Thermo Intro

temperature

Fahrenheit   Celsius

Rankine   Kelvin

Room temp 20°C (~70°F)
↓
293K

$\Delta F = \frac{9}{5} \Delta C$

$T_F = \frac{9}{5} T_c + 32$

$T_K = T_c + 273.15$

Number, moles, molar mass, density

1 mole of things = $6.022 \cdot 10^{23}$ things
particle
atom
molecule

$N \rightarrow$ number of particles

$n \rightarrow$ number of moles

C^{12} $\to$ 12 protons and neutrons

$\to$ 6 protons

1 mole is a gram of protons and neutrons

Ex. mass of one proton?

mass of one proton × number of proton = mass of collection

$M \to$ total mass of a collection

$m \to$ mass of one particle

$$m \times N = M$$

$$m = \frac{M}{N} = \frac{1\,\text{gram}}{N_A}$$

$N_A \to$ Avogadro's number $6.022 \cdot 10^{23}$

$$m = 1.7 \cdot 10^{-24} \text{ grams}$$

$$= 1.7 \cdot 10^{-27} \text{ kg}$$

What about N_2?

$$1 \text{ mole of } N_2 = 2 \cdot \left(14 \tfrac{g}{mol}\right) = 28\, g$$

Dry air?

78% N_2, 21% O_2, 1% Ar

$$0.78 \cdot (28\, g/mol) + 0.21 (32\, g/mol) + 0.01 (40\, g/mol) = 29\, g/mol$$

Ideal Gas Law → an equation of state → state variables

- experimental law

number of particles

(microscopic) $\boxed{PV = N k_B T}$ → Boltzmann's constant

$$\boxed{k_B = 1.38 \cdot 10^{-23}\, J/K}$$

$P = \frac{F}{\text{Area}}$ $[Pa] = \left[\frac{N}{m^2}\right]$ $[m^3]$ → Temperature $[K]$

Alternative form $\rightarrow$ $PV = nRT$ (macroscopic)

$\hookrightarrow$ universal gas constant

$$N k_B = n \cdot R$$

$$N = n \cdot N_A \quad \leftarrow \text{definition of moles}$$

$$\cancel{n} \cdot N_A \cdot k_B = \cancel{n} \cdot R$$

$$N_A \cdot k_B = R = 6.022 \cdot \cancel{10^{23}} \frac{\text{part}}{\text{mol}} \; 1.38 \cdot \cancel{10^{-23}} \, J/K$$

$$R = 8.31 \, \frac{J}{K \cdot mol}$$

Volume of 1 mole of air at room temp and atmospheric pressure

$\hookrightarrow$ 293K or 300K

$\hookrightarrow$ 1 atm = $1.013 \cdot 10^5$ Pa

$$V = \frac{nRT}{P} = \frac{1\,mol \cdot 8.31\,J/K\,mol \cdot 300K}{10^5\,Pa} = 0.024\ m^3$$

$$V_{cube} = s^3$$

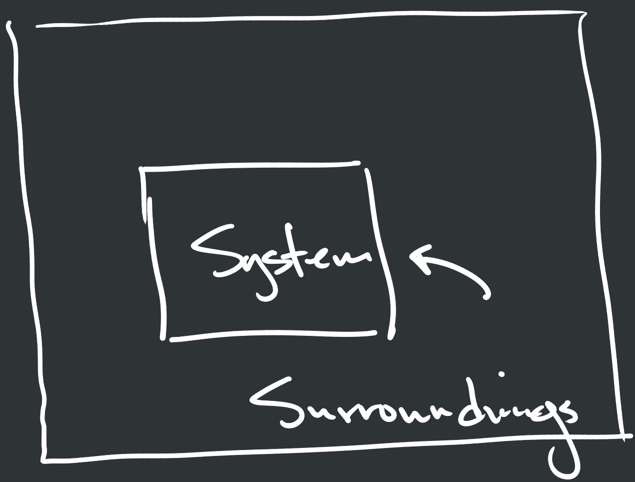
$$\sqrt[3]{0.024\ m^3} = s = 0.288\ m$$

$$\sim 30\ cm$$

Laws of Thermodynamics

0. Thermometers work
1. Conservation of Energy
2. Heat flows from high temp to low temp
   Entropy is maximized
   No perpetual motion machine
3. You can't reach absolute zero

1st Law → Conservation of Energy



Energy of a system can change

Work + **Heat**

Work → Force applied over distance

Heat → spontaneous flow of energy due to a difference in temperature

$$dU = Q + W$$

$$\rightarrow dU = đQ + đW$$

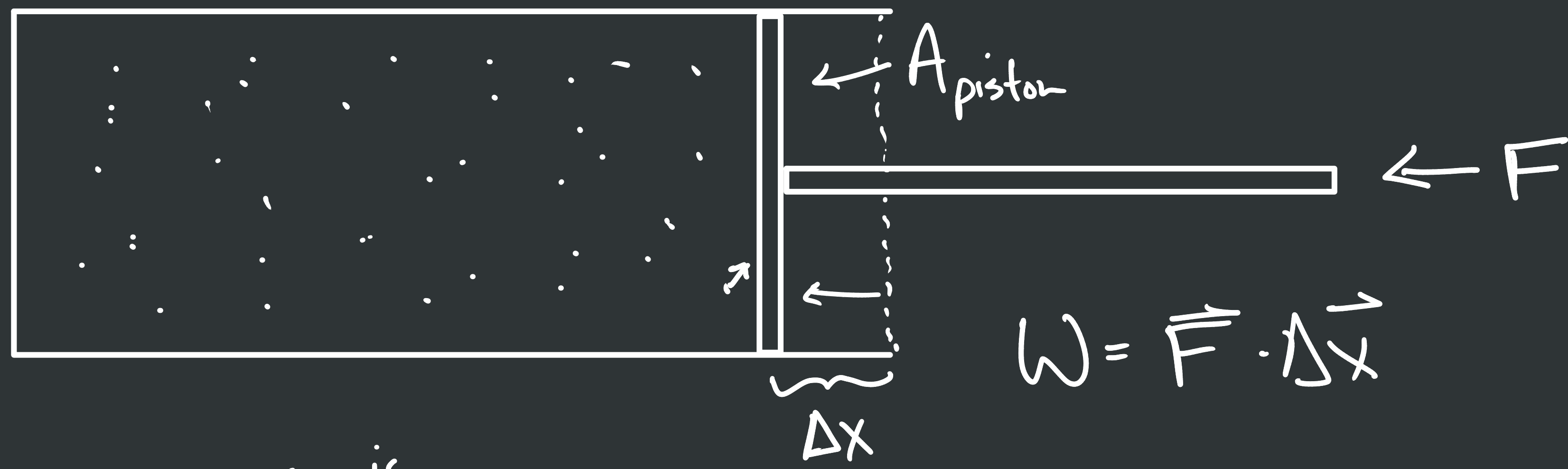
inexact differential

$$\cancel{dQ = Q(x+dx) - Q(x)}$$

$$dU = \delta Q + \delta W$$

↳ internal energy of the system, heat is added or work is done to the system they are positive

Compression Work (or Expansion)



$$W = \vec{F} \cdot \vec{\Delta x}$$

$$P_{gas} = \frac{F_{gas}}{A_{piston}}$$

assuming pressure is uniform throughout the chamber

↳ compression (expansion) must happen quasistatically

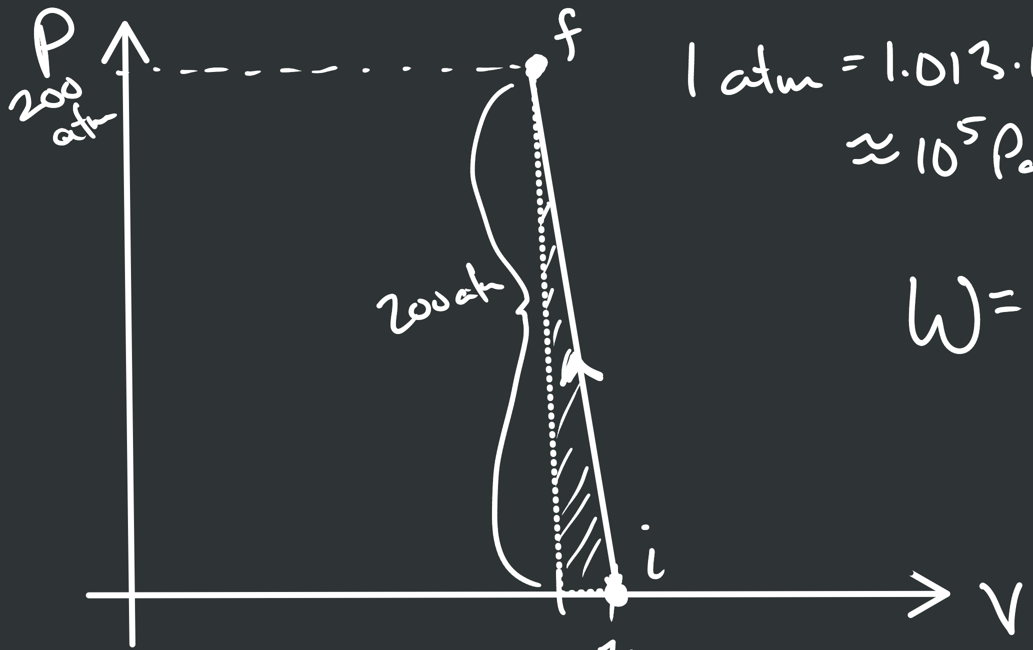
$$W = P_{gas} \cdot \underbrace{A_{piston} \cdot \Delta x}_{\Delta V}$$

$$W = -P\Delta V$$

ΔV is (−) in compression but energy is increasing

$$dU = đQ + đW \qquad đW = -P\,dV \qquad \underline{W = \int -P\,dV}$$

Problem 1.32. By applying a pressure of 200 atm, you can compress water to 99% of its usual volume. Sketch this process (not necessarily to scale) on a *PV* diagram, and estimate the work required to compress a liter of water by this amount. Does the result surprise you?



$\Delta L = 0.01L$ $\qquad$ 0.99 $1L$

$$1\text{ atm} = 1.013\cdot10^5\text{ Pa} \approx 10^5\text{ Pa}$$

$$P_{abs} = P_{gauge} + P_{atm}$$

$$W = -\int p\,dV$$

$$\Delta V = 0.01L = 10^{-5}\,m^3$$

$$0.01L\cdot\frac{1m^3}{1000L} = 10^{-5}\,m^3$$

$$A_\Delta = \frac{1}{2}bh \Rightarrow \underline{W = \frac{1}{2}P\Delta V}$$

$$W = \frac{1}{2}(200\cdot10^5\text{ Pa})\cdot10^{-5}\,m^3 = \underline{100\text{ J}}$$

$$1L\cdot\frac{\overset{10^3}{1000\text{ mL}}}{1\text{ L}}\cdot\frac{1\text{ cm}^3}{1\text{ mL}}\cdot\frac{(1\text{ m})^3}{(100\text{ cm})^3} = 1\cdot10^{-3}\,m^3 = 0.001\,m^3$$

$(10^2)^3 = 10^6$

$$\boxed{1000L = 1\,m^3}$$

slope-intercept

$y = mx + b$

$m = \frac{\Delta p}{\Delta V}$

y-int

point-slope

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200}{-0.01} = -2 \cdot 10^4 \, atm/L$

$p - 0 = -2 \cdot 10^4 \frac{atm}{L}(V - 1L)$

$p = -2 \cdot 10^4 \frac{atm}{L} \cdot V + 2 \cdot 10^4 \, atm$

$\underline{1.33|}$

1.33

Problem 1.33. An ideal gas is made to undergo the cyclic process shown in Figure 1.10(a). For each of the steps A, B, and C, determine whether each of the following is positive, negative, or zero: (a) the work done on the gas; (b) the change in the energy content of the gas; (c) the heat added to the gas. Then determine the sign of each of these three quantities for the whole cycle. What does this process accomplish?

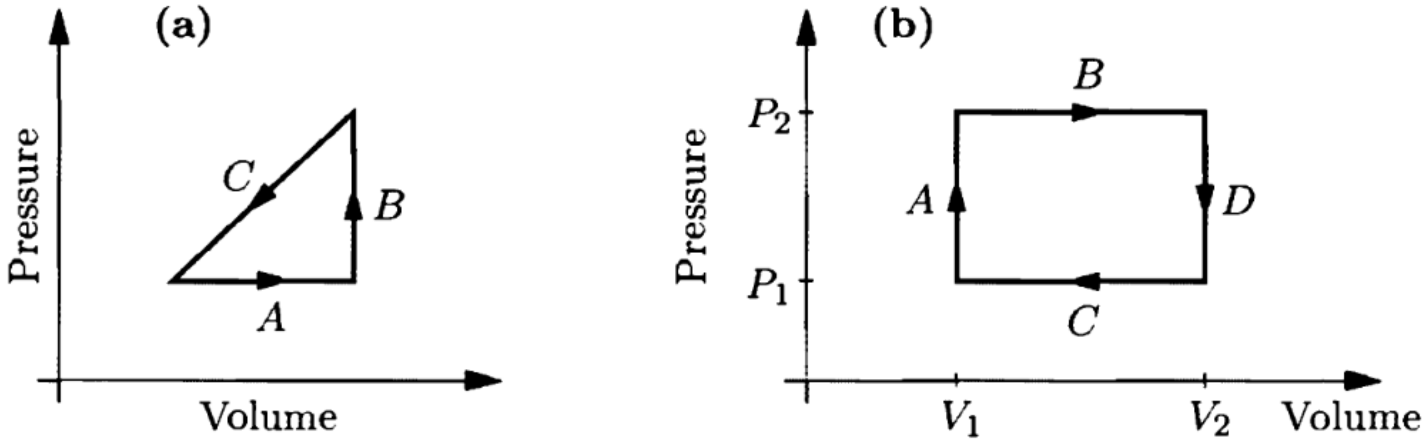


Figure 1.10. *PV* diagrams for Problems 1.33 and 1.34.

Come back!

Proportinality

$$PV = Nk_BT$$

$$P_1V = Nk_BT_1$$

$$P_2V = Nk_BT_2$$

$$P = \frac{Nk_B}{V}T$$

$$\frac{P_2\cancel{V} = \cancel{N}\cancel{k_B}T_2}{P_1\cancel{V} = \cancel{N}\cancel{k_B}T_1}$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 2$$

$$\boxed{P \propto T}$$

$$P = C \cdot T$$

constant of proportinality

$$\frac{P_2}{P_1} = \frac{N_2}{N_1} \rightarrow P \propto N^1 \rightarrow P = C_2 N$$

$$\hookrightarrow \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^1$$

$$PV = N k_B T$$

$$P = \frac{N k_B T}{V}$$

$$P \propto N^1 T^1 V^{-1}$$

$$P \propto V^{-1} \longrightarrow P = C_2 V^{-1} = \frac{C_2}{V}$$

$$\hookrightarrow \frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-1} = \frac{V_1}{V_2}$$

$$F_G \propto r^{-2} \longrightarrow F_G = C_3 \cdot r^{-2}$$

$$= \frac{C_3}{r^2}$$

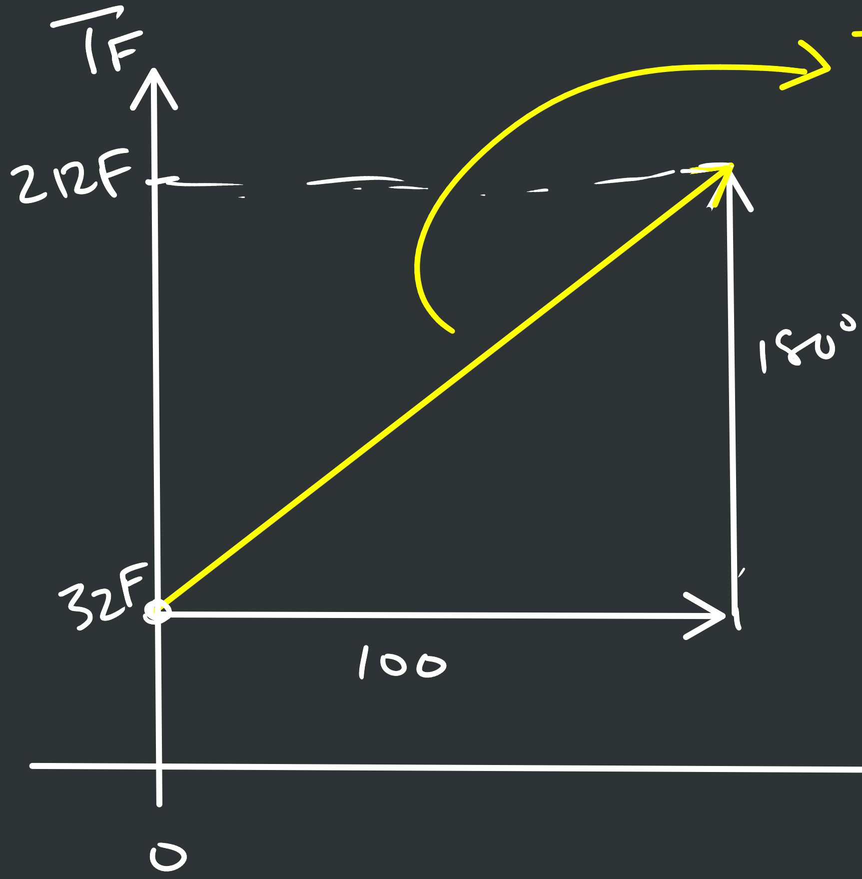
$$\frac{F_2}{F_1} = \left(\frac{r_2}{r_1}\right)^{-2}$$

$$\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$$

$$T = \frac{PV}{k_B N}$$

$$p \propto N^1 T^1 V^{-1}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^1 \cdot \left(\frac{V_2}{V_1}\right)^{-1}$$



$$T_F = m T_c + 32°F$$

$$m = \frac{180 F°}{100 C°} = \frac{9 F°}{5 C°}$$

$$T_F = \frac{9F°}{5C°} T_c + 32F°$$

Heat → flow of energy due to a difference in temperature

$\Delta U = Q + \cancel{W}^{0}$

[Joules] [calorie] 1 cal = 4.186 J

vs.

[Calorie] ← food

1 Cal = 1000 calorie

$\frac{Q}{\Delta T} = \mathbb{C}$ ← heat capacity $\left[\frac{J}{K}\right]$

1

$\frac{\mathbb{C}}{m} = c$ ← specific heat capacity $\left[J/gK\right] = \left[kJ/kgK\right]$ $\boxed{Q = mc\Delta T}$

2

$\frac{\mathbb{C}}{n} = C_{v,p}$ ← molar heat capacity at constant volume

molar heat capacity at constant pressure

$Q = nC_v \Delta T$

(3) $\frac{C}{V}$ ← volumetric heat capacity

Phase change

$\frac{Q}{m} = L$ ← Latent Heat

→ Latent Heat of fusion - L_f
solid ⟷ liquid

→ Latent Heat of vaporization - L_v
liquid ⟷ gas

Calorimetry - when multiple object are in thermal contact at what temperature is thermal equilibrium

$$\Delta U = 0 = \underbrace{Q_1 + Q_2 + Q_3 + \dots}_{\text{objects changing temp or phase transitions occuring}}$$

Ex: 5kg of Al at 500°C | 5kg of Pb at 75°C | What is equil temp?

$\hookrightarrow$ 490° How much heat was given off?

$C_{Al} = 0.9\ \mathrm{kJ/kgK}$ $\quad C_{Pb} = 0.13\ \mathrm{kJ/kgK}$

$$Q = mc\Delta T = mc(T_f - T_i)$$
$$= 5\,\mathrm{kg} \cdot 0.9\,\mathrm{kJ/kgK}\,(490°\mathrm{C} - 500°\mathrm{C})$$
$$= 5\,\mathrm{kg} \cdot 0.9\,\mathrm{kJ/kgK} \cdot (-10\,\mathrm{K})$$
$$Q = -45\,\mathrm{kJ} \longrightarrow +45\,\mathrm{kJ} = 5\,\mathrm{kg} \cdot 0.13\,\mathrm{kJ/kgK} \cdot \Delta T$$

$$\Delta T = 69.2\,\mathrm{K}$$
$$T_f = 145°\mathrm{C}$$

$$Q_1 + Q_2 = 0$$

$$m_{Al} \cdot c_{Al} \cdot \Delta T_{Al} + m_{Pb} \cdot c_{Pb} \cdot \Delta T_{Pb} = 0$$

$$m_{Al} \cdot c_{Al} \cdot (T_f - 500) + m_{Pb} \cdot c_{Pb} \cdot (T_f - 75) = 0$$

solve for T_f

$$T_f = 446 \text{ ish } {}^\circ C$$

HW: 1.7, 1.16, 1.17

$\boxed{1.7}$

$$\beta = \frac{\frac{\Delta V}{V}}{\Delta T}$$

$$\beta = \frac{1}{5500} K^{-1} = 1.81 \cdot 10^{-4} K^{-1}$$

$$r = \frac{1}{2} cm$$

$$A = \pi \left(\frac{1}{2} cm\right)^2 = \frac{\pi}{4} cm^2$$

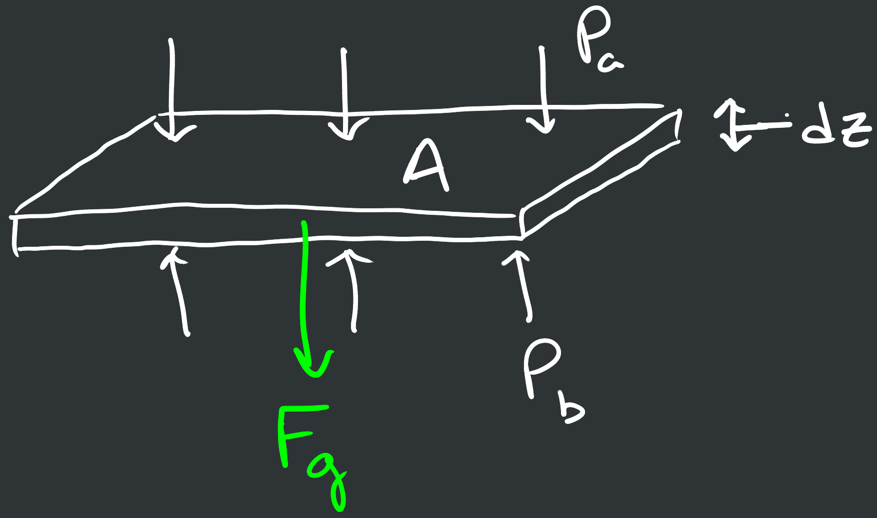
$$V = A \cdot h = \frac{\pi}{2} cm^3$$

$$\frac{\Delta V}{V} = \beta \Delta T$$

$$\Delta z = 1 mm \approx 1 K$$

1.16 $\rho = \frac{M}{V}$

"rho"
volumetric mass density



a) $P = \frac{F}{A}$

$$+P_b \cdot A - P_a \cdot A - M \cdot g = 0$$

$$\frac{dP}{dz} = \underbrace{\qquad\quad}_{\rho}$$

b) $\overset{\text{pressure}}{\downarrow}$
$PV = N k_B T$

$$\rho = \underbrace{\qquad\qquad\qquad}_{P,\ T,\ m}$$ ← avg mass of air molecules
$m = 29$ g/mol

$M = m \cdot n$

$\boxed{1.17}$ c) $\left(P + \frac{an^2}{V^2}\right)\underbrace{\left(V - nb\right)}_{V\left(1 - \frac{nb}{V}\right)} = nRT$

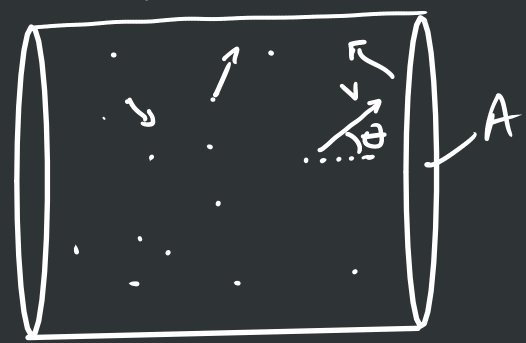
$\left(1 - \frac{nb}{V}\right)^{-1} \rightsquigarrow$ apply the binomial expansion

Kinetic Theory and Equipartition of Energy

pressure $\longleftrightarrow$ kinetic energy $\longleftrightarrow$ temperature

$$\frac{\text{pressure}}{\text{collision}} = \frac{F}{A} = \frac{\Delta p}{A\,\Delta t} \longleftarrow 2mv\cos\theta$$

$2mv\cos\theta$: Δp for each collision w/ the wall in Δt amount of time



$$d(\text{pressure}) = \frac{2mv\cos\theta}{dA \cdot dt} \cdot \left.\begin{array}{l}\text{number of particles hitting area } dA \\ \text{w/ } v \text{ velocity in } dt\end{array}\right\} \text{integrate over all velocities + } \theta$$

$$\underbrace{\text{number of particles hitting area } dA \text{ w/ } v \text{ velocity in } dt}_{\text{number of atoms traveling in a particular direction w/ a particular speed} \;\cdot\; \text{fraction of them that are within striking distance of the surface of } dA}$$

number of atoms traveling in a particular direction w/ a particular speed $\cdot$ fraction of them that are within striking distance of the surface of dA

Probability

$$P(x) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$N(x) \leftarrow$ number of desired outcomes

total outcomes $\rightarrow$ $$N = \sum_{x=0}^{\infty} N(x)$$

$$P(x) = \frac{N(x)}{N}$$

rearrange

$$N(x) = P(x) \cdot N$$

normalized

$$\sum_{x=0}^{\infty} P(x) = 1$$

$$\langle x \rangle = \underbrace{\sum_{x=0}^{\infty} x \cdot P(x)}_{\text{weighted average by probability}}$$

average value

expectation value

(not the same as the most probable)

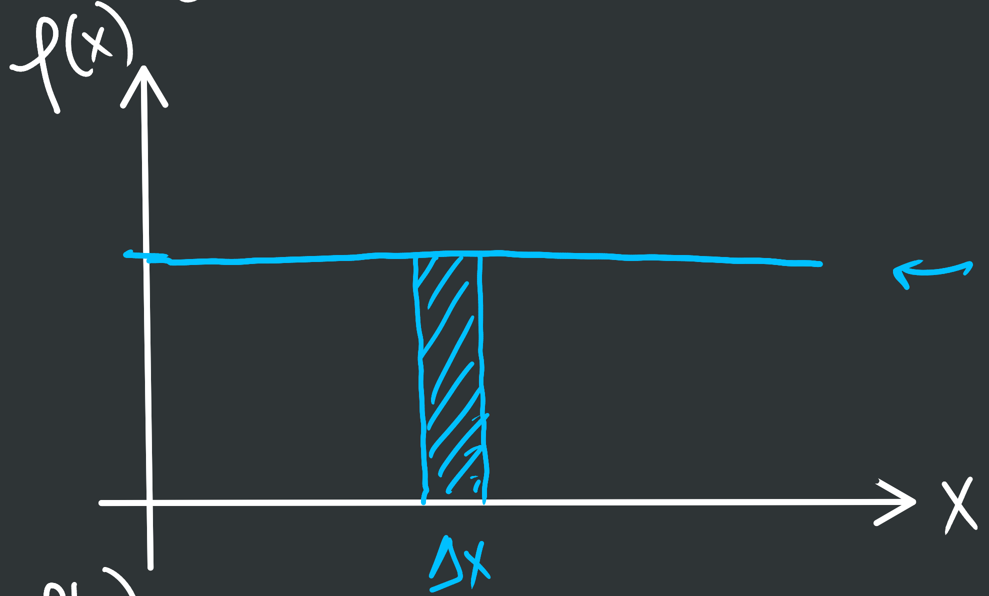
$$\langle x^2 \rangle = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

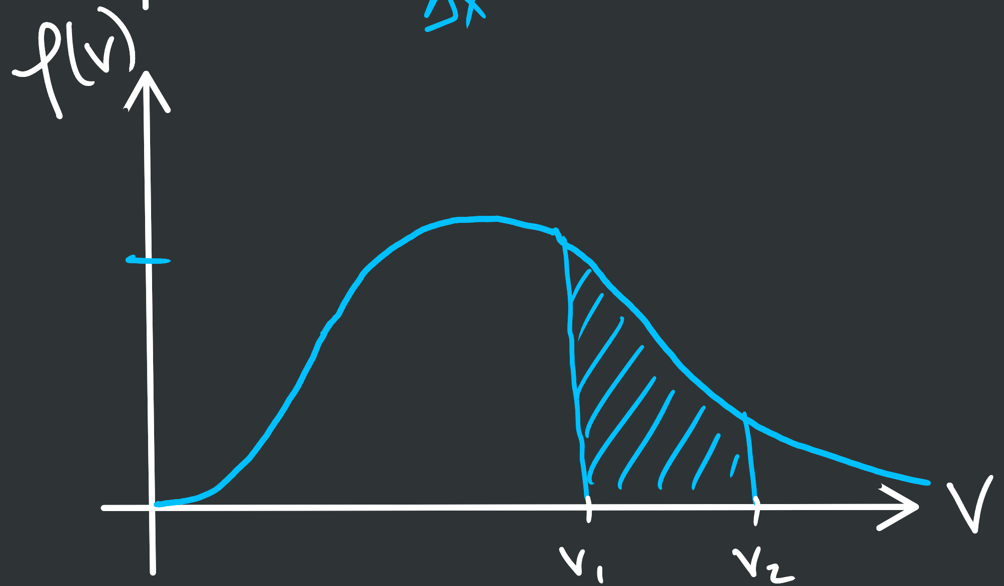
$$\langle f(x) \rangle = \sum_{x=0}^{\infty} f(x) \cdot P(x)$$

Continuous variable

$$\left\{ \begin{array}{l} \text{probability that an individual} \\ \text{value lies between } x \text{ and } x+dx \end{array} \right\} = \rho(x) \cdot dx$$

- probability density function
- probability distribution





$$P(v_1, v_2) = \int_{v_1}^{v_2} \rho(v) \cdot dv$$

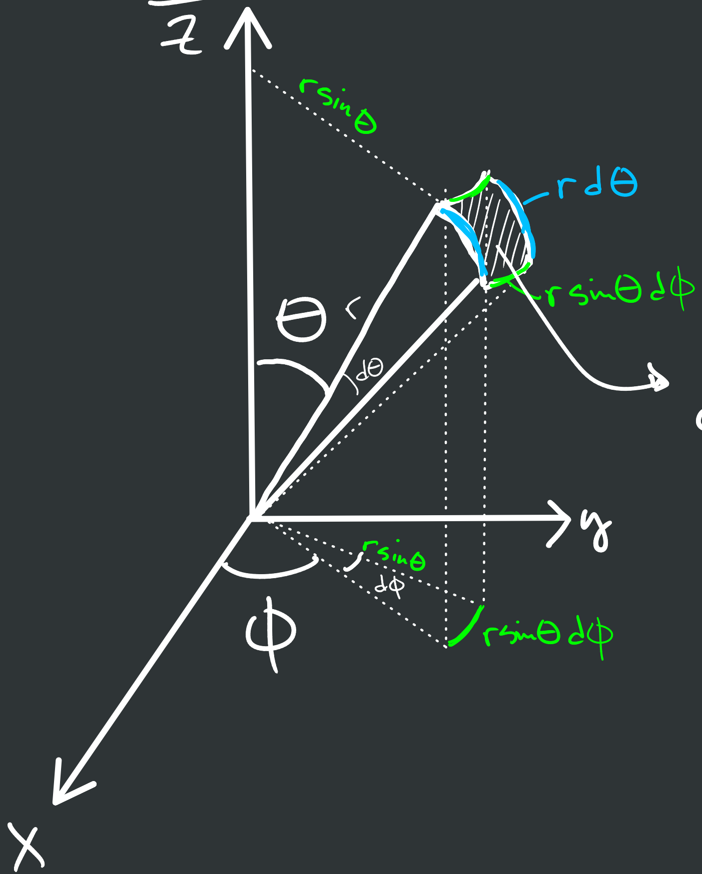
$$\int_{-\infty}^{+\infty} \rho(v) \, dv = 1$$ ← normalization

$$\langle v \rangle = \int_{-\infty}^{\infty} v \cdot \rho(v) \cdot dv$$

$$\langle v^2 \rangle = \int_{-\infty}^{\infty} v^2 \cdot \rho(v) \cdot dv$$

$$\langle f(v) \rangle = \int_{-\infty}^{\infty} f(v) \cdot \rho(v) \cdot dv$$

number of atoms traveling in a particular direction w/ a particular speed

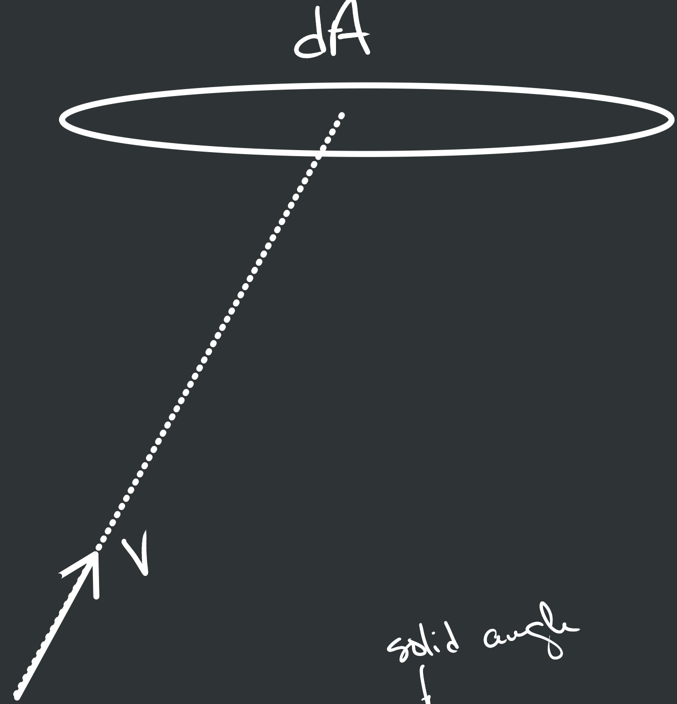


$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

$$d\Omega = \frac{dA}{r^2}$$

$$= \frac{\cancel{r^2} \sin\theta \, d\theta \, d\phi}{\cancel{r^2}}$$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

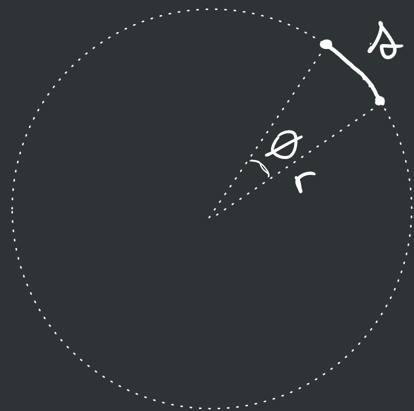


solid angle

$$d\Omega = \frac{dA}{r^2}$$

$A = 4\pi r^2$

total solid angle of a sphere 4π (steradians)



$$\theta = \frac{s}{r}$$

$$d\theta = \frac{ds}{r}$$

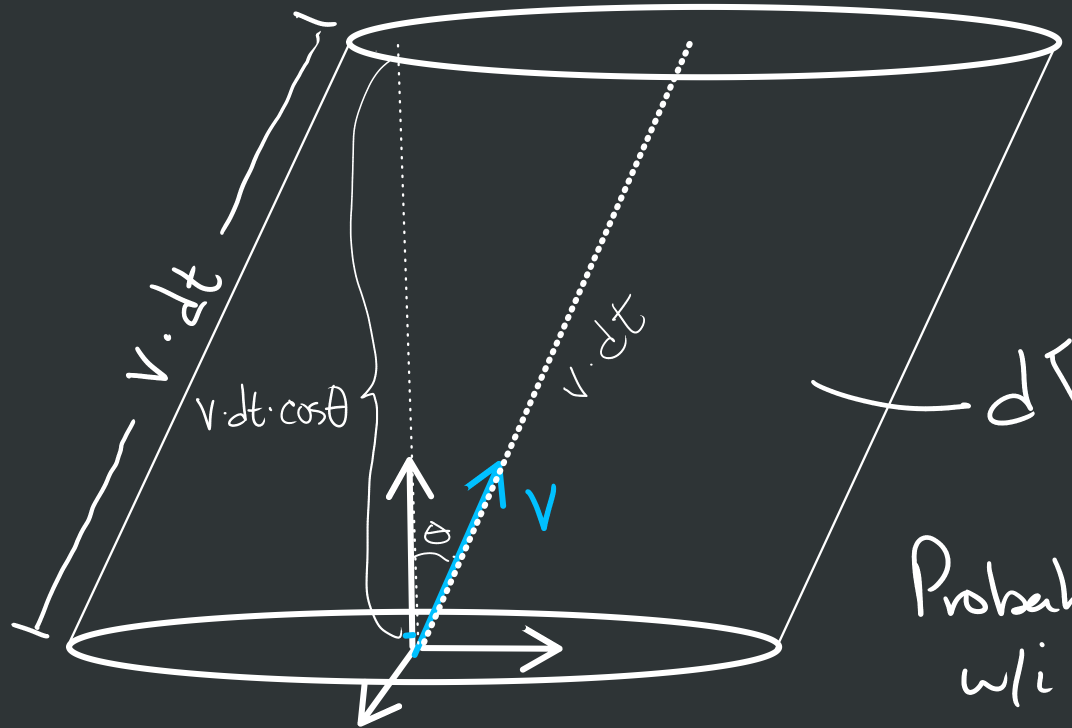
$C = 2\pi r$

Probability of a particle moving in a direction of θ, ϕ $\longrightarrow$ $\dfrac{d\Omega}{4\pi} = \dfrac{\sin\theta \, d\theta \, d\phi}{4\pi}$

Number of particles travelling w/ speed v in the direction of θ, ϕ $\rightarrow \frac{\sin\theta \, d\theta \, d\phi}{4\pi} \cdot f(v) \cdot dv \cdot N$

NEXT PIECE

dA of the wall



$$d\mathbb{V} = v \cdot dt \cdot dA \cdot \cos\theta$$

Probability of being w/i striking distance $= \frac{d\mathbb{V}}{\mathbb{V}}$

$$= \frac{v \, dt \cdot dA \cos\theta}{\mathbb{V}}$$

$$d(\text{pressure}) = \frac{\cancel{2}mv\cos\theta}{\cancel{dA}\,\cancel{dt}} \cdot \frac{\sin\theta\, d\theta\, d\phi}{\cancel{4}_{2}\pi} \cdot f(v)\cdot dv \cdot N \cdot \frac{v\,\cancel{dt}\cdot\cancel{dA}\cos\theta}{\mathbb{V}}$$

$$p = \frac{N\cdot m}{2\pi\mathbb{V}} \underbrace{\int_0^\infty v^2 f(v)\,dv}_{\langle v^2\rangle} \underbrace{\int_0^{\pi/2} \cos^2\theta\cdot\sin\theta\, d\theta}_{1/3} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$\cos^2 = 1-\sin^2$

$$p = \frac{N m}{\cancel{2\pi}\,\mathbb{V}} \cdot \langle v^2\rangle \cdot \frac{1}{3} \cdot \cancel{2\pi}$$

$$p\mathbb{V} = N\frac{m\langle v^2\rangle}{3} \longleftrightarrow p\mathbb{V} = N k_B T$$

$$\frac{m\langle v^2\rangle}{3} = k_B T$$

$$\langle v\rangle^2 \stackrel{?}{=} \langle v^2\rangle$$

$$\langle K\rangle = \frac{1}{2} m \langle v^2\rangle$$

$$m\langle v^2\rangle = 3k_B T \rightarrow \langle v^2\rangle = \frac{3k_B T}{m}$$

$$\frac{1}{2} m\langle v^2\rangle = \frac{3}{2} k_B T$$

$$\langle v\rangle \neq \sqrt{\langle v^2\rangle} = \sqrt{\frac{3k_B T}{m}}$$

$$\langle K\rangle = \frac{3}{2} k_B T$$

$$V_{rms} \equiv \sqrt{\langle v^2\rangle} = \sqrt{\frac{3k_B T}{m}}$$

$$U = N\langle K\rangle$$

- $U = \frac{3}{2} N k_B T$
- $U = \frac{3}{2} n R T$

Ex: "average" speed of air

air $\rightarrow$ 29 g/mol = 0.029 kg/mol

$$m = \frac{0.029\ \text{kg}}{N_A}$$

$$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

$$= \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right)$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_y^2 \rangle + \frac{1}{2} m \langle v_z^2 \rangle$$

$$= \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_x^2 \rangle + \frac{1}{2} m \langle v_x^2 \rangle$$

$$= \frac{3}{2} m \langle v_x^2 \rangle = \frac{3}{2} k_B T \qquad \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} k_B T$$

→ 3 could be thought of as the # of degrees of freedom of a monatomic ideal gas

ways that energy can be possessed

$$v_{rms,air} = \sqrt{\frac{3 \overset{R}{N_A k_B} \cdot 293\text{K}}{0.029\text{kg}}}$$

$$= \sqrt{\frac{3 \cdot 8.31 \cdot 293}{0.029}}$$

$$v_{rms,air} = 501 \text{ m/s}$$

Equipartition Theorem

$\rightarrow U_{thermal} = N \cdot f \cdot \frac{1}{2} k_B T$

degrees of freedom (quadratic)

- $\frac{1}{2}mv_x^2$, $\frac{1}{2}mv_y^2$, $\frac{1}{2}mv_z^2$ (translational)
- $\frac{1}{2}I\omega_x^2$, $\frac{1}{2}I\omega_y^2$
- $\frac{1}{2}kx^2$, ...

monatomic $\rightarrow f = 3$

diatomic $\rightarrow f = 5$ (near room temperature)

<u>solid</u> $\longrightarrow f = 6$

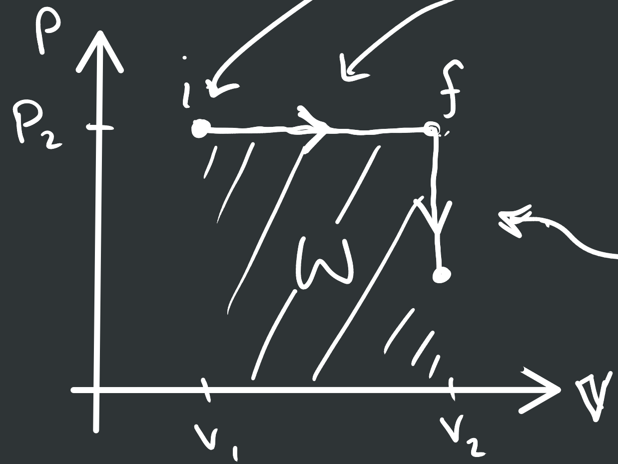
revisit : 1^{st} Law and P-V diagrams | Heat Capacity

$$dU = đQ + đW$$

$$dU = đQ - p\,dV \quad (1^{st} \text{ Law})$$

state $P_2 V_1 = N k_B T_1$

isobaric process (constant pressure)



$$W = -P_2(V_2 - V_1)$$

isochoric process (constant volume)

$$U = \frac{f}{2} N k_B T$$

$$dU = \frac{f}{2} N k_B \, dT$$

$$U_f - U_i = \frac{f}{2} N k_B T_f - \frac{f}{2} N k_B T_i$$

$$\Delta U = \frac{f}{2} N k_B \Delta T$$

$$dU = \frac{f}{2} N k_B \, dT$$

$$PV = N k_B T$$

$$P_f V_f - P_i V_i = N k_B T_f - N k_B T_i$$

isobaric process $\rightarrow P_2 \Delta V = N k_B \Delta T$

$$\Delta U = \frac{f}{2} N k_B \Delta T = \underbrace{\frac{f}{2} P \Delta V}_{\text{only true for isobaric}}$$

by extension:

$$\Delta U = \frac{f}{2} N k_B \Delta T = \underbrace{\frac{f}{2} \Delta P \cdot V}_{\text{only true for isochoric}}$$

1.47

$$Q_{tea} = m_{tea} c \Delta T_{tea}$$

$$Q_{tea} = 200\text{g} \cdot \left(1 \frac{\text{cal}}{\text{g} \cdot \text{K}}\right) \cdot (-35\text{ K})$$

$$Q_{tea} = -7000 \text{ cal}$$

$$Q_{ice} = +7000 \text{ cal} = m_{ice} \cdot c_{ice} \cdot (+15\text{K}) + m_{ice} \cdot L_f + m_{ice} c_{water} (65\text{K})$$