Chapter 11 - Fraunhofer Diffraction

From ch. 5

for coherent light

$$E_0^2 = N^2 E^2$$

incoherent

$$E_0^2 = N E^2$$

Go to chapter 7

$$E_e \rightarrow I = \frac{1}{2} \epsilon_0 c^2 E_0^2$$

$$I \propto N^2$$

so for double slit 4 times the irradiance
from one source → constructive interference

In general for any obstruction.

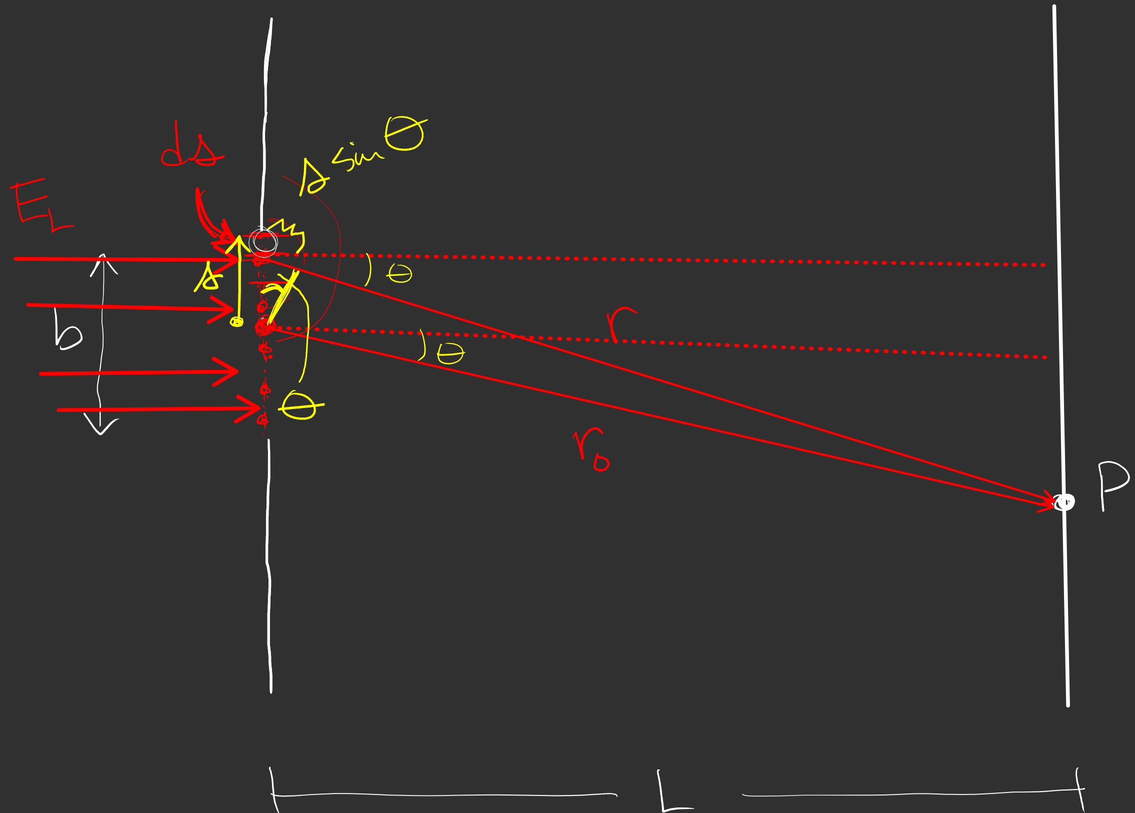
Huygens - Fresnel Principle

any point on a wavefront can be considered as a source of spherical wavelets

→ actual field beyond the wavefront is the superposition of the wavelets, considering their phase + amplitude diffraction → **far** field approximation → Fraunhofer Diffraction

diffraction → **near** field approximation → Fresnel Diffraction

↳ distance from the source to the screen.



$$dE_p = \frac{E_L\,ds}{r} e^{i(kr-\omega t)}$$

$$r = r_0 + s\sin\theta$$

$$dE_p = \frac{E_L\,ds}{r_0 + \underbrace{\cancel{s\sin\theta}}_{\text{tiny}} \to 0} e^{i(k(r_0+s\sin\theta)-\omega t)}$$

$$dE_p = \frac{E_L\,ds}{r_0} e^{i(kr_0-\omega t)} e^{iks\sin\theta}$$

$$E_p = \int_{-b/2}^{b/2} \frac{E_L}{r_0} e^{i(kr_0-\omega t)} e^{iks\sin\theta}\,ds$$

$$E_p = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \int_{-b/2}^{b/2} e^{iks\sin\theta}\, ds$$

$$E_p = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \left[\frac{e^{iks\sin\theta}}{ik\sin\theta} \right]_{-b/2}^{b/2}$$

$$E_p = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \cdot \cancel{2}\left(\frac{b}{\cancel{2}}\right) \underbrace{\underbrace{\frac{e^{ik\frac{b}{2}\sin\theta} - e^{-ik\frac{b}{2}\sin\theta}}{\left(\frac{b}{2}\right) 2ik\sin\theta}}_{\frac{\sin\left(\frac{kb}{2}\sin\theta\right)}{\frac{kb}{2}\sin\theta}}}_{\operatorname{sinc}\left(\frac{kb}{2}\sin\theta\right)}$$

$\sin\left(\frac{kb}{2}\sin\theta\right)$

$$E_p = \underbrace{\frac{E_L b}{r_0}\,\mathrm{sinc}\left(\frac{kb}{2}\sin\theta\right)}_{\text{amplitude at point P}} e^{i(kr_0-\omega t)}$$

$$\hookrightarrow E_0 = \frac{E_L b}{r_0}\,\mathrm{sinc}\left(\frac{kb}{2}\sin\theta\right)$$

Where will this be equal to zero?

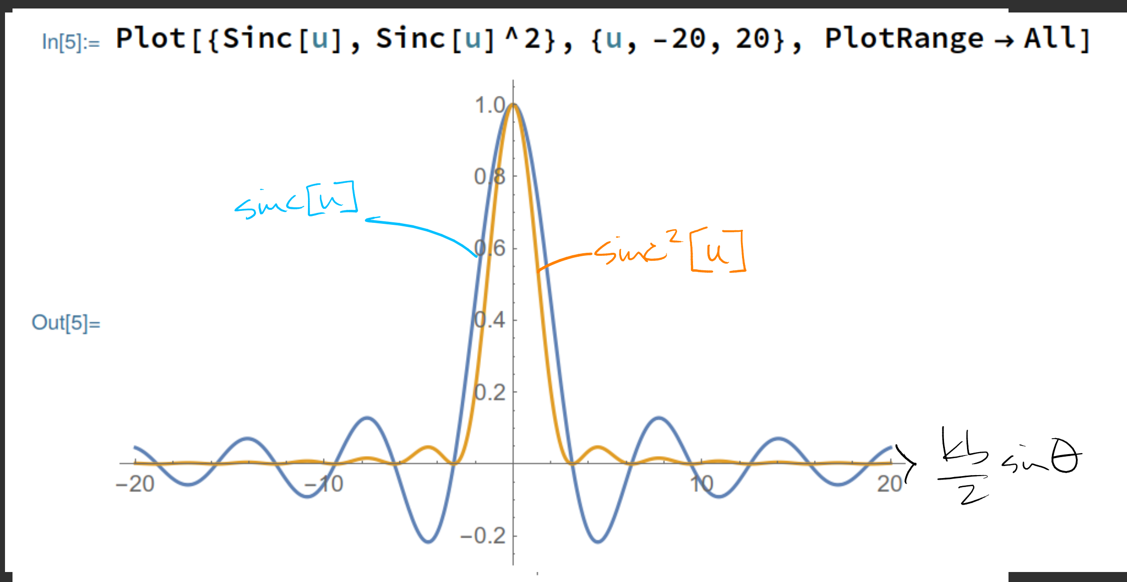
$$\underbrace{\frac{kb}{2}\sin\theta}_{\substack{\text{what is this}\\ \text{phase difference}\\ \text{of light from the}\\ \text{edge and light}\\ \text{from the center}\\ \text{of the slit}}} = m\pi \qquad m = \pm 1, \pm 2, \dots$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{\cancel{2}\cancel{\pi}}{\lambda}\cdot\frac{b}{\cancel{2}}\sin\theta = m\cancel{\pi}$$

$$\boxed{b\sin\theta = m\lambda}$$

Condition for destructive interference



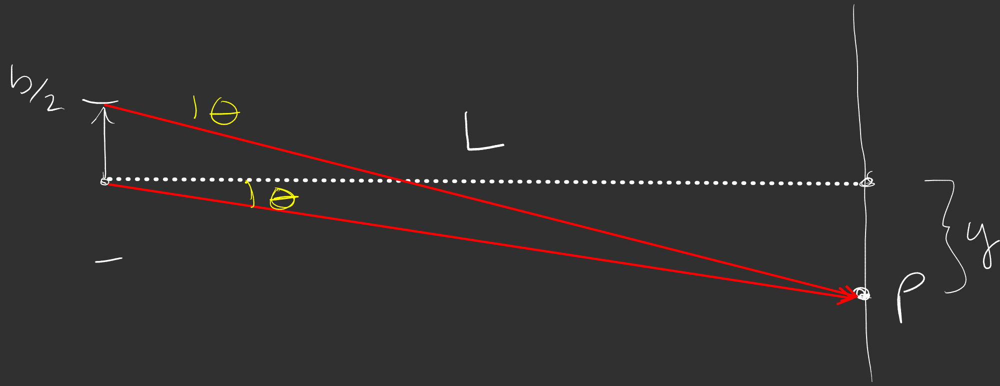
$$\beta = \frac{kb}{2}\sin\theta$$

$$\sin\left(k(x+\Delta) - \omega t\right)$$

$$\sin\left(kx - \omega t + \underbrace{k\Delta}_{\text{phase difference}}\right)$$

$$E_0 = \frac{E_L b}{r_0}\,\mathrm{sinc}\,\beta$$

$$I = \frac{1}{2}\epsilon_0 c^2 E_0^2 = \frac{1}{2}\epsilon_0 c^2\left(\frac{E_L b}{r_0}\right)^2 \mathrm{sinc}^2\beta$$



$$\tan\theta = \frac{y}{L}$$

far field approx.

$$\tan\theta \approx \sin\theta \approx \theta$$

$$b\sin\theta = m\lambda$$

$$b\,\frac{y}{L} = m\lambda$$

destr. interf. $\longrightarrow$ $\boxed{y = \dfrac{m\lambda L}{b}}$ $\quad m = \pm 1, \pm 2, \ldots$