Week 9 - Rotational Motion and Equilibrium

After this you can
- discuss the connection between translational motion and rotational motion.
- discuss the new quantities that describe rotational motion and calculate them.

Translational Motion

displacement, Δx

velocity, $v = \frac{\Delta x}{\Delta t}$

acceleration, $a = \frac{\Delta v}{\Delta t}$

→ mass (inertia)

$\Delta = r\Delta\theta$

$v = r\omega$

$a = r\alpha$

Rotation Motion

angular displacement, $\Delta\theta$

angular velocity, $\omega = \frac{\Delta\theta}{\Delta t}$

angular acceleration, $\alpha = \frac{\Delta\omega}{\Delta t}$

→ rotational inertia, I

$\Sigma\vec{F} = \vec{F}_{NET} = m\vec{a}$ ← Newton's 2nd for translation

$\Sigma\tau = \tau_{NET} = I\cdot\alpha$ ← Newton's 2nd for rotation

$W = F\cos\theta\,\Delta x$

$K = \frac{1}{2}mv^2$

$W_r = \tau\cdot\Delta\theta$

$K_r = \frac{1}{2}I\omega^2$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\Delta t$$

$$\vec{p}_i = \vec{p}_f$$

<u>angular momentum</u>

$$L = I \cdot \omega$$

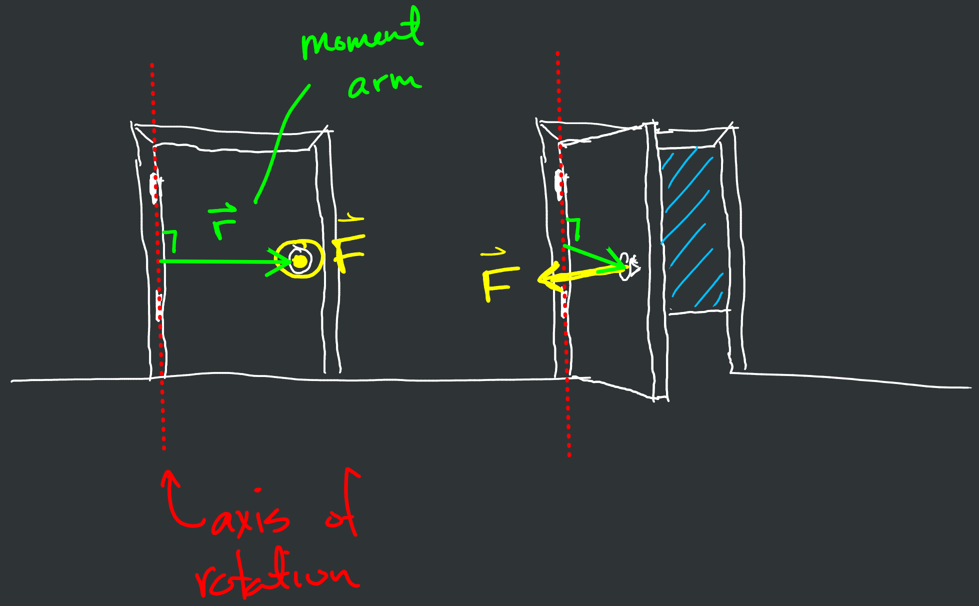
$$\Delta L = \tau \cdot \Delta t$$

$$L_i = L_f$$

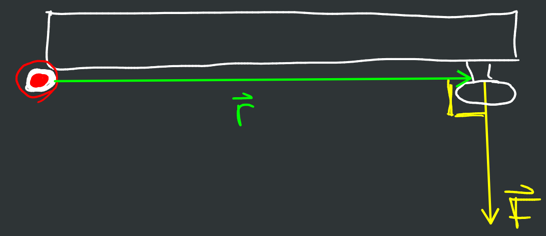
conservation of angular momentum

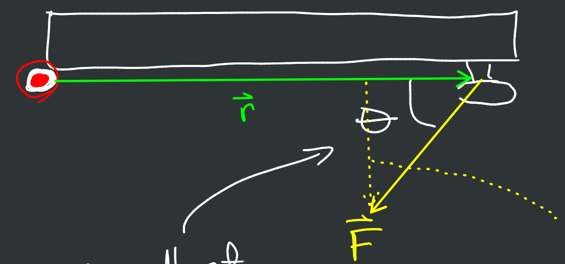
After this you can
- define torque and calculate it under any conditions.
- discuss Newton's second law for rotations.
- calculate rotational inertia for a range of shapes or combinations of shapes.

$$\tau_{net} = I \cdot \alpha$$



view from above





smallest angle between $\vec{r}$ + $\vec{F}$

component of $\vec{F}$ that is perpendicular to moment arm

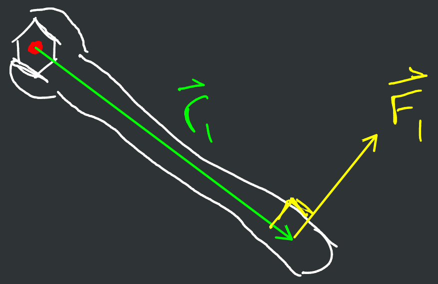
often $\theta = 90^\circ$

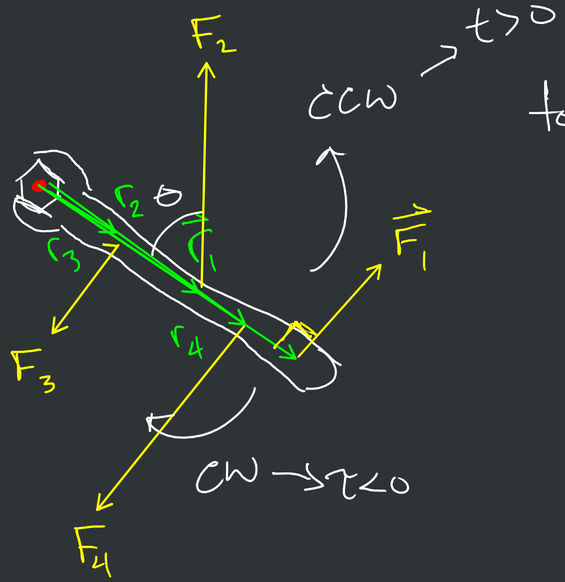
$\sin 90^\circ = 1$

$|\tau| = |F||r|$

magnitude torque → $\underline{|\tau| = |F| \sin\theta\, |r|}$

$[\tau] = [Nm]$





to add torques we need to consider the direction of the torque

counterclockwise → (+) torque $\tau > 0$

Ex. $F_1 r_1 + F_2 \sin\theta \, r_2$

clockwise → (−) torque $\tau < 0$

$F_3 \cdot r_3 + F_4 \cdot r_4$

$$\sum \tau = \tau_{NET} = +F_1 r_1 + F_2 \sin\theta \, r_2 - F_3 r_3 - F_4 r_4$$

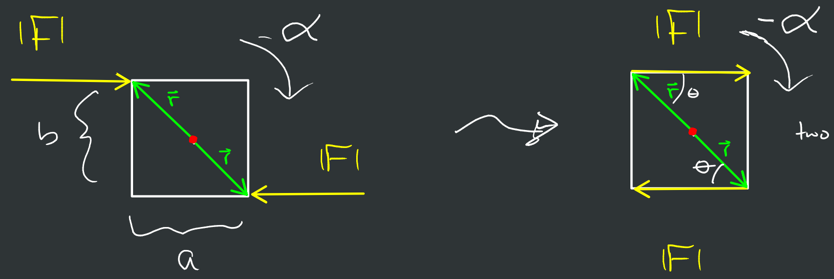
What does τ_{NET} cause? → rotational acceleration up until now:

$F_{NET} = 0$

motion continues

$\frac{F_{NET}}{a} = m$ ← inertia

now:



two clockwise torques

$\tau_{NET} < 0$

$\alpha < 0$

$$\underbrace{\frac{\tau_{NET}}{\alpha} = I}_{\text{do this in lab.}}$$

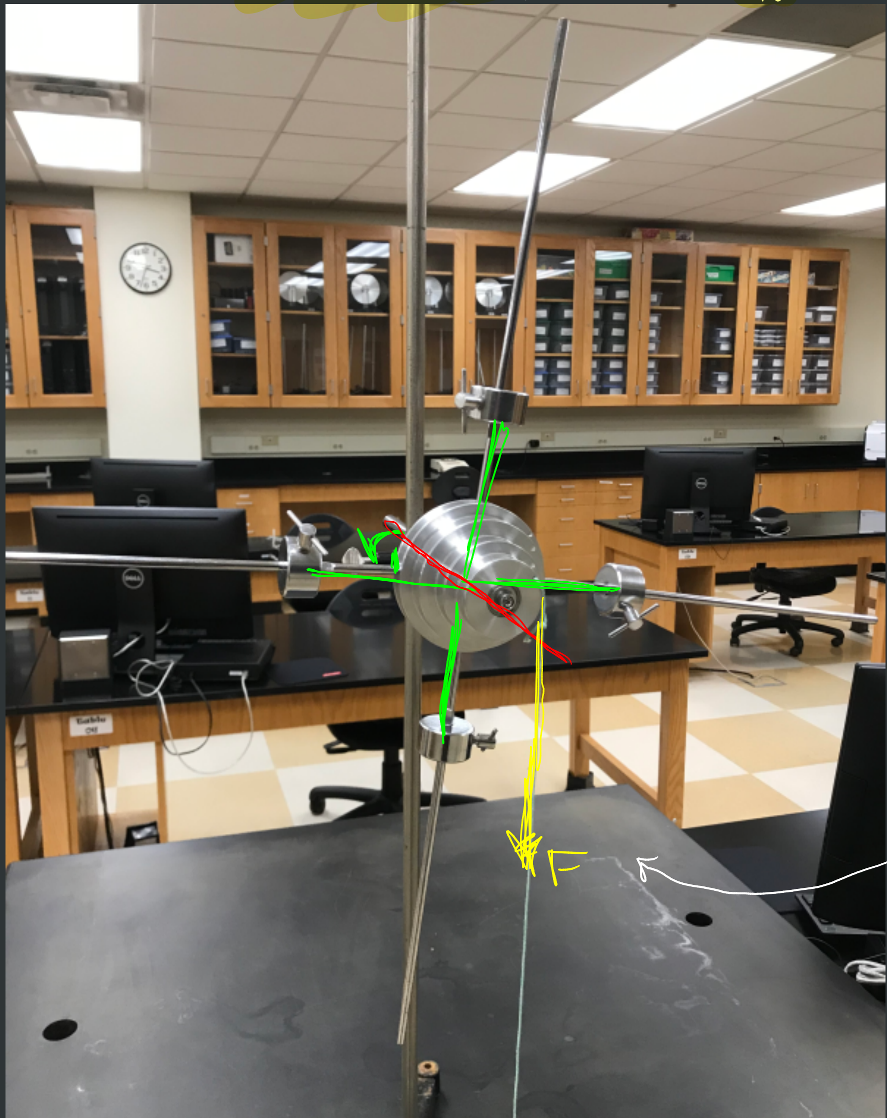
rotational inertia

↳ depends on mass, and distribution of the mass around the axis of rotation

Table 8.1 Rotational Inertia for Uniform Objects with Various Geometrical Shapes

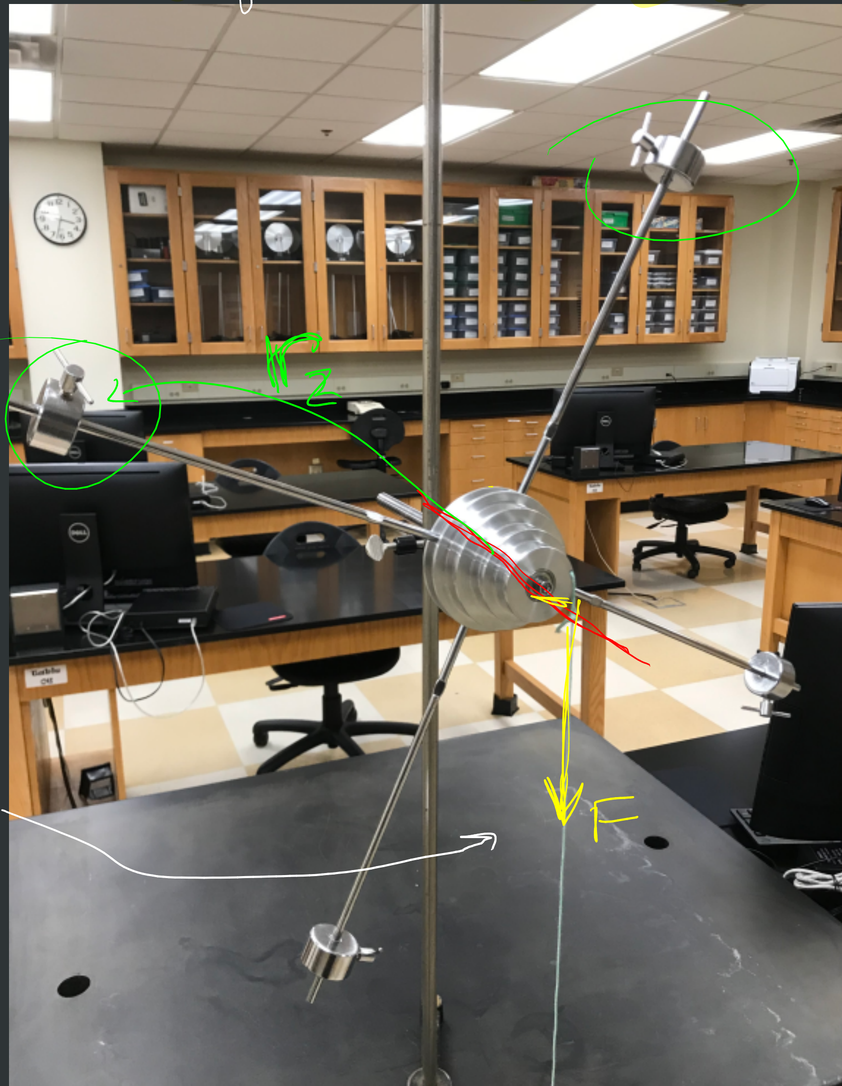
Shape	Axis of Rotation	Rotational Inertia	Shape	Axis of Rotation	Rotational Inertia
Thin hollow cylindrical shell (or hoop)	Central axis of cylinder	MR^2	Solid sphere	Through center	$\frac{2}{5}MR^2$
Solid cylinder (or disk)	Central axis of cylinder	$\frac{1}{2}MR^2$	Thin hollow spherical shell	Through center	$\frac{2}{3}MR^2$
Hollow cylindrical shell or disk (Top view)	Central axis of cylinder	$\frac{1}{2}M(a^2+b^2)$	Thin rod (or rectangular plate)	Perpendicular to rod through end (or along edge of plate)	$\frac{1}{3}ML^2$
Rectangular plate	Perpendicular to plate through center	$\frac{1}{12}M(a^2+b^2)$	Thin rod (or rectangular plate)	Perpendicular to rod through center (or parallel to edge of plate through center)	$\frac{1}{12}ML^2$

smaller rotational inertia



larger α

larger rotational inertia



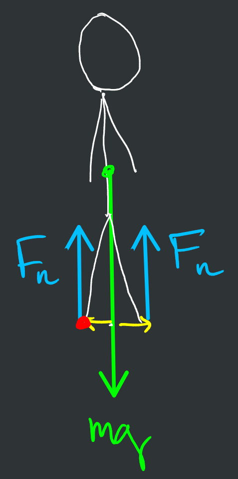
smaller α

same torque

Rule of Thumb for Rotational Inertia:

the farther the mass is from the axis of rotation, the larger the rotational inertia

After this you can
- define the conditions for an object or system to be in equilibrium.
- use the conditions of equilibrium to solve a balance problem for an unknown quantity.

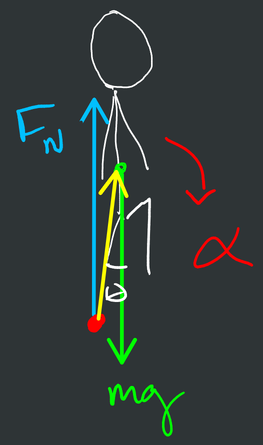


$$\Sigma F = 0$$

$$F_n + F_n - mg = 0$$

$$2F_n = mg$$

$$F_n = \frac{mg}{2}$$

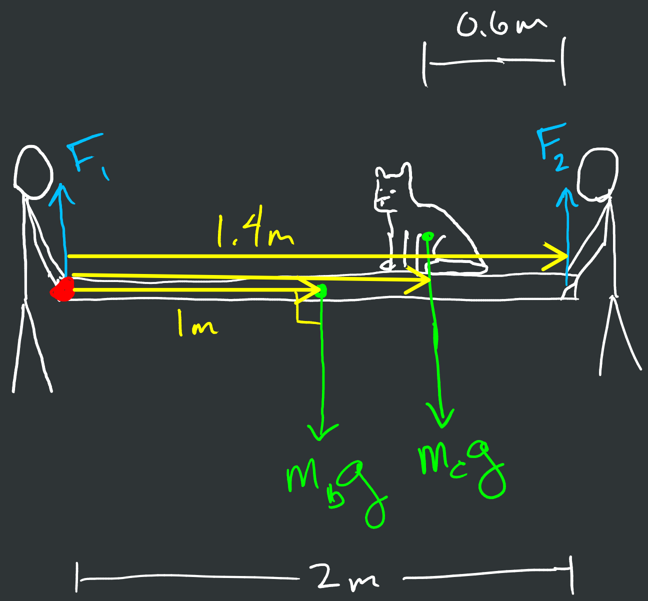


$$\tau_n = 0 = F_n \cdot \sin\theta \cdot \underbrace{r_n}_{0}$$

$$\tau_{mg} = mg \sin\theta \cdot r_{mg} \neq 0$$

$$\Sigma F = 0 \qquad \Sigma \tau \neq 0$$

So equilibrium: $\Sigma F = 0$ + $\Sigma \tau = 0$



know:

$m_b = 10\,kg$

$m_c = 10\,kg$

want:

$F_1 = ?$

$F_2 = ?$

$$\Sigma F = 0$$

$$F_1 + F_2 - m_b g - m_c g = 0$$

$$\Sigma \tau = 0$$

$$\underbrace{\cancel{F_1(0m)}}_{=0} - m_b g \cdot (1m) - m_c g(1.4m) + \boxed{F_2}(2m) = 0$$

$$\underbrace{-10kg(9.8\tfrac{N}{kg})(1m)}_{-98\,Nm} - \underbrace{(10kg)(9.8\tfrac{N}{kg})(1.4m)}_{-137.2\,Nm} + F_2(2m) = 0$$

$$-98\,Nm - 137.2\,Nm + (2m)F_2 = 0$$

$$(2m)F_2 = 235.2\,Nm$$

$$\boxed{F_2 = 117.6\,N}$$

$$F_1 + 117.6 - 10kg(9.8\tfrac{N}{kg}) - 10kg(9.8\tfrac{N}{kg}) = 0$$

$$F_1 - 78.4\,N = 0$$

$$\boxed{F_1 = 78.4\,N}$$