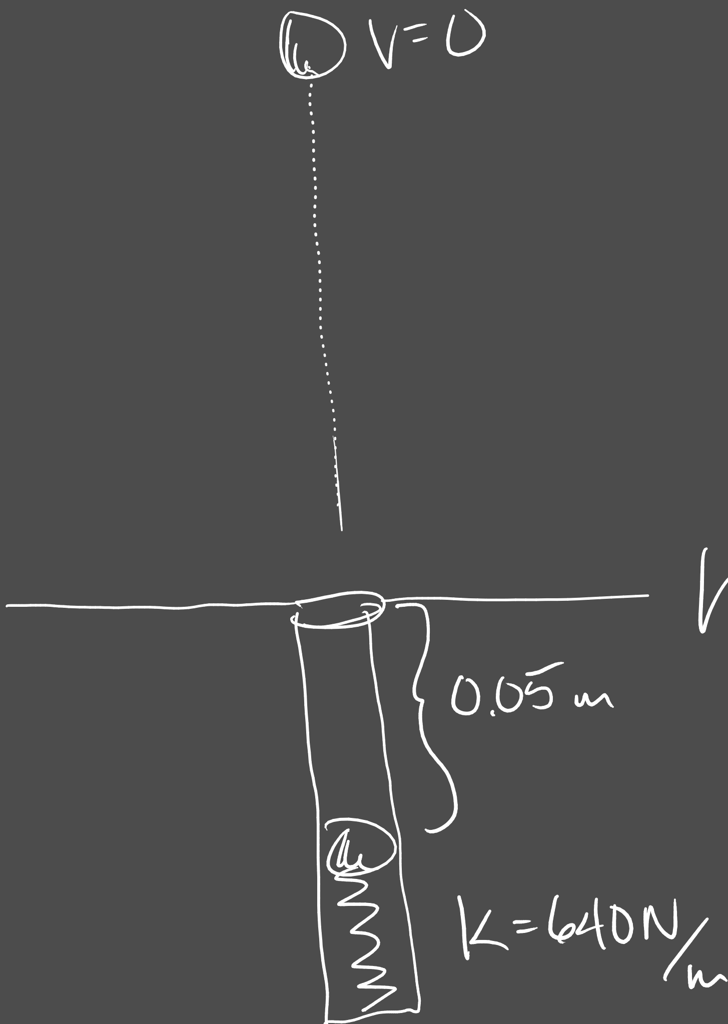
$$U_i = K_f$$

$$U_i = 0.8\,J$$

$$U_i = \frac{1}{2}kx^2$$

$$0.8J = \frac{1}{2}k\cdot(0.05m)^2$$

$$k = 640\,N/m$$



$h=0$

$$\cancel{K_i}^{\,0} + \underbrace{U_i}_{U_{spring}} = \cancel{K_f}^{\,0} + \underbrace{U_f}_{U_{Grav}}$$

$$\underbrace{\frac{1}{2}kx^2}_{} = mgh$$

$$0.8J = mgh$$

$$0.8J = 0.1kg(9.8 m/s^2)h$$

$$\boxed{h = .82\ m}$$

Power → rate of change of energy

→ rate of work

lift 200, 1 kg bricks
2m high in
10 minutes

$W_{brick} = F_{lift} \cdot \Delta y$

$W_{brick} = m \cdot g \cdot \Delta y$

$W_{brick} = 1kg \cdot 9.8 \cdot 2m =$

lift 150, 3kg bricks
1.5m in 20 min

$W = 3kg \cdot 9.8 \cdot 1.5m \cdot 150$

$W = 6615\ J$

$W_{brick} = 20\,J$

$W_{total} = 20\,J/\text{brick} \cdot 200\ \text{bricks}$

$Work = 4000\,J$

$$Power = \frac{Work}{time}$$

$$Power = \frac{4000\,J}{10\ min}$$

$Power = 400\,J/min$

$$Power = \frac{6615\,J}{20\ min}$$

$Power = 330\,J/min$

$$Power = \frac{\Delta Energy}{\Delta t}$$

$\left[\frac{Joules}{sec}\right] \rightarrow$ [Watt]

[horsepower]

746 Watts = 1 hp