Chapter 6

Magnetic Field

$$\underbrace{\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}}_{\text{The Lorentz Force Law}}$$

$\vec{B} \rightarrow$ magnetic field that causes a moving particle to experience a force $\perp$ its velocity

$\rightarrow$ arises from the motion of charges (or current)

$\vec{F}_B = q\vec{v} \times \vec{B}$

$B \rightarrow$ units $\left[\frac{Ns}{m \cdot C}\right] \rightarrow$ [Tesla]

$\rightarrow$ Cause $\rightarrow$ motion of other charges

$\rightarrow$ a current

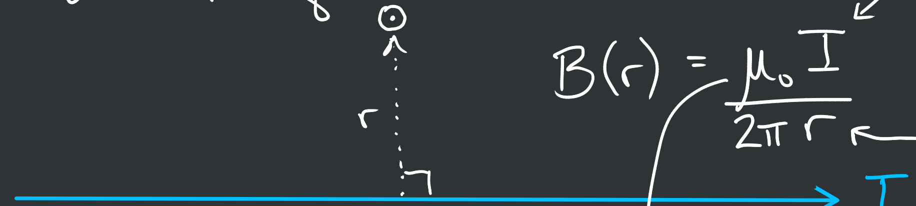
$\vec{F}_B = I\vec{\ell} \times \vec{B}$

thumb

forefinger or hand

other fingers

Ex: a straight line current



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

current

distance away from the current

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A}$$

something to prove later

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

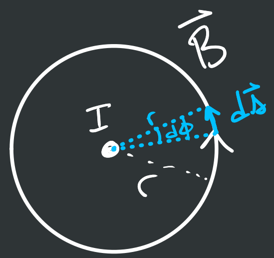
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Path integral in magnetic fields

$$\left(\oint \vec{E} \cdot d\vec{s} = \frac{q_{encl}}{\epsilon_0} \right)$$

$\oint \vec{B} \cdot d\vec{s} = 0$ along a closed path

But what if we <u>enclose</u> the current



$$\oint \vec{B}\cdot d\vec{s}$$

$$d\vec{s} = r\,d\phi\,\hat{\phi}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$$

$$= \oint \frac{\mu_0 I}{2\pi r}\cdot r\,d\phi\,\underbrace{\hat{\phi}\cdot\hat{\phi}}_{1}$$

$$= \oint \frac{\mu_0 I}{2\pi \cancel{r}}\cdot \cancel{r}\,d\phi$$

$$= \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\phi$$

$$= \frac{\mu_0 I}{\cancel{2\pi}}\cdot \cancel{2\pi}$$

$$\boxed{\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}}$$

<u>Ampere's Law</u>
(steady current) $\frac{d\rho}{dt} = 0 = \vec{\nabla}\cdot\vec{J}$

<u>assertion</u>: this will apply to any shape of current

: superposition lets us put many straight lines together

$$I_{enclosed} = \int \vec{J} \cdot d\vec{a}$$

$\rightarrow$ boundary that ds is making

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

$\downarrow$ Stokes Theorem

$$\oint \vec{B} \cdot d\vec{s} = \int (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \int \mu_0 \vec{J} \cdot d\vec{a}$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

Ampere's Law in Differential Form

$\vec{\nabla} \cdot \vec{E}$

$\vec{\nabla} \times \vec{E}$

$\vec{\nabla} \cdot \vec{B}$

$\vec{\nabla} \times \vec{B}$