Chapter 4

- **Heat Engines** → absorbs heat, produces work

$\text{heat} \xrightarrow{\text{cannot convert all of the heat to work}} \text{work}$

cannot convert
all of the heat to work

$$\left.\begin{array}{l}\text{Heat comes in}\\ \text{increases the}\\ \text{entropy of}\\ \text{the engine}\end{array}\right\} \longrightarrow \underbrace{\text{to start the cycle over entropy must be taken out}}_{\text{heat exhausted}}$$

$$\Delta U_{cycle} = 0 = Q + W_{gas}$$

$$= \underset{\text{heat in}}{Q_h} - \underset{\text{heat out}}{Q_c} + W_{gas}$$

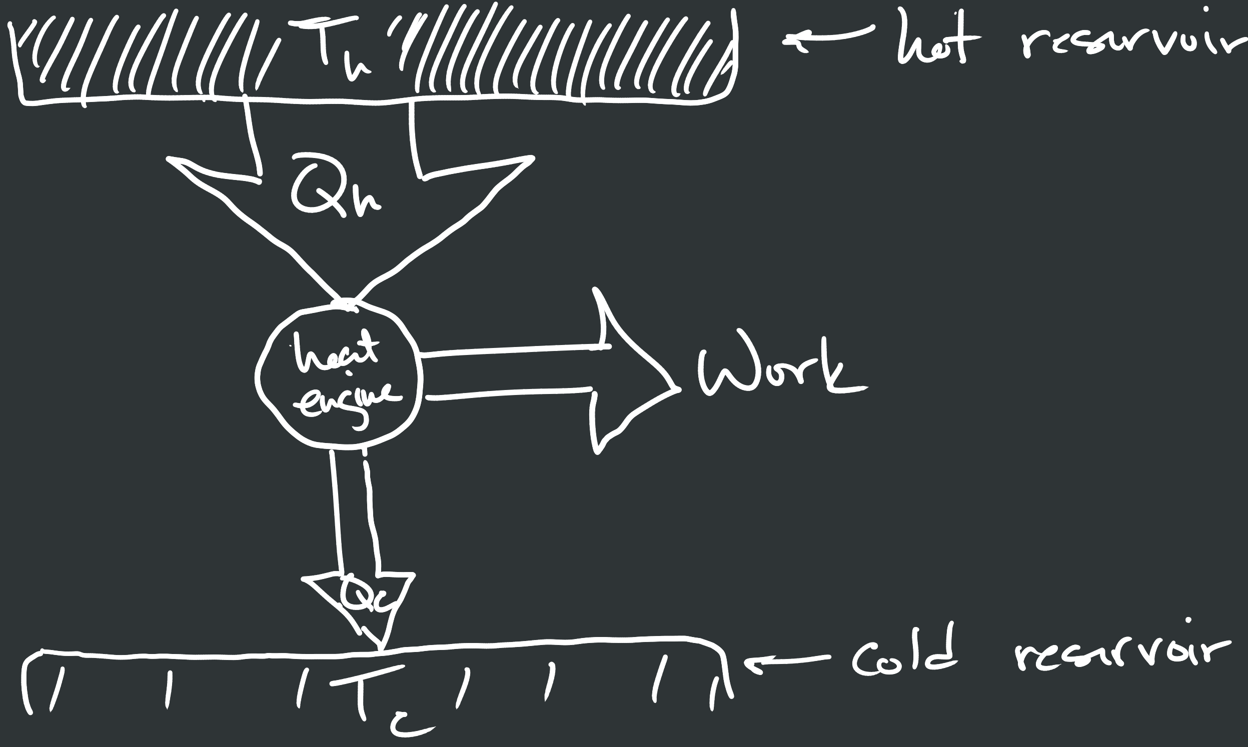
$$-W_{gas} = Q_h - Q_c$$

$$W_{surr} = -W_{gas}$$

$$\rightarrow W_{surr} = Q_h - Q_c$$

$$\text{efficiency} \equiv \frac{\text{what you get}}{\text{what you pay for}}$$

$$e = \frac{W_{surr}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$



$$\Delta S_{in} = \frac{Q_h}{T_h}$$ ← higher than

$$\Delta S_{out} = \frac{Q_c}{T_c}$$ ← lower than

$W_{surr} = Q_h - Q_c$

realistically, $\frac{Q_h}{T_h} \leq \frac{Q_c}{T_c}$

usually creating more entropy through the engine. More heat is dumped into the cold reservoir and less energy is available for work

$$\frac{T_c}{T_h} \leq \frac{Q_c}{Q_h}$$

$e = 1 - \frac{Q_c}{Q_h}$, then $e \leq \underbrace{1 - \frac{T_c}{T_h}}_{\text{so what is the max efficiency?}}$

for max efficiency:

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

But we need to break this up into a couple of steps:

$$\underbrace{\frac{Q_h}{T_h}}_{\substack{\text{entropy}\\\text{that is}\\\text{removed from}\\\text{the hot}\\\text{reservior}}} = \underbrace{\frac{Q_h}{T_{gas}}}_{\substack{\text{entropy}\\\text{gained by}\\\text{the engine}}}$$

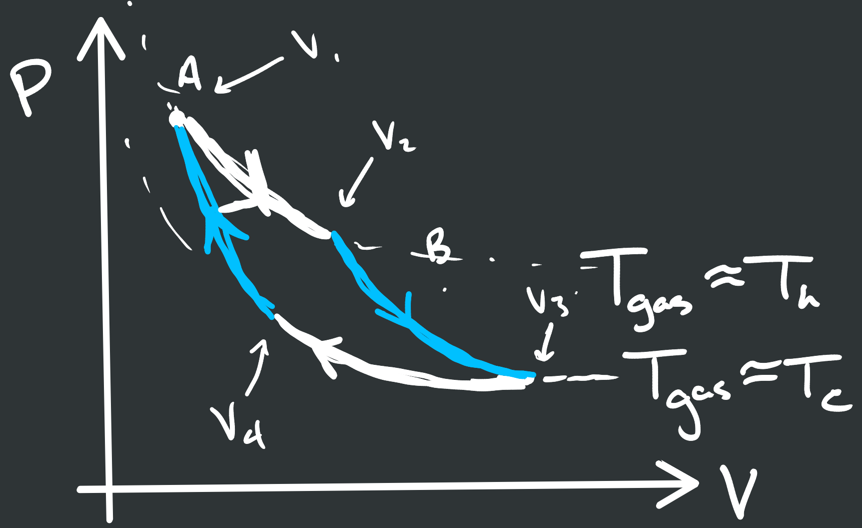
$T_{gas} = T_h$ to avoid any new entropy

$$\underbrace{\phantom{T_{gas} = T_h}}_{T_{gas} + dT = T_h}$$

so that heat will actually flow

So we expand the volume to keep the temperature the same, T_{gas} constant

↳ isothermal expansion



on the exhaust:
(isothermal compression)

$$\frac{Q_c}{T_c} = \frac{Q_c}{T_{gas}}$$

$$T_{gas} \approx T_c + dT$$

So to connect these two: adiabatic (isentropic)

$Q = 0$

no heat flows in or out

This cycle is called the Carnot Cycle.

$$e = 1 - \frac{T_c}{T_h}$$

only acheived by Carnot cycle

Prove this

4.5 isothermal expansion

$$\cancel{dU}^{0} = dQ + dW$$

$$đQ = -đW = p\,dV \neq p\Delta V$$

$$Q = \int_{V_i}^{V_f} p\,dV \quad , \quad p = \frac{Nk_BT}{V}$$

$$Q = Nk_BT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$Q = Nk_BT \ln\left(\frac{V_f}{V_i}\right)$$ ← heat in if $V_f > V_i$ (expansion)

$$Q_h = Nk_BT_h \ln\left(\frac{V_2}{V_1}\right)$$

$$Q_c = -Nk_BT_c \ln\left(\frac{V_4}{V_3}\right)$$ ← heat out since $V_3 < V_4$

$$Q_c = Nk_BT_c \ln\left(\frac{V_3}{V_4}\right)$$

$$e = 1 - \frac{Q_c}{Q_h} \stackrel{?}{=} 1 - \frac{T_c}{T_h}$$

$$e = 1 - \frac{\cancel{Nk}_B T_c \ln\left(\frac{V_3}{V_4}\right)}{\cancel{Nk}_B T_h \ln\left(\frac{V_2}{V_1}\right)}$$

if $\ln\left(\frac{V_3}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right)$

$\hookrightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1}$ then proven.

<u>adiabatic</u> <u>expansion</u>

$$V_f T_f^{f/2} = V_i T_i^{f/2}$$

$$V_3 T_c^{f/2} = V_2 T_h^{f/2}$$

$$\left(\frac{T_c}{T_h}\right)^{f/2} = \frac{V_2}{V_3}$$

<u>adiabatic</u> <u>compression</u>

$$V_1 T_h^{f/2} = V_4 \cdot T_c^{f/2}$$

$$\frac{V_1}{V_4} = \left(\frac{T_c}{T_h}\right)^{f/2}$$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} \; !$$

So $e = 1 - \frac{T_c}{T_h}$ for <u>Carnot</u> Cycle