

At the end of this worksheet you should be able to

- discuss the quantities of displacement, velocity, and acceleration.
- interpret the meaning of the sign of the above quantities.
- discuss the cause of the change of those quantities.
- take a given state of motion of an object and predict its future state of motion.
- identify key features of a graph of these quantities over time.
- create a qualitative plot of these quantities over time given an interesting physical problem.
- solve a range of problems under the conditions of constant acceleration.

1. I move from a place marked $x = +2 \text{ m}$ to a place marked $x = +10 \text{ m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative? Plot this displacement and velocity over time.

"displacement" = change in position
 $\Delta \vec{x} = \vec{x}_f - \vec{x}_i$

$\Delta \vec{x} = +10 \text{ m} - (+2 \text{ m})$
 $\Delta \vec{x} = +8 \text{ m}$
positive direction

$\vec{v}_{\text{avg}} = \frac{\Delta \vec{x}}{\Delta t}$
 $\vec{v}_{\text{avg}} = \frac{+8 \text{ m}}{4 \text{ s}}$
 $\vec{v}_{\text{avg}} = +2 \text{ m/s}$

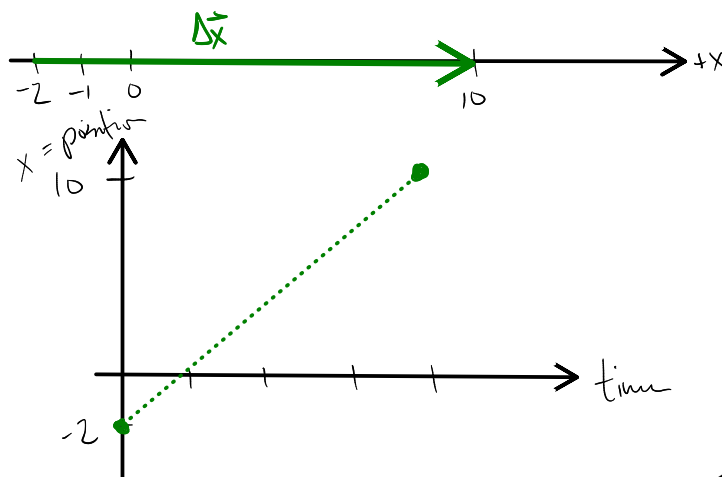
The diagram for problem 1 consists of two vertically aligned graphs. The top graph is a displacement vs. position plot. The horizontal axis is labeled $+x$ and has tick marks at 0, 2, and 10. A green arrow labeled $\Delta \vec{x}$ points from the position 2 to the position 10. The bottom graph is a velocity vs. time plot. The vertical axis is labeled $x = \text{position}$ and has tick marks at 2 and 10. The horizontal axis is labeled time and has four tick marks. A green dotted line starts at the point (0, 2) and ends at the point (4, 10). A green arrow points to this line with the text *slope = avg velocity = +2 m/s*.

2. I move from a place marked $x = -2 \text{ m}$ to a place marked $x = 10 \text{ m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative? Plot this displacement and velocity over time.

$$\Delta \vec{x} = +10 \text{ m} - (-2 \text{ m})$$

$$= +12 \text{ m}$$

$$\vec{v}_{\text{avg}} = \frac{+12 \text{ m}}{4 \text{ s}} = +3 \text{ m/s}$$

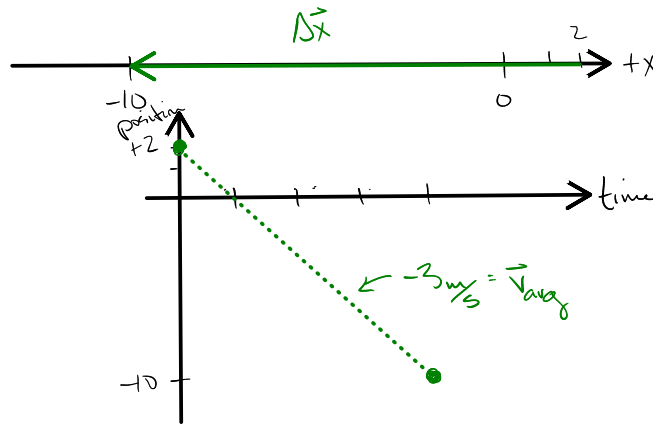


3. I move from a place marked $x = 2 \text{ m}$ to a place marked $x = -10 \text{ m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative? Plot this displacement and velocity over time.

$$\Delta x = -10 \text{ m} - (+2 \text{ m})$$

$$= -12 \text{ m}$$

$$v_{\text{avg}} = \frac{-12 \text{ m}}{4 \text{ s}} = -3 \text{ m/s}$$

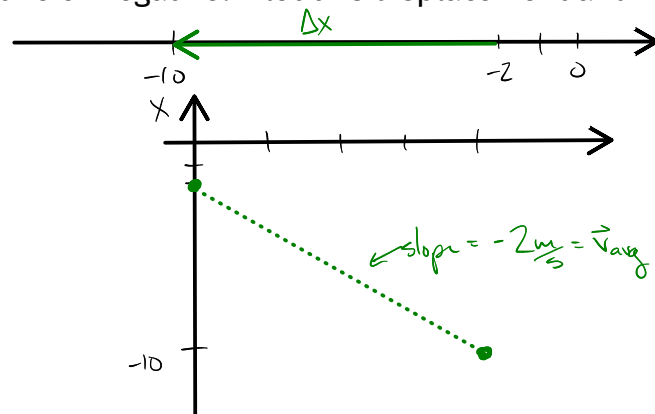


4. I move from a place marked $x = -2 \text{ m}$ to a place marked $x = -10 \text{ m}$ in 4 sec. What is my total displacement? Is it a positive or negative displacement? What is my average velocity over this interval? Is it positive or negative? Plot this displacement and velocity over time.

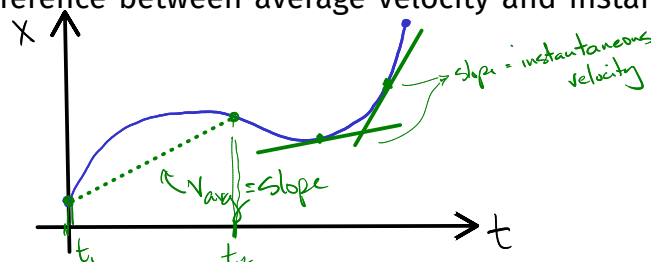
$$\Delta x = -10 \text{ m} - (-2 \text{ m})$$

$$= -8 \text{ m}$$

$$v_{\text{avg}} = \frac{-8 \text{ m}}{4 \text{ s}} = -2 \text{ m/s}$$



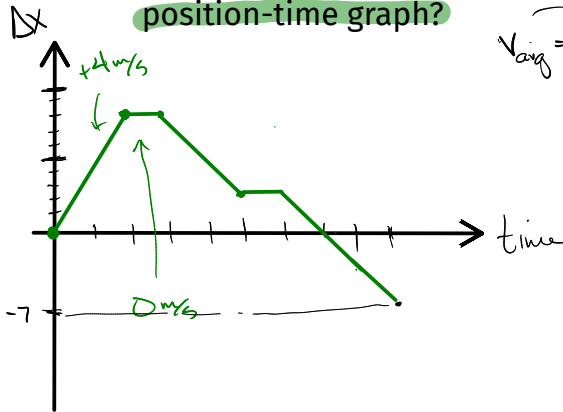
5. What is the difference between average velocity and instantaneous velocity? Give an example.



6. If the rate of change of position is called velocity, and the rate of change of velocity is called acceleration, then what is the rate of change of acceleration called? (this is not in your book, you'll have to search for it) Does the rate of change of *that* have a name?

“jerk” → “snap” → “crackle” → “pop”

7. Can you generate a **displacement graph** if you know an object's velocity-time graph? I move with a constant velocity $+4 \text{ m/s}$ for 2 seconds. Then I stand still for 1 second and then move with a constant velocity -3 m/s for 2 seconds. I rest another second before continuing at -3 m/s for 3 seconds. Do we need to know anything else to generate the **position-time graph**?



$$v_{\text{avg}} = v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v \cdot \Delta t$$

$$\Delta x = +4 \frac{\text{m}}{\text{s}} \cdot 2 \text{ s}$$

$$= +8 \text{ m}$$

$$\Delta x = v \cdot \Delta t$$

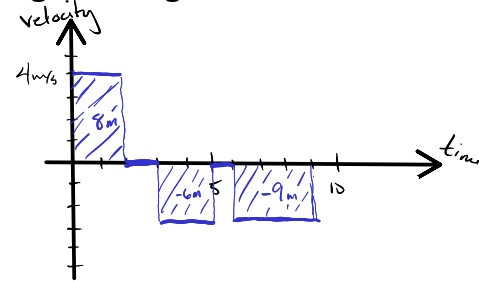
$$= -3 \frac{\text{m}}{\text{s}} \cdot 2 \text{ s}$$

$$\Delta x = -6 \text{ m}$$

$$x_f - x_i$$

$$x_f - 8 \text{ m} = -6 \text{ m}$$

$$x_f = +2 \text{ m}$$



8. An object has a velocity of 10 m/s and then 5 seconds later has a velocity of 15 m/s . What is its average acceleration over this time interval? In what direction did the velocity change?

$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i$$

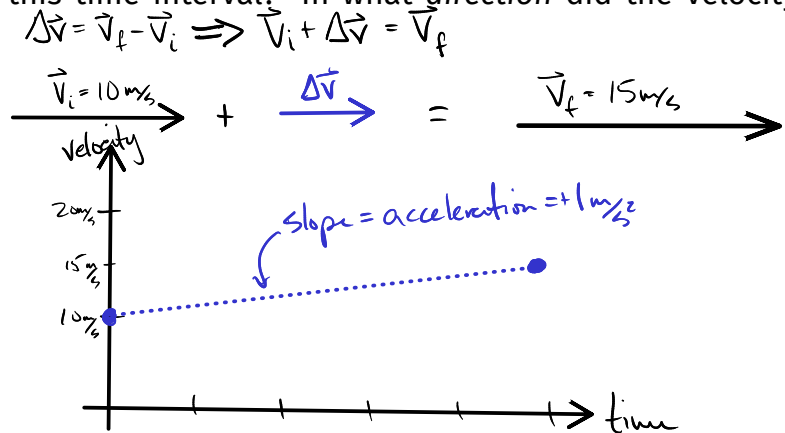
$$\Delta \vec{v} = +15 \frac{\text{m}}{\text{s}} - (+10 \frac{\text{m}}{\text{s}})$$

$$\Delta \vec{v} = +5 \frac{\text{m}}{\text{s}}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \frac{+5 \frac{\text{m}}{\text{s}}}{5 \text{ s}}$$

$$\vec{a} = +1 \frac{\text{m}}{\text{s}^2}$$



9. An object has a velocity of 15 m/s and then 5 seconds later it came to a stop. What is its average acceleration over this time interval? What is the significance of this sign?

$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i$$

$$= 0 \frac{\text{m}}{\text{s}} - (+15 \frac{\text{m}}{\text{s}})$$

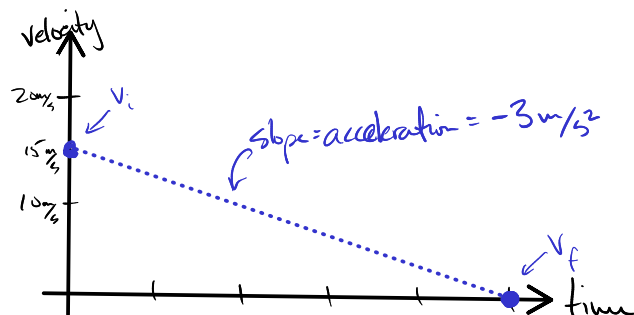
$$= -15 \frac{\text{m}}{\text{s}}$$

$$\vec{v}_i = 15 \frac{\text{m}}{\text{s}} + \Delta \vec{v} = v_f$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{-15 \frac{\text{m}}{\text{s}}}{5 \text{ s}}$$

$$= -3 \frac{\text{m}}{\text{s}^2}$$

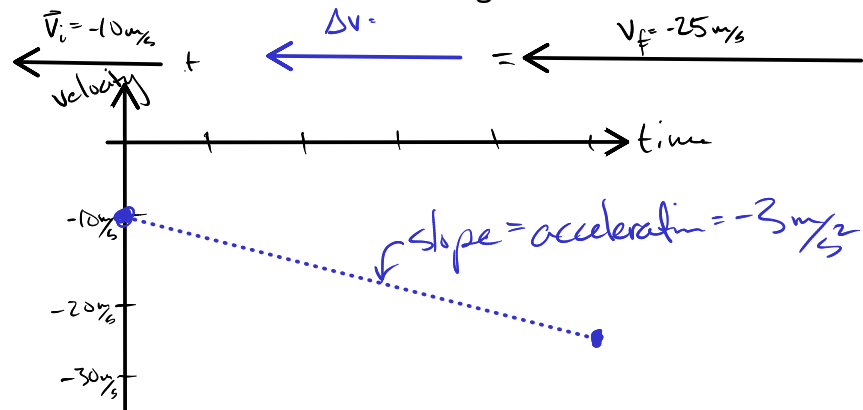


$$\Sigma \vec{F} = m \vec{a}$$

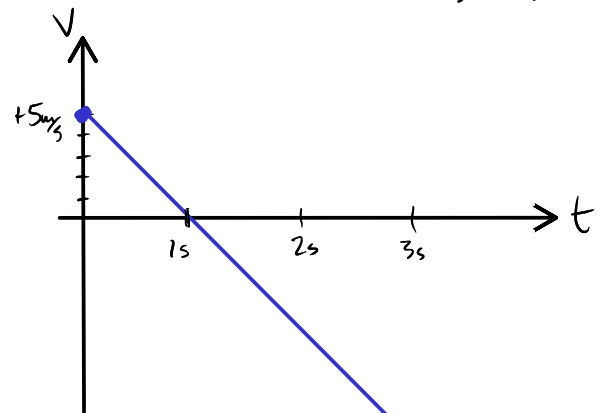
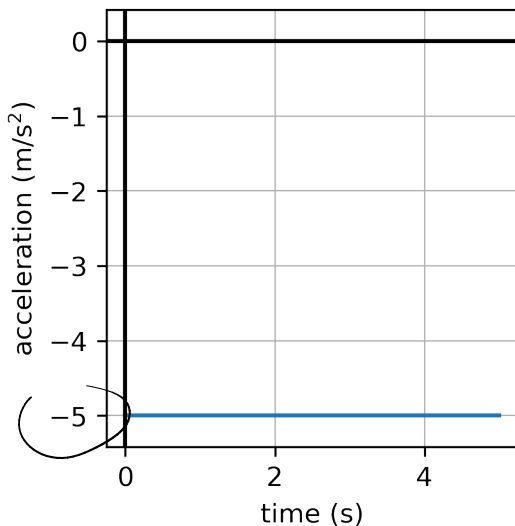
$\underbrace{\Sigma \vec{F}}_{\text{same sign}} \quad \underbrace{m \vec{a}}_{\text{same sign}}$

10. An object has a velocity of -10 m/s and then 5 seconds later it has a velocity of -25 m/s . What is its average acceleration over this time interval? What is the significance of this sign?

$$\begin{aligned} \Delta \vec{v} &= \vec{v}_f - \vec{v}_i \\ &= -25 \text{ m/s} - (-10 \text{ m/s}) \\ &= -15 \text{ m/s} \\ \vec{a} &= \frac{\Delta \vec{v}}{\Delta t} = \frac{-15 \text{ m/s}}{5 \text{ s}} = -3 \text{ m/s}^2 \end{aligned}$$



11. The following plot shows constant acceleration over time. If the initial velocity is $v_i = 5 \text{ m/s}$, then sketch a graph of velocity vs. time.



$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a \cdot \Delta t$$

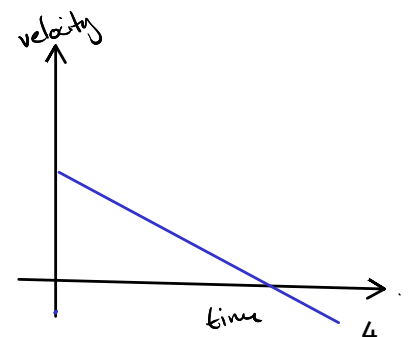
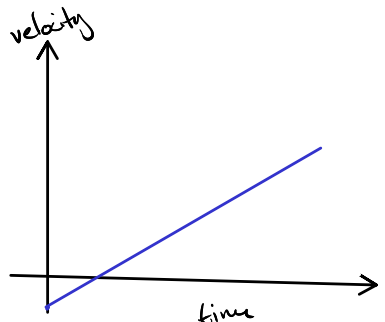
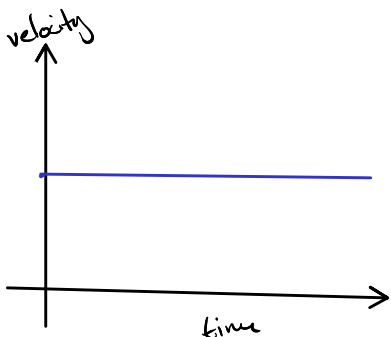
$$v_f - v_i = a \cdot \Delta t$$

$$v_f = v_i + a \cdot \Delta t$$

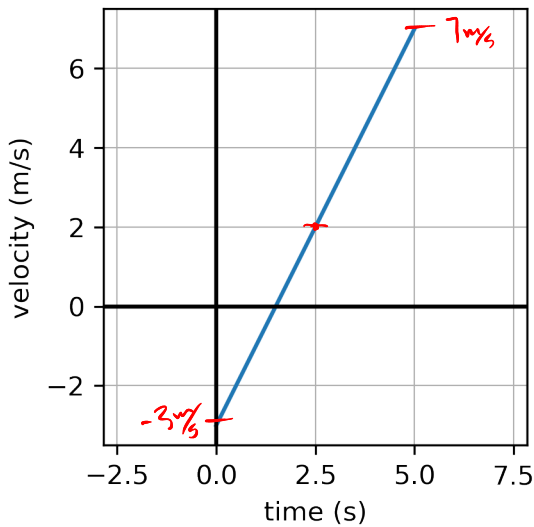
$$0 \text{ m/s} = 5 \text{ m/s} + (-5 \text{ m/s}^2) \cdot t$$

$\underline{t = 1 \text{ s}}$

12. Draw a plot of velocity vs time for a case of 0 acceleration. Positive acceleration? Negative acceleration?



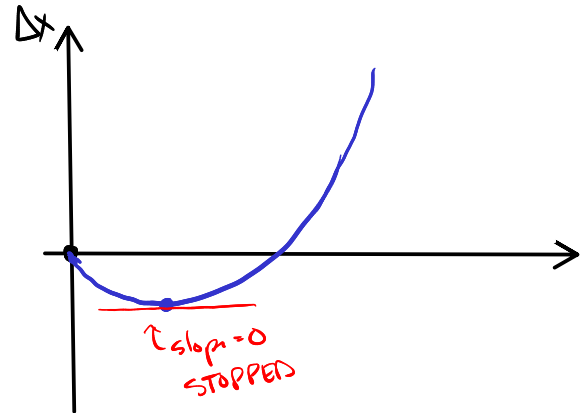
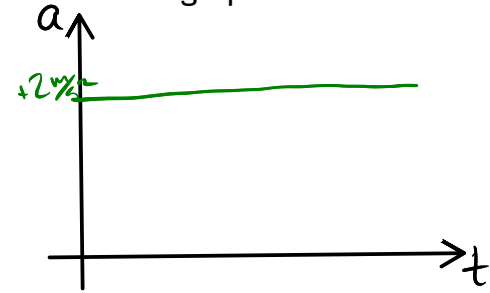
13. The following plot shows velocity vs. time. What is this acceleration? What is the initial velocity? Draw the acceleration vs. time graph. Draw the position vs time graph? What do you not know about the position graph?



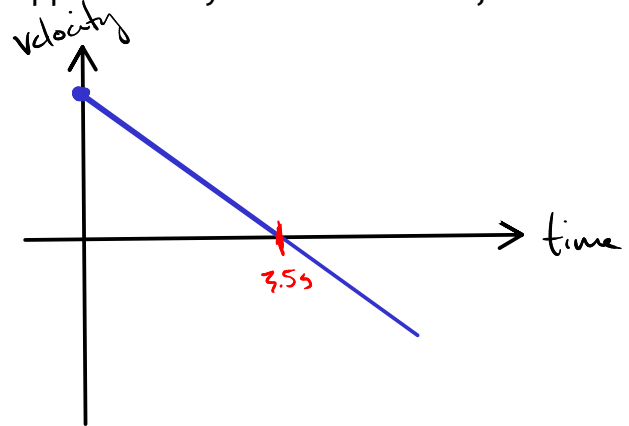
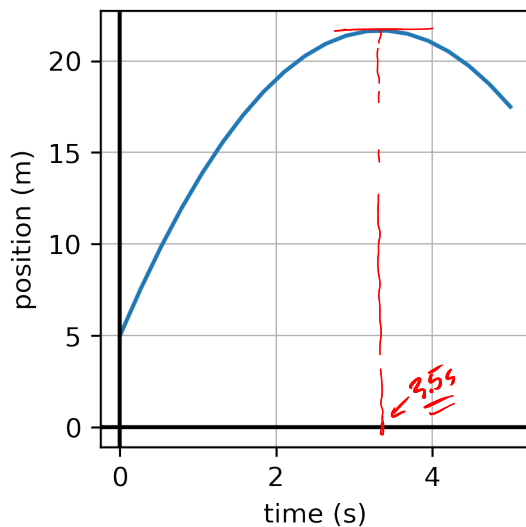
$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{7 \text{ m/s} - (-3 \text{ m/s})}{5 \text{ s}}$$

$$a = +2 \text{ m/s}^2$$

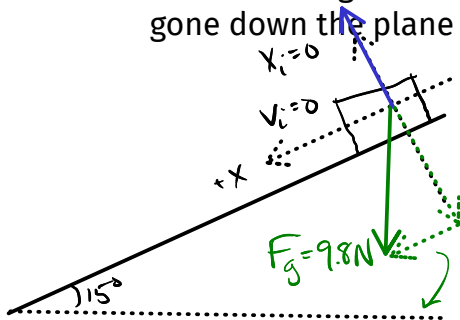


14. The following plot shows the position of an object over time. What is the sign of the acceleration? What is the sign of the initial velocity? Approximately when does the object come to a stop?



15. Draw a plot of velocity vs time for an object that is initially moving forward, but then stops and starts to move backward. Assume constant acceleration. What is the sign of the acceleration and what does that mean for the slope of this plot?
16. What does a plot of position vs time look like for the same case as before (initially moving forward, but then stopping and moving backward)?

17. A 1 kg object slides down a friction-less inclined plane starting from rest. The plane has an incline angle of 15° . What is the acceleration of the object? How far does the object go down the plane in 3 seconds?



$$F_{gx} = mg \sin \theta = \Sigma F_x = ma_x$$

$$mg \sin \theta = ma_x$$

$$a = g \sin \theta$$

$$a = +2.53 \text{ m/s}^2$$

$$\Delta x = 0 \text{ m/s} (3 \text{ s}) + \frac{1}{2} (+2.53 \text{ m/s}^2) (3 \text{ s})^2$$

$$\Delta x = 11.4 \text{ m}$$

Kinematic equations

$$\bullet v(t) = v_i + at$$

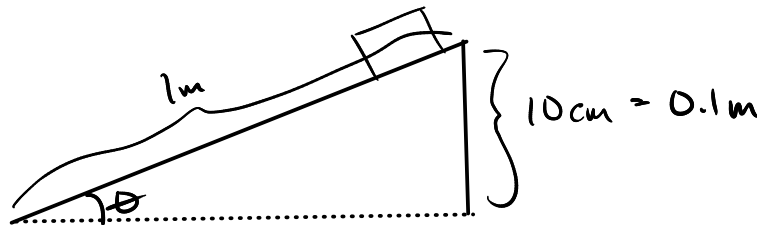
$$\bullet x(t) = x_i + v_i \cdot t + \frac{1}{2} at^2$$

$$\hookrightarrow x_f - x_i = \Delta x = v_i t + \frac{1}{2} at^2$$

$$\bullet v_f^2 = v_i^2 + 2a\Delta x$$

Bonus: how fast? $v(3 \text{ s}) = 0 \text{ m/s} + (2.53 \text{ m/s}^2)(3 \text{ s}) = 7.6 \text{ m/s}$

18. A 1 kg object slides down a friction-less inclined plane. The length of the plane is 1 m, and the height above the horizontal is 10 cm. How long does it take the object to reach the bottom and how fast it is going when it gets there?



$$\sin \theta = \frac{0.1}{1}$$

$$\theta = \underline{\quad ? \quad}$$

$$\theta = 5.7^\circ$$

19. Examples 4.1, 4.2 and 4.3 from the text book (pgs. 127-130) are excellent and you should use the space below to work those out for yourself.

20. Here is an example of turning a problem inside out. In the lecture video, I worked an example of a person throwing an object up and off the edge of a bridge. The object goes up and then comes down and splashes in the water below, 44.1 m below the place where it was released. In the example, we knew the time and found the initial velocity to be +8.66 m/s. In this problem, use this information to find the time. (You should get 4 seconds)

21. I am standing at the very edge of a cliff and I want to know how far down it is to the ground or the water below me. To do this, I drop a rock off the edge, and count how long it takes to hit the ground. What is a general expression for the height of the cliff based on the amount of time it takes to hit the ground? If it takes 4 seconds to do this, how high is the cliff?

known:
 $y_i = 0$
 $v_{iy} = 0$
 $t = 4\text{ s}$
 $v_{iy} = 0\text{ m/s}$
 $a_y = -9.8\text{ m/s}^2$

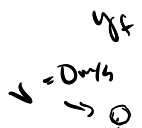
want:
 y_f

How fast?
 $v_f^2 = v_i^2 + 2a\Delta y$
 $v_f = v_i + at$
 $v_f = -9.8\text{ m/s}^2 \cdot 4\text{ s}$
 $v_f = -39.2\text{ m/s}$

$y_f = y_i + v_i t + \frac{1}{2}at^2$
 $y_f = \frac{1}{2}(-9.8\text{ m/s}^2)(4\text{ s})^2$
 $y_f = -78.4\text{ m}$
78.4 m high

$\downarrow -mg = F_g = \Sigma F_y = ma_y$
 $-mg = ma_y$
 $a_y = -g$

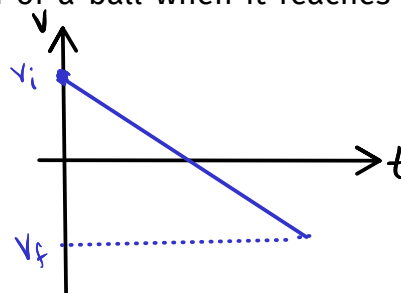
22. (a) What is the velocity of a ball when it reaches its highest height after being thrown straight up?



$$v_{\text{top}} = 0\text{ m/s}$$

- (b) What is the acceleration of a ball when it reaches its highest height after being thrown straight up?

$$a = -9.8\text{ m/s}^2$$



- (c) If I throw a ball upward, what is an expression for the amount of time it takes to reach its highest height? If the initial velocity is 15 m/s, then how much time does it take?

known:

v_i

a

$v_f = 0$

want:

$t = ?$

$$v_f = v_i + at$$

$$\frac{v_f - v_i}{a} = \frac{at}{a}$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{0 - (+15 \text{ m/s})}{-9.8 \text{ m/s}^2}$$

$$t = 1.5 \text{ s}$$

- (d) How much time does it take for the ball thrown straight up to come back down?

$$t = 1.5 \text{ s}$$

- (e) So if I throw a ball straight up and its total travel time from when it leaves my hand to when it comes back down is 4 seconds, then what was the original speed with which I threw it?

$$v(2\text{s}) = 0 = v_i + (-9.8 \text{ m/s}^2)(2\text{s})$$

$$v_i = +19.6 \text{ m/s}$$

$$v_f = v_i + at$$

$$-v = v + (-9.8 \text{ m/s}^2)(4\text{s})$$

$$-2v = -$$

$$v = +19.6 \text{ m/s}$$

$$y_f = y_i + v_i t + \frac{1}{2} at^2$$

$$0 = 0 + v_i(4\text{s}) + \frac{1}{2}(-9.8)(4)^2$$

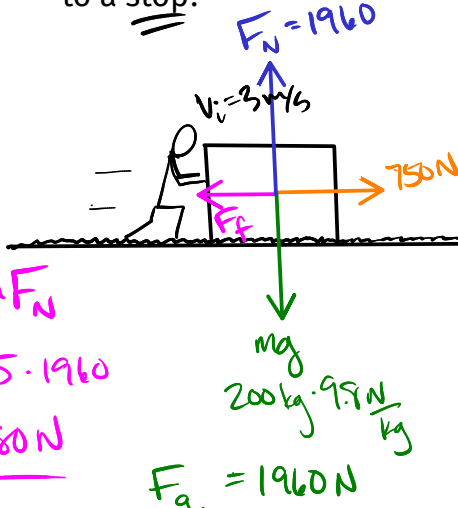
$$v_i = +19.6 \text{ m/s}$$

- (f) What is the final ~~speed~~ ^{velocity} when the ball in the previous problem comes back down? (Assume final position is the same height from which I threw it.)

$$v_f = -19.6 \text{ m/s}$$

23. The moon has a gravitational field strength $g = 1.6 \text{ N/kg}$, so objects feel lighter on the moon in terms of lifting them vertically, but what about pushing horizontally? If you wanted to accelerate an 10 kg object horizontally (on a frictionless surface) on the moon with an acceleration of 2 m/s^2 , what force would you need to provide? Would this force be different on the earth? Why?

24. I am pushing a 200 kg box across a friction-full surface with 750 N force. The coefficient of friction between the box and the floor is $\mu = 0.5$. What is the net force on the box? What is the acceleration? What does the sign mean here? If the box was moving at 3 m/s when I began to encounter the friction patch on the floor, how long does it take to come to a stop?



$F_N = 1960$
 $v_i = 3 \text{ m/s}$
 750 N
 F_f
 mg
 $200 \text{ kg} \cdot 9.8 \text{ N/kg}$
 $F_g = 1960 \text{ N}$

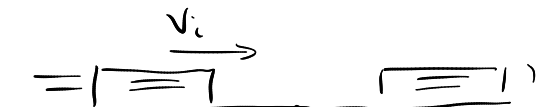
$\Sigma F_x = +750 + (-980)$
 $\Sigma F_x = -230 \text{ N}$
 $\Sigma F_x = ma_x$
 $-230 \text{ N} = 200 \text{ kg} \cdot a_x$
 $a_x = -1.15 \text{ m/s}^2$

$F_f = \mu F_N$
 $F_f = 0.5 \cdot 1960$
 $= 980 \text{ N}$

$\rightarrow t$
 $V_i = 3 \text{ m/s}$
 $a = -1.15 \text{ m/s}^2$
 $V_f = 0 \text{ m/s}$
 $V_f = V_i + at$
 $0 = 3 \text{ m/s} + (-1.15 \text{ m/s}^2) \cdot t$
 $t = 2.6 \text{ s}$

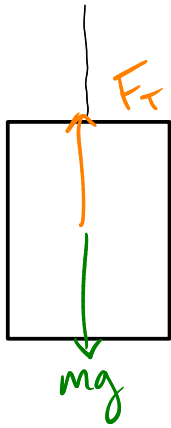
how far?
 $\Delta x = 3.9 \text{ m}$

25. I give a 2 kg book a quick push to start it sliding across a table at 10 m/s initially after my push it done, and it slides about 1 m across the table before it comes to a stop, then what is the coefficient of friction between the book and table?



$\rightarrow \mu = ?$
 $\rightarrow v_f = 0$

$\mu = 5.1$



$$\Sigma F_y = ma_y$$

$$a_y = - (\quad)$$

$$\boxed{y_i = ?}$$
 ← first positive number

$$\underline{v_f = ?}$$