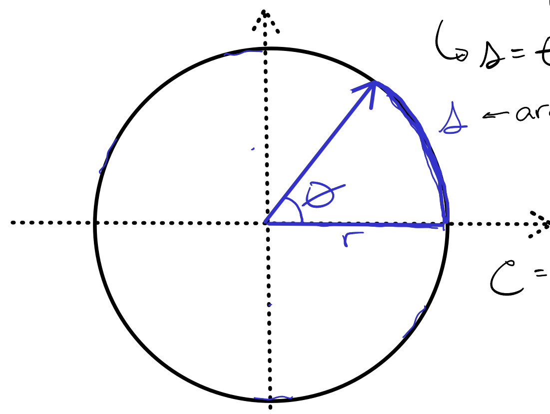
At the end of this worksheet you should be able to

- apply the relationships between angle and motion at the edge of a circle to describe the motion of an object in circular motion.
- apply Newton's 2nd law in the radial direction to solve interesting problems involving motion of objects in a circular path.
- apply the principles of radial net force and circular motion to planetary orbits and satellites as well horizontal and vertical paths near earth's surface.

$\theta = \frac{s}{r}$

$\hookrightarrow s = \theta \cdot r$

$s \leftarrow$ arc length




$C = 2\pi r$

1. How many degrees are in 1 rad?

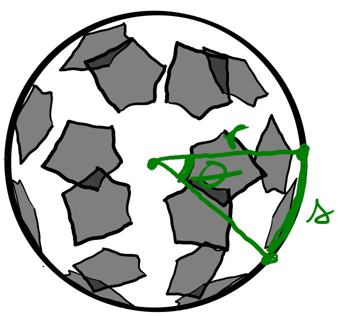
$1 \text{ rev} = 2\pi \text{ rad}$

$360^\circ = 2\pi \text{ rad}$

$\rightarrow 180^\circ = \pi \text{ rad}$

$1 \cancel{\text{rad}} \cdot \frac{180^\circ}{\pi \cancel{\text{rad}}} = 57.3^\circ$

2. A soccer ball of radius 10 cm spins through an angle of 20°, then how many radians is that? What distance has a point on the equator of the ball traveled? What if it spins through 750°, then what distance has a point on the edge traveled?




$20^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = 0.35 \text{ rad}$

$s = \theta \cdot r$

$s = 0.35 \text{rad} \cdot 10 \text{cm}$

$s = 3.5 \text{ cm}$

$750^\circ \cdot \frac{\pi}{180^\circ} = 13.1 \text{ rad}$

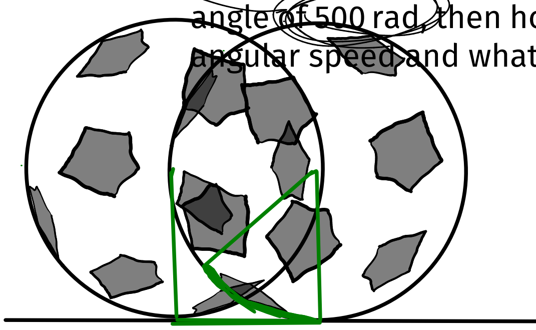
$s = 13.1 \text{rad} \cdot 10 \text{cm}$

$s = 131 \text{ cm}$

$500 \text{ rad} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} = 79.6 \text{ rev}$

3. When you roll something along the ground, it is spinning of course, but it is also moving linearly (its center of mass is moving). It turns out that the distance the edge of a soccer ball moves as it spins is equal to the linear distance the ball moves, as long as it does not slip. So if a soccer ball of radius 10 cm rolls at constant angular speed through an angle of 500 rad, then how far has it rolled? If it takes 10 seconds to do this, what was its angular speed and what was its linear speed?

$V = \frac{\Delta x}{\Delta t} = \frac{50 \text{m}}{10 \text{s}} = 5 \text{ m/s}$



$s = \theta \cdot r$

$s = 500 \text{rad} \cdot 10 \text{cm}$

$s = 5000 \text{cm}$

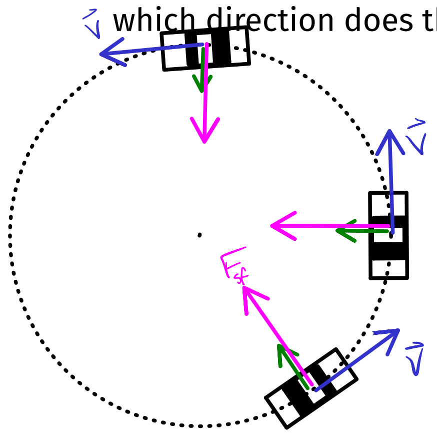
$s = 50 \text{ m} = \Delta x$

"omega" $\rightarrow \omega = \frac{\Delta\theta}{\Delta t} = \frac{500 \text{ rad}}{10 \text{ s}} = 50 \frac{\text{rad}}{\text{s}}$

$\hookrightarrow$ angular velocity

angular speed

4. When a car turns at constant speed, it travels along an approximately circular path. In which direction does the net force act and what provides this net force?

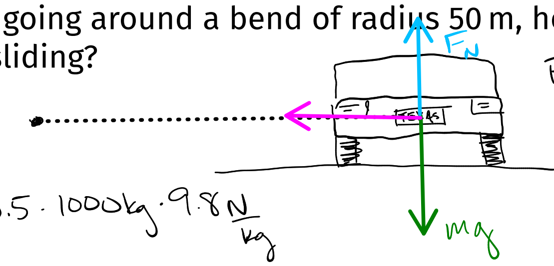



$a_R = \frac{v^2}{r} = \omega^2 r$ } applies to all circular motion

$\Sigma F_R = m a_R$

$\Sigma F_R = F_{s.f.}$ ← specific to this problem

5. For a 1000 kg car turning like in the previous problem, if the coefficient of static friction between the tires and the road is $\mu = 0.5$, then what is the maximum static force of friction that the road could provide to the car? If the car is going around a bend of radius 50 m, how how fast could it go around the bend without sliding? → $v = ?$




$F_N = mg$

$\Sigma F_R = F_{sf,max}$

$\Sigma F_R = m a_R$

$a_R = \frac{v^2}{r}$

$F_{sf} \leq \mu_s F_N$

$F_{sf,max} = \mu \cdot mg = 0.5 \cdot 1000kg \cdot 9.8 \frac{N}{kg}$

$F_{sf,max} = 4900N$

$F_{sf\,max} = \frac{m v^2}{r} \rightarrow 4900N = \frac{1000kg\, v^2}{50m} \rightarrow v = \sqrt{\frac{50 \cdot 4900}{1000}} = 15.7 \,m/s$

Bonus: what is the acceleration?

$\Sigma F_R = m a_R$

$\frac{4900N}{1000kg} = a_R = 4.9 \,m/s^2$

6. If the same 1000 kg car is attempting to go around a bend of radius 20 m, at 20 m/s, then can it do this safely without sliding? ($\mu = 0.5$ still)

$F_{sf,max} = 4900N$ ←

NO!

$F_{sf} = \frac{1000kg \cdot (20 m/s)^2}{20m}$

$= 20{,}000 N$

$4900N = \frac{1000kg \cdot (v)^2}{20m}$

$v = 9.89 \,m/s$

NO!

$\Sigma F_R = m a_R$

$\Sigma F_R = \frac{m v^2}{r}$

— or —

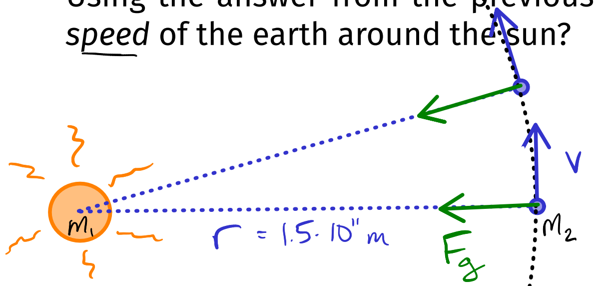
$\Sigma F_R = m \omega^2 r$

7. The earth orbits the sun, and while its path around the sun is not exactly circular, its close enough to treat that way here. What is the angular velocity of the earth around the sun? To do this, think about how long it takes to go one full revolution around the sun. How many radians is a revolution? So now how many radians per second does the earth travel around the sun?

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{3.154 \cdot 10^7 s} = 2 \cdot 10^{-7} \frac{rad}{s}$

$\Delta t = 1 \text{ yr} \cdot \frac{365 \text{ da}}{1 \text{ yr}} \cdot \frac{24 \text{ hr}}{1 \text{ da}} \cdot \frac{3600 s}{1 \text{ hr}} = 3.154 \cdot 10^7 s$

8. What is the radius between the earth and the sun? (look this up in your book or google) Using the answer from the previous problem, what does this mean for the *tangential speed* of the earth around the sun?




$$\omega = \frac{v}{r}$$

$$v = \omega \cdot r$$

$$v = 2 \cdot 10^{-7}\,\frac{rad}{s} \cdot 1.5 \cdot 10^{11}\,m$$

$$v = 3 \cdot 10^{4}\,\frac{m}{s} = 30{,}000\,m/s$$

9. Now without looking it up, how could we use this information to determine the mass of the sun? The formula for the force of gravity between to masses can be written as, $F_g = \frac{Gm_1m_2}{r^2}$ ($G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$). Note that this not the form of the force gravity that we have been using. Why is that? Now look up the mass of the sun and see how how close we got.

mass of sun, mass of earth

$$F_G = \frac{Gm_1m_2}{r^2}$$

mass of the earth

$$F_G = \Sigma F_R = \frac{m_2 v^2}{r}$$

$$\frac{Gm_1\cancel{m_2}}{r^{\cancel{2}}} = \frac{\cancel{m_2}v^2}{\cancel{r}}$$

$$m_1 = \frac{r \cdot v^2}{G} = \frac{1.5 \cdot 10^{11}\,m\,(3 \cdot 10^4\,m/s)^2}{6.67 \cdot 10^{-11}\,Nm^2/kg^2}$$

mass of sun

$$\boxed{m_1 = 2 \cdot 10^{30}\,kg}$$

10. By the way, how can we use free fall to get a measure of the mass of the earth? If we got to the lab and measure an acceleration of a 1 kg mass to be $9.82\,\mathrm{m/s^2}$, then how can we calculate the mass of the earth?

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$\Sigma F = m_2 a \quad a = 9.82\,m/s^2$$

mass of earth, mass of object

$$\frac{Gm_1\cancel{m_2}}{r^2} = \cancel{m_2} \cdot 9.82\,m/s^2$$

$r = 6.371 \cdot 10^{6}\,m$

radius of earth

mass of earth

$$m_1 = \frac{r_e^2 \cdot 9.82\,m/s^2}{G} = \boxed{5.98 \cdot 10^{24}\,kg = m_1}$$

11. In order to put a satellite into orbit around the earth, it needs to be traveling at a specific distance with a specific velocity, otherwise the force of gravity from the earth may be too

$\Sigma F_R = \frac{mv^2}{r} = m\omega^2 r$ $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{time}$



$10^6$ m

large, and it will crash, or too small and it will fly away into space. So suppose you wanted to put a 1000 kg satellite in orbit around the earth at a distance of 1000 km above the *surface of the earth*. How fast would this satellite need to be going in order to have this orbit?

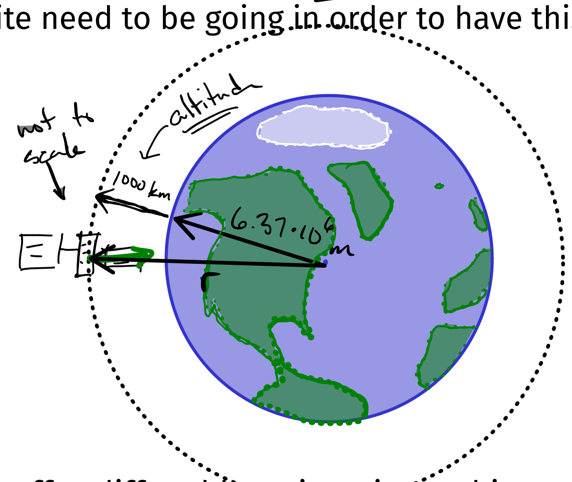
mass of earth mass of sat

$$F_G = \frac{G m_1 m_2}{r^2} = \Sigma F_R = \frac{m_2 v^2}{r} = \cancel{m\omega^2 r}$$

$r = r_e + alt$
$r = 6.37\cdot10^6\,m + 10^6$
$r = 7.37\cdot10^6\,m$

$$(r)\ \frac{G m_1 \cancel{m_2}}{r^{\cancel{2}}} = \frac{\cancel{m_2} v^2}{\cancel{r}}\ (r)$$

$$v = \sqrt{\frac{Gm_1}{r}} = \sqrt{\frac{6.67\cdot10^{-11}\cdot 6\cdot10^{24}\,kg}{7.37\cdot10^6\,m}} = 7368\,m/s$$
$$= 7370\,m/s$$



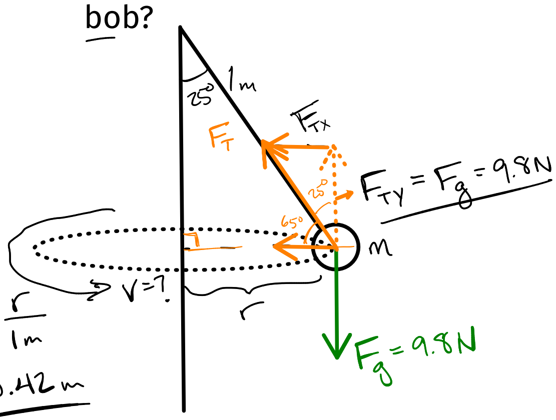

12. If you wanted to kick a soccer ball horizontally off a cliff and have it go into orbit near the surface of the earth, then what velocity would you need to give it to achieve this?

$$F_G = \frac{G M_e \cancel{m_b}}{r_e^2} = \frac{\cancel{m_b} v^2}{r_e} \longrightarrow v = \sqrt{\frac{GM_e}{r_e}} = 7900\,m/s$$

$g = 9.8\,N/kg$

$$g = \frac{v^2}{r_e}$$
$$v = \sqrt{g\cdot r_e} = 7900\,m/s$$

13. A pendulum is swinging in a horizontal circle. The length of the pendulum is 1 m. If the angle of the pendulum string is 25°, then what is the radius of travel of the pendulum bob?



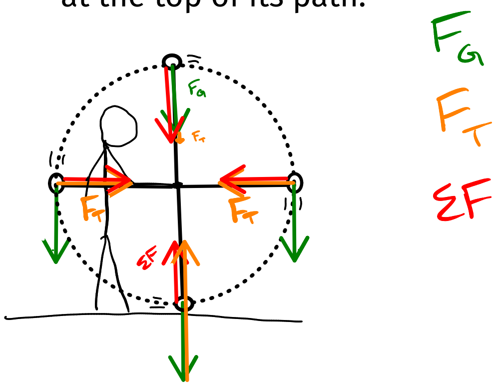

$\sin 25° = \frac{r}{1m}$
$r = 0.42\,m$

$\tan 25° = \frac{F_{Tx}}{9.8N}$
$F_{Tx} = 9.8N\cdot\tan25°$
$F_{Tx} = 4.6N$

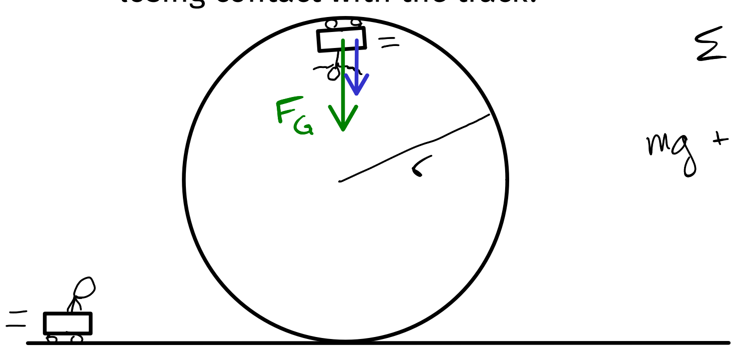
$\Sigma F_R = F_{Tx} = 4.6N$
$\Sigma F_R = \frac{mv^2}{r}$
$4.6N = \frac{1kg\cdot v^2}{0.42m}$
$v = 1.39\,m/s$

14. The mass of the pendulum bob from the previous problem is 1 kg. What upward force is necessary to keep the pendulum from moving up and down? What does this imply about the tension in the string? What does this mean for the radial tension force? How fast must this pendulum bob be moving?

15. When you are swinging a ball at the end of a string in a *vertical* circle, explain why the tension in the string is higher when the ball is at the bottom of its path, than when it is at the top of its path.




16. A roller coaster cart is doing a loop-the-loop. When the cart is at the top, what forces are acting on the cart to keep it in its circular path? What is the minimum force that would still technically mean that the cart is still in contact with the track? For a 30 m radius loop, what is the minimum speed that the cart must be going to make the loop without losing contact with the track?



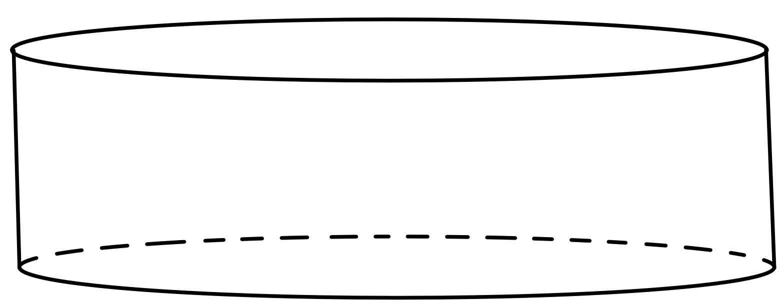

$$\Sigma F = \frac{mv^2}{r}$$

$$mg + \cancel{F_N}^{\to 0} = \frac{mv^2}{r}$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{gr}$$

Homework #7



**Homework: (#9)**

$$\omega = \frac{2\pi \text{ rad}}{\text{time}} \quad 7.8\text{ hr} \rightarrow \text{sec}$$

$$F_G = \frac{G m_1 m_2}{r^2} = \Sigma F_R = \frac{\cancel{m_2} v^2}{\cancel{r}} = m_2 \omega^2 r$$

mass of earth

$$(\cancel{r^2}) \frac{G m_1 \cancel{m_2}}{\cancel{r^2}} = \cancel{m_2} \omega^2 r \, (r^2)$$

$$G m_1 = \omega^2 r^3$$

$$r = \sqrt[3]{\frac{G m_1}{\omega^2}} \rightarrow r = r_e + \text{alt}$$

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**#11**

$$\Sigma F_R = F_g = m a_R$$

satellite

$$\frac{G m_1 \cancel{m_2}}{r^2} = \cancel{m_2} \omega^2 r$$

$$\omega = \left(\frac{G m_1}{r^3}\right)^{1/2}$$

$$\omega \propto r^{-3/2}$$

$$\frac{\omega_2}{\omega_1} = \left(\frac{r_2}{r_1}\right)^{-3/2}$$

$$\frac{r_2}{r_1} = 4$$

$$\frac{\omega_2}{\omega_1} = (4)^{-3/2} = 0.125$$

$$\boxed{G m_1} = \omega^2 r^3$$

$$\omega_1^2 r_1^3 = \omega_2^2 r_2^3$$

Sarah only

$$\omega_1 = \frac{2\pi}{T_1}$$

$$\omega_2 = 0.125 \cdot \omega_1$$

$$\omega_2 = \frac{2\pi}{T_2}$$

$$T_2 = \frac{2\pi}{\omega_2} \quad ✓$$

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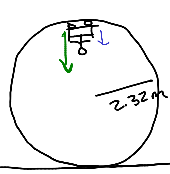
**#12**




$$\Sigma F_R = \frac{m v^2}{r}$$

normal force

$$+mg + \cancel{F_N}^{\,0} = \frac{m v^2}{r}$$

$$\cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$v = \sqrt{g r}$$

$$\frac{N}{m} \cdot m = \frac{m}{s^2} \cdot m$$