ice frame

1st throw → $0 = 0.5(-15) + 124.5\,V_1$

$$V_1 = \frac{0.5(15)}{124.5}$$

2nd throw →

$$0.5(-15) + 124.5\left(\frac{0.5(15)}{124.5}\right) = 0 = 0.5(-15) + 0.5\left(-15 + \frac{0.5(15)}{124.5}\right) + 124\,V_2$$

$0.5(-15) + 124.5\left(\frac{0.5(15)}{124.5}\right) = 0.5(-15) + 0.5\left(-15 + \frac{0.5(15)}{124.5}\right) + 124\,V_2$

$$\frac{124.5\left(\frac{0.5(15)}{124.5}\right) - 0.5\left(-15 + \left(\frac{0.5(15)}{124.5}\right)\right)}{124} = V_2$$

$$\frac{0.5(15) + 0.5(15) - \frac{0.5\cdot 0.5\,(15)}{124.5}}{124} = V_2$$

$$\underbrace{\frac{0.5(15)}{124}}_{\Delta V_1 \text{ from other method}} + \underbrace{\frac{0.5(15)}{124} - \frac{0.5\cdot 0.5\,(15)}{124\cdot 124.5}}_{\text{must be equal to } \frac{0.5(15)}{124.5}} = V_2$$

$$\frac{0.5(15)}{124} + \frac{0.5(15)}{124} - \frac{0.5\cdot 0.5\,(15)}{124.5\cdot 124} = V_2$$

exactly equal to that!

but how?

$$\frac{+0.5(15)}{124} - \frac{0.5\cdot 0.5\,(15)}{124\cdot 124.5}$$

$$\frac{0.5(15)}{124}\left(1 - \frac{0.5}{124.5}\right)$$

$$\frac{0.5(15)}{124}\left(\frac{124.5 - 0.5}{124.5}\right)$$

$$\frac{0.5(15)}{124.5}\ V_1!$$

$$\underbrace{\frac{0.5(15)}{124}}_{\Delta V_2} + \underbrace{\frac{0.5(15)}{124.5}}_{\Delta V_1} = V_2$$

So V_3

$$V_3 = \underbrace{\frac{0.5(15)}{123.5}}_{\Delta V_3} + \frac{0.5(15)}{124} + \frac{0.5(15)}{124.5}$$

$$\Delta V_4 = \frac{0.5(15)}{123}$$ and so on

my frame

1st throw → $p_i = p_f$

$$0 = 0.5(-15) + 124.5\,\Delta V_1$$

$$\Delta V_1 = \frac{0.5(-15)}{124.5} \rightarrow V_1 = \frac{0.5(-15)}{124.5}$$

2nd throw → reset reference frame

new initial momentum

$$0 = 0.5(-15) + 124\,\Delta V_2$$

$$\Delta V_2 = \frac{0.5(15)}{124}$$

$$V_2 = \frac{0.5(15)}{124} + V_1$$

$$V_2 = \frac{0.5(15)}{124} + \frac{0.5(15)}{124.5}$$

$$= 0.5(15)\left(\underbrace{\frac{1}{124} + \frac{1}{124.5}}\right)$$

$$\frac{124.5 + 124}{(124)\,124.5}$$

$$\frac{\frac{124.5 + 124}{124.5}}{\cdot 124} \qquad \frac{\frac{124.5 + 124}{124}}{124.5}$$

$$\frac{1 + \frac{124}{124.5}}{124} \qquad \frac{\frac{124.5}{124} + 1}{124.5}$$