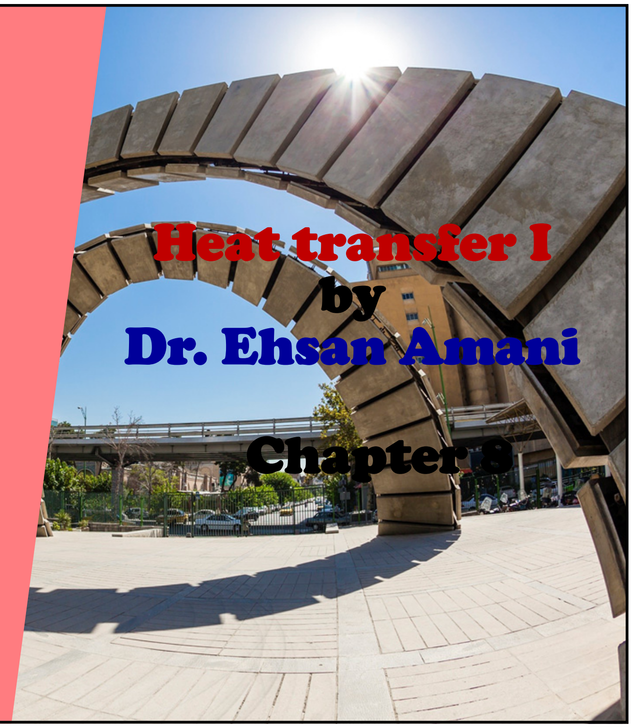
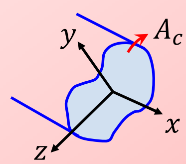


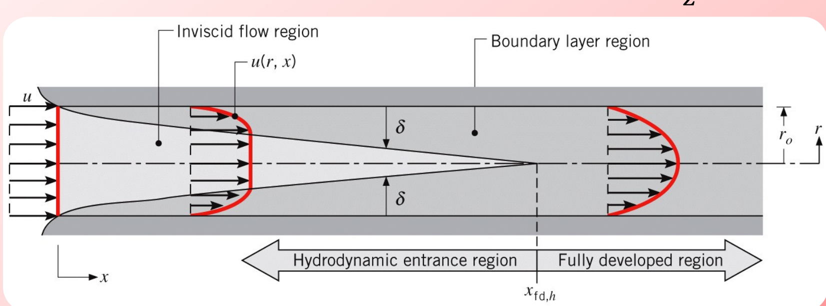
# Heat transfer I
by
**Dr. Ehsan Amani**

**Chapter 8**

## 1.8 Internal flow hydrodynamics

- **Entrance vs. fully-developed**
  - If $A_c, V = cte$, a fully-developed region is established
  - $x > x_{fd}: v = w = 0$




Chapter 8 By E. Amani

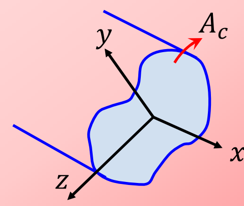
## 1.8 Internal flow hydrodynamics

- **Entrance vs. fully-developed**
  - Exercise: Show that for an incompressible flow in the fully-developed region of channel of arbitrary cross-section:

$$u = u(y,z,t) \xrightarrow{\text{steady}} u = u(y,z)$$

$$p = u(x,t) \xrightarrow{\text{steady}} p = p(x)$$

$$\frac{dp}{dx} = f(t) \xrightarrow{\text{steady}} \frac{dp}{dx} = cte$$

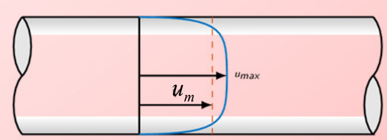





## 1.8 Internal flow hydrodynamics

- **Mean (or bulk) velocity**

$$u_m(x,t) = \frac{\overbrace{\int_{A_c} \rho u dA_c}^{\dot{m}(x,t)}}{\int_{A_c} \rho dA_c} \qquad (1.8)$$




  - For incompressible flow:

$$u_m(x,t) = \frac{\dot{m}(t)}{\rho A_c} = \frac{\int_{A_c} u dA_c}{A_c} \qquad (2.8) \xrightarrow[A_c = cte]{\text{Steady}} u_m = cte$$

- **Relevant characteristic parameters:**

$V \equiv u_m$ $\qquad L \equiv D_H$ Hydraulic diameter

$$D_H \equiv \frac{4A_c}{P} \quad (3.8) \xrightarrow[\text{pipe}]{\text{Circular}} D_H = \frac{4\frac{\pi}{4}D^2}{\pi D} = D \quad (3.8)'$$

Wetted perimeter

# 1.8 Internal flow hydrodynamics

- **Relevant Reynolds number**

$$\mathrm{Re}_D \equiv \frac{u_m D_H}{\nu} \quad (4.8)$$

  - For incompressible flows:

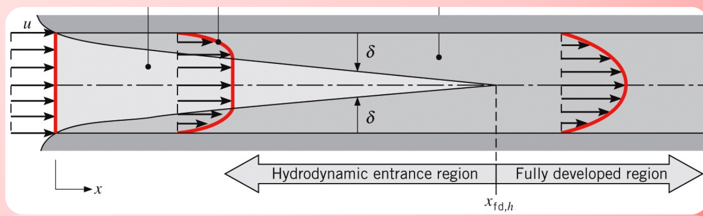
$$\mathrm{Re}_D = \frac{\dot{m} D_H}{\mu A_c} = \frac{4\dot{m}}{\mu P} \quad (5.8)$$

**Steady, $A_c = cte$:** $\mathrm{Re}_D = cte$

**Circular pipe:** $\mathrm{Re}_D = \dfrac{4\dot{m}}{\pi D \mu} \quad (5.8)'$

- **Laminar vs. turbulent:**

$\mathrm{Re}_D < \mathrm{Re}_{D,c}$

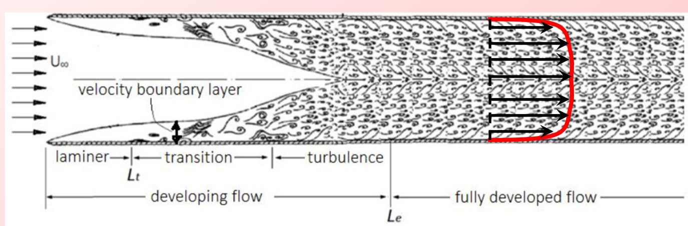





# 1.8 Internal flow hydrodynamics

- **Laminar vs. turbulent:**

$\mathrm{Re}_D > \mathrm{Re}_{D,c}$




$$\mathrm{Re}_{D,c} \sim 2300 \quad (2000\text{-}10\,000) \qquad (6.8)$$

  - Hydrodynamic entry length:

$$\text{Turbulent Flow: } 10 < \left(x_{fd,h}/D\right) < 60 \qquad (7.8)$$
$$\text{Laminar Flow: } \left(x_{fd,h}/D\right) \approx 0.05\, Re_D$$

## 1.8 Internal flow hydrodynamics

- **Pressure drop and friction:**

**Friction coefficient or Fanning friction factor**

$$c_f \equiv \frac{\tau_s}{\rho u_m^2/2} \quad (8.8)$$

**friction factor**

$$f \equiv -\frac{(dp/dx)D_H}{\rho u_m^2/2} \quad (9.8)$$

- Exercise: show that for steady fully-developed flow:

$$c_f = \frac{f}{4} \quad (10.8)$$

- All equations of this chapter are valid considering the elevation change by:

$$p \rightarrow p_e = p + (C - \rho g_x x - \rho g_y y - \rho g_z z) \qquad \frac{\partial p}{\partial x} \rightarrow \frac{\partial p_e}{\partial x} = \frac{\partial p}{\partial x} - \rho g_x$$

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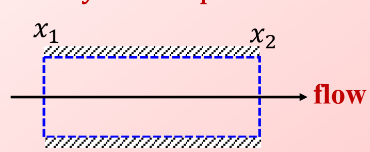
## 1.8 Internal flow hydrodynamics

- **Pressure drop and friction:**
  - Note that for fully-developed flow




$$\Delta p = -\int_1^2 dp = -\int_1^2 \underbrace{\frac{dp}{dx}}_{f(t)} dx = -\underbrace{\frac{dp}{dx}}_{(9.8)} \int_1^2 dx = f\frac{\rho u_m^2}{2D_H}(x_2 - x_1)$$

?

$$\Delta p = f\frac{\rho u_m^2}{2D_H}(x_2 - x_1) \quad (13.8)$$

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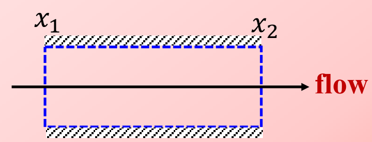
# 1.8 Internal flow hydrodynamics

- **Power requirement: Mechanical power loss**
  - To be provided by pump, blower, compressor, etc.
  - Assumptions:
    1. Incompressible
    2. Steady
    3. Channel flow




$$\dot{W}_{\text{loss}} = \int_{x_1} \left(\frac{p}{\rho} + \frac{v^2}{2}\right) \rho u dA_c - \int_{x_2} \left(\frac{p}{\rho} + \frac{v^2}{2}\right) \rho u dA_c \qquad (11.8)$$

4. Fully-developed flow

$$\dot{W}_{\text{loss}} = Q(p_1 - p_2) = Q\Delta p \qquad (12.8)$$

$$\Delta p = f \frac{\rho u_m^2}{2D_H}(x_2 - x_1)$$

?

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# 1.8 Internal flow hydrodynamics

- **Friction factor calculation:**
  - Laminar, steady, incompressible flow with constant properties in circular pipes (fluid mechanics II):

$$u(r) = -\frac{r_0^2}{4\mu}\left(\frac{dp}{dx}\right)\left[1 - \left(\frac{r}{r_0}\right)^2\right] \qquad (14.8)$$

✓ Exercise: Show that

$$u_m = \frac{\dot{m}}{\rho A_c} = -\frac{r_0^2}{8\mu}\left(\frac{dp}{dx}\right) \qquad (15.8) \qquad u_m \leftrightarrow \dot{m} \leftrightarrow \frac{dp}{dx}$$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_0}\right)^2\right] \qquad (16.8)$$

$$c_f = \frac{16}{\text{Re}_D} \qquad (17.8) \qquad f = \frac{64}{\text{Re}_D} \qquad (18.8)$$

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# 1.8 Internal flow hydrodynamics

- **Friction factor calculation:**
  - Laminar, steady, incompressible flow with constant properties in non-circular pipes:

$$f\mathrm{Re}_D = cte \quad (19.8)$$

TABLE 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

| Cross Section | $\frac{b}{a}$ | $Nu_D \equiv \frac{hD_h}{k}$ (Uniform $q_s''$) | $Nu_D \equiv \frac{hD_h}{k}$ (Uniform $T_s$) | $f Re_{D_h}$ |
|---|---|---|---|---|
| | — | 4.36 | 3.66 | 64 |
| a b | 1.0 | 3.61 | 2.98 | 57 |
| a b | 1.43 | 3.73 | 3.08 | 59 |
| a b | 2.0 | 4.12 | 3.39 | 62 |
| a b | 3.0 | 4.79 | 3.96 | 69 |
| a b | 4.0 | 5.33 | 4.44 | 73 |
| a b | 8.0 | 6.49 | 5.60 | 82 |
| | ∞ | 8.23 | 7.54 | 96 |
| Heated / Insulated | ∞ | 5.39 | 4.86 | 96 |
| | — | 3.11 | 2.47 | 53 |

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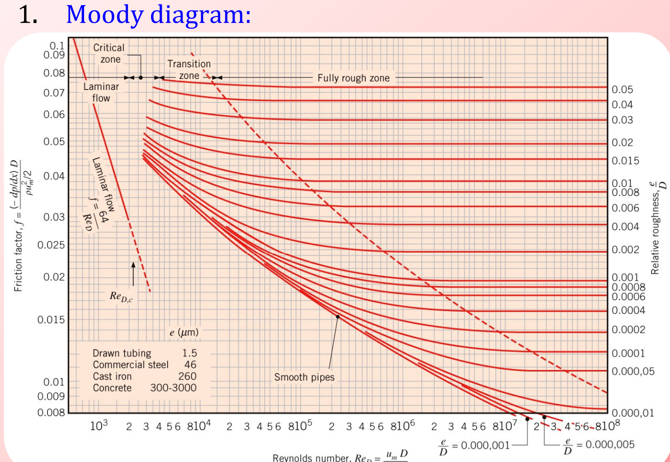
# 1.8 Internal flow hydrodynamics

- **Friction factor calculation:**
  - Turbulent, steady, incompressible flow:
    1. Moody diagram:

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# 1.8 Internal flow hydrodynamics

- **Friction factor calculation:**
  - Turbulent, steady, incompressible flow:
    1. Moody diagram:
    2. Curve-fits to the Moody diagram:

| | | |
|---|---|---|
| $\frac{1}{\sqrt{f}} = -2.0 \log\left[\frac{e/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right]$ | (8.20)[b] | Turbulent, fully developed |
| $f = (0.790 \ln Re_D - 1.64)^{-2}$ | (8.21)[b] | Turbulent, fully developed, smooth walls, $3000 \lesssim Re_D \lesssim 5 \times 10^6$ |

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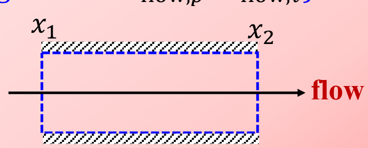
# 2.8 The mean and reference temperatures

- **Thermomechanical energy balance on a channel:**
  - Eq. (21.6)′ for a steady incompressible fluid with assumption 5 chap 2 (neglecting $KE, PE, \dot{W}_{\text{flow},p}, \dot{W}_{\text{flow},\tau}$):

$$\cancelto{0}{(\dot{U}_t)_{\text{st}}} = \int_{A_c:\text{in}} u_t \, d\dot{m} - \int_{A_c:\text{out}} u_t \, d\dot{m} + q + \dot{W}_s + \int_V \dot{q} \, dV \qquad (20.8)$$

steady




  - Eq. (21.6)′ for a steady ideal gas with assumption 4 chap 2 (neglecting $KE, PE, \dot{W}_{\text{flow},\tau}$):

$$\cancelto{0}{(\dot{U}_t)_{\text{st}}} = \int_{A_c:\text{in}} h_t \, d\dot{m} - \int_{A_c:\text{out}} h_t \, d\dot{m} + q + \dot{W}_s + \int_V \dot{q} \, dV \qquad (21.8)$$

steady

  - The mean temperature is defined to facilitate the computation of thermal energy (for incompressible fluid) or enthalpy (for ideal gas) integral fluxes

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# 2.8 The mean and reference temperatures

- **The mean temperature:**
  - No phase-change + $c_p = cte$:

$$\dot{m} c_p T_m \equiv \int_{A_c} c_p\, T d\dot{m} \begin{cases} \int_{A_c} u_t\, d\dot{m} & \text{; incompressible fluid} \\ \int_{A_c} h_t\, d\dot{m} & \text{; ideal gas} \end{cases} \qquad (22.8)$$

$$T_m = \frac{\int_{A_c} T d\dot{m}}{\dot{m}} = \frac{\int_{A_c} \rho u T dA_c}{\int_{A_c} \rho u dA_c} \quad (23.8)' \xrightarrow[\text{flow}]{\text{Incompressible}} T_m = \frac{\int_{A_c} u T dA_c}{u_m A_c} \quad (23.8)''$$

  - $T_m$ is , in fact, the mass-weighted averaging of temperature over the channel cross-section

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# 2.8 The mean and reference temperatures

- **The mean temperature:**
  - The characteristic temperature for internal flows:

$$T_{\text{ref}} = T_m; \quad q_s'' = h(T_s - T_m) \qquad (24.8)$$

  - Note: The value of $h$ depends on the definition of $T_{\text{ref}}$

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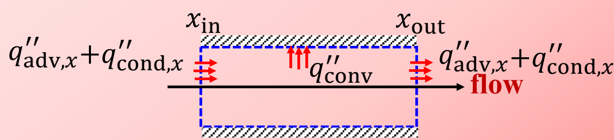
## 3.8 The computation of $T_m$

- **Assumptions:**
  - Ideal gas with 4 or incompressible fluid with 5:
  - Steady flow
  - No phase-change
  - No thermal energy generation
  - Negligible viscous dissipation
  - $c_p = cte$
  - Negligible streamwise conduction, i.e., $\mathrm{Pe}_D \gg 1$ [3]

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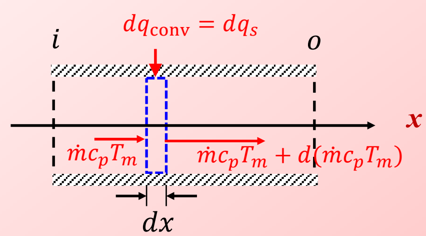
## 3.8 The computation of $T_m$

- **Energy balance, Eq. (20.8) or (21.8):**




**Lecture Notes**

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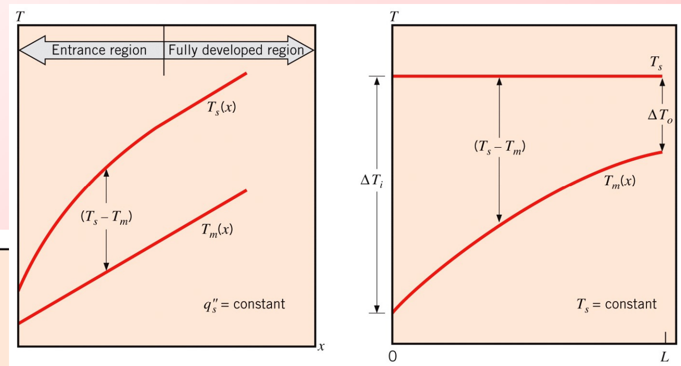
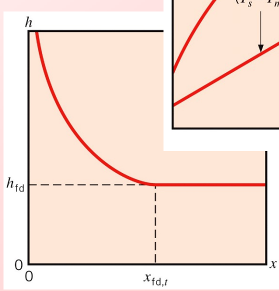
## 3.8 The computation of $T_m$

- **The mean temperature variation along the pipe:**




$$q_s'' = h(T_s - T_m)$$

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## 4.8 Definitions of $h_{ave}$

- **For internal flows and constant wall heat flux: Eqs. (11.6) and (15.6)**

$$q_s \equiv h_{ave} A_s (\bar{T}_s - \bar{T}_m) \quad (11.6) \qquad h_{ave} = (\overline{h^{-1}})^{-1} \quad (15.6)$$

$$\longrightarrow \bar{T}_s = \bar{T}_m + q_s''/h_{ave} \quad (31.8)$$

$$\bar{T}_m = \frac{1}{L}\int_0^L (27.8)dx = T_{m,i} + \frac{q_s'' PL}{2\dot{m}c_p} \quad (32.8)$$

- **For internal flows and constant wall temperature: Eqs. (11.6) is not useful !?**

⇨ **Lecture Notes**

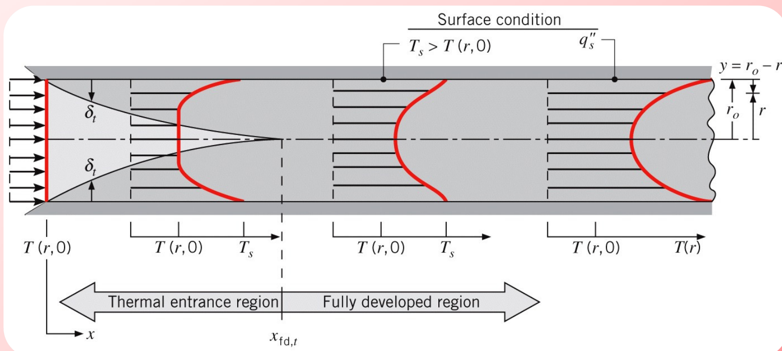
**Chapter 8** **By E. Amani**

# 5.8 Thermally fully-developed condition

- **Experiment:**

Turbulent Flow: $10 < \left(x_{fd,t} / D\right) < 60$

Laminar Flow: $\left(x_{fd,t} / D\right) \approx 0.05\, Re_D Pr$ (37.8)

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# 5.8 Thermally fully-developed condition

- **Condition:**

$$\frac{\partial T}{\partial x} \neq 0 \ !$$

- Assuming negligible streamwise conduction, or $Pe_D \gg 1$ [3]:

$$\theta \equiv \frac{T_s(x) - T(x,r)}{T_s(x) - T_m(x)} \quad (39.8) \longrightarrow \boxed{\frac{\partial \theta}{\partial x} = 0} \quad \boxed{\theta = \theta(r)} \quad (38.8)$$

- Corollary:

$$h(x) = \frac{q_s''(x)}{(T_s(x) - T_m(x))} = \frac{-k \left.\frac{dT}{dy}\right|_{y=0}}{(T_s(x) - T_m(x))} = \frac{k \left.\frac{dT}{dr}\right|_{r=r_0}}{(T_s(x) - T_m(x))} = -k \left.\frac{d\theta}{dr}\right|_{r=r_0}$$

$$\xrightarrow[(38.8)]{} h(x) = h = cte$$

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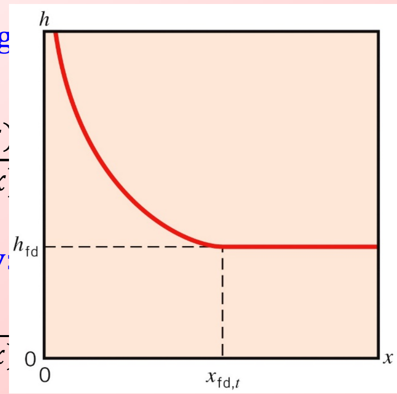
## 5.8 Thermally fully-developed condition

- **Condition:**

$$\frac{\partial T}{\partial x} \neq 0 \ !$$

- Assuming ... uction, or $\mathrm{Pe}_D \gg 1$ [3]:

$$\theta \equiv \frac{T_s(x) ...}{T_s(x) ...} \quad \boxed{... \frac{\partial\theta}{\partial x} = 0} \quad \boxed{... = \theta(r)} \quad (38.8)$$




- Corollary: ...

$$h(x) = \frac{q_s''(x)}{(T_s(x) - T_m(x) ...} ... \left.\frac{\partial T}{\partial r}\right|_{r=r_0} ... - T_m(x)) = -k \left.\frac{d\theta}{dr}\right|_{r=r_0}$$

$$\xrightarrow{(38.8)} h(x) = h = cte$$

**Chapter 8** **By E. Amani**

## 6.8 The computation of $h$

- **Laminar fully-developed flow in circular pipes:**
  - There is an analytical solution for $q_s'' = cte$ and semi-analytical solution for $T_s = cte$
  - See file "chap8-proofs.pdf" and references [1,3] for the derivations

**TABLE 8.4** Summary of convection correlations for flow in a circular tube[a,d]

| Correlation | | Conditions |
|---|---|---|
| $f = 64/Re_D$ | (8.19) | Laminar, fully developed |
| $Nu_D = 4.36$ | (8.53) | Laminar, fully developed, uniform $q_s''$ |
| $Nu_D = 3.66$ | (8.55) | Laminar, fully developed, uniform $T_s$ |

**Chapter 8** **By E. Amani**

## 6.8 The computation of $h$

- **Laminar entrance flow in circular pipes:**

| | | |
|---|---|---|
| $\overline{Nu}_D = 3.66 + \dfrac{0.0668\, Gz_D}{1 + 0.04\, Gz_D^{2/3}}$ | (8.57) | Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform $T_s$, $Gz_D = (D/x)\, Re_D\, Pr$ |
| $\overline{Nu}_D = \dfrac{\dfrac{3.66}{\tanh[2.264\, Gz_D^{-1/3} + 1.7\, Gz_D^{-2/3}]} + 0.0499\, Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\, Pr^{1/6}\, Gz_D^{-1/6})}$ | (8.58) | Laminar, combined entry, $Pr \gtrsim 0.1$, uniform $T_s$, $Gz_D = (D/x)\, Re_D\, Pr$ |

?

$$\mathrm{Gz}_D \equiv \left[\frac{x/D}{\mathrm{Re}_D \mathrm{Pr}}\right]^{-1} = \left[\frac{x/D}{\mathrm{Pe}_D}\right]^{-1} \qquad (53.8)$$

Graetz number

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## 6.8 The computation of $h$

- **Turbulent fully-developed flow in circular pipes:**

| | | |
|---|---|---|
| $Nu_D = 0.023\, Re_D^{4/5}\, Pr^n$ | (8.60)[c] | Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$ |
| $Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3} \left(\dfrac{\mu}{\mu_s}\right)^{0.14}$ | (8.61)[c] | Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$ |
| $Nu_D = \dfrac{(f/8)(Re_D - 1000)\, Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$ | (8.62)[c] | Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$ |
| $Nu_D = 4.82 + 0.0185(Re_D\, Pr)^{0.827}$ | (8.64) | Liquid metals, turbulent, fully developed, uniform $q_s''$, $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2}$, $10^2 \lesssim Re_D\, Pr \lesssim 10^4$ |
| $Nu_D = 5.0 + 0.025(Re_D\, Pr)^{0.8}$ | (8.65) | Liquid metals, turbulent, fully developed, uniform $T_s$, $Re_D\, Pr \gtrsim 100$ |

[a]Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on $T_m$; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f = (T_s + T_m)/2$; properties in Equations 8.57 and 8.58 are based on $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.

[b]Equation 8.20 pertains to smooth or rough tubes. Equation 8.21 pertains to smooth tubes.

[c]As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.

[d]For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

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# 6.8 The computation of $h$

- **Fully-developed flow in non-circular pipes:**
  - For turbulent flows with $\Pr \geq 0.7$, the circular pipe correlations can be used $(D \rightarrow D_H)$
  - For laminar flows:

**TABLE 8.1** Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

| Cross Section | $\frac{b}{a}$ | $Nu_D = \frac{hD_h}{k}$ (Uniform $q_s''$) | (Uniform $T_s$) | $f Re_{D_h}$ |
|---|---|---|---|---|
| Circle | — | 4.36 | 3.66 | 64 |
| Rectangle $a \times b$ | 1.0 | 3.61 | 2.98 | 57 |
| Rectangle $a \times b$ | 1.43 | 3.73 | 3.08 | 59 |
| Rectangle $a \times b$ | 2.0 | 4.12 | 3.39 | 62 |
| Rectangle $a \times b$ | 3.0 | 4.79 | 3.96 | 69 |
| Rectangle $a \times b$ | 4.0 | 5.33 | 4.44 | 73 |
| Rectangle $a \times b$ | 8.0 | 6.49 | 5.60 | 82 |
| Parallel plates | ∞ | 8.23 | 7.54 | 96 |
| Heated / Insulated | ∞ | 5.39 | 4.86 | 96 |
| Triangle | — | 3.11 | 2.47 | 53 |

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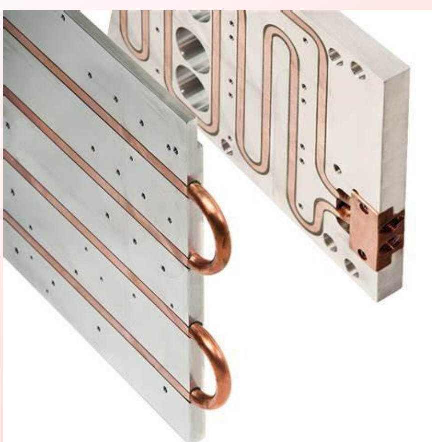
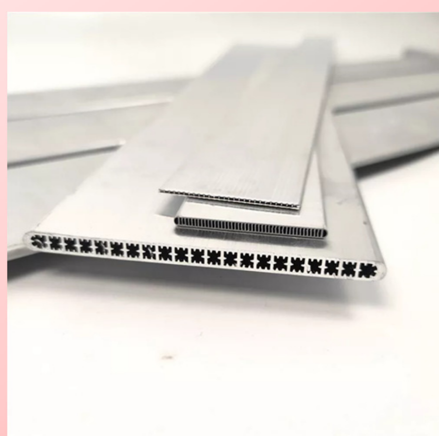
# 7.8 Sample problem: Cold plate





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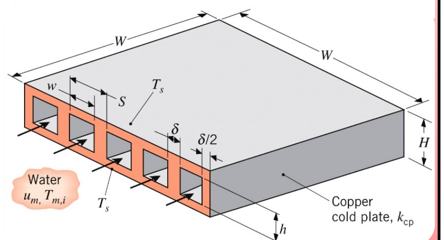
# 7.8 Sample problem: Cold plate

## Sample problem

**8.82** A *cold plate* is an active cooling device that is attached to a heat-generating system in order to dissipate the heat while maintaining the system at an acceptable temperature. It is typically fabricated from a material of high thermal conductivity, $k_{cp}$, within which channels are machined and a coolant is passed. Consider a copper cold plate of height $H$ and width $W$ on a side, within which water passes through square channels of width $w = h$. The transverse spacing between channels $\delta$ is twice the spacing between the sidewall of an outer channel and the sidewall of the cold plate.
Consider conditions for which *equivalent* heat-generating systems are attached to the top and bottom of the cold plate, maintaining the corresponding surfaces at the same temperature $T_s$. The mean velocity and inlet temperature of the coolant are $u_m$ and $T_{m,i}$, respectively.

Assume 1D conduction in vertical direction. The side walls are insulated. The inner channel surface temperature is assumed constant at $T_s = 360$ (large wall conductivity).

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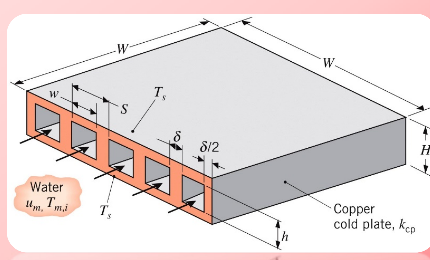
(a) Assuming fully developed turbulent flow throughout each channel, obtain a system of equations that may be used to evaluate the total rate of heat transfer to the cold plate, $q$, and the outlet temperature of the water, $T_{m,o}$, in terms of the specified parameters.

(b) Consider a cold plate of width $W = 100$ mm and height $H = 10$ mm, with 10 square channels of width $w = 6$ mm and a spacing of $\delta = 4$ mm between channels. Water enters the channels at a temperature of $T_{m,i} = 300$ K and a velocity of $u_m = 2$ m/s. If the top and bottom cold plate surfaces are at $T_s = 360$ K, what is the outlet water temperature and the total rate of heat transfer to the cold plate? The thermal conductivity of the copper is 400 W/m·K, while average properties of the water may be taken to be $\rho = 984$ kg/m$^3$, $c_p = 4184$ J/kg·K, $\mu = 489 \times 10^{-6}$ N·s/m$^2$, $k = 0.65$ W/m·K, and $Pr = 3.15$. Is this a good cold plate design? How could its performance be improved?

(c) For improving the accuracy of the calculations, determine the temperature for evaluating the properties.
(d) Calculate the power required to flow the water inside the cold plate.

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