Newton's Laws

1. Obj at rest remains at rest.....

2. $\vec{F} = \dfrac{d\vec{p}}{dt} = m\vec{a}$

velocity
↓

$\vec{a} = \dfrac{d\vec{v}}{dt} \approx \dfrac{\Delta\vec{v}}{\Delta t}$
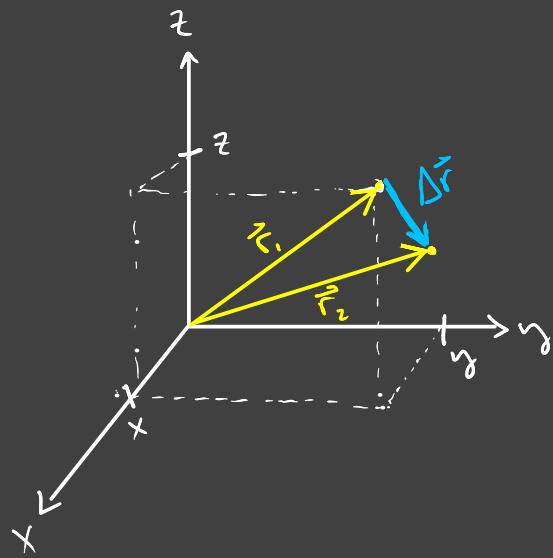
3. Action - Reaction

$\vec{p} = m\vec{v}$

$\dfrac{d\vec{x}}{dt} \approx \dfrac{\Delta\vec{x}}{\Delta t}$

$\vec{F}_{NET}$

$x \Rightarrow$ position

$jerk = \dfrac{da}{dt}$

$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$

position ↑ $\underset{\text{displacement}}{}$

$\vec{A} + \vec{B}$

$\Delta r_1 \quad \Delta r_2 \quad \Delta r_3 \quad \Delta r_4 \quad \Delta r_5$
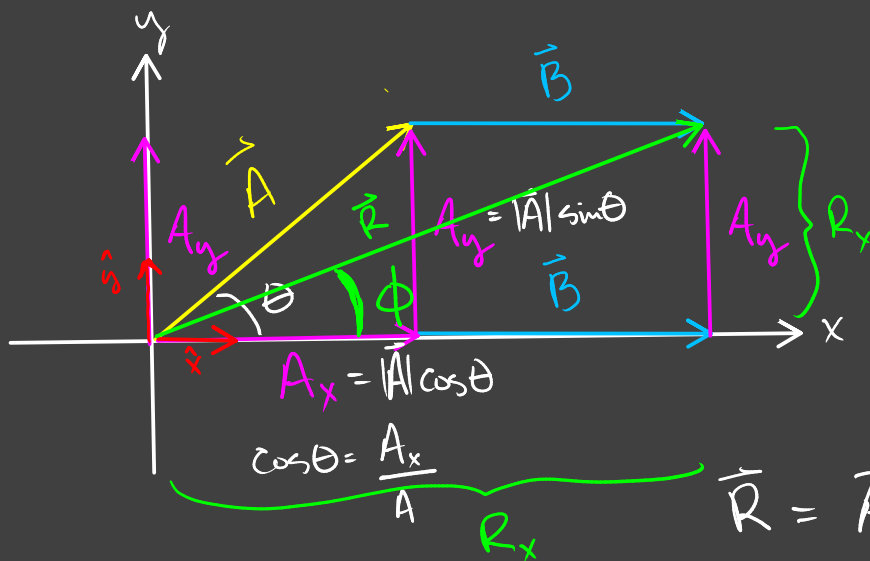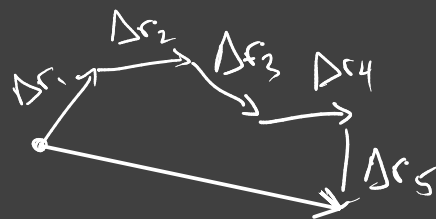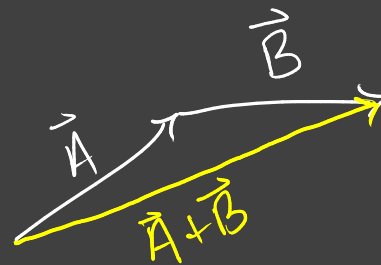
$\vec{A} = A_x \hat{x} + A_y \hat{y}$

$\vec{B} = B_x \hat{x} + B_y \hat{y}$

$$\vec{R} = \vec{A} + \vec{B} = \overbrace{(A_x + B_x)}^{R_x} \hat{x} + \overbrace{(A_y + B_y)}^{R_x} \hat{y}$$

$A_y = |A|\sin\theta$

$A_x = |A|\cos\theta$

$\cos\theta = \dfrac{A_x}{A}$
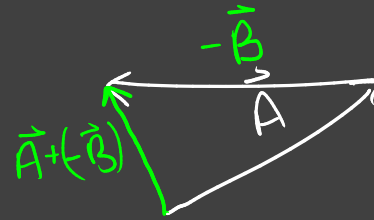
R_x

$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$

$\tan\phi = \dfrac{R_y}{R_x} \sim \phi = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

	X	y
A:	A_x	A_y
B:	B_x	B_y
R:	R_x	R_y

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$
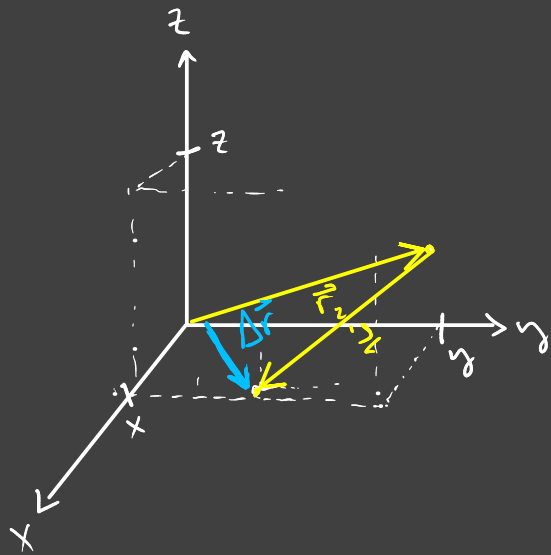
$$\vec{A} - \vec{B}$$

$$\vec{A} - \vec{B} = \left(A_x - B_x \right) \hat{x} + \left(A_y - B_y \right) \hat{y}$$
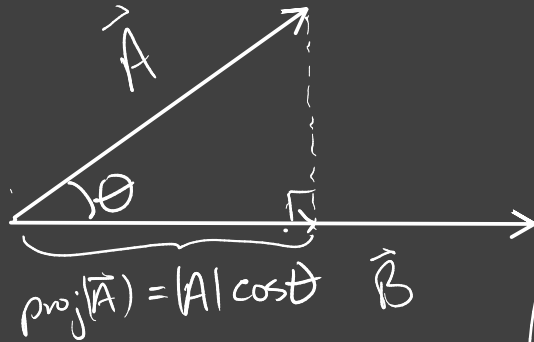
- scalar multiplication

$$c\vec{A} = c A_x \hat{x} + c A_y \hat{y} + c A_z \hat{z}$$

HW: 9, 23, 24, 26, 27, 30, 46, 48

Friday

- dot product / scalar product

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\text{proj}(\vec{A}) = |A|\cos\theta$$

$$\vec{A} \cdot \vec{B} = |\vec{A}|\cos\theta \cdot |\vec{B}|$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y \cdot B_y + A_z \cdot B_z = \vec{B} \cdot \vec{A}$$

$$|\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}} \qquad \vec{r}^2 = \vec{r} \cdot \vec{r}$$

- cross product

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - B_y A_z)\hat{x} - (A_x B_z - B_x A_z)\hat{y}$$
$$+ (A_x B_y - B_x A_y)\hat{z}$$

Derivative of Vectors

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$$\frac{d(\vec{r} + \vec{s})}{dt} = \frac{d\vec{r}}{dt} + \frac{d\vec{s}}{dt}$$

$$\frac{d(f\vec{r})}{dt} = \vec{r}\frac{df}{dt} + f \cdot \frac{d\vec{r}}{dt}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} + \frac{dz}{dt}\hat{z} + x\underbrace{\frac{d\hat{x}}{dt}}_{0} + y\underbrace{\frac{d\hat{y}}{dt}}_{0} + z\underbrace{\frac{d\hat{z}}{dt}}_{0}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} + \frac{dz}{dt}\hat{z}$$

$$\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$$

$$V_x = \frac{dx}{dt} \qquad V_y = \frac{dy}{dt} \qquad V_z = \frac{dz}{dt}$$

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1. If $\vec{F} = 0$, then $\vec{V}$ is constant

2. $\vec{F} = \frac{d\vec{p}}{dt}$ ⟵ $\vec{p} = $ momentum

3. $\vec{F}_{12} = -\vec{F}_{21}$

1st – Newton's Law hold in *inertial* reference frames

2nd – $\vec{F} = \dfrac{d\vec{p}}{dt}$ ——— $\vec{p} = m\vec{v}$

mass does not change

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

$$= m\vec{a}$$

$$\frac{d\vec{r}}{dt} = \dot{\vec{r}} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = \dot{\vec{v}} = \ddot{\vec{r}} = \vec{a}$$

$$\frac{d\vec{p}}{dt} = \dot{\vec{p}} = \vec{F} = m\dot{\vec{v}} = m\ddot{\vec{r}}$$

$$\vec{F} = m\ddot{\vec{r}}$$

Ex:

$$\vec{F} = F_x \hat{x}$$

$$\ddot{\vec{r}} = \frac{F_x \hat{x}}{m}$$

$$\frac{d^2x}{dt^2} = \frac{F_x}{m}$$

$$\ddot{x} = \frac{F_x}{m}$$

$$F_y = 0$$

$$F_z = 0$$

$$\ddot{y} = 0$$

$$\dot{y} = \int \ddot{y}\, dt = \int 0\, dt$$

$$\dot{y} = V_{y0} \quad \longleftarrow \text{y vel is constant}$$

$$y = \int \dot{y}\, dt = \int V_{y0}\, dt$$

$$y = V_{y0}\, t + y_0$$

$$\ddot{z} = 0$$

$$z = V_{z0}\, t + z_0$$

$$\dot{x} = \int \ddot{x}\, dt = \int \frac{F_x}{m}\, dt = \frac{F_x}{m} \int 1\, dt = \frac{F_x}{m} t + C$$

$$\underbrace{}_{t}$$

$$\dot{x} = \frac{F_x}{m} t + C$$

$$\dot{x}(t=0) = V_{x0}$$
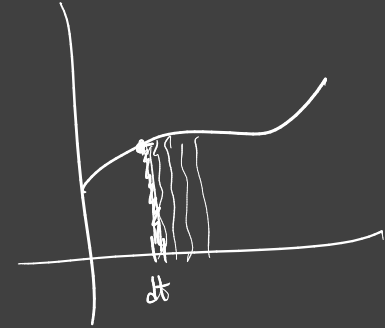
$$\boxed{\dot{X} = \frac{F_x}{m} t + V_{xo}}$$

$$X = \int \frac{dx}{dt} dt$$

$$X = \int \dot{x} \, dt = \int \left(\frac{F_x}{m} t + V_{xo} \right) dt$$

$$\rightarrow \quad X = \frac{F_x}{m} \int t \, dt + V_{xo} \int dt$$

$$X = \frac{1}{2} \cdot \frac{F_x}{m} \cdot t^2 + V_{xo} t + D$$

$$\curvearrowleft X_o$$

$$\rightarrow \quad \boxed{X(t) = \frac{1}{2} \frac{F_x}{m} t^2 + V_{xo} \cdot t + X_o}$$

$$X_o + V_i t + \frac{1}{2} a t^2$$

$$\ddot{\vec{r}} = \ddot{x} \hat{x} + \ddot{y} \hat{y} + \ddot{z} \hat{z}$$

$$\frac{\vec{F}}{m} = \ddot{\vec{r}} \begin{cases} \dfrac{F_x}{m} = \ddot{x} \\[2mm] \dfrac{F_y}{m} = \ddot{y} \\[2mm] \dfrac{F_z}{m} = \ddot{z} \end{cases}$$

3rd $\vec{F}_{12} = -\vec{F}_{21}$; force occur in pairs

F_{nb}

F_{fb}

F_{ft} on the table
by the book

F_{gb} F_{nt}

2

1

$\vec{F}_{21}$

$\vec{F}_{12}$
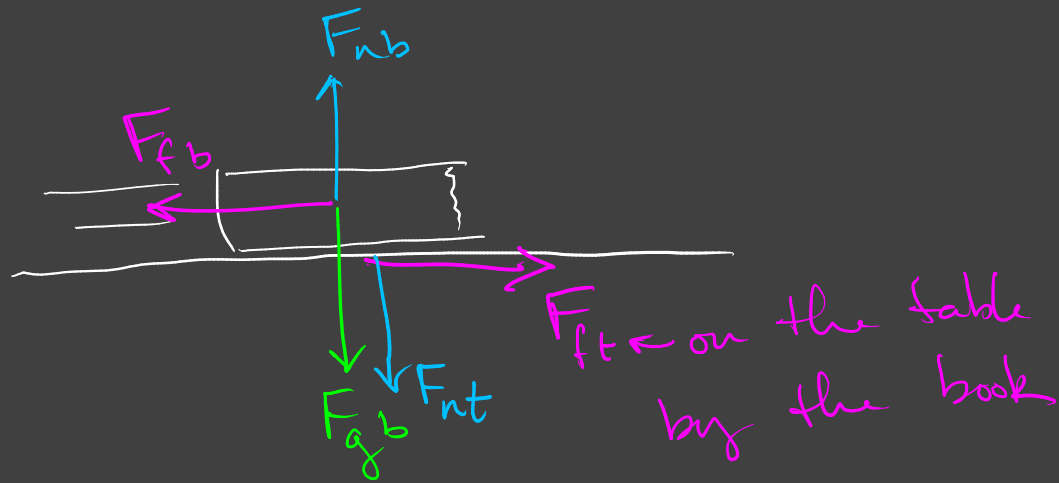
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_1^{ext} = \dot{p}_1 \qquad \vec{F}_2 = \vec{F}_{21} + \vec{F}_2^{ext} = \dot{p}_2$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\dot{\vec{P}} = \dot{\vec{p}}_1 + \dot{\vec{p}}_2$$

$$\dot{\vec{P}} = \vec{F}_{12} + \vec{F}_1^{ext} + \vec{F}_{21} + \vec{F}_2^{ext}$$

$$\hookrightarrow -\vec{F}_{21}$$

$$\dot{\vec{P}} = \vec{F}_1^{ext} +. \vec{F}_2^{ext}$$

$$\dot{\vec{P}} = \vec{F}^{ext}$$

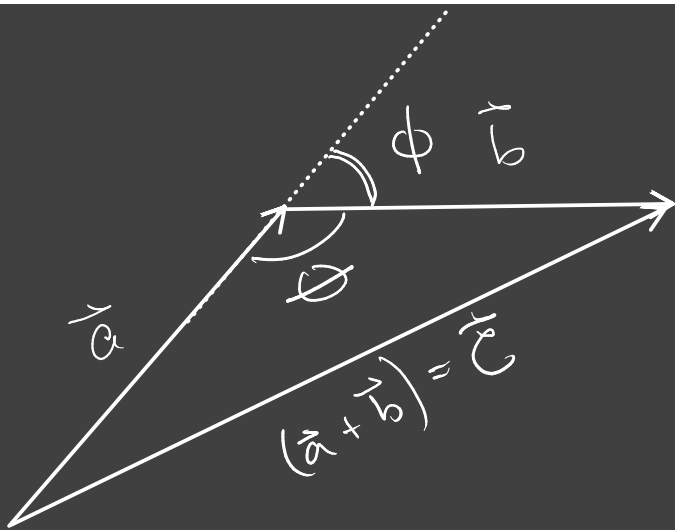$$so, \quad if \quad \vec{F}^{ext} = 0$$

$$\dot{\vec{P}} = 0$$

$$\vec{P} = constant$$

$$\vec{p}_1(t_1) + \vec{p}_2(t_1) = \vec{p}_1(t_2) + \vec{p}_2(t_2)$$

HW: 9, 23, 24, 26, 27, 30, 46, 48

9

1.9 ★ In elementary trigonometry, you probably learned the law of cosines for a triangle of sides a, b, and c, that $c^2 = a^2 + b^2 - 2ab\cos\theta$, where θ is the angle between the sides a and b. Show that the law of cosines is an immediate consequence of the identity $(\mathbf{a} + \mathbf{b})^2 = a^2 + b^2 + 2\mathbf{a} \cdot \mathbf{b}$.

$$\left(\vec{a} + \vec{b}\right)^2 = \left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} + \vec{b}\right)$$

$$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + 2\vec{a} \cdot \vec{b}$$

$$c^2 = a^2 + b^2 + 2\vec{a} \cdot \vec{b}$$

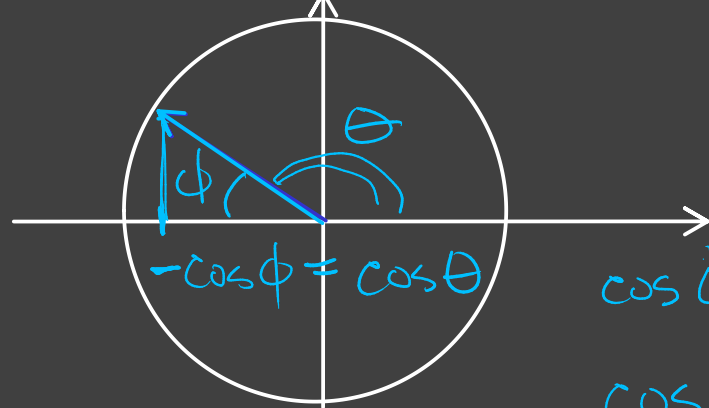$$= a^2 + b^2 + 2|\vec{a}||\vec{b}|\cos\phi$$

$$\phi = \pi - \theta$$

$$= a^2 + b^2 + 2|\vec{a}||\vec{b}|\cos(\pi - \theta)$$

$$\rightarrow \cos\left(\alpha \pm \beta\right) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\alpha = \pi \quad \beta = \theta$$

$$\cos(\pi - \theta) = \underbrace{\cos\pi}_{-1}\cos\theta + \underbrace{\sin\pi}_{0}\sin\theta$$

$$= -\cos\theta \checkmark$$

$$-\cos\phi = \cos\theta$$

$$\cos\phi = -\cos\theta$$

$$\cos(\pi-\theta) = -\cos\theta$$

23|

1.23 ★★ The unknown vector **v** satisfies **b** · **v** = λ and **b** × **v** = **c**, where λ, **b**, and **c** are fixed and known. Find **v** in terms of λ, **b**, and **c**.

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

$$\vec{b} \cdot \vec{v} = \lambda$$

$$\underset{A\times}{\vec{b}} \times (\underset{B\times C}{\vec{b} \times \vec{v}}) = \underbrace{(\vec{b} \cdot \vec{v})}_{\lambda}\vec{b} - \underbrace{(\vec{b} \cdot \vec{b})}_{b^2}\vec{v} = \vec{b} \times \vec{c}$$

$$b^2 \vec{v} = \lambda \vec{b} - \vec{b} \times \vec{c}$$

$$\vec{v} = \frac{\lambda\vec{b} - \vec{b} \times \vec{c}}{b^2}$$

24

1.24 ★ In case you haven't studied any differential equations before, I shall be introducing the necessary ideas as needed. Here is a simple excercise to get you started: Find the general solution of the first-order equation $df/dt = f$ for an unknown function $f(t)$. [There are several ways to do this. One is to rewrite the equation as $df/f = dt$ and then integrate both sides.] How many arbitrary constants does the general solution contain? [Your answer should illustrate the important general theorem that the solution to any nth-order differential equation (in a very large class of "reasonable" equations) contains n arbitrary constants.]

1.25 ★ Answer the same questions as in Problem 1.24, but for the differential equation $df/dt = -3f$.

$$\boxed{\frac{df}{dt} = f}$$

$$\int \frac{1}{f}\,df = \int dt$$

$$\ln f = t + C$$

$$e^{\ln f} = e^{(t+C)}$$

$$f = e^{(t+C)}$$

$$f = e^{t} \cdot e^{C}$$

$$\boxed{f = A e^{t}}$$

$$f(t=0) = A$$

$$\uparrow$$

initial condition

$$\frac{df}{f} = -3\,dt$$

$$= -3t + C$$

$$e^{-3t+C}$$

$$f = A e^{-3t}$$

$$\boxed{\frac{df}{dt} = At} \longrightarrow f = \frac{A}{2}t^2 + C$$

$$a^t \quad e^t$$

$$\frac{d}{dt}(e^t) = e^t$$

$$\rightarrow \frac{df}{dt} = f$$

$$\Delta f = \frac{df}{dt} \cdot \Delta t \qquad df = \frac{df}{dt} \cdot dt \qquad \Bigg| \qquad \frac{d^2 f}{dt^2} = c \cdot f$$

$$f_1 = f_0 + \Delta f$$
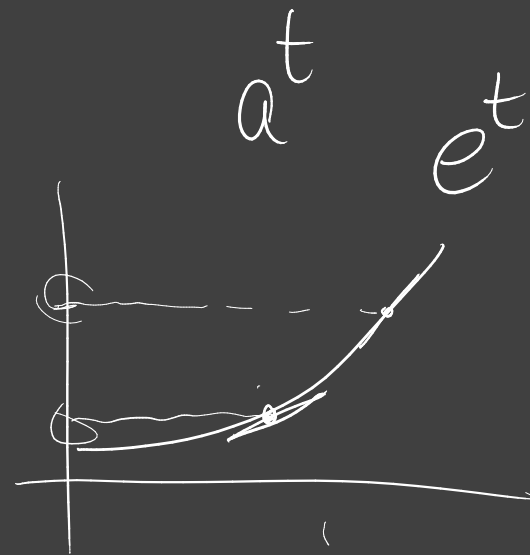
$$\ddot{x} = \frac{F}{m} \qquad\qquad\qquad\qquad\qquad f = Ae^{1t} \rightarrow Ae^{5t}$$

S

$X = 0$

$y = u_y \cdot t$

S'

$X' = \cancel{X}^{\nearrow 0} - V_x \cdot t$

$y' = y - \cancel{V_y \cdot t}^{\nearrow 0}$

$S' = S - change\ between\ frame$

S''

Polar Coordinates

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
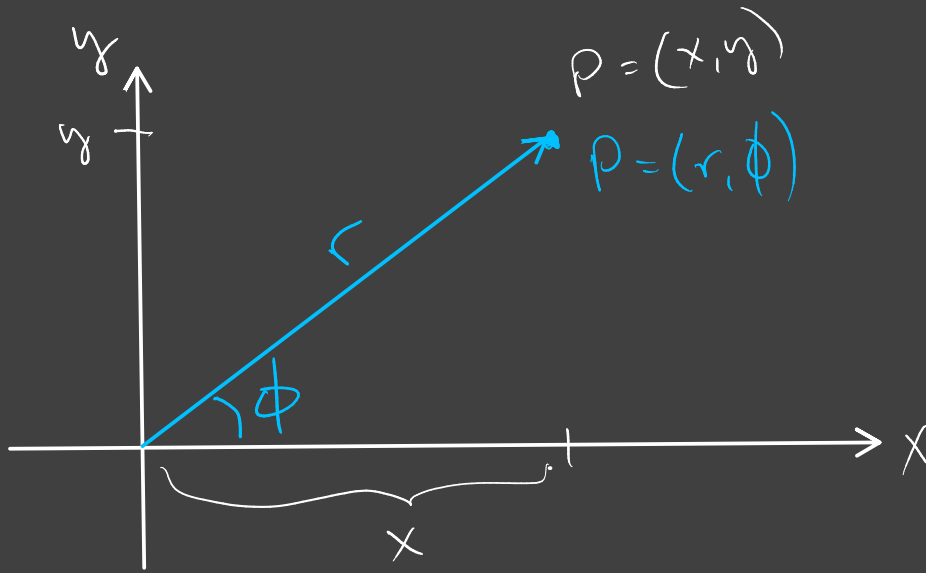
$$\longleftrightarrow \begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$$

$$\vec{F} = F_x \hat{x} + F_y \hat{y}$$
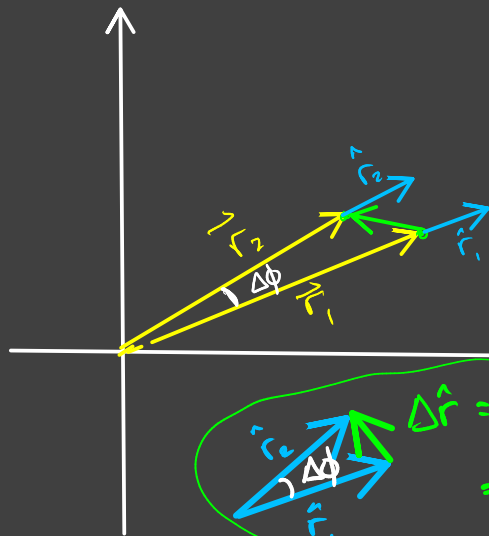
$$\vec{F} = F_r \hat{r} + F_\phi \hat{\phi}$$
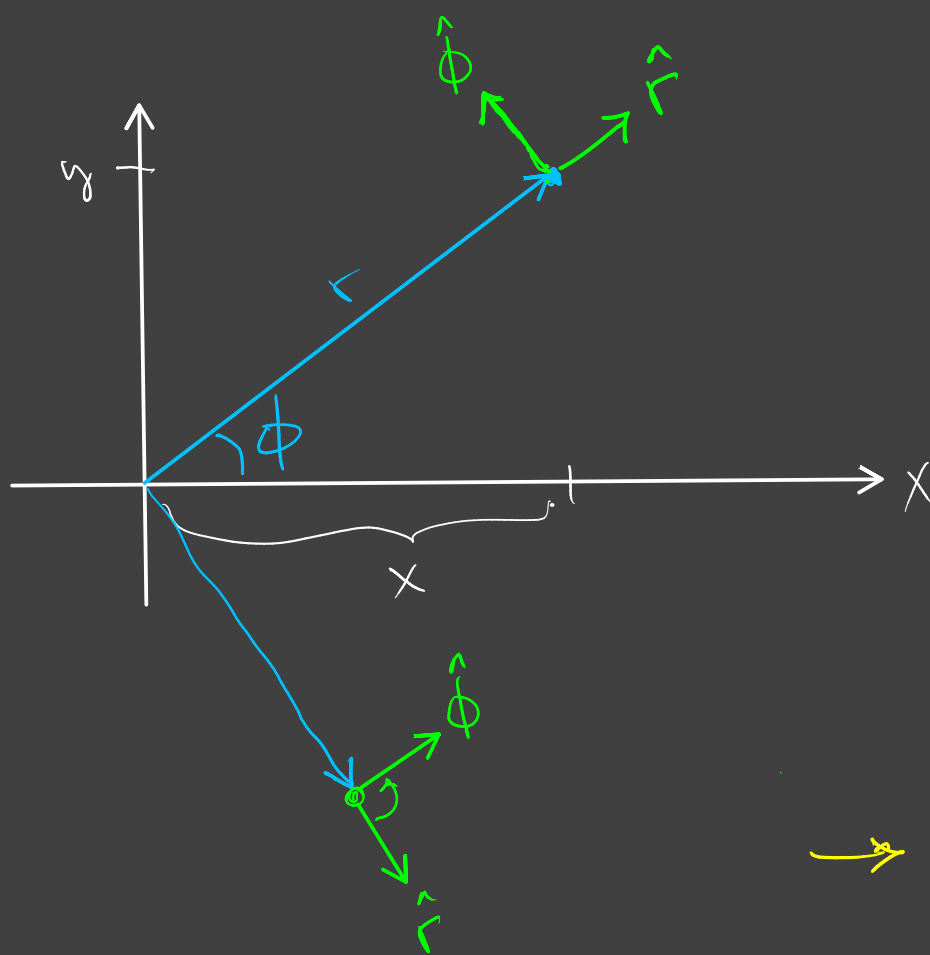
$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

in cartesian

$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$\vec{r} = r\,\hat{r}$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{d(r\hat{r})}{dt} = \hat{r}\frac{dr}{dt} + r\frac{d\hat{r}}{dt}$$

$$\boxed{\vec{V} = \dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\phi}\,\hat{\phi}}$$

$$\vec{V} = V_r\,\hat{r} + V_\phi\,\hat{\phi}$$

ω

angular velocity

$$\Delta\hat{r} = \Delta\phi \cdot 1\,\hat{\phi}$$
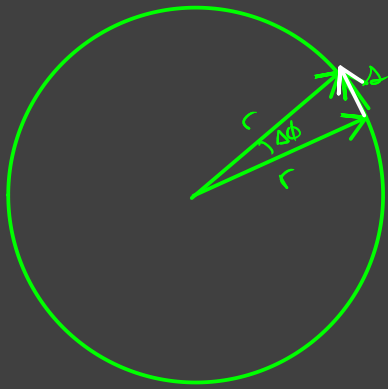$$= \Delta\phi\,\hat{\phi}$$

$$\frac{\Delta\hat{r}}{\Delta t} = \frac{\Delta\phi}{\Delta t}\hat{\phi} \rightsquigarrow \frac{d\hat{r}}{dt} = \dot{\hat{r}} = \frac{d\phi}{dt}\hat{\phi} = \dot{\phi}\hat{\phi}$$

$s = \phi \cdot r$
$V = \omega \cdot r$
$a = \alpha \cdot r$
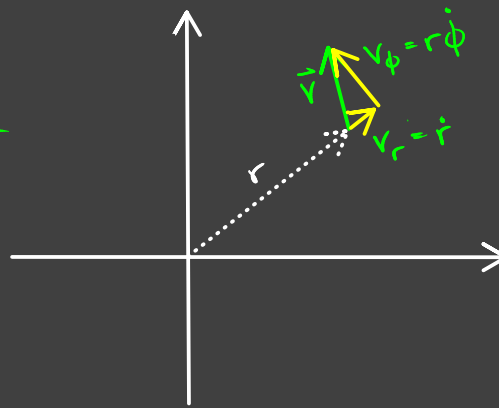
circle
Constant r

$\dot{r} = 0$

$\Delta \phi = \frac{s}{r}$

$\Delta s = \Delta \phi r$

$V_\phi = r\dot{\phi}$

$\vec{r}$

$V_r = \dot{r}$

r

Now for acceleration

$$\vec{a} = \ddot{\vec{r}} = \frac{d}{dt}\left(\dot{\vec{r}}\right) = \frac{d}{dt}\left(\dot{r}\,\hat{r} + r\dot{\phi}\,\hat{\phi}\right)$$

$$= \frac{d}{dt}\left(\dot{r}\,\hat{r}\right) + \frac{d}{dt}\left(r\dot{\phi}\,\hat{\phi}\right)$$

$$= \ddot{r}\,\hat{r} + \dot{r}\,\dot{\hat{r}} + \dot{r}\dot{\phi}\,\hat{\phi} + r\ddot{\phi}\,\hat{\phi} + r\dot{\phi}\,\dot{\hat{\phi}}$$

$\dot{\hat{r}} = \dot{\phi}\,\hat{\phi}$

$\dot{\hat{\phi}} = -\dot{\phi}\,\hat{r}$

$$\vec{a} = (\qquad\qquad)\hat{r} + (\qquad\qquad)\hat{\phi}$$

$\Delta\hat{\phi} = \Delta\phi \cdot 1 \cdot (-\hat{r})$

magnitude

$\frac{\Delta\hat{\phi}}{\Delta t} = \dot{\hat{\phi}} = -\frac{\Delta\phi}{\Delta t}\hat{r}$

$\dot{\hat{\phi}} = -\frac{d\phi}{dt}\hat{r} = -\dot{\phi}\,\hat{r}$