

Chapter 4

Heat Engines → absorbs heat, produces work

cannot convert
all of the heat to work

Heat comes in
increases the
entropy of
the engine

to start the cycle over
entropy must be taken
out

heat exhausted

$$\Delta U_{\text{cycle}} = 0 = Q + W_{\text{gas}}$$

$$= \underset{\substack{\uparrow \\ \text{heat in}}}{Q_h} - \underset{\substack{\uparrow \\ \text{heat out}}}{Q_c} + W_{\text{gas}}$$

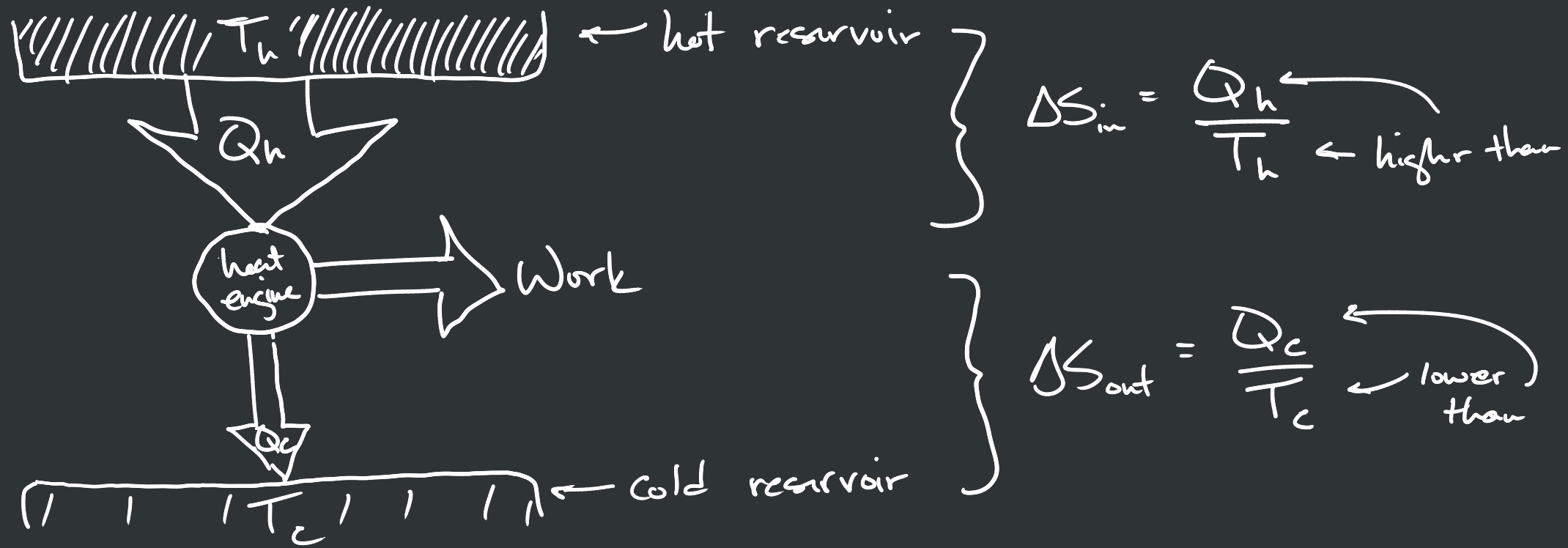
$$-W_{\text{gas}} = Q_h - Q_c$$

$$W_{\text{surr}} = -W_{\text{gas}}$$

$$\rightarrow W_{\text{surr}} = Q_h - Q_c$$

$$\text{efficiency} \equiv \frac{\text{what you get}}{\text{what you pay for}}$$

$$e = \frac{W_{\text{surr}}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$



$$W_{surv} = Q_h - Q_c$$

realistically, $\frac{Q_h}{T_h} \leq \frac{Q_c}{T_c}$

$$\frac{T_c}{T_h} \leq \frac{Q_c}{Q_h}$$

$$e = 1 - \frac{Q_c}{Q_h}, \text{ then } e \leq 1 - \frac{T_c}{T_h}$$

so what is the max efficiency?

usually creating more entropy through the engine. More heat is dumped into the cold reservoir and less energy is available for work

for max efficiency:

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

But we need to break this up into a couple of steps:

$$\frac{Q_h}{T_h} = \frac{Q_h}{T_{\text{gas}}}$$

entropy
that is
removed from
the hot
reservoir

entropy
gained by
the engine

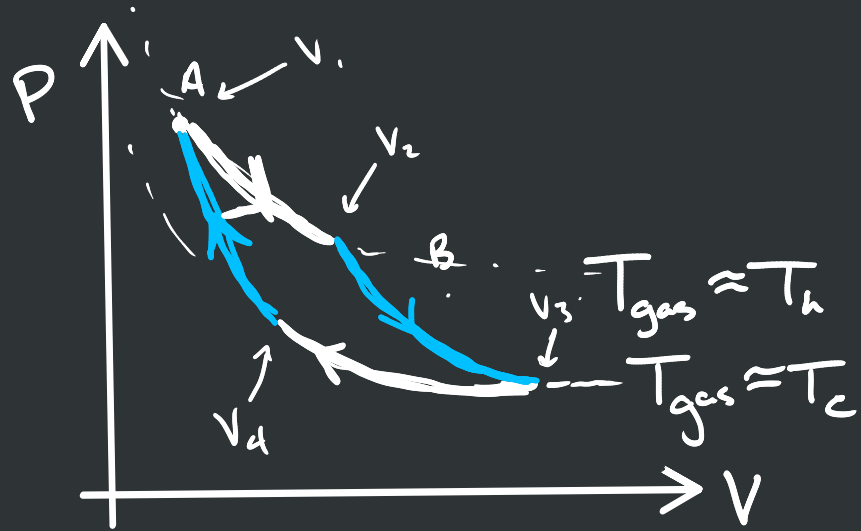
$T_{\text{gas}} = T_h$
to avoid any
new entropy

$$T_{\text{gas}} + dT = T_h$$

so that heat will
actually flow

So we expand the volume to
keep the temperature the same, T_{gas} constant

↳ isothermal expansion



This cycle is called the Carnot Cycle.

$$e = 1 - \frac{T_c}{T_h}$$

only achieved by Carnot cycle

on the exhaust:
(isothermal compression)

$$\frac{Q_c}{T_c} = \frac{Q_c}{T_{gas}}$$

$$T_{gas} \approx T_c + dT$$

So to connect these two: adiabatic
(isentropic)
 $Q = 0$
no heat flows in or out

Prove this

4.5 isothermal expansion

$$dU = \pm Q + \pm W$$

$$\pm Q = -\pm W = p dV \neq p \Delta V$$

$$Q = \int_{V_i}^{V_f} p dV, \quad p = \frac{N k_B T}{V}$$

$$Q = N k_B T \int_{V_i}^{V_f} \frac{dV}{V}$$

$$Q = N k_B T \ln\left(\frac{V_f}{V_i}\right) \leftarrow \begin{array}{l} \text{heat in if} \\ V_f > V_i \text{ (expansion)} \end{array}$$

$$Q_h = N k_B T_h \ln\left(\frac{V_2}{V_1}\right)$$

$$Q_c = -N k_B T_c \ln\left(\frac{V_4}{V_3}\right) \leftarrow \begin{array}{l} \text{heat out} \\ \text{since } V_3 < V_4 \end{array}$$

$$Q_c = N k_B T_c \ln\left(\frac{V_3}{V_4}\right)$$

$$e = 1 - \frac{Q_c}{Q_h} \stackrel{?}{=} 1 - \frac{T_c}{T_h}$$

$$e = 1 - \frac{N k_B T_c \ln\left(\frac{V_3}{V_4}\right)}{N k_B T_h \ln\left(\frac{V_2}{V_1}\right)}$$

$$\text{if } \ln\left(\frac{V_3}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right)$$

$$\hookrightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} \text{ then } \underline{\underline{\text{proven}}}.$$

adiabatic expansion

$$V_f T_f^{f/2} = V_i T_i^{f/2}$$

$$V_3 T_c^{f/2} = V_2 T_h^{f/2}$$

$$\left(\frac{T_c}{T_h}\right)^{f/2} = \frac{V_2}{V_3}$$

adiabatic compression

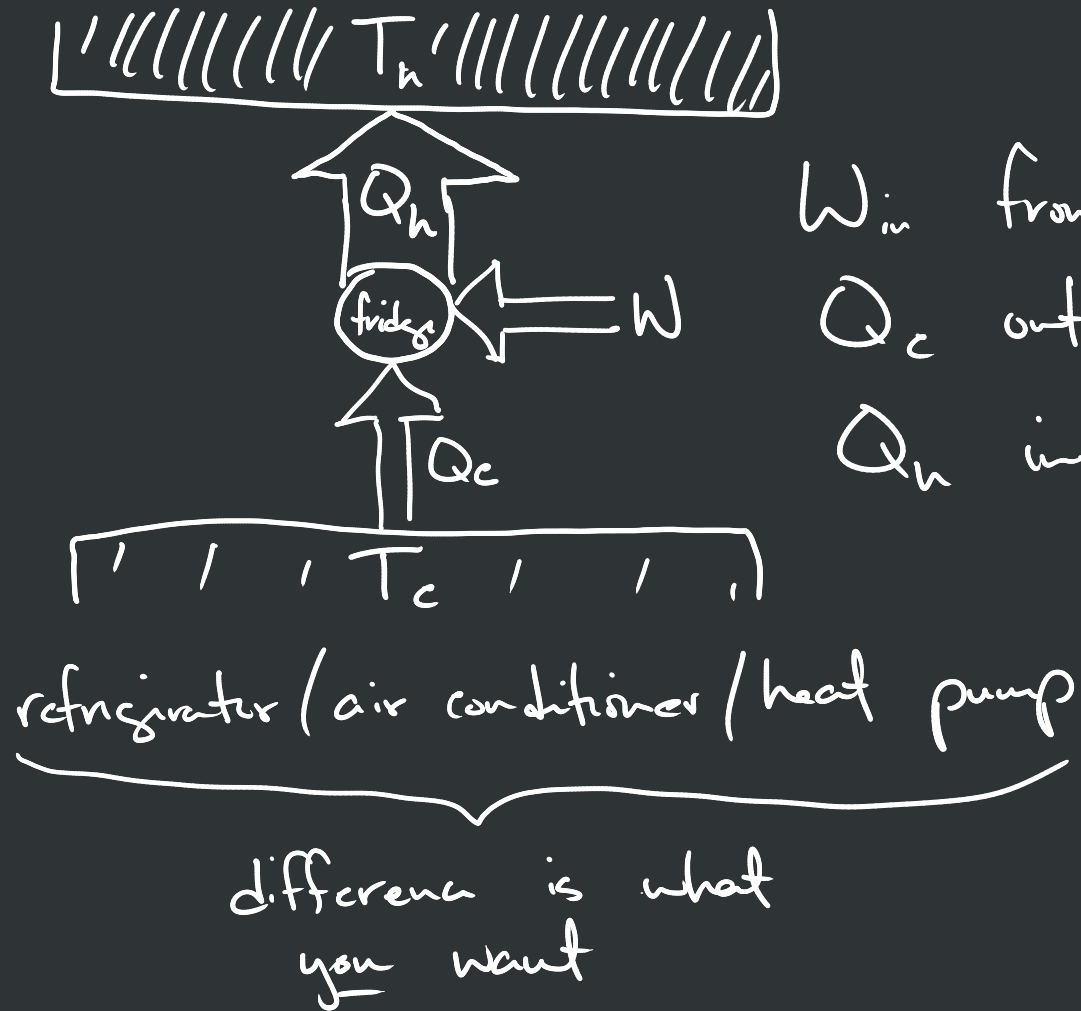
$$V_1 T_h^{f/2} = V_4 T_c^{f/2}$$

$$\frac{V_1}{V_4} = \left(\frac{T_c}{T_h}\right)^{f/2}$$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} !$$

$$\text{So } e = 1 - \frac{T_c}{T_h} \text{ for } \underline{\underline{\text{Carnot Cycle}}}$$

4.2) Refrigerators - operates oppositely to heat engine



W_{in} from environment

Q_c out of cold res.

Q_h into hot res.

efficiency \Rightarrow coefficient of performance

$$COP = \frac{\text{what you get}}{\text{what you pay for}}$$

refrigerator

$$COP = \frac{Q_c}{W}$$

heat pump

$$COP = \frac{Q_h}{W}$$

$$\text{COP} = \frac{Q_c}{W}$$

$$\Delta U = 0 = -(Q_h - Q_c) + W \quad \left. \begin{array}{l} \text{use 1st law} \\ \text{to change variables} \end{array} \right\}$$

$$W = Q_h - Q_c$$

$$\text{COP} = \frac{Q_c}{Q_h - Q_c}$$

$$\text{COP} = \frac{1}{\frac{Q_h}{Q_c} - 1}$$

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c}$$

$$\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$$

second law to
establish a
limit

$$\text{COP} \leq \frac{1}{\frac{T_h}{T_c} - 1}$$

$$\boxed{\text{COP} \leq \frac{T_c}{T_h - T_c}}$$

Ex. kitchen freezer $\rightarrow -20^\circ\text{C}$

$$T_h = 293\text{ K} \quad T_c = 253\text{ K}$$

$$\text{COP} \leq \frac{253\text{ K}}{40\text{ K}} = 6.3 = \frac{Q_c}{W}$$

How much heat is dumped
into kitchen?

$$Q_h = Q_c + W = 7.3\text{ J}$$

for one
joule of
electrical
energy out of
the wall, 6.3 J
of heat more out

4.10) assume a $COP = 6.3$

$$COP = \frac{Q_c}{W} = \frac{Q_c / \Delta t}{W / \Delta t} = \frac{300W}{P_{wall}} = 6.3$$

$$P_{wall} = \frac{300W}{6.3} = \underline{\underline{47.6W}}$$

4.15) a) $COP = \frac{\text{what you get}}{\text{what you pay for}} = \frac{Q_c}{Q_f}$
 $\leftarrow \text{heat removed}$
 $\leftarrow \text{heat you pay for}$

b) $Q_r = Q_c + Q_f$

$$COP = \frac{Q_c}{Q_f} = \frac{Q_r - Q_f}{Q_f} = \frac{Q_r}{Q_f} - 1$$

$\frac{Q_r}{Q_f} > 2$ to have $COP > 1$ } Conservation of Energy does not exclude this

c) you do

