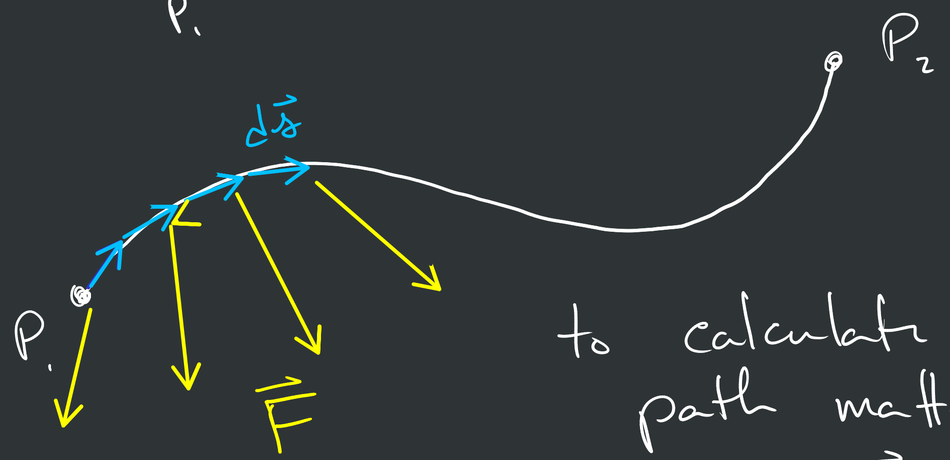
Chapter 2 - Electric Potential

$$W_{ME} = - \int_{P_1}^{P_2} \vec{F} \cdot d\vec{s} \qquad (d\vec{r}, d\vec{\ell})$$



to calculate the work done path matters.

BUT, if $\vec{F}$ is conservative then it is path-independent

$$\vec{F} = q \cdot \vec{E}$$

$$W_{ME} = - \int_{P_1}^{P_2} q\vec{E} \cdot d\vec{s}$$

$$\frac{W_{ME}}{q} = - \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s}$$

$$\frac{U(P_2)-U(P_1)}{q} = \frac{\Delta U}{q} \equiv \Delta\phi \equiv \phi_{21} = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{s}$$

$\Delta K = 0$

electric potential difference

book's notation

$$\frac{[\text{Joules}]}{[\text{Coulomb}]} \quad \frac{U}{q} = \phi \quad [\text{Volts}]$$

sometimes $P_1 \to \infty$, I can set $\phi_1 = 0$.

sometimes P_1 is somewhere else, usually $\phi_1 = 0$ at that place

BUT, it is really only $\Delta\phi$ that matters.

Closed loop

$$\oint \vec{E}\cdot d\vec{s} = 0$$

line integral over a closed loop

What about the potential around a point charge?

→ point charge is located at the origin.

→ $\phi(\infty) = 0$

$$\phi(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}\,'$$

$dr'\,\hat{r}$

$$= -\int_{\infty}^{r} \frac{q_0}{4\pi\epsilon_0 r'^2}\,\hat{r} \cdot d\vec{r}\,'$$

$$= -\frac{q_0}{4\pi\epsilon_0}\int_{\infty}^{r} \frac{1}{r'^2}\,dr'$$

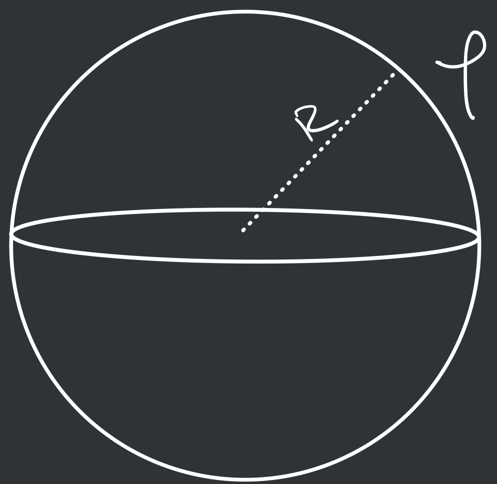
$$\phi(r) = +\frac{q_0}{4\pi\epsilon_0 r}$$

$$U(r) = \frac{q_0 q_1}{4\pi\epsilon_0 r} = q_1\,\phi(r)$$

SEE Example on p. 62

— line integral, means the path matters, so you need to break up the path in some way.

Ex: Find ϕ inside and outside a uniformly charged sphere.



outside $r > R$

$$\phi_{outside} = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r}$$

$$E = \frac{\rho R^3}{3\epsilon_0 r^2} = \boxed{\frac{Q}{4\pi\epsilon_0 r^2}}$$

$$\phi_{outside} = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{r} \frac{1}{r^2}\hat{r}\cdot dr\,\hat{r}$$

$$Q = \rho \cdot V$$

$$Q = \rho \cdot \frac{4}{3}\pi R^3$$

$$= \frac{Q}{4\pi\epsilon_0 r} = \rho\,\frac{4}{3}\pi R^3 \cdot \frac{1}{4\pi\epsilon_0 r}$$

$$\phi_{outside} = \frac{\rho R^3}{3\epsilon_0 r} \quad ✓$$

inside $r < R$

$$E = \frac{\rho r}{3\epsilon_0}$$

$$\phi(r)_{inside} = -\int \vec{E}\cdot d\vec{r}$$

$$= \underbrace{-\int_{R}^{r} \frac{\rho r}{3\epsilon_0}\,dr}_{\text{inside}} \; \underbrace{- \int_{\infty}^{R} \frac{\rho R^3}{3\epsilon_0 r^2}\,dr}_{\text{outside}}$$

$$\frac{\rho R^{\cancel{3}\,2}}{3\epsilon_0 \cancel{R}} = \frac{\rho R^2}{3\epsilon_0}$$

$$= -\int_R^r \frac{\rho r}{3\epsilon_0}\,dr$$

$$= -\frac{\rho}{3\epsilon_0}\int_R^r r\,dr$$

$$= -\frac{\rho}{6\epsilon_0} r^2 \Big|_R^r = -\frac{\rho}{6\epsilon_0}\left(r^2 - R^2\right)$$

$$\phi_{inside}(r) = -\frac{\rho}{6\epsilon_0}\left(r^2 - R^2\right) + \frac{\rho R^2}{3\epsilon_0}$$

$$= -\frac{\rho}{6\epsilon_0} r^2 + \frac{\rho}{6\epsilon_0} R^2 + \frac{2\rho R^2}{2\cdot 3\epsilon_0}$$

$$\phi_{inside}(r) = -\frac{\rho}{6\epsilon_0} r^2 + \frac{\rho R^2}{2\epsilon_0} \checkmark$$

Summing up what we have so far:

- potential energy for point charges

$$U = \frac{1}{2} \sum_{j=1}^{N} \sum_{k \neq j} \frac{1}{4\pi\epsilon_0} \frac{q_j q_k}{r_{jk}}$$