Chapter 2 - air resistance

$$\sum \vec{F} = \vec{F}_{NET} = m\ddot{\vec{r}}$$

$$\vec{F}_D = -f(v)\hat{v}$$

$$f(v) = a + bv + cv^2$$

a: 0

bv: linear drag

cv^2: quadratic drag

$$f_{lin} = bv \qquad b \propto D \qquad b = \beta D$$

β: related to viscocity

$$f_{quad} = cv^2 \qquad c \propto D^2 \qquad c = \gamma D^2$$

γ: density

$$\frac{f_{quad}}{f_{lin}} \propto \frac{D^2 v^2}{D v} = D v$$

density

$$\frac{f_{quad}}{f_{lin}} \approx \text{Reynolds number} = \frac{\rho D v}{\eta}$$

viscosity

$$\vec{F}_{NET} = m\ddot{\vec{r}}$$

$$m\vec{g} - b\vec{v} = m\ddot{\vec{r}}$$

$$\underbrace{m\vec{g} - b\vec{v} = m\dot{\vec{v}}}_{\text{really two equations}}$$

$$-b v_x = m\dot{v}_x \quad \Big| \quad mg - b v_y = m\dot{v}_y$$

down $\oplus$ y-dir

$$\dot{v}_x = -\frac{b}{m} \cdot v_x$$

$$\boxed{\frac{dv_x}{dt} = -\frac{b}{m} \cdot v_x}$$

$$\frac{dv_x}{v_x} = -\frac{b}{m} dt$$

$$\int \frac{dv_x}{v_x} = -\frac{b}{m}\int dt$$

$$e^{\ln v_x} = e^{\left(-\frac{b}{m}t + C\right)} = e^{-\frac{b}{m}t} \cdot e^{C}$$

$$v_x = A \cdot e^{-\frac{b}{m}t}$$

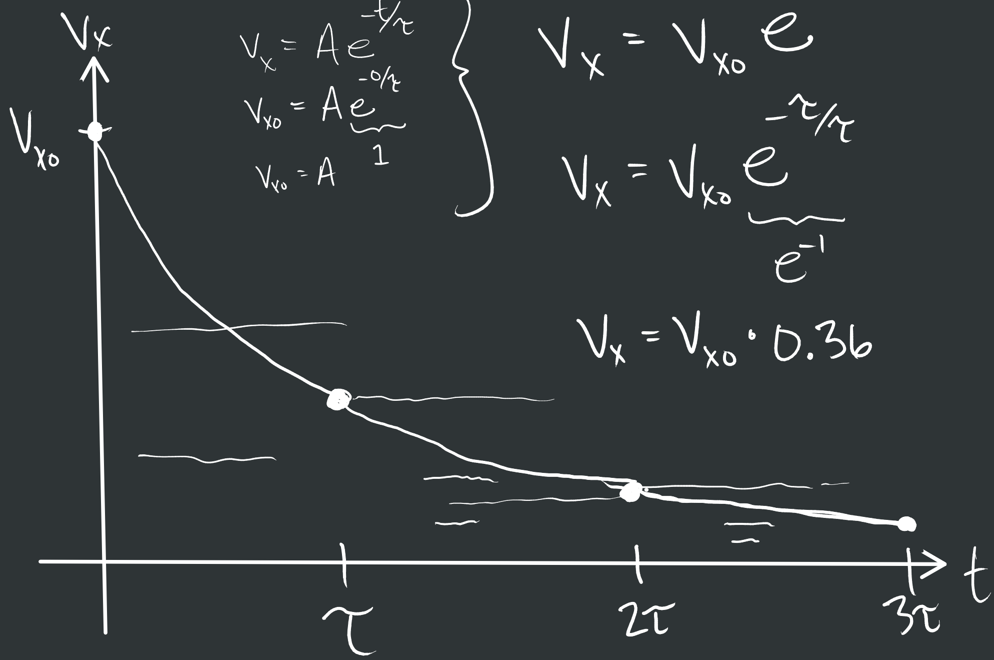
$$\frac{b}{m} = \left[\frac{1}{s}\right] = \left[s^{-1}\right]$$

$$\tau = \frac{m}{b} = [s] \rightarrow \text{time constant}$$

$$v_x = A e^{-t/\tau}$$

exponential decay

$$\dot{x} = v_{x0} e^{-t/\tau}$$



$$v_x = A e^{-t/\tau}$$

$$v_{x0} = A \underbrace{e^{-0/\tau}}_{1}$$

$$v_{x0} = A$$

$$v_x = v_{x0} e^{-t/\tau}$$

$$v_x = v_{x0} \underbrace{e^{-\tau/\tau}}_{e^{-1}}$$

$$v_x = v_{x0} \cdot 0.36$$

$$\frac{dx}{dt} = v_{x0}\, e^{-t/\tau}$$

$$\int dx = v_{x0} \int e^{-t/\tau}\, dt$$

$$\int_{x_0}^{x} dx' = v_{x0} \underbrace{\int_0^{t} e^{-t'/\tau}\, dt'}$$

function

$$x' \Big|_{x_0}^{x} = v_{x0}(-\tau)\, e^{-t'/\tau} \Big|_0^{t}$$

$$(x - x_0) = -v_{x0}\, \tau \cdot \left(e^{-t/\tau} - \cancelto{1}{e^{-0/\tau}} \right)$$

$$x - x_0 = v_{x0}\, \tau \left(1 - e^{-t/\tau} \right)$$

$$\boxed{x(t) = v_{x0}\, \tau \left(1 - e^{-t/\tau} \right) + x_0}$$

Linear Drag Vertically

$$mg - bv_y = m\dot{v}_y$$

if $\dot{v}_y = 0$

then $mg - bv_y = 0$

$$v_y = \frac{mg}{b} \equiv v_t$$

velocity when $\dot{v} = 0$

$$m\dot{v}_y = mg - bv_y$$

$$= -b\left(-\frac{mg}{b} + v_y\right)$$

$$m\dot{v}_y = -b(-v_t + v_y)$$

$$u = (-v_t + v_y)$$

$$\dot{u} = \frac{du}{dt} = 0 + \dot{v}_y$$

$$\dot{u} = \dot{v}_y$$

$$m\dot{u} = -b \cdot u$$

$$\dot{u} = -\frac{b}{m} \cdot u$$

$$\tau = \frac{m}{b}$$

$$u = Ae^{-t/\tau}$$

$$-v_t + v_y = Ae^{-t/\tau}$$

$$V_y = A e^{-t/\tau} + V_t$$

$$V_y(t=0) = V_{y0}$$

$$V_{y0} = A \underbrace{e^{-0/\tau}}_{1} + V_t$$

$$V_{y0} = A + V_t$$

$$A = V_{y0} - V_t$$

$$V_y = (V_{y0} - V_t) e^{-t/\tau} + V_t$$

$$\boxed{V_y = (V_{y0} - V_t) e^{-t/\tau} + V_t}$$

$$V_y = \underbrace{V_{y0} e^{-t/\tau}}_{t \to \infty,\ \to 0} + \underbrace{V_t\left(1 - e^{-t/\tau}\right)}_{t \to \infty}$$

$$\cancel{dt}\,\frac{dy}{\cancel{dt}} = \left((v_{y0} - v_t) e^{-t/\tau} + v_t \right) dt$$

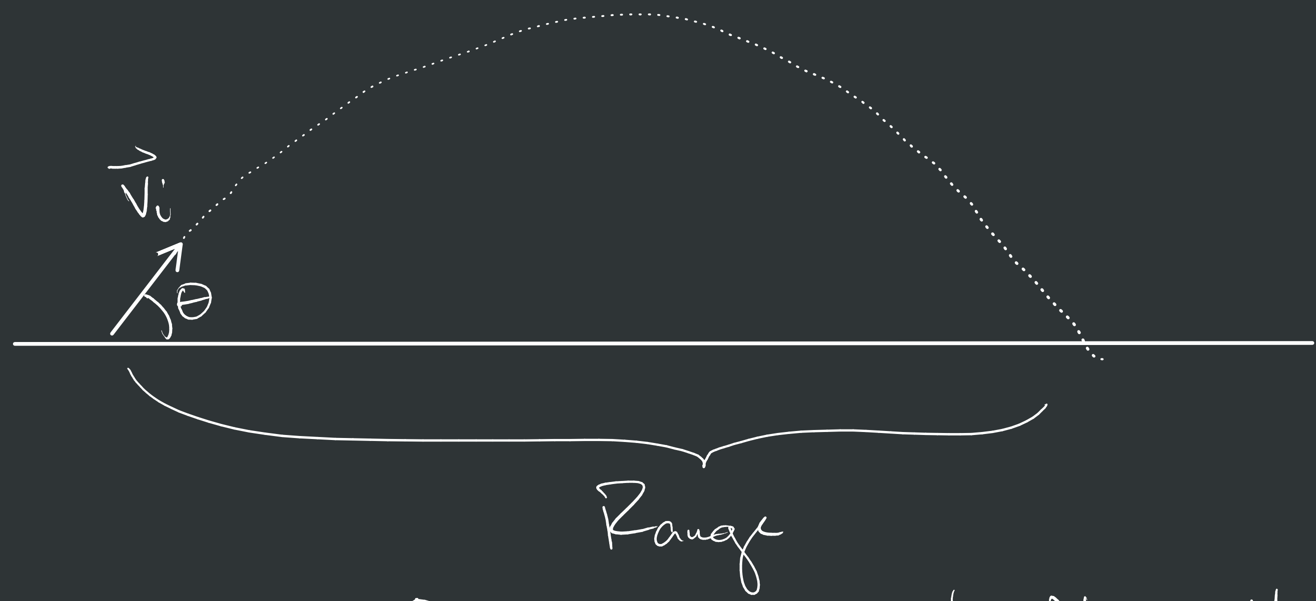
$$\int_{y_0}^{y} dy' = \int_0^t \left((v_{y0} - v_t) e^{-t'/\tau} + v_t \right) dt'$$

$$y' \Big|_{y_0}^{y} = \left[(v_{y0} - v_t)(-\tau) e^{-t'/\tau} + v_t t' \right]_0^t$$

$$y - y_0 = -(v_{y0} - v_t)\tau e^{-t/\tau} + v_t t - \left[-(v_{y0} - v_t)\tau \cdot 1 + 0 \right]$$

algebra

$$\boxed{y = (v_{y0} - v_t)\tau \left(1 - e^{-t/\tau}\right) + v_t t + y_0}$$



$$V_x = v_i \cos\theta$$

$$x = V_x \cdot t = v_i \cos\theta \cdot t$$

$$V_{yo} = V_i \sin\theta$$

$$y = y_i + V_{yo} \cdot t + \frac{1}{2} a_y t^2$$

$$\boxed{\text{Range} \to \text{solve } y=0 \text{ for } t, \text{ plug } t \text{ in for } x}$$

$$0 = V_{yo} t + \frac{1}{2} a_y t^2$$

$$0 = t\left(V_{yo} + \frac{1}{2} a_y t\right)$$

$$t = 0 \quad \text{or} \quad t = -\frac{2V_{yo}}{a_y}$$

$$\text{Range} = V_x \left(\frac{-2V_{yo}}{a_y}\right) \qquad a_y = -g$$

$$R = \frac{2 V_x V_{yo}}{g}$$

$$x = v_{x0}\tau\left(1 - e^{-t/\tau}\right) + \cancel{x_0}^{\,0} \quad \Big| \quad y = (v_{y0} - v_t)\tau\left(1 - e^{-t/\tau}\right) + v_t t + \cancel{y_0}^{\,0}$$

$$x = v_{x0}\tau\left(1 - e^{-t/\tau}\right)$$

solve for t

$$y = (v_{y0} - v_t)\tau\left(1 - e^{-t/\tau}\right) + v_t t$$

$$= (v_{y0} - v_t)\tau\left(1 - e^{-t/\tau}\right) + v_t t$$

⋮

lots of algebra (to eliminate t)

⋮

$$y = \left(\frac{v_{y0} + v_t}{v_{x0}}\right)x + v_t\tau\ln\left(1 - \frac{x}{v_{x0}\tau}\right)$$

$$0 = \left(\frac{V_{y0}+V_t}{V_{x0}}\right) R + V_t \tau \ln\left(1-\frac{R}{V_{x0}\tau}\right)$$

$$-\left(\frac{V_{y0}+V_t}{V_{x0}}\right)\frac{R}{V_t\tau} = \ln\left(1-\frac{R}{V_{x0}\tau}\right)$$

$$e^{-\left(\frac{V_{y0}+V_t}{V_{x0}}\right)\frac{R}{V_t\tau}} = 1-\frac{R}{V_{x0}\tau}$$

$$\boxed{\frac{R}{V_{x0}\tau} = 1 - e^{-\left(\frac{V_{y0}+V_t}{V_{x0}}\right)\frac{R}{V_t\tau}}}$$

$$\cancel{x = e^{x}}$$

$$\left\{\begin{array}{l} \to x = e^{-x} \\ \to x = (1-e^{-x}) \end{array}\right.$$