Chapter 3 - Momentum + Angular Momentum

Recall: $\vec{p} = m\vec{v}$ $\qquad \sum \vec{F}_n = \dot{\vec{p}}$

$$\mathbb{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \dots = \sum \vec{p}_n$$

$$\dot{\vec{\mathbb{P}}} = \vec{F}^{ext}$$

If $\vec{F}^{ext} = 0$

$$\dot{\vec{\mathbb{P}}} = 0$$

$$\mathbb{P}_i = \mathbb{P}_f$$ Conservation of Momentum

If two objects,

really 2 equations

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

If collision is perfectly inelastic

↳ stick together

then,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_f + m_2\vec{v}_f$$

$$= \underbrace{(m_1 + m_2)}_{\text{mass add up!}}\vec{v}_f$$

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} \checkmark$$

two equations!

Rockets + Explosions

At the beginning

$$\mathbb{P} = 0$$

Round 1

$$\mathbb{P}_i = 0 = \mathbb{P}_f = p_{ME} + p_{ball}$$

$$0 = \underbrace{(75\,kg + 99(0.5\,kg))}_{124.5\,kg} v + (0.5\,kg)(-15\,m/s)$$

$$0 = 124.5\,v - 7.5\,kg\,m/s$$

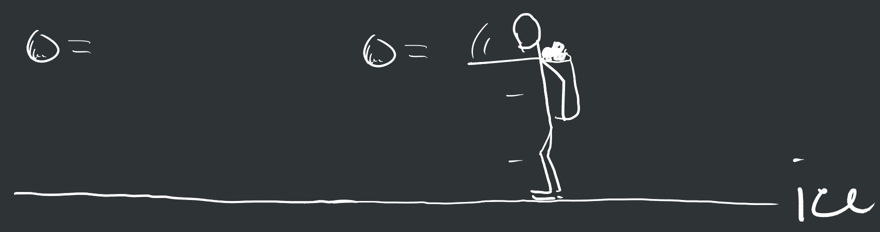
$$v = \frac{7.5\,kg\,m/s}{124.5\,kg} = 0.0602\,m/s$$

Round 2

$$V' = V_{ice} - V_{s'}$$

$$-15\,m/s = V_{ice} - 0.0602\,m/s$$

$$-15 + 0.06 = V_{ice} = -14.94\,m/s$$



$$P_i = P_f$$

$$\underbrace{(124.5)(0.0602\,m/s) + 0.5(-15)}_{=0} = 124\cdot v + \underbrace{0.5(-15) + 0.5(-14.94)}$$

$$-0.5(-15 - 14.94) = 124\cdot v$$

$-15 + 0.0602$

$$V = \frac{-0.5(-15 - 14.94)}{124} = \frac{-0.5(-15 - 15 + 0.0602}{124}$$

$$= 0.1207\,m/s = \frac{0.5\cdot 15 + 0.5\cdot 15 + 0.5\left(\frac{0.5(15)}{124.5}\right)}{124}$$

Round 1 Redux – my frame

$$\mathbb{P}_i = 0 = \mathbb{P}_f = p_{ME} + p_{ball}$$

$$0 = \big(\underbrace{75\,kg + 99(0.5\,kg)}_{124.5\,kg}\big)v + (0.5\,kg)(-15\,m/s)$$

$$0 = 124.5\,v - 7.5\,\frac{kg\,m}{s}$$

$$v = \frac{7.5\,kg\,m/s}{124.5\,kg} = 0.0602\,m/s$$

Round 2 Redux – my frame

$$\overset{\mathbb{P}_i'}{0} = 124.0 \cdot v + 0.5(-15)$$

$$v = \frac{0.5(15)}{124} = 0.06048\,m/s$$

Round 3 Redux - my frame

$$\overset{\mathbb{P}_i''}{0} = 123.5v + 0.5(-15)$$

$$v = \frac{0.5(15)}{123.5} = 0.0607 \text{ m/s}$$

0.18

Generalize

$$dV = \frac{dP_{ex}}{m}$$

variable

$$\int_{V_i}^{V_f} dV = \int_{m_o+m_f}^{m_o} \frac{V_{ex}\,dm}{m} = V_{ex}\int_{m_o+m_f}^{m_o} \frac{dm}{m}$$

$$v\Big|_{v_i}^{v_f} = v_{ex} \ln(m)\Big|_{m_0+m_f}^{m_0}$$

$$v_f - v_i = v_{ex}\left[\ln(m_0) - \ln(m_0+m_f)\right]$$

$$v_f = v_{ex} \ln\left(\frac{m_0}{m_0+m_f}\right) + v_i$$

$$\hookrightarrow \underline{\underline{v(t)}} = v_{ex} \ln\left(\frac{m(t)}{m_0+m_f}\right) + v_i$$

test w baseballs

$$v_f = -15 \ln\left(\frac{75\,kg}{125\,kg}\right)$$

$$= \underline{\underline{+7.66\ m/s}}$$