Week 2 covers sections of chapter 14 in the textbook. Topics include:

- heat, energy, and power
- heat capacity, specific heat capacity, and molar heat capacity
- latent heat of fusion or vaporization

internal energy

$$\Delta U = Q + W$$

25°C → $4.184 \frac{J}{gK}$ vs. $4.186$ ← 20°C or 15°C

?

$4.184 \frac{kJ}{kg\,K}$

1. Suppose you have 2 kg of water in a cup and you put it in the microwave. What form of heating is this (conduction, convection, or radiation)? It takes 1 minute to heat up from 20 °C to 40 °C. What is the temperature change in Celsius ($\Delta T$)? What is the change in temperature change in Kelvin? Look up the specific heat of water and be careful to specify the units. How much heat would it take to accomplish this temperature change?

$$\Delta T = 40°C - 20°C = 20°C = 20K$$

$$Q = M \cdot C \cdot \Delta T$$

heat (J), mass, specific heat

$$Q = 2kg \cdot 4.184 \frac{kJ}{kgK} \cdot 20K$$

$$Q = 167\,kJ$$

2. What are the units of power? What is another way to express those units? To follow up the previous problem, what is the power delivered to the water by the microwave? If this rate of heat delivery continues, how long will it take for the water to reach its boiling point?

$$P = \frac{\Delta U}{\Delta t} = \frac{Q}{\Delta t}$$

$$[Watts] = \frac{[Joule]}{[sec]}$$

$$P = \frac{167{,}000J}{60s}$$

$$P = 2783W$$

$$P = \frac{Q}{\Delta t} \leftarrow ? \longrightarrow \Delta t = \frac{Q}{P} = \frac{502{,}000\,J}{2783W}$$

$$\Delta t = 180s$$

$$Q = 2kg \cdot 4.184 \frac{kJ}{kgK} \cdot 60K$$

$$= 502\,kJ \rightarrow 502{,}000J$$

3. You have three samples of material, all 1 kg each. The materials are gold, copper, and aluminum. Put these in order of how much heat is necessary to change the temperature 1 degree.

$c_{Au} = 0.128 \frac{kJ}{kgK}$

$c_{Cu} = 0.385 \frac{kJ}{kgK}$

$c_{Al} = 0.900 \frac{kJ}{kgK}$

least ⟶ greatest

$Au \rightarrow Cu \rightarrow Al$

4. You have the same materials as in the last problem. If you put the same amount of heat into each sample, put them in order of which would heat up the most (remember that when we say "heat up" we mean increase in temperature).

$Q = mc\Delta T$

$\hookrightarrow \Delta T = \frac{Q}{mc}$

least $\Delta T$ ⟶ most $\Delta T$

$Al \rightarrow Cu \rightarrow Au$

5. How much heat does it take to bring 2 kg of aluminum from 25 °C to 50 °C?

6. If it takes 5000 J to bring an ingot of gold from 25 °C to 50 °C, then what mass of gold is the ingot?

7. If 5000 J of heat goes into a 1.5 kg glass dish that was initially at 20 °C, then what is its final temperature?

8. If a 5 kg block of aluminum has a temperature of 500 °C, how much heat does it give off to cool down to 490 °C? What should the sign of heat be in this case?

$$c_{Al} = 0.9 \frac{kJ}{kgK}$$

$$Q = mc\Delta T$$

$$= 5kg \cdot 0.9 \frac{kJ}{kgK} \cdot \underbrace{(490 - 500)}_{-10}$$

$$Q = -45 kJ$$

9. If the aluminum block in the above problem gave up this heat because it was put in contact with a cooler 5 kg block of lead, then by how much does the temperature of the lead rise? If the lead was originally 75 °C, then what would be its new temperature?

$$c_{Pb} = 0.13 \frac{kJ}{kgK}$$

$$+45kJ \rightarrow \underset{\checkmark}{Q} = \underset{\checkmark}{m}\underset{\checkmark}{c}\underset{?}{\Delta T}$$

$$\Delta T = \frac{Q}{mc} = 69.2 K = 69.2°C$$

$$\Delta T = 69.2°C = T_f - 75°C$$

$$T_f = 144°C$$

10. Keep this process going, the aluminum cooling off by 10 degress and the lead warming up by whatever you found in the above problem. Approximately what temperature would they meet? This temperature is known as the *equilibrium temperature* since

after this heat flow stops, and they remain at the same temperature (ignoring heat that they lose to the surroundings).

| $T_{Al}$ | $T_{Pb}$ |
|---|---|
| 500 | 75° |
| 490 | 144 |
| 480 | 213 |
| 470 | 282 |
| 460 | 351 |
| 450 | 420 |
| ~~440~~ | ~~489~~ |

← What is the equilibrium temp?

11. Lets do this in one step now. If the aluminum has an initial temperature of 500 °C and an *unknown final temperature*, and the lead starts at 75 °C and has an unknown final temperature but the same final temperature as the aluminum, since that is the equilibrium temperature, then how can we find this with one expression? (*Hint: the overall energy of the system does not change. So any change in energy of one plus the change in energy of the other must be zero.*)

$$\Delta U_{system} = 0$$

$$Q_{Al} + Q_{Pb} = 0$$

$$m_{Al} c_{Al} \Delta T_{Al} + m_{Pb} \cdot c_{Pb} \cdot \Delta T_{Pb} = 0$$

$$m_{Al} c_{Al} (\underline{T_f} - T_{iAl}) + m_{Pb} c_{Pb} (\underline{T_f} - T_{iPb}) = 0$$

$$5 \cdot 0.9 (T_f - 500) + 5 \cdot 0.13 (T_f - 75) = 0$$

$$4.5T_f - 2250 + 0.65T_f - 48.75 = 0$$

$$T_f = 446°C$$

12. The exact same logic that applied to the above problem, applies to mixing two substances together. You can still treat them as separate substances with one or more giving up energy in the form of heat to the others. It is always assumed that the materials are kept in a well insulated container so that no heat is lost to the surroundings. This is a good way of measuring the heat capacity of an unknown material by mixing it with a material of known initial temperature and heat capacity and then measuring the equilibrium temperature that results. So suppose you start with 100 g of water at 90 °C, and you pour in 50 g of unknown metal at 20 °C. You stir the mixture and notice that the temperature of the mixture comes down to 40 °C and

then remains at that temperature. What is the specific heat of the unknown material?

Heat capacity at constant volume.

monatomic → $12.5 \frac{J}{K\,mol} = \frac{3}{2}R$

diatomic → $20.8 \frac{J}{K\,mol} = \frac{5}{2}R$

$$Q_{water} + Q_{metal} = 0$$

$$\underbrace{100g \cdot 4.184 \frac{J}{gK}(40-90)}_{} + 50g \cdot C \cdot (40-20) = 0$$

$$C = 20.9 \frac{J}{g \cdot K}$$

13. A monatomic gas is in a container that keeps the gas at constant volume of 0.01 m$^3$. The gas is at room temperature and the pressure is 1 atm. How much heat do you have to add to increase the temperature by 10 °C. What is the pressure in the container at that temperature?

molar heat capacity at constant volume

$$Q = n \cdot C_v \cdot \Delta T$$

# of moles

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{10^5 Pa \cdot 0.01 m^3}{8.31 \frac{J}{K\,mol} \cdot 293K}$$

$$n = 0.41\ mol$$

$$Q = 0.41\ mol \cdot 12.5 \frac{J}{K\,mol} \cdot 10K$$

$$\boxed{Q = 51J}$$

$$\boxed{PV = nRT} \rightarrow P \propto T \rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{303}{293} = 1.03$$

$$\boxed{P_2 = 1.03 atm}$$

14. A monatomic gas is in a container that keeps the gas at constant pressure of 1 atm. The gas is at room temperature and the volume is 0.01 m$^3$. How much heat do you have to add to increase the temperature by 10 °C. What is the volume of the container at that temperature?

molar heat capacity at constant pressure

$$Q = n\, C_p \cdot \Delta T$$

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{10^5 Pa \cdot 0.01 m^3}{8.31 \frac{J}{K\,mol} \cdot 293K}$$

$$n = 0.41\ mol$$

$$Q = 0.41\ mol \cdot 20.8 \frac{J}{K \cdot mol} \cdot 10K$$

$$\boxed{Q = 85J}$$

Heat Capacity at constant pressure

monatomic → $20.8 \frac{J}{K\,mol}$

diatomic → $29.1 \frac{J}{K \cdot mol}$

15. Why is there a difference between the heat needed to change the temperature of an ideal gas at constant volume vs at constant pressure?

the constant pressure case must do work on the surroundings

16. How much heat is needed to melt a block of ice that is 10 kg at 0 °C? What if the ice starts at −10 °C?

$Q = m \cdot L_f$

$= 10\,\text{kg} \cdot 333.7 \frac{\text{kJ}}{\text{kg}}$

$Q = 3337\,\text{kJ}$

$Q = Q_{\text{warm ice}} + Q_{\text{melt ice}} \left( + Q_{\text{warm the water that was ice}} \right)$

$Q = \underbrace{10\,\text{kg} \cdot \left(2.1 \frac{\text{kJ}}{\text{kgK}}\right)(+10\,\text{K})}_{210\,\text{kJ}} + 3337\,\text{kJ}$

$Q = 3547\,\text{kJ}$

change these numbers

17. You drop a 0.1 kg ice cube at 0 °C into a cup with 200 g water at room temperature (20 °C). Before you work this problem, think about the possible outcomes. How much heat does it take to melt the ice cube completely? How much heat would it take to cool the water to 0 °C? Given this information which outcome is the one that happens? Find the equilibrium temperature.

To melt the ice completely

$Q = m L_f$

$= 0.1\,\text{kg} \cdot 333.7 \frac{\text{kJ}}{\text{kg}}$

$= 33.4\,\text{kJ}$

To cool the water to 0°

$Q = mc\Delta T$

$= 0.2\,\text{kg} \cdot 4.184 \cdot (-20\,\text{K})$

$= -16.7\,\text{kJ}$

So, the water will cool to 0° and the process will stop.

$Q = m_{ice} L_f$

$+16.7\,\text{kJ} = m_{ice} \cdot 333.7 \frac{\text{kJ}}{\text{kg}}$

amount of ice that melts $\rightarrow m_{ice} = 0.05\,\text{kg}$

ice → 0.1kg @ −10°C

water → 0.5kg @ 50°C

To warm the ice to 0°C:

$Q = m_{ice} c_{ice} \Delta T$

$= 0.1\,\text{kg} \cdot 2.1 \cdot 10\,\text{K}$

$Q = 2.1\,\text{kJ}$

To melt the ice:

$Q = 33.4\,\text{kJ}$

$35.5\,\text{kJ}$ ← ice → water @ 0°C

$Q = M_{water} \cdot c_{water} \cdot \Delta T$

$Q = 0.5 \cdot 4.184 \cdot (-50\,\text{K})$

$= 104.6\,\text{kJ}$

$Q_{\text{warm ice}} + Q_{\text{melt ice}} + Q_{\text{warm cold water that used to be ice}} + Q_{\text{cool the warm water}} = 0$

$2.1\,\text{kJ} + 33.4\,\text{kJ} + 0.1\,\text{kg} \cdot 4.184 \cdot (T_f - 0) + 0.5\,\text{kg} \cdot 4.184 \cdot (T_f - 50) = 0$

$\boxed{T_f = 27.5°C}$

18. A 60 kg hiker wished to climb to the summit of Mt. Ogden, an ascent of 5000 verticalfeet (1500 m).

- How much work will it take for her to reach this height?

$$\cancel{K_i}^{0} + U_i + W_{NC} = \cancel{K_f}^{0} + U_f$$

$$W_{NC} = U_f - \cancel{U_i}^{0}$$

$$W_{NC} = mgh = 60\,kg \cdot 9.8\,\frac{N}{kg} \cdot 1500\,m$$

$$= 882{,}000\,J$$

- Assuming that she is only 25% efficient at converting chemical energy from food into mechanical work, and that essentially all of the mechanical work is used to climb vertically, roughly how many bowls of corn flakes should the hiker eat before setting out? (standard serving size 1 oz, 100 Calories)

$$E_{bowl} = 100\,\text{Calories} \cdot \frac{1000\,cal}{1\,Cal} \cdot \frac{4.184\,J}{1\,cal} = \underline{\underline{418{,}400\,J}}$$

$$E_{climb} = 0.25 \cdot E_{total}$$

$$882{,}000\,J = 0.25 \cdot E_{total}$$

$$E_{total} = 3{,}528{,}000\,J$$

$$\frac{E_{total}}{E_{bowl}} = \text{bowls} = \frac{3528000\,J}{418{,}400\,J} = \underline{8.4\ \text{bowls}}$$

- As the hiker climbs the mountain the other 75% of the energy from the corn flakes is converted into thermal energy. If there were no way to dissipate this energy, by how many degrees would her body temperature increase? (Assume the human body is mostly water so that it has the same specific heat as water.)

$$0.75 \cdot 3{,}528{,}000\,J = 2{,}646{,}000\,J = \underline{2{,}646\,kJ}$$

$$Q = mc\Delta T$$

$$2{,}646\,kJ = 60\,kg \cdot 4.184\,\frac{kJ}{kg\,K} \cdot \Delta T$$

$$\Delta T = 10.5^\circ C$$

$$37 + 10.5 = 47.5^\circ C \longrightarrow 117^\circ F \quad \underline{\text{dead}}$$

- In fact, the extra energy does not warm the hiker's body significantly; instead, it goes (mostly) into evaporating water from her skin and within her lungs. How many liters of water should she drink during the hike to replace the lost fluids? (At 25 °C, a reasonable outdoor temperature to assume, the latent heat of vaporization of water is 580 cal/g, 8% more that at 100 °C.)

$\rightarrow 2427 \frac{kJ}{kg}$

$$Q = m L_v$$

$$2646\,kJ = m \cdot 2427 \frac{kJ}{kg}$$

$$m = 1.09\,kg \rightarrow \underline{\underline{1.09\,L}}$$