1. We have gone through several kinds of equations now and lets sum up some of these *as proportions*:

   $F_{NET} = m \cdot a$

   - acceleration is <u>directly prop</u> net force.

     $a \propto F_{NET}$

     $F_{NET} \propto a$

     $a \propto F_{NET}$

   - assuming constant acceleration and beginning at rest, an object's velocity is <u>prop to sqr rt of</u> the displacement.

     $v_f$

     $v_i = 0$

     $v_f = v_i + at \quad | \quad \Delta x = v_i t + \frac{1}{2}at^2 \quad | \quad v_f^2 = v_i^2 + 2a\Delta x$

     $v_f^2 = 2a\Delta x$

     $v_f = \sqrt{2a\Delta x}$

     $v_f \propto \sqrt{\Delta x}$

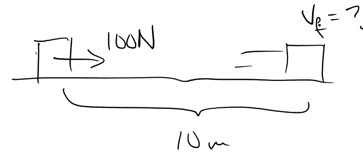
     $v_f \propto \Delta x^{1/2}$

   - assuming constant acceleration and beginning at rest, an object's displacement is ______ the elapsed time.

   - for an object that has been dropped, the distance it has fallen is ______ its velocity at that distance.

2. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N over a distance of 10 m. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 m?

   Some starters: $v_f$

   - What is the net force on the box?
   - What is the acceleration of the box?
   - What is the final velocity after 10 m

$v_f$ ← $a, \Delta x, v_i = 0$    $v_f^2 = v_i^2 + 2a\Delta x$

← $a, t, v_i = 0$    $v_f = v_i + at$

$a$ ← ~~$v_f, v_i, t$~~

← $F_{NET}, m$    $F_{NET} = ma$

$F_{NET} = ? = F_{A,x}$

$F_{NET} = 100N \Rightarrow a = \frac{100N}{100kg} = 1\,m/s^2$

$= \frac{F_A}{m} = a$

$v_f^2 = 2\left(\frac{F_{NET}}{m}\right)\Delta x$

$v_f = \sqrt{\frac{2F_{NET}\Delta x}{m}}$

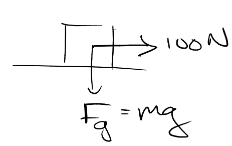
$v_{f,100kg} = \sqrt{\frac{2(100) \cdot 10}{100}}$

$= \sqrt{20}$

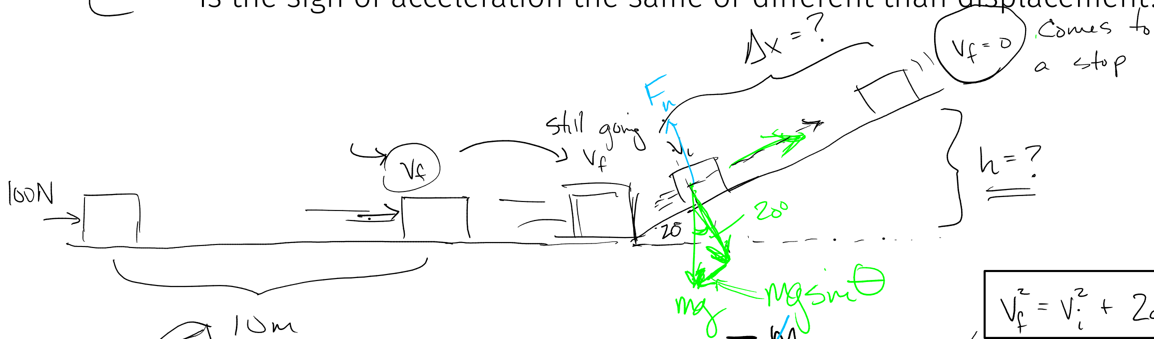
$\boxed{v_{f,100kg} = 4.5\,m/s}$

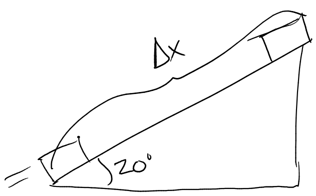
$v_{f,200kg} = \sqrt{\frac{2(100N)(10m)}{200kg}}$

$\boxed{v_{f,200kg} = 3.16\,m/s}$



3. Following up on the previous problem, if I stopped pushing after 10 m and the box continued with its speed, and then at then started sliding up a 20° ramp, then how far along the length of the ramp would the box rise? What height is this above the horizontal? Do the 100 kg and the 200 kg box rise to the same height?
Some starters:
   - What is the net force on the box as it goes up the inclined plane?
   - What is the acceleration of the box as it goes up the inclined plane?
   - What is the sign of the displacement of the box going up the plane?
   - Is the sign of acceleration the same or different than displacement?




$h = \Delta x \cdot \sin\theta$

$h = \frac{v_i^2}{2g\cancel{\sin\theta}} \cdot \cancel{\sin\theta}$

$\frac{(4.5\,m/s)^2}{2(9.8)} = \boxed{1.03\,m}$ $\boxed{h = \frac{v_i^2}{2g}}$ ← independent of incline angle

$-\cancel{m}g\sin\theta = \cancel{m}a$

$a = -g\sin\theta$

$\boxed{v_f^2 = v_i^2 + 2a\Delta x}$

0 m/s

$0 = v_i^2 + 2(-g\sin\theta)\Delta x$

$\boxed{\frac{+v_i^2}{+2g\sin\theta} = \Delta x}$

length along the plane

100 kg → $\frac{4.5^2}{2(9.8)\sin 20°} = \boxed{3.02\,m}$

200 kg → $\frac{3.11^2}{2(9.8)\sin 20°} = \boxed{1.49\,m}$

4. We want to examine the idea of *mechanical advantage*.
   - How much force would it take to lift a 100 kg box straight up at constant speed a distance of 5 meters.

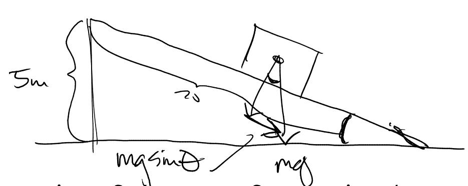
$a = 0 \Rightarrow F_{NET} = 0$

$F_L = mg$

$\rightarrow F_L = 980\,N$

   - How much force would it take to push a box up an inclined plane that was 20 m long up to the same height? (again at constant speed)

$\boxed{\sin\theta = \frac{5}{20}} = .25 = \frac{1}{4}$ $\theta = \sin^{-1}\left(\frac{1}{4}\right)$ $\theta = 14.5°$




$F_p = mg\sin\theta$

$= 980 \cdot .25 = 245\,N$

   - What is the ratio of the two forces in these two cases? Which would you rather do? This ratio is known as *mechanical advantage*

$\frac{F_p}{F_L} = \frac{1}{4}$

$\frac{F_L}{F_p} = \frac{4}{1}$

- What is the ratio of the displacement in these two cases?


$$\frac{X_L}{X_P} = \frac{1}{4} \Rightarrow \frac{X_P}{X_L} = \frac{4}{1} = \frac{F_L}{F_P}$$

- How can you use this idea to quickly figure out the force it would take you to push the box up a 100 m ramp that goes up the same height?

$$980\,\text{N} \rightarrow \boxed{\frac{F_L}{F_P} = \frac{X_P}{X_L}} \quad \xrightarrow{\text{solve for } F_P} \quad \boxed{F_P = F_L \cdot \frac{X_L}{X_P}} = 980\left(\frac{5}{100}\right) = 49\,\text{N}$$

100, 5

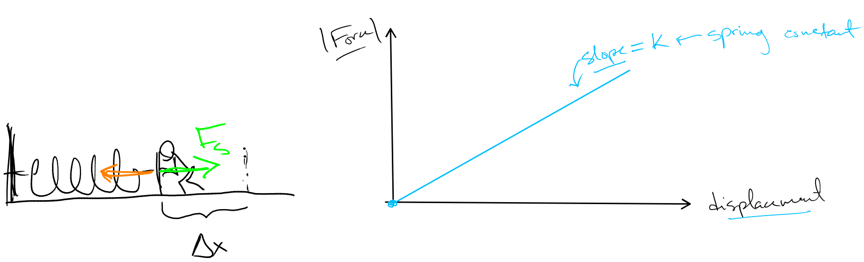
$$\frac{F_P}{F_L} = \frac{X_L}{X_P} \Rightarrow F_P = F_L \cdot \frac{X_L}{X_P}$$

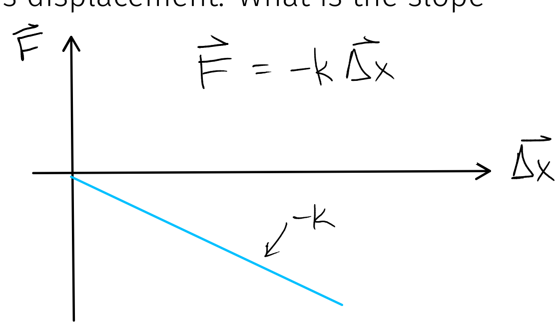
5. Let's talk about springs. *Hooke's Law* is the relationship between the force exerted by a spring and amount the spring has been stretched or compressed. The amount a spring has been stretched or compressed is the displacement of the end of the spring. Hooke's law says the magnitude of the force exerted is directly proportional to the displacement of the end of the spring. Write this in terms of a proportionality statement and again as an equation with a *constant of proportionality*. What are the units of the constant of proportionality?

$$F_s \propto \Delta x$$

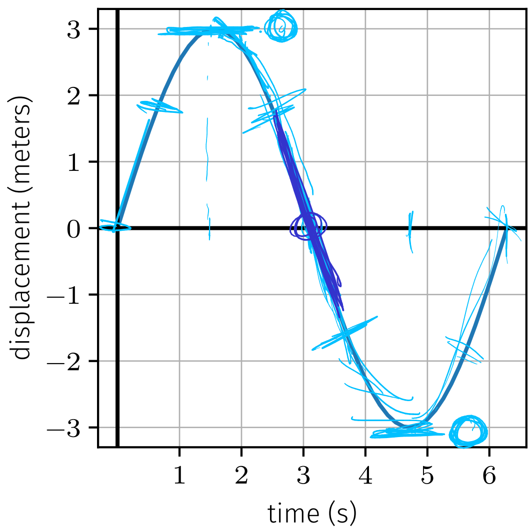
$$\boxed{|F_s| = k|\Delta x|} \qquad [N] = \left[\frac{N}{m}\right][m]$$

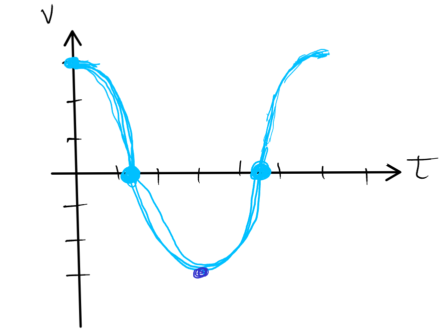
Draw a qualitative plot of the magnitude of this force vs displacement. What is the slope of this graph?

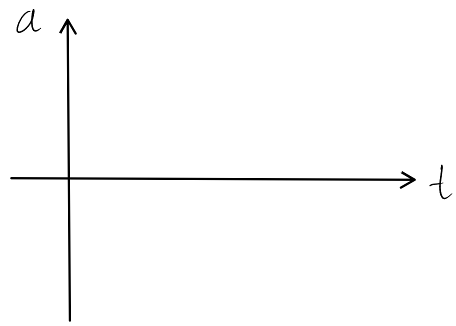


6. Consider the graph below of displacement under condition of *not constant* acceleration. What would a graph of velocity vs. time look like for this case? What about acceleration vs time.

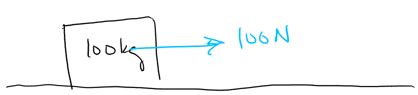



7. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N for 10 s. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 s?

time




$F_{NET,x} = ma_x$

$\frac{100N}{100kg} = \boxed{a_x = 1\,m/s^2}$

200kg box

$\left[\left(\frac{100\,N}{200\,kg}\right) = 0.5\,m/s^2\right]$

- $V_{fx} = V_{ix} + a_x t$

$V_f^2 = V_i^2 + 2a\Delta x$

$V_{fx} = 0 + 1\,m/s^2(10s)$

$\boxed{V_{fx} = 10\,m/s}$

$\Delta x = V_i t + \frac{1}{2}at^2$

$[V_{fx} = 5\,m/s^2]$

8. A follow up to the previous question. The *momentum* of an object is defined as the product of an object's mass and its velocity. In this case the momentum of this object changes because a force external to the object was exerted on it. What is the momentum of the

momentum $\rightarrow p = m \cdot V$

$p_f = 1000\,kg\,m/s$

object initially? What is the momentum of the object at the end? It is said that the change in momentum of an object is equal to the *impulse*, where *impulse* is defined as the product of constant force and the amount of time the force is acting. Is that the case here? How could you use this to quickly find the final velocity of an object if the same force were pushing it for 100 s?

$$\Delta p = p_f - p_i = 1000\,kg\,m/s - 0\,kg\,m/s = 1000\,kg\,m/s$$

constant

$$\Delta p = \text{Impulse} = F \cdot \Delta t$$
$$= 100\,N \cdot (10)\,s$$
$$= 1000\,N \cdot s$$

$$\left[\frac{kg\,m}{s^2}\right][s]$$

$$\frac{kg\,m}{s}$$

$$\boxed{mv_f - mv_i = F\Delta t}$$

$$\frac{m(v_f - \cancel{v_i}^{\,0})}{m} = \frac{F\Delta t}{m}$$

$$v_f = \frac{F\Delta t}{m}$$

9. If I launch a cannonball from ground level with a speed of 1000 m/s, then there are a variety of angles to choose from. If I fire it at 90°, then how long will it take to come down? How high will it go? How far will it go horizontally? What about 80°? What about 70°? Let's just make a table...

1000 sin 90°

1000 m/s

time in air

| Angle | Time | Max Vertical Height | Horizontal Distance |
|---|---|---|---|
| 90° | 204 s | $5.1 \times 10^4$ m | 0 m |
| 80° | 201 s | $4.95 \cdot 10^4$ m | $3.4 \cdot 10^4$ m |
| 70° | | | |
| 60° | | | |
| 50° | | | |
| 40° | | | |
| 30° | | | |
| 20° | | | |
| 10° | | | |
| 0° | 0 s | 0 m | 0 m |

$V_x = 173\,m/s$ $V_{iy} = 985$

max horizontal 45°

90°

$$\boxed{V_{fy}^2 = V_{iy}^2 + 2a_y\Delta y}$$

0 m/s 1000 m/s $-9.80\,m/s^2$ max height

max height

$$\boxed{\Delta y = \frac{-V_{iy}^2}{2a_y}} = \frac{-(1000\,m/s^2)^2}{2(-9.80)}$$
$$= 5.1 \times 10^4\,m$$

$$\boxed{V_{fy} = V_{iy} + at}$$

0 1000 −9.80

$t = 102\,s$ time to go up!

double it to get total time

$$\boxed{\Delta y = V_i t + \frac{1}{2}at^2}$$

total displ. 0 m

$$0 = V_y t + \frac{1}{2}a_y t^2$$
$$-V_{iy}\cancel{t} = \frac{1}{2}a_y t^{\cancel{2}}$$

$$\boxed{t = \frac{-2V_{iy}}{a_y}}$$

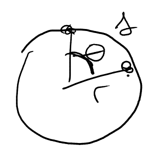
Do NOT DOUBLE

$$V_x = V_i \cos\theta$$
$$V_{iy} = V_i \sin\theta$$

$$\boxed{\Delta x = V_x t}$$

10. If a soccer ball with a radius of 10 cm is rolls along the ground without slipping at 5 m/s, then how many revolutions does it roll through in 10 s and what distance has a point on the edge of the ball traveled? Some starters:

v




$$\theta = \frac{s}{r}$$

definition

$5\,m/s$

$\omega = \frac{\Delta\theta}{\Delta t}$ but also $\omega = \frac{v}{r}$

$\boxed{\omega = 50\,rad/sec}$ ← 10cm = 0.1m

- How fast is it *spinning*? By that we mean *angular speed*.
- How many radians does the ball rotate through in this time? What is that in revolutions?
- How far does it roll in this time? Is this the same distance as the distance of a point on the edge of the ball? Why or why not?

two ways

$\boxed{\omega = \frac{\Delta\theta}{\Delta t}} \Rightarrow \Delta\theta = \omega \cdot \Delta t$

$= 50\,\frac{rad}{sec} \cdot 10\,sec$

$\boxed{\Delta\theta = 500\,rad}$

$500\,rad \cdot \frac{1\,rev}{2\pi\,rad} = 79.6$ revolutions

$\boxed{s = r \cdot \theta}$

$= 0.10\,m \cdot (500\,rad) = 50\,m$

same!

$\Delta x = v \cdot \Delta t$

$= 5\,m/s \cdot 10\,s = 50\,m$

YES, the entire circumference of the ball touches the ground in one revolution, so the ball must move that distance

11. Following up on the previous problem, how many seconds does it take for the ball to complete one revolution? This amount of time is referred to as the *period* of its rotation, and this is a similar characteristic time for the motion of the ball as the *period of a pendulum* was in the first lab.

$\frac{time}{revolution}$ = period

$\frac{10\,s}{79.6\,rev} = \boxed{0.126\,s/rev}$

another way

$\omega = \frac{\Delta\theta}{\Delta t}$

1 rev

$\Delta t = \frac{\Delta\theta}{\omega} = \frac{2\pi\,rad}{50\,rad/sec} = \boxed{0.126\,\frac{sec}{1\,rev}}$

12. Another follow up. How many revolutions does the ball travel through *per second*? You could convert this from angular speed $\omega$ that you would have calculated in the first instance of this problem, but if all you new was the period of the ball's rotation, how could you calculate it from there? *(Hint: what is the difference between revolutions per second and seconds per revoution?)* This quantity of revolutions per unit of time is sometimes called *frequency*.

$\omega = 50\,\frac{rad}{sec} \cdot \frac{1\,rev}{2\pi\,rad} = 7.96\,\frac{rev}{sec}$

or

period $= 0.126\,\frac{rev}{sec} \Rightarrow \frac{1}{0.126\,\frac{rev}{sec}} = 7.96\,\frac{sec}{rev}$

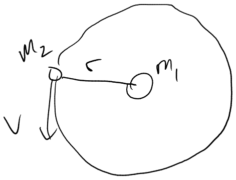
take the reciprocal of the period

13. Suppose a satellite is in orbit around a distant planet. You observe the the satellite to be 5000 km from the center of the planet, and rotating the planet once every 2 days. What is the mass of the planet you have discovered? What is the period of the satellites motion?

2 days

$\frac{1}{2\,\text{days}}$

What is its frequency? How fast is the satellite moving around the planet? What is the angular speed?




$$\frac{Gm_1 \cancel{m_2}}{\cancel{r^2}} = \boxed{\frac{\cancel{m_2} v^2}{\cancel{r}}} \leftarrow m a_R \leftarrow a_R = \frac{v^2}{r} \text{ or } \omega^2 r$$

$$\frac{Gm_1}{r} = v^2$$

$$m_1 = \frac{rv^2}{G}$$

$$m_1 = \frac{5000\cdot 10^3\,\text{m}\,(18\,\text{m/s})^2}{\left(6.67\cdot 10^{-11}\,\text{N}\,\frac{\text{m}^2}{\text{kg}^2}\right)}$$

$$\boxed{m_1 = 2.4\cdot 10^{19}\,\text{kg}}$$

meter

$$v = \frac{2\pi r}{t}$$

2 days in seconds

$$v = \frac{2\pi\,(5000\cdot 10^3\,\text{km})}{\left(2\,\text{d}\cdot 24\,\frac{\text{hr}}{\text{d}}\cdot 3600\,\frac{\text{s}}{\text{hr}}\right)}$$

$$\boxed{v = 18\,\text{m/s}}$$

period $= 2\,\text{d} = 1.7\cdot 10^5\,\text{s}$

frequency $= \frac{1\,\text{rev}}{2\,\text{day}} = 5.7\cdot 10^{-6}\,\frac{\text{rev}}{\text{sec}}$

$$\omega = \frac{v}{r}$$

$y_i = 0$ $\quad$ $C = 0$

A projectile is launched from the ground with a velocity of 45.0 m/s at an angle of 30.0 degrees above the horizontal. How far away does it land?

$V_{ix} = 45\cos 30 = 39\,m/s \longrightarrow y_f = 0$ $\quad$ $\Delta y = 0$

$V_{iy} = 45\sin 30 = 22.5\,m/s$

$\Delta y = V_{iy}\cdot t + \frac{1}{2}a_y t^2$

$0 = 22.5t + \frac{1}{2}(-9.8)t^2$

$4.9t^{\cancel{2}} = 22.5\cancel{t}$

$4.9t = 22.5 \longrightarrow t = \frac{22.5}{4.9} = 4.6\,s$

$\Delta x = V_x \cdot t = 39\,m/s \cdot 4.6\,s = 179\,m$

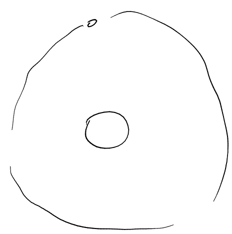
A arrow is released from a bow (ignore the height of the archer, so just assume it starts at ground level) with an initial velocity of 700.0 m/s at an angle of 30.0 degrees above the horizontal. How long does it take to land if it was fired over level ground?

$F_{fs} = \mu F_n \qquad F_n = mg$

$F_f = \mu\cdot m\cdot a_c = m a_R$

$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$

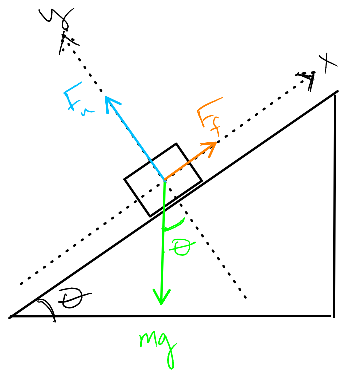
$\mu = \frac{v^2}{gr}$



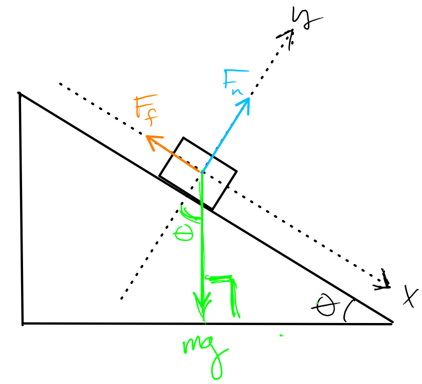
$\omega = 1.5\cdot 10^{-4}\,rad/s = \frac{\Delta\theta}{\Delta t}$ $\quad$ $2\pi$

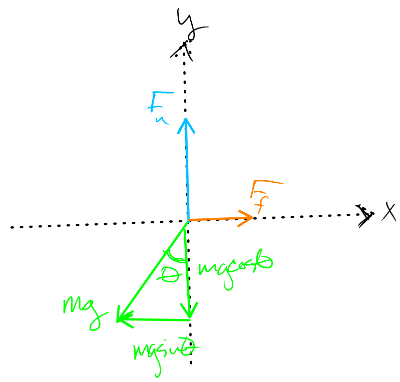
$t = \frac{2\pi}{\omega} = \quad s$

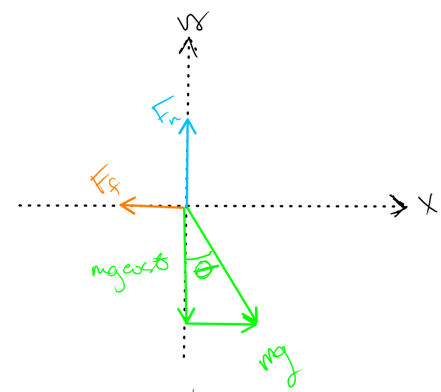
12 hr.




$F_{st} \leq \mu_s F_n$




| x | y |
|---|---|
| $+F_f$ | 0 |
| 0 | $+F_n$ |
| $-mg\sin\theta$ | $-mg\cos\theta$ |
| 0 | 0 |

$F_{NET,x}$ $F_{NET,y}$

$$F_f - mg\sin\theta = 0$$
$$F_f = +mg\sin\theta$$

$$0 = F_n - mg\cos\theta$$
$$F_n = mg\cos\theta$$

| x | y |
|---|---|
| $-F_f$ | 0 |
| 0 | $+F_n$ |
| $+mg\sin\theta$ | $-mg\cos\theta$ |
| 0 | 0 |

$$-F_f + mg\sin\theta = 0$$
$$-F_f = -mg\sin\theta$$
$$F_f = mg\sin\theta$$
$$\underbrace{F_f = 79.4\,N}$$