Chapter 2 - 2^{nd} Law of Thermodynamics

↳ heat spontaneously flows from high temp to low temp

Einstein Solid → $\boxed{\text{Entropy}}$ ← Ideal Gas

Combinatorics

one coin

$$P(\text{heads}) = \frac{1}{2}$$

multiple coins

$$P(n) = \frac{n}{N}$$

$$P(3 \text{ heads}) = \frac{\Omega(3)}{\Omega(\text{all})} = \frac{\Omega(3)}{\sum_{n=0}^{n} \Omega(n)}$$

← microstate in the macrostate of 3

← all of the microstates

5 coins

microstates $\left\{ \begin{array}{l} H\,H\,T\,T\,H \;-\; 3H \\ T\,H\,H\,H\,H \;-\; 4H \end{array} \right\}$ macrostates

how many microstates are in a macrostate?

↳ multiplicity

$$\Omega(n) = \frac{5!}{n!(5-n)!}$$

number of heads

$\Omega(0) = 1$
$\Omega(1) = 5$
$\Omega(2) = 10$
$\Omega(3) = 10$
$\Omega(4) = 5$
$\Omega(5) = 1$

$$\Omega(1) = \frac{5!}{1!(5-1)!} = \frac{5!}{1!4!}$$

$$= \frac{5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{1 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

$$= 5$$

$$\Omega(2) = \frac{5!}{2!3!} = 10$$

$$\Omega(N, n) = \frac{N!}{n!(N-n)!} \leftarrow \text{Notation: } \binom{N}{n}$$

# of coins

10 atoms each w/ 0 or 1 packets of energy (energy unit)

How many possible ways are there to distribute 4 energy units

← microstate

$$\Omega(10, 4) = \frac{10!}{4!\,6!} = 210$$

4 energy packets ← macrostate

What if an atom can have more than one energy packet at a time?

microstates

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!\,(N-1)!} \qquad \binom{q+N-1}{q}$$

# of atoms

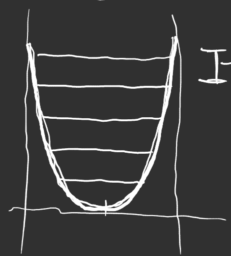
# of energy packets

q = macrostate

4 energy packets ← macrostate

This model of a collection of atoms w/ equal size energy quanta distributed among them is the <u>Einstein Solid</u>.

$\underbrace{\text{Einstein Solid}}$ ↪ <u>Debye</u> Model



I ~ hf

$$\underline{\underline{\frac{1}{2}kx^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

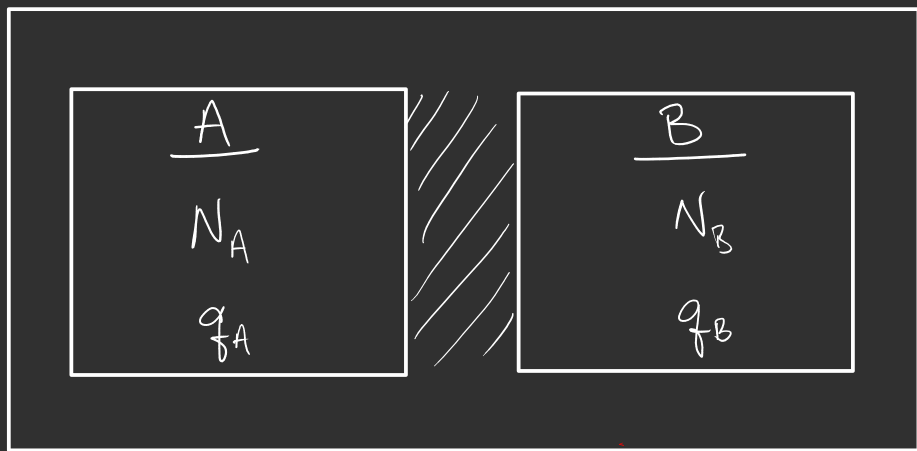
<u>Large Number</u> → addition of small numbers is not important

$$10^{23} + 23 = 10^{23}$$

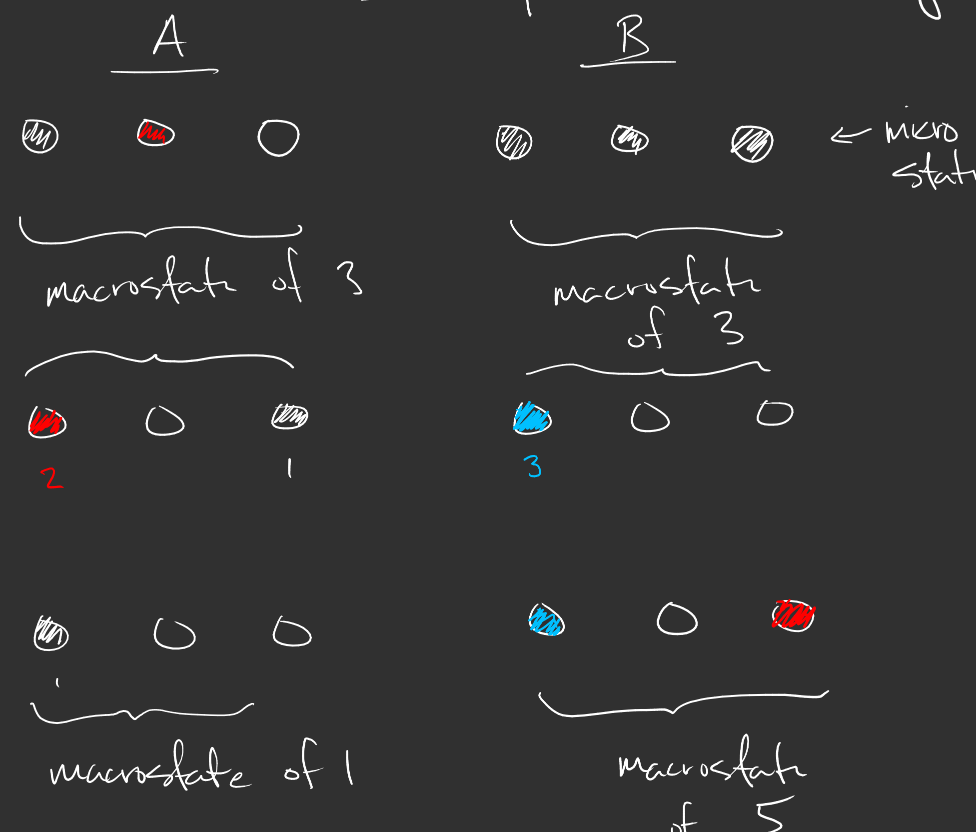
<u>Very Large Number</u>

$$10^{10^{23}} \times 10^{23} = 10^{10^{23}+23} \approx 10^{10^{23}}$$

Two Systems



Ex. Each system has 3 particles and 6 total quanta of energy



q_A	mult_A	q_B	mult_B	mult_total
0	1	6	28	28
1	3	5	21	63
2	6	4	15	90
3	10	3	10	100
4	15	2	6	90
5	21	1	3	63
6	28	0	1	28

macrostate w/ the most microstates

total number of microstates

462

$\Omega(3,1) = 3$

$\Omega(3,5) = \frac{7!}{5!\,2!} = 21$

Fundamental Assumption of Stat Mech:

all microstates are possible and equally probable

But that does not mean that every microstate will occur.

Not all macrostates are equally probable.

$$P_A(q) = \frac{\Omega_{total}(q_A)}{\sum_{q_A} \Omega_{total}(q_A)}$$

We could find the total # of microstates:

$$\Omega(\underset{\overset{\uparrow}{N}}{6}, \underset{\overset{\uparrow}{q}}{6}) = \frac{(6+6-1)}{6!\,(6-1)!} = \underline{\underline{462}}$$