Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being 0 °C when water freezes and 100 °C when water boils at 1 atm of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. 105 °F, 98.6 °F, 72 °F, 32 °F, 0 °F. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

$$T_F = \frac{9}{5}\cdot T_C + 32$$

$$\frac{9}{5}T_C = T_F - 32$$

$$T_C = \frac{5}{9}(T_F - 32)$$

T_F	T_C
105°F	40.6°C
98.6°F	37°C
72°F	22°C
32	0
0	-18

$$T = \frac{9}{5}\cdot T + 32$$

$$-\frac{4}{5}T = 32$$

$$T = -\left(\frac{5}{4}\right)\cdot 32$$

$$\boxed{T = -40}$$

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by 30 °F, then by how much does the temperature change in Celsius? What does this mean about the "size" of a Celsius degree vs. the "size" of a Fahrenheit degree? Which one represents a larger change in temperature?

$$\Delta T_F = \frac{9}{5}\Delta T_C$$

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

$T_K = T_C + 273.15$

$T_C = T_K - 273.15$

T_C	T_K
−273.15°C	0 K
0°C	273.15K
20°C	293.15K
−203.15°C	70K
37°C	310K

$\Delta T_C = \Delta T_K$

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

5. What is the ~~molecular~~ atomic ~~weight~~ mass of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

$12\frac{g}{mol}$

6p 6p 6n 8n

atomic mass − # of protons = # of neutrons

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

$M_{He} = 4g$ 1 mole of He = $6.022 \cdot 10^{23}$ atoms 12g

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

$12g + 2(16) = 44g$

$\frac{44\,g}{6.022 \cdot 10^{23}\text{ molecules}} = 7.3 \cdot 10^{-23} \frac{g}{\text{molecule}} = 7.3 \cdot 10^{-26}\,kg$

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

$\hookrightarrow 28g \cdot 0.78 =$ ___

$32g \cdot 0.21 =$ ___

$40g \cdot 0.01 =$ ___

29 g/mol

9. A balloon is filled with 0.4 mol of helium so that its volume is $0.010\,m^3$.

- Find the number of atoms.

$N = n \cdot N_A$

# of particles; number of moles

$N = 0.4\,mol \cdot 6.022 \cdot 10^{23} \frac{\text{particles}}{mol} = 2.4 \cdot 10^{23}$ He atoms

- Find the number density.

$\text{number density} = \frac{N}{V} = \frac{2.4 \cdot 10^{23}\text{ atoms}}{0.010\,m^3} = 2.4 \cdot 10^{25}\text{ He atoms}/m^3$

- Find the mass density.

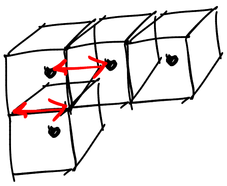
$4 \frac{g}{\cancel{mol}} \cdot 0.4\,\cancel{mol} = 1.6\,g = 0.0016\,kg$

"rho"

$\text{volumetric mass density} = \rho = \frac{M}{V} = \frac{0.0016\,kg}{0.010\,m^3} = 0.16\,kg/m^3$

- Estimate the average distance between atoms. To do this, ~~fine~~ find the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

$$\text{number density} = \frac{\text{particle}}{\text{volume}} \rightarrow \frac{\text{volume}}{\text{particle}} = \frac{1}{2.4\cdot 10^{25}\ \text{He atoms}/\text{m}^3}$$



$$V_{cube} = s^3 \qquad = 4.17\cdot 10^{-26}\ \frac{\text{m}^3}{\text{atom}}$$

$$s = \sqrt[3]{V} = \sqrt[3]{4.17\cdot 10^{-26}\ \frac{\text{m}^3}{\text{atom}}} = 3.5\cdot 10^{-9}\ \text{m}$$

$$= 3.5\ \text{nm}$$

$$= 35\ \text{Å}$$

10. You have a pound of feathers and a pound of lead.

    - Which one weighs more? same
    - Which one has more mass? same
    - Which one has the greater volume? feathers
    - Which one contains a larger number of moles? trick
    - Which one contains a larger number of atoms? feathers
    - Which one contains a larger number of protons and neutrons? Same

11. You check your car tire pressure and see that the pressure is 25 lb/in^2. What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

$$25\ \cancel{\text{psi}}\cdot\frac{1\ \cancel{\text{atm}}}{14.7\ \cancel{\text{psi}}}\cdot\frac{1.013\cdot 10^5\ \text{Pa}}{1\ \cancel{\text{atm}}} = 1.7\cdot 10^5\ \text{Pa}$$

$$P_{abs} = P_{gauge} + P_{atm}$$

$$= 1.7\cdot 10^5\ \text{Pa} + 1.013\cdot 10^5$$

$$P_{abs} = 2.7\cdot 10^5\ \text{Pa}$$

288K

$+ 14.7\,psi = 39.7\,psi$ ← absolute pressure

12. You check you car tire pressure when it is 15 °C and it is 25 lb/in^2. By what factor do you increase the number of particles in the tire so that the pressure becomes that 30 lb/in^2? (*Hint: The volume and temperature do not change.*)

↳ 44.7 psi

$PV = Nk_BT$

$P = \frac{k_BT}{V} \cdot N$

$P \propto N$

$\frac{P_2}{P_1} = \frac{N_2}{N_1}$

$\frac{44.7}{39.7} = \frac{N_2}{N_1} = \underline{1.13}$

$\%\Delta = \frac{(N_2 - N_1)}{N_1} \times 100$

$\%\Delta = \left(\frac{N_2}{N_1} - 1\right) \times 100$

13% increase

20°C = 293K

13. The gas pressure inside of a 1 liter sealed container at room temperature is 1 atm. How many molecules are inside? How many moles of molecules?

$V = 1\,L \cdot \frac{1000\,mL}{1\,L} \cdot \frac{1\,cm^3}{1\,mL} \cdot \frac{1\,m^3}{(100\,cm)^3} = 0.001\,m^3$

$(10^2)^3$ $\quad 10^{-3}\,m^3$

10^6

$P = 1\,atm = 1.013 \cdot 10^5\,Pa$

$PV = Nk_BT$

$N = \frac{PV}{k_BT} = \frac{(10^{-3}\,m^3)(10^5\,Pa)}{(1.38 \cdot 10^{-23}\,\frac{J}{K})(293K)} = 2.5 \cdot 10^{22}$ particles

↳ $\div N_A = 0.041$ mol

Ideal Gas Law

microscopic

$PV = Nk_BT$

Pa, m^3

↳ # of particles

↳ k_B = Boltzmann's constant

$k_B = 1.38 \cdot 10^{-23}\,\frac{J}{K}$

macroscopic

$PV = nRT$

atm → Pa

liters → m^3

↳ moles

Kelvin

$0.0821\,\frac{atm \cdot L}{K\,mol}$

↓

$8.31\,\frac{J}{K\,mol}$

14. If the pressure inside a tank is 1 atm when the temperature is 100 K, then what is the pressure when the temperature rises to 200 K?

$PV = Nk_BT$

$P = \frac{Nk_BT}{V}$

$P \propto T^1$

$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^1$

$\frac{P_2}{P_1} = \left(\frac{200}{100}\right)^1 = 2$ $\quad P_2 = 2\,atm$

P_1 ← 1 atm

15. If the pressure inside a tank is 1 atm when the temperature is 100 °C, then what is the pressure when the temperature rises to 200 °C? *CAREFUL!*

↳ 373K

↳ 473K

$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{473}{373} = 1.26$

$P_2 = 1\,atm \cdot 1.26 = 1.26\,atm$

16. A gas is in a sealed container. By what factor does the pressure change if

- the volume is doubled? $\rightarrow \frac{V_2}{V_1} = 2$ $\quad V_2 = 2 \cdot V_1$

$$PV = Nk_BT$$
$$P = Nk_BT\, V^{-1} \rightarrow P \propto V^{-1}$$
$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = 2^{-1} = \frac{1}{2} = 0.5 = \frac{P_2}{P_1}$$

- the temperature is tripled?

$$P = Nk_BT\, V^{-1}$$
$$P \propto T^1$$
$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = 3$$

- the volume is double and the temperature is tripled?

$$P = Nk_BT\, V^{-1} \rightarrow P \propto T \cdot V^{-1}$$
$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \cdot \left(\frac{V_2}{V_1}\right)^{-1} = 3 \cdot (2)^{-1} = \frac{3}{2}$$

- the volume is halved?

$$P = Nk_BT\, V^{-1} \qquad P \propto V^{-1}$$
$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$$

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

$$V = l \cdot w \cdot h = 10\,\text{m} \cdot 15\,\text{m} \cdot 2\,\text{m} = 300\,\text{m}^3$$
$$PV = nRT$$
$$n = \frac{PV}{RT} = \frac{10^5 \cdot 300\,\text{m}^3}{8.31\,\frac{J}{K\,mol} \cdot 293\,K} = 12{,}300\ \text{mol}$$

18. RT, 1atm, 1mol, how big? show me

$$V = \frac{nRT}{P} = \frac{1\,\text{mol} \cdot 8.31 \cdot 293\,K}{10^5\,Pa} = 0.024\,\text{m}^3$$
$$s = \sqrt[3]{V} = \sqrt[3]{0.024\,\text{m}^3}$$
$$s = 0.29\,\text{m}$$
$$\sim 30\,\text{cm}$$

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

↳ mass of <u>one</u> <u>particle</u> in kg!