1. We have gone through several kinds of equations now and lets sum up some of these *as proportions*:

$\Sigma F = ma$

- acceleration is directly prop to the net force.

$a = \frac{\Sigma F}{m}$ $k = \frac{1}{m}$

- assuming constant acceleration and beginning at rest, an object's final velocity is prop to the square root of the displacement.

$v_f^2 = \cancel{v_i^2}^{0} + 2a\Delta x$

$v_f = \sqrt{2a\Delta x}$ $k = \sqrt{2a}$

- assuming constant acceleration and beginning at rest, an object's displacement is prop to the square of the elapsed time.

$\Delta x = \cancel{v_i t}^{0} + \frac{1}{2}at^2$

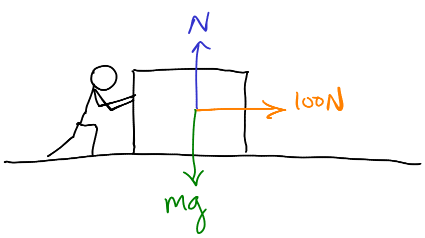
$\Delta x = \frac{1}{2}at^2$ $k = \frac{1}{2}a$

- for an object that has been dropped, the distance it has fallen is prop to the square of its velocity at that distance.

$v_f^2 = \cancel{v_i^2}^{0} + 2a\Delta y$

$\frac{\Delta y_2}{\Delta y_1} = \left(\frac{v_{f2}}{v_{f1}}\right)^2$ proportion equation $\boxed{\Delta y = \frac{v_f^2}{2a}}$ $k = \frac{1}{2a}$

2. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N for 10 s. How fast is the box going at this point? If I did the same thing to a 200 kg box, then how fast is it going after 10 s?



$\Sigma F_x = ma_x$

$100N = 100kg\, a_x$

$a_x = 1\ m/s^2$

$v_f = \cancel{v_i}^{0} + a \cdot t$

$v_f = 1 m/s \cdot 10s$

$v_f = 10 m/s$

$100N = 200kg \cdot a$

$a = 0.5\ m/s^2$

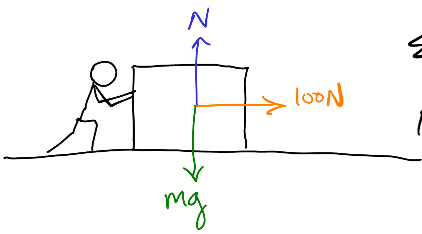
$v_f = \cancel{v_i}^{0} + 0.5 \cdot 10s$

$v_f = 5\ m/s$

3. I push a 100 kg box starting at rest along a friction-less floor, with a force of 100 N over a distance of 10 m. How fast is the box going at this point? If I did the same thing to a

200 kg box, then how fast is it going after 10 m?
Some starters:

- What is the net force on the box?
- What is the acceleration of the box?
- What is the final velocity after 10 m



$\Sigma F_x = m a_x$

$100N = 100kg\, a_x$

$a_x = 1\ m/s^2$

$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\,\Delta x$

$v_f = \sqrt{2(1\ m/s^2)(10\ m)}$

$v_f = 4.5\ m/s$

for 200kg

$a = 0.5\ m/s^2$

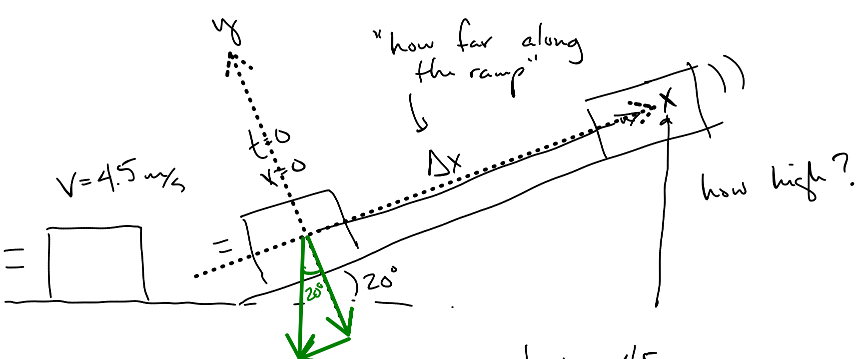
$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\,\Delta x$

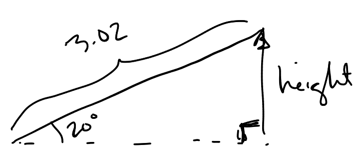
$v_f = \sqrt{2a\Delta x}$

$v_f = 3.2\ m/s$

4. Following up on the previous problem, if I stopped pushing after 10 m and the box continued with its speed, and then at then started sliding up a 20° ramp, then how far along the length of the ramp would the box rise? What height is this above the horizontal? Do the 100 kg and the 200 kg box rise to the same height?
Some starters:

- What is the net force on the box as it goes up the inclined plane?
- What is the acceleration of the box as it goes up the inclined plane?
- What is the sign of the displacement of the box going up the plane?
- Is the sign of acceleration the same or different than displacement?





$\sin 20° = \frac{height}{3.02}$

$height = 1.03\ m$

$a_x = -g\sin\theta$

$a_x = -9.8\ \frac{m}{s^2} \cdot \sin 20° = -3.35\ m/s^2$

$v_i = 4.5\ m/s$

$v_f = 0\ m/s$

$v_f^2 = v_i^2 + 2a\Delta x$

$0^2 = (4.5)^2 + 2(-3.35)\Delta x$

$\Delta x = 3.02\ m$

5. A 50-kg crate of apples is lowered by a rope straight down and has an acceleration of $1.0\,\mathrm{m/s^2}$ in the downward direction. What is the tension in the rope? State its magnitude and direction.

F_T

$m = 50\,kg$

$a = -1\,\frac{m}{s^2}$

$-mg = -50\,kg\left(9.8\,\frac{N}{kg}\right)$

$= -490\,N$

$\Sigma F_y = m a_y$

$\Sigma F_y = 50\,kg \cdot (-1\,m/s^2)$

$\Sigma F_y = -50\,N$

$F_T + F_G = \Sigma F_y$

$F_T - 490\,N = -50\,N$

$F_T = +440\,N$ up

6. A 900 kg elevator moves downward 11.0 m in 4 s. The tension in the supporting cable is 7800 N. What are the initial and final velocities of the elevator?

$+y$, $-y$

$7800\,N$

$900\,kg$

$mg = 900 \cdot 9.8 = 8820\,N$

$\Delta y = -11\,m$

$\Sigma F_y = -8820 + 7800 = -1020\,N = m a_y$

$-1020\,N = 900\,kg \cdot a_y$

$a_y = -1.13\,m/s^2$

- $v_f = v_i + at$
- $\Delta y = v_i t + \frac{1}{2} a_y t^2$
- $v_f^2 = v_i^2 + 2 a_y \Delta y$

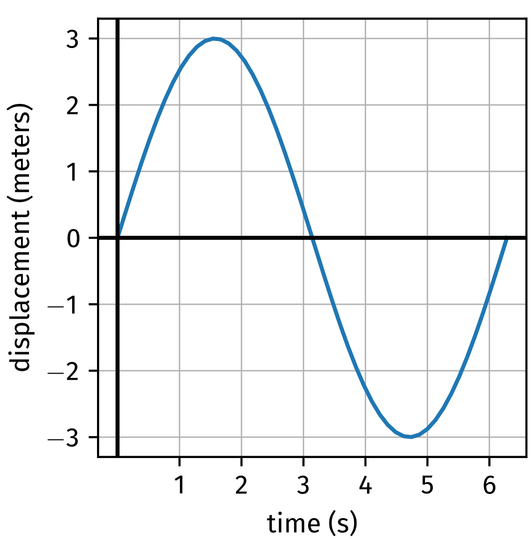
$-11\,m = v_i(4) + \frac{1}{2}(-1.13\,m/s^2)(4)^2$

$v_i = -0.49\,m/s$

$v_f = -0.49 + (-1.13)(4)$

$v_f = -5.01\,m/s$

7. Consider the graph below of displacement under condition of *not constant* acceleration. What would a graph of velocity vs. time look like for this case? What about acceleration vs time.



8. If a soccer ball with a radius of 10 cm is rolls along the ground without slipping at 5 m/s, then how many revolutions does it roll through in 10 s and what distance has a point on the edge of the ball traveled? Some starters:

   - How fast is it *spinning*? By that we mean *angular speed*.
   - How many radians does the ball rotate through in this time? What is that in revolutions?
   - How far does it roll in this time? Is this the same distance as the distance of a point on the edge of the ball? Why or why not?

$\omega = \frac{\Delta\theta}{\Delta t}$ $\boxed{\omega = \frac{v}{r}}$

$\omega = \frac{5\,m/s}{0.1\,m} = 50\,rad/s$

$\Delta\theta = \omega \cdot \Delta t$

$= 50\,rad/s \cdot 10\,s$

$\Delta\theta = 500\,rad$

$\boxed{s = \theta \cdot r}$ $s = 500\,rad \cdot 0.10\,m$

$s = 50\,m$

$v = \frac{\Delta x}{\Delta t}$

$\Delta x = v \cdot \Delta t$

$= 5\,m/s \cdot 10\,s$

$= 50\,m$ ✓

$\Delta\theta = 500\,rad \cdot \frac{1\,rev}{2\pi\,rad} = 79.6\,rev$

9. Following up on the previous problem, how many seconds does it take for the ball to complete one revolution? This amount of time is referred to as the *period* of its rotation, and this is a similar characteristic time for the motion of the ball as the *period of a pendulum* was in the first lab.

$\omega = \frac{\Delta\theta}{\Delta t}$

$\Delta t = \frac{\Delta\theta}{\omega}$ ← 1 rev = 2π

$\Delta t = \frac{2\pi\,rad}{50\,rad/s}$

$T = \Delta t = 0.126\,s$

$\frac{79.6\,rev}{10\,sec}$ $\frac{10\,sec}{79.6\,rev} = 0.126\,s$

10. Another follow up. How many revolutions does the ball travel through *per second*? You could convert this from angular speed ω that you would have calculated in the first instance of this problem, but if all you new was the period of the ball's rotation, how could you calculate it from there? *(Hint: what is the difference between revolutions per second and seconds per revoution?)* This quantity of revolutions per unit of time is sometimes called *frequency*.

using ω from the previous problem:

$$\underbrace{\omega}_{\text{angular velocity}} = 50\,\frac{rad}{s} \cdot \frac{1\ rev}{2\pi\ rad} = \underbrace{7.96\,\frac{rev}{s}}_{\text{frequency}}$$

using only period:

$$\text{period} \rightarrow T = 0.126\,\frac{sec}{rev}$$

$$\frac{10\,sec}{79.6\,rev} \rightarrow \frac{79.6\,rev}{10\,sec}$$

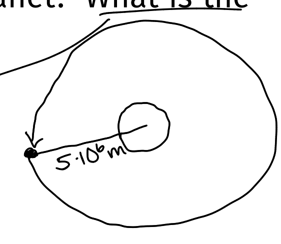
$$f = \frac{1}{T} = \frac{1}{0.126\,\frac{sec}{rev}} = 7.96\,\frac{rev}{sec}$$

✓✓

$$(\cancel{r^2})\,\frac{Gm_1m_2}{\cancel{r^2}} = \frac{mv^2}{\cancel{r}}\,(\cancel{r^2})$$

$$v = \omega\cdot r = 3.6\cdot10^{-5}\,\frac{rad}{s}\cdot 5\cdot10^{6}\,m =$$

11. Suppose a satellite is in orbit around a distant planet. You observe the the satellite to be 5000 km from the center of the planet, and rotating the planet once every 2 days. What is the mass of the planet you have discovered? What is the period of the satellites motion? What is its frequency? How fast is the satellite moving around the planet? What is the angular speed?



planet's mass

satellite's mass

$$\frac{Gm_1\cancel{m_2}}{r^2} = \Sigma F_R = \cancel{m_2}\,\omega^2\cdot r$$

$$(r^2)\,\frac{Gm_1\cancel{m_2}}{r^2} = \cancel{m_2}\,\omega^2\cdot r\,(r^2)$$

$$m_1 = \frac{\omega^2\cdot r^3}{G} = \frac{(3.6\cdot10^{-5})^2\cdot(5\cdot10^{6})^3}{6.67\cdot10^{-11}} = 2.4\cdot10^{21}\ kg$$

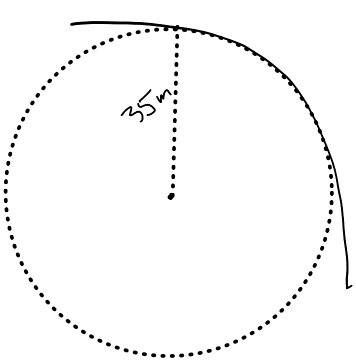
$$2\,days\cdot\frac{24\,hr}{1\,day}\cdot\frac{3600\,s}{1\,hr} = 172{,}800\,s$$

$$\omega = \frac{2\pi}{172{,}800\,s} = 3.6\cdot10^{-5}\,\frac{rad}{s}$$

$$\text{period} \rightarrow T = 2\,day = 172{,}800\,s$$

$$\text{frequency} = \frac{1}{T} = \frac{1}{172{,}800\,s} =$$

12. A car travels at 17 m/s without skidding around a level curved road with a radius of 35m. What is the coefficient of static friction between the tires and the road if this speed is the fastest it can go around this curve without skidding?



$$\mu mg = \Sigma F_R = \frac{mv^2}{r} = \frac{m\cdot17^2}{35}$$

$$\mu \cancel{m} g = 8.26\,\cancel{m}$$

$$\mu = 0.84$$

#9

$$\boxed{\frac{Gm_1 \cancel{m_2}}{r^2} = \cancel{m_2}\omega^2 r}$$

$$\omega_1 = \frac{2\pi}{time_1}$$

$$\omega = \left(\frac{Gm_1}{r^3}\right)^{1/2}$$

$$r_2 = 4r$$

$$r_1 = r$$

$$\boxed{\frac{\omega_2}{\omega_1} = \left(\frac{r_2}{r_1}\right)^{-3/2}} \leftarrow \omega \propto r^{-3/2}$$

$$\boxed{\frac{r_2}{r_1} = 4}$$

$$\omega_2 = \omega_1 \cdot (4)^{-3/2} = \frac{\omega_1}{8}$$

$$\omega_2 = \frac{2\pi}{time_2}$$

$$time_2 = \frac{2\pi}{\omega_2}$$

$$V = \frac{4}{3}\pi r^3$$

$$V \propto r^3$$

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3$$

2.7

$$\frac{V_2}{V_1} = 2.7^3 = 19.7$$