

Chapter 4 \leadsto Energy

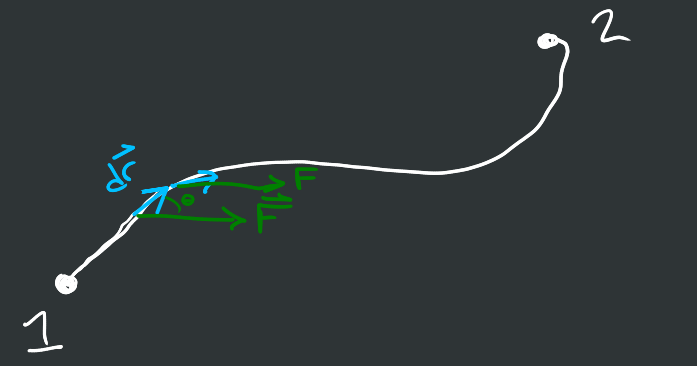
- Kinetic Energy

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v} \cdot \vec{v} = \frac{p^2}{2m}$$

- Work is a transfer of energy to the system

$$dW = \vec{F} \cdot d\vec{r} \Rightarrow W = \int_1^2 \vec{F} \cdot d\vec{r}$$

$|\vec{F}| \cdot |d\vec{r}| \cdot \cos\theta$



path integral means work depends on the path between points, unless it doesn't

- Amount of work done (total) is the same as the change in kinetic energy

$$W = \Delta T$$

← Work-Kinetic energy theorem

- If the work done does not depend on the path taken, (it depends on the end points (positions)), that force is a conservative force

→ work done by a conservative force \Rightarrow potential energy

$$\Delta U = - W_{\text{conservative force}}$$

Conservative forces

- gravitational force
- electric force
- spring force

non conservative force

- friction
- drag
- "applied force"

↓ potential energy is defined by a reference point
(\vec{r} when $U=0$)

• conservation of energy

$$W = \Delta K$$

$$\hookrightarrow W_{nc} = \underbrace{\Delta K + \Delta U}_{\text{mechanical energy}}$$

$$U_g = mgh$$

$$W_{nc} = T_f - T_i + U_f - U_i$$

$$T_i + U_i + W_{nc} = T_f + U_f$$

$$\bullet \text{ if } \Delta U = - \int_i^f \vec{F} \cdot d\vec{r} \Rightarrow$$

$$dU = -W = -\vec{F} \cdot d\vec{r}$$

~~$$\frac{dU}{d\vec{r}} = -\vec{F}$$~~

$$\vec{F} = -\vec{\nabla} U = \frac{\partial U}{\partial x} \hat{x} + \frac{\partial U}{\partial y} \hat{y} + \frac{\partial U}{\partial z} \hat{z}$$

\uparrow gradient \rightarrow cartesian

$$F = -\frac{dU}{dx} \leftarrow \text{one dimension}$$

• What make a F conservative

- F depends only on position
(not v, t, a)

• Work is path-independent

$$\hookrightarrow \vec{\nabla} \times \vec{F} = 0$$

\uparrow curl

$$F_s = -kx$$

$$\rightarrow F_g = mg$$

$$F_G = \frac{G m_1 m_2}{r^2}$$

$$F_0 = b \cdot v$$

$$= c v^2$$

• everything above can be done in spherical coords,
or cylindrical coords \rightarrow often this will be easier

central forces

$$\vec{F}(\vec{r}) = f(r) \hat{r}$$

\hat{r} radial force

$$\vec{F}_G(\vec{r}) = -\frac{Gm_1 m_2}{r^2} \hat{r} \quad \leftarrow \text{in spherical coords.}$$

$$\vec{r} = x\hat{x} + y\hat{y} \quad \hat{r} = \frac{\vec{r}}{r}$$

$$r = \sqrt{x^2 + y^2}$$

$$= -\left(\frac{Gm_1 m_2}{x^2 + y^2}\right) \cdot \frac{x\hat{x} + y\hat{y}}{\sqrt{x^2 + y^2}}$$

$$= -\frac{Gm_1 m_2}{(x^2 + y^2)^{3/2}} \cdot (x\hat{x} + y\hat{y}) \quad \leftarrow$$

$$\vec{\nabla} \times \vec{F}_G =$$

\curvearrowright what is curl in spherical?

- All of this applies multiple particle

$$T = T_1 + T_2 + T_3 + \dots$$

$$T = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} \underset{\substack{\uparrow \\ \text{rot. inertia}}}{I} \omega_{cm}^2$$

Recall: $T = \frac{1}{2} m v^2 = \frac{1}{2} m \vec{v} \cdot \vec{v}$

$$\frac{dT}{dt} = \frac{1}{2} m \cdot \frac{d(\vec{v} \cdot \vec{v})}{dt}$$

$$= \frac{1}{2} m \left(\frac{d(\vec{v})}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d(\vec{v})}{dt} \right)$$

$$\Delta T = W$$

$$= \frac{1}{2} m (2 \dot{\vec{v}} \cdot \vec{v})$$

$$= m \dot{\vec{v}} \cdot \vec{v}$$

$$\vec{p} = \vec{F}$$

$$\frac{dT}{dt} = \vec{F} \cdot \vec{v}$$

$$\frac{dT}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

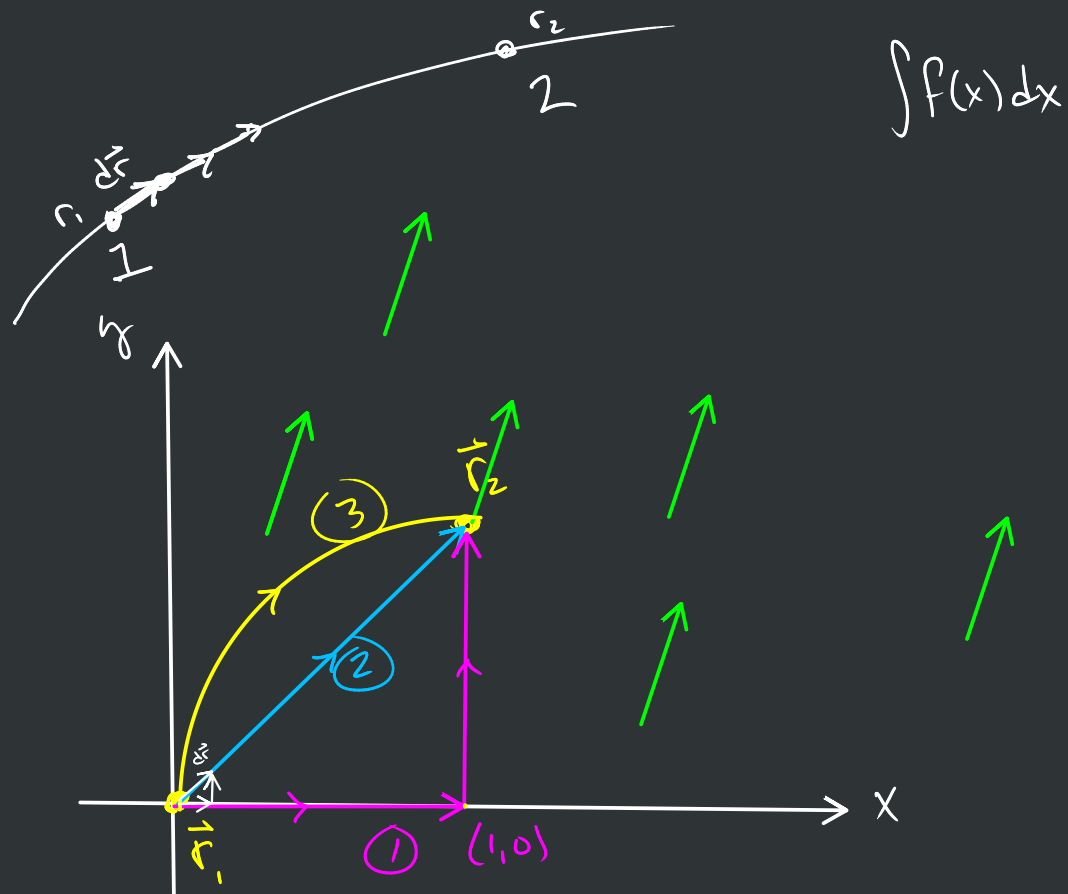
$$\rightarrow dT = \vec{F} \cdot d\vec{r}$$

define this
as a small amount
of work

dW does not make
sense.
 $\rightarrow dW$ (in thermo)

$$\Delta T = T_2 - T_1 = \underbrace{\int_1^2 \vec{F} \cdot d\vec{r}}_{\text{line integral}}$$

Ex: $\vec{F} = 1\hat{x} + 2\hat{y}$
 $\vec{r}_1 = 0$
 $\vec{r}_2 = 1\hat{x} + 1\hat{y}$



Path ①

$$\int_1^2 \vec{F} \cdot d\vec{r} = \underbrace{\int_{(0,0)}^{(1,0)} \vec{F} \cdot d\vec{r}_1}_{\text{Path 1}} + \underbrace{\int_{(1,0)}^{(1,1)} \vec{F} \cdot d\vec{r}_2}_{\text{Path 2}}$$

$$\begin{aligned} d\vec{r}_1 &= d\vec{x} = dx \hat{x} \\ \vec{F} \cdot d\vec{x} &= (1\hat{x} + 2\hat{y}) \cdot (dx \hat{x} + 0\hat{y}) \\ &= 1 dx \end{aligned}$$

$$\begin{aligned} d\vec{r}_2 &= dy \hat{y} \\ \vec{F} \cdot dy \hat{y} &= (1\hat{x} + 2\hat{y}) \cdot (0\hat{x} + dy \hat{y}) \\ &= 2 dy \end{aligned}$$

$$\int_1^2 \vec{F} \cdot d\vec{r} = \int_{x=0}^{x=1} dx + \int_{y=0}^{y=1} 2 dy$$

$$= x|_0^1 + 2y|_0^1$$

$$= 1 + 2 = 3$$

Work along path ①

Path ②

$$\int_1^2 \vec{F} \cdot d\vec{r} =$$

$$d\vec{r} = dx \hat{x} + dy \hat{y}$$

$$\vec{F} \cdot d\vec{r} = (1\hat{x} + 2\hat{y}) \cdot (dx\hat{x} + dy\hat{y})$$

$$= dx + 2dy$$

$$\int_1^2 (dx + 2dy) = \int_0^1 dx + \int_0^1 2dy = 3$$

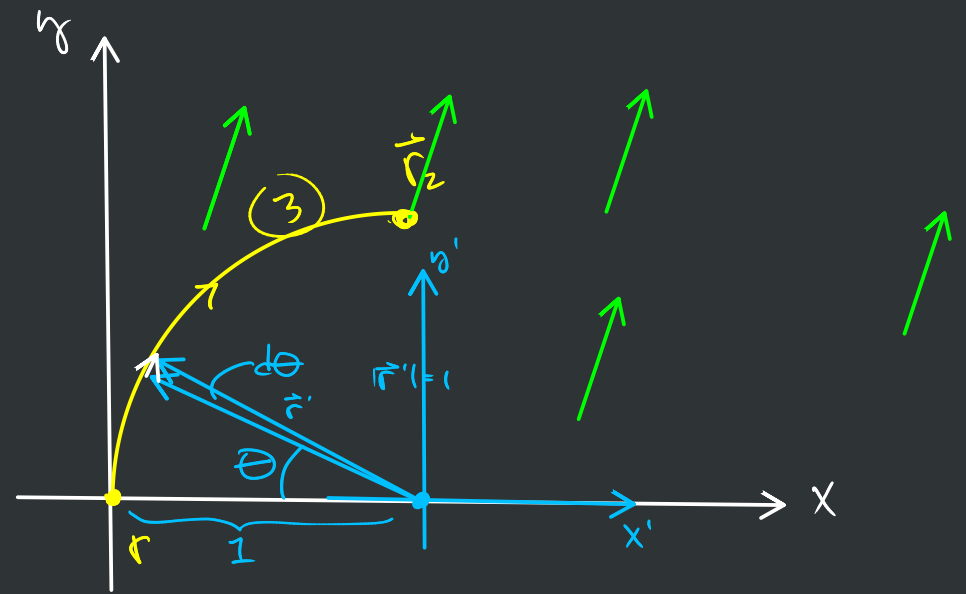
Path (3)

$$\int_1^2 \underline{\vec{F}} \cdot d\underline{\vec{r}}$$

$$\int_1^2 (1\hat{x} + 2\hat{y}) \cdot (\sin\theta d\theta\hat{x} + \cos\theta d\theta\hat{y})$$

$$\int_1^2 (\sin\theta d\theta + 2\cos\theta d\theta)$$

$$\int_0^{\pi/2} \sin\theta d\theta + \int_0^{\pi/2} 2\cos\theta d\theta = 3$$



$$\vec{r}' = -\cos\theta \hat{x}' + \sin\theta \hat{y}'$$

↓ change coordinate systems

$$\vec{r} = (1 - \cos\theta)\hat{x} + \sin\theta\hat{y}$$

$$d\vec{r} = \sin\theta d\theta \hat{x} + \cos\theta d\theta \hat{y}$$

Homework: Same 3 paths

$$\vec{F} = y\hat{x} + 2x\hat{y}$$

