

Chapter 11 - Fraunhofer Diffraction

From ch. 5

for coherent light

$$E_o^2 = N^2 E^2$$

incoherent

$$E_o^2 = N E^2$$

Go to chapter 7

$$E_e \rightarrow I = \frac{1}{2} \epsilon_0 c^2 E_o^2$$

$$I \propto N^2$$

so for double slit 4 times the irradiance
from one source \rightarrow constructive interference

In general for any obstruction.

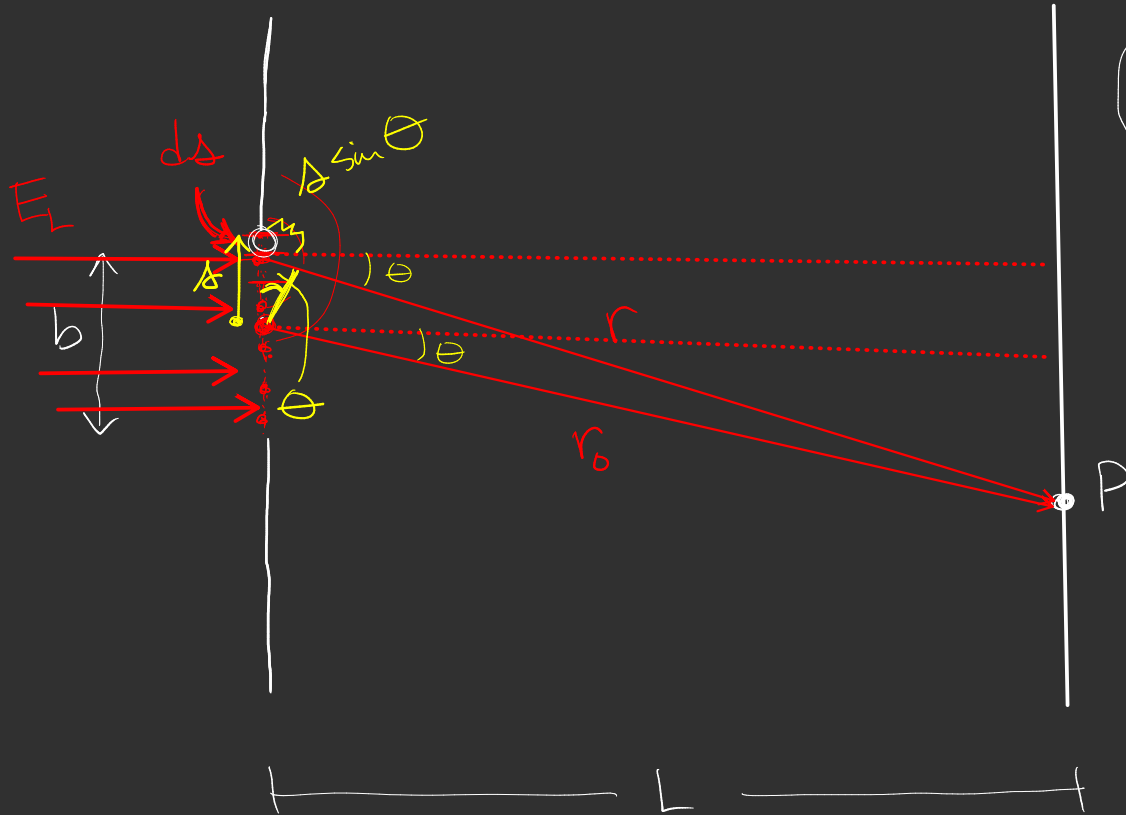
Huygens - Fresnel Principle

any point on a
wavefront can be
considered as a source
of spherical wavelets

actual field beyond the wavefront
is the superposition of the
wavelets, considering their phase
& amplitude

diffraction \rightarrow far field approximation \rightarrow Fraunhofer Diffraction
 \rightarrow near field approximation \rightarrow Fresnel Diffraction

\hookrightarrow distance from the source to the screen.



$$dE_p = \frac{E_L ds}{r} e^{i(kr - \omega t)}$$

$$r = r_0 + s \sin \theta$$

$$dE_p = \frac{E_L ds}{\underbrace{r_0 + s \sin \theta}_{\text{tiny}}} e^{i(k(r_0 + s \sin \theta) - \omega t)}$$

$$dE_p = \frac{E_L ds}{r_0} e^{i(kr_0 - \omega t)} e^{iks \sin \theta}$$

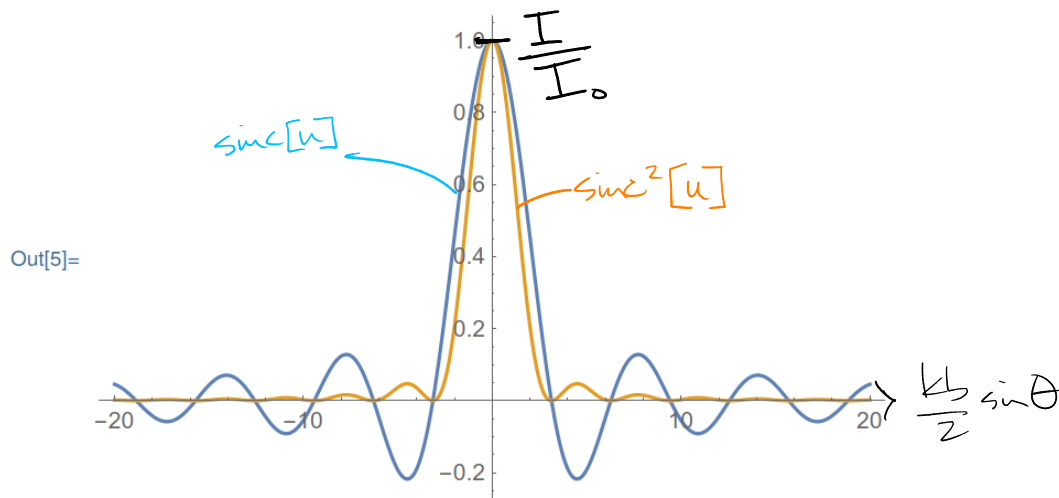
$$E_p = \int_{-b/2}^{b/2} \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} e^{iks \sin \theta} ds$$

$$E_p = \underbrace{\frac{E_L b}{r_o} \operatorname{sinc}\left(\frac{k b}{2} \sin \theta\right)}_{\text{amplitude at point P}} e^{i(k r_o - \omega t)}$$

amplitude at point P

$$\rightarrow E_o = \frac{E_L b}{r_o} \operatorname{sinc}\left(\frac{k b}{2} \sin \theta\right)$$

In[5]:= Plot[{Sinc[u], Sinc[u]^2}, {u, -20, 20}, PlotRange -> All]



$$\begin{aligned} &\sin(k(x+\Delta) - \omega t) \\ &\sin(kx - \omega t + k\Delta) \\ &\quad \underbrace{\hspace{1cm}}_{\text{phase difference}} \end{aligned}$$

Where will this be equal to zero?

$$\rightarrow \underbrace{\frac{k b}{2} \sin \theta}_{\text{phase difference of light from the edge and light from the center of the slit}} = m\pi \quad m = \pm 1, \pm 2, \dots$$

what is this
phase difference
of light from the
edge and light
from the center
of the slit

$$\rightarrow k = \frac{2\pi}{\lambda}$$

$$\frac{2\pi}{\lambda} \cdot \frac{b \sin \theta}{2} = m\pi$$

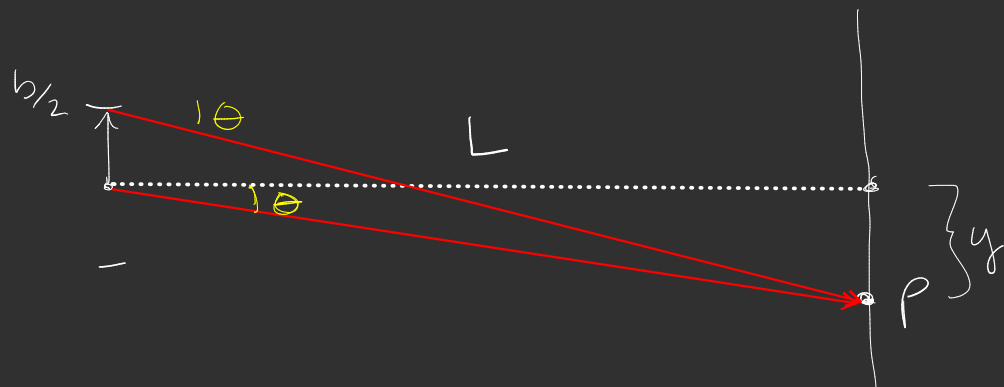
$$\underline{\underline{b \sin \theta = m \lambda}}$$

Condition for
destructive
interference

$$\beta = \frac{k b}{2} \sin \theta$$

$$\rightarrow \boxed{E_o = \frac{E_L b}{r_o} \operatorname{sinc} \beta}$$

$$I = \frac{1}{2} \epsilon_0 c^2 E_o^2 = \frac{1}{2} \epsilon_0 c^2 \left(\frac{E_L b}{r_o} \right)^2 \operatorname{sinc}^2 \beta$$



$$\tan \theta = \frac{y}{L}$$

far field approx.

$$\hookrightarrow \tan \theta \approx \sin \theta \approx \theta$$

$$b \sin \theta = m \lambda$$

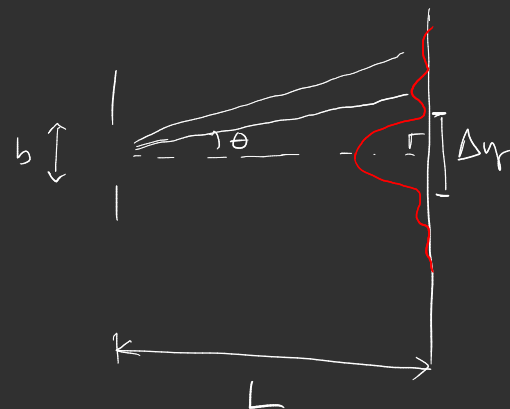
$$b \frac{y}{L} = m \lambda$$

destr. interf. $\rightarrow \boxed{y = \frac{m \lambda L}{b}} \quad m = \pm 1, \pm 2, \dots$

What about maxima?

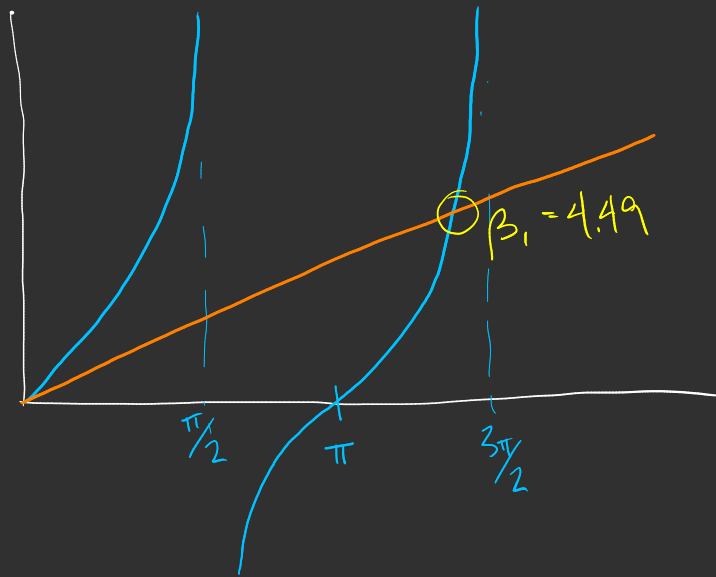
$$E_0 = \frac{E_L b}{r_0} \text{sinc} \beta$$

$$\beta = \frac{k b}{2} \sin \theta \quad \leftarrow \text{what values of } \theta \text{ produce maxima}$$



$$\frac{\partial \text{sinc} \beta}{\partial \beta} = \frac{\partial}{\partial \beta} \left(\frac{\sin \beta}{\beta} \right) = \frac{\cos \beta}{\beta} - \frac{\sin \beta}{\beta^2} = 0$$

$$\frac{\cos \beta}{\beta} = \frac{\sin \beta}{\beta^2}$$



$$\beta = \frac{\sin \beta}{\cos \beta}$$

$$\rightarrow \beta = \tan \beta$$

$$\beta_1 = 4.49$$

$$\beta_2 = 7.72$$

$$\beta_3 = 10.9$$

$$\beta = 2 \quad \tan 2 =$$

$$\beta = 1 \quad \tan(1) = 1.55$$

$$\beta = 0 \quad \tan(0) = 0$$

$$\beta_1 = \frac{kb}{2} \sin \theta$$

$$\sin \theta_1 = \frac{2\beta_1}{kb}$$

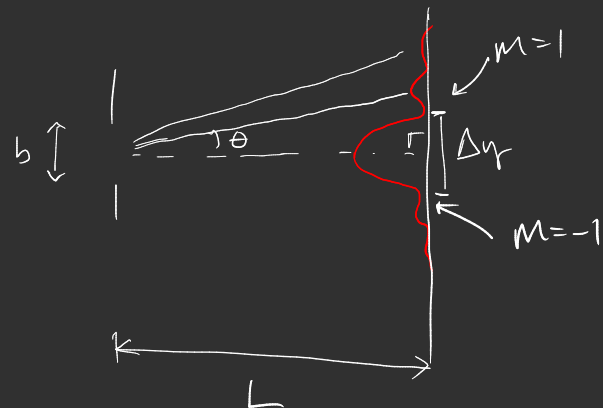
What about width of the central peak?

$$y = \frac{m\lambda L}{b} \quad m = \pm 1, \pm 2, \dots$$

$$\Delta y = y_{m=1} - y_{m=-1}$$

$$W = \Delta y = \frac{1\lambda L}{b} - \frac{(-1)\lambda L}{b}$$

$$W = \frac{2\lambda L}{b}$$

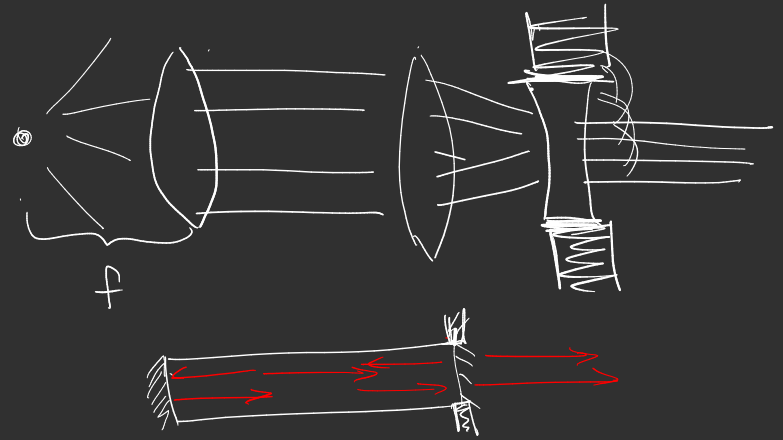


↓
 $w \rightarrow b$ the smaller L gets

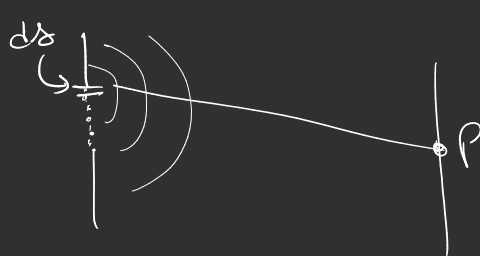
$$w = b = \frac{2\lambda L_{\min}}{b}$$

$$L_{\min} = \frac{b^2}{2\lambda}$$

$L \gg \frac{b^2}{\lambda}$ } criteria for far field approximation



$$dE = \frac{E_L ds}{r} e^{i(kr - \omega t)}$$



$$\frac{dp}{dt} = F = qE$$

$$\frac{F}{q} = E = \left[\frac{N}{C} \right]$$

$$\psi(r, t) = \frac{A}{r} e^{i(kr - \omega t)}$$

$$E(r, t) = \frac{E_0}{r} e^{i(kr - \omega t)}$$

Volts

$$E(r, t) = \frac{E_0 \cdot 1m}{r} e^{i(kr - \omega t)}$$

$$E(r) \rightarrow \frac{E_2}{E_1} = \frac{r_1}{r_2}$$

← 1m ← any r

← E(1m)

$$E(r) = \frac{E_0(1m)}{r} e^{i(kr - \omega t)}$$

$$E = \frac{E_0}{r} e^{i(kr - \omega t)}$$

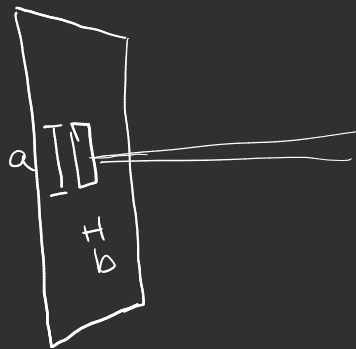
→ source strength

Ids ← how many oscillations are in ds ?

$b \left\{ \right.$

$$- \frac{N}{b} ds$$

$$dE = \frac{E_0}{r} e^{i(kr - \omega t)} \cdot \frac{N}{b} ds$$



$$\mathcal{E}_L = \lim_{N \rightarrow \infty} \frac{\mathcal{E}_0 N}{b}$$

$$d\mathcal{E} = \frac{\mathcal{E}_L ds}{r} e^{i(kr - \omega t)}$$

$$\begin{cases} I_b = I_{0b} \text{sinc}^2 \beta \\ I_a = I_{0a} \text{sinc}^2 \alpha \end{cases}$$

$$\beta = \frac{k b}{2} \sin \theta$$

$$\alpha = \frac{k a}{2} \sin \phi$$

$$\rightarrow I = I_0 \text{sinc}^2 \alpha \text{sinc}^2 \beta$$

