Week 3 covers sections of chapter 14 in the textbook. Topics include:

- First Law of Thermodynamics

- thermodynamic processes and cycles

- efficiency

- heat engines vs heat pumps

$$\underline{1^{st} \text{ Law of Thermodynamics}}$$

$$\Delta U = Q + W \quad \Big\} \text{ Conservation of energy}$$

$$\left. \begin{array}{c} mc\Delta T \\ \text{or} \\ nC_v\Delta T \\ \text{or} \\ nC_p\Delta T \end{array} \right\} = \Delta U = Q + W$$

ΔU is proportional to ΔT

1. On a cold day, you rub your hands together to warm them up. If you press your hands together with a force of $5\,\text{N}$ and the coefficient of friction between your hands is $\mu = 0.45$ and your hands move an average distance of $12\,\text{cm}$, then how much has the internal energy of your hands changed?

2. Express the first law of thermodynamics as a statement about power and the rate of energy change instead of the total energy changes.

3. When you exercise, you do work on something else. What does this mean about the sign of the work done on you? When you exercise, your body gives off heat to the surroundings (through

evaporation mostly, but also some radiation and convection). what does this mean about the sign of the heat flow from you? If you do work at a rate of 200 W and you radiate heat at a rate of 800 W, the what is the rate of internal energy change of your body? If you exercise for 30 minutes, then how much has your internal energy changed? How many bowls of cornflakes is this?

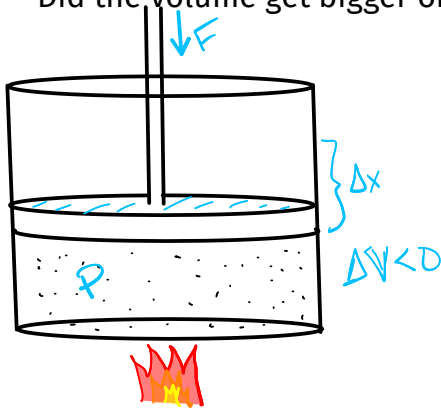4. In the notes I claim that Work =-Pressure × ΔVolume. Show clearly how the units here work out to be equal.

assuming constant force →
$$\text{Work} = F \cdot \Delta x$$
$$= [N] \cdot [m]$$
$$= [Nm]$$
$$= [\text{Joule}]$$

$$\text{Work} = - \text{pressure} \cdot \Delta V$$
assuming constant pressure
$$[Pa] \cdot [m^3]$$
$$\left[\frac{N}{m^2}\right] \cdot [m^3]$$
$$[N \cdot m]$$
$$[\text{Joule}]$$

5. 14 kJ of heat flow into a gas cylinder with a piston and the internal energy of the gas increases by 42 kJ. How is this possible? Did the volume get bigger or smaller?

↓F

Δx
ΔV<0
P

$$\Delta U = Q + W$$
$$+42kJ \quad +14 kJ$$
$$42kJ = 14kJ + W$$
$$W = +28kJ$$

Pressure
final state
$$P_2$$
initial state
$$P_1$$
W
$$V_2 \quad V_1$$ Volume

$$W = -p\Delta V$$ only applies to constant pressure

6. Rank the thermodynamic processes represented in the figure below in order of the work done from least to greatest. Rank

positive work as higher than negative work.
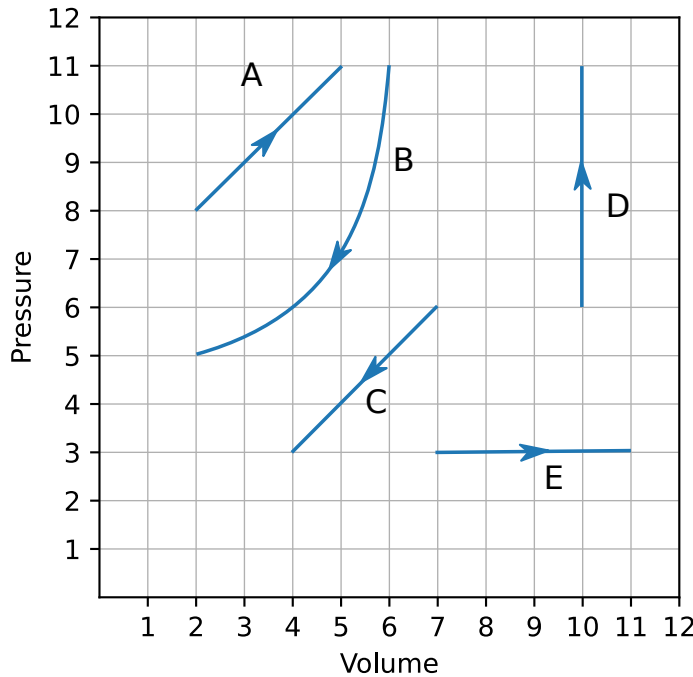
most negative
work on
the gas $\longrightarrow$ most positive
work on
the gas

A, E, D, C, B

$U = \frac{3}{2}RnT$

$\Delta U = \frac{3}{2}Rn\Delta T$
$\underbrace{\qquad}_{C_v}$

7. A monatomic ideal gas initially at room temperature (20 °C) undergoes an isobaric expansion from an initial volume of 1 L to 2 L at a pressure of 2 atm. Then the gas goes through an isochoric process where the pressure lowers to 1 atm. Draw a PV diagram of this process, and then calculate the total work done on the gas in these two processes.

1st : isobaric

$\Delta U = Q + W$

$\frac{3}{2}Rn\Delta T = Q - p\Delta V$

$Q = \frac{3}{2}Rn\Delta T + p\Delta V$

$= \frac{3}{2}p\Delta V + p\Delta V$

$\boxed{Q = \frac{5}{2}p\Delta V} = \frac{5}{2}(2\cdot10^5 Pa)(+0.001m^3) = 500 J$

$= \frac{5}{2}Rn\Delta T$ monatomic, isobaric
$\underbrace{\qquad}_{C_p}$

isobaric

$W = -p\Delta V$

$= -(2\cdot10^5 Pa)\cdot(+0.001\,m^3)$

$= -200\,J$

$PV_2 = nRT_2$

$- PV_1 = nRT_1$

$\overline{P\Delta V = nR\Delta T}$

$V_i = 1L = 0.001\,m^3$

$V_2 = 2L = 0.002\,m^3$

$\Delta V = +0.001\,m^3$

2nd : isochoric

$\Delta U = Q + \cancel{W}^0$
$W = 0$

$\frac{3}{2}Rn\Delta T = Q$
$\underbrace{\qquad}_{C_v}$

$P_2V = nRT_2$
$- P_1V = nRT_1$
$\overline{\Delta P\cdot V = nR\Delta T}$

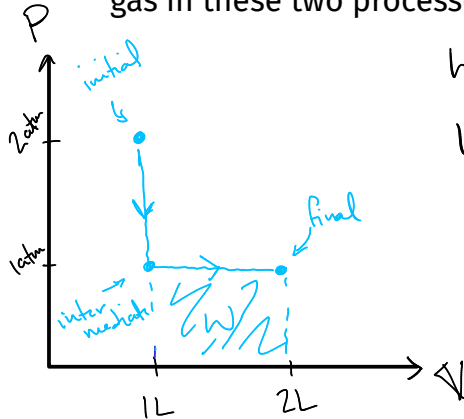$\boxed{\frac{3}{2}\Delta P\cdot V = Q}$ monatomic, isochoric

$Q = \frac{3}{2}(-1\cdot10^5 Pa)\cdot(0.002\,m^3) = \boxed{-300\,J}$

carefully!

8. A monatomic ideal gas initially at room temperature (20 °C) undergoes an isochoric process where the pressure changed from 2 atm to 1 atm. Then the gas goes through an isobaric expansion from an initial volume of 1 L to 2 L. Draw a PV diagram

of this process, and then calculate the total work done on the gas in these two processes.
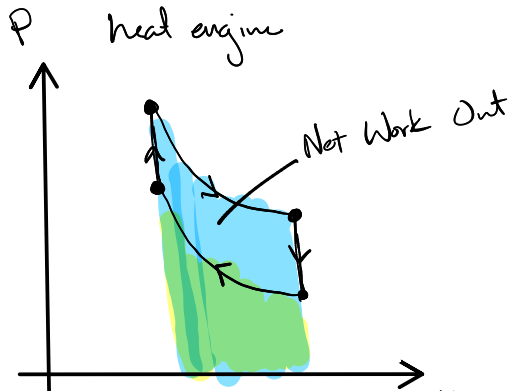
$$W = -p \Delta V$$

$$W = -(1 \cdot 10^5 \, Pa) \cdot (+0.001 \, m^3)$$

$$W = -100 J$$

9. Sketch a PV diagram of a thermodynamic cycle that contains an two isothermal ~~expansions~~ processes and and two isochoric processes. First do this as a heat engine, then do it as a refrigerator. Specify for each step, when heat is going in or out, and when work is being done on the gas or on the surroundings.

heat engine

Net Work Out

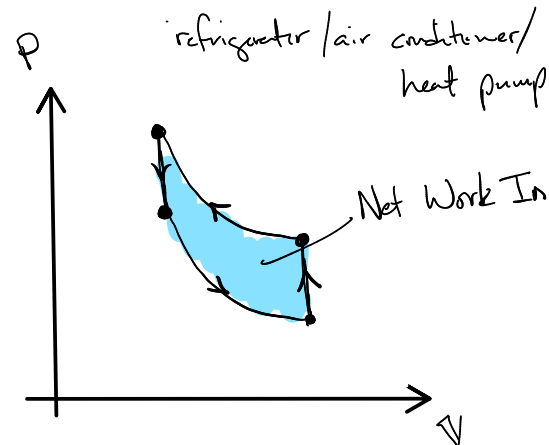refrigerator / air conditioner / heat pump

Net Work In

$$\underline{\Delta U = Q + W}$$

for a complete cycle

$$\Delta U = 0 = Q + W$$

$$W = -Q$$

10. The following PV diagram is a wrong representation of the Otto Cycle, which is a model of a gasoline engine. What are the
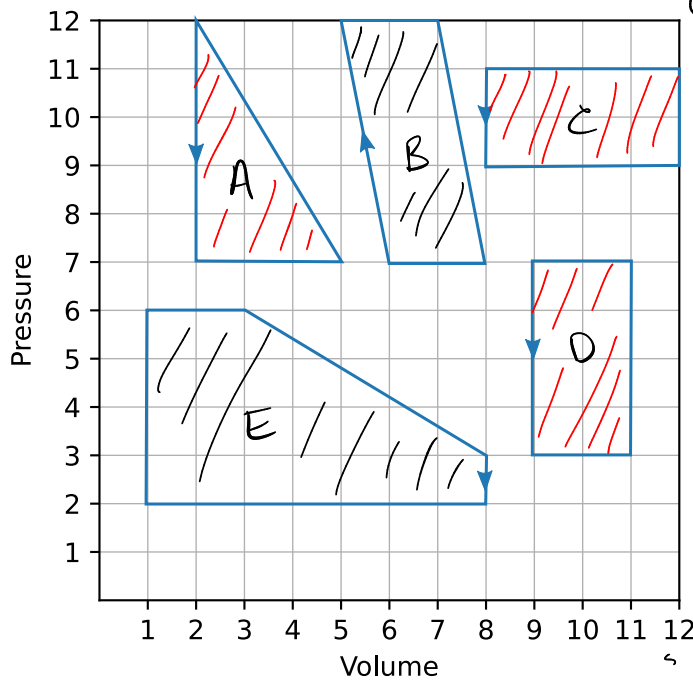
problems in the diagram below.

11. Rank the following cycles by the net work done by the system onto the surroundings from least to greatest. Rank positive work on the surroundings as higher than negative work (which would be done on the engine like in a heat pump). Which of

these are heat engines and which are heat pumps?

most $\ominus$ work done on surroundings $\longrightarrow$ most $\oplus$ work done on surroundings

$$C=D, \; A, \qquad B, \; E$$

Work done on gas

$$E, \; B, \qquad A, \; C=D$$

A. isochoric

$$\Delta u = Q + \cancel{W}^{0}$$
$$\downarrow \quad \therefore W = 0$$
$$\Delta u = Q$$
$$\Delta u = \tfrac{3}{2}\Delta P \cdot V = Q$$
— or —
$$\Delta u = \tfrac{3}{2} nR \Delta T$$

B. isobaric

$$\Delta u = Q + W$$
$$\downarrow$$
$$\tfrac{3}{2} nR\Delta T = Q - P\Delta V$$
$$\tfrac{3}{2} P\Delta V = Q - P\Delta V$$

$$\Big\{ Q = \tfrac{3}{2}(3\cdot10^5 \text{ Pa})\cdot(0.2 \text{ m}^3)$$
$$Q = 9 \cdot 10^4 \leftarrow \text{heat to the gas}$$

$$W = -P\Delta V$$
$$= -(4\cdot10^5 \text{ Pa})\cdot(0.6 \text{ m}^3)$$
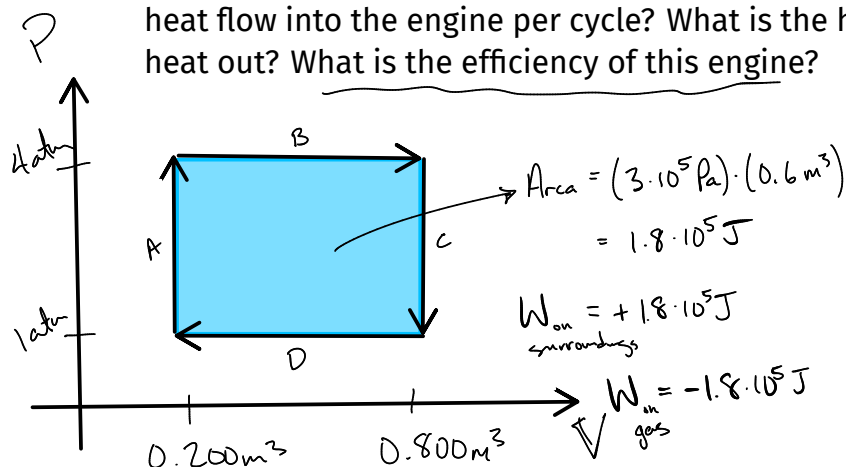$$W = -2.4 \cdot 10^5 \text{ J}$$
$$\approx (-) \text{ work on the gas}$$

$$Q = \tfrac{5}{2} P\Delta V$$
$$Q = \tfrac{5}{2}(4\cdot10^5 \text{ Pa})\cdot(+0.6 \text{ m}^3)$$
$$Q = +6\cdot10^5 \text{ Pa} \leftarrow \text{more heat into gas}$$

C. isochoric

$$W = 0$$
$$Q = -3.6\cdot10^5 \text{ J} \leftarrow \text{heat removed from gas}$$

D. isobaric

$$W = 6\cdot10^4 \text{ J}$$
$$Q = -1.5\cdot10^5 \text{ J}$$
b/c both are $(-)$

work done on the gas

12. A heat engine operates with two isobaric processes and two isochoric processes. The pressure varies from 1 atm and 4 atm and the volume varies from 0.200 m³ to 0.800 m³. Draw a PV diagram and specify the directions of these transitions. How much work is done by the engine in one cycle? What is the net heat flow into the engine per cycle? What is the heat in and the heat out? What is the efficiency of this engine?

Total Work done on the gas: $-1.8\cdot10^5$ J

Heat into gas: $6.9\cdot10^5$ J

Heat out of gas: $-5.1\cdot10^5$ J

Total Heat = $+1.8\cdot10^5$ J

So this engine is bringing in heat and converting it to work on the surroundings

$$\text{Area} = (3\cdot10^5 \text{ Pa})\cdot(0.6 \text{ m}^3)$$
$$= 1.8\cdot10^5 \text{ J}$$

$$W_{on} = +1.8\cdot10^5 \text{ J} \quad \text{surroundings}$$

$$W_{on} = -1.8\cdot10^5 \text{ J} \quad \text{gas}$$

what we get $\rightarrow$

what we pay for $\rightarrow$

$$\text{efficiency} = \frac{W_{NET}}{Q_{in}} = \frac{1.8\cdot10^5 \text{ J}}{6.9\cdot10^5 \text{ J}} = 0.26 \text{ or } 26\%$$

13. What is the difference between a heat pump and an air conditioner?

14. The definition of efficiency from the notes is:

$$e = \frac{W_{out}}{Q_{in}}$$

Also from the first law of thermo, we had:

$$Q_{in} = W_{out} + Q_{out}$$

Combine these two equations to get an expression for efficiency only in terms of heat in and out (*Hint: substitute for Work.*)

15. Follow up on the previous problem. Heat engines can operate at many different efficiencies depending on the cycles and how much heat goes in and out, but there is a cycle that represents an ideal heat engine that is the best efficiency possible. It is called the *Carnot Cycle*. The details of this cycle are not important but the fundamental take away is that the heat that comes in (Q_{in}) comes from a source that is at temperature T_H, and the heat that goes out (Q_{out}) goes to a temperature of the surroundings at T_C. For a heat engine that operates between these temperatures (like the Carnot cycle) the ratio of the Heat

out and in is equal to the ratio of the temperatures:

$$\frac{Q_{out}}{Q_{in}} = \frac{T_C}{T_H}$$

This is a long lead up to the task of use this proportion to find an expression for the efficiency of an ideal heat engine in terms of the temperatures between which the engine operates. (*Hint: Make a substitution in the equation from the answer to the previous problem.*)

16. In the engine of a car, suppose the combustion of gasoline and air that takes place produces temperatures that are around 3000 °C while the exhaust gas exits the engine at a temperature of around 1000 °C. Forget about the actual cycle that a gasoline engine undergoes, if this were an ideal process what is the efficiency that this engine can achieve. If you would improve the design to the extent that the exhaust gasses left the engine at room temperature and it still operated as an ideal heat engine, then what would the efficiency be?

fish Q:

$\downarrow$ ___ ___ $\downarrow V_i$ V_f

If the pressure on a fish increases from 1.10 atm to 1.20 atm, its swim bladder decreases in volume from 8.24 mL to 7.32 mL while the temperature of the air inside remains constant. How much work is done on the air in the bladder?

| 0.1087 ± 2% | J

$$W = 1.1 \times 10^5 \cdot 8.24 \cdot 10^{-6} \ln\left(\frac{8.24 \times 10^{-6}}{(7.32 \times 10^{-6})}\right)$$

isothermal

$P_1 V_1 = Nk_B T$

$P_2 V_2 = Nk_B T$

$$W = Nk_B T \ln\left(\frac{V_i}{V_f}\right)$$

$$PV = Nk_B T$$

$$W = PV \ln\left(\frac{V_i}{V_f}\right)$$
$\quad\uparrow \quad \uparrow$
$\quad [Pa] \; [m^3]$

8.24 mL
$8.24 \cdot 10^{-6} m^3$

$$W = 0.1073$$

efficiency

$$e = \frac{W_{NET}}{Q_{in}}$$

$\checkmark$

$\checkmark$

$$W_{NET} = Q_{NET} = Q_{in} + Q_{out}$$

$\uparrow$
find this

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