

Chapter 4

Heat Engines → absorbs heat, produces work

↗
cannot convert
all of the heat to work

Heat comes in
increases the
entropy of
the engine

→
to start the cycle over
entropy must be taken
out

heat exhausted

$$\Delta U_{\text{cycle}} = 0 = Q + W_{\text{gas}}$$

$$= \underset{\substack{\uparrow \\ \text{heat in}}}{Q_h} - \underset{\substack{\uparrow \\ \text{heat out}}}{Q_c} + W_{\text{gas}}$$

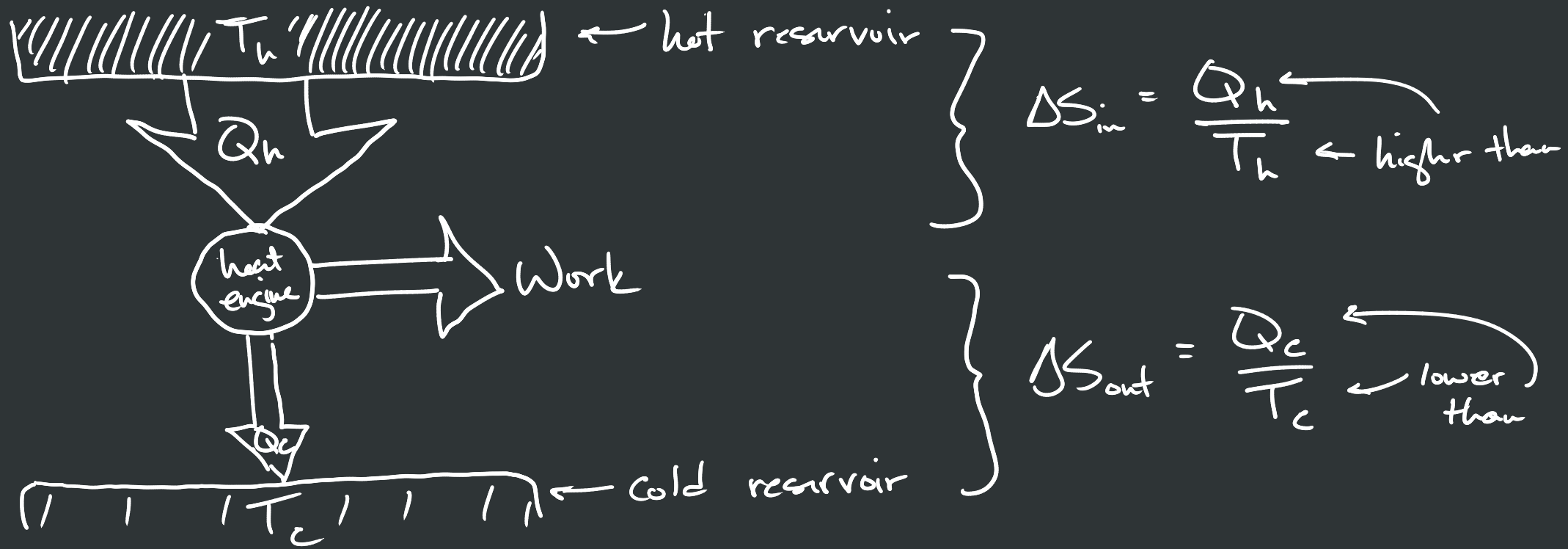
$$-W_{\text{gas}} = Q_h - Q_c$$

$$W_{\text{surr}} = -W_{\text{gas}}$$

$$\rightarrow W_{\text{surr}} = Q_h - Q_c$$

$$\text{efficiency} \equiv \frac{\text{what you get}}{\text{what you pay for}}$$

$$e = \frac{W_{\text{surr}}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$



$$W_{surv} = Q_h - Q_c$$

realistically, $\frac{Q_h}{T_h} \leq \frac{Q_c}{T_c}$

$$\frac{T_c}{T_h} \leq \frac{Q_c}{Q_h}$$

$$e = 1 - \frac{Q_c}{Q_h}, \text{ then } e \leq 1 - \frac{T_c}{T_h}$$

so what is the max efficiency?

usually creating more entropy through the engine. More heat is dumped into the cold reservoir and less energy is available for work

for max efficiency:

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

But we need to break this up into a couple of steps:

$$\underbrace{\frac{Q_h}{T_h}} = \underbrace{\frac{Q_h}{T_{\text{gas}}}}$$

entropy
that is
removed from
the hot
reservoir

entropy
gained by
the engine

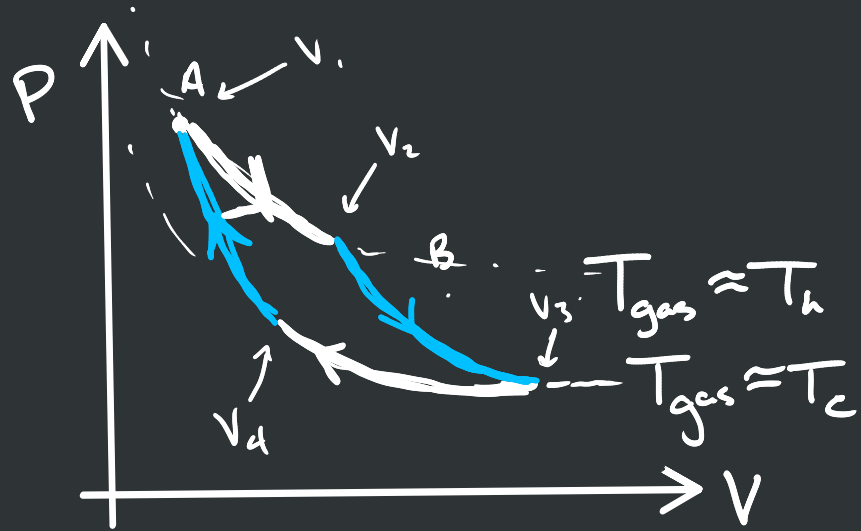
$T_{\text{gas}} = T_h$
to avoid any
new entropy

$$T_{\text{gas}} + dT = T_h$$

so that heat will
actually flow

So we expand the volume to
keep the temperature the same, T_{gas} constant

↳ isothermal expansion



This cycle is called the Carnot Cycle.

$$e = 1 - \frac{T_c}{T_h}$$

only achieved by Carnot cycle

on the exhaust:
(isothermal compression)

$$\frac{Q_c}{T_c} = \frac{Q_c}{T_{\text{gas}}}$$

$$T_{\text{gas}} \approx T_c + dT$$

So to connect these two: adiabatic
(isentropic)
 $Q = 0$
no heat flows in or out

Prove this

4.5 isothermal expansion

$$dU = \pm Q + \pm W$$

$$\pm Q = -\pm W = p dV \neq p \Delta V$$

$$Q = \int_{V_i}^{V_f} p dV, \quad p = \frac{Nk_B T}{V}$$

$$Q = Nk_B T \int_{V_i}^{V_f} \frac{dV}{V}$$

$$Q = Nk_B T \ln\left(\frac{V_f}{V_i}\right) \leftarrow \begin{array}{l} \text{heat in if} \\ V_f > V_i \text{ (expansion)} \end{array}$$

$$Q_h = Nk_B T_h \ln\left(\frac{V_2}{V_1}\right)$$

$$Q_c = -Nk_B T_c \ln\left(\frac{V_4}{V_3}\right) \leftarrow \begin{array}{l} \text{heat out} \\ \text{since } V_3 < V_4 \end{array}$$

$$Q_c = Nk_B T_c \ln\left(\frac{V_3}{V_4}\right)$$

$$e = 1 - \frac{Q_c}{Q_h} \stackrel{?}{=} 1 - \frac{T_c}{T_h}$$

$$e = 1 - \frac{N k_B T_c \ln\left(\frac{V_3}{V_4}\right)}{N k_B T_h \ln\left(\frac{V_2}{V_1}\right)}$$

$$\text{if } \ln\left(\frac{V_3}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right)$$

$$\hookrightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} \text{ then } \underline{\underline{\text{proven}}}.$$

adiabatic expansion

$$V_f T_f^{f/2} = V_i T_i^{f/2}$$

$$V_3 T_c^{f/2} = V_2 T_h^{f/2}$$

$$\left(\frac{T_c}{T_h}\right)^{f/2} = \frac{V_2}{V_3}$$

adiabatic compression

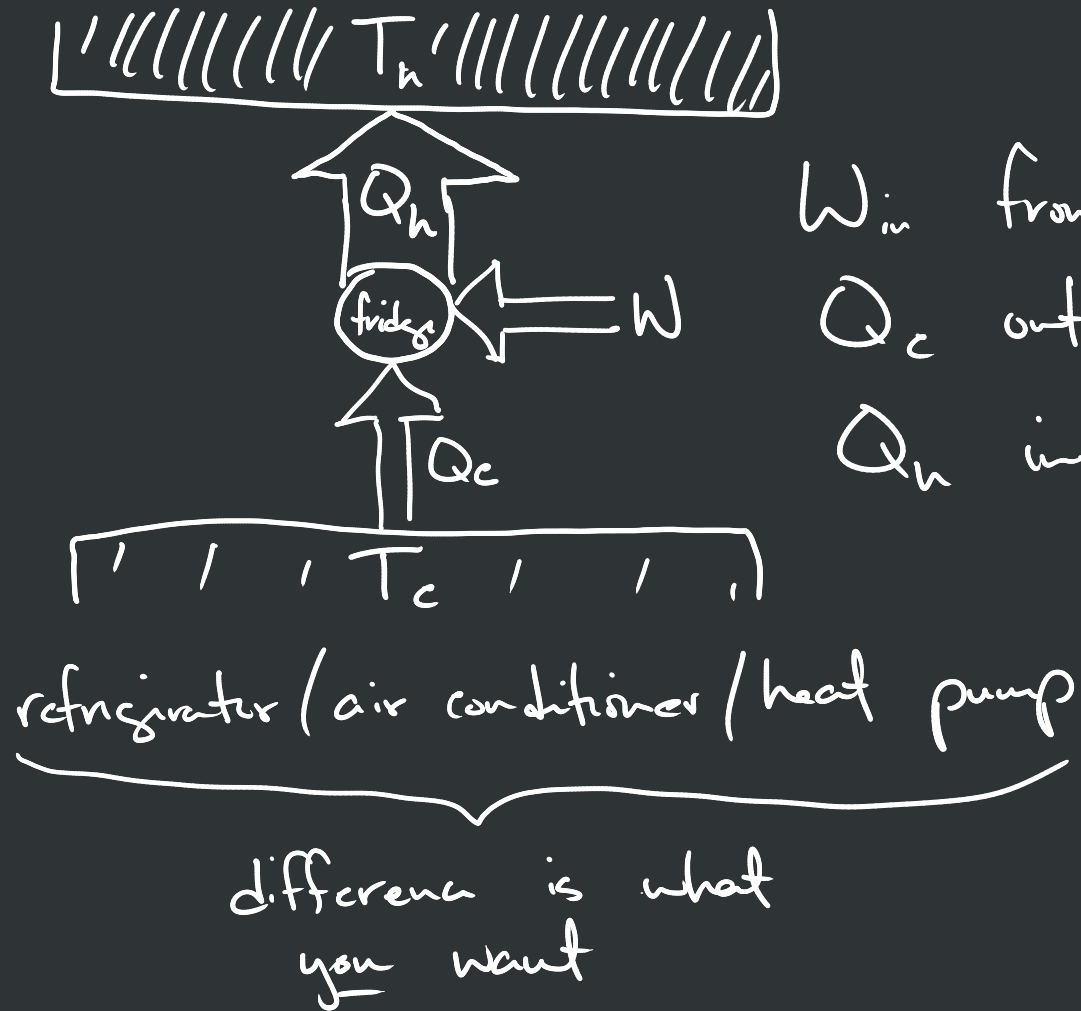
$$V_1 T_h^{f/2} = V_4 T_c^{f/2}$$

$$\frac{V_1}{V_4} = \left(\frac{T_c}{T_h}\right)^{f/2}$$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} !$$

$$\text{So } e = 1 - \frac{T_c}{T_h} \text{ for } \underline{\underline{\text{Carnot Cycle}}}$$

4.2) Refrigerators - operates oppositely to heat engine



W_{in} from environment

Q_c out of cold res.

Q_h into hot res.

efficiency \Rightarrow coefficient of performance

$COP = \frac{\text{what you get}}{\text{what you pay for}}$

refrigerator

$$COP = \frac{Q_c}{W}$$

heat pump

$$COP = \frac{Q_h}{W}$$

$$\text{COP} = \frac{Q_c}{W}$$

$$\Delta U = 0 = -(Q_h - Q_c) + W \quad \left. \begin{array}{l} \text{use 1st law} \\ \text{to change variables} \end{array} \right\}$$

$$W = Q_h - Q_c$$

$$\text{COP} = \frac{Q_c}{Q_h - Q_c}$$

$$\text{COP} = \frac{1}{\frac{Q_h}{Q_c} - 1}$$

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c}$$

$$\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$$

second law to
establish a
limit

$$\text{COP} \leq \frac{1}{\frac{T_h}{T_c} - 1}$$

$$\boxed{\text{COP} \leq \frac{T_c}{T_h - T_c}}$$

Ex. kitchen freezer $\rightarrow -20^\circ\text{C}$

$$T_h = 293\text{ K} \quad T_c = 253\text{ K}$$

$$\text{COP} \leq \frac{253\text{ K}}{40\text{ K}} = 6.3 = \frac{Q_c}{W}$$

How much heat is dumped
into kitchen?

$$Q_h = Q_c + W = 7.3\text{ J}$$

for one
joule of
electrical
energy out of
the wall, 6.3 J
of heat more out

4.10) assume a $COP = 6.3$

$$COP = \frac{Q_c}{W} = \frac{Q_c / \Delta t}{W / \Delta t} = \frac{300W}{P_{wall}} = 6.3$$

$$P_{wall} = \frac{300W}{6.3} = \underline{\underline{47.6W}}$$

4.15) a) $COP = \frac{\text{what you get}}{\text{what you pay for}} = \frac{Q_c}{Q_f}$
 $\leftarrow \text{heat removed}$
 $\leftarrow \text{heat you pay for}$

b) $Q_r = Q_c + Q_f$

$$COP = \frac{Q_c}{Q_f} = \frac{Q_r - Q_f}{Q_f} = \frac{Q_r}{Q_f} - 1$$

$\frac{Q_r}{Q_f} > 2$ to have $COP > 1$ } Conservation of Energy does not exclude this

c) you do

