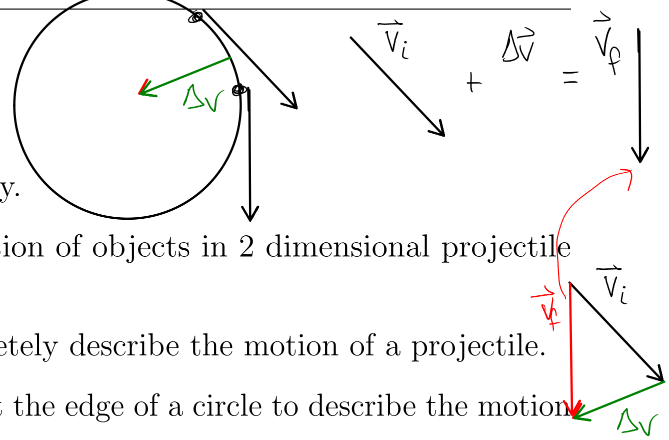
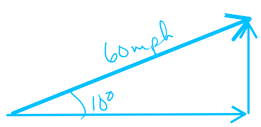


At the end of this worksheet you should be able to

- apply the concepts of vector components to velocity.
- use Newton's 1st and 2nd laws to discuss the motion of objects in 2 dimensional projectile motion.
- apply the kinematic equations to be able to completely describe the motion of a projectile.
- apply the relationships between angle and motion at the edge of a circle to describe the motion of an object in circular motion.
- apply Newton's 2nd law in the radial direction to solve interesting problems involving motion of objects in a circular path.
- apply the principles of radial net force and circular motion to planetary orbits and satellites as well horizontal and vertical paths near earth's surface.

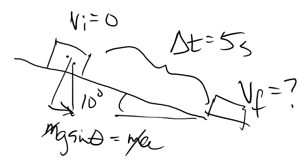
1. I initially throw a baseball at an angle of 10° with respect to horizontal. The initial speed of the ball is 60 miles per hour. What is the x- and y- component of the initial velocity?



$V_y = 60\text{mph} \cdot \sin 10° = 10.4$ mph

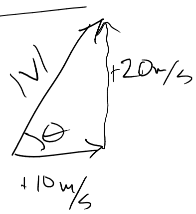
$V_x = 60 \cos 10°$

$V_x = 59$ mph



$V_f = 0 + a \cdot \Delta t$

2. If the x-component of a velocity vector is +10 m/s and the y-component is +20 m/s, then what is the speed of the ball and what is the angle of its initial trajectory with respect to the horizontal?

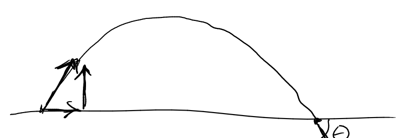


$|V| = \sqrt{10^2 + 20^2}$

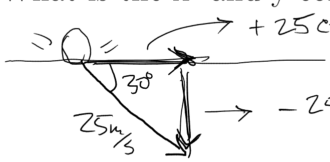
$|V| = 22.3$ m/s

$\theta = \tan^{-1}\left(\frac{20}{10}\right)$

$\theta = 63.4°$



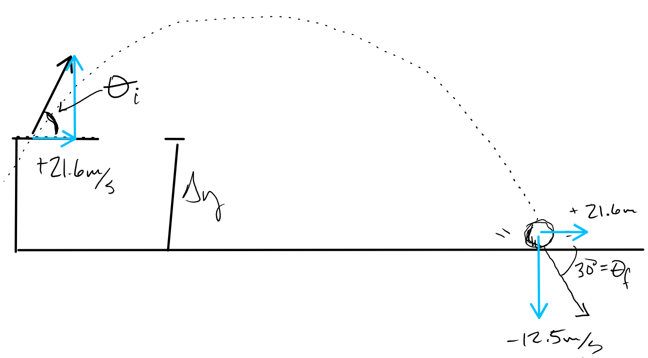
3. A baseball is thrown and then lands with a speed of 25 m/s at an angle of 30° degrees to the horizontal. If I described this as an angle with respect to the vertical what would I have said. What is the x- and y-components of the final velocity of this ball?



$+25\cos 30° = +21.7$ m/s

$-25\sin 30 = -12.5$ m/s

4. In the last problem, what is the initial horizontal velocity of the baseball? What information would you need in order to find what the initial vertical velocity component was?



$t \rightarrow v_{fy} = v_{iy} + a_y t$

$\Delta y \rightarrow v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$

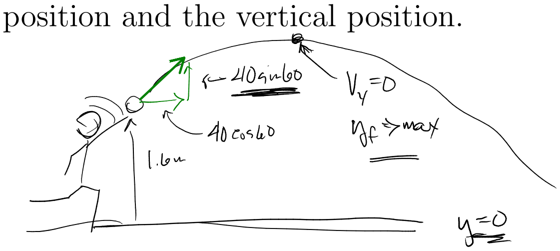
$\Delta x \rightarrow \Delta x = v_x t$

$\theta_i \rightarrow v_{iy} = v_{ix} \tan\theta$

$\Delta y = v_i t + \frac{1}{2} a_y t^2$

5. I throw a baseball with a speed of 40 m/s at an angle of 60° degrees above the horizontal. The ball is released at a height of 1.6 meters. For each second that goes by, calculate the horizontal velocity, the vertical velocity, the horizontal position and the vertical position.

time	x-vel	y-vel	x-pos	y-pos
0 s	+20 m/s	+34.6 m/s	0 m	1.6 m
1	20	24.8	20	31.3 m
2	20	15	40	51.3 m
3	20	5.2	60	61.3 m
4	20	−4.6	80	61.6 m ← Come down
5	20	−14.4	100	52.3 m
6	20	−24.2	⋮	33.0 m
7	20	−34	⋮	3.9 m
8	20	−44		−34 m



$v_y(t) = v_{iy} + a_y t$

$y(t) = y_i + v_{iy} t + \frac{1}{2} a_y t^2$

$= 1.6 + 34.6(1) + \frac{1}{2}(-9.8\ m/s^2)(1)^2$

the ball lands when $y = 0$

↳ time = 7.1 s

y_f

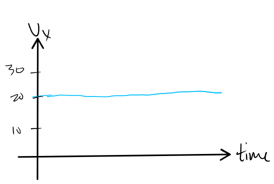
$0 = 1.6 + 34.6t + \frac{1}{2}(-9.8)t^2$

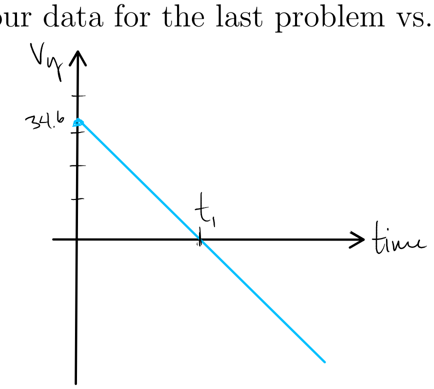
$= -4.9t^2 + 34.6t + 1.6$

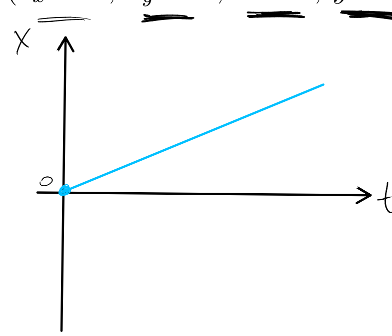
$At^2 + Bt + C$

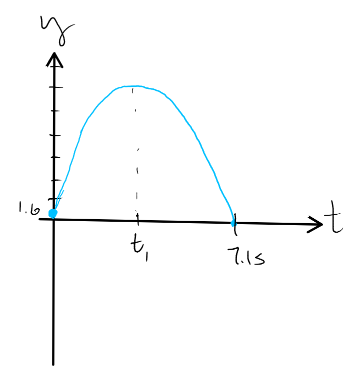
$$\boxed{t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}}$$

6. Plot your data for the last problem vs. time. (v_x vs t, v_y vs t, x vs t, y vs t)









7. I throw a baseball with a speed of 40 m/s at an angle of 60° degrees above the horizontal. The ball is released at a height of 1.6 meters. What is the vertical displacement when the baseball hits the ground? When will the ball reach its maximum height, and when will it hit the ground?

$\hookrightarrow \Delta y = -1.6$

max height

$V_{yf} = 0$

$V_{iy} = +34.6$

$0 = V_{iy} + at$

$\boxed{t = 3.5\,s}$

hits the ground

$\Delta y = V_i t + \frac{1}{2}at^2$

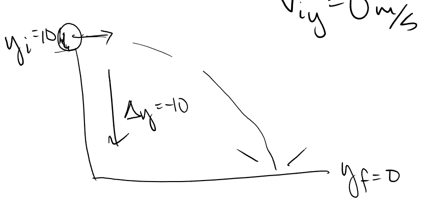
$-1.6 = 34.6t + \frac{1}{2}(-9.8)t^2$

$\hookrightarrow$ quadratic eq solver

$\boxed{t = 7.1}$

8. For a soccer ball that is kicked horizontally off the edge of a 10 m cliff with an initial speed of 20 m/s, what are the x- and y- components? When does the ball land?

$V_{iy} = 0\,m/s$



first approach

$y_f = y_i + V_{iy}t + \frac{1}{2}a_y t^2$

0 10

$\Delta y = -10 = \frac{1}{2}a_y t^2$

$\sqrt{\frac{-10 \cdot 2}{-9.8}} = \boxed{t = 1.4\,s}$

second approach

$-10\,m$

$\rightarrow V_{fy}^2 = V_{iy}^2 + 2a_y\Delta y$

$V_{fy}^2 = 2a_y\Delta y$

$V_{fy} = \pm\sqrt{2a_y\Delta y}$

careful w/ choosing correct sign!

$-14\,m/s \rightarrow V_{fy} = V_{iy} + a_y t$

solve for time

$t = 1.4\,s$

9. If I just dropped the soccer ball off the 10 m cliff, how long would that take to land? Compare this answer to the previous problem. Is that surprising? What if you kicked the ball with 100 m/s initial speed horizontal velocity? Surely that would matter...

$V_f^2 = 2a\Delta y$

$V_f = \pm\sqrt{2(9.8\,m/s^2)(10\,m)}$

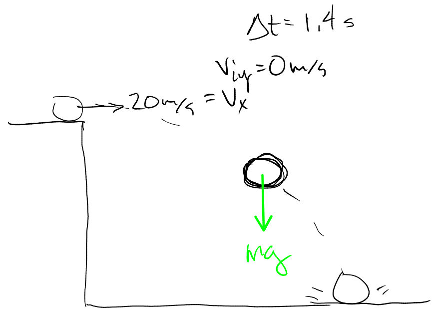
$V_f = \pm 14\,m/s \leftarrow \boxed{-14\,m/s}$

$V_f = at$

$t = \frac{V_f}{a} = \frac{-14\,m/s}{9.8\,m/s^2} = 1.4\,s$

$\theta_i = 0^\circ$

10. OK back to the soccer ball that is kicked horizontally off the edge of a 10 m cliff with an initial speed of 20 m/s. How far from the base of the cliff will it land? ($\Delta x = 28$ s)



$\boxed{\Delta x = v_x t}$

$a_y = -9.80\ m/s^2$

$a_x = 0\ m/s^2$

$$\boxed{\begin{array}{l} \circ\ \Delta y = v_i t + \frac{1}{2} a_y t^2 \\ \cdot\ v_f = v_i + at \\ \circ\ v_f^2 = v_i^2 + 2a\Delta y \end{array}}$$

$v_x = v_i \cos\theta$

$v_{iy} = v_i \sin\theta$

11. For the previous problem, what is the *speed* of the ball when it lands and at what angle with respect to the horizontal does it hit the ground?

$$v_{fy}^2 = v_{iy}^2 + 2a\underbrace{\Delta y}_{10\,m} \longrightarrow v_{fy} = -14\ m/s$$

$v_x = +20\ m/s$

$|v| = 24.4\ m/s$

$\theta = \tan^{-1}\left(\frac{14}{20}\right) = 35^\circ$ ← below horizontal

12. Let's do the last problem inside out. So take how far away from the base of the cliff the soccer ball lands, and work backwards to find its initial horizontal speed. The only assumption is that it is initially kicked exactly horizontally.

given $\Delta x = 28$ m and $v_{iy} = 0$, find v_x. Should get $+20$ m/s.

13. Now let's do the previous problem *in general.* If I kick a ball directly horizontally off a cliff of height h, and its lands a distance x away from the base of the cliff, then what was the initial velocity of the ball? (We will use this result in lab soon.)

14. How many degrees are in 1 rad?

$$1\,\cancel{\text{rad}} \cdot \frac{180^\circ}{\pi\,\cancel{\text{rad}}} = 57.3^\circ$$

15. A soccer ball of radius 10 cm spins through an angle of 20°, then how many radians is that? What distance has a point on the equator of the ball traveled? What if it spins through 750°, then what distance has a point on the edge traveled?

$$20^\circ \cdot \frac{\pi\ \text{rad}}{180^\circ} = 0.34\ \text{rad}$$

$$\theta = \frac{s}{r}$$

$$\boxed{s = \theta \cdot r}$$

$$= 0.34\,\text{rad} \cdot 0.1\,\text{m} = 0.034\ \text{m}$$

3.4 cm

$$750^\circ \cdot \frac{\pi}{180^\circ} = 13.1\ \text{rad}$$

$$s = 13.1\,\text{rad} \cdot (0.1\,\text{m}) = \underline{\underline{1.31\,\text{m}}}$$

16. When you roll something along the ground, it is spinning of course, but it is also moving linearly (its center of mass is moving). It turns out that the distance the edge of a soccer ball moves as it spins is equal to the linear distance the ball moves, as long as it does not slip. So if a soccer ball of radius 10 cm rolls at constant angular speed through an angle of 500 rad, then how far has it rolled? If it takes 10 seconds to do this, what was its angular speed and what was its linear speed?

distance its center of mass has move = arc length ← $s = r\theta$

$$V_{\text{center of mass}} = V_{\text{edge}}$$

$$\rightarrow V_{\text{edge}} = \frac{\Delta s}{\Delta t} = \underbrace{\omega}_{\text{angular speed}} \cdot r$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

17. When a car turns at constant speed, it travels along an approximately circular path. In which direction does the net force act and what provides this net force?

radially

static friction

18. For a 1000 kg car turning like in the previous problem, if the coefficient of friction between the tires and the road is $\mu = 0.5$, then what is the maximum static force of friction that the road could provide to the car? If the car is going around a bend of radius 50 m, how fast could it go around the bend without sliding?

$$F_{sf} \leq \mu_s F_n$$

$$F_{sf} = F_{NET,R} = ma_R = \frac{mv^2}{r}$$

$\mu_s F_n \rightarrow$ $\boxed{F_{sf} = \frac{mv^2}{r}}$ solve for v $V = 15.7\,m/s$

4900 N

19. If the same 1000 kg car is attempting to go around a bend of radius 20 m, at 20 m/s, then can it do this safely without sliding? ($\mu = 0.5$ still)

$$F_{max,fs} = 4900\,N \overset{?}{\longleftrightarrow} F_{NET,R} = \frac{mv^2}{r} = \frac{1000kg(20\frac{m}{s})^2}{20m} = 20000\,N$$

no!

20. The earth orbits the sun, and while its path around the sun is not exactly circular, its close enough to treat that way here. What is the angular velocity of the earth around the sun? To do this, think about how long it takes to go one full revolution around the sun. How many radians is a revolution? So now how many radians per second does the earth travel around the sun?

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,rad}{3.15\cdot10^7\,s} = 2\cdot10^{-7}\,rad/s$$

$$1\,yr\cdot\frac{365.25\,d}{1\,yr}\cdot\frac{24\,hr}{1\,d}\cdot\frac{3600\,s}{1\,hr} = 3.15\cdot10^7\,s$$

$r = 1.5 \cdot 10^{8}\ \mathrm{km} = 1.5 \cdot 10^{11}\ \mathrm{m}$

21. What is the radius between the earth and the sun? (look this up in your book or google) Using the answer from the previous problem, what does this mean for the *tangential speed* of the earth around the sun?

$$\boxed{v = r \cdot \omega}$$

$$= 1.5 \cdot 10^{11}\ \mathrm{m} \cdot 2 \cdot 10^{-7}\ \mathrm{rad/s} = 3 \cdot 10^{4}\ \mathrm{m/s} = 30{,}000\ \mathrm{m/s}$$

$$F_g = m_2 g_1 \qquad \boxed{g_1 = \frac{G m_1}{r^2}}$$

22. Now without looking it up, how could we use this information to determine the mass of the sun? Remember that the formula for the force of gravity between to masses can be written as, $\boxed{F_g = \frac{Gm_1m_2}{r^2}}$ ($G = 6.67 \times 10^{-11}\ \mathrm{Nkg^2/m^2}$). Now look up the mass of the sun and see how how close we got?

$$\frac{G m_1 m_2}{r^2} = F_{NET,R} = \frac{m_2 v^2}{r}$$

$$\frac{G m_1 \cancel{m_2}}{r^{\cancel{2}}} = \frac{\cancel{m_2} v^2}{\cancel{r}}$$

solve m_1

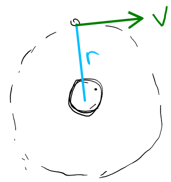
$$\frac{G m_1}{r} = v^2$$

distance b/t earth + sun $\rightarrow r$

$$m_1 = \frac{v^2 \cdot r}{G} = 2 \cdot 10^{30}\ \mathrm{kg}$$

23. By the way, how can we use free fall to get a measure of the mass of the earth? If we got to the lab and measure an acceleration of a 1 kg mass to be $9.82\ \mathrm{m/s^2}$, then how can we calculate the mass of the earth?

24. In order to put a satellite into orbit around the earth, it needs to be traveling at a specific distance with a specific velocity, otherwise the force of gravity from the earth may be too large, and it will crash, or too small and it will fly away into space. So suppose you wanted to put a



earth

sat.

$$\frac{G m_1 \cancel{m_2}}{r^{\cancel{2}}} = \frac{\cancel{m_2} v^2}{\cancel{r}}$$

$$\boxed{\frac{G m_1}{r} = v^2}$$

needs to be satisfied for stable orbit

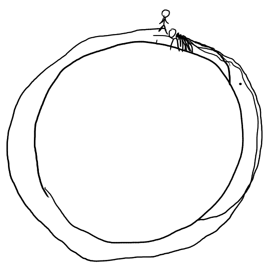
6.37×10^6 m

$r = r_e + \text{altitude}$

1000 kg satellite in orbit around the earth at a distance of 1000 km above the *surface of the earth*. How fast would this satellite need to be going in order to have this orbit?

$v = ?$

$m_e = 5.97 \cdot 10^{24}$ kg $= m_1$



$$V = \sqrt{\frac{Gm_1}{r}} = 7350 \text{ m/s}$$

$$\left.\begin{array}{l} 6.37 \cdot 10^6 \text{ m} \\ +1 \cdot 10^6 \text{ m} \end{array}\right\} 7.37 \cdot 10^6 \text{ m}$$

25. If you wanted to kick a soccer ball horizontally off a cliff and have it go into orbit near the surface of the earth, then what velocity would you need to give it to achieve this?

$$9.8 \frac{N}{kg} = g \text{ — } \frac{Gm_1 m_2}{r_e^2} = \frac{m_2 v^2}{r_e}$$

$$\cancel{m_2} g = \frac{\cancel{m_2} v^2}{r}$$

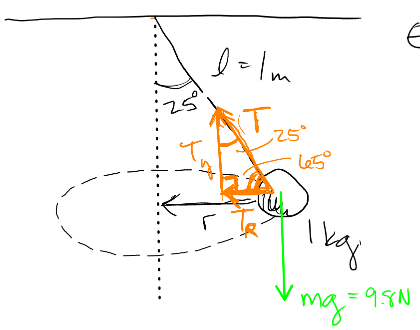
$6.37 \cdot 10^6$ m

$$gr = v^2 \rightsquigarrow v = \sqrt{g r_e} \rightsquigarrow v = \sqrt{9.8 \cdot 6.37 \cdot 10^6} = 7{,}900 \text{ m/s}$$

26. A pendulum is swinging in a horizontal circle. The length of the pendulum is 1 m. If the angle of the pendulum string is 25°, then what is the radius of travel of the pendulum bob?

from the vertical

$\theta = 25°$



find r

$r = l \cdot \sin\theta$

$r = 1\text{m} \cdot \sin 25°$

$r = 0.42$ m

27. The mass of the pendulum bob from the previous problem is 1 kg. What upward force is necessary to keep the pendulum from moving up and down? What does this imply about the tension in the string? What does this mean for the radial tension force? How fast must this pendulum bob be moving?

$T_y = 9.8$ N

$$\sin 65° = \frac{T_y}{T}$$

$$T = \frac{T_y}{\sin 65°} = \boxed{10.8 \text{ N} = T}$$

$$\frac{T_x}{T} = \cos 65°$$

$$T_x = T \cos 65° = 10.8 \cos 65°$$

$$\boxed{T_R = 4.56 \text{ N}}$$

$$T_R = F_{NET,R} = ma_R$$

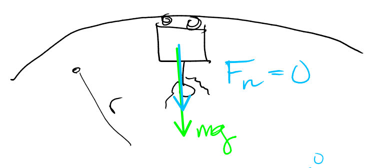
$$T_R = \frac{mv^2}{r}$$

$$F_{NET,R} = ma_R \qquad a_R = \frac{v^2}{r} = \omega^2 r$$

$$\boxed{v = \sqrt{\frac{T_R \cdot r}{m}} = 1.38 \text{ m/s}}$$

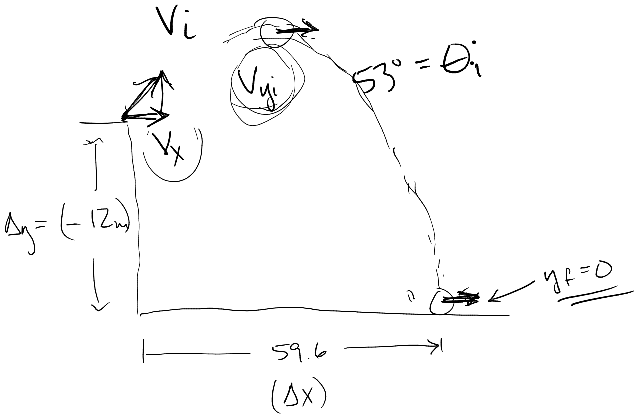
28. When you are swinging a ball at the end of a string in a *vertical* circle, explain why the tension in the string is higher when the ball is at the bottom of its path, than when it is at the top of its path.

29. A roller coaster cart is doing a loop-the-loop. When the cart is at the top, what forces are acting on the cart to keep it in its circular path? What is the minimum force that would still technically mean that the cart is still in contact with the track? For a 30 m radius loop, what is the minimum speed that the cart must be going to make the loop without losing contact with the track?



$$F_{NET} = \cancel{F_n} + mg = \frac{mv^2}{r}$$

#6 After being assaulted by flying cannonballs, the knights on the castle walls (12.0 m above the ground) respond by propelling flaming pitch balls at their assailants. One ball lands on the ground at a distance of 59.6 m from the castle walls. If it was launched at an angle of 53.0° above the horizontal, what was its initial speed?



$$v_{fy}^2 = v_i^2 + 2a\Delta y$$

? ?

horizontal

- $\Delta x = v_x t$
- $\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$

vertical

$$v_x = v_i \cos\theta$$

$$v_{yi} = v_i \sin\theta$$

$$\Delta x = \underbrace{v_i \cos\theta}_{v_x} \cdot t$$

$$t = \frac{\Delta x}{v_i \cos\theta}$$

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$

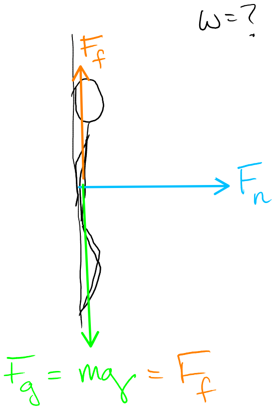
$$\Delta y = v_i t + \frac{1}{2} a t^2$$

$$\Delta y = \underbrace{\cancel{v_i} \sin\theta}_{v_{yi}} \cdot \underbrace{\left(\frac{\Delta x}{\cancel{v_i} \cos\theta}\right)}_{t} + \frac{1}{2}(-9.8)\underbrace{\left(\frac{\Delta x}{v_i \cos\theta}\right)^2}_{t}$$

$$\Delta y = \Delta x \cdot \tan\theta + \frac{-9.8}{2} \cdot \left(\frac{\Delta x}{\cos\theta}\right)^2 \cdot \frac{1}{v_i^2}$$

If the coefficient of static friction between a person and the wall of the cylinder is 0.200 and the cylinder has a radius of 1.60 m, what is the minimum angular speed of the cylinder so that the people don't fall out?

$\omega = ?$



$$F_f = \mu F_n$$

$$\frac{F_f}{\mu} = F_n$$

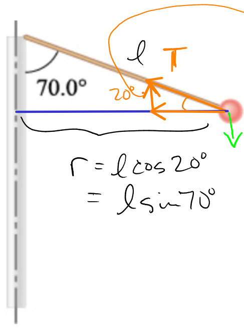
$$\frac{mg}{\mu} = F_n = F_{NET,R} = m a_R$$

$$a_R = \frac{v^2}{r} = \omega^2 r$$ since we want ω

$$\frac{\cancel{m} g}{\mu} = \cancel{m} \omega^2 r$$

$$\sqrt{\frac{g}{\mu r}} = \omega$$

A 0.700-kg ball is on the end of a rope that is 1.80 m in length. The ball and rope are attached to a pole and the entire apparatus, including the pole, rotates about the pole's symmetry axis. The rope makes a constant angle of 70.0° with respect to the vertical. What is the tangential speed of the ball?



Axis of rotation

$r = \ell \cos 20° = \ell \sin 70°$

$mg = T_y$

want:

$T_y \longrightarrow T \longrightarrow T_x \longrightarrow F_{NET,R} \longrightarrow v$

$F_{NET,R} = ma_c$

$F_{NET,R} = \frac{mv^2}{r}$

$a_c = \frac{v^2}{r} = \omega^2 r$