Chapter 6

Magnetic Field

$$\underbrace{\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}}_{\text{The Lorentz Force Law}}$$

$\vec{B} \to$ magnetic field that causes a moving particle to experience a force $\perp$ its velocity

$\to$ arises from the motion of charges (or current)

$\vec{F}_B = q\vec{v} \times \vec{B}$ $\to$ $B \to$ units $\left[\frac{Ns}{m \cdot C}\right] \to$ [Tesla]

$\vec{F}_B = I\vec{l} \times \vec{B}$

thumb ($\vec{F}_B$)

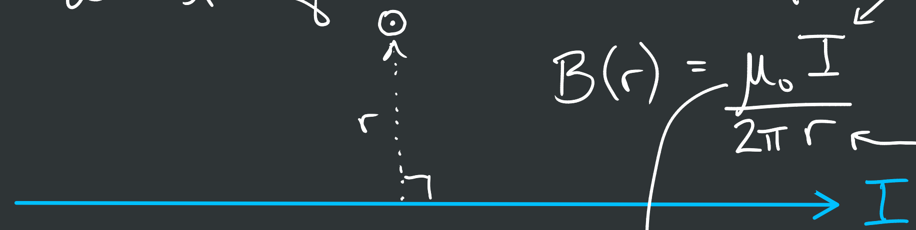
forefinger or hand ($I\vec{l}$)

other fingers ($\vec{B}$)

$\to$ Cause $\to$ motion of other charges

$\to$ a current

Ex: a straight line current



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

current

distance away from the current

$$\mu_0 = 4\pi \cdot 10^{-7}\ \frac{T\cdot m}{A}$$

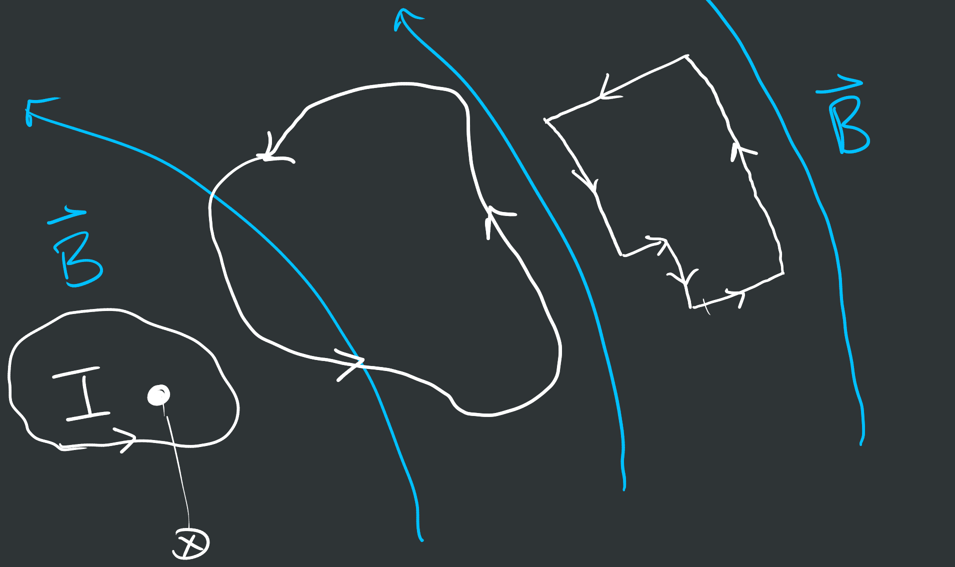
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

something to prove later

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{\ell} \times \vec{r}}{r^3}$$

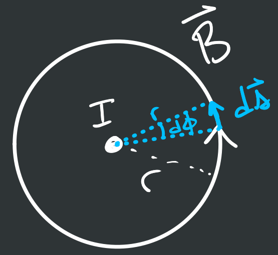
Path integral in magnetic fields



$$\left(\oint \vec{E}\cdot d\vec{s} = \frac{q_{encl}}{\epsilon_0}\right)$$

$$\oint \vec{B}\cdot d\vec{s} = 0 \quad \text{along a closed path}$$

But what if we enclose the current



$$\oint \vec{B}\cdot d\vec{s}$$

$$d\vec{s} = r\,d\phi\,\hat{\phi}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

$$= \oint \frac{\mu_0 I}{2\pi r}\cdot r\,d\phi\,\underbrace{\hat{\phi}\cdot\hat{\phi}}_{1}$$

$$= \oint \frac{\mu_0 I}{2\pi \cancel{r}}\cdot\cancel{r}\,d\phi$$

$$= \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\phi$$

$$= \frac{\mu_0 I}{\cancel{2\pi}}\cdot\cancel{2\pi}$$

$$\boxed{\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}}$$

Ampere's Law (steady current) $\frac{d\rho}{dt} = 0 = \vec{\nabla}\cdot\vec{J}$

assertion: this will apply to any shape of current

: superposition lets us put many straight lines together

$$I_{enclosed} = \int \vec{J} \cdot d\vec{a}$$

→ boundary that ds is making

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

↓ Stokes Theorem

$$\oint \vec{B} \cdot d\vec{s} = \int (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \int \mu_0 \vec{J} \cdot d\vec{a}$$

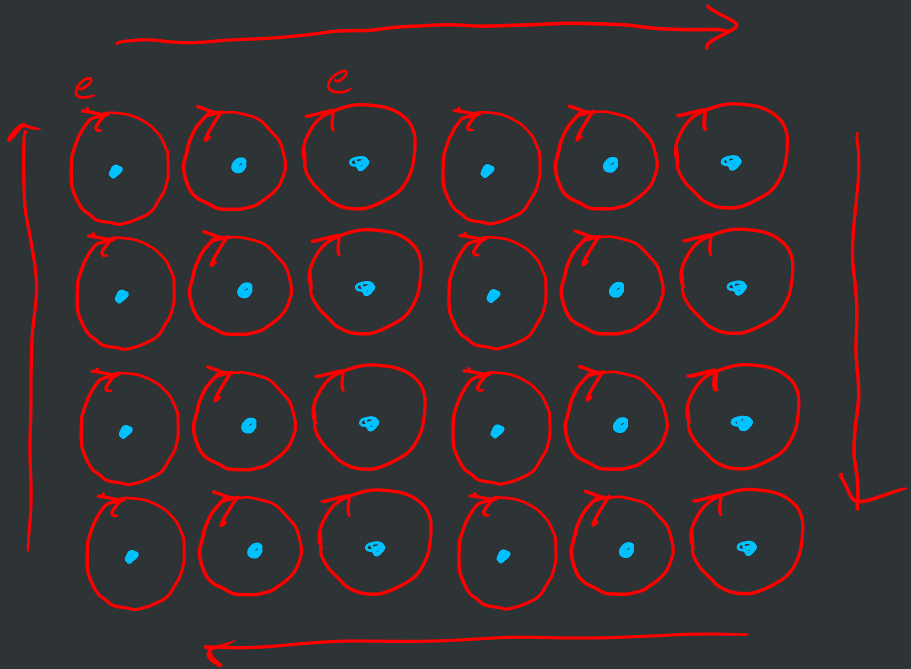
$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

Ampere's Law in Differential Form

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

Helmholtz theorem → any vector field can be uniquely determined by its curl + divergence

$$\vec{\nabla} \times \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$



$$\vec{E} = -\vec{\nabla}\phi(x,y,z)$$

does something like this exist in magnetic field

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

vector potential

We want to figure out an easy way to calculate $\vec{A}$ for some current distribution, $\vec{J}$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0}_{\text{true!}}$$

similar to

$$\vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\times(\vec{\nabla}\phi) = 0$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\nabla^2\vec{A} = (\nabla^2 A_x)\hat{x} + (\nabla^2 A_y)\hat{y} + (\nabla^2 A_z)\hat{z}$$

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2}$$

$$\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\underbrace{\vec{\nabla}\times(\vec{\nabla}\times\vec{A})} = \mu_0\vec{J}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J}$$

$\hookrightarrow$ choose $\vec{\nabla}\cdot\vec{A} = 0$

$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}}$$

$$\nabla^2 A_x = -\mu_0 J_x$$

$$\nabla^2 A_y = -\mu_0 J_y$$

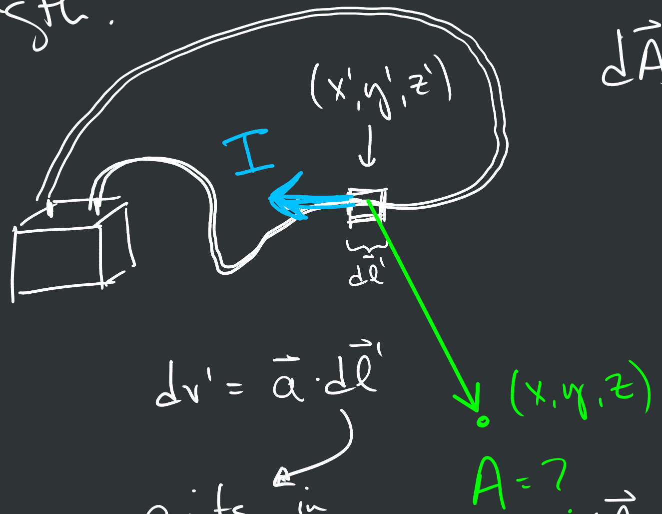
$$\nabla^2 A_z = -\mu_0 J_z$$

$$\boxed{A_x(x,y,z) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(x',y',z')\,dv'}{r}}$$

r → the distance b/t (x,y,z) and (x',y',z')

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(x',y',z')\,dv'}{r} \quad \text{(eq 2.18 in book)}$$

these equations are limited to current distributions that are finite in length.



$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{J}}{r}dv'$$

$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{J}\,dv'}{r}$$

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\vec{J} = \frac{I}{\vec{a}}$$

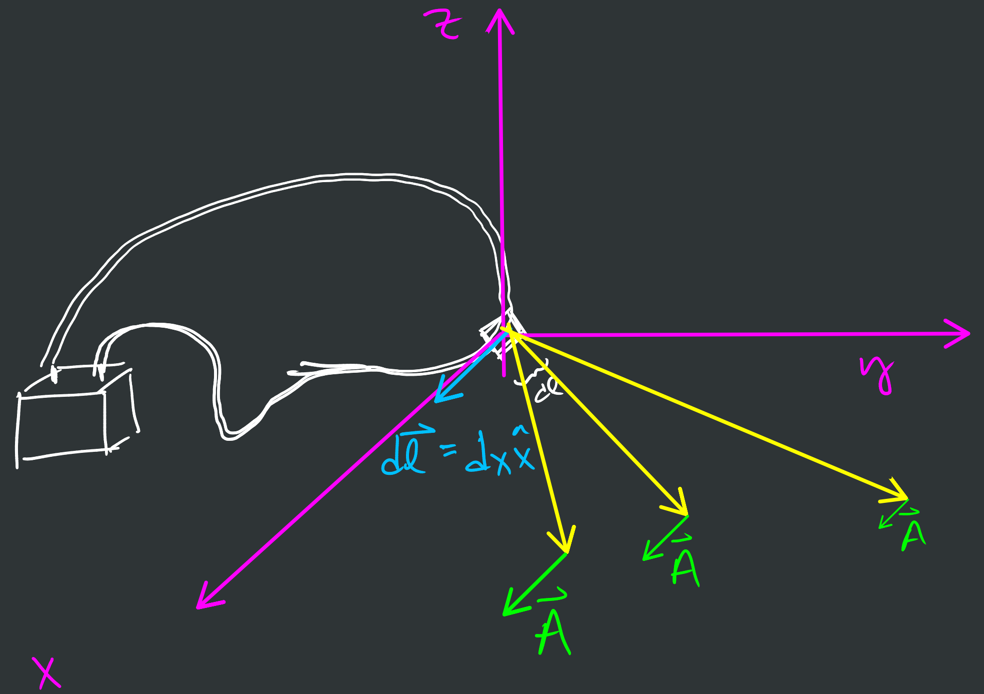
$$dv' = \vec{a}\cdot d\vec{l}'$$

points in the direction of positive current

$$d\vec{A} = \frac{\mu_0}{4\pi}\cdot\frac{1}{r}\cdot\frac{I}{\cancel{\vec{a}}}\cdot\cancel{\vec{a}}\cdot d\vec{l}'$$

$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}'}{r}$$

$$\vec{A}(x,y,z) = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}'}{r}$$



$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{I}{r}\,d\vec{l}'$$

$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{I\,dx\,\hat{x}}{\sqrt{x^2+y^2}} = dA_x$$

$$d\vec{B} = \vec{\nabla}\times d\vec{A} \longleftarrow \left(\vec{B} = \vec{\nabla}\times\vec{A}\right)$$

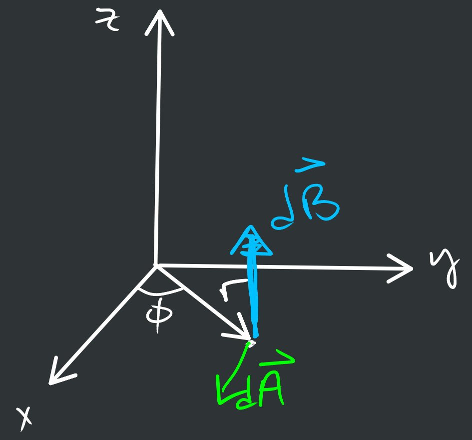
$$d\vec{B} = -\frac{\partial A_x}{\partial y}\,\hat{z}$$

$$= -\frac{\mu_0 I}{4\pi}\,dx\,\frac{\partial}{\partial y}\left(\left(x^2+y^2\right)^{-1/2}\right)\hat{z}$$

$$= +\frac{\mu_0 I}{4\pi}\,dx\left(+\frac{1}{\cancel{2}}\right)\left(x^2+y^2\right)^{-3/2}\cdot\cancel{2}y\,\hat{z}$$

$$= \frac{\mu_0 I}{4\pi}\frac{y\,dx}{\left(x^2+y^2\right)^{3/2}}\,\hat{z} \qquad dx \to dl$$

$$x^2+y^2 = r^2 \Rightarrow \left(x^2+y^2\right)^{3/2} = r^3$$



$$= \frac{\mu_0 I}{4\pi} \frac{y\, dl}{r \cdot r^2} \hat{z}$$

$\sin\phi$

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\sin\phi\, dl}{r^2} \hat{z}$$

works according to the right hand rule

$$|A \times B| = |A||B| \sin\theta$$

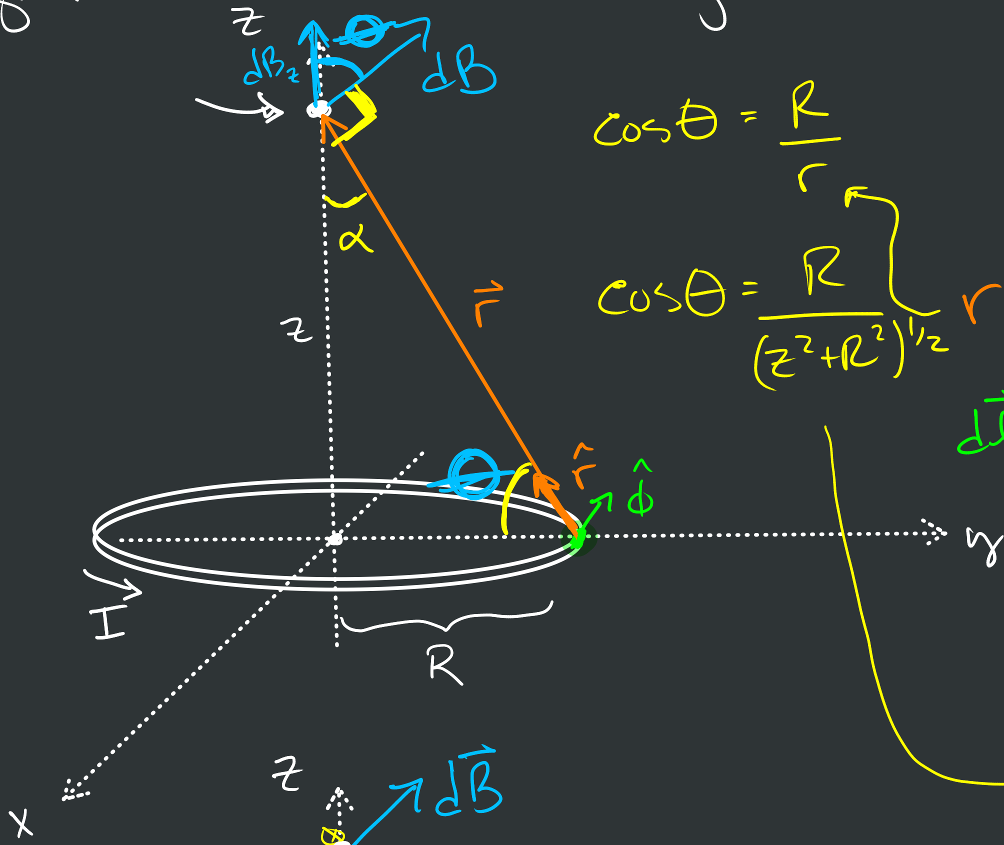
$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\sin\phi\, dl}{r^2} \cdot \frac{r}{r} \hat{z}$$

$|dl||r| \sin\phi$

$$\boxed{\begin{aligned} \vec{dB} &= \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}}{r^3} \\ \vec{dB} &= \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \hat{r}}{r^2} \end{aligned}}$$

Biot-Savart Law

Magnetic field for a ring of current:



$$\cos\theta = \frac{R}{r}$$

$$\cos\theta = \frac{R}{(z^2+R^2)^{1/2}} \qquad r = (z^2+R^2)^{1/2}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

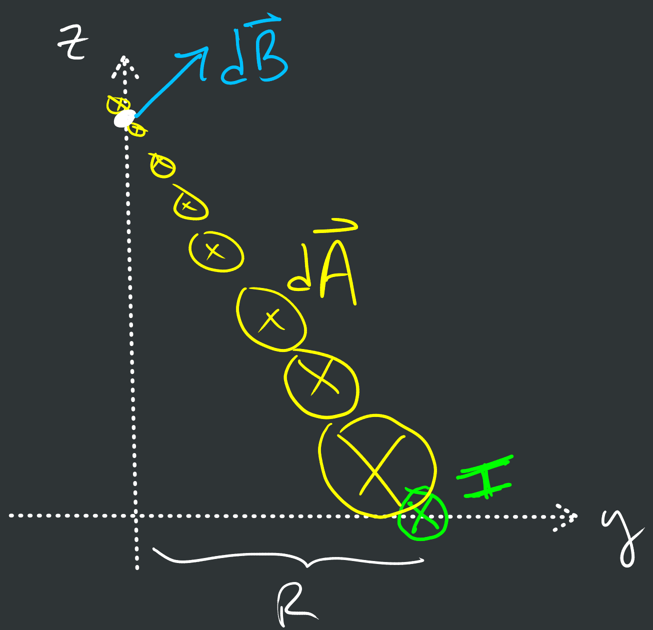
r^2 → distance from source current to where you want to know the mag field

$$d\vec{l} = R\,d\phi\,\hat{\phi}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R\,d\phi\,\boxed{\hat{\phi}\times\hat{r}}}{(z^2+R^2)}$$

perpendicular

1



$$dB_z = \frac{\mu_0 I}{4\pi}\frac{R\,d\phi}{(z^2+R^2)}\cdot\cos\theta$$

$$dB_z = \frac{\mu_0 I}{4\pi}\cdot\frac{R^2\,d\phi}{(z^2+R^2)^{3/2}}$$

$$B_z = \int dB_z = \int_0^{2\pi}\frac{\mu_0 I}{4\pi}\frac{R^2}{(z^2+R^2)^{3/2}}\,d\phi$$

$$\boxed{B_z = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}}$$

what about at the center of the ring?

$z=0$

$$\boxed{B(0) = \frac{\mu_0 I}{2R}}$$

what about N turns of wire at radius R

$$B(z) = \frac{\mu_0 N I R^2}{2(z^2+R^2)^{3/2}}$$