Chapter 3 - Momentum & Angular Momentum

Recall: $\vec{p} = m\vec{v}$ $\quad \Sigma \vec{F}_n = \dot{\vec{p}}$

$$\mathbb{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \cdots = \Sigma \vec{p}_n$$

$$\dot{\vec{\mathbb{P}}} = \vec{F}_{ext}$$

If $\vec{F}_{ext} = 0$

$$\dot{\vec{\mathbb{P}}} = 0$$

$$\mathbb{P}_i = \mathbb{P}_f$$ Conservation of Momentum

If two objects,

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

really 2 equations

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

If collision is perfectly inelastic

↳ stick together

then,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_f + m_2\vec{v}_f$$

$$= (m_1 + m_2)\vec{v}_f$$

mass add up!

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}$$ ✓ ← two equations!

Rockets + Explosions

At the beginning

$$P = 0$$

Round 1

$$P_i = 0 = P_f = p_{ME} + p_{ball}$$

$$0 = \underbrace{(75\,kg + 99(0.5\,kg))}_{124.5\,kg} V + (0.5\,kg)(-15\,m/s)$$

$$0 = 124.5\,V - 7.5\,kg\,m/s$$

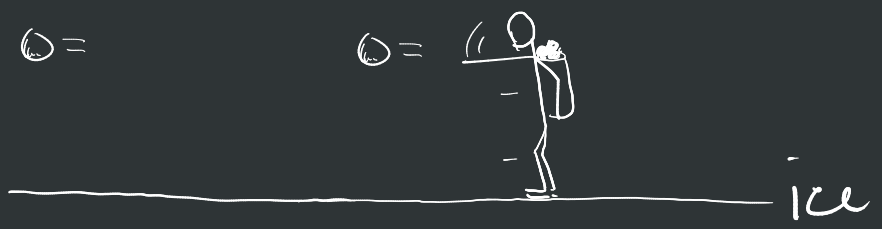
$$V = \frac{7.5\,kg\,m/s}{124.5\,kg} = 0.0602\,m/s$$

Round 2

$$V' = V_{ice} - V_s'$$

$$-15\,m/s = V_{ice} - 0.0602\,m/s$$

$$-15 + 0.06 = V_{ice} = -14.94\,m/s$$



$$P_i = P_f$$

$$\underbrace{(124.5)(0.0602\,m/s) + 0.5(-15)}_{=0} = 124 \cdot V + 0.5(-15) + 0.5(-14.94)$$

$$-0.5(-15 - 14.94) = 124 \cdot V$$

$$V = \frac{-0.5(-15-14.94)}{124} \overset{-15+0.0602}{=} \frac{-0.5(-15-15+0.0602)}{124}$$

$$= 0.1207\,m/s$$

but we need a pattern to do this numerically – this way is not giving that away

$(m+\Delta m)(v+\Delta v) = mv + \Delta m v + m\Delta v + \Delta m \Delta v$

$\Delta P_{me} = \Delta P_{ball}$

$124.5(0.0602) - 124v = 0.5(-14.94)$

$m\Delta v \quad mv \quad \Delta mv$

$0.5(15) + 0.5 \cdot 0.0602$

$\frac{0.5(14.94)}{124}$ $\quad$ $\frac{0.5(15)}{124.5}$ $\quad$ IGNORE

0.06024910 $\quad$ 0.06024096

THESE

$\frac{0.5}{124}\left(15 - \frac{0.5(15)}{124.5}\right)$ $\quad$ $\frac{0.5}{124}(15-x) - x = ?$

NOTES, REMINGTON AT WORK

$$V = \frac{-0.5(-15-14.94)}{124} = \frac{0.5(15+14.94)}{124}$$

$$= \underbrace{\frac{0.5(15)}{124}}_{0.06048} + \underbrace{\frac{0.5(14.94)}{124}}_{0.0602}$$

$\frac{0.5(14.94)}{124} \overset{?}{=} \frac{0.5(15)}{124.5}$

$0.0602 = 0.0602$

Round 1 Redux – my frame

$$P_i = 0 = P_f = p_{ME} + p_{ball}$$

$$0 = \underbrace{(75\,kg + 99(0.5kg))}_{124.5kg} V + (0.5kg)(-15\,m/s)$$

$$0 = 124.5\,V - 7.5\,kg\,m/s$$

$$V = \frac{7.5\,kg\,m/s}{124.5\,kg} = 0.0602\,m/s$$

Round 2 Redux – my frame

P_i'

$$0 = 124.0 \cdot V + 0.5(-15)$$

$$V = \frac{0.5(15)}{124} = 0.06048\,m/s$$

$$\frac{0.5(15 - 0.602)}{124} =$$

$$\frac{0.15}{124}\left(15 - \frac{0.5(15)}{124.5}\right) = \frac{0.5(15)}{124.5}$$

$$\frac{1}{124} - \frac{0.5}{124.5} = \frac{1}{124.5}$$

Round 3 Redux – my frame

$$\vec{P}_i'' \quad 0 = 123.5\,V + 0.5(-15)$$

$$V = \frac{0.5(15)}{123.5} = 0.0607\ m/s \qquad 0.18$$

Generalize

$$dV = \frac{dP_{ex}}{m} \leftarrow \text{variable}$$

$$\int_{V_i}^{V_f} dV = \int_{m_0+m_f}^{m_0} \frac{V_{ex}\,dm}{m} = V_{ex} \int_{m_0+m_f}^{m_0} \frac{dm}{m}$$

$$V\Big|_{V_i}^{V_f} = V_{ex} \ln(m)\Big|_{m_0+m_f}^{m_0}$$

$$V_f - V_i = V_{ex}\left[\ln(m_0) - \ln(m_0+m_f)\right]$$

$$V_f = V_{ex} \ln\left(\frac{m_0}{m_0+m_f}\right) + V_i$$

↳ $$V(t) = V_{ex} \ln\left(\frac{m(t)}{m_0+m_f}\right) + V_i$$

test w beachball

$$V_f = -15 \ln\left(\frac{75kg}{125kg}\right)$$

$$= +7.66\ m/s$$

$$\rightarrow V(t) = V_{ex} \ln\left(\frac{m(t)}{m_0 + m_f}\right) + V_0$$

How many baseballs to stop from 10 m/s

$$0 = +15 \ln\left(\frac{m_0}{m_0 + 0.5n}\right) + 10$$

$$-10 = +15 \ln\left(\frac{m_0}{m_0 + 0.5n}\right)$$

$$\frac{-10}{+15} = \ln\left(\frac{m_0}{m_0 + 0.5n}\right)$$

$$e^{\frac{-10}{15}} = \frac{m_0}{m_0 + 0.5n}$$

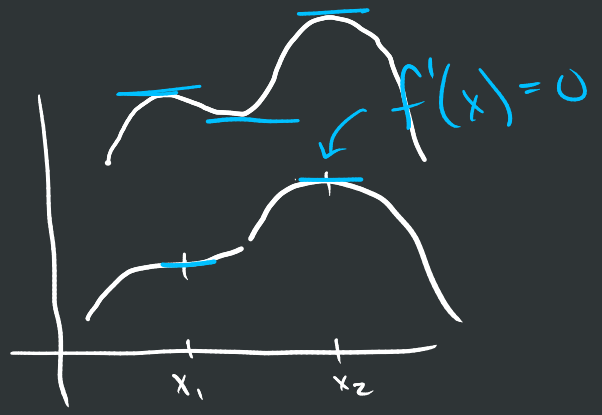
$$e^{\frac{-10}{15}}(m_0 + 0.5n) = m_0$$

$$m_0 + 0.5n = m_0 e^{10/15}$$

$$n = \frac{m_0(e^{10/15} - 1)}{0.5}$$

$m_0 = 75\,kg$

$n = 142.16$ ← # of baseballs



Center of Mass

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$m_1 + m_2 + \cdots$ → total, M

$$\vec{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha$$

$\vec{R}$ → center of mass is a position

for continuous medium

$m_\alpha \to dm$

$$\vec{R} = \frac{1}{M}\int \vec{r}\, dm$$

volume

$$\vec{R} = \frac{1}{M}\int \vec{r}\, \rho\, dV$$

$$\rho = \frac{M}{V}$$

$$X = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha x_\alpha$$

X → x-component (x position)

$$Y = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha y_\alpha$$

$$Z = \vdots$$

$$\dot{\vec{R}} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \dot{\vec{r}}_\alpha$$

$$\vec{P} = M\dot{\vec{R}}$$

$$\vec{P} = M\left(\frac{1}{M}\sum_\alpha m_\alpha \dot{\vec{r}}_\alpha\right)$$

$$= \sum_\alpha m_\alpha \dot{\vec{r}}_\alpha$$

$$\vec{P} = \sum_\alpha \vec{p}_\alpha \leftarrow \text{already knew from Ch. 1.}$$

$$\dot{\vec{P}} = M\ddot{\vec{R}} = \vec{F}^{ext} \leftarrow \text{can treat a collection as a single particle at the center of mass}$$