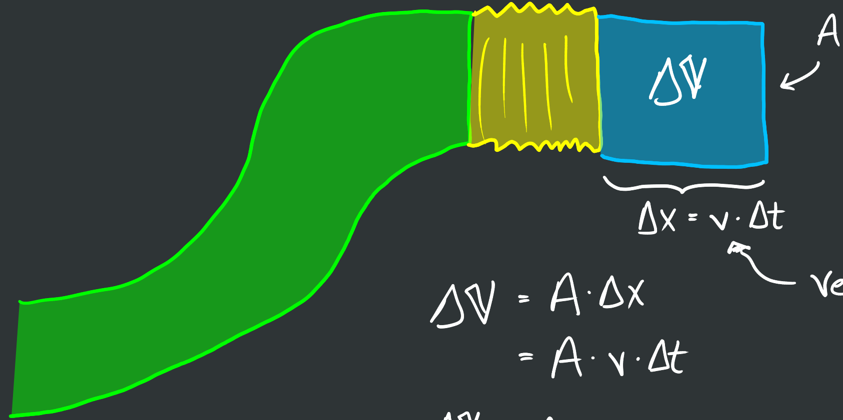
Current - Chapter 4

↳ flow of charge



$$\Delta x = v \cdot \Delta t$$

velocity

$$\Delta V = A \cdot \Delta x$$
$$= A \cdot v \cdot \Delta t$$
$$\frac{\Delta V}{\Delta t} = A \cdot v$$

volumetric flow

$$\frac{\Delta V}{\Delta t} = A \cdot v$$

$$\rho_m = \frac{m}{V}$$
$$V = \frac{m}{\rho_m}$$

$$\frac{\Delta (m/\rho_m)}{\Delta t} = A \cdot v$$

$$\frac{1}{\rho_m} \frac{\Delta m}{\Delta t} = A \cdot v$$

$$\frac{\Delta m}{\Delta t} = \rho_m \cdot A \cdot v$$

mass flow

$$\frac{\Delta m}{\Delta t} = \rho_m \cdot A \cdot v$$

charge flow

$$\frac{\Delta q}{\Delta t} = \rho_f A \cdot v_D$$

"drift velocity"
↳ average velocity of charge carrier.

free charge density

Current

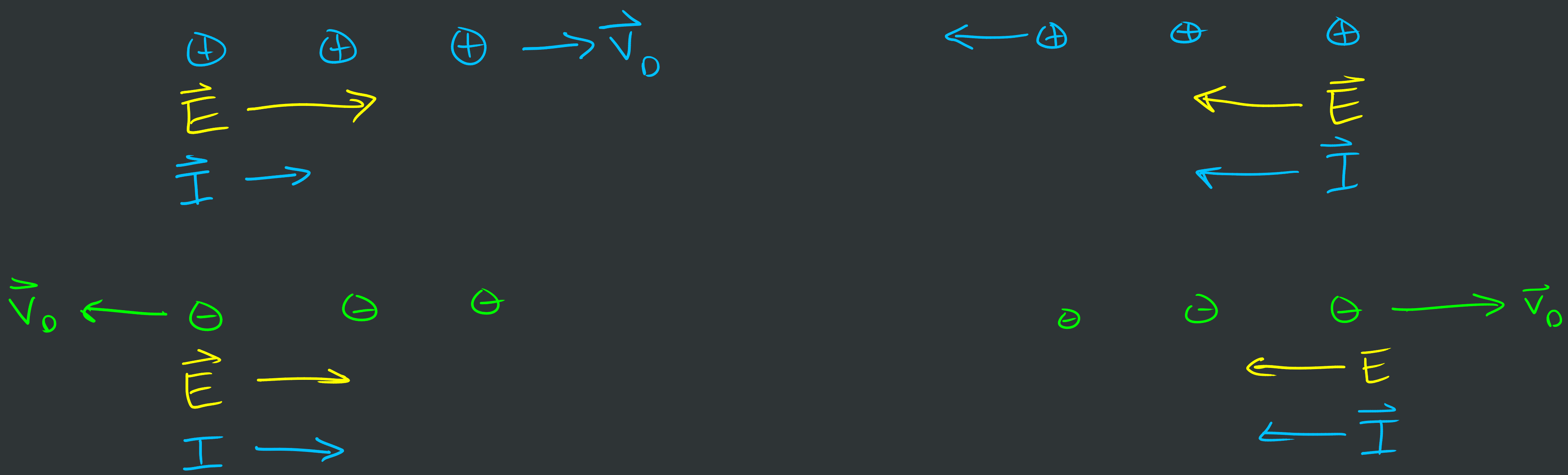
$$I \equiv \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

$$I = q \cdot n \cdot A \cdot v_D$$ } current flowing in a wire

$\frac{\text{charge}}{\text{time}} \Rightarrow$ [Ampere]

charge of a single charge carrier (e, or ion)

number density
density of charged particles



current density

$$\vec{J} \equiv \frac{I}{\vec{A}} = qn\vec{v}$$

$\frac{\text{current}}{\text{cross sectional Area}}$

multiple charge carries

$$\vec{J} = \sum_k n_k q_k \vec{v}_k$$

$k \rightarrow$ number of charge carrier types

$$I = \int_S \vec{J} \cdot d\vec{a}$$

can vary over area

For steady flow of charges into and out of a closed boundary:

$$\oiint_S \vec{J} \cdot d\vec{a} = 0$$

no charge is created or destroyed in any surface on net (at least created in pairs)

But what about unsteady flow

$\oint\!\!\oint \vec{J}\cdot d\vec{a} > 0$ ← rate at which charge is leaving

ρ ← charge density inside the boundary

$-\frac{dq}{dt}$ ← rate at which charge is leaving

$$q = \int_V \rho \, dV$$

$$\oint\!\!\oint \vec{J}\cdot d\vec{a} = -\frac{d}{dt}\int \rho \, dV$$

$$\oint\!\!\oint \vec{J}\cdot d\vec{a} = -\int \frac{d\rho}{dt} \, dV$$

now apply the divergence theorem

$$\oint\!\!\oint \vec{J}\cdot d\vec{a} = \int_V \vec{\nabla}\cdot\vec{J} \, dV = -\int_V \frac{d\rho}{dt} \, dV$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}} \longrightarrow \text{continuity equation}$$