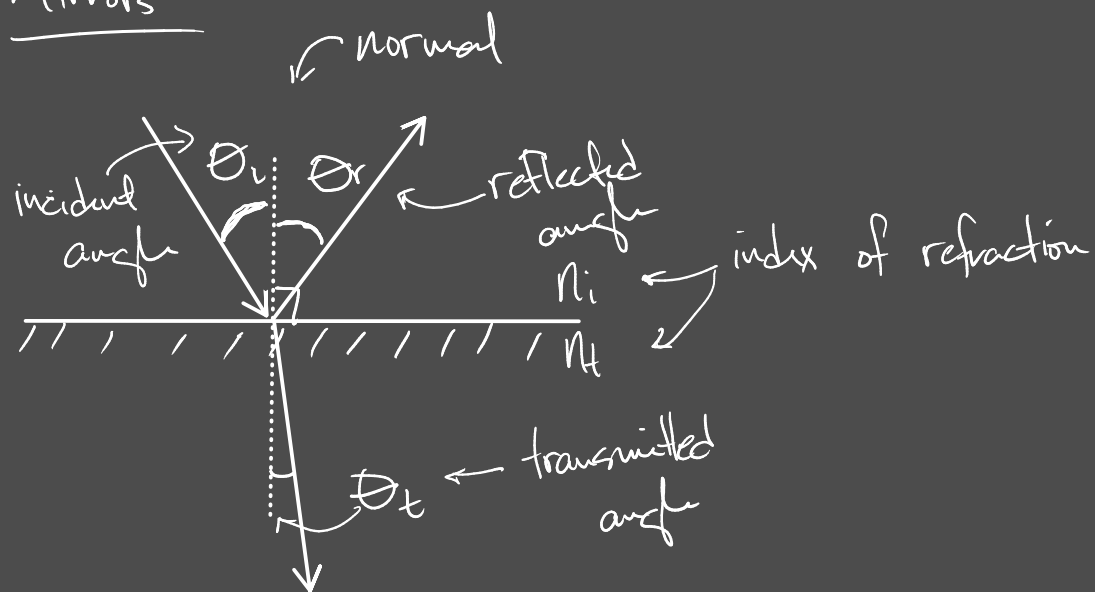
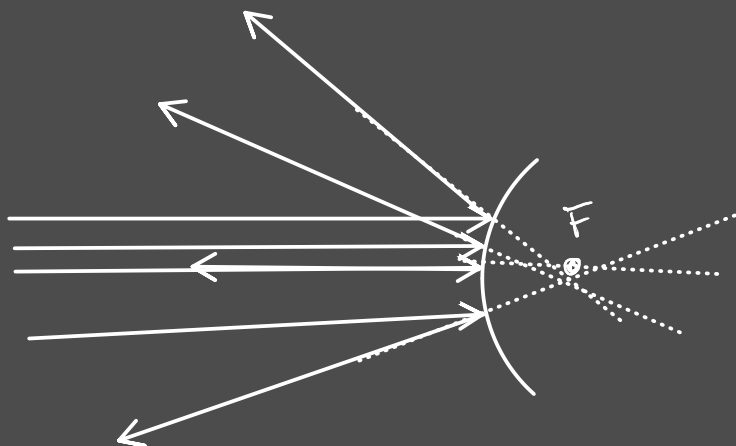
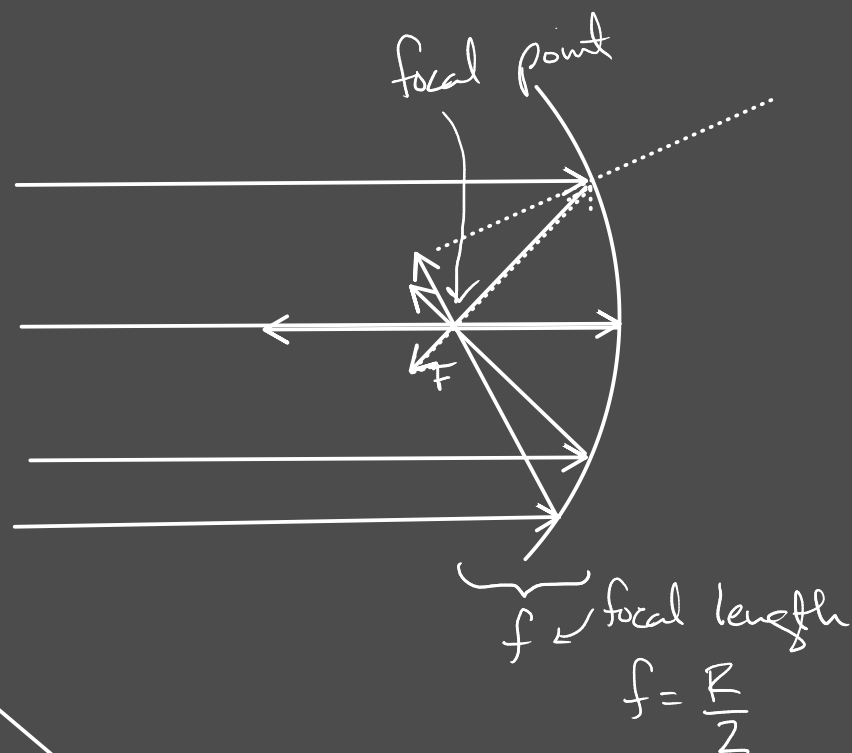
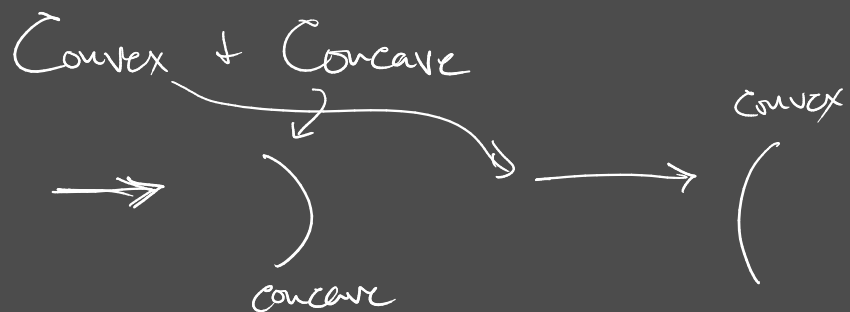


Brain storm review

Mirrors



$$n_i \sin \theta_i = n_t \sin \theta_t$$



Photon → a particle of light

Light → ray
→ photon
→ wave → (electromagnetic)
charge + current

Planck's Equation

$$E = h \nu$$

↑
↑ "nu"

Planck constant

$$h = 6.63 \cdot 10^{-34} \text{ Js}$$

de Broglie

$$\lambda = \frac{h}{p}$$

Basic relativistic mechanics

$$\rightarrow E = \gamma m c^2$$

↑
total

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$$

kinetic energy

$$E_k = m c^2 (\gamma - 1)$$

↓

$$E_k \approx \frac{1}{2} m v^2$$

$$v \ll c$$

rest energy

$$E_r = m c^2$$

↑
Energy - mass
equivalence

Energy-momentum relation

$$\rightarrow E^2 = (pc)^2 + (m c^2)^2$$

Solve for momentum

↓

relativistic momentum

$$p = \gamma m v$$

$$E_k = \gamma m c^2 - m c^2$$

$$E = E_k + m c^2 = \gamma m c^2 - \cancel{m c^2} + m c^2$$

$$p = \frac{\sqrt{E^2 - m^2 c^4}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m^2 c^4}} = \lambda$$

$$p = \gamma m v \Rightarrow v = \frac{p}{\gamma m} = \frac{c^2 p}{E} = c \sqrt{1 - \frac{m^2 c^2}{E^2}} = v$$

γm
 $E = \gamma m c^2$
 $\frac{E}{c^2} = \gamma m$

for a massless photon: $m=0$

$$p = \frac{E}{c} \quad \left| \quad \lambda = \frac{h}{p} = \frac{hc}{E} \quad \right| \quad v = c$$

$$\rightarrow E = \frac{hc}{\lambda}$$

$$\boxed{E = h\nu}$$

wave \swarrow frequency

$$v = \lambda \cdot \nu$$

$$v = c$$

$$c = \lambda \cdot \nu$$

$$\frac{c}{\lambda} = \nu$$

An electron is accelerated to a kinetic energy E_K of 2.5 MeV. (a) Determine its relativistic momentum, de Broglie wavelength, and speed. (b) Determine the same properties for a photon having the same total energy as the electron.

$$E_K = 2.5 \text{ MeV}$$

$$E_K = 2.5 \text{ MeV} \cdot \frac{1.6 \cdot 10^{-19} \text{ J}}{1 \text{ eV}} \cdot \frac{10^6}{1 \text{ M}} \\ = 2.5 \cdot 1.6 \cdot 10^{-13} \text{ J} \\ = 4 \cdot 10^{-13} \text{ J}$$

$$E_{\text{total}} = 4 \cdot 10^{-13} \text{ J} + 9.11 \cdot 10^{-31} \cdot 9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2} \\ = 4 \cdot 10^{-13} \text{ J} + 8.2 \cdot 10^1 \cdot 10^{-15} \\ + 8.2 \cdot 10^{-14} \\ + 0.82 \cdot 10^{-13} = 4.82 \cdot 10^{-13} \text{ J}$$

$$p = \frac{\sqrt{E^2 - m^2 c^4}}{c} = \frac{\sqrt{(4.82 \cdot 10^{-13})^2 - (8.2 \cdot 10^{-14})^2}}{3 \cdot 10^8} = \frac{\sqrt{23.23 \cdot 10^{-26} - 67.24 \cdot 10^{-28}}}{3 \cdot 10^8}$$

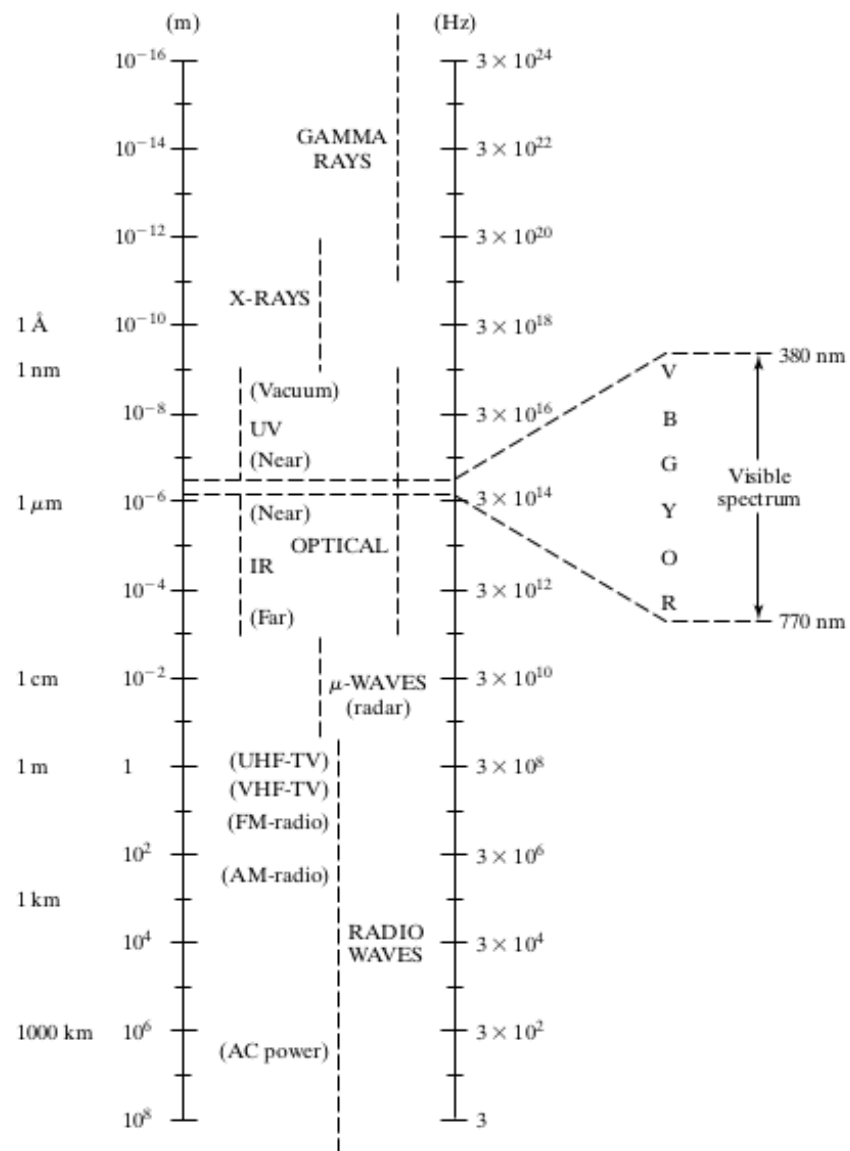
$$1 \text{ eV} \Rightarrow 1 e = 1.6 \cdot 10^{-19} \text{ C}$$

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

$$\bullet 1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ C} \cdot 1 \frac{\text{J}}{\text{C}} = 1.6 \cdot 10^{-19} \text{ J}$$

$$\bullet m_e = 9.11 \cdot 10^{-31} \text{ kg}$$

$$\bullet c = 3 \cdot 10^8 \text{ m/s}$$



A certain sensitive radar receiver detects an electromagnetic signal of frequency 100 MHz and power (energy/time) 6.63×10^{-16} J/s.

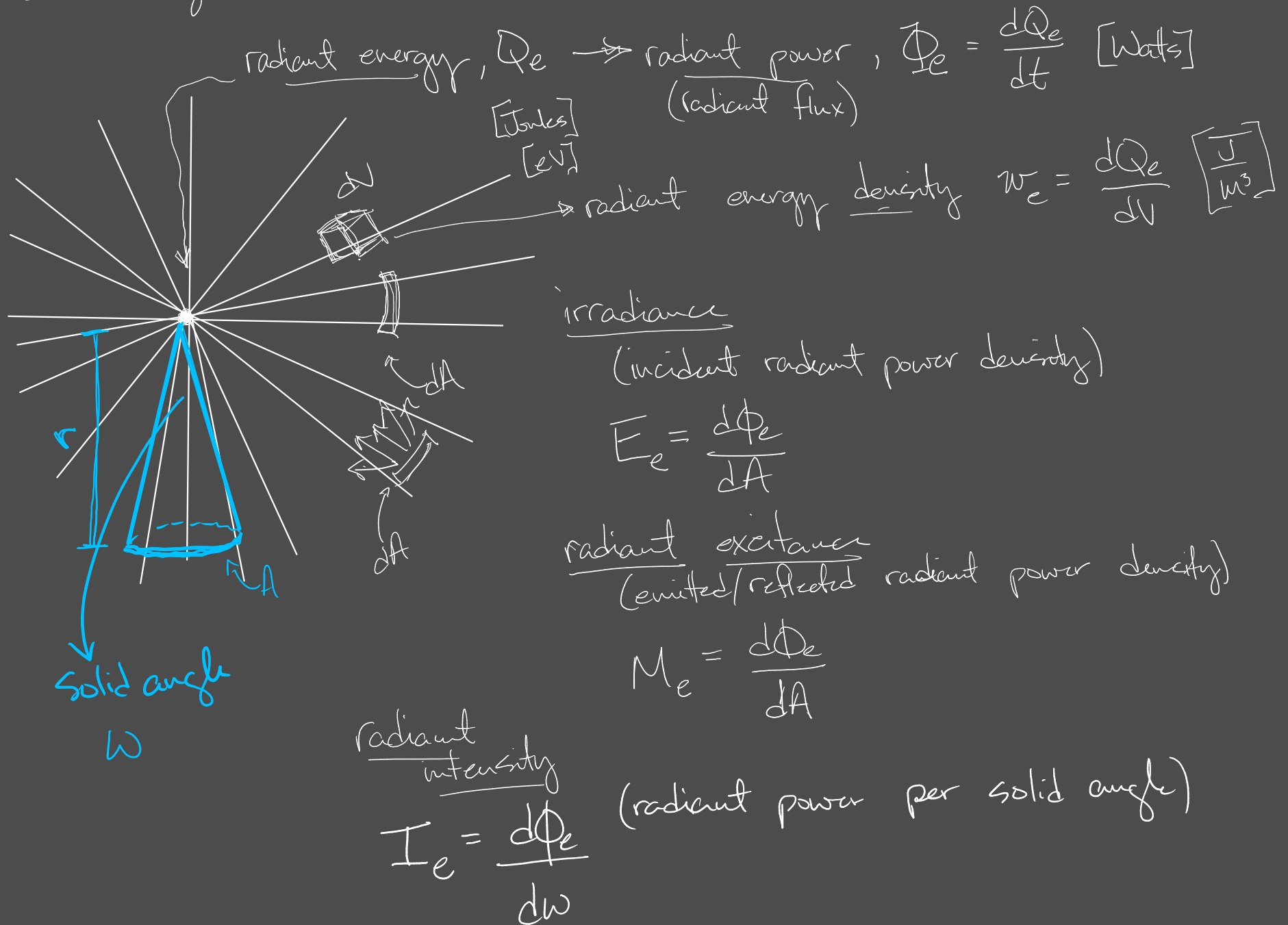
- What is the wavelength of a photon with this frequency?
- What is the energy of a photon in this signal? Express this energy in J and in eV.
- How many photons/s would arrive at the receiver in this signal?
- What is the energy (in J and in eV) of a visible photon of wavelength 555 nm?
- How many visible ($\lambda = 555$ nm) photons/s would correspond to a detected power of 6.63×10^{-16} J/s?
- What is the energy (in J and in eV) of an X-ray of wavelength 0.1 nm?
- How many X-ray ($\lambda = 0.1$ nm) photons/s would correspond to a detected power of 6.63×10^{-16} J/s?

a. $c = \lambda \cdot \nu$

b. $E = h\nu$

c.
$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{n \cdot \text{energy/photon}}{\text{time}} \Rightarrow \frac{\text{power}}{\text{energy/photon}} = \frac{n \text{ photons}}{\text{time}}$$

Radiometry



Irradiance from a point source.

$$E_e = \frac{d\Phi_e}{dA}$$

$$E_e = \frac{\Phi_e}{4\pi r^2}$$

$$E_e = \frac{I \cdot \cancel{4\pi}}{\cancel{4\pi} r^2}$$

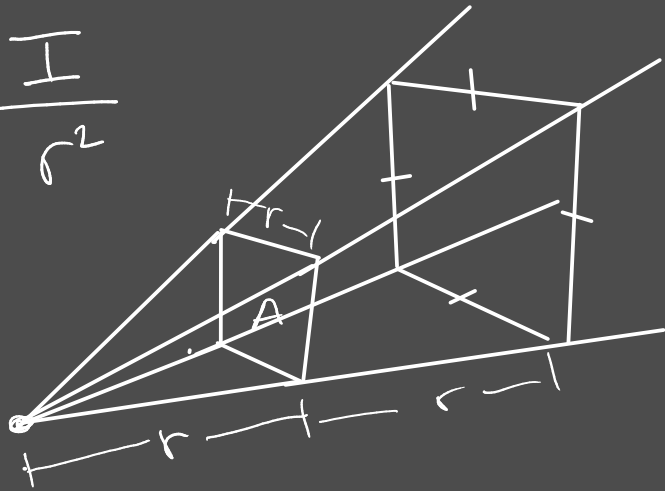
$$E_e = \frac{I}{r^2}$$

$$I_e = \frac{\Phi_e}{4\pi}$$

specific
to pt. source

constant

$$\Phi_e = I \cdot 4\pi$$



$$A = r \cdot \theta$$

arc
length

$$\theta = \frac{\Delta}{r}$$

in
radians

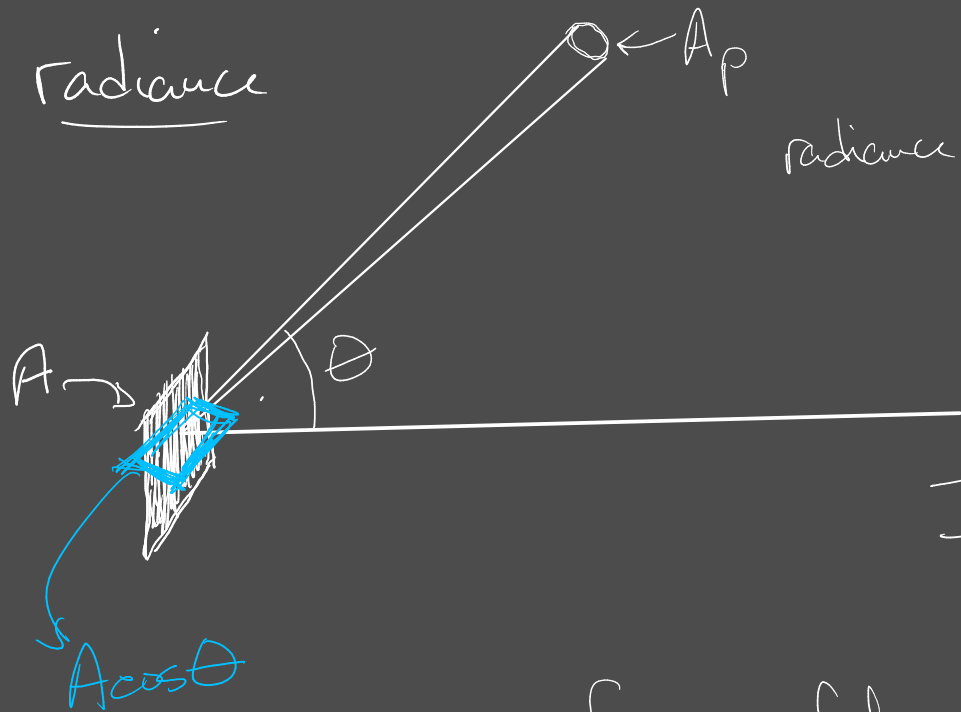
$$\omega = \frac{A}{r^2}$$

Solid angle
[steradians]

full sphere

$$\omega = 4\pi$$

Radiance



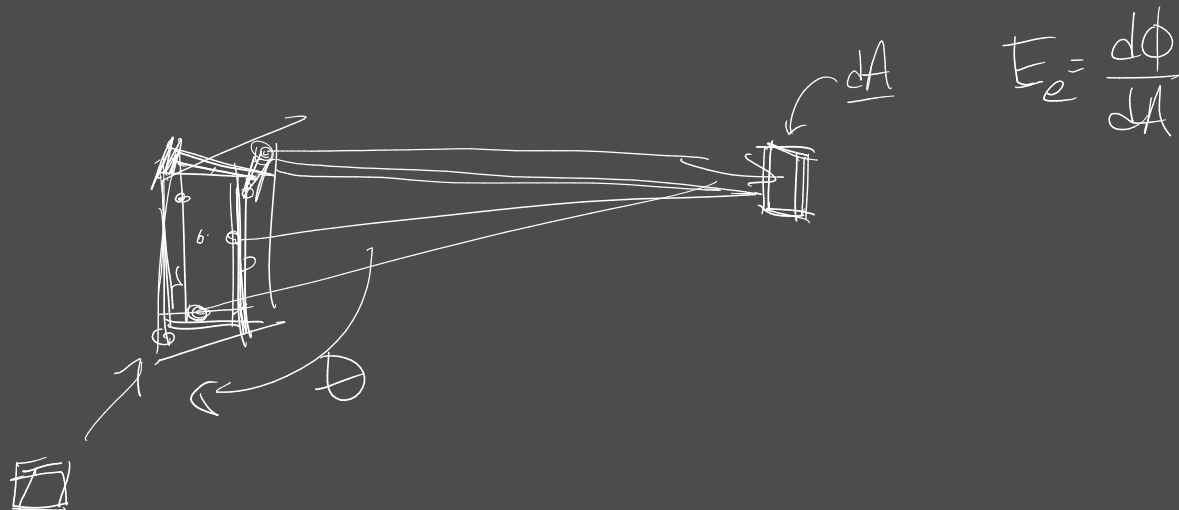
radiance, $L_e = \frac{dI_e}{d(A \cos \theta)}$

$I(\theta) = I(0) \cos \theta \leftarrow \text{Lambert's Law}$

↑
max
intensity

for a perfectly
diffuse emitter

$L_e = \frac{I_0 \cancel{\cos \theta}}{A \cancel{\cos \theta}} = \frac{I_0}{A} \leftarrow \text{constant}$



HW: 1, 2, 3, 7, 8, 10

8 Show that the relativistic kinetic energy,

$$E_K = mc^2(\gamma - 1)$$

reduces to the classical expression $\frac{1}{2}mv^2$, when $v \ll c$.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2}$$

$$E_K = \gamma mc^2 - mc^2 \longrightarrow E_K = \frac{1}{2}mv^2$$

$$= \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} mc^2 - mc^2$$

$$v \ll c$$

$$\frac{v}{c} \ll 1$$

$$\left(1 \pm x\right)^n = 1 + \underbrace{\frac{nx}{1!}} + \frac{n(n-1)x^2}{2!} + \dots$$

$$\approx \left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right) mc^2 - mc^2$$

$$\cancel{mc^2} + \frac{1}{2} m \cancel{v^2} \frac{c^2}{c^2} - \cancel{mc^2} = \frac{1}{2} mv^2$$

