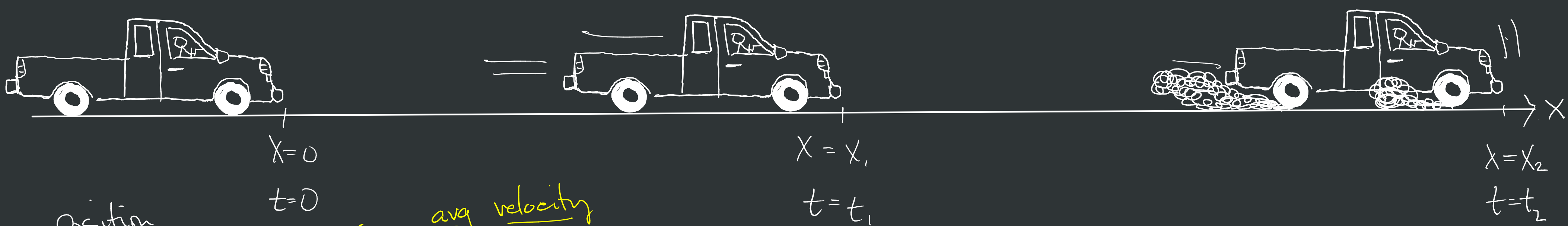
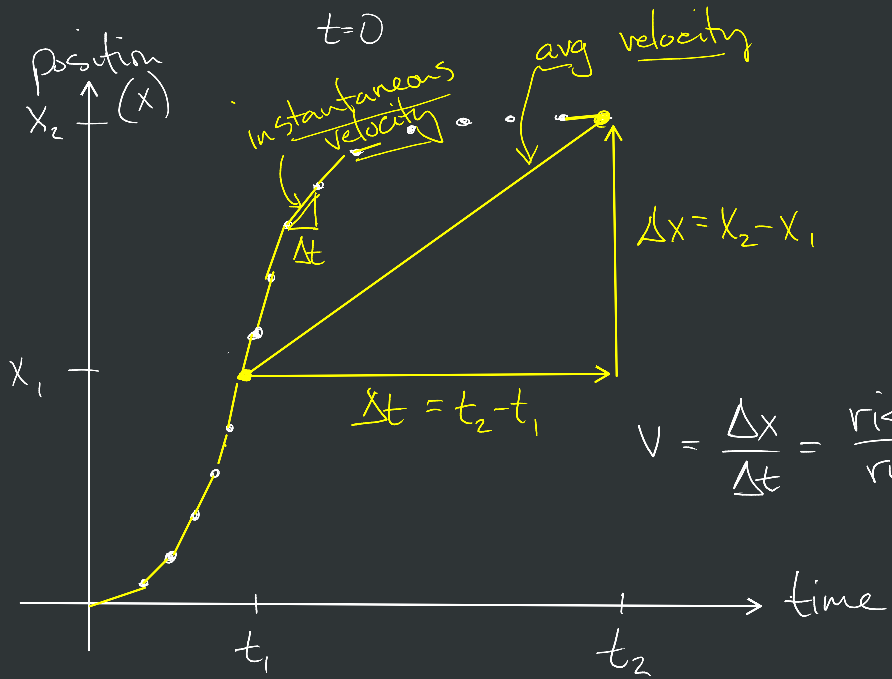
Week 4 - Newton's second law and the kinematic equations

At the end of this you can
- interpret and discuss the important features of graphs of position, velocity, and acceleration vs. time.
- use a graph of position vs time to find the instantaneous velocity at any time
- use a graph of velocity vs time to find the instantaneous acceleration at any time





displacement → change in position
↳ sign of displacement indicates the direction of travel

velocity → rate of change of position
→ rate of displacement
↳ $\frac{\Delta x}{\Delta t}$ – ratio w/ time interval

$$V = \frac{\Delta x}{\Delta t} = \frac{\text{rise}}{\text{run}} = \text{slope}$$

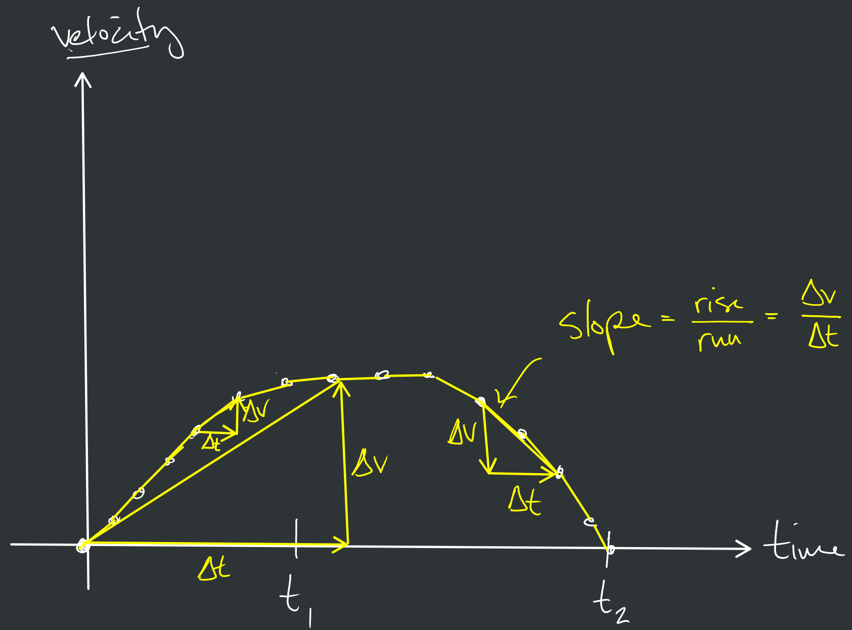
average velocity – ratio of displacement to time over a large time interval instantaneous velocity ⇒ velocity over the shortest time interval measurable

acceleration - rate of change of velocity

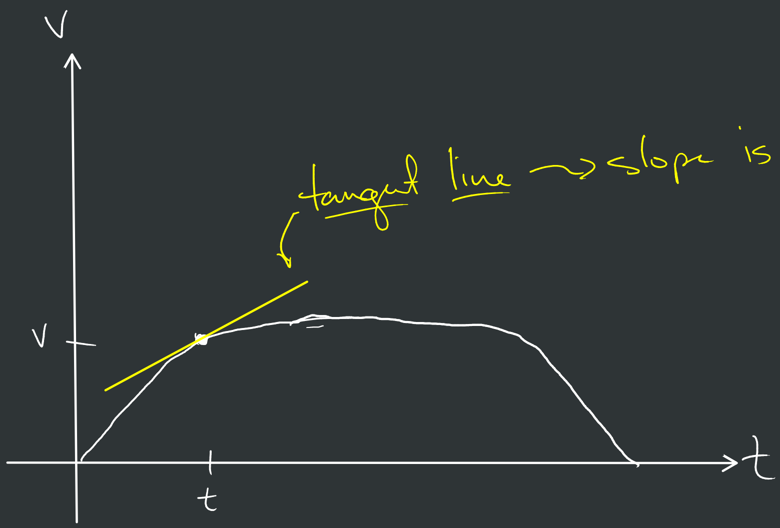
$$\frac{\Delta v}{\Delta t}$$

average acceleration — over a long time interval

instantaneous — over the shortest time interval



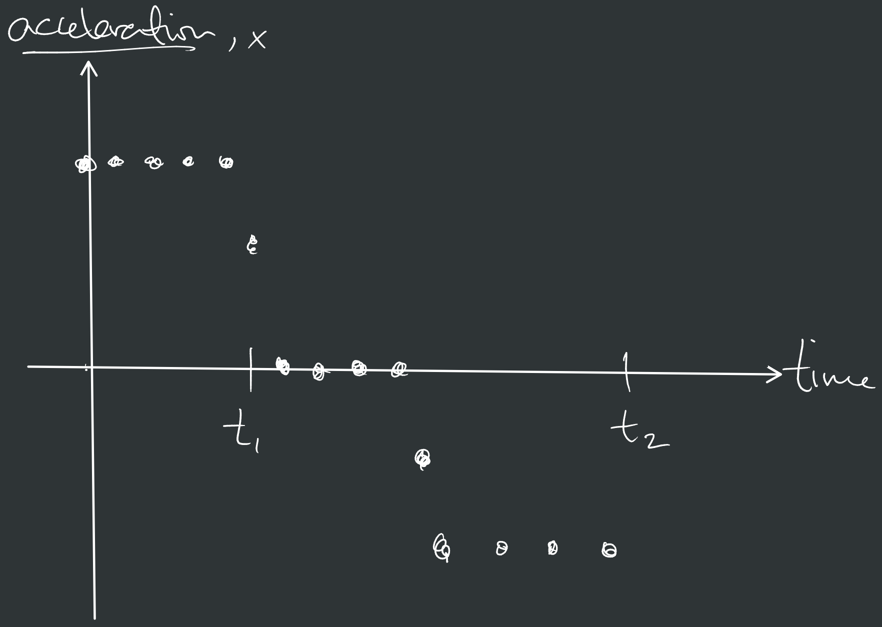
slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta v}{\Delta t} = a$



tangent line → slope is instantaneous acceleration

Slope of position vs. time → velocity

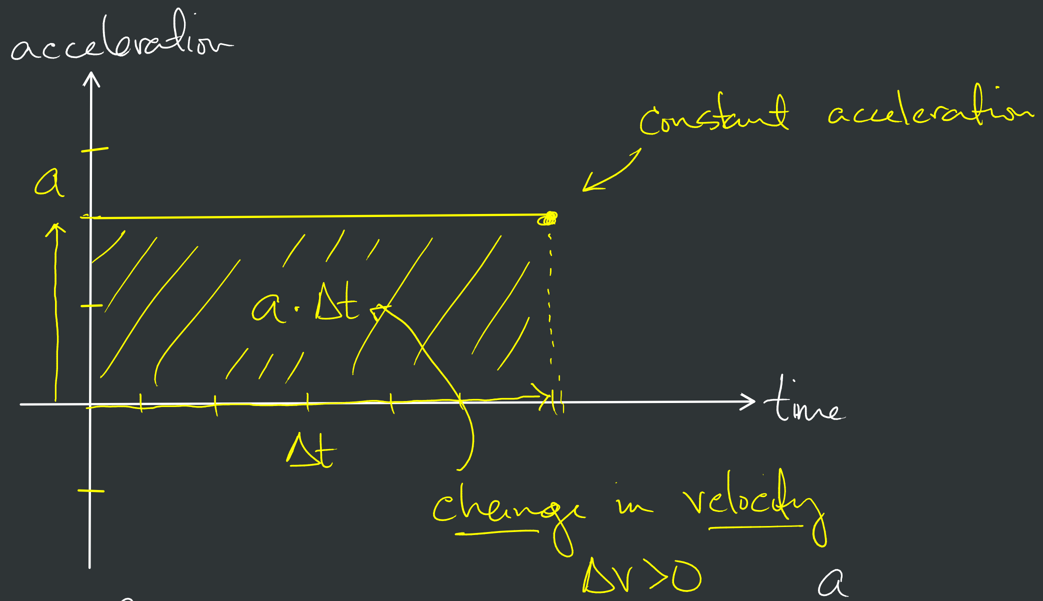
slope of velocity vs. time → acceleration



$$\vec{F}_{NET} = m \cdot \vec{a} \quad \left(F_{NET,x} = m \cdot a_x \right)$$

Constant acceleration

After this you can
- discuss what limitations we will impose on acceleration in this class and why
- apply a geometrical argument for how to use a plot of acceleration vs time
    to find the change in velocity
- apply the same for using a plot of velocity vs time to find the change in position
- discuss the entire subject of calculus and its uses

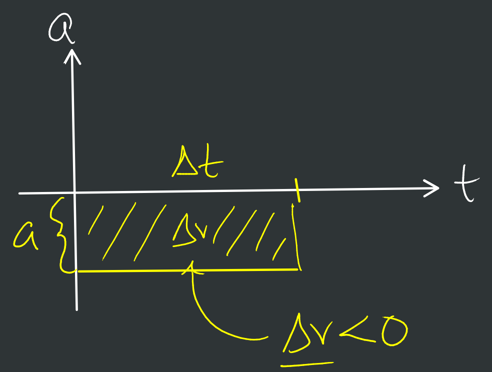


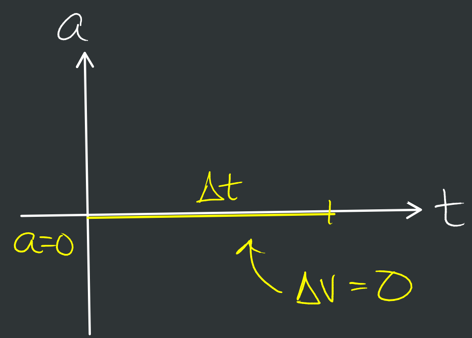
$$a = \frac{\Delta v}{\Delta t}$$

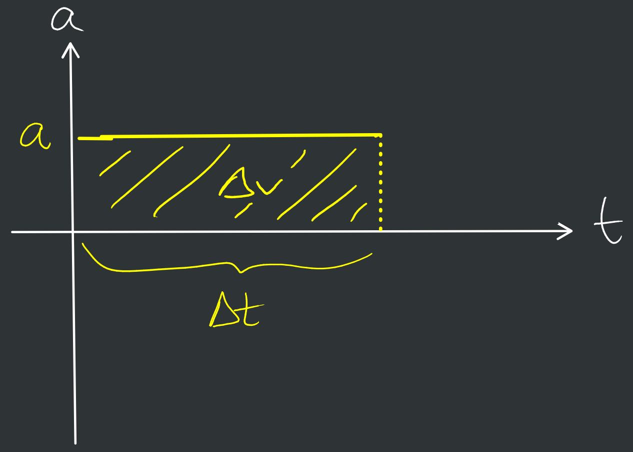
$$a \cdot \Delta t = \Delta v$$

vel vs. time $\xrightarrow{\text{slope}}$ acceleration

change in vel. $\xleftarrow{\text{area}}$ acceleration vs. time







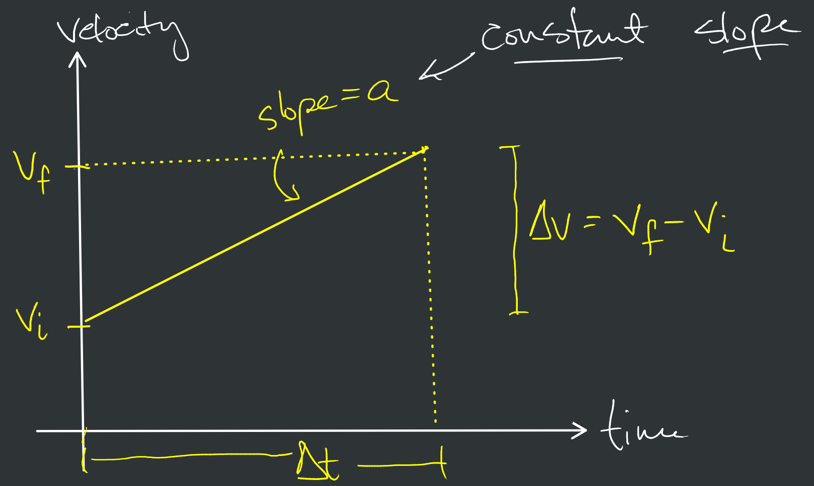
$$\Delta V = a \cdot \Delta t$$

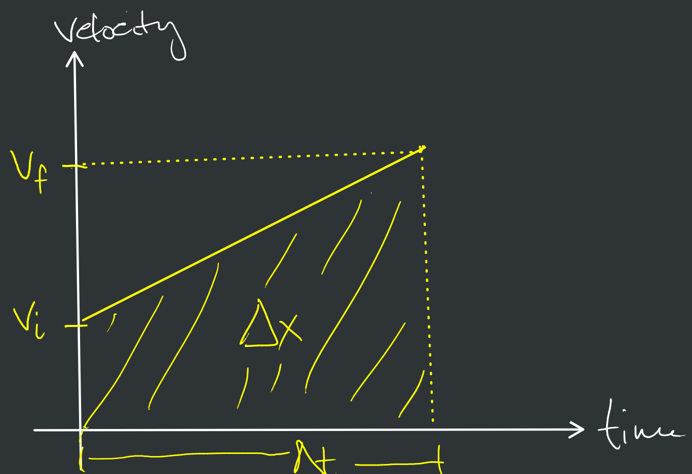
$$V_f - V_i = a \cdot \Delta t$$

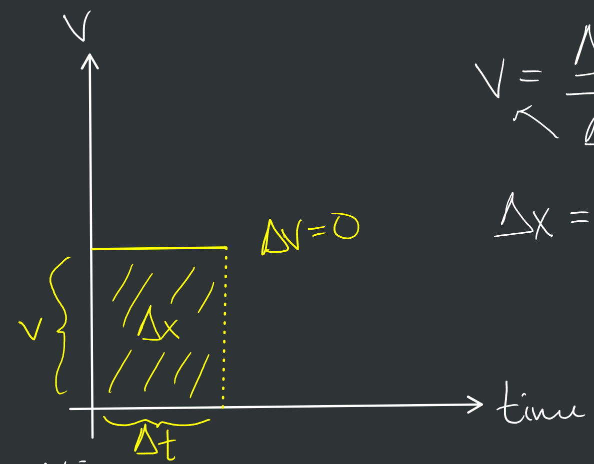
$$\boxed{V_f = V_i + \underbrace{a \cdot \Delta t}_{\Delta v}}$$

$$t_i = 0$$

$$v(t) = V_i + a \cdot t$$ ← linear function



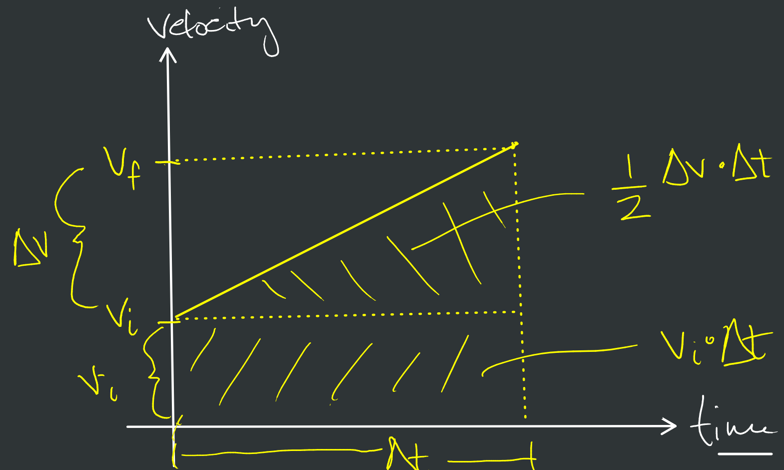




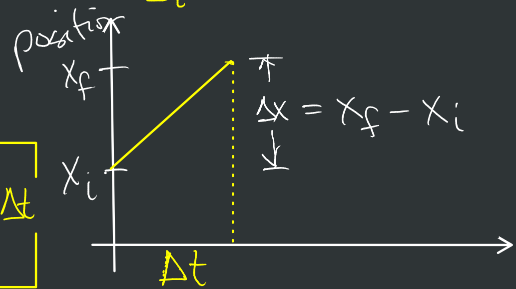
$$V = \frac{\Delta x}{\Delta t}$$

$$\Delta x = V \cdot \Delta t$$

↑ constant velocity



$$\boxed{\Delta x = V_i \cdot \Delta t + \frac{1}{2}\Delta V \cdot \Delta t}$$



$$\Delta x = V_i \cdot \Delta t + \frac{1}{2}\Delta v \cdot \Delta t$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a \cdot \Delta t$$

$$\boxed{\Delta x = V_i \cdot \Delta t + \frac{1}{2} a \Delta t^2}$$

$$\Delta x = X_f - X_i = V_i \cdot \Delta t + \frac{1}{2} a \Delta t^2$$

$$X_f = X_i + V_i \cdot \Delta t + \frac{1}{2} a \Delta t^2$$

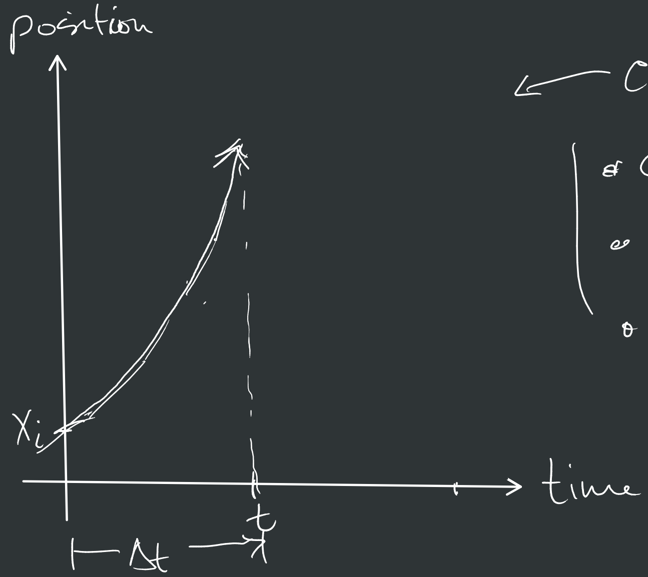
$t_i = 0$

$\Delta t = t_f - t_i$

$\Delta t = t$

$t_i = 0$

$$\boxed{X(t) = X_i + V_i \cdot t + \frac{1}{2} a \cdot t^2}$$



Conditions

- constant acceleration
- linear velocity
- quadratic position

$$y(x) = C + Bx + Ax^2$$

$$y(x) = \underbrace{Ax^2 + Bx + C}_{\text{parabola}}$$

quadratic function

kinematic equations

↳ describing motion

$$\boxed{\begin{array}{l} \bullet\ V_f = V_i + a \cdot \Delta t \\ \bullet\ X_f = X_i + V_i \cdot \Delta t + \frac{1}{2} a \Delta t^2 \\ \bullet\ V_f^2 = V_i^2 + 2a\Delta X \end{array}}$$

$(\Delta x = V_i \cdot \Delta t + \frac{1}{2} a \Delta t^2)$

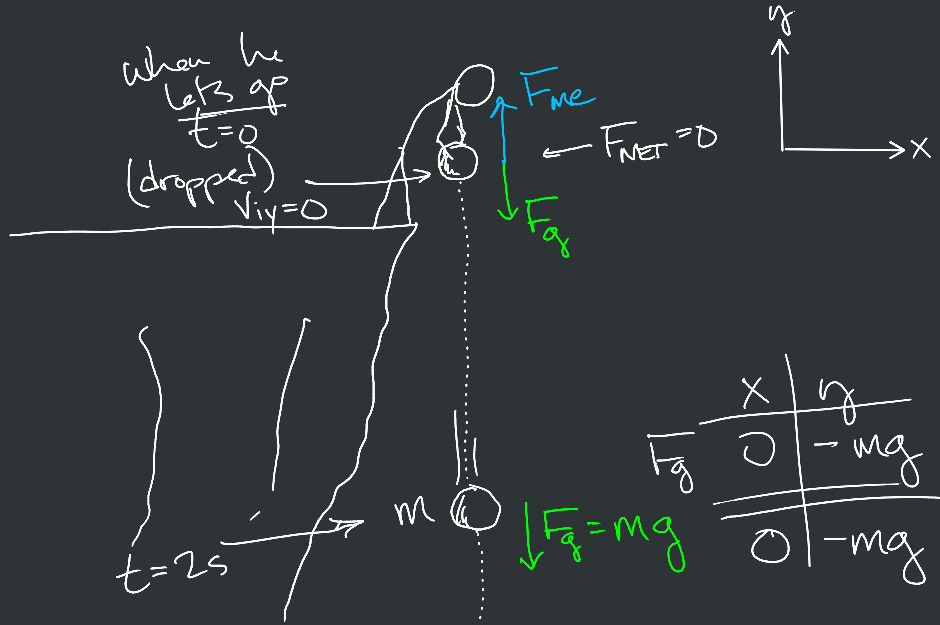
← constant acceleration

After this you can

- discuss the conditions of free fall and why the kinematic equations apply
- differentiate these from the conditions where free fall DOES NOT apply
- draw qualitative plots of displacement, velocity, and acceleration during freefall

constant acceleration

v vs. time



	x	y
F_g	0	$-mg$
	0	$-mg$

$$\Sigma F = F_{NET} = ma$$

$$-\cancel{m}g = \cancel{m}a$$

$$-g = a$$

$$\boxed{a = -9.8\ m/s^2}$$

for free fall the acceleration is known $|a| = 9.8\ m/s^2$ downward

$$g = 9.8\ \frac{N}{kg} = 9.8\ m/s^2$$

Ex. • how far has the ball fallen?
• how fast is it going 2s later?

$$? = \Delta y = v_{iy}t + \frac{1}{2}at^2$$

$$\Delta y = 0\ m/s\,(2s) + \frac{1}{2}(-9.8\ m/s^2)(2s)^2$$

$$\Delta y \approx -5\ m/\cancel{s^2}\,(4\cancel{s^2})$$

$$\Delta y = -20\ m$$

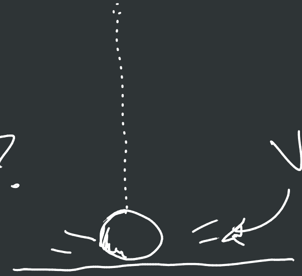
known:
$t = 2s$
$a = -9.8\ m/s^2$
$v_{iy} = 0\ m/s$

$$v_f = v_i + at$$

$$v_f = 0\ m/s + (-9.8\ m/s^2)(2s)$$

$$v_f = -20\ m/s$$

$\Delta y = ?$



$V_f = ?$

$V_f \neq 0$!!

moment before it hits the ground.

what if we know Δy and not t?

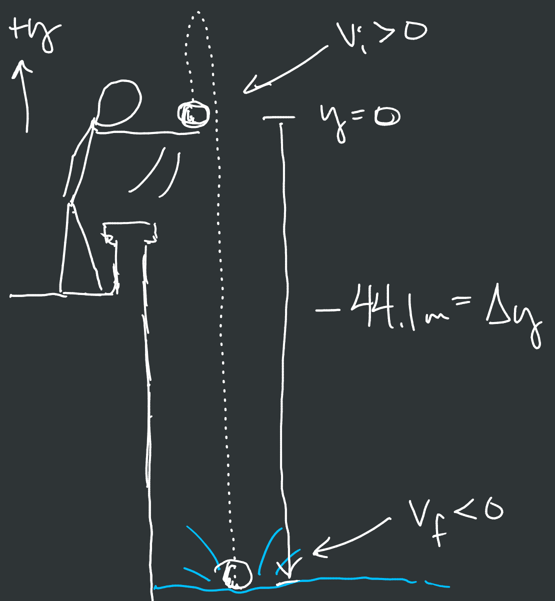
$\Delta y = V_i t + \frac{1}{2} a t^2$

solve for t. (quadratic formula)

or

$V_f^2 = V_i^2 + 2a\Delta y$

Standing on a bridge, you throw a stone straight upward.
The stone hits a stream, 44.1 m below the point at which you release it, 4.00 s later.
- Sketch graphs of y(t) and v(t). The positive y-axis points up.
- What is the velocity of the stone just after it leaves your hand?
- What is the velocity of the stone just before it hits the water?



known:

$t = 4s$

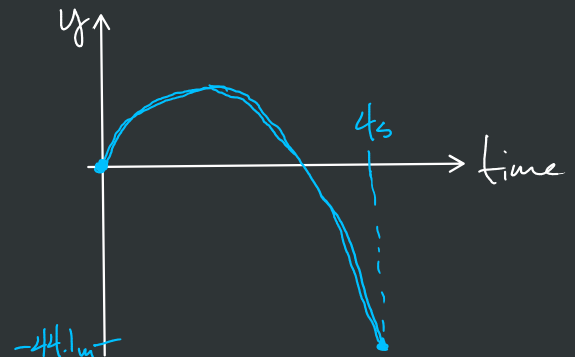
$\Delta y = -44.1\,m$

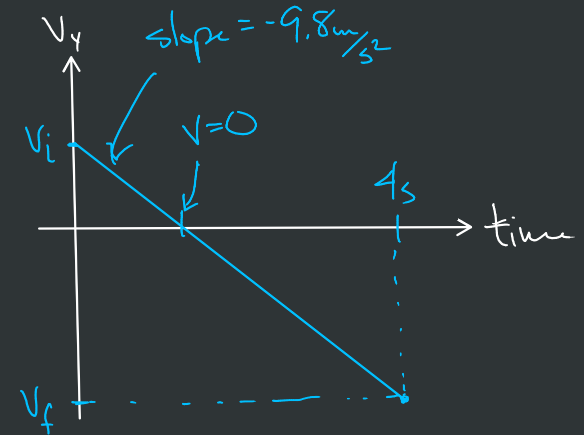
$a = -9.8\,m/s^2$

want:

V_i

V_f





$\Delta y = V_i t + \frac{1}{2} a t^2$

$-44.1\,m = V_i (4s) + \frac{1}{2}(-9.8\,m/s^2)(4s)^2$

$\boxed{V_i = +8.6\,m/s}$

$V_f = V_i + at$

$\boxed{V_f = -30.6\,m/s}$

$V_f^2 = V_i^2 + 2a\Delta y$

$V_f = -30.6\,m/s$

(other way)