Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being 0 °C when water freezes and 100 °C when water boils at 1 atm of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. 105 °F, 98.6 °F, 72 °F, 32 °F, 0 °F. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

$$T = \frac{9}{5}T + 32$$

$$-\frac{4}{5}T = 32$$

$$T = 32\left(-\frac{5}{4}\right) = -40$$

$$T_F = \frac{9}{5}T_c + 32$$

$$\frac{9}{5}T_c = T_F - 32$$

$$T_c = \frac{5}{9}(T_F - 32)$$

T_F	T_c
105°F	40.6°C
98.6°F	37°C
72	22
32	0
0	−18

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by 30 °F, then by how much does the temperature change in Celsius? What does this mean about the "size" of a Celsius degree vs. the "size" of a Fahrenheit degree? Which one represents a larger change in temperature?

$$\Delta T_F = \frac{9}{5}\Delta T_c$$

$$T_{F,2} - T_{F1}$$

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

$\Delta T_c = \Delta T_K$

$T_K = T_c + 273.15$

$T_c = T_K - 273.15$

T_c	T_K
−273.15°C	0 K
0°C	273.15 K
20°C	293.15 K
−203.15°C	70 K
37°C	310 K

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

5. What is the ~~molecular weight~~ atomic mass of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

6p

6p

8n

$12 - 6p = 6n$

atomic mass − # of protons = # of neutrons

4_{amu} ← mass of one atom

$4 \frac{g}{mol}$

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

$M_{He} = 4g = 0.004kg$ 1 mole of He $= 6.022 \cdot 10^{23}$ atoms

$m_C = 12g = 0.012kg$

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

$12\frac{g}{mol} + 2\left(16\frac{g}{mol}\right) = 44\frac{g}{mol} = 0.044kg$

$$\frac{44\frac{g}{mol}}{6.022 \cdot 10^{23}\frac{molecules}{mole}} = 7.3 \cdot 10^{-23}\frac{g}{molecule} = 7.3 \cdot 10^{-26}\frac{kg}{molecule}$$

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

$28g \times 0.78 =$ ______

$32g \times 0.21 =$ ______

$40g \times 0.01 =$ ______

$29\ g/mol$

9. A balloon is filled with 0.4 mol of helium so that its volume is 0.010 m^3.

- Find the number of atoms.

$N = n \cdot N_A$

\# of particle number of moles

$N = 0.4\ mol \cdot 6.022 \cdot 10^{23}\frac{atoms}{mol} = 2.4 \cdot 10^{23}$ atoms

- Find the number density.

Number density $= \frac{N}{V} = \frac{2.4 \cdot 10^{23} \text{ atoms}}{0.010 \text{ m}^3} = 2.4 \cdot 10^{25} \text{ atoms/m}^3$

- Find the mass density.

$He \rightarrow 4 \frac{g}{mol} \cdot 0.4 \text{ mol} = 1.6 \text{ g} = 0.0016 \text{ kg}$

"rho"

volumetric mass density

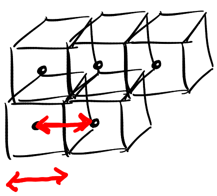
$\rho = \frac{M}{V} = \frac{0.0016 \text{ kg}}{0.010 \text{ m}^3} = 0.16 \frac{\text{kg}}{\text{m}^3}$

- Estimate the average distance between atoms. To do this, ~~fine~~ find the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

Number density $= \frac{N}{V}$

$\frac{V}{N} = \left(2.4 \cdot 10^{25} \text{ atoms/m}^3\right)^{-1}$

$V_{\text{of one}} = 4.17 \cdot 10^{-26} \frac{\text{m}^3}{\text{atom}}$



$V = s^3$

$s = \sqrt[3]{V}$

$= \sqrt[3]{4.17 \cdot 10^{-26} \text{ m}^3}$

$= 3.47 \cdot 10^{-9} \text{ m}$

$= 3.47 \text{ nm} \rightarrow 34.7 \text{ Å}$ ✓

10. You have a pound of feathers and a pound of lead.

- Which one weighs more? same
- Which one has more mass? same
- Which one has the greater volume? feathers
- Which one contains a larger number of moles? trick
- Which one contains a larger number of atoms? feathers
- Which one contains a larger number of protons and neutrons? same

11. You check your car tire pressure and see that the pressure is $25\,\mathrm{lb/in^2}$. What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

$$25\,\cancel{psi} \cdot \frac{1\,atm}{14.7\,\cancel{psi}} \cdot \frac{1.013\cdot 10^5\,Pa}{1\,\cancel{atm}} = 1.7\cdot 10^5\,Pa$$

gauge pressure

$$P_{abs} = P_{gauge} + P_{atm}$$
$$= 1.7\cdot 10^5\,Pa + 1.013\cdot 10^5$$
$$P_{abs} = 2.7\cdot 10^5\,Pa$$

$$P_{gauge} + P_{atm} = P_{abs}$$
$$25\,psi + 14.7\,psi = 39.7\,psi$$

12. You check you car tire pressure when it is 15 °C and it is $25\,\mathrm{lb/in^2}$. By what factor do you increase the number of particles in the tire so that the pressure becomes that $30\,\mathrm{lb/in^2}$? (*Hint: The volume and temperature do not change.*)

$$30\,psi + 14.7\,psi = 44.7\,psi$$

$$p\cancel{V} = N\cancel{k_B T}$$

$$\%\Delta = \left(\frac{N_2 - N_1}{N_1}\right) \times 100$$

$$\frac{P_2}{P_1} = \frac{N_2}{N_1} \rightarrow \frac{44.7\,psi}{39.7\,psi} = \frac{N_2}{N_1} = 1.13$$

$$\boxed{\%\Delta = \left(\frac{N_2}{N_1} - 1\right) \times 100}$$

$\rightarrow$ 13% increase

13. The gas pressure inside of a 1 liter sealed container at room temperature is 1 atm. How many molecules are inside? How many moles of molecules?

$$n = \frac{pV}{RT}$$

$$n = \frac{(1\,atm)(1\,L)}{(0.0821)(293\,K)} = 0.042\ mole$$

$\times N_A$

$2.5\cdot 10^{22}$ particles

$V = 1L = 0.001\,m^3$

$p = 1\,atm = 10^5\,Pa$

$$N = \frac{pV}{k_B T} = \frac{(10^5)\cdot(10^{-3})}{(1.38\cdot 10^{-23})(293)} = 2.5\cdot 10^{22}\ \text{particles}$$

$\div N_A = 0.42$ mol

Ideal Gas Law

microscopic

$pV = N k_B T$

$p \rightarrow$ Pascal, $V \rightarrow m^3$, $N \rightarrow$ # of particles, $T \rightarrow$ Kelvin

$k_B \rightarrow$ Boltzmann's constant

$k_B = 1.38\cdot 10^{-23}\ \frac{J}{K}$

macroscopic

$pV = n\cdot R\cdot T$

$p \rightarrow$ atm $\rightarrow$ Pa, $V \rightarrow$ liters $\rightarrow m^3$, $n \rightarrow$ moles, $T \rightarrow$ Kelvin

$R \rightarrow 0.0821\ \frac{atm\cdot L}{mol\cdot K} \rightarrow 8.31\ \frac{J}{K\,mol}$

14. If the pressure inside a tank is 1 atm when the temperature is 100 K, then what is the pressure when the temperature rises to 200 K?

$$pV = Nk_BT$$

$$p = \left(\frac{Nk_B}{V}\right)T$$

$$p \propto T$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

$$\frac{p_2}{p_1} = \frac{200K}{100K} = 2$$

$$p_2 = 2 \cdot p_1 = \underline{2\,atm}$$

another way:

$$\frac{p_2 \cancel{V_2} = \cancel{N_2 k_B} T_2}{p_1 \cancel{V_1} = \cancel{N_1 k_B} T_1}$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

15. If the pressure inside a tank is 1 atm when the temperature is 100 °C, then what is the pressure when the temperature rises to 200 °C? *CAREFUL!*

373K

473K

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = \frac{473}{373} = 1.27$$

16. A gas is in a sealed container. By what factor does the pressure change if

- the volume is doubled?

$$V_2 = 2 \cdot V_1$$

$$\frac{V_2}{V_1} = 2$$

$$pV = Nk_BT$$

$$p = \frac{Nk_BT}{V}$$

$$p = \underbrace{(Nk_BT)}_{\text{constant}}V^{-1}$$

$$p \propto V^{-1} \longrightarrow \frac{p_2}{p_1} = \left(\frac{V_2}{V_1}\right)^{-1}$$

$$\frac{p_2}{p_1} = (2)^{-1} = \frac{1}{2}$$

- the temperature is tripled?

$$p = Nk_BTV^{-1}$$

$$p \propto T \rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} = 3$$

- the volume is double and the temperature is tripled?

$$p = Nk_BTV^{-1} \qquad p \propto TV^{-1} \rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} \cdot \left(\frac{V_2}{V_1}\right)^{-1} = 3 \cdot (2)^{-1}$$

$$= \frac{3}{2}$$

- the volume is halved?

$$\frac{V_2}{V_1} = \frac{1}{2} \qquad \frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$$

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

$$V = 2 \cdot 10 \cdot 15 = 300\,m^3$$

\# of moles

$$\rightarrow n = \frac{PV}{RT} = \frac{10^5\,Pa \cdot 300\,m^3}{8.31\,J/mK \cdot 293K} = \underline{12{,}300\ mol}$$

18. RT, 1atm, 1mol; how big? show me!

$$V = \frac{nRT}{P} = \frac{1\,mol \cdot 8.31 \cdot 293}{10^5\,Pa} = 0.024\,m^3$$

$$s = \sqrt[3]{V} = \sqrt[3]{0.024}\,m = 0.29\,m$$

↳ ~30 cm

↳ 1 ft

$$V_{rms} = \sqrt{\frac{3k_BT}{m}}$$

↳ mass of one particle in kg!

$$V = s^3 = (0.33\,m)^3 = 0.0359\,m^3$$

If 2.00 mol of nitrogen gas (N_2) are placed in a cubic box, $\underline{33\ \mathrm{cm}}$ on each side, at $\underline{2.600\ \mathrm{atm}}$ of absolute pressure, what is the rms speed of the nitrogen molecules?

712 ± 2% m/s

$\hookrightarrow P$ (in Pa) $= 2.6 \cdot 10^5$ Pa

2 mol of N_2

$$2 \cdot \left(14 \tfrac{g}{mol}\right) = \frac{28\,g}{N_A} = 4.6 \cdot 10^{-23}\,g$$

$\underline{4.6 \cdot 10^{-26}}$ kg/particle

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{2.6 \cdot 10^5\,Pa \cdot 0.0359\,m^3}{2\,mol \cdot 8.31}$$

$$\underline{T = 561\,K}$$

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

$$V_{rms} = \sqrt{\frac{3(1.38 \cdot 10^{-23}\,J/K)(561\,K)}{4.6 \cdot 10^{-26}}}$$

$$\boxed{V_{rms} = 711\,m/s}$$

one interesting thing:

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

$$m = \frac{M}{N_A} \leftarrow \text{mass per mol}$$

$$V_{rms} = \sqrt{\frac{3 (k_B \cdot N_A) \cdot T}{M}}$$

$$k_B \cdot N_A = R = 8.31\,J/K\,mol$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

so you can use this if you are careful