At the end of this worksheet you should be able to

- describe the quantities involved in discussing rotational motion and the connection of them to translational motion.
- calculate the quantities describing rotational motion.
- use the conditions of equilibrium to solve for an unknown quantity.
- use the principle of conservation of angular momentum to solve for an unknown quantity.

$$\Sigma F = ma \longrightarrow \Sigma \tau = I \cdot \alpha$$

1. In the lecture video, I talked about the analogy between linear (translational) motion and rotational motion. All of the most important equations that we have used have a corresponding version in rotational motion. I wrote down in the notes many of these rotational version, but I skipped over the kinematic equations so let's do them here. I am not interested in you working with these equations since we have conservation of energy and conservation of angular momentum now, but do this as an exercise in understanding the analogy.

$\Delta x \rightarrow \Delta\theta \leftarrow$ angular displacement

$v \rightarrow \omega \leftarrow$ angular velocity

$a \rightarrow \alpha \leftarrow$ angular acc.

$F \rightarrow \tau \leftarrow$ torque

$p \rightarrow L \leftarrow$ angular momentum

$$\left.\begin{aligned} v_f &= v_i + a\cdot t \\ v_f^2 &= v_i^2 + 2a\Delta x \\ \Delta x &= v_i t + \tfrac{1}{2}at^2 \end{aligned}\right\} \quad \begin{aligned} \omega_f &= \omega_i + \alpha\cdot t \\ \omega_f^2 &= \omega_i^2 + 2\alpha\Delta\theta \\ \Delta\theta &= \omega_i t + \tfrac{1}{2}\alpha t^2 \end{aligned}$$

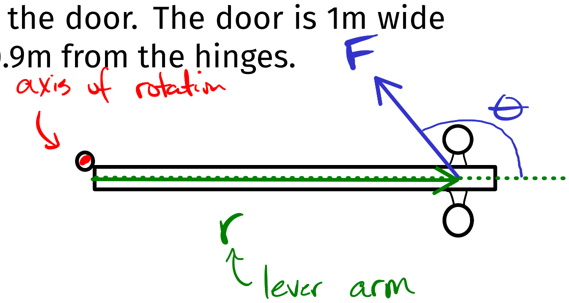
$K = \frac{1}{2}mv^2$

$K_{rot} = \frac{1}{2}I\omega^2$

$p = mv$

$L = I \cdot \omega$

2. You exert a force of 100N to open a door. To get the door swinging you exert this force for 1 second and you apply this force perpendicularly to the door. The door is 1m wide and 2m tall and has a mass of 30kg. The door handle is 0.9m from the hinges.



- What torque do you exert on the door?

$$\tau = |F| \sin\theta \, |r|$$

$$\tau = 100\text{N} \cdot \sin 90° \cdot 0.9\text{m} = 90\,\text{Nm}$$

- What is the rotational inertia of the door? $I = \frac{1}{3}ML^2$

$I \rightarrow$ moment of inertia

$$I = \tfrac{1}{3}(30\text{kg})(1\text{ m})^2$$

$$I = 10\,\text{kgm}^2$$

- What is the angular acceleration of the door?

$$\Sigma\tau = 90\,Nm = I \cdot \alpha$$

$$90\,Nm = 10\,kgm^2 \cdot \alpha$$

$$\alpha = \frac{90\,N\cancel{m}}{10\,kgm^2} = 9\,\frac{\cancel{kgm}}{s^2\,\cancel{kgm}} = 9\,\frac{rad}{s^2}$$

- How fast is the door moving when you stop pulling and let it swing?

$$\omega_f = \cancel{\omega_i}^{\,0} + \alpha t$$

$$\omega_f = 9\,\frac{rad}{\cancel{s^2}} \cdot 1\cancel{s} = 9\,\frac{rad}{s}$$

$$\Delta p = F \cdot \Delta t$$

- What is the initial angular momentum of the door? What angular impulse did you give the door?

$$L_i = I_i \cdot \cancel{\omega_i}^{\,0}$$

$$L_i = 0\,kgm^2/s$$

$$\Delta L = \tau \cdot \Delta t$$

$$= 90\,Nm \cdot 1s$$

$$= 90\,Nms = 90\,\frac{kgm}{\cancel{s^2}} \cdot m \cdot \cancel{s}$$

$$= 90\,kgm^2/s$$

- What is the door's final angular momentum? Use this to find the angular speed when you stop changing the door's momentum.

$$\Delta L = 90\,kgm^2/s = L_f - \cancel{L_i}^{\,0}$$

$$L_f = 90\,kgm^2/s$$

$$L_f = I_f \cdot \omega_f$$

$$= 10\,kgm^2 \cdot 9\,\frac{rad}{s}$$

$$= 90\,kgm^2/s$$

- What is the door's kinetic energy after you stop pulling?

$$K_f = \frac{1}{2} I_f \omega_f^2$$

$$= \frac{1}{2} \cdot 10\,kgm^2 \cdot \left(9\,\frac{rad}{s}\right)^2 = 405\,\frac{kgm^2}{s^2} = 405\,J$$

- What was its initial kinetic energy? How much work did you do to open the door?

$$K_i = 0\,J$$

$$\underbrace{K_i}_{0J} + \cancel{U_i}^{\,0} + W_{NC} = K_f + \cancel{U_f}^{\,0}$$

$$W_{NC} = K_f = 405\,J$$

- Over what angular displacement did you exert this 100N force?

$$\Delta\theta = \cancel{\omega_i t}^{\,0} + \frac{1}{2}\alpha t^2$$

$$= \frac{1}{2}\left(9\,\frac{rad}{s^2}\right)(1s)^2$$

$$= 4.5\,rad$$

$$W_{NC} \overset{?}{=} \tau \cdot \Delta\theta \;\checkmark$$

$$\Delta\theta \overset{?}{=} \frac{W_{NC}}{\tau} = \frac{405\,J}{90\,Nm} = 4.5\,rad$$

- I can't think of anything else to calculate about this; can you?

3. To generate torque with a wrench, you exert a force at the end of the wrench, 30 cm away from the bolt. You push with 500 N in the direction to loosen the bolt, but it will not budge. Why is it not rotating? In order to generate more torque, you go get a longer wrench, this one 55 cm.

   (a) Calculate the torque each wrench applies to the bolt.

   (b) By what factor have you changed the length of the lever arm?

   (c) By what factor have you changed the torque?

a) $\tau_1 = F \sin\theta \cdot r$

$= 500\,N \cdot \sin 90^\circ \cdot (0.30\,m)$

$= 150\,Nm$

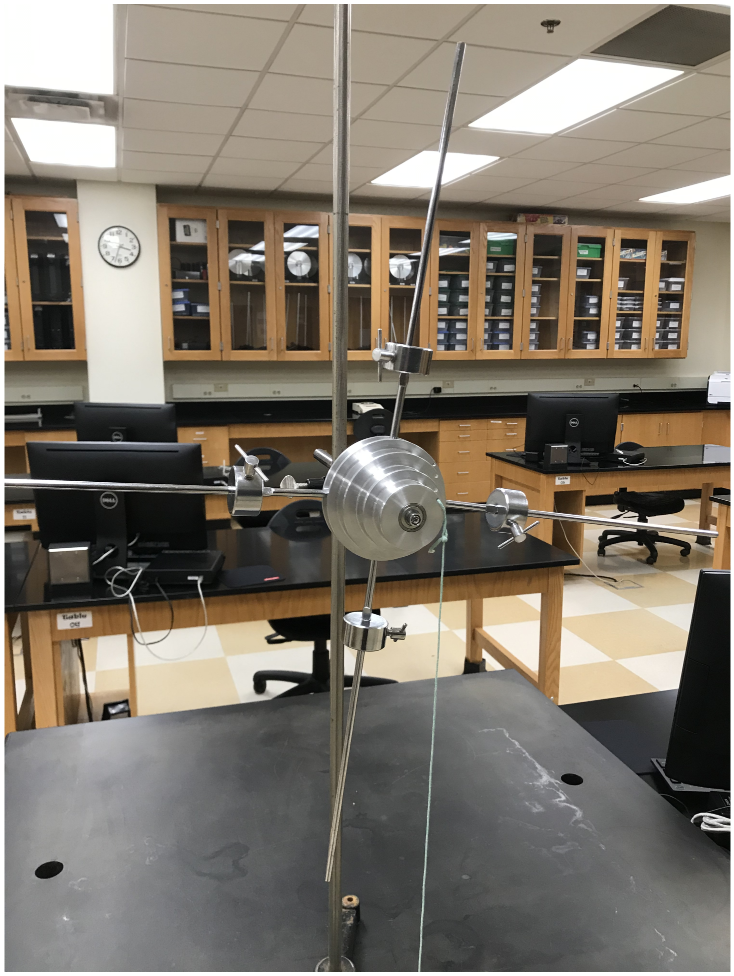
$\tau_2 = 500\,N \cdot \sin 90^\circ \cdot (0.55\,m)$

$\tau_2 = 275\,Nm$

b) + c) $\tau \propto r \rightarrow \frac{\tau_2}{\tau_1} = \frac{r_2}{r_1} = \frac{0.55\,m}{0.30\,m} = \underline{1.83}$

$\frac{\tau_2}{\tau_1} = 1.83$

4. Determine the rotational inertia of the compound pulley shown. Compare it to its theoretical value.



- $r_1 = 0.0202$ m, $r_2 = 0.028\,65$ m, $r_3 = 0.038\,52$ m
- $I_{disk} = 0.000\,58\ \mathrm{kg\,m^2}$, $4I_{rods} = 0.0127\ \mathrm{kg\,m^2}$, 1 mass $= 0.185$ kg
- $L_{rods} = 0.11$ m and 0.34 m
- $I_{total} \approx 0.098\ \mathrm{kg\,m^2}$ for fully extended, $0.013\ \mathrm{kg\,m^2}$

Trial	**mass**	r_{pulley}	r_{masses}	**Time (s)**	**a (m/s²)**
1	0.200	0.02865	0.34		
2	0.200	0.0202	0.34		
3	0.400	0.02865	0.34		
4	0.200	0.02865	0.11		

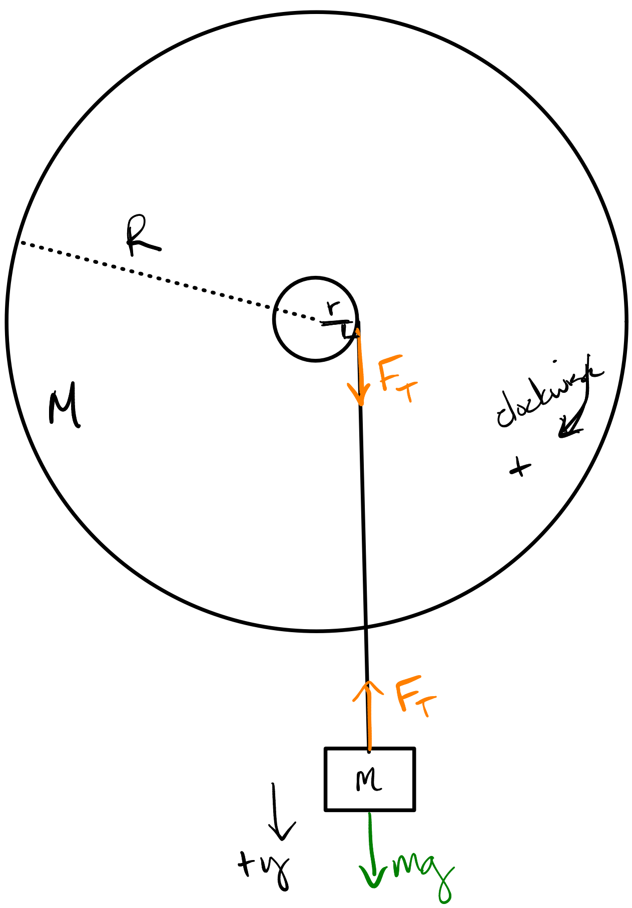
Be careful here! The force applied to the pulley is not simply mg. The hanging mass is accelerating, but not with $9.8\ \mathrm{m/s^2}$.

5. In lab this week, we will measure the rotational inertia of a disk and also of a hollow cylinder. To do this we will apply a constant torque to a pulley below the spinning object and measure the angular acceleration of the spinning object. To apply a constant force to the pulley, we will use a mass that will hang down from a string and have a constant force of gravity applied to it. Lets do this problem *in general* and develop an equation to calculate the rotational inertia from knowing the hanging mass, the radius at which the string is applying a torque to the pulley. *Be careful here! The force applied to the pulley is not mg. The hanging mass is accelerating, but not with* $9.8\,\mathrm{m/s^2}$
Some starters:

   - Draw a free body diagram on the hanging mass. You know the weight but you do not know the tension.
   - In lab we will measure the angular acceleration, so how can we write the acceleration fo the hanging mass in terms of the angular acceleration of the spinning pulley its attached to?

$I = MR^2$

compare to



• $\sum \tau = I \cdot \alpha$

$F_T \cdot r = I \cdot \alpha$

$\hookrightarrow F_T = \frac{I \cdot \alpha}{r}$

$s = \theta \cdot r$

$v = \omega \cdot r$

$a = \alpha \cdot r$

• $\sum F = ma$

$mg - F_T = ma$

$mg - F_T = m \cdot \alpha \cdot r$

$mg - \frac{I\alpha}{r} = m\alpha r$

$\left(-\frac{r}{\alpha}\right)\left[-\frac{I\alpha}{r} = m\alpha r - mg\right]\left(-\frac{r}{\alpha}\right)$

$I = m\alpha r\left(-\frac{r}{\alpha}\right) - mg\left(-\frac{r}{\alpha}\right)$

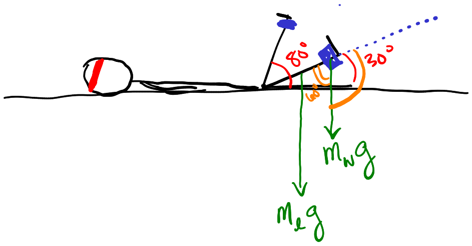
$I = -mr^2 + \frac{mgr}{\alpha}$

$\boxed{I = m\left(\frac{gr}{\alpha} - r^2\right)}$ ← use in lab

$I = MR^2$

6. A person laying on the floor is doing a leg lift with an ankle weight on. The ankle weight is 50 N. Explain why the exercise is hard at first when the leg is horizontal but very easy when the leg gets to vertical. Find the torque on the persons leg when their leg is at 30° with respect to the horizontal. The ankle weight is essentially at the end of their leg which is 1m long, and the mass of their leg is 12.75kg. Take the center of mass of the leg to be the geometric center of the leg. Calculate this again when their leg is at an 80° with respect to the horizontal.

$\Sigma\tau = \tau_l + \tau_w$



$F_{gl} \cdot \sin\theta \cdot r_l + F_{gw} \cdot \sin\theta \cdot r_w$

center of gravity

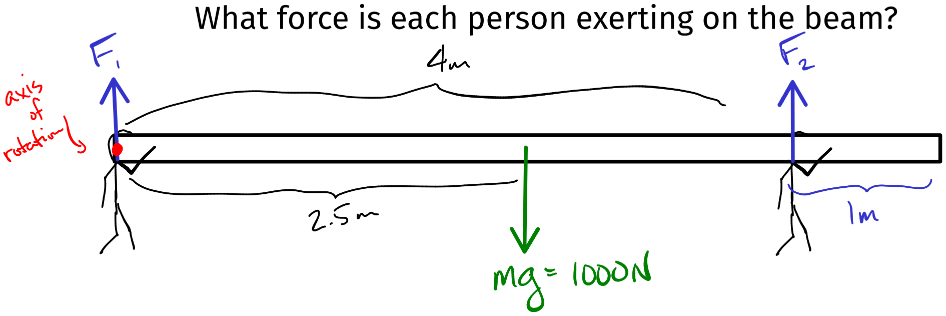
$= m_l g \sin\theta \cdot (0.5m) + \underbrace{m_w g}_{50N} \cdot \sin\theta \cdot (1m)$

$\Sigma\tau = 12.75 \cdot 9.8 \sin 120° \cdot 0.5 + 50N \cdot \sin 120° \cdot 1m$

$\Sigma\tau = 97.4 \, Nm$

$\Sigma\tau = 12.75 \cdot 9.8 \sin(170°) \cdot 0.5 + 50N \sin 170° \cdot 1m = 19.5 \, Nm$

7. Two people are carrying a 1000N wooden beam that is 5m long. One person is positioned at one end of the beam, and the other person is positioned 1m away from the other end. What force is each person exerting on the beam?



$\Sigma F = 0$

$F_1 + F_2 - 1000N = 0$

$\Sigma\tau = 0$

$F_1 \cdot 0m - 1000N \cdot 2.5m + F_2 \cdot 4m = 0$

8. Let's do the previous problem inside out. A question might read, "Where should the second person be positioned so that his force on the beam is ______________ ?" Provide the force from the previous question and solve for his position from one end.

9. Another followup, where would the second person need to be so that his force was 750 N?

10. Chris and Jamie are carrying Wayne on a horizontal stretcher. The uniform stretcher is 2.00 m long and weighs 100 N. Wayne weighs 800 N. Wayne's center of gravity is 75.0 cm from Chris. Chris and Jamie are at the ends of the stretcher. What force is Chris providing to support the stretcher? What force is Jamie providing to support the stretcher?

11. A pole-vaulter holds out a 4.75m pole horizontally in front of him. Assuming the pole is uniform in construction, and that he holds the pole with one hand at the very end, and one hand 0.75m from the end, what is the ratio of the force applied by the hand on the end of the pole to the weight of the pole?

12. A 80 kg child stands on a merry-go-round that spins with an angular velocity of 0.6 rev/s. The platform of the merry-go-round is 3 m in diameter and has a mass of 60 kg. The child initially stands 1.2 m from the axis of rotation. He moves closer to the center, ending up 0.3 m from the axis while the platform is spinning.

    (a) Calculate the rotational inertia of the child-platform system when the child is 1.2 m from the axis of rotation.

    (b) Calculate the rotational inertia of the child-platform system when the child is 0.3 m from the axis of rotation.

    (c) What is the final angular speed of the platform when the child is 0.3 m from the center? Express your answer in revolutions per second.

13. Three objects roll down an incline. Determine the final speed of each object using energy.

    (a) A solid sphere

(b) A solid cylinder

(c) A hollow cylinder (ring)

(d) How do these values compare to an object sliding down a frictionless incline?