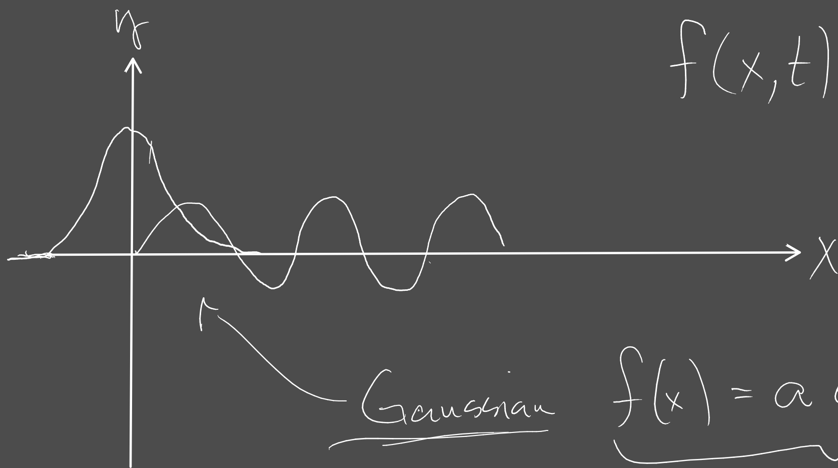
Chapter 4



$$f(x,t) = f(x - vt)$$

speed of wave

Gaussian $\underbrace{f(x) = a e^{-kx^2}}$

Gaussian wave $\rightarrow f(x,t) = a e^{-k(x-vt)^2}$

Sine wave

$f(x) = a \sin(kx) \rightarrow f(x,t) = a \sin\left(k(x-vt)\right)$

move to the right in time

$$\frac{dy}{dx} = y(x)$$

$$y = e^x$$

$$\frac{d^2y}{dx^2} = -y(x)$$

$$\boxed{F_{NET} = ma}$$

$$F_{NET} = m\frac{d^2x}{dt^2}$$

$$\frac{Gm_1m_2}{x^2} = m_2\frac{d^2x}{dt^2}$$

$x(t) =$ ______

Poisson's Eq

$$\nabla^2 f = g(r)$$

$$\nabla^2 f = 0$$

Laplace: $\frac{d^2f}{dx^2} = 0$

Heat: $\frac{d^2f}{dx^2} = \frac{df}{dt}$

Wave: $\frac{\partial^2 f}{\partial x^2} = \mp\frac{1}{v^2}\cdot\frac{\partial^2 f}{\partial t^2}$

$$y = f(x \mp vt) = f(x') \qquad x' = x - vt$$

1st derivation:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1} = \frac{\partial f}{\partial x'}$$

2nd derivation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'}\right)$$

$$\frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial x}}_{1}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v} = \frac{\partial f}{\partial x'} \cdot (\mp v)$$

$$= \mp v \cdot \frac{\partial f}{\partial x'}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial\left(\frac{\partial f}{\partial t}\right)}{\partial x'} \cdot \underbrace{\frac{\partial x'}{\partial t}}_{\mp v}$$

$$\mp v \frac{\partial f}{\partial x'}$$

$$= \mp v \frac{\partial\left(\frac{\partial f}{\partial x'}\right)}{\partial x'} \cdot \mp v$$

$$\frac{\partial^2 f}{\partial t^2} = \mp v^2 \frac{\partial^2 f}{\partial x'^2}$$

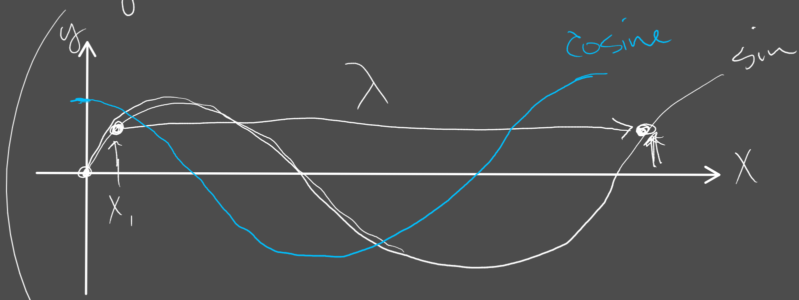
$$\boxed{\frac{\partial^2 f}{\partial x^2} = \mp \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}}$$

Harmonic Waves

$$y = A\sin(k(x-vt))$$ → form a complete basis set

$$f(x,t) = \sum_{n=0}^{\infty} A_n \sin(k_n(x-vt))$$

Fourier Series



$$A\sin(k(x+\lambda-vt)) = A\sin(k(x-vt)+2\pi)$$

$$k(x-vt)+k\lambda$$

$$k\lambda = 2\pi$$

$$\boxed{k = \frac{2\pi}{\lambda}} \Rightarrow \lambda = \frac{2\pi}{k}$$

propogation constant

wave number → spacial frequency

$$\boxed{\kappa = \frac{1}{\lambda}}$$

kappa

similar thing w/ time → T (period)

$$kvT = 2\pi$$

$$v = \underbrace{\frac{2\pi}{k}}_{\lambda} \cdot \underbrace{\frac{1}{T}}_{\nu}$$

← frequency "nu"

$$\boxed{\nu = \frac{1}{T}}$$

$$\boxed{v = \lambda\nu}$$

also,

$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

← angular frequency

$$\boxed{v = \frac{\omega}{k}}$$

$$y = A\sin(k(x-vt)) \leftarrow$$

$$y = A\sin\left(2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right) \leftarrow$$

$$y = A\sin(\underbrace{kx-\omega t}_{\text{argument or phase}}) \leftarrow$$

$$y = A\sin(kx-\omega t+\phi_0)$$

← initial phase or phase shift

Complex Numbers

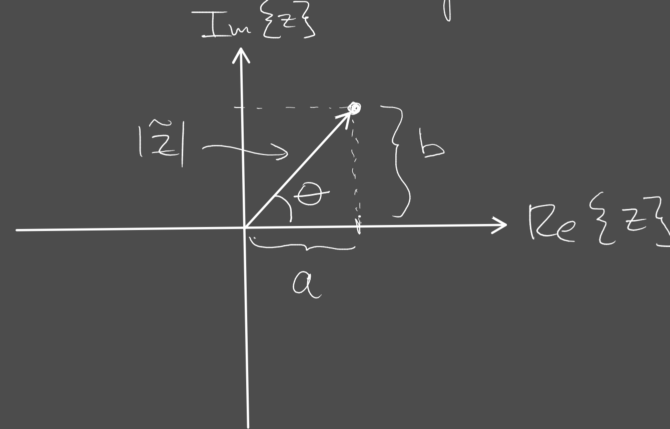
$i = \sqrt{-1}$ or $i^2 = -1$

also imaginary $\to j$ and in python

$\to \tilde{z} = a + ib$

real part

imaginary part



$|\tilde{z}|^2 = a^2 + b^2$

$a = |\tilde{z}|\cos\theta$

$b = |\tilde{z}|\sin\theta$

$\tilde{z} = |\tilde{z}|(\cos\theta + i\sin\theta)$

$\boxed{e^{i\theta}} = \cos\theta + i\sin\theta$

$\tilde{z} = \underbrace{|\tilde{z}|e^{i\theta}}_{\text{polar coordinates}}$

3blue1brown, numberphile

$2^3 = 2\cdot 2\cdot 2$

$2^{3/2} = \sqrt{(2\cdot 2\cdot 2)}$

$e^x \to \underbrace{1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{6} \ldots \frac{x^n}{n!}}$

$x \to$ complex

$e^1 \cdot e^2 = e^3$

Euler Formula

$\theta = \pi$

$e^{i\pi} = -1$

$\boxed{e^{i\pi} + 1 = 0}$ Euler's identity

$\tilde{z}^* = a - bi \leftarrow$ complex conjugate $\longrightarrow \tilde{z}^* = |\tilde{z}|e^{-i\theta}$

$|\tilde{z}|e^{-i\theta} \cdot |\tilde{z}|e^{i\theta}$

$|\tilde{z}|^2$

$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 \leftarrow$ real # + magnitude squared

$$\rightarrow \quad Re\{\tilde{z}\} = \frac{\tilde{z} + \tilde{z}^*}{2}$$

$$\cos\theta = Re\{e^{i\theta}\} = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\rightarrow \quad Im\{\tilde{z}\} = \frac{\tilde{z} - \tilde{z}^*}{2i}$$

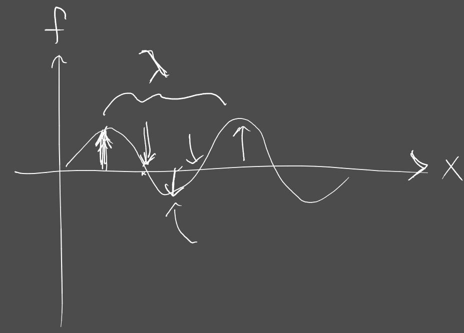
$$\sin\theta = Im\{e^{i\theta}\} = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

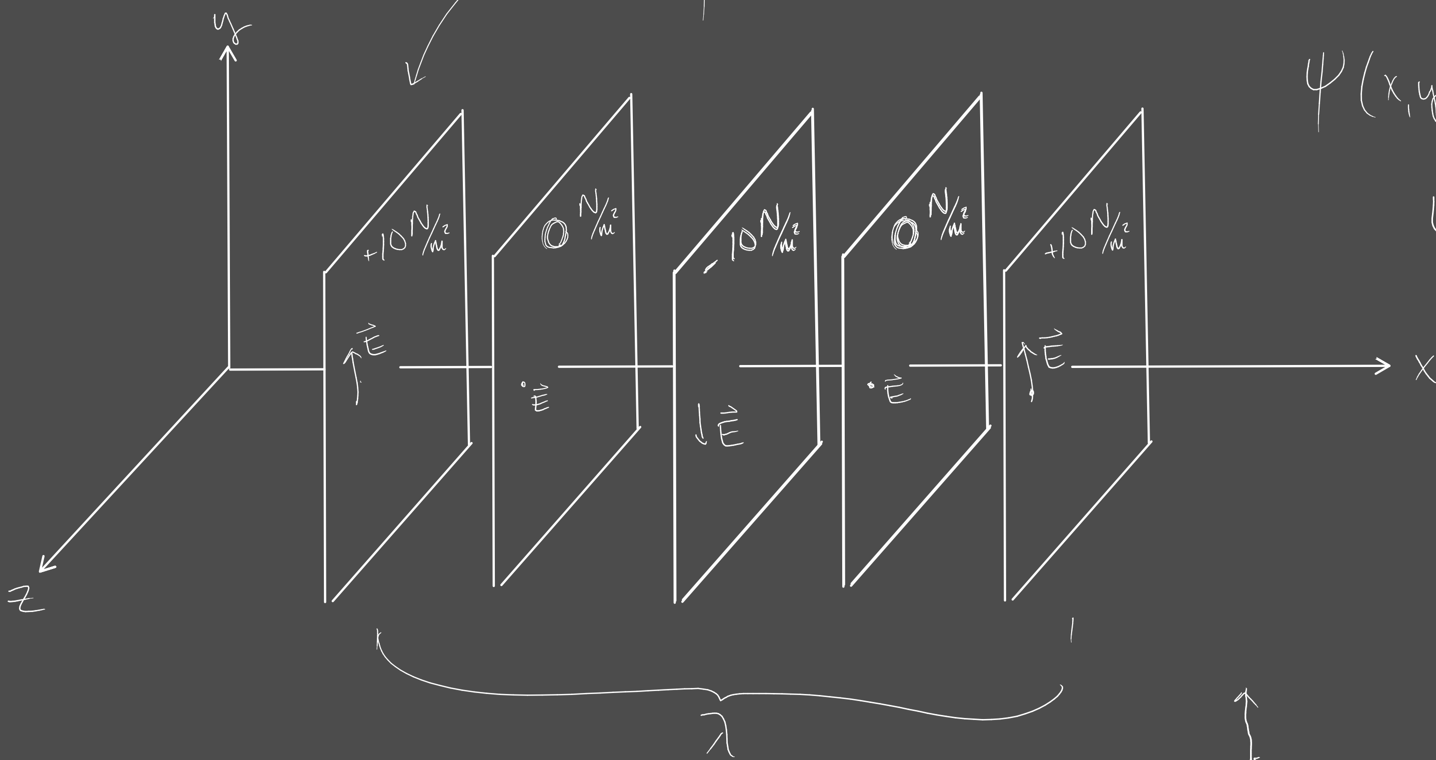
$$\tilde{y} = Ae^{i(kx-\omega t)} = A\cos(kx-\omega t) + Ai\sin(kx-\omega t)$$

$$y = Re\{\tilde{y}\} = A\cos(kx-\omega t) \quad \text{or} \quad y = Im\{\tilde{y}\} = A\sin(kx-\omega t)$$

Plane waves

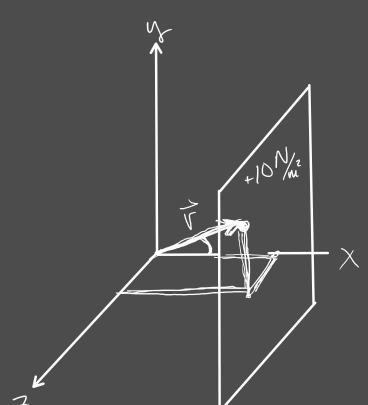
$$\underbrace{f(x,t) = A\sin(kx - \omega t)}$$





$$\psi(x,y,z,t) = A\sin(? - \omega t)$$

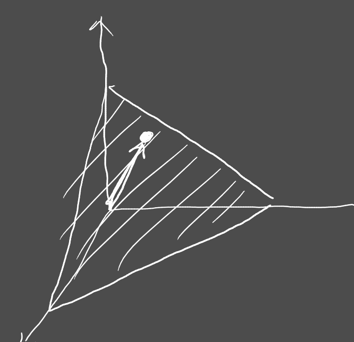
$$\psi(x,y,z,t=1) = A\sin(\_\_)$$



$$\sin(\boxed{kx})$$

$$kx = \underbrace{kr\cos\theta}$$

$$\sin(ky$$



$$|\vec{k}| = \frac{2\pi}{\lambda} \qquad \vec{k} = k_x\hat{x} + k_y\hat{y} + k\hat{z}$$

$$\psi(\vec{r},t) = A\sin(\vec{k}\cdot\vec{r} - \omega t)$$

plane wave

$$\psi(\vec{r},t) = A e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\frac{\partial^2 f}{\partial x^2} = \mp\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} = \underbrace{\vec{\nabla} f}_{\text{vector function}}$$

scalar function

gradient

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \nabla^2 f$$

Laplacian

$$\vec{\nabla}\cdot\vec{F}$$

divergence $\vec{\nabla}\cdot\vec{\nabla} f = \nabla^2 f$

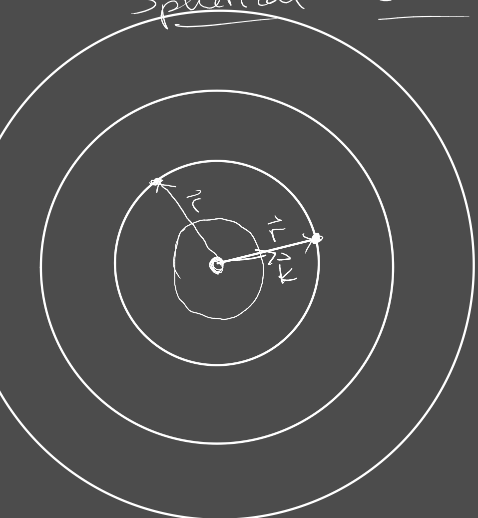
$$\vec{\nabla}\times\vec{F}$$

curl

$$\underbrace{\vec{\nabla}\times\vec{\nabla} f}_{\text{vector}} \neq \underbrace{\nabla^2 f}_{\text{scalar}}$$

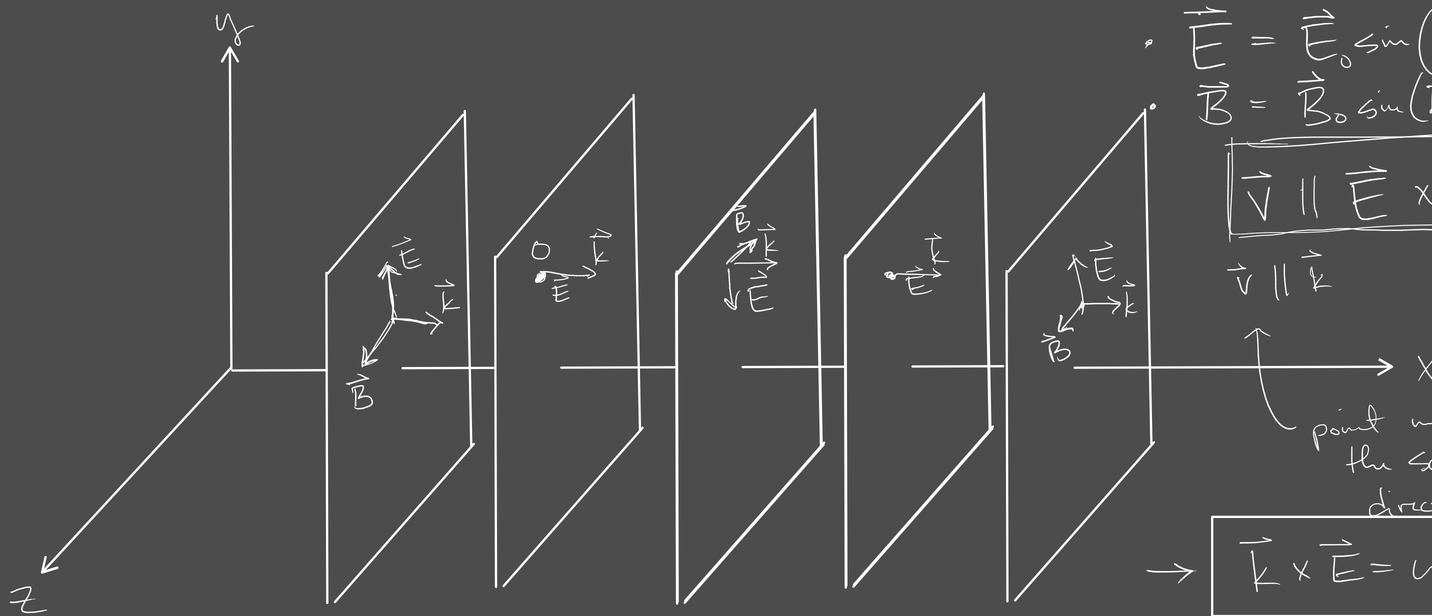
$$\boxed{\nabla^2\psi = \mp\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}}$$

Spherical Wave



$$\psi(\vec{r},t) = \frac{A}{r}\sin(kr-\omega t)$$

$$\psi(\vec{r},t) = \underbrace{\frac{A}{r}e^{i(kr-\omega t)}}_{\text{consistent w/ } 1/r^2 \text{ irradiance}}$$



- $\vec{E} = \vec{E}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$
- $\vec{B} = \vec{B}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$

$$\boxed{\vec{v} \parallel \vec{E} \times \vec{B}}$$

$\vec{v} \parallel \vec{k}$

point in the same direction

$$\boxed{c = \frac{|E|}{|B|}}$$

$E = c \cdot B$

$$\boxed{\vec{k} \times \vec{E} = \omega \vec{B}}$$

$$|k||E|\underbrace{\sin\theta}_{1} = \omega |B|$$

$$\frac{k}{\omega} = \frac{B}{E}$$

$$c = v = \frac{\omega}{k} = \frac{E}{B}$$

$c = 2.998 \cdot 10^8 \, m/s$

$$\left.\begin{aligned} \epsilon_0 &= 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \\ \mu_0 &= 4\pi \cdot 10^{-7} \frac{A^2}{Nm^2} \end{aligned}\right\} E+M$$

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}}$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

Maxwell's Eqn

$$\underbrace{\nabla^2 E = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E}{\partial t^2}}$$

Energy carried by a wave

$$\boxed{u_E = \frac{1}{2}\epsilon_0 E^2}$$

energy density

$$\boxed{u_B = \frac{1}{2}\cdot\frac{B^2}{\mu_0}}$$

$$E = cB$$

$$u_E = \frac{1}{2}\epsilon_0 (cB)^2$$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$u_E = \frac{1}{2}\cancel{\epsilon_0}\cdot\frac{1}{\cancel{\epsilon_0}\mu_0}B^2$$

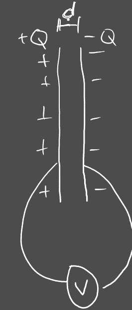
$$u_E = \frac{1}{2}\frac{B^2}{\mu_0} = u_B$$

$$\boxed{u_E = u_B}$$

$$\boxed{u_{Total} = u_E + u_B = 2u_E = 2u_B}$$

$$\boxed{u_{Total} = \epsilon_0 E^2 = \frac{B^2}{\mu_0}}$$

ideal capacitor



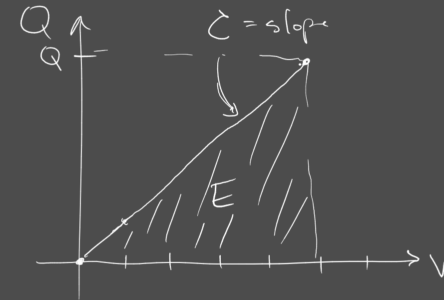
$$C = \frac{Q}{V}$$

$$\int_0^d \Delta V = -\int \nabla E$$

$$|E| = V\,\underbrace{\Delta x}$$

$$= V\cdot d$$

$$C = \epsilon_0\frac{A}{d}$$



$$U = \frac{1}{2}\Delta V\cdot Q$$

$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}C\,d^2\cdot E^2$$

$$U = \frac{1}{2}\epsilon_0\frac{A}{\cancel{d}}\cdot d^{\cancel{2}}\cdot E^2$$

$$\underbrace{}_{\text{Volume } \mathcal{V}}$$

$$U = \frac{1}{2}\epsilon_0 E^2\cdot\mathcal{V}$$

$$u = \frac{U}{\mathcal{V}} = \frac{1}{2}\epsilon_0 E^2$$

$$u_T = \sqrt{u}\cdot\sqrt{u}$$

$$= \sqrt{\epsilon_0}\,E\cdot\frac{B}{\sqrt{\mu_0}}$$

$$= \sqrt{\frac{\epsilon_0}{\mu_0}}\cdot E\cdot B$$

$$\frac{\sqrt{\epsilon_0}}{\sqrt{\epsilon_0}} \rightarrow = \frac{\epsilon_0}{\sqrt{\epsilon_0\mu_0}}E\cdot B$$

$$\boxed{u_T = \epsilon_0\, c\, E\cdot B}$$

$$\frac{\sqrt{\mu_0}}{\sqrt{\mu_0}} \rightarrow = \frac{\sqrt{\epsilon_0\mu_0}}{\mu_0}EB$$

$$\boxed{u_T = \frac{1}{c\,\mu_0}EB}$$

Power = rate of energy transfer

$$\text{power} = \frac{\text{energy}}{\text{time}} = \frac{u V}{t} = \frac{u \cdot \overset{\text{Area}}{A} \cdot \Delta x}{t} = \frac{u \cdot A \cdot v \cdot \cancel{t}}{\cancel{t}}$$

radiant flux
(radiant power)

Φ_E

$$\frac{\text{power}}{A} = \frac{u \cdot \cancel{A} \cdot v}{\cancel{A}}$$

close to irradiance

$$\frac{\Phi_E}{\text{Area}} = (E_E)$$

$$S = \epsilon_0 c E B \cdot c$$

$$S = \epsilon_0 c^2 E B$$

$\vec{E} \times \vec{B} \parallel \hat{k}$

$|\vec{E} \times \vec{B}| = EB$

$$\underbrace{\vec{S} = \epsilon_0 c^2 \vec{E} \times \vec{B} \quad \text{or} \quad \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}_{\text{Poynting Vector}}$$

$\vec{E} + \vec{B}, \therefore \vec{E} \times \vec{B}$ is a waveform

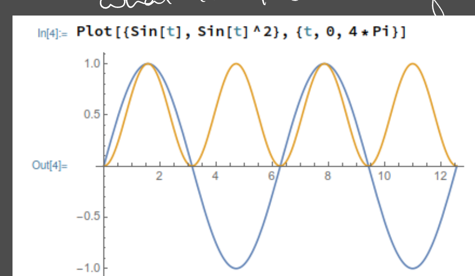
depend on time $\rightarrow \vec{S}$ depends on time

$$\langle f(t) \rangle_T = \frac{1}{T} \int_0^T f(t)\, dt$$

$$E_e = \langle |S| \rangle = \epsilon_0 c^2 E_0 B_0 \langle \underbrace{\sin^2(\vec{k} \cdot \vec{r} - \omega t)}_{\text{plane wave}} \rangle$$

notation for time average

what is the average of $\sin^2 x$ (or $\cos^2 x$)



$$E_e = \frac{1}{2} \epsilon_0 c^2 E_0 B_0$$

or

$$E_e = \frac{1}{2} \epsilon_0 c E_0^2$$

What about light going through a medium?

$$n = \frac{c}{v}$$

adjustments to equation

$$c \rightsquigarrow \frac{c}{n} \quad (v)$$

$$\epsilon_0 \rightsquigarrow \epsilon = n^2 \epsilon_0$$

permittivity of medium

relative permittivity

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

What about spherical wave?

$$\psi(\vec{r}, t) = \frac{A}{r} \sin(kr - \omega t)$$

$$|E| = \frac{E_0}{r} \sin(kr - \omega t)$$

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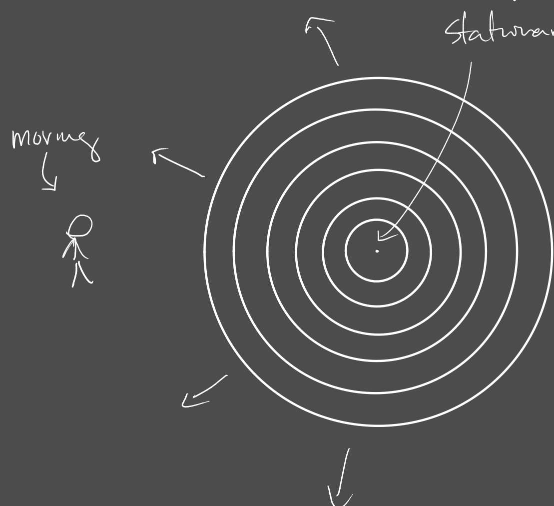
stationary



observed (shifted)

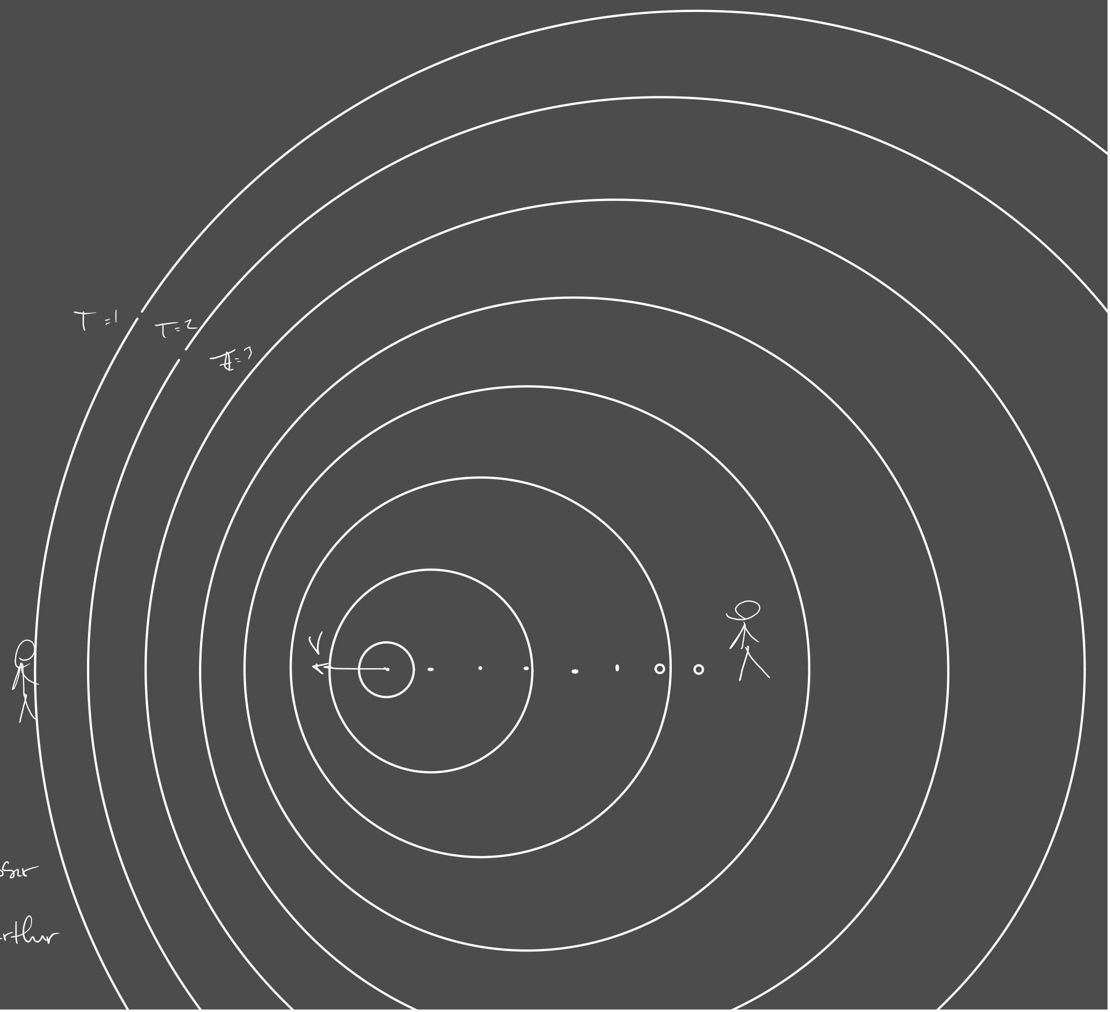
$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

source

relative velocity

$v > 0$ getting closer

$v < 0$ getting farther



$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$R = 8{,}31$

$k_B = 1{,}38 \cdot 10^{-24}$

M — molar mass – mass of one mole of a gas