

Thermo Intro

temperature

Fahrenheit

Celcius

Rankine

Kelvin

Room temp 20°C ($\sim 70^{\circ}\text{F}$)

\downarrow
293K

$$\Delta F = \frac{9}{5} \Delta C$$

$$T_F = \frac{9}{5} T_C + 32$$

$$T_K = T_C + 273.15$$

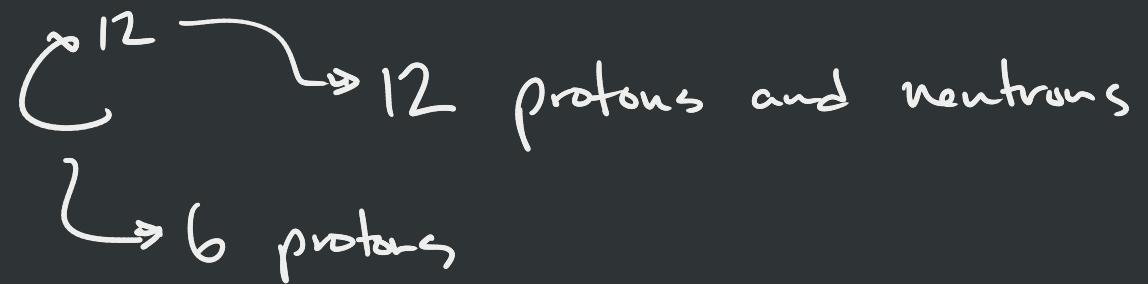
Number, moles, molar mass, density

1 mole of things = $6.022 \cdot 10^{23}$ things

particle
atom
molecule

$N \rightarrow$ number of particles

$n \rightarrow$ number of moles



Ex. mass of one proton?

$$\text{mass of one proton} \times \text{number of proton} = \frac{\text{mass of collection}}{\text{collection}}$$

1 mole is a gram of protons and neutrons

M → total mass of a collection

m → mass of one particle

$$m \times N = M$$

$$m = \frac{M}{N} = \frac{1 \text{ gram}}{N_A}$$

$N_A \approx \text{Avogadro's number } 6.022 \cdot 10^{23}$

$$m = 1.7 \cdot 10^{-24} \text{ grams}$$

$$= 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2 ?

$$1 \text{ mole of } N_2 = 2 \cdot (14 \frac{\text{g}}{\text{mol}}) = 28 \text{ g}$$

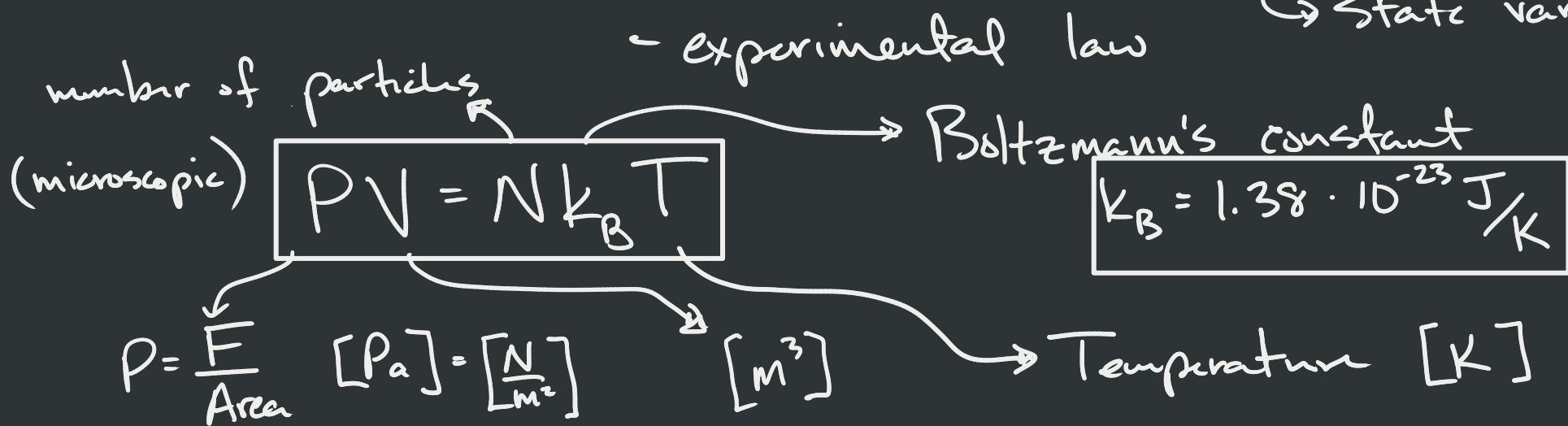
Dry air?

78% N_2 , 21% O_2 , 1% Ar

$$0.78 \cdot (28 \frac{\text{g/mol}}{\text{mol}}) + 0.21 (32 \frac{\text{g/mol}}{\text{mol}}) + 0.01 (40 \frac{\text{g/mol}}{\text{mol}}) = 29 \frac{\text{g/mol}}{\text{mol}}$$

Ideal Gas Law \rightarrow an equation of state

\hookrightarrow state variables



Alternative form $\rightarrow PV = nRT$ (macroscopic)

↗ universal gas constant

$$N k_B = n \cdot R$$

$$\left. \begin{array}{l} \\ N = n \cdot N_A \end{array} \right\} \leftarrow \text{definition of moles}$$

$$\cancel{n} \cdot N_A \cdot k_B = \cancel{n} \cdot R$$

$$N_A \cdot k_B = R = 6.022 \cdot 10^{23} \frac{\cancel{\text{part}}}{\cancel{\text{mol}}} 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

$$R = 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}$$

Volume of 1 mole of air at room temp and atmospheric pressure

293K or 300K

$$\hookrightarrow 1 \text{ atm} = 1.013 \cdot 10^5 \frac{\text{Pa}}{\text{bar}}$$

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot 300 \text{ K}}{10^5 \text{ Pa}} = 0.024 \text{ m}^3$$

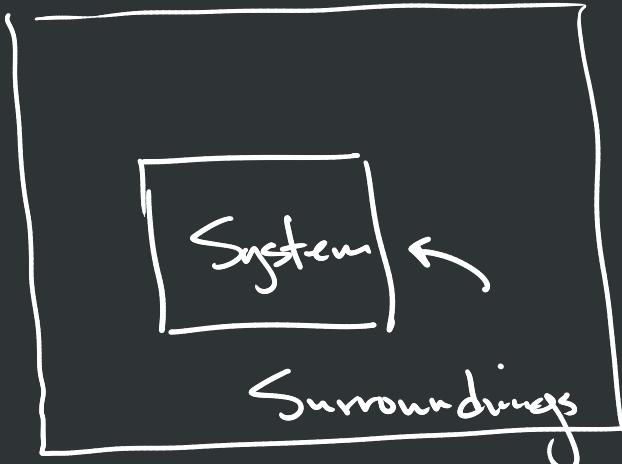
$$\nabla_{\text{cub}} = \delta^3$$
$$\sqrt[3]{0.024 \text{ m}^3} = \delta = 0.288 \text{ m}$$

$\sim 30 \text{ cm}$

Laws of Thermodynamics

0. Thermometers work
1. Conservation of Energy
2. Heat flows from high temp to low temp
Entropy is maximized
No perpetual motion machine
3. You can't reach absolute zero

1st Law \rightarrow Conservation of Energy



Energy of a system can change

Work

Heat

Force applied
over distance

Spontaneous flow
of energy due
to a difference
in temperature

$$dU = Q + W$$

$$\rightarrow dU = \delta Q + \delta W$$

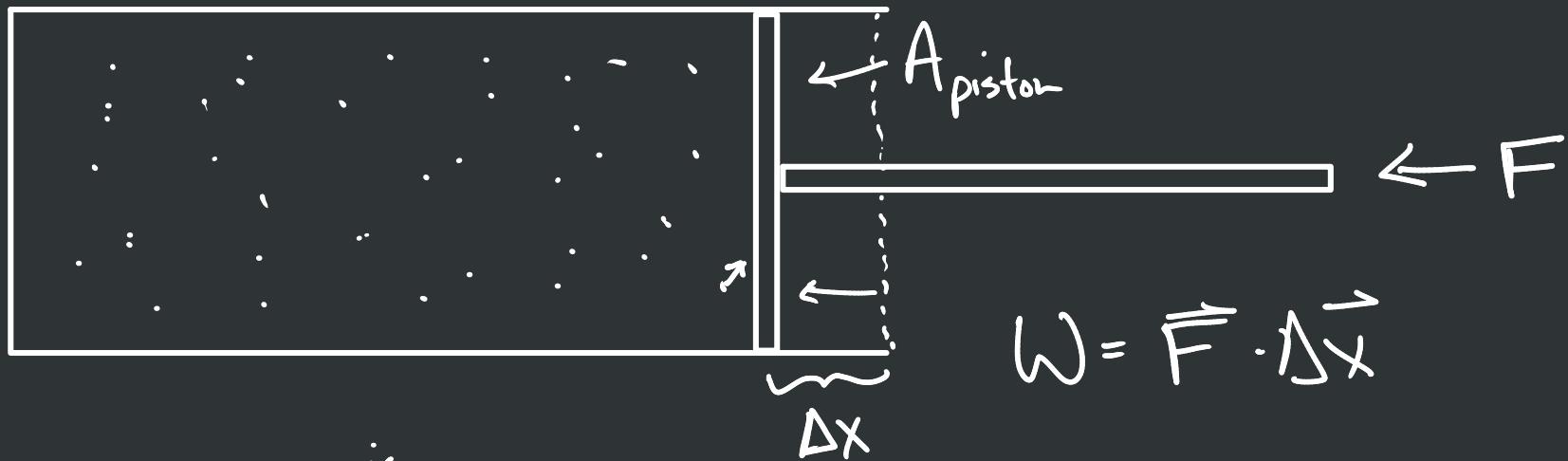
↑ inexact differential

~~$$dQ = Q(x+dx) - Q(x)$$~~

$$dU = \delta Q + \delta W$$

↳ internal energy of the system, heat is added
or work is done to the system they are positive

Compression Work (or Expansion)



$$W = \vec{F} \cdot \vec{\Delta x}$$

assuming pressure is
uniform throughout the
chamber
↳ compression (expansion)
must happen quasistatically

$$P_{\text{gas}} = \frac{F_{\text{gas}}}{A_{\text{piston}}}$$

$$W = P_{\text{gas}} \cdot A_{\text{piston}} \cdot \Delta x$$



ΔV
 ΔV is (-) in
compression but
energy is
increasing

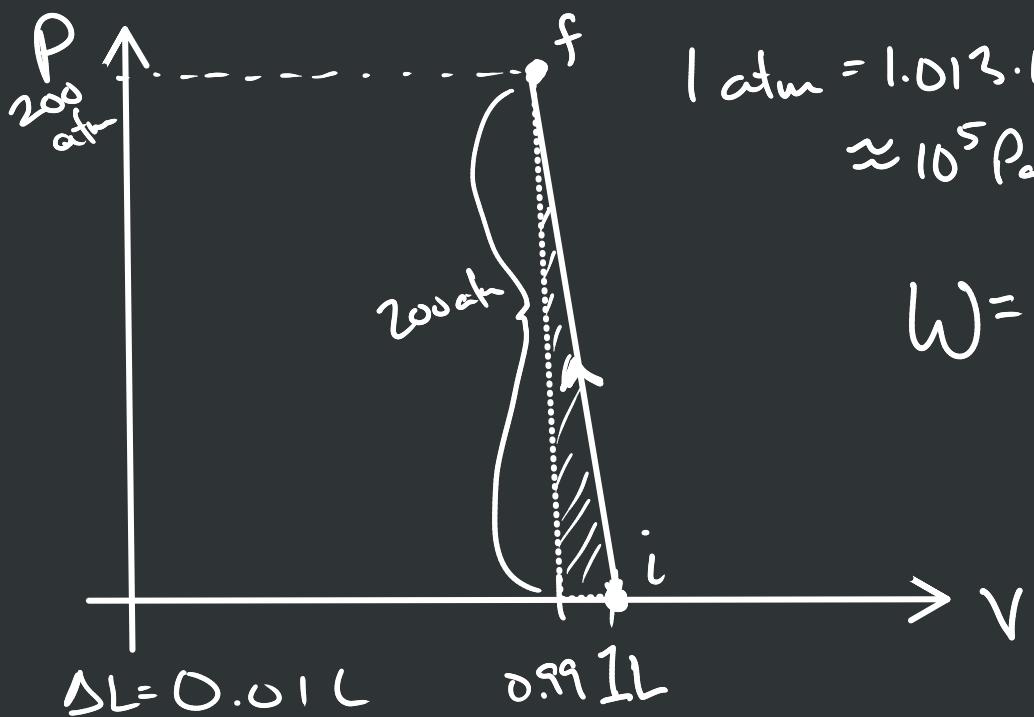
$$W = -P \Delta V$$

$$dU = dQ + dW$$

$$dW = -PdV$$

$$\underline{W = \int -PdV}$$

Problem 1.32. By applying a pressure of 200 atm, you can compress water to 99% of its usual volume. Sketch this process (not necessarily to scale) on a PV diagram, and estimate the work required to compress a liter of water by this amount. Does the result surprise you?



$$1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$$

$$\approx 10^5 \text{ Pa}$$

$$W = -\int P dV$$

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$$

$$\Delta V = 0.01 \text{ L} = 10^{-5} \text{ m}^3$$

$$0.01 \text{ L} \cdot \frac{1 \text{ m}^3}{1000 \text{ L}} = 10^{-5} \text{ m}^3$$

$$A_D = \frac{1}{2} b h \Rightarrow W = \underline{\underline{\frac{1}{2} P \Delta V}}$$

$$W = \frac{1}{2} (200 \cdot 10^5 \text{ Pa}) \cdot 10^{-5} \text{ m}^3 = \underline{\underline{100 \text{ J}}}$$

$$1 \text{ L} \cdot \frac{10^3 \text{ mL}}{1 \text{ L}} \cdot \frac{1 \text{ cm}^3}{1 \text{ mL}} \cdot \frac{(1 \text{ m})^3}{(100 \text{ cm})^3} = 1 \cdot 10^{-3} \text{ m}^3 = 0.001 \text{ m}^3$$

$$(10^2)^3 = 10^6$$

$$\boxed{1000 \text{ L} = 1 \text{ m}^3}$$

Slope - intercept

$$y = mx + b$$

$m = \frac{\Delta p}{\Delta V}$

↑
y-int

point - slope

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200}{-0.01} = -2 \cdot 10^4 \text{ atm/L}$$

$$P - P_0 = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}} (V - V_0)$$

$$P = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}} \cdot V + 2 \cdot 10^4 \text{ atm}$$

1.33]

1.33

Problem 1.33. An ideal gas is made to undergo the cyclic process shown in Figure 1.10(a). For each of the steps *A*, *B*, and *C*, determine whether each of the following is positive, negative, or zero: (a) the work done on the gas; (b) the change in the energy content of the gas; (c) the heat added to the gas. Then determine the sign of each of these three quantities for the whole cycle. What does this process accomplish?

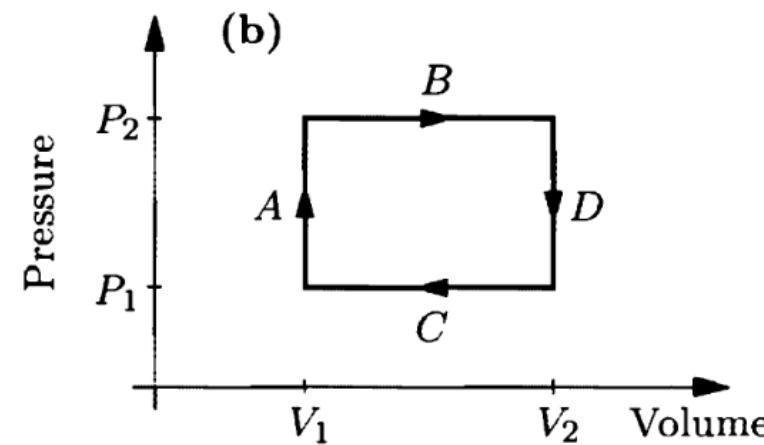
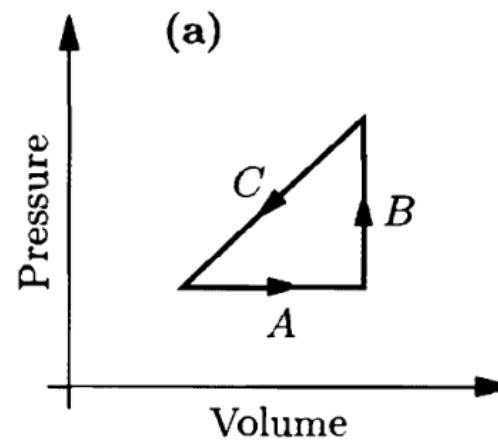


Figure 1.10. *PV* diagrams for Problems 1.33 and 1.34.

Come back!

Proportionality

$$PV = Nk_B T$$

$$P_1 V = Nk_B T_1$$

$$\frac{P_2 V = Nk_B T_2}{P_1 V = Nk_B T_1}$$

$$P_2 V = Nk_B T_2$$



$$P = \frac{Nk_B}{V} T$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 2$$

$$\begin{array}{|c|}\hline P \propto T \\ \hline \end{array}$$

$$P = C \cdot T$$

↳ constant of proportionality

$$\frac{P_2}{P_1} = \frac{N_2}{N_1} \rightarrow P \propto N^{\prime} \rightarrow P = C_2 N$$

$$\hookrightarrow \frac{P_2}{P_1} = \left(\frac{N_2}{N_1} \right)^{\prime}$$

$$PV = N k_B T$$

$$P = \frac{N k_B T}{V}$$

$$P \propto N' T' V^{-1}$$

$$P \propto V^{-1} \rightarrow P = C_2 V^{-1} = \frac{C_2}{V}$$

$$\hookrightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{-1} = \frac{V_1}{V_2}$$

$$F_G \propto r^{-2} \rightarrow F_G = C_3 \cdot r^{-2}$$

↓

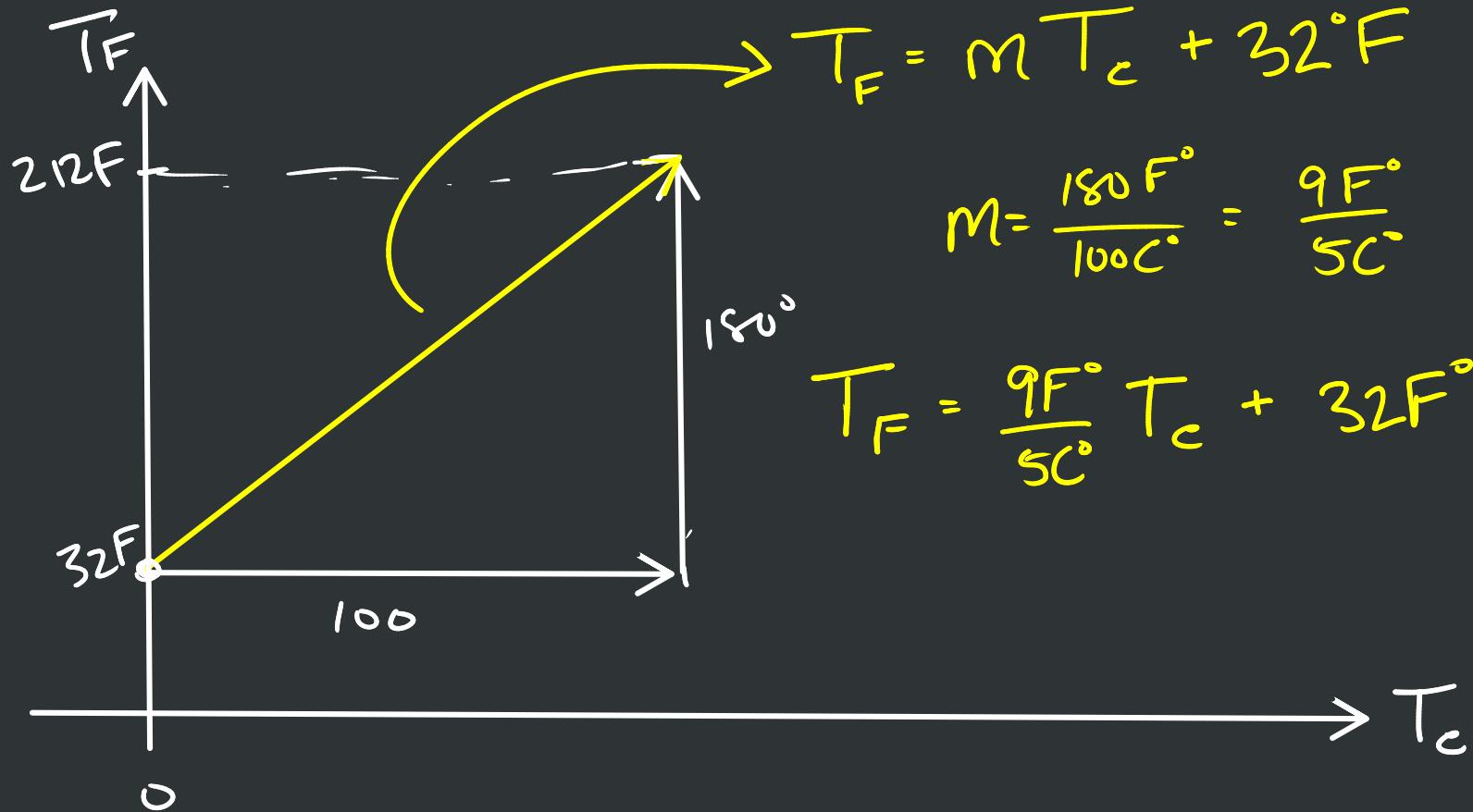
$$\frac{F_2}{F_1} = \left(\frac{r_2}{r_1}\right)^{-2}$$

$$\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$$

$$P \propto N^{\gamma} T^{\beta} V^{-\delta}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\beta} \cdot \left(\frac{V_1}{V_2}\right)^{-\delta}$$

$$T = \frac{PV}{k_B N}$$



Heat \rightarrow flow of energy due to a difference in temperature

$$\Delta U = Q + \cancel{W} \quad [\text{Joules}] \quad [\text{calorie}] \quad 1 \text{ cal} = 4.186 \text{ J}$$

vs.

Calorie \leftarrow food

$$\frac{Q}{\Delta T} = C \leftarrow \text{heat capacity} \quad \left[\frac{\text{J}}{\text{K}} \right] \quad | C_{\text{cal}} = 1000 \text{ calorie}$$

①

$$\frac{C}{m} = c \leftarrow \text{specific heat capacity} \quad \left[\frac{\text{J}}{\text{gK}} \right] = \left[\frac{\text{kJ}}{\text{kgK}} \right] \quad | \overline{Q = mc \cdot \Delta T}$$

②

$$\frac{C}{n} = C_{v,p} \leftarrow \text{molar heat capacity at constant volume}$$

molar heat capacity at constant pressure

$$Q = nC_v \cdot \Delta T$$

③ $\frac{C}{V} \leftarrow$ volumetric heat capacity

Phase change

$$\frac{Q}{m} = L \leftarrow \text{Latent Heat}$$

→ Latent Heat of fusion - L_f
solid \leftrightarrow liquid

→ Latent Heat of vaporization - L_v
liquid \leftrightarrow gas

Calorimetry - when multiple objects are in thermal contact
at what temperature is thermal equilibrium

$$\Delta U = Q = Q_1 + Q_2 + Q_3 + \dots$$

{

objects changing temp or phase transitions occurring

Ex: $\left| \begin{array}{l} 5\text{kg of Al at } 500^\circ\text{C} \\ 5\text{kg of Pb at } 75^\circ\text{C} \end{array} \right|$ What is equil temp?

$\hookrightarrow 490^\circ$ How much heat was given off?
 \equiv

$$C_{Al} = 0.9 \frac{\text{kJ}}{\text{kgK}}$$

$$C_{Pb} = 0.13 \frac{\text{kJ}}{\text{kgK}}$$

$$Q = mc\Delta T = mc(T_f - T_i)$$

$$= 5\text{kg} \cdot 0.9 \frac{\text{kJ}}{\text{kgK}} (490^\circ\text{C} - 500^\circ\text{C})$$

$$= 5\text{kg} \cdot 0.9 \frac{\text{kJ}}{\text{kgK}} \cdot (-10\text{K})$$

$$Q = -45 \text{ kJ} \quad \xrightarrow{+45 \text{ kJ}} +45 \text{ kJ} = 5\text{kg} \cdot 0.13 \frac{\text{kJ}}{\text{kgK}} \cdot \Delta T$$

$$\Delta T = 69.2 \text{ K}$$

$$T_f = 145^\circ\text{C}$$

$$Q_1 + Q_2 = 0$$

$$m_{Al} \cdot C_{Al} \cdot \Delta T_{Al} + m_{Pb} \cdot C_{Pb} \cdot \Delta T_{Pb} = 0$$

$$m_{Al} \cdot C_{Al} \cdot (T_f - 500) + m_{Pb} \cdot C_{Pb} \cdot (T_f - 75) = 0$$

solve for T_f

$$T_f = 446 \text{-ish } {}^{\circ}\text{C}$$

HW: 1.7, 1.16, 1.17

$$1.7 \quad \beta = \frac{\Delta V}{V} / \Delta T$$

$$\beta = \frac{1}{5500} \text{ K}^{-1} = 1.81 \cdot 10^{-4} \text{ K}^{-1}$$

$$r = \frac{1}{2} \text{ cm}$$

$$A = \pi \left(\frac{1}{2} \text{ cm} \right)^2 = \frac{\pi}{4} \text{ cm}^2$$

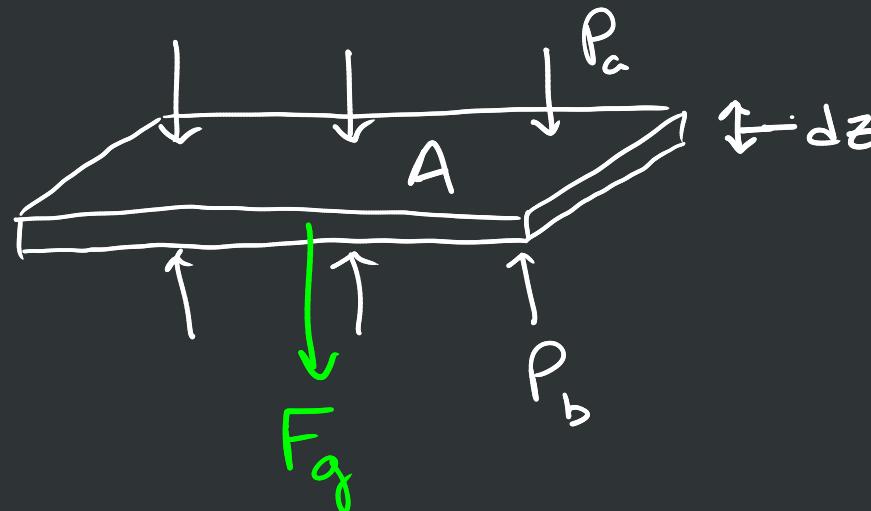
$$V = A \cdot h = \frac{\pi}{2} \text{ cm}^3$$

$$\frac{\Delta V}{V} = \beta \Delta T$$

$$\Delta z = 1 \text{ mm} \approx 1 \text{ K}$$

$$1.16 \quad \rho = \frac{M}{V}$$

"rho"
Volumetric mass density



a) $P = \frac{F}{A}$

$$+ P_b \cdot A - P_a \cdot A - M \cdot g = 0$$



$$\frac{dP}{dz} = \underbrace{\rho}_{\rho}$$

b) $PV = Nk_B T$

$$\rho = \underbrace{\frac{P}{T}}_{m} \leftarrow \text{Avg Mass of air molecules}$$

$$m = 29 \text{ g/mol}$$

$$N = m \cdot n$$

1.17] c) $\left(P + \frac{an^2}{V^2}\right) \underbrace{\left(V - nb\right)}_{V \left(1 - \frac{nb}{V}\right)} = nRT$

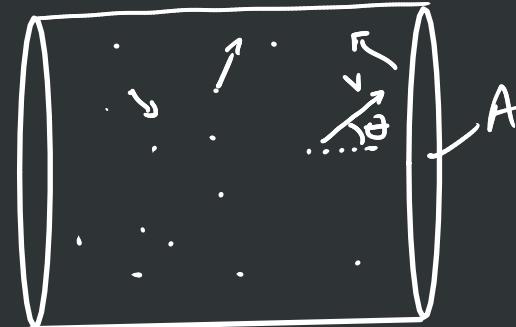
$\left(1 - \frac{nb}{V}\right)^{-1} \rightarrow$ apply the binomial expansion

1.2b

Kinetic Theory and Equipartition of Energy

pressure \longleftrightarrow kinetic energy \longleftrightarrow temperature

$$\text{pressure}_{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \xleftarrow[\Delta p \text{ for each collision w/ the wall in } \Delta t \text{ amount of time}]{2mv \cos \theta}$$



$$d(\text{pressure}) = \frac{2mv \cos \theta}{dA \cdot dt} \cdot \underbrace{\text{number of particles hitting area } dA}_{w/ v \text{ velocity in } dt} \} \text{ integrate over all velocities + } \theta \}$$

Number of atoms traveling in a particular direction w/ a particular speed fraction of them that are within striking distance of the surface of dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

total outcomes $\rightarrow N = \sum_{x=0}^{\infty} N(x)$

average $P(x) = \frac{N(x)}{N}$

$$N(x) = P(x) \cdot N$$

normalized

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle X \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

↑

average value
expectation value
(not the same
as the most probable)

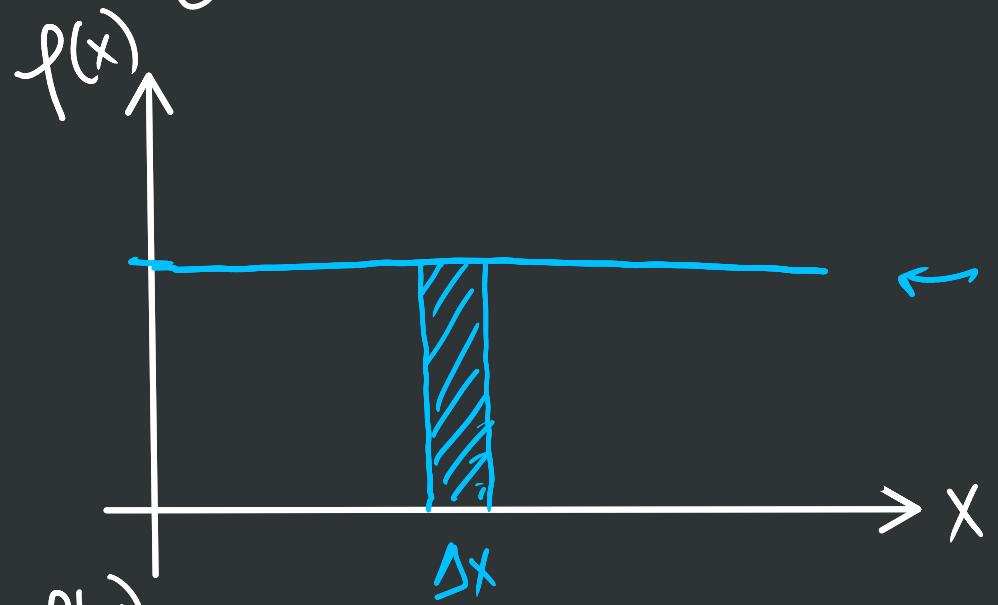
$$\langle X^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

weighted average
by probability

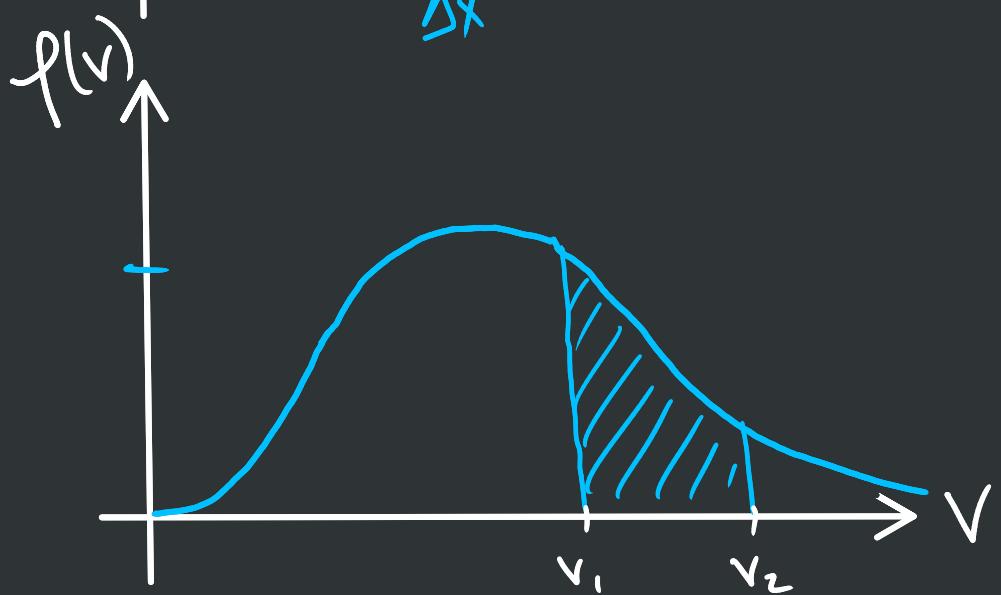
Continuous variable

$$\left\{ \begin{array}{l} \text{probability that an individual} \\ \text{value lies between } x \text{ and } x+dx \end{array} \right\} = f(x) \cdot dx$$



← uniform
prob
function

- probability density function
- probability distribution



$$P(v_1, v_2) = \int_{v_1}^{v_2} f(v) \cdot dv$$

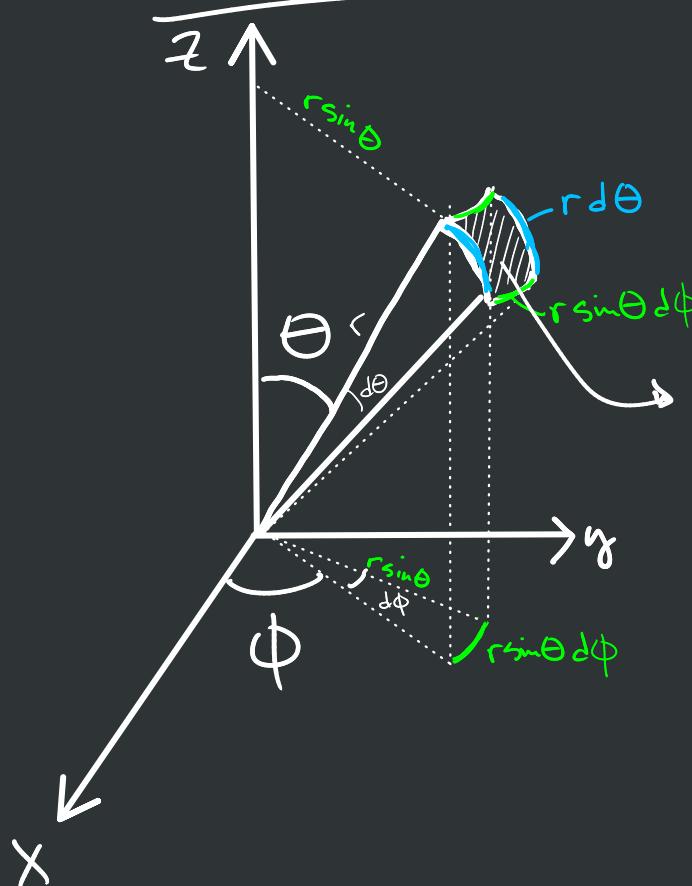
$$\int_{-\infty}^{+\infty} f(v) dv = 1 \quad \leftarrow \text{normalization}$$

$$\langle v \rangle = \int_{-\infty}^{\infty} v \cdot f(v) \cdot dv$$

$$\langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 \cdot f(v) \cdot dv$$

$$\langle f(v) \rangle = \int_{-\infty}^{\infty} f(v) \cdot f(v) \cdot dv$$

number of atoms
traveling in a
particular direction
w/ a particular
speed



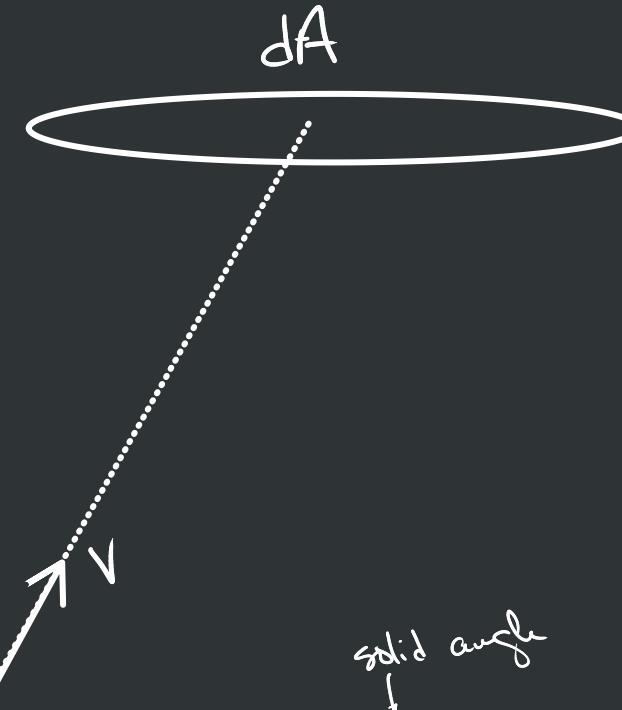
$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

$$\begin{aligned} d\Omega &= \frac{dA}{r^2} \\ &= \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2} \end{aligned}$$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

Probability of a particle
moving in a direction of Θ, Φ

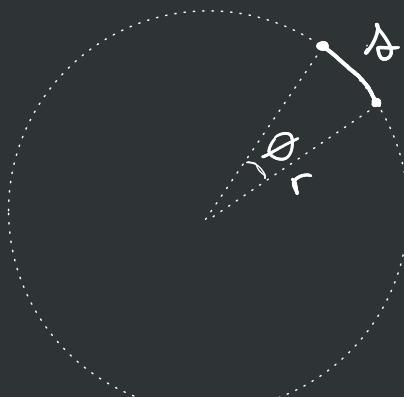
$$\frac{d\Omega}{4\pi} = \frac{\sin\theta \, d\theta \, d\phi}{4\pi}$$



solid angle

$$d\Omega = \frac{dA}{r^2}$$

$A = 4\pi r^2$
total solid angle
of a sphere 4π
(steradians)

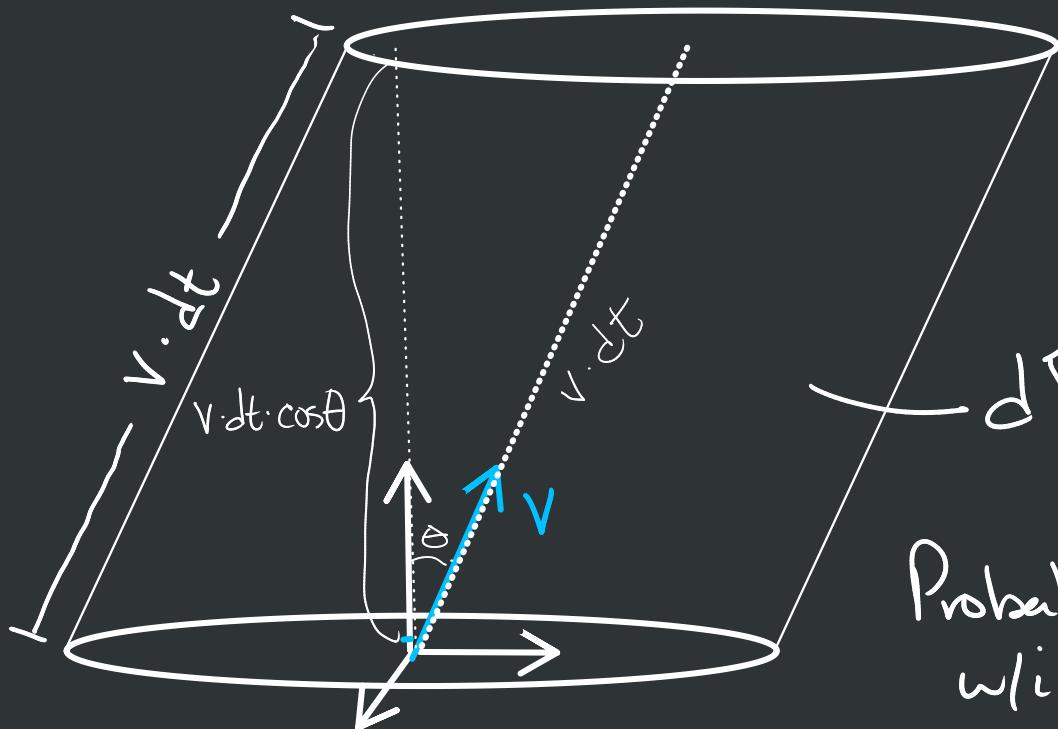


$$\begin{aligned} \Theta &= \frac{\Delta}{r} \\ d\Theta &= \frac{d\Delta}{r} \\ C &= 2\pi r \end{aligned}$$

Number of particles
travelling w/ speed v in the direction of Θ, ϕ $\rightarrow \frac{\sin\Theta d\Theta d\phi}{4\pi} \cdot \rho(v) \cdot dv \cdot N$

NEXT PIECE

dA of the wall



$$dV = v \cdot dt \cdot dA \cdot \cos\theta$$

Probability of being w/i striking distance = $\frac{dV}{V}$

$$= \frac{v \cdot dt \cdot dA \cdot \cos\theta}{V}$$

$$d\text{pressure} = \frac{2mv \cos \theta}{dA dt} \cdot \frac{\sin \theta d\theta d\phi}{2\pi} \cdot \varphi(v) \cdot dv \cdot N \cdot v \frac{dt \cdot dA \cos \theta}{V}$$

$$P = \frac{N \cdot m}{2\pi V} \int_0^\infty r^2 \varphi(v) dv \underbrace{\int_0^{\pi/2} \cos^2 \theta \cdot \sin \theta d\theta}_{\gamma_3} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$\langle v^2 \rangle$

$$P = \frac{Nm}{2\pi V} \cdot \langle v^2 \rangle \cdot \frac{1}{3} \cdot 2\pi$$

$$PV = \frac{Nm \langle v^2 \rangle}{3} \quad \longleftrightarrow \quad PV = Nk_B T$$

$$\frac{m \langle v^2 \rangle}{3} = k_B T$$

$$\langle v \rangle^2 \stackrel{?}{=} \langle v^2 \rangle$$

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$m \langle v^2 \rangle = 3k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle K \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3k_B T}{m}$$

$$\langle v \rangle \neq \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}}$$

$$U = N \langle K \rangle$$

$$V_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}}$$

• $U = \frac{3}{2} N k_B T$ monatomic ideal gas

• $U = \frac{3}{2} n R T$

{ $\begin{aligned} & \text{Ex: "average" speed of air} \\ & \text{air} \rightarrow 29 \text{ g/mol} = 0.029 \text{ kg/mol} \\ & m = \frac{0.029 \text{ kg}}{N_A} \end{aligned}$ }

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

$$= \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right)$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

$$= \frac{3}{2} m \langle v_x^2 \rangle = \frac{3}{2} k_B T \quad \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} k_B T$$

→ 3 could be thought of as the
of degrees of freedom of a monatomic ideal
gas
ways that energy can be possessed

$$v_{rms, \text{air}} = \sqrt{\frac{3 N_A k_B \cdot 293 K}{0.029 \text{ kg}}}$$

$$= \sqrt{\frac{3 \cdot 8.31 \cdot 293}{0.029}}$$

$$v_{rms, \text{air}} = 501 \text{ m/s}$$

1.3 Equipartition Theorem

$$\rightarrow U_{\text{thermal}} = N \cdot f \cdot \frac{1}{2} k_B T$$

↑ degrees of freedom (quadratic)

- $\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2$ (translational)

- $\frac{1}{2}I\omega_x^2, \frac{1}{2}I\omega_y^2$

- $\frac{1}{2}kx^2, \dots$

ideal gases { monatomic $\rightarrow f=3$
diatomic $\rightarrow f=5$ (near room temperature)

solid $\rightarrow f=6$

revisit 1.4 + 1.5: 1st Law and P-V diagrams

$$dU = dQ + dW$$

$$dU = dQ - pdV$$

(1st Law)

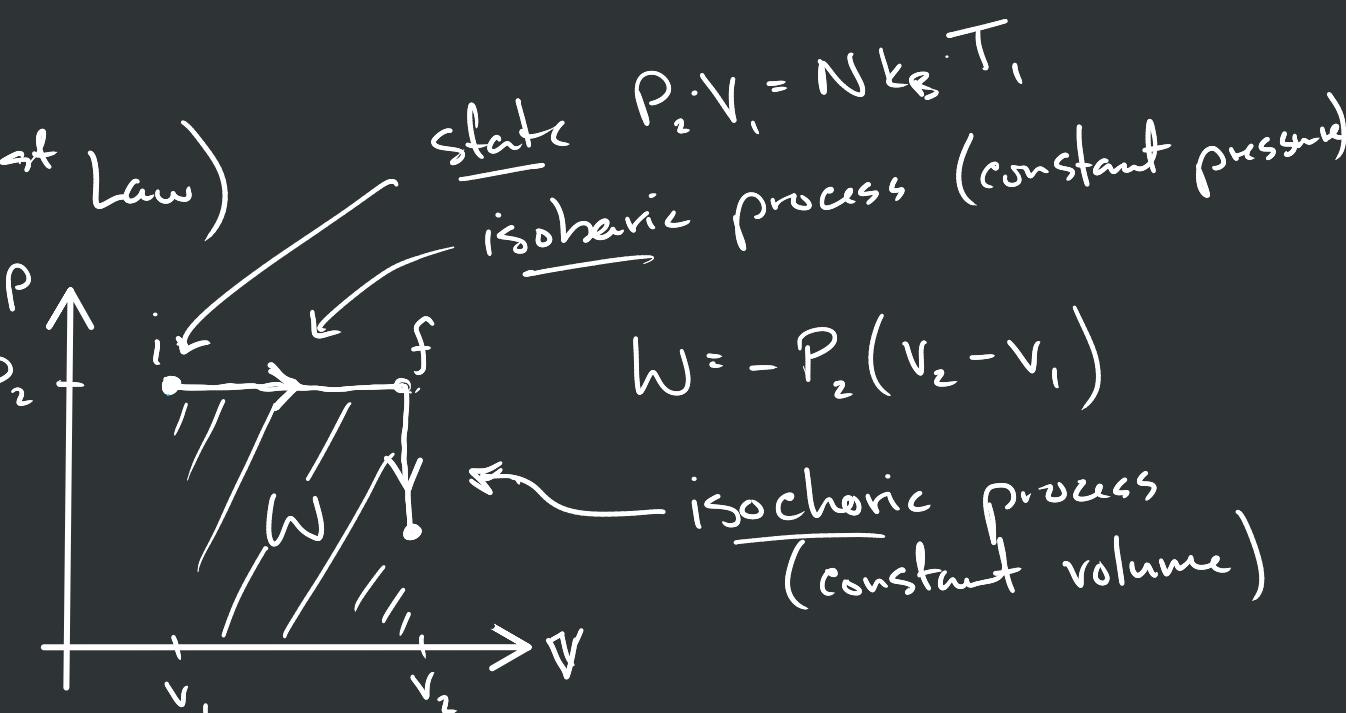
$$U = \frac{f}{2} N k_B T$$

$$dU = \frac{f}{2} N k_B dT$$

$$U_f - U_i = \frac{f}{2} N k_B T_f - \frac{f}{2} N k_B T_i$$

$$\Delta U = \frac{f}{2} N k_B \Delta T$$

$$dU = \frac{f}{2} N k_B dT$$



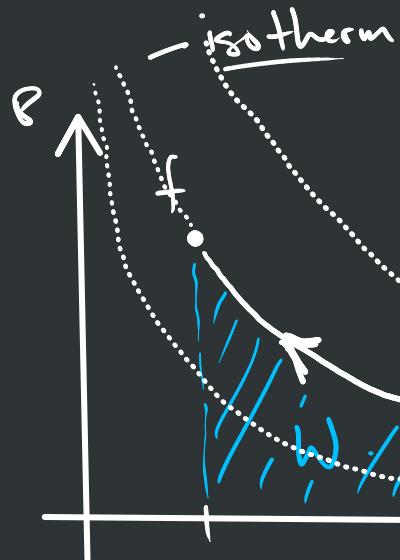
$$PV = N k_B T$$

$$P_f V_f - P_i V_i = N k_B T_f - N k_B T_i$$

isobaric process $\rightarrow P_2 \Delta V = N k_B \Delta T$

Isothermal

→ compression



$$\delta U = \delta Q + \delta W$$

$$\delta W = -\delta Q$$

$$PV = Nk_B T$$

↓ all constant

$$P = \frac{Nk_B T}{V}$$

$$\Delta U = \frac{f}{2} N k_B \Delta T = \frac{f}{2} P \Delta V$$

only true for
isobaric

by extension:

$$\Delta U = \frac{f}{2} N k_B \Delta T = \frac{f}{2} \Delta P \cdot V$$

only true
for isochoric

$$\delta W = -P dV$$

$$W = - \int_{V_i}^{V_f} P dV$$

$$W = - \int_{V_i}^{V_f} \frac{Nk_B T}{V} dV$$

$$dW = - \frac{1}{c} dQ$$

$$W = - Nk_B T \int_{V_i}^{V_f} \frac{dV}{V}$$

$$Q = - W$$

$$W = - Nk_B T \ln(V) \Big|_{V_i}^{V_f}$$

$$Q = - PV \ln\left(\frac{V_i}{V_f}\right)$$

$$W = - Nk_B T (-\ln V_f + \ln V_i)$$

$$Q = PV \ln\left(\frac{V_f}{V_i}\right)$$

$$W = Nk_B T (\ln V_i - \ln V_f)$$

$$W = Nk_B T \ln\left(\frac{V_i}{V_f}\right)$$

Work done
during an
isothermal
process

$$PV = Nk_B T$$

$$W = PV \ln\left(\frac{V_i}{V_f}\right)$$

Adiabatic Process (isentropic) → no heat added/removed
 compression → still quasistatic
 → super insulated
 → quick

first law: $dU = \cancel{dQ} + dW$

$$dU = dW$$

$$\Delta U = W$$

$$U = \frac{f}{2} N k_B T \Rightarrow \Delta U = \frac{f}{2} N k_B \Delta T = W$$

↑
 what is the final temp?

$$dW = -pdV \quad \text{not const}$$

$$W = - \int \frac{N k_B T}{V} dV \quad \text{not const}$$

$$dU = dW$$

$$U = \frac{f}{2} N k_B T$$

$$\frac{f}{2} N k_B dT = -pdV$$

$$\frac{f}{2} N k_B dT = - \frac{N k_B T}{V} dV$$

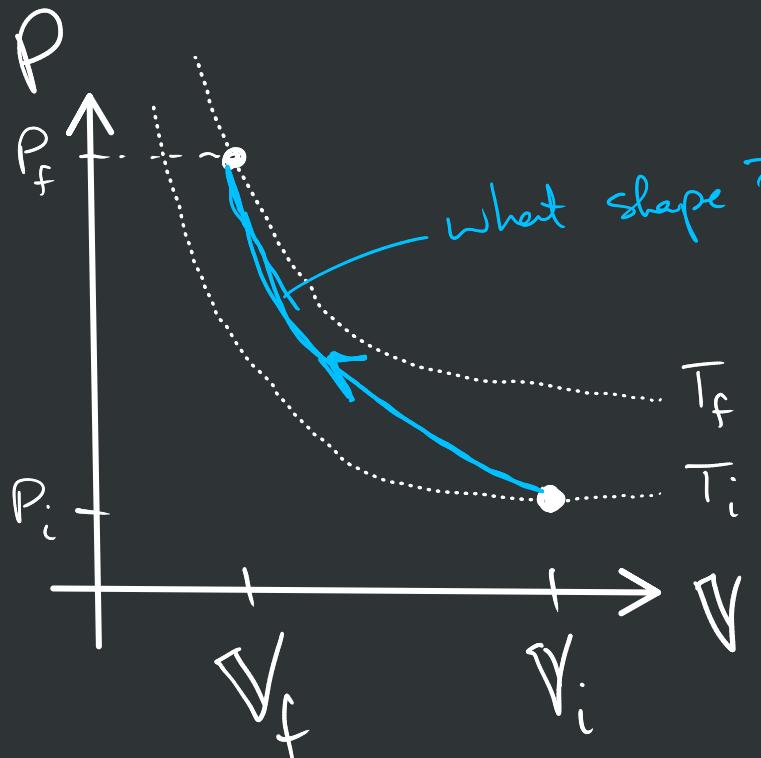
$$\frac{f}{2} \int_{T_i}^{T_f} \frac{dT}{T} = - \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = - \ln\left(\frac{V_f}{V_i}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\ln\left(\left(\frac{T_f}{T_i}\right)^{\frac{f}{2}}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\rightarrow \underbrace{\left| \left(\frac{T_f}{T_i}\right)^{\frac{f}{2}} = \frac{V_i}{V_f} \right|}_{\text{constant}} \Rightarrow V \propto T^{-f/2}$$

$\hookrightarrow V = \text{constant} \cdot T^{-f/2}$
 $\hookrightarrow V \cdot T^{f/2} = \text{constant}$



$$P(V) \quad P_f(v_f)$$

$$V \cdot T^{f/2} = \text{constant} \iff \left(\frac{T_f}{T_i} \right)^{\frac{f}{2}} = \frac{V_i}{V_f}$$

$$PV = Nk_B T$$

$$T_i = \frac{P_i V_i}{N k_B}$$

$$\left(\frac{P_f V_f}{P_i V_i} \right)^{f/2} = \frac{V_i}{V_f}$$

$$\frac{T_f = \frac{P_f V_f}{N k_B}}{T_i = \frac{P_i V_i}{N k_B}}$$

$$\frac{P_f V_f}{P_i V_i} \propto \left(\frac{V_i}{V_f} \right)^{f/2}$$

$$P(V) = P_i \left(\frac{V_i}{V} \right)^{\gamma}$$

ideal gas: $f = 3 \leftarrow \text{monatomic} \rightarrow \gamma = 5/3$
 $f = 5 \leftarrow \text{diatomic} \rightarrow \gamma = 7/5$

$$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f} \right)^{\gamma} \left(\frac{V_i}{V_f} \right)^{f/2}$$

$$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f} \right)^{1 + f/2}$$

$$d\omega = -p dV$$

$$\omega = - \int p dV$$

$$= - \int p_i \left(\frac{V_i}{V} \right)^\gamma dV \quad \text{and } \underline{\text{integrate}}$$

$$\gamma = 1 + \frac{2}{f}$$

$$\gamma = \frac{f+2}{f}$$

Revisit section 1.6 | - Heat Capacity

$$\text{1st Law} \rightarrow dU = \cancel{\pm Q} + \underbrace{\pm W}_{-\rho dV}$$

$$dU = \pm Q - \rho dV$$

$$\rightarrow \pm Q = dU + \rho dV$$

\downarrow $U(T, V)$ or $U(T, P)$

$$dU(T, V) = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

↑
constant
volume

$$\pm Q = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + \rho dV$$

$$\frac{\pm Q}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + \rho \right] \frac{dV}{dT}$$

Two cases : ① constant volume
 ② constant pressure

① constant volume

$$dV = 0$$

$$C_v = \frac{dQ}{dT} = \left(\frac{\partial U}{\partial T} \right)_V$$

heat capacity at constant volume

② constant pressure

$$C_p = \left(\frac{dQ}{dT} \right)_P = \underbrace{C_v}_{C_v} + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{dV}{dT} \right)_P$$

$$C_p = C_v + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{dV}{dT} \right)_P$$

for an ideal

$$U = \frac{f}{2} N k_B T$$

$$\rightarrow U = \frac{f}{2} n \cdot R \cdot T$$

$$\frac{\partial U}{\partial T} = \frac{f}{2} n R$$

$$\frac{C_v}{n} = \frac{f}{2} R$$

$$\frac{\frac{3}{2} R}{K_{mol}} = \frac{12.5 \text{ J}}{K_{mol}}$$

$$\left| \begin{array}{l} \text{sol. id} \\ f = 6 \end{array} \right.$$

diatomic

$$\frac{5}{2} R = \frac{20.8 \text{ J}}{K_{mol}}$$

$$\left| \begin{array}{l} \frac{6}{2} R = 25 \text{ J} \\ K_{mol} \end{array} \right.$$

$$C_p = C_v + \left[\underbrace{\left(\frac{\partial U}{\partial V} \right)_T}_{U = \frac{f}{2} n_m R T} + P \right] \left(\frac{dV}{dT} \right)_P$$

$$\left(\frac{\partial U}{\partial V} \right)_T = 0$$

$$\left(\frac{dV}{dT} \right)_P = \frac{n R}{P}$$

$$C_p = C_v + n R$$

$$\left| \begin{array}{l} \text{monatomic} \\ C_p = \frac{3}{2} R + R = \frac{5}{2} R \\ \frac{C_p}{n} = \frac{3}{2} R + R = \frac{5}{2} R \end{array} \right.$$

$$\left| \begin{array}{l} \text{diatomic} \\ C_p = \frac{7}{2} R = 29 \text{ J/K mol} \end{array} \right.$$

$$C_p - C_v = \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

end 1.b | - Enthalpy

$$dU = \pm Q + \pm W$$

$$\underbrace{- P dV}_{\text{other}} + W_{\text{other}}$$

$$dU + P dV = \pm Q + W_{\text{other}}$$

enthalpy \rightarrow heat content

$$U + PV = H$$

$$\Delta U + P \Delta V = \Delta H \leftarrow \text{at } \underline{\text{constant pressure}}$$

1.47

$$Q_{\text{tea}} = m_{\text{tea}} \cdot c \cdot \Delta T_{\text{tea}}$$

$$Q_{\text{tea}} = 200 \text{ g} \cdot \left(1 \frac{\text{cal}}{\text{g} \cdot \text{K}} \right) \cdot (-35 \text{ K})$$

$$Q_{\text{tea}} = -7000 \text{ cal}$$

$$\left\{ \begin{array}{l} Q_{\text{ice}} = +7000 \text{ cal} = M_{\text{ice}} \cdot C_{\text{ice}} \cdot (+15 \text{ K}) + M_{\text{ice}} \cdot L_f + M_{\text{ice}} \cdot C_{\text{water}} \cdot (65 \text{ K}) \\ \\ \end{array} \right.$$

1.17 a) $P \cdot V = nRT \left(1 + \frac{B}{V/n} \right)$

\uparrow
2nd term
in the expansion

calculate $\frac{B}{V/n}$

$T \text{ (K)}$	$B \frac{\text{cm}^3}{\text{mol}}$	$\frac{n}{V} \cdot B$
100	-160	?
200	-35	
300	-4.2	

$$P \cdot V = nRT \left(1 + \frac{n}{V} \cdot B \right)$$

$$P = \frac{n}{V} RT \left(1 + \frac{n}{V} \cdot B \right) \rightarrow \text{solve for } \frac{n}{V}$$

$$P = \left(\frac{n}{V}\right)RT + \left(\frac{n}{V}\right)^2 BRT$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$0 = \left(\frac{n}{V}\right)^2 BRT + \left(\frac{n}{V}\right)RT - P$$

$$-RT \pm \sqrt{(RT)^2 + 4BRT \cdot P}$$

\uparrow \uparrow
 83.1 1 atm
 $=$

$$PV = nRT$$

$$\underbrace{P_{\text{atm}} \cdot 10^5}_{\substack{\uparrow \\ \uparrow}} \cdot \underbrace{V_{\text{cm}^3} \cdot 10^{-6}}_{\substack{\uparrow \\ \uparrow}} = nRT$$

$8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}$

$$\frac{P_{\text{atm}} \cdot V_{\text{cm}^3}}{10} = n \cdot R \cdot T$$

$$P_{\text{atm}} \cdot V_{\text{cm}^3} = n \cdot \underbrace{10 \cdot R}_{\substack{\uparrow \\ \uparrow}} T$$

$$R_{\text{atm} \cdot \text{cm}^3} = 83.1 \frac{\text{atm} \cdot \text{cm}^3}{\text{K} \cdot \text{mol}}$$

$$c) \left(P + \frac{an^2}{V^2} \right) (V - nb) = nRT \quad | \quad PV = nRT \left(1 + \frac{n}{V} \cdot B + \left(\frac{n}{V} \right)^2 C \right)$$



goal $\rightarrow \underline{\underline{B(a,b)}} + \underline{\underline{C(a,b)}}$

$$V \left(1 - \frac{nb}{V} \right)$$

$$\left(P + \frac{an^2}{V^2} \right) V \cdot \underbrace{\left(1 - \frac{nb}{V} \right)}_{\rightarrow} = nRT$$

$$PV + \frac{an^2}{V} = nRT \left(1 - \frac{nb}{V} \right)^{-1}$$

$$PV = nRT \left(1 - \frac{nb}{V} \right)^{-1} - \frac{an^2}{V} \cdot \frac{nRT}{nRT}$$

$$= nRT \left(\left(1 - \frac{nb}{V} \right)^{-1} - \frac{n}{V} \frac{a}{RT} \right)$$

Hint! $(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2$

$$B(a, b, T)$$

