At the end of this worksheet you should be able to:

- convert units within the metric system.
- apply how to correctly convert area and volume units.
- apply scientific notation.
- apply the definition of percent change.
- convert between a ratio statement and percent change statement.
- apply the two forms of proportionality statements to make predictions from dependent variable change.

1. How many meters are in a decameter and how do you know?

   10 m = 1 decameter

2. How many decameters are in a meter and why?

   0.1 decameters = 1 m

3. How many centimeters are in a meter? Draw on your paper approximately a centimeter.

   100 cm = 1 m

4. How many millimeters are in a meter and how many meters are in a millimeter?

   1000 mm = 1 m

   $\underbrace{0.001\,m}_{10^{-3}\,m} = 1\,mm$

5. The diameter of the earth is 6,380,000 m. What is this in kilometers?

   6,380 km | $\frac{6,380,000\,\cancel{m}}{1} \cdot \frac{1\ km}{1000\ \cancel{m}} = \underline{6,380\ km}$

6. How many centimeters are in 3.2 km?

$$3.2\,\text{km} \cdot \frac{\overset{10^3}{1000\,\text{m}}}{1\,\text{km}} \cdot \frac{\overset{10^2}{100\,\text{cm}}}{1\,\text{m}} = 3.2 \cdot 10^5\,\text{cm} \rightarrow 320{,}000\,\text{cm}$$

7. How many inches are in 1 meter? Do this using the conversion of 1 in = 2.54 cm.

$1\,\text{m} \rightarrow ?\,\text{in}$

$$1\,\text{m} \cdot \frac{100\,\text{cm}}{1\,\text{m}} \cdot \frac{1\,\text{in}}{2.54\,\text{cm}} = \underline{39.4\,\text{in}}$$

8. If I say your desk has an area of 0.5 meters, then what is wrong with that statement?

$A = l \cdot w$ $\qquad 0.5\,\text{m}^2$

9. Convert miles per hour to meters per second.

1

$$\frac{1\,\cancel{\text{mi}}}{1\,\cancel{\text{hour}}} \cdot \frac{5280\,\cancel{\text{ft}}}{1\,\cancel{\text{mi}}} \cdot \frac{12\,\cancel{\text{in}}}{1\,\cancel{\text{ft}}} \cdot \frac{1\,\text{m}}{39.4\,\cancel{\text{in}}} \cdot \frac{1\,\cancel{\text{hour}}}{3600\,\text{second}} = 0.45\,\text{m/s}$$

10. List as many formulas for the area of different shapes as you can remember (or look some up). What do these all have in common? What about volume formulas? What does this tell you about the units of these kinds of quantities?

$A_{rect} = l \cdot w$ | $A_{squ} = s^2$

$A_{\Delta} = \frac{1}{2} b \cdot h$

$A_{\circ} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

$A_{sphere} = 4\pi r^2$

$V = l \cdot w \cdot h = s^3$

$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{cyl.} = \pi r^2 \cdot h$

11. If your desk has an area of 0.5 meters^2, then what is its area in centimeters^2? ?

$$0.5\,\text{m}^2 \cdot \frac{\overset{10^4}{(100\,\text{cm})^2}}{(1\,\text{m})^2} = 0.5 \cdot 10^4\,\text{cm}^2$$

$$= 5 \cdot 10^3\,\text{cm}^2 = 5000\,\text{cm}^2$$

12. If a ball has a diameter of 18 cm, then what is the volume of the ball in meters^3?

$r = 9 cm$

$V = \frac{4}{3}\pi r^3$ $r = 0.09$ m

$V = 3.05 \cdot 10^{-3}$ m^3

13. Calculate your age in seconds on your last birthday.

$37 \text{ years} \cdot \frac{365.25 \text{ day}}{1 \text{ yr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} = 1.17 \cdot 10^9 \text{ sec}$

1.2 billion seconds

14. Make up your own unit conversion problem that involves at least two conversions chained together.

$60 \frac{\text{miles}}{\text{hour}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in}}{\text{ft}} \cdot \frac{2.54 \text{ cm}}{1 \text{ in}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = 96.6 \frac{\text{km}}{\text{hour}}$

$1 \text{ gallon} \cdot \frac{16 \text{ cups}}{1 \text{ gal}} \cdot \frac{8 \text{ oz}}{1 \text{ cup}} \cdot \frac{30 \text{ mL}}{1 \text{ oz}} \cdot \frac{\text{L}}{1000 \text{ mL}} = 3.84 \text{ L}$

15. Put the number 21,345,000,000 kg in scientific notation?

$\cdot 10^{10}$

$2.1345 \cdot 10^{10}$ kg

16. Put the number 0.0000000234 km in scientific notation?

$\underline{2.34 \cdot 10^{-8} \text{ km}}$ $2.34 \cdot 10^{-2}$ mm μm

0.0234 mm

17. Correct the scientific notation of 140×10^{-3} seconds.

$1.40 \cdot 10^{-1}$ s

18. Correct the scientific notation of 0.012×10^{-3} meters.

$1.2 \cdot 10^{-5}$ m

19. What is 0.000 345 meter in micrometers μm?

$3.45 \cdot 10^{-4}\,\text{m} \cdot \frac{10^{6}\,\mu\text{m}}{1\,\text{m}} = 3.45 \cdot 10^{2}\,\mu\text{m} = 345\,\mu\text{m}$

or

$3.45 \cdot 10^{-4}\,\text{m} \cdot \frac{1\,\mu\text{m}}{10^{-6}\,\text{m}} = 345\,\mu\text{m}$

20. What is 3.4×10^{5} m in kilometers?

$3.4 \cdot 10^{5}\,\text{m} \cdot \frac{1\,\text{km}}{10^{3}\,\text{m}} = 3.4 \cdot 10^{2}\,\text{km} = 340\,\text{km}$

$\text{my} = 3 \times \text{your}$ $\quad \frac{\text{my}}{\text{yours}} = 3$

21. If I have three times as many marbles as you do then what is the ratio of my marbles to yours? What is the ratio of your marbles to mine?

$\frac{\text{my}}{\text{your}} = \frac{3}{1}$ $\quad \frac{\text{your}}{\text{my}} = \frac{1}{3}$

22. If I increase in speed from 25 mph to 40 mph, then by what percent has my speed changed? By what ratio has the speed changed?

$\%\Delta = \frac{\text{final} - \text{initial}}{\text{initial}} \times 100$

$= \frac{40\,\text{mph} - 25\,\text{mph}}{25\,\text{mph}} \times 100 = 60\%$

→ 0.6

$\frac{\text{final}}{\text{initial}} = \frac{40\,\text{mph}}{25\,\text{mph}} = 1.6$

23. If my speed changes from 29 mph to 10 mph, then by what percent has my speed changed? What is the negative sign in the answer tell you? By what factor has your speed changed from initially to finally? Could a negative ratio make sense here?

$\%\Delta = \frac{10\,\text{mph} - 29\,\text{mph}}{29\,\text{mph}} \times 100 = -65.5\%$

$\frac{\text{initial}}{\text{final}} = \frac{29}{10} = 2.9$

24. If the generic variable y is inversely proportional to the a variable x, then write out this statement mathematically ~~in two~~ ways.

statement

$y \propto x^{-1}$ $\longrightarrow$ $y = k \cdot x^{-1}$ ← function

proportion equation

$$\boxed{\frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^{-1}} = \frac{x_1}{x_2}$$

$$y = \frac{k}{x}$$

25. Since the area of a triangle is directly proportional to both the base and height, then how would an *equation* for this look like ~~and what~~ is the constant of proportionality?

$A \propto b^1 \cdot h^1$ → function

$$\frac{A_2}{A_1} = \left(\frac{b_2}{b_1}\right)^1 \left(\frac{h_2}{h_1}\right)^1$$

$$A = k \cdot b \cdot h$$

$$k = \frac{1}{2}$$

26. The formula for the volume of a cylinder is $V = \pi r^2 h$. How would you write out a proportionality statement that was consistent with this formula? What is the constant of proportionality?

π

$V = \pi r^2 h$ $\qquad$ $V \propto r^2 h$

$$\rightarrow \left(\frac{V_2}{V_1}\right) = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{h_2}{h_1}\right)$$

$r_2 = 2.7 r_1$

27. If the radius of a sphere changes by a factor of 2.7, then by what factor does the volume change?

$V = \frac{4}{3}\pi r^3 \longrightarrow \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3$ $\qquad$ $\frac{r_2}{r_1} = 2.7$

$$\frac{V_2}{V_1} = (2.7)^3 = 19.68 \qquad \frac{V_2}{V_1} = 19.68$$

28. How would you turn the previous problem "inside out"?

A sphere's volume increases by a factor of 19.68. By what factor does the radius change for this to happen?

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 \qquad \frac{V_2}{V_1} = 19.68 \qquad 19.68 = \left(\frac{r_2}{r_1}\right)^3$$

29. By what percent does ~~the volume change in~~ the previous two problems?

$$\frac{r_2}{r_1} = (19.68)^{1/3} = 2.7$$

$$\%\Delta = \left(\frac{\text{final} - \text{initial}}{\text{initial}}\right) \times 100 \qquad \%\Delta = \left(\frac{\text{final}}{\text{initial}} - 1\right) \times 100$$

$$= \left(\frac{\text{final}}{\text{initial}} - \cancel{\frac{\text{initial}}{\text{initial}}}^{\,1}\right) \times 100 \qquad = (19.68 - 1) \times 100 = 1868\%$$

$A = \pi r^2$

→ If I increase the radius of a circle by 10%, then by what factor does the area change?

$\% \Delta = \left(\frac{r_2}{r_1} - 1 \right) \times 100$

$+\frac{10\%}{100} = \frac{r_2}{r_1} - 1$

$\frac{r_2}{r_1} = 1.1$

$A \propto r^2$

$\frac{A_2}{A_1} = \left(\frac{r_2}{r_1} \right)^2$

$\frac{A_2}{A_1} = (1.1)^2 = 1.21$

21%

In order to double the volume of a sphere, by what factor must the radius change?

$\frac{V_2}{V_1} = 2$

$V \propto r^3$

$\boxed{\frac{V_2}{V_1} = \left(\frac{r_2}{r_1} \right)^3}$

$2 = \left(\frac{r_2}{r_1} \right)^3$

$\frac{r_2}{r_1} = \sqrt[3]{2}$

If you had rope that wrapped around the entire world,
and then you wanted to raise the height of
the rope off the surface of the earth by one foot,
then how much more rope would you need?

$C \propto r$

$C = 2\pi r$

$V = \frac{4}{3}\pi r^3$

$C_2 - C_1 = 2\pi r_2 - 2\pi r_1$

$C_2 - C_1 = 2\pi \underbrace{(r_2 - r_1)}_{1\,\text{ft}}$

$\frac{V_2 = \frac{4}{3}\pi r_2^3}{V_1 = \frac{4}{3}\pi r_1^3}$

$\frac{V_2}{V_1} = \frac{r_2^3}{r_1^3}$

$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1} \right)^3$

$\Delta C = 6.28\,\text{ft}$