Chapter 5 - Superposition

<u>superposition</u> – waves at the same place and time, displacements add up.

$$\psi = \psi_1 + \psi_2 + \dots$$

- superposition of harmonic waves

$$E_1(x_1,t) = E_1 \cos(kx_1 - \omega t + \phi_1)$$

$$E_2(x_2,t) = E_2 \cos(\underbrace{kx_2 - \omega t + \phi_2})$$

$$\alpha_1 = ks_1 + \phi_1$$

$$\alpha_2 = ks_2 + \phi_2$$

$$\underbrace{\alpha_2 - \alpha_1}_{\text{phase difference}} = k(s_2 - s_1) + (\phi_2 - \phi_1)$$

what if $\alpha_2 - \alpha_1 = 2\pi m$ } even multiple of π

$$E_R = E_1 + E_2 = E_1 \cos(\alpha_1 - \omega t) + E_2 \cos(\alpha_2 - \omega t)$$

$\alpha_2 = \alpha_1 + 2\pi m$

$\cos(\alpha_1 + 2\pi m - \omega t)$

$\cos\theta = \cos(\theta + 2\pi m)$

constructive interference

$$\hookrightarrow E_R = (E_1 + E_2)\cos(\alpha_1 - \omega t)$$

what if $\alpha_2 - \alpha_1 = (2m-1)\pi$ odd multiple of π

$$E_R = E_1 + E_2 = E_1 \cos(\alpha_1 - \omega t) + E_2 \cos(\alpha_2 - \omega t)$$

$$\alpha_2 = \alpha_1 + (2m-1)\pi$$

$$-\cos\theta = \cos(\theta + (2m-1)\pi)$$

destructive interference

$$E_R = (E_1 - E_2)\cos(\alpha_1 - \omega t)$$

what about others?

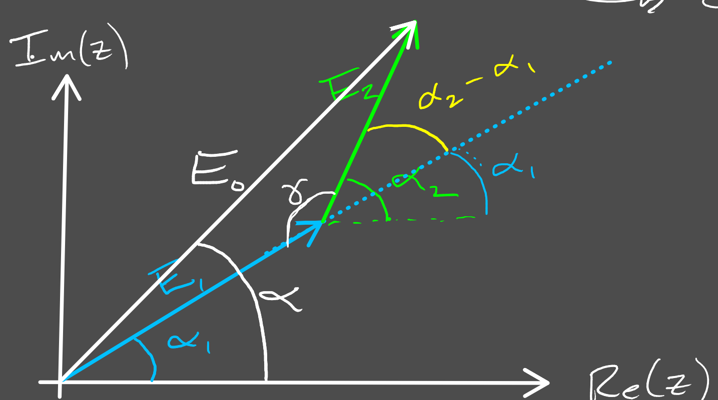
$$E_R = \mathrm{Re}\left(E_1 e^{i(\alpha_1 - \omega t)} + E_2 e^{i(\alpha_2 - \omega t)}\right)$$

$$E_1 e^{i\alpha_1} e^{-i\omega t} + E_2 e^{i\alpha_2} e^{-i\omega t}$$

$$E_R = \mathrm{Re}\left(e^{-i\omega t}\left(E_1 e^{i\alpha_1} + E_2 e^{i\alpha_2}\right)\right)$$

"phasor diagram"

$\hookrightarrow$ complex as vector

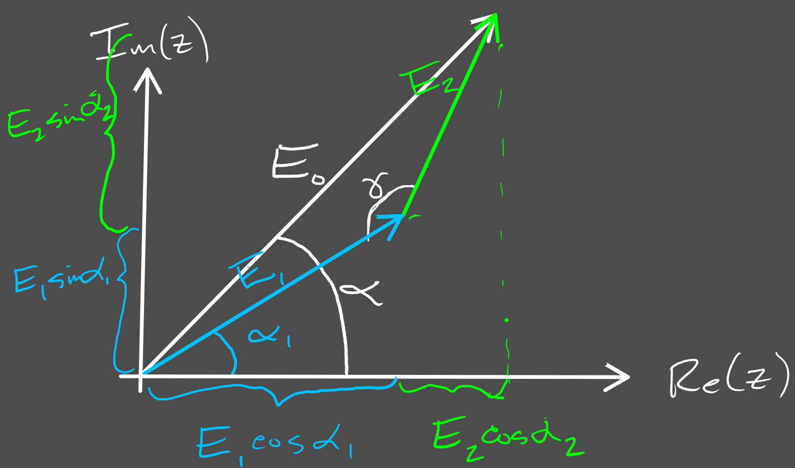


$$E_0^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\gamma$$

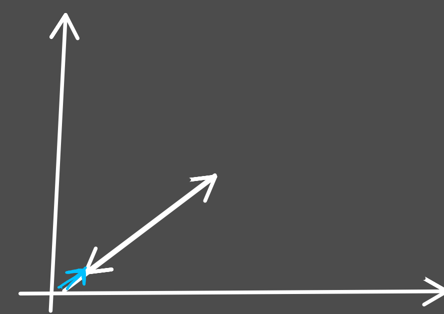
$$\gamma = \pi - (\alpha_2 - \alpha_1)$$
$$= \pi - \alpha_2 + \alpha_1$$
$$= \pi + \alpha_1 - \alpha_2$$

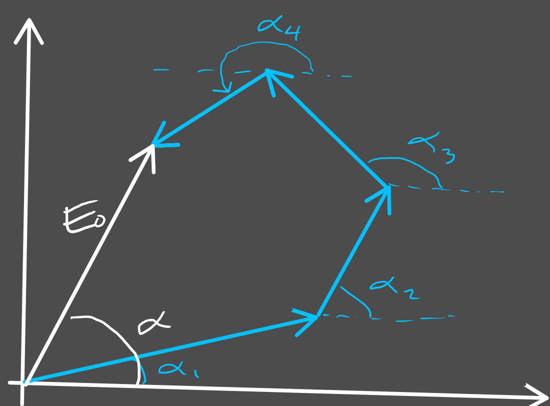
$$\cos(\pi + \alpha_1 - \alpha_2)$$
$$= -\cos(\alpha_1 - \alpha_2)$$

$$E_0^2 = E_1^2 + E_2^2 + 2E_1E_2\cos(\alpha_1-\alpha_2)$$



$$\tan\alpha = \frac{E_1\sin\alpha_1 + E_2\sin\alpha_2}{E_1\cos\alpha_1 + E_2\cos\alpha_2}$$





$$\tan\alpha = \frac{\sum_1^n E_i \sin\alpha_i}{\sum_i^N E_i \cos\alpha_i}$$

$$E_0^2 = \left(\sum_i^N E_i \cos\alpha_i\right)^2 + \left(\sum_1^n E_i \sin\alpha_i\right)^2$$

$$\left(\sum_i^N E_i \cos\alpha_i\right)^2 = \sum_i^N E_i^2\cos^2\alpha_i + \sum_i^N 2E_i\cos\alpha_i \sum_{j>i}^N E_j\cos\alpha_j$$

$$2\sum_i^N \sum_{j>i}^N E_iE_j\cos\alpha_i\cos\alpha_j$$

$$\left(\sum_1^n E_i \sin\alpha_i\right)^2 = \sum_i^N E_i^2\sin^2\alpha_i + 2\sum_i^N\sum_{j>i}^N E_iE_j\sin\alpha_i\sin\alpha_j$$

$$E_0^2 = \sum_i^N E_i^2 \cos^2\alpha_i + \sum_i^N E_i^2 \sin^2\alpha_i + \square$$

$$= \sum_i^N E_i^2 (\underbrace{\cos^2\alpha_i + \sin^2\alpha_i}_{1})$$

$$= \sum_i^N E_i^2 + 2\sum_i^N \sum_{j>i}^N E_i E_j (\underbrace{\cos\alpha_i \cos\alpha_j + \sin\alpha_i \sin\alpha_j}_{\cos(\alpha_j - \alpha_i)})$$

$$\boxed{E_0^2 = \sum_i^N E_i^2 + 2\sum_i^N \sum_{j>i}^N E_i E_j \cos(\alpha_j - \alpha_i)}$$

For random sources $\longrightarrow$ if these α_i are random what does this sum approach? $\rightarrow 0$

$$E_0^2 = \sum_i^N E_i^2$$

<u>if</u> they are equal magnitude sources

$$E_0^2 = N E_1^2 \rightarrow E_0 = E_1\sqrt{N}$$

irradiance $\rightarrow$ $E_e = \frac{1}{2}\epsilon_0 c^2 E_0 B_0$ <u>or</u> $E_e = \frac{1}{2}\epsilon_0 c E_0^2$

$$E_e = \frac{1}{2}\epsilon_0 c N E_1^2 = N\left(\frac{1}{2}\epsilon_0 c E_1^2\right)$$

$$E_e \propto N$$

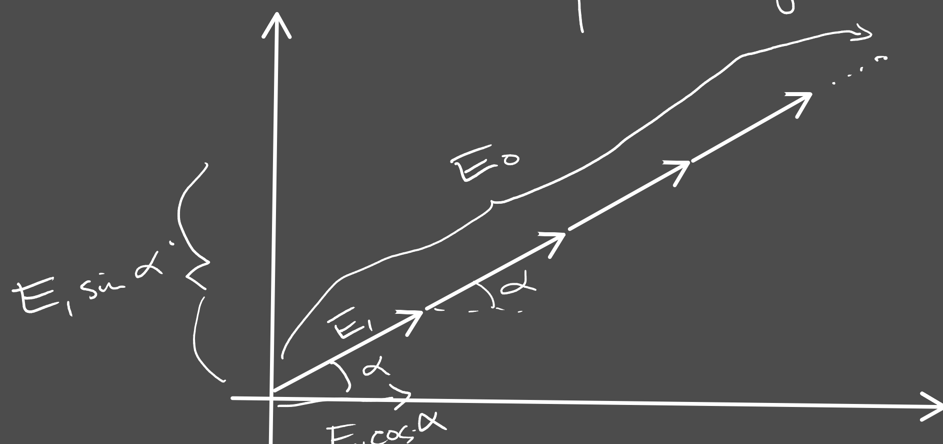
now what about coherent sources

↳ same frequency and waveform and phase

$$\boxed{E_0^2 = \sum_i^N E_i^2 + 2\sum_i^N \sum_{j>i}^N E_i E_j \underbrace{\cos(\alpha_j - \alpha_i)}_{\cos(\alpha_j-\alpha_i) = \cos(0) = 1}}$$

$$E_0^2 = \sum_i^N E_i^2 + 2\sum_i^N \sum_{j>i}^N E_i E_j$$

– if sources of equal magnitude



$$E_0^2 = (NE_1\cos\alpha)^2 + (NE_1\sin\alpha)^2 = N^2E_1^2\underbrace{(\cos^2\alpha + \sin^2\alpha)}_{1}$$

$$E_0^2 = N^2E_1^2 \rightsquigarrow E_0 = NE_1$$

back to irradiance

$$E_e = \frac{1}{2}\epsilon_0 c N^2 E_1^2$$

$$E_e \propto N^2$$

$$\frac{E_{e,coher}}{E_{e,rand}} = \frac{N^2}{N} = N$$

$$\cos(-x) = \cos(x)$$

$$\sin(-x) = -\sin(x)$$

Standing waves

two waves traveling in opposite directions

$$E_1 = E_0 \sin(-kx + \omega t + \phi_R) \leftarrow \text{to right}$$

$$E_2 = E_0 \sin(kx + \omega t) \leftarrow \text{to left}$$

$$E_R = E_0\left(\sin(kx + \omega t) + \sin(-kx + \omega t + \phi_R)\right)$$

$$\sin\beta_1 + \sin\beta_2 = 2\sin\left(\tfrac{1}{2}(\beta_1 + \beta_2)\right)\cos\left(\tfrac{1}{2}(\beta_1 - \beta_2)\right)$$

$$E_R = 2E_0 \sin\left(\omega t + \frac{\phi_R}{2}\right)\cos\left(kx + \phi_R/2\right)$$

$$\phi_R = \pi$$

$$E_R = 2E_0 \sin\left(\omega t + \frac{\pi}{2}\right)\cos\left(kx + \pi/2\right)$$

$$E_R = \underbrace{2E_0 \sin(kx)}_{\text{specially varying amplitude}} \underbrace{\cos(\omega t)}_{\text{variation in amplitude in time}}$$

$= 0$ always in certain places $\longrightarrow$ <u>nodes</u>

$kx = m\pi \qquad m = 0, \pm 1, \pm 2, \ldots$

$k = \frac{2\pi}{\lambda}$

$\frac{2\cancel{\pi}}{\lambda} \cdot x = m\cancel{\pi}$

$x = m\frac{\lambda}{2} \qquad m = 0, \pm 1, \pm 2, \ldots$

$\Delta x = \frac{\lambda}{2}$ ← distance between adjacent nodes

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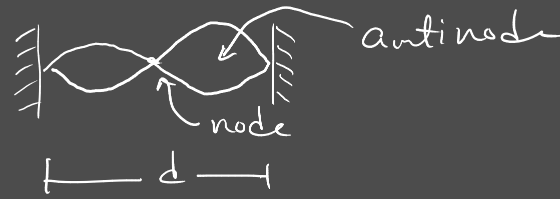
$\Delta t = \frac{T}{2}$

$\omega = 2\pi\nu \qquad \nu = \frac{1}{T}$

$\omega = \frac{2\pi}{T}$

laser cavity



$$d = m\left(\frac{\lambda}{2}\right)$$

$$\lambda = \frac{2d}{m} \qquad m = 1, 2, 3 \ldots$$

<u>Frequency Beating</u>