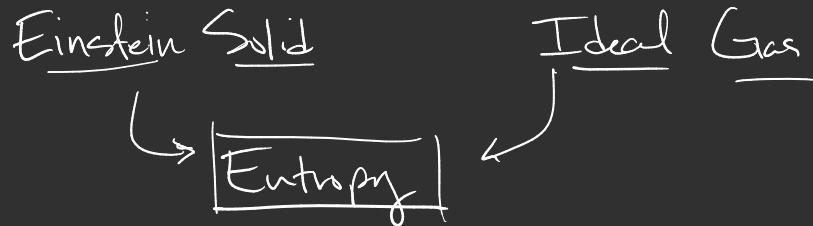


Chapter 2 - 2nd Law of Thermodynamics

↳ heat spontaneously flows from high temp to low temp



→ Anaconda python
install language
Monty Python

Combinatorics

$$\frac{\text{one coin}}{P(\text{heads})} = \frac{1}{2}$$

multiple coins

$$P(n) = \frac{n}{N}$$

$$P(3 \text{ heads}) = \frac{\Omega(3)}{\Omega(\text{all})}$$

$$= \frac{\Omega(3)}{\sum_{n=0}^5 \Omega(n)}$$

$$\begin{array}{c} 5 \text{ coins} \\ \text{microstates} \left\{ \begin{array}{l} \text{HHTTH} - 3H \\ \text{THHHH} - 4H \end{array} \right. \right\} \text{macrostates} \end{array}$$

how many microstates are in a macrostate?
 ↗ multiplicity

$$\Omega(n) = \frac{5!}{n!(5-n)!}$$

number of heads
$\Omega(0) = 1$
$\Omega(1) = 5$
$\Omega(2) = 10$
$\Omega(3) = 10$
$\Omega(4) = 5$
$\Omega(5) = 1$

$$\Omega(1) = \frac{5!}{1!(5-1)!} = \frac{5!}{1!4!}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 5$$

$$\Omega(2) = \frac{5!}{2!3!} = 10$$

$$\Omega(N, n) = \frac{N!}{n!(N-n)!} \leftarrow \text{Notation: } \binom{N}{n}$$

↑
 # of coins

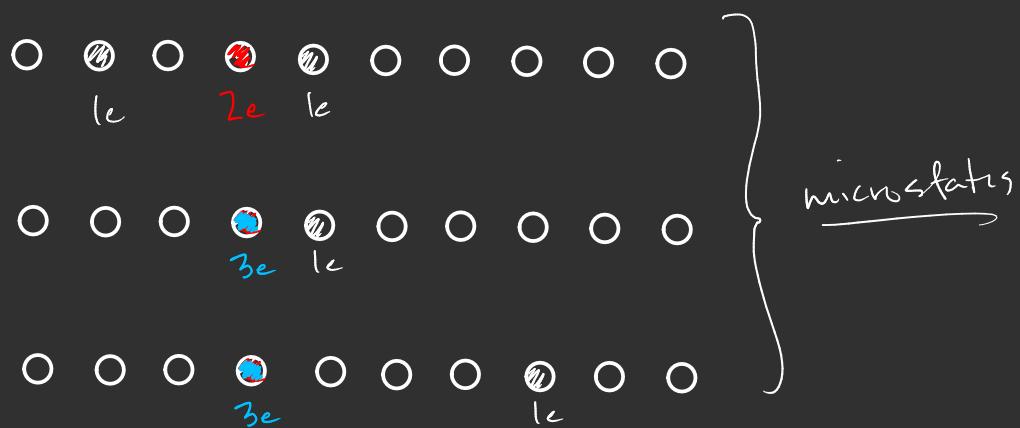
10 atoms each w/ 0 or 1 packets of energy (energy unit)

How many possible ways are there to distribute 4 energy units

$$0 \otimes 0 \otimes 0 \otimes 0 \otimes 0 \otimes 0 \leftarrow \text{microstate} \quad \Omega(10, 4) = \frac{10!}{4! 6!} = 210$$

4 energy packets \leftarrow macrostate

What if an atom can have more than one energy packet at a time?



$$\Omega(N, q_f) = \frac{(q_f + N - 1)!}{q_f! (N - 1)!} \binom{q_f + N - 1}{q_f}$$

↑
 # of atoms
 ↑
 # of energy packets
 $q_f = \text{macrostate}$

4 energy packets \leftarrow macrostate

This model of a collection of atoms w/ equal size energy quanta distributed among them is the Einstein Solid.



- Large Number → addition of small numbers is not important

$$10^{23} + 23 = 10^{23}$$

- Very Large Number

$$10^{10^{23}} \times 10^{23} = 10^{10^{23} + 23} \approx 10^{10^{23}}$$

Stirling's Approximation

$$N! \approx N^N e^{-N} \sqrt{2\pi N} = \underbrace{\frac{N^N}{e^N}}_{\substack{\text{very large numbers} \\ \downarrow \\ \ln N! \approx N \ln N - N}} \sqrt{2\pi N}$$

we leave this off

Two Systems

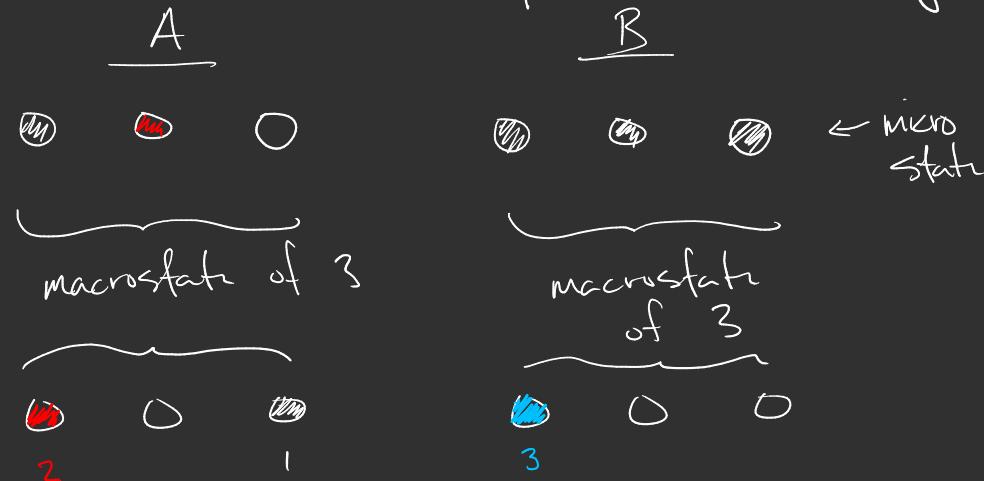


q_A	mult_A	q_B	mult_B	mult_total
0	1	6	28	28
1	3	5	21	63
2	6	4	15	90
3	10	3	10	100
4	15	2	6	90
5	21	1	3	63
6	28	0	1	28

(462) ← total number of microstates

macrostate of the most w/ microstates

Ex. Each system has 3 particles and 6 total quanta of energy



$$\Omega(3,1) = 3$$

$$\Omega(3,5) = \frac{7!}{5!2!} = 21$$

Fundamental Assumption of Stat Mech:

all microstates are possible and equally probable

But that does not mean that every microstate will occur.

Not all macrostates are equally probable.

$$P_A(q_f) = \frac{\Omega_{\text{total}}(q_f)}{\sum_{q_f} \Omega_{\text{total}}(q_f)}$$

We could find the total # of microstates:

$$\mathcal{L}(6, 6) = \frac{(6+6-1)}{6! (6-1)!} = 462$$

$\uparrow \quad \uparrow$
 $N \quad q$

So lets apply Stirling's Approx to Multiplicity $\left[\ln N! = N \ln N - N \right]$

$$\mathcal{L}(N, q) = \frac{(q+N-1)!}{q! (N-1)!} \approx \underbrace{\frac{(q+N)!}{q! N!}}_{\text{Very large number}}$$

$$\ln \mathcal{L} = \underbrace{\ln(q+N)!}_{\rightarrow q \ln q - q} - \underbrace{\ln q!}_{\rightarrow q \ln q - q} - \underbrace{\ln N!}_{\rightarrow N \ln N - N} \rightarrow N \ln N - N$$

$$\ln(q+N)! = (q+N) \ln(q+N) - (q+N)$$

$$\ln \mathcal{L} = (q+N) \ln(q+N) - q - N - q \ln q + q - N \ln N + N$$

$$\rightarrow \boxed{\ln \mathcal{L} = (q+N) \ln(q+N) - q \ln q - N \ln N}$$

high temperature limit $\rightarrow q \gg N$

$$\begin{aligned}
 \ln \mathcal{Q} &= (q_f + N) \ln(q_f + N) - q_f \ln q_f - N \ln N \\
 &= q_f \underbrace{\ln(q_f + N)}_{\ln q_f + \ln(1 + \frac{N}{q_f})} + N \underbrace{\ln(q_f + N)}_{\ln(1 + \frac{N}{q_f})} - q_f \ln q_f - N \ln N \\
 &= \ln \left[q_f \cdot \left(1 + \frac{N}{q_f} \right) \right] \\
 &= \ln q_f + \underbrace{\ln \left(1 + \frac{N}{q_f} \right)}_{\ln(1+x) \approx x \text{ for small } x} \\
 &\quad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \\
 &\quad = \ln q_f + \frac{N}{q_f} \approx \ln(q_f + N)
 \end{aligned}$$

$$\begin{aligned}
 \ln \mathcal{Q} &= \cancel{q_f \ln q_f} + N + \cancel{N \ln q_f} + \cancel{\frac{N^2}{q_f}} - \cancel{q_f \ln q_f} - \cancel{N \ln N} \\
 &= N \ln \left(\frac{q_f}{N} \right) + N + \underbrace{\frac{N^2}{q_f}}_{\text{small}}
 \end{aligned}$$

$$\ln \mathcal{Q}(q_f \gg N) \approx N \ln \left(\frac{q_f}{N} \right) + N$$

$$\begin{aligned}
 \mathcal{Q}(q_f \gg N) &= e^{N \ln \left(\frac{q_f}{N} \right) + N} = e^{N \ln \left(\frac{q_f}{N} \right)} \cdot e^N = \underbrace{\left(e^{\ln \left(\frac{q_f}{N} \right)} \right)^N}_{\left(\frac{q_f}{N} \right)^N} \cdot e^N \\
 \mathcal{Q}(q_f \gg N) &= \left(\frac{q_f}{N} \right)^N e^N = \overline{\left(\frac{e^{q_f}}{N} \right)^N}
 \end{aligned}$$

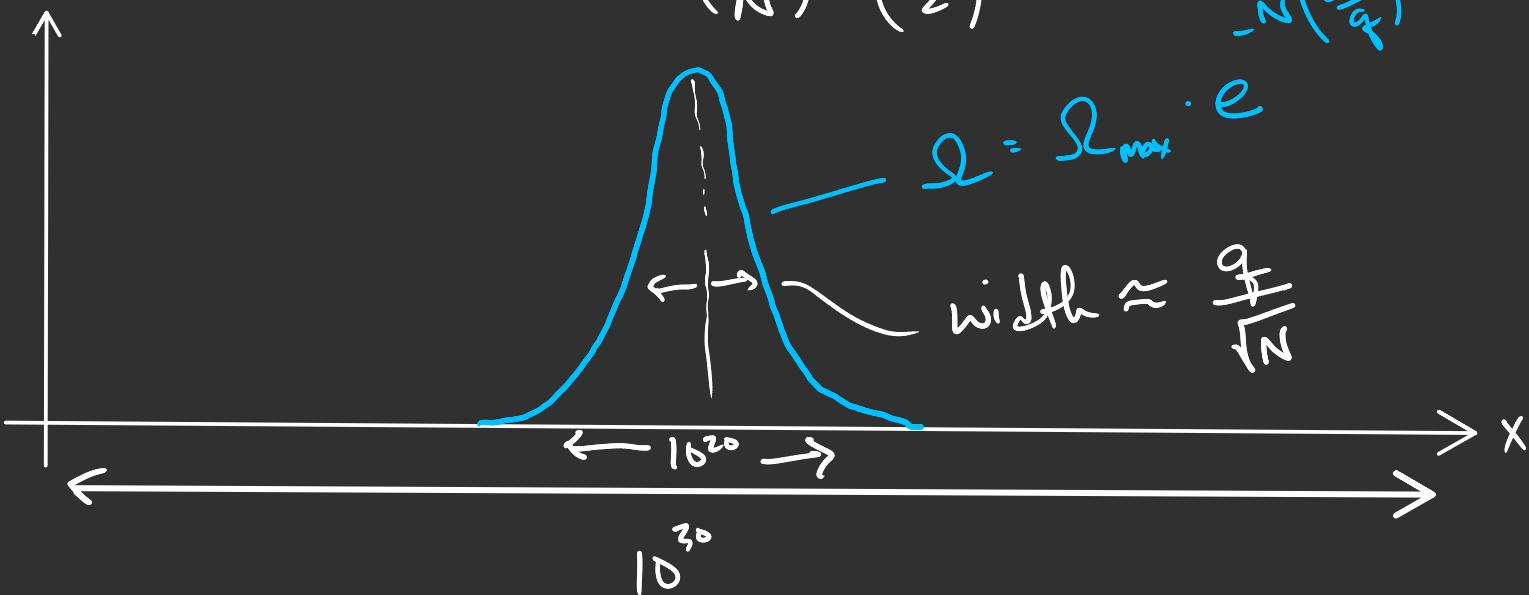
- 2 Einstein solids (high temperature limit) ($g \gg N$)

$$\mathcal{L}_A = \left(\frac{e g_f}{N_A} \right)^{N_A} \quad \mathcal{L}_B = \left(\frac{e g_f}{N_B} \right)^{N_B}$$

$$g_f = g_{fA} + g_{fB}$$

if $N_A = N_B$, multiplicity max will occur $\frac{g_f}{2} = g_{fA} = g_{fB}$

$$\mathcal{L} = \mathcal{L}_A \mathcal{L}_B = \left(\frac{e}{N} \right)^{2N} \cdot \left(\frac{g_f}{2} \right)^{2N}$$



$$g_f = 10^{30}$$

$$N = 10^{20}$$

$$\frac{g_f}{\sqrt{N}} = 10^{20}$$

Entropy + the 2nd Law:

Any large system in equilibrium will be found in the macrostate with the greatest ~~multiplicity~~ entropy

2nd Law of Thermodynamics

~~Multiplicity~~ tends to increase.
Entropy

Multiplicities are very large! Take the natural log of them.

$$\text{entropy} \rightarrow S = k_B \cdot \ln \Omega$$

$$\frac{S}{k_B} = \ln \Omega$$

Ex: Entropy of an Einstein solid

$$\mathcal{I} = \left(\frac{eq}{N}\right)^N \quad q \gg N \quad \text{Einstein solid}$$

homework

$$(q \approx N)$$

$$N = 10^{23} \quad q = 10^{25}$$

$$S = k_B \ln \left(\left(\frac{eq}{N} \right)^N \right) = N \cdot k_B \ln \left(\frac{eq}{N} \right) = N k_B \left(1 + \ln \left(\frac{q}{N} \right) \right)$$

$\overset{10^{23}}{\uparrow} \cdot 1.38 \cdot 10^{-23}$

$$= 1.38 \left(1 + \ln(10^2) \right)$$

$$S = 7.7 \text{ J/K} \quad \underbrace{4.6}_{\text{4.6}}$$

Entropy of a composite system

$$\mathcal{I}_{\text{total}} = \mathcal{I}_A \mathcal{I}_B$$

$$S = k_B \ln \mathcal{I}_A \cdot \mathcal{I}_B = k_B \ln \mathcal{I}_A + k_B \ln \mathcal{I}_B$$

$$S_{\text{f}} = S_A + S_B$$

Chapter 3

Thermal equilibrium

For the Einstein solid

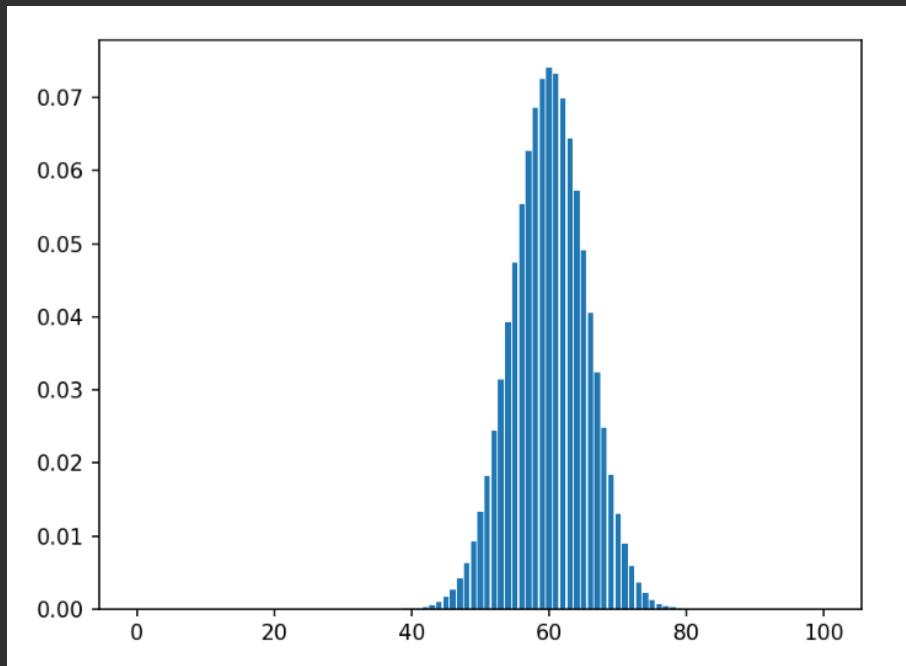
$$\frac{\delta S_{\text{total}}}{\delta q_A} = 0$$

↓ generalize

$$\frac{\delta S_{\text{total}}}{\delta U_A} = 0$$

$$\frac{\delta S_A + S_B}{\delta U_A} = 0$$

$$\frac{\delta S_A}{\delta U_A} + \frac{\delta S_B}{\delta U_A} = 0$$



$$U_B = U - U_A$$

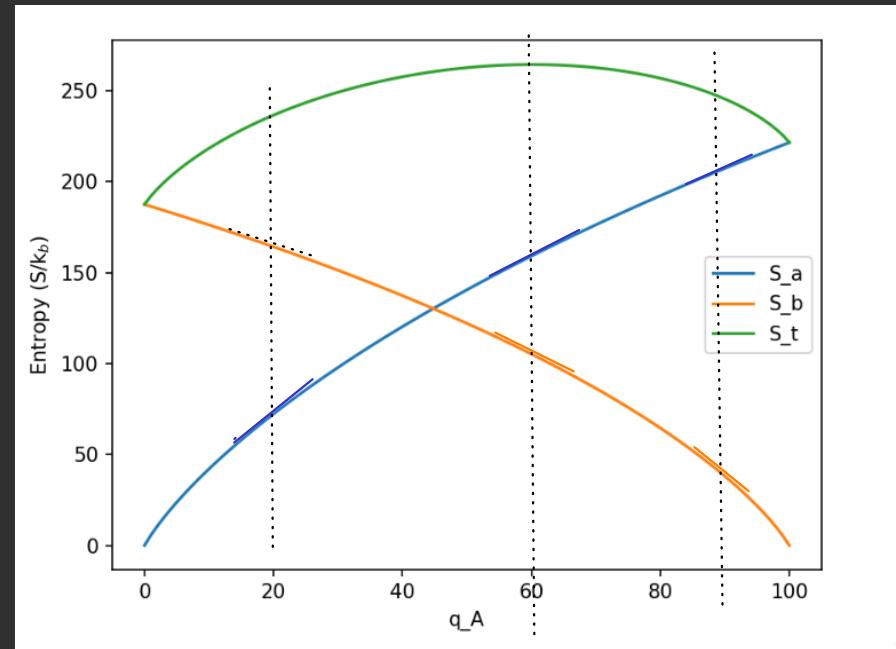
$$\left\{ \begin{array}{l} dU_B = -dU_A \end{array} \right.$$

$$\frac{\partial S_A}{\partial U_A} - \frac{\partial S_B}{\partial U_B} = 0$$

$$\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B}$$

$$\frac{1}{T} = \frac{\partial S}{\partial U}$$

$$T = \left(\frac{\partial S}{\partial U} \right)_{N,V}^{-1}$$



Now apply to our Einstein solid ($q \gg N$)

$$U = q \cdot E \Rightarrow q = \frac{U}{E}$$

of energy packets in one packet

energy packets

$$\Omega = \left(\frac{e^q}{N!} \right)^N$$

$$S = k_B \ln \Omega$$

$$S = k_B \ln \left(\left(\frac{eq}{N} \right)^N \right) = N \cdot k_B \ln \left(\frac{eq}{N} \right) = N k_B \left(1 + \ln \left(\frac{q}{N} \right) \right)$$

$$S = N k_B \left(1 + \ln \left(\frac{U}{N\epsilon} \right) \right)$$

$$S = N k_B + N k_B \ln U - N k_B \ln (N\epsilon)$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{N,V} = \frac{\partial (N k_B \ln U)}{\partial U} = \frac{N k_B}{U}$$

$$U = N k_B T \leftrightarrow U = \frac{f}{2} N k_B T$$

$$f = 2 \quad \} \text{ Einstein solid}$$

Heat Capacity of Einstein solid:

$$C_V = \left(\frac{\partial U}{\partial T} \right)_{N,V} = N k_B \leftarrow \begin{array}{l} \text{experimentally} \\ \text{testable} \\ \text{quantity} \end{array}$$

↳ 1D kinetic energy

↳ 1D spring pot. energy

Review our process so far:

1. Used combinatorics and QM to find Ω in terms of N, V, U etc.
 2. $S = k_B \ln \Omega$ to get entropy
 3. $T = \left(\frac{\partial S}{\partial U} \right)_{N,V,\text{etc}}$
 4. Solve step #3 for U as a function of T
 5. $C_v = \frac{\partial U}{\partial T}$
- Stat mech
give an alternative
to arriving
at #4

But, going backward to measure the entropy is easy

$$dS = \underbrace{\frac{\partial S}{\partial U} \cdot dU}_{\{}}$$

$$dS = \frac{dU}{T}$$

→ Use constant volume process (isochoric)

$$\left. \begin{array}{l} dV = 0 \\ dW = 0 \end{array} \right\} dU = \pm Q + \pm W$$

$$dU = \pm Q$$

$$dS = \frac{\pm Q}{T}$$

important! used to be
the definition of entropy!

$$dS = \frac{C_v \cdot dT}{T}$$

$\Rightarrow T$ does not change much w/
a small addition of heat!

$$\Delta S = \int_{T_i}^{T_f} \frac{C_v}{T} dT$$

\$\Rightarrow C_v\$ can be constant, or a function of temperature

$$S(T_f) - S(T_i) = \int_{T_i}^{T_f} \frac{C_v}{T} dT$$

$$S(T) - S(0) = \int_0^T \frac{C_v}{T} dT$$

\Rightarrow but you need to know this function all the way to zero kelvin!

$C_v \rightarrow 0$ as $T \rightarrow 0$

\Rightarrow could be 0 or residual entropy but some constant

\Rightarrow much experimental data has gone into this for many substances

3rd Law
of Thermodynamics

$$L = \frac{N!}{q!(N-q)!}$$

$$\begin{aligned}
 L(1000, 500) &= \frac{1000!}{(500!)^2} \approx \frac{1000^{1000} e^{-1000} \sqrt{2\pi 1000}}{(500^{500} e^{-500} \sqrt{2\pi 500})^2} = \frac{1000^{1000} \sqrt{2\pi 1000}}{500^{1000} (\sqrt{2\pi 500})^2} \\
 &= \left(\frac{1000}{500}\right)^{1000} \frac{\sqrt{2\pi 1000}}{\sqrt{2^2 \pi^2 500^2}} \\
 &= 2^{1000} \cdot \sqrt{\frac{2 \cdot 1000}{(2 \cdot 500)^2 \pi}} \\
 &= 2^{1000} \cdot \sqrt{\frac{2}{1000 \pi}} \\
 &= \frac{2^{1000}}{\sqrt{500 \pi}}
 \end{aligned}$$

2^{1000}
 $\sum_{q=1}^{1000} \frac{1000!}{q!(1000-q)!}$??

2.5 Ideal Gas Multiplicity

Multiplicity is the number of microstates in a macrostate.

total energy, volume
what defines macrostate
for ideal gas

$\Omega =$ distribution of particles in space . distribution of energy among particles

For a single particle,

$\Omega_1 \propto V$. combination of momentums that has some energy
volume of momentum space

$$I_1 \propto V \cdot V_p$$

Heisenberg Uncertainty Principle

$$\Delta x \cdot \Delta p_x \approx h \leftarrow \text{Plank constant}$$

uncertainty in position uncertainty in momentum
inversely proportional

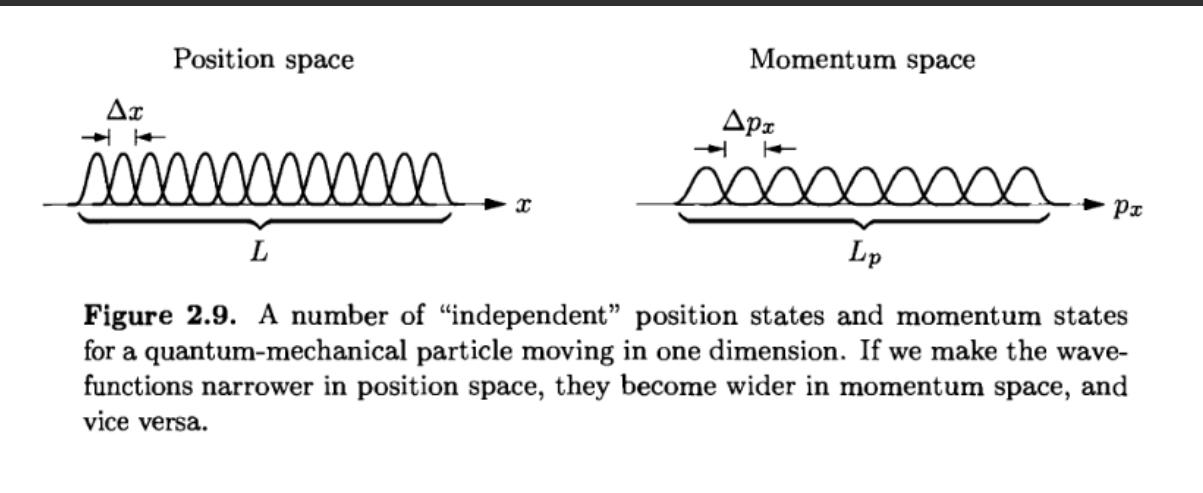


Figure 2.9. A number of “independent” position states and momentum states for a quantum-mechanical particle moving in one dimension. If we make the wavefunctions narrower in position space, they become wider in momentum space, and vice versa.

$$I_1 = \frac{L_x L_y L_z}{\Delta x \Delta y \Delta z} \cdot \frac{L_{p_x} L_{p_y} L_{p_z}}{\Delta p_x \Delta p_y \Delta p_z}$$

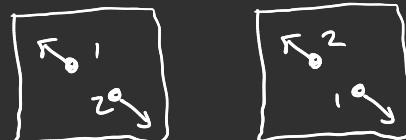
$$I_1 = \frac{V \cdot V_p}{h \cdot h \cdot h} = \frac{V \cdot V_p}{h^3}$$

So what about two particles?

$$I_2 = I_A \cdot I_B$$

cannot distinguish
the particles

$$= \frac{V^2 \cdot V_p^2}{(h^3)^2} \cdot \frac{1}{2}$$



$$I_N = \frac{1}{N!} \cdot \frac{V^N V_p^N}{(h^3)^N}$$

So lets go back to one particle that has all of the energy of the system, U ,

$$U = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) \cdot \frac{m}{m}$$

$$U = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

$$p_x^2 + p_y^2 + p_z^2 = 2mU = (\sqrt{2mU})^2$$

equation of a sphere
in momentum space that
has a radius, $r = \sqrt{2mU}$

So the "volume" of momentum space that represents all of the possible momenta that the particle can have and still have U amount of energy is really the surface area of the sphere in momentum space $\sim 4\pi r^2 \sim 4\pi(\sqrt{2mU})^2$

For two particles constrained to U total energy,

then

$$P_{1x}^2 + P_{1y}^2 + P_{1z}^2 + P_{2x}^2 + P_{2y}^2 + P_{2z}^2 = (\sqrt{2mU})^2$$

6 dimensional sphere? hypersphere!

"Surface area" = $\frac{2\pi^{d/2} r^{d-1}}{(\frac{d}{2}-1)!}$

3N dimensional sphere

"Surface area" = $\frac{2\pi^{\frac{3N}{2}} r^{\frac{3N-1}{2}}}{(\frac{3N}{2}-1)!}$

$$\sqrt{2mU} = (2mU)^{\frac{1}{2}}$$

TEST: d=2

$$2\pi r \stackrel{?}{=} \frac{2\pi^{\frac{2}{2}} r^{2-1}}{(\frac{2}{2}-1)!}$$

$$= 2\pi r \checkmark$$

TEST: d=3

$$4\pi r^2 \stackrel{?}{=} \frac{2\pi^{\frac{3}{2}} r^{\frac{3-1}{2}}}{(\frac{3}{2}-1)!}$$

$$= 2 \left[\underbrace{\frac{\pi^{\frac{3}{2}}}{(\frac{1}{2})!}}_{2\pi} \right] r^2$$

$$\Omega_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \cdot \frac{2\pi^{\frac{3N}{2}} \cdot (2mU)^{\frac{3N-1}{2}}}{(\frac{3N}{2}-1)!}$$

$$\frac{1}{(\frac{1}{2})!} = \frac{2}{\sqrt{\pi}}$$

$$= 4\pi r^2 \checkmark$$

so let's throw away what does not matter for large numbers

$$\Omega_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \cdot \frac{\pi^{3N/2}}{\left(\frac{3N}{2}\right)!} (2mU)^{\frac{3N}{2}}$$

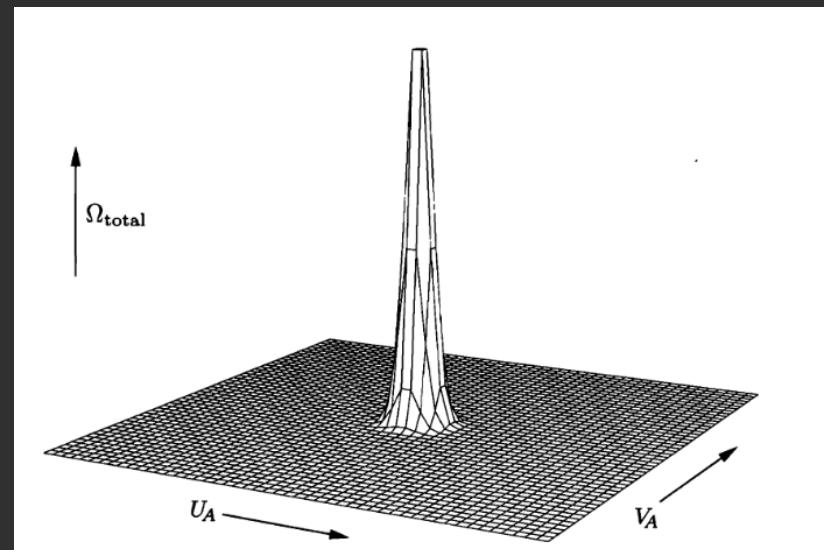
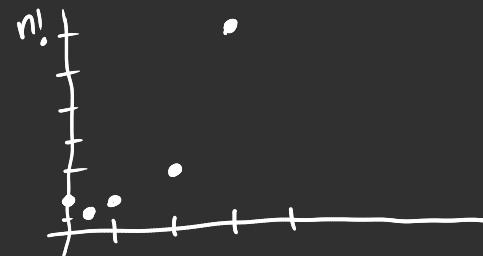
$$\Omega_N = f(N) \cdot V^N \cdot U^{\frac{3N}{2}}$$

Put two ideal gasses in thermal contact
(barrier exchanges energy + volume)

$$\begin{aligned}\Omega_{\text{total}} &= f(N) V_A^N U_A^{\frac{3N}{2}} \cdot f(N) V_B^N U_B^{\frac{3N}{2}} \\ &= (f(N))^2 (V_A V_B)^N (U_A \cdot U_B)^{\frac{3N}{2}}\end{aligned}$$

$$2! = 2 \cdot 1$$

$$\frac{1}{2}! = \frac{\sqrt{\pi}}{2} = 0.8862$$



Entropy of an ideal gas

$$\Omega_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \cdot \frac{(2\pi m u)^{\frac{3N}{2}}}{(\frac{3N}{2})!}$$

$$S = k_B \ln \Omega = k_B \ln \left[\frac{1}{N!} \frac{V^N}{h^{3N}} \cdot \frac{(2\pi m u)^{\frac{3N}{2}}}{(\frac{3N}{2})!} \right]$$

$$S = k_B \left[-\ln(N!) - \ln\left(\frac{3N}{2}!\right) + N \ln V + \frac{3N}{2} \ln(2\pi m u) - \frac{3N}{2} \ln(h^2) \right]$$

\$-\ln(N!)\$ \$\ln\left(\frac{3N}{2}!\right)\$ \$N \ln V\$ \$\frac{3N}{2} \ln(2\pi m u)\$ \$-\frac{3N}{2} \ln(h^2)\$
\$N \ln \frac{1}{N} + N\$ \$-\frac{3N}{2} \ln\left(\frac{3N}{2}\right) + \frac{3N}{2}\$
\$\frac{3N}{2} \ln\left(\frac{2}{3N}\right) + \frac{3N}{2}\$

$$S = k_B N \left(\ln \left[\frac{V}{N} \left(\frac{2\pi m u}{h^2} \cdot \frac{2}{3N} \right)^{\frac{3}{2}} \right] + 1 + \frac{3}{2} \right)$$

$$S = k_B N \left(\ln \left[\frac{V}{N} \left(\frac{4\pi m U}{3N h^2} \right)^{3/2} \right] + \frac{5}{2} \right) \quad \text{Sackur-Tetrod Equation}$$

$\frac{V}{N} \rightarrow$ volume
per particle

$\frac{U}{N} \rightarrow$ energy
per particle

apply to one mole of N_2 gas
at room temp and atmospheric pressure

Find entropy: $n_m = 1$, or $N = N_A = n_m$

$$U = \frac{5}{2} n_m RT = \frac{5}{2} N k_B T$$

$$V = \frac{n_m R T}{P} = \frac{N k_B T}{P}$$

$$\hookrightarrow V = \frac{RT}{P} = \frac{N_A k_B T}{P}$$

$$N_2 = 28 \frac{\text{g}}{\text{mol}} = 0.028 \frac{\text{kg}}{\text{mol}} = \frac{0.028 \text{ kg}}{6.022 \cdot 10^{23} \text{ atoms}} = 4.65 \cdot 10^{-26} \text{ kg}$$

$$\begin{aligned} S &= k_B N \left(\ln \left[\frac{V}{N} \left(\frac{4\pi m U}{3N h^2} \right)^{3/2} \right] + \frac{5}{2} \right) \\ &= k_B N_A \left(\ln \left[\frac{N_A k_B T}{N_A \cdot P} \left(\frac{4\pi (4.65 \cdot 10^{-26}) \cdot \frac{5}{2} N_A k_B \cdot 293 K}{3 N_A h^2} \right)^{3/2} \right] + \frac{5}{2} \right) \\ &= 8.31 \left(\ln \left[\frac{1.38 \cdot 10^{-23} \cdot 293 K}{10^5 P_a} \left(\frac{4\pi (4.65 \cdot 10^{-26}) \cdot \frac{5}{2} \cdot 1.38 \cdot 10^{-23} \cdot 293}{3 (6.626 \cdot 10^{-34})^2} \right)^{3/2} \right] + \frac{5}{2} \right) = 135.5 \text{ J/K} \end{aligned}$$

HW: Ch 2.26, 32

$$\mathcal{L}(\text{all}) = \sum_{n=0}^N \frac{N!}{n!(N-n)!} = \cancel{\text{N}} \cdot 2^N$$

HW: Ch 3.1, 10, 14 ←

2.26] $\mathcal{L} = \frac{A \cdot A_p}{h^2}$

so how
do you adapt for
2D gas

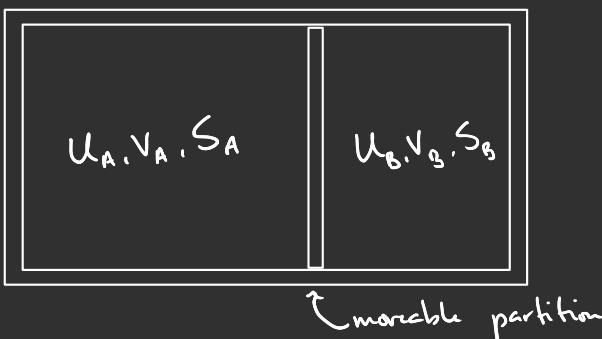
$$\mathcal{L}_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \times (\text{area of the momentum hypersphere})$$

3.4 Mechanical Equilibrium + Pressure

Exchange of energy is governed by temperature difference

Exchange of volume is governed by pressure difference.

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{N,V} \quad \leftarrow \text{is the pressure related to entropy too?}$$



equilibrium { $\frac{\delta S_{\text{total}}}{\delta U_A} = 0$
this is defined
for same temperature
for A & B

$$\underbrace{\frac{\delta S_{\text{total}}}{\delta V_A} = 0}_{\text{new}}$$

$$S_{\text{total}} = S_A + S_B$$

$$\frac{\partial(S_A + S_B)}{\partial V_A} = 0$$

$$\frac{\partial S_A}{\partial V_A} + \frac{\partial S_B}{\partial V_A} = 0$$

$$dV_A = -dV_B$$

$$\frac{\partial S_A}{\partial V_A} = \frac{\partial S_B}{\partial V_B}$$

at equilibrium
constant U, N

At equilibrium, pressure is the same, b/c otherwise the partition would move.

$$P = T \left(\frac{\partial S}{\partial V} \right)_{U,N}$$

$$\underbrace{\text{units?}}_{\text{J/Km}^3} = \frac{N \cdot m}{K \cdot m^{3/2}} = \frac{Pa}{K}$$

is this right?

CHECK

$$\Omega = f(N) \cdot V^N \cdot U^{\frac{3N}{2}} \leftarrow \text{multiplicity of ideal gas}$$

$$S = k_B \left(\ln f(N) + \ln V^N + \ln U^{\frac{3N}{2}} \right)$$

$$S = k_B \left(\ln f(N) + N \ln V + \frac{3N}{2} \ln U \right)$$

$$P = T \left(\frac{\partial S}{\partial V} \right)_{N,T} = k_B T \cdot N \cdot \frac{\partial (\ln V)}{\partial V}$$

$$P = \frac{N k_B T}{V}$$

$$\underline{PV = N k_B T} \quad \checkmark \checkmark$$

$$dS = \left(\frac{\partial S}{\partial U}\right)_V dU + \left(\frac{\partial S}{\partial V}\right)_U dV$$

$$dS = \frac{1}{T} dU + \frac{P}{T} dV$$

$$dU = TdS - PdV$$

thermodynamic identity

} true for any infinitesimal change
as long as T & P are the
same throughout the system

Compare this to 1st Law

$$dU = \delta Q + \delta W$$

$$\delta Q \stackrel{?}{=} TdS$$

$$\delta W \stackrel{?}{=} -PdV$$

- dV happen quasistatically
w/ no friction

- no non-compressive sources of work \rightarrow like electric fields
- no other relevant variables change \rightarrow like N

if these are true,

$$\delta Q = T \delta S$$

Examples:

so an adiabatic and quasistatic process, is called isentropic

$$\delta Q = D = T \delta S$$

$$D = \delta S$$

Example: even when the volume of a system is changing

$$\delta Q = T \delta S$$

$$\delta S = \frac{\delta Q}{T} \quad \left. \right\} \begin{array}{l} 1 \text{ L of water boils} \rightarrow 2260 \text{ kJ} \\ \text{are added} \end{array}$$

$$\Delta S = \frac{Q}{T} = \frac{2260 \text{ kJ}}{373 \text{ K}} = \frac{2,260,000 \text{ J}}{373 \text{ K}}$$

$$\Delta S = 6060 \text{ J/K}$$

Exampk: $(\Delta S)_p = \int_{T_i}^{T_f} \frac{C_p}{T} dT \} = \pm Q$



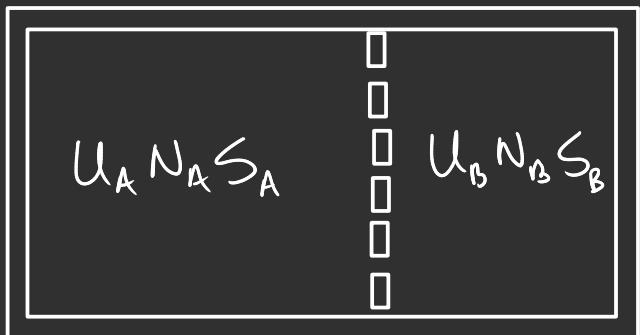
much more practical
than the constant
volume version

3.5] Diffusive Equilibrium + Chemical Potential

thermal equilibrium \rightarrow temperature is the same

mechanical equilibrium \rightarrow pressure is the same

diffusive equilibrium ?



can exchange
energy and particles
(can not exchange volume)
same kind of particle
(indistinguishable)

$$\left(\frac{\partial S_{\text{total}}}{\partial N_A} \right)_{U,V} = 0 \quad \leftarrow \text{at equilibrium}$$

$$\frac{\partial S_A}{\partial N_A} + \frac{\partial S_B}{\partial N_A} = 0$$

$$\frac{\partial S_A}{\partial N_A} = \frac{\partial S_B}{\partial N_B}$$

$$-T \frac{\partial S_A}{\partial N_A} = -T \frac{\partial S_B}{\partial N_B}$$

$$\mu_A = \mu_B$$

chemical potential $\leftarrow \mu = -T \frac{\partial S_A}{\partial N_A}$

modify thermodynamic identity

$$dS(U,V,N) = \underbrace{\left(\frac{\partial S}{\partial U}\right)_{V,N} dU}_{\frac{1}{T}} + \underbrace{\left(\frac{\partial S}{\partial V}\right)_{U,N} dV}_{\frac{P}{T}} + \underbrace{\left(\frac{\partial S}{\partial N}\right)_{U,V} dN}_{-\frac{\mu}{T}}$$

$$TdS = \underbrace{dU + PdV - \mu dN}_{=}$$

$$\boxed{dU = TdS - PdV + \underbrace{\mu dN}_{\text{associated w/ "chemical work"}}$$

if we hold S and V constant

$$dU = \mu dN$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$$

$\rightarrow \mu$ is the energy add/removed to add a particle & keep entropy constant
(Usually you have to remove energy, so μ is negative)

Use ideal gas multiplicity to find the internal energy as a function of temperature.

$$I_N = f(N) \cdot V^N \cdot U^{\frac{3N}{2}}$$

$$S = k_B \left[\ln(f(N)) + N \ln V + \frac{3N}{2} \ln U \right]$$

$$\underbrace{\frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_N = \frac{3}{2} N k_B \frac{1}{U}}$$

Solve for U

$$\underbrace{U = \frac{3}{2} N k_B T}$$

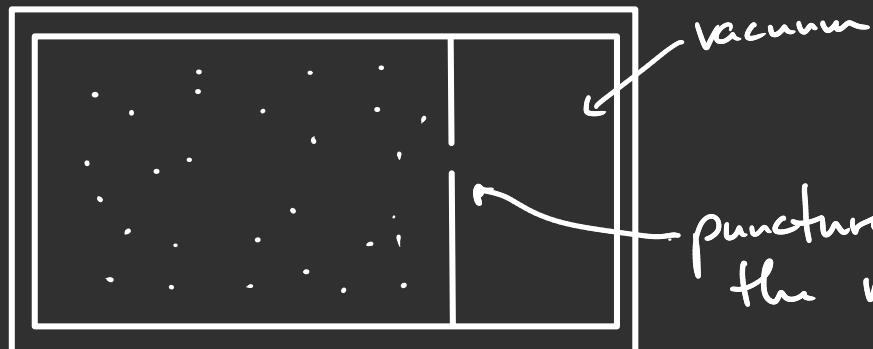
equipartition theorem

$$\begin{aligned}\Delta U &= \underbrace{\frac{3}{2} N k_B \Delta T}_{= \Delta P \cdot V} \\ &= P \cdot \Delta V \\ &\neq \cancel{\Delta P \cdot V}\end{aligned}$$

Now for all the weird cases:

Joule Expansion (Free Expansion)

↳ new process



$\dot{W} = ?$ no push/pull

$\dot{W} = 0$

$\dot{Q} = ?$ isolated/insulated

$\dot{Q} = 0$

put these together

$\Delta U = ? \rightarrow$ no change in temperature

$\Delta U = 0$

$$\downarrow \uparrow \quad PV = nRT$$

But! $\Delta S = S_f - S_i$

$$= k_B \ln\left(\frac{S_f}{S_i}\right)$$

$$\left(S = f(N) \cdot V^N \cdot U^{3N/2} \right)$$

$$\Delta S = k_B N \ln\left(\frac{V_f}{V_i}\right)$$

(remove the partition) (and it
very fast)

So what if the volume
change was small? dV

$$dU = 0$$

$$dU = \cancel{\delta Q} + \cancel{\delta W}$$

$$dU = TdS - pdV$$

$$TdS = pdV > 0$$

Compare to below ↴

If I draw back the partition I do \ominus work, and I add heat to keep energy the same.

$$\Delta S = k_B N \ln\left(\frac{V_f}{V_i}\right)$$

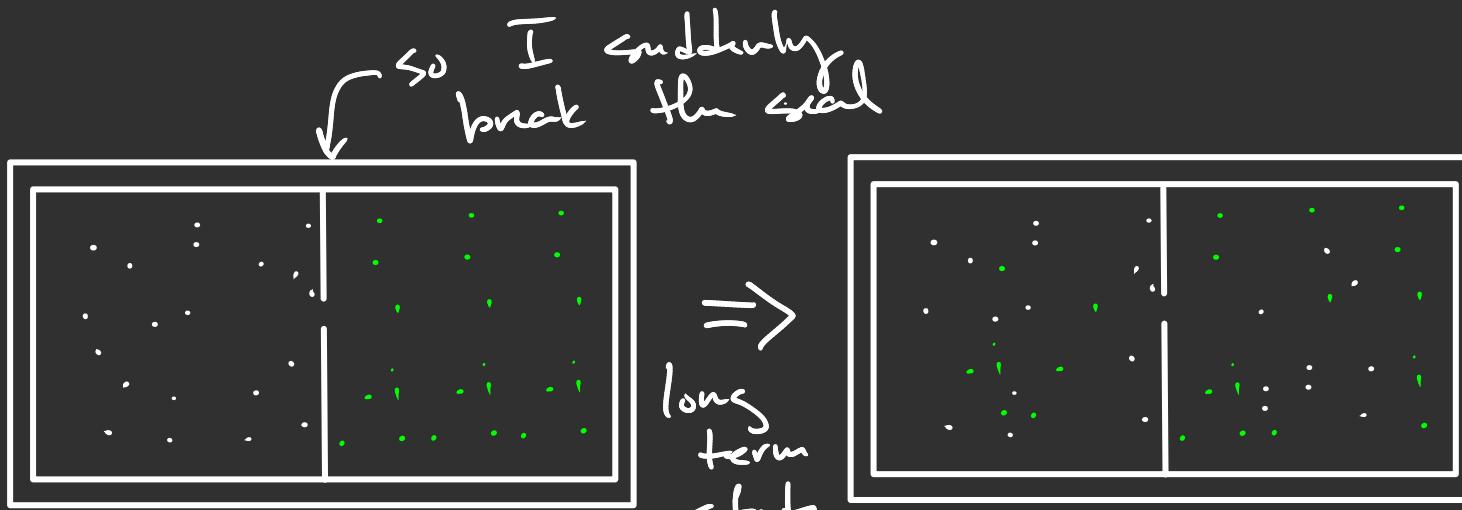
} . Constant $N + U$ process
} . Isothermal expansion

$$\Delta U = 0$$

$$Q = -W$$

↑
putting into the system
always causes an increase
in entropy

Now what if instead of vacuum, there was another gas



is that they are equally mixed
But the volume has changed!

$$\Delta S_A = k_B N \ln \underbrace{\left(\frac{V_f}{V_i} \right)}_{2 \text{ for equally sized containers}}$$

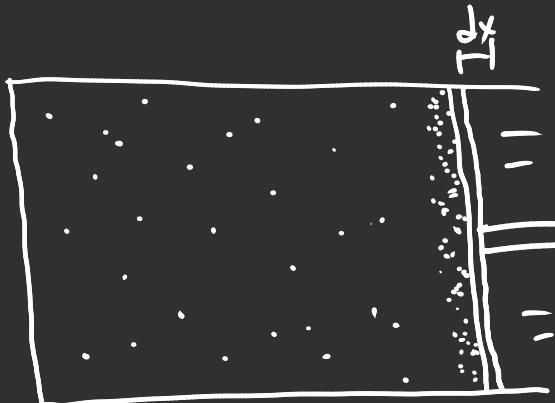
$$\Delta S_A = k_B N \ln 2$$

$$\Delta S_B = k_B N \ln 2$$

$$\Delta S_{\text{total}} = 2 k_B N \ln 2$$

\hookrightarrow entropy of mixing

What about another non-quasistatic process?



$$\delta W > p\delta V$$

I have to do
more work to move
aside all the molecules
quickly

greater than

$$\delta W + \delta Q = \delta U = T\delta S - p\delta V$$

must be less than

$$\delta Q \leq T\delta S \text{ when } \delta W \geq p\delta V$$

$$\delta S \geq \frac{\delta Q}{T}$$

extra entropy that was created
since more work was done to
change the volume

will not change much
for a small δV change.

$$3.32 \quad \text{a) } W = F \cdot \Delta x \\ = 2000 \text{ N} \cdot 0.001 \text{ m} = 2 \text{ J} \quad \left. \right\} \text{ work done by } \underline{\text{me}}.$$

$$\text{b) } Q = 0 \quad (\text{constantly})$$

$$\text{c) } \Delta U = \xi Q + \xi W \quad \left. \right\} \text{ from the perspective of the gas} \\ \Delta U = 2 \text{ J}$$

$$\text{d) } \Delta S = ?$$

$$\Delta U = T \Delta S - P \Delta V \quad \Delta x \cdot A \\ \left. \begin{array}{l} \downarrow 300 \text{ K} \\ \downarrow 10^5 \text{ Pa} \end{array} \right\} \rightarrow -0.001 \text{ m} \times 0.01 \text{ m}^2 \\ 10^{-3} \quad 10^{-2} \\ = -0.00001 \text{ m}^3 = -10^{-5} \text{ m}^3$$

$$\Delta S = \frac{2 \text{ J} - 1 \text{ J}}{300 \text{ K}} = 0.01 \text{ J/K}$$