

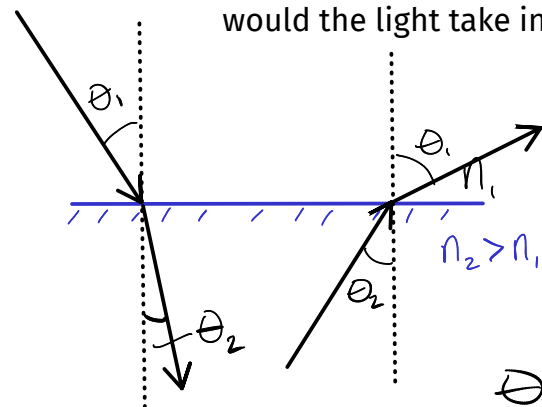
Week 13 covers sections 3,4, and 9 of chapter 23 in the textbook. Topics include:

- refraction and Snell's Law
- dispersion (color separation by a prism)
- total internal reflection and critical angle
- image formation and converging and diverging lenses

1. Snell's law relates the angle of incidents and transmission to the indices of refraction of the material. Here is a formula for it:

$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2} \quad \longrightarrow \quad n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Let material 1 be air  $n_1 = 1$  and material 2 be water  $n_2 = 1.33$ . When light is traveling from air to water at an incident angle of  $30^\circ$ , then what is the transmitted angle? Draw a picture of this. What angle of incidence would make the transmitted angle be  $20^\circ$ ? If light were traveling from the water out to the air at an incident angle of  $10^\circ$  what angle would the light take in the air? What if this angle under the water were  $20^\circ$ ?



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \frac{n_1}{n_2} \cdot \sin \theta_1$$

$$\theta_2 = \sin^{-1} \left( \frac{n_1}{n_2} \sin \theta_1 \right)$$

$$\theta_2 = \sin^{-1} \left( \frac{1}{1.33} \sin 30^\circ \right)$$

$$\theta_1 = 30^\circ \rightarrow \theta_2 = 22.1^\circ$$

$$\theta_1 = \sin^{-1} \left( \frac{n_2}{n_1} \sin \theta_2 \right)$$

$$\theta_2 = 20^\circ ; \theta_1 = 27.1^\circ$$

$$\theta_1 = \sin^{-1} \left( \frac{n_2}{n_1} \sin \theta_2 \right)$$

$$\theta_2 = 10^\circ ; \theta_1 = 13.35^\circ$$

$$(\theta_2 = 20^\circ ; \theta_1 = 27.1^\circ)$$

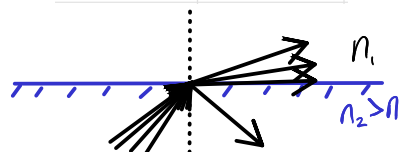
2. Make a table of the incident angles and transmitted angles for light traveling from air into water. What angle does the transmitted angle approach? Then revert the direction of light and calculate a table for the light going from water into air. What happens?

air -> water	
theta 1	theta 2
0	0
10	7.502100527
20	14.90149465
30	22.08241319
40	28.90108454
50	35.1678191
60	40.62813065
70	44.95375334
80	47.77035587
90	48.75346663

water -> air	
theta 1	theta 2
0	0
10	13.35313
20	27.05765
30	41.68233
40	58.74952
50	#NUM!
60	#NUM!
70	#NUM!
80	#NUM!
90	#NUM!

40	58.74952
41	60.75733
42	62.86617
43	65.10188
44	67.50241
45	70.12761
46	73.08195
47	76.58136
48	81.25795
49	#NUM!

48	81.25795
48.1	81.86377
48.2	82.51679
48.3	83.23069
48.4	84.02723
48.5	84.94535
48.6	86.06933
48.7	87.68138
48.8	#NUM!
48.9	#NUM!



48.753...  $\rightarrow 90^\circ$

3. What you have found is an angle for which the light does not emerge from the material, but rather reflects inside the material. This is known as total internal reflection and it is how fiber optic cables work. The angle for which this occurs is known as the critical angle. Now use Snell's Law to derive an expression for the critical angle in terms of the index of refraction of the two materials. Be careful about which index of refraction is for the material that the light is in, and which index of refraction that the light does not go into.

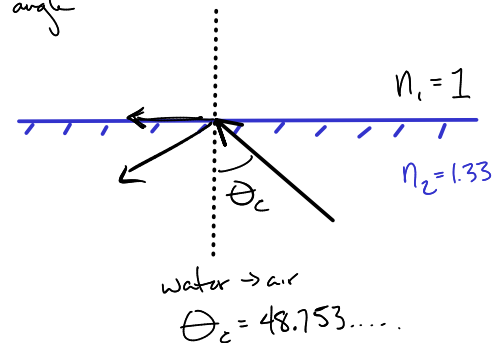
$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{"critical" angle}$$

$\theta_2 \rightarrow \theta_c$

$\uparrow$   
 $90^\circ$   
 $\sin 90^\circ = 1$

$$n_1 = n_2 \sin \theta_c$$

$$\theta_c = \sin^{-1} \left( \frac{n_1}{n_2} \right)$$

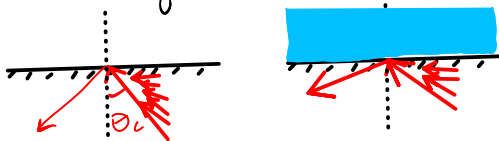


4. What is the critical angle for light in glass surrounded by air? If you surround the glass with water, what does the critical angle become?

$$n_{\text{glass}} = 1.5 \quad | \quad n_{\text{water}} = 1.33 \quad | \quad n_{\text{air}} = 1$$

glass surr. by air:  $\theta_c = 41.8^\circ$

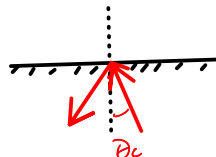
glass surr. by water:  $\theta_c = 62.5^\circ$



$$n_{\text{diamond}} = 2.41$$

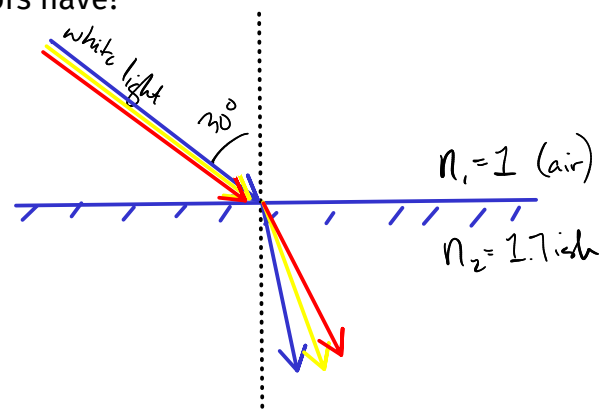
diamond surr by air:  $\theta_c = 24.5^\circ$

diamond surr by water:  $\theta_c = 33.5^\circ$



5. I mentioned in the notes that white light is composed of all the colors in the visible spectrum at once and that the index of refraction of most materials (*slightly*) depends on the frequency of the light itself. Below is a table of three colors and the index of refraction in flint glass. What is the speed of these three wavelengths in flint glass? If light consisting of these colors arrives in a single beam at an incident angle of  $30^\circ$ , then what angle of transmission do each of these colors have?

$\lambda$ (nm)	color	$n$ (flint glass)	$\theta_t$
486.1	blue	1.7328	16.77
589.2	yellow	1.7205	16.89
656.3	red	1.7076	17.03



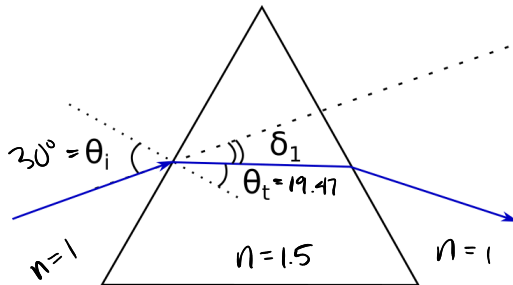
6. When light passes through a prism its path changes once on the way into the prism and then again on the way out. Assume the prism is an equilateral triangle of glass ( $n = 1.5$ ).

(a) For an exterior incident angle of  $30^\circ$ , what is the transmitted angle in the prism?

$$n_{i1} \sin \theta_{i1} = n_{t1} \sin \theta_{t1}$$

$$\theta_{t1} = \sin^{-1} \left( \frac{n_{i1}}{n_{t1}} \cdot \sin \theta_{i1} \right) = \sin^{-1} \left( \frac{1}{1.5} \cdot \sin(30^\circ) \right)$$

(b) Inside the prism, what is the deviation  $\delta_1$  of the light's path?  $\theta_{t1} = 19.47^\circ$



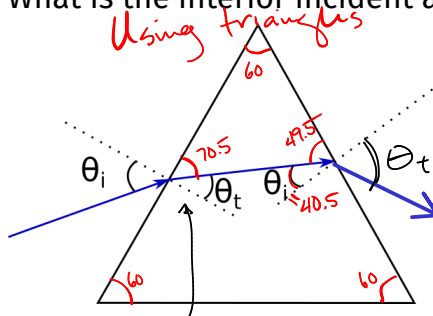
$$\theta_{i1} = \delta_1 + \theta_{t1}$$

$$\delta_1 = \theta_{i1} - \theta_{t1}$$

$$\delta_1 = 30 - 19.47$$

$$\delta_1 = 10.53^\circ$$

(c) What is the interior incident angle as the light is leaving the prism?



$$90 - 19.47 = 70.5$$

$$180 - 60 - 70.5 = 49.5$$

$$90 - 49.5 = 40.5^\circ$$

generalize

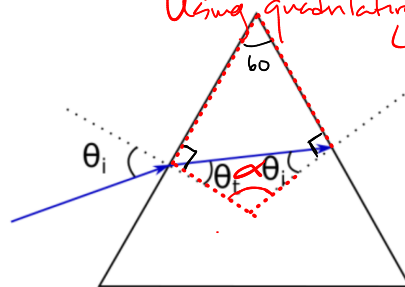
$$90 - \theta_{t1}$$

$$180 - 60 - (90 - \theta_{t1})$$

$$90 - (180 - 60 - (90 - \theta_{t1})) \Rightarrow 90 - 180 + 60 + 90 - \theta_{t1}$$

$60 - \theta_{t1}$  ← useful to solve!

Using quadrilateral



$$360^\circ = 90 + 90 + 60 + \alpha$$

$$\alpha = 120$$

$$180^\circ = \theta_{t1} + \theta_{i2} + \alpha$$

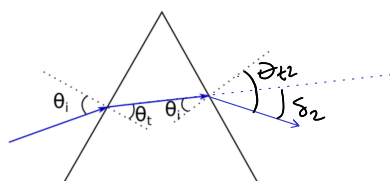
$$180^\circ = 19.47^\circ + \theta_{i2} + 120^\circ$$

$$\theta_{i2} = 40.53^\circ$$

(d) What is the final transmitted angle as the light leaves the prism?

$$\theta_{t2} = \sin^{-1} \left( \frac{n_{i2}}{n_{t2}} \cdot \sin \theta_{i2} \right) = 77.1^\circ$$

(e) What is the second deviation of the light's path?

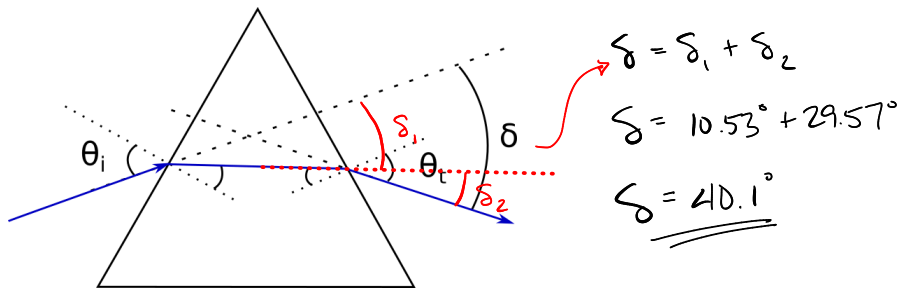


$$\theta_{i2} = \delta_2 + \theta_{t2}$$

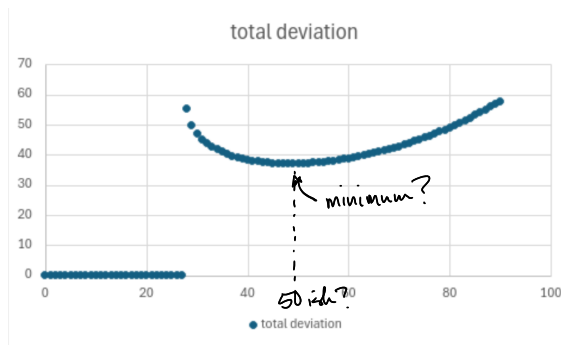
$$\delta_2 = \theta_{i2} - \theta_{t2}$$

$$\delta_2 = 77.1 - 40.53 = 29.57^\circ$$

(f) What is the total deviation of the light's path?



(g) Using excel or google sheets, do this procedure with many exterior incident angles (all integer angles). Don't forget that excel's trig functions only accept radians and inverse trig functions will return angles in radians so you will have to include some conversion factors in the formula ( $\pi \text{ rad} = 180^\circ$ ). Plot the total deviation as a function incident angle and see if there are any interesting features.



7. For a converging lens with a focal length of  $\pm 5$  cm, plot the image distance as a function of object distance. Do a ray tracing for objects at three different object distances, one farther than 2 focal lengths away, one between 1 and 2 focal lengths away, and one less than a focal length away from the lens.

thin lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

magnification equation

$$-\frac{q}{p} = m = \frac{h'}{h}$$

$$f = +5 \text{ cm}$$

$$p = +15 \text{ cm}$$

$$q = ? \rightarrow 7.5 \text{ cm}$$

$$h = 2 \text{ cm}$$

$$h' = ? \rightarrow -1 \text{ cm}$$

$$m = ? \rightarrow -0.5$$

$$q = \frac{1}{\frac{1}{f} - \frac{1}{p}}$$

$$q = \frac{1}{\frac{1}{+5} - \frac{1}{+15}}$$

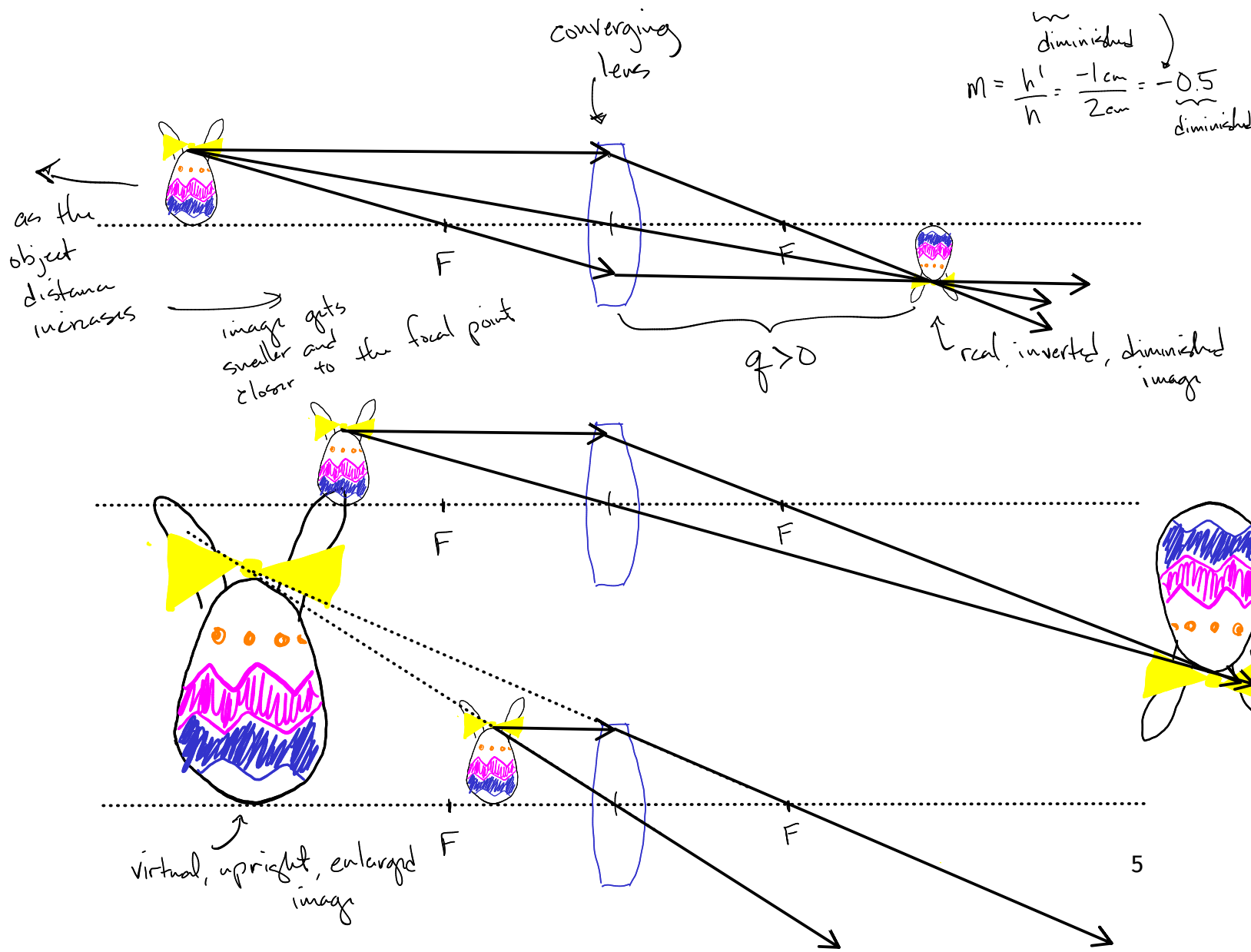
$$q = +7.5 \text{ cm}$$

$$h' = h \cdot \left(-\frac{q}{p}\right)$$

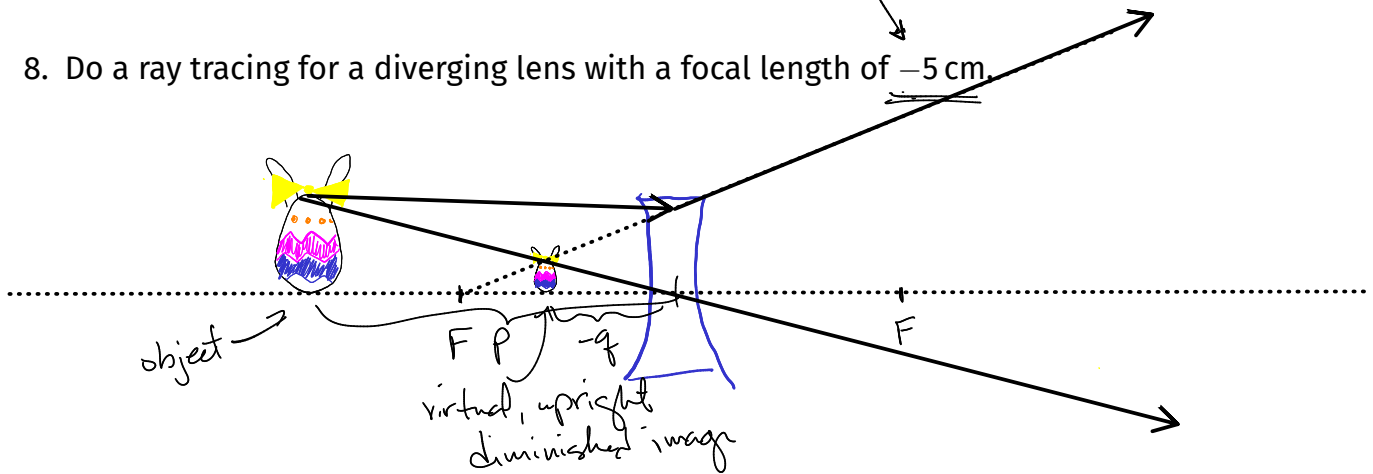
$$h' = 2 \text{ cm} \cdot \left(-\frac{+7.5}{+15}\right)$$

$$h' = -1 \text{ cm}$$

$$m = \frac{h'}{h} = \frac{-1 \text{ cm}}{2 \text{ cm}} = -0.5$$



8. Do a ray tracing for a diverging lens with a focal length of  $-5\text{ cm}$ .



9. In order to magnify an object by 5 times and form a real image with a  $100\text{ mm}$  focal length lens, where should the object be placed in front of the lens?

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{+100\text{ mm}}$$

tricky  $\downarrow$   
 $m = -5 = -\frac{q}{p}$

$$\left(\frac{5}{5}\right)\frac{1}{p} + \frac{1}{5p} = \frac{1}{100}$$

$$\frac{5}{5p} + \frac{1}{5p} = \frac{1}{100}$$

$$\frac{6}{5p} = \frac{1}{100}$$

$$\frac{5p}{6} = 100$$

$$p = +120\text{ mm}$$

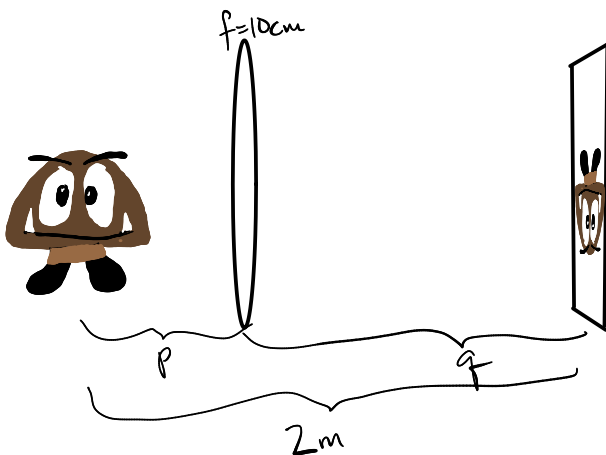
$$q = +600\text{ mm}$$

$$-5 = -\frac{q}{p}$$

$$5 = \frac{q}{p}$$

$$5p = q$$

10. You make a work of art that consists of an object  $2\text{ m}$  away from where you want a real image formed. Where should you place a  $10\text{ cm}$  lens in order to make the image?



$$\bullet \frac{1}{p} + \frac{1}{q} = \frac{1}{0.10\text{ m}}$$

$$\bullet p + q = 2\text{ m}$$

$$q = 2 - p$$

$$\frac{1}{p} + \frac{1}{2-p} = \frac{1}{0.10}$$

$$\frac{(2-p)}{p(2-p)} + \frac{p}{p(2-p)} = 10$$

$$\frac{2-p+p}{p(2-p)} = 10$$

$$\frac{2}{p(2-p)} = 10$$



$$Z = 10p(2-p)$$

$$Z = 20p - 10p^2$$

$$0 = -10p^2 + 20p - 2$$

$$\frac{0}{2} = \frac{10p^2 - 20p + 2}{2}$$

$$0 = 5p^2 - 10p + 1$$

$a=5 \quad b=-10 \quad c=1$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for artistic  
reasons

$p = 0.1055$	$q = 1.8944$	$m = -\frac{q}{p} = -17.9$
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— or —

$$p = 1.89$$

$$q = 0.1055 \quad m = -0.055$$