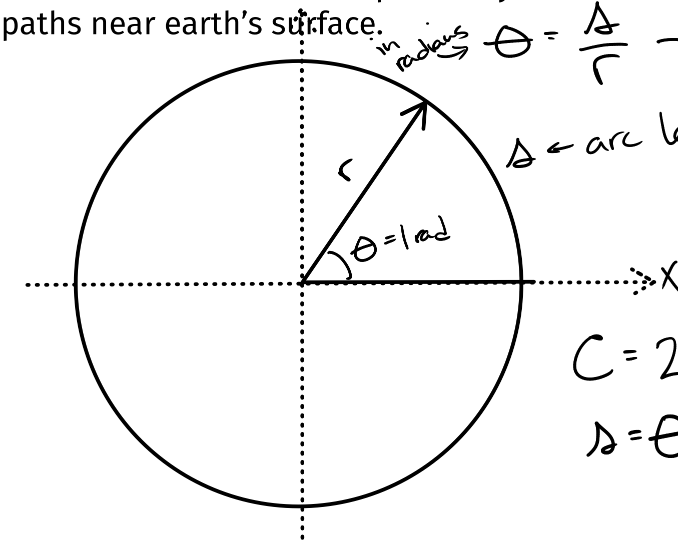
At the end of this worksheet you should be able to

- apply the relationships between angle and motion at the edge of a circle to describe the motion of an object in circular motion.
- apply Newton's 2nd law in the radial direction to solve interesting problems involving motion of objects in a circular path.
- apply the principles of radial net force and circular motion to planetary orbits and satellites as well horizontal and vertical paths near earth's surface.

in radians → $\theta = \frac{s}{r} \rightarrow s = \theta \cdot r$



s ← arc length

$C = 2\pi \cdot r$

$s = \theta \cdot r$

1. How many degrees are in 1 rad?

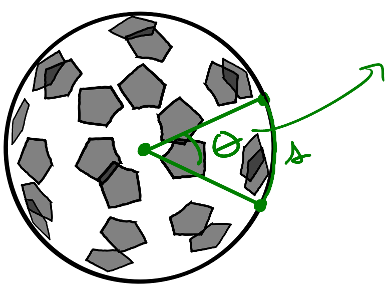
1 rev = 2π rad

360° = 2π rad

180° = π rad

$1\,\cancel{\text{rad}} \cdot \frac{360^\circ}{2\pi\,\cancel{\text{rad}}} = 57.3^\circ$

2. A soccer ball of radius 10 cm spins through an angle of 20°, then how many radians is that? What distance has a point on the equator of the ball traveled? What if it spins through 750°, then what distance has a point on the edge traveled?



$20^\circ \cdot \frac{\pi\ \text{rad}}{180^\circ} = 0.35\ \text{rad}$

$750^\circ \cdot \frac{\pi\ \text{rad}}{180^\circ} = 13.1\ \text{rad}$

$s = \theta \cdot r$ ← radians!

$s = 0.35\,\text{rad} \cdot 10\,\text{cm}$

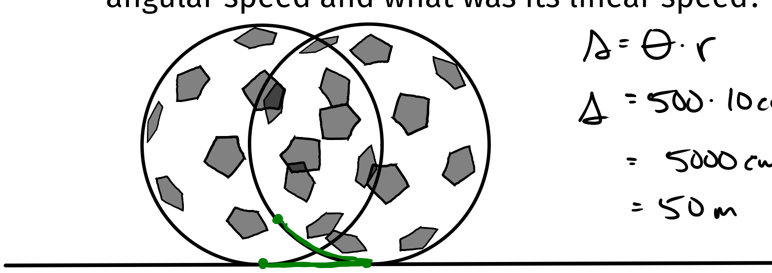
$s = 3.5\ \text{cm}$

$s = 13.1\,\text{rad} \cdot 10\,\text{cm}$

$s = 131\ \text{cm}$

how many revolutions? $500\ \text{rad} \cdot \frac{1\ \text{rev}}{2\pi\ \text{rad}} = 79.6$ revolutions

3. When you roll something along the ground, it is spinning of course, but it is also moving linearly (its center of mass is moving). It turns out that the distance the edge of a soccer ball moves as it spins is equal to the linear distance the ball moves, as long as it does not slip. So if a soccer ball of radius 10 cm rolls at constant angular speed through an angle of 500 rad, then how far has it rolled? If it takes 10 seconds to do this, what was its angular speed and what was its linear speed?



$s = \theta \cdot r$

$s = 500 \cdot 10\,\text{cm}$

$= 5000\,\text{cm}$

$= 50\,\text{m}$

$v = \frac{\Delta x}{\Delta t} = \frac{50\,\text{m}}{10\,\text{s}} = 5\,\text{m/s}$

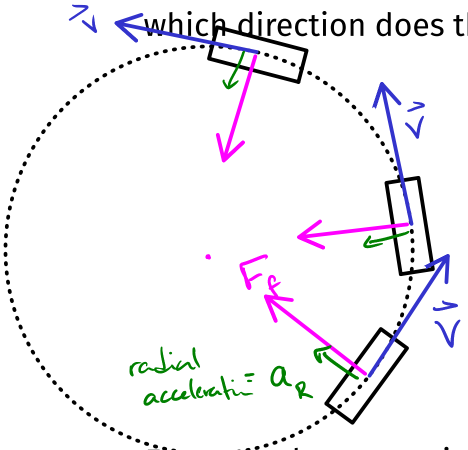
$\omega = \frac{\Delta\theta}{\Delta t} = \frac{500\,\text{rad}}{10\,\text{s}} = 50\,\text{rad/s}$

"omega" → angular velocity

$\theta = \frac{s}{r}$ $\quad$ $\omega = \frac{v}{r}$ $\quad \longleftarrow \quad$ $s = \theta \cdot r$, $v = \omega \cdot r$

4. When a car turns at constant speed, it travels along an approximately circular path. In which direction does the net force act and what provides this net force? → friction

ΣF acts radially



5. For a 1000 kg car turning like in the previous problem, if the coefficient of friction between the tires and the road is $\mu = 0.5$, then what is the maximum static force of friction that the road could provide to the car? If the car is going around a bend of radius 50 m, how how fast could it go around the bend without sliding?

$F_{sf} \leq \mu F_N$

$F_{sf,max} = \mu F_N = \mu \cdot m \cdot g = 0.5 \cdot 1000\,kg \cdot 9.8\frac{N}{kg} = 4900\,N$ ← max force of friction

in this case

$\Sigma F_R = F_{sf}$

$\Sigma F_R = m a_R$

$4900\,N = 1000\,kg \cdot a_R$

$a_R = 4.9\,m/s^2$

$a_R \neq \frac{\Delta v}{\Delta t}$

$\boxed{a_R = \frac{v^2}{r} \leftarrow \text{constant speed}}$

$4.9\,m/s^2 = \frac{v^2}{50\,m} \Rightarrow v = \sqrt{4.9 \cdot 50}$

$\boxed{v = 15.7\,m/s}$

6. If the same 1000 kg car is attempting to go around a bend of radius 20 m, at 20 m/s, then can it do this safely without sliding? ($\mu = 0.5$ still)

$\Sigma F_R = \frac{mv^2}{r}$

$= \frac{1000\,kg \cdot (20\,m/s)^2}{20}$

$= 20{,}000\,N > 4900\,N$

No!

$4900\,N = \frac{1000\,kg \cdot v_{max}^2}{20\,m}$

$v_{max} = 9.9\,m/s < 20\,m/s$

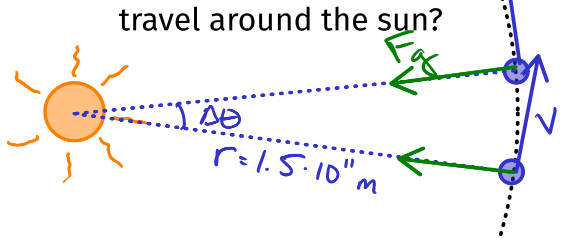
No!

$\Sigma F_R = \frac{mv^2}{r}$

$v = \omega \cdot r$

$\Sigma F_R = m\omega^2 r$

7. The earth orbits the sun, and while its path around the sun is not exactly circular, its close enough to treat that way here. What is the angular velocity of the earth around the sun? To do this, think about how long it takes to go one full revolution around the sun. How many radians is a revolution? So now how many radians per second does the earth travel around the sun?



$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,rad}{1\,yr} \cdot \frac{1\,yr}{365\,da} \cdot \frac{1\,da}{24\,hr} \cdot \frac{1\,hr}{3600\,s} = 2 \cdot 10^{-7}\,\frac{rad}{s}$

← 2π for 1 revolution

$\Delta t = \omega \cdot \Delta\theta$

8. What is the radius between the earth and the sun? (look this up in your book or google) Using the answer from the previous problem, what does this mean for the *tangential speed* of the earth around the sun?

linear-ish

$$\rightarrow V = \omega \cdot r$$

$$V = 2 \cdot 10^{-7} \frac{rad}{s} \cdot 1.5 \cdot 10^{11} \text{ m}$$

$$V = 3 \cdot 10^{4} \frac{m}{s} = \underline{30{,}000 \, m/s}$$

9. Now without looking it up, how could we use this information to determine the mass of the sun? The formula for the force of gravity between to masses can be written as, $F_g = \frac{Gm_1m_2}{r^2}$ ($G = 6.67 \times 10^{-11}$ ~~Nkg²/m²~~ $\mathrm{Nm^2/kg^2}$). Note that this not the form of the force gravity that we have been using. Why is that? Now look up the mass of the sun and see how how close we got.

$$F_g = \frac{G m_s m_e}{r^2} = \quad \Sigma F_R = \frac{m_e V^2}{r}$$

$$\frac{G m_s \cancel{m_e}}{\cancel{r^2}} = \frac{\cancel{m_e} V^2}{\cancel{r}}$$

$$m_s = \frac{r \cdot V^2}{G}$$

$$m_s = \frac{1.5 \cdot 10^{11} \text{ m} \cdot (3 \cdot 10^4 \, m/s)^2}{6.67 \cdot 10^{-11} \, \mathrm{Nm^2/kg^2}}$$

$$\rightarrow m_s = \underline{2 \cdot 10^{30} \text{ kg}}$$

10. By the way, how can we use free fall to get a measure of the mass of the earth? If we got to the lab and measure an acceleration of a 1 kg mass to be 9.82 m/s², then how can we calculate the mass of the earth?

$$F_g = \frac{G m_e m_{object}}{r^2} \qquad F_g = m_{object} \cdot g^{\,9.8} \Big\} \text{ on earth!}$$

$$\frac{G m_e \cancel{m_o}}{r^2} = \cancel{m_o} \cdot 9.8 \, m/s^2 \Rightarrow \underline{m_e} = \frac{r^2 \cdot 9.8}{G}$$

r: $6.371 \cdot 10^{6}$ m

$$= \frac{(6.371 \cdot 10^6 \text{ m})^2 \cdot 9.82 \, m/s^2}{6.67 \cdot 10^{-11} \, \mathrm{Nm^2/kg^2}}$$

$$m_e = \underline{5.98 \cdot 10^{24} \text{ kg}} \checkmark$$

11. In order to put a satellite into orbit around the earth, it needs to be traveling at a specific distance with a specific velocity, otherwise the force of gravity from the earth may be too

NOT TO SCALE

large, and it will crash, or too small and it will fly away into space. So suppose you wanted to put a 1000 kg satellite in orbit around the earth at a distance of 1000 km above the *surface of the earth*. How fast would this satellite need to be going in order to have this orbit?

altitude

$$F_g = \frac{G M_e m_s}{r^2} = \Sigma F_R = \frac{m_s v^2}{r}$$

$r = r_e + \text{alt.} = 6.37\cdot10^6\,m + 10^6\,m = 7.37\cdot10^6\,m$ from the center of earth

$$\frac{G M_e \cancel{m_s}}{r^{\cancel{2}}} = \frac{\cancel{m_s} v^2}{\cancel{r}} \Rightarrow v^2 = \frac{G M_e}{r}$$

$$\rightarrow v = \sqrt{\frac{G M_e}{r}} = \sqrt{\frac{6.67\cdot10^{-11}\cdot 6\cdot10^{24}}{7.37\cdot10^{6}}} = 7368\,m/s$$

$$= 7400\,m/s$$

12. If you wanted to kick a soccer ball horizontally off a cliff and have it go into orbit near the surface of the earth, then what velocity would you need to give it to achieve this?

$$F_G = \frac{G M_e m_o}{r_e^2} = \frac{m_o v^2}{r} \rightarrow v = \sqrt{\frac{G M_e}{r_e}} = 7900\,\frac{m}{s}$$

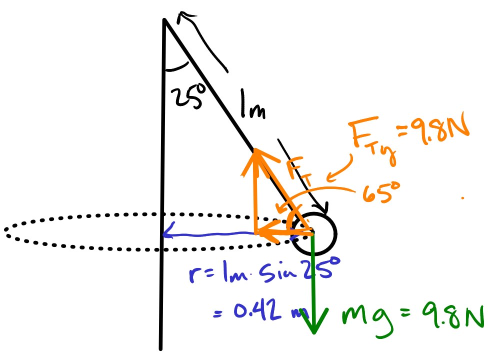
(or) very near earth's surface

$g = 9.8\,\frac{N}{kg}$

$$g\cancel{m_o} = \frac{\cancel{m_o} v^2}{r_e}$$

$$v = \sqrt{g r_e} = 7900\,m/s$$

13. A pendulum is swinging in a horizontal circle. The length of the pendulum is 1 m. If the angle of the pendulum string is 25°, then what is the radius of travel of the pendulum bob?



$$F_T = \frac{9.8N}{\sin 65°} = 10.8N$$

$$F_{T,R} = 10.8N\cdot\cos 65° = \underline{\underline{4.5\,N}}$$

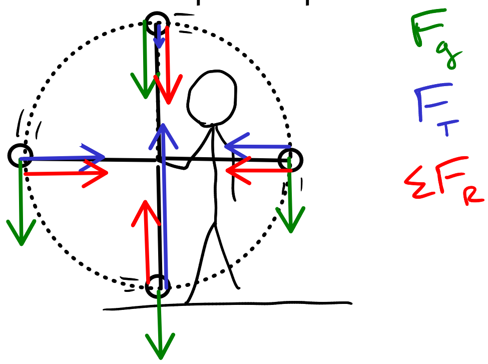
$$\Sigma F_R = \frac{mv^2}{r}$$

$$4.5N = \frac{1kg\cdot v^2}{0.42}$$

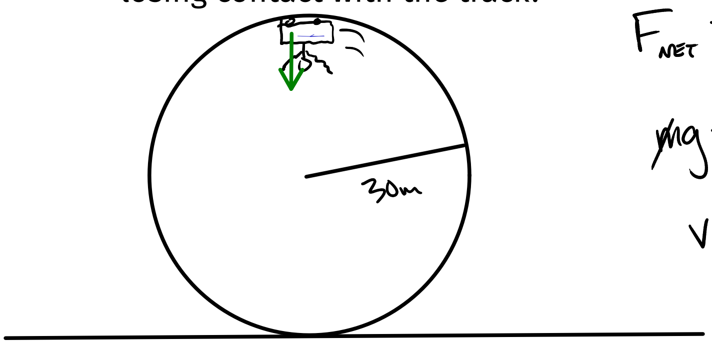
$$\underline{\underline{v = 1.39\,m/s}}$$

14. The mass of the pendulum bob from the previous problem is 1 kg. What upward force is necessary to keep the pendulum from moving up and down? What does this imply about the tension in the string? What does this mean for the radial tension force? How fast must this pendulum bob be moving?

15. When you are swinging a ball at the end of a string in a *vertical* circle, explain why the tension in the string is higher when the ball is at the bottom of its path, than when it is at the top of its path.



16. A roller coaster cart is doing a loop-the-loop. When the cart is at the top, what forces are acting on the cart to keep it in its circular path? What is the minimum force that would still technically mean that the cart is still in contact with the track? For a 30 m radius loop, what is the minimum speed that the cart must be going to make the loop without losing contact with the track?

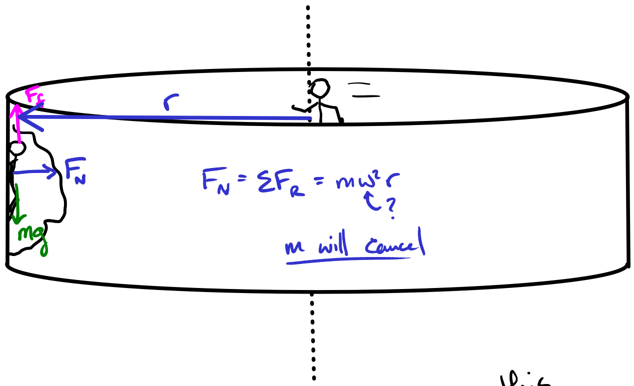


$$F_{NET} = \frac{mv^2}{r}$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{gr}\,?$$

#7



#9

use this

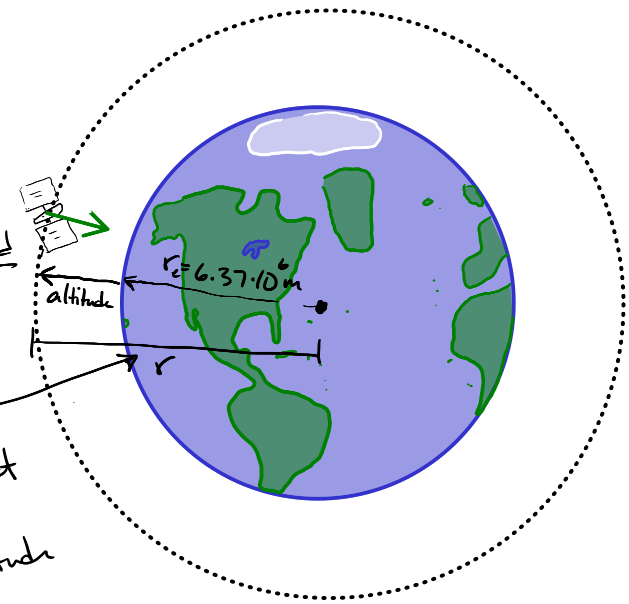
$$F_G = \Sigma F_R = \frac{mv^2}{r} = m\omega^2 r$$

$$\omega = \frac{1\,\text{rev}}{720\,\text{hr}} \cdot \frac{2\pi\,\text{rad}}{1\,\text{rev}} \cdot \frac{1\,\text{hr}}{3600\,\text{s}} = 2.42 \cdot 10^{-6}\,\frac{\text{rad}}{\text{s}}$$

$$\frac{GM_e \cancel{m}}{r^2} = \cancel{m}\omega^2 r$$

$$\underbrace{r = \sqrt[3]{\frac{GM_e}{\omega^2}}}_{\text{but this is total } r!}$$

So subtract r_e to get altitude



But! how do you get v?

$v = \omega \cdot r$!