At the end of this worksheet you should be able to

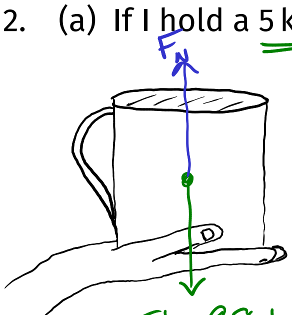
- discuss Newton's laws and provide examples of the application of each.
- apply Newton's first law to solve interesting physical problems.
- apply Newton's second law to solve interesting physical problems for objects that accelerate.
- apply Newton's third law to situations involving the motion of multiple objects.

1. What base units are the composite force units of Newtons equal to?

$$\Sigma F = m \cdot a$$

$$[\text{Newtons}] = [\text{kg}]\left[\frac{\text{m}}{\text{s}^2}\right]$$

2. (a) If I hold a 5 kg cup motionless in my hand, what force do I provide to the cup?

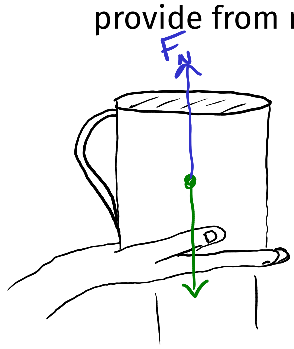


$5\text{kg} \cdot 9.8\frac{N}{kg} = 49N$

	x	y
F_G	0	$-49N$
F_N	0	F_N
	0	0

$F_N = 49N$

(b) If I raise a 5 kg cup with my hand, *at constant speed*, then what force do I need to provide from my hand to the cup?

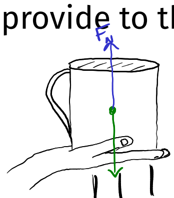


$5\text{kg} \cdot 9.8\frac{N}{kg} = 49N$

	x	y
F_G	0	$-49N$
F_N	0	F_N
	0	0

$F_N = 49N$

(c) If I accelerate the cup upwards with an acceleration of $+1\,\text{m/s}^2$, then what force does my hand need to provide to the cup?



$5\text{kg} \cdot 9.8\frac{N}{kg} = 49N$

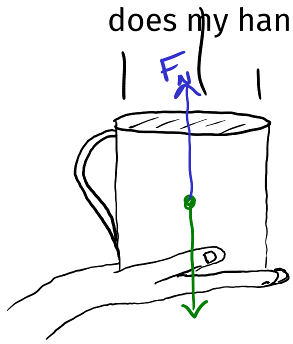
$$\Sigma F_y = m a_y$$
$$= 5\text{kg} \cdot (+1\,\text{m/s}^2)$$
$$\Sigma F_y = 5N$$

	x	y
F_G	0	$-49N$
F_N	0	F_N
	0	$5N$

$-49N + F_N = 5N$

$F_N = 54N$

(d) If I accelerate the cup downwards with an acceleration of $-1\,\mathrm{m/s^2}$, then what force does my hand need to provide to the cup?



$\Sigma F_y = m a_y$

$= 5\text{kg} \cdot (-1\,\text{m/s}^2)$

$\Sigma F_y = -5\text{N}$

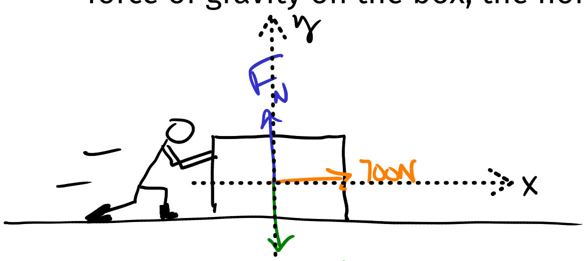
	x	y
F_G	0	-49N
F_N	0	F_N
	0	-5N

$-49\text{N} + F_N = -5\text{N}$

$F_N = 44\text{N}$

$5\text{kg} \cdot 9.8\frac{\text{N}}{\text{kg}} = 49\text{N}$

3. If I push a 100 kg box with an applied force of 700 N along a friction-less surface. Find the force of gravity on the box, the normal force, and the net force.



$F_G = 100\text{kg} \cdot 9.8\frac{\text{N}}{\text{kg}}$

$F_G = 980\text{N}$

	x	y
F_G	0	-980
F_N	0	F_N
F_A	$+700\text{N}$	0
ΣF	$+700\text{N}$	0

$-980 + F_N = 0$

$F_N = 980\text{N}$

vector addition

Newton's 2nd

$a_y = 0 \therefore \Sigma F_y = 0$

Bonus: what is the acceleration?

$\Sigma F_x = m a_x$

$700\text{N} = 100\text{kg} \cdot a_x \Rightarrow a_x = 7\,\text{m/s}^2$

coefficient of friction

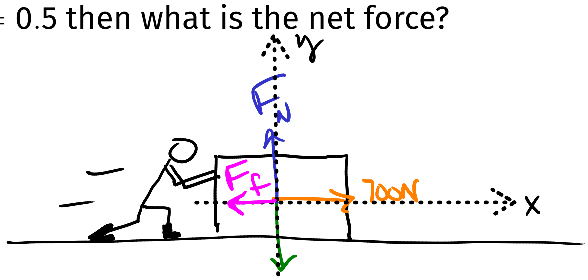
$\mu = \frac{|F_f|}{|F_N|}$

$|F_f| = \mu \cdot |F_N|$

$F_f = 0.5 \cdot 980\text{N}$

$= 490\text{N}$

4. Take the same problem from above and now add friction. The coefficient of friction is $\mu = 0.5$ then what is the net force?



$F_G = 100\text{kg} \cdot 9.8\frac{\text{N}}{\text{kg}}$

$F_G = 980\text{N}$

	x	y
F_G	0	-980N
F_N	0	F_N
F_A	$+700\text{N}$	0
F_f	-490N	0
ΣF	$+210\text{N}$	0

$\rightarrow +980\text{N}$

What is the acceleration now?

$210\text{N} = 100\text{kg} \cdot a_x \Rightarrow a_x = 2.1\,\text{m/s}^2$ ✓

5. For both of the previous problem, what happens when I stop pushing?

frictionless #3

$a_x \Rightarrow 0\,m/s^2$

frictionfull #4

	x	y
F_G	0	-980N
F_N	0	F_N
F_A	0	0
F_f	-490N	0
ΣF	-490N	0

$\Sigma F_x = m a_x$

$-490N = 100kg \cdot a_x$

$a_x = -4.9\,\frac{m}{s^2}$

6. What force do I need to exert to push the box at constant speed?

frictionless #3

$F_A = 0\,N$

frictionfull #4

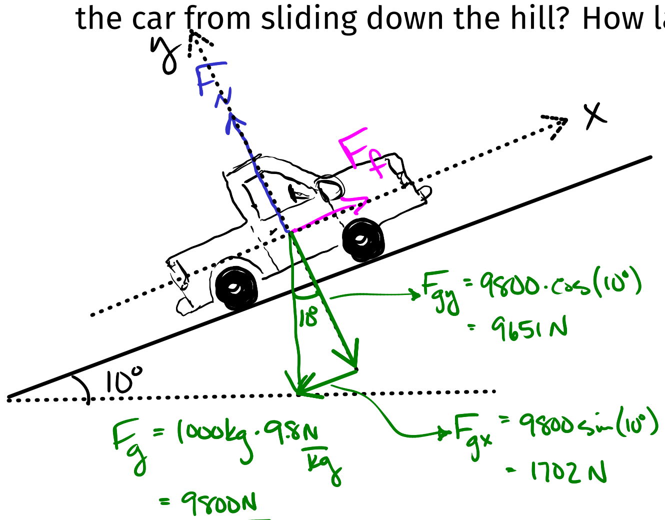
$F_A = 490N$

$\hookrightarrow a_x = 0$

	x	y
F_G	0	-980N
F_N	0	F_N
F_A	+490	0
F_f	-490N	0
ΣF	0	0

7. If I need to provide a 1000 N force to keep a 100 kg box moving at constant speed along a level floor, then what is the coefficient of friction between the floor and the box?

8. A 1000 kg car is parked on a hill that has an angle of 10° with respect to the horizontal. What is the weight of the car? What is the normal force on the car? What force is keeping the car from sliding down the hill? How large is that force?



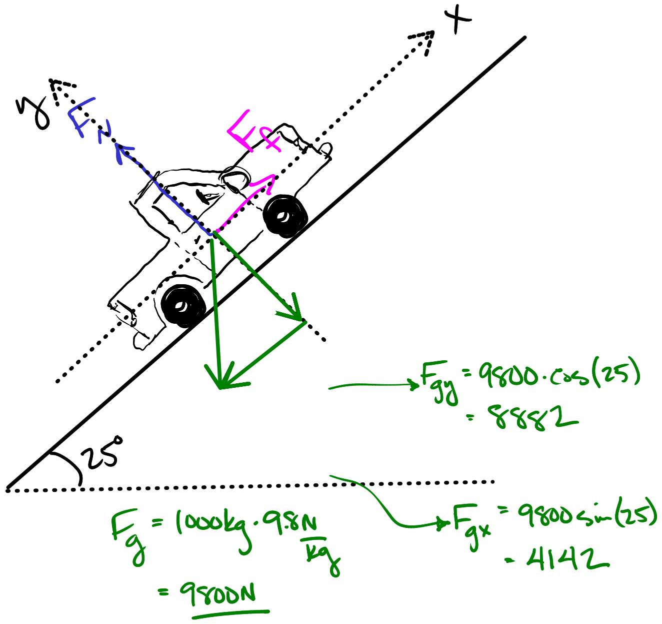
	x	y
F_G	-1702N	-9651N
F_N	0	F_N
F_f	F_f	0
ΣF	0	0

$F_N = +9651\,N$

$F_f = +1702\,N$

$F_f \leq \mu_s F_N$

9. If the maximum incline that the car can be parked on without sliding is 25°, then what is the coefficient of friction between the tires and the road?



	x	y
F_G	−4142	−8882
F_N	0	F_N
F_f	F_f	0
ΣF	0	0

$F_f = 4142$ N

$F_N = 8882$ N

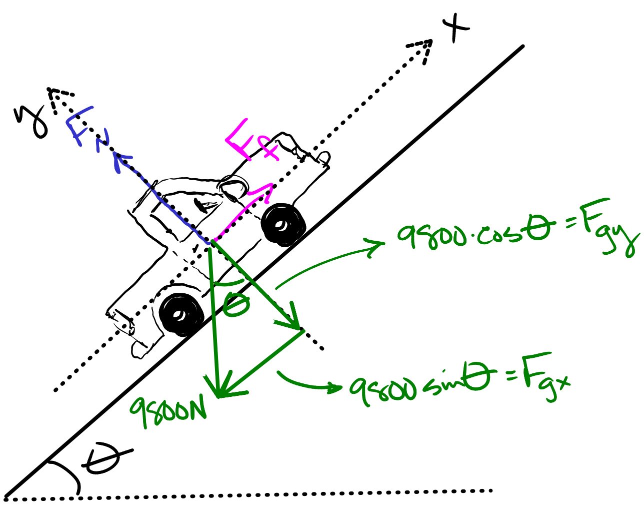
$$\mu_s = \frac{F_f}{F_N} \rightarrow \mu_s = \frac{4142}{8882} = \underline{0.47}$$

10. Work the previous problem inside-out.

$\mu = 0.47$

$\theta = ?$

if a 1000kg car is parked on an incline with a cof of 0.47, what is the steepest incline it can be parked on and not slide?



	x	y
F_G	$-9800 \sin\theta$	$-9800 \cos\theta$
F_N	0	F_N
F_f	F_f	0
ΣF	0	0

$F_N = 9800 \cos\theta$

$F_f = 9800 \sin\theta$

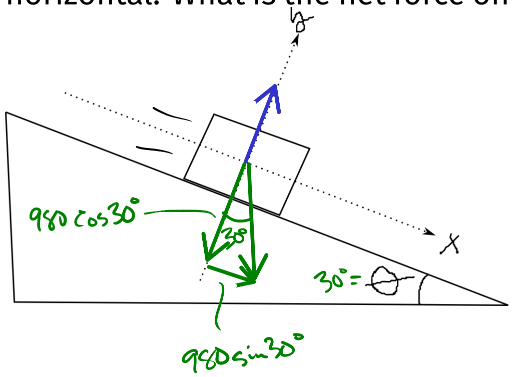
$$\mu = 0.47 = \frac{F_f}{F_N}$$

$$0.47 = \frac{\cancel{9800} \sin\theta}{\cancel{9800} \cos\theta}$$

$$0.47 = \tan\theta$$

$$\theta = 25°$$

11. A 100 kg box slides down a friction-less inclined plane that has an angle of 30° to the horizontal. What is the net force on the box and then what is the acceleration of the box?



	x	y	
F_G	+490N	-849N	
F_N	0	F_N	+849N
ΣF	+490N	0	

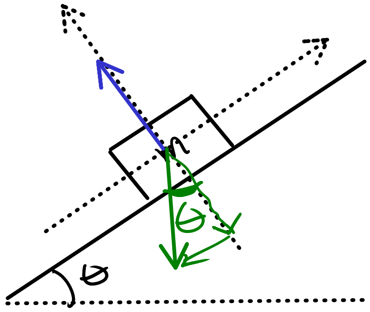
$\Sigma F_x = ma_x$

$a_x = +4.9 \ m/s^2$

12. A 100 kg box slides down a friction-full inclined plane that has an angle of 30° to the horizontal and a coefficient of friction of $\mu = 0.1$. What is the net force on the box and then what is the acceleration of the box?

$a_x = 4.05 \ m/s^2$

13. Let's do the friction-less inclined plane problem *in general* for any mass and incline. Follow the same procedure as before but with the variable m for mass and θ for incline angle. Find an expression for the net force on the mass as a function of θ and for the acceleration as a function of θ.



	x	y
F_G	$-mg\sin\theta$	$-mg\cos\theta$
F_N	0	F_N
ΣF	$-mg\sin\theta$	0

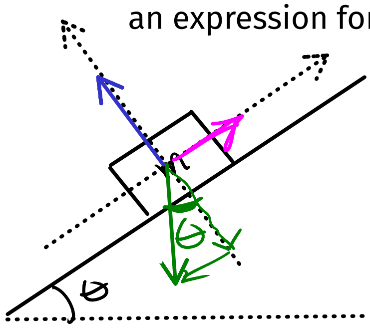
$F_N = mg\cos\theta$

$$\Sigma F_x = ma_x$$

$$\frac{-\cancel{m}g\sin\theta}{\cancel{m}} = \frac{\cancel{m}a_x}{\cancel{m}}$$

$$\boxed{a_x = -g\sin\theta}$$

14. Now let's do the inclined plane with friction *in general*. Just like the previous problem, use m for mass, θ for angle, and now use μ as a variable for coefficient of friction. Find an expression for the acceleration of the mass as a function of θ, m, and μ.



	x	y
F_G	$-mg\sin\theta$	$-mg\cos\theta$
F_N	0	F_N
F_f	$\mu mg\cos\theta$	0
ΣF	$\mu mg\cos\theta - mg\sin\theta$	0

$F_N = mg\cos\theta$

$F_f = \mu F_N$

$F_f = \mu mg\cos\theta$

Where is acceleration $a_x = 0$?
↳ What θ?

$0 = g(\mu\cos\theta - \sin\theta)$

$0 = \mu\cos\theta - \sin\theta$

$\sin\theta = \mu\cos\theta \Rightarrow \mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\theta = \tan^{-1}(\mu)$

$$\Sigma F_x = ma_x$$

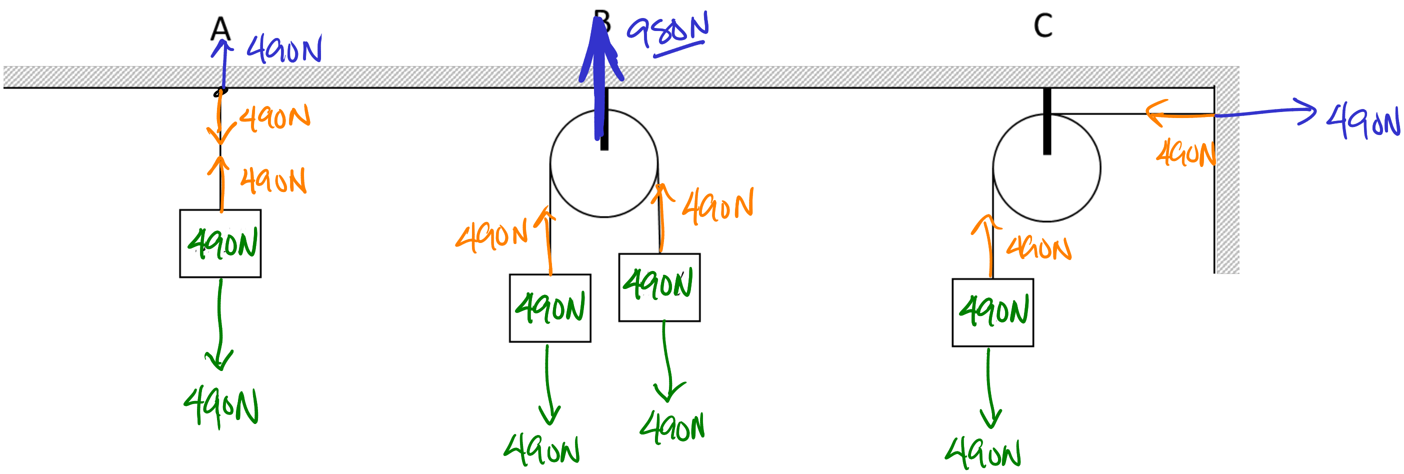
$$\mu mg\cos\theta - mg\sin\theta = ma_x$$

$$\cancel{m}(\mu g\cos\theta - g\sin\theta) = \cancel{m}a_x$$

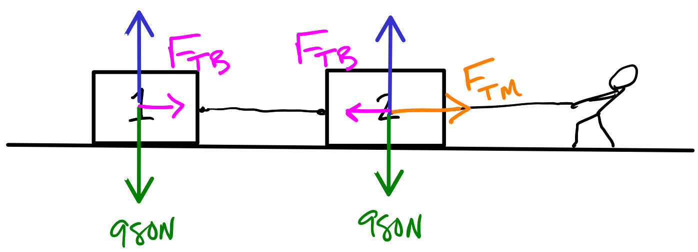
$$\mu g\cos\theta - g\sin\theta = a_x \Rightarrow g(\mu\cos\theta - \sin\theta) = a_x$$

15. In order to *hold* a box on a friction-less inclined plane, that is kept motionless, what force would be necessary to do that? Is there a difference between the force to hold it motionless on the incline and the force to push it up the incline *at constant speed*?

16. A 50 kg weight is suspended in three different ways shown below. What is the tension in each rope?



17. I am pulling two objects connected by a string. They are both 100 kg. There is no friction between the objects and the surface, and they are accelerating at $1\,\mathrm{m/s^2}$. What is my pulling force and what tension force is between the boxes?



$F_{TB} = m\,a$

$F_{TB} = 100\text{kg} \cdot 1\frac{m}{s^2}$

$F_{TB} = 100\text{N}$

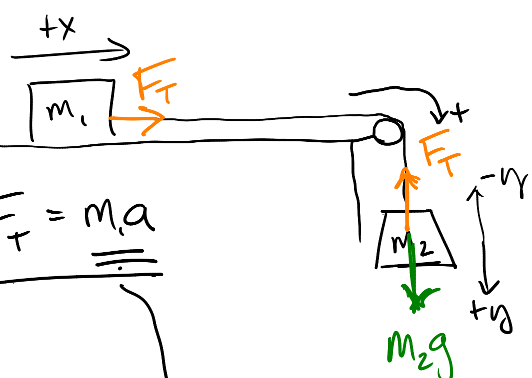
$F_{TM} - F_{TB} = m\,a$

$F_{TM} = ma + F_{TB}$

$F_{TM} = 100\text{kg} \cdot 1\frac{m}{s^2} + 100\text{N}$

$F_{TM} = 200\text{N}$

18. One object m_1 connected by a string to another object m_2 that is hanging off the edge of the table. There is a pulley at the edge of the table so the string is not introducing any friction to the system. If this is a friction-less table, then what is an expression for the acceleration of the objects?



$F_T = m_1 a$

$-F_T + m_2 g = m_2 a$

$-m_1 a + m_2 g = m_2 a$

$m_2 g = m_1 a + m_2 a$

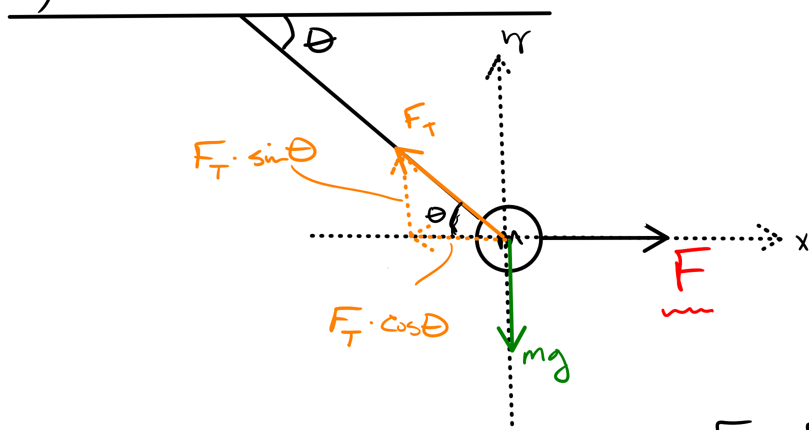
$= (m_1 + m_2) a$

$$\boxed{a = \frac{m_2 g}{m_1 + m_2}}$$

$$\boxed{F_T = \frac{m_1 m_2 g}{m_1 + m_2}}$$

19. Now do the previous problem again, but with a coefficient of friction μ between the sliding mass (m_1) and the surface.

9) + 10)



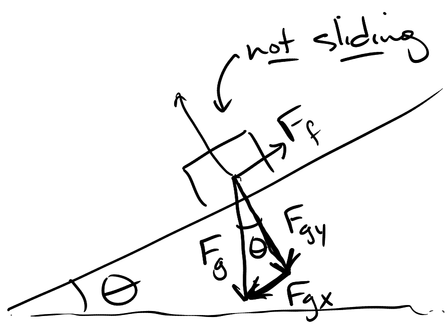
	x	y
F_g	0	$-mg$
F	F	0
F_T	$-F_T\cos\theta$	$+F_T\sin\theta$
F_{NET}	0	0

$F - F_T\cos\theta = 0$ | $-mg + F_T\sin\theta = 0$

?

$F_T = $ ___

#5)



μ_s

$\cancel{\mu_k}$

$\cancel{F_{f\,max} = \mu_s F_N}$

$\cancel{F_f \leq \mu_s F_N}$

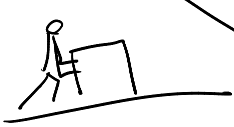
$F_{gx} = F_g \cdot \sin\theta$

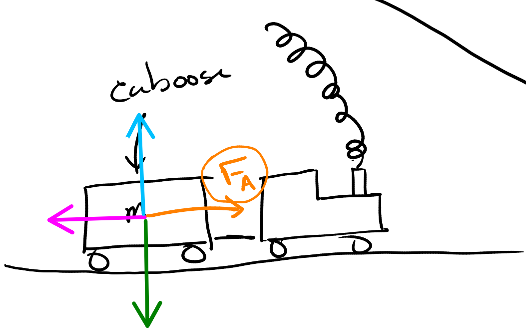
$F_{gy} = F_g \cdot \cos\theta$

	x	y
F_g	$-F_{gx}$	$-F_{gy}$
F_N	0	F_N
F_f	F_f	0
ΣF	0	0

$-F_{gx} + F_f = 0$

$F_f = F_{gx}$





$F_A + F_f = 2.1\,N$

given

$F_A = ?$ ←

	x	y
F_A	F_A	0
F_g	0	$-mg$
F_N	0	F_N
F_f	?	0
ΣF	2.1N	0

$F_N = mg$

F_f ← given

$\Sigma F_x = ma_x$

$= 1\,kg\,(2.1\,m/s^2)$

$= 2.1\,N$