Fluid mechanics II
by
Dr. Ehsan Amani

Chapter 7

Thermodynamics background

- For isentropic process (adiabatic + entropy constant),
- of an ideal gas,
- with constant c_p and c_v,

it can be shown that

$$\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{p_2}{p_1}\right), \quad k = \frac{c_p}{c_v}$$

$$\boxed{\frac{p}{\rho^k} = \text{constant}}$$

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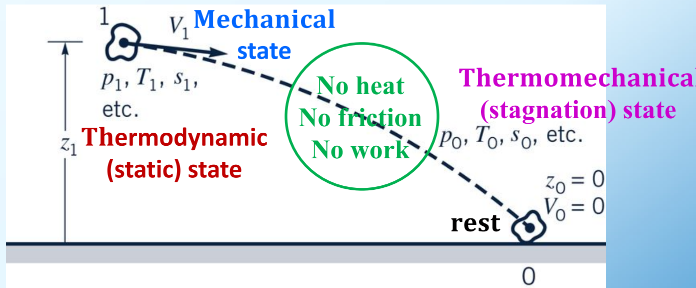
Thermodynamics background

- **Stagnation (total) properties**



$$\check{h}_0 \equiv \check{h} + \frac{V^2}{2} + \cancel{gz}$$

neglected

ideal gas, constant c_p $\Rightarrow$ $\boxed{T_0 = T + \frac{V^2}{2c_p}}$

$$\boxed{\frac{T_0}{T} = 1 + \frac{V^2}{2c_pT}.} \quad \boxed{\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k(k-1)}} \quad \boxed{\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}.}$$

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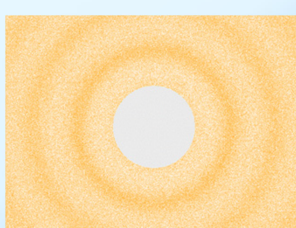
Sound wave and Mach number

- Sound (acoustic) wave:
  the propagation of a pressure disturbance in a medium via molecular interactions



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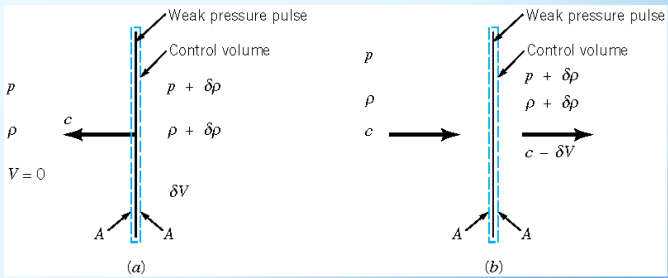
Sound wave and Mach number

- Speed of sound derivation:

  assuming an infinitesimal pressure change and resulted infinitesimal density and velocity change in the medium by a sound (pressure) wave



- Continuity: $\rho A c = (\rho + \delta\rho) A (c - \delta V)$
- Momentum:

$$-c\rho c A + (c - \delta V)(\rho + \delta\rho)(c - \delta V)A = pA - (p + \delta p)A$$

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Sound wave and Mach number

- Removing δV and solving for c in terms of δp and $\delta\rho$:

$$c = \sqrt{\frac{\delta p}{\delta\rho}}$$

- In practice

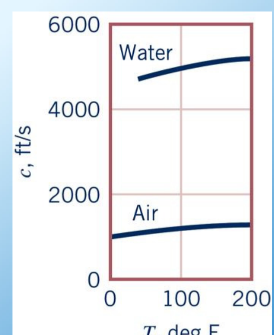
$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$

- For ideal gas:

$$c = \sqrt{kRT}$$

- Mach number:

$$Ma \equiv \frac{V}{c}$$



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Sound wave and Mach number

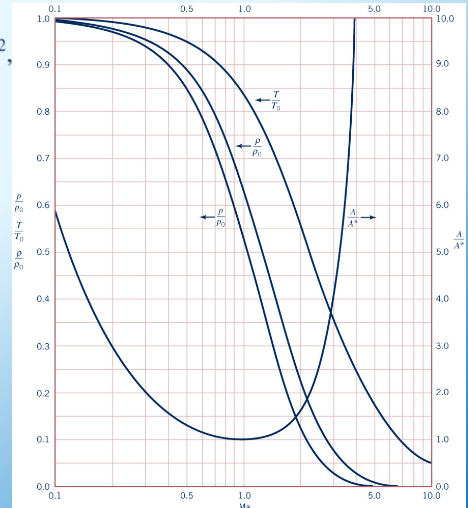
- Stagnation properties ratio versus Mach number:

$$\frac{V^2}{2c_pT} = \frac{k-1}{2}\left(\frac{V^2}{c^2}\right) = \left(\frac{k-1}{2}\right)\mathrm{Ma}^2,$$

$$\boxed{\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{k/(k-1)}}$$

$$\boxed{\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{1/(k-1)}.}$$

$$\boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2}\mathrm{Ma}^2.}$$



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Sound wave and Mach number

- Stagnation proper

Isentropic Flow Functions for an Ideal Gas with $k = 1.4$

Ma	T/T_0	p/p_0	ρ/ρ_0	A/A^*	Ma	T/T_0	p/p_0	ρ/ρ_0	A/A^*
0.0000	1.0000	1.0000	1.0000	∞	2.0500	0.5433	0.1182	0.2176	1.7600
0.0500	0.9995	0.9983	0.9988	11.5914	2.1000	0.5313	0.1094	0.2058	1.8369
0.1000	0.9980	0.9930	0.9950	5.8218	2.1500	0.5196	0.1011	0.1946	1.9185
0.1500	0.9955	0.9844	0.9888	3.9103	2.2000	0.5081	0.0935	0.1841	2.0050
0.2000	0.9921	0.9725	0.9803	2.9635	2.2500	0.4969	0.0865	0.1740	2.0964
0.2500	0.9877	0.9575	0.9694	2.4027	2.3000	0.4859	0.0800	0.1646	2.1931
0.3000	0.9823	0.9395	0.9564	2.0351	2.3500	0.4752	0.0740	0.1556	2.2953
0.3500	0.9761	0.9188	0.9413	1.7780	2.4000	0.4647	0.0684	0.1472	2.4031
0.4000	0.9690	0.8956	0.9243	1.5901	2.4500	0.4544	0.0633	0.1392	2.5168
0.4500	0.9611	0.8703	0.9055	1.4487	2.5000	0.4444	0.0585	0.1317	2.6367
0.5000	0.9524	0.8430	0.8852	1.3398	2.5500	0.4347	0.0542	0.1246	2.7630
0.5500	0.9430	0.8142	0.8634	1.2549	2.6000	0.4252	0.0501	0.1179	2.8960
0.6000	0.9328	0.7840	0.8405	1.1882	2.6500	0.4159	0.0464	0.1115	3.0359
0.6500	0.9221	0.7528	0.8164	1.1356	2.7000	0.4068	0.0430	0.1056	3.1830
0.7000	0.9107	0.7209	0.7916	1.0944	2.7500	0.3980	0.0398	0.0999	3.3377
0.7500	0.8989	0.6886	0.7660	1.0624	2.8000	0.3894	0.0368	0.0946	3.5001
0.8000	0.8865	0.6560	0.7400	1.0382	2.8500	0.3810	0.0341	0.0896	3.6707
0.8500	0.8737	0.6235	0.7136	1.0207	2.9000	0.3729	0.0317	0.0849	3.8498
0.9000	0.8606	0.5913	0.6870	1.0089	2.9500	0.3649	0.0293	0.0804	4.0376
0.9500	0.8471	0.5595	0.6604	1.0021	3.0000	0.3571	0.0272	0.0762	4.2346
1.0000	0.8333	0.5283	0.6339	1.0000	3.1000	0.3422	0.0234	0.0685	4.6573
1.0500	0.8193	0.4979	0.6077	1.0020	3.2000	0.3281	0.0202	0.0617	5.1210
1.1000	0.8052	0.4684	0.5817	1.0079	3.3000	0.3147	0.0175	0.0555	5.6286
1.1500	0.7908	0.4398	0.5562	1.0175	3.4000	0.3019	0.0151	0.0501	6.1837
1.2000	0.7764	0.4124	0.5311	1.0304	3.5000	0.2899	0.0131	0.0452	6.7896
1.2500	0.7619	0.3861	0.5067	1.0468	3.6000	0.2784	0.0114	0.0409	7.4501
1.3000	0.7474	0.3609	0.4829	1.0663	3.7000	0.2675	0.0099	0.0370	8.1691
1.3500	0.7329	0.3370	0.4598	1.0890	3.8000	0.2572	0.0086	0.0335	8.9506
1.4000	0.7184	0.3142	0.4374	1.1149	3.9000	0.2474	0.0075	0.0304	9.7990
1.4500	0.7040	0.2927	0.4158	1.1440	4.0000	0.2381	0.0066	0.0277	10.7188
1.5000	0.6897	0.2724	0.3950	1.1762	4.5000	0.1980	0.0035	0.0174	16.5622
1.5500	0.6754	0.2533	0.3750	1.2116	5.0000	0.1667	0.0019	0.0113	25.0000
1.6000	0.6614	0.2353	0.3557	1.2502	5.5000	0.1418	0.0011	0.0076	36.8690
1.6500	0.6475	0.2184	0.3373	1.2922	6.0000	0.1220	0.0006	0.0052	53.1798
1.7000	0.6337	0.2026	0.3197	1.3376	6.5000	0.1058	0.0004	0.0036	75.1343
1.7500	0.6202	0.1878	0.3029	1.3865	7.0000	0.0926	0.0002	0.0026	104.1429
1.8000	0.6068	0.1740	0.2868	1.4390	7.5000	0.0816	0.0002	0.0019	141.8415
1.8500	0.5936	0.1612	0.2715	1.4952	8.0000	0.0725	0.0001	0.0014	190.1094
1.9000	0.5807	0.1492	0.2570	1.5553	8.5000	0.0647	0.0001	0.0011	251.0862
1.9500	0.5680	0.1381	0.2432	1.6193	9.0000	0.0581	0.0000	0.0008	327.1893
2.0000	0.5556	0.1278	0.2300	1.6875	10.0000	0.0476	0.0000	0.0005	535.9375

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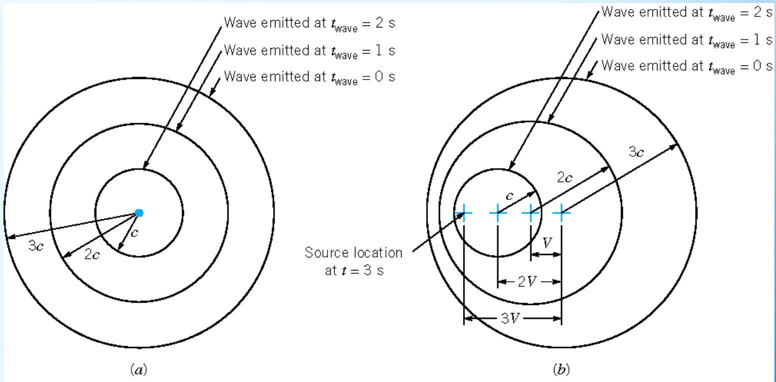
Compressible flow regimes

- **Mach cone**

A moving source of disturbance with velocity $V < c$:



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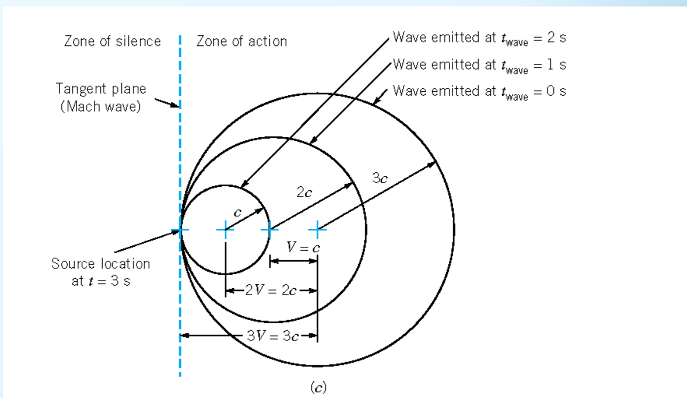
Compressible flow regimes

- **Mach cone**

A moving source of disturbance with velocity $V = c$:



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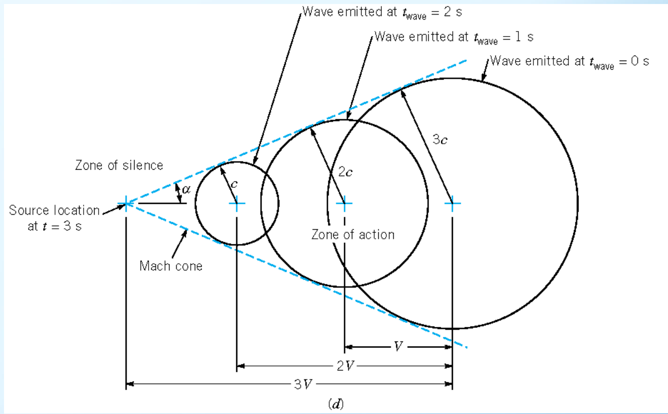
Compressible flow regimes

- **Mach cone**

A moving source of disturbance with velocity $V > c$:



$$\sin \alpha = \frac{c}{V} = \frac{1}{\text{Ma}}$$

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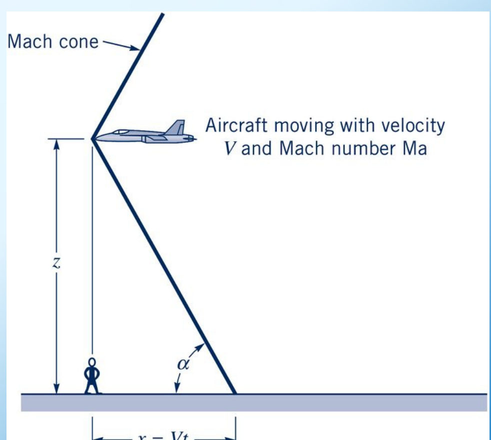
Compressible flow regimes

- **Mach cone**

A moving source of disturbance with velocity $V > c$:



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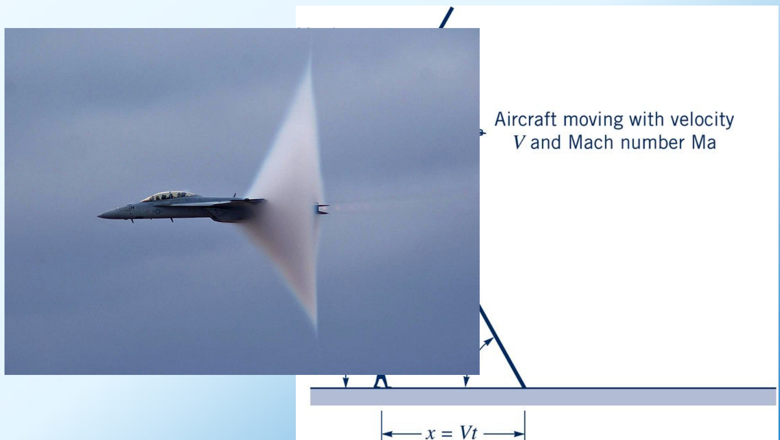
Compressible flow regimes

- **Mach cone**

A moving source of disturbance with velocity $V > c$:



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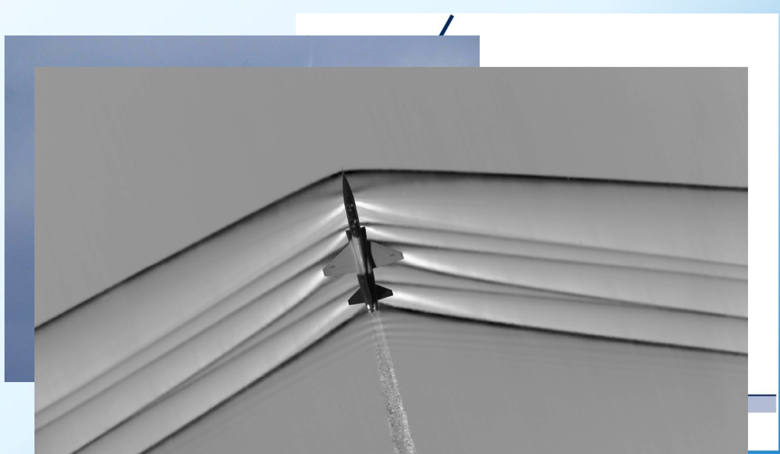
Compressible flow regimes

- **Mach cone**

A moving source of disturbance with velocity V > c:



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Compressible flow regimes

- **Regimes:**
- Incompressible (Ma < 0.3)
- Compressible subsonic (0.3 < Ma < 1)
- Compressible supersonic (Ma > 1)

- Transonic (0.9 < Ma < 1.2)
- Hypersonic (Ma > 5)

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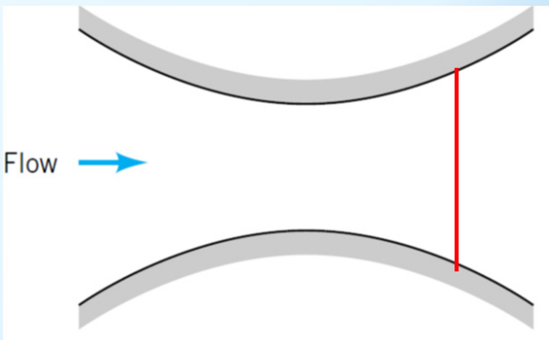
Compressible flow regimes

- **Shock waves:**
- Normal shocks



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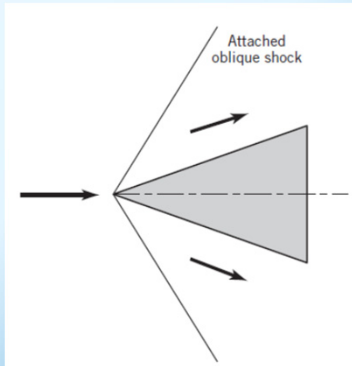
Compressible flow regimes

- **Shock waves:**
- Normal shocks
- Oblique shocks



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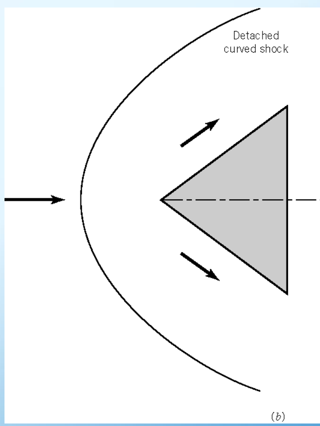
Compressible flow regimes

- **Shock waves:**
- Normal shocks
- Oblique shocks
- Curved shocks



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Compressible flow regimes

- **Shock waves**

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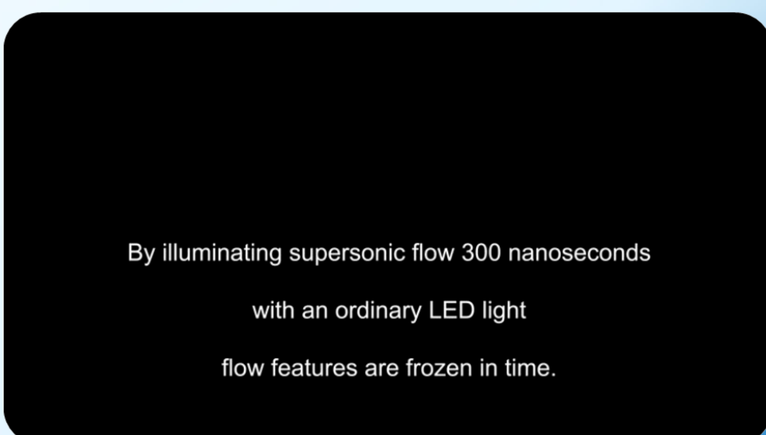
Compressible flow regimes

- **Shock waves**



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Compressible flow regimes

- **Shock waves**

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Compressible flow regimes

- **Shock waves**

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Compressible flow regimes

- **Shock waves:**
- Normal shocks → **1D**
- Oblique shocks
- Curved shocks

$\left.\begin{array}{l}\text{Oblique shocks}\\ \text{Curved shocks}\end{array}\right\}$ **2D, 3D**

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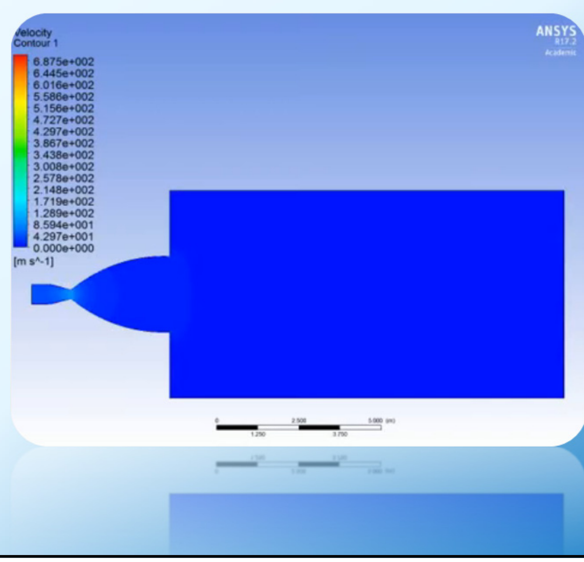
CFD of compressible flows



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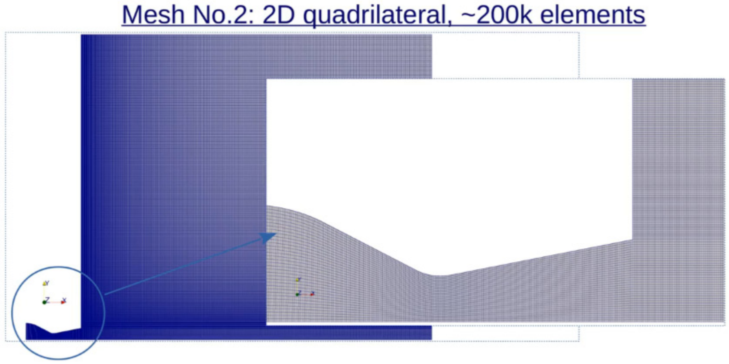
CFD of compressible flows

Mesh No.2: 2D quadrilateral, ~200k elements



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CFD of compressible flows

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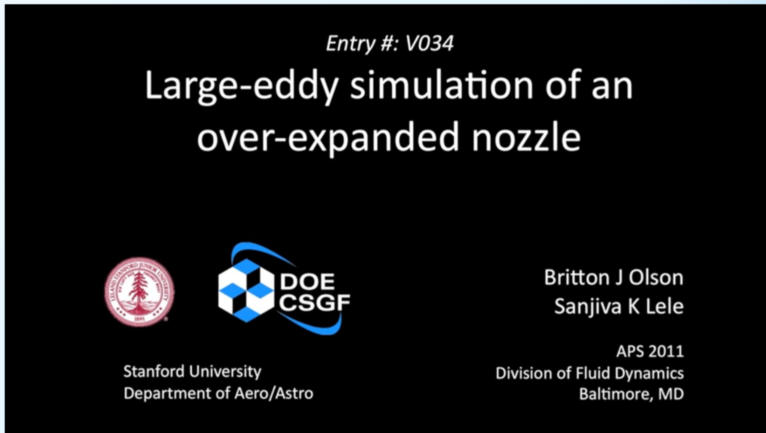
CFD of compressible flows



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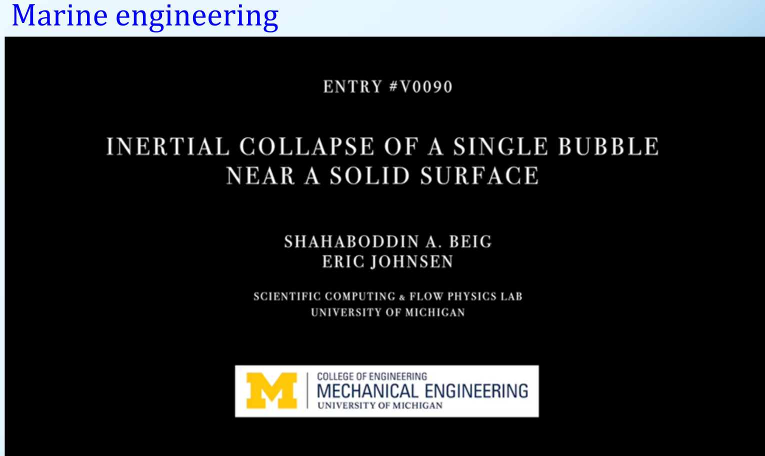
An application of compressible flow

- **Wall erosion**
- Bioengineering
- Marine engineering
-



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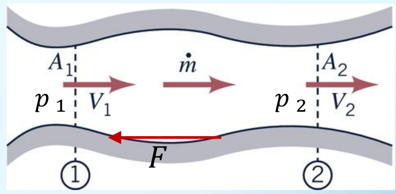
General 1D compressible flow equations

- **Finite control volume approach:**



- Mass: $\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \dot{m}$
- Momentum: $p_1 A_1 - p_2 A_2 - F + \rho_1 V_1^2 A_1 - \rho_2 V_2^2 A_2 = 0$
- Energy: $h_1 + \frac{V_1^2}{2} + \frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}}{\dot{m}} = h_2 + \frac{V_2^2}{2}$
- 2nd law: $s_2 - s_1 = \frac{1}{\dot{m}} \int_1^2 \frac{d\dot{Q}}{T} + \frac{\dot{S}_{gen}}{\dot{m}}$
- 2 state equations

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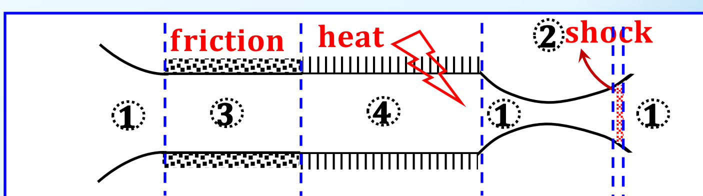
1D compressible flow problems

- **A 1D problem can be broken down into several coupled sub-problems:**



1. Isentropic variable area
2. Normal shock
3. Constant area with friction (Fanno line)
4. Constant area with heat transfer (Rayleigh line)
5. …

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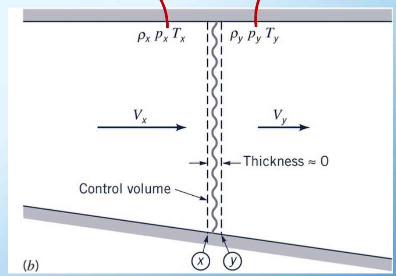
Normal shock

- **Assumptions: 1D + zero-thickness**
- **Equations:**

Mass ($A_1 = A_2 = A$): (1)
Momentum ($F = 0$): (2)
Energy ($\dot{Q} = \dot{W} = 0$): (3)
2^{nd} law ($s_2 - s_1 = \dot{S}_{gen}/\dot{m} > 0$): $(*)$
Eqs. of state ($h = h(s,\rho)$,
$\rho = \rho(s,p)$): (4), (5)

1 2



- **Unknowns (5):** $\rho_2, V_2, p_2, h_2, s_2$

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Normal shock

- **Analytical solution for ideal gas with a constant c_p**

$h \rightarrow c_p(T - T_0)$ $V \rightarrow \mathrm{Ma}c = \mathrm{Ma}\sqrt{kRT}$

Equations (4): $(1),(2),(3), p = \rho RT$
Unknowns (4): $\rho_2, \mathrm{Ma}_2, p_2, T_2$
2 solutions:

Shock

$$\boxed{\mathrm{Ma}_y^2 = \mathrm{Ma}_x^2} \quad \boxed{\mathrm{Ma}_y^2 = \frac{\mathrm{Ma}_x^2 + 2/(k-1)}{[2k/(k-1)]\mathrm{Ma}_x^2 - 1}.}$$

if $\mathrm{Ma}_x > 1$ and shock: It can be shown $s_2 > s_1$: $(*)$ ✓
if $\mathrm{Ma}_x < 1$ and shock: It can be shown $s_2 < s_1$: $(*)$ ×

Proof [1] P. 624

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Normal shock

- **Analytical solution for ideal gas with a constant c_p**

The shock solution ($\mathrm{Ma}_x > 1$):

$$\mathrm{Ma}_y^2 = \frac{\mathrm{Ma}_x^2 + 2/(k-1)}{[2k/(k-1)]\mathrm{Ma}_x^2 - 1}.$$

$$\frac{p_y}{p_x} = \frac{2k}{k+1}\mathrm{Ma}_x^2 - \frac{k-1}{k+1}$$

$$\frac{T_y}{T_x} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_x^2\right)\left(\frac{2k}{k-1}\mathrm{Ma}_x^2 - 1\right)\left[\frac{2(k-1)}{(k+1)^2\mathrm{Ma}_x^2}\right].$$

$$\frac{p_{0_y}}{p_{0_x}} = \left(\frac{k+1}{2}\mathrm{Ma}_x^2\right)^{\frac{k}{k-1}}\left(1 + \frac{k-1}{2}\mathrm{Ma}_x^2\right)^{\frac{-k}{(k-1)}}\left(\frac{2k}{k+1}\mathrm{Ma}_x^2 - \frac{k-1}{k+1}\right)^{\frac{-1}{(k-1)}}.$$

$$\frac{T_{0_y}}{T_{0_x}} = 1.$$

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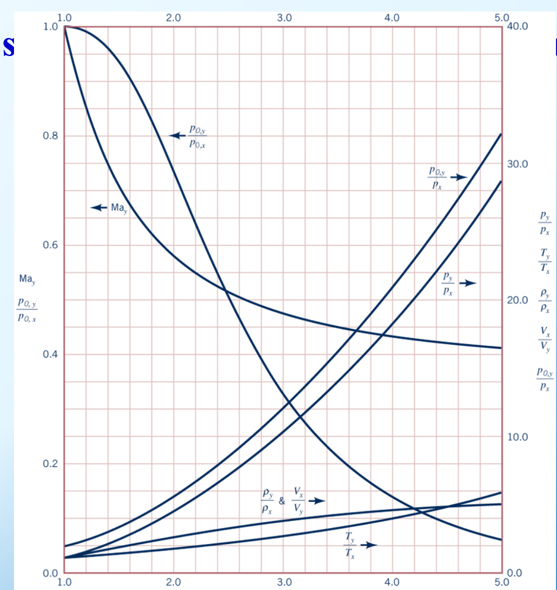
Normal shock

- **Analytical s... nt c_p**



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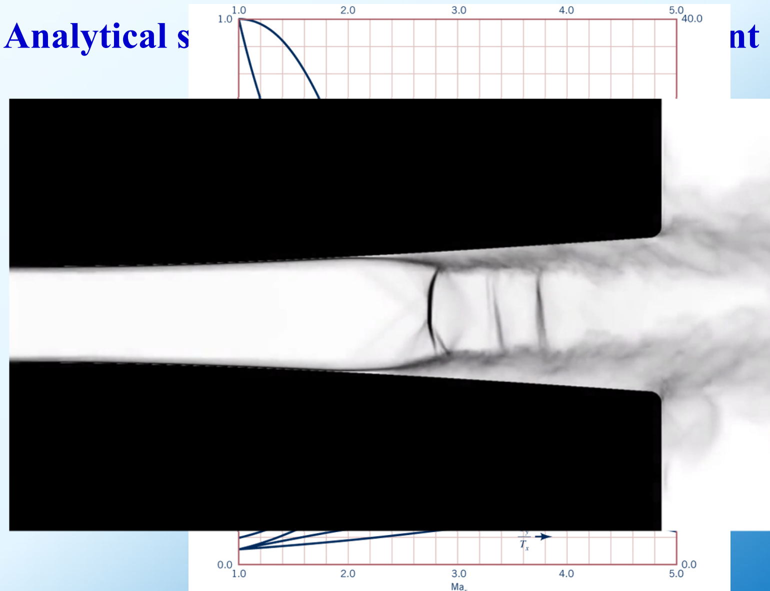
Normal shock

- **Analytical s** ... **nt c_p**



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Normal shock

- **Analytical s** ... **t c_p**

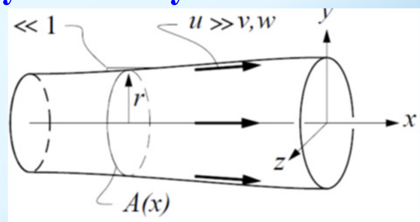
Normal Shock Functions for an Ideal Gas with $k = 1.4$

Ma_x	Ma_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = V_x/V_y$	p_{0y}/p_{0x}	Ma_x	Ma_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = V_x/V_y$	p_{0y}/p_{0x}
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	3.05	0.4723	10.6863	2.7383	3.9025	0.3145
1.05	0.9531	1.1196	1.0328	1.0840	0.9999	3.10	0.4695	11.0450	2.7986	3.9466	0.3012
1.10	0.9118	1.2450	1.0649	1.1691	0.9989	3.15	0.4669	11.4096	2.8598	3.9896	0.2885
1.15	0.8750	1.3763	1.0966	1.2550	0.9967	3.20	0.4643	11.7800	2.9220	4.0315	0.2762
1.20	0.8422	1.5133	1.1280	1.3416	0.9928	3.25	0.4619	12.1563	2.9851	4.0723	0.2645
1.25	0.8126	1.6563	1.1594	1.4286	0.9871	3.30	0.4596	12.5383	3.0492	4.1120	0.2533
1.30	0.7860	1.8050	1.1909	1.5157	0.9794	3.35	0.4573	12.9263	3.1142	4.1507	0.2425
1.35	0.7618	1.9596	1.2226	1.6028	0.9697	3.40	0.4552	13.3200	3.1802	4.1884	0.2322
1.40	0.7397	2.1200	1.2547	1.6897	0.9582	3.45	0.4531	13.7196	3.2472	4.2251	0.2224
1.45	0.7196	2.2863	1.2872	1.7761	0.9448	3.50	0.4512	14.1250	3.3151	4.2609	0.2129
1.50	0.7011	2.4583	1.3202	1.8621	0.9298	3.55	0.4492	14.5363	3.3839	4.2957	0.2039
1.55	0.6841	2.6363	1.3538	1.9473	0.9132	3.60	0.4474	14.9533	3.4537	4.3296	0.1953
1.60	0.6684	2.8200	1.3880	2.0317	0.8952	3.65	0.4456	15.3763	3.5245	4.3627	0.1871
1.65	0.6540	3.0096	1.4228	2.1152	0.8760	3.70	0.4439	15.8050	3.5962	4.3949	0.1792
1.70	0.6405	3.2050	1.4583	2.1977	0.8557	3.75	0.4423	16.2396	3.6689	4.4262	0.1717
1.75	0.6281	3.4063	1.4946	2.2791	0.8346	3.80	0.4407	16.6800	3.7426	4.4568	0.1645
1.80	0.6165	3.6133	1.5316	2.3592	0.8127	3.85	0.4392	17.1263	3.8172	4.4866	0.1576
1.85	0.6057	3.8263	1.5693	2.4381	0.7902	3.90	0.4377	17.5783	3.8928	4.5156	0.1510
1.90	0.5956	4.0450	1.6079	2.5157	0.7674	3.95	0.4363	18.0363	3.9694	4.5439	0.1448
1.95	0.5862	4.2696	1.6473	2.5919	0.7442	4.00	0.4350	18.5000	4.0469	4.5714	0.1388
2.00	0.5774	4.5000	1.6875	2.6667	0.7209	4.10	0.4324	19.4450	4.2048	4.6245	0.1276
2.05	0.5691	4.7363	1.7285	2.7400	0.6975	4.20	0.4299	20.4133	4.3666	4.6749	0.1173
2.10	0.5613	4.9783	1.7705	2.8119	0.6742	4.30	0.4277	21.4050	4.5322	4.7229	0.1080
2.15	0.5540	5.2263	1.8132	2.8823	0.6511	4.40	0.4255	22.4200	4.7017	4.7685	0.0995
2.20	0.5471	5.4800	1.8569	2.9512	0.6281	4.50	0.4236	23.4583	4.8751	4.8119	0.0917
2.25	0.5406	5.7396	1.9014	3.0186	0.6055	4.60	0.4217	24.5200	5.0523	4.8532	0.0846
2.30	0.5344	6.0050	1.9468	3.0845	0.5833	4.70	0.4199	25.6050	5.2334	4.8926	0.0781
2.35	0.5286	6.2763	1.9931	3.1490	0.5615	4.80	0.4183	26.7133	5.4184	4.9301	0.0721
2.40	0.5231	6.5533	2.0403	3.2119	0.5401	4.90	0.4167	27.8450	5.6073	4.9659	0.0667
2.45	0.5179	6.8363	2.0885	3.2733	0.5193	5.00	0.4152	29.0000	5.8000	5.0000	0.0617
2.50	0.5130	7.1250	2.1375	3.3333	0.4990	5.50	0.4090	35.1250	6.8218	5.1489	0.0424
2.55	0.5083	7.4196	2.1875	3.3919	0.4793	6.00	0.4042	41.8333	7.9406	5.2683	0.0297
2.60	0.5039	7.7200	2.2383	3.4490	0.4601	6.50	0.4004	49.1250	9.1564	5.3651	0.0211
2.65	0.4996	8.0262	2.2902	3.5047	0.4416	7.00	0.3974	57.0000	10.4694	5.4444	0.0154
2.70	0.4956	8.3383	2.3429	3.5590	0.4236	7.50	0.3949	65.4583	11.8795	5.5102	0.0113
2.75	0.4918	8.6562	2.3966	3.6119	0.4062	8.00	0.3929	74.5000	13.3867	5.5652	0.0085
2.80	0.4882	8.9800	2.4512	3.6636	0.3895	8.50	0.3912	84.1250	14.9911	5.6117	0.0064
2.85	0.4847	9.3096	2.5067	3.7139	0.3733	9.00	0.3898	94.3333	16.6927	5.6512	0.0050
2.90	0.4814	9.6450	2.5632	3.7629	0.3577	9.50	0.3886	105.1250	18.4915	5.6850	0.0039
2.95	0.4782	9.9862	2.6206	3.8106	0.3428	10.00	0.3876	116.5000	20.3875	5.7143	0.0030
3.00	0.4752	10.3333	2.6790	3.8571	0.3283	∞	0.3780	∞	∞	6.0000	0.0000

Chapter 7 Appendix D.S [1] By E. Amani

Isentropic flow (differential analysis)

- Assumptions: 1D + steady + no body force + no friction + no heat



- Using continuity:

$$\rho A V = const \ \rightarrow \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

- Momentum (compressible Bernoulli):

$$dp + \frac{1}{2}\rho d(V^2) = 0$$

- and the definition: $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$

- Results in:

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}}$$

$$\boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

Chapter 7 By E. Amani

Isentropic flow (differential analysis)

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}}$$

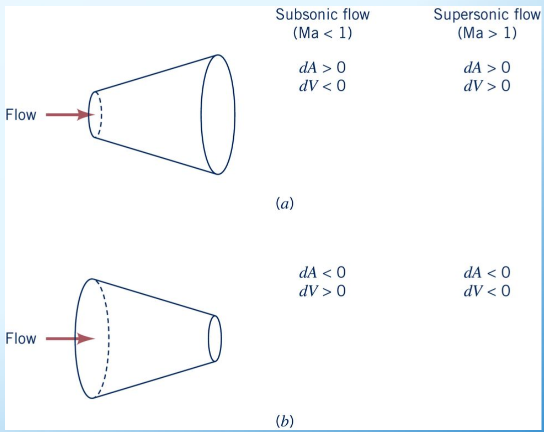
$$\boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

Chapter 7 By E. Amani

Isentropic flow (differential analysis)

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}} \quad \boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

- **Conclusions:**



Chapter 7 By E. Amani

Isentropic flow (differential analysis)

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}} \quad \boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

- **Conclusions:**

The critical state can only occur at a throat ($dA = 0$):

$\mathrm{Ma} = 1 \ \rightarrow dA = 0$

$dA = 0 \ \rightarrow \mathrm{Ma} = 1$ or $dV = 0$

$\mathrm{Ma} < 1, A \downarrow (dA < 0) \rightarrow p \downarrow, V \uparrow \rightarrow \frac{p}{p_0} \downarrow$

constant stagnation properties

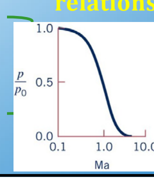
Proof: e.g. based on ideal gas isentropic relations

Maximum of 1

$\mathrm{Ma} \uparrow,\ T \downarrow,\ \rho \downarrow$

$\mathrm{Ma} > 1, A \downarrow (dA < 0) \rightarrow p \uparrow, V \downarrow \rightarrow \frac{p}{p_0} \uparrow$

constant stagnation properties



$\mathrm{Ma} \downarrow,\ T \uparrow,\ \rho \uparrow$

Minimum of 1

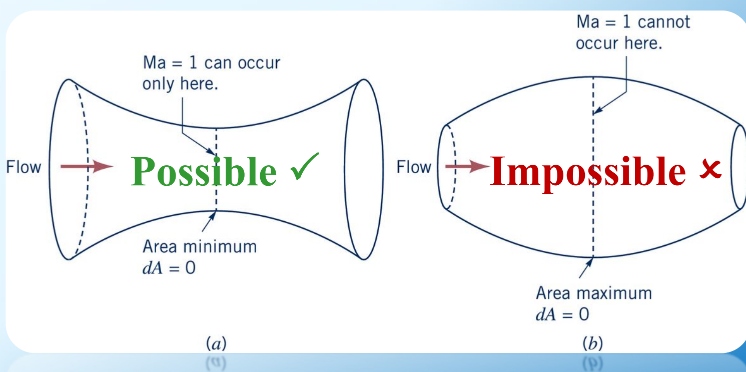
Chapter 7 By E. Amani

Isentropic flow (differential analysis)

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}} \quad \boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

- **Conclusions:**

The critical state can only occur at a throat ($dA = 0$):



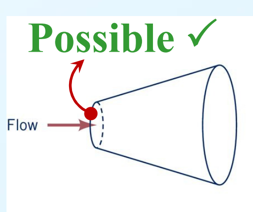
Chapter 7 **By E. Amani**

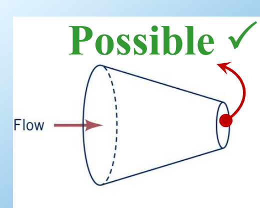
Isentropic flow (differential analysis)

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}} \quad \boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

- **Conclusions:**

The critical state can only occur at a throat ($dA = 0$):





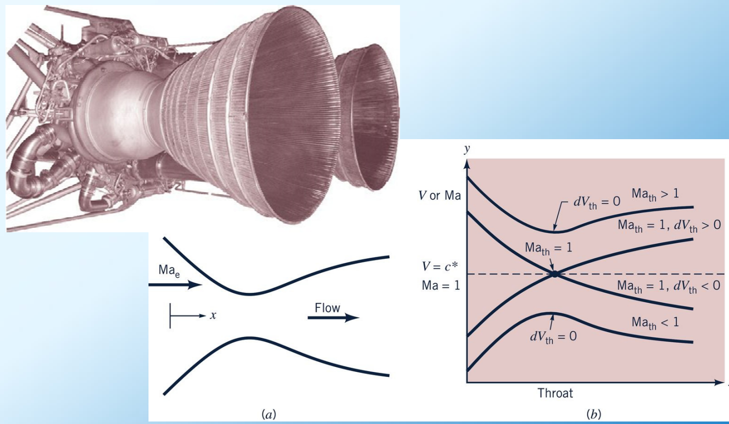
Cannot pass Ma = 1 state only with a converging or diverging nozzle.

Chapter 7 **By E. Amani**

Isentropic flow (differential analysis)

$$\boxed{\frac{dp}{\rho V^2} = -\frac{dV}{V}} \quad \boxed{\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}}$$

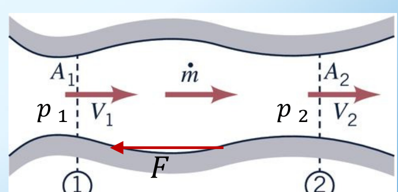
- **Conclusions:**



Chapter 7 **By E. Amani**

Isentropic flow (control volume analysis)

- **Assumptions: 1D + steady + no body force + no friction + no heat + no shock + no work + ideal gas + constant c_p**
- **Equations:**



Mass: (1)

Momentum: (2)

Energy ($\dot{Q} = \dot{W} = 0$): (3)

2^{nd} law ($s_2 = s_1$): (6)

Eqs. of state ($h = c_p(T - T_0), p = \rho RT$): (4), (5)

Mach number ($V = \mathrm{Ma}\sqrt{kRT}$): (7)

- **Unknowns (7) from the 8 parameters:** $\rho_2, V_2, p_2, h_2, s_2, F, A_2, Ma_2$

Chapter 7 **By E. Amani**

Isentropic flow (with area variation)

- **Analytical solution (relations)**

$$T_{0_1} = T_1 + \frac{V_1^2}{2c_p} = T_2 + \frac{V_2^2}{2c_p} = T_{0_2} \quad \textbf{or} \quad \frac{T_2}{T_1} = \frac{1 + [(k-1)/2]\mathrm{Ma}_1^2}{1 + [(k-1)/2]\mathrm{Ma}_2^2}.$$

$$p_1\left(1 + \frac{V_1^2}{2c_pT_1}\right)^{k/(k-1)} = p_2\left(1 + \frac{V_2^2}{2c_pT_2}\right)^{k/(k-1)} \quad \textbf{or} \quad \frac{p_2}{p_1} = \frac{\{1 + [(k-1)/2]\mathrm{Ma}_1^2\}^{k/(k-1)}}{\{1 + [(k-1)/2]\mathrm{Ma}_2^2\}^{k/(k-1)}}$$

If ρ_2 is given or sought:

$$p_2 = \rho_2 R T_2$$

If A_2 is given or sought:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad \textbf{or} \quad \frac{A_2}{A_1} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left\{\frac{1 + [(k-1)/2]\mathrm{Ma}_2^2}{1 + [(k-1)/2]\mathrm{Ma}_1^2}\right\}^{(k+1)/2(k-1)}$$

If F is given or sought:

$$F = p_1A_1 - p_2A_2 + \rho_1V_1^2A_1 - \rho_2V_2^2A_2$$

Chapter 7 By E. Amani

Isentropic flow (with area variation)

- **Analytical solution (graph)**

$$\frac{T_2}{T_1} = \left.\frac{T}{T_0}\right)_{\mathrm{Ma}_2} \Big/ \left.\frac{T}{T_0}\right)_{\mathrm{Ma}_1}$$

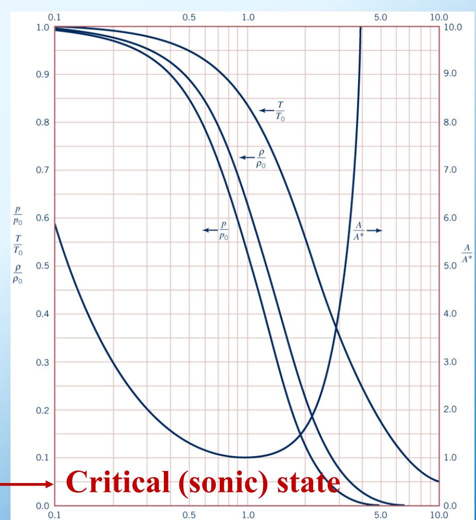
$$\left.\frac{T}{T_0}\right)_{\mathrm{Ma}}$$

$$\frac{p_2}{p_1} = \left.\frac{p}{p_0}\right)_{\mathrm{Ma}_2} \Big/ \left.\frac{p}{p_0}\right)_{\mathrm{Ma}_1}$$

$$\left.\frac{p}{p_0}\right)_{\mathrm{Ma}}$$

$$\frac{A_2}{A_1} = \left.\frac{A}{A^*}\right)_{\mathrm{Ma}_2} \Big/ \left.\frac{A}{A^*}\right)_{\mathrm{Ma}_1}.$$

$$\left.\frac{A}{A^*}\right)_{\mathrm{Ma}}$$

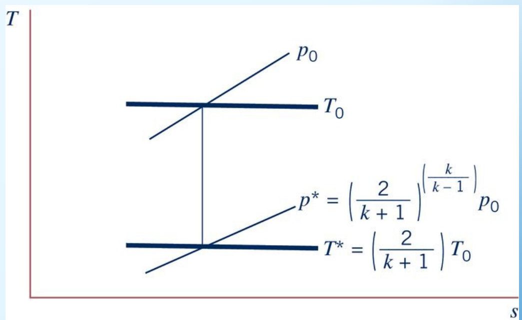


$*$ → Critical (sonic) state

Chapter 7 Appendix D.I [1] By E. Amani

Isentropic flow (with area variation)

- **Analytical solution (graph)**



* ⟶ **Critical (sonic) state**

Chapter 7 By E. Amani

Isentropic flow (with area variation)

- **Analytical solution (table)**

$$\frac{T_2}{T_1} = \left.\frac{T}{T_0}\right)_{\mathrm{Ma}_2} \Big/ \left.\frac{T}{T_0}\right)_{\mathrm{Ma}_1}$$

$$\left.\frac{T}{T_0}\right)_{\mathrm{Ma}}$$

$$\frac{p_2}{p_1} = \left.\frac{p}{p_0}\right)_{\mathrm{Ma}_2} \Big/ \left.\frac{p}{p_0}\right)_{\mathrm{Ma}_1}$$

$$\left.\frac{p}{p_0}\right)_{\mathrm{Ma}}$$

$$\frac{A_2}{A_1} = \left.\frac{A}{A^*}\right)_{\mathrm{Ma}_2} \Big/ \left.\frac{A}{A^*}\right)_{\mathrm{Ma}_1}.$$

$$\left.\frac{A}{A^*}\right)_{\mathrm{Ma}}$$

Isentropic Flow Functions for an Ideal Gas with $k = 1.4$

Ma	T/T_0	p/p_0	ρ/ρ_0	A/A^*	Ma	T/T_0	p/p_0	ρ/ρ_0	A/A^*
0.0000	1.0000	1.0000	1.0000	∞	2.0500	0.5433	0.1182	0.2176	1.7600
0.0500	0.9995	0.9983	0.9988	11.5914	2.1000	0.5313	0.1094	0.2058	1.8369
0.1000	0.9980	0.9930	0.9950	5.8218	2.1500	0.5196	0.1011	0.1946	1.9185
0.1500	0.9955	0.9844	0.9888	3.9103	2.2000	0.5081	0.0935	0.1841	2.0050
0.2000	0.9921	0.9725	0.9803	2.9635	2.2500	0.4969	0.0865	0.1740	2.0964
0.2500	0.9877	0.9575	0.9694	2.4027	2.3000	0.4859	0.0800	0.1646	2.1931
0.3000	0.9823	0.9395	0.9564	2.0351	2.3500	0.4752	0.0740	0.1556	2.2953
0.3500	0.9761	0.9188	0.9413	1.7780	2.4000	0.4647	0.0684	0.1472	2.4031
0.4000	0.9690	0.8956	0.9243	1.5901	2.4500	0.4544	0.0633	0.1392	2.5168
0.4500	0.9611	0.8703	0.9055	1.4487	2.5000	0.4444	0.0585	0.1317	2.6367
0.5000	0.9524	0.8430	0.8852	1.3398	2.5500	0.4347	0.0542	0.1246	2.7630
0.5500	0.9430	0.8142	0.8634	1.2549	2.6000	0.4252	0.0501	0.1179	2.8960
0.6000	0.9328	0.7840	0.8405	1.1882	2.6500	0.4159	0.0464	0.1115	3.0359
0.6500	0.9221	0.7528	0.8164	1.1356	2.7000	0.4068	0.0430	0.1056	3.1830
0.7000	0.9107	0.7209	0.7916	1.0944	2.7500	0.3980	0.0398	0.0999	3.3377
0.7500	0.8989	0.6886	0.7660	1.0624	2.8000	0.3894	0.0368	0.0946	3.5001
0.8000	0.8865	0.6560	0.7400	1.0382	2.8500	0.3810	0.0341	0.0896	3.6707
0.8500	0.8737	0.6235	0.7136	1.0207	2.9000	0.3729	0.0317	0.0849	3.8498
0.9000	0.8606	0.5913	0.6870	1.0089	2.9500	0.3649	0.0293	0.0804	4.0376
0.9500	0.8471	0.5595	0.6604	1.0021	3.0000	0.3571	0.0272	0.0762	4.2346
1.0000	0.8333	0.5283	0.6339	1.0000	3.1000	0.3422	0.0234	0.0685	4.6573
1.0500	0.8193	0.4979	0.6077	1.0020	3.2000	0.3281	0.0202	0.0617	5.1210
1.1000	0.8052	0.4684	0.5817	1.0079	3.3000	0.3147	0.0175	0.0555	5.6286
1.1500	0.7908	0.4398	0.5562	1.0175	3.4000	0.3019	0.0151	0.0501	6.1837
1.2000	0.7764	0.4124	0.5311	1.0304	3.5000	0.2899	0.0131	0.0452	6.7896
1.2500	0.7619	0.3861	0.5067	1.0468	3.6000	0.2784	0.0114	0.0409	7.4501
1.3000	0.7474	0.3609	0.4829	1.0663	3.7000	0.2675	0.0099	0.0370	8.1691
1.3500	0.7329	0.3370	0.4598	1.0890	3.8000	0.2572	0.0086	0.0335	8.9506
1.4000	0.7184	0.3142	0.4374	1.1149	3.9000	0.2474	0.0075	0.0304	9.7990
1.4500	0.7040	0.2927	0.4158	1.1440	4.0000	0.2381	0.0066	0.0277	10.7188
1.5000	0.6897	0.2724	0.3950	1.1762	4.5000	0.1980	0.0035	0.0174	16.5622
1.5500	0.6754	0.2533	0.3750	1.2116	5.0000	0.1667	0.0019	0.0113	25.0000
1.6000	0.6614	0.2353	0.3557	1.2502	5.5000	0.1418	0.0011	0.0076	36.8690
1.6500	0.6475	0.2184	0.3373	1.2922	6.0000	0.1220	0.0006	0.0052	53.1798
1.7000	0.6337	0.2026	0.3197	1.3376	6.5000	0.1058	0.0004	0.0036	75.1343
1.7500	0.6202	0.1878	0.3029	1.3865	7.0000	0.0926	0.0002	0.0026	104.1429
1.8000	0.6068	0.1740	0.2868	1.4390	7.5000	0.0816	0.0002	0.0019	141.8415
1.8500	0.5936	0.1612	0.2715	1.4952	8.0000	0.0725	0.0001	0.0014	190.1094
1.9000	0.5807	0.1492	0.2570	1.5553	8.5000	0.0647	0.0001	0.0011	251.0862
1.9500	0.5680	0.1381	0.2432	1.6193	9.0000	0.0581	0.0000	0.0008	327.1893
2.0000	0.5556	0.1278	0.2300	1.6875	10.0000	0.0476	0.0000	0.0005	535.9375

Appendix D.I [1]

Chapter 7 By E. Amani

Isentropic flow (with area variation)

- **Analytical solution (mass flow rate)**

$$\dot m = \rho V A.$$

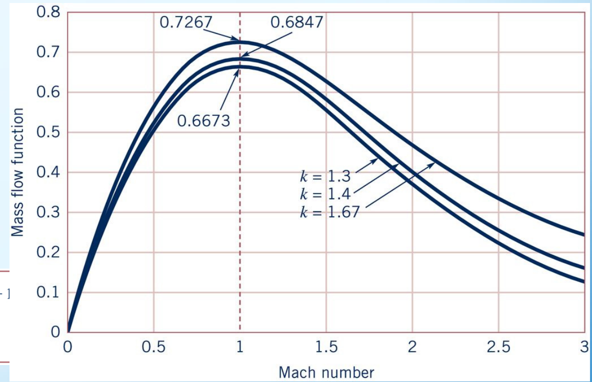
or

$$\dot m = \sqrt{\frac{k}{R}}\left(\frac{pA\mathrm{Ma}}{\sqrt{T}}\right).$$

or

$$\dot m = \frac{p_0 A^*}{\sqrt{RT_0}}\sqrt{k}\left(\frac{2}{k+1}\right)^{(k+1}$$

or



$$\frac{\dot m\sqrt{RT_0}}{p_0 A} = \sqrt{k}\mathrm{Ma}\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-(k+1)/2(k-1)}.$$

Chapter 7 **By E. Amani**

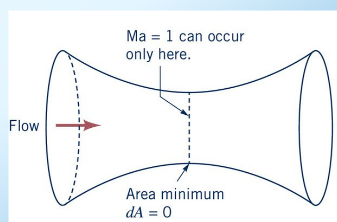
Isentropic flow (with area variation)

- **Maximum mass flow rate (choked flow)**

$$\rho V A = \rho_{th} V_{th} A_{th} = \dot m$$

$$\frac{\dot m\sqrt{RT_0}}{A_{th} p_0} = f(\mathrm{Ma}_{th}), T_0, p_0, k = cte$$

$$\text{If } \mathrm{Ma}_{th} = f(p_0, p_B) = 1 \rightarrow \left.\frac{\dot m}{A_{th}}\right)_{max}$$

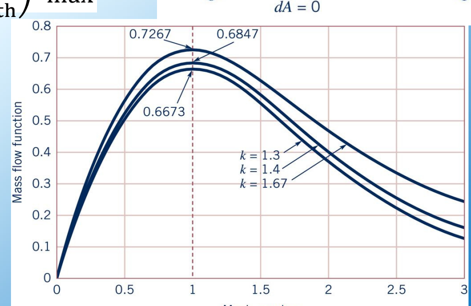


- **Fixed-geometry duct:**

$$\dot m)_{max} = cte$$

- **Variable-geometry duct:**

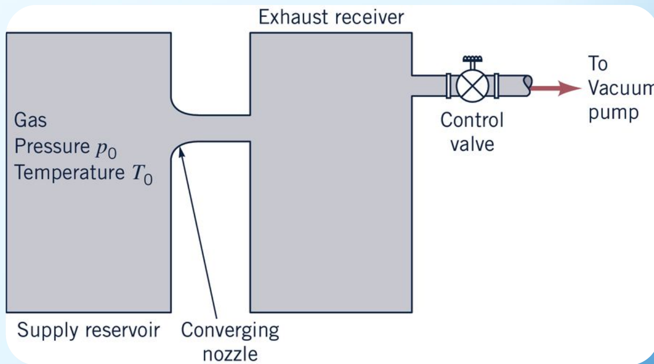
$$\dot m)_{max} = cte * A_{th}$$



Chapter 7 **By E. Amani**

Converging nozzles

- **Operation**



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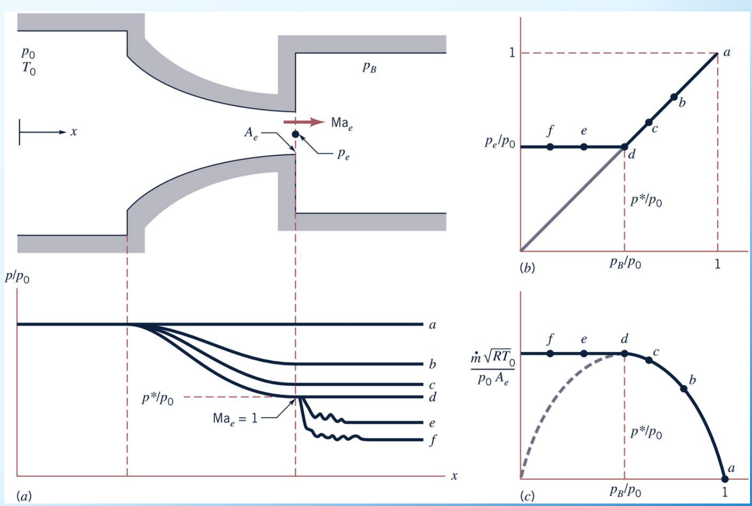
Converging nozzles

- **Operation**



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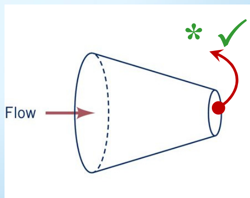
Converging nozzles

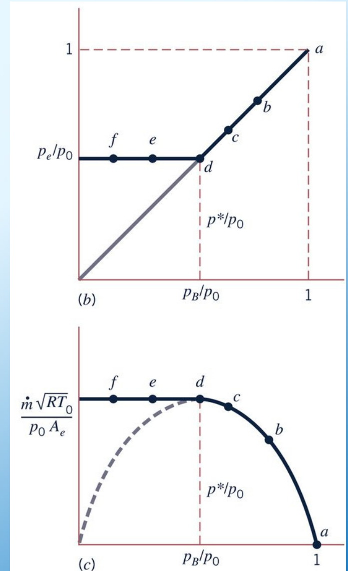
- **Calculations**

1. **Regime:**

$$\frac{p_B}{p_0} > \frac{p^*}{p_0} \longrightarrow \textbf{Unchoked}\ (p_e = p_B)$$

$$\frac{p_B}{p_0} \le \frac{p^*}{p_0} \longrightarrow \textbf{Choked}\ (A_e = A^*, \mathrm{Ma}_e = 1)$$





2. **Use isentropic and/or shock relations**

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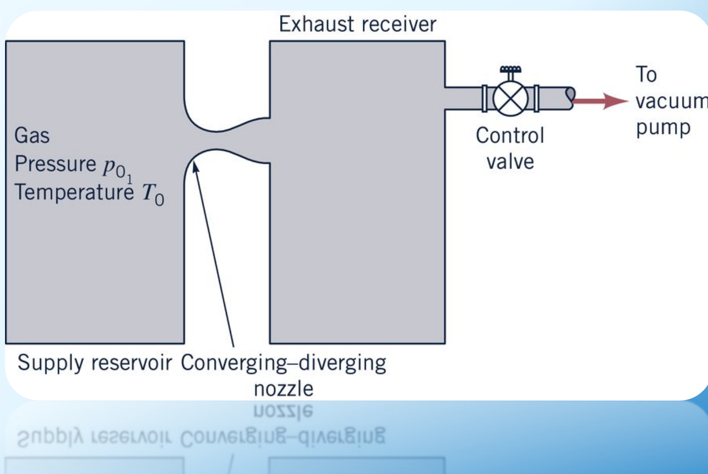
Converging-diverging nozzles

- **Operation**



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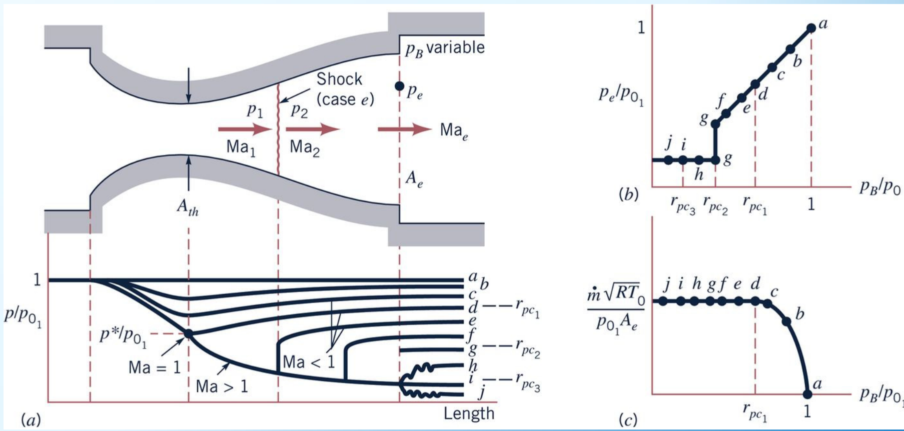
Converging-diverging nozzles

- **Operation**



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Converging-diverging nozzles

- **Calculations**

1. **Regime: compare p_B/p_0 with r_{pc1}, r_{pc2}, and r_{pc3}**

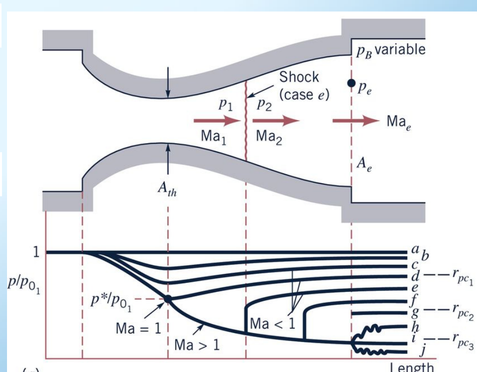
$$\mathrm{Ma}_{\mathrm{pc1}} = \mathrm{Ma}\big)_{A/A^*=A_e/A_{\mathrm{th}},\mathrm{Ma}<1}$$

$$r_{\mathrm{pc1}} = \left.\frac{p}{p_0}\right)_{\mathrm{Ma}_{\mathrm{pc1}}}$$

$$\mathrm{Ma}_{\mathrm{pc3}} = \mathrm{Ma}\big)_{A/A^*=A_e/A_{\mathrm{th}},\mathrm{Ma}>1}$$

$$r_{\mathrm{pc3}} = \left.\frac{p}{p_0}\right)_{\mathrm{Ma}_{\mathrm{pc3}}}$$

$$r_{\mathrm{pc2}} = \left.\frac{p_y}{p_x}\right)_{\mathrm{Ma}_{\mathrm{pc3}}} \times r_{\mathrm{pc3}}$$

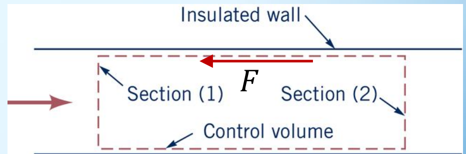


2. **Use isentropic and/or shock relations**

Chapter 7 By E. Amani

Fanno line

- **Assumptions: 1D + steady + no body force + no heat (adiabatic) + no work + constant area**
- **Equations:**



Mass ($A_1 = A_2 = A$): (1)

Momentum: (2)

Energy ($\dot{Q} = \dot{W} = 0$): (3)

2nd law ($s_2 - s_1 = \dot{S}_{gen}/\dot{m} > 0$): (*)

Eqs. of state: (4), (5)

- **For different given F**

  and unknowns (5): $\rho_2, V_2, p_2, h_2, s_2$

  The solutions can be graphed as a line called the Fanno line

Chapter 7 **By E. Amani**

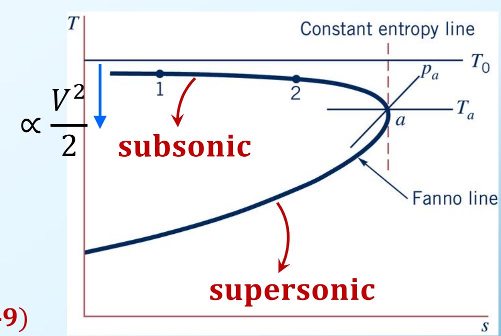
Fanno line

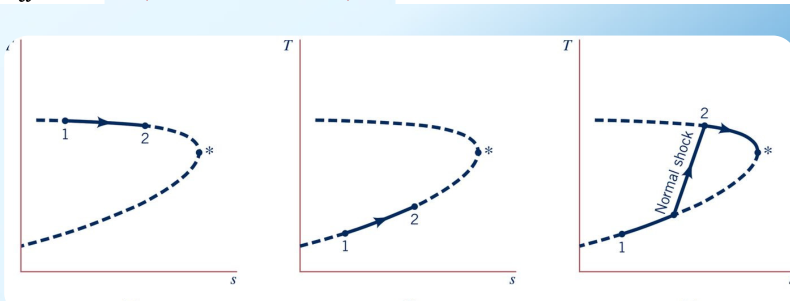
- **Conclusions:**

$$\check{h} + \frac{V^2}{2} = \check{h}_0 = \text{constant}$$

$$\rho V = G = \text{constant}$$

$$\text{Ma}_a = 1$$ **(Proof: [1] P. 649)**





(a) (b) (c)

Chapter 7 **By E. Amani**

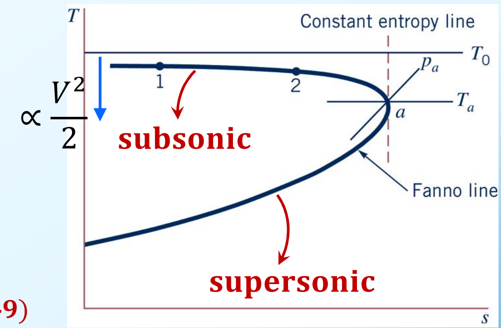
Fanno line

- **Conclusions:**

$$\breve{h} + \frac{V^2}{2} = \breve{h}_0 = \text{constant}$$

$$\rho V = G = \text{constant}$$

$$\text{Ma}_a = 1 \quad \textbf{(Proof: [1] P. 649)}$$



- **Subsonic Fanno flow:**

$$\rho = \frac{p}{RT} \Rightarrow \rho V = cte \qquad c_v(T - T_0) + \overbrace{\frac{p}{\rho}}^{RT} + \frac{V^2}{2} = cte$$

- **Supersonic Fanno flow:**

Viscous dissipation

$$c_v(T - T_0) + RT + \frac{V^2}{2} = cte \Rightarrow \rho V = cte \Rightarrow p = \rho RT$$

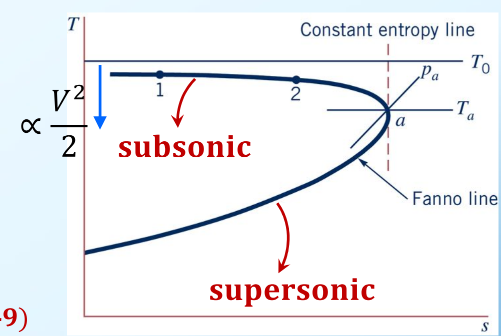
Fanno line

- **Conclusions:**

$$\breve{h} + \frac{V^2}{2} = \breve{h}_0 = \text{constant}$$

$$\rho V = G = \text{constant}$$

$$\text{Ma}_a = 1 \quad \textbf{(Proof: [1] P. 649)}$$

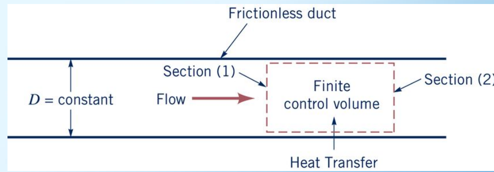


Summary of Fanno Flow Behavior

	Flow	
Parameter	**Subsonic Flow**	**Supersonic Flow**
Stagnation temperature	Constant	Constant
Ma	Increases (maximum is 1)	Decreases (minimum is 1)
Friction	Accelerates flow	Decelerates flow
Pressure	Decreases	Increases
Temperature	Decreases	Increases

Rayleigh line

- **Assumptions: 1D + steady + no body force + no friction + no work + constant area**
- **Equations:**



Mass ($A_1 = A_2 = A$): (1)

Momentum ($F = 0$): (2)

Energy ($\dot{W} = 0$): (3)

2nd law ($s_2 - s_1 = \frac{1}{\dot{m}}\int_1^2 \frac{d\dot{Q}}{T} > 0$; heating, < 0; cooling): $(*)$

Eqs. of state: (4), (5)

- **For different given $\dot{Q}$**

and unknowns (5): $\rho_2, V_2, p_2, h_2, s_2$

The solutions can be graphed as a line called the Rayleigh line

Chapter 7 By E. Amani

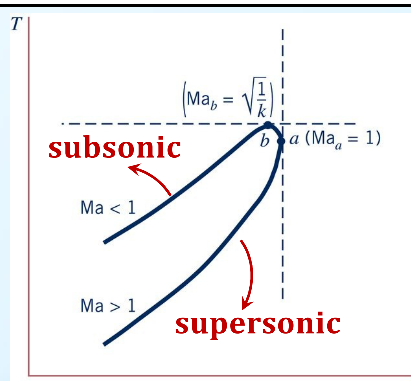
Rayleigh line

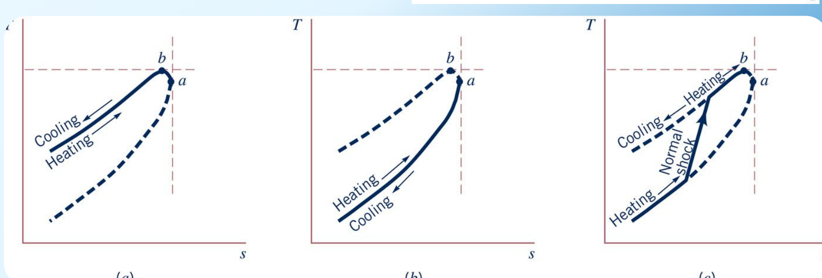
- **Conclusions:**

$$p + \frac{G^2}{\rho} = \text{constant} = I$$

$$\rho V = G = \text{constant}$$

$\text{Ma}_a = 1$ **(Proof: [1] P. 661)**





Chapter 7 By E. Amani

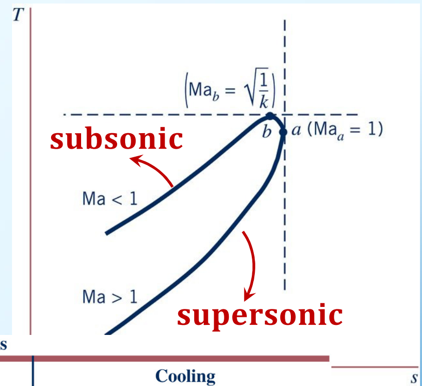
Rayleigh line

- **Conclusions:**

$$p + \frac{G^2}{\rho} = \text{constant} = I$$

$$\rho V = G = \text{constant}$$

$\mathrm{Ma}_a = 1$ **(Proof: [1] P. 661)**



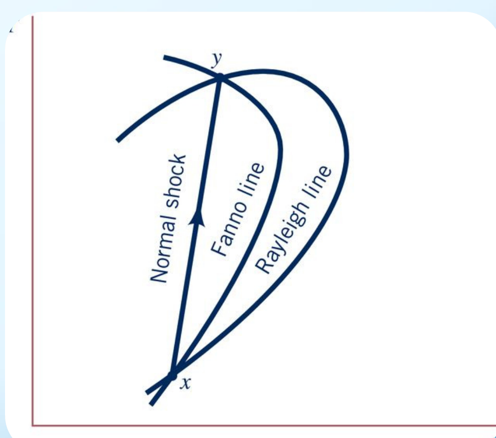
Summary of Rayleigh Flow Characteristics

	Heating		Cooling	
	Subsonic	Supersonic	Subsonic	Supersonic
V	Increase	Decrease	Decrease	Increase
Ma	Increase	Decrease	Decrease	Increase
T	Increase for $0 \le \mathrm{Ma} \le \sqrt{1/k}$ Decrease for $\sqrt{1/k} \le \mathrm{Ma} \le 1$	Increase	Decrease for $0 \le \mathrm{Ma} \le \sqrt{1/k}$ Increase for $\sqrt{1/k} \le \mathrm{Ma} \le 1$	Decrease
T_0	Increase	Increase	Decrease	Decrease
p	Decrease	Increase	Increase	Decrease
p_0	Decrease	Decrease	Increase	Increase

Chapter 7 **By E. Amani**

Normal shock

- **Intersections of the two lines:**



Chapter 7 **By E. Amani**

The Fanno line for constant c_p ideal gas

- **For Ma < 1:** $\text{Ma}^2 kRT = \text{Ma}^2 kp/\rho$

$$dF = \tfrac{1}{8}\rho V^2 f\pi D_H dx = \tfrac{\pi}{8} p D_H \text{Ma}^2 k f dx$$

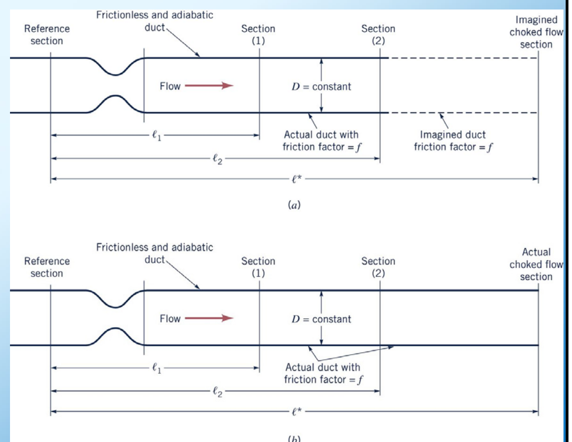
$$F = \int_{\ell_1}^{\ell_2} dF$$

$$dF = \tau_w \pi D_H dx$$

(31.4) $\tau_W = \frac{1}{8}\rho V^2 f$

Assuming turbulent fully-developed pipe flow from Moody chart

$$f\left(Re, Ma, \frac{\varepsilon}{D}\right) \sim f(Re, Ma = 0, \varepsilon/D)$$



Chapter 7 By E. Amani

The Fanno line for constant c_p ideal gas

- **Analytical solution relations (averaged f):**

$$\frac{1}{\ell_2 - \ell_1}\int_{\ell_1}^{\ell_2} f dx \to f$$

Chapter 7 By E. Amani

The Fanno line for constant c_p ideal gas

- **Analytical solution relations (averaged f):**

$$\frac{1}{k}\frac{(1-\mathrm{Ma}^2)}{\mathrm{Ma}^2}+\frac{k+1}{2k}\ln\left\{\frac{[(k+1)/2]\mathrm{Ma}^2}{1+[(k-1)/2]\mathrm{Ma}^2}\right\}=\frac{f(\ell^*-\ell)}{D}$$

$$\frac{f(\ell^*-\ell_2)}{D}-\frac{f(\ell^*-\ell_1)}{D}=\frac{f}{D}(\ell_1-\ell_2)$$

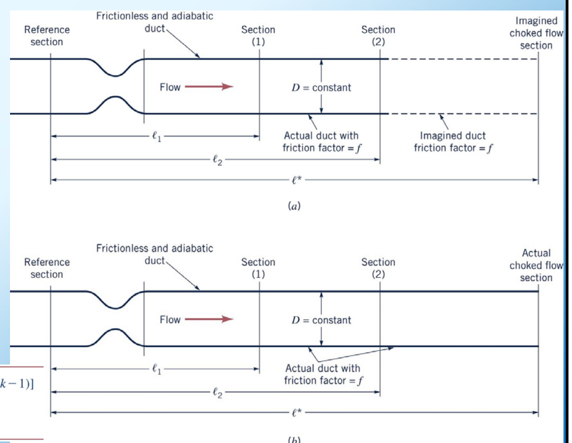
$$\frac{X}{X_1}=\frac{X/X^*)_{\mathrm{Ma}}}{X/X^*)_{\mathrm{Ma}_1}}$$

$$\frac{T}{T^*}=\frac{(k+1)/2}{1+[(k-1)/2]\mathrm{Ma}^2}$$

$$\frac{p}{p^*}=\frac{1}{\mathrm{Ma}}\left\{\frac{(k+1)/2}{1+[(k-1)/2]\mathrm{Ma}^2}\right\}^{1/2}$$

$$\frac{V}{V^*}=\left\{\frac{[(k+1)/2]\mathrm{Ma}^2}{1+[(k-1)/2]\mathrm{Ma}^2}\right\}^{1/2}$$

$$\frac{p_0}{p_0^*}=\frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{[(k+1)/2(k-1)]}$$



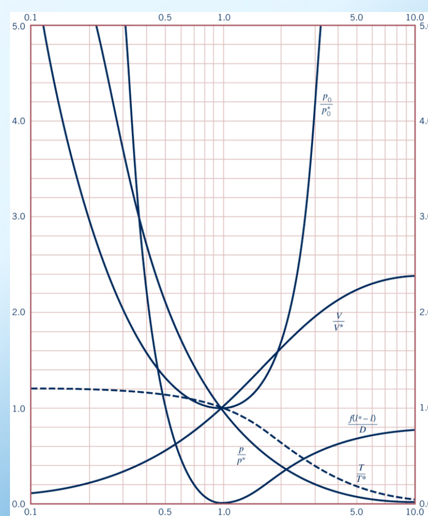
The Fanno line for constant c_p ideal gas

- **Analytical solution relations (graph):**

$$\frac{f(\ell^*-\ell_2)}{D}-\frac{f(\ell^*-\ell_1)}{D}=\frac{f}{D}(\ell_1-\ell_2)$$

$$\frac{X}{X_1}=\frac{X/X^*)_{\mathrm{Ma}}}{X/X^*)_{\mathrm{Ma}_1}}$$

$$\left.\frac{X}{X^*}\right)_{\mathrm{Ma}}$$



Appendix D. F [1]

The Fanno line for constant c_p ideal gas

- **Analytical solution relations (table):**

$$\boxed{\frac{f(\ell^* - \ell_2)}{D} - \frac{f(\ell^* - \ell_1)}{D} = \frac{f}{D}(\ell_1 - \ell_2)}$$

$$\frac{X}{X_1} = \frac{X/X^*)\ \mathrm{Ma}}{X/X^*)\ \mathrm{Ma_1}}$$

$$\left.\frac{X}{X^*}\right)\ \mathrm{Ma}$$

Fanno Flow Functions for an Ideal Gas with $k = 1.4$

Ma	$f(\ell^*-\ell)/D$	p/p^*	T/T^*	p_0/p_0^*	$\rho/\rho^* = V^*/V$	Ma	$f(\ell^*-\ell)/D$	p/p^*	T/T^*	p_0/p_0^*	$\rho/\rho^* = V^*/V$
0.00	∞	∞	1.2000	∞	∞	1.60	0.1724	0.5568	0.7937	1.2502	0.7016
0.05	280.0203	21.9034	1.1994	11.5914	18.2620	1.65	0.1902	0.5342	0.7770	1.2922	0.6876
0.10	66.9216	10.9435	1.1976	5.8218	9.1378	1.70	0.2078	0.5130	0.7605	1.3376	0.6745
0.15	27.9320	7.2866	1.1946	3.9103	6.0995	1.75	0.2250	0.4929	0.7442	1.3865	0.6624
0.20	14.5333	5.4554	1.1905	2.9635	4.5826	1.80	0.2419	0.4741	0.7282	1.4390	0.6511
0.25	8.4834	4.3546	1.1852	2.4027	3.6742	1.85	0.2583	0.4562	0.7124	1.4952	0.6404
0.30	5.2993	3.6191	1.1788	2.0351	3.0702	1.90	0.2743	0.4394	0.6969	1.5553	0.6305
0.35	3.4525	3.0922	1.1713	1.7780	2.6400	1.95	0.2899	0.4234	0.6816	1.6193	0.6211
0.40	2.3085	2.6958	1.1628	1.5901	2.3184	2.00	0.3050	0.4082	0.6667	1.6875	0.6124
0.45	1.5664	2.3865	1.1533	1.4487	2.0693	2.10	0.3339	0.3802	0.6376	1.8369	0.5963
0.50	1.0691	2.1381	1.1429	1.3398	1.8708	2.20	0.3609	0.3549	0.6098	2.0050	0.5821
0.55	0.7281	1.9341	1.1315	1.2549	1.7092	2.30	0.3862	0.3320	0.5831	2.1931	0.5694
0.60	0.4908	1.7634	1.1194	1.1882	1.5753	2.40	0.4099	0.3111	0.5576	2.4031	0.5580
0.65	0.3246	1.6183	1.1065	1.1356	1.4626	2.50	0.4320	0.2921	0.5333	2.6367	0.5477
0.70	0.2081	1.4935	1.0929	1.0944	1.3665	2.60	0.4526	0.2747	0.5102	2.8960	0.5385
0.75	0.1273	1.3848	1.0787	1.0624	1.2838	2.70	0.4718	0.2588	0.4882	3.1830	0.5301
0.80	0.0723	1.2893	1.0638	1.0382	1.2119	2.80	0.4898	0.2441	0.4673	3.5001	0.5225
0.85	0.0363	1.2047	1.0485	1.0207	1.1489	2.90	0.5065	0.2307	0.4474	3.8498	0.5155
0.90	0.0145	1.1291	1.0327	1.0089	1.0934	3.00	0.5222	0.2182	0.4286	4.2346	0.5092
0.95	0.0033	1.0613	1.0165	1.0021	1.0440	3.50	0.5864	0.1685	0.3478	6.7896	0.4845
1.00	0.0000	1.0000	1.0000	1.0000	1.0000	4.00	0.6331	0.1336	0.2857	10.7188	0.4677
1.05	0.0027	0.9443	0.9832	1.0020	0.9605	4.50	0.6676	0.1083	0.2376	16.5622	0.4559
1.10	0.0099	0.8936	0.9662	1.0079	0.9249	5.00	0.6938	0.0894	0.2000	25.0000	0.4472
1.15	0.0205	0.8471	0.9490	1.0175	0.8926	5.50	0.7140	0.0750	0.1702	36.8690	0.4407
1.20	0.0336	0.8044	0.9317	1.0304	0.8633	6.00	0.7299	0.0638	0.1463	53.1798	0.4357
1.25	0.0486	0.7649	0.9143	1.0468	0.8367	6.50	0.7425	0.0548	0.1270	75.1343	0.4317
1.30	0.0648	0.7285	0.8969	1.0663	0.8123	7.00	0.7528	0.0476	0.1111	104.1429	0.4286
1.35	0.0820	0.6947	0.8794	1.0890	0.7899	7.50	0.7612	0.0417	0.0980	141.8415	0.4260
1.40	0.0997	0.6632	0.8621	1.1149	0.7693	8.00	0.7682	0.0369	0.0870	190.1094	0.4239
1.45	0.1178	0.6339	0.8448	1.1440	0.7503	8.50	0.7740	0.0328	0.0777	251.0862	0.4221
1.50	0.1361	0.6065	0.8276	1.1762	0.7328	9.00	0.7790	0.0293	0.0698	327.1893	0.4207
1.55	0.1543	0.5808	0.8105	1.2116	0.7166	∞	0.8215	0.0000	0.0000	∞	0.4082

Appendix D. F [1]

Chapter 7 By E. Amani

The Rayleigh line for constant c_p ideal gas

- **Analytical solution relations:**

$$\frac{X}{X_1} = \frac{X/X^*)\ \mathrm{Ma}}{X/X^*)\ \mathrm{Ma_1}}$$

$$\boxed{\frac{p}{p^*} = \frac{1+k}{1+k\mathrm{Ma}^2}}$$

$$\boxed{\frac{T}{T^*} = \left[\frac{(1+k)\mathrm{Ma}}{1+k\mathrm{Ma}^2}\right]^2}$$

$$\boxed{\frac{\rho^*}{\rho} = \frac{V}{V^*} = \mathrm{Ma}\left[\frac{(1+k)\mathrm{Ma}}{1+k\mathrm{Ma}^2}\right]}$$

$$q_{1-2} = c_p(T_{02} - T_{01})$$

$$\boxed{\frac{T_0}{T_{0,}{}^*} = \frac{2(k+1)\mathrm{Ma}^2\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)}{(1+k\mathrm{Ma}^2)^2}}$$

$$\boxed{\frac{p_0}{p_{0,}{}^*} = \frac{(1+k)}{(1+k\mathrm{Ma}^2)}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{k/(k-1)}}$$

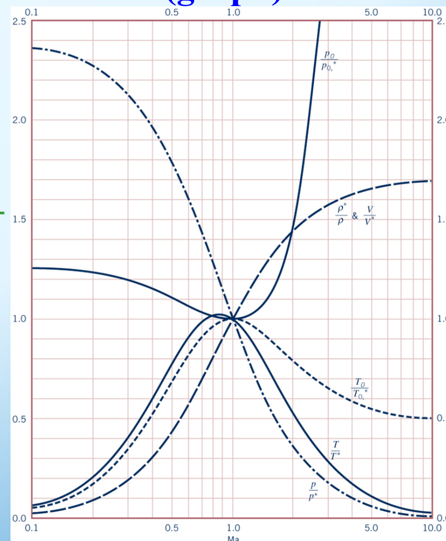
Chapter 7 By E. Amani

The Rayleigh line for constant c_p ideal gas

- **Analytical solution relations (graph):**

$$\frac{X}{X_1} = \frac{X/X^*)_{\mathrm{Ma}}}{X/X^*)_{\mathrm{Ma}_1}}$$

$$\left.\frac{X}{X^*}\right)_{\mathrm{Ma}}$$



Chapter 7 Appendix D. R [1] By E. Amani

The Rayleigh line for constant c_p ideal gas

- **Analytical solution relations (table):**

$$\frac{X}{X_1} = \frac{X/X^*)_{\mathrm{Ma}}}{X/X^*)_{\mathrm{Ma}_1}}$$

$$\left.\frac{X}{X^*}\right)_{\mathrm{Ma}}$$

Raleigh Flow Functions for an Ideal Gas with $k = 1.4$

Ma	T_0/T_0^*	p/p^*	T/T^*	p_0/p_0^*	$\rho/\rho^* = V^*/V$	Ma	T_0/T_0^*	p/p^*	T/T^*	p_0/p_0^*	$\rho/\rho^* = V^*/V$
0.00	0.0000	2.4000	0.0000	1.2679	∞	1.60	1.7699	0.5236	0.7017	1.1756	0.7461
0.05	0.0105	2.3916	0.0143	1.2657	167.2500	1.65	1.8422	0.4988	0.6774	1.2066	0.7364
0.10	0.0370	2.3669	0.0560	1.2591	42.2500	1.70	1.9161	0.4756	0.6538	1.2402	0.7275
0.15	0.0741	2.3267	0.1218	1.2486	19.1019	1.75	1.9915	0.4539	0.6310	1.2767	0.7194
0.20	0.1181	2.2727	0.2066	1.2346	11.0000	1.80	2.0685	0.4335	0.6089	1.3159	0.7119
0.25	0.1667	2.2069	0.3044	1.2177	7.2500	1.85	2.1472	0.4144	0.5877	1.3581	0.7051
0.30	0.2181	2.1314	0.4089	1.1985	5.2130	1.90	2.2275	0.3964	0.5673	1.4033	0.6988
0.35	0.2713	2.0487	0.5141	1.1779	3.9847	1.95	2.3096	0.3795	0.5477	1.4516	0.6929
0.40	0.3257	1.9608	0.6151	1.1566	3.1875	2.00	2.3934	0.3636	0.5289	1.5031	0.6875
0.45	0.3807	1.8699	0.7080	1.1351	2.6409	2.10	2.5663	0.3345	0.4936	1.6162	0.6778
0.50	0.4360	1.7778	0.7901	1.1141	2.2500	2.20	2.7466	0.3086	0.4611	1.7434	0.6694
0.55	0.4915	1.6860	0.8599	1.0940	1.9607	2.30	2.9344	0.2855	0.4312	1.8860	0.6621
0.60	0.5471	1.5957	0.9167	1.0753	1.7407	2.40	3.1299	0.2648	0.4038	2.0451	0.6557
0.65	0.6029	1.5080	0.9608	1.0582	1.5695	2.50	3.3333	0.2462	0.3787	2.2218	0.6500
0.70	0.6587	1.4235	0.9929	1.0431	1.4337	2.60	3.5448	0.2294	0.3556	2.4177	0.6450
0.75	0.7148	1.3427	1.0140	1.0301	1.3241	2.70	3.7644	0.2142	0.3344	2.6343	0.6405
0.80	0.7710	1.2658	1.0255	1.0193	1.2344	2.80	3.9923	0.2004	0.3149	2.8731	0.6365
0.84515	0.8221	1.2000	1.0286	1.0116	1.1667	2.90	4.2286	0.1879	0.2969	3.1359	0.6329
0.85	0.8276	1.1931	1.0285	1.0109	1.1600	3.00	4.4734	0.1765	0.2803	3.4245	0.6296
0.90	0.8845	1.1246	1.0245	1.0049	1.0977	3.50	5.8276	0.1322	0.2142	5.3280	0.6173
0.95	0.9420	1.0603	1.0146	1.0012	1.0450	4.00	7.4050	0.1026	0.1683	8.2268	0.6094
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	4.50	9.2111	0.0818	0.1354	12.5023	0.6039
1.05	1.0587	0.9436	0.9816	1.0012	0.9613	5.00	11.2500	0.0667	0.1111	18.6339	0.6000
1.10	1.1181	0.8909	0.9603	1.0049	0.9277	5.50	13.5244	0.0554	0.0927	27.2113	0.5971
1.15	1.1784	0.8417	0.9369	1.0109	0.8984	6.00	16.0362	0.0467	0.0785	38.9459	0.5949
1.20	1.2395	0.7958	0.9118	1.0194	0.8727	6.50	18.7870	0.0399	0.0673	54.6830	0.5932
1.25	1.3017	0.7529	0.8858	1.0303	0.8500	7.00	21.7778	0.0345	0.0583	75.4138	0.5918
1.30	1.3649	0.7130	0.8592	1.0437	0.8299	7.50	25.0095	0.0301	0.0509	102.2875	0.5907
1.35	1.4292	0.6758	0.8323	1.0594	0.8120	8.00	28.4827	0.0265	0.0449	136.6235	0.5898
1.40	1.4947	0.6410	0.8054	1.0777	0.7959	8.50	32.1979	0.0235	0.0399	179.9236	0.5891
1.45	1.5615	0.6086	0.7787	1.0983	0.7815	9.00	36.1557	0.0210	0.0356	233.8840	0.5885
1.50	1.6296	0.5783	0.7525	1.1215	0.7685	10	44.8000	0.0170	0.0290	381.6149	0.5875
1.55	1.6990	0.5500	0.7268	1.1473	0.7568	∞	∞	0.0000	0.0000	∞	0.5833

Chapter 7 Appendix D. R [1] By E. Amani

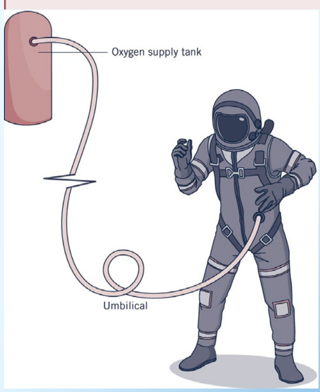
An example problem [1]

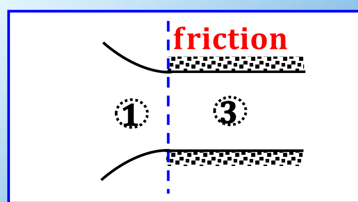
- **Breaking down into two coupled sub-problems**

EXAMPLE 11.12 Duct Sizing in Fanno Flow

GIVEN An astronaut breathes oxygen supplied to a spacesuit through an umbilical cord (Fig. E11.12*a*). The umbilical is 7 m long. The oxygen is supplied from a storage tank at 100 kPa and 283 K. The suit pressure is 20 kPa.

FIND The required inside diameter of the umbilical to supply 0.05 kg/s of oxygen. The inside of the umbilical is lined with a material with roughness of 0.01 mm.





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An online compressible flow solver

- **CFLOW**

https://onlineflowcalculator.com/pages/CFLOW/calculator.html

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The end of chapter 7

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