

# Thermo Intro

## temperature

Fahrenheit

Celsius

Rankine

Kelvin

Room temp  $20^{\circ}\text{C}$  ( $\sim 70^{\circ}\text{F}$ )

↓

$293\text{K}$

$$\Delta F = \frac{9}{5} \Delta C$$

$$T_F = \frac{9}{5} T_C + 32$$

$$T_K = T_C + 273.15$$

Number, moles, molar mass, density

1 mole of things =  $6.022 \cdot 10^{23}$  things  
particle  
atom  
molecule

$N \rightarrow$  number of particles

$n \rightarrow$  number of moles

12  $\rightarrow$  12 protons and neutrons  
 $\rightarrow$  6 protons

1 mole is a gram  
of protons and neutrons

Ex. mass of one proton?

mass of one proton  $\times$  number of proton = mass  
of collection

$M \rightarrow$  total mass of  
a collection

$m \rightarrow$  mass of one  
particle

$$m \times N = M$$

$$m = \frac{M}{N} = \frac{1 \text{ gram}}{N_A} \rightarrow \text{Avogadro's number } 6.022 \cdot 10^{23}$$

$$m = 1.7 \cdot 10^{-24} \text{ grams} \\ = 1.7 \cdot 10^{-27} \text{ kg}$$

What about  $N_2$ ?

$$1 \text{ mole of } N_2 = 2 \cdot \left(14 \frac{\text{g}}{\text{mol}}\right) = 28 \text{ g}$$

Dry air?

78%  $N_2$ , 21%  $O_2$ , 1% Ar

$$0.78 \cdot (28 \text{ g/mol}) + 0.21 (32 \text{ g/mol}) + 0.01 (40 \text{ g/mol}) = 29 \text{ g/mol}$$

Ideal Gas Law  $\rightarrow$  an equation of state

- experimental law

$\rightarrow$  state variables

number of particles  
(microscopic)

$PV = Nk_B T$

$\rightarrow$  Boltzmann's constant

$k_B = 1.38 \cdot 10^{-23} \text{ J/K}$

$\rightarrow$  Temperature [K]

$P = \frac{F}{\text{Area}} \quad [Pa] = \left[\frac{N}{m^2}\right] \quad [m^3]$

Alternative form  $\rightarrow PV = nRT$  (macroscopic)

$\rightarrow$  universal gas constant

$$N k_B = n \cdot R$$

$$\uparrow N = n \cdot N_A \leftarrow \text{definition of moles}$$

$$n \cdot N_A \cdot k_B = n \cdot R$$

$$N_A \cdot k_B = R = 6.022 \cdot 10^{23} \frac{\text{part}}{\text{mol}} \cdot 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

$$R = 8.31 \frac{\text{J}}{\text{K} \cdot \text{mol}}$$

Volume of 1 mole of air at room temp and atmospheric pressure

$\hookrightarrow 293\text{K}$  or  $300\text{K}$

$\hookrightarrow 1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$

$$V = \frac{nRT}{P} = \frac{1 \text{ mol} \cdot 8.31 \text{ J/Kmol} \cdot 300\text{K}}{10^5 \text{ Pa}} = 0.024 \text{ m}^3$$

$$\begin{aligned} V_{\text{cube}} &= s^3 \\ \sqrt[3]{0.024 \text{ m}^3} &= s = 0.288 \text{ m} \\ &\sim 30 \text{ cm} \end{aligned}$$

## Laws of Thermodynamics

0. Thermometers work

1. Conservation of Energy

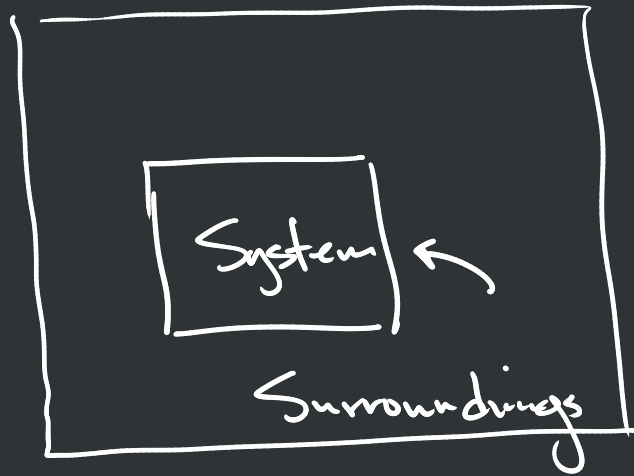
2. Heat flows from high temp to low temp

Entropy is maximized

No perpetuated motion machine

3. You can't reach absolute zero

# 1<sup>st</sup> Law $\rightarrow$ Conservation of Energy



Energy of a system can change

Work + Heat

$\downarrow$   
Force applied  
over distance

$\downarrow$   
Spontaneous flow  
of energy due  
to a difference  
in temperature

$$dU = Q + W$$

$$\rightarrow dU = \pm Q + \pm W$$

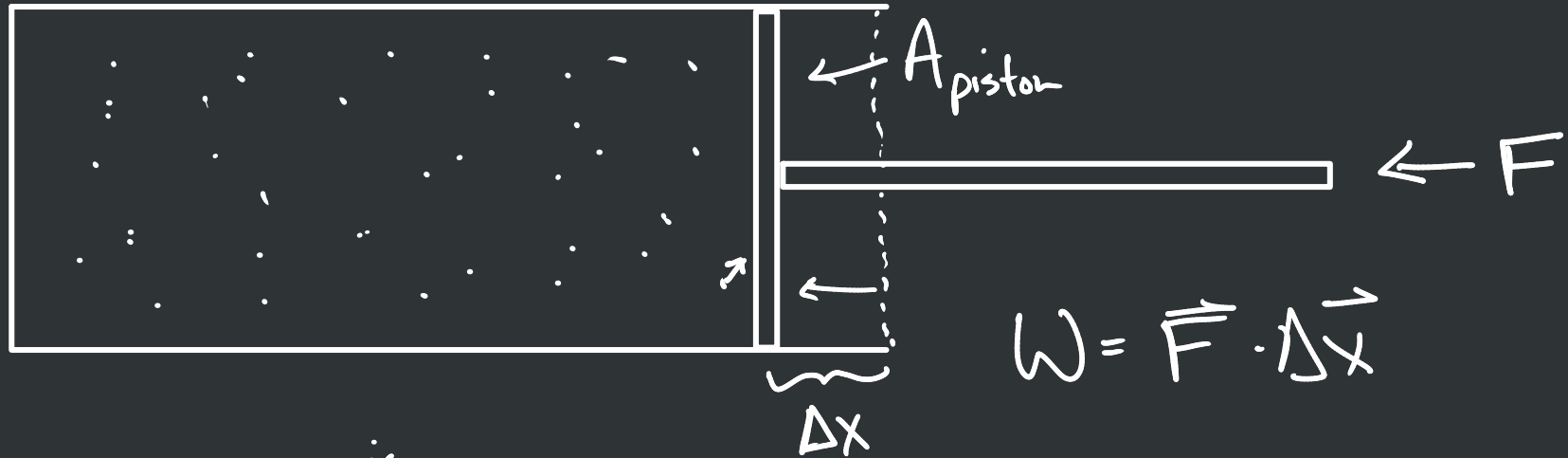
$\uparrow$  inexact differential

$$dU = \delta Q + \delta W$$

$\hookrightarrow$  internal energy of the system, heat is added  
or work is done to the system they are positive

~~$$dQ = Q(x+dx) - Q(x)$$~~

# Compression Work (or Expansion)



$$W = \vec{F} \cdot \Delta \vec{x}$$

$$P_{\text{gas}} = \frac{F_{\text{gas}}}{A_{\text{piston}}}$$

$$W = P_{\text{gas}} \cdot \underbrace{A_{\text{piston}} \Delta x}_{\Delta V}$$

↓

$$W = -P \Delta V$$

$\Delta V$  is (-) in compression but energy is increasing

assuming pressure is uniform throughout the chamber

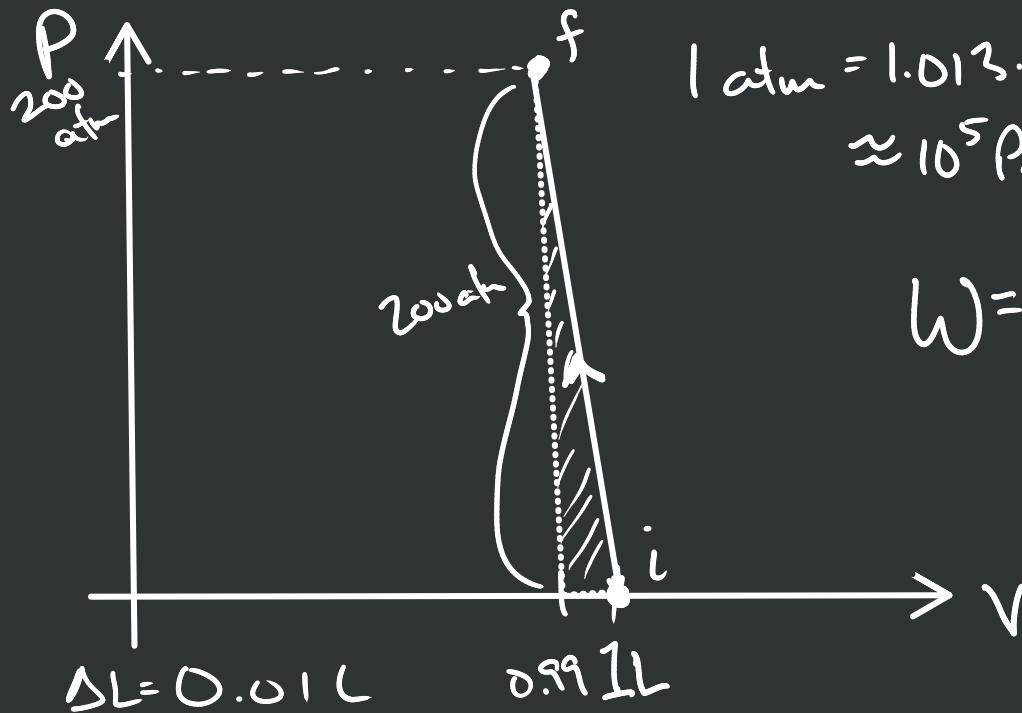
compression (expansion) must happen quasistatically

$$dU = \delta Q + \delta W$$

$$\delta W = -P dV$$

$$\underline{W = \int -P dV}$$

**Problem 1.32.** By applying a pressure of 200 atm, you can compress water to 99% of its usual volume. Sketch this process (not necessarily to scale) on a  $PV$  diagram, and estimate the work required to compress a liter of water by this amount. Does the result surprise you?



$$1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa} \approx 10^5 \text{ Pa}$$

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$$

$$W = -\int P dV$$

$$\Delta V = 0.01 \text{ L} = 10^{-5} \text{ m}^3$$

$$0.01 \text{ L} \cdot \frac{1 \text{ m}^3}{1000 \text{ L}} = 10^{-5} \text{ m}^3$$

$$A_{\Delta} = \frac{1}{2} b h \Rightarrow \underline{W = \frac{1}{2} P \Delta V}$$

$$W = \frac{1}{2} (200 \cdot 10^5 \text{ Pa}) \cdot 10^{-5} \text{ m}^3 = \underline{100 \text{ J}}$$

$$1 \text{ L} \cdot \frac{10^3}{1000 \text{ mL}} \cdot \frac{1 \text{ cm}^3}{1 \text{ mL}} \cdot \frac{(1 \text{ m})^3}{(100 \text{ cm})^3} = 1 \cdot 10^{-3} \text{ m}^3 = 0.001 \text{ m}^3$$

$$\boxed{1000 \text{ L} = 1 \text{ m}^3}$$



slope-intercept

$$y = mx + b$$

$\uparrow$   
 $m = \frac{\Delta p}{\Delta V}$

$\leftarrow y\text{-int}$

point-slope

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200}{-0.01} = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}}$$

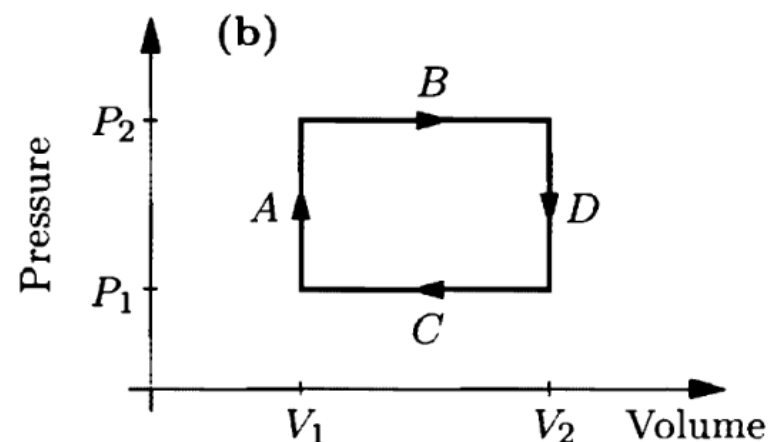
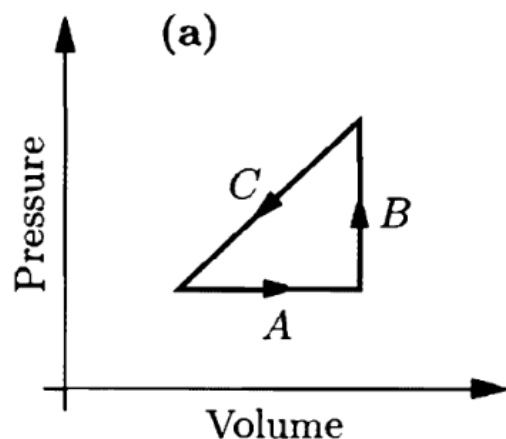
$$p - 0 = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}} (V - 1 \text{ L})$$

$$p = -2 \cdot 10^4 \frac{\text{atm}}{\text{L}} \cdot V + 2 \cdot 10^4 \text{ atm}$$

1.33

1.33

**Problem 1.33.** An ideal gas is made to undergo the cyclic process shown in Figure 1.10(a). For each of the steps  $A$ ,  $B$ , and  $C$ , determine whether each of the following is positive, negative, or zero: (a) the work done on the gas; (b) the change in the energy content of the gas; (c) the heat added to the gas. Then determine the sign of each of these three quantities for the whole cycle. What does this process accomplish?



**Figure 1.10.**  $PV$  diagrams for Problems 1.33 and 1.34.


Come back!

# Proportionality

$$PV = Nk_B T$$

$$P_1 V = Nk_B T_1$$


$$P_2 V = Nk_B T_2$$


$$P = \left( \frac{Nk_B}{V} \right) T$$

$$\frac{P_2 V = Nk_B T_2}{P_1 V = Nk_B T_1}$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 2$$


$$P \propto T$$


$$P = C \cdot T$$

↳ constant of proportionality

$$\frac{P_2}{P_1} = \frac{N_2}{N_1} \rightarrow P \propto N' \rightarrow P = C_2 N$$

$$\hookrightarrow \frac{P_2}{P_1} = \left( \frac{N_2}{N_1} \right)'$$

$$PV = N k_B T$$

$$P = \frac{N k_B T}{V}$$

$$P \propto N' T' V^{-1}$$

$$P \propto V^{-1} \rightarrow P = C_2 V^{-1} = \frac{C_2}{V}$$

$$\hookrightarrow \frac{P_2}{P_1} = \left( \frac{V_2}{V_1} \right)^{-1} = \frac{V_1}{V_2}$$

$$F_G \propto r^{-2}$$



$$F_G = C_3 \cdot r^{-2}$$

$$= \frac{C_3}{r^2}$$

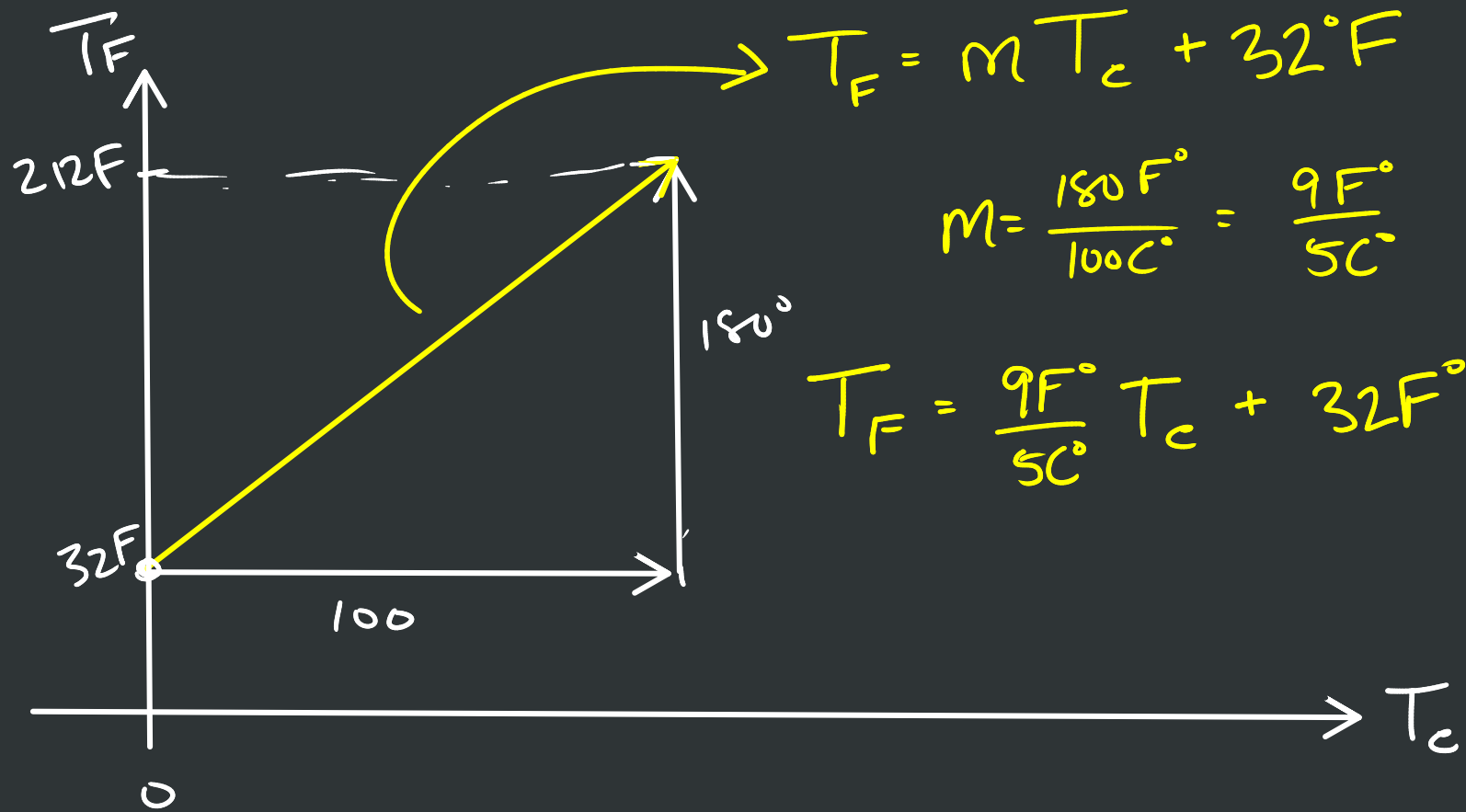
$$\hookrightarrow \frac{F_2}{F_1} = \left( \frac{r_2}{r_1} \right)^{-2}$$

$$\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$$

$$T = \frac{pV}{k_B N}$$

$$p \propto N' T' V^{-1}$$

$$\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right) \cdot \left( \frac{V_2}{V_1} \right)^{-1}$$



Heat  $\rightarrow$  flow of energy due to a difference in temperature

$$\Delta U = Q + \cancel{W}^{\rightarrow 0}$$

[Joules]

[calorie]

$$1 \text{ cal} = 4.186 \text{ J}$$

vs.

[Calorie]  $\leftarrow$  food

$$\frac{Q}{\Delta T} = C \leftarrow \text{heat capacity} \left[ \frac{\text{J}}{\text{K}} \right]$$

$$1 \text{ Cal} = 1000 \text{ calorie}$$

①  $\frac{C}{m} = c \leftarrow \text{specific heat capacity} \left[ \frac{\text{J}}{\text{gK}} \right] = \left[ \frac{\text{kJ}}{\text{kgK}} \right] \quad \boxed{Q = mc \Delta T}$

②  $\frac{C}{n} = C_{V,P} \leftarrow \begin{array}{l} \text{molar heat capacity at constant volume} \\ \text{molar heat capacity at constant pressure} \end{array}$

$$Q = n C_V \Delta T$$

③  $\frac{C}{V} \leftarrow$  volumetric heat capacity

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Phase change

$$\frac{Q}{m} = L \leftarrow \text{Latent Heat}$$

$\rightarrow$  Latent Heat of fusion -  $L_f$   
solid  $\leftrightarrow$  liquid

$\rightarrow$  Latent Heat of vaporization -  $L_v$   
liquid  $\leftrightarrow$  gas

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Calorimetry - when multiple objects are in thermal contact  
at what temperature is thermal equilibrium

$$\Delta U = 0 = Q_1 + Q_2 + Q_3 + \dots$$

$\underbrace{\hspace{10em}}$   
objects changing temp or phase transitions occurring



Ex: 5kg of Al at 500°C | 5kg of Pb at 75°C | What is equil temp?

↳ 490° How much heat was given off?

$$C_{Al} = 0.9 \frac{\text{kJ}}{\text{kg K}}$$

$$C_{Pb} = 0.13 \frac{\text{kJ}}{\text{kg K}}$$

$$Q = mc\Delta T = mc(T_f - T_i)$$
$$= 5\text{kg} \cdot 0.9 \frac{\text{kJ}}{\text{kg K}} (490^\circ\text{C} - 500^\circ\text{C})$$

$$= 5\text{kg} \cdot 0.9 \frac{\text{kJ}}{\text{kg K}} \cdot (-10\text{K})$$

$$Q = -45\text{kJ} \longrightarrow +45\text{kJ} = 5\text{kg} \cdot 0.13 \frac{\text{kJ}}{\text{kg K}} \cdot \Delta T$$

$$\Delta T = 69.2\text{K}$$

$$T_f = 145^\circ\text{C}$$

$$Q_1 + Q_2 = 0$$

$$m_{Al} \cdot C_{Al} \cdot \Delta T_{Al} + m_{Pb} \cdot C_{Pb} \cdot \Delta T_{Pb} = 0$$

$$m_{Al} \cdot C_{Al} \cdot (T_f - 500) + m_{Pb} \cdot C_{Pb} \cdot (T_f - 75) = 0$$

solve for  $T_f$

$$T_f = 446 \text{ ish } ^\circ\text{C}$$

HW: 1.7, 1.16, 1.17

1.7]  $\beta = \frac{\frac{\Delta V}{V}}{\Delta T}$

$$\beta = \frac{1}{5500} \text{ K}^{-1} = 1.81 \cdot 10^{-4} \text{ K}^{-1}$$

$$r = \frac{1}{2} \text{ cm}$$

$$A = \pi \left( \frac{1}{2} \text{ cm} \right)^2 = \frac{\pi}{4} \text{ cm}^2$$

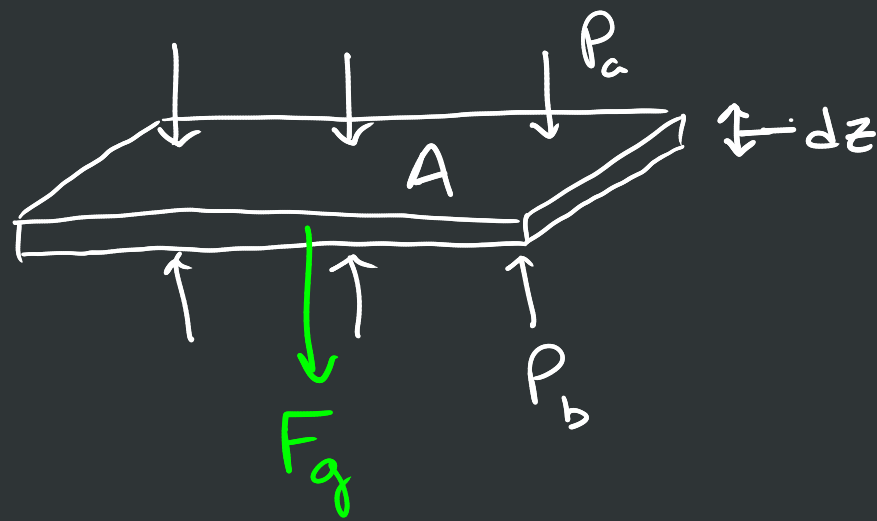
$$V = A \cdot h = \frac{\pi}{2} \text{ cm}^3$$

$$\frac{\Delta V}{V} = \beta \Delta T$$

$$\Delta z = 1 \text{ mm} \approx 1 \text{ K}$$

$$1.16] \quad \rho = \frac{M}{V}$$

"rho"  
volumetric mass density



$$a) \quad P = \frac{F}{A} \quad + P_b \cdot A - P_a \cdot A - M \cdot g = 0$$

$$\frac{dP}{dz} = \rho$$

pressure  
↓

$$b) \quad P V = N k_B T$$

$$\rho = \underbrace{P, T, m}_{\leftarrow \text{avg mass of air molecules } m = 29 \text{ g/mol}}$$

$$N = m \cdot n$$

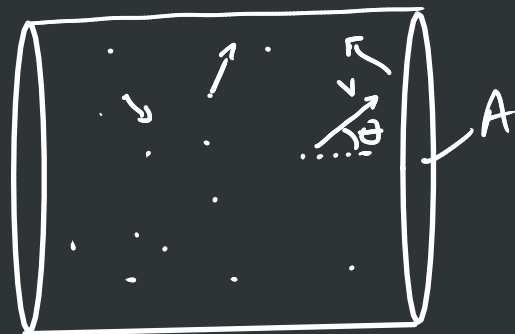
$$\underline{1.17} \quad c) \left( P + \frac{an^2}{V^2} \right) \underbrace{\left( V - nb \right)}_{V \left( 1 - \frac{nb}{V} \right)} = nRT$$

$$\left( 1 - \frac{nb}{V} \right)^{-1} \rightarrow \text{apply the binomial expansion}$$

# Kinetic Theory and Equipartition of Energy

pressure  $\longleftrightarrow$  kinetic energy  $\longleftrightarrow$  temperature

$$\text{pressure}_{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \leftarrow \frac{2mv \cos \theta}{\Delta p \text{ for each collision w/ the wall in } \Delta t \text{ amount of time}}$$



$$d(\text{pressure}) = \frac{2mv \cos \theta}{dA \cdot dt} \left\{ \begin{array}{l} \text{number of particles hitting area } dA \\ \text{w/ } v \text{ velocity in } dt \end{array} \right\} \left\{ \begin{array}{l} \text{integrate over all} \\ \text{velocities + } \theta \end{array} \right\}$$

number of atoms  
traveling in a  
particular direction  
w/ a particular  
speed

fraction of them  
that are within  
striking distance  
of the surface  
of  $dA$

# Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$  number of desired outcomes

total outcomes  $\rightarrow$   $N = \sum_{x=0}^{\infty} N(x)$

rearrange  $\rightarrow P(x) = \frac{N(x)}{N}$

$$N(x) = P(x) \cdot N$$

normalized

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \sum_{x=0}^{\infty} x \cdot P(x)$$

$\uparrow$   
average value  
expectation value

weighted average  
by probability

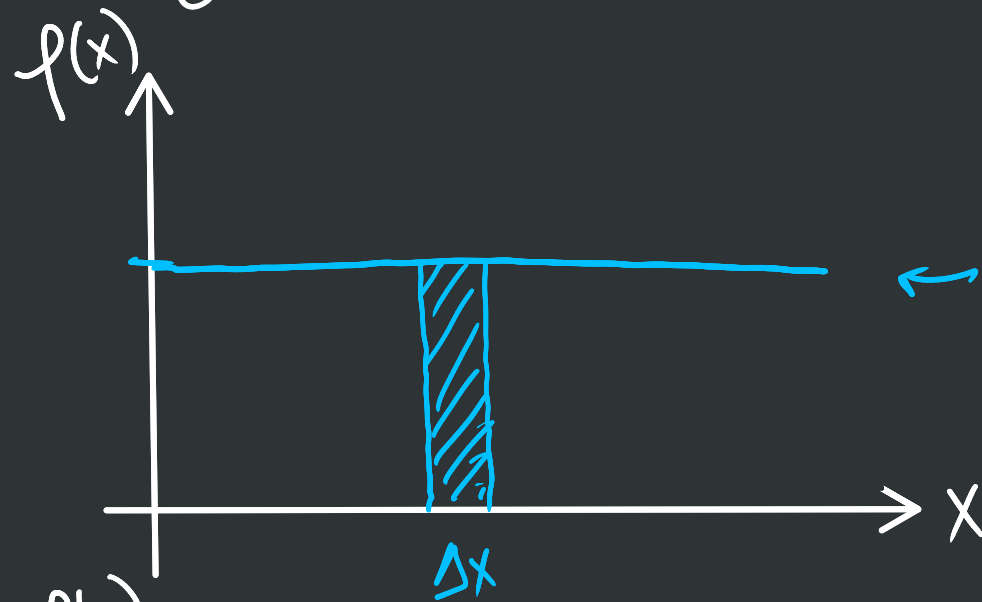
(not the same  
as the most probable)

$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

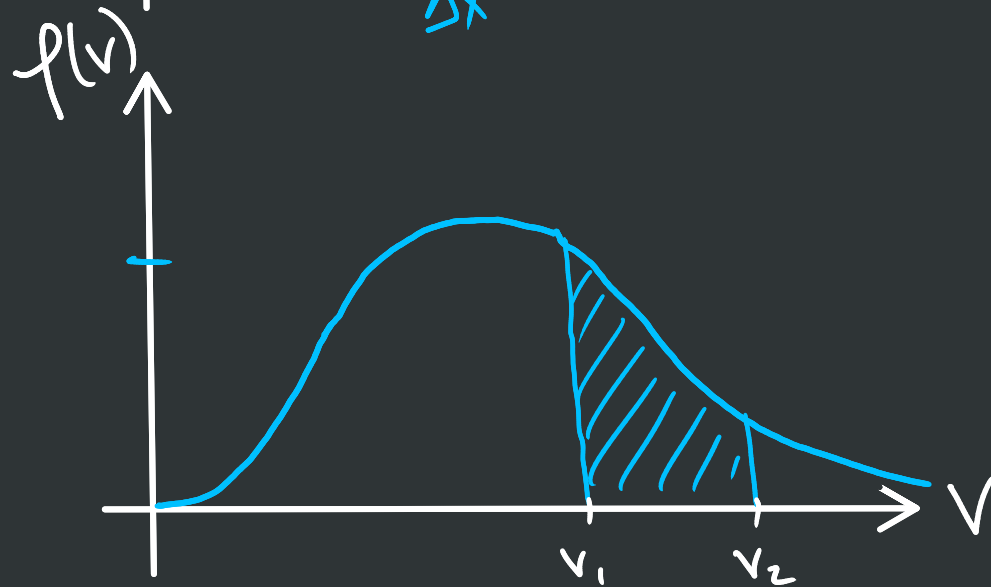
## Continuous variable

$$\left\{ \begin{array}{l} \text{probability that an individual} \\ \text{value lies between } x \text{ and } x+dx \end{array} \right\} = f(x) \cdot dx$$



← uniform  
prob  
function

↑  
- probability density function  
- probability distribution



$$P(v_1, v_2) = \int_{v_1}^{v_2} f(v) \cdot dv$$

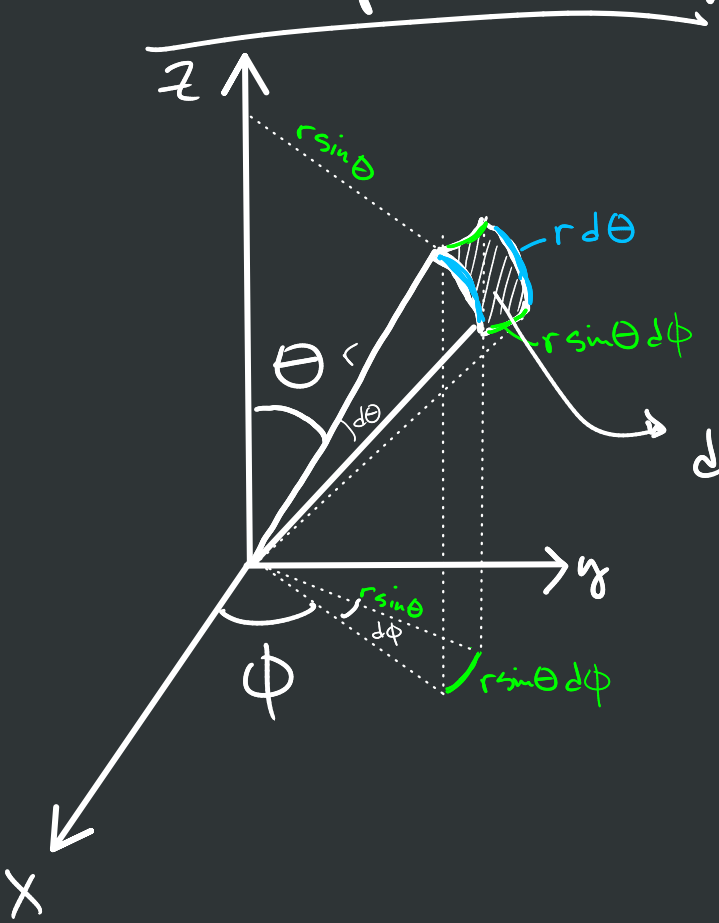
$$\int_{-\infty}^{+\infty} f(v) dv = 1 \quad \leftarrow \text{normalization}$$

$$\langle v \rangle = \int_{-\infty}^{+\infty} v \cdot f(v) \cdot dv$$

$$\langle v^2 \rangle = \int_{-\infty}^{+\infty} v^2 \cdot f(v) \cdot dv$$

$$\langle f(v) \rangle = \int_{-\infty}^{+\infty} f(v) \cdot f(v) \cdot dv$$

Number of atoms traveling in a particular direction w/ a particular speed

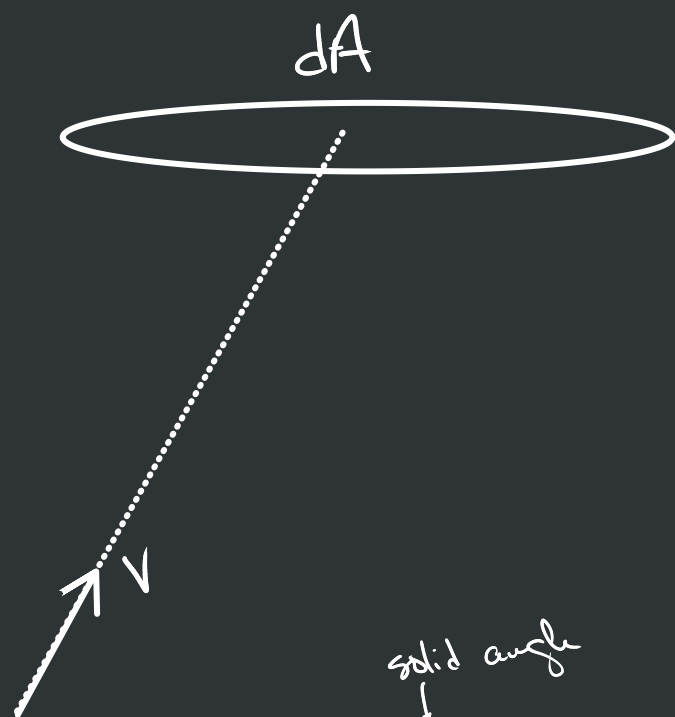


$$dA = r^2 \sin \theta d\theta d\phi$$

$$d\Omega = \frac{dA}{r^2}$$

$$= \frac{\cancel{r^2} \sin \theta d\theta d\phi}{\cancel{r^2}}$$

$$d\Omega = \sin \theta d\theta d\phi$$

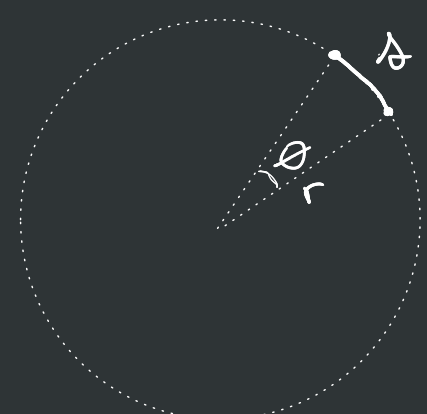


solid angle

$$d\Omega = \frac{dA}{r^2}$$

$$A = 4\pi r^2$$

total solid angle of a sphere  $4\pi$  (steradians)



$$\theta = \frac{s}{r}$$

$$d\theta = \frac{ds}{r}$$

$$C = 2\pi r$$

Probability of a particle moving in a direction of  $\theta, \phi \longrightarrow \frac{d\Omega}{4\pi} = \frac{\sin \theta d\theta d\phi}{4\pi}$

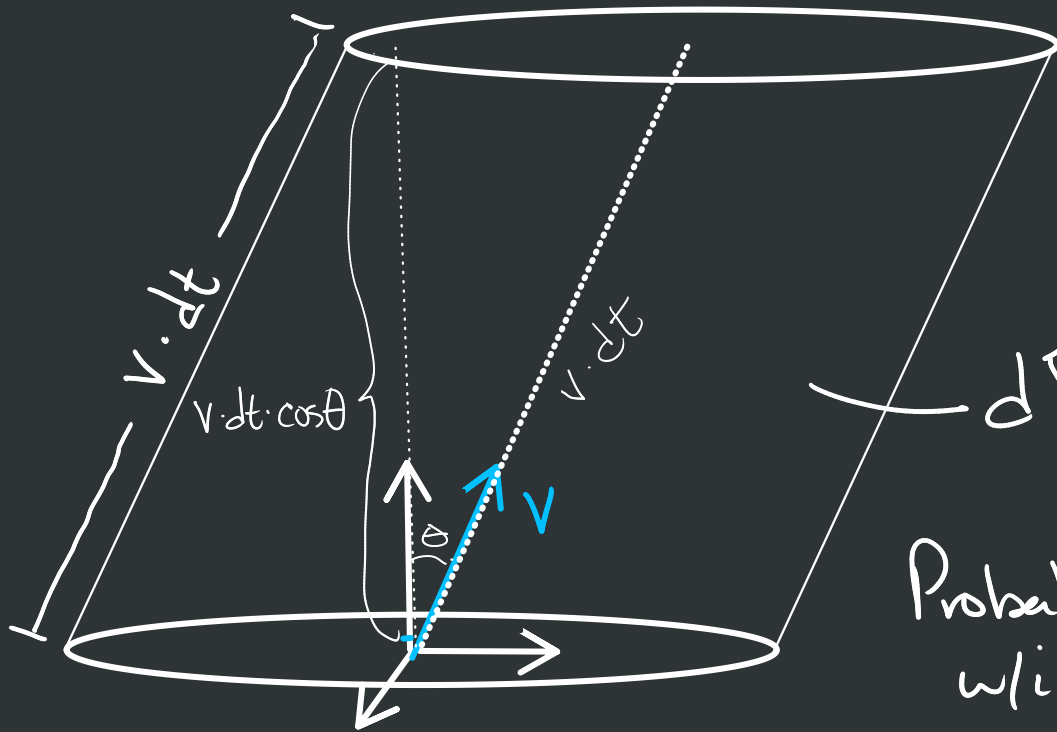


Number of particles  
travelling w/ speed  $v$   
in the direction of  $\Theta, \Phi$

$$\rightarrow \frac{\sin\Theta d\Theta d\Phi}{4\pi} \cdot f(v) \cdot dv \cdot N$$

NEXT PIECE

$dA$  of the wall



$$dV = v \cdot dt \cdot dA \cdot \cos\Theta$$

Probability of being  
w/ striking distance =  $\frac{dV}{V}$

$$= \frac{v \cdot dt \cdot dA \cos\Theta}{V}$$

$$d(\text{pressure}) = \frac{2mv \cos \theta}{dA dt} \cdot \frac{\sin \theta d\theta d\phi}{\frac{4\pi}{2}} \cdot f(v) \cdot dv \cdot N \cdot \frac{v dA \cos \theta}{V}$$

$$p = \frac{N \cdot m}{2\pi V} \underbrace{\int_0^{\infty} v^2 f(v) dv}_{\langle v^2 \rangle} \underbrace{\int_0^{\pi/2} \cos^2 \theta \cdot \sin \theta d\theta}_{\frac{1}{3}} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$\cos^2 = 1 - \sin^2$

$$p = \frac{Nm}{2\pi V} \cdot \langle v^2 \rangle \cdot \frac{1}{3} \cdot 2\pi$$

$$pV = N \frac{m \langle v^2 \rangle}{3} \longleftrightarrow pV = N k_B T$$

$$\frac{m \langle v^2 \rangle}{3} = k_B T$$

$$\langle v \rangle^2 \stackrel{?}{=} \langle v^2 \rangle$$

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$m \langle v^2 \rangle = 3 k_B T$$

$$\langle v^2 \rangle = \frac{3 k_B T}{m}$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle K \rangle = \frac{3}{2} k_B T$$

$$\langle v \rangle \neq \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$

$$U = N \langle K \rangle$$

$$V_{rms} \equiv \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$

$$\bullet U = \frac{3}{2} N k_B T$$

$$\underline{\text{air}} \rightarrow 29 \text{ g/mol} = 0.029 \text{ kg/mol}$$

$$\bullet U = \frac{3}{2} n R T$$

$$m = \frac{0.029 \text{ kg}}{N_A}$$

$$V_{\text{rms,air}} = \sqrt{\frac{3 \cdot \textcircled{N_A k_B^R} \cdot 293\text{K}}{0.029\text{kg}}}$$

$$= \sqrt{\frac{3 \cdot 8.31 \cdot 293}{0.029}}$$

$$\underline{V_{\text{rms,air}}} = 501 \text{ m/s}$$

1.47

$$Q_{\text{tea}} = m_{\text{tea}} c \Delta T_{\text{tea}}$$

$$Q_{\text{tea}} = 200\text{g} \cdot \left(1 \frac{\text{cal}}{\text{g} \cdot \text{K}}\right) \cdot (-35\text{K})$$

$$Q_{\text{tea}} = -7000 \text{ cal}$$

$$Q_{\text{ice}} = +7000 \text{ cal} = m_{\text{ice}} \cdot C_{\text{ice}} \cdot (+15\text{K}) + m_{\text{ice}} \cdot L_f + m_{\text{ice}} C_{\text{water}} (65\text{K})$$





