Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being 0 °C when water freezes and 100 °C when water boils at 1 atm of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. 105 °F, 98.6 °F, 72 °F, 32 °F, 0 °F. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

$$T_F = \frac{9}{5} \cdot T_c + 32$$

$$\frac{9}{5} T_c = T_F - 32$$

$$T_c = \frac{5}{9}(T_F - 32)$$

T_F	T_c
105°F	40.6°C
98.6°F	37°C
72°F	22°C
32	0
0	-18

$$T = \frac{9}{5} \cdot T + 32$$

$$-\frac{4}{5} T = 32$$

$$T = -\left(\frac{5}{4}\right) \cdot 32$$

$$\boxed{T = -40}$$

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by 30 °F, then by how much does the temperature change in Celsius? What does this mean about the ”size” of a Celsius degree vs. the ”size” of a Fahrenheit degree? Which one represents a larger change in temperature?

$$\Delta T_F = \frac{9}{5} \Delta T_c$$

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

$T_K = T_C + 273.15$

$T_C = T_K - 273.15$

T_C	T_K
−273.15°C	0 K
0°C	273.15K
20°C	293.15K
−203.15°C	70K
37°C	310K

$\Delta T_C = \Delta T_K$

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

atomic mass $12\frac{g}{mol}$

5. What is the ~~molecular~~ ~~weight~~ of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

→ 6p → 6p → 6n → 8n

atomic mass − # of protons = # of neutrons

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

$m_{He} = 4g$ 1 mole of He = $6.022 \cdot 10^{23}$ atoms → 12g

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

$12g + 2(16) = 44g$

$\frac{44\,g}{6.022 \cdot 10^{23}\text{ molecules}} = 7.3 \cdot 10^{-23}\ \frac{g}{\text{molecule}} = 7.3 \cdot 10^{-26}\ kg$

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

$28g \cdot 0.78 =$ ____

$32g \cdot 0.21 =$ ____

$40g \cdot 0.01 =$ ____

$29\,g/mol$

9. A balloon is filled with 0.4 mol of helium so that its volume is $0.010\,m^3$.

- Find the number of atoms.

$N = n \cdot N_A$

# of particles; number of moles

$N = 0.4\text{ mol} \cdot 6.022 \cdot 10^{23}\ \frac{\text{particles}}{\text{mol}} = 2.4 \cdot 10^{23}$ He atoms

- Find the number density.

$\text{number density} = \frac{N}{V} = \frac{2.4 \cdot 10^{23}\text{ atoms}}{0.010\,m^3} = 2.4 \cdot 10^{25}$ He atoms$/m^3$

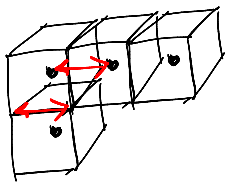
- Find the mass density. $4\frac{g}{\cancel{mol}} \cdot 0.4\,\cancel{mol} = 1.6\,g = 0.0016\,kg$

"rho"

volumetric mass density $= \rho = \frac{M}{V} = \frac{0.0016\,kg}{0.010\,m^3} = 0.16\,kg/m^3$

- Estimate the average distance between atoms. To do this, ~~fine~~ find the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

number density $= \frac{\text{particle}}{\text{volume}} \rightarrow \frac{\text{volume}}{\text{particle}} = \frac{1}{2.4 \cdot 10^{25} \text{ He atoms}/m^3}$



$V_{cube} = s^3$

$= 4.17 \cdot 10^{-26} \frac{m^3}{\text{atom}}$

$s = \sqrt[3]{V} = \sqrt[3]{4.17 \cdot 10^{-26} \frac{m^3}{\text{atom}}} = 3.5 \cdot 10^{-9} \, m$

$= 3.5 \, nm$

$= 35 \, \text{Å}$

10. You have a pound of feathers and a pound of lead.

- Which one weighs more? same
- Which one has more mass? same
- Which one has the greater volume? feathers
- Which one contains a larger number of moles? trick
- Which one contains a larger number of atoms? feathers
- Which one contains a larger number of protons and neutrons? Same

11. You check your car tire pressure and see that the pressure is 25 lb/in^2. What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

$25 \, \cancel{psi} \cdot \frac{1 \, \cancel{atm}}{14.7 \, \cancel{psi}} \cdot \frac{1.013 \cdot 10^5 \, Pa}{1 \, \cancel{atm}} = 1.7 \cdot 10^5 \, Pa$

$P_{abs} = P_{gauge} + P_{atm}$

$= 1.7 \cdot 10^5 \, Pa + 1.013 \cdot 10^5 \, Pa$

$P_{abs} = 2.7 \cdot 10^5 \, Pa$

$P_{abs} = 25\,psi + 14.7\,psi = 39.7\,psi$

12. You check you car tire pressure when it is 15 °C and it is 25 lb/in^2. By what factor do you increase the number of particles in the tire so that the pressure becomes that 30 lb/in^2? (*Hint: The volume and temperature do not change.*)

$\to 30\,psi + 14.7\,psi = 44.7\,psi$

$PV = Nk_BT$

$P = \frac{k_BT}{V} \cdot N$

$P \propto N$

$\frac{P_2}{P_1} = \frac{N_2}{N_1}$

$\frac{P_2}{P_1} = \frac{44.7}{39.7} = 1.13$

$\%\Delta = \frac{(N_2 - N_1)}{N_1} \times 100$

$\%\Delta = \left(\frac{N_2}{N_1} - 1\right) \times 100$

13% increase

$20°C = 293K$

13. The gas pressure inside of a 1 liter sealed container at room temperature is 1 atm. How many molecules are inside? How many moles of molecules?

$V = 1L \cdot \frac{1000\,mL}{1\,L} \cdot \frac{1\,cm^3}{1\,mL} \cdot \frac{1\,m^3}{(100\,cm)^3} = 0.001\,m^3$

$(10^2)^3$ 10^6 $10^{-3}\,m^3$

$P = 1\,atm = 1.013 \cdot 10^5\,Pa$

$PV = Nk_BT$

$N = \frac{PV}{k_BT} = \frac{(10^{-3}\,m^3)(10^5\,Pa)}{(1.38 \cdot 10^{-23}\,J/K)(293K)} = 2.5 \cdot 10^{22}$ particles

$\to \div N_A = 0.041$ mol

Ideal Gas Law

microscopic

$PV = Nk_BT$

P: Pa, V: m^3, N: # of particles, k_B = Boltzmann's constant

$k_B = 1.38 \cdot 10^{-23}\,J/K$

macroscopic

$PV = nRT$

P: atm $\to$ Pa, V: liters $\to$ m^3, n: moles, T: Kelvin

R: $0.0821\,\frac{atm \cdot L}{K\,mol} \to 8.31\,\frac{J}{K\,mol}$

14. If the pressure inside a tank is 1 atm when the temperature is 100 K, then what is the pressure when the temperature rises to 200 K?

$PV = Nk_BT$

$P = \frac{Nk_B}{V} T$

$P \propto T$

$\frac{P_2}{P_1} = \frac{T_2}{T_1}$

$\frac{P_2}{P_1} = \frac{200\,K}{100\,K} = 2$

$P_2 = (1\,atm) \cdot 2 = 2\,atm$

15. If the pressure inside a tank is 1 atm when the temperature is 100 °C, then what is the pressure when the temperature rises to 200 °C? *CAREFUL!*

$\to 373K$

$\to 473K$

$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{473}{373}$

$\frac{P_2}{P_1} = 1.27$

16. A gas is in a sealed container. By what factor does the pressure change if

- the volume is doubled?

$$\frac{V_2}{V_1} = 2 \qquad V_2 = 2 \cdot V_1$$

$$PV = Nk_BT$$

$$P = \underbrace{Nk_BT}_{\text{constant}} \cdot V^{-1} \qquad P \propto V^{-1}$$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = (2)^{-1} = \frac{1}{2}$$

- the temperature is tripled?

$$PV = Nk_BT \qquad \frac{P_2}{P_1} = \frac{T_2}{T_1} = 3$$

$$P \propto T$$

- the volume is double and the temperature is tripled?

$$P = Nk_BT \cdot V^{-1} \qquad \frac{P_2}{P_1} = \frac{T_2}{T_1} \cdot \left(\frac{V_2}{V_1}\right)^{-1} = 3 \cdot (2)^{-1} = \frac{3}{2}$$

- the volume is halved?

$$\frac{V_2}{V_1} = \frac{1}{2} \qquad \frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$$

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

$$V = 10\,\text{m} \times 15\,\text{m} \times 2\,\text{m} = 300\,\text{m}^3$$

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{10^5\,\text{Pa} \cdot 300\,\text{m}^3}{8.31\,\frac{\text{J}}{\text{K mol}} \cdot 293\,\text{K}} = 12{,}300\,\text{mol}$$

18. RT, 1 atm, 1 mol; how big? show me

$$V = \frac{nRT}{P} = \frac{1\,\text{mol} \cdot 8.31 \cdot 293\,\text{K}}{10^5\,\text{Pa}} = 0.024\,\text{m}^3$$

$$V = s^3$$

$$s = \sqrt[3]{0.024\,\text{m}^3} = 0.29\,\text{m}$$

$\to \sim 30\,\text{cm}$

~12 in

$$V_{rms} = \sqrt{\frac{3k_BT}{m}}$$

mass of one particle in kg!

O_2 at RT, how fast is each molecule moving?

$2 \cdot 16 g/_{mol} = 32 g/_{mol}$ $\quad 0.032 \frac{kg}{mol} \cdot \frac{1 \text{ mol}}{6.022 \cdot 10^{23} \text{ part}} = 5.31 \cdot 10^{-26}$ kg

$$V_{rms} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 293K}{5.31 \cdot 10^{-26} kg}}$$

$$= 478 \text{ m/s}$$

$$V_{rms} = \left(\frac{3k_BT}{m}\right)^{1/2} = (3k_B)^{1/2}\, T^{1/2}\, m^{-1/2}$$

$V_{rms} \propto T^{1/2}$ for same m

$$\frac{V_2}{V_1} = \left(\frac{T_2}{T_1}\right)^{1/2}$$

$V_{rms} \propto m^{-1/2}$ for constant T

$$\frac{V_{O_2}}{V_{N_2}} = \left(\frac{M_{O_2}}{M_{N_2}}\right)^{-1/2}$$

For your HW problem:

2 mol, N_2, $(31 cm)^3$, 2.9 atm

$PV = nRT$

$$T = \frac{PV}{nR} = \frac{2.9 \cdot 10^5 \cdot (0.31 m)^3}{2 \text{ mol} \cdot 8.31}$$

$$= 519.8 K$$

$$V_{rms} = \sqrt{\frac{3 k_BT}{m}}$$

$$V_{rms} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 519.8}{4.65 \cdot 10^{-26}}}$$

$V_{rms} = 680$ m/s

$m = .028 \frac{kg}{mol} \cdot \frac{1 \text{ mol}}{6.02 \cdot 10^{23} \text{ parts}} = 4.65 \cdot 10^{-26}$ kg