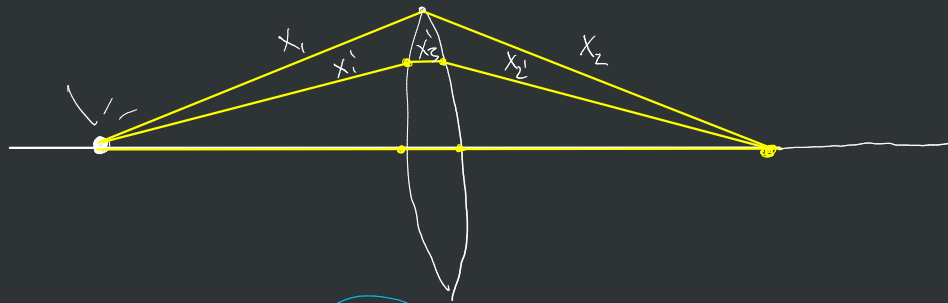
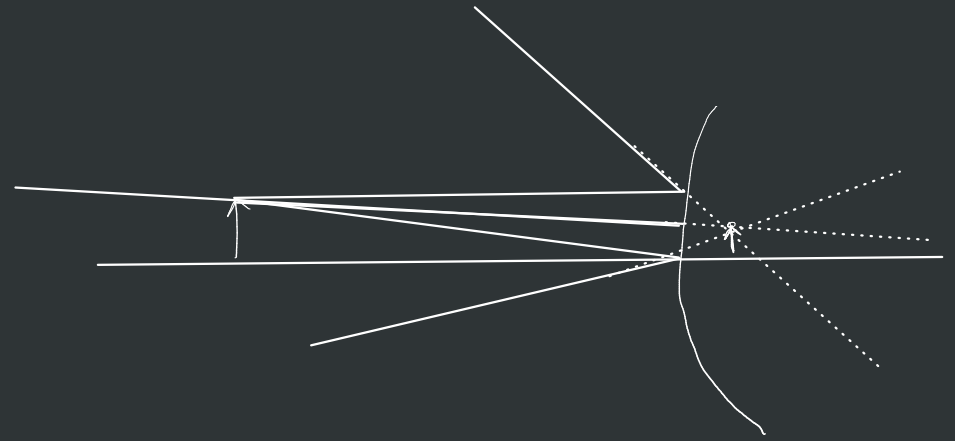
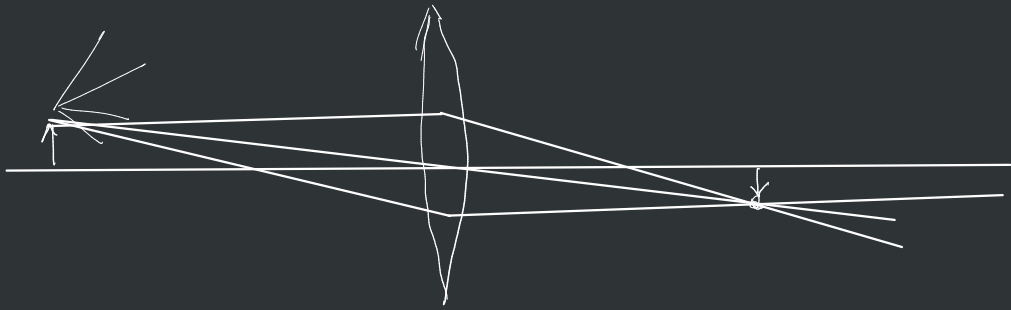


Chapter 2 problems: 3, 4, 9, 11, 13, 19, 22, one more



$$\rightarrow t = \frac{x}{v} = \frac{n \cdot x}{c}$$

$v = \frac{c}{n}$

$$\frac{n x_1}{c} + \frac{n x_2}{c} = \frac{n x_1'}{c} + \frac{n_2 x_3'}{c} + \frac{n x_2'}{c}$$

$$n x_1 + n x_2 = n x_1' + n x_2' + \underline{n_2 x_3'}$$

through the lens

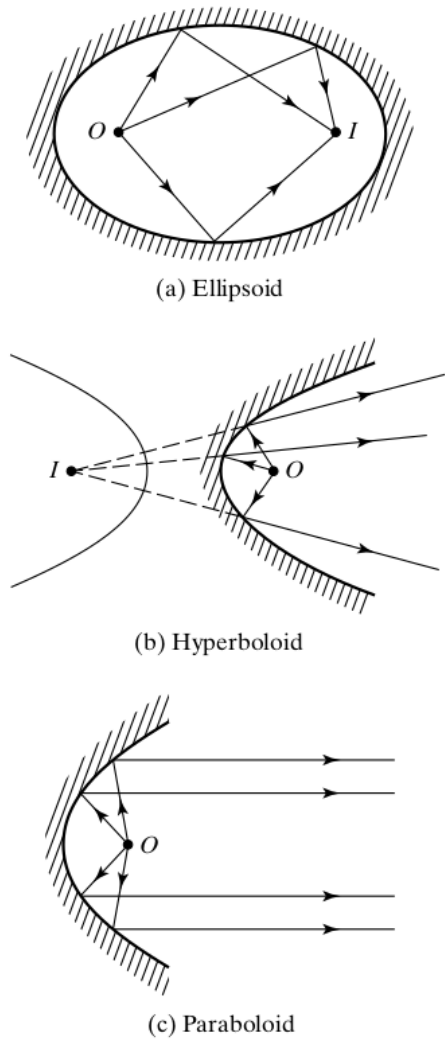
$$\rightarrow \# \text{ of wavelengths} = \frac{x_1}{\lambda} = \frac{n \cdot x_1}{\lambda_0}$$

$\lambda = \frac{\lambda_0}{n}$  ← in vacuum

$$\frac{n x_1}{\lambda_0} + \frac{n x_2}{\lambda_0} = \frac{n x_1'}{\lambda_0} + \frac{n x_2'}{\lambda_0} + \frac{n_2 x_3'}{\lambda_0}$$

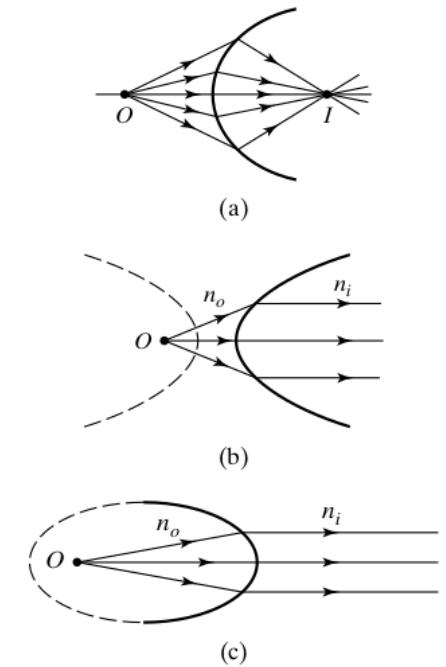
optical path length

## REFLECTION



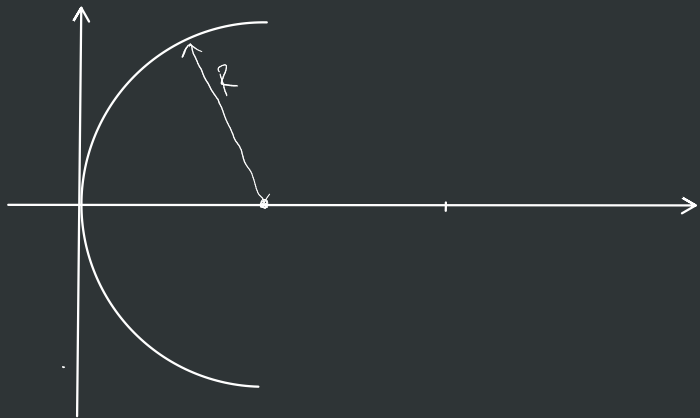
**Figure 11** Cartesian reflecting surfaces showing conjugate object and image points.

## REFRACTION



**Figure 13** Cartesian refracting surfaces. (a) Cartesian ovoid images  $O$  at  $I$  by refraction. (b) Hyperbolic surface images object point  $O$  at infinity when  $O$  is at one focus and  $n_i > n_o$ . (c) Ellipsoid surface images object point  $O$  at infinity when  $O$  is at one focus and  $n_o > n_i$ .

# → Reflection from Spherical Surfaces



$$y^2 + (x - R)^2 = R^2$$

$$y^2 + x^2 - 2xR + \cancel{R^2} - \cancel{R^2} = 0$$

$$x^2 - 2Rx + y^2 = 0$$

$$a=1 \quad b=-2R \quad c=y^2$$

$$x = \frac{+2R \pm \sqrt{4R^2 - 4y^2}}{2}$$

$$x = R \pm \sqrt{R^2 - y^2}$$

$$x < R$$

$$x = R - (R^2 - y^2)^{1/2}$$

$$x = R - (R^2 (1 - \frac{y^2}{R^2}))^{1/2}$$

$$x = R - R(1 - \frac{y^2}{R^2})^{1/2}$$

$$x \approx R - R(1 - \frac{1}{2} \cdot \frac{y^2}{R^2})$$

$$x \approx R - R + \frac{1}{2} \frac{y^2}{R}$$

$$x \approx \frac{1}{2} \frac{y^2}{R}$$

compare to

$$x = \frac{y^2}{4f}$$

$$2R = 4f \rightarrow f = \frac{R}{2}$$

binomial series

$$(1+x)^n = 1 + nx + \frac{1}{2}n(n-1)x^2 + \frac{1}{6}n(n-1)(n-2)x^3 + \dots$$

$$= \sum_{k=0}^{\infty} \binom{n}{k} x^k$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

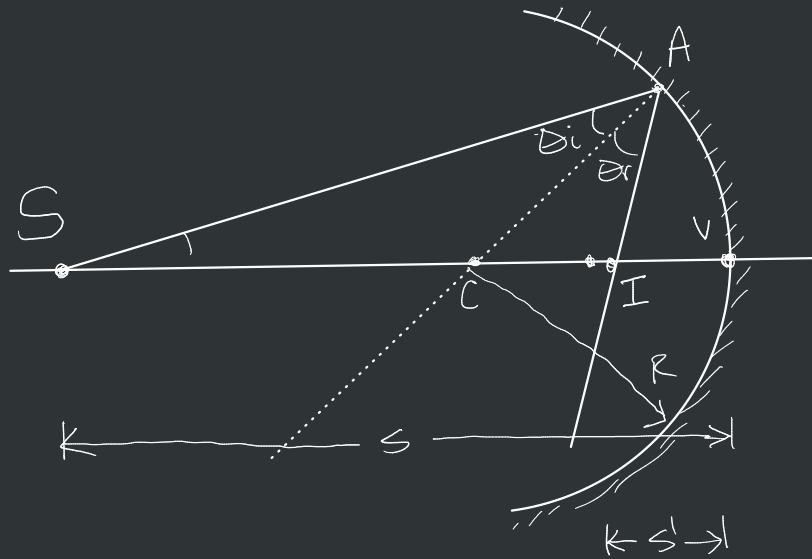
combination  
multiplicity!

paraxial region

$$x = \frac{y^2}{2R} + \Delta x$$

$$\Delta x = \frac{y^4}{8R^3} + \frac{y^6}{16R^5}$$

only significant for large values of y.



CA is an angle bisector  
 $\angle SAI$

$$\frac{SC}{SA} = \frac{CI}{IA}$$

$$SA \approx s$$

$$IA \approx s'$$

$$SC \approx s - |R|$$

$$SC = s + R$$

$$CI \approx |R| - s'$$

$$CI = -R - s'$$

$$\frac{s + R}{s} = - \frac{R + s'}{s'}$$

$$1 + \frac{R}{s} = - \frac{R}{s'} - 1$$

$$\frac{R}{s} + \frac{R}{s'} = -2$$

$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R}$$

$$f = \lim_{s \rightarrow \infty} s'$$

$$\frac{1}{s'} = -\frac{2}{R} \equiv \frac{1}{f}$$

mirror  
equation

$$f = -\frac{R}{2}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Sign convention for mirrors

- light travels from left to right
- mirror surfaces point leftward

real object,  $s > 0$ , to the left of mirror vertex

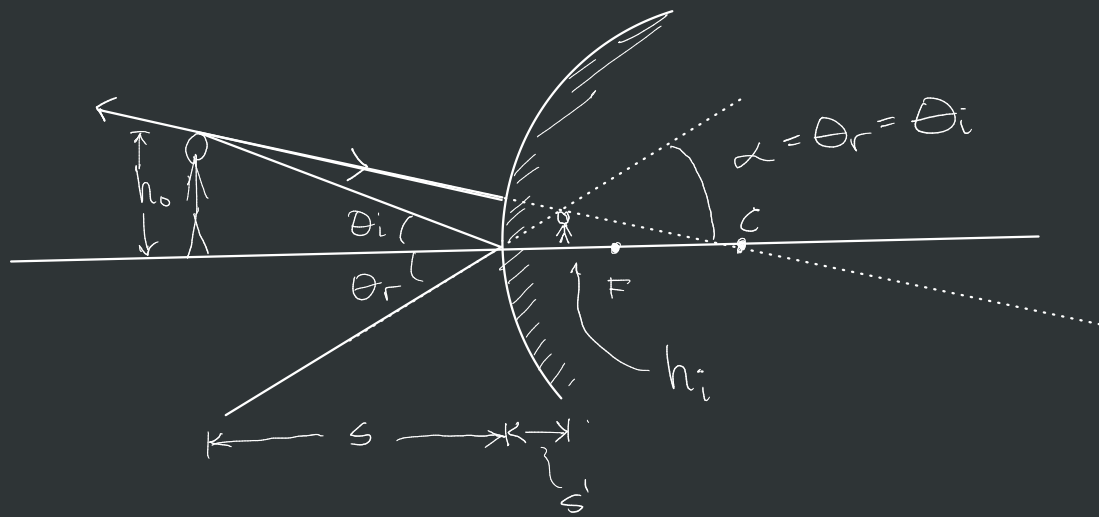
real image,  $s' > 0$ , to the left of mirror vertex

radius of curvature,  $R > 0$ , to the right of mirror vertex (convex)

$R < 0 \Leftarrow$  concave

$f > 0$  for concave

$f < 0$  for convex



$$\frac{h_o}{s} = \frac{h_i}{s'} \quad \left\{ \begin{array}{l} \text{all magnitudes} \end{array} \right.$$

$$m = \frac{h_i}{h_o} = -\frac{s'}{s} \quad \left\{ \begin{array}{l} \text{incorporates sign} \\ \text{convention} \end{array} \right.$$

magnification

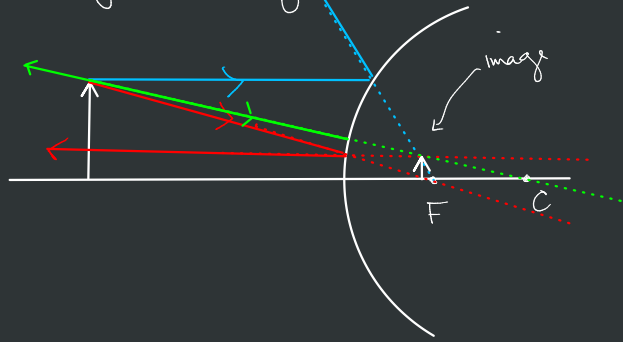
$m > 0 \rightarrow$  upright, erect

$m < 0 \rightarrow$  inverted

$0 < |m| < 1 \rightarrow$  diminished, smaller

$|m| > 1 \rightarrow$  enlarged, larger

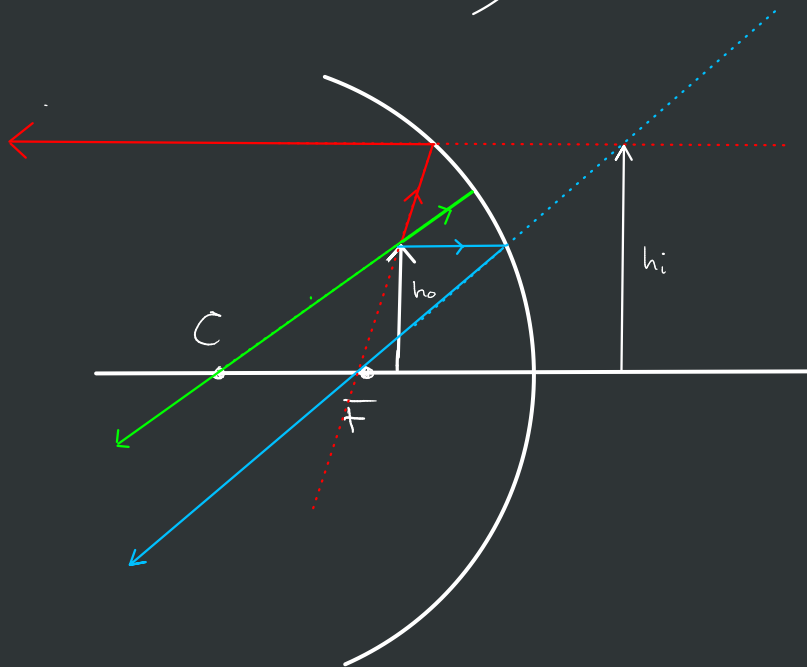
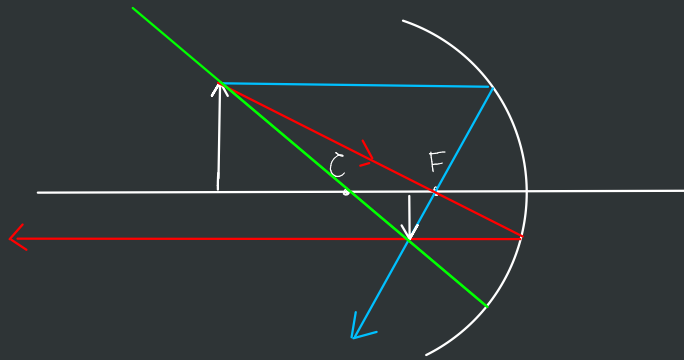
# Ray tracings



Ray 1 - parallel to axis  
goes through the focal point

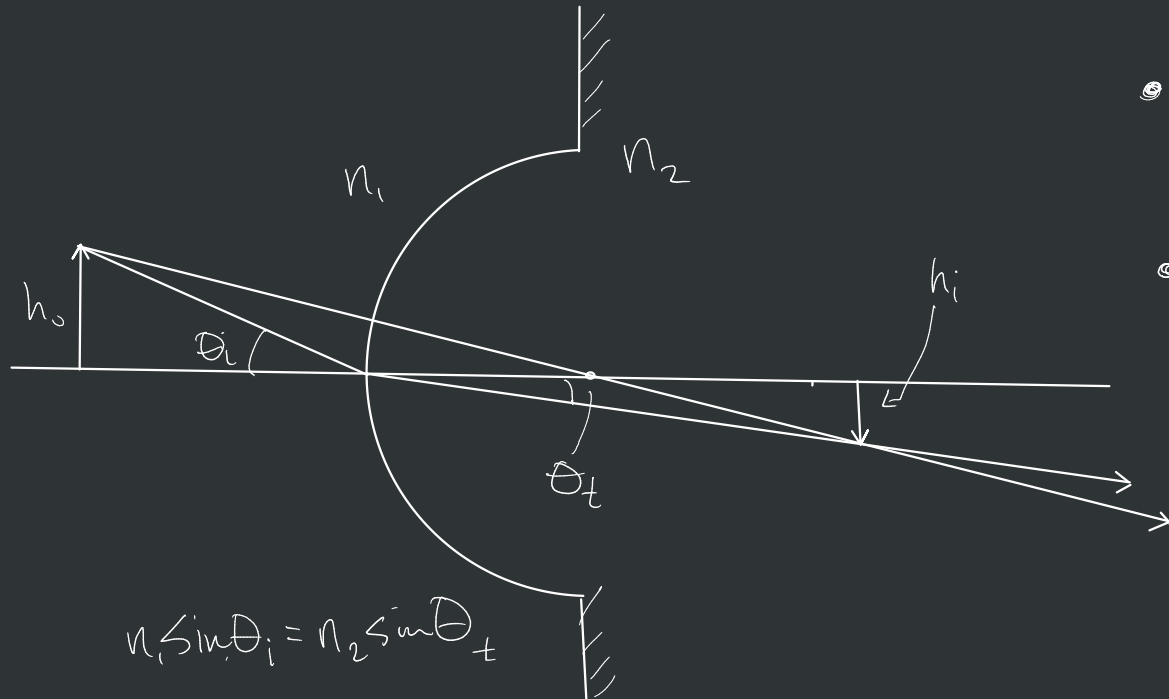
Ray 2 - go through the focal point and reflect  
parallel to the optical axis

Ray 3 - toward center of circle  
reflects on itself





# Refraction at a Spherical Surface



$$n_1 s \sin \theta_i = n_2 s' \sin \theta_t$$

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

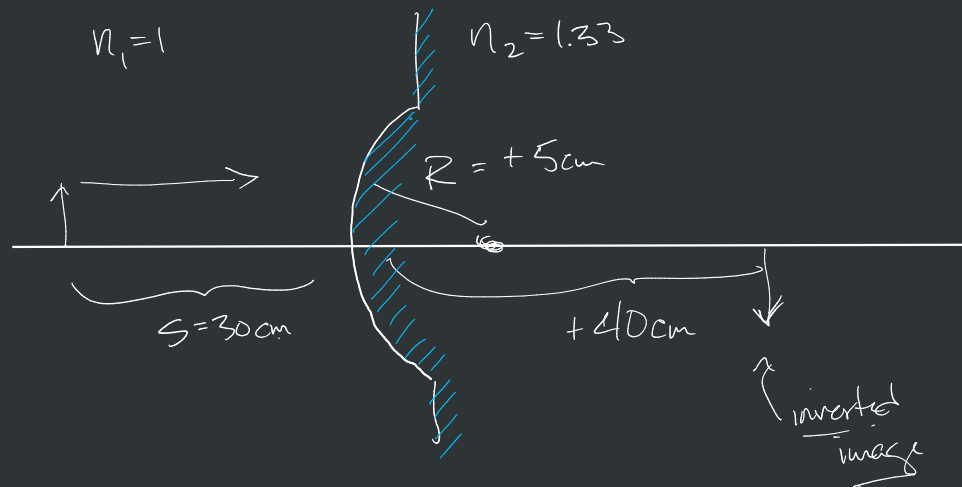
$$m = \frac{h_i}{h_0} = -\frac{n_1 s'}{n_2 s}$$

what if  $R \rightarrow \infty$

$$\frac{n_1}{s} + \frac{n_2}{s'} = 0$$

$$s' = -\left(\frac{n_2}{n_1}\right)s$$

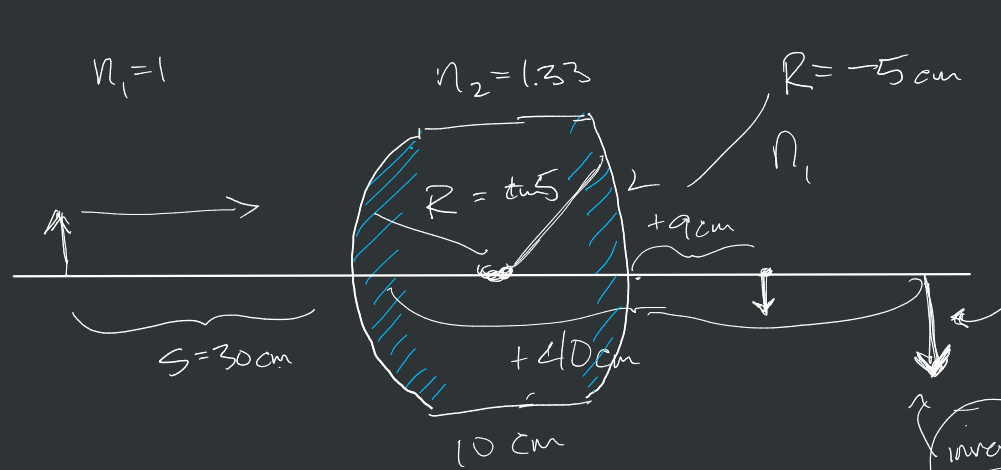
apparent depth



$$\frac{1}{30 \text{ cm}} + \frac{1.33}{s'} = \frac{1.33 - 1}{5 \text{ cm}}$$

$$s' = +40 \text{ cm}$$

$$m = -\frac{n_1 s'}{n_2 s} = \frac{-1 \cdot 40 \text{ cm}}{1.33 \cdot 30 \text{ cm}} = -1$$



$$\frac{n_1}{S} + \frac{n_2}{S'} = \frac{n_2 - n_1}{R}$$

object n: 1.33, image n: 1  
 $S = -30\text{cm}$ ,  $R = -5\text{cm}$

$$S' = +9\text{cm}$$

inverted image  
 object for the second refraction

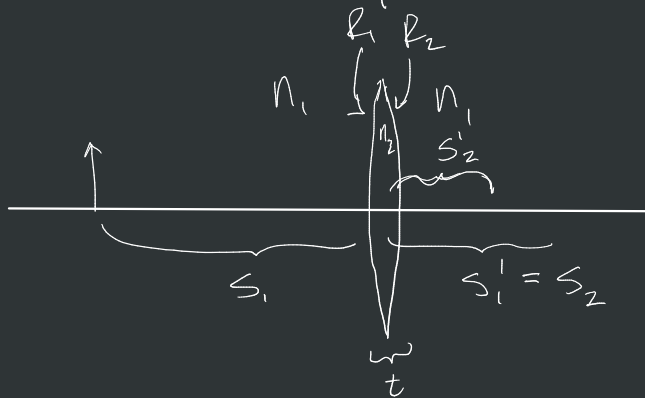
$$m_2 = -\frac{n_2}{n_1} \cdot \frac{h_i}{h_o} = +0.4$$

wrt. sign of object height

$$m_2 = \frac{h_i}{h_o} = +0.4$$

$h_i < 0$ ,  $h_o < 0$

Thin lens equation



$$\frac{n_1}{S_1} + \frac{n_2}{S'_1} = \frac{n_2 - n_1}{R_1} \quad \#1$$

$$\frac{n_2}{S_2} + \frac{n_1}{S'_2} = \frac{n_1 - n_2}{R_2}$$

$$S_2 = t - S'_1$$

thin lens  $t \approx 0$

$$S_2 = -S'_1$$

$$-\frac{n_2}{S'_1} + \frac{n_1}{S'_2} = \frac{n_1 - n_2}{R_2} \quad \#2$$

Add #1 + #2

$$\frac{n_1}{s_1} + \frac{n_1}{s'_1} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2} = \frac{n_2 - n_1}{R_1} - \frac{n_2 - n_1}{R_2} = (n_2 - n_1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{n_1}{s} + \frac{n_1}{s'} = (n_2 - n_1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

↑ object distance      ↑ final image distance

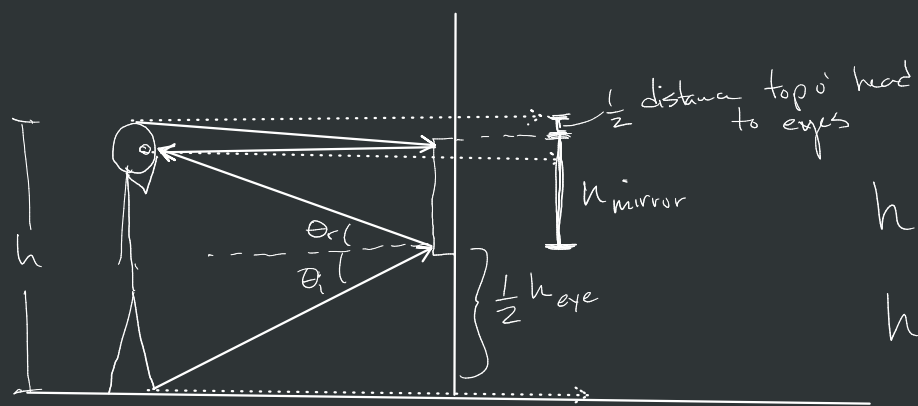
$$\frac{1}{s} + \frac{1}{s'} = \frac{(n_2 - n_1)}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

thin lens equation

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

lens makers equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$



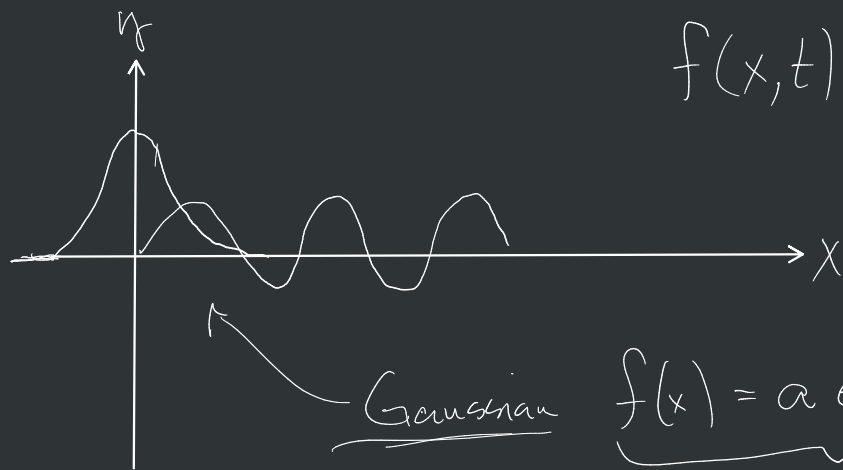
$$h = \frac{1}{2} h_{eye} + h_{mirror} + \frac{1}{2} \text{top of head to eyes}$$

$$h = \frac{1}{2} (h_{eye} + \text{top of head to eye}) + h_{mirror}$$

$\underbrace{\hspace{10em}}_h$

$$h = \frac{1}{2} h + h_{mirror}$$

$$\boxed{\frac{1}{2} h = h_{mirror}}$$



$$f(x, t) = f(x - vt)$$

↑  
speed of wave

Gaussian  $f(x) = a e^{-kx^2}$

Gaussian wave  $\rightarrow f(x, t) = a e^{-k(x-vt)^2}$

Sine wave  
 $f(x) = a \sin(kx) \rightarrow f(x, t) = a \sin(k(x-vt))$

↑  
move to the right in time

