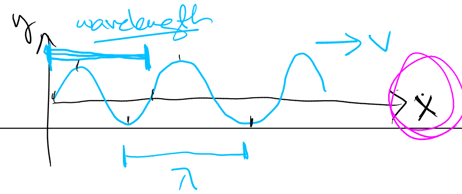
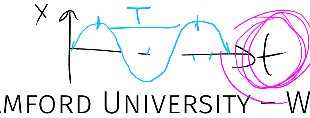




At the end of this worksheet you should be able to

$$\boxed{V = \frac{\Delta x}{t}}$$

- use the properties of waves to solve for an unknown quantity.
- use the mathematical description of a wave to plot waves motion over time.
- use the conditions of a standing wave on a string to solve for an unknown quantity.

$V \propto F^{1/2} L^{1/2} m^{-1/2} \rightsquigarrow \boxed{V = \sqrt{\frac{FL}{m}}}$ $\mu = \frac{m}{L}$ ← linear mass density $\boxed{V = \sqrt{\frac{F}{\mu}}}$

1. The speed of a wave on a string is proportional to the square root of the tension F in the string and the length L, and inversely proportional to the square root the mass m of the string.

$\boxed{V \propto F^{1/2} \mu^{-1/2}}$

- By what factor does the velocity of the wave change if the tension doubles?

$V \propto F^{1/2}$ $F_2 = 2 \cdot F_1$

$\frac{V_2}{V_1} = \left(\frac{F_2}{F_1}\right)^{1/2}$ $\frac{V_2}{V_1} = \sqrt{2} = 1.4$ $\frac{F_2}{F_1} = 2$

$V_2 = 1.4 \cdot V_1$

- What about if the length halves?

half length, but mass stays the same

$V \propto L^{1/2}$

$\frac{V_2}{V_1} = \left(\frac{L_2}{L_1}\right)^{1/2} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7$

cut string, half mass

$V \propto L^{1/2} m^{-1/2}$

$\frac{V_2}{V_1} = \left(\frac{1}{2}\right)^{1/2} \left(\frac{1}{2}\right)^{-1/2} = \left(\frac{1}{2}\right)^{1/2} \left(\frac{2}{1}\right)^{1/2} = 1$

- What if the mass triples?

$V \propto m^{-1/2}$

$\frac{V_2}{V_1} = \left(\frac{m_2}{m_1}\right)^{-1/2} = 3^{-1/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.58$ $\frac{V_2}{V_1} = 0.58 \Rightarrow V_2 = 0.58\, V_1$

- What if the mass density doubles?

$V \propto (\mu)^{-1/2}$ $\frac{\mu_2}{\mu_1} = 2 \Rightarrow \frac{\mu_1}{\mu_2} = \frac{1}{2}$

$\frac{V_2}{V_1} = \left(\frac{\mu_2}{\mu_1}\right)^{-1/2} = \left(\frac{\mu_1}{\mu_2}\right)^{1/2} = \left(\frac{1}{2}\right)^{1/2} = 0.7$

- If the force doubles and the length triples, by what factor does the velocity change?

$\frac{V_2}{V_1} = \left(\frac{F_2}{F_1}\right)^{1/2} \left(\frac{L_2}{L_1}\right)^{1/2} = 2^{1/2} \cdot 3^{1/2} = \sqrt{6} = 2.45$

- What if the mass density triples and the force doubles?

$\frac{V_2}{V_1} = (2)^{1/2} (3)^{-1/2} = 0.816$

- By what factor does the force need to change to double the velocity?

$$\frac{V_2}{V_1} = \left(\frac{F_2}{F_1}\right)^{1/2} \Rightarrow \left(\frac{V_2}{V_1}\right)^2 = \frac{F_2}{F_1} \qquad \boxed{\frac{V_2}{V_1} = 2} \qquad \frac{F_2}{F_1} = 4$$

$$(2)^2 = \frac{F_1}{F_2} = 4$$

- By what factor does the length need to change to double the velocity?

$$\frac{V_2}{V_1} = \left(\frac{L_2}{L_1}\right)^{1/2} \Rightarrow \left(\frac{V_2}{V_1}\right)^2 = \frac{L_2}{L_1}$$

$$2^2 = \frac{L_2}{L_1} = 4$$

- By what factor does the length need to change to quarter the velocity?

$$\frac{V_2}{V_1} = \frac{1}{4} \qquad \frac{L_2}{L_1} = \left(\frac{V_2}{V_1}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} \qquad L_2 = \frac{1}{16}L_1$$

- If the velocity doubles and the length halves, by what factor does the force need to change?

$$\frac{V_2}{V_1} = 2$$

$$V \propto F^{1/2} L^{1/2}$$

$$\frac{V_2}{V_1} = \left(\frac{F_2}{F_1}\right)^{1/2}\left(\frac{L_2}{L_1}\right)^{1/2}$$

$$\left(\frac{L_1}{L_2}\right)^{1/2}\left(\frac{V_2}{V_1}\right) = \left(\frac{F_2}{F_1}\right)^{1/2}$$

$$\frac{F_2}{F_1} = \left(\frac{V_2}{V_1}\right)^2\left(\frac{L_1}{L_2}\right) \qquad \frac{L_2}{L_1} = \frac{1}{2} \Rightarrow \frac{L_1}{L_2} = 2$$

$$\frac{F_2}{F_1} = (2^2)(2)$$

$$\frac{F_2}{F_1} = 8$$

2. A string is 2 m long and has a mass of 10 g. What is its mass density? If you cut the string in half, what is its mass density then? If you exerted a force of 10 N, then what is the velocity of waves on the string? What would the force need to be to make the velocity 100 m/s?

$\to$ kg

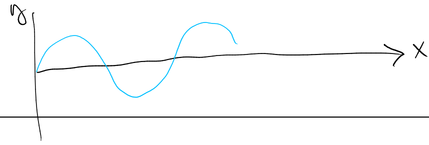
$$\mu = \frac{m}{L} = \frac{0.010\,kg}{2\,m} = 0.005\,kg/m$$ $\to$ cut in half, μ stays the same

$$V = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{10\,N}{0.005\,kg/m}} = 44.7\,m/s$$

$$V = \sqrt{\frac{F}{\mu}}$$

$$V^2 = \frac{F}{\mu}$$

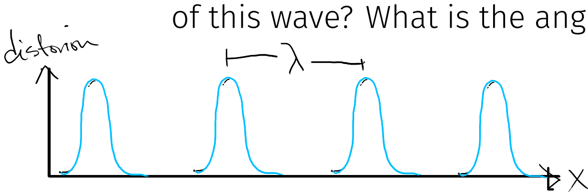
$$F = \mu V^2 = 0.005\,kg/m\,(100\,m/s)^2 = 50\,N$$



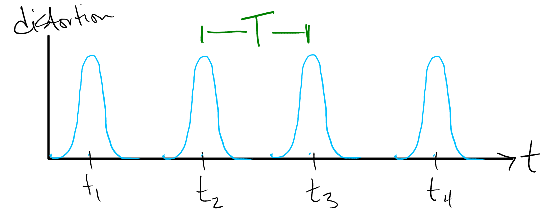
$\sin\theta \rightarrow y = A\sin(\omega t)$ $\quad \omega = \frac{2\pi}{T}$

$y(x,t) = A\sin(kx - \omega t)$

3. A string with a mass density of 10 g/m has a tension of 100 N. A periodic waveform that has a period of 0.1 s is put into this string. What is the frequency? What is the wavelength of this wave? What is the angular frequency and what is the wavenumber?



$V = \frac{\lambda}{T} = \lambda \cdot f$



$V = \sqrt{\frac{100\,N}{.010\,kg/m}} = 100\,m/s$

$\omega = \frac{2\pi}{T} = 2\pi f = 62.8\,rad/s$

$f = \frac{1}{T}$

$f = \frac{1}{0.1s} = 10\,Hz$

$100\,m/s = \lambda \cdot 10\,Hz$

$\boxed{\lambda = 10\,m}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{10\,m} = 0.628\,rad/m$

4. The wavenumber of a wave is 20 rad/meter, and the period is 0.1 seconds. $\rightarrow k$ $\rightarrow T$

   - What is the frequency, angular frequency, wavelength, and velocity?

     $f = 10\,Hz$

     $\omega = 62.8\,rad/s$

     $\lambda = 0.31\,m$

     $V = 3.1\,m/s$

   - If the force on the string that is carrying this wave is 100 N, then what is the linear mass density?

     $\mu = 10.14\,kg/m$ $\qquad \mu = \frac{F}{V^2}$

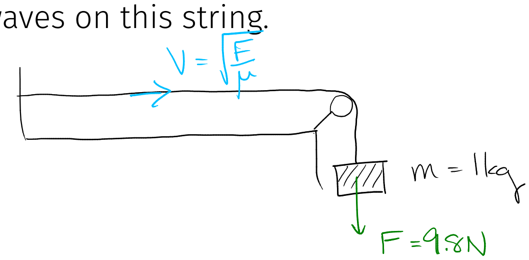
   - If the string is 10 grams then what is the length of the string?

     $\mu = \frac{m}{L}$ $\qquad L = \frac{.010\,kg}{10.14\,kg/m} = 9.86 \cdot 10^{-4}\,m$

     $L = \frac{m}{\mu}$

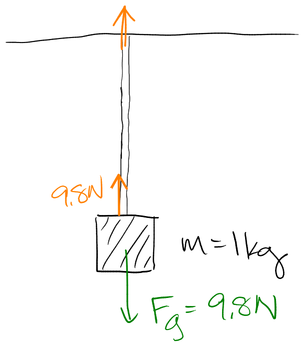
5. A 1 meter long string with a linear mass density of 10 g/m is oriented horizontally and a pulley at one end allows a 1 kg to hang down and put tension in the string. What is the speed of waves on this string.

$\mu = 0.010\,kg/m$



$V = \sqrt{\frac{9.8N}{0.01\,kg/m}} = 31.3\,m/s$

6. The same string from above is hanging vertically from a support with the 1 kg mass tied on the end. What is the speed of a wave *near the mass*? What is the speed of a wave *near the top*?

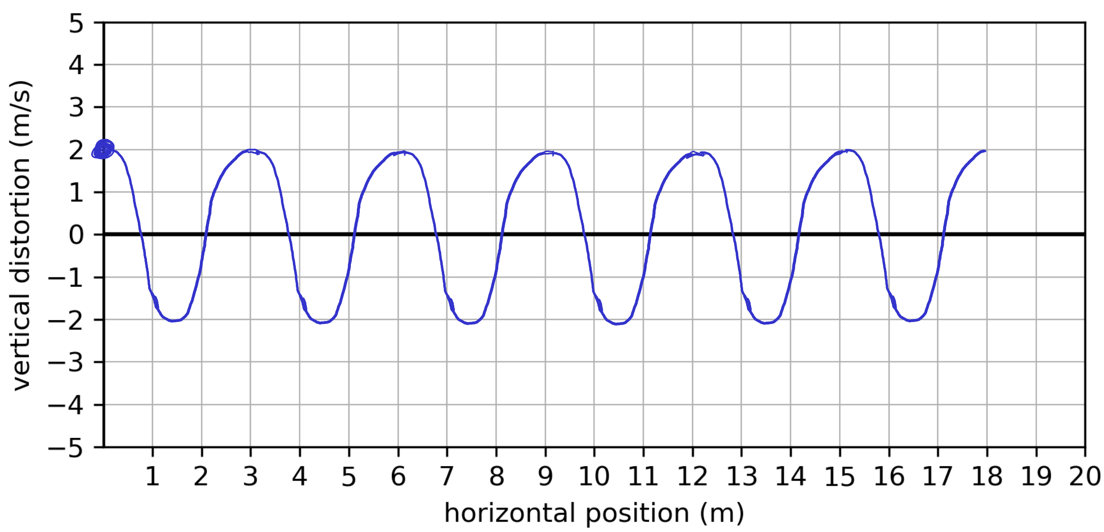


$v_1 = \sqrt{\frac{(1.010\,kg)9.8N/m}{0.01\,kg/m}} = 31.46\ m/s$

$v_2 = 31.3\ m/s$

travelling harmonic

7. A wave has a wavelength of 3 m and an amplitude of 2 m. It travels with a speed of 5 m/s. If the wave has its maximum at the horizontal position of 0 m when $t = 0$ s, then sketch a plot of the wave at this time below:



$v = \frac{\lambda}{T}$ $T = \frac{\lambda}{v}$

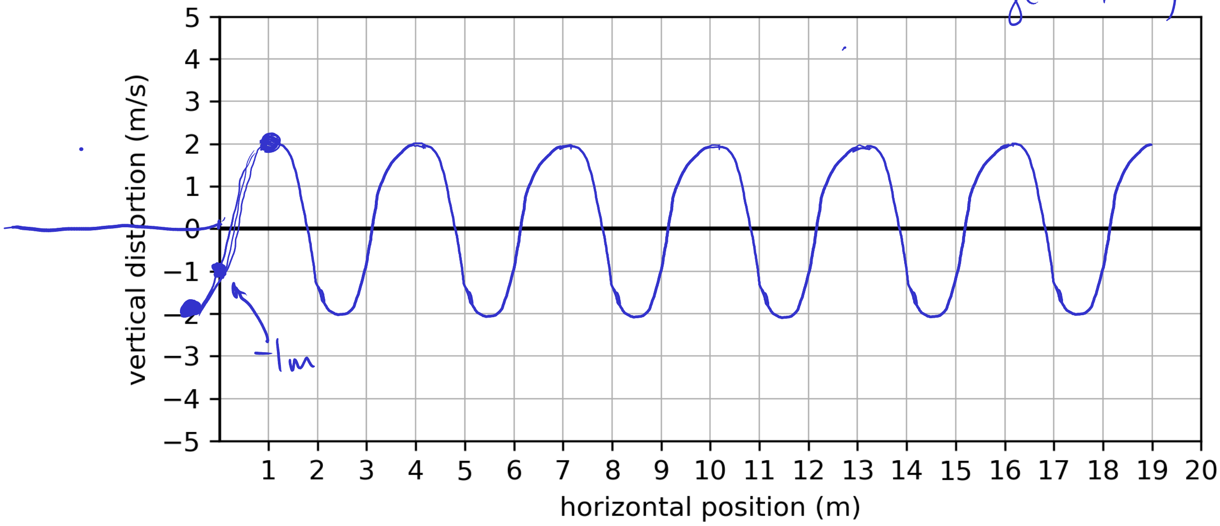
$$y(x,t) = y_{max}\cos(kx - \omega t)$$

$$y(x,t) = y_{max}\cos\left(\frac{2\pi}{\lambda}\cdot x - \frac{2\pi}{T}t\right) \rightarrow y(x,t) = y_{max}\cos\left(\frac{2\pi}{3}x - \frac{2\pi}{.6}t\right)$$

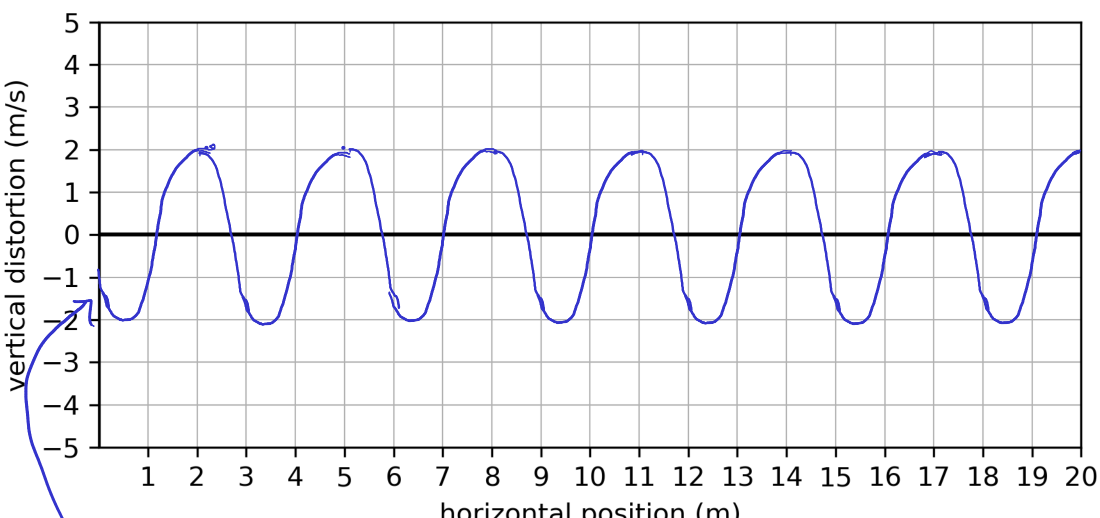
8. Now sketch the wave after 0.2 seconds have gone by.

$$y(0\,m, 0.2s) = 2m\cos\left(-\frac{2\pi}{0.6}\cdot 0.2\right) = -1m$$

360°



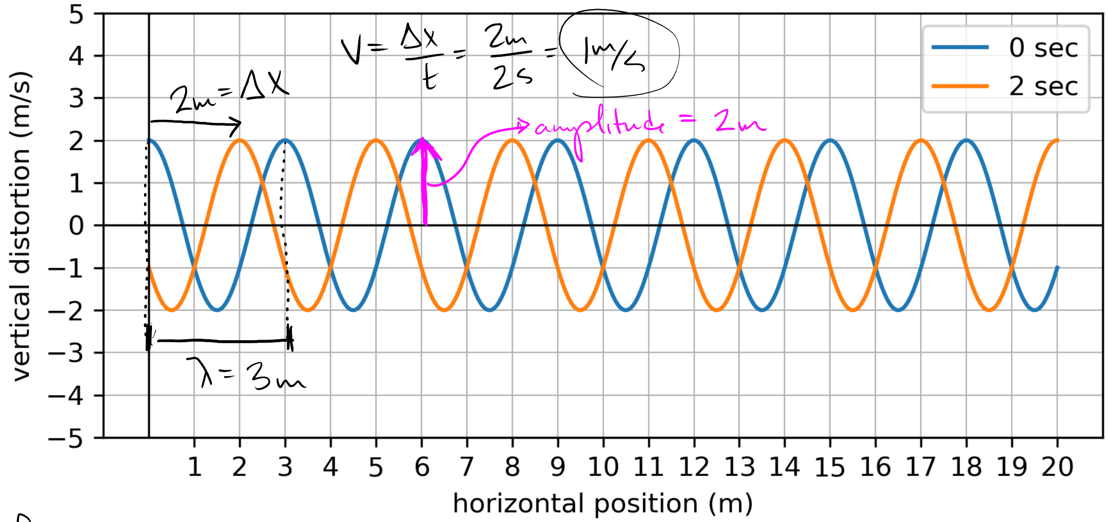
9. Now sketch the wave 1 second after the begining.



$$y(0m, 1s) = (2m)\cos\left(-\frac{2\pi}{0.6}(1s)\right)$$

$$= -1m$$

10. The following plot shows a wave at $t = 0\,\text{s}$ and then later when $t = 2\,\text{s}$. What is the wavelength, amplitude, period, angular frequency, wavenumber, and velocity?



$v = \lambda f$

$f = \frac{v}{\lambda} = \frac{1\,m/s}{3\,m} = 0.33\,Hz$

$T = \frac{1}{f} = 3\,s$

$\omega = 2\pi f$

$\omega = 2.09\,rad/s$

$k = \frac{2\pi}{\lambda}$

$k = 2.09\,rad/m$

$6\pi = \frac{2\pi}{\lambda}$

$6\pi\lambda = 2\pi$

$\lambda = \frac{2\cancel{\pi}}{6\cancel{\pi}}$

$\lambda = 1/3\,m = .33\,m$

11. A wave on a string is described by the equation below. Plot this function for $t = 0\,\text{s}$ and again for $t = 2\,\text{s}$.

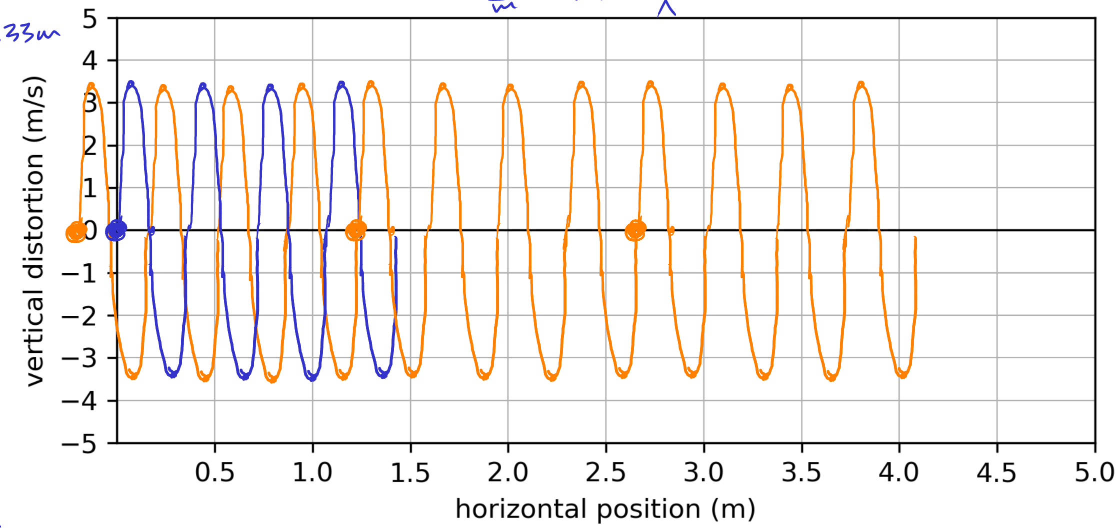
$$y(x,t) = 3.5\,\text{m}\sin\left((6\pi\,\text{rad/m})x - (8\pi\,\text{rad/s})t\right)$$

$6\pi\,\frac{rad}{m} = k = \frac{2\pi}{\lambda}$ $\qquad \hookrightarrow \omega$

$\omega = 8\pi\,rad/s$

$8\pi = \frac{2\pi}{T} = \omega$

$T = \frac{2\pi}{8\pi} = 1/4 = 0.25$ s



$V = \frac{\lambda}{T} = \lambda \cdot f$

$V = \frac{\omega}{k}$

$V = \frac{0.33\,m}{0.25\,s}$

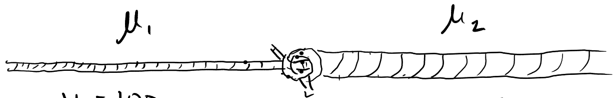
$V = 1.32\,m/s$

$V = \frac{8\pi}{6\pi} = 1.32\,m/s$

$\Delta x = V \cdot t$

$= 1.32\,m/s \cdot (2s) = 2.67\,m$

12. A string has a tension of 100 N. But this string has two sections that are knotted together. One side has a linear mass density of 10 g/m and the other side has a linear mass density of 35 g/m. What is the velocity of the wave on the first side of the knot? What is the velocity on the second side? If a wave with a wavelength of 0.2 meters is traveling on the first side, what is its frequency? When the crest of this wave from the first side reaches the knot, this crest simply passes through the knot. But the velocity has changed. So how much time goes by to when the next crest passes the knot? How far has the first crest traveled in the second side of the string? What does this mean about the wavelength on the second side? What is the frequency of wave crests coming into the knot? What is the frequency of crests leaving the knot? On the basis of this analysis, when the speed of a wave changes suddenly at an interface does the wavelength change or does the frequency or do both change?

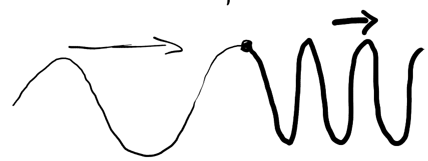


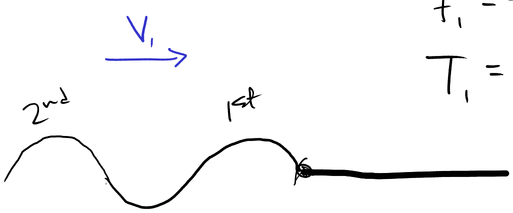
$v_1 = 100\ m/s$ $\quad v_2 = 53.5\ m/s$

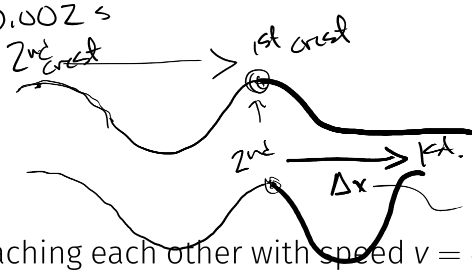
$f_1 = 500\ Hz$

$T_1 = 0.002\ s$

$V = \lambda \cdot f$







$\frac{V_1}{\lambda_1} = f = \frac{V_2}{\lambda_2}$

$\Delta x = V_2 \cdot T$

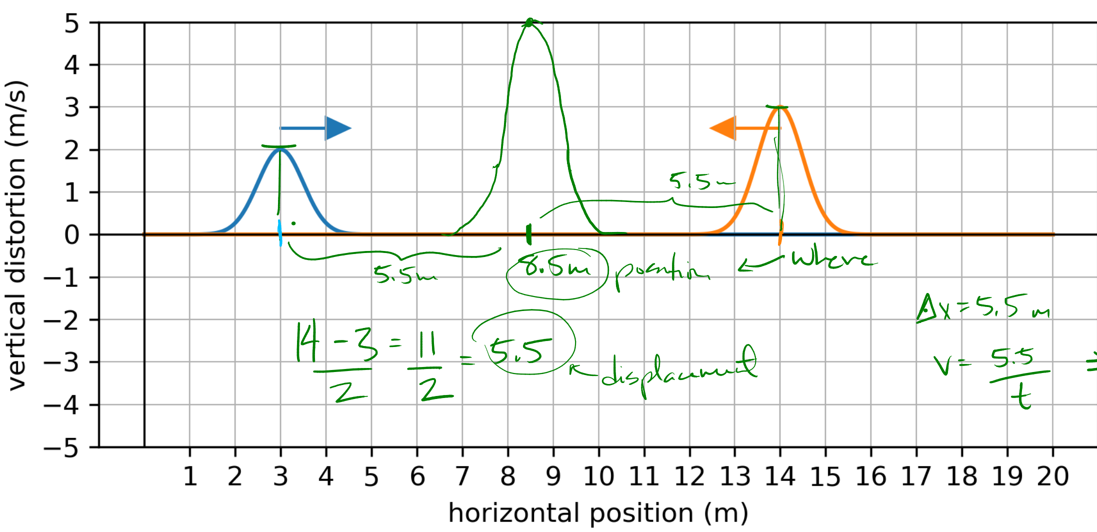
$\Delta x = 53.5\ m/s\ (0.002\ s)$

$\Delta x = 0.107\ m \leftarrow \lambda$

$\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2}$

$\frac{V_2}{V_1} = \frac{\lambda_2}{\lambda_1}$

13. Two waves are approaching each other with speed $v = 4$ m/s. At what time and where will these waves fully overlap (their max peaks line up) and what will the wave look like at that time?

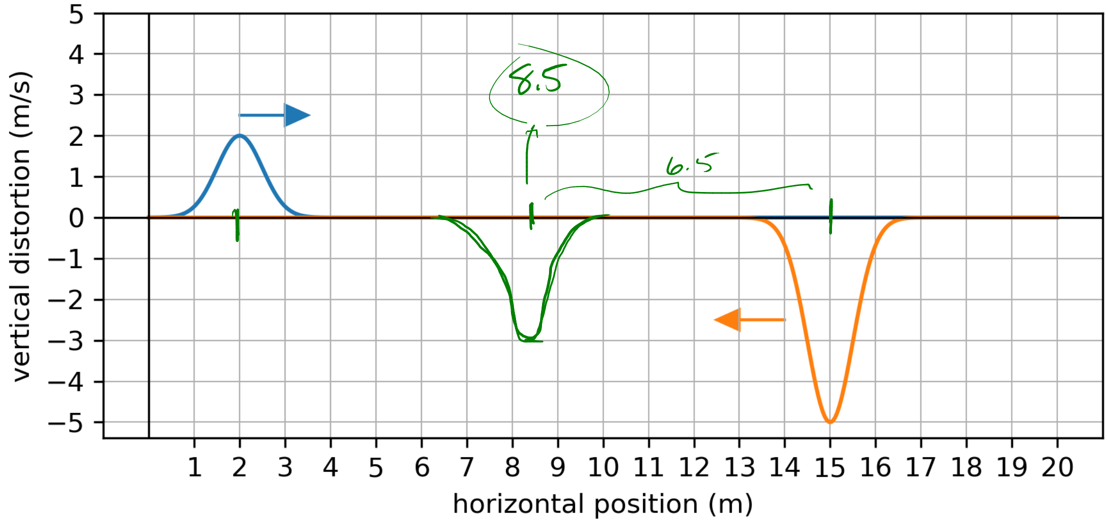


when

$\Rightarrow t = \frac{5.5}{4\ m/s} = 1.375\ s$

14. Again, two waves approach each other with speed $v = 6\,\mathrm{m/s}$. At what time and where will these waves fully overlap and what will the wave look like at that time?

$\frac{13}{2} = 6.5$



when

$\frac{6.5\,\mathrm{m}}{6\,\mathrm{m/s}} = 1.08\,\mathrm{s}$

3 or 5

$n = 4$

15. The plot below represents a standing wave in a string. How many antinodes are there? How many nodes are there? What is the wavelength of the string. If the speed of the wave on the string is $v = 40\,\mathrm{m/s}$, then what input frequency produces this standing wave? What period of oscillation of the input frequency?

$\lambda = 1\,\mathrm{m}$

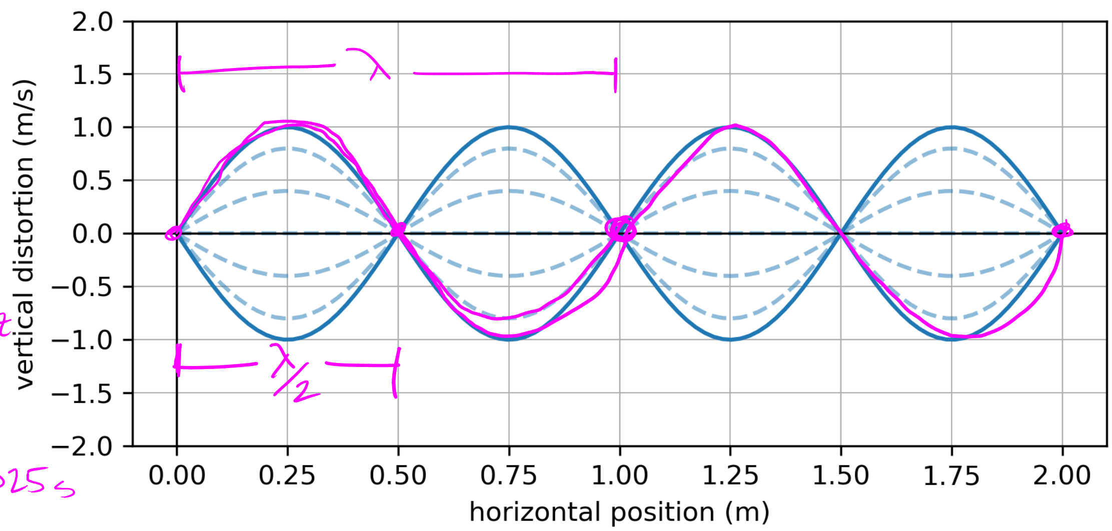
$v = \lambda f$

$\frac{v}{\lambda} = f = \frac{40\,\mathrm{m/s}}{1\,\mathrm{m}}$

$f = 40\,\mathrm{Hz}$

$T = \frac{1}{f} = \frac{1}{40}\,\mathrm{s}$

$T = 0.025\,\mathrm{s}$

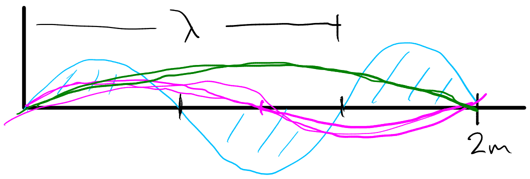


$\left.\begin{array}{l} \lambda = 1\,\mathrm{m} \\ v = 40\,\mathrm{m/s} \end{array}\right\}$ 4 antinodes

$v = \lambda f$

$\rightarrow f = \frac{v}{\lambda} = 40\,\mathrm{Hz}$ $T = \frac{1}{40\,\mathrm{Hz}} = 0.025\,\mathrm{s}$

16. If the string from the above problem has three antinodes, then what is the wavelength? If the velocity of the wave is still $v = 40\,m/s$, then what is the frequency? What is the wavelength and frequency when there are two antinodes? What about when there is one antinode? What is the fundamental frequency? What is the 10th harmonic frequency? What is the 10th harmonic wavelength?



$\lambda_n = \frac{2L}{n}$ $\frac{v}{\lambda_n} = f_n$ $f_n = nf_1$

$\lambda_3 = \frac{2(2m)}{3} = 1.33\,m$ | $\frac{v}{\lambda_3} = f_3 = 30\,Hz$ $40\,Hz = 4f_1$

$f_1 = 10\,Hz$

$\lambda_2 = 2m$ | $f_2 = 20\,Hz$ $f_3 = 3 \cdot 10\,Hz = 30\,Hz$

$\lambda_1 = 4m$ | $f_1 = 10\,Hz$

fundamental

$\lambda_n = \frac{2L}{n}$ ← number of antinodes $f = \frac{v}{\lambda}$ $f_{10} = f_1 \cdot 10 = 100\,Hz$

$\lambda_3 = \frac{2(2m)}{3} = 1.33m \rightarrow f_3 = \frac{40m/s}{1.33m} = 30\,Hz = f_1 \cdot 3$

$\lambda_2 = \frac{2(2m)}{2} = 2m \rightarrow f_2 = \frac{40m/s}{2m} = 20\,Hz = f_1 \cdot 2$

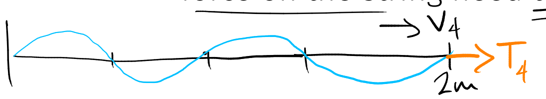
$\lambda_1 = \frac{2(2m)}{1} = 4m \rightarrow f_1 = \frac{40m/s}{4m} = 10\,Hz$

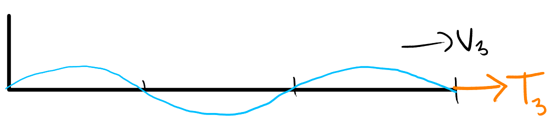
1st harmonic ⟷ fundamental

$\lambda_{10} = \frac{2(2m)}{10} = 0.4\,m$ | $f_{10} = 10 \cdot 10\,Hz = 100\,Hz$

17. Instead of changing the frequency to make a fewer anti-nodes, we could change the velocity instead, and the easiest way to do that is to adjust the force in the string. So if the standing wave goes from 4 antinodes to 3 antinodes, by what ratio does the *speed* of the wave change? If the speed of the wave changes by this ratio, then by what ratio does the force on the string need to change?

$\frac{X_{final}}{X_{initial}}$





f is constant $\rightarrow v \propto F^{1/2}$

$v = \lambda \cdot f \rightarrow \frac{v_3}{v_4} = \frac{\lambda_3}{\lambda_4} = 1.33$ $\frac{v_3}{v_4} = \left(\frac{F_3}{F_4}\right)^{1/2}$

$\lambda_4 = 1m$ $1.33 = \left(\frac{F_3}{F_4}\right)^{1/2}$

$\lambda_3 = 1.33m$ $\frac{v_3}{v_4} = 1.33$

$\lambda_n = \frac{2L}{n}$ $v_3 = 1.33 v_4$ $\frac{F_3}{F_4} = 1.33^2 = 1.77$

$\lambda_4 = \frac{2L}{4}$ $\lambda_3 = \frac{2L}{3}$ $\frac{\lambda_3}{\lambda_4} = \frac{\cancel{2L}/3}{\cancel{2L}/4} = \frac{4}{3} = 1.33$ larger

18. Suppose a wave has a 4th harmonic frequency of 200 Hz then what is the fundamental frequency?

4 antinodes

$f_4 = 200\,Hz$

$V = \lambda f$

$f_n = f_1 \cdot n$ fundamental

$200\,Hz = f_1 \cdot 4$

$\boxed{f_1 = 50\,Hz}$

$n=8$ $n=7$

19. If a standing wave is produced a frequency of 98 Hz and a the next standing wave frequency is 112 Hz. What is the fundamental frequency and how many antinodes were there for these two standing waves?

$$f_{n+1} - f_n = (n+1)f_1 - nf_1$$
$$= \cancel{nf_1} + f_1 - \cancel{nf_1}$$
$$f_{n+1} - f_n = f_1$$

$$112\,Hz - 98\,Hz = 14\,Hz = f_1$$

20. If you increase the tension in a string by a factor of 1.3, then by what factor do you change the fundamental frequency of a wave in that string? If you increase the tension in string by 10%, then by what percent do you change the fundamental frequency?

$\frac{F_2}{F_1}$ $\frac{f_{final}}{f_{initial}}$

what is not changing?
- L
- μ
- n
- $\lambda = \frac{2L}{n}$

What is changing
- f
- F
- $v = \sqrt{\frac{F}{\mu}}$

$\boxed{v \propto F^{1/2}}$ $\boxed{v \propto f}$

$\boxed{f \propto F^{1/2}}$ $F^{1/2} \propto f$

$$\frac{f_2}{f_1} = \left(\frac{F_2}{F_1}\right)^{1/2}$$

$$\frac{F_2}{F_1} = 1.3$$

$$\frac{f_2}{f_1} = (1.3)^{1/2} = 1.14$$

$$f_2 = 1.14 f_1$$

$$\%\Delta = \frac{F_2 - F_1}{F_1} \times 100$$

$$\%\Delta = \left(\frac{F_2}{F_1} - 1\right) \times 100$$

$$10\% = \left(\frac{F_2}{F_1} - 1\right) \times 100$$

$$0.1 = \frac{F_2}{F_1} - 1$$

$$\boxed{1.1 = \frac{F_2}{F_1}}$$

$$\frac{F_2}{F_1} = 1.1$$

$$\frac{f_2}{f_1} = \left(\frac{F_2}{F_1}\right)^{1/2}$$

$$\frac{f_2}{f_1} = (1.1)^{1/2}$$

$$\frac{f_2}{f_1} = 1.0488$$

$$\%\Delta = \left(\frac{f_2}{f_1} - 1\right) \times 100$$

$$\boxed{\%\Delta = 4.88\%}$$

$$V = \sqrt{\frac{FL}{m}} \qquad \mu = \frac{m}{L}$$

$$\mu \cdot L = m$$

$$V = \sqrt{\frac{F\cancel{L}}{\mu \cancel{L}}}$$

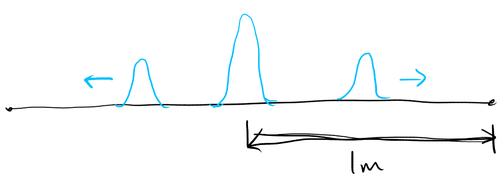
$$V = \sqrt{\frac{F}{\mu}}$$

$$V = \sqrt{\frac{F}{\mu}}$$

$$V^2 = \frac{F}{\mu}$$

$$\mu = \frac{F}{V^2} = 0.002765 \ kg/m$$

A string 2.00 m long is held fixed at both ends. If a sharp blow is applied to the string at its center, it takes 0.0300 s for the pulses to travel to the ends of the string and return to the middle.



$$\frac{2m}{0.03s} = V = \_\_\_\_$$

$$f_n = \frac{nv}{2L} \qquad \lambda = \frac{2L}{n}$$

$n=1$ $\qquad \lambda =$

$$f_n = n \cdot f_1$$