Chapter 4

· **Heat Engines** → absorbs heat, produces work

cannot convert all of the heat to work

Heat comes in increases the entropy of the engine → to start the cycle over entropy must be taken out

heat exhausted

$$\Delta U_{cycle} = 0 = Q + W_{gas}$$

$$= \underset{\uparrow \text{ heat in}}{Q_h} - \underset{\nwarrow \text{ heat out}}{Q_c} + W_{gas}$$

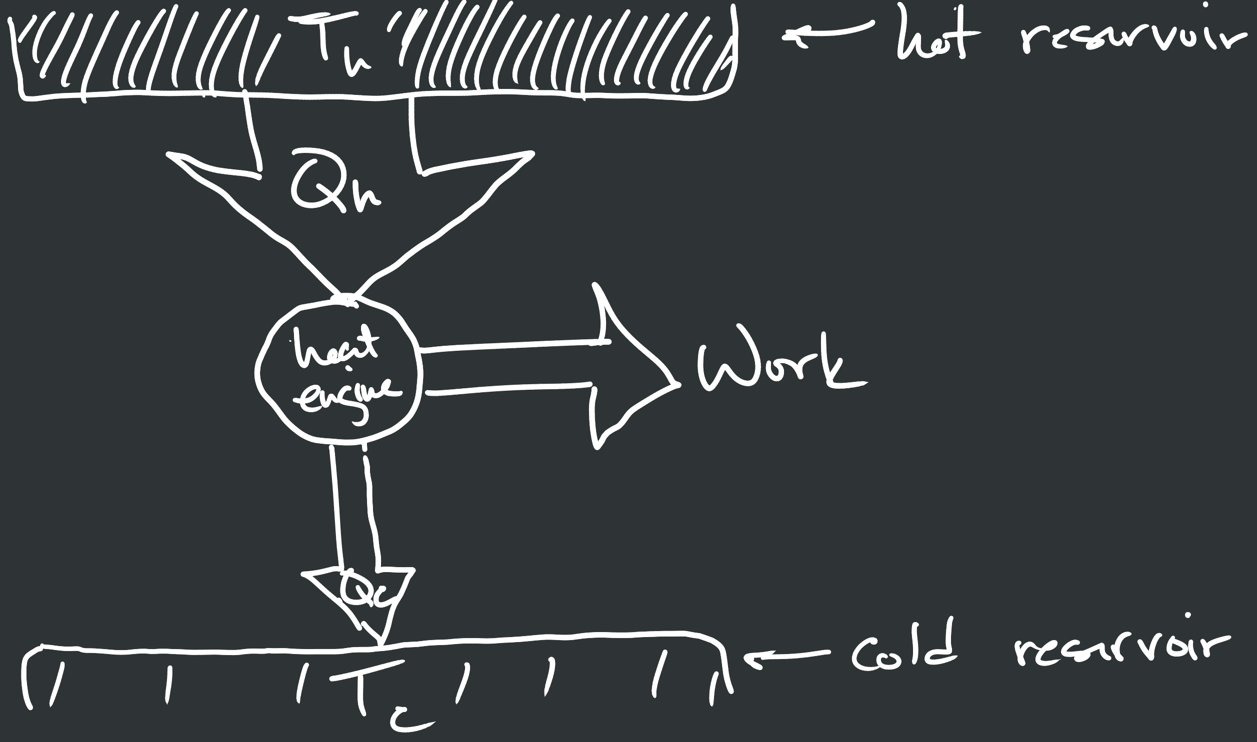
$$-W_{gas} = Q_h - Q_c$$

$$W_{surr} = -W_{gas}$$

$$\rightarrow W_{surr} = Q_h - Q_c$$

$$\text{efficiency} \equiv \frac{\text{what you get}}{\text{what you pay for}}$$

$$e = \frac{W_{surr}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$



$$\Delta S_{in} = \frac{Q_h}{T_h}$$ ← higher than

$$\Delta S_{out} = \frac{Q_c}{T_c}$$ ← lower than

$W_{surr} = Q_h - Q_c$

realistically, $\frac{Q_h}{T_h} \leq \frac{Q_c}{T_c}$

usually creating more entropy through the engine. More heat is dumped into the cold reservoir and less energy is available for work

$$\longrightarrow \frac{T_c}{T_h} \leq \frac{Q_c}{Q_h}$$

$e = 1 - \frac{Q_c}{Q_h}$, then $\underbrace{e \leq 1 - \frac{T_c}{T_h}}_{\text{so what is the max efficiency?}}$

for max efficiency:

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

But we need to break this up into a coupl of steps:

$$\underbrace{\frac{Q_h}{T_h}}_{\text{entropy that is removed from the hot reservior}} = \underbrace{\frac{Q_h}{T_{gas}}}_{\text{entropy gained by the engine}}$$

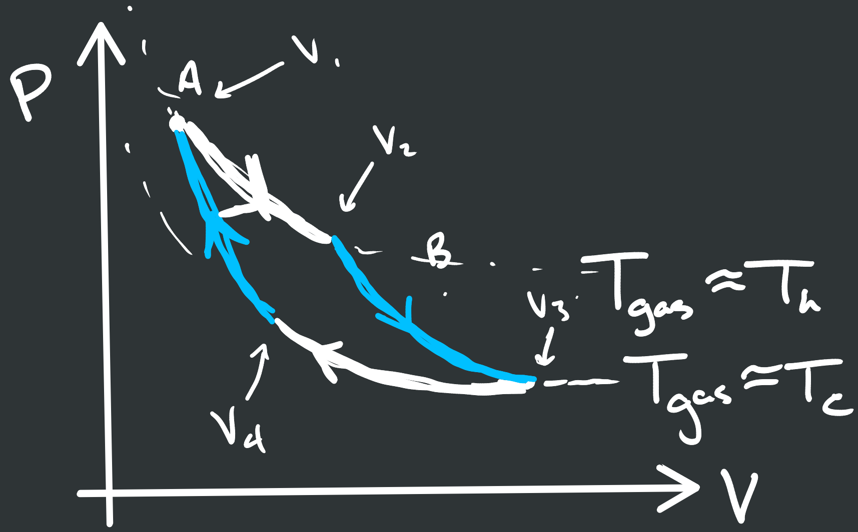
$T_{gas} = T_h$ to avoid any new entropy

$$\underbrace{}_{T_{gas} + dT = T_h}$$

so that heat will actually flow

So we expand the volume to keep the temperature the same, T_{gas} constant

↳ isothermal expansion



on the exhaust:
(isothermal compression)

$$\frac{Q_c}{T_c} = \frac{Q_c}{T_{gas}}$$

$$T_{gas} \approx T_c + dT$$

So to connect these two: adiabatic (isentropic)

$Q = 0$
no heat flows in or out

This cycle is called the Carnot Cycle.

$$e = 1 - \frac{T_c}{T_h}$$

only acheived by Carnot cycle

Prove this

4.5 isothermal expansion

$$\cancel{dU}^{0} = \bar{d}Q + \bar{d}W$$

$$đQ = -đW = p\,dV \neq p\Delta V$$

$$Q = \int_{V_i}^{V_f} p\,dV \quad , \quad p = \frac{Nk_BT}{V}$$

$$Q = Nk_BT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$Q = Nk_BT \ln\left(\frac{V_f}{V_i}\right)$$ ← heat in if $V_f > V_i$ (expansion)

$$Q_h = Nk_BT_h \ln\left(\frac{V_2}{V_1}\right)$$

$$Q_c = -Nk_BT_c \ln\left(\frac{V_4}{V_3}\right)$$ ← heat out since $V_3 < V_4$

$$Q_c = Nk_BT_c \ln\left(\frac{V_3}{V_4}\right)$$

$$e = 1 - \frac{Q_c}{Q_h} \stackrel{?}{=} 1 - \frac{T_c}{T_h}$$

$$e = 1 - \frac{\cancel{Nk}_B T_c \ln\left(\frac{V_3}{V_4}\right)}{\cancel{Nk}_B T_h \ln\left(\frac{V_2}{V_1}\right)}$$

if $\ln\left(\frac{V_3}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right)$

$\frac{V_3}{V_4} = \frac{V_2}{V_1}$ then proven.

adiabatic expansion

$$V_f T_f^{f/2} = V_i T_i^{f/2}$$

$$V_3 T_c^{f/2} = V_2 T_h^{f/2}$$

$$\left(\frac{T_c}{T_h}\right)^{f/2} = \frac{V_2}{V_3}$$

adiabatic compression

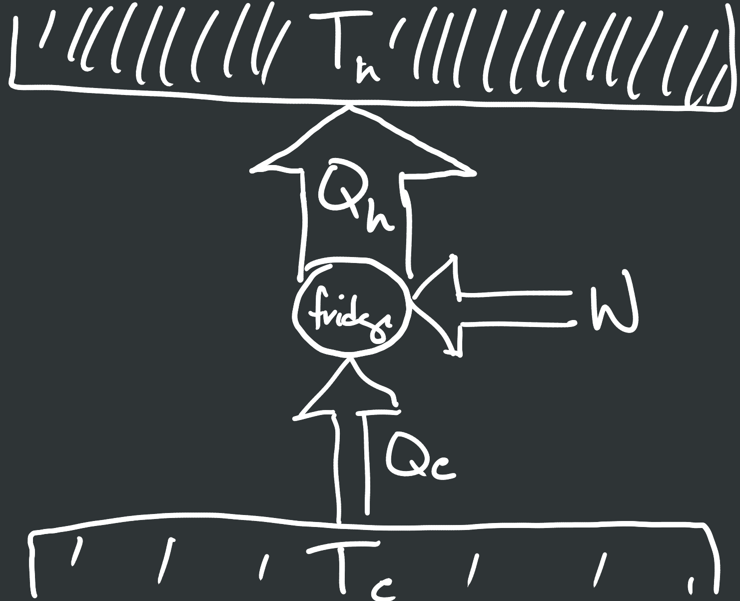
$$V_1 T_h^{f/2} = V_4 \cdot T_c^{f/2}$$

$$\frac{V_1}{V_4} = \left(\frac{T_c}{T_h}\right)^{f/2}$$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} \; !$$

so $e = 1 - \frac{T_c}{T_h}$ for Carnot Cycle

4.2) Refrigerators – operates oppositely to heat engine



W_{in} from environment

Q_c out of cold res.

Q_h into hot res.

refrigerator / air conditioner / heat pump

difference is what you want

efficiency => coefficient of performance

$$COP = \frac{\text{what you get}}{\text{what you pay for}}$$

refrigerator

$$COP = \frac{Q_c}{W}$$

heat pump

$$COP = \frac{Q_h}{W}$$

$$COP = \frac{Q_c}{W}$$

$$\Delta U = 0 = -(Q_h - Q_c) + W$$

$$W = Q_h - Q_c$$

use 1st law to change variables

$$COP = \frac{Q_c}{Q_h - Q_c}$$

$$COP = \frac{1}{\frac{Q_h}{Q_c} - 1}$$

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c}$$

$$\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$$

second law to establish a limit

$$COP \leq \frac{1}{\frac{T_h}{T_c} - 1}$$

$$\boxed{COP \leq \frac{T_c}{T_h - T_c}}$$

Ex. kitchen freezer → −20°C

$T_h = 293\,K \quad T_c = 253\,K$

$$COP \leq \frac{253\,K}{40\,K} = 6.3 = \frac{Q_c}{W}$$

for one joule of electrical energy out of the wall, 6.3 J of heat move out

How much heat is dumped into kitchen?

$$Q_h = Q_c + W = 7.3\,J$$

4.10) assume a $COP = 6.3$

$$COP = \frac{Q_c}{W} = \frac{Q_c/\Delta t}{W/\Delta t} = \frac{300\,W}{P_{wall}} = 6.3$$

$$P_{wall} = \frac{300\,W}{6.3} = \underline{\underline{47.6\,W}}$$

4.15) a) $COP = \dfrac{\text{what you get}}{\text{what you pay for}} = \dfrac{Q_c}{Q_f}$ ← heat removed; ← heat you pay for

b) $Q_r = Q_c + Q_f$

$$COP = \frac{Q_c}{Q_f} = \frac{Q_r - Q_f}{Q_f} = \frac{Q_r}{Q_f} - 1$$

$\dfrac{Q_r}{Q_f} > 2$ to have $COP > 1$ } Conservation of Energy does not exclude this

c) you do