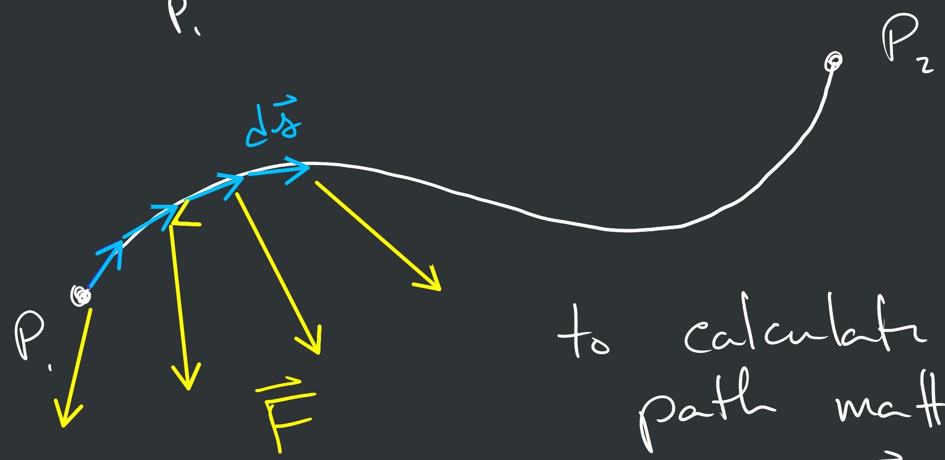
Chapter 2 - Electric Potential

$$W_{ME} = -\int_{P_1}^{P_2} \vec{F} \cdot d\vec{s} \qquad (d\vec{r}, d\vec{\ell})$$



to calculate the work done path matters.

BUT, if $\vec{F}$ is conservative then it is path-independent

$$\vec{F} = q \cdot \vec{E}$$

$$W_{ME} = -\int_{P_1}^{P_2} q\vec{E} \cdot d\vec{s}$$

$$\frac{W_{ME}}{q} = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{s}$$

$$\frac{U(P_2)-U(P_1)}{q} = \frac{\overset{\Delta K = 0}{\Delta U}}{q} \equiv \Delta\phi \equiv \phi_{21} = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{s}$$

electric potential difference

book's notation

$$\frac{[\text{Joules}]}{[\text{Coulomb}]} \quad \frac{U}{q} = \phi \quad [\text{Volts}]$$

sometimes $P_1 \to \infty$, I can set $\phi_1 = 0$.

sometimes P_1 is somewhere else, usually $\phi_1 = 0$ at that place

BUT, it is really only $\Delta\phi$ that matters.

Closed loop

$$\oint \vec{E}\cdot d\vec{s} = 0$$

line integral over a closed loop

What about the potential around a point charge?

→ point charge is located at the origin.

→ $\phi(\infty) = 0$

$$\phi(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}\,'$$

$$= -\int_{\infty}^{r} \frac{q_0}{4\pi\epsilon_0 r'^2}\hat{r} \cdot \underbrace{d\vec{r}\,'}_{dr'\,\hat{r}}$$

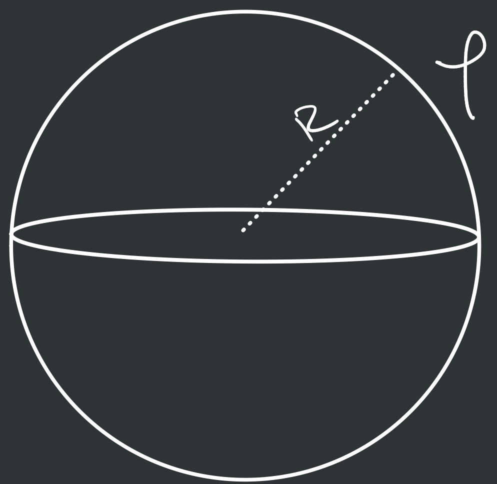
$$= -\frac{q_0}{4\pi\epsilon_0}\int_{\infty}^{r} \frac{1}{r'^2}\,dr'$$

$$\phi(r) = +\frac{q_0}{4\pi\epsilon_0 r}$$

$$U(r) = \frac{q_0 q_1}{4\pi\epsilon_0 r} = q_1\,\phi(r) \quad \checkmark$$

$\boxed{\text{SEE Example on p. 62}}$ — line integral means the path matters, so you need to break up the path in some way.

Ex: Find ϕ inside and outside a uniformly charged sphere.



outside $r > R$

$$\phi_{outside} = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r}$$

$$E = \frac{\rho R^3}{3\epsilon_0 r^2} = \boxed{\frac{Q}{4\pi\epsilon_0 r^2}}$$

$$Q = \rho \cdot V$$

$$Q = \rho \cdot \frac{4}{3}\pi R^3$$

$$\phi_{outside} = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{r} \frac{1}{r^2}\hat{r}\cdot dr\,\hat{r}$$

$$= \frac{Q}{4\pi\epsilon_0 r} = \rho\,\frac{4}{3}\pi R^3 \cdot \frac{1}{4\pi\epsilon_0 r}$$

$$\phi_{outside} = \frac{\rho R^3}{3\epsilon_0 r}$$

inside $r < R$

$$E = \frac{\rho r}{3\epsilon_0}$$

$$\phi(r)_{inside} = -\int \vec{E}\cdot d\vec{r}$$

$$= -\underbrace{\int_{R}^{r} \frac{\rho r}{3\epsilon_0}\,dr}_{\text{inside}} - \underbrace{\int_{\infty}^{R} \frac{\rho R^3}{3\epsilon_0 r^2}\,dr}_{\text{outside}}$$

$$\frac{\rho R^{\cancel{3}\,2}}{3\epsilon_0 \cancel{R}}$$

$$= \frac{\rho R^2}{3\epsilon_0}$$

$$= -\int_R^r \frac{\rho r}{3\epsilon_0}\,dr$$

$$= -\frac{\rho}{3\epsilon_0}\int_R^r r\,dr$$

$$= -\frac{\rho}{6\epsilon_0} r^2\Big|_R^r = -\frac{\rho}{6\epsilon_0}\left(r^2 - R^2\right)$$

$$\phi_{inside}(r) = -\frac{\rho}{6\epsilon_0}\left(r^2 - R^2\right) + \frac{\rho R^2}{3\epsilon_0}$$

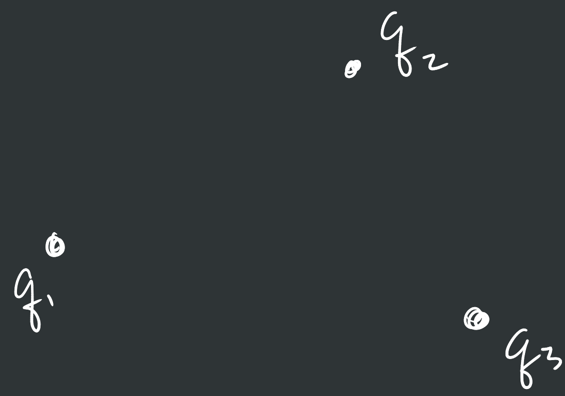
$$= -\frac{\rho}{6\epsilon_0} r^2 + \frac{\rho}{6\epsilon_0} R^2 + \frac{2\rho R^2}{2\cdot 3\epsilon_0}$$

$$\phi_{inside}(r) = -\frac{\rho}{6\epsilon_0} r^2 + \frac{\rho R^2}{2\epsilon_0} \checkmark$$

Summing up what we have so far:

- potential energy for point charges

$$U = \frac{1}{2} \sum_{j=1}^{N} \sum_{k \neq j} \frac{q_j q_k}{4\pi\epsilon_0 r_{jk}}$$



- electric potential for point charges

$$\phi = \sum_{k}^{N} \frac{q_k}{4\pi\epsilon_0 r_k}$$

- electric potential of a continuous distribution

$$\phi = \int_{\substack{\text{all} \\ \text{sources}}} \frac{\rho \, dV}{4\pi\epsilon_0 r} \leftarrow \text{limited to sources of finite extent}$$

Making a comparison now w/ the first two equations:

$$\mathcal{U} = \underbrace{\frac{1}{2}\sum_{j}^{N} q_j}_{\text{adding up all the charge}} \underbrace{\sum_{k \neq j}^{N} \frac{q_k}{4\pi\epsilon_0 r_{jk}}}_{\text{like the sum for } \phi}$$

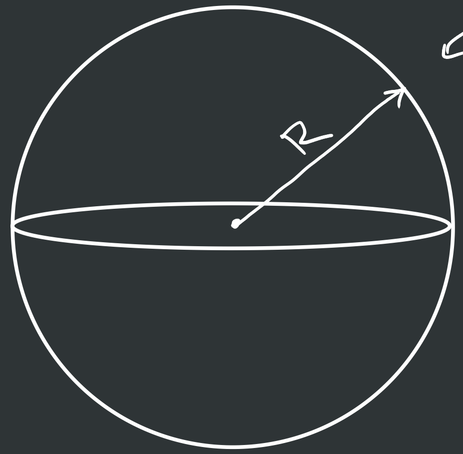
go to continuous charge distribution

$$\boxed{\mathcal{U} = \frac{1}{2}\int \rho \phi \, dV}$$

relates the energy to build a charge distribution to the potential of that distribution

another way from Chapter 1 (eq. 1.53)

$$\mathcal{U} = \frac{\epsilon_0}{2}\int_{\text{entire field}} E^2 \, dV$$



a shell of charge, Q

$$\vec{E}_{inside} = 0$$

$$\vec{E}_{outside} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$U = \frac{\epsilon_0}{2}\int E^2\,dV$$

$$U = \frac{\epsilon_0}{2}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\left(\frac{Q}{4\pi\epsilon_0 r^2}\right)^2 \underbrace{r^2\sin\theta\,dr\,d\theta\,d\phi}$$

$$= \frac{\cancel{\epsilon_0}\,Q^2}{2\;4^2\pi^2\epsilon_0^{\cancel{2}}}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\frac{1}{r^2}\sin\theta\,dr\,d\theta\,d\phi$$

$$= \frac{Q^2}{2\cdot 4^{\cancel{2}}\cdot\pi^{\cancel{2}}\epsilon_0}\left(\cancel{4\pi}\right)\underbrace{\int_R^{\infty}\frac{1}{r^2}\,dr}_{-\left(\frac{1}{r}\right)_R^{\infty} = \frac{1}{R}}$$

$$U = \underbrace{\frac{1}{2}\int \rho\,\phi\,dV}$$

U (direct integration)

add dq shells to a radius of R until you have Q.

$$dU = \frac{q}{4\pi\epsilon_0 R}\cdot dq$$

work to add dq to q at a radius of R

$$U = \int_0^Q \frac{q\,dq}{4\pi\epsilon_0 R}$$

$$U = \frac{Q^2}{8\pi\epsilon_0 R}$$

$$\boxed{U = \frac{Q^2}{8\pi\epsilon_0 R}}$$

$$U = \frac{1}{2}\int \rho\,\phi\, dV$$

$$\rightarrow \infty$$

$$\rho = ?$$

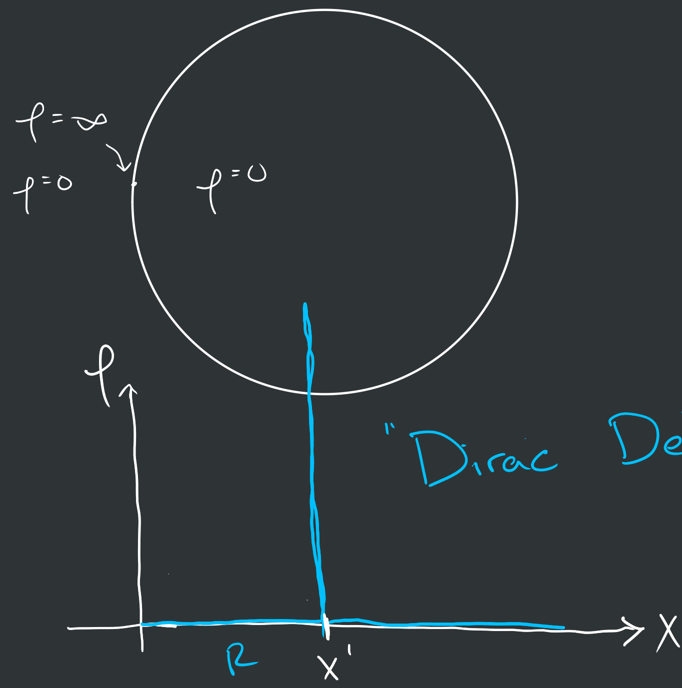
$$\rho = \frac{Q}{V}, \quad V \rightarrow 0$$

$$\rho(r) = \rho\cdot\delta(R-r)$$

$$Q = \int \rho(r)\,\underbrace{dV}$$

$$Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} \rho\,\delta(R-r)\cdot r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$Q = 4\pi\rho\int_0^{\infty} r^2\,\delta(R-r)\, dr$$



"Dirac Delta" function

$$\delta(x'-x) = \begin{cases} 0 & x \neq x' \\ \infty & x = x' \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x'-x)\, dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x'-x)\, dx = f(x')$$

$$Q = 4\pi\rho R^2$$

$$\rho = \frac{Q}{4\pi R^2}$$

surface charge density

$$U = \frac{1}{2}\int \rho\cdot\delta(R-r)\,\Phi(r)\,dV$$

$$U = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \rho\,\delta(R-r)\,\Phi(r)\,r^2\,\underbrace{\sin\theta}\,dr\,\underbrace{d\theta\,d\phi}$$

$$U = \frac{4\pi}{2}\int_0^{\infty}\rho\,r^2\,\Phi(r)\,\delta(R-r)\,dr$$

$$= \frac{4\pi}{2}\underbrace{\rho}_{\frac{Q}{4\pi R^2}} R^2\,\underbrace{\Phi(R)}_{\frac{Q}{4\pi\epsilon_0 R}}$$

$$= \frac{\cancel{4\pi}}{2}\cdot\frac{Q}{\cancel{4\pi}\cancel{R^2}}\cdot\cancel{R^2}\cdot\frac{Q}{4\pi\epsilon_0 R} = \boxed{\frac{Q^2}{8\pi\epsilon_0 R}}$$ ✓✓

So given E we can find ϕ.

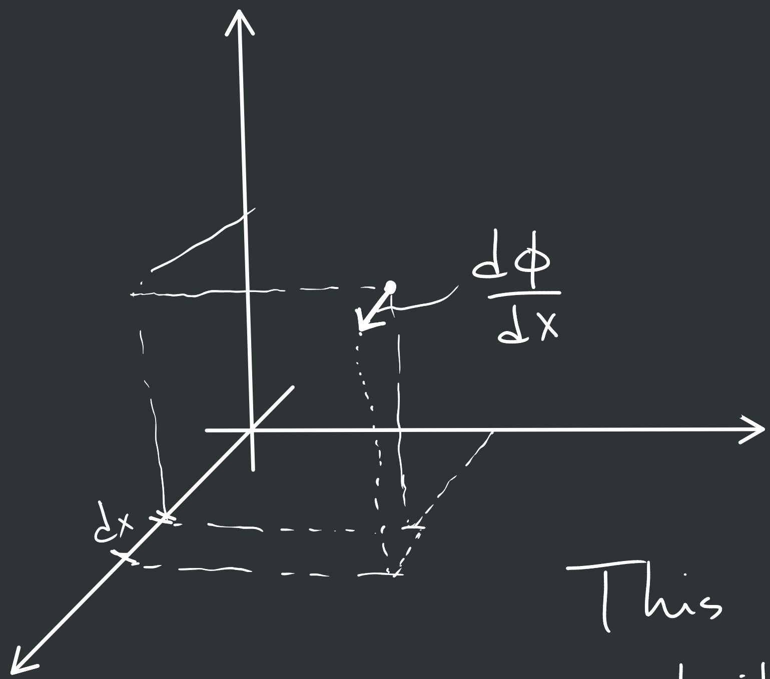
$$\Delta\phi = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{s}$$

How do we go backwards?

looks like $\rightarrow \dfrac{d\phi}{ds} = -E$, but we are missing vectors!

We will do Cartesian coordinates first.

$$\underbrace{\phi(x,y,z)}_{\text{function of 3 variables}} \rightsquigarrow \frac{\partial\phi}{\partial x} \leftarrow$$ partial derivative of ϕ w.r.t x, holding y and z fixed!



Since we can do the same in the y & z directions, we can construct a vector

$$\frac{\partial \phi}{\partial x}\hat{x} + \frac{\partial \phi}{\partial y}\hat{y} + \frac{\partial \phi}{\partial z}\hat{z}$$

This vector is called the gradient of ϕ and it produces a vector field

$$\vec{\nabla}\phi = \frac{\partial \phi}{\partial x}\hat{x} + \frac{\partial \phi}{\partial y}\hat{y} + \frac{\partial \phi}{\partial z}\hat{z}$$

Now, a small change in any scalar function can be written as (mathematically):

$$\hookrightarrow d\phi = \underbrace{\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz}_{} \leftarrow$$

but also there is physics

$$d\phi = -\vec{E}\cdot d\vec{s}$$

$$\to d\vec{s} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz = \vec{\nabla}\phi\cdot d\vec{s} = -\vec{E}\cdot d\vec{s}$$

$$\boxed{\vec{E} = -\vec{\nabla}\phi}$$

$$W = \int \vec{F}\cdot d\vec{s}$$

$$\frac{W}{q} = \int \frac{\vec{F}}{q}\cdot d\vec{s}$$

$$\phi = -\int \vec{E}\cdot d\vec{s}$$

$$\boxed{\phi = -\int_{P_a}^{P_b} \vec{E}\cdot d\vec{s}}$$

Ex p.62

$$\phi = -Kxy$$

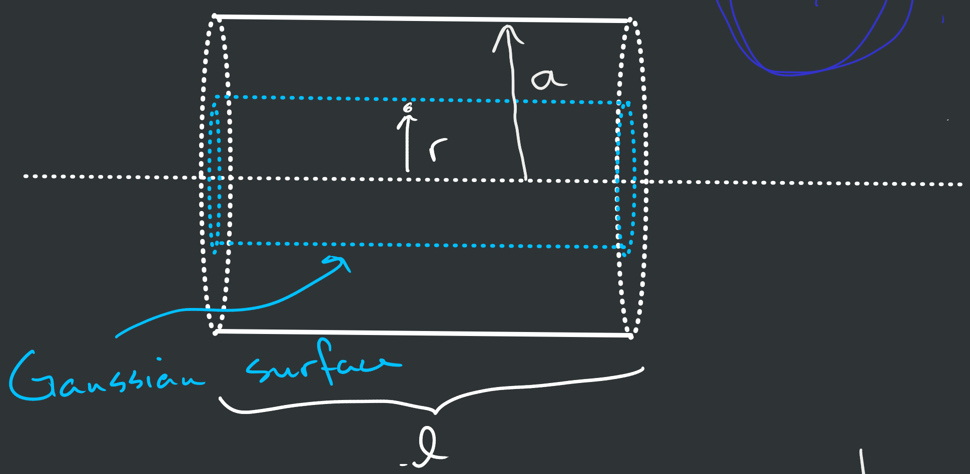
$$\vec{E} = -\vec{\nabla}(-Kxy)$$

$$= K\vec{\nabla}(xy) = K\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}\right)(xy)$$

$$= K(y\hat{x} + x\hat{y})$$ ✓

42) $E_{in} = \dfrac{\rho r}{2\epsilon_0}$ ⟵ show this w/ Gauss's Law

$E_{out} = \dfrac{\rho a^2}{2\epsilon_0 r}$



$$\int \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0}\int \rho \, dV$$

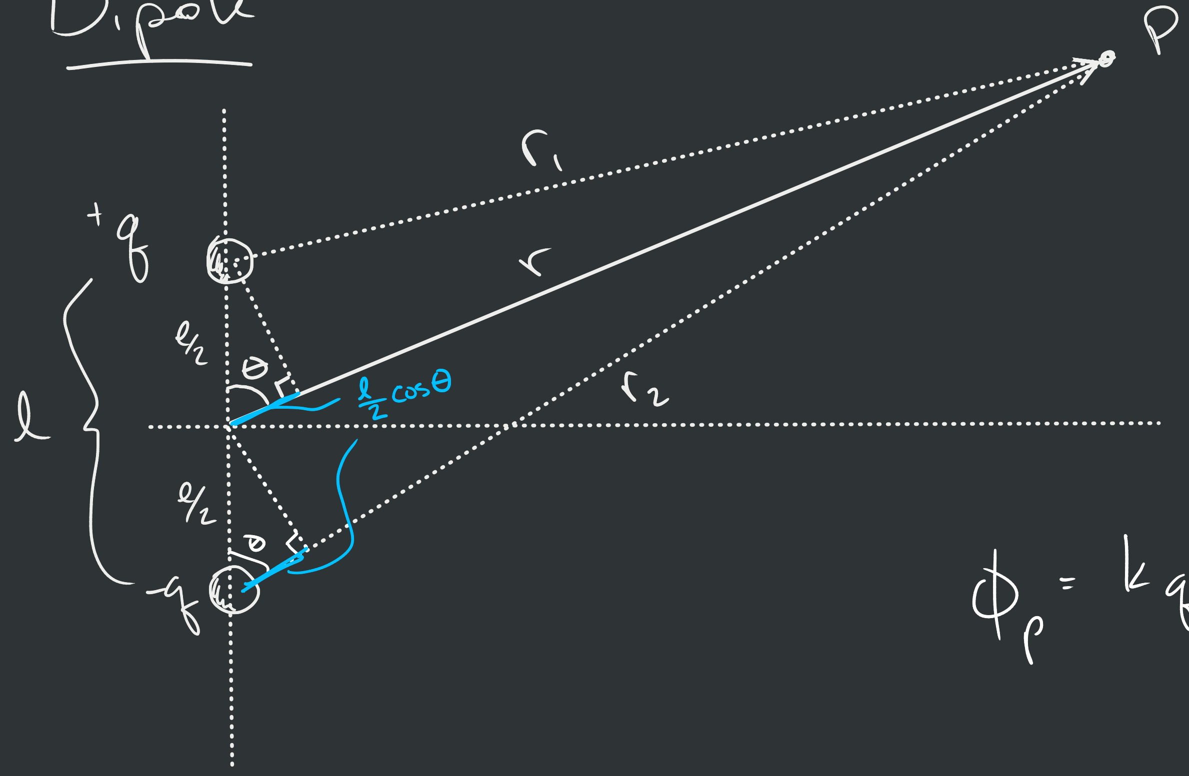
$$E\cdot \cancel{\ell}\cdot 2\cancel{\pi}\cancel{r} = \frac{1}{\varepsilon_0}\rho\cancel{\pi}\cancel{r}^2\cancel{\ell}$$

$$\underline{\vec{E} = \frac{\rho r}{2\varepsilon_0}\hat{r}}$$

$$\phi = \int_{P_a}^{P_b} \vec{E}\cdot \underbrace{d\vec{s}}_{\underline{dr\,\hat{r}}}$$

$$\phi = \frac{\rho a^2}{2e}\ln(\infty$$

Dipole



$$\phi_p = \frac{kq}{r_1} + \frac{k(-q)}{r_2}$$

$$r_1 = r - \frac{\ell}{2}\cos\theta$$

$$r_2 = r + \frac{\ell}{2}\cos\theta$$

$$\phi_p = kq\left[\frac{1}{r\left(\frac{r}{r} - \frac{\ell}{2}\frac{\cos\theta}{r}\right)} - \frac{1}{r\left(\frac{r}{r} + \frac{\ell}{2}\frac{\cos\theta}{r}\right)}\right]$$

$$= \frac{kq}{r}\left[\frac{1}{\underbrace{1 - \frac{\ell\cos\theta}{2r}}} - \frac{1}{1 + \frac{\ell\cos\theta}{2r}}\right]$$

$$\left(1 - \frac{\ell\cos\theta}{2r}\right)^{-1}$$

$$(1 \pm x)^{-1} \simeq 1 \mp x$$

Taylor expansion to 1st order

$$\phi_p = \frac{kq}{r}\left[\cancel{1} + \frac{\ell\cos\theta}{2r} - \cancel{1} + \frac{\ell\cos\theta}{2r}\right]$$

$$\phi_p = \frac{kq}{r}\left(\frac{l\cos\theta}{r}\right)$$

$$\phi_p = \frac{kq\, l\cos\theta}{r^2}$$

$ql \rightsquigarrow$ dipole moment $\equiv p$

plot equipotenials

$$\boxed{\phi_p = \frac{kp\cos\theta}{r^2}}$$

$\vec{E} = ?$

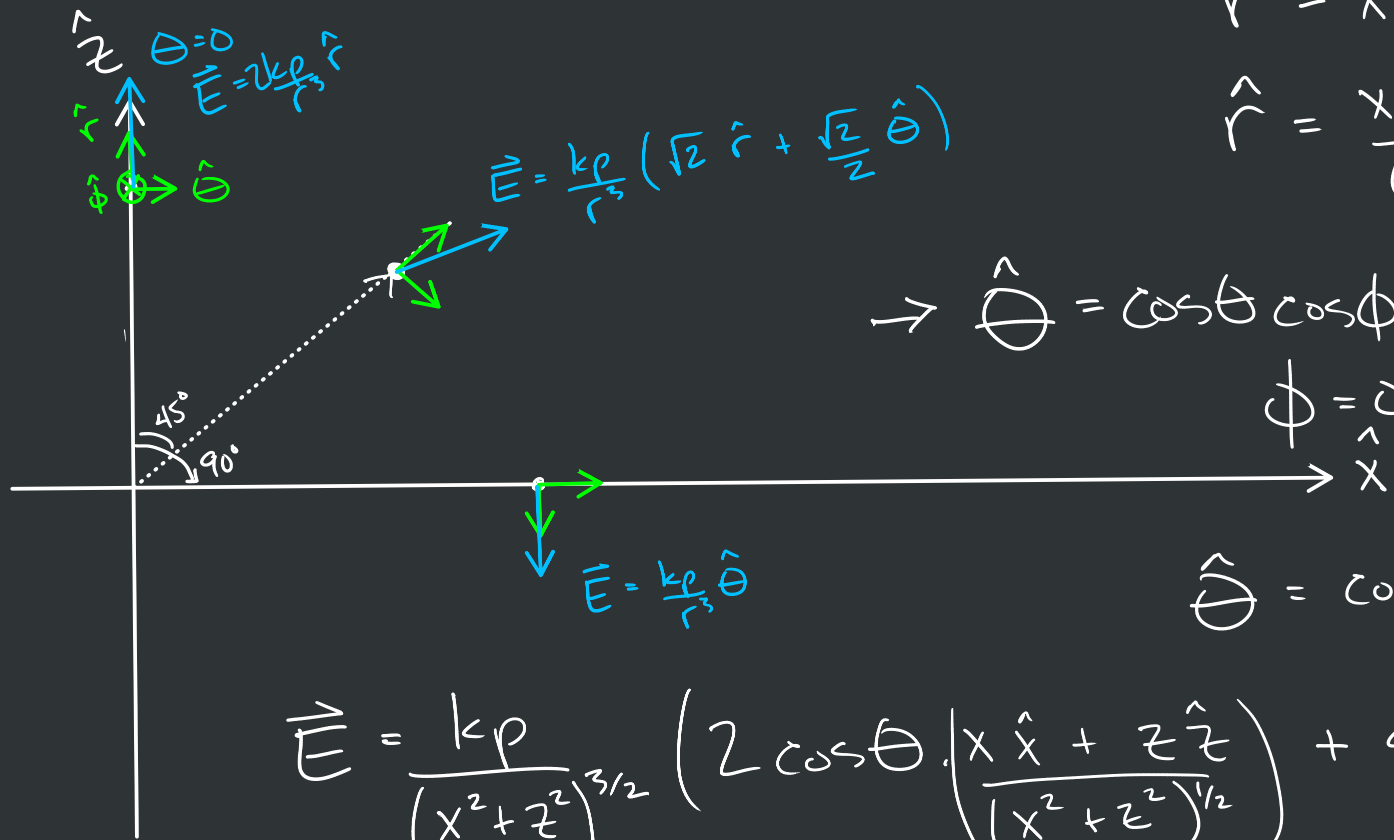
$$\vec{E} = -\vec{\nabla}\Phi = -\left(\frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\left(\frac{\partial\Phi}{\partial\theta}\right)\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\hat{\phi}\right)$$

$$= -\frac{\partial\left(\frac{kp\cos\theta}{r^2}\right)}{\partial r}\hat{r} - \frac{1}{r}\left(\frac{\partial\left(\frac{kp\cos\theta}{r^2}\right)}{\partial\theta}\right)$$

$$= -\left(-\frac{2kp\cos\theta}{r^3}\hat{r}\right) - \frac{1}{r}\left(-\frac{kp\sin\theta}{r^2}\hat{\theta}\right) + 0\hat{\phi}$$

$$= \frac{2kp\cos\theta}{r^3}\hat{r} + \frac{kp\sin\theta}{r^3}\hat{\theta}$$

$$\vec{E} = \frac{kp}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$



$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$\hat{r} = \frac{x\,\hat{x} + y\,\hat{y} + z\,\hat{z}}{(x^2 + y^2 + z^2)^{1/2}}$$

$$\rightarrow \hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$

$$\phi = 0$$

$$\hat{\theta} = \cos\theta\,\hat{x} - \sin\theta\,\hat{z}$$

$$\vec{E} = \frac{kp}{(x^2+z^2)^{3/2}}\left(2\cos\theta\cdot\left(\frac{x\,\hat{x} + z\,\hat{z}}{(x^2+z^2)^{1/2}}\right) + \sin\theta\cdot\left(\cos\theta\,\hat{x} - \sin\theta\,\hat{z}\right)\right)$$

$$\cos\theta = \frac{z}{r} = \frac{z}{(x^2+z^2)^{1/2}}$$

$$\sin\theta = \frac{x}{r} = \frac{x}{(x^2+z^2)^{1/2}}$$

(after some algebra)

$\downarrow$

$$\boxed{\vec{E} = k p \left(\frac{3xz}{(x^2+z^2)^{5/2}} \hat{x} + \frac{2z^2 - x^2}{(x^2+z^2)^{5/2}} \hat{z} \right)}$$

$$\Phi_p = \frac{k p \cos\theta}{r^2} \qquad r^2 = x^2 + z^2$$

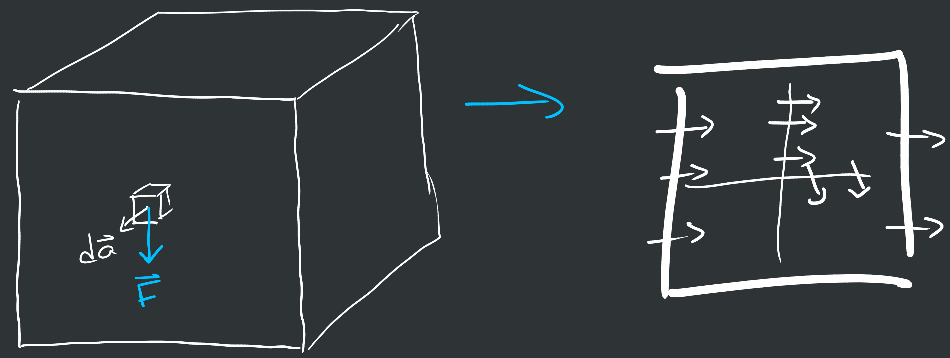
$$\cos\theta = \frac{z}{r} = \frac{z}{(x^2+z^2)^{1/2}}$$

$$\Phi_p = \frac{k p z}{(x^2+z^2)^{3/2}}$$

Divergence → flux density

$$\oint = \int_S \vec{F} \cdot d\vec{a}$$

flux



$$\oint = \int_{S_1} \vec{F} \cdot d\vec{a}_1 + \int_{S_2} \vec{F} \cdot d\vec{a}_2$$

$$\oint = \int_S \vec{F} \cdot d\vec{a} = \sum_{i=1}^{N} \int_{S_i} \vec{F} \cdot d\vec{a}_i$$

$$\lim_{V_i \to 0} \frac{\int_{S_i} \vec{F} \cdot d\vec{a}_i}{V_i} = \mathrm{div}(\vec{F})$$

$$= \vec{\nabla} \cdot \vec{F}$$

"del" - operator notation

$$\sum_{i=1}^{N} \frac{V_i}{V_i} \int_{S_i} \vec{F} \cdot d\vec{a}_i = \int_S \vec{F} \cdot d\vec{a}$$

$$\sum_{i=1}^{N} V_i \vec{\nabla} \cdot \vec{F} =$$

$$\underbrace{\boxed{\int_V \vec{\nabla} \cdot \vec{F} \, dv = \int_S \vec{F} \cdot d\vec{a}} \boxed{= \Phi}}_{\text{divergence theorem}}$$

divergence theorem (Green's Theorem, Gauss's Theorem)

So how will we use this?

Gauss's Law $\quad \int_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int_V \rho \, dv$

$$\int_V \vec{\nabla}\cdot\vec{E}\,dv = \int_V \frac{\rho}{\epsilon_0}\,dv$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}} \leftarrow \text{Gauss's Law (in differential form)}$$

$$\text{del} = \vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \leftarrow \text{in Cartesian coordinates}$$

$$\vec{\nabla}\phi = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\phi$$

$$\vec{\nabla}\cdot\vec{E} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(E_x\hat{x} + E_y\hat{y} + E_z\hat{z}\right)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot(-\vec{\nabla}\phi) = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{\nabla}\phi = -\frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2\phi = -\frac{\rho}{\epsilon_0}}$$ Poisson's Equation

↓ special case

$$\boxed{\nabla^2\phi = 0}$$ Laplace's Equation

$$\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)$$

Laplacian → $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ } in Cartesian coordinates

$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Phi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \Phi}{\partial \phi^2}$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\phi = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

In review

$\vec{E} = -\nabla \phi$ (gradient)

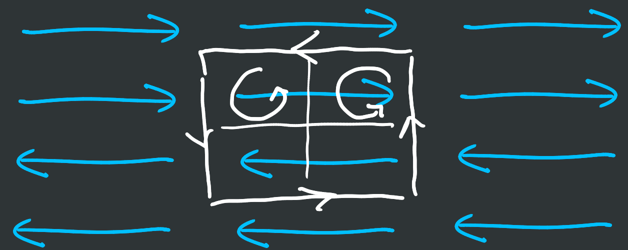
$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (divergence)

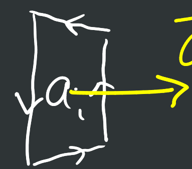
$\vec{\nabla} \times \vec{E}$

Curl

capital gamma → $\Gamma = \oint_C \vec{F} \cdot d\vec{s}$

circulation





$\vec{a}_i = a_i \hat{n}$

$$\mathrm{curl}(\vec{F}) \cdot \hat{n} = \lim_{a_i \to 0} \frac{\Gamma_i}{a_i} = \lim_{a_i \to 0} \frac{\oint_c \vec{F} \cdot d\vec{s}}{a_i} \hat{n}$$

$$\underline{\vec{\nabla} \times \vec{F}} = \mathrm{curl}(\vec{F}) \cdot \hat{n}$$

in cartesian

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{x} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) \hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{z}$$

The circulation around a large path is the sum of the curls within that area.

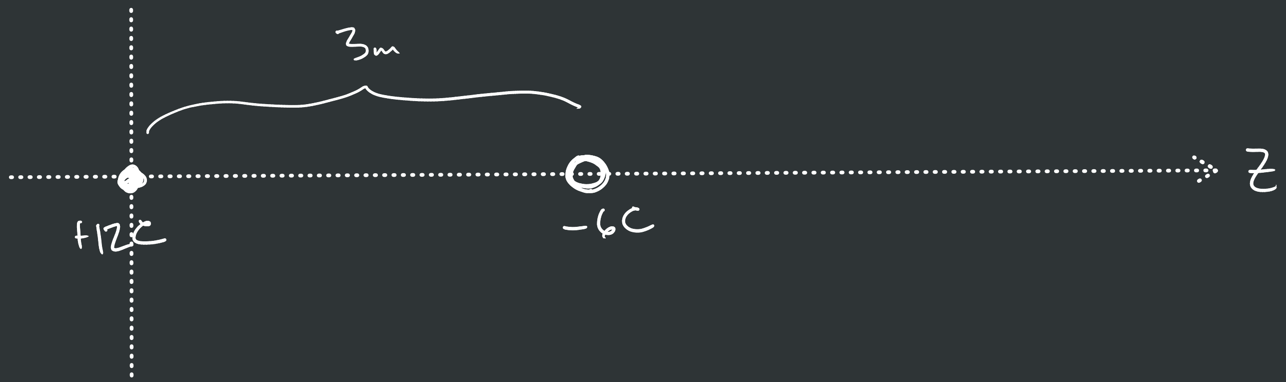
$$\boxed{\underbrace{\oint \vec{F} \cdot d\vec{s}}_{\Gamma} = \int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{a}} \rightarrow \text{Stoke's Theorem}$$

The path integral around a closed loop is zero for electric fields

$\vec{\nabla} \times \vec{E} = 0$, then the field is conservative.

this is true for <u>static</u> <u>electric</u> <u>fields</u>

33)



$$\phi = \frac{q}{4\pi\epsilon_0 r}$$

$$\phi_{12C} = \frac{12C}{4\pi\epsilon_0 |z|}$$

$$\phi_{-6C} = \frac{-6C}{4\pi\epsilon_0 |z-3|}$$

$$\phi(z) = \frac{12\,C}{4\pi\epsilon_0 |z|} + \frac{-6C}{4\pi\epsilon_0 |z-3|}$$

$$4\pi\epsilon_0 \phi(z) = \frac{12}{|z|} + \frac{-6}{|z-3|}$$

12)