

Thermo Intro

temperature

Fahrenheit

Celsius

Rankine

Kelvin

Room temp 20°C ($\sim 70^{\circ}\text{F}$)

↓

293K

$$\Delta F = \frac{9}{5} \Delta C$$

$$T_F = \frac{9}{5} T_C + 32$$

$$T_K = T_C + 273.15$$

Number, moles, molar mass, density

1 mole of things = $6.022 \cdot 10^{23}$ things
particle
atom
molecule

$N \rightarrow$ number of particles

$n \rightarrow$ number of moles

$\overset{12}{\curvearrowright} \rightarrow 12$ protons and neutrons
 $\hookrightarrow 6$ protons

1 mole is a gram
of protons and neutrons

Ex. mass of one proton?

mass of one proton \times number of proton = mass
of collection

$M \rightarrow$ total mass of
a collection

$m \rightarrow$ mass of one
particle

$$m \times N = M$$

$$m = \frac{M}{N} = \frac{1 \text{ gram}}{N_A} \rightarrow \text{Avogadro's number } 6.022 \cdot 10^{23}$$

$$m = 1.7 \cdot 10^{-24} \text{ grams} \\ = 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2 ?

$$1 \text{ mole of } N_2 = 2 \cdot \left(14 \frac{\text{g}}{\text{mol}}\right) = 28 \text{ g}$$

Dry air?

78% N_2 , 21% O_2 , 1% Ar

$$0.78 \cdot (28 \text{ g/mol}) + 0.21 (32 \text{ g/mol}) + 0.01 (40 \text{ g/mol}) = 29 \text{ g/mol}$$

Ideal Gas Law \rightarrow an equation of state

- experimental law

\rightarrow state variables

number of particles
(microscopic)

$PV = Nk_B T$

\rightarrow Boltzmann's constant

$k_B = 1.38 \cdot 10^{-23} \text{ J/K}$

$P = \frac{F}{\text{Area}}$ $[Pa] = \left[\frac{N}{m^2}\right]$ $[m^3]$ \rightarrow Temperature $[K]$

Alternative form $\rightarrow PV = nRT$ (macroscopic)

\rightarrow universal gas constant

$$N k_B = n \cdot R$$

$$\uparrow N = n \cdot N_A \leftarrow \text{definition of moles}$$

$$n \cdot N_A \cdot k_B = n \cdot R$$

$$N_A \cdot k_B = R = 6.022 \cdot 10^{23} \frac{\text{part}}{\text{mol}} \cdot 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

$$R = 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}$$

Volume of 1 mole of air at room temp and atmospheric pressure

$\hookrightarrow 293\text{K}$ or 300K

$\hookrightarrow 1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \text{ J/Kmol} \cdot 300\text{K}}{10^5 \text{ Pa}} = 0.024 \text{ m}^3$$

$$\begin{aligned} V_{\text{cube}} &= s^3 \\ \sqrt[3]{0.024 \text{ m}^3} &= s = 0.288 \text{ m} \\ &\sim 30 \text{ cm} \end{aligned}$$

Laws of Thermodynamics

0. Thermometers work

1. Conservation of Energy

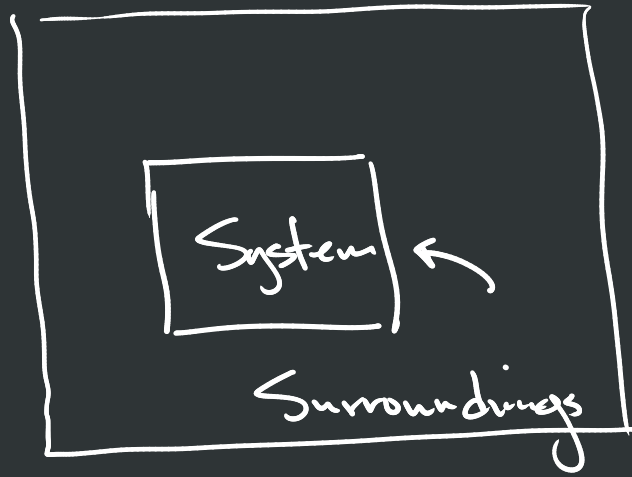
2. Heat flows from high temp to low temp

Entropy is maximized

No perpetuated motion machine

3. You can't reach absolute zero

1st Law \rightarrow Conservation of Energy



Energy of a system can change

Work + Heat

\downarrow
Force applied
over distance

\downarrow
Spontaneous flow
of energy due
to a difference
in temperature

$$dU = Q + W$$

$$\rightarrow dU = \pm Q + \pm W$$

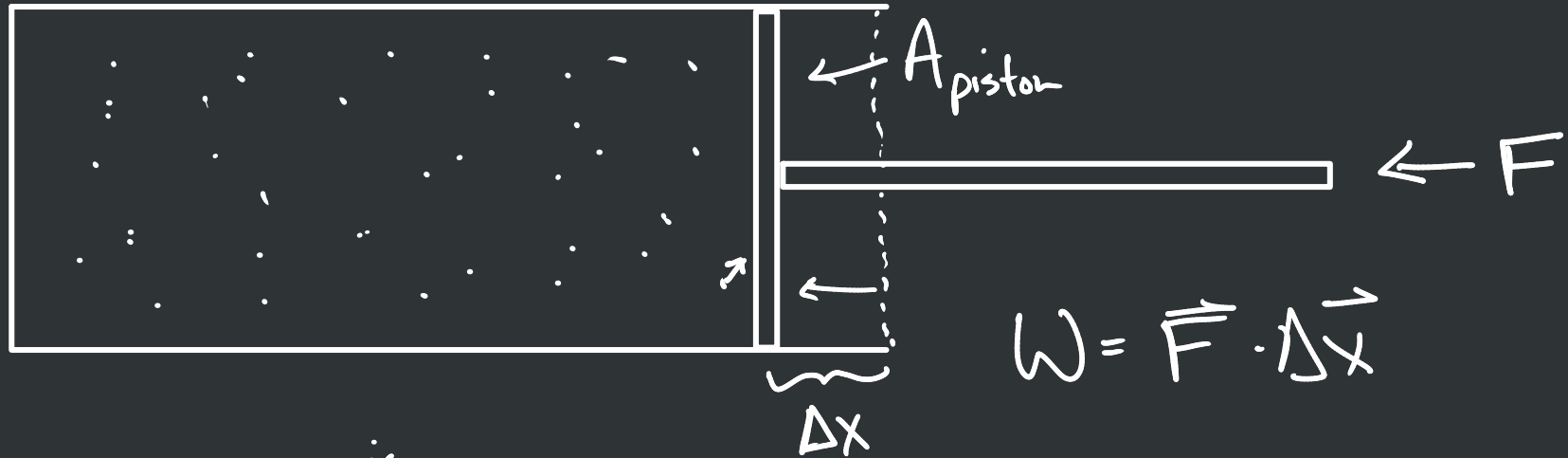
\uparrow inexact differential

$$dU = \delta Q + \delta W$$

\hookrightarrow internal energy of the system, heat is added
or work is done to the system they are positive

~~$$dQ = Q(x+dx) - Q(x)$$~~

Compression Work (or Expansion)



$$W = \vec{F} \cdot \Delta \vec{x}$$

$$P_{\text{gas}} = \frac{F_{\text{gas}}}{A_{\text{piston}}}$$

$$W = P_{\text{gas}} \cdot \underbrace{A_{\text{piston}} \cdot \Delta x}_{\Delta V}$$

\downarrow

$$W = -P \Delta V$$

ΔV is (-) in compression but energy is increasing

assuming pressure is uniform throughout the chamber

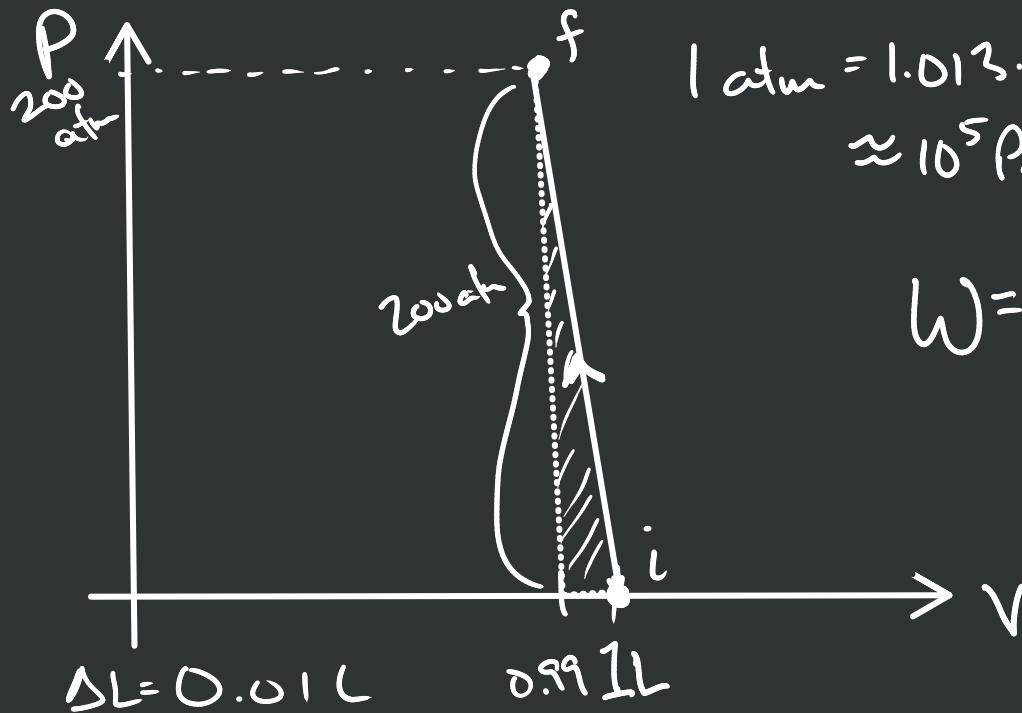
compression (expansion) must happen quasistatically

$$dU = \delta Q + \delta W$$

$$\delta W = -P dV$$

$$\underline{W = \int -P dV}$$

Problem 1.32. By applying a pressure of 200 atm, you can compress water to 99% of its usual volume. Sketch this process (not necessarily to scale) on a PV diagram, and estimate the work required to compress a liter of water by this amount. Does the result surprise you?



$$1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa} \approx 10^5 \text{ Pa}$$

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$$

$$W = -\int P dV$$

$$\Delta V = 0.01 \text{ L} = 10^{-5} \text{ m}^3$$

$$0.01 \text{ L} \cdot \frac{1 \text{ m}^3}{1000 \text{ L}} = 10^{-5} \text{ m}^3$$

$$A_{\Delta} = \frac{1}{2} b h \Rightarrow \underline{W = \frac{1}{2} P \Delta V}$$

$$W = \frac{1}{2} (200 \cdot 10^5 \text{ Pa}) \cdot 10^{-5} \text{ m}^3 = \underline{100 \text{ J}}$$

$$1 \text{ L} \cdot \frac{10^3}{1 \text{ L}} \cdot \frac{1 \text{ cm}^3}{1 \text{ mL}} \cdot \frac{(1 \text{ m})^3}{(100 \text{ cm})^3} = 1 \cdot 10^{-3} \text{ m}^3 = 0.001 \text{ m}^3$$

$$\boxed{1000 \text{ L} = 1 \text{ m}^3}$$

slope-intercept

$$y = mx + b$$

\uparrow
 $m = \frac{\Delta p}{\Delta V}$

\nwarrow y-int

point-slope

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200}{-0.01} = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}}$$

$$p - 0 = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}} (V - 1 \text{ L})$$

$$p = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}} \cdot V + 2 \cdot 10^4 \text{ atm}$$

1.33]

1.33

Problem 1.33. An ideal gas is made to undergo the cyclic process shown in Figure 1.10(a). For each of the steps A , B , and C , determine whether each of the following is positive, negative, or zero: (a) the work done on the gas; (b) the change in the energy content of the gas; (c) the heat added to the gas. Then determine the sign of each of these three quantities for the whole cycle. What does this process accomplish?

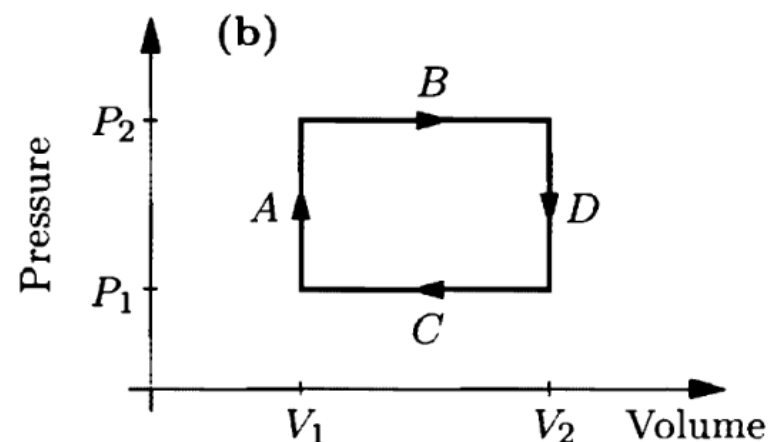
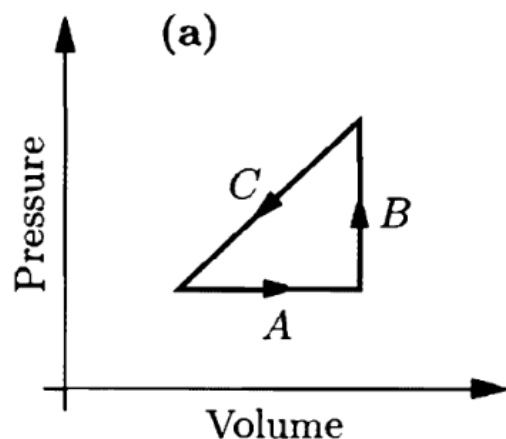


Figure 1.10. PV diagrams for Problems 1.33 and 1.34.


Come back!

Proportionality

$$PV = Nk_B T$$

$$P_1 V = Nk_B T_1$$


$$P_2 V = Nk_B T_2$$


$$P = \left(\frac{Nk_B}{V} \right) T$$

$$\frac{P_2 V = Nk_B T_2}{P_1 V = Nk_B T_1}$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 2$$


$$P \propto T$$


$$P = C \cdot T$$

↳ constant of proportionality

$$\frac{P_2}{P_1} = \frac{N_2}{N_1} \rightarrow P \propto N' \rightarrow P = C_2 N$$

$$\hookrightarrow \frac{P_2}{P_1} = \left(\frac{N_2}{N_1} \right)'$$

$$PV = N k_B T$$

$$P = \frac{N k_B T}{V}$$

$$P \propto N' T' V^{-1}$$

$$P \propto V^{-1} \rightarrow P = C_2 V^{-1} = \frac{C_2}{V}$$

$$\hookrightarrow \frac{P_2}{P_1} = \left(\frac{V_2}{V_1} \right)^{-1} = \frac{V_1}{V_2}$$

$$F_G \propto r^{-2}$$



$$F_G = C_3 \cdot r^{-2}$$

$$= \frac{C_3}{r^2}$$

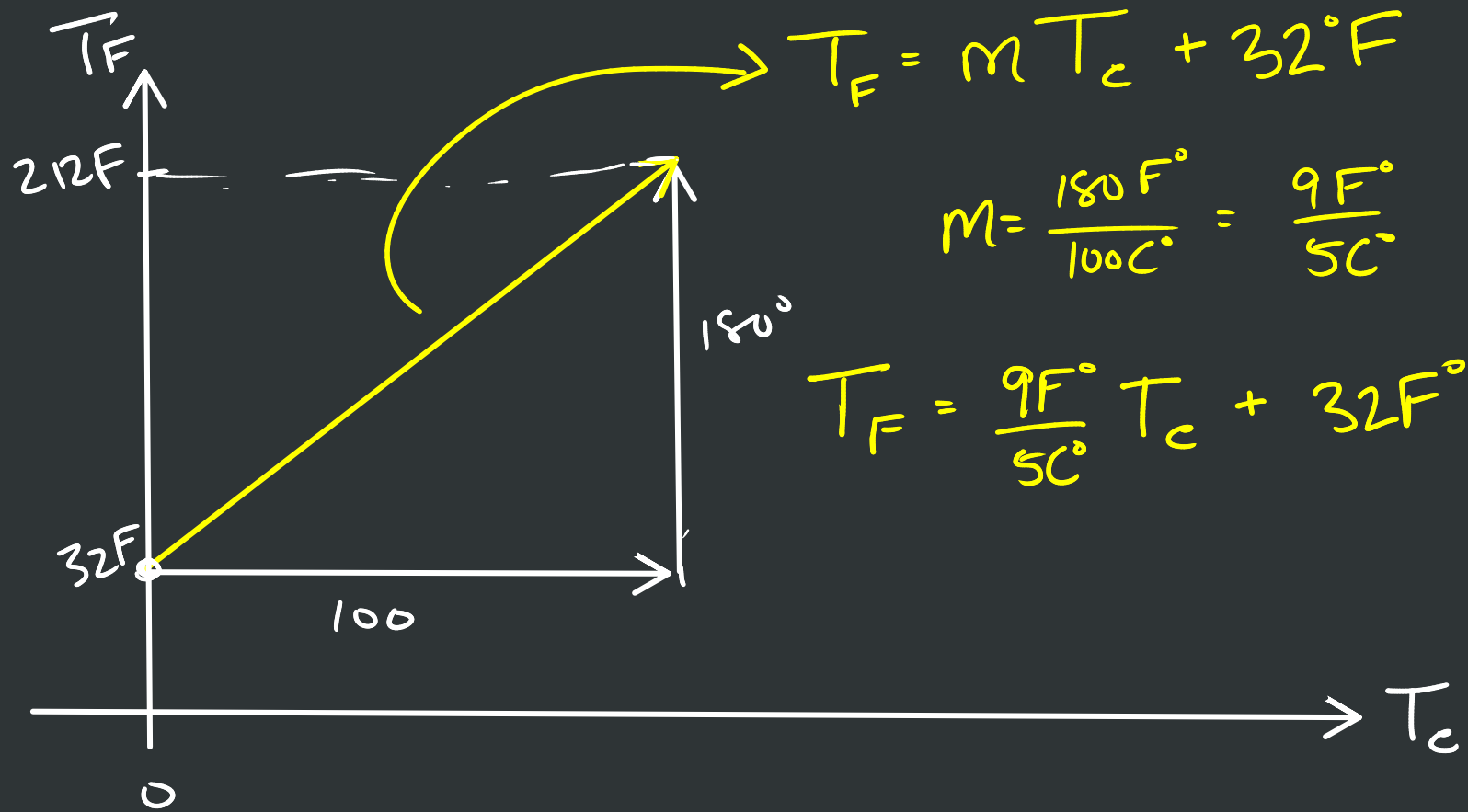
$$\hookrightarrow \frac{F_2}{F_1} = \left(\frac{r_2}{r_1} \right)^{-2}$$

$$\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$$

$$T = \frac{pV}{k_B N}$$

$$p \propto N' T' V^{-1}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right) \cdot \left(\frac{V_2}{V_1} \right)^{-1}$$



Heat \rightarrow flow of energy due to a difference in temperature

$$\Delta U = Q + \cancel{W}^{\rightarrow 0}$$

[Joules]

[calorie]

$$1 \text{ cal} = 4.186 \text{ J}$$

vs.

[Calorie] \leftarrow food

$$\frac{Q}{\Delta T} = C \leftarrow \text{heat capacity} \left[\frac{\text{J}}{\text{K}} \right]$$

$$1 \text{ Cal} = 1000 \text{ calorie}$$

① $\frac{C}{m} = c \leftarrow \text{specific heat capacity} \left[\frac{\text{J}}{\text{gK}} \right] = \left[\frac{\text{kJ}}{\text{kgK}} \right] \quad \boxed{Q = mc \Delta T}$

② $\frac{C}{n} = C_{V,P} \leftarrow \begin{array}{l} \text{molar heat capacity at constant volume} \\ \text{molar heat capacity at constant pressure} \end{array}$

$$Q = n C_V \Delta T$$

③ $\frac{C}{V} \leftarrow$ volumetric heat capacity

Phase change

$$\frac{Q}{m} = L \leftarrow \text{Latent Heat}$$

\rightarrow Latent Heat of fusion - L_f
solid \leftrightarrow liquid

\rightarrow Latent Heat of vaporization - L_v
liquid \leftrightarrow gas

Calorimetry - when multiple objects are in thermal contact
at what temperature is thermal equilibrium

$$\Delta U = 0 = Q_1 + Q_2 + Q_3 + \dots$$

$\underbrace{\hspace{10em}}$
objects changing temp or phase transitions occurring

Ex: 5kg of Al at 500°C | 5kg of Pb at 75°C | What is equil temp?

↳ 490° How much heat was given off?

$$C_{Al} = 0.9 \frac{\text{kJ}}{\text{kg K}}$$

$$C_{Pb} = 0.13 \frac{\text{kJ}}{\text{kg K}}$$

$$Q = mc\Delta T = mc(T_f - T_i)$$
$$= 5\text{kg} \cdot 0.9 \frac{\text{kJ}}{\text{kg K}} (490^\circ\text{C} - 500^\circ\text{C})$$

$$= 5\text{kg} \cdot 0.9 \frac{\text{kJ}}{\text{kg K}} \cdot (-10\text{K})$$

$$Q = -45\text{kJ} \longrightarrow +45\text{kJ} = 5\text{kg} \cdot 0.13 \frac{\text{kJ}}{\text{kg K}} \cdot \Delta T$$

$$\Delta T = 69.2\text{K}$$

$$T_f = 145^\circ\text{C}$$

$$Q_1 + Q_2 = 0$$

$$m_{Al} \cdot C_{Al} \cdot \Delta T_{Al} + m_{Pb} \cdot C_{Pb} \cdot \Delta T_{Pb} = 0$$

$$m_{Al} \cdot C_{Al} \cdot (T_f - 500) + m_{Pb} \cdot C_{Pb} \cdot (T_f - 75) = 0$$

solve for T_f

$$T_f = 446 \text{ ish } ^\circ\text{C}$$

HW: 1.7, 1.16, 1.17

1.7] $\beta = \frac{\frac{\Delta V}{V}}{\Delta T}$

$$\beta = \frac{1}{5500} \text{ K}^{-1} = 1.81 \cdot 10^{-4} \text{ K}^{-1}$$

$$r = \frac{1}{2} \text{ cm}$$

$$A = \pi \left(\frac{1}{2} \text{ cm} \right)^2 = \frac{\pi}{4} \text{ cm}^2$$

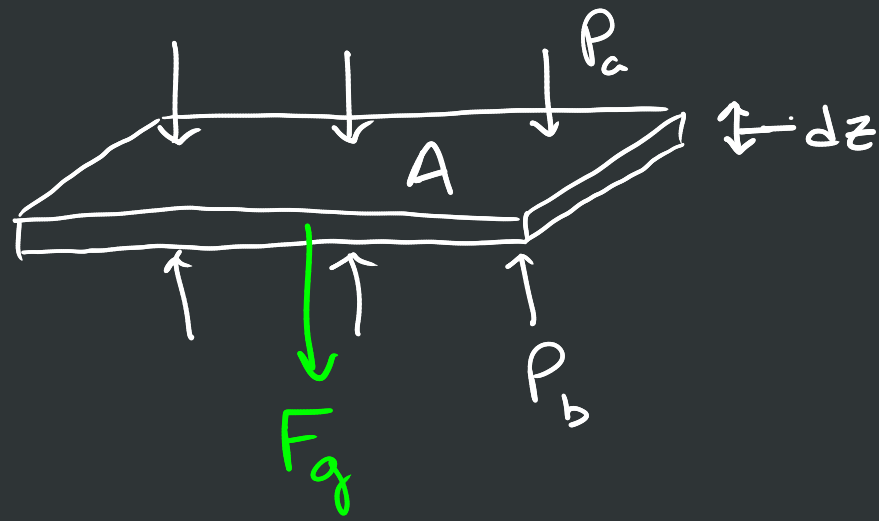
$$V = A \cdot h = \frac{\pi}{2} \text{ cm}^3$$

$$\frac{\Delta V}{V} = \beta \Delta T$$

$$\Delta z = 1 \text{ mm} \approx 1 \text{ K}$$

1.16] $\rho = \frac{M}{V}$

"rho"
volumetric mass density



a) $P = \frac{F}{A}$ $+ P_b \cdot A - P_a \cdot A - M \cdot g = 0$

$\frac{dP}{dz} = \rho$

b) $PV = Nk_B T$

pressure
↓

$\rho = \underbrace{P, T, m}_{\leftarrow \text{avg mass of air molecules } m = 29 \text{ g/mol}}$

$$N = m \cdot n$$

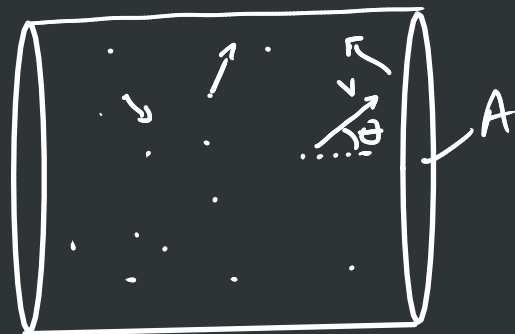
$$\underline{1.17} \quad c) \left(P + \frac{an^2}{V^2} \right) \underbrace{\left(V - nb \right)}_{V \left(1 - \frac{nb}{V} \right)} = nRT$$

$$\left(1 - \frac{nb}{V} \right)^{-1} \rightarrow \text{apply the binomial expansion}$$

Kinetic Theory and Equipartition of Energy

pressure \longleftrightarrow kinetic energy \longleftrightarrow temperature

$$\text{pressure}_{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \leftarrow \frac{2mv \cos \theta}{\Delta p \text{ for each collision w/ the wall in } \Delta t \text{ amount of time}}$$



$$\text{pressure} = \frac{\Delta p}{A \Delta t} \cdot \text{number of particles hitting area } A \text{ w/ some velocity in } \Delta t$$

integrate over all velocities + θ

total change in momentum	=	change in momentum per collision	·	number of atoms traveling in a particular direction w/ a particular speed	·	fraction of them that are within striking distance of the surface of A
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Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

total outcomes $\rightarrow N = \sum_{x=0}^{\infty} N(x)$

$$P(x) = \frac{N(x)}{N}$$

normalized

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

\uparrow
average value
expectation value

$\underbrace{\hspace{10em}}_{\text{weighted by probability}}$ average probability

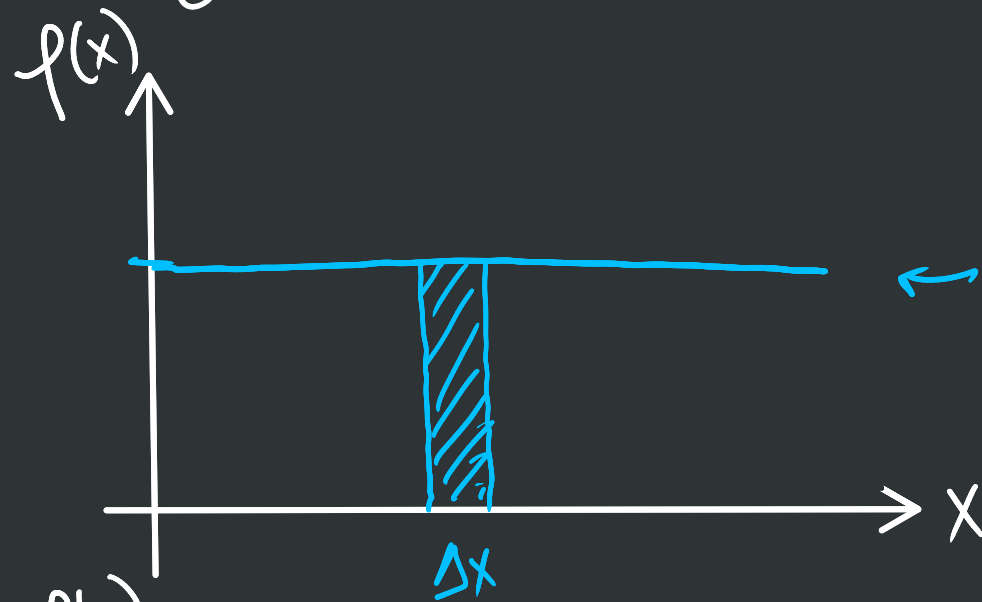
(not the same
as the most probable)

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

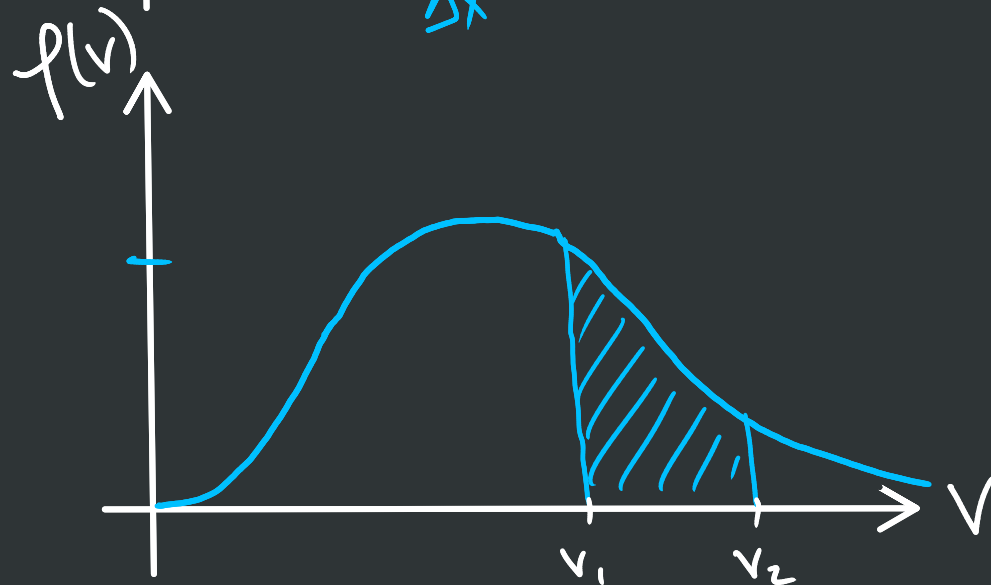
Continuous variable

$$\left\{ \begin{array}{l} \text{probability that an individual} \\ \text{value lies between } x \text{ and } x+dx \end{array} \right\} = f(x) \cdot dx$$



← uniform
prob
function

↑
- probability density function
- probability distribution



$$P(v_1, v_2) = \int_{v_1}^{v_2} f(v) \cdot dv$$

$$\int_{-\infty}^{+\infty} f(v) dv = 1 \quad \leftarrow \text{normalization}$$

$$\langle v \rangle = \int_{-\infty}^{+\infty} v \cdot f(v) \cdot dv$$

$$\langle v^2 \rangle = \int_{-\infty}^{+\infty} v^2 \cdot f(v) \cdot dv$$

$$\langle f(v) \rangle = \int_{-\infty}^{+\infty} f(v) \cdot f(v) \cdot dv$$

1.47

$$Q_{\text{tea}} = m_{\text{tea}} c \Delta T_{\text{tea}}$$

$$Q_{\text{tea}} = 200\text{g} \cdot \left(1 \frac{\text{cal}}{\text{g} \cdot \text{K}}\right) \cdot (-35\text{K})$$

$$Q_{\text{tea}} = -7000 \text{ cal}$$

$$Q_{\text{ice}} = +7000 \text{ cal} = m_{\text{ice}} \cdot C_{\text{ice}} \cdot (+15\text{K}) + m_{\text{ice}} \cdot L_f + m_{\text{ice}} C_{\text{water}} (65\text{K})$$

