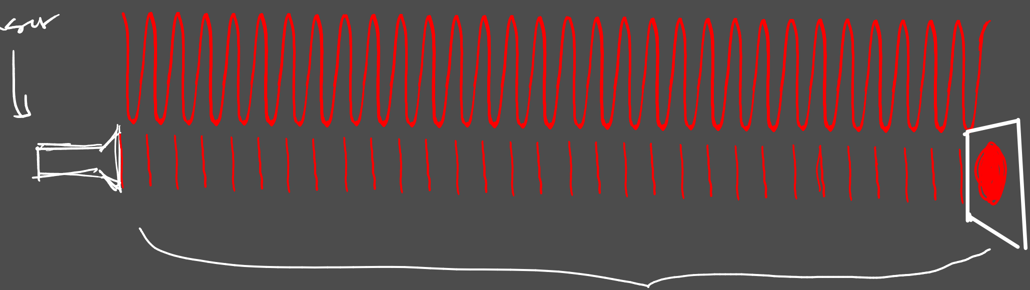
Diffraction and Interference - Chapter 25

Coherent light - single wavelength
- every part of the beam is "in phase" or lined up with itself

laser



10.1 m

$\lambda = 0.33\ m$

How many λ's fit b/t the laser + the screen?

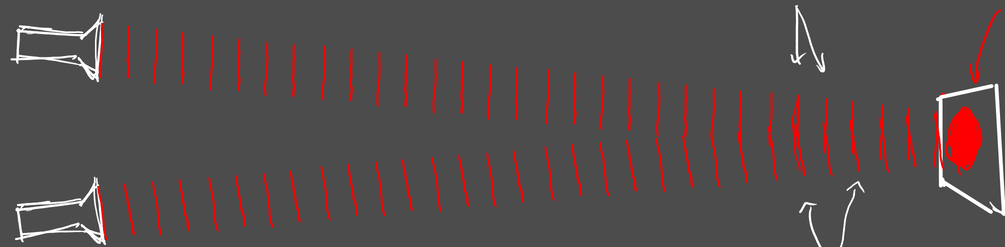
$$\frac{\text{total distance}}{\text{a wavelength}} = \#\text{ of wavelengths} = \frac{10.1\ m}{0.33\ m} = 30.6 \approx 31$$

counted

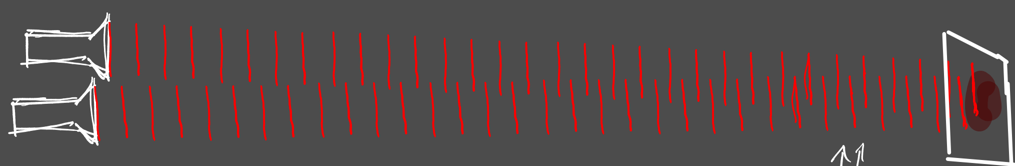
0 1 2 3 4 5 6 7 8 9 10 11 12

"in phase"

twice as bright



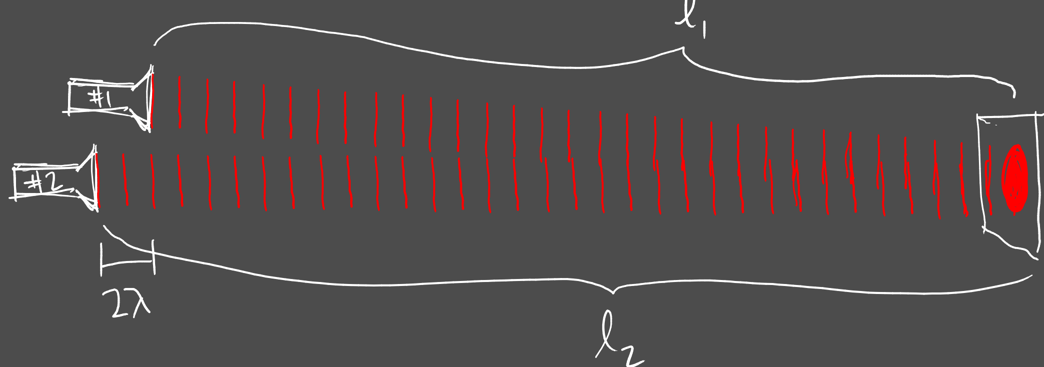
constructive interference



spot on the screen is diminished or totally dark.

"180° out of phase"

destructive interference



Constructive interference

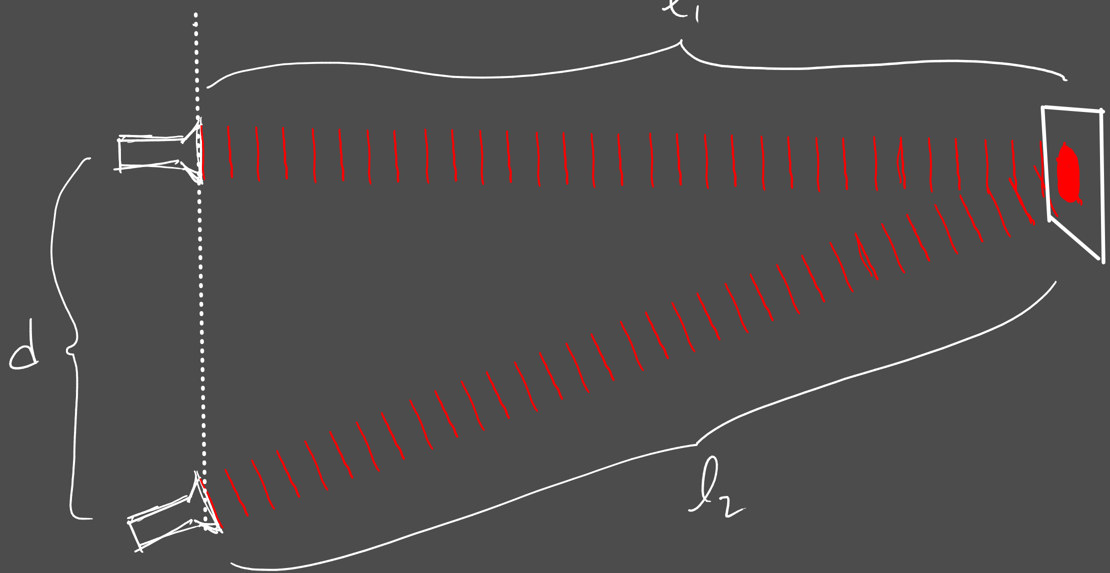
$$\Delta l = l_2 - l_1 = m \cdot \lambda$$

$m = 0, 1, 2, 3, \ldots$

destructive interference

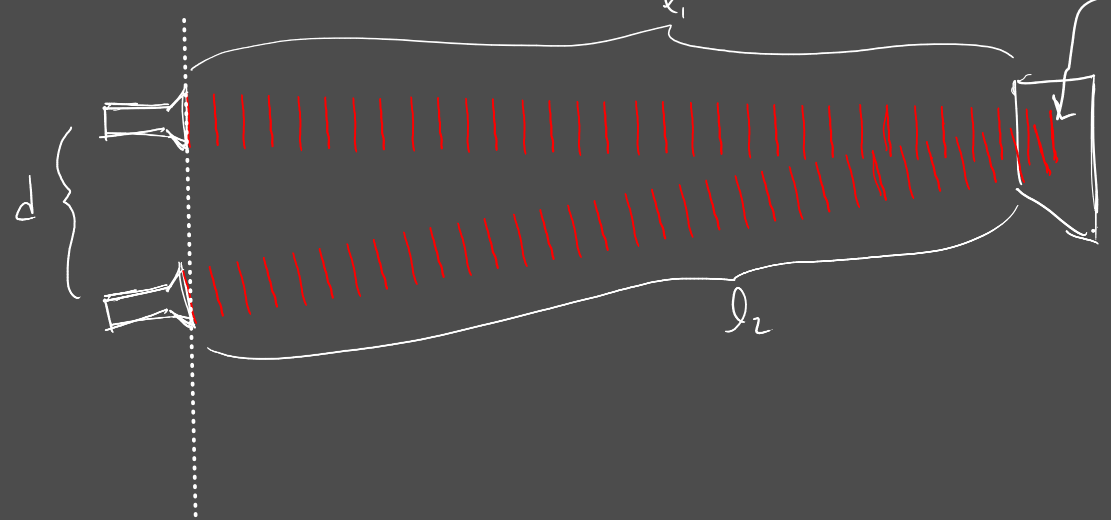
$$\Delta l = l_2 - l_1 = \left(m + \tfrac{1}{2}\right) \cdot \lambda$$

$m = 0, 1, 2, 3 \ldots$



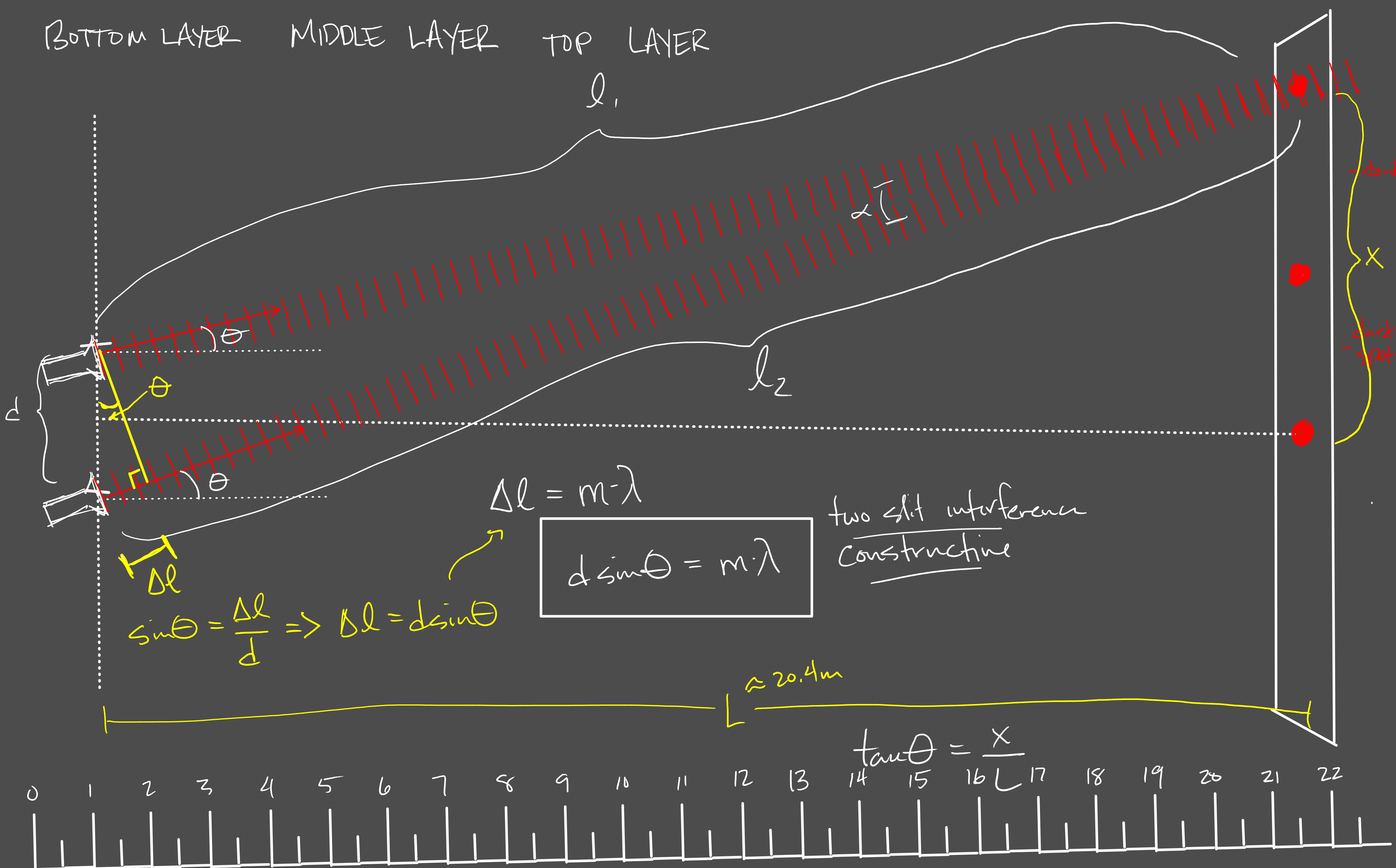
constructive interference

$$\ell_2 - \ell_1 = m \cdot \lambda$$



destructive interference

$$\ell_1 - \ell_2 = (m + \tfrac{1}{2})\lambda$$



$x = L\tan\theta$

$x = 20.4\tan(15.3^\circ)$

$x = 5.6\text{ m} \approx 5.9\text{m}$

$\sin\theta = \frac{m\lambda}{d}$

$\theta = \sin^{-1}\left(\frac{2\cdot .33\text{m}}{2.5\text{m}}\right) = 15.3^\circ$

$l_1 = 20.8\text{ m}$ $\quad l_2 = 21.5$

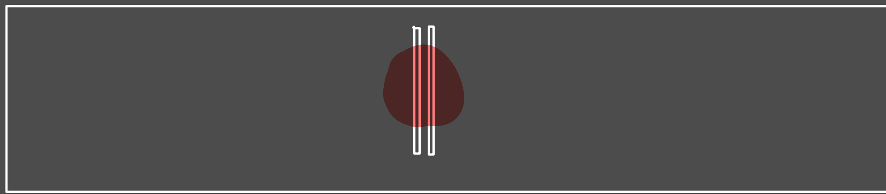
$l_2 - l_1 = 0.7\text{m} \stackrel{?}{=} 2\cdot\lambda$

$3\lambda = 1\text{m}$

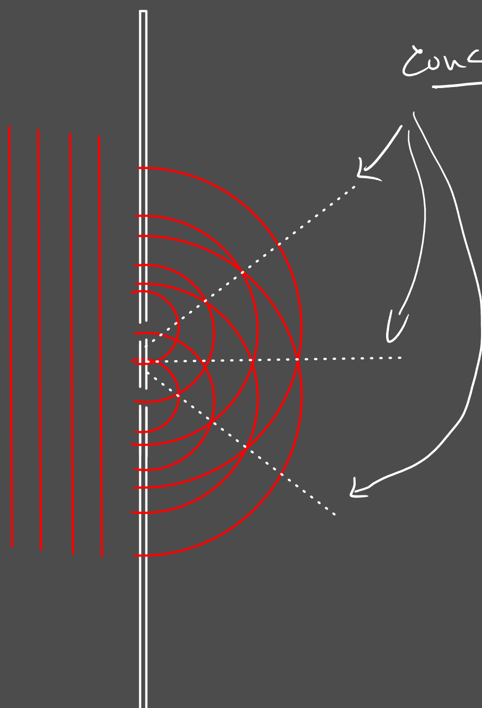
$\lambda = \frac{1}{3}\text{m} = .33$

$.7 \approx .67$

double slit



constructive interference



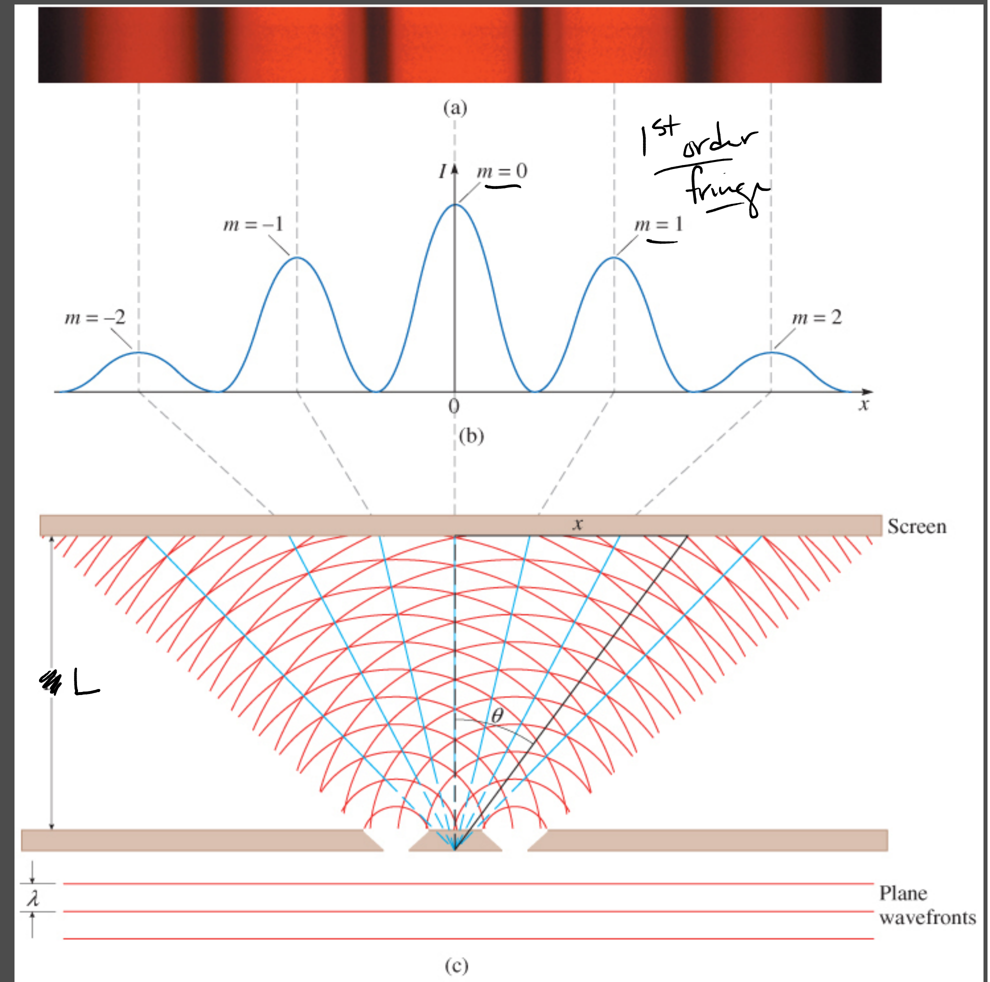
$$d \sin\theta = m\lambda$$

$$\tan\theta = \frac{x}{L}$$

small angle approximation

when θ is small

$$\tan\theta \approx \sin\theta \approx \theta$$

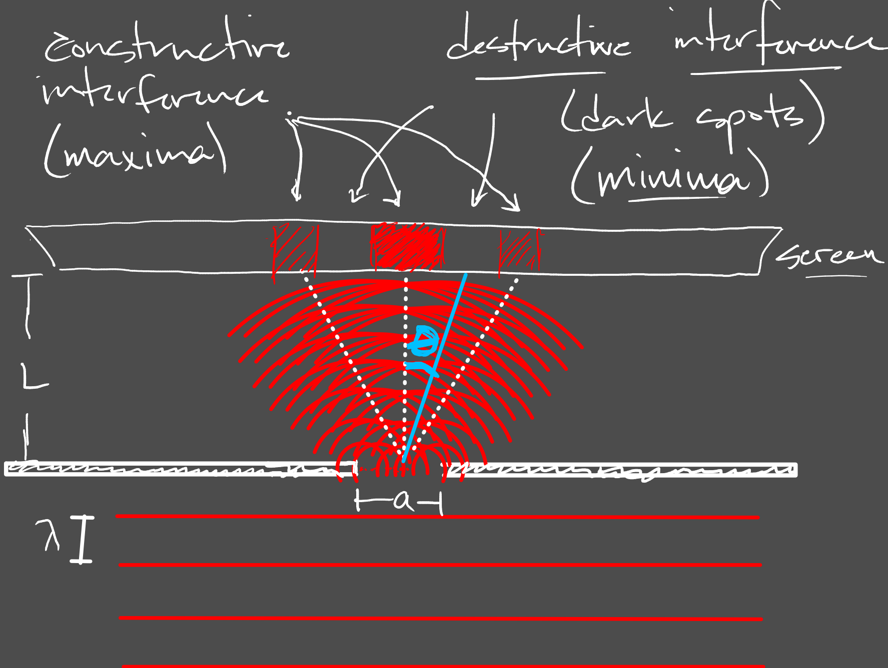


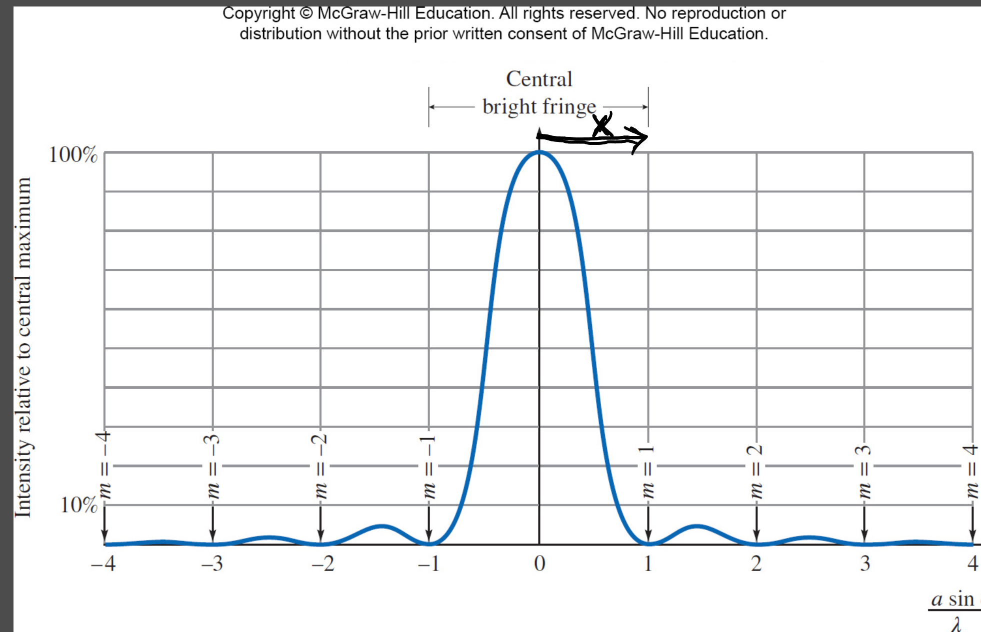
$$d \sin\theta = m\lambda$$

$$\boxed{d \cdot \frac{x}{L} = m\lambda}$$

double slit constructive interference small angle approximation

Single Slit Diffraction / Interference





This pattern also results from a small obstruction to the light beam

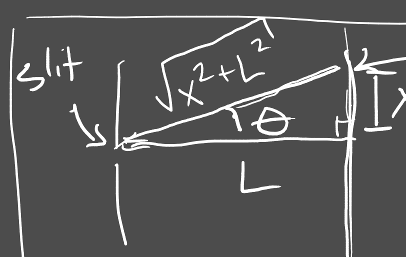
locations of the minima:

$$\boxed{a \cdot \sin\Theta = m\lambda}$$

$\sin\Theta$ → location from central axis to minima

m → fringe from the center

a → slit width



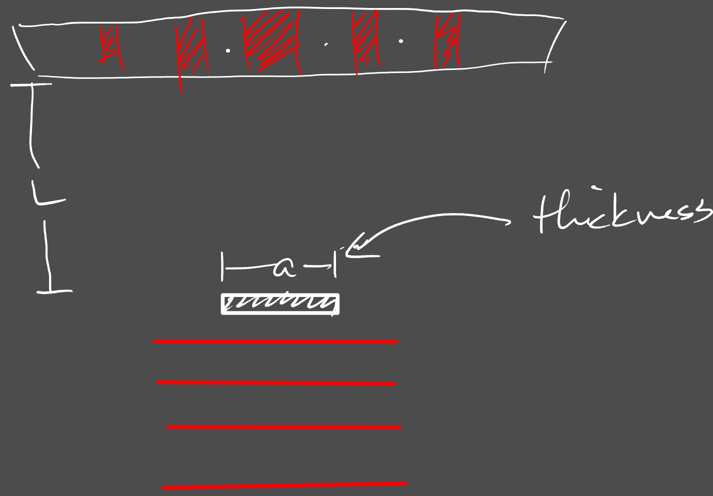
small angle approximation

$$\left.\sin\Theta = \frac{x}{L}\right\} \quad \frac{a\,x}{L} = m\lambda$$

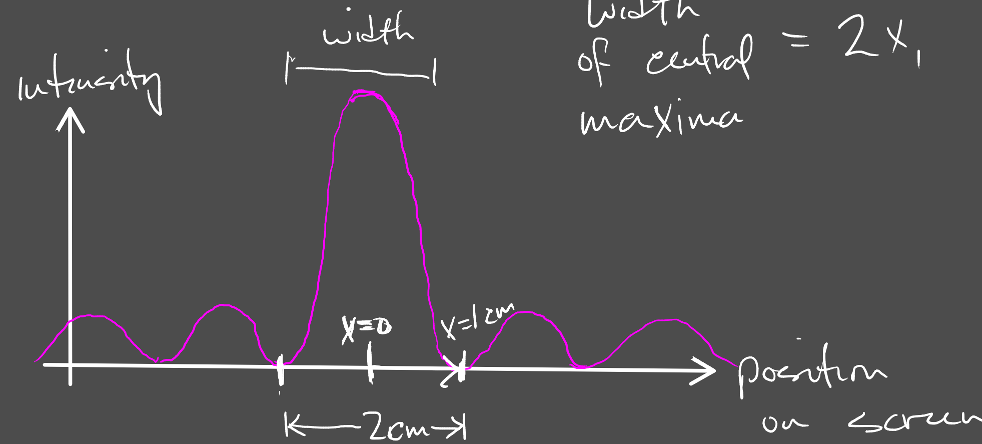
not small angle approximation

$$\sin\Theta = \frac{x}{\sqrt{x^2+L^2}}$$

$$\frac{a \cdot x}{\sqrt{x^2+L^2}} = m\lambda$$



The diffraction pattern from a single slit is viewed on a distant screen. Using violet light, the width of the central maximum is 2.0 cm. (a) Would the central maximum be narrower or wider if red light is used instead? (b) If the violet light has wavelength 0.43 μm and the red light has wavelength 0.70 μm, what is the width of the central maximum when red light is used?



width of central maxima $= 2x_1$

$$\frac{ax}{L} = m\lambda$$

$$x = \frac{m\lambda \cdot L}{a}$$

location of minima

from violet to red

λ increases

400 nm → 700 nm

width $\propto x \propto \lambda$

$w \propto x \propto \lambda$

$w \propto \lambda$

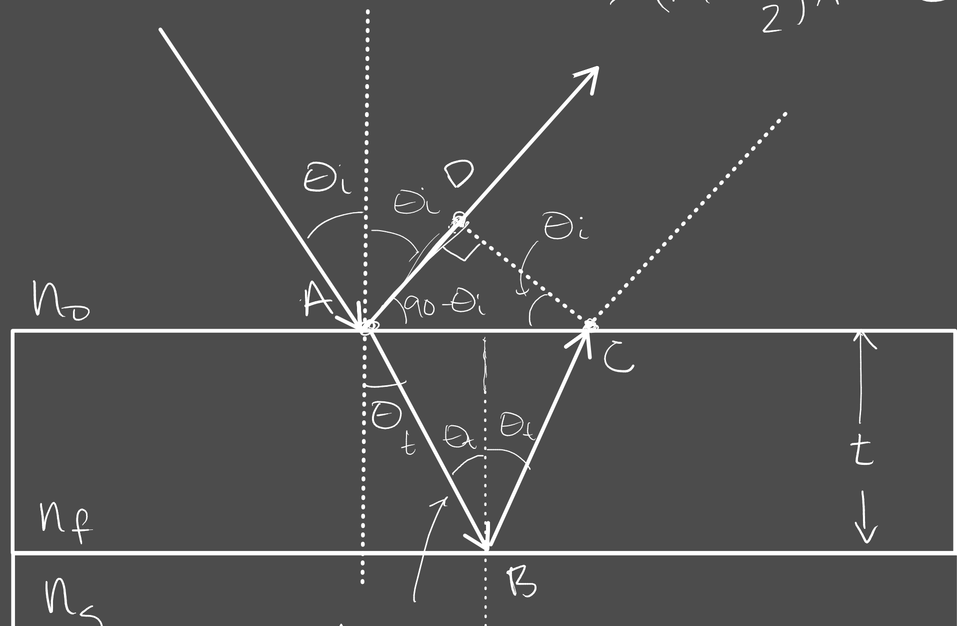
$$\frac{W_{red}}{W_v} = \frac{\lambda_{red}}{\lambda_v}$$

$$W_{red} = 2.0\,\text{cm} \cdot \left(\frac{.70\,\cancel{\mu m}}{.43\,\cancel{\mu m}}\right)$$

$$\underline{\underline{W_{red} = 3.2\,\text{cm}}}$$

path length difference $\rightarrow m\lambda$ constructive interference

path length difference $\rightarrow (m+\frac{1}{2})\lambda$ destructive interference



$$\frac{t}{path} = \cos\theta_t$$

$$path = \frac{t}{\cos\theta_t}$$

$$\frac{AC}{2\cdot t} = \tan\theta_t$$

$$AC = 2t\cdot\tan\theta_t$$

optical path length $\rightarrow n\cdot l$

$$\Delta \text{ path length} = n\left(\frac{2t}{\cos\theta_t}\right) - n_0(\overline{AD})$$

$$AD = AC\sin\theta_i$$

Snell's $\rightarrow$ $n_0\sin\theta_i = n_f\sin\theta_t$

$$\sin\theta_i = \frac{n_f}{n_0}\sin\theta_t$$

$$AD = AC\,\frac{n_f}{n_0}\sin\theta_t$$

$$AD = \frac{n_f}{n_0}\cdot 2t\cdot\sin\theta_t\tan\theta_t$$

$$AD = \frac{n_f}{n_0}\cdot 2t\cdot\frac{\sin^2\theta_t}{\cos\theta_t}$$

$$\Delta = \frac{2n_f t}{\cos\theta_t} - \cancel{n_0}\left(\frac{n_f}{\cancel{n_0}}\cdot 2t\cdot\frac{\sin^2\theta_t}{\cos\theta_t}\right)$$

$$\Delta = \frac{2n_f t}{\cos\theta_t}\underbrace{\left(1 - \sin^2\theta_t\right)}_{\cos^2\theta_t}$$

optical path length difference

$$\Delta = 2nt\cos\theta_t$$

$$\Delta_p + \Delta_R = \begin{cases} m\lambda & \text{const} \\ (m+\frac{1}{2})\lambda & \text{destr.} \end{cases}$$

phase shift from reflection

<u>BUT</u> <u>WAIT</u>!

- $\frac{\lambda}{2}$ → ~~phase~~ path <u>shift</u> from reflection higher index to a lower index material

- π → phase shift (relative) from internal and external reflections

$$\sin\left(k(x - vt) + \phi_R\right)$$

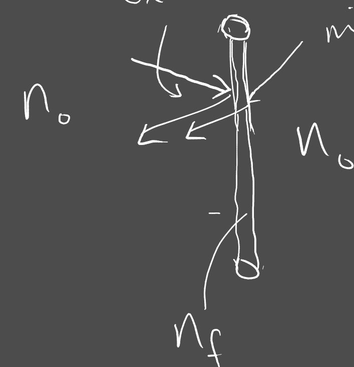
$$\sin\left(kx - \omega t + \phi_R\right)$$

$$\phi_R = \pi$$

$n_o < n_f$

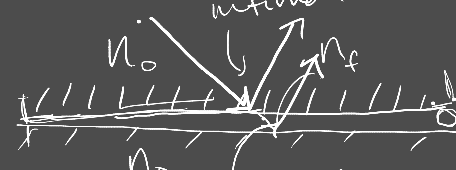
external reflection

internal reflection



$\underbrace{\text{internal reflection}}$. $\frac{\lambda}{2}$ shift

internal reflection



$n_o > n_f$

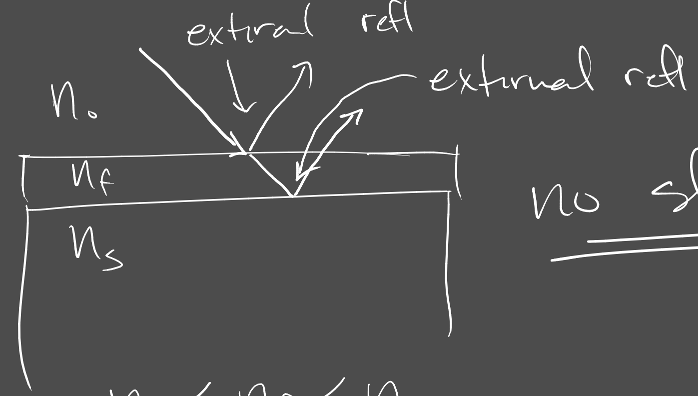
→ $\frac{\lambda}{2}$ shift

$$\sin(kx - \omega t + \pi)$$

$$\frac{k}{k} \qquad k \leftarrow \frac{2\pi}{\lambda}$$

$$\sin\left(kx - \omega t + \frac{k\cancel{\pi}\lambda}{2\cancel{\pi}}\right)$$

$$\sin\left(k\left(x + \frac{\lambda}{2}\right) - \omega t\right)$$



$n_0 < n_f < n_s$

no shift