Week 1 covers sections 1-5 of chapter 13 in the textbook. Topics include

- temperature and measurement scales
- measurements of amount and density
- the ideal gas law
- kinetic theory of gas

1. The Celsius temperature scale is based on the *triple point* of water, but it is more common to think of it as being 0 °C when water freezes and 100 °C when water boils at 1 atm of pressure. But the Fahrenheit scale is more well known to us so lets do some conversion of common Fahrenheit temperatures. 105 °F, 98.6 °F, 72 °F, 32 °F, 0 °F. Keep going down in Fahrenheit, and see if you can find a Fahrenheit temperature that gives you the same number in Celsius. Make sure you can go backwards and convert some Celsius temperatures back to Fahrenheit.

$$T = \frac{9}{5}T + 32$$

$$-\frac{4}{5}T = 32$$

$$T = 32\left(-\frac{5}{4}\right) = -40$$

$$T_F = \frac{9}{5}T_c + 32$$

$$\frac{9}{5}T_c = T_F - 32$$

$$T_c = \frac{5}{9}(T_F - 32)$$

T_F	T_c
105°F	40.6°C
98.6°F	37°C
72	22
32	0
0	−18

2. If I only tell you a *change* in Fahrenheit temperature of a substance but not the actual temperature, then you can figure out the corresponding change in Celsius, but still not the actual temp. A change in temperature measured in Fahrenheit is 1.8 times bigger than the change measured in Celsius. So if the temperature increased by 30 °F, then by how much does the temperature change in Celsius? What does this mean about the "size" of a Celsius degree vs. the "size" of a Fahrenheit degree? Which one represents a larger change in temperature?

$$\Delta T_F = \frac{9}{5}\Delta T_c$$

$$T_{F,2} - T_{F1}$$

3. The kelvin temperature scale is designed as an *absolute* temperature scale, meaning the lowest temperature any object could theoretically be is set to 0 K. The size of a Kelvin degree is the same as the size of a Celsius degree, so that a 20 °C change in temperature is the same as a 20 K temperature change. Absolute zero in the Kelvin Scale is set to −273.15 °C. So, what is 0 °C in Kelvin? What is 20 °C in Kelvin. What is 70 K in Celsius? What is normal human body temperature in K?

$\Delta T_c = \Delta T_K$

$T_K = T_c + 273.15$

$T_c = T_K - 273.15$

T_c	T_K
−273.15 °C	0 K
0 °C	273.15 K
20 °C	293.15 K
−203.15 °C	70 K
37 °C	310 K

4. What is absolute zero in the Fahrenheit temperature scale? Find this by using $T_C = -273.15$ first if you want, but then try using a substitution for T_C that will give you an expression for finding any Fahrenheit temperature given a Kelvin one.

atomic mass

5. What is the ~~molecular weight~~ of Carbon-12? Find a periodic table to help. How many protons are in Carbon-12? How many neutrons? What about the number of protons in Carbon-14? What about the number of neutrons in Carbon-14?

6p

8n

6p

$12 - 6p = 6n$

atomic mass − # of protons = # of neutrons

4_{amu} ← mass of one atom

$4 \frac{g}{mol}$

6. How many atoms are in a mole of Helium? How many atoms are in a mole of Carbon-12? What is the mass of a mole of Helium? What is the mass of a mole of Carbon-12?

$m_{He} = 4g = 0.004 kg$ 1 mole of He $= 6.022 \cdot 10^{23}$ atoms

$m_C = 12g = 0.012 kg$

7. What is the mass of a single CO_2 molecule? What is the mass of a mole of CO_2?

$12 \frac{g}{mol} + 2\left(16 \frac{g}{mol}\right) = 44 \frac{g}{mol} = 0.044 kg$

$$\frac{44 \frac{g}{mol}}{6.022 \cdot 10^{23} \frac{molecules}{mole}} = 7.3 \cdot 10^{-23} \frac{g}{molecule} = 7.3 \cdot 10^{-26} \frac{kg}{molecule}$$

8. What is the mass of a mole of dry air which is 78% N_2, 21% O_2, and 1% Ar?

$28g \times 0.78 =$ ______

$32g \times 0.21 =$ ______

$40g \times 0.01 =$ ______

29 g/mol

9. A balloon is filled with 0.4 mol of helium so that its volume is 0.010 m^3.

   - Find the number of atoms.

$N = n \cdot N_A$

# of particle

number of moles

$N = 0.4 \text{ mol} \cdot 6.022 \cdot 10^{23} \frac{atoms}{mol} = 2.4 \cdot 10^{23}$ atoms

- Find the number density.

$$\text{Number density} = \frac{N}{V} = \frac{2.4 \cdot 10^{23}\ \text{atoms}}{0.010\ m^3} = 2.4 \cdot 10^{25}\ \text{atoms}/m^3$$

- Find the mass density.

$$He \rightarrow 4\,\frac{g}{mol} \cdot 0.4\,mol = 1.6\,g = 0.0016\,kg$$

"rho"

volumetric mass density

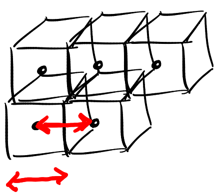
$$\rho = \frac{M}{V} = \frac{0.0016\ kg}{0.010\ m^3} = 0.16\ \frac{kg}{m^3}$$

- Estimate the average distance between atoms. To do this, find ~~fine~~ the *volume per particle*, and then treat that volume like a cube and find the side length of the cube. Draw a picture of this model and use that to justify your approximation.

$$\text{Number density} = \frac{N}{V}$$

$$\frac{V}{N} = \left(2.4 \cdot 10^{25}\ \text{atoms}/m^3\right)^{-1}$$

$$V_{\text{of one}} = 4.17 \cdot 10^{-26}\ \frac{m^3}{\text{atom}}$$



$$V = s^3$$

$$s = \sqrt[3]{V} = \sqrt[3]{4.17 \cdot 10^{-26}\ m^3} = 3.47 \cdot 10^{-9}\ m = 3.47\ nm \rightarrow 34.7\ \text{Å}\ \checkmark$$

10. You have a pound of feathers and a pound of lead.

- Which one weighs more? same
- Which one has more mass? same
- Which one has the greater volume? feathers
- Which one contains a larger number of moles? trick
- Which one contains a larger number of atoms? feathers
- Which one contains a larger number of protons and neutrons? same

11. You check your car tire pressure and see that the pressure is 25 lb/in^2. What is this in Pascal? (You'll need to look up a conversion factor). This is a gauge pressure, so what is the absolute pressure in the tire?

$$25\,\cancel{psi}\cdot\frac{1\,atm}{14.7\,\cancel{psi}}\cdot\frac{1.013\cdot10^5\,Pa}{1\,\cancel{atm}} = 1.7\cdot10^5\,Pa$$

gauge pressure

$P_{abs} = P_{gauge} + P_{atm}$

$= 1.7\cdot10^5\,Pa + 1.013\cdot10^5$

$P_{abs} = 2.7\cdot10^5\,Pa$

$P_{gauge} + P_{atm} = P_{abs}$

$25\,psi + 14.7\,psi = 39.7\,psi$

12. You check you car tire pressure when it is 15 °C and it is 25 lb/in^2. By what factor do you increase the number of particles in the tire so that the pressure becomes that 30 lb/in^2? (*Hint: The volume and temperature do not change.*)

$\to 30 + 14.7 = 44.7\,psi$

$pV = Nk_BT$

$p \propto N$

$$\frac{p_2}{p_1} = \frac{N_2}{N_1} \Rightarrow \frac{44.7}{39.7} = \frac{N_2}{N_1} = 1.13$$

$$\%\Delta = \left(\frac{N_2 - N_1}{N_1}\right)\times100$$

$$\boxed{\%\Delta = \left(\frac{N_2}{N_1} - 1\right)\times100}$$

13% increase

13. The gas pressure inside of a 1 liter sealed container at room temperature is 1 atm. How many molecules are inside? How many moles of molecules?

$$n = \frac{pV}{RT}$$

$$n = \frac{(1\,atm)(1\,L)}{(0.0821)(293\,K)} = 0.042\,mole$$

$\times N_A$

$2.5\cdot10^{22}$ particles

$V = 1\,L = 0.001\,m^3$

$p = 1\,atm = 10^5\,Pa$

$$N = \frac{pV}{k_BT} = \frac{(10^5)\cdot(10^{-3})}{(1.38\cdot10^{-23})(293)} = 2.5\cdot10^{22}\text{ particles}$$

$\div N_A = 0.42\,mol$

Ideal Gas Law

microscopic

$pV = Nk_BT$

Pascal, m^3, # of particles, Kelvin

$k_B \to$ Boltzmann's constant

$k_B = 1.38\cdot10^{-23}\,\frac{J}{K}$

macroscopic

$pV = n\cdot R\cdot T$

atm, liters, moles, Kelvin

atm $\to$ Pa

liters $\to$ m^3

$0.0821\,\frac{atm\cdot L}{mol\cdot K} \to 8.31\,\frac{J}{K\,mol}$

14. If the pressure inside a tank is 1 atm when the temperature is 100 K, then what is the pressure when the temperature rises to 200 K?

$$\frac{T_2}{T_1} = \frac{200K}{100K} = 2$$

$$\frac{p_2 \cancel{V_2} = \cancel{n_2}\cancel{R}T_2}{p_1 \cancel{V_1} = \cancel{n_1}\cancel{R}T_1}$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = 2$$

$$\frac{p_2}{p_1} = 2 \Rightarrow p_2 = 2\,atm$$

1 atm $\rightarrow p_1$

constant of proportionality

$$p = \frac{Nk_B}{V} T$$

$$p \propto T$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

15. If the pressure inside a tank is 1 atm when the temperature is 100 °C, then what is the pressure when the temperature rises to 200 °C? *CAREFUL!*

373K

473K

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = \frac{473}{373} = 1.27$$

$$p_2 = 1.27 \cdot p_1$$

16. A gas is in a sealed container. By what factor does the pressure change if

- the volume is doubled?

$$\frac{V_2}{V_1} = 2$$

$$pV = Nk_BT$$

$$p = \frac{Nk_BT}{V}$$

$$p = Nk_BT \cdot V^{-1}$$

$$p \propto V^{-1} \rightarrow \frac{p_2}{p_1} = \left(\frac{V_2}{V_1}\right)^{-1} = (2)^{-1} = \frac{1}{2} \Rightarrow p_2 = \frac{1}{2}p_1$$

- the temperature is tripled?

$$p = Nk_B\underline{\underline{T}}\,V^{-1}$$

$$p \propto T^1 \qquad \frac{p_2}{p_1} = \frac{T_2}{T_1} = 3$$

- the volume is double and the temperature is tripled?

$$p = Nk_B\underline{\underline{T}}\,\underline{\underline{V}}^{-1} \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} \cdot \left(\frac{V_2}{V_1}\right)^{-1}$$

$$= 3 \cdot (2)^{-1} = \frac{3}{2} = \frac{p_2}{p_1}$$

- the volume is halved?

$$\frac{p_2}{p_1} = \left(\frac{V_2}{V_1}\right)^{-1} = 2$$

17. You are standing in a room at atmospheric pressure and room temperature. You estimate the room to be 10 m wide by 15 m long by 2 m high. How many moles of gas are in the room?

$$V = 2 \cdot 10 \cdot 15 = 300\,m^3$$

\# of moles

$$n = \frac{pV}{RT} = \frac{10^5\,Pa \cdot 300\,m^3}{8.31\,\frac{J}{mK} \cdot 293\,K} = \underline{12{,}300\,mol}$$

18) RT, 1 atm, 1 mol; how big? show me!

$$V = \frac{nRT}{p} = \frac{1\,mol \cdot 8.31 \cdot 293}{10^5\,Pa} = 0.024\,m^3$$

$$s = \sqrt[3]{V} = \sqrt[3]{0.024\,m^3} = 0.29\,m$$

→ 30 cm

→ 12 in

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

→ mass of one particle in kg!

$$V = s^3 = (0.33\,m)^3 = 0.0359\,m^3$$

If 2.00 mol of nitrogen gas (N_2) are placed in a cubic box, 33 cm on each side, at 2.600 atm of absolute pressure, what is the rms speed of the nitrogen molecules?

712 ± 2% m/s

$\rightarrow P$ (in Pa) $= 2.6 \cdot 10^5$ Pa

2 mol of N_2

$$2 \cdot \left(14 \frac{g}{mol}\right) = 28\,g$$

$$\frac{28\,g}{N_A} = 4.6 \cdot 10^{-23}\,g$$

$4.6 \cdot 10^{-26}$ kg/particle

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

$$V_{rms} = \sqrt{\frac{3(1.38 \cdot 10^{-23}\,J/K)(561\,K)}{4.6 \cdot 10^{-26}}}$$

$$\boxed{V_{rms} = 711\,m/s}$$

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{2.6 \cdot 10^5\,Pa \cdot 0.0359\,m^3}{2\,mol \cdot 8.31}$$

$$T = 561\,K$$

one interesting thing:

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

$$m = \frac{M}{N_A}$$ ← mass per mol

$$V_{rms} = \sqrt{\frac{3 (k_B \cdot N_A) \cdot T}{M}}$$

$k_B \cdot N_A = R = 8.31\,J/K\,mol$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

so you can use this if you are careful