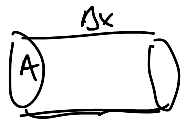
problems

Sunday, September 26, 2021 8:37 PM

4, 5, 10

4)



$\rightarrow \dfrac{m}{\Delta t}$

$V = A \cdot \Delta x$

$= A v \Delta t$

$m = \rho V$

$m = \rho A v \Delta t$

$\dfrac{m}{\Delta t} = \rho A v$ ✓

$dm v = F \Delta t$

$\dfrac{dm}{dt} \cdot v = F$

$\rho A v \cdot v = F$

$\rho A v^2 = F$ ✓

$$F = K \rho A v^2$$

$$K = 1/4$$

$$(\text{Eq } 2.4) \rightarrow C = \gamma D^2$$

$$F = \frac{1}{4} \rho \pi R^2 v^2$$

$$= \frac{1}{4} \rho \pi \left(\frac{D}{2}\right)^2 v^2$$

$$= \underbrace{\frac{1}{16} \rho \pi}_{\gamma} D^2$$

$$F = \underbrace{\gamma D^2}_{C} v^2 \quad ✓$$

$\rho = 1.29\ kg/m^3$ so $\frac{1}{16}(\pi)(1.29\ kg/m^3)$

$= 0.25$

compare to book value

$\gamma = 0.25\ Ns^2/m^4$

5] from notes

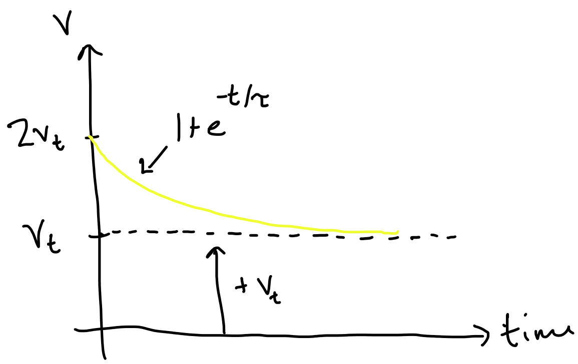
$$v_y = \underbrace{(v_{y0} - v_t)e^{-t/\tau}}_{\substack{\text{bigger than} \\ v_t \text{ but decrease} \\ \text{to zero over time}}} + v_t$$

if $v_{y0} = 2v_t$

$$v_y = (2v_t - v_t)e^{-t/\tau} + v_t$$

$$= v_t e^{-t/\tau} + v_t$$

$$= v_t\left(1 + e^{-t/\tau}\right)$$



10] $D = 2mm$ $\rho_{ball} = 7.8\, gm/cm^3$

$\rho_{fl} = 1.3\, g/cm^3$

$\eta_{fl} = 12\, Ns/m^2$

(a) $F_{lin} = 3\pi\eta D v$

$\tau = \frac{m}{b}$ $v_{ter} = \frac{mg}{b} = \tau g$

$$b = 3\pi\eta D$$

$$= 3\pi \cdot 12\frac{Ns}{m^2} \cdot \overset{2\cdot10^{-3}}{(0.002\text{ m})}$$

$$= 226.2 \cdot 10^{-3}$$

$$\underline{b = 0.226\ Ns/m}$$

$$\mathbb{V}_{ball} = \frac{4}{3}\pi\frac{D^3}{8} = \frac{\pi D^3}{6} = \frac{\pi\cdot(0.2\,cm)^3}{6}$$

$$= \frac{\pi\cdot 8\cdot 10^{-3}}{6}$$

$$= 4.19\cdot 10^{-3}\,cm^3$$

$$m = \rho\,\mathbb{V}$$

$$= 7.8\frac{g}{cm^3}\cdot 4.19\cdot 10^{-3}\,cm^3 = 3.267\cdot 10^{-2}\,g$$

$$\underline{m = 3.267\cdot 10^{-5}\,kg}$$

$$\tau = \frac{m}{b} = \frac{3.267\cdot 10^{-5}\,\cancel{kg}}{0.226\,\frac{\cancel{kg}\,\cancel{m}}{\cancel{s^2}}\cdot\frac{\cancel{s}}{\cancel{m^2}}\cdot\cancel{m}} = \frac{\rho_{ball}\,\pi D^3}{6\cdot 3\pi\eta D} = \frac{\rho_{ball}\,D^2}{18\pi\,\eta}$$

$$\boxed{\tau = 1.44\cdot 10^{-4}\,s}$$

$\underline{V_{ter}}$

buoyant force

$$0 = mg - bV_{ter} - F_b$$

terminal velocity

equal to the weight of fluid displaced

$$\rho_{fl}\,\mathbb{V}_{ball}\cdot g$$

$$0 = mg - bV_{ter} - \rho_{fl}\,\mathbb{V}\cdot g$$

$$V_{ter} = \frac{m_{ball}\,g - \rho_{fl}\,\mathbb{V}\cdot g}{b}$$

$$= \frac{(\rho_{ball} - \rho_{fl})\,\mathbb{V}\cdot g}{b}$$

$$= \frac{\left(7.8\,\cancel{g}/\cancel{cm^3} - 1.3\,\cancel{g}/\cancel{cm^3}\right)\cdot 4.19\cdot 10^{-3}\,\cancel{cm^3}\cdot 10^{-3}\,\cancel{kg}/\cancel{g}\cdot 9.8\,\cancel{N}/\cancel{kg}}{0.266\,\cancel{N}s/m}$$

$$\boxed{V_{ter} = 0.001\,m/s = 1\,mm/s}$$

How long to reach 95% of terminal velocity?

$$m\dot{v} = mg - bv - \rho_{fl}\,\mathbb{V}g$$

$$m\dot{v} = -b\left(-\frac{mg}{b} + v + \frac{\rho_{fl} V g}{b}\right)$$

$$= -b\left(v - \underbrace{\left(\frac{mg}{b} - \frac{\rho_{fl} V g}{b}\right)}_{v_{ter}}\right)$$

$$\dot{v} = -\frac{b}{m}\left(v - v_{ter}\right)$$

same as notes

$$v = v_{y0} e^{-t/\tau} + v_{ter}\left(1 - e^{-t/\tau}\right)$$

"dropped" $\longrightarrow v_{y0} = 0$

$$v = v_{ter}\left(1 - e^{-t/\tau}\right)$$

$$\underbrace{\frac{v}{v_{ter}}}_{95\%} = 1 - e^{-t/\tau}$$

$$0.95 = 1 - e^{-t/\tau} \quad \leftarrow \text{Solve for } t$$

$$e^{-t/\tau} = 0.05$$

$$-\frac{t}{\tau} = \ln(0.05) = -2.995 = -3$$

$$t = 3\tau = 3 \cdot \left(1.44 \cdot 10^{-4}\,s\right)$$

$$= 4.32 \cdot 10^{-4}\,s$$

(b) $$\frac{F_{quad}}{F_{lin}} = \frac{K \rho_{fl} A v^2}{3\pi \eta D v} = \frac{\rho_{fl} \cdot \frac{\cancel{\pi} D^{\cancel{2}}}{4} \cdot v_{ter}^{\cancel{2}}}{4 \cdot 3\cancel{\pi} \eta \cancel{D} \cancel{v_{ter}}}$$

1/4

$$= \frac{\rho_{fl} D v_{ter}}{48 \cdot \eta} = \frac{R}{48}$$

$$= \frac{\overset{\rho_{fl}}{1.3 \, g/cm^3} \cdot \overset{kg/g}{10^{-3} \frac{kg}{g}} \cdot \overset{cm^3/m^3}{(10^2)^3 \frac{cm^3}{m^3}} \cdot \overset{D}{(0.002\,m)} \cdot \overset{v_{ter}}{(0.001\,m/s)}}{48 \cdot 12 Ns/m^2}$$

$$= \frac{1.3 \cdot 10^{-3} \cdot 10^5 \cdot 2 \cdot 10^{-3} \cdot 1 \cdot 10^{-3}}{4.8 \cdot 1.2 \cdot 10^2}$$

$$= \frac{1.3 \cdot 2}{4.8 \cdot 1.2} \cdot 10^{-6}$$

$$= 0.45 \cdot 10^{-6}$$

$$\boxed{\frac{F_{quad}}{F_{lin}} = 4.5 \cdot 10^{-7}}$$

Also since I was curious
we have from above

$$v_{ter} = \frac{(\rho_{ball} - \rho_{fl}) V \cdot g}{6}$$

So using
that to simplify

$$\frac{F_{quad}}{F_{lin}} = \frac{R}{48} = \frac{\rho_{fl} \cdot \cancel{D} \cdot (\rho_{ball} - \rho_{fl}) \overset{\frac{\cancel{\pi} D^3}{6}}{V} \cdot g}{48 \cdot \eta \; 3\cancel{\pi} \eta \cancel{D}} = \frac{\rho_{fl}(\rho_{ball} - \rho_{fl}) D^3 \cdot g}{864 \cdot \eta^2}$$

$$\text{so} = \frac{1.3\,g/cm^3 (7.8\,g/cm^3 - 1.3\,g/cm^3) \cdot (0.2\,cm)^3 \cdot 9.8 \frac{m}{s^2} \cdot 10^2 \, cm/m \cdot (10^2\,cm)^2/m^2 \, (10^{-3}\,kg/g)^2}{864 \cdot (12\,Ns/m^2)^2}$$

Doesn't seem right