Chapter 2 - air resistance

$$\sum \vec{F} = \vec{F}_{NET} = m\ddot{\vec{r}}$$

$$\vec{F}_D = -f(v)\hat{v}$$

$$f(v) = a + bv + cv^2$$

a: 0

bv: linear drag

cv^2: quadratic drag

$$f_{lin} = bv \qquad b \propto D \qquad b = \beta D$$

related to viscocity

$$f_{quad} = cv^2 \qquad c \propto D^2 \qquad c = \gamma D^2$$

density

$$\frac{f_{quad}}{f_{lin}} \propto \frac{D^2 v^2}{D v} = D v$$

density

$$\frac{f_{quad}}{f_{lin}} \approx \text{Reynolds number} = \frac{\rho D v}{\eta}$$

viscosity

$$\vec{F}_{NET} = m\ddot{\vec{r}}$$

$$m\vec{g} - b\vec{v} = m\ddot{\vec{r}}$$

$$\underbrace{m\vec{g} - b\vec{v} = m\dot{\vec{v}}}_{\text{really two equations}}$$

$$-bv_x = m\dot{v}_x$$

$$\dot{v}_x = -\frac{b}{m} \cdot v_x$$

$$\boxed{\frac{dv_x}{dt} = -\frac{b}{m} \cdot v_x}$$

$$\frac{dv_x}{v_x} = -\frac{b}{m} dt$$

$$mg - bv_y = m\dot{v}_y$$

down ⊕ y-dir

$$\int \frac{dv_x}{v_x} = -\frac{b}{m}\int dt$$

$$e^{\ln v_x} = e^{\left(-\frac{b}{m}t + C\right)} = e^{-\frac{b}{m}t} \cdot e^{C}$$

$$v_x = A \cdot e^{-\frac{b}{m}t}$$

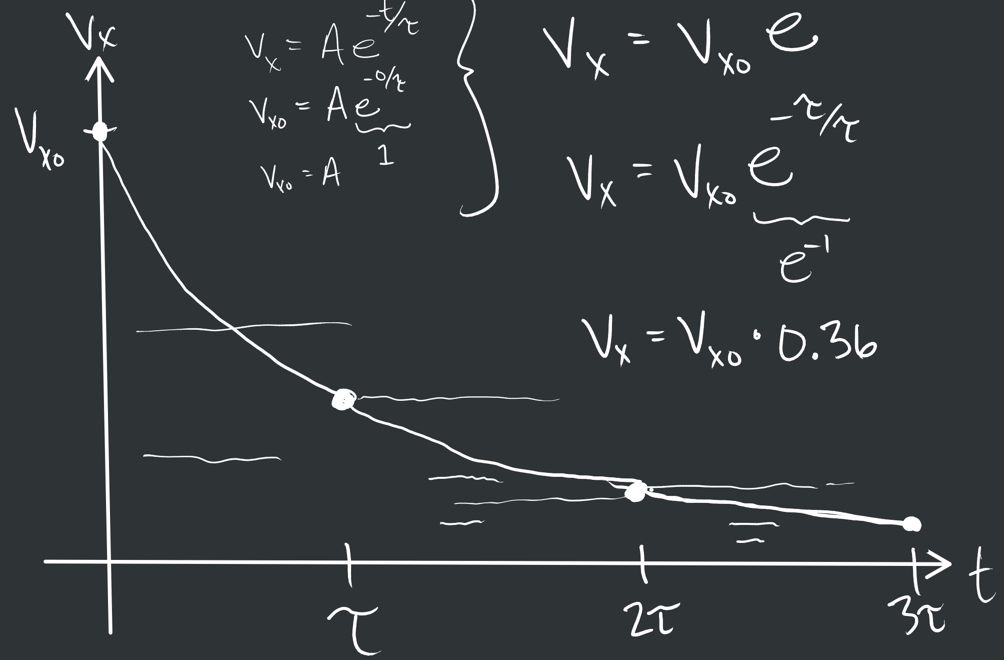
$$\frac{b}{m} = \left[\frac{1}{s}\right] = \left[s^{-1}\right]$$

$$\tau = \frac{m}{b} = [s] \rightarrow \text{time constant}$$

$$v_x = A e^{-t/\tau}$$

exponential decay

$$\dot{x} = v_{x0} e^{-t/\tau}$$



$$v_x = A e^{-t/\tau}$$
$$v_{x0} = A \underbrace{e^{-0/\tau}}_{1}$$
$$v_{x0} = A$$

$$v_x = v_{x0} e^{-t/\tau}$$

$$v_x = v_{x0} \underbrace{e^{-\tau/\tau}}_{e^{-1}}$$

$$v_x = v_{x0} \cdot 0.36$$

$$\frac{dx}{dt} = v_{x0} e^{-t/\tau}$$

$$\int dx = v_{x0} \int e^{-t/\tau} \, dt$$

$$\int_{x_0}^{x} dx' = v_{x0} \underbrace{\int_0^t e^{-t'/\tau} \, dt'}$$

function

$$x' \Big|_{x_0}^{x} = v_{x0} (-\tau) \, e^{-t'/\tau} \Big|_0^t$$

$$(x - x_0) = -v_{x0} \tau \cdot \left(e^{-t/\tau} - \cancelto{1}{e^{-0/\tau}} \right)$$

$$x - x_0 = v_{x0} \tau \left(1 - e^{-t/\tau} \right)$$

$$x(t) = v_{x0} \tau \left(1 - e^{-t/\tau} \right) + x_0$$

Linear Drag Vertically

$$mg - bv_y = m\dot{v}_y$$

if $\dot{v}_y = 0$

then $mg - bv_y = 0$

$$v_y = \frac{mg}{b} \equiv v_t$$

velocity when $\dot{v} = 0$

$$m\dot{v}_y = mg - bv_y$$

$$= -b\left(-\frac{mg}{b} + v_y\right)$$

$$m\dot{v}_y = -b(-v_t + v_y)$$

$$u = (-v_t + v_y)$$

$$\dot{u} = \frac{du}{dt} = 0 + \dot{v}_y$$

$$\dot{u} = \dot{v}_y$$

$$m\dot{u} = -b \cdot u$$

$$\dot{u} = -\frac{b}{m} \cdot u \leftarrow$$

$$\tau = \frac{m}{b}$$

$$u = Ae^{-t/\tau}$$

$$-v_t + v_y = Ae^{-t/\tau}$$

$$V_y = Ae^{-t/\tau} + V_t$$