Chapter 5 - Superposition

superposition – waves at the same place and time, displacements add up.

$$\psi = \psi_1 + \psi_2 + \dots$$

- superposition of harmonic waves

$$E_1(x_1,t) = E_1 \cos(kx_1 - \omega t + \phi_1)$$

$$E_2(x_2,t) = E_2 \cos(\underbrace{kx_2 - \omega t + \phi_2})$$

$$\alpha_1 = ks_1 + \phi_1$$

$$\alpha_2 = ks_2 + \phi_2$$

$$\underbrace{\alpha_2 - \alpha_1}_{\text{phase difference}} = k(s_2 - s_1) + (\phi_2 - \phi_1)$$

what if $\alpha_2 - \alpha_1 = 2\pi m$ } even multiple of π

$$E_R = E_1 + E_2 = E_1 \cos(\alpha_1 - \omega t) + E_2 \cos(\alpha_2 - \omega t)$$

$\downarrow$ $\alpha_2 = \alpha_1 + 2\pi m$

$\cos(\alpha_1 + 2\pi m - \omega t)$

$\cos\theta = \cos(\theta + 2\pi m)$

constructive interference

$$\hookrightarrow E_R = (E_1 + E_2)\cos(\alpha_1 - \omega t)$$

what if $\alpha_2 - \alpha_1 = (2m-1)\pi$ odd multiple of π

$$E_R = E_1 + E_2 = E_1 \cos(\alpha_1 - \omega t) + E_2 \cos(\alpha_2 - \omega t)$$

$$\alpha_2 = \alpha_1 + (2m-1)\pi$$

$$-\cos\theta = \cos(\theta + (2m-1)\pi)$$

destructive interference

$$E_R = (E_1 - E_2)\cos(\alpha_1 - \omega t)$$

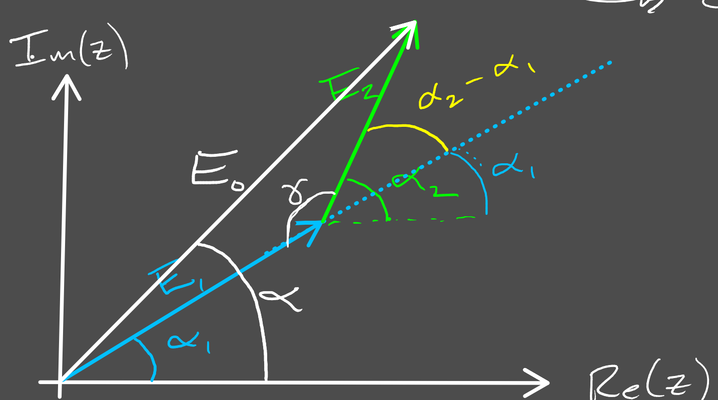
what about others?

$$E_R = \mathrm{Re}\left(E_1 e^{i(\alpha_1 - \omega t)} + E_2 e^{i(\alpha_2 - \omega t)}\right)$$

$$E_1 e^{i\alpha_1} e^{-i\omega t} + E_2 e^{i\alpha_2} e^{-i\omega t}$$

$$E_R = \mathrm{Re}\left(e^{-i\omega t}\left(E_1 e^{i\alpha_1} + E_2 e^{i\alpha_2}\right)\right)$$

"phasor diagram" → complex as vector

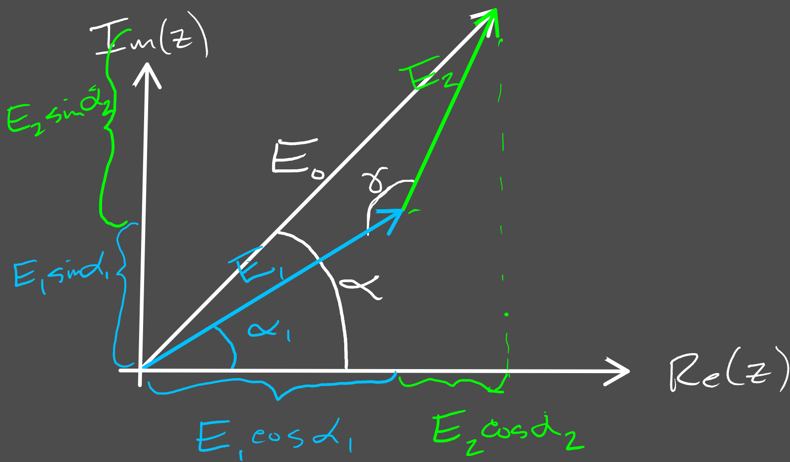


$$E_0^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\gamma$$

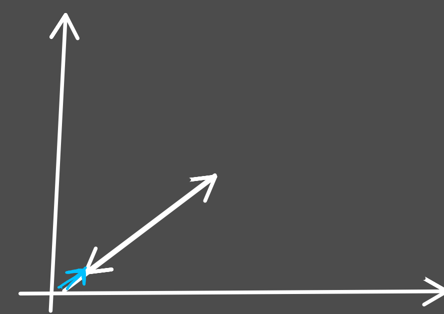
$$\gamma = \pi - (\alpha_2 - \alpha_1)$$
$$= \pi - \alpha_2 + \alpha_1$$
$$= \pi + \alpha_1 - \alpha_2$$

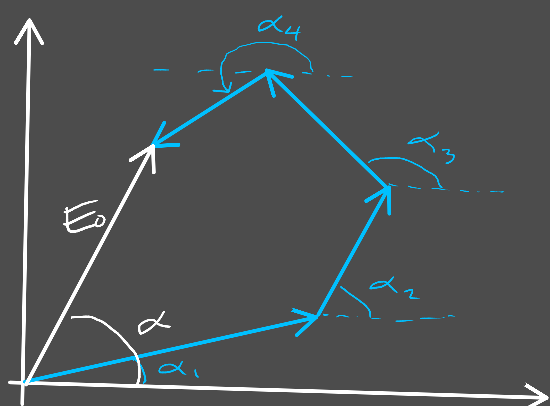
$$\cos(\pi + \alpha_1 - \alpha_2)$$
$$= -\cos(\alpha_1 - \alpha_2)$$

$$E_0^2 = E_1^2 + E_2^2 + 2E_1E_2\cos(\alpha_1-\alpha_2)$$



$$\tan\alpha = \frac{E_1\sin\alpha_1 + E_2\sin\alpha_2}{E_1\cos\alpha_1 + E_2\cos\alpha_2}$$





$$\tan\alpha = \frac{\sum_i^n E_i\sin\alpha_i}{\sum_i^N E_i\cos\alpha_i}$$

$$E_0^2 = \left(\sum_i^N E_i\cos\alpha_i\right)^2 + \left(\sum_i^n E_i\sin\alpha_i\right)^2$$

$$\left(\sum_i^N E_i\cos\alpha_i\right)^2 = \sum_i^N E_i^2\cos^2\alpha_i + \sum_i^N 2E_i\cos\alpha_i \sum_{j>i}^N E_j\cos\alpha_j$$

$$2\sum_i^N\sum_{j>i}^N E_iE_j\cos\alpha_i\cos\alpha_j$$

$$\left(\sum_i^n E_i\sin\alpha_i\right)^2 = \sum_i^N E_i^2\sin^2\alpha_i + 2\sum_i^N\sum_{j>i}^N E_iE_j\sin\alpha_i\sin\alpha_j$$

$$E_o^2 = \sum_i^N E_i^2 \cos^2\alpha_i + \sum_i^N E_i^2 \sin^2\alpha_i + \boxed{}$$

$$= \sum_i^N E_i^2 \underbrace{(\cos^2\alpha_i + \sin^2\alpha_i)}_{1}$$

$$= \sum_i^N E_i^2 + 2\sum_i^N \sum_{j>i}^N E_i E_j \underbrace{(\cos\alpha_i \cos\alpha_j + \sin\alpha_i \sin\alpha_j)}_{\cos(\alpha_j - \alpha_i)}$$

$$\boxed{E_o^2 = \sum_i^N E_i^2 + 2\sum_i^N \sum_{j>i}^N E_i E_j \cos(\alpha_j - \alpha_i)}$$