After this you can
- discuss the quantity of momentum and how it changes.
- calculate the momentum of an object within a system or a sytem as a whole.
- discuss the connection of conservation of momentum to Newton's Laws.
- use the principle of conservation of momentum to solve for an unknown quantity.

Momentum → quantity of motion

scalar   vector

→ simple product of mass + velocity

→ vector quantity

$$\vec{p} = m\vec{v} \qquad [kg]\left[\frac{m}{s}\right] = \frac{kg\,m}{s}$$

$p_x = mv_x$

~~$p_y = mv_y$~~

$-p_1 \qquad +p_2$

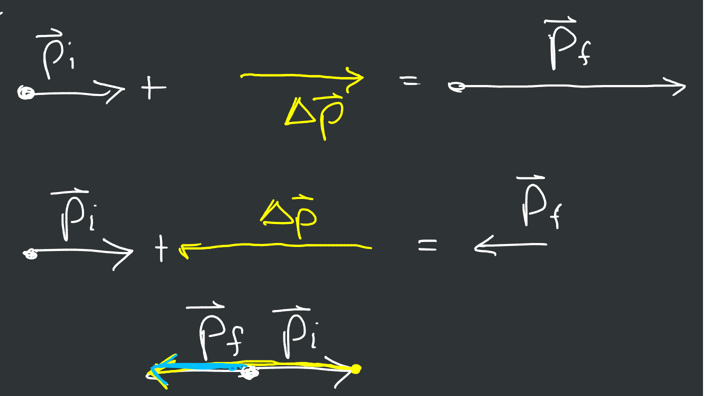
$+x$

multiple objects in a system

$$\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$$

change in momentum

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i \longrightarrow \vec{p}_i + \Delta\vec{p} = \vec{p}_f$$



$$\frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{m\,\Delta\vec{v}}{\Delta t} = m\vec{a} = \vec{F}_{NET}$$

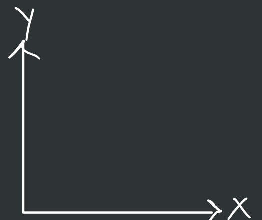
$m\vec{v}$

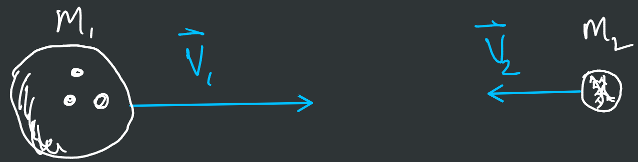
rate of change of momentum

mass has not changed

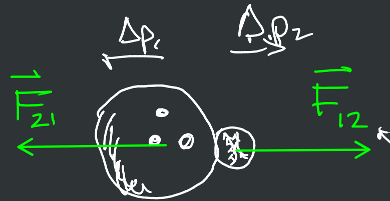
acceleration

$$\boxed{\frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{NET}} \longrightarrow \text{Newton's } 2^{nd} \text{ Law}$$





} system



force from baseball onto the bowling ball

force from bowling onto baseball

} internal forces

not constant ; short duration

$$\vec{F}_{21} = -\vec{F}_{12} \quad \text{or} \quad F_{12} = -F_{21}$$

$$\frac{\Delta \vec{p}_1}{\cancel{\Delta t}} = -\frac{\Delta \vec{p}_2}{\cancel{\Delta t}}$$

$$\Delta \vec{p}_1 = -\Delta \vec{p}_2$$

$$\vec{p}_{1f} - \vec{p}_{1i} = -(\vec{p}_{2f} - \vec{p}_{2i})$$

before/after the collision

$$\vec{p}_{1f} - \vec{p}_{1i} = -\vec{p}_{2f} + \vec{p}_{2i}$$

$$\underbrace{\vec{p}_{1f} + \vec{p}_{2f}}_{\text{total momentum after collision}} = \underbrace{\vec{p}_{1i} + \vec{p}_{2i}}_{\text{total momentum before the collision}}$$

- $m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$

After this you can
- discuss impulse and what it does.
- model non-constant forces as average forces over an interval.
- discuss applications of the impulse principle.

what cause a change in momentum → net force → rate of change of $\vec{p}$.

single objects

external net force

$$\frac{\Delta\vec{p}}{\Delta t} = \vec{F}_{NET}$$

time the force is acting

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\Delta\vec{p} = \underbrace{\vec{F}_{NET}\cdot\Delta t}$$

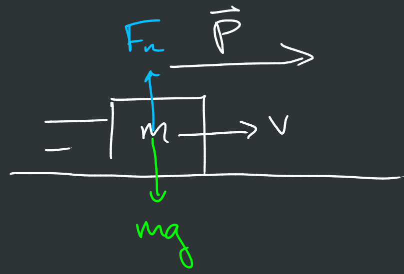
impulse ; assumes constant force

$$\Delta\vec{p} = \vec{J} = \vec{F}_{NET}\cdot\Delta t$$

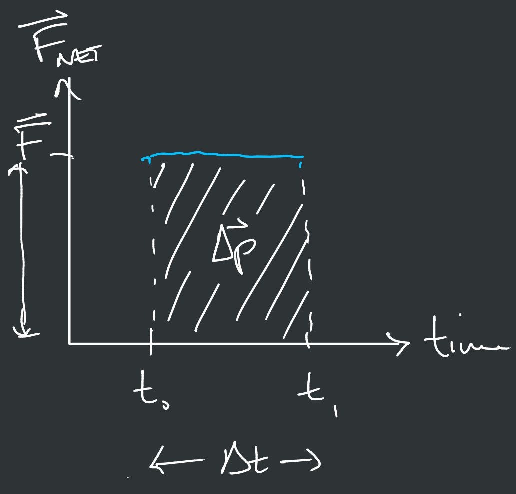
impulse causes a change in momentum

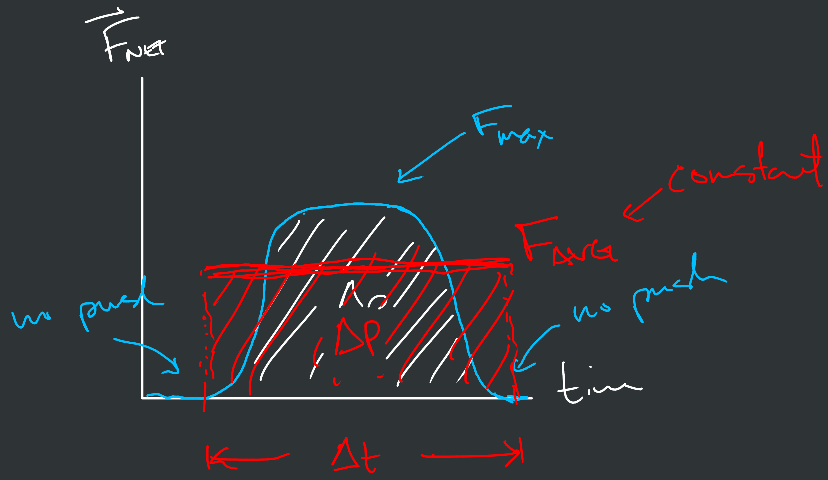
net force acting over time

$$\Delta\vec{p} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots)\Delta t$$



What if $\vec{F}_{NET}$ is not constant?





$$\Delta p = F_{AVG} \cdot \Delta t$$

Application

in a collision w/ something fixed

momentum goes zero.

$$\vec{p}_i = (1000\,kg)(+20\,m/s) = +20{,}000\,kg\,m/s$$

$$\vec{p}_f = (1000\,kg)(0\,m/s) = 0\,kg\,m/s$$

$$\Delta\vec{p} = 0 - (+20000\,kg\,m/s) = -20{,}000\,kg\,m/s$$

make time larger?

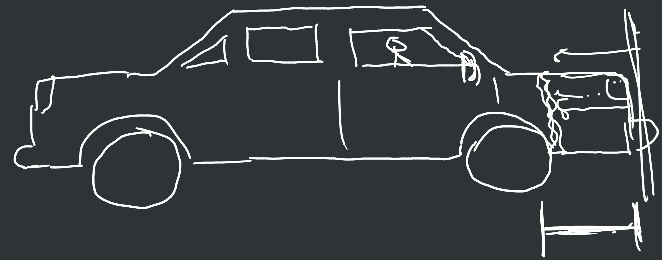
collision time

$$\Delta p = F_{AVG} \cdot \Delta t$$

fixed

this hurts

how can we make this less



crush zone

Δt increases w/ a larger crush zone

Δt increases

$$\frac{\Delta p}{\Delta t} = F_{AVG}$$

After this you can

- differentiate types of collisions based on the conservation of kinetic energy

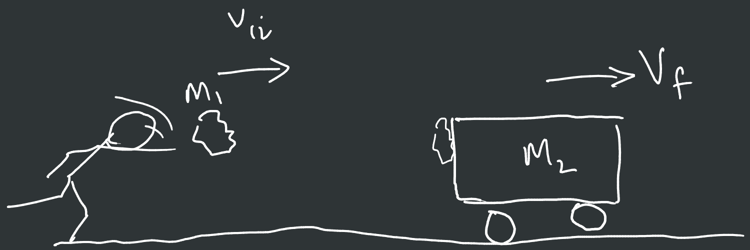
types of collisions

elastic collisions → kinetic energy is conserved
- perfect bounce

inelastic collisions → some kinetic energy
$K_f < K_i$ → where does it go?
- heat → internal energy
- sound
- deformation

perfectly inelastic collisions → objects stick together
- form a single object
- combined mass
- single velocity

Perfectly Inelastic Collision



conservation of momentum

$$\underbrace{m_1v_{1i} + \cancel{m_2v_{2i}}^{\,0}}_{\text{before}} = m_1v_{1f} + m_2v_{2f}$$

$$v_{1f} = v_{2f} = v_f$$

knowns	want
$v_{2i}=0$	v_f
v_{1i}	
m_1	
m_2	

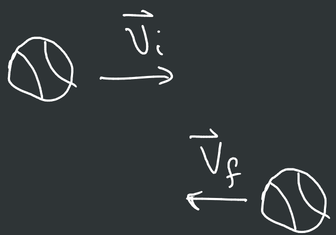
$$m_1v_{1i} = m_1v_f + m_2v_f$$

$$m_1v_{1i} = \underbrace{(m_1 + m_2)}_{\text{combined}}v_f$$

$$\frac{m_1v_{1i}}{(m_1+m_2)} = v_f$$

Inelastic Collision

fixed wall



$$\frac{K_f}{K_i} = ?$$

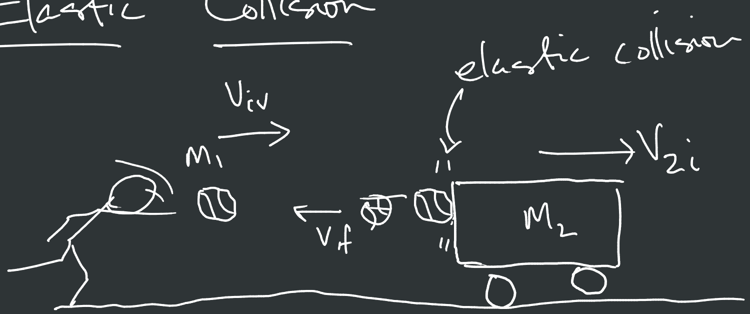
$K \propto v^2$

$$\frac{K_f}{K_i} = \left(\frac{v_f}{v_i}\right)^2$$

$$\underline{\Delta\vec{p}} = m\vec{v}_f - m\vec{v}_i$$

$|v_f| < |v_i|$

Elastic Collision



know:
- M_1
- M_2
- $\vec{V}_{1i}$
- $\vec{V}_{2i}$

want
- $\vec{V}_{1f}$
- $\vec{V}_{2f}$

- $M_1V_{1i} + M_2V_{2i} = M_1V_{1f} + M_2V_{2f}$

- $\underbrace{\frac{1}{2}m_1V_{1i}^2 + \frac{1}{2}m_2V_{2i}^2}_{\text{before}} = \underbrace{\frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2}_{\text{after}}$

Results: $V_{1f} = \frac{(M_1 - M_2)}{(M_1 + M_2)} \cdot V_{1i} + \frac{2m_2}{(m_1 + m_2)} \cdot V_{2i}$

$V_{2f} = V_{1f} + V_{1i} - V_{2i}$

- $M_1V_{1i} + M_2V_{2i} = M_1V_{1f} + M_2V_{2f}$

$-M_1V_{1f} + M_1V_{1i} = M_2V_{2f} - M_2V_{2i}$

(A) $\boxed{-m_1(V_{1f} - V_{1i}) = M_2(V_{2f} - V_{2i})}$

- $\cancel{\frac{1}{2}}m_1V_{1i}^2 + \cancel{\frac{1}{2}}m_2V_{2i}^2 = \cancel{\frac{1}{2}}m_1V_{1f}^2 + \cancel{\frac{1}{2}}m_2V_{2f}^2$

$-m_1V_{1f}^2 + m_1V_{1i}^2 = m_2V_{2f}^2 - m_2V_{2i}^2$

$-m_1\underbrace{(V_{1f}^2 - V_{1i}^2)}_{\text{difference of square}} = m_2\underbrace{(V_{2f}^2 - V_{2i}^2)}$

(B) $\boxed{-m_1(V_{1f} + V_{1i})(V_{1f} - V_{2i}) = M_2(V_{2f} + V_{2i})(V_{2f} - V_{2i})}$

$$\frac{eq\ B}{eq\ A} \qquad \frac{-\cancel{m_1}(v_{1f}+v_{1i})\cancel{(v_{1f}-v_{2i})}}{-\cancel{m_1}\cancel{(v_{1f}-v_{1i})}} = \frac{\cancel{m_2}(v_{2f}+v_{2i})\cancel{(v_{2f}-v_{2i})}}{\cancel{m_2}\cancel{(v_{2f}-v_{2i})}}$$

$$v_{1f}+v_{1i} = v_{2f}+v_{2i}$$

$$\boxed{v_{2f} = v_{1f} + v_{1i} - v_{2i}}$$

plug in and substitute in eq A

$$\frac{-m_1}{m_2}(v_{1f}-v_{1i}) = \frac{m_2(v_{2f}-v_{2i})}{\cancel{m_2}}$$

$$\frac{-m_1}{m_2}(v_{1f}-v_{1i}) = v_{2f}-v_{2i}$$

$$-\frac{m_1}{m_2}v_{1f} + \frac{m_1}{m_2}v_{1i} = v_{1f} + v_{1i} \underbrace{- v_{2i} - v_{2i}}_{-2v_{2i}}$$

$$m_2\left[\frac{m_1}{m_2}v_{1i} - v_{1i} + 2v_{2i}\right] = \left[v_{1f} + \frac{m_1}{m_2}v_{1f}\right]m_2$$

$$m_1 v_{1i} - m_2 v_{1i} + 2 m_2 v_{2i} = m_2 v_{1f} + m_1 v_{1f}$$

$$\frac{v_{1i}(m_1 - m_2) + 2 m_2 v_{2i}}{(m_1 + m_2)} = \frac{v_{1f}\cancel{(m_1 + m_2)}}{\cancel{(m_1 + m_2)}}$$

$$\boxed{v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{2 m_2}{(m_1 + m_2)} \cdot v_{2i}}$$

$$v_{2f} = v_{1f} + v_{1i} - v_{2i}$$

$$v_{2f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{2 m_2}{(m_1 + m_2)} \cdot v_{2i} + v_{1i} - v_{2i}$$

$$v_{2f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + v_{1i} + \frac{2 m_2}{(m_1 + m_2)} \cdot v_{2i} - v_{2i}$$

$$\frac{m_1 + m_2}{m_1 + m_2} \qquad \frac{m_1 + m_2}{m_1 + m_2}$$

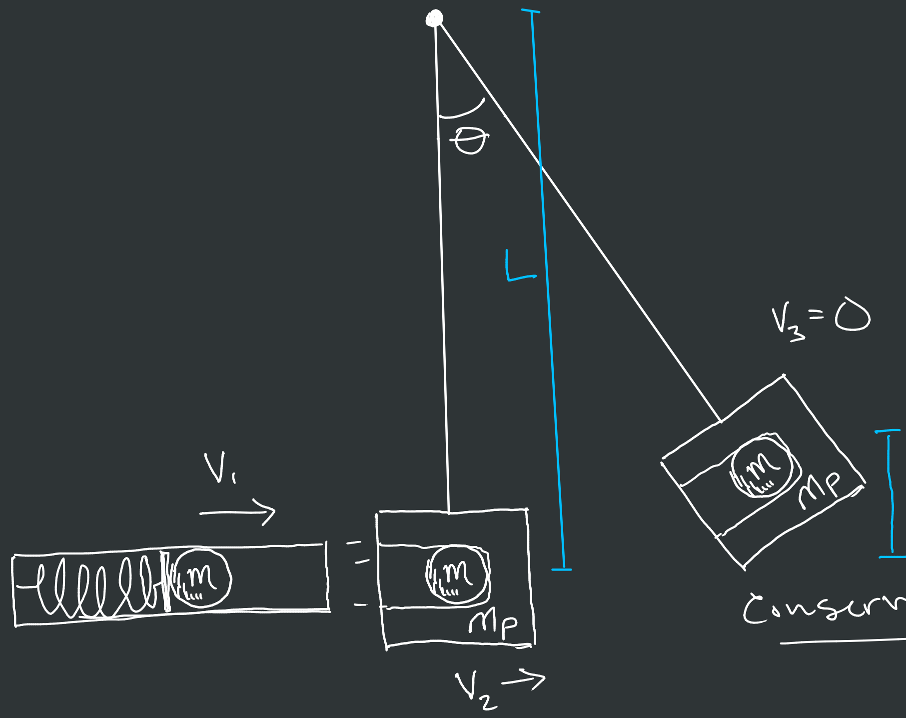
$$v_{2f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{m_1 + m_2}{(m_1 + m_2)} v_{1i} + \frac{2 m_2}{(m_1 + m_2)} \cdot v_{2i} - \frac{m_1 + m_2}{(m_1 + m_2)} v_{2i}$$

$$V_{2f} = \frac{m_1 - \cancel{m_2} + m_1 + \cancel{m_2}}{m_1 + m_2} \cdot V_{1i} + \frac{\cancel{2}m_2 - m_1 - \cancel{m_2}}{m_1 + m_2} \cdot V_{2i}$$

$$\boxed{V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_1 + m_2} V_{2i}}$$

$$\boxed{V_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i} + \frac{2m_2}{(m_1 + m_2)} \cdot V_{2i}}$$

Week 8 Lab - Ballistic Pendulum



$$h = L(1 - \cos\theta)$$

Conservation of energy

$$U_{gf} = K_i$$

$$\cancel{(m + m_p)} gh = \frac{1}{2}\cancel{(m + m_p)} v_2^2$$

$$\sqrt{2gh} = V_2$$

Conservation of momentum

$$\vec{p}_i = \vec{p}_f$$

$$mV_1 = mV_2 + m_p V_2$$

$$mV_1 = (m + m_p)V_2$$

$$V_1 = \frac{(m + m_p)}{m} V_2$$

horizontal

$$\Delta x = V_1 \Delta t$$

vertical

$$\Delta y = \cancel{V_{iy}\Delta t}^{\,0} + \frac{1}{2} a_y \Delta t^2$$

$$\Delta y = \frac{1}{2}(-9.8\,m/s^2)\,\Delta t^2$$

$$\sqrt{\frac{2 \cdot \Delta y}{g}} = \Delta t$$

$$\Delta x = V_1 \sqrt{\frac{2\Delta y}{g}}$$

$$\boxed{\Delta x \sqrt{\frac{g}{2\Delta y}} = V_1}$$