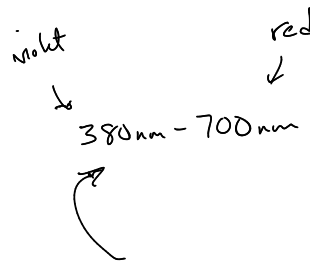


Week 11 covers sections of chapter 22 in the textbook. Topics include:

- electromagnetic waves
- index of refraction
- doppler shift



1. What is the range of wavelengths of human vision? What is the range of frequencies? Approximately what wavelength is red? Green? Violet? A wave that has a wavelength of 1 nm could be used for what purpose? What is its frequency? Wifi usually works around a frequency of 2.4 GHz which is the same as a consumer microwave oven. What wavelength is this? WBHM is the Public Radio station in Birmingham and you can find it on the radio dial at 90.3 MHz. What wavelength is this?

$$f = \frac{3 \cdot 10^8 \text{ m/s}}{1 \cdot 10^{-9}} = 3 \cdot 10^{17} \text{ Hz} \quad \text{X-RAYS}$$

$$v = \lambda \cdot f$$

$$\lambda = \frac{v}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{2.4 \cdot 10^9 \text{ Hz}} = 0.125 \text{ m}$$

$$\lambda = \frac{3 \cdot 10^8 \text{ m/s}}{90.3 \cdot 10^6 \text{ Hz}} = 3.32 \text{ m}$$

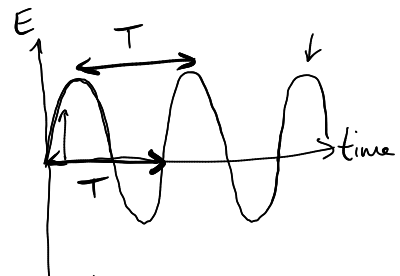
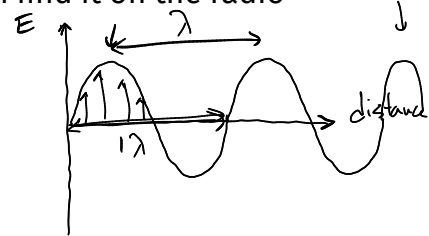
$$\text{frequency} \rightarrow f = \frac{1}{T} \leftarrow \text{sec}$$

$$[s^{-1}] = [Hz]$$

$$v = \frac{\lambda}{T} \leftarrow \text{wavelength}$$

$$\text{velocity, speed} \rightarrow v = \lambda \cdot f$$

$$v_{\text{vacuum}} = c = 3 \cdot 10^8 \text{ m/s}$$



$$f_{\text{violet}} = \frac{v}{\lambda_v} = \frac{3 \cdot 10^8 \text{ m/s}}{380 \cdot 10^{-9} \text{ m}} = 7.8 \cdot 10^{14} \text{ Hz}$$

$$f_{\text{red}} = \frac{3 \cdot 10^8 \text{ m/s}}{700 \cdot 10^{-9} \text{ m}} = 4.3 \cdot 10^{14} \text{ Hz}$$

pico = $10^{-12}$
nano = $10^{-9}$
micro = $10^{-6}$
milli = $10^{-3}$
base = 1
kilo = $10^3$
Mega = $10^6$
Giga = $10^9$
Tera = $10^{12}$

2. What frequency has the same magnitude as its wavelength? What region is this in?

$$v = \lambda \cdot f \Rightarrow 3 \cdot 10^8 \text{ m/s} = x^2$$

$$x = \sqrt{3 \cdot 10^8}$$

$$x = 1.7 \cdot 10^4$$

$$1.7 \cdot 10^4 \text{ Hz} = f$$

$$1.7 \cdot 10^4 \text{ m} = \lambda$$

radio waves!

3. The sun is 93 000 000 miles away from earth. If the sun suddenly burned out, how long would it take for us to know?

$$\boxed{v = \lambda \cdot f} \quad \left| \quad v = \frac{\text{distance}}{\text{time}} \right|$$
  
 true!                      also true!                       $t = 8.3 \text{ min}$

4. One light-year is the distance that light travels in a year. How many meters is this?

$$\begin{aligned}
 d &= 9.46 \cdot 10^{15} \text{ m} \\
 &= \underline{\underline{5.87 \cdot 10^{12} \text{ miles}}}
 \end{aligned}$$

5. What is the speed of light in water where  $n = 1.33$ ? What about the speed of light in diamond where  $n = 2.42$ ?

$$n = \frac{c}{v} \quad \begin{array}{l} \leftarrow \text{in vacuum} \\ \leftarrow \text{in material } n \end{array}$$
  

$$v_n = \frac{c}{n}$$
  

$$v_{H_2O} = \frac{3 \cdot 10^8 \text{ m/s}}{1.33} = \underline{\underline{2.26 \cdot 10^8 \text{ m/s}}} \quad \left| \quad v_c = \frac{3 \cdot 10^8 \text{ m/s}}{2.42} = \underline{\underline{1.24 \cdot 10^8 \text{ m/s}}}$$

6. The speed of light in some unknown material is measured to be  $1.3 \times 10^6 \text{ m/s}$ . What is the index of refraction?

$$n = \frac{c}{v} = \frac{3 \cdot 10^8 \text{ m/s}}{1.3 \cdot 10^6 \text{ m/s}} = 231$$

7. If the frequency of light in water is  $1 \times 10^{14}$  Hz then what is it in diamond? What is the wavelength in water? What is the wavelength in diamond?

$$\begin{aligned}
 v_{H_2O} &= \lambda \cdot f \\
 \lambda_{H_2O} &= \frac{2.26 \cdot 10^8 \text{ m/s}}{1 \cdot 10^{14} \text{ Hz}} \\
 \lambda_{H_2O} &= 2.26 \cdot 10^{-6} \text{ m}
 \end{aligned}
 \quad \left| \quad \begin{aligned}
 \lambda_c &= \frac{v_c}{f} \\
 &= \frac{1.24 \cdot 10^8 \text{ m/s}}{1 \cdot 10^{14} \text{ Hz}} \\
 \lambda_c &= 1.24 \cdot 10^{-6} \text{ m}
 \end{aligned}
 \right.
 \quad \left| \quad \begin{aligned}
 \frac{c}{n} &= v = \lambda \cdot f \\
 \frac{c}{n} &= \lambda f \\
 \lambda &= \frac{c}{fn} \\
 \lambda &= \frac{c}{f} \cdot \frac{1}{n}
 \end{aligned}
 \right.
 \quad \rightarrow \quad \lambda \propto n^{-1}$$

$$\boxed{\frac{\lambda_2}{\lambda_1} = \left(\frac{n_2}{n_1}\right)^{-1}}$$

8. Show that the quantity and units work out for the relationship between the speed of light in vacuum and the permittivity and permeability of free space:

$$c = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} \rightarrow 4\pi \cdot 10^7 \frac{\text{Tm}}{\text{A}}$$

$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$

9. The simplest model for an electromagnetic wave to take is a sine wave. The general form for using the sine wave and having the parameters match those of wavelength and period that we have discussed can be written like this:

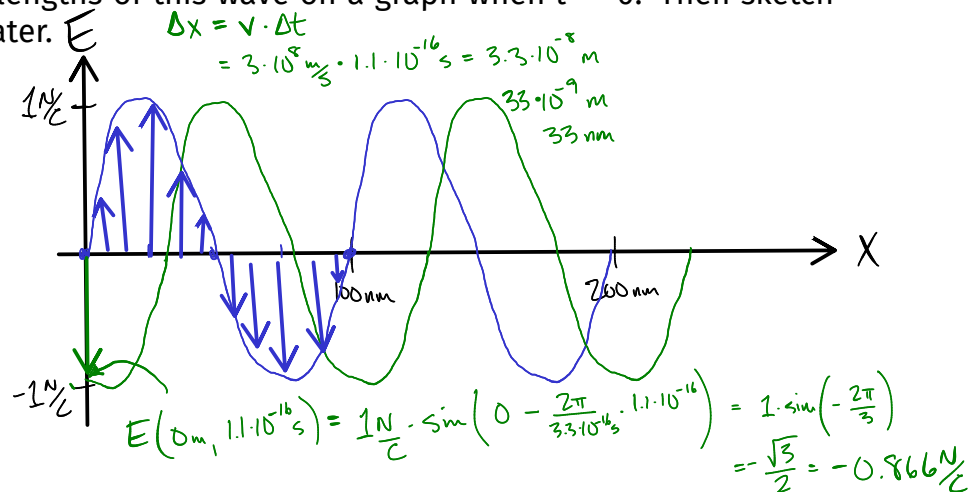
$$E(x, t) = E_0 \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

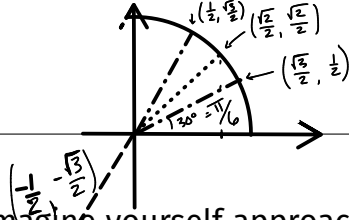
$k = \frac{2\pi}{\lambda}$

This is just a function that gives you the value of the electric field at a particular place (x) at a particular time (t). For an electromagnetic wave of wavelength 100 nm, what is the period? Sketch two wavelengths of this wave on a graph when  $t = 0$ . Then sketch another plot 1/3 of a period later.

$$\begin{aligned}
 v &= \lambda \cdot f & f &= \frac{1}{T} & \frac{c}{n} &= v \\
 f &= \frac{v}{\lambda} = \frac{3 \cdot 10^8 \text{ m/s}}{100 \cdot 10^{-9} \text{ m}} = 3 \cdot 10^{15} \text{ Hz} \\
 T &= \frac{1}{f} = 3.3 \cdot 10^{-16} \text{ s} \\
 \frac{T}{3} &= 1.1 \cdot 10^{-16} \text{ s}
 \end{aligned}$$

$t = 0$   
 $t = 1.1 \cdot 10^{-16} \text{ s}$





10. **Doppler Effect.** Imagine yourself approaching a source of light and think of the light as waves. You are traveling in the opposite direction as the light propagation. You can see from some of the above graphs of waves traveling that if you are doing this, then the period of time between when crests of the light are detected by you will be shorter than for a stationary observer. This means the frequency you observe will be higher than the frequency of the source. The faster you go the higher this frequency shift will be. The opposite effect will happen if you are traveling away from the source of light; you will observe a lower frequency. The equation relating the frequency you observe,  $f_o$ , the frequency of the source  $f_s$  and the *relative velocity* between source and observer  $v_{rel}$  is

$$\text{observed frequency } f_o = f_s \sqrt{\frac{1 + \frac{v_{rel}}{c}}{1 - \frac{v_{rel}}{c}}}$$

source frequency

In this equation  $v_{rel}$  is negative when the source and observer are moving away from each other and positive when they are approaching each other.

Using this equation, calculate how fast you would have to be going in your car in order for a red light to appear green.

$$\lambda_{red} = 700 \text{ nm} \quad f_{red} = 4.3 \cdot 10^{14} \text{ Hz}$$

$$\lambda_{green} = 550 \text{ nm} \quad f_{green} = 5.4 \cdot 10^{14} \text{ Hz}$$

$$\left(\frac{f_o}{f_s}\right)^2 = \left(\frac{5.4 \cdot 10^{14} \text{ Hz}}{4.3 \cdot 10^{14} \text{ Hz}}\right)^2 = 1.577$$

$$f_o = f_s \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

$$\left(\frac{f_o}{f_s}\right)^2 = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}$$

$$\left(\frac{f_o}{f_s}\right)^2 - 1 = \left(\frac{f_o}{f_s}\right)^2 \cdot \frac{v}{c} + \frac{v}{c}$$

$$\left(\frac{f_o}{f_s}\right)^2 - 1 = \left(\left(\frac{f_o}{f_s}\right)^2 + 1\right) \frac{v}{c}$$

$$\frac{v}{c} = \frac{\left(\frac{f_o}{f_s}\right)^2 - 1}{\left(\frac{f_o}{f_s}\right)^2 + 1} = \frac{0.577}{2.577} = 0.224 = \frac{v}{c}$$

$$\boxed{\begin{array}{l} v = 0.224c \\ v = 6.7 \cdot 10^7 \text{ m/s} \end{array}}$$

$$\left(1 - \frac{v}{c}\right) \left(\frac{f_o}{f_s}\right)^2 = 1 + \frac{v}{c}$$

$$\left(\frac{f_o}{f_s}\right)^2 - \left(\frac{f_o}{f_s}\right)^2 \cdot \frac{v}{c} = 1 + \frac{v}{c}$$

$$-1 + \left(\frac{f_o}{f_s}\right)^2 \cdot \frac{v}{c} + \left(\frac{f_o}{f_s}\right)^2 \cdot \frac{v}{c} = 1$$

11. The speed of light is very very fast, and many times the relative velocities between source and observer are much smaller than that. The equation above can be simplified in cases

when  $v_{rel} \ll c$  to a much easier equation:

$$f_o = f_s \left( 1 + \frac{v_{rel}}{c} \right)$$

The radar of a police officer's radar gun emits microwave radiation at about  $3 \times 10^{10}$  Hz. The officer is driving at 35 mph some very reckless person is driving toward him at 55 mph in a 35 mph zone.

(a) What is the relative velocity between the officer and the driver?

$$v_{rel} = 35 \text{ mph} + 55 \text{ mph} = 90 \text{ mph}$$

$$\frac{90 \text{ miles}}{\text{hour}} \cdot \frac{1609 \text{ meter}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{3600 \text{ s}} = 40.2 \text{ m/s}$$

(b) What is the frequency of the radar that the speeding car observes?

$$\text{speeder} \rightarrow f_o = f_s \left( 1 + \frac{v}{c} \right) \leftarrow \text{officer}$$

$$f_o = 3 \cdot 10^{10} \text{ Hz} \left( 1 + \frac{40.2}{3 \cdot 10^8} \right)$$

$$f_o = 3.0000004023 \cdot 10^{10} \text{ Hz}$$

(c) When this radiation hits the speeding car, it reflects back to the officer, but the reflected light is essentially re-emitted from the speeding car at the doppler shifted frequency it "observed". So now the speeding car is emitting radar (it is a now source) and the officer is observing this radiation, but since they are moving toward each other, it is doppler shifted again. What frequency of radar does the officer observe now?

$$\text{officer} \rightarrow f_o = f_s \left( 1 + \frac{v}{c} \right) \leftarrow \text{speeder}$$

$$f_o = 3.0000004023 \cdot 10^{10} \text{ Hz} \left( 1 + \frac{40.2}{3 \cdot 10^8} \right)$$

$$f_o = 3.0000008046 \cdot 10^{10} \text{ Hz}$$

$$f_{\text{officer receives}} = f_{\text{officer sends}} \left( 1 + \frac{v}{c} \right)^2$$

- (d) Actually measuring this frequency is hard, since they are so close together, but what can be done is measuring the *beat frequency* of the officer's emitted radar, and the observed beam reflected from the speeder. What is the beat frequency of these two waves?

frequency of amplitude change  $\rightarrow$

$$f_b = |f_2 - f_1|$$

$$f_b = \underline{\underline{8046 \text{ Hz}}}$$

12. At what relative speed does the approximate form of the doppler shift equation give an error of 1%. You should make a table (or even better an Excel sheet) and try out several values for  $v_{rel}$  to find out. Choose any  $f_s$  or just use the ratio of  $f_o/f_s$ .

relative velocity	approximate	exact	percent error	source frequency
1	30000000100	30000000100	0	3E+10
10	30000001000	30000001000	3.81E-14	
100	30000010000	30000010000	5.56E-12	
1000	30000100000	30000100000	5.56E-10	
10000	30001000000	30001000017	5.56E-08	
100000	30010000000	30010001667	5.56E-06	
1000000	30100000000	30100167224	0.000556	
10000000	31000000000	31017236587	0.055571	
20000000	32000000000	32071349029	0.22247	
30000000	33000000000	33166247904	0.501256	
40000000	34000000000	34306312494	0.892875	
50000000	35000000000	35496478699	1.39867	
60000000	36000000000	36742346142	2.02041	
70000000	37000000000	38050309946	2.760319	
80000000	38000000000	39427724440	3.621118	
90000000	39000000000	40883108632	4.60608	
100000000	40000000000	42426406871	5.719096	

$4.5 \cdot 10^7 \text{ m/s}$

Solve for  $v$

$$\hookrightarrow f_o = f_s \left(1 + \frac{v}{c}\right)$$

$$\frac{f_o}{f_s} = 1 + \frac{v}{c}$$

$$\frac{v}{c} = \frac{f_o}{f_s} - 1$$

$$v = c \left( \frac{f_o}{f_s} - 1 \right)$$