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AMATH 584 Yale Faces Notebook

In [1]:
```
import numpy as np
from matplotlib import pyplot
import re
import os
```

The function that I'm using is one I took from stack overflow

I wasn't aware of how to read in a pgm image, so I took this function from here: https://stackoverflow.com/questions/7368739/numpy-and-16-bit-pgm/7369986 (https://stackoverflow.com/questions/7368739/numpy-and-16-bit-pgm/7369986)

In [2]:
```
def read_pgm(filename, byteorder='>'):
    """Return image data from a raw PGM file as numpy array.

    Format specification: http://netpbm.sourceforge.net/doc/pgm.html

    """
    with open(filename, 'rb') as f:
        buffer = f.read()
    try:
        header, width, height, maxval = re.search(
            b"(^P5\s(?:\s*#.*[\r\n])*"
            b"(\d+)\s(?:\s*#.*[\r\n])*"
            b"(\d+)\s(?:\s*#.*[\r\n])*"
            b"(\d+)\s(?:\s*#.*[\r\n]\s)*)", buffer).groups()
    except AttributeError:
        raise ValueError("Not a raw PGM file: '%s'" % filename)
    return np.frombuffer(buffer,
                            dtype='u1' if int(maxval) < 256 else byteord
                            count=int(width)*int(height),
                            offset=len(header)
                            ).reshape((int(height), int(width)))
```

In [3]:
```
image = read_pgm("CroppedYale/yaleB01/yaleB01_P00A+000E+00.pgm", byteord
```

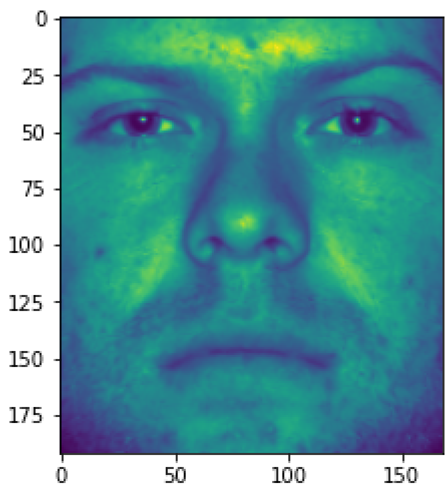
this gives us the image pixel data in 2-dimensions. We want to put it into one dimension, and so this section is where I looped through to get my image in a list

In [4]:
```
print(image)
```

```
[[80 80 84 ... 63 67 71]
 [81 79 78 ... 60 61 64]
 [80 79 82 ... 61 59 62]
 ...
 [12 12 19 ... 21 18 14]
 [12 12 15 ... 14 14 14]
 [12 12 12 ... 12 12 11]]
```

In [5]:
```
pyplot.imshow(image)
```

Out[5]: <matplotlib.image.AxesImage at 0x7f7a77b64b90>



In [6]:
```
templistofpixels = []
for i in image:
    for j in i:
        templistofpixels.append(j)
```

putting list into an array, making it a column vector, and learning how to append column vectors in numpy

In [7]:
```
bigmatrix = np.matrix([templistofpixels]).T
```

In [8]:
```
image=read_pgm("CroppedYale/yaleB01/yaleB01_P00A+000E+20.pgm", byteorder
```

In [9]:
```
templistofpixels = []
for i in image:
    for j in i:
        templistofpixels.append(j)
temparrayofpixels = np.matrix([templistofpixels]).T
bigmatrix = np.append(bigmatrix,temparrayofpixels,1)
```

In [10]:
```
temparrayofpixels
```

Out[10]:
```
matrix([[94],
        [94],
        [96],
        ...,
        [ 6],
        [ 7],
        [ 8]], dtype=uint8)
```

adding column vectors to my matrix iteratively until all of my column vectors for all of my cropped faces are in one big matrix

In [11]:
```
for i in os.listdir('CroppedYale/'):
    for j in os.listdir('CroppedYale/' + i):
        try:
            tempimage = read_pgm('CroppedYale/' + i + '/' + j)
            templistofpixels=[]
            for k in tempimage:
                for l in k:
                    templistofpixels.append(l)
            temparrayofpixels = np.matrix([templistofpixels]).T
            bigmatrix = np.append(bigmatrix,temparrayofpixels,1)
        except:
            continue
```

In [12]:
```
print(bigmatrix.shape)
bigmatrix
```

```
(32256, 2429)
```

Out[12]:
```
matrix([[ 80,  94,   3, ...,  12, 165, 103],
        [ 80,  94,   3, ...,  16, 165, 107],
        [ 84,  96,   3, ...,  19, 173, 111],
        ...,
        [ 12,   6, 100, ...,  10,  60,  99],
        [ 12,   7,  91, ...,  11,  62,  94],
        [ 11,   8,  83, ...,  12,  64, 109]], dtype=uint8)
```

finding the average face

In [13]:
```
image.shape
```
Out[13]:
```
(192, 168)
```

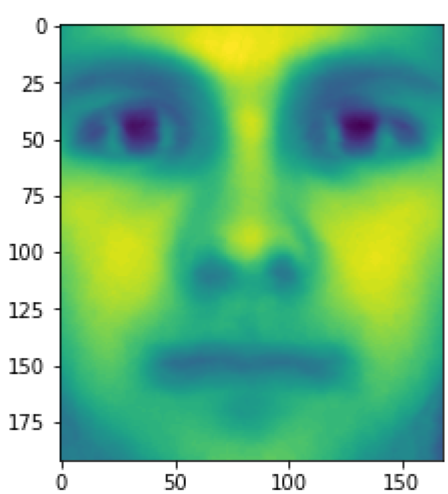
In [14]:
```
print(bigmatrix.shape)
summed = np.sum(bigmatrix, axis=1)
print(summed.shape)
sumreshape = np.reshape(summed,(-1,168))
print(sumreshape.shape)
```
```
(32256, 2429)
(32256, 1)
(192, 168)
```

In [15]:
```
avfacevec = (summed / 2429).astype(int)
```

In [16]:
```
avfacevec
```
Out[16]:
```
matrix([[60],
        [62],
        [62],
        ...,
        [43],
        [41],
        [41]])
```

In [17]:
```
avface = (sumreshape / 2429).astype(int)
```

In [18]:
```
pyplot.imshow(avface)
```
Out[18]:
```
<matplotlib.image.AxesImage at 0x7f7aa07ab590>
```



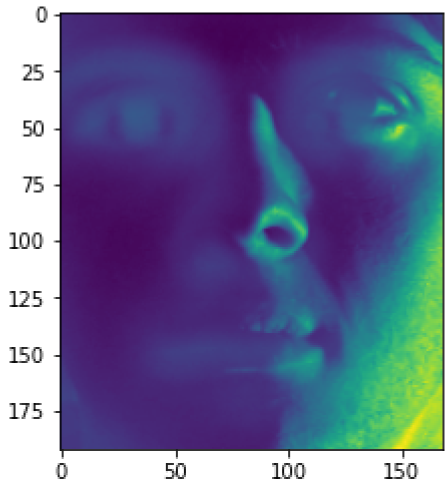
In [19]:
```
print(bigmatrix)
print(avfacevec)
print(bigmatrix-avfacevec)
normedfacemat = bigmatrix -avfacevec
```

```
[[ 80  94   3 ...  12 165 103]
 [ 80  94   3 ...  16 165 107]
 [ 84  96   3 ...  19 173 111]
 ...
 [ 12   6 100 ...  10  60  99]
 [ 12   7  91 ...  11  62  94]
 [ 11   8  83 ...  12  64 109]]
[[60]
 [62]
 [62]
 ...
 [43]
 [41]
 [41]]
[[ 20  34 -57 ... -48 105  43]
 [ 18  32 -59 ... -46 103  45]
 [ 22  34 -59 ... -43 111  49]
 ...
 [-31 -37  57 ... -33  17  56]
 [-29 -34  50 ... -30  21  53]
 [-30 -33  42 ... -29  23  68]]
```

visualizing mean-subtracted face of a picture vs the original picture. His eyes and mouth become less prominent, while the shape of his nose retains prominence. The dark shadows along the sides of his face lose some prominence, but not all.

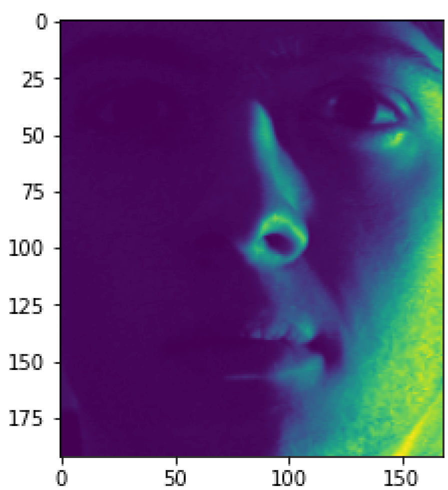
In [20]:
```
pyplot.imshow(np.reshape(normedfacemat.T[75],(-1,168)))
```

Out[20]: <matplotlib.image.AxesImage at 0x7f7a74c20d10>



In [21]:
```
pyplot.imshow(np.reshape(bigmatrix.T[75],(-1,168)))
```

Out[21]: <matplotlib.image.AxesImage at 0x7f7a74c15c90>



it's beautiful, now time to do analysis. First I will do the svd

In [22]:
```
u, s, vt = np.linalg.svd(normedfacemat)
```

In [23]:
```
print(u.shape)
print(s.shape)
print(vt.shape)
```

```
(32256, 32256)
(2429,)
(2429, 2429)
```

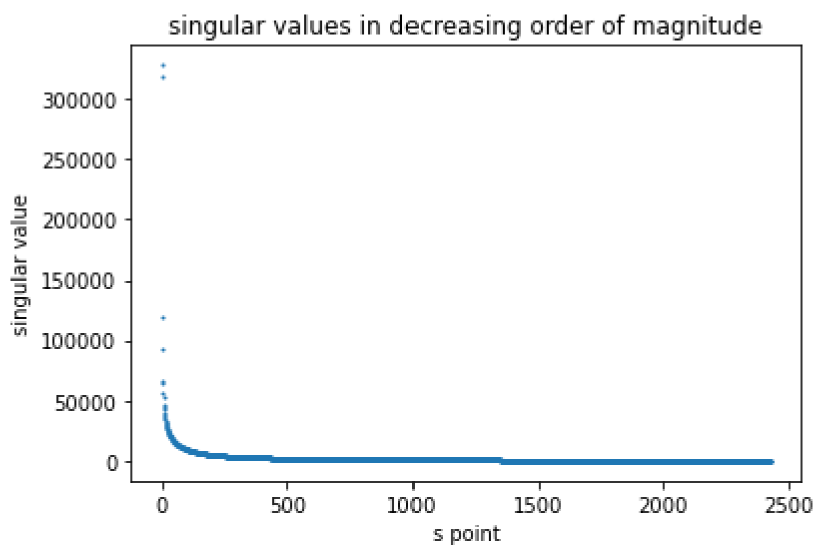
In [24]:
```
s
```

Out[24]:
```
array([3.27966346e+05, 3.18072926e+05, 1.19774020e+05, ...,
       8.08433527e+00, 1.45646903e-10, 1.93629060e-12])
```

Plotting s will tell me how relatively important or useful each eigenface is. The higher the value, the more useful the eigenfaces are.

In [25]:
```
pyplot.title('singular values in decreasing order of magnitude')
pyplot.xlabel('s point')
pyplot.ylabel('singular value')
pyplot.plot(s, 'o', markersize = 1)
```

Out[25]:
```
[<matplotlib.lines.Line2D at 0x7f7a743221d0>]
```



From this, it can be seen that the first two eigenfaces are the most important, followed by the next handful. The most important eigenvectors for face image

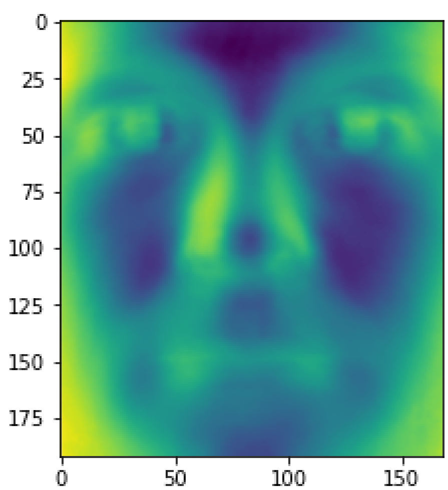
reconstruction/compression are thus stored in first 500 eigenfaces.

the columns of u represent the 'eigenfaces' of our set. they are arranged in order of importance. The values in s represent the weight of each eigenface -- a smaller value of s means that that column of u is less important to forming the faces. Columns of the transposed V matrix would give weights of eigenfaces with which to form the columns of the original matrix -- like a recipe for how to make any one given face from the eigenfaces.

now i will look at eigenfaces

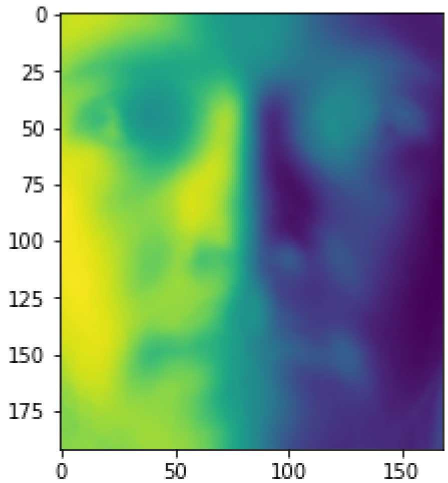
In [26]:
```
pyplot.imshow(np.reshape(u.T[0],(-1,168)))
```

Out[26]:
```
<matplotlib.image.AxesImage at 0x7f7a7430d650>
```



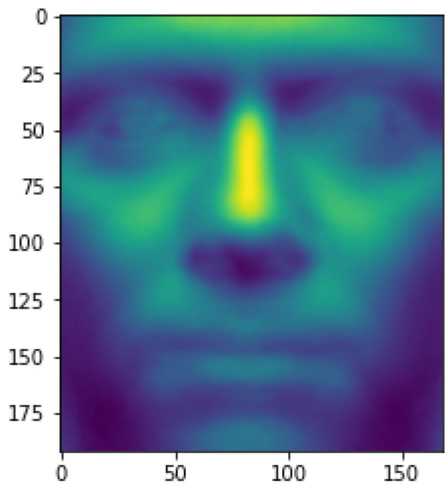
In [27]:
```
pyplot.imshow(np.reshape(u.T[1],(-1,168)))
```

Out[27]: <matplotlib.image.AxesImage at 0x7f7a7427e590>



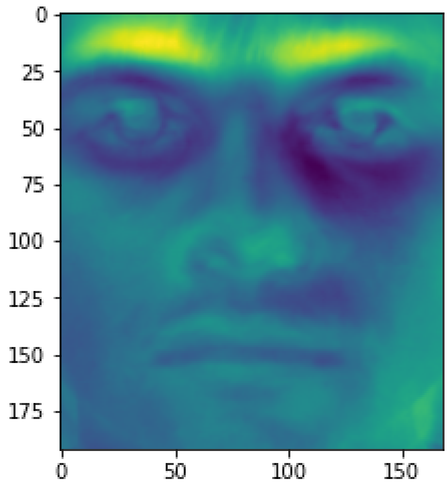
In [28]:
```
pyplot.imshow(np.reshape(u.T[2],(-1,168)))
```

Out[28]: <matplotlib.image.AxesImage at 0x7f7a741f3510>



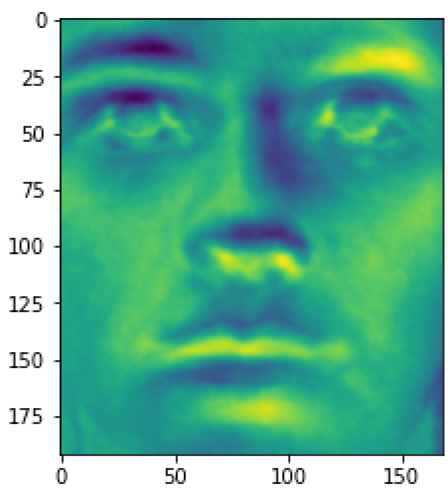
In [29]:
```
pyplot.imshow(np.reshape(u.T[5],(-1,168)))
```

Out[29]:
```
<matplotlib.image.AxesImage at 0x7f7a74166490>
```



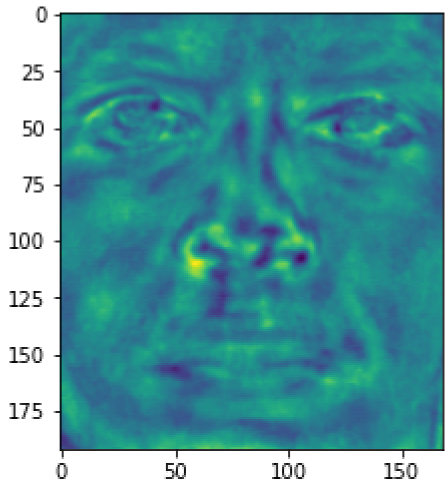
In [30]:
```
pyplot.imshow(np.reshape(u.T[15],(-1,168)))
```

Out[30]:
```
<matplotlib.image.AxesImage at 0x7f7a7415b410>
```



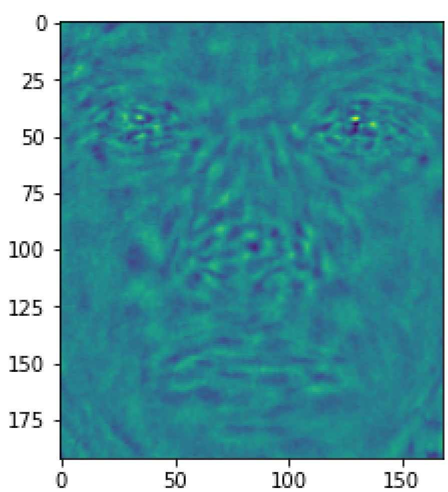
In [31]:
```
pyplot.imshow(np.reshape(u.T[150],(-1,168)))
```

Out[31]: <matplotlib.image.AxesImage at 0x7f7a74b62210>



In [32]:
```
pyplot.imshow(np.reshape(u.T[500],(-1,168)))
```

Out[32]: <matplotlib.image.AxesImage at 0x7f7a74023890>



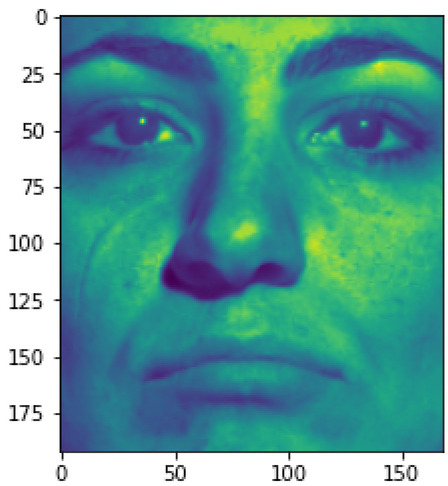
very scary. now let's try to find out if we can recreate faces well using only the first r values

of the u matrix

In [33]:
```
testface = normedfacemat.T[300]
```

In [34]:
```
pyplot.imshow(np.reshape(testface,(-1,168)))
```

Out[34]:
```
<matplotlib.image.AxesImage at 0x7f7a45f77990>
```

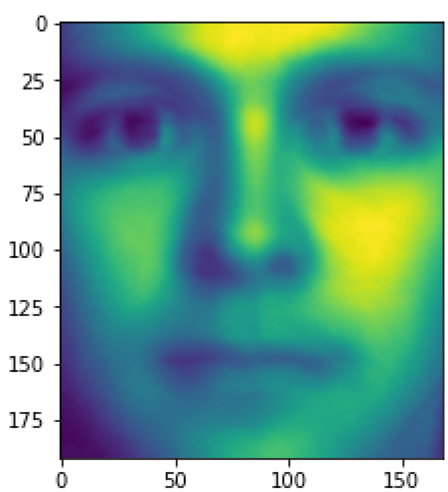


trying to recreate with 5 eigenfaces

In [35]:
```
recreatedface = u[:, :5] * u[:,:5].T * testface.T
```

In [36]:
```
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

Out[36]:
```
<matplotlib.image.AxesImage at 0x7f7a45eeac10>
```

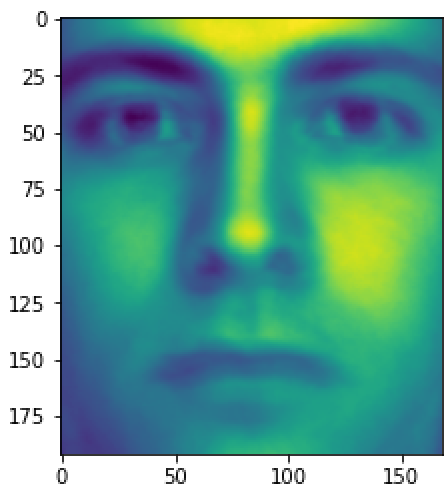


trying to recreate with 10 eigenfaces

In [37]:
```
recreatedface = u[:, :10] * u[:,:10].T * testface.T
```

In [38]:
```
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

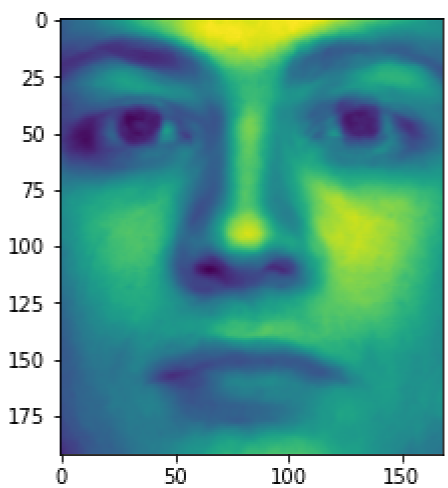
Out[38]: <matplotlib.image.AxesImage at 0x7f7a45e60d90>



trying to recreate with 20 eigenfaces

In [39]:
```
recreatedface = u[:, :20] * u[:,:20].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

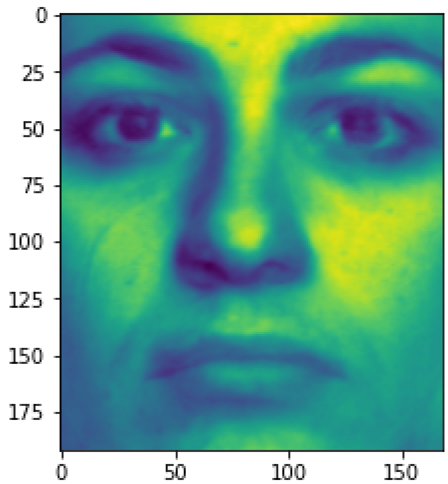
Out[39]: <matplotlib.image.AxesImage at 0x7f7a45ddc210>



trying to recreate with 40 eigenfaces

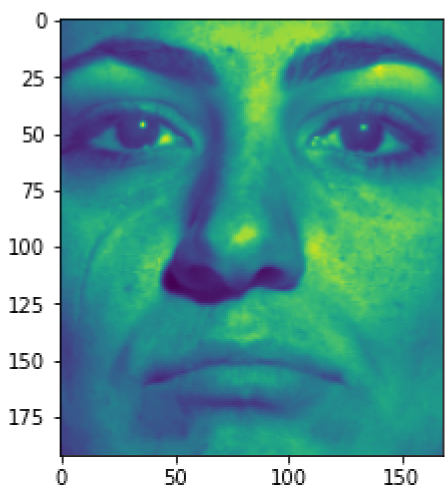
In [40]:
```
recreatedface = u[:, :40] * u[:,:40].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

Out[40]: <matplotlib.image.AxesImage at 0x7f7a45d50150>



In [41]:
```
# original face for reference
pyplot.imshow(np.reshape(testface, (-1,168)))
```

Out[41]: <matplotlib.image.AxesImage at 0x7f7a45d3cb10>

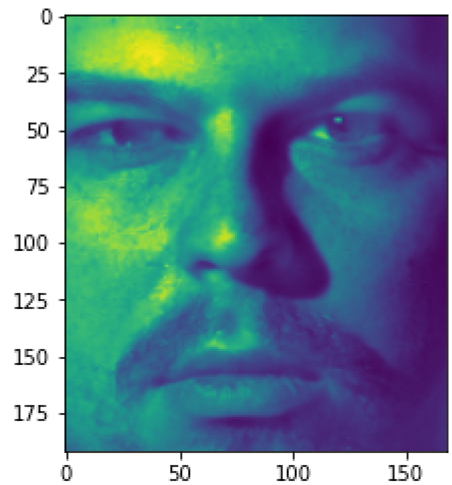


i think that looks pretty good with 40 eigenfaces actually. Let me try it again with some other testfaces.

In [42]:
```
testface = normedfacemat.T[800]
```

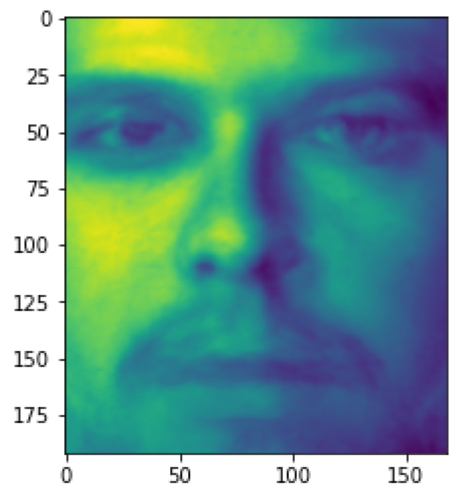
In [43]:
```
pyplot.imshow(np.reshape(testface, (-1,168)))
```

Out[43]: <matplotlib.image.AxesImage at 0x7f7a45cafcd0>



In [44]:
```
recreatedface = u[:, :40] * u[:,:40].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

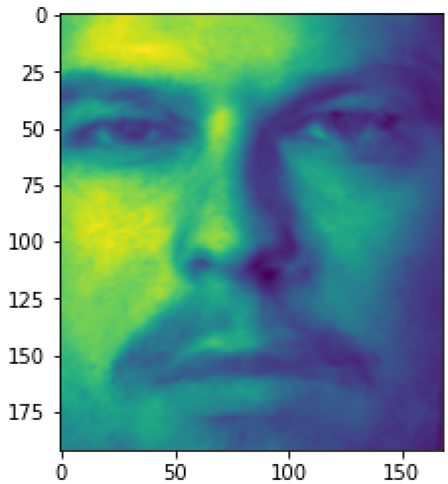
Out[44]: <matplotlib.image.AxesImage at 0x7f7a45c2c1d0>



he looks less good than the previous test face. It's possible that his face is more anomalous to the average face, and so it is less well captured by the first 40 eigenfaces. I can increase the number of eigenfaces and then try again

In [45]:
```
recreatedface = u[:, :80] * u[:,:80].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

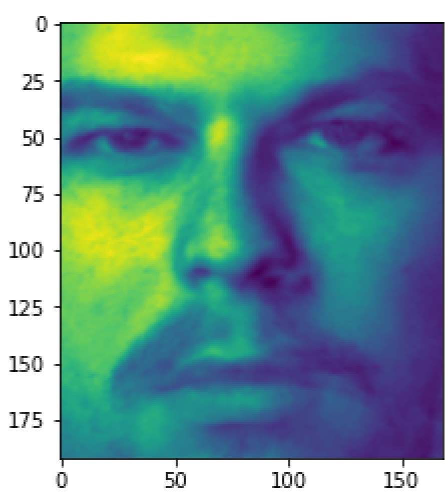
Out[45]:
```
<matplotlib.image.AxesImage at 0x7f7a45ba0110>
```



I'll try one more with 120 eigenfaces

In [46]:
```
recreatedface = u[:, :120] * u[:,:120].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

Out[46]:
```
<matplotlib.image.AxesImage at 0x7f7a45ea8c50>
```

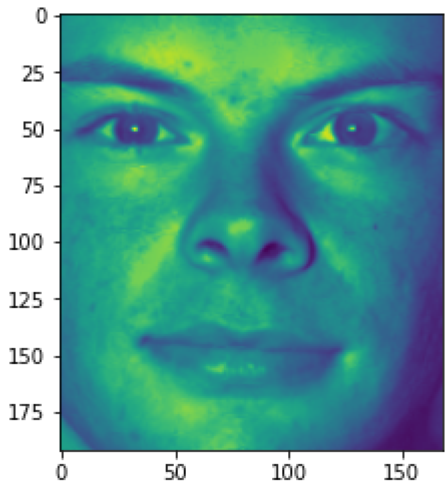


I'll consider that a good approximation! Trying again with another test face:

In [47]:
```
testface = normedfacemat.T[350]
```

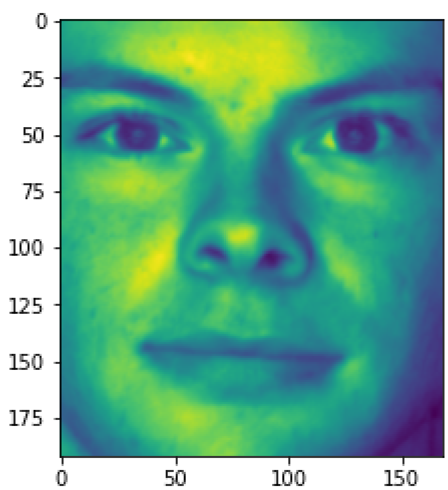
In [48]:
```
pyplot.imshow(np.reshape(testface, (-1,168)))
```

Out[48]: <matplotlib.image.AxesImage at 0x7f7a45aec590>



In [49]:
```
recreatedface = u[:, :120] * u[:,:120].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

Out[49]: <matplotlib.image.AxesImage at 0x7f7a45a56a50>

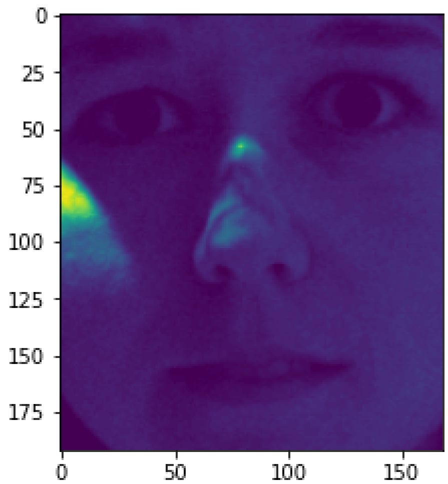


looks good! One more face ... (this one is very dark, so I visualize his original face without mean values subtracted)

In [50]:
```
testface = bigmatrix.T[1738]
```

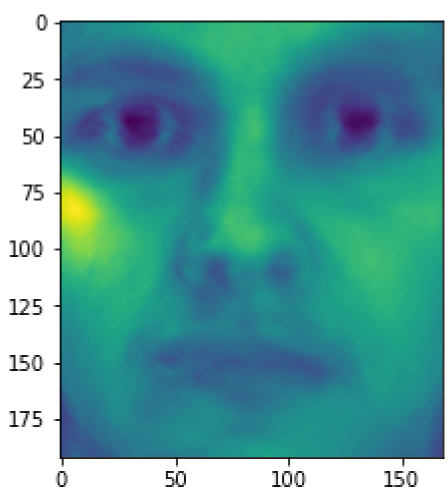
In [51]:
```
pyplot.imshow(np.reshape((testface), (-1,168)))
```

Out[51]: <matplotlib.image.AxesImage at 0x7f7a459ca6d0>



In [52]:
```
recreatedface = u[:, :120] * u[:,:120].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

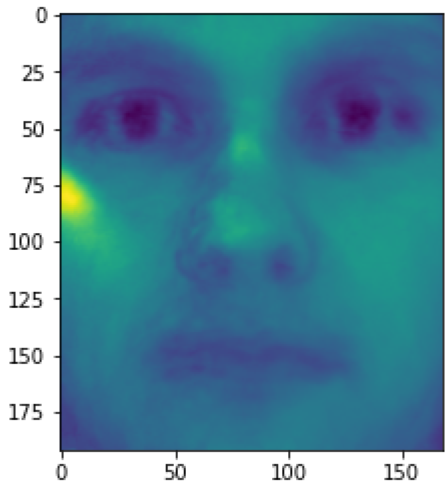
Out[52]: <matplotlib.image.AxesImage at 0x7f7a459bcb50>



the strange lighting and angle in this one might make it harder to recreate. I'll try with more eigenfaces

In [53]:
```
recreatedface = u[:, :400] * u[:,:400].T * testface.T
pyplot.imshow(np.reshape(recreatedface + avfacevec,(-1,168)))
```

Out[53]: <matplotlib.image.AxesImage at 0x7f7a45930b10>



That at least looks more cleanly like his face. So, the rank r = 400 seems to be a good amount of eigenfaces to represent the majority of faces reasonably well, though some faces may be represented well with significantly less eigenfaces.

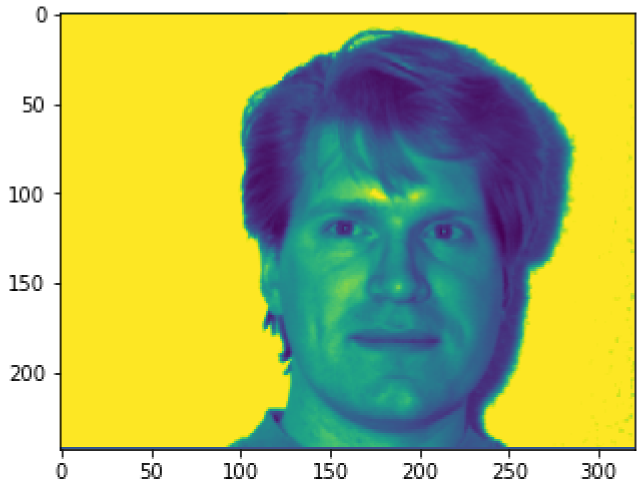
I think that's all my analysis for the cropped faces! Now moving on to the uncropped dataset.

In [54]:
```
from PIL import Image
image = Image.open('yalefaces/subject01.centerlight','r')
```

In [55]:
```
pix_val = list(image.getdata())
```

In [56]:
```
pyplot.imshow(np.reshape(pix_val,(-1,320)))
```

Out[56]: <matplotlib.image.AxesImage at 0x7f7a4582d2d0>



In [57]:
```
temparrayofpixels = np.matrix([pix_val]).T
```

In [58]:
```
bigmatrix = temparrayofpixels
```

In [59]:
```
bigmatrix
```

Out[59]:
```
matrix([[130],
        [130],
        [130],
        ...,
        [ 68],
        [ 68],
        [ 68]])
```

In [60]:
```
for i in os.listdir('yalefaces/'):
    try:
        tempimage = Image.open('yalefaces/' + i, 'r')
        pix_val = list(tempimage.getdata())
        temparrayofpixels = np.matrix([pix_val]).T
        bigmatrix = np.append(bigmatrix,temparrayofpixels,1)
    except:
        continue
```

In [61]:
```
bigmatrix.shape
```

Out[61]:
```
(77760, 166)
```

In [62]:
```
bigmatrix
```

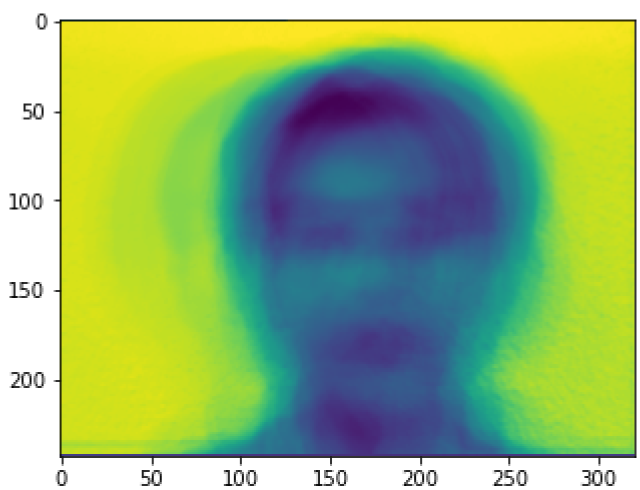
Out[62]:
```
matrix([[130, 130, 124, ..., 108, 130, 130],
        [130, 130, 116, ..., 116, 130, 130],
        [130, 130, 102, ..., 117, 130, 130],
        ...,
        [ 68,  68,  68, ...,  68,  68,  68],
        [ 68,  68,  68, ...,  68,  68,  68],
        [ 68,  68,  68, ...,  68,  68,  68]])
```

finding the average face

In [63]:
```
summed = np.sum(bigmatrix, axis=1)
```

In [64]:
```
pyplot.imshow(np.reshape(summed,(-1,320)))
```

Out[64]:
```
<matplotlib.image.AxesImage at 0x7f7a4579e410>
```



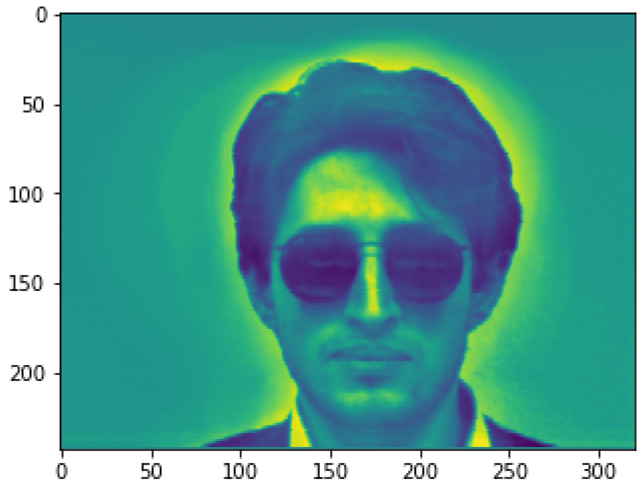
In [65]:
```
avfacevec = (summed / 166).astype(int)
```

In [66]:
```
normedfacemat = bigmatrix - avfacevec
```

finding mean subtracted faces

In [67]:
```
pyplot.imshow(np.reshape(normedfacemat.T[5],(-1,320)))
```

Out[67]: <matplotlib.image.AxesImage at 0x7f7a6b50a610>



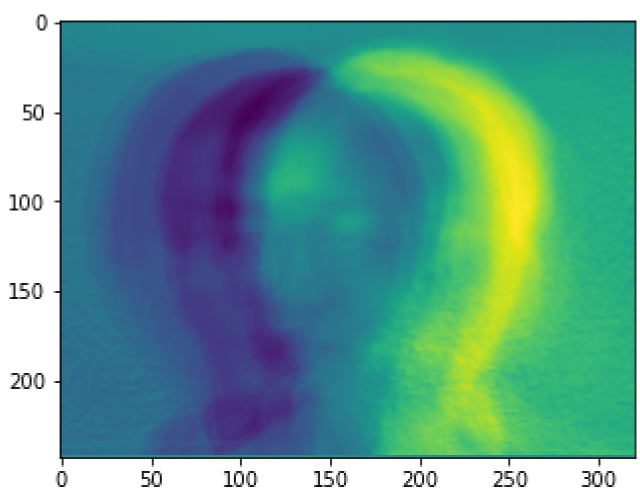
lookin good! time to do my svd

In [68]:
```
u, s, vt = np.linalg.svd(normedfacemat,full_matrices=False)
```

looking at eigenfaces

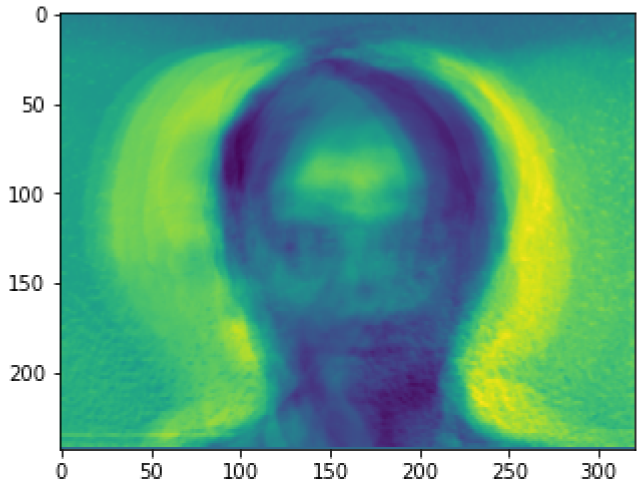
In [69]:
```
pyplot.imshow(np.reshape(u.T[0],(-1,320)))
```

Out[69]: <matplotlib.image.AxesImage at 0x7f7a6b538890>



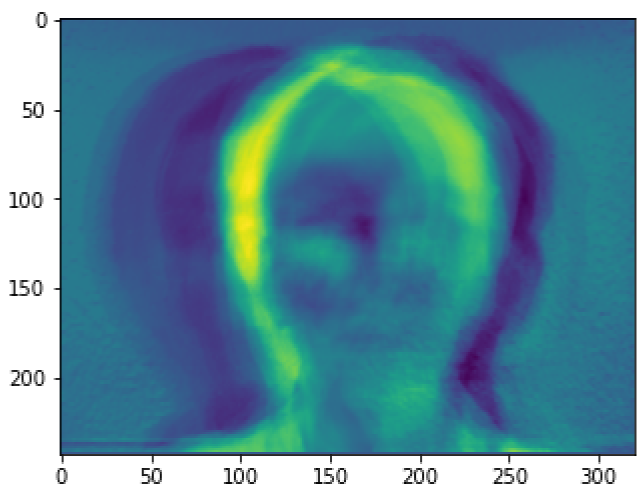
In [70]:
```
pyplot.imshow(np.reshape(u.T[1],(-1,320)))
```

Out[70]: <matplotlib.image.AxesImage at 0x7f7a6b4b89d0>



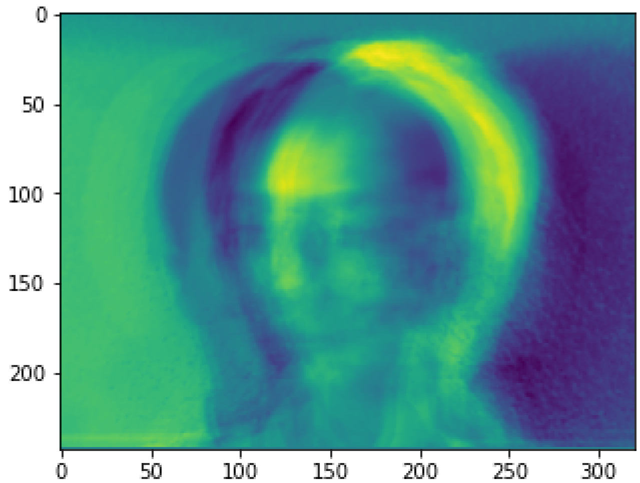
In [71]:
```
pyplot.imshow(np.reshape(u.T[2],(-1,320)))
```

Out[71]: <matplotlib.image.AxesImage at 0x7f7a6b422650>



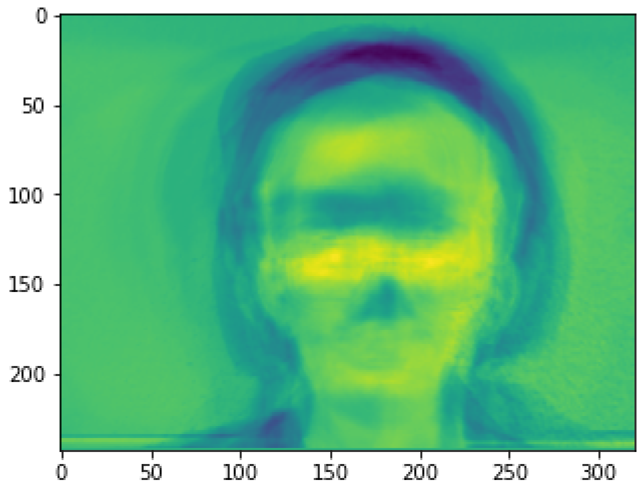
In [72]:
```
pyplot.imshow(np.reshape(u.T[3],(-1,320)))
```

Out[72]: <matplotlib.image.AxesImage at 0x7f7a45a99650>



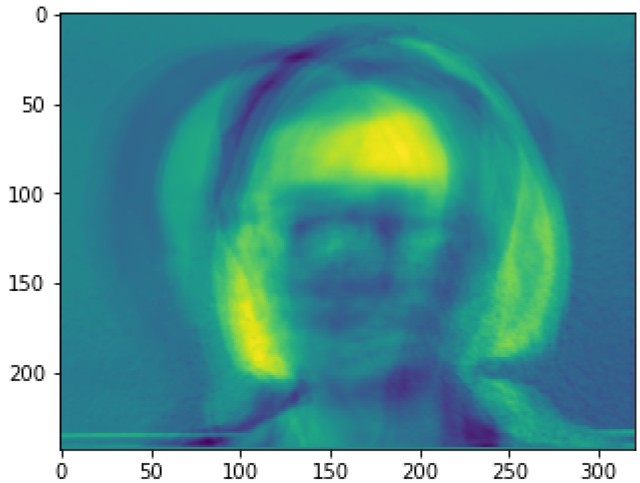
In [73]:
```
pyplot.imshow(np.reshape(u.T[4],(-1,320)))
```

Out[73]: <matplotlib.image.AxesImage at 0x7f7a6b3c7d50>



In [74]:
```
pyplot.imshow(np.reshape(u.T[5],(-1,320)))
```

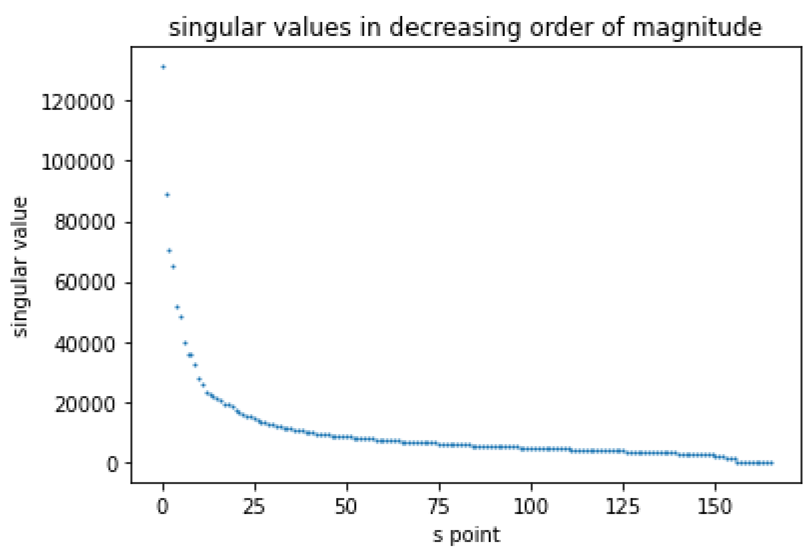
Out[74]: <matplotlib.image.AxesImage at 0x7f7a6b32fcd0>



wow these are all very terrifying, now let's look at the single values

In [75]:
```
pyplot.title('singular values in decreasing order of magnitude')
pyplot.xlabel('s point')
pyplot.ylabel('singular value')
pyplot.plot(s, 'o', markersize = 1)
```

Out[75]: [<matplotlib.lines.Line2D at 0x7f7a6b297e10>]



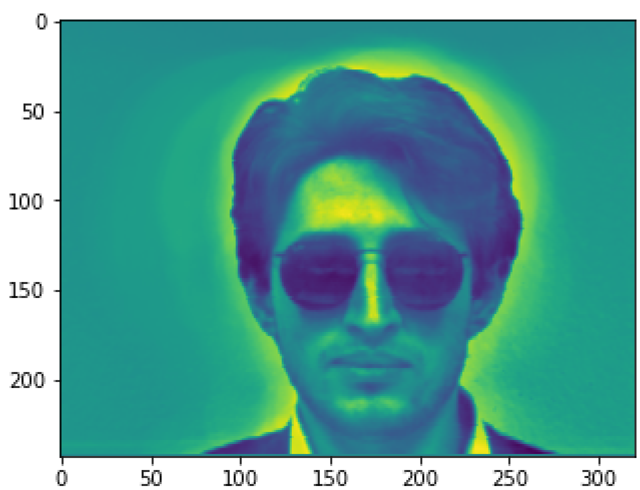
As was expected, the highest single values for the uncropped faces have much lower

the uncropped faces have much lower magnitudes than those of the cropped faces. This is because the faces in this set are less aligned, and features that showed up in the eigenfaces of the aligned faces (like defined eyes, nose, and mouth) were not present in the eigenfaces here. Since the heads are not in the same part of each photo, each eigenface also only contributes well to the faces that are in a similar position of the photograph to it. Still, I will try to use the eigenfaces to reconstruct with.

In [76]:
```
testface = normedfacemat.T[5]
```

In [77]:
```
pyplot.imshow(np.reshape(testface,(-1,320)))
```

Out[77]: <matplotlib.image.AxesImage at 0x7f7a6b20c890>

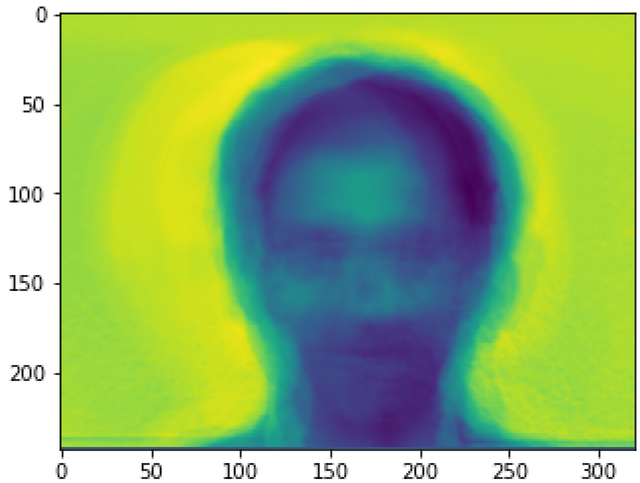


trying with 5 eigenfaces

In [78]:
```
recface = np.matmul(u[:,:5], np.matmul(u[:,:5].T,testface.T))
```

In [79]:
```
pyplot.imshow(np.reshape(recface + avfacevec,(-1,320)))
```

Out[79]: <matplotlib.image.AxesImage at 0x7f7a6b1f87d0>

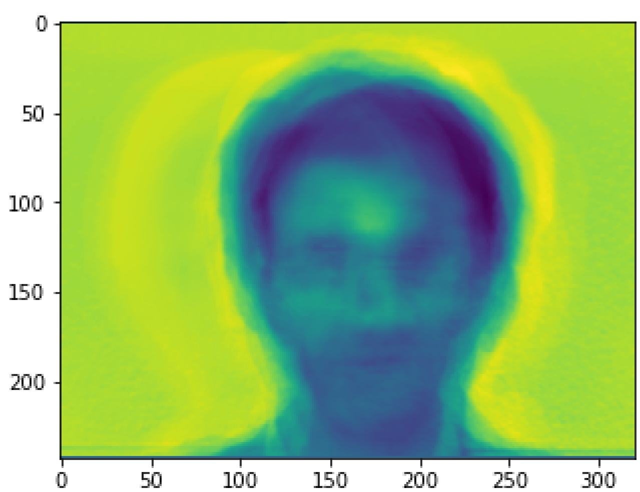


trying with 10 eigenfaces

In [80]:
```
recface = np.matmul(u[:,:10], np.matmul(u[:,:10].T,testface.T))
```

In [81]:
```
pyplot.imshow(np.reshape(recface + avfacevec,(-1,320)))
```

Out[81]: <matplotlib.image.AxesImage at 0x7f7a6b161610>

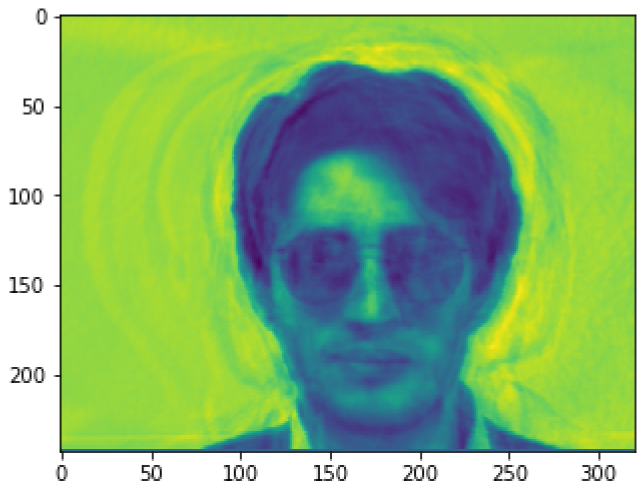


trying with 50 eigenfaces

In [82]:
```
recface = np.matmul(u[:,:50], np.matmul(u[:,:50].T,testface.T))
```

In [83]:
```
pyplot.imshow(np.reshape(recface + avfacevec,(-1,320)))
```

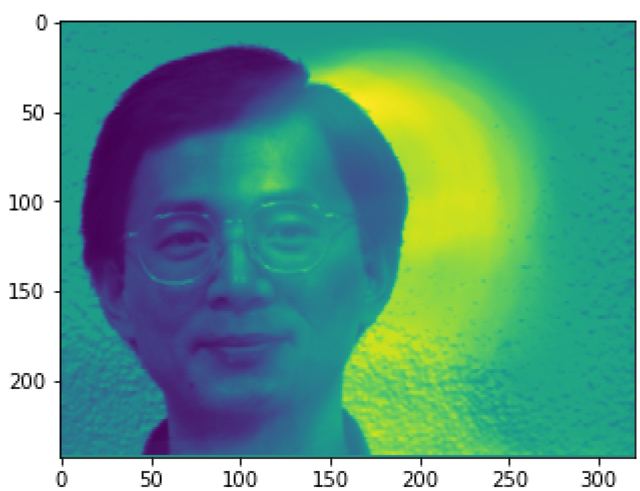
Out[83]: <matplotlib.image.AxesImage at 0x7f7a6b0cb4d0>



trying with another face

In [84]:
```
testface = normedfacemat.T[44]
pyplot.imshow(np.reshape(testface,(-1,320)))
```

Out[84]: <matplotlib.image.AxesImage at 0x7f7a6b0b6290>

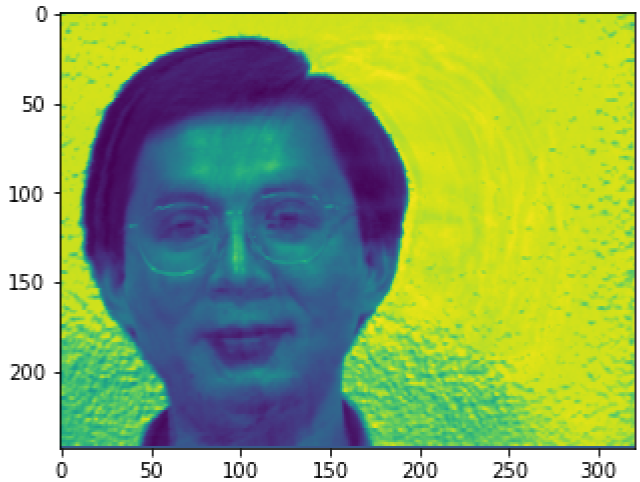


compression with 50 eigenfaces

In [85]:
```
recface = np.matmul(u[:,:50], np.matmul(u[:,:50].T,testface.T))
```

In [89]:
```
pyplot.imshow(np.reshape(recface + avfacevec,(-1,320)))
```

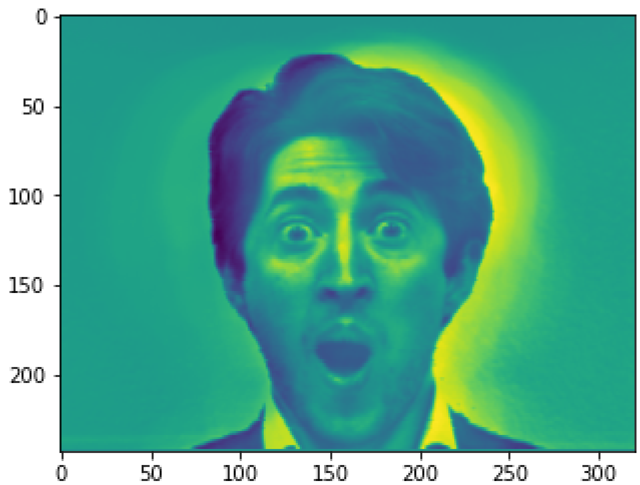
Out[89]:
```
<matplotlib.image.AxesImage at 0x7f7a6af5e790>
```



These generally, actually, look pretty good. I will try some with weird facial expressions to confirm that 50 is enough eigenfaces for this set, though I already have suspicions as to why this set may seem easier to recreate with, despite the lack of alignment.

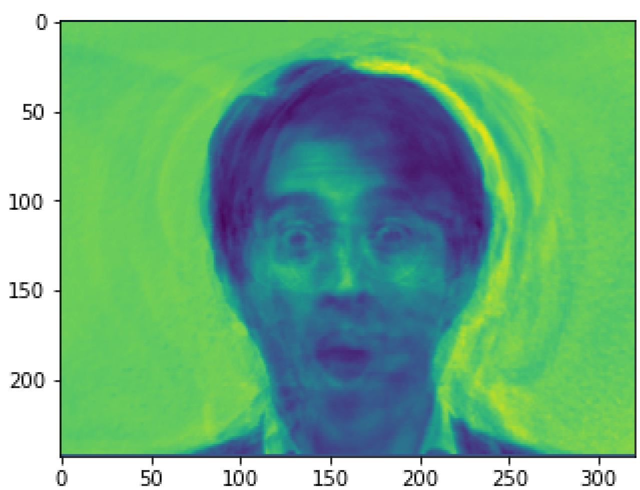
In [93]:
```
testface = normedfacemat.T[22]
pyplot.imshow(np.reshape(testface,(-1,320)))testface = normedfacemat.T[2
pyplot.imshow(np.reshape(testface,(-1,320)))
```

Out[93]: <matplotlib.image.AxesImage at 0x7f7a6ae03390>



In [94]:
```
recface = np.matmul(u[:,:50], np.matmul(u[:,:50].T,testface.T))
pyplot.imshow(np.reshape(recface + avfacevec,(-1,320)))
```

Out[94]: <matplotlib.image.AxesImage at 0x7f7a6b1b6d10>



that still looks pretty good, all things considered! Even though the eigenfaces for this group are less strong in terms of their singular values than the other set, none of the photos in this set are as difficult to emulate as the most difficult photos to emulate in the cropped set. These photos all have similar lighting

conditions, and are generally better lit, with people who still generally face forward. The ability to recreate an image from eigenfaces therefore seems easier for this set, as no faces in this set are as poorly lit as the worst lit photos from the first set. Faces can be recreated reasonably well for even strange facial expressions with a rank of r=50, or from the first 50 most important eigenfaces from the svd.

In [ ]:

```

```