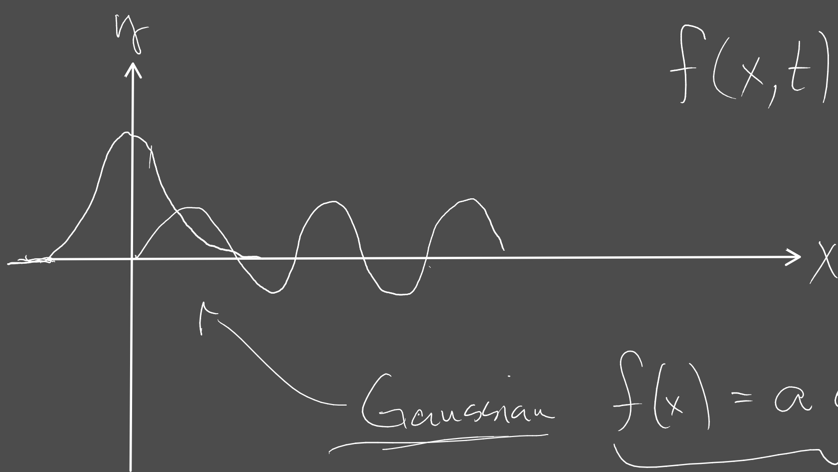
Chapter 4



$$f(x,t) = f(x - vt)$$

speed of wave

Gaussian $\underbrace{f(x) = a e^{-kx^2}}$

Gaussian wave $\rightsquigarrow f(x,t) = a e^{-k(x-vt)^2}$

Sine wave

$$f(x) = a \sin(kx) \rightsquigarrow f(x,t) = a\sin\left(k(x-vt)\right)$$

move to the right in time

$$\frac{dy}{dx} = y(x)$$

$$y = e^x$$

$$\frac{d^2y}{\partial x^2} = -y(x)$$

$$\boxed{F_{NET} = ma}$$

$$F_{NET} = m\frac{d^2x}{dt^2}$$

$$\frac{Gm_1m_2}{x^2} = m_2\frac{d^2x}{dt^2}$$

$x(t) =$ ______

Poisson's Eq

$$\nabla^2 f = g(r)$$

$$\nabla^2 f = 0$$

Laplace: $\frac{d^2f}{dx^2} = 0$

Heat: $\frac{d^2f}{dx^2} = \frac{df}{dt}$

Wave: $\frac{\partial^2 f}{\partial x^2} = \mp\frac{1}{v^2}\cdot\frac{\partial^2 f}{\partial t^2}$

$$y = f(x \mp vt) = f(x') \qquad x' = x - vt$$

1st derivation:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1} = \frac{\partial f}{\partial x'}$$

2nd derivation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'}\right)$$

$$\frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v} = \frac{\partial f}{\partial x'} \cdot (\mp v)$$

$$= \mp v \cdot \frac{\partial f}{\partial x'}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$$\mp v \frac{\partial f}{\partial x'}$$

$$= \mp v \frac{\partial\left(\partial f / \partial x'\right)}{\partial x'} \cdot \mp v$$

$$\frac{\partial^2 f}{\partial t^2} = \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

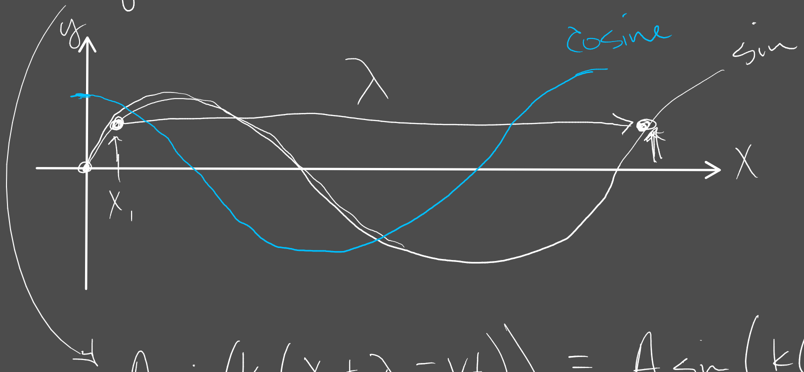
$$\boxed{\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}}$$

Harmonic Waves

$$y = A\sin(k(x - vt))$$ → form a complete basis set

$$f(x,t) = \sum_{n=0}^{\infty} A_n \sin(k_n(x - vt))$$

Fourier Series



$$A\sin(k(x + \lambda - vt)) = A\sin(k(x - vt) + 2\pi)$$

$$k(x - vt) + k\lambda$$

$$k\lambda = 2\pi$$

$$\boxed{k = \frac{2\pi}{\lambda}} \Rightarrow \lambda = \frac{2\pi}{k}$$

← propogation constant

wave number → special frequency

$$\boxed{\kappa = \frac{1}{\lambda}}$$

↑ kappa

similar thing w/ time → T (period)

$$kvT = 2\pi$$

$$v = \underbrace{\frac{2\pi}{k}}_{\lambda} \cdot \underbrace{\frac{1}{T}}_{\nu}$$

$$\boxed{\nu = \frac{1}{T}}$$

← frequency "nu"

$$\boxed{v = \lambda\nu}$$

also,

$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

↖ angular frequency

$$\boxed{v = \frac{\omega}{k}}$$

$$y = A\sin(k(x - vt)) \leftarrow$$

$$y = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right) \leftarrow$$

$$y = A\sin(\underbrace{kx - \omega t}_{\text{argument or phase}}) \leftarrow$$

$$y = A\sin(kx - \omega t + \phi_0)$$

↑ initial phase or phase shift

Complex Numbers

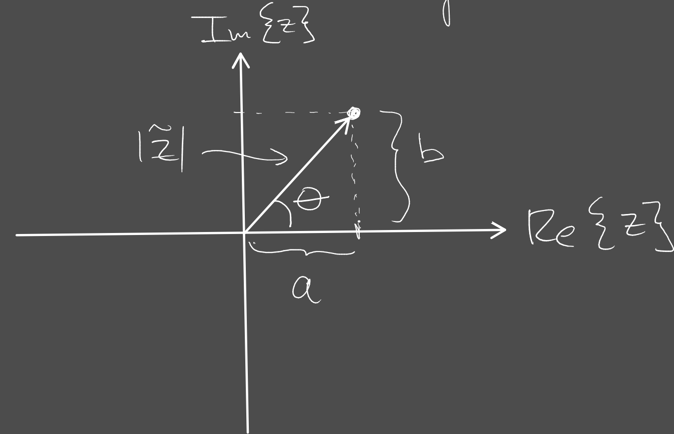
$i = \sqrt{-1}$ or $i^2 = -1$

also imaginary $\hookrightarrow j$ and in python

$\rightarrow \tilde{z} = a + ib$

real part

imaginary part



$|\tilde{z}|^2 = a^2 + b^2$

$a = |\tilde{z}|\cos\theta$

$b = |\tilde{z}|\sin\theta$

$\tilde{z} = |\tilde{z}|(\cos\theta + i\sin\theta)$

$\boxed{e^{i\theta}} = \cos\theta + i\sin\theta$

$\tilde{z} = \underbrace{|\tilde{z}|e^{i\theta}}_{\text{polar coordinates}}$

3blue 1 brown, numberphile

$2^3 = 2 \cdot 2 \cdot 2$

$2^{3/2} = \sqrt{(2 \cdot 2 \cdot 2)}$

$e^x \rightarrow \underbrace{1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{6} \cdots \frac{x^n}{n!}}$

$x \rightarrow$ complex

Euler Formula

$\theta = \pi$

$e^{i\pi} = -1$

$\boxed{e^{i\pi} + 1 = 0}$ Euler's identity

$e^1 \cdot e^2 = e^3$

$\tilde{z}^* = a - bi \leftarrow$ complex conjugate $\longrightarrow \tilde{z}^* = |\tilde{z}|e^{-i\theta}$

$|\tilde{z}|e^{-i\theta} \cdot |\tilde{z}|e^{i\theta}$

$|\tilde{z}|^2$

$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 \leftarrow$ real # + magnitude squared

$$\rightarrow \quad \mathrm{Re}\{\tilde{z}\} = \frac{\tilde{z} + \tilde{z}^*}{2} \qquad \cos\theta = \mathrm{Re}\{e^{i\theta}\} = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad e^{i\theta} = \cos\theta + i\sin\theta$$

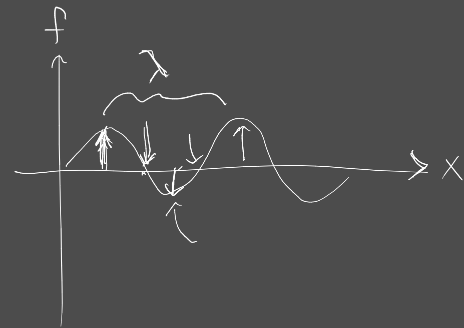
$$\rightarrow \quad \mathrm{Im}\{\tilde{z}\} = \frac{\tilde{z} - \tilde{z}^*}{2i} \qquad \sin\theta = \mathrm{Im}\{e^{i\theta}\} = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\tilde{y} = Ae^{i(kx-\omega t)} = A\cos(kx-\omega t) + Ai\sin(kx-\omega t)$$

$$y = \mathrm{Re}\{\tilde{y}\} = A\cos(kx-\omega t) \quad \text{or} \quad y = \mathrm{Im}\{\tilde{y}\} = A\sin(kx-\omega t)$$

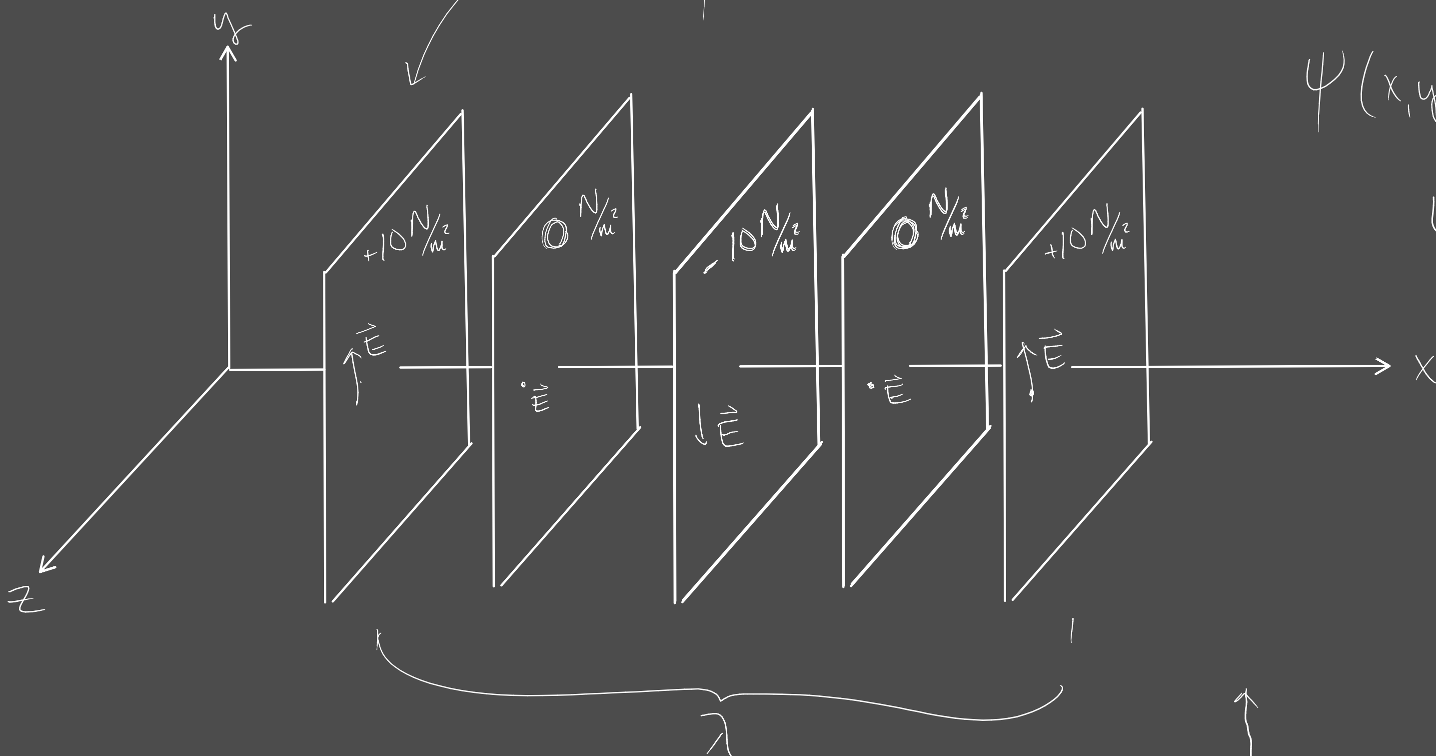
Plane waves

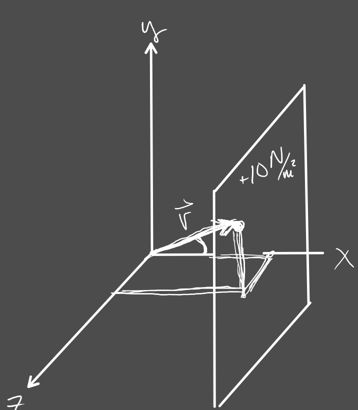
$$\underbrace{f(x,t) = A\sin(kx - \omega t)}$$



$$\psi(x,y,z,t) = A\sin(? - \omega t)$$

$$\psi(x,y,z,t=1) = A\sin(\_)$$

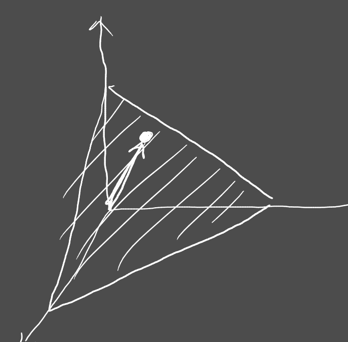




$\sin(\boxed{kx})$

$\sin(ky$

$kx = \underbrace{kr\cos\theta}$



$$|\vec{k}| = \frac{2\pi}{\lambda} \qquad \vec{k} = k_x\hat{x} + k_y\hat{y} + k\hat{z}$$

$$\psi(\vec{r},t) = A\sin(\vec{k}\cdot\vec{r} - \omega t) \leftarrow \text{plane wave}$$

$$\psi(\vec{r},t) = A e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\boxed{\nabla^2 \psi = \mp \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}}$$

scalar function

$$\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} = \underbrace{\vec{\nabla} f}_{\text{vector function}} \leftarrow \text{gradient}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \nabla^2 f$$

Laplacian

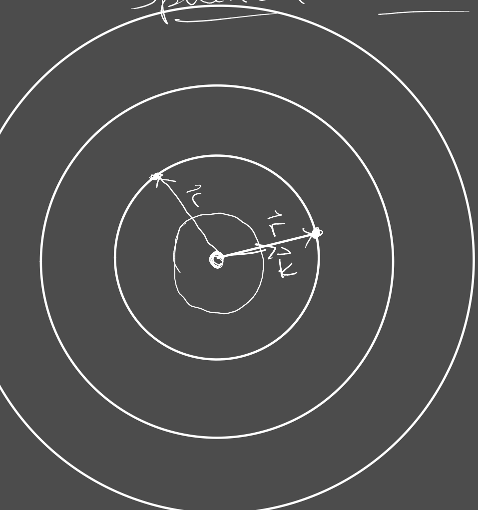
$$\vec{\nabla} \cdot \vec{F}$$

divergence $\vec{\nabla} \cdot \vec{\nabla} f = \nabla^2 f$ ✓

$$\vec{\nabla} \times \vec{F} \leftarrow \text{curl}$$

$$\underbrace{\vec{\nabla} \times \vec{\nabla} f}_{\text{vector}} \neq \underbrace{\nabla^2 f}_{\text{scalar}}$$

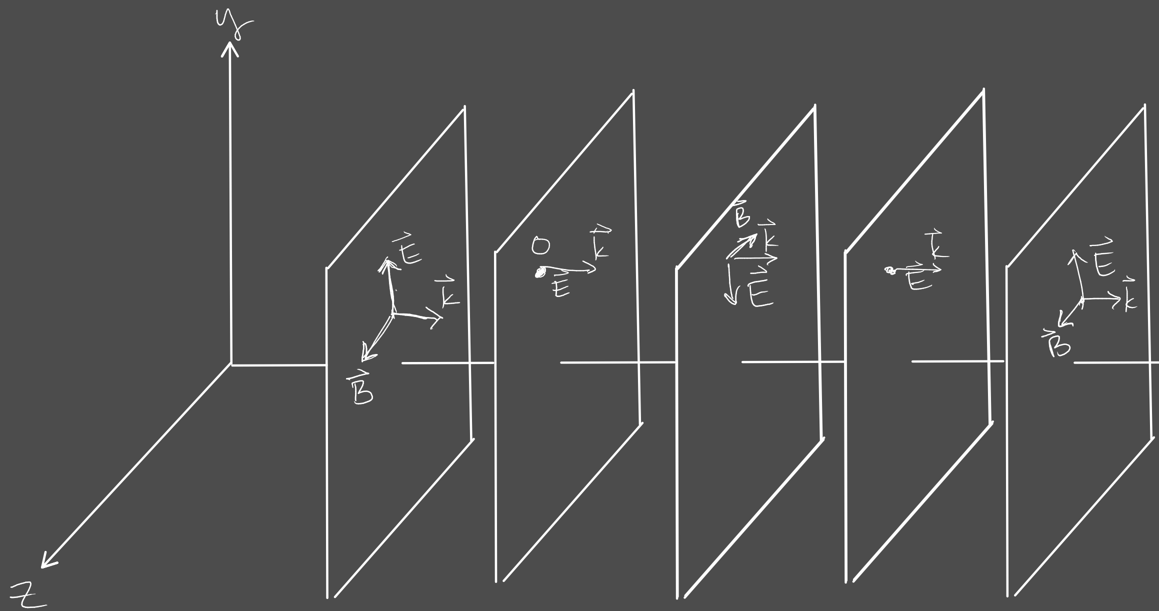
Spherical Wave



$$\psi(\vec{r}, t) = \frac{A}{r} \sin(kr - \omega t)$$

$$\psi(\vec{r}, t) = \underbrace{\frac{A}{r} e^{i(kr - \omega t)}}$$

consistent w/ $1/r^2$ irradiance



$$\vec{E} = E_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$\vec{B} = B_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$\boxed{\vec{v} \parallel \vec{E} \times \vec{B}}$$

$\vec{v} \parallel \vec{k}$

point in the same direction

$$c = \frac{|E|}{|B|}$$

$$E = c \cdot B$$

$$c = 2.998 \cdot 10^8 \, m/s$$

$$\left.\begin{aligned} \epsilon_0 &= 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \\ \mu_0 &= 4\pi \cdot 10^{-7} \frac{A^2}{Nm^2} \end{aligned}\right\} E + M$$

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}}$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

Maxwell's Eqn

$$\underbrace{\nabla^2 E = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E}{\partial t^2}}$$