Week 1

Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being $0\,^{\circ}\mathrm{C}$ when water freezes and $100\,^{\circ}\mathrm{C}$ when water boils at $1\,\mathrm{atm}$ of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. $105\,^{\circ}\mathrm{F}$, $98.6\,^{\circ}\mathrm{F}$, $72\,^{\circ}\mathrm{F}$, $32\,^{\circ}\mathrm{F}$, $0\,^{\circ}\mathrm{F}$. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by $30\,^{\circ}\mathrm{F}$, then by how much does the temperature change in Celsius? What does this mean about the ”size” of a Celsius degree vs. the ”size” of a Fahrenheit degree? Which one represents a larger change in temperature?

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

5. What is the molecular weight of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

9. A balloon is filled with 0.4 mol of helium so that its volume is $0.010\,\mathrm{m}^3$.
   - Find the number of atoms.
   - Find the number density.
   - Find the mass density.

- Estimate the average distance between atoms. To do this, fine the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

10. You have a pound of feathers and a pound of lead.
    - Which one weighs more?
    - Which one has more mass?
    - Which one has the greater volume?
    - Which one contains a larger number of moles?
    - Which one contains a larger number of atoms?
    - Which one contains a larger number of protons and neutrons?

11. You check your car tire pressure and see that the pressure is $25\,\mathrm{lb/in^2}$. What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

12. You check you car tire pressure when it is $15\,^{\circ}\mathrm{C}$ and it is $25\,\mathrm{lb/in^2}$. By what factor do you increase the number of particles in the tire so that the pressure becomes that $30\,\mathrm{lb/in^2}$? (*Hint: The volume and temperature do not change.*)

13. The gas pressure inside of a 1 liter sealed container at room temperature is $1\,\mathrm{atm}$. How many molecules are inside? How many moles of molecules?

14. If the pressure inside a tank is $1\,\mathrm{atm}$ when the temperature is $100\,\mathrm{K}$, then what is the pressure when the temperature rises to $200\,\mathrm{K}$?

15. If the pressure inside a tank is $1\,\mathrm{atm}$ when the temperature is $100\,^{\circ}\mathrm{C}$, then what is the pressure when the temperature rises to $200\,^{\circ}\mathrm{C}$? *CAREFUL!*

16. A gas is in a sealed container. By what factor does the pressure change if
    - the volume is doubled?
    - the temperature is tripled?
    - the volume is double and the temperature is tripled?
    - the volume is halved?

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

Week 2

Week 2 covers sections of chapter 14 in the textbook. Topics include:

- heat, energy, and power
- heat capacity, specific heat capacity, and molar heat capacity
- latent heat of fusion or vaporization

1. Suppose you have 2 kg of water in a cup and you put it in the microwave. What form of heating is this (conduction, convection, or radiation)? It takes 1 minute to heat up from 20 °C to 40 °C. What is the temperature change in Celsius (ΔT)? What is the change in temperature change in Kelvin? Look up the specific heat of water and be careful to specify the units. How much heat would it take to accomplish this temperature change?

2. What are the units of power? What is another way to express those units? To follow up the previous problem, what is the power delivered to the water by the microwave? If this rate of heat delivery continues, how long will it take for the water to reach its boiling point?

3. You have three samples of material, all 1 kg each. The materials are gold, copper, and aluminum. Put these in order of how much heat is necessary to change the temperature 1 degree.

4. You have the same materials as in the last problem. If you put the same amount of heat into each sample, put them in order of which would heat up the most (remember that when we say ”heat up” we mean increase in temperature).

5. How much heat does it take to bring 2 kg of aluminum from 25 °C to 50 °C?

6. If it takes 5000 J to bring an ingot of gold from 25 °C to 50 °C, then what mass of gold is the ingot?

7. If 5000 J of heat goes into a 1.5 kg glass dish that was initially at 20 °C, then what is its final temperature?

8. If a 5 kg block of aluminum has a temperature of 500 °C, how much heat does it give off to cool down to 490 °C? What should the sign of heat be in this case?

9. If the aluminum block in the above problem gave up this heat because it was put in contact with a cooler 5 kg block of lead, then by how much does the temperature of the lead rise? If the lead was originally 75 °C, then what would be its new temperature?

10. Keep this process going, the aluminum cooling off by 10 degress and the lead warming up by whatever you found in the above problem. Approximately what temperature would they meet? This temperature is known as the *equilibrium temperature* since after this heat flow stops, and they remain at the same temperature (ignoring heat that they lose to the surroundings).

11. Lets do this in one step now. If the aluminum has an initial temperature of 500 °C and an *unknown final temperature*, and the lead starts at 75 °C and has an unknown final temperature but the same final temperature as the aluminum, since that is the equilibrium temperature, then how can we find this with one expression? (*Hint: the overall energy of*

the system does not change. So any change in energy of one plus the change in energy of the other must be zero.)

12. The exact same logic that applied to the above problem, applies to mixing two substances together. You can still treat them as separate substances with one or more giving up energy in the form of heat to the others. It is always assumed that the materials are kept in a well insulated container so that no heat is lost to the surroundings. This is a good way of measuring the heat capacity of an unknown material by mixing it with a material of known initial temperature and heat capacity and then measuring the equilibrium temperature that results. So suppose you start with 100 g of water at 90 °C, and you pour in 50 g of unknown metal at 20 °C. You stir the mixture and notice that the temperature of the mixture comes down to 40 °C and then remains at that temperature. What is the specific heat of the unknown material?

13. A monatomic gas is in a container that keeps the gas at constant volume of 0.01 m^3. The gas is at room temperature and the pressure is 1 atm. How much heat do you have to add to increase the temperature by 10 °C. What is the pressure in the container at that temperature?

14. A monatomic gas is in a container that keeps the gas at constant pressure of 1 atm. The gas is at room temperature and the volume is $0.01\,\text{m}^3$. How much heat do you have to add to increase the temperature by $10\,^\circ\text{C}$. What is the volume of the container at that temperature?

15. Why is there a difference between the heat needed to change the temperature of an ideal gas at constant volume vs at constant pressure?

16. How much heat is needed to melt a block of ice that is 10 kg at $0\,^\circ\text{C}$? What if the ice starts at $-10\,^\circ\text{C}$?

17. You drop a 0.1 kg ice cube at $0\,^\circ\text{C}$ into a cup with 200 g water at room temperature ($20\,^\circ\text{C}$). Before you work this problem, think about the possible outcomes. How much heat does it take to melt the ice cube completely? How much heat would it take to cool the water to $0\,^\circ\text{C}$? Given this information which outcome is the one that happens? Find the equilibrium temperature.

18. A 60 kg hiker wished to climb to the summit of Mt. Ogden, an ascent of 5000 verticalfeet (1500 m).

    - How much work will it take for her to reach this height?

    - Assuming that she is only 25% efficient at converting chemical energy from food into mechanical work, and that essentially all of the mechanical work is used to climb vertically, roughly how many bowls of corn flakes should the hiker eat before setting out? (standard serving size 1 oz, 100 Calories)

    - As the hiker climbs the mountain the other 75% of the energy from the corn flakes is converted into thermal energy. If there were no way to dissipate this energy, by how many degrees would her body temperature increase? (Assume the human body is mostly water so that it has the same specific heat as water.)

    - In fact, the extra energy does not warm the hiker's body significantly; instead, it goes (mostly) into evaporating water from her skin and within her lungs. How many liters of water should she drink during the hike to replace the lost fluids? (At 25 °C, a reasonable outdoor temperature to assume, the latent heat of vaporization of water is 580 cal/g, 8% more that at 100 °C.)

Week 3

Week 3 covers sections of chapter 14 in the textbook. Topics include:

- First Law of Thermodynamics
- thermodynamic processes and cycles
- efficiency
- heat engines vs heat pumps

1. On a cold day, you rub your hands together to warm them up. If you press your hands together with a force of 5 N and the coefficient of friction between your hands is $\mu = 0.45$ and your hands move an average distance of 12 cm, then how much has the internal energy of your hands changed?

2. Express the first law of thermodynamics as a statement about power and the rate of energy change instead of the total energy changes.

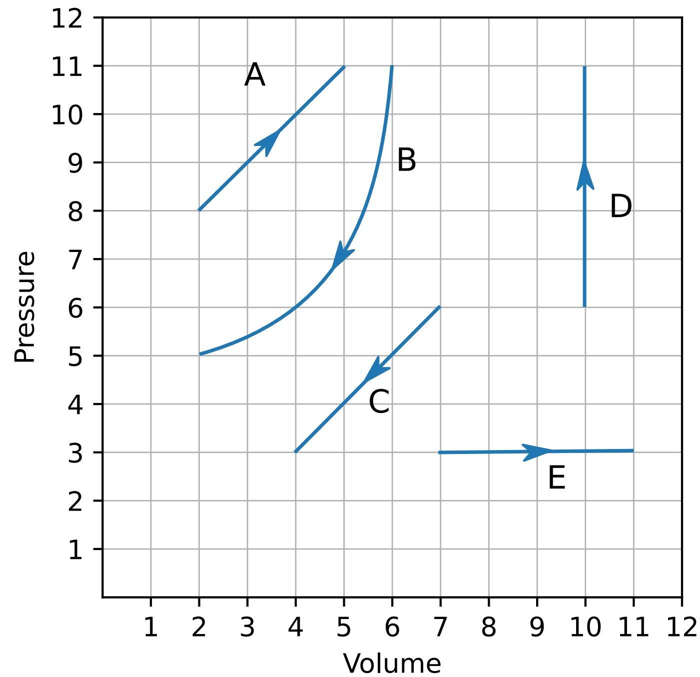
3. When you exercise, you do work on something else. What does this mean about the sign of the work done on you? When you exercise, your body gives off heat to the surroundings (through evaporation mostly, but also some radiation and convection). what does this mean about the sign of the heat flow from you? If you do work at a rate of 200 W and

you radiate heat at a rate of 800 W, the what is the rate of internal energy change of your body? If you exercise for 30 minutes, then how much has your internal energy changed? How many bowls of cornflakes is this?

4. In the notes I claim that Work $=$ Pressure $\times$ ΔVolume. Show clearly how the units here work out to be equal.

5. 14 kJ of heat flow into a gas cylinder with a piston and the internal energy of the gas increases by 42 kJ. How is this possible? Did the volume get bigger or smaller?

6. Rank the thermodynamic processes represented in the figure below in order of the work done from least to greatest. Rank positive work as higher than negative work.

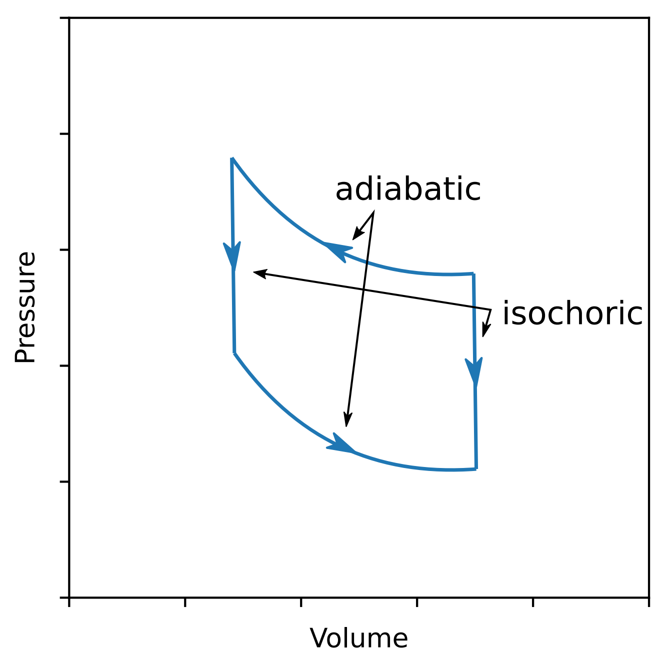


7. A monatomic ideal gas initially at room temperature (20 °C) undergoes an isobaric expansion from an initial volume of 1 L to 2 L at a pressure of 2 atm. Then the gas goes through an isochoric process where the pressure lowers to 1 atm. Draw a PV diagram of this process, and then calculate the total work done on the gas in these two processes.

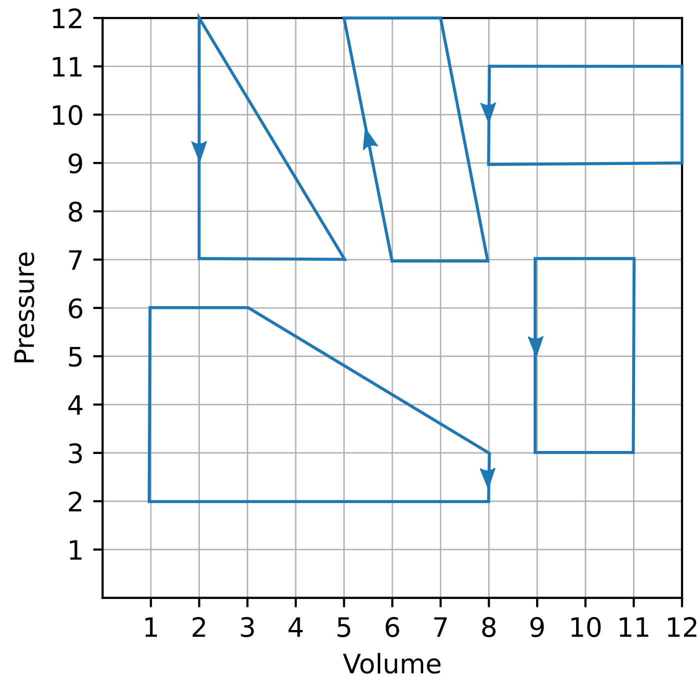
8. A monatomic ideal gas initially at room temperature (20 °C) undergoes an isochoric process where the pressure changed from 2 atm to 1 atm. Then the gas goes through an isobaric expansion from an initial volume of 1 L to 2 L. Draw a PV diagram of this process, and then calculate the total work done on the gas in these two processes.

9. Sketch a PV diagram of a thermodynamic cycle that contains an two isothermal expansions and and two isochoric processes. First do this as a heat engine, then do it as a refrigerator. Specify for each step, when heat is going in or out, and when work is being done on the gas or on the surroundings.

10. The following PV diagram is a wrong representation of the Otto Cycle, which is a model of a gasoline engine. What are the problems in the diagram below.



11. Rank the following cycles by the net work done by the system onto the surroundings from least to greatest. Rank positive work on the surroundings as higher than negative work (which would be done on the engine like in a heat pump). Which of these are heat engines and which are heat pumps?



12. A heat engine operates with two isobaric processes and two isochoric processes. The pressure varies from 1 atm and 4 atm and the volume varies from $0.200\,\mathrm{m^3}$ to $0.800\,\mathrm{m^3}$. Draw a PV diagram and specify the directions of these transitions. How much work is done by the engine in one cycle? What is the net heat flow into the engine per cycle? What is the heat in and the heat out? What is the efficiency of this engine?

13. What is the difference between a heat pump and an air conditioner?

14. The definition of efficiency from the notes is:

$$e = \frac{W_{out}}{Q_{in}}$$

Also from the first law of thermo, we had:

$$Q_{in} = W_{out} + Q_{out}$$

Combine these two equations to get an expression for efficiency only in terms of heat in and out (*Hint: substitute for Work.*)

15. Follow up on the previous problem. Heat engines can operate at many different efficiencies depending on the cycles and how much heat goes in and out, but there is a cycle that represents an ideal heat engine that is the best efficiency possible. It is called the *Carnot Cycle*. The details of this cycle are not important but the fundamental take away is that the heat that comes in (Q_{in}) comes from a source that is at temperature T_H, and the heat that goes out (Q_{out}) goes to a temperature of the surroundings at T_C. For a heat engine that operates between these temperatures (like the Carnot cycle) the ratio of the Heat out and in is equal to the ratio of the temperatures:

$$\frac{Q_{out}}{Q_{in}} = \frac{T_C}{T_H}$$

This is a long lead up to the task of use this proportion to find an expression for the efficiency of an ideal heat engine in terms of the temperatures between which the engine operates. (*Hint: Make a substitution in the equation from the answer to the previous problem.*)

16. In the engine of a car, suppose the combustion of gasoline and air that takes place produces temperatures that are around 3000 °C while the exhaust gas exits the engine at a temperature of around 1000 °C. Forget about the actual cycle that a gasoline engine undergoes, if this were an ideal process what is the efficiency that this engine can achieve. If you would improve the design to the extent that the exhaust gasses left the engine at room temperature and it still operated as an ideal heat engine, then what would the efficiency be?

Week 4

Week 4 covers sections of chapter 16 in the textbook. Topics include:

- charge
- electric field and electric force
- motion of particles within a constant electric field

1. Let's work on units.
    - What are the units of electric field? Use the equation $\vec{F} = q \cdot \vec{E}$
    - What are the units of the Coulomb constant, k? Use the equation $E = \frac{kq_0}{r^2}$ or $F = \frac{kq_o q_1}{r^2}$.
    - What are the units of the vacuum permittivity constant, ε_0? Use the equation $k = \frac{1}{4\pi\varepsilon_0}$.

2. An elementary charge is the charge of either an electron or proton, and interestingly they are the same amount but different sign. How many elementary charges are in one Coulomb of charge? How many Coulombs of charge is one elementary charge? How many electrons would there be in 1 μC of charge? 1 nC?

3. What is the electric field of a 10 μC point charge 1 m away from the point charge?

4. Where is the electric field of a 10 μC point charge 50 N/C?

5. How much charge is in a point charge if the electric field is 1 N/C 1 m away?

6. If you double the distance away from a point charge, by what factor do you change the electric field strength? Triple? Half? Quarter?

7. If the electric field doubles, by what factor do the distances between these locations differ? What if the electric field is 10 times smaller?

8. A 3 μC and a 5 μC are 5 m away from each other. What is the magnitude of the electric field produced by the 3 μC charge 5 m away from it? What is the force on the 5 μC charge? If the 5 μC charge was instead a 9 μC charge, what is the force on it? In what direction does this force point?

9. Lets see if Newton's Third Law applies to the electric force. In the last problem you found the electric field of the 3 μC charge and then multiplied by the other charge. But this time find the electric field of the 5 μC charge 5 m away? Is it the same? What is the force on the 3 μC charge? Is it the same? What is its direction? Is this consistent with Newton's Third Law?

10. A $+20$ μC charge is 2 m away from a -5 μC charge. What is the electric field halfway between the two charges? What is it 1 m away from the $+20$ μC charge on the side opposite the -5 μC charge. What is the electric field 1 m away from the -5 μC charge on the oppo-

site side as the $+20\,\mu C$ charge? Draw a picture of all of this to keep the locations straight.

11. For the charge distribution above, can you find where the electric field would equal zero? Think first about which of the three possibilities along the axis where these charges exist. Then set up an equation and solve for the distance away from the negative charge where the field is zero.

12. I have collected $0.001\,C$ of charge on a sheet that has an area of $100\,cm^2$. What is the electric field 1 mm away from this sheet? What is it 1 cm away? Is is the same value 1 m away? (*Thing about the inside out versions of this problem too.*)

13. When I put charge on a large sheet, it is common to refer to the *charge density* of the sheet rather than the total charge and the total area. The charge density, σ is defined like this:
$$\sigma = \frac{Q}{A}.$$
Substitute this into the equation for the electric field of a large charged sheet.

14. You have two large circular sheets of diameter 15 cm. You orient the sheets so they are parallel to one another. Draw a picture of this. Now imagine moving electrons from on plate and placing them on the other plate. If you do this with a billion electrons then how much charge will be on each plate? This kind of device is called a *capacitor* and we will talk more about this next week. What will be the electric field between the plates? What will be the electric field outside of the positive plate? What will be the electric field outside of the negative plate?

15. A capacitor has a charge density of 1×10^{-6} C/m^2. If the positively charged plate is on the left, and the negatively charged plate is on the right and they are separated by a distance of 1 cm. You place a proton in the center between the plates. What electric force does this charge experience? In what direction does it experience the force? What about if this were an electron?

16. Look up the mass of a proton and an electron (book, google, etc). If you were to let go of either the proton or electron in the previous problem then what would the acceleration of that particle be? Write a general expression for the acceleration of a charged point particle if the electric force is the only force acting on it.

17. You have a capacitor where a total charge of 10 μC has moved between the plates, and the plates have an area of $0.01\,\mathrm{m}^2$. The plates are separated by a distance of 1 cm. If a proton is released from rest from the positive plate, then how fast is it going when it gets to the negative plate?

EXTRAS!

18. A positive and a negative charge will clearly attract on another until they collide, but if you put them perfectly lined up in an electric field, then they will maintain some distance from each other. Draw a uniform electric field that points to the right. Place a negative and positive charge within this field in an arrangement so that the charges will not move (and the force on them is zero). What distance apart are they. Assume it is a proton and an electron.

Week 5

Week 5 covers sections of chapter 17 in the textbook. Topics include:

- work and electric potential energy
- electric potential or voltage
- conservation of energy
- electric field lines and equipotential surfaces
- energy storage in capacitors

1. An uniform electric field has a strength of 1000 N/C and points in the positive x-direction. It doesn't really matter what is causing the electric field but you can imagine we are inside of a capacitor. A 1 μC charge is moved around within this field by an external force.

   (a) How much work does it take to move the charge from the position $x = 5\,\text{cm}$ to a position of $x = 1\,\text{cm}$?

   (b) How much work does it take to move the charge from the position $x = 5\,\text{cm}$ to a position of $x = 9\,\text{cm}$

   (c) How much work does it take to move the charge from the position $x = -5\,\text{cm}$ to a position of $x = 1\,\text{cm}$

   (d) How much work does it take to move the charge from the position $y = 0\,\text{cm}$ to a position of $y = 5\,\text{cm}$

2. For the above problem, what is the amount of work *per unit of charge* that was done in each step?

3. A fixed point charge of 10 μC creates an electric field throughout space around it.

   (a) How much energy would it take to move a 1 μC charge from infinitely far away to a position of 100 m away?

   (b) How much energy would it take to move a 1 μC charge from infinitely far away to a position of 90 m away?

   (c) How much energy would it take to move a 1 μC charge from a position 100 m away to a position of 90 m away?

   (d) To move 10 m closer, how much more energy would it take?

(e) How much energy would it take for each 10 m displacement closer to the fixed charge?

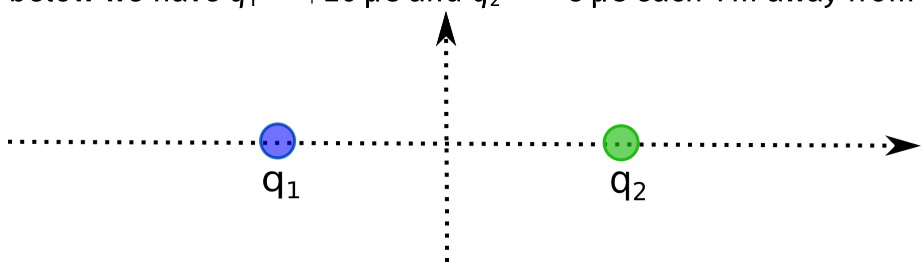
4. Work the steps in the above problem for the voltage change rather than the energy change. Also plot a graph of voltage vs. radius and illustrate on the graph what it looks like to move from say 50 m to 40 m in terms of the voltage difference.

5. Now imagine that the source charge is negative. What does this change about all of the answers to the previous problem?

6. What are some inside out versions of the previous problems. (Write out the wording and I'll feature some of them in the space below.)

7. In order to find the voltage at a place around multiple charges, you simply find the voltage at a location due to each charge individually and then add them up. Same goes for finding the potential energy of putting a charge at a place around other charges. So in the figure

below we have $q_1 = +20\,\mu C$ and $q_2 = -5\,\mu C$ each 1 m away from the origin.



(a) What is the voltage at the location of $x = +2$ m on the x-axis relative to infinitely far away?

(b) What is the voltage at the location of $x = -2$ m on the x-axis relative to infinitely far away?

(c) What is the voltage at the location of $x = -2$ m on the x-axis relative to the location of $x = +2$ m on the x-axis?

(d) What is the voltage at the location of $x = +1\,\text{m}, y = +1$ m (again relative to infinitely far away; if I ever forget to say this then this is what I mean)?

(e) Where along the x-axis would the voltage be equal to zero? What does this mean for the amount of work it would take to move a charge to that location from infinitely far away?

8. Now we need to put together the change in potential energy with the kinetic energy so that we can use the conservation of energy. To review some equations that we used last semester, lets start with a statement of the conservation of energy

$$K_f + U_f = K_i + U_i + W_{non\text{-}conservative}$$

The only non-conservative forces we will have are the ones from outside that increase the potential energy of a charge, but there will be no friction. So if a charge starts from rest at some place you move it to another place and it arrives there at rest then the equation becomes:

$$0 + U_f = 0 + U_i + W_{nc}$$
$$\Delta U = W_{nc}$$

which is what we have been using this entire time.

But, if instead of doing work to move it, what if we just let go? The charge would begin to move in response to the electric field that it was in and the potential energy would go down, and in its place the kinetic energy would increase since the charge is speeding up. There are several ways to represent this:

$$K_f + U_f = K_i + U_i + 0$$
$$\Delta K = -\Delta U$$

Ok now for an actual question. Take this last expression and do a substitution to express the change in kinetic energy as a change in voltage.

9. A $-10\,\mu C$ charge is in a uniform electric field of $1000\,N/C$ that points to the right. If the charge travels a distance of 1 cm, then how much has the voltage changed within the field? How much has the potential energy changed? How much has the kinetic energy changed? The charge has a mass of 1 mg. How fast is the charge going after it has traveled this distance?

10. Draw a capacitor and an electric field within it pointing to the right. What do the equipotential surfaces look like within this capacitor? If the electric field within the capacitor is 500 N/C and the capacitor plates are 1 cm apart, then what is the voltage between the two plates? Divide this voltage difference into a sensible number of “steps” of voltage. Make these the equipotential surfaces and then find the distance between surfaces. Add all of this to your drawing.

11. Refer to your drawing in the above problem. If you put a positive charge in between the plates of this capacitor which way will it accelerate? Is this in the direction of increasing or decreasing voltage or neither? What about a negative charge? The equation for relating the potential energy and the voltage does need a sign on the charge.

12. IN PROGRESS...

Week 6

Week 6 covers sections of chapter 18 in the textbook. Topics include:

- resistive circuits and Ohm's Law
- power use in circuits
- networks of resistors

1. Charges are flowing through a wire like water through a pipe. If the wire has 5 A of current flowing through it, then how much charge passes a point in the wire in 10 s?

2. Batteries are often have a rating written on them. A Energizer AA battery has a rating of 3000 milliAmp · hours (mA h).

   (a) What kind of quantity is a milliAmp · hour? What base unit is this related to?

   (b) If the battery discharged all of this charge at 1.5 V, then how much potential energy was initially in the battery?

   (c) If the battery discharges at a rate of 25 mA, then how long will the battery last?

(d) What is the power output of the battery?

3. A simple particle accelerator can be designed from a capacitor in a vacuum. The negatively charged plate has a heater near it, and when electrons have enough thermal energy, then they are released from the plate and are accelerated in the electric field between the plates. If the positive plate has a hole in it, then the electrons will fly through and now you have an electron gun. If the potential on the plates is maintained, then a steady stream of electrons can result. If a mole of electrons passes through the hole in the positively charged plate per second, then what is the current of the gun? Bonus question from last week, if voltage between the plates is 5000 V then how fast will the electrons emerge from the gun?

4. The resistance of a wire is directly proportional to the length and inversely proportional to the cross-sectional area. But measuring the area of a wire really comes from measuring the diameter of a wire and then using that to find the area. So use the area of a circle formula to substitute into the resistance of a wire equation and make sure you express this in terms of diameter. If the diameter of one wire is twice the diameter of another, then by what factor are the resistances related?

5. A copper wire is 100 m long and has a diameter of 1 mm. It is connected to a 12 V battery.
   (a) What is the resistance of the wire?
   (b) How much current goes through the wire?
   (c) What is the power supplied by the battery?
   (d) What power is dissipated by the wire?
   (e) How much heat would this give off in 10 minutes?
   (f) How much would this heat up a liter of water?

(g) If the length of the wire doubled, what would happen to all of the above quantities?

6. 100 m of a certain wire has a resistance of 5 Ω. What is the resistance of the same wire if it is 12 m long?

7. If you stretch a wire to three times its original length, then the wire will get thinner, since it is the volume that remains the same. So by what factor does the cross sectional area change (assume as with all wires that it is a cylinder)? By what factor does the resistance change?

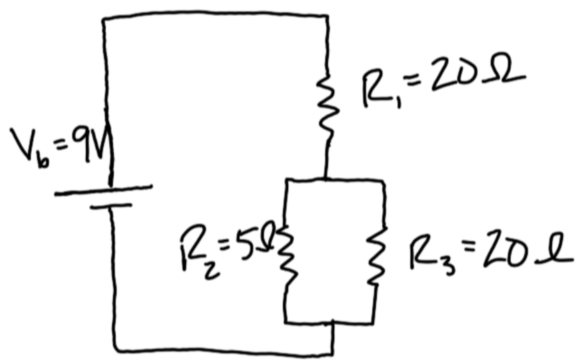
8. Combine the equations for the power dissipated by a resistor, $P = IV$ and Ohm's Law $V = IR$, in two ways and get two equations for power out. One should express power in terms of current and resistance, and the other should express power in terms of voltage and resistance.

9. You have a 12 V battery connected to a 120 Ω resistor. Draw a picture of this. If you measure the voltage from the negative terminal to the positive terminal of the battery, what would you read? If you measured from the positive to the negative, what would you read? If you measured from the positive terminal of the battery to the side of the resistor near to it, what would you measure? If you measure across the resistor from the side nearer to the positive terminal to the other side, what would you measure? How much current is going through the resistor? What power is dissipated by the resistor?

10. Now let's make two changes to the previous problem. Instead of a 12 V battery you have a 9 V and two 1.5 V batteries *in series*. Then instead of a single resistor, there are 3: one that is 20 Ω, one 40 Ω and one 60 Ω all in series. Draw a picture of this first. What is the equivalent resistance of this set of resistors? How much current flows through each one of these resistors? What is the voltage drop across the entire group of resistors? What is the voltage drop across each one? How much power is dissipated by the entire group of resistors? How much power is dissipated by each one?

11. Now lets take the same $20\,\Omega$, $40\,\Omega$ and $60\,\Omega$ resistors and connect them each *in parallel*, that is they each have their own connection to the 12 V battery. Draw this out as a circuit diagram. What is the potential drop across each resistor? What is the current through each resistor? What is the total current supplied by the battery? What is the equivalent resistance of this circuit? How much power is supplied by the battery? How much power is dissipated by each resistor?

12. For the resistor network below, what is the equivalent resistance? What is the current through R_1? What is the voltage drop across R_1? What is the voltage drop across R_2 and R_3? What is the current through R_2 and R_3?



13. A light bulb is rated as 75 W. If it is connected to a standard 120 V then what current does it draw from the outlet? What is the resistance of the filament in the bulb?

14. Household wiring is done so that each outlet is in parallel to the others. A group of several outlets are all connected to a circuit breaker, which will typically break the circuit when more than 20 A flow down the wire to that group of outlets. The voltage provided to the circuit is 120 volts (although unlike the constant voltage provided by a battery, the voltage out of the wall is alternating and looks like a sine wave, but this is not important for us in this problem). Most appliances are rated by the power that they do work, so

for example a vacuum cleaner may operate at 700 W and a coffee maker at 900 W and a toaster might be 1000 W. If these are all plugged in to outlets on the same circuit, how much current would be supplied through the breaker? Will it trip and stop the current?

15. When you pay your power bill, you pay for each kiloWatt · hour of energy that you use. How much energy in Joules is one kW · h?

16. Assume each atom of copper in a wire contributes one electron to be able to move freely in a current. The mass density of copper is $\rho_m = 8.92\,\text{g}/\text{cm}^3$ and the atomic mass of copper is $63.5\,\text{g}/\text{mole}$. What is the number density of conduction electrons in copper (electrons per m^3)? If 5 A of current are flowing through a copper wire that has a diameter of 0.1 mm, then what is the drift velocity of electrons in the wire?

Week 7

Week 7 covers section 7 of chapter 17, the second half of section 6 in chapter 18 about capacitors, and section 10 of chapter 18. Topics include:

- capacitors as energy storage devices
- capacitors in circuits
- circuits with resistors and capacitors called the RC circuit

1. Draw a capacitor and an electric field within it pointing to the right. What do the equipotential surfaces look like within this capacitor? If the electric field within the capacitor is 500 N/C and the capacitor plates are 1 cm apart, then what is the voltage between the two plates? Divide this voltage difference into a sensible number of "steps" of voltage. Make these the equipotential surfaces and then find the distance between surfaces. Add all of this to your drawing.

2. Suppose the capacitor plates in the problems above has an area of $0.1\,\mathrm{m}^2$. What is the capacitance? How much charge is stored on the capacitor? How much energy is stored in the capacitor?

3. Start with the equation for the energy stored in a capacitor and the definition of capacitance, and using substitution derive two other expressions for the energy stored on a capacitor.

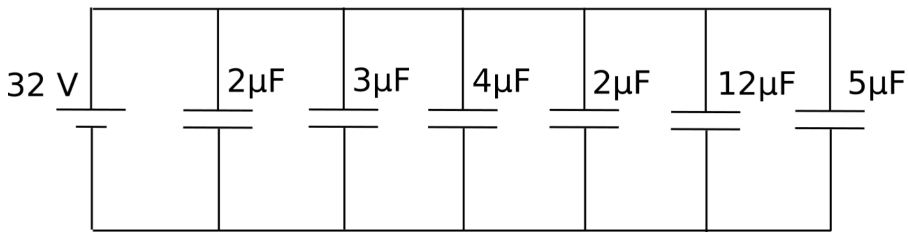
4. Derive an expression for the energy density of a capacitor. Start with one of the expressions above for the energy of the capacitor and divide it by the volume of the capacitor. Use some clever substitution to get an expression for the energy density of the capacitor in terms of the electric field inside the capacitor. Design you're own capacitor and then determine its energy density. Compare the energy density of your capacitor to that of an Li-ion battery as well as gasoline (use google).

5. Let's throw in one more conceptual idea here. We have assumed that the gap between the plates of a capacitor is vacuum within which the electric field can exist apparently. But if we filled this space with an insulating material, then the capacitance will actually be larger sometimes much larger, depending on the material. Insulating materials are often call *dielectric materials* and have a *dielectric constant* that corresponds to the amount that they increase the capacitance above vacuum. It appears in the parallel plate capacitor formula like this:
$$C = \kappa \frac{\varepsilon_0 A}{d}$$
Where that constant κ is the dielectric constant. It varies in size from 1 (vacuum) and 1.00054 (air) to 3-4 for rubber or even 6000 for barium titanate. This means that with a higher capacitance, more charge can be stored on the capacitor per volt that it is charged with. However, all dielectrics have a limit to how high of an electric field they can withstand before they themselves break down. This is know as the dielectic strength and above this electric field the material becomes a conductor and shorts out the capacitor.

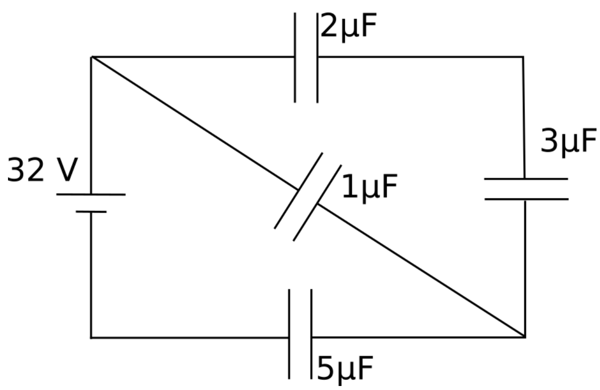
You can see this easily in air when a spark from static electricity shocks you. This shock occurs due to a build up of charge that produces an electric field higher than 3 kV/mm (for air). There is not really a question here, but we will see this effect in lab.

6. Three capacitors are in series and connected to a 12 V, 4 μF, 10 μF, and 15 μF. What is the equivalent capacitance of this circuit? How much charge is stored on each capacitor? What is the potential drop across each capacitor? How much energy is stored on each capacitor and what is the total?

7. For the circuit depicted below, find the equivalent capacitance, the charge stored by each capacitor, and the total charge stored.



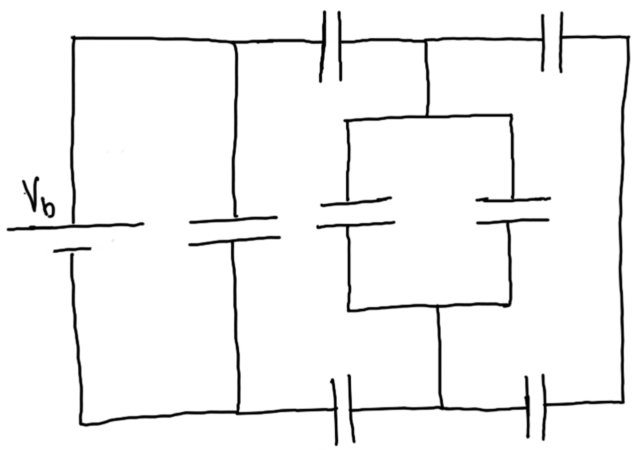
8. What is the equivalent capacitance of the circuit below?



9. In the above problem, suppose the equivalent capacitance was known (make up a number here) but the capacitance on the far right side was unknown. Using the made up number, what is the unknown capacitor?

10. A 6.0 µF capacitor is needed to construct a circuit, but all that is available from the capacitor store is 9.0 µF capacitors. How could you use these to construct a 6.0 µF circuit?

11. This is just mean but see if you can solve this puzzle for the equivalent capacitance if all capacitors are 11 µF



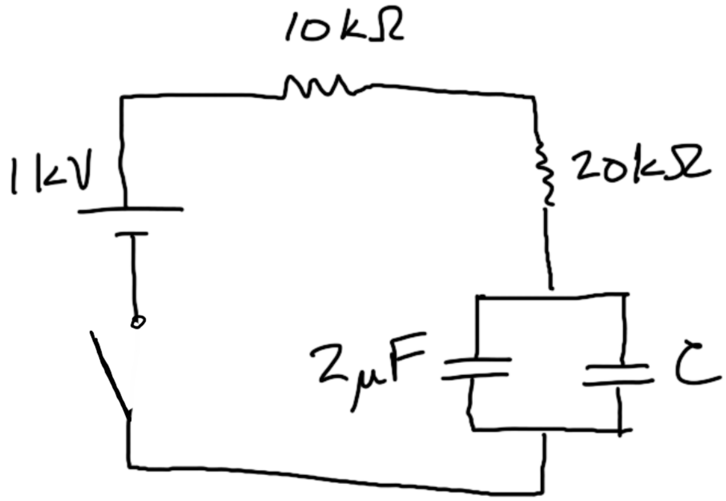
12. Show that the units in the equation for time constant $\tau = RC$ work out.

13. A 100 V battery is connected with an open switch to a 1 MΩ resistor and a 1 μF capacitor. When the switch is closed at $t = 0$ s current flows through the resistor and charge accumulates in the capacitor. What is the initial current in the resistor? What is the initial charge in the capacitor? What is the current in the resistor after a long time has passed? What is the charge in the capacitor after a long time has passed? What is the time constant, τ of this circuit? What is the current, charge, and voltage across the capacitor after 3 seconds have passed? At what time is the current in the resistor 1 μA? At what time is the voltage across the capacitor 50 V? When one time constant worth of time has passed, what is the ratio of the current at that time to the initial current in the resistor?

14. A defibrillator is a way of delivering a large amount of energy in a controlled way to a person's heart to reset the electrical signals that are carefully timed to contract different parts of the heart in a specific order. This discharge is essentially a capacitor and the person's body is a resistor in the pathway between the leads that are in contact with the patient.

    Suppose that a defibrillator has a capacitance of $100\,\mu\mathrm{F}$ and the effective resistance of the body is $300\,\Omega$ and the capacitor is initially charged to a voltage of $5\,\mathrm{kV}$. What is the initial energy stored in the capacitor? What is the initial charge? If the discharge lasts exactly $1\,\mathrm{ms}$ and is stopped at that time by the other electronics in the device, what is the charge on the capacitor at that time? What is the energy still left in the capacitor? How much energy has been delivered to the patient?

15. In the circuit below, the current in the 10 kΩ resistor has a value of 10 mA after 10 s have passed since the switch was closed. What is the time constant of this circuit? What is the equivalent capacitance? What is the value of the unknown capacitor?

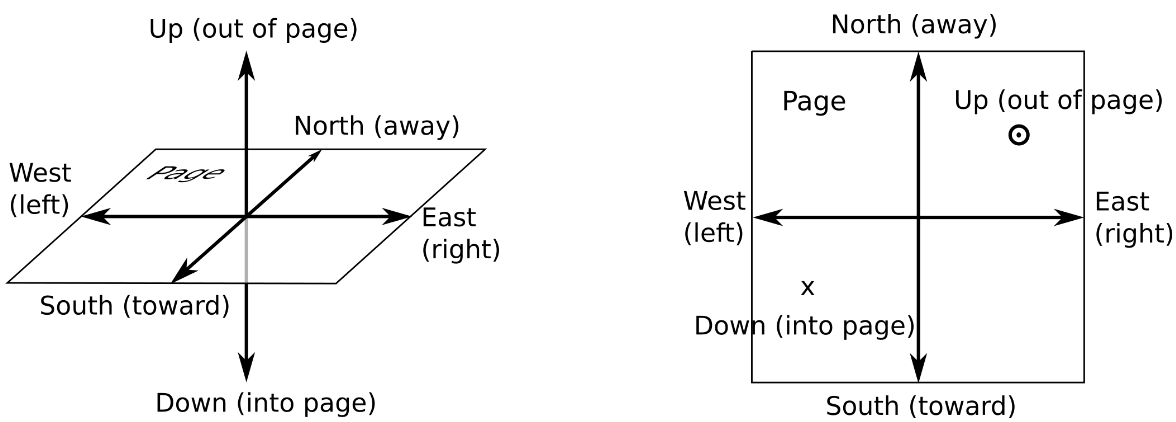


Week 9

Week 9 covers sections 1, 8, and 9 of chapter 19 in the textbook. Topics include:

- magnetic field and everyday phenomena
- magnetic field from a line current
- magnetic field from a coil or dipole
- solenoids
- the right hand rule

1. The hardest part of describing magnetic fields is describing their direction in relation to the direction of other quantities. So the guide below will hopefully help us all use the same terminology so that we can use a consistent system to describe things. There are two common ways of describing the direction of things: the *cardinal directions* (North, South, East, West, Up, Down) and relative directions (Away, Toward, Right, Left, Up Down). Both of these are displayed in the figures below so take some time to see how these two perspectives, and two descriptions work.



2. Draw a picture of a straight line current going to the right. What is the direction of the magnetic field south of the wire? What is the direction of the magnetic field north of the current? What is the direction of the magnetic field directly above the current (out of the page)? What is the direction of the magnetic field directly below the current (into the page)? Now repeat this but with a current that is flowing south (toward you on the page)? If you put a compass in these locations, in what direction would the compass point?

3. Draw a plot of the strength of the magnetic field at a function of distance away from a long straight current. First, what current would have to be in the wire so that the magnetic field strength was 1 T at a distance 1 cm away from the wire? Plot this in 0.5 cm increments.

4. How far away from the wire in the above problem do you have to be to have a magnetic field strength of 1% of the strength at 1 cm away? Work this as a proportion problem.

5. Draw two parallel wires that have current traveling in the same direction (draw them both going upward). The wires are separated by a distance of 1 cm and they each carry a current of 10 A. Find the magnitude and direction of the magnetic field 0.5 cm to the left of the wire on the left and midway between the wires, and 0.5 cm to the right of the wire on the right. What is the magnetic field produced by the wire on the left at the location of the wire on the right?

6. Draw the same problem as above, but with the currents traveling in opposite directions. Make the current on the left go north, and the current on the right go south.

7. Now draw a 10 A current loop and have the current go counter clockwise. What is the direction of the magnetic field at the center of the loop? Use both versions of the right hand rule to show yourself this. In what direction does the magnetic field point in the region outside of the loop. Draw the current with some perspective so that the magnetic field points to the right. What radius of loop is necessary to produce 1 T at the center? If the radius of the loop was 20 cm then how many loops would you have to have to make 1 T of magnetic field at the center?

8. Now draw a loop of current, but draw it so that the magnetic field point northward. On the east side of this loop, in what direction is the current going? What about the west side? Now draw in magnetic field lines, two loops for each side of this current and draw an axis through the center of the loop. Draw in three different locations how a compass would orient itself in this magnetic field. This pattern is known as a dipole and we will need this again next week to hold onto this drawing.

9. Two separate current loops are positioned so that they form concentric circles. The radius of the smaller loop is 1 cm and the radius of the larger is 1.5 cm. The larger loop has a current of 10 A. If the magnetic field at the center of the loops is 0 T, then what is the magnitude and direction of the current in the inner loop.

Week 10

Week 10 covers sections of chapter 19 in the textbook. Topics include:

- magnetic force on a moving charge
- magnetic force on a loop
- circular path of a charge in a uniform magnetic field
- mass spectrometers and cyclotrons
- torque on a dipole and electric motors

1. Draw several examples of a magnetic field pointing in a certain direction. Populate each example with several charges moving in a certain direction and for each one find the direction of the magnetic force on that particle. Make sure you put in both positive and negative charges. Put in some cases where the magnetic force is zero.

2. Now think backwards. If the force on a particle moving to the right is upward, then what direction is the magnetic field it is within? What about a particle moving up with a force to the left? If a particle in in a magnetic field that points down, and it is experiencing a force south, then in which direction is its velocity? Think of some more variations on this.

3. If an electron is experiencing a magnetic force to the right from a field pointed north, what direction is it moving? Think of some other combinations of this problem?

4. How fast would a proton have to be traveling to experience a 1 N force in a magnetic field of 1 T? How fast would 1 C of charge need to travel to experience the same?

5. In Birmingham, Earth's magnetic field has a magnitude of 50 μT and is directed 70° below the horizontal and pointing north. Find the magnetic force on an oxygen ion moving east at 250 m/s. Compare the magnitude of the magnetic force with the ion's weight, 5.2×10^{-25} N, and to the electric force on it due to the Earth's fair weather electric field (150 N/C downward).

6. How much current would be necessary to exert a 1 N force on 5 cm of wire within a magnetic field of 0.5 mT if the current was directed at a 30° angle to the magnetic field? Choose a direction of the magnetic field and draw a picture of this, indicating the direction of the current and the direction of the force on the current. If the current were rotated, what is the maximum force that could be exerted on it?

7. If a 100 m long power line is carrying 150 A of current south in Birmingham (see the description above), then what is the magnetic force on this line? Since current is really electrons flowing in the opposite direction as the "current" does this change the direction of the force on the line?

8. Charged particles in magnetic fields tend to travel in circular paths. A 1 µC particle with a 0.01 µg mass is traveling with a speed of 1 km/s in a magnetic field. The particle travels in a circular path that has a radius of 0.5 m

   (a) What is its radial acceleration?

   (b) What radial force is needed to cause this particle to travel in this particular path at this speed?

   (c) If the magnetic force is the only force acting on this particle, then how large is that force?

   (d) What magnetic field is necessary to provide this large of a magnetic force?

(e) If the particle is making counterclockwise loops within the magnetic field when viewed from above then in which direction does the magnetic field point?

(f) How long does this particle take to complete one full circle of its path around? (This is known as the period of motion.)

(g) How many times does the particle complete a full circle in one second? (This is known as the frequency.)

9. A mass spectrometer is a device for accelerating ions with parallel plates of different voltage and sending them into a magnetic field. The ions travel through a velocity selector so that they all have the same speed entering the magnetic field. Draw a picture of

this with the ions accelerated to the right between plates and then into a magnetic field that is into the page. What will the path of the particles look like when they enter the magnetic field? Suppose we have two types of particles with different masses like $^{6}Li^{+}$ and $^{7}Li^{+}$ which are isotopes (same number of protons but different number of neutrons). For these particles going through the same magnetic field, what is the ratio of the radii of these two particles? If the radius of orbit of $^{6}Li^{+}$ is 8.4 cm what is the radius of orbit of $^{7}Li^{+}$? If the speed of the particles entering the magnetic field is 1×10^{6} m/s, then what is the magnitude of the magnetic field?

10. Two wires are separated by 1 cm and carry 100 A each, but in opposite directions. Draw these going North/South with the North current on the left, just so we all have the same picture. What is the magnetic field created by the northward current at the location of the southward current (magnetude and direction)? What is the force on the southward current due to the northward currents magnetic field? What about the force of the southward current on the northward current? Does Newton's Third Law apply here?

11. What about currents that travel in the same direction? Do they attract or repel one another and why?

12. A 100 A current is flowing northward. A 0.5 μC charge has a velocity of 100 m/s northward. What is the magnetic force on the particle? What if the particle is traveling south? What if the particle's charge is negative?

13. If you threw a conducting metal rod to the left through a uniform magnetic field that also pointed left, so that the rod was perpendicular to the magnetic field as it flew through it, then what would happen to the negative charges in the rod that are free to move around?

14. An electric field and a magnetic field are arranged in space so that they cross each other at a right angle. Draw this so with the electric field pointing north and the magnetic field pointing east. A positive proton is moving into the page. Sketch the directions of the electric force on the particle and the magnetic force on the particle. The electric field is 100 N/C and the magnetic field is 50 mT. How fast would the particle have to travel for the magnetic force and electric force to be equal to each other? This device is known as a velocity selector since only the particles with a specific velocity will pass through the center of a hole and out of the device. What will happen to particles that are traveling too quickly or too slowly?

Week 11

Week 11 covers sections of chapter 22 in the textbook. Topics include:

- electromagnetic waves
- index of refraction
- doppler shift

1. What is the range of wavelengths of human vision? What is the range of frequencies? Approximately what wavelength is red? Green? Violet? A wave that has a wavelength of 1 nm could be used for what purpose? What is its frequency? Wifi usually works around a frequency of 2.4 GHz which is the same as a consumer microwave oven. What wavelength is this? WBHM is the Public Radio station in Birmingham and you can find it on the radio dial at 90.3 MHz. What wavelength is this?

2. What frequency has the same magnitude as its wavelength? What region is this in?

3. The sun is 93 000 000 miles away from earth. If the sun suddenly burned out, how long would it take for us to know?

4. One light-year is the distance that light travels in a year. How many meters is this?

5. What is the speed of light in water where $n = 1.33$? What about the speed of light in diamond where $n = 2.42$?

6. The speed of light in some unknown material is measured to be 1.3×10^6 m/s. What is the index of refraction?

7. If the frequency of light in water is 1×10^{14} Hz in water then what is it in diamond? What is the wavelength in water? What is the wavelength in diamond?

8. Show that the quantity and units work out for the relationship between the speed of light in vacuum and the permittivity and permeability of free space:

$$c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$$

9. The simplest model for an electromagnetic wave to take is a sine wave. The general form for using the sine wave and having the parameters match those of wavelength and period that we have discussed can be written like this:

$$E(x, t) = E_0 \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

This is just a function that gives you the value of the electric field at a particular place (x) at a particular time (t). For an electromagnetic wave of wavelength 100 nm, what is the period? Sketch two wavelengths of this wave on a graph when $t = 0$. Then sketch another plot 1/3 of a period later.

10. *Doppler Effect.* Imagine yourself approaching a source of light and think of the light as waves. You are traveling in the opposite direction as the light propagation. You can see from some of the above graphs of waves traveling that if you are doing this, then the period of time between when crests of the light are detected by you will be shorter than for a stationary observer. This means the frequency you observe will be higher than the frequency of the source. The faster you go the higher this frequency shift will be. The opposite effect will happen if you are traveling *away* from the source of light; you will observe a lower frequency. The equation relating the frequency you observe, f_o, the frequency of the source f_s and the *relative velocity* between source and observer v_{rel} is

$$f_o = f_s\sqrt{\frac{1+\frac{v_{rel}}{c}}{1-\frac{v_{rel}}{c}}}$$

In this equation v_{rel} is negative when the source and observer are moving away from each other and positive when they are approaching each other.

Using this equation, calculate how fast you would have to be going in your car in order for a red light to appear green.

11. The speed of light is very very fast, and many times the relative velocities between source and observer are much smaller than that. The equation above can be simplified in cases

when $v_{rel} << c$ to a much easier equation:

$$f_s = f_o\left(1 + \frac{v_{rel}}{c}\right)$$

The radar of a police officer's radar gun emits microwave radiation at about 3×10^{10} Hz. The officer is at driving at 35 mph some very reckless person is driving toward him at 55 mph in a 35 mph zone.

(a) What is the relative velocity between the officer and the driver?

(b) What is the frequency of the radar that the speeding car observes?

(c) When this radiation hits the speeding car, it reflects back to the officer, but the reflected light is essentially re-emitted from the speeding car at the doppler shifted frequency it "observed". So now the speeding car is emitting radar (it is a now source) and the officer is observing this radiation, but since they are moving toward each other, it is doppler shifted again. What frequency of radar does the officer observe now?

(d) Actually measuring this frequency is hard, since they are so close together, but what can be done is measuring the *beat frequency* of the officer's emitted radar, and the observed beam reflected from the speeder. What is the beat frequency of these two waves?

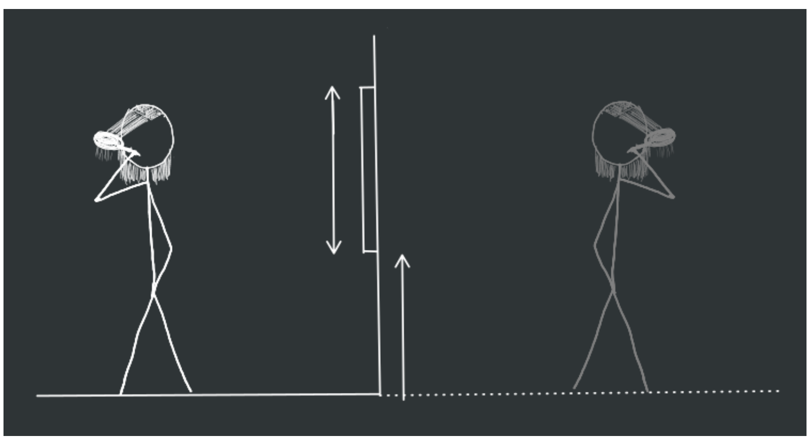
12. At what relative speed does the approximate form of the doppler shift equation give an error of 1%. You should make a table (or even better an Excel sheet) and try out several values for v_{rel} to find out. Choose any f_s or just use the ratio of f_o/f_s.

Week 12

Week 12 covers sections of chapter 23 in the textbook. Topics include:

- law of reflection
- reflection from plane mirrors
- reflection from convex and concave mirrors
- image formation
- ray tracing

1. When a plane mirror is hung on a wall, then what conditions must be met about it size and placement so that you can see your entire body in the mirror? Draw the path that light takes from your feet to your eyes. Draw in the path that light takes from the top of your head to your eyes.



2. Two mirrors are perpendicular to each other and meet at a corner, one horizontal and one vertical. Light is incident at the horizontal mirror at an angle of 30 degrees from the normal. Draw this picture and then find at what angle the light leaves the other mirror?

3. The focal length of a convex mirror is the point midway between the center of the circle that defines the mirror and the vertex of the mirror or where it intersects the principle optical axis. Draw this out and identify the focal point. Now place an object somewhere along the optical axis. Now let's do a ray tracing to find the image. Only two rays are needed but the third can confirm.

   (a) Draw a ray parallel to the principle axis from the top of your object to the mirror. This will reflect as if it came from the focal point.

   (b) Draw a ray from the top of your object toward the center of the circle to the mirror. This ray will reflect on itself as if it came from the center of the circle.

   (c) (Optional) Draw a toward the focal point. It will reflect parallel to the principle axis.

   Find the point where all of these lines appear to intersect. Is your image upright or inverted? Would its image distance be positive or negative? Is this a real or virtual image? Is the focal length be positive or negative?

4. Now let's work a similar example with the mirror and magnification equations. Draw out a convex mirror with a radius of curvature of 4 cm and put an object 10 cm away from the mirror. The object has a height of 2 cm. Use both a ray tracing and the equations to find the image distance, the image height?

5. For a convex mirror with a focal length of $-2\,\text{cm}$, plot image distance as a function of object distance. Will there ever be a real image according to your graph? What value does the image distance approach?

6. Now let's work with a concave mirror. Draw a circle and identify its center, the mirrored surface, the principle axis, and the focal point. Draw an object on the principle axis farther away from the mirror than the center of the circle. Now let's do a ray tracing to find the location and size of this image.

   (a) Draw a ray of light from the top of the object to the mirror parallel to the principle axis. This ray will reflect through the focal point.

   (b) Draw a ray of light from the top of the object through the focal point to the mirror. This ray will reflect parallel to the optical axis.

   (c) (Optional) Draw a ray through the center of the circle to the mirror. This radial ray will reflect on itself backwards.

   Find the point where these rays intersect. This is the location of the image. Draw the image from the principle axis to the intersection point. Is the image upright or inverted? Would this image distance be positive or negative? Is this a real or virtual image? Is the focal length be positive or negative?

7. Now work out the above example using the mirror equation and magnification equation. For a concave mirror that has a radius of curvature of 4 cm, what is the focal length? If the object is 10 cm away from the vertex of the mirror and 2 cm in height, then where is the image formed and what is it height?

8. Draw a plot of the image distance as a function of object distance for a concave mirror that has a focal length of 2 cm. What can you say about the image distance for object distances larger than the radius of the mirror? What about image distances for object distances less than the radius of the mirror but more than the focal point? What happens to the image distance for object distances less than the focal length? What does the image distance approach for very large object distances?


0 cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Week 13

Week 13 covers sections 3,4, and 9 of chapter 23 in the textbook. Topics include:

- refraction and Snell's Law
- dispersion (color separation by a prism)
- total internal reflection and critical angle
- image formation and converging and diverging lenses

1. Snell's law relates the angle of incidents and transmission to the indices of refraction of the material. Here is a formula for it:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Let material 1 be air $n_1 = 1$ and material 2 be water $n_2 = 1.33$. When light is traveling from air to water at an incident angle of 30°, then what is the transmitted angle? Draw a picture of this. What angle of incidence would make the transmitted angle be 20°? If light were traveling from the water out to the air at an incident angle of 10° what angle would the light take in the air? What if this angle under the water were 20°?

2. Make a table of the incident angles and transmitted angles for light traveling from air into water. What angle does the transmitted angle approach? Then revert the direction of light and calculate a table for the light going from water into air. What happens?

air ->	water
θ_1	θ_2
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	

water ->	air
θ_2	θ_1
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	

3. What you have found is an angle for which the light does not emerge from the material, but rather reflects inside the material. This is known as *total internal reflection* and it is how fiber optic cables work. The angle for which this occurs is known as the *critical angle*. Now use Snell's Law to derive an expression for the critical angle in terms of the index of refraction of the two materials. Be careful about which index of refraction is for the material that the light is in, and which index of refraction that the light *does not* go into.

4. What is the critical angle for light in glass surrounded by air? If you surround the glass with water, what does the critical angle become?

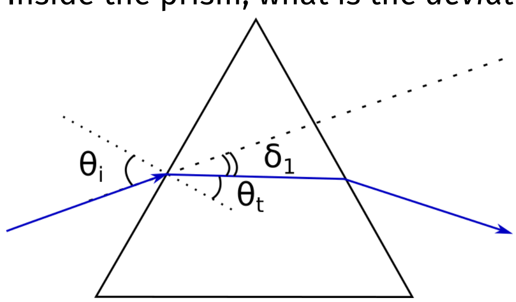
5. I mentioned in the notes that white light is composed of all the colors in the visible spectrum at once and that the index of refraction of most materials (*slightly*) depends on the frequency of the light itself. Below is a table of three colors and the index of refraction in flint glass. What is the speed of these three wavelengths in flint glass? If light consisting of these colors arrives in a single beam at an incident angle of 30°, then what angle of transmission do each of these colors have?

λ (nm)	color	n (flint glass)
486.1	blue	1.7328
589.2	yellow	1.7205
656.3	red	1.7076

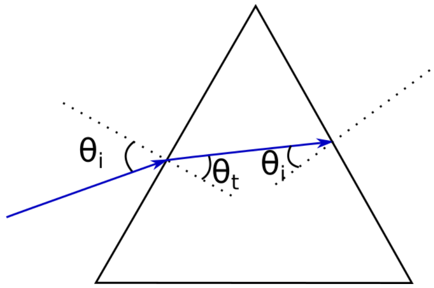
6. When light passes through a prism its path changes once on the way into the prism and then again on the way out. Assume the prism is an equilateral triangle of glass ($n = 1.5$).

   (a) For an exterior incident angle of 30°, what is the transmitted angle in the prism?

   (b) Inside the prism, what is the *deviation* δ_1 of the light's path?



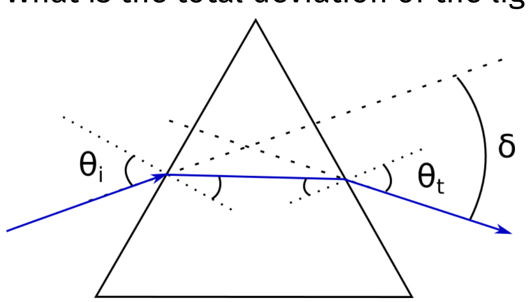
   (c) What is the interior incident angle as the light is leaving the prism?



(d) What is the final transmitted angle as the light leaves the prism?

(e) What is the second deviation of the light's path?

(f) What is the total deviation of the lights path?



(g) Using excel or google sheets, do this procedure with many exterior incident angles (all integer angles). Don't forget that excel's trig functions only accept radians and inverse trig functions will return angles in radians so you will have to include some conversion factors in the formula (π rad $= 180^\circ$). Plot the total deviation as a function incident angle and see if there are any interesting features.

7. For a converging lens with a focal length of $+5\,\text{cm}$, plot the image distance as a function of object distance. Do a ray tracing for objects at three different object distances, one farther that 2 focal lengths away, one between 1 and 2 focal lengths away, and one less than a focal length away from the lens.

8. Do a ray tracing for a diverging lens with a focal length of -5 cm.

9. In order to magnify an object by 5 times and form a real image with a 100 mm focal length lens, where should the object be placed in front of the lens?

10. You make a work of art that consists of an object 10 meters away from where you want a real image formed. Where should you place a 10 cm lens in order to make the image?

Week 14

Week 14 covers sections of chapter 24 in the textbook. Topics include:

- compound systems of lenses
- the optics of the eye and corrective lenses

1. When two or more thin lenses are used in series, an image is formed by the system as a whole, but predicting where based on the geometry of the system can seem daunting. The key to understanding how to approach such a system is to know that each lenses forms an image, and that images formed from a previous lens serve as the object for the next lens. This can obviously become a very tedious problem with many lenses since you have to work a separate image/object problem for each lens, but lets just stick with two for the moment. Lets use two converging lenses of focal lengths $f_1 = +100$ mm and $f_2 = +150$ mm and space them 300 mm apart ($s = 300$ mm). An object that is 125 mm tall and upright is located 400 mm from the first lens. Let's start a ray tracing first:

   (a) Where is the image formed from the first lens only?

   (b) Is this a real or virtual image and is it upright or inverted?

   (c) How far away is this image from the second lens? Is there a *general* equation you can use to find this? Is this a real or virtual object? Is it upright or inverted?

(d) Where is the image formed as a result of the second lens acting on the first image as an object?

(e) Is this a real image? Is it upright or inverted?

(f) What is the magnification from each of these stages and what are the image heights? $m_1 = \frac{h_1'}{h_1}$ and $m_2 = \frac{h_2'}{h_2 = h_1'}$

(g) What is the overall magnification? Compare this to the product of the individual magnifications.

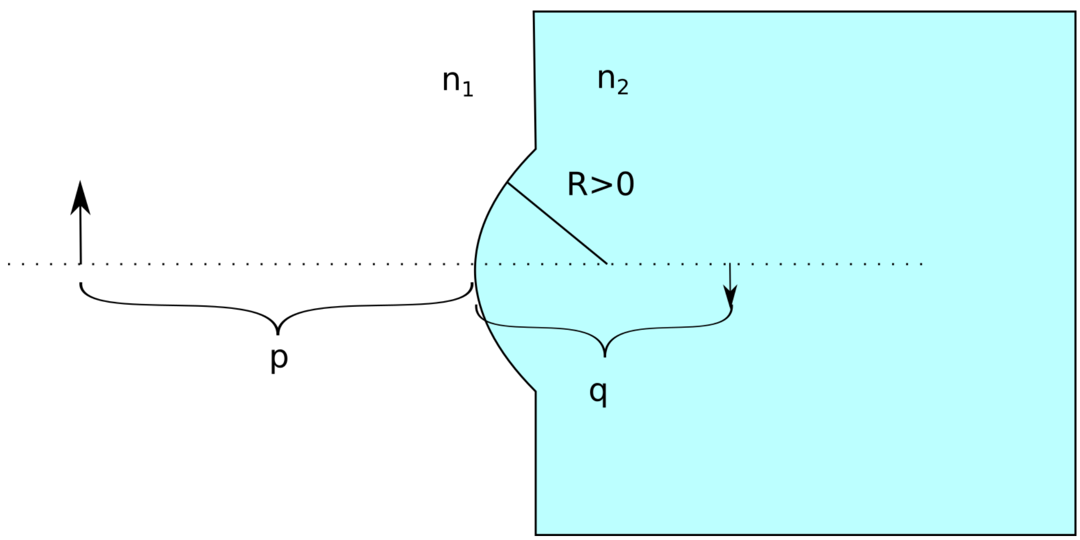
(h) What happens to the final image distance as the lenses get farther apart?

(i) What happens to the final image distance as the lenses get closer together? Is it possible to get a virtual final image?

(j) What happens if you get the lenses even closer together? Is it possible to get a virtual object? What is the image when you have a virtual object?

2. Let's apply some of what we have learned to a model of the eye. The typical eye is around 22.5 mm in diameter. We will draw a picture of the eye with an object very far away and to the left so that the light is traveling right from the object and into the eye. The cornea and lens of the eye work together to form an image on the retina (which is the inside surface on the right in our drawing). The cornea is a spherical bulge on the surface of the eye that has a radius of curvature of about 8 mm.

Much of the convergence of light from an object to the retina is actually accomplished by the cornea, rather than the lens, which is for fine adjustment and close objects. In order to model the focusing power of the cornea, we need a slightly different equation that handles images formed by only one spherical surface (in other words half of a lens) where the image is formed *inside the material of the lens.* Here is a picture of what I mean:



This equation for this:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

where p and q have their usual meaning but $n_1 = 1$ since in this case it is air and $n_2 = 1.33$ since the inside of the eye is mostly waterish.

Now assume that the object is very far away.

(a) Where would the image be formed if only the cornea is doing any refracting?

(b) Does this image form on the retina of the eye? ($D_{\text{eye}} = 22.5\,\text{mm}$) What else needs to be in the eye?

So we need additional convergence to make the image be formed exactly on the retina, but the cornea has done a lot of work.

(c) What focal length would we need from a thin lens that is positioned 5.4 mm away from the cornea in order to take the image formed by the cornea alone and instead form the image on the retina?

The lens in the eye is not symmetric but has a different radius of curvature on each side, and it has an index of refraction $n = 1.45$ but it is immersed in the waterishness of the eye and not air, so we need to use the *lens makers equation* to see what focal length of a relaxed lens is:

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

But note that a convex radius is positive and a concave radius is negative *when approached from the left to the right*. The left side of the lens has a radius of curvature of $+10\,\text{mm}$ and the right side of the lens has a radius of curvature of $-6\,\text{mm}$. Update your drawing with this info and then work the following problem.

(d) What is the focal length of a lens with this particular geometry?

3. Now all of that was for an object very far away. The *closest* that people with normal vision can see an object clearly is about 25 cm. The cornea's shape does not change but

the lens can, so what does the focal length of the lens need to be in order to form and image from this very close object?