At the end of this worksheet you should be able to

- discuss the quantities of displacement, velocity, and acceleration.
- interpret the meaning of the sign of the above quantities.
- discuss the cause of the change of those quantities.
- take a given state of motion of an object and predict its future state of motion.
- identify key features of a graph of these quantities over time.
- create a qualitative plot of these quantities over time given an interesting physical problem.
- solve a range of problems under the conditions of constant acceleration.

$\longrightarrow +x$

1. I move from a place marked $x = 2\,\text{m}$ to a place marked $x = 10\,\text{m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

displacement $\Delta x = x_f - x_i = +10\text{m} - (+2\text{m}) = +8\text{m}$ $\quad V = \frac{\Delta x}{\Delta t} = \frac{+8\text{m}}{4\text{s}} = +2\ \text{m/s}$

"change in"

2. I move from a place marked $x = -2\,\text{m}$ to a place marked $x = 10\,\text{m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

$\Delta x = +10\text{m} - (-2\text{m}) = +12\text{m}$ $\quad V = \frac{12\text{m}}{4\text{s}} = +3\ \text{m/s}$

3. I move from a place marked $x = 2\,\text{m}$ to a place marked $x = -10\,\text{m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

$\Delta x = -10\text{m} - (+2\text{m}) = -12\text{m}$ $\quad V = \frac{-12\text{m}}{4\text{s}} = -3\ \text{m/s}$

left $\quad$ left

4. I move from a place marked $x = -2\,\text{m}$ to a place marked $x = -10\,\text{m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative?

$\Delta x = -10\text{m} - (-2\text{m}) = -8\text{m}$ $\quad V = \frac{-8\text{m}}{4\text{s}} = -2\ \text{m/s}$

5. What is the difference between average velocity and instantaneous velocity? Give an example.

6. If the rate of change of position is called velocity, and the rate of change of velocity is called acceleration, then what is the rate of change of acceleration called? (this is not in your book, you'll have to search for it) Does the rate of change of *that* have a name?

\+ \+

7. An object has a velocity of 10 m/s and then 5 seconds later has a velocity of 15 m/s. What is its average acceleration over this time interval? In what *direction* did the velocity change?

$\Delta v = v_f - v_i$ $\qquad a = \frac{\Delta v}{\Delta t} = \frac{+5\,m/s}{5\,s} = +1\,m/s^2$

$\Delta v = 15\,m/s - 10\,m/s = +5\,m/s$

$\vec{v}_i = +10\,m/s \;+\; \Delta\vec{v} = +5\,m/s \;=\; v_f = +15\,m/s$

8. An object has a velocity of 15 m/s and then 5 seconds later it came to a stop. What is its average acceleration over this time interval? What is the significance of the sign?

$\Delta v = 0\,m/s - (+15\,m/s) = -15\,m/s$ $\qquad a = \frac{-15\,m/s}{5\,s} = -3\,m/s^2$

$v_i = +15\,m/s \;+\; \Delta v = -15\,m/s \;=\; v_f = 0\,m/s$

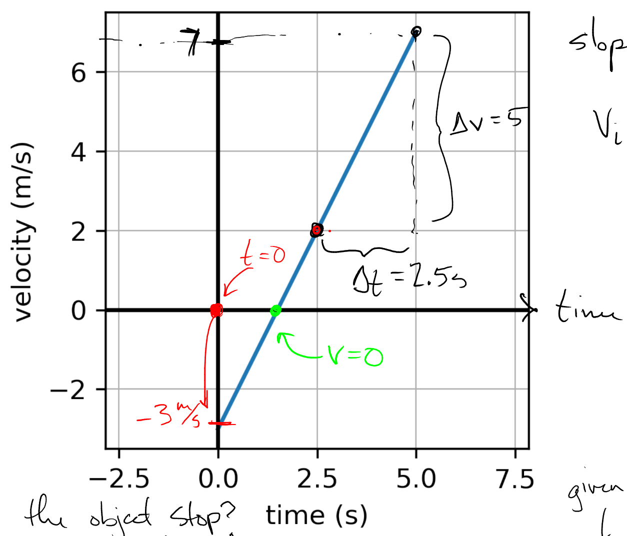
9. An object has a velocity of −10 m/s and then 5 seconds later it has a velocity of −25 m/s. What is its average acceleration over this time interval? What is the significance of the sign?

$\Delta v = -25\,m/s - (-10\,m/s) = -15\,m/s$ $\qquad a = \frac{-15\,m/s}{5\,s} = -3\,m/s^2$

$v_i = -10\,m/s \;+\; \Delta v = -15\,m/s \;=\; v_f = -25\,m/s$

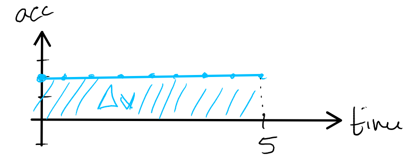
10. The following plot shows constant velocity vs. time. What is this acceleration? What is the initial velocity? Draw the acceleration vs. time graph. Draw the position vs time graph? What do you not know about the position graph?

$a = ?$ $\quad v_i = ?$ $\quad a$ vs. t $\quad x$ vs. t



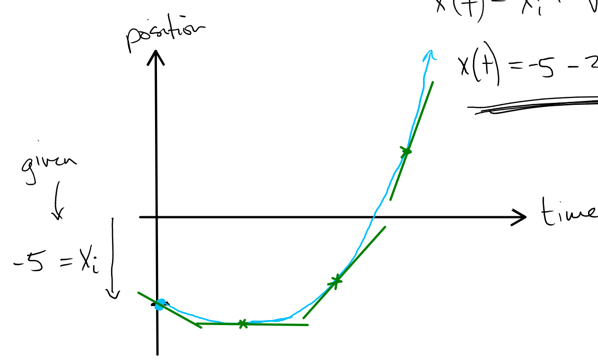
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = a = \frac{5\,\text{m/s}}{2.5\,\text{s}} = 2\,\text{m/s}^2$$

$V_i = -3\,\text{m/s}$



$x(t) = x_i + v_i t + \frac{1}{2} a t^2$

$x(t) = -5 - 3t + t^2$



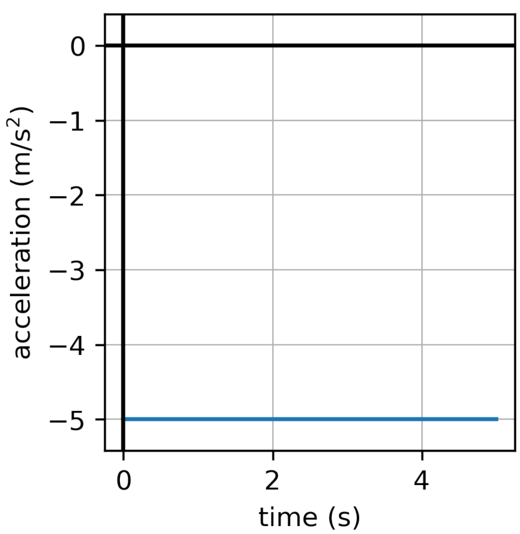
when does the object stop? $(v_f = 0)$

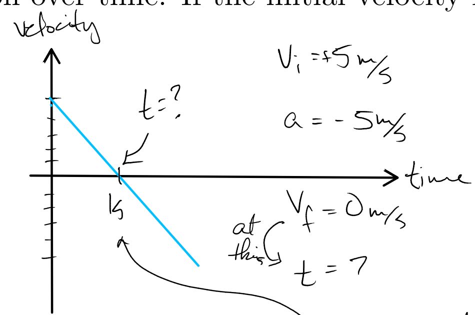
$a = \frac{\Delta v}{\Delta t}$

$2\,\text{m/s}^2 = \frac{v_f - v_i}{\Delta t} \Rightarrow 2\,\text{m/s}^2 = \frac{0 - (-3\,\text{m/s})}{\Delta t}$

$2\,\text{m/s}^2 \cdot \Delta t = 3\,\text{m/s} \Rightarrow \Delta t = \frac{3\,\text{m/s}}{2\,\text{m/s}^2} = 1.5\,\text{s}$

11. The following plot shows constant acceleration over time. If the initial velocity is $v_i = 5\,\text{m/s}$, then sketch a graph of velocity vs. time.





$V_i = 5\,\text{m/s}$

$a = -5\,\text{m/s}^2$

$V_f = 0\,\text{m/s}$

at this $t = ?$

$a = \frac{\Delta v}{\Delta t} = \frac{V_f - V_i}{\Delta t}$

$\boxed{V_f = V_i + a\Delta t}$

$0\,\text{m/s} = 5\,\text{m/s} + (-5\,\text{m/s}^2)\Delta t$

$0\,\text{m/s} = 5\,\text{m/s} - 5\,\text{m/s}^2 \cdot \Delta t$

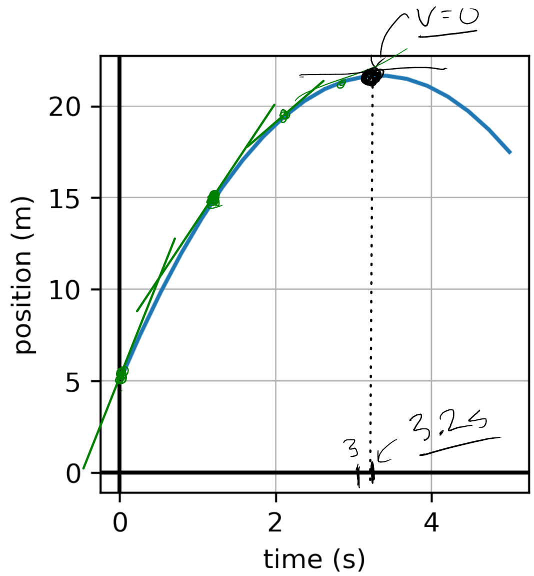
$\Delta t = 1\,\text{s}$

what is my v_f when $t = 5$?

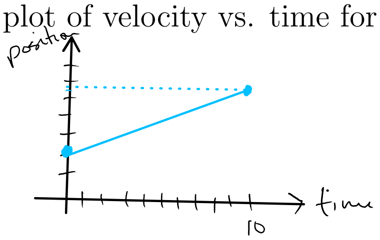
$V_f = 5\,\text{m/s} - \underbrace{5\,\text{m/s}^2 \cdot 5\,\text{s}}_{-25\,\text{m/s}}$

$V_f = -20\,\text{m/s}$

12. The following plot shows the position of an object over time. What is the sign of the acceleration? What is the sign of the initial velocity? Approximately when does the object come to a stop?



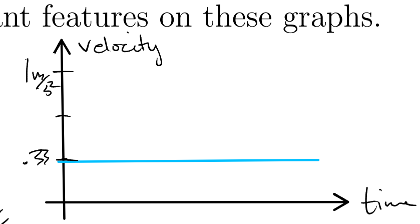
13. A turtle starts walking in a straight line at a steady speed from a position $x = 2.1\,\mathrm{m}$ to a position of $x = 5.4\,\mathrm{m}$ in 10 seconds. Draw a plot of position vs time for this motion. Draw a plot of velocity vs. time for this motion. Label all the important features on these graphs.



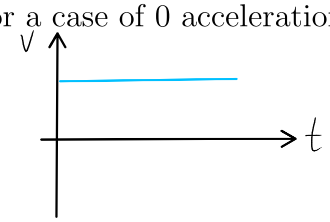
$x(t) = \underbrace{x_i + v_i t}_{\text{linear}} + \cancel{\frac{1}{2}at^2} \to 0$ $(a=0)$

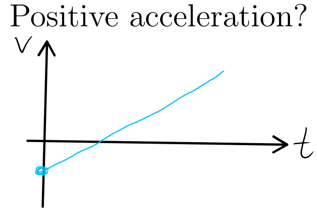
$v = \frac{\Delta x}{\Delta t}$

$v = 0.33\ \mathrm{m/s}$



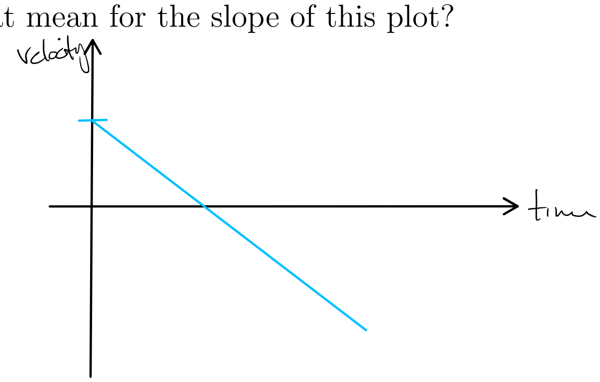
14. Draw a plot of velocity vs time for a case of 0 acceleration. Positive acceleration? Negative acceleration?



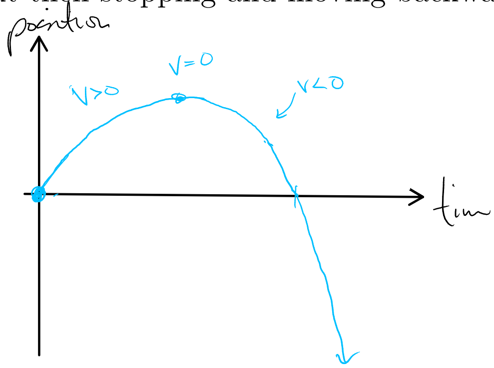


15. Draw a plot of velocity vs time for an object that is initially moving forward, but then stops and starts to move backward. Assume constant acceleration. What is the sign of the acceleration and what does that mean for the slope of this plot?

$a < 0$



16. What does a plot of position vs time look like for the same case as before (initially moving forward, but then stopping and moving backward)? $x_i = 0$



17. Since $g = 9.8\,\mathrm{N/kg}$ also happens to correspond to acceleration due to gravity when no other forces are acting on the object, many people say $g = 9.8\,\mathrm{m/s^2}$. What is g in feet/second^2? ($32\,\mathrm{ft/s^2}$)

18. A 1 kg object slides down a friction-less inclined plane. The plane has an incline angle of 15°. What is the acceleration of the object? How far does the object gone down the plane in 3 seconds?

t

$F_{NET,x} = mg \cdot \sin\theta = ma$

$g\sin\theta = a$

$a = g \cdot \sin\theta$
- down plane
- frictionless
- no other forces

$\Delta x = ?$ $\vert\; v_i = 0\,\mathrm{m/s} \;\vert$ ← given

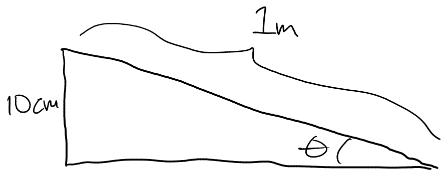
$\boxed{\Delta x = 11.43\,\mathrm{m}}$

- $\boxed{\Delta x = v_i t + \frac{1}{2} a t^2}$
- $v_f = v_i + at$ ← $a = \frac{\Delta v}{\Delta t}$
- $v_f^2 = v_i^2 + 2 a \cdot \Delta x$

begins to slide ← $v_i = 0$

19. A 1 kg object ~~slides~~ down a friction-less inclined plane. The *length of the plane* is 1 m, and the *height above the horizontal* is 10 cm. How long does it take the object to reach the bottom and how fast it is going when it gets there?

$\Delta t = ?$ $\vert\; v_i = 0\,\mathrm{m/s} \;\vert$ ← given



$\sin\theta = \frac{0.1}{1}$

$\theta = \sin^{-1}\left(\frac{0.1}{1}\right)$

$\theta = 5.74°$

$a = g\sin\theta$

$a = g(0.1) = 0.98\,\mathrm{m/s^2}$

$a = g\sin\theta$

$a = 0.98\,\mathrm{m/s^2}$

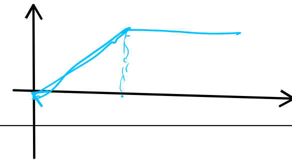
$\Delta x = 1\,\mathrm{m}$

$\Delta x = v_i t + \frac{1}{2} a t^2$?

$0\,\mathrm{m/s}$

$1 = \frac{1}{2}(0.98)t^2 \Rightarrow t = \sqrt{\frac{2(1)}{.98}}$

$\boxed{t = 1.43\,\mathrm{s}}$



20. Examples 4.1, 4.2 and 4.3 from the text book (pgs. 127-130) are excellent and you should use the space below to work those out for yourself.

21. Here is an example of turning a problem inside out. In the lecture video, I worked an example of a person throwing an object up and off the edge of a bridge. The object goes up and then comes down and splashes in the water below, 44.1 m below the place where it was released. In the example, we knew the time and found the initial velocity to be +8.66 m/s. In this problem, use this information to find the time. (You should get 4 seconds)

22. I am standing at the very edge of a cliff and I want to know how far down it is to the ground or the water below me. To do this, I drop a rock off the edge, and count how long it takes to hit the ground. What is a general expression for the height of the cliff based on the amount of time it takes to hit the ground? If it takes 4 seconds to do this, how high is the cliff?

Know:
t = given
$a = -9.8\,m/s^2$
"drop" $\rightarrow v_i = 0\,m/s$

want:
Δy

$\Delta y = \cancel{v_i t}^{\,0} + \frac{1}{2}at^2$

$\Delta y = \frac{1}{2}(-9.80\,m/s^2)t^2$

$t = 4$

$\Delta y = \frac{1}{2}(-9.8)(4\,s)^2$

$\boxed{\Delta y = -78.4\,m}$

$y_f = 0 \quad y_i = h$

$0 = h + \cancel{v_i t}^{\,0} + \frac{1}{2}at^2$

$-h = \frac{1}{2}(-9.8)t^2$

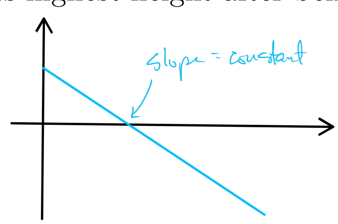
$h = \frac{1}{2}(9.8)t^2$

23. What is the velocity of a ball when it reaches its highest height after being thrown straight up?

$v_f = 0\,m/s$

24. What is the acceleration of a ball when it reaches its highest height after being thrown straight up?

$a = -9.8\,m/s^2$



25. If I throw a ball upward, what is an expression for the amount of time it takes to reach its highest height? If the initial velocity is 15 m/s, then how much time does it take?

Know:
v_i
$a = -9.8\,m/s^2$
$v_f = 0\,m/s$

want:
t

$v_f = v_i + at$

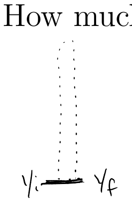
$\frac{v_f - v_i}{a} = t = \frac{-v_i}{a}$

$t = \frac{-v_i}{-9.80\,m/s^2} = \frac{v_i}{9.8\,m/s^2}$

$v_i = 15\,m/s$

$t = \frac{15\,m/s}{9.8\,m/s^2} = 1.5\,s$

26. How much time does it take for the ball thrown straight up to come back down?



$\Delta y = y_f - y_1 = 0$

$\Delta y = v_i t + \frac{1}{2}at^2$

$0 = v_i t + \frac{1}{2}at^2$

$\frac{-v_i \cancel{t}}{\cancel{t}} = \frac{\frac{1}{2}at^{\cancel{2}}}{\cancel{t}}$

$\frac{-v_i}{a} = \frac{\frac{1}{2}at}{a}$

$\boxed{t = \frac{-2v_i}{a}}$

$V_f = 0$
$t = 2s$

27. So if I throw a ball straight up and its total travel time from when it leaves my hand to when it comes back down is 4 seconds, then what was the original speed with which I threw it?

know: t, a, $\Delta y = 0$ want: V_i

$t = \frac{-2V_i}{a}$

$V_i = \frac{-a \cdot t}{2}$

V_i

$V_i = \frac{-at}{2} = \frac{-(-9.8 m/s)(4s)}{2} = +19.6 m/s$

another way

$\frac{4s}{2} = t = 2s$

$V_f = 0$

$V_f = V_i + at$

$V_i = +19.6 m/s$

28. What is the final speed when the ball in the previous problem comes back down? (Assume final position is the same height from which I threw it.)

$\Delta t = 4s$

$V_f = V_i + at$

$V_f = +19.6 + (-9.8 m/s)(4)$

$V_f = -19.6 m/s$

$V_f^2 = V_i^2 + 2a\Delta y$

$V_f^2 = V_i^2$ $\Delta y = 0$

$\pm V_f = V_i$ find V_i then assume V_f

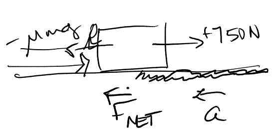
29. The moon has a gravitational field strength $g = 1.6\,\mathrm{N/kg}$, so objects feel lighter on the moon in terms of lifting them vertically, but what about pushing horizontally? If you wanted to accelerate an 10 kg object horizontally (on a frictionless surface) on the moon with an acceleration of $2\,\mathrm{m/s^2}$, what force would you need to provide? Would this force be different on the earth? Why?

30. I am pushing a 200 kg box across a friction-full surface with 750 N force. The coefficient of friction between the box and the floor is $\mu = 0.5$. What is the net force on the box? What is the acceleration? What does the sign mean here? If the box was moving at 3 m/s when I began to encounter the friction patch on the floor, how long does it take to come to a stop?

$|F_g| = 200kg(9.8)N/kg = mg$

$|F_n| = mg$

$|F_f| = \mu|F_n| = \mu mg$



$F_{NET} = 750N - \mu mg = -230N$

$F_{NET} = ma$

$a = \frac{F_{NET}}{m} = \frac{-230N}{200kg} = -1.15 m/s^2$

$V_i = +3 m/s$

$V_f = 0 m/s$

$V_f = V_i + at$

31. I give a 2 kg book a quick push to start it sliding across a table at 10 m/s initially after my push it done, and it slides about 1 m across the table before it comes to a stop, then what is the coefficient of friction between the book and table?

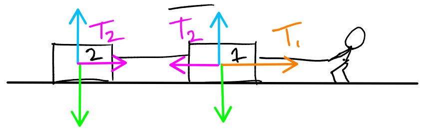
$a \rightarrow F_{NET} \rightarrow F_f \rightarrow \mu$

$V_f^2 = V_i^2 + 2a\Delta x$

V_f: 0 m/s, V_i: 10 m/s, a: ?, Δx: 1 m

The next two problems are here to set up the lab next week and are more in the vein of last week's problems, but still apply and are good to work through.

32. I am pulling two objects connected by a string. They are both 100 kg. There is no friction between the objects and the surface, and they are accelerating at $1\,\text{m/s}^2$. What is my pulling force and what tension force is between the ~~strings~~? boxes



$F_{NET,1,x} = T_1 - T_2$

$F_{NET,1,x} = m \cdot a_x = 100\,\text{kg} \cdot 1\,\text{m/s}^2 = 100\,\text{N} = T_1 - T_2$

$100\,\text{N} = T_1 - 100\,\text{N}$

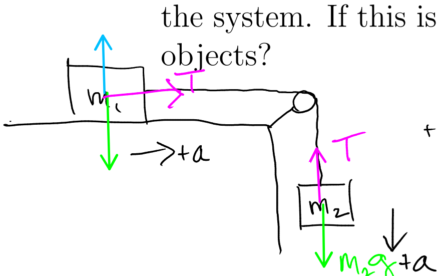
$+100\,\text{N} \quad +100\,\text{N}$

$T_1 = 200\,\text{N}$

$F_{NET,2,x} = T_2$

$F_{NET,2,x} = 100\,\text{kg} \cdot 1\,\text{m/s}^2 = 100\,\text{N} = T_2$

33. One object m_1 connected by a string to another object m_2 that is hanging off the edge of the table. There is a pulley at the edge of the table so the string is not introducing any friction to the system. If this is a friction-less table, then what is an expression for the acceleration of the objects?



m_1

$+T = F_{NET\,1} = m_1 a$

$\boxed{T = m_1 a}$

m_2

$+m_2 g - T = F_{NET,2} = m_2 a$

$\boxed{m_2 g - T = m_2 a}$

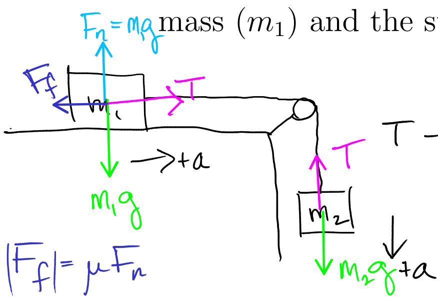
$m_2 g - m_1 a = m_2 a$

$m_2 g = m_1 a + m_2 a$

$m_2 g = a(m_1 + m_2)$

$\boxed{a = \frac{m_2 g}{m_1 + m_2}}$

34. Now do the previous problem again, but with a coefficient of friction μ between the sliding mass (m_1) and the surface.



$|F_f| = \mu F_n$

$|F_f| = \mu m_1 g$

m_1

$T - F_f = F_{NET\,1} = m_1 a$

$T - F_f = m_1 a$

$\boxed{T - \mu m_1 g = m a}$

m_2

$+m_2 g - T = F_{NET,2} = m_2 a$

$\boxed{m_2 g - T = m_2 a}$

$m_2 g - m_2 a = T$

$m_2 g - m_2 a - \mu m_1 g = m_1 a$

$m_2 g - m_2 a - m_1 a = \mu m_1 g$

$$\frac{m_2 g - a(m_1 + m_2)}{m_1 g} = \frac{\mu m_1 g}{m_1 g}$$

$$\boxed{\mu = \frac{m_2 g - a(m_1 + m_2)}{m_1 g}}$$

in lab, unknowns:
- μ
- T

In lab, knowns
- m_1
- m_2
- a

μ → depends on surfaces

→ does not depend on surface area

→ does not depend on normal force

$\xrightarrow{+x}$

$15N \leftarrow \boxed{4.0\,kg} \rightarrow 42N$

$a = ?$

$F_{NET,x} = m \cdot a$

$\Sigma F_{,x} = +42N + (-15N) = 27N = 4kg \cdot a$

$a = \frac{27N}{4kg} = +6.75$ right

$v_f^2 = \cancel{v_i^2}^{\,0} + 2ax \Rightarrow v_f^2 = 2ax$

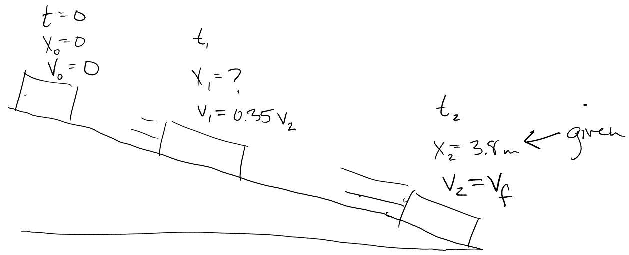
HW#7

$$\boxed{\frac{v_1}{v_2} = \left(\frac{x_1}{x_2}\right)^{1/2}}$$

$x_2 = 3.8\,m$

$v_1 = 0.35\, v_2$

$$\boxed{\frac{v_1}{v_2} = 0.35}$$



#10 $T - mg = F_{NET} = ma$

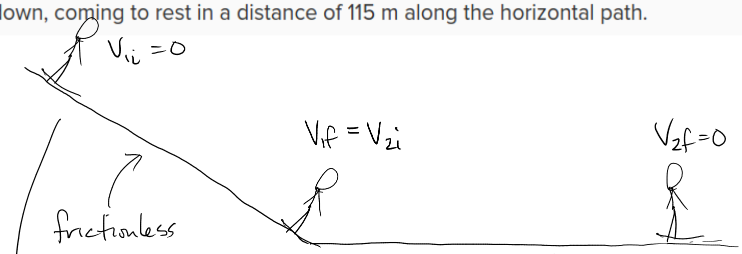
$a = \frac{F_{NET}}{m}$ ← negative

Δt $\quad$ $V_f = ?$

Δx $\quad$ $V_f = V_i + at$

$\Delta x = V_i t + \frac{1}{2}at^2$

A skier with a mass of 63.0 kg starts from rest and skis down an icy (frictionless) slope that has a length of 90.0 m at an angle of 32.0° with respect to the horizontal. At the bottom of the slope, the path levels out and becomes horizontal, the snow becomes less icy, and the skier begins to slow down, coming to rest in a distance of 115 m along the horizontal path.



$F_{NET} \rightarrow a \rightarrow V_{1f}$ $\qquad$ $V_i + V_f \rightarrow a \rightarrow F_{NET}$ frictionfull

$\hookrightarrow F_f \rightarrow \mu$