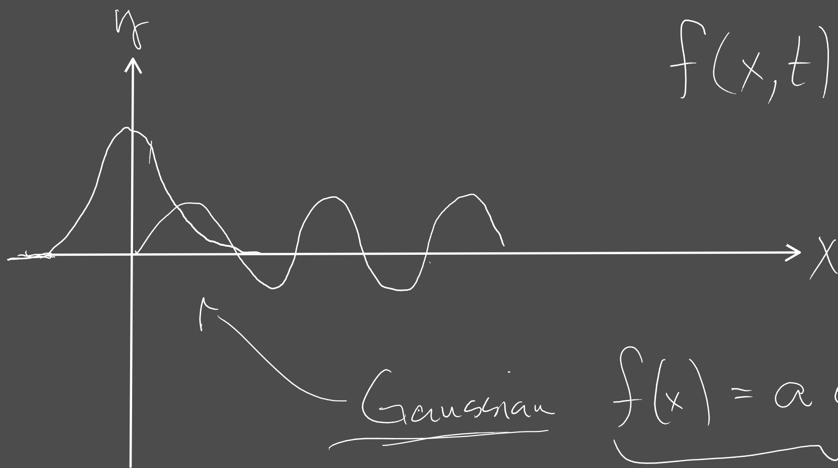
# Chapter 4




$$f(x,t) = f(x - vt)$$

speed of wave

Gaussian $\underbrace{f(x) = a e^{-kx^2}}$

Gaussian wave $\rightsquigarrow f(x,t) = a e^{-k(x-vt)^2}$

Sine wave

$$f(x) = a \sin(kx) \rightsquigarrow f(x,t) = a \sin\left(k(x - vt)\right)$$

move to the right in time

Poisson's Eq

$$\nabla^2 f = g(r)$$

$$\nabla^2 f = 0$$

$$\frac{dy}{dx} = y(x)$$

$$y = e^x$$

$$\frac{d^2y}{\partial x^2} = -y(x)$$

$$\boxed{F_{NET} = ma}$$

$$F_{NET} = m\frac{d^2x}{dt^2}$$

$$\frac{Gm_1m_2}{x^2} = m_2\frac{d^2x}{dt^2}$$

$x(t) =$ ______

Laplace: $\frac{d^2f}{dx^2} = 0$

Heat: $\frac{d^2f}{dx^2} = \frac{df}{dt}$

Wave: $\frac{\partial^2 f}{\partial x^2} = \mp\frac{1}{v^2}\cdot\frac{\partial^2 f}{\partial t^2}$

$$y = f(x \mp vt) = f(x') \qquad x' = x - vt$$

<u>1st derivation</u> :

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1} = \frac{\partial f}{\partial x'}$$

2nd derivation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'}\right)$$

$$\frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v} = \frac{\partial f}{\partial x'} \cdot (\mp v)$$

$$= \mp v \cdot \frac{\partial f}{\partial x'}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$$\mp v \frac{\partial f}{\partial x'}$$

$$= \mp v \frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \mp v$$

$$\frac{\partial^2 f}{\partial t^2} = \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}}$$

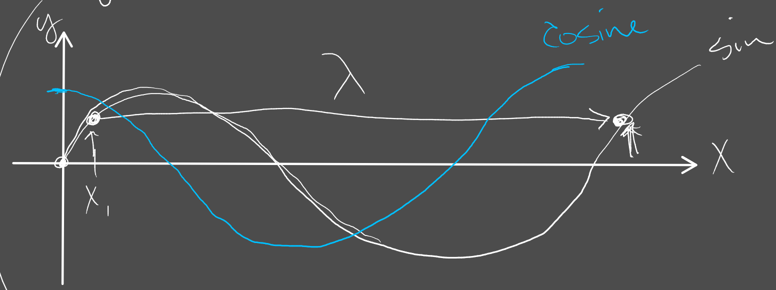
## Harmonic Waves

$$y = A\sin(k(x - vt))$$

→ form a complete basis set

$$f(x,t) = \sum_{n=0}^{\infty} A_n \sin(k_n(x - vt))$$

**Fourier Series**




$$A\sin(k(x + \lambda - vt)) = A\sin(k(x - vt) + 2\pi)$$

$$k(x - vt) + k\lambda$$

$$k\lambda = 2\pi$$

$$\boxed{k = \frac{2\pi}{\lambda}} \Rightarrow \lambda = \frac{2\pi}{k}$$

propogation constant

wave number → spacial frequency

$$\boxed{\kappa = \frac{1}{\lambda}}$$

kappa

similar thing w/ time → T (period)

$$kvT = 2\pi$$

$$v = \underbrace{\frac{2\pi}{k}}_{\lambda} \cdot \underbrace{\frac{1}{T}}_{\nu}$$

← frequency "nu"

$$\boxed{\nu = \frac{1}{T}}$$

$$\boxed{v = \lambda\nu}$$

also,

$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

angular frequency

$$\boxed{v = \frac{\omega}{k}}$$

$$y = A\sin(k(x - vt)) \leftarrow$$

$$y = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right) \leftarrow$$

$$y = A\sin(\underbrace{kx - \omega t}_{\text{argument or phase}}) \leftarrow$$

$$y = A\sin(kx - \omega t + \phi_0)$$

← initial phase or phase shift

# Complex Numbers

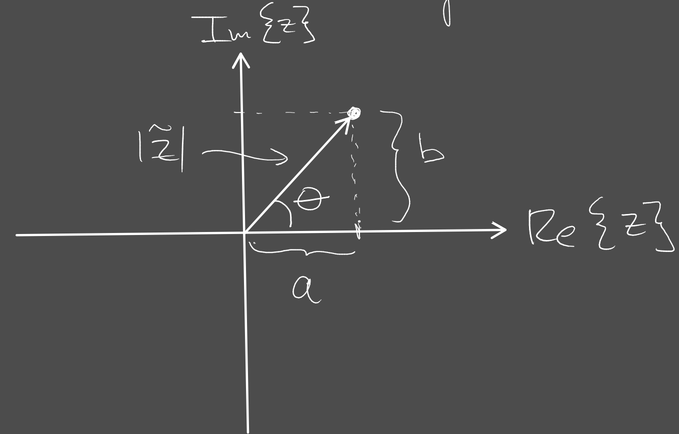
$i = \sqrt{-1}$ or $i^2 = -1$

also imaginary $j$ and in python

$\rightarrow \tilde{z} = a + ib$

real part

imaginary part




$$|\tilde{z}|^2 = a^2 + b^2$$

$$a = |\tilde{z}|\cos\theta$$

$$b = |\tilde{z}|\sin\theta$$

$$\tilde{z} = |\tilde{z}|(\cos\theta + i\sin\theta)$$

$$\boxed{e^{i\theta}} = \cos\theta + i\sin\theta$$

Euler Formula

$\theta = \pi$

$e^{i\pi} = -1$

$$\boxed{e^{i\pi} + 1 = 0}$$

Euler's identity

$$\tilde{z} = \underbrace{|\tilde{z}|e^{i\theta}}_{\text{polar coordinates}}$$

$\tilde{z}^* = a - bi \leftarrow$ complex conjugate $\longrightarrow \tilde{z}^* = |\tilde{z}|e^{-i\theta}$

$|\tilde{z}|e^{-i\theta} \cdot |\tilde{z}|e^{i\theta}$

$|\tilde{z}|^2$

$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 \leftarrow$ real # + magnitude squared

$$\tilde{y} = A e^{i(kx-\omega t)} = A\cos(kx-\omega t) + Ai\sin(kx-\omega t)$$

$$y = \mathrm{Re}\{\tilde{y}\} = A\cos(kx-\omega t) \quad \text{or} \quad y = \mathrm{Im}\{\tilde{y}\} = A\sin(kx-\omega t)$$