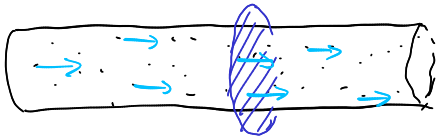


Week 6 covers sections of chapter 18 in the textbook. Topics include:

- resistive circuits and Ohm's Law
- power use in circuits
- networks of resistors

1. Charges are flowing through a wire like water through a pipe. If the wire has 5 A of current flowing through it, then how much charge passes a point in the wire in 10 s?



$$I = \frac{\Delta q}{\Delta t}$$

$$5 \text{ A} = \frac{\Delta q}{10 \text{ s}}$$

$$\Delta q = 50 \text{ C}$$

Current
 ↳ [Ampères]
 [Amps]

$$I = \frac{\Delta q}{\Delta t}$$

2. Batteries are often have a rating written on them. A Energizer AA battery has a rating of 3000 milliAmp · hours (mA h).

- (a) What kind of quantity is a milliAmp · hour? What base unit is this related to?

$$\text{mA} \cdot \text{hr} \rightarrow \text{A} \cdot \text{hr} \rightarrow \text{A} \cdot \text{s} \rightarrow \text{Coulomb}$$

$$3000 \text{ mA} \cdot \text{hr} \cdot \frac{1 \text{ A}}{1000 \text{ mA}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = 10,800 \text{ C}$$

- (b) If the battery discharged all of this charge at 1.5 V, then how much potential energy was initially in the battery?

$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta U = q \cdot \Delta V = 10,800 \text{ C} \cdot 1.5 \text{ V} = \underline{16,200 \text{ J}}$$

- (c) If the battery discharges at a rate of 25 mA, then how long will the battery last?

$$\checkmark \quad \checkmark \quad I = \frac{\Delta q}{\Delta t} \Rightarrow \Delta t = \frac{\Delta q}{I} = \frac{10,800 \text{ C}}{0.025 \text{ A}} = 432,000 \text{ s} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{1 \text{ day}}{24 \text{ hr}} = 5 \text{ days}$$

$$\begin{aligned} \Delta t &= \frac{\Delta q}{I} \\ &= \frac{3000 \text{ mA} \cdot \text{hr}}{25 \text{ mA}} \\ &= 120 \text{ hr} \end{aligned}$$

(d) What is the power output of the battery?

$$\begin{aligned}
 P_b &= \frac{\Delta U_b}{\Delta t} = \frac{q \cdot \Delta V}{\Delta t} = I_b \cdot \Delta V_b \\
 &= \frac{16,200 \text{ J}}{432,000 \text{ s}} = 0.025 \text{ A} \cdot 1.5 \text{ V} \\
 &= 0.0375 \text{ Watts} = 0.0375 \text{ Watts}
 \end{aligned}$$

(e) What is the resistance of this light bulb?

3. A simple particle accelerator can be designed from a capacitor in a vacuum. The negatively charged plate has a heater near it, and when electrons have enough thermal energy, then they are released from the plate and are accelerated in the electric field between the plates. If the positive plate has a hole in it, then the electrons will fly through and now you have an electron gun. If the potential on the plates is maintained, then a steady stream of electrons can result. If a mole of electrons passes through the hole in the positively charged plate per second, then what is the current of the gun? Bonus question from last week, if voltage between the plates is 5000V then how fast will the electrons emerge from the gun?

<p><u>Ohm's Law</u></p> $R = \frac{\Delta V}{I} \Rightarrow \Delta V = IR$ $I = \frac{\Delta V}{R}$ <p>$[Ohm] = \frac{[Volts]}{[Amps]}$</p> <p>$[Ω]$</p>	<p><u>Resistance of a wire</u></p> $R \propto l A^{-1}$ $R = \frac{\rho l}{A}$ <p>resistance</p> <p>resistivity (material dependent)</p>	<p><u>Area of a wire</u></p> $A = \pi r^2$ $r = \frac{d}{2}$ $A = \pi \left(\frac{d}{2}\right)^2$ $A = \frac{\pi d^2}{4}$	<p>$R = \frac{\rho l}{\frac{\pi d^2}{4}}$ ← <u>vinculum</u></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $R = \frac{4 \rho l}{\pi d^2}$ </div> $R \propto l d^{-2}$ $\frac{R_2}{R_1} = \left(\frac{d_2}{d_1}\right)^{-2} = 2^{-2} = \frac{1}{4}$
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4. The resistance of a wire is directly proportional to the length and inversely proportional to the cross-sectional area. But measuring the area of a wire really comes from measuring the diameter of a wire and then using that to find the area. So use the area of a circle formula to substitute into the resistance of a wire equation and make sure you express this in terms of diameter. If the diameter of one wire is twice the diameter of another, then by what factor are the resistances related?

$$\rho_{\text{Cu}} = 1.67 \cdot 10^{-8} \Omega \text{m}$$

5. A copper wire is 100 m long and has a diameter of 1 mm. It is connected to a 12 V battery.

(a) What is the resistance of the wire?

$$R = \frac{\rho l}{\pi d^2} = \frac{1.67 \cdot 10^{-8} \Omega \text{m} \cdot 100 \text{m}}{\pi \cdot (0.001 \text{m})^2} = 2.12 \Omega$$

(b) How much current goes through the wire?

$$\frac{\Delta V}{I} = R \Rightarrow I = \frac{\Delta V}{R} = \frac{12 \text{V}}{2.12 \Omega} = 5.66 \text{A}$$

(c) What is the power supplied by the battery?

$$P_b = I_b \cdot \Delta V_b = 5.66 \text{A} \cdot 12 \text{V} = 67.9 \text{W}$$

(d) What power is dissipated by the wire?

wire heats up ΔT

Ohm's Law

$$R = \frac{\Delta V}{I}$$

$$\Delta V = I \cdot R$$

$$I = \frac{\Delta V}{R}$$

$$P_e = I_e \cdot \Delta V_e = 5.66 \text{A} \cdot 12 \text{V} = 67.9 \text{W}$$

$$P_e = I_e \cdot I_e \cdot R$$

$$P_e = I_e^2 \cdot R$$

$$P_e = \frac{\Delta V_e}{R} \cdot \Delta V_e$$

$$P_e = \frac{\Delta V_e^2}{R}$$

(e) How much heat would this give off in 10 minutes?

$$P = I \cdot \Delta V \quad P = \frac{\Delta V}{\Delta t} \quad P = \frac{W}{\Delta t} \quad P = \frac{Q}{\Delta t}$$

$$10 \text{ min} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 600 \text{ s}$$

$$Q = P \cdot \Delta t$$

$$= 67.9 \text{W} \cdot 600 \text{s} = 40,700 \text{J}$$

(f) How much would this heat up a liter of water?

ΔT

$1 \text{L} = 1 \text{kg}$

$$Q = mc \Delta T$$

$$40,700 \text{J} = 1 \text{kg} \cdot 4.184 \frac{\text{kJ}}{\text{kgK}} \cdot \Delta T$$

$$\Delta T = 9.74 \text{K}$$

$$T_i = 20^\circ \text{C} \rightarrow T_f = 29.74^\circ \text{C}$$

- (g) If the length of the wire doubled, what would happen to all of the above quantities?

	l double
$R \propto l$	R double
$I \propto R^{-1}$	I half
tricky! $\Rightarrow P \propto I$	P half
$Q \propto P$	Q half
$\Delta T \propto Q$	ΔT half

6. 100 m of a certain wire has a resistance of $5\ \Omega$. What is the resistance of the same wire if it is 12 m long?

7. If you stretch a wire to three times its original length, then the wire will get thinner, since it is the volume that remains the same. So by what factor does the cross sectional area change (assume as with all wires that it is a cylinder)? By what factor does the resistance change?

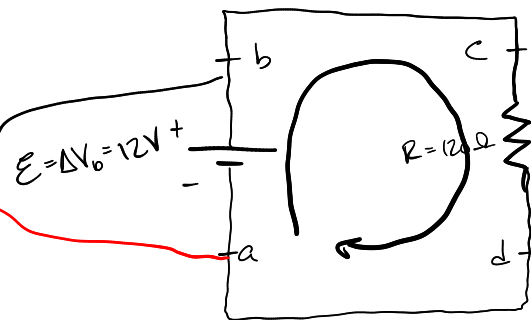
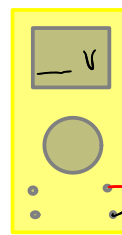
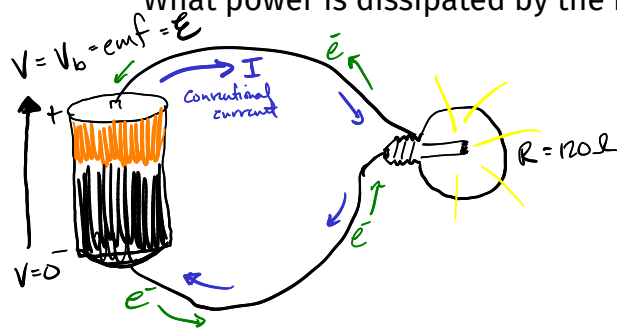
$$\delta m \tau$$

8. Combine the equations for the power dissipated by a resistor, $P = IV$ and Ohm's Law $V = IR$, in two ways and get two equations for power out. One should express power in terms of current

and resistance, and the other should express power in terms of voltage and resistance.

$$P = IV = I^2 R = \frac{\Delta V^2}{R}$$

9. You have a 12V battery connected to a $120\ \Omega$ resistor. Draw a picture of this. If you measure the voltage from the negative terminal to the positive terminal of the battery, what would you read? If you measured from the positive to the negative, what would you read? If you measured from the positive terminal of the battery to the side of the resistor near to it, what would you measure? If you measure across the resistor from the side nearer to the positive terminal to the other side, what would you measure? How much current is going through the resistor? What power is dissipated by the resistor?



$$\Delta V_{ab} = +12\text{ V}$$

$$\Delta V_{ba} = -12\text{ V}$$

$$\Delta V_{bc} = 0\text{ V}$$

$$\Delta V_{da} = 0\text{ V}$$

$$\Delta V_{cd} = -12\text{ V}$$

Loop Rule

$$+ \Delta V_B - \Delta V_R = 0$$

$$\Delta V_B = \Delta V_R = I \cdot R$$

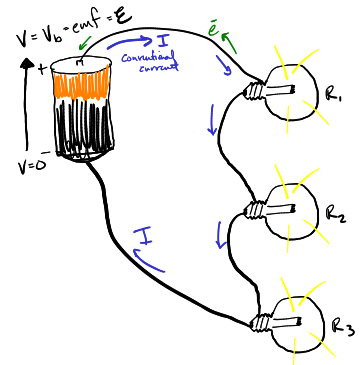
$$12\text{ V} = I \cdot (120\ \Omega)$$

$$I = 0.1\text{ A}$$

Power Dissipated

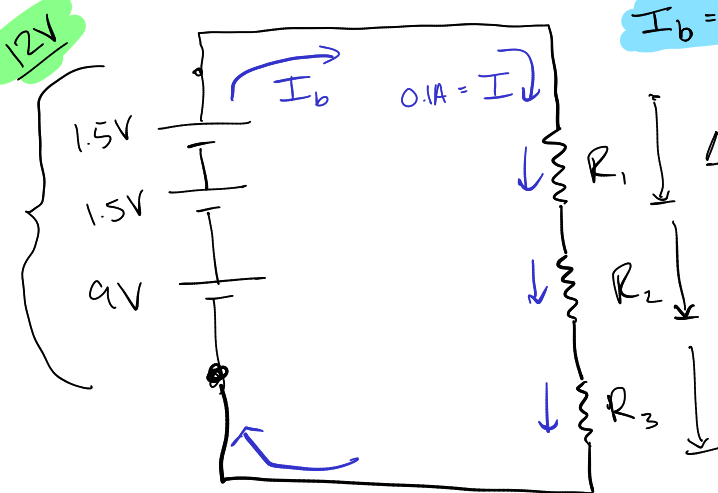
$$P_R = I_R \cdot \Delta V_R = I_R^2 \cdot R = \frac{\Delta V_R^2}{R} = 0.1\text{ A} \cdot 12\text{ V} = \underline{\underline{1.2\text{ W}}}$$

10. Now let's make two changes to the previous problem. Instead of a 12V battery you have a 9V and two 1.5V batteries *in series*. Then instead of a single resistor, there are 3: one that is 20Ω , one 40Ω and one 60Ω all in series. Draw a picture of this first. What is the equivalent resistance of this set of resistors? How much current flows through each one of these resistors? What is the voltage drop across the entire group of resistors? What is the voltage drop across each one? How much power is dissipated by the entire group of resistors? How much power is dissipated by each one?



For resistors in series

$$I_b = I_1 = I_2 = I_3 = I$$



$$\Delta V_1 = I \cdot R_1 = 0.1A \cdot 20\Omega = 2V$$

$$\Delta V_2 = I \cdot R_2 = 0.1A \cdot 40\Omega = 4V$$

$$\Delta V_3 = I \cdot R_3 = 0.1A \cdot 60\Omega = 6V$$

$$12V$$

Loop Rule

$$\Delta V_{B1} + \Delta V_{B2} + \Delta V_{B3} - \Delta V_{R1} - \Delta V_{R2} - \Delta V_{R3} = 0$$

$\underbrace{\quad}_{12V} \quad \quad \quad \begin{matrix} "IR_1" & "IR_2" & "IR_3" \end{matrix}$

$$+12V - IR_1 - IR_2 - IR_3 = 0$$

$$+12V - I(R_1 + R_2 + R_3) = 0$$

$$R_E = R_1 + R_2 + R_3 + \dots$$

$$R_E = 20\Omega + 40\Omega + 60\Omega$$

$$R_E = 120\Omega$$

$$12V = I \cdot R_E$$

$$12V = I \cdot 120\Omega$$

$$I = \frac{12V}{120\Omega} = 0.1A$$

Power Dissipated

$$P_{total} = I^2 \cdot R \leftarrow R_E$$

$$= (0.1A)^2 \cdot 120\Omega$$

$$= 1.2W$$

$$P_1 = I^2 \cdot R_1 = (0.1A)^2 \cdot 20\Omega = 0.2W$$

$$P_2 = I^2 \cdot R_2 = (0.1A)^2 \cdot 40\Omega = 0.4W$$

$$P_3 = I^2 \cdot R_3 = (0.1A)^2 \cdot 60\Omega = 0.6W$$

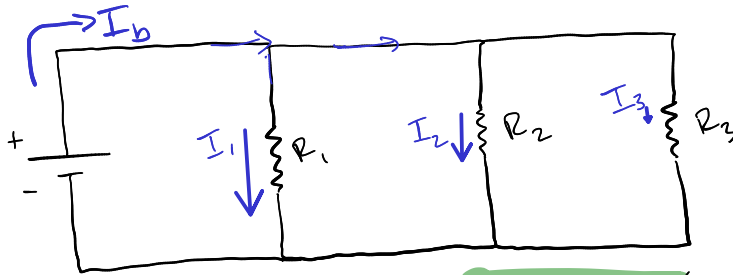
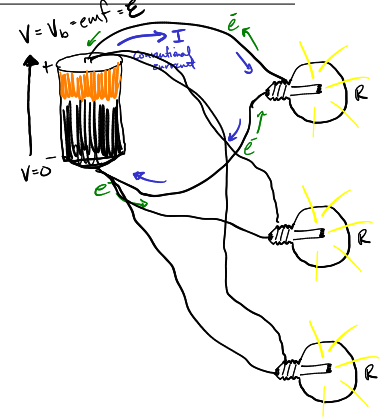
$$1.2W$$

Power Supplied

$$P_b = I_b \cdot V_b$$

$$= 0.1A \cdot 12V = 1.2W \quad \checkmark \quad 6$$

11. Now let's take the same $20\ \Omega$, $40\ \Omega$ and $60\ \Omega$ resistors and connect them each *in parallel*, that is they each have their own connection to the 12 V battery. Draw this out as a circuit diagram. What is the potential drop across each resistor? What is the current through each resistor? What is the total current supplied by the battery? What is the equivalent resistance of this circuit? How much power is supplied by the battery? How much power is dissipated by each resistor?



$$\Delta V_B = \Delta V_{R_1} = 12\text{ V}$$

$$\Delta V_{R_1} = I_1 R_1$$

$$I_1 = \frac{\Delta V_{R_1}}{R_1} = \frac{12\text{ V}}{20\ \Omega}$$

$$I_1 = 0.6\text{ A}$$

$$\Delta V_B = \Delta V_{R_2} = 12\text{ V}$$

$$I_2 = \frac{\Delta V_{R_2}}{R_2}$$

$$I_2 = \frac{12\text{ V}}{40\ \Omega}$$

$$I_2 = 0.3\text{ A}$$

$$\Delta V_B = \Delta V_{R_3} = 12\text{ V}$$

$$I_3 = \frac{\Delta V_{R_3}}{R_3}$$

$$I_3 = \frac{12\text{ V}}{60\ \Omega}$$

$$I_3 = 0.2\text{ A}$$

$$I_b = I_1 + I_2 + I_3 + \dots$$

$$I_b = 0.6\text{ A} + 0.3\text{ A} + 0.2\text{ A} = 1.1\text{ A}$$

$$\Delta V_B = I_b \cdot R_E$$

$$R_E = \frac{12\text{ V}}{1.1\text{ A}} = 10.9\ \Omega$$

Power supplied = Total power dissipated

$$P_b = I_b \cdot V_b = 1.1\text{ A} \cdot 12\text{ V} = 13.2\text{ W}$$

Power dissipated

$$P_{R_1} = \frac{\Delta V_{R_1}^2}{R_1} = \frac{(12\text{ V})^2}{20\ \Omega} = 7.2\text{ W}$$

$$P_{R_2} = \frac{(12\text{ V})^2}{40\ \Omega} = 3.6\text{ W}$$

$$P_{R_3} = \frac{(12\text{ V})^2}{60\ \Omega} = 2.4\text{ W}$$

$$13.2\text{ W}$$

for resistors in parallel

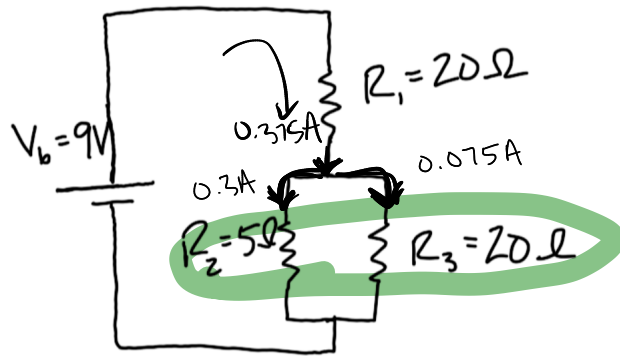
$$R_E = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\right)}$$

$$\text{book: } \frac{1}{R_E} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

$$R_E = \frac{1}{\frac{1}{20} + \frac{1}{40} + \frac{1}{60}}$$

$$R_E = 10.9\ \Omega$$

12. For the resistor network below, what is the equivalent resistance? What is the current through R_1 ? What is the voltage drop across R_1 ? What is the voltage drop across R_2 and R_3 ? What is the current through R_2 and R_3 ?

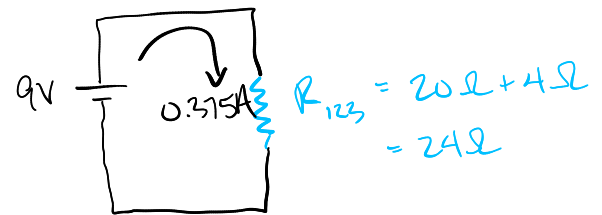
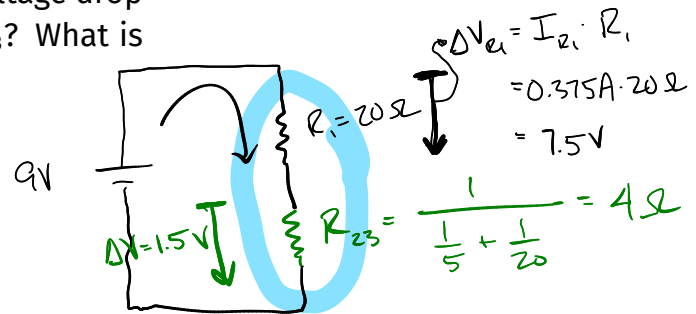


$$\Delta V_{R_2} = 1.5V = I_2 \cdot R_2$$

$$I_2 = \frac{1.5V}{5\Omega} = 0.3A$$

$$\Delta V_{R_3} = 1.5V = I_3 \cdot R_3$$

$$I_3 = \frac{1.5V}{20\Omega} = 0.075A$$



$$\Delta V_b = \Delta V_{R_{123}}$$

$$9V = I_b \cdot 24\Omega$$

$$I_b = \frac{9V}{24\Omega} = 0.375A$$

13. A light bulb is rated as 75W. If it is connected to a standard 120V then what current does it draw from the outlet? What is the resistance of the filament in the bulb?

$$P = I \cdot V = I^2 \cdot R = \frac{\Delta V^2}{R}$$

$$75W = \frac{(120V)^2}{R}$$

$$R = \frac{120^2}{75} = 192\Omega$$

What is the current?

$$P = I \cdot V$$

$$75W = I \cdot 120V$$

$$I = 0.625A$$

14. Household wiring is done so that each outlet is in parallel to the others. A group of several outlets are all connected to a circuit breaker, which will typically break the circuit when more than

20 A flow down the wire to that group of outlets. The voltage provided to the circuit is 120 volts (although unlike the constant voltage provided by a battery, the voltage out of the wall is alternating and looks like a sine wave, but this is not important for us in this problem). Most appliances are rated by the power that they do work, so for example a vacuum cleaner may operate at 700 W and a coffee maker at 900 W and a toaster might be 1000 W. If these are all plugged in to outlets on the same circuit, how much current would be supplied through the breaker? Will it trip and stop the current?

15. When you pay your power bill, you pay for each kilowatt · hour of energy that you use. How much energy in Joules is one kW · h?

16. Assume each atom of copper in a wire contributes one electron to be able to move freely in a current. The mass density of copper is $\rho_m = 8.92 \text{ g/cm}^3$ and the atomic mass of copper is 63.5 g/mole. What is the number density of conduction electrons in copper (electrons per m^3)? If 5 A of current are flowing through a copper wire that has a diameter of 0.1 mm, then what is the drift velocity of electrons in the wire?

