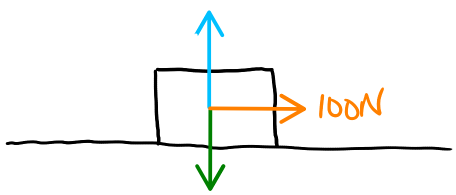
HW week 7-8 → I said things

1. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N for 10 s. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 s?



$\Sigma F_x = ma_x$

$\Sigma F_x = 100N = 100kg\, a_x$

$a_x = 1\, m/s^2$

$v_f = \cancel{v_i}^{\,0} + at$

$v_f = 1\, m/s^2 \cdot 10 s = 10\, m/s$

$a \propto (m)^{-1} \leftarrow$

$a_x = 0.5\, m/s^2$

$v_f = 5\, m/s$

2. A follow up to the previous question. The *momentum* of an object is defined as the product of an object's mass and its velocity. In this case the momentum of this object changes because a force external to the object was exerted on it. What is the momentum of the object initially? What is the momentum of the object at the end? It is said that the change in momentum of an object is equal to the *impulse*, where *impulse* is defined as the product of constant force and the amount of time the force is acting. Is that the case here? How could you use this to quickly find the final velocity of an object if the same force were pushing it for 100 s?

$p = mv$

$p_i = 0\, kgm/s$

$p_f = 100kg \cdot 10\, m/s$
$= 1000\, kgm/s$

$E_i = K_i = \frac{1}{2}mv_i^2 = 0 J$

$E_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(100kg)(10\, m/s)^2 = 5000 J$

$\Delta p = \underbrace{F_{AVG} \cdot \Delta t}_{\text{impulse}}$

$\Delta E = \underbrace{\boxed{F_{AVG} \cdot \Delta x}}_{\text{work}} \cdot \cos\theta$

$\Delta p = 1000\, kgm/s \overset{?}{=} 100N \cdot 10s = 1000 Ns$ ✓
$= 1000\, kg/s$

$\Delta p = F_{AVG} \cdot \Delta t$

$p_f - \cancel{p_i}^{\,0} = F_{AVG} \cdot \Delta t$

$\hookrightarrow v_f = \frac{F_{AVG} \cdot \Delta t}{m} = \frac{100N \cdot 100s}{100 kg}$
$= 100\, m/s$

3. Write down the equations for the velocity of two objects after an elastic collision. Then suppose the objects have the same mass. What do the equations look like now and what interpretation can you draw from that?

4. A 50 kg person runs at 10 m/s and jumps onto a 20 kg cart initially at rest. How quickly does the cart and person travel after the collision. How much kinetic energy is lost in this collision?

$p_i = p_f$

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$m_1 v_{1i} + m_2 v_{2i} = v_f (m_1 + m_2)$ ← inelastic collision

$50\,kg \cdot 10\,m/s + \cancel{20\,kg \cdot 0\,m/s}^{0} = 70\,kg \cdot v_f$

$v_f = \frac{50 \cdot 10}{70} = 7.14\,m/s$

5. A horse pulls a 250 kg cart a distance of 1.5 km. The frictional force on the cart is a constant 250 N. The horse eats oats in the morning to prepare for this trip. Each gram of oats provides 10 kJ of energy, but only 10% of this energy can go into work pulling the cart. How many grams of oats must the horse eat?

$W_{horse} = F_{horse} \cdot \Delta x \cdot \cos 0°$

$= 250\,N \cdot 1500\,m$

$= 375{,}000\,J$

$10\% \cdot E_{horse} = W_{horse}$

$0.1 \cdot E_{horse} = 375{,}000\,J$

$E_{horse} = 3{,}750{,}000\,J$

$M \cdot 10{,}000 \frac{J}{g} = E_{horse}$

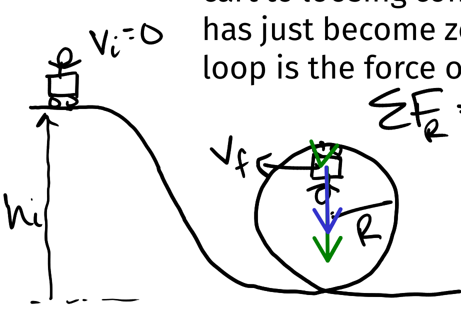
$M = \frac{3{,}750{,}000}{10{,}000}$

$M = 375\,g$

6. I want to try a weight loss program that involves repeatedly lifting a 50 kg barbell from the ground to over my head at a height of 2 m. I can do this about 5 times per minute. How long will it take me to burn 0.5 kg of fat? "Burning" fat means I have used it to supply energy to do work. Each gram of fat has roughly 39 kJ of energy to the body, but the muscles can only use 10% of this to do work.

$t = 2.76$ days

7. What is the minimum height *h* that a roller coaster needs to start at rest in order to do a loop-the-loop of radius *r* and not lose contact with the track at the top of the loop? The cart to loosing contact with the track means that the normal force of the track on the cart has just become zero, and that the only force acting on the cart at the top of the circular loop is the force of gravity.



$\Sigma F_R = mg + \cancel{F_N}^{0} = \frac{mv^2}{R}$

$\cancel{m}g = \frac{\cancel{m}v^2}{R}$

$v = \sqrt{gR}$

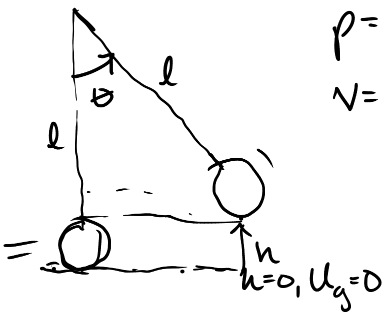
$\cancel{K_i}^{0} + U_i + \cancel{W_{NC}}^{0} = K_f + U_f$

$mgh_i = \frac{1}{2} m v_f^2 + mg(2R)$

$\cancel{m}gh_i = \frac{1}{2}\cancel{m}gR + 2\cancel{m}gR$

$h_i = \frac{R}{2} + 2R = 2.5R = \boxed{\frac{5}{2}R = h}$

8. A 10 kg pendulum has a maximum momentum of 100 kgm/s. How fast is this and where does it occur? If the pendulum has a length of 10 m then how high does it rise, and what angle from the vertical does it attain?



$p = mv$

$K = \frac{p^2}{2m}$

$v = \frac{p}{m} = \frac{100\,\text{kgm/s}}{10\,\text{kg}} = 10\,\text{m/s}$

$K_i + \cancel{U_i}^{0} + W_{nc} = \cancel{K_f}^{0} + U_f$

$\frac{1}{2}(\cancel{10\,\text{kg}})(10\,\text{m/s})^2 = (\cancel{10\,\text{kg}})(9.8)h_f$

$\frac{10^2}{2 \cdot 9.8} = h_f = 5.1\,\text{m}$

$h = l(1-\cos\theta)$

$\frac{h}{l} = 1-\cos\theta \Rightarrow \cos\theta = 1 - \frac{h}{l}$

$\theta = 60.6°$

9. A pendulum is an example of simple harmonic motion, an oscillation that if you graphed it out would resemble a sine/cosine pattern over time. The formula for the angular frequency of a pendulum is $\omega = \sqrt{\frac{g}{l}}$. Use this to calculate the natural frequency and period of oscillation of a pendulum. If a pendulum has a period of 6 s, then how long is it?

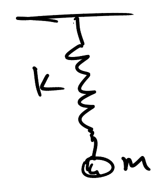
$\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{10\,\text{m}}} = 1\,\frac{\text{rad}}{\text{s}}$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{1\,\text{rad/s}}{2\pi} = 0.16\,\text{Hz}$

$T = \frac{1}{f} = \frac{1}{0.16\,\text{Hz}} = 6.25\,\text{s}$

spring-mass system



$\omega = \sqrt{\frac{k}{m}}$

$\downarrow$

f

$\downarrow$

T

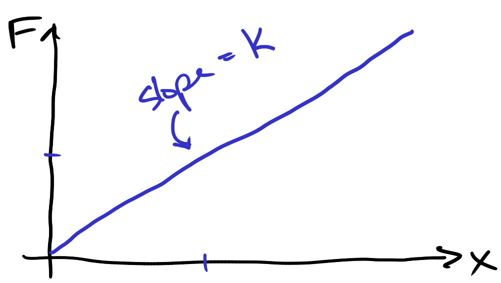
10. Let's talk about springs. *Hooke's Law* is the relationship between the force exerted by a spring and amount the spring has been stretched or compressed. The amount a spring has been stretched or compressed is the displacement of the end of the spring. Hooke's law says the magnitude of the force exerted is directly proportional to the displacement of the end of the spring. Write this in terms of a proportionality statement and again as an equation with a *constant of proportionality*. What are the units of the constant of proportionality?

$F \propto x$

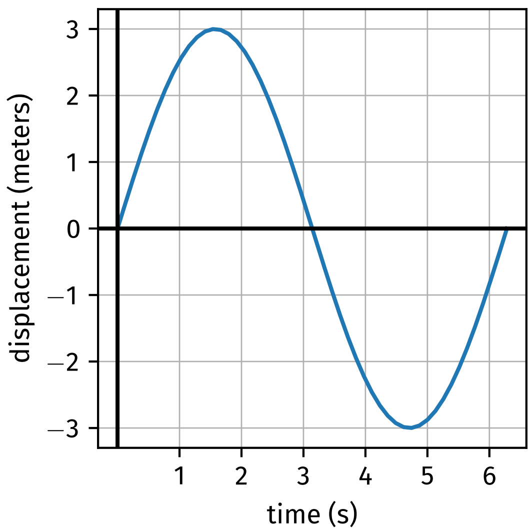
$F = kx$

↳ constant of proportionality

Draw a qualitative plot of the magnitude of this force vs displacement. What is the slope of this graph?



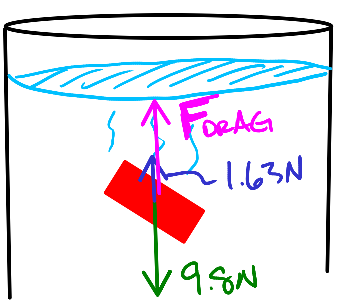
11. Consider the graph below of displacement under condition of *not constant* acceleration. What would a graph of velocity vs. time look like for this case? What about acceleration vs time. What is the period of oscillation of this object? What other information would you need in order to calculate the spring constant?



$$\frac{6\,g}{cm^3}\cdot\frac{1\,kg}{1000\,g}\cdot\left(\frac{100\,cm}{1\,m}\right)^3 = 6000\,kg/m^3$$

in water

12. An brick with a density $\rho = 6\,\mathrm{g/cm^3}$ and a mass of 1 kg is sinking. What is the buoyant force on the brick? Bonus problem: if the brick is sinking at constant velocity, then what other force must be on the brick and what is its value?



$\rho = \frac{m}{V}$

$F_b = m_f g$

$F_b = \rho_f V_f \cdot g$

$\hookrightarrow \underbrace{V_o = V_f}_{\text{brick is entirely submerged}}$

$V_o = \frac{m_o}{\rho_o} = \frac{1\,kg}{6000\,kg/m^3}$

$V_o = 1.67\cdot 10^{-4}\,m^3$

$F_{DRAG} - 9.8N + 1.63 = 0$

$F_{DRAG} = 9.8 - 1.63 = 8.17\,N$

$F_b = \overset{10^3}{1000}\,kg/m^3 \cdot 1.67\cdot 10^{-4}\cdot 9.8 = 1.63\,N$

13. If a 1 kg object is floating in the water with 1/4 of its volume above the surface of the water, then what is its density?

$\hookrightarrow 1000\,kg/m^3$

$\frac{V_{sub}}{V_o} = \frac{\rho_o}{\rho_f}$

$V_{sub} = \frac{3}{4} V_o$

$\frac{V_{sub}}{V_o} = \frac{3}{4}$

14. A typical bath tub is about 42 gallons which is $0.16\,\mathrm{m}^3$. Suppose it take 5 minutes to fill the tub from the faucet, then what is the velocity of water out of the faucet if it has a cross sectional area of $3\,\mathrm{cm}^2$. How high must the water in the nearest water tower be above the level of your faucet? If you turn your faucet off (an all other faucets in the neighborhood are off) then what is the pressure (absolute and gauge) in the pipes of your house?