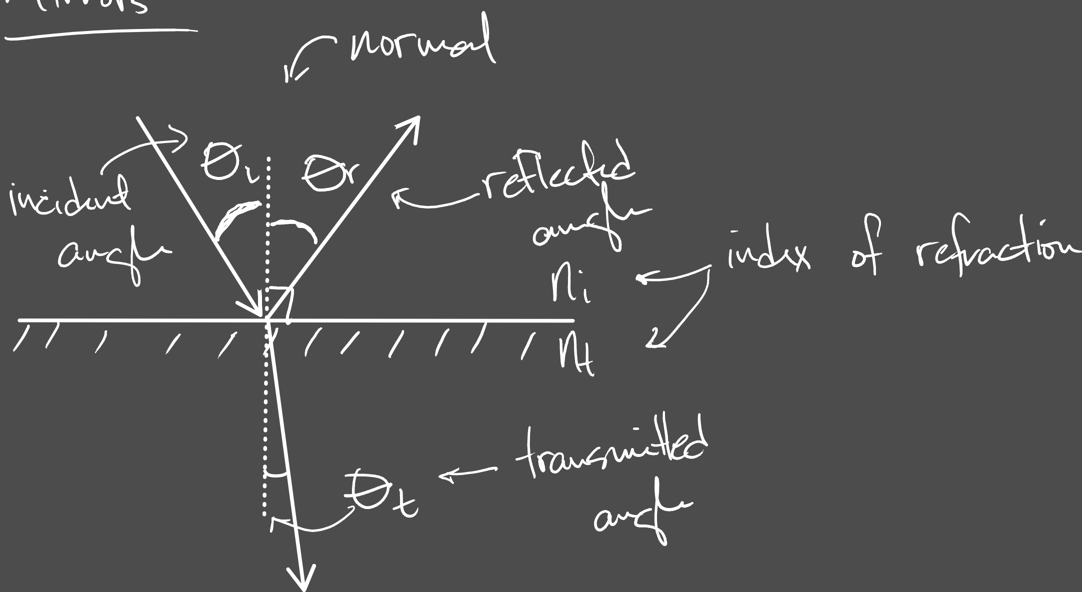
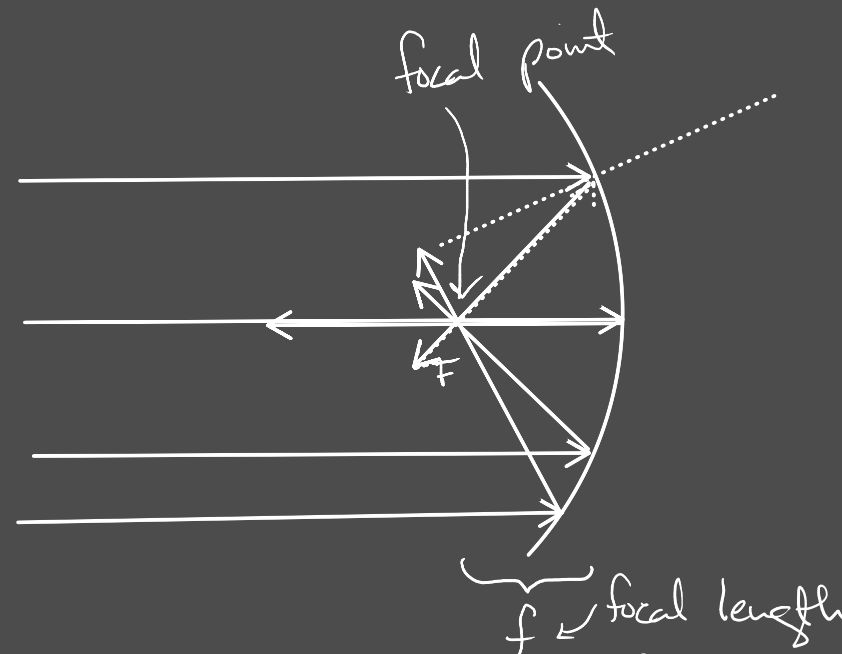
Brainstorm review

Mirrors

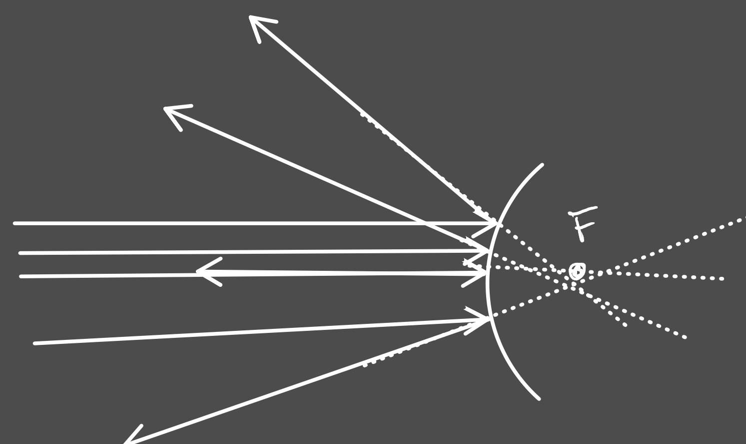


$$n_i \sin\theta_i = n_t \sin\theta_t$$



$$f = \frac{R}{2}$$

Convex + Concave



Photon → a particle of light

Light → ray
Light → photon
Light → wave → (electromagnetic)

charge + current

Planck's Equation

$$E = h\nu$$

"nu"

Planck constant

$h = 6.63 \cdot 10^{-34}\,Js$

de Broglie

$$\lambda = \frac{h}{p}$$

Basic relativistic mechanics

$$\rightarrow E = \gamma m c^2$$

total

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\gamma = (1 - v^2/c^2)^{-1/2}$$

kinetic energy

$$E_k = mc^2(\gamma - 1)$$

$$\downarrow$$

$$E_k \approx \frac{1}{2} m v^2$$

$$v \ll c$$

rest energy

$$E_r = mc^2$$

Energy - mass equivalence

Energy-momentum relation

$$\rightarrow E^2 = (pc)^2 + (mc^2)^2$$

solve for momentum

$$\downarrow$$

relativistic momentum

$$p = \gamma m v$$

$$E_k = \gamma mc^2 - mc^2$$

$$E = E_k + mc^2 = \gamma mc^2 \cancel{- mc^2} \cancel{+ mc^2}$$

$$\boxed{p = \frac{\sqrt{E^2 - m^2c^4}}{c}}$$

$$\lambda = \frac{h}{p} = \boxed{\frac{hc}{\sqrt{E^2 - m^2c^4}} = \lambda}$$

$$p = \gamma m v \Rightarrow v = \frac{p}{\gamma m} = \frac{c^2 p}{E} = \boxed{c\sqrt{1 - \frac{m^2c^2}{E^2}} = v}$$

$$E = \gamma m c^2$$

$$\frac{E}{c^2} = \gamma m$$

wave

frequency

$$v = \lambda \cdot \nu$$

$$v = c$$

$$c = \lambda \cdot \nu$$

$$\frac{c}{\lambda} = \nu$$

for a massless photon: $m=0$

$$p = \frac{E}{c} \quad \Bigg| \quad \lambda = \frac{h}{p} = \frac{hc}{E} \quad \Bigg| \quad v = c$$

$$E = \frac{hc}{\lambda}$$

$$\boxed{E = h\nu}$$

An electron is accelerated to a kinetic energy E_K of 2.5 MeV. (a) Determine its relativistic momentum, de Broglie wavelength, and speed. (b) Determine the same properties for a photon having the same total energy as the electron.

$$E_K = 2.5\ MeV$$

$$E_K = 2.5\ MeV \cdot \frac{1.6 \cdot 10^{-19}\ J}{1\ eV} \cdot \frac{10^{6}}{1\ M}$$

$$= 2.5 \cdot 1.6 \cdot 10^{-13}\ J$$

$$= 4 \cdot 10^{-13}\ J$$

$$1\ eV \Rightarrow 1e = 1.6 \cdot 10^{-19}\ C$$

$$1\ V = 1\ \frac{J}{C}$$

- $1\ eV = 1.6 \cdot 10^{-19}\ \cancel{C} \cdot 1\ \frac{J}{\cancel{C}} = 1.6 \cdot 10^{-19}\ J$
- $m_e = 9.11 \cdot 10^{-31}\ kg$
- $C = 3 \cdot 10^{8}\ m/s$

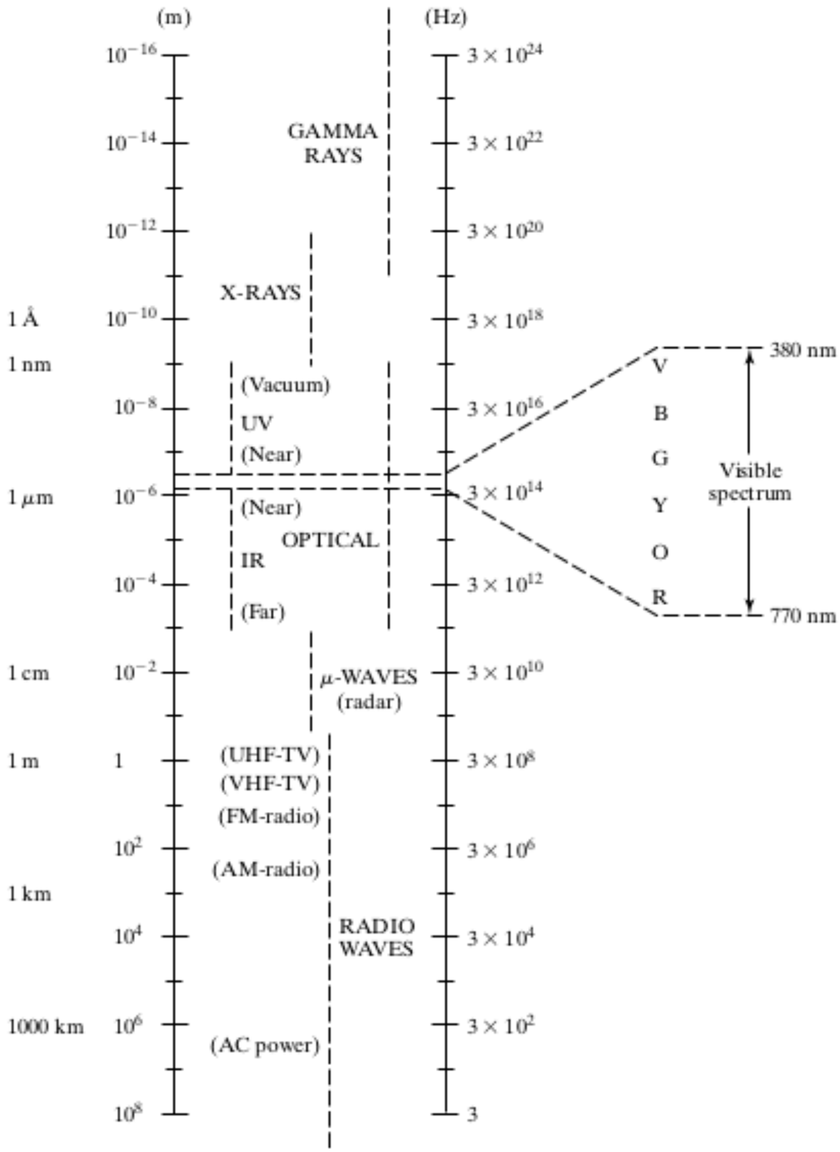
$$E_{total} = 4 \cdot 10^{-13}\ J + 9.11 \cdot 10^{-31} \cdot 9 \cdot 10^{16}\ m^2/s^2$$

$$= 4 \cdot 10^{-13}\ J + 8.2 \cdot 10^{1} \cdot 10^{-15}$$

$$+ 8.2 \cdot 10^{-14}$$

$$+ 0.82 \cdot 10^{-13} = 4.82 \cdot 10^{-13}\ J$$

$$p = \frac{\sqrt{E^2 - m^2c^4}}{c} = \frac{\sqrt{(4.82 \cdot 10^{-13})^2 - (8.2 \cdot 10^{-14})^2}}{3 \cdot 10^{8}} = \frac{\sqrt{23.23 \cdot 10^{-26} - 67.24 \cdot 10^{-28}}}{}$$



(m)
(Hz)
10^{-16}
3×10^{24}
10^{-14}
GAMMA RAYS
3×10^{22}
10^{-12}
3×10^{20}
X-RAYS
1 Å
10^{-10}
3×10^{18}
1 nm
(Vacuum)
10^{-8}
UV
3×10^{16}
(Near)
1 μm
10^{-6}
3×10^{14}
(Near)
OPTICAL
IR
10^{-4}
3×10^{12}
(Far)
1 cm
10^{-2}
μ-WAVES (radar)
3×10^{10}
1 m
1
(UHF-TV)
(VHF-TV)
3×10^{8}
(FM-radio)
10^{2}
3×10^{6}
(AM-radio)
1 km
10^{4}
RADIO WAVES
3×10^{4}
1000 km
10^{6}
3×10^{2}
(AC power)
10^{8}
3
380 nm
V
B
G
Y
O
R
Visible spectrum
770 nm

A certain sensitive radar receiver detects an electromagnetic signal of frequency 100 MHz and power (energy/time) 6.63×10^{-16} J/s.

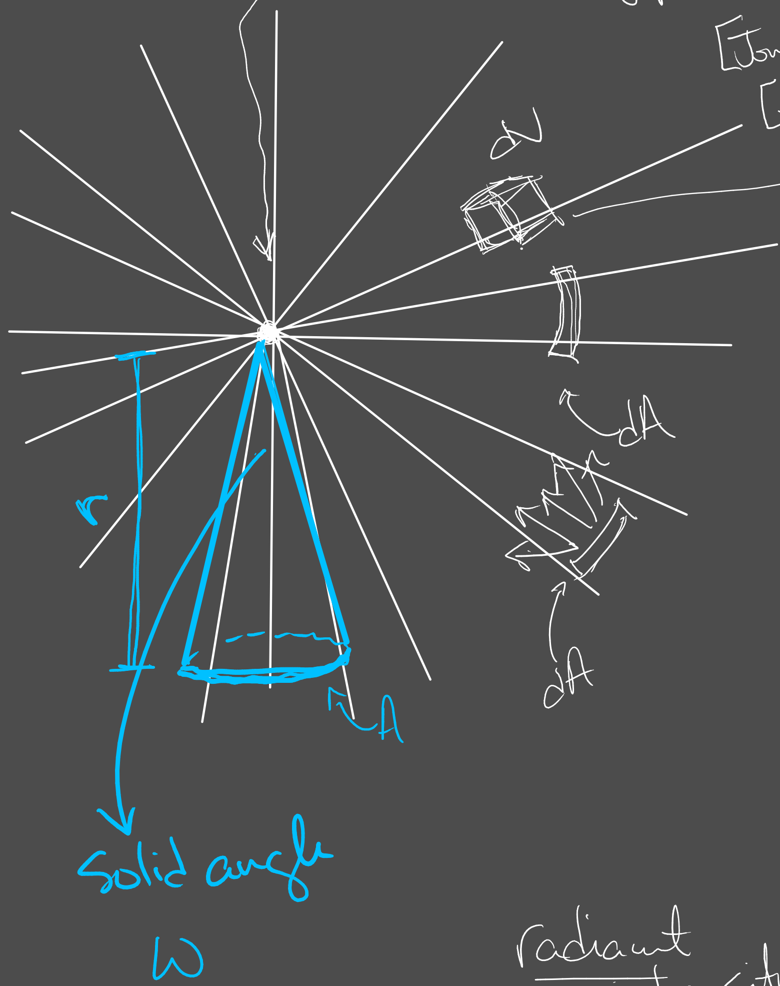
a. What is the wavelength of a photon with this frequency?
b. What is the energy of a photon in this signal? Express this energy in J and in eV.
c. How many photons/s would arrive at the receiver in this signal?
d. What is the energy (in J and in eV) of a visible photon of wavelength 555 nm?
e. How many visible ($\lambda = 555$ nm) photons/s would correspond to a detected power of 6.63×10^{-16} J/s?
f. What is the energy (in J and in eV) of an X-ray of wavelength 0.1 nm?
g. How many X-ray ($\lambda = 0.1$ nm) photons/s would correspond to a detected power of 6.63×10^{-16} J/s?

a. $c = \lambda \cdot \nu$

b. $E = h\nu$

c. $\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{n \cdot \text{energy/photon}}{\text{time}} \Rightarrow \frac{\text{power}}{\text{energy/photon}} = \frac{n \text{ photons}}{\text{time}}$

Radiometry



radiant energy, Q_e [Joules] [eV] $\rightarrow$ radiant power (radiant flux), $\Phi_e = \frac{dQ_e}{dt}$ [Watts]

radiant energy density $w_e = \frac{dQ_e}{dV}$ $\left[\frac{J}{m^3}\right]$

irradiance (incident radiant power density)

$$E_e = \frac{d\Phi_e}{dA}$$

radiant excitance (emitted/reflected radiant power density)

$$M_e = \frac{d\Phi_e}{dA}$$

radiant intensity (radiant power per solid angle)

$$I_e = \frac{d\Phi_e}{d\omega}$$

Irradiance from a point source.

$$E_e = \frac{d\Phi_e}{dA}$$

$$I_e = \frac{\Phi_e}{4\pi}$$

specific to pt. source

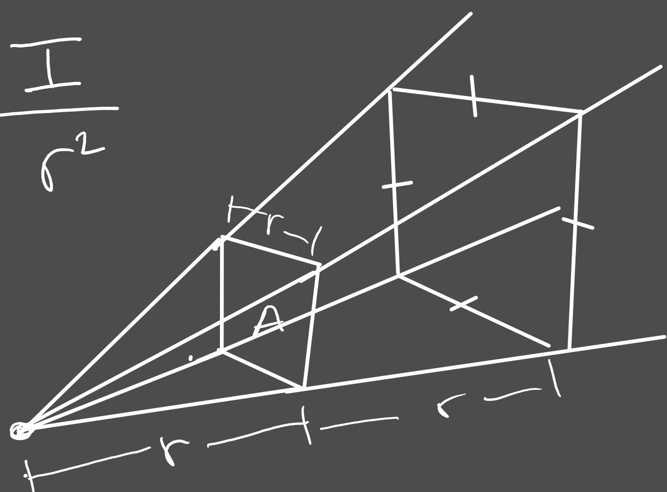
constant

$$E_e = \frac{\Phi_e}{4\pi r^2}$$

$$E_e = \frac{I \cdot \cancel{4\pi}}{\cancel{4\pi} r^2}$$

$$\Phi_e = I \cdot 4\pi$$

$$E_e = \frac{I}{r^2}$$



$$s = r \cdot \theta$$

arc length

$$\theta = \frac{s}{r}$$

in radians

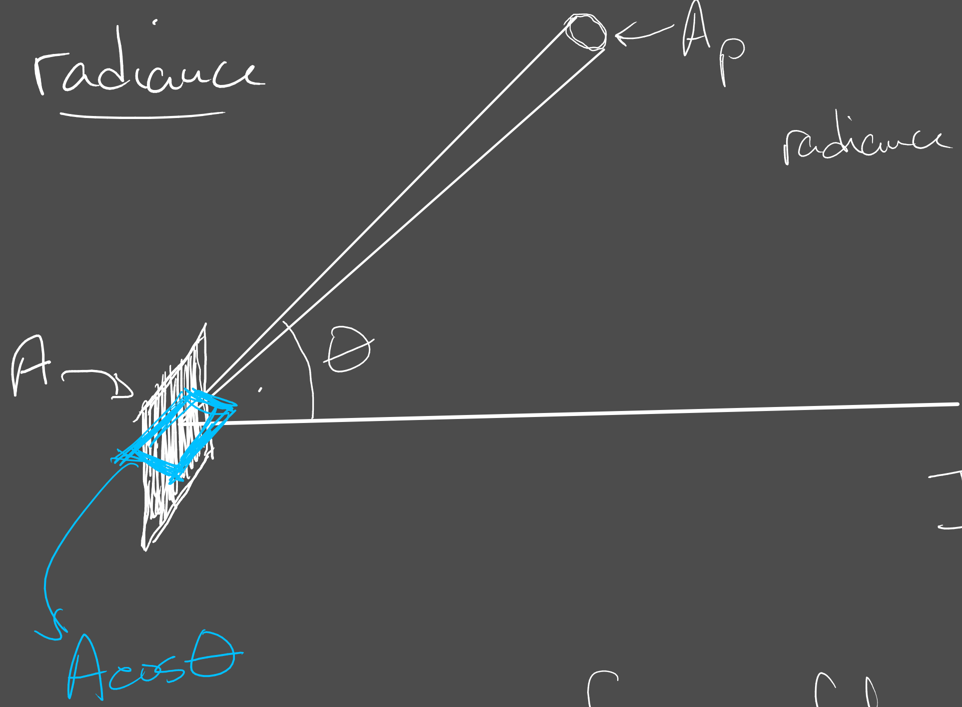
$$\omega = \frac{A}{r^2}$$

solid angle [steradians]

full sphere

$$\omega = 4\pi$$

radiance



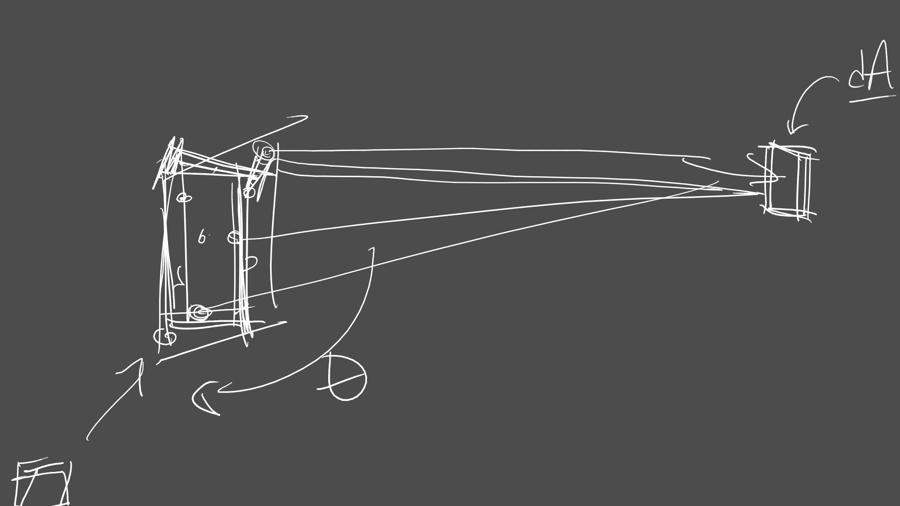
radiance, $L_e = \dfrac{dI_e}{d(A\cos\theta)}$

$$I(\theta) = I(0)\cos\theta$$ ← Lambert's Law

max intensity

for a perfectly diffuse emitter

$$L_e = \frac{I_0 \cancel{\cos\theta}}{A\cancel{\cos\theta}} = \frac{I_0}{A}$$ ← constant



$$E_e = \frac{d\phi}{dA}$$

HW: 1, 2, 3, 7, 8, 10

8 Show that the relativistic kinetic energy,

$$E_K = mc^2(\gamma - 1)$$

reduces to the classical expression $\frac{1}{2}mv^2$, when $v \ll c$.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2}$$

$$E_k = \gamma mc^2 - mc^2 \longrightarrow E_k = \frac{1}{2}mv^2$$

$v \ll c$

$\frac{v}{c} \ll 1$

$$= \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} mc^2 - mc^2$$

$$\underbrace{(1 \pm x)^n = 1 + \frac{nx}{1!}}_{} + \cancel{\frac{nx^2}{2!}} + \cdots$$

$$\approx \left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right) mc^2 - mc^2$$

$$\cancel{mc^2} + \frac{1}{2}mv^2\frac{\cancel{c^2}}{\cancel{c^2}} - \cancel{mc^2} = \frac{1}{2}mv^2$$