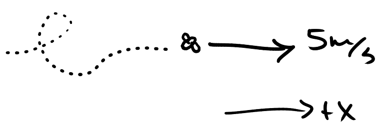
At the end of this worksheet you should be able to

- find the momentum of an object or collection of objects.
- find the change in momentum of an object or collections of objects due to an impulse.
- use the conservation of momentum to solve for an unknown quantity.
- use the principle of relative velocity to solve for an unknown in elastic collisions.

Note that at in every problem this week we will be ignoring friction and air resistance. Its not that momentum can't work with those quantities, but momentum does not really help tell us anything new about them, so our problems will not involve friction unless explicitly specified. When using the conservation of momentum in this way, we really restrict ourselves to talking about the motion of the objects *just before* they collide as well as *just after* they have stopped colliding.

1. In this problem a bug and car collide. Assume the car is coasting frictionlessly. Also assume you can measure quantities with perfect accuracy.

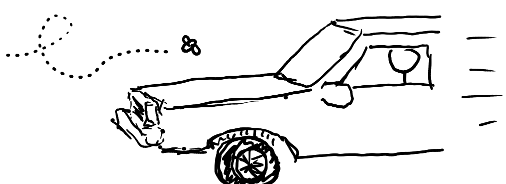
   - A 1 g bug is flying east through the air along a road at 5 m/s. What is the bug's momentum? Remember that momentum is a *vector* so if you are calling the momentum positive that means east.



$$\vec{p}_{bug} = m_{bug}\vec{v}_{bug}$$

$$\vec{p} = 0.001\,kg \cdot (5\,m/s) = 0.005\,kg\,m/s$$

   - A 1000 kg car is traveling west along this same road at a speed of 20 m/s. What is the car's momentum. Remember the sign!



$$\vec{p}_{car} = m_{car}\vec{v}_{car} = 1000\,kg \cdot (-20\,m/s) = -20{,}000\,kg\,m/s$$

   - What is the total momentum of this system?

$$\vec{P}_{total} = \vec{p}_{bug} + \vec{p}_{car} = 0.005\,kg\,m/s - 20{,}000\,kg\,m/s$$

$$\vec{P}_{total} = -19999.995\,kg\,m/s$$

   - When the bug collides with the windshield of the car, what is the momentum of the bug-car system? What is the total mass of the car now? What is the speed of the car?

$$m_{b+c} = 1000.001\,kg$$

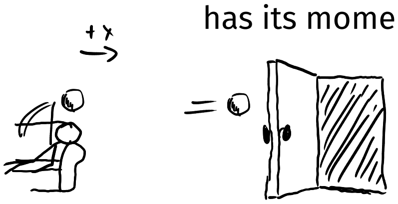
$$\vec{P}_{total} = -19999.995\,kg\,m/s = \vec{p}_{b+c} = m_{b+c}\vec{v}_{b+c} = (1000.001\,kg)\,\vec{v}_{b+c}$$

$$\vec{v}_{b+c} = \underline{-19.999975\,m/s}$$

2. You want to close a door but you don't want to get up. You look around and see that there is a bouncy ball that looks like you could throw it pretty fast. For some reason there is also a wad of clay that you know would stick to the door if you threw it. The ball and the clay have the same mass. Which one should you throw against the door to close it most effectively?
Some starters: $m = 2\,kg$ $\quad v = 10\,m/s$

- Choose a mass for your ball and clay, and choose a velocity. Or leave it as *m* and *v* and work it *in general*.
- If the ball hits the door and bounces back perfectly, its speed should be basically the same after it bounces off the door. How has its velocity changed? How much has its momentum changed?

+x →



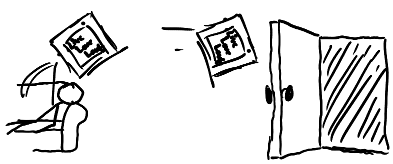
$\vec{p}_i = m_{bb} v_i = 2\,kg \cdot 10\,m/s = 20\,kg\,m/s$

$\vec{p}_f = 2\,kg \cdot (-10\,m/s) = -20\,kg\,m/s$

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -20\,kg\,m/s - (+20\,kg\,m/s)$

$\Delta\vec{p} = -40\,kg\,m/s$

- If the clay hits the door and sticks, then what is its change in momentum? Its speed is basically zero after the collision.



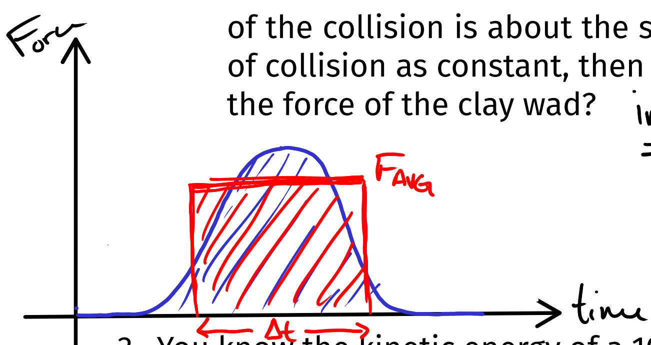
$\vec{p}_i = 2\,kg \cdot (10\,m/s) = 20\,kg\,m/s$

$\vec{p}_f = 2\,kg \cdot (0^{ish}\,m/s) = 0^{ish}\,kg\,m/s$

$\Delta\vec{p} = 0 - (+20\,kg\,m/s)$

$\Delta\vec{p} = -20\,kg\,m/s$

- Which one of these cases would close the door more effectively? If the time interval of the collision is about the same for both of these cases, and we model the force of collision as constant, then what is the ratio of the force from the bouncy ball, to the force of the clay wad? impulse: $\Delta p = F_{AVG} \cdot \Delta t$



$$\frac{\Delta p_{bb} = F_{bb} \cdot \cancel{\Delta t}}{\Delta p_{LLL} = F_{LLL} \cdot \cancel{\Delta t}} = \frac{-40}{-20} = \boxed{2 = \frac{F_{bb}}{F_{LLL}}}$$

3. You know the kinetic energy of a 10 kg object is 100 J, then what is the momentum of the object? Now do this *inside out*. Now do it *in general*. In other words show where $K = \dfrac{p^2}{2m}$ comes from.

$K = \frac{1}{2} m v^2$

$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot 100}{10}} = 4.47\,m/s$

$p = mv = 10\,kg \cdot 4.47\,m/s = 44.7\,kg\,m/s$

$v = \sqrt{\frac{2K}{m}}$

$p = mv$

$p = m\sqrt{\frac{2K}{m}}$

$p = \sqrt{\frac{2Km^{\cancel{2}}}{\cancel{m}}}$

$p = \sqrt{2Km}$

$p^2 = 2Km \Rightarrow \boxed{K = \frac{p^2}{2m}}$ ✓

4. - If I am are standing motionless, what is my momentum? I have a mass of 75 kg.

$p = 0$

- What if I am wearing a backpack with 100 - 0.5 kg baseballs while standing on ice (with skates)? $p = 0$
- I throw one of the baseballs with a velocity at −15 m/s. What is its momentum?

$$p = mv = (0.5\,kg)(-15\,m/s) = -7.5\,kg\,m/s$$

- What is my mass after I throw it (think about the mass of my backpack)? What is my momentum after I throw the baseball? What is my velocity?

$$M = 124.5\,kg$$

$$0 = p_{bb} + p_{me}$$

$$0 = -7.5\,kg\,m/s + p_{ME}$$

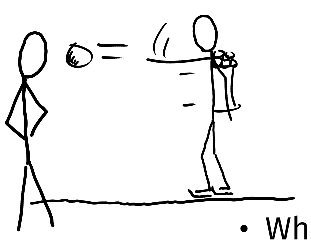
$$p_{ME} = +7.5\,kg\,m/s$$

$$p_{ME} = M_{ME} \cdot V_{ME}$$

$$7.5\,kg\,m/s = 124.5\,kg \cdot V_{ME}$$

$$\boxed{V_{ME} = 0.0602\,m/s}$$

- If I throw another baseball with a velocity of −15 m/s *relative to my current velocity*, then what is the velocity of the ball relative to the ice?



$$\vec{V}_{ice} = \vec{V}_{bb} + \vec{V}_{ME}$$

← velocity of the moving frame relative to the stationary frame

objects velocity in the stationary frame

object's velocity in the moving frame

$$V_{ice} = -15\,m/s + 0.0602\,m/s$$

$$V_{ice} = -14.94\,m/s$$

- What is my new velocity after throwing this second baseball? How much did my velocity change from before?

$$p_i = p_f$$

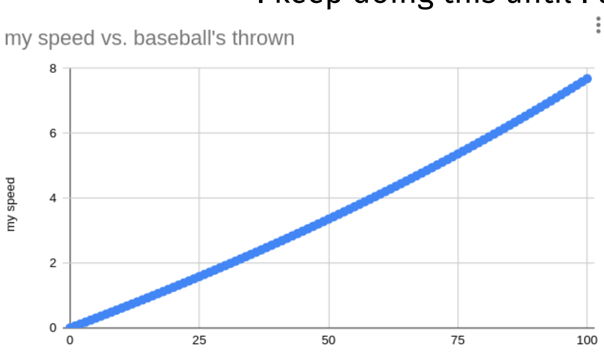
$$\cancel{0.5\,kg(-15\,m/s)} + 124.5\,kg(0.0602\,m/s) = \cancel{0.5\,kg(-15\,m/s)} + 0.5\,kg(-14.94\,m/s) + 124 \cdot v$$

$$\frac{124.5\,kg(0.0602\,m/s) - 0.5\,kg(-14.94\,m/s)}{124\,kg} = \boxed{V = 0.1207\,m/s}$$

$$\Delta v = 0.0605$$

- If I did this again, throwing a baseball with a velocity of −15 m/s *relative to my current velocity*, what is my velocity after I throw it?

- I keep doing this until I am out of baseballs. How fast am I going?



$$\rightarrow V_f = 7.68\,m/s$$

rocket equation

$$V_f = V_{ex} \cdot \ln\left(\frac{m_f}{m_i}\right) + V_i$$

$$= (-15\,m/s) \cdot \ln\left(\frac{75}{125}\right) + 0\,m/s$$

$$V_f = 7.66\,m/s$$

$$m_1 v_{1i} + \cancel{m_2 v_{2i}}^{\,0} = m_1 v_{1f} + m_2 \underset{?}{v_{2f}}$$

5. An empty 10 kg wagon is rolling past me at a speed of 5 m/s while I am holding a 30 kg bag of concrete. If I drop the concrete bag into the wagon right at it gets to me, what is its speed immediately after that?

Conservation of momentum:

$$p_i = p_f$$

$$m_1 v_{1i} + \cancel{m_2 v_2}^{\,0} = m_1 v_{1f} + m_2 v_{2f} = m_1 v_f + m_2 v_f$$

$$m_1 v_{1i} = (m_1 + m_2) v_f$$

$$v_f = \frac{10\,\text{kg} \cdot 5\,\text{m/s}}{(10\,\text{kg} + 30\,\text{kg})} = \underline{1.25\,\text{m/s}}$$

6. I drop a 1 kg basketball from a height of 1 m and it bounces off the floor and rises to a height of 0.75 m. How much energy was converted to internal energy?
Some starters:

- What is the basketball's velocity right before it hits the ground?
- What is the basketball's velocity right after it hits the ground? Think about what it must be for it to rise to 0.75 m.
- What is the ratio of the change in kinetic energy?
- What is the change in kinetic energy?
- What is the change in momentum of the ball?
- Is momentum conserved here? Why or why not?

$$\boxed{v_f = \sqrt{2(-9.8)\Delta y}}$$

OR $\cancel{K_i}^{\,0} + U_i + \cancel{W_{nc}}^{\,0} = K_f + \cancel{U_f}^{\,0}$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_f^2$$

$$\boxed{v_f = \sqrt{2gh}} = 4.43\,\text{m/s}$$

$$K = \frac{1}{2} m v^2 = 9.8\,\text{J}$$

$$v_f^2 = v_i^2 + 2a\Delta y$$

OR $K_i + \cancel{U_i}^{\,0} + \cancel{W_{nc}} = \cancel{K_f}^{\,0} + U_f$

$$\frac{1}{2} \cancel{m} v_i^2 = \cancel{m} g h$$

$$v_i = \sqrt{2gh} = 3.8\,\text{m/s}$$

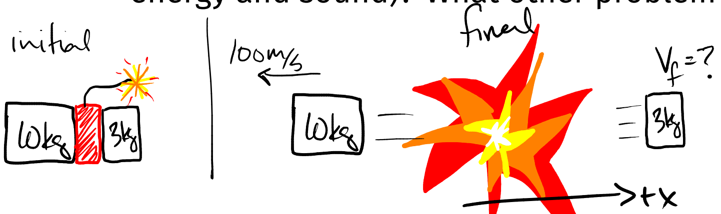
$$K = \frac{1}{2} m v^2 = 7.3\,\text{J}$$

$$\frac{K_{after}}{K_{before}} = \frac{7.3\,\text{J}}{9.8\,\text{J}} = 0.75 = \frac{h_a}{h_b}$$

$$\Delta K = K_{after} - K_{before} = \underline{\underline{-2.5\,\text{J}}}$$

$$\Delta p = +3.8\,\text{kg m/s} - (-4.43\,\text{kg m/s}) = \underline{8.23\,\text{kg m/s}}$$

7. Explosions are actually a kind of collision, but in reverse. Let's work a problem to see. Two indestructible objects are tied together with a stick of dynamite between them and everything is at rest. When it explodes, they fly away from each other in opposite directions. One of the objects has a mass of 10 kg and the other a mass of 3 kg. The 10 kg object flies away with a speed of 100 m/s. What is the velocity of the 3 kg object? At least how much energy was the explosion (some of it was probably also put into internal energy and sound)? What other problem in this worksheet is this similar to?



$p_i = p_f$

$0 = p_1 + p_2$

$0 = 10kg(-100\,m/s) + 3kg \cdot v_f$

$v_f = 333\ m/s$

8. A 5 kg gun fires a 10 g bullet at a velocity of 500 m/s. The explosion that causes this shot takes place in 1 millisecond. What force is exerted on the gun and what is the gun's recoil velocity?

$p_i = p_f$

$0\,kg\,m/s = 0.010\,kg(500\,m/s) + 5kg \cdot v$

$\frac{-0.010\,kg \cdot 500}{5} = v = -1\ m/s$

$\Delta p_{gun} = F_{AVG_{gun}} \cdot \Delta t$

$\frac{\Delta p}{\Delta t} = F_{AVG} = \frac{p_f - p_i}{\Delta t} = \frac{5kg(-1\,m/s) - 0}{0.001\,s}$

$F_{AVG} = -5000\ N$

9. Two objects collide elastically, one has a mass of 1 kg and the second has a mass of 3 kg. The first object is traveling to the right at a speed of 3 m/s and the second is traveling in front of it initially at a speed of 1 m/s. What is the velocity of the objects after the collision. Think of and work as many variations of this problem as you can.

$$\boxed{v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{2m_2}{(m_1 + m_2)} v_{2i}}$$

$$\boxed{v_{2f} = v_{1f} + v_{1i} - v_{2i}}$$

⇓ alternative (from the notes)

$$v_{2f} = \frac{2m_1}{(m_1 + m_2)} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}$$

$m_1 = 1kg$ $v_{1i} = 3\ m/s$

$m_2 = 3kg$ $v_{2i} = 1\ m/s$

$v_{1f} = 0\ m/s$

$v_{2f} = 2\ m/s$

10. ~~Two objects collide elastically.~~ Two objects collide elastically, one has a mass of 1 kg and the second has a mass of 3 kg. The first object is traveling to the right at a speed of 3 m/s. With what velocity would the second object need to be travelling so that after the collision, the first object was motionless?

$m_1 = 1\,kg$ $V_1 = 3\,m/s$ $V_{1f} = 0\,m/s$ $V_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i} + \frac{2m_2}{(m_1+m_2)} V_{2i}$

$m_2 = 3\,kg$ $V_{2i} = ?$?

$V_{2i} = +1\,m/s$

Bonus: $V_{2f} = +2\,m/s$

11. A massive object moving with a velocity v and is going to collide elastically with a very small object that is initially at rest. What is the velocity of these two objects after they collide? (*By massive and very small I mean when you add or subtract m_1 and m_2 they are indistinguishable from m_1 due to the rules of significant figures.*)

$V_{1f} = \frac{\overbrace{(m_1 - m_2)}^{m_1}}{\underbrace{(m_1 + m_2)}_{m_1}} V_{1i} + \cancel{\frac{2m_2}{(m_1+m_2)}} V_{2i}$ m_1 $V_{2f} = \frac{2\cancel{m_1}}{\underbrace{(m_1+m_2)}_{\cancel{m_1}}} V_{1i} + \cancel{\frac{(m_2 - m_1)}{(m_2+m_1)}} V_{2i}$

$V_{1f} = V_{1i}$ m_2 $V_{2f} = 2 \cdot V_{1i}$

12. Reverse the previous problem. A tiny object is colliding with velocity v with a huge object that is at rest. What is the final velocity of the two objects?

$V_{1f} = \frac{\overbrace{(m_1 - m_2)}^{-\cancel{m_2}}}{\underbrace{(m_1 + m_2)}_{\cancel{m_2}}} V_{1i} + \cancel{\frac{2m_2}{(m_1+m_2)}} V_{2i}$ m_1 $V_{2f} = \frac{2m_1}{\underbrace{(m_1+m_2)}_{m_2}} V_{1i} + \cancel{\frac{(m_2 - m_1)}{(m_2+m_1)}} V_{2i}$

$V_{1f} = -V_{1i}$ m_2 $V_{2f} = 0\,m/s$

13. Let's work the ballistic pendulum problem like we do in the lab and do it *in general*. So we have a bullet of initial velocity v_{bi} and mass m_b and a target initially at rest with mass m_t. The collision is perfectly inelastic, and the pendulum (with the bullet inside) rises to a height h after the collision takes place. What is the initial velocity of the bullet?

14. Now in question #2 of the lab write-up it asks could this experiment be done with an *elastic* collision, given that we only measure the same things as we did in the inelastic collision case. So same parameters as before, but this time when the ball bounces off the target it has a speed v_{bf} and the pendulum rises to a height h but this time without the bullet embedded in it.

relative to stationary ship

A 68.0-kg astronaut is in space, far from any objects that would exert a significant gravitational force on him. He would like to move toward his spaceship, but his jet pack is not functioning. He throws a 0.720-kg socket wrench with a velocity of 5.00 m/s in a direction away from the ship. After 0.500 s, he throws a 0.800-kg spanner in the same direction with a speed of 8.00 m/s. After another 9.90 s, he throws a mallet with a speed of 6.00 m/s in the same direction. The mallet has a mass of 1.20 kg. How fast is the astronaut moving after he throws the mallet? (assume all speeds are relative to the spaceship).

0.253 ± 2% m/s

$$\underbrace{0}_{p_i} = \underbrace{0.72\,kg(-5\,m/s)}_{\text{throws socket wrench}} + \underbrace{(68\,kg + 0.8\,kg + 1.2\,kg)}_{\text{still has a spanner + a mallet}} V_{after\ socket}$$

$$V_{after\ socket} = \frac{0.72(5\,m/s)}{68 + 0.8 + 1.2} = 0.051\,m/s$$

$$\underbrace{(68 + 0.8 + 1.2)\,0.051\,m/s}_{\text{new initial momentum}} = \underbrace{0.8\,kg(-8\,m/s)}_{\text{throws spanner}} + \underbrace{(68\,kg + 1.2\,kg)}_{\text{still has a mallet}} V_{after\ spanner}$$

$$V_{after\ spanner} = \frac{(68 + 0.8 + 1.2)\,0.051\,m/s - 0.8\,kg(-8\,m/s)}{68 + 1.2}$$

$$V_{after\ spanner} = 0.144\,m/s$$

new initial momentum

$$(68 + 1.2)(0.144\,m/s) = \underbrace{1.2\,kg(-6.00\,m/s)}_{\text{throws mallet}} + 68\,kg \cdot V_{final}$$

$$V_{final} = \frac{(68 + 1.2)(0.144\,m/s) - 1.2\,kg(-6.00\,m/s)}{68}$$

$$V_{final} = 0.253\,m/s$$