At the end of this worksheet you should be able to

- calculate the new quantities of pressure, gauge pressure, and density.
- apply the new principles of Pascal, Archimedes, and Bernoulli.
- use the continuity principle to solve for an unknown quantity.
- use the hydrostatic pressure principle to solve for an unknown quantity.

You will want to keep the following table of densities from your textbook handy.

mass density

"rho"

$$\rho = \frac{m}{V}$$

Table 9.1 Densities of Common Substances (at 0°C and 1 atm unless otherwise indicated)

Gases	Density (kg/m^3)	Liquids	Density (kg/m^3)	Solids	Density (kg/m^3)
Hydrogen	0.090	Gasoline	680	Polystyrene	100
Helium	0.18	Ethanol	790	Cork	240
Steam (100°C)	0.60	Oil	800–900	Wood (pine)	350–550
Methane	0.72	Water (20°C)	998.21	Wood (oak)	600–900
Air (20°C)	1.20	Water (0°C)	999.84	Ice	917
Nitrogen	1.25	Water (3.98°C)	999.98	Wood (ebony)	1000–1300
Carbon monoxide	1.25	Seawater	1025	Bone	1500–2000
Air (0°C)	1.29	Blood (37°C)	1060	Concrete	2000
Oxygen	1.43	Mercury	13 600	Quartz, granite	2700
Carbon dioxide	1.98			Aluminum	2702
Argon	1.66			Iron, steel	7860
Xenon	5.86			Copper	8920
Radon	9.73			Lead	11 300
				Gold	19 300
				Platinum	21 500

1000

$$\frac{g}{cm^3}$$

1. If 500 N person stands on one foot that has an area 50.0 cm^2, what is the pressure on the floor? If this person stands on their heel in a high heeled shoe that has an area of 1.00 cm^2, what pressure is there? What if this person stands on a diamond that is cut to have a bottom facet that is 10 000 μm^2 ($1\,\mu m = 10^{-6}$ m)? *(Careful with unit conversions!).*

$$P = \frac{F}{A}$$

$$\frac{[\text{Newton}]}{[m^2]} = [\text{Pascal}]$$

$$A_1 = 50.0\,\cancel{cm^2} \cdot \frac{(1\,m)^2}{(100\,\cancel{cm})^2} = 0.005\,m^2$$

$$P_1 = \frac{500N}{0.005\,m^2} = 100{,}000\,Pa = 10^5\,Pa$$

$$(10^2)^2 = 10^4$$

$$A_2 = 1\,cm^2 \cdot \frac{(1\,m)^2}{(100\,cm)^2} = 10^{-4}\,m^2$$

$$P_2 = \frac{500N}{10^{-4}\,m^2} = 500 \cdot 10^4\,Pa = 5 \cdot 10^6\,Pa = 5\,MPa$$

$$A_3 = 10{,}000\,\mu m^2 \cdot \frac{(10^{-6}\,m)^2}{(1\,\mu m)^2} = 10^{-8}\,m^2$$

$$P_3 = \frac{500N}{10^{-8}\,m^2} = 500 \cdot 10^8\,Pa = 5 \cdot 10^{10}\,Pa = 50 \cdot 10^9\,Pa = 50\,GPa$$

2. A patient's ~~blood~~ systolic blood pressure when resting is 160 mmHg. What is this pressure in pascals, psi, and atm? torr bar

$760\ mmHg = 1\ atm$ | $14.7\ psi = 1\ atm$ | $101.3\ kPa = 1\ atm$

$101.3 \cdot 10^3\ Pa =$

$1.013 \cdot 10^5\ Pa =$

$10^5\ Pa = 1\ atm$

$160\ \cancel{mmHg} \cdot \frac{1\ atm}{760\ \cancel{mmHg}} = 0.21\ atm$

$0.21\ atm \cdot \frac{14.7\ psi}{1\ atm} = 3.09\ psi$

$0.21\ atm \cdot \frac{10^5\ Pa}{1\ atm} = 2.1 \cdot 10^4\ Pa$

3. Throughout this worksheet and the homework *gauge pressure* is a way of expressing the pressure measured by an instrument relative to the atmospheric pressure. So 1 atm of absolute pressure is set to 0 atm of gauge pressure. Perfect vacuum is absolute zero pressure, so that would be −1 atm of gauge pressure. So what is 2 atm of gauge pressure as ~~atmospheric~~ absolute pressure? 30 kPa of absolute pressure is what gauge pressure? What about a gauge pressure of 1000 kPa as absolute pressure?

$P_{abs} = P_{gauge} + P_{atm}$

$P_{gauge} = P_{abs} - P_{atm}$

$P_{abs} = 2\ atm + 1\ atm = 3\ atm$

$P_{gauge} = 30\ kPa - 101.3\ kPa = -70\ kPa$

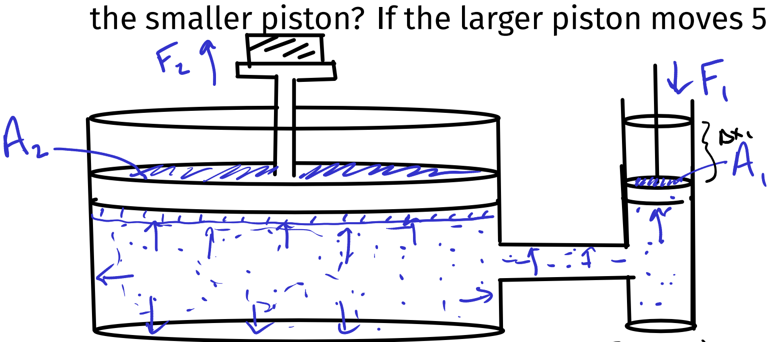
$P_{abs} = 1000\ kPa + 101.3\ kPa = 1100\ kPa$

$F_2 \cdot \Delta x_2 = W = F_1 \cdot \Delta x_1$

$\rightarrow 25{,}000\,N \cdot (0.05\,m) = 250\,N \cdot \Delta x_1$

$\Delta x_1 = 5\,m$

4. In a hydraulic lift, the radius of ~~a~~ the small piston is 2 cm and the radius of the larger piston is 20 cm, what weight can the larger piston support when a force of 250 N is applied to the smaller piston? If the larger piston moves 5 cm, how far does the small piston move?



$\frac{F_2}{A_2} = P = \frac{F_1}{A_1}$ → $A \propto r^2$

$\frac{F_2}{A_2} = \frac{F_1}{A_1}$

$\frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2$

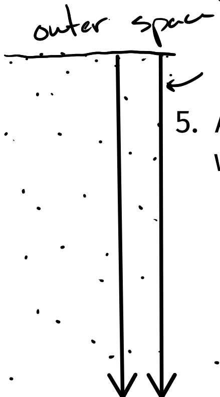
$\frac{F_2}{F_1} = \frac{A_2}{A_1}$

$F_2 = \left(\frac{A_2}{A_1}\right) F_1$

$F_2 = \left(\frac{r_2}{r_1}\right)^2 \cdot F_1$

$F_2 = \left(\frac{20\,cm}{2\,cm}\right)^2 \cdot 250\,N$

$= 25{,}000\,N$



5. At the surface of a freshwater lake, the air pressure is 1 atm. At what depth under the water is the absolute pressure 4 atm?

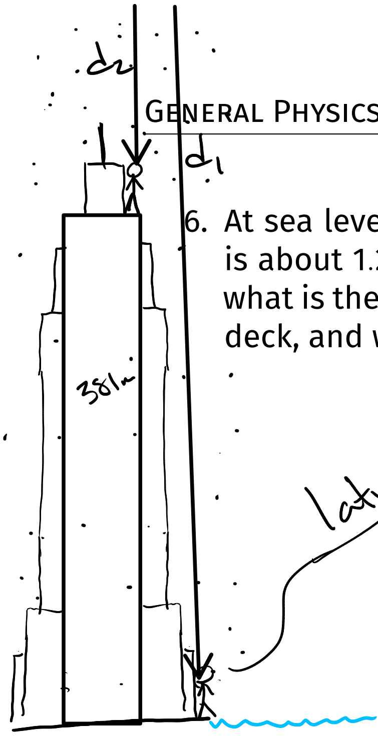
$P_{abs} = \rho_f \cdot g \cdot \underset{\text{depth}}{d} + \underbrace{P_{atm}}_{\text{if air is above the water}}$

$P_{gauge} = \rho_f g d$

$P_{abs} = \rho_f \cdot g \cdot d + P_{atm}$

$4 \cdot 10^5\ Pa = 1000 \frac{kg}{m^3} \cdot 9.8 \frac{N}{kg} \cdot d + 1 \cdot 10^5\ Pa$

$d = 30.6\,m$



6. At sea level, the average atmospheric pressure is 1 atm. The density of air at this level is about 1.29 kg/m^3. Assuming the density of air is constant (its not but just go with it), what is the air pressure at the Empire State Building that has a height of 381 m at the top deck, and we will just assume that its base is at sea level?

$$P = \rho_f \cdot g \cdot d$$

$$1.013 \cdot 10^5 \, Pa = (1.29 \, kg/m^3) \cdot (9.8 \, \tfrac{N}{kg}) \cdot d_1$$

$$d_1 = 8013 \, m$$

$$d_2 = 8013 \, m - 381 \, m = 7632 \, m$$

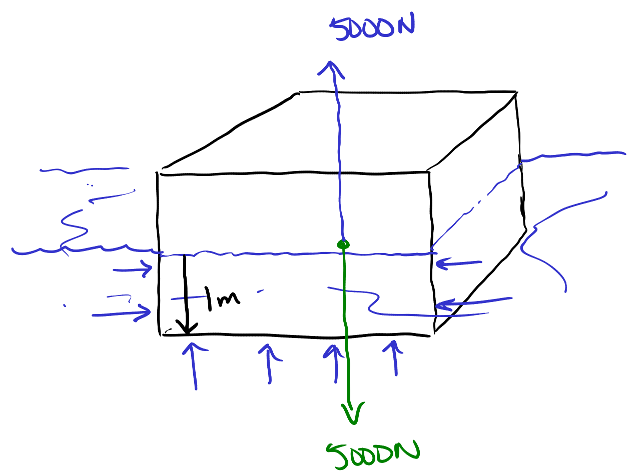
$$P_2 = \rho_f \cdot g \cdot d_2$$

$$= 1.29 \cdot 9.8 \cdot 7632$$

$$P_2 = 0.965 \cdot 10^5 \, Pa$$

7. A diver swims to a depth of 10 meters in a lake. What is the pressure on the diver's body? What is the force on the diver's eardrums from the water if the area of the eardrum is 0.60 cm^2? ~~What would this be if the diver was swimming in sea water?~~ (Ignore the fact that there is atmospheric pressure inside the eardrum, but also think about this problem if you didn't ignore that fact.)

8. A 5000 N object is floating in fresh water.



- What is the net force on the object?

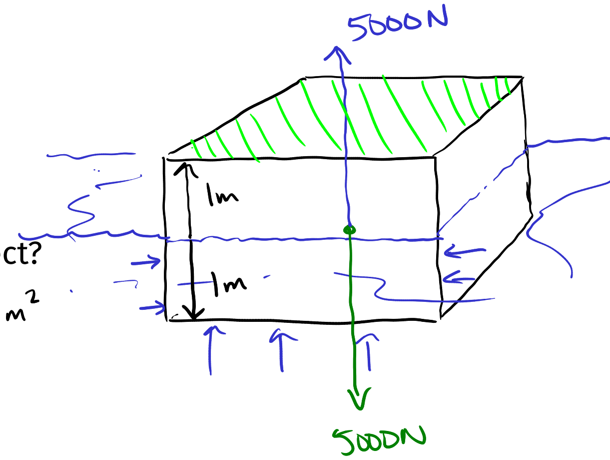
$$F_{NET} = 0$$

- What is the magnitude of the buoyant force?

$$F_b = 5000N \text{ upward}$$

- If ~~the~~ the bottom of the object is 1 meter below the surface of the water, then what pressure is on the bottom of the object?

$$P = \rho_f \cdot g \cdot d$$
$$= 1000\,\frac{kg}{m^3} \cdot 9.8\,\frac{N}{kg} \cdot 1\,m = \underline{9800\,Pa}$$



- What is the area of the bottom surface of the object?

$$P = \frac{F}{A} \Rightarrow A = \frac{F}{P} = \frac{5000\,N}{9800\,Pa} = 0.51\,m^2$$

- If the object has another 1 m sticking out of the water, then what is the volume of the object?

$$V = A \cdot \text{height}$$
$$= 0.51\,m^2 \cdot 2\,m = 1.02\,m^3$$

- What is its density~~?~~ of the object?

$$\rho = \frac{m}{V} \qquad F_g = 5000N = m \cdot g$$
$$m = 510\,kg$$
$$\rho = \frac{510\,kg}{1.02\,m^3} = 500\,kg/m^3$$

9. The following cylindrical barrels are filled to the brim with fluids of the given density. Put these in order from smallest to largest pressure at the bottom of the barrel.

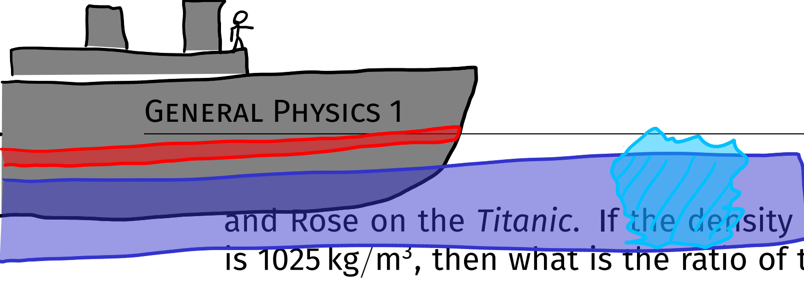
(a) $R = 40$ cm, $h = 80$ cm, $\rho = 1000$ kg/m^3
(b) $R = 40$ cm, $h = 100$ cm, $\rho = 1000$ kg/m^3
(c) $R = 50$ cm, $h = 100$ cm, $\rho = 800$ kg/m^3
(d) $R = 50$ cm, $h = 80$ cm, $\rho = 800$ kg/m^3
(e) $R = 50$ cm, $h = 125$ cm, $\rho = 800$ kg/m^3

small $\longrightarrow$ large
d, a=c, e=b

10. Let's do a problem based on the famous Archimedes legend. The story goes that a King Hiero II of Syracuse commissioned an ornate golden crown to be made, but when he got it, he was suspicious that silver had been mixed in with the gold. He charged Archimedes to figure out how to determine the density of the crown without damaging it. Archimedes' "Eureka" moment came when he realized he could determine the volume of a complex shape by submerging it in a tub of water that had been filled to the brim. The amount of water that spilled over the edge would be equal to the volume of the crown as long as you could catch all of the spilled water and measure its volume. So with all of that said, suppose the amount of water that spilled over the edge weighed 1.0 N but there was still some water stuck to the sides of the bucket that didn't get weighed so maybe call it 1.1 N I don't know this is pretty messy. The crown itself weighed 24.1 N. So what is the density of the crown and how does it compare to gold? *Hint: what is the volume of water that spilled over the edge?*

11. The above story is ridiculous. What if the water that spilled over the edge had weighed 1.2 N or 1.0 N? This is far too imprecise for something as serious as an allegation of cheating the King out of a pure gold crown. So instead we want to measure the buoyant force on the crown. So, if you weight the crown to be 24.1 N and then you lower it into the water fully submerged and weight it then (by tying a string to it and lowering it into the water but the string is attached to a balance) and then its "weight" is 22.85 N. What is the buoyant force upward on the crown? What volume of water has been displaced? What is the volume of the crown? What is the density of the crown? What is better about this method?

12. "ICEBERG DEAD AHEAD!" It is sometimes said that only 10% of an iceberg's volume actually sticks out above the surface of the water which is what made it so deadly to poor Jack



and Rose on the *Titanic*. If the density of ice is 917 kg/m^3 and the density of seawater is 1025 kg/m^3, then what is the ratio of the volume of ice under the water to the entire volume of ice? What is the ratio of the volume of ice above the water to the entire volume of ice? Next work these *in general* for any density of object floating in any density of fluid.

floating

$$F_b = F_g$$

$$F_b = \rho_f \cdot g \cdot V_f \quad \Big| \quad F_g = m_o \cdot g = \rho_o V_o \cdot g$$

$$\rho_f \cdot \cancel{g} \cdot V_f = \rho_o V_o \cdot \cancel{g}$$

$$\rho_f V_{sub} = \rho_o V_o$$

$$\frac{V_{sub}}{V_o} = \frac{\rho_o}{\rho_f} = \frac{917\,\text{kg/m}^3}{1025\,\text{kg/m}^3} = 0.89$$

$$V_{sub} = 0.89 \cdot V_o$$

$$\frac{V_o - V_{not\ sub}}{V_o} = 0.89 = 1 - \frac{V_{not\ sub}}{V_{sub}} \Rightarrow \frac{V_{not\ sub}}{V_o} = 0.11$$

13. An artery has an inner diameter of 1.5 mm, but narrows to an inner diameter of 1.0 mm due to a build up of plaque. By what percent does the speed of blood flow change when it enters the narrow section?

percent change ↓ (percent difference)

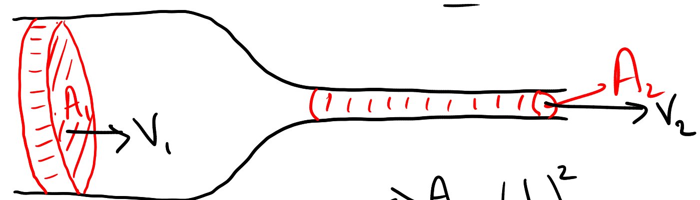
$$\%\Delta = \left(\frac{v_2}{v_1} - 1\right) \times 100$$

Continuity equation

$$A_1 v_1 = A_2 v_2$$

$$\boxed{\frac{v_2}{v_1} = \frac{A_1}{A_2}}$$

$$v \propto A^{-1}$$



$$A = \pi r^2$$
$$A \propto r^2$$
$$A = \frac{\pi}{4} d^2 \Rightarrow A \propto d^2$$

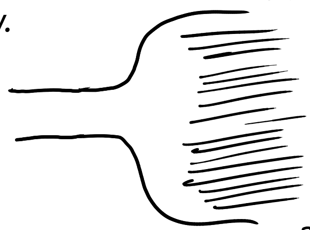
$$\frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2$$

$$\boxed{\frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2}$$

$$\%\Delta = \left[\left(\frac{d_1}{d_2}\right)^2 - 1\right] \times 100$$

$$\%\Delta = 125\%$$

14. What if we have a case where a single pipe splits into many pipes? Following an example from the textbook, the aorta is the artery from your heart that feeds 32 other major arteries. The aorta has an inner radius of 1 cm and assume each of the other arteries has an inner radius of 0.21 cm. If blood in the aorta has an average speed of 28 cm/s, then what is the average speed of blood in the major arteries? *Hint: you can just treat the major arteries as one big pipe that has an area 32 times bigger than the area of a single artery.*



$$A_{aorta} = \pi r^2 = \pi (0.01\,\text{m})^2 = 3.14 \cdot 10^{-4}\,\text{m}^2$$

$$A_{artery} = \pi (0.0021\,\text{m})^2 = 1.38 \cdot 10^{-5}\,\text{m}^2$$

$$\times 32$$

$$4.43 \cdot 10^{-4}\,\text{m}^2 = A_{arteries}$$

$$A_{aorta} v_{aorta} = A_{arteries} v_{arteries}$$

$$3.14 \cdot 10^{-4} \cdot 0.28\,\text{m/s} = 4.43 \cdot 10^{-4} \cdot v_{arteries} \Rightarrow v_{arteries} = 0.2\,\text{m/s} = 20\,\text{cm/s}$$

$$\frac{V_2}{V_1} = \frac{A_1}{A_2}$$

$$\frac{V_2}{V_1} = 2 \Rightarrow V_2 = 2 \cdot V_1$$

15. Suppose it takes one minute to fill a 5 gallon bucket with water from your garden hose that is open all the way. The diameter of your hose is 1 inch. How fast is water traveling out of the end of the hose? How fast does it travel if you hold your thumb over the end of the hose and cover half of the area of the hose? (1 inch=0.0254 m, 1 gallon=0.003 78 m^3, 1 min=60 s)

$A_1 V_1 = A_2 V_2$ ← continuity eq

$\frac{\Delta V}{\Delta t} = A \cdot v$ ← def of volumetric flow

$5 \text{ gallon} \cdot \frac{0.00378\,m^3}{1 \text{ gal}} = 0.0189\,m^3$

1 minute = 60 sec

1 inch = 0.0254 m

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.0254\,m)^2$

$A = 0.000506\,m^2$

$5.06 \cdot 10^{-4}\,m^2$

$\frac{0.0189\,m^3}{60\text{ sec}} = 5.06 \cdot 10^{-4}\,m^2 \cdot v$

$v = 0.62\,m/s$

$\frac{0.0189}{60\text{ sec}} = 2.53 \cdot 10^{-4} \cdot v$

$v = 1.24\,m/s$

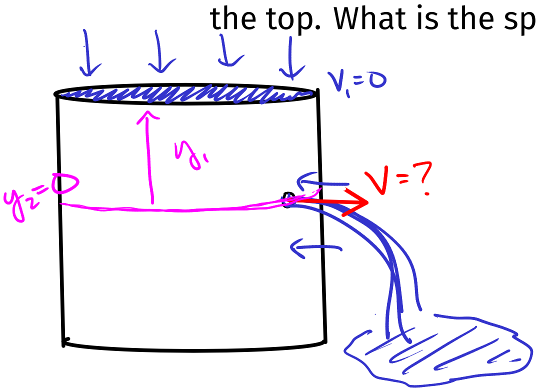
16. Show that Bernoulli's Principle really just reduces to the equation for pressure as a result of gravity when the fluid is not flowing (so $v_1 = v_2 = 0$). This equation is now the same as the *hydrostatic pressure* equation.

$$\cancel{\tfrac{1}{2}\rho v_1^2}^{\,0} + \rho g y_1 + P_1 = \cancel{\tfrac{1}{2}\rho v_2^2}^{\,0} + \cancel{\rho g y_2}^{\,0} + P_2$$

$$P_2 - P_1 = \Delta P = \rho g y_1$$

gauge pressure (often)

17. A cylindrical container of water is full to the brim when a hole is punctured 0.5 m from the top. What is the speed of the water as it comes out of the hole?



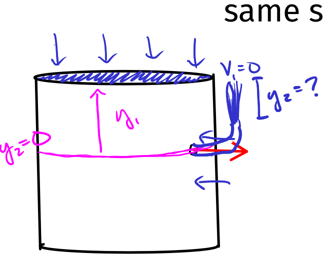
$$\cancel{\tfrac{1}{2}\rho v_1^2}^{\,0} + \rho g y_1 + \cancel{P_1}^{\,1\,atm} = \tfrac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}^{\,0} + \cancel{P_2}^{\,1\,atm}$$

$v_2 = ?$

$$\cancel{\rho} g y_1 = \tfrac{1}{2}\cancel{\rho} v_2^2$$

$$V_2 = \sqrt{2 \cdot g \cdot y_1} = 3.13\,m/s$$

18. Following up on the previous problem, if you redirected this water straight up with the same speed, how high would it rise?



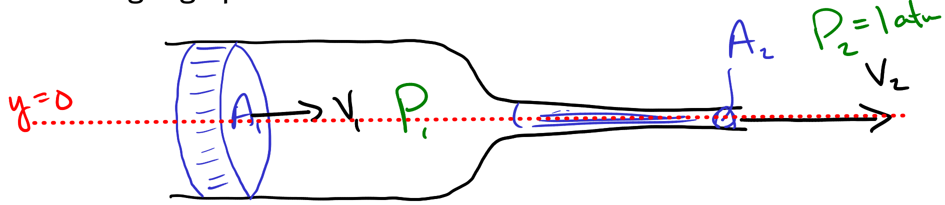
$$\cancel{\tfrac{1}{2}\rho v_1^2}^{\,0} + \rho g y_1 + \cancel{P_1}^{\,1\,atm} = \cancel{\tfrac{1}{2}\rho v_2^2}^{\,0} + \rho g y_2 + \cancel{P_2}^{\,1\,atm}$$

$$\cancel{\rho g} y_1 = \cancel{\rho g} y_2$$

$$y_1 = y_2$$

19. ~~If~~ water flows horizontally through a hose that has a radius of 1 cm at a speed of 2 m/s. If the nozzle of the hose narrows to 0.25 cm as the water sprays out, then what is the pressure inside the hose? What is it as a gauge pressure?

$\rho = 1000\ \mathrm{kg/m^3}$



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$$\frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1}^{\,0} + \underbrace{P_1}_{?} = \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}^{\,0} + P_2 \quad \leftarrow \text{Bernoulli}$$

$1\,\mathrm{atm} = 10^5\ \mathrm{Pa}$

$$A_1 v_1 = A_2 v_2 \quad \leftarrow \text{continuity}$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

$$v_2 = \left(\frac{r_1}{r_2}\right)^2 \cdot v_1 = 32\ \mathrm{m/s}$$

$$\frac{1}{2}\left(1000\,\frac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(2\,\mathrm{m/s}\right)^2 + P_1 = \frac{1}{2}\left(1000\,\frac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(32\,\mathrm{m/s}\right)^2 + 10^5\ \mathrm{Pa}$$

absolute $\rightarrow P_1 = 6.1 \cdot 10^5\ \mathrm{Pa}$

$\qquad\qquad -1 \cdot 10^5\ \mathrm{Pa}$

gauge $\rightarrow P_1 = 5.1 \cdot 10^5\ \mathrm{Pa}$

A water tower supplies water through the plumbing in a house. A 3.16-cm-diameter faucet in the house can fill a cylindrical container with a diameter of 42.3 cm and a height of 52.0 cm in 12.0 s. How high above the faucet is the top of the water in the tower? (Assume that the diameter of the tower is so large compared to that of the faucet that the water at the top of the tower does not move.)

$\hookrightarrow v_1 = 0$

$$\frac{\Delta \mathbb{V}}{t} = A_{faucet} \cdot v_{faucet}$$

faucet at 0 height

$$\cancel{\tfrac{1}{2}\rho v_1^2}^{\,0} + \rho g y_1 + \cancel{P_1}^{\,1atm} = \tfrac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}^{\,0} + \cancel{P_2}^{\,1atm}$$

?

$v_2 = v_{faucet}$

$$P = \rho g h = P_{blood}$$

A nozzle of inner radius 1.03 mm is connected to a hose of inner radius 8.70 mm. The nozzle shoots out water moving at 25.0 m/s.

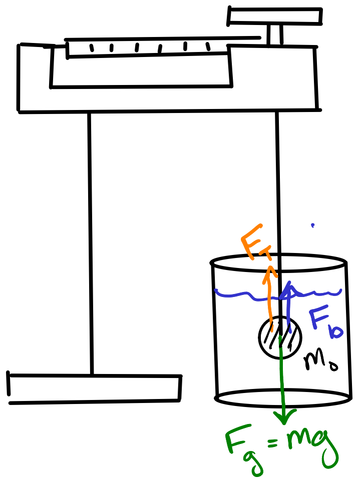
0.00103 m 0.00870 m

$$mfr = \rho \cdot A \cdot v$$

$$= 1000 \frac{kg}{m^3} \cdot \underbrace{(0.00870 m)^2 \cdot \pi}_{A} \cdot \underbrace{0.35 m/s}_{v}$$

$$= 0.0832 kg/s$$

$$= 83.2 g/s$$



$$\Sigma F = 0$$

$$F_b + F_T - m_o \cdot g = 0$$

$$F_T \rightarrow m_a \cdot g$$

$$F_b = m_f \cdot g$$

$$m_f \cdot \cancel{g} + m_a \cdot \cancel{g} - m_o \cdot \cancel{g} = 0$$

$$m_f + m_a - m_o = 0$$

$$\rho_f = \frac{m_f}{V_f} \qquad \rho_o = \frac{m_o}{V_o}$$

$$V_f = V_o \Rightarrow \underline{\text{constant}}$$

$$\rho \propto m$$

$$\frac{\rho_f}{\rho_o} = \frac{m_f}{m_o} \Rightarrow \boxed{m_f = \frac{m_o}{\rho_o} \cdot \rho_f}$$

Solve for ρ_o

$$\left\{ \frac{m_o}{\rho_o} \cdot \rho_f + m_a - m_o = 0 \right.$$

$$\rho_o = \left(\frac{m_o}{m_o - m_a} \right) \rho_f$$