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B. proof (3D):

$\vec{x}(u,v) = x(u,v)\,\hat{i} + y(u,v)\,\hat{j} + z(u,v)\,\hat{k}$ $\quad t \to$ is implied

$$\begin{cases} \vec{x}_u = \partial\vec{x}/\partial u & (1.B3) \\ \vec{x}_v = \partial\vec{x}/\partial v & (2.B3) \\ \hat{n} = \dfrac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|} & (3.B3) \end{cases} \qquad \begin{cases} \hat{n} \to (3.B3) \\ \hat{t} = d\hat{x}_e^{(s)}/ds & \vec{x}_e : \text{tangent to edge} \\ \hat{p} = \hat{t} \times \hat{n} & (4.B3) \end{cases}$$

For volume fied operation

surface del $\vec{\nabla}_s \equiv (\underline{I} - \hat{n}\hat{n}) \cdot \vec{\nabla} = \vec{\nabla} - \hat{n}(\hat{n}\cdot\vec{\nabla})$ (5.B3)

surface identity tensor $\underline{I}_s \equiv \underline{I} - \hat{n}\hat{n}$ (6.B3)

$\underline{I}_s \cdot \vec{A} = \vec{A}_s$ (7.B3) $\longrightarrow \underline{I}_s \cdot \vec{A}_s = 0$, $\underline{I}_s \cdot \vec{A}_n = 0$

more general $\vec{\nabla}_s$ definition for both surface and volume fields

Consider a non-orthogonal curvilinear Coordinates [remon]

and a surface $\vec{x}(q^\alpha, t)$ $\quad \alpha = 1 \to q^1 = u$ $\quad$ 2D

$\alpha = 1,2 \to q^1 = u,\ q^2 = v$ $\quad$ 3D

$\vec{g}_\alpha = \dfrac{\partial\vec{x}}{\partial q^\alpha}$, $g_{\alpha\beta} = \vec{g}_\alpha \cdot \vec{g}_\beta$, $g = |g_{\alpha\beta}|$, $g^{\alpha\beta} = g_{\alpha\beta}^{-1}$, $dS = \sqrt{g}\, dq^\alpha$

Covariant base vector; $\underline{g} = [g_{\alpha\beta}]$ covariant metric tensor; determinant; Counter Covariant metric tensor; surface element

$\vec{\nabla}_s \equiv g^{\alpha\beta} \vec{g}_\alpha \dfrac{\partial}{\partial q^\beta}$ $\xrightarrow{2D}$ $(q^\alpha = u)$ $g^{\alpha\beta} = 1$, $\vec{g}_\alpha = \hat{t} \longrightarrow \vec{\nabla}_s = \hat{t}\dfrac{\partial}{\partial u}$

$\xrightarrow{3D}$ (A.27) [1]

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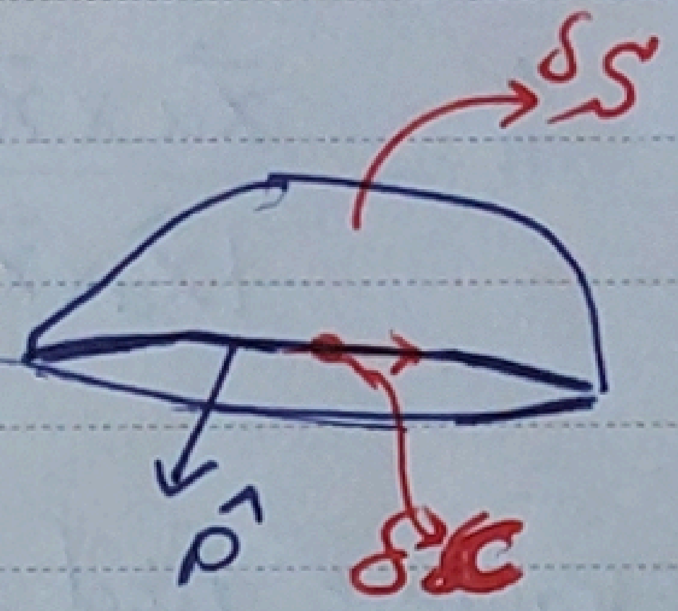
$$K = K_1 + K_2 = \underbrace{-\vec{\nabla}_s \cdot \hat{n} = -\vec{\nabla} \cdot \hat{n}}_{\downarrow \text{proof}} \qquad (8.B3)$$

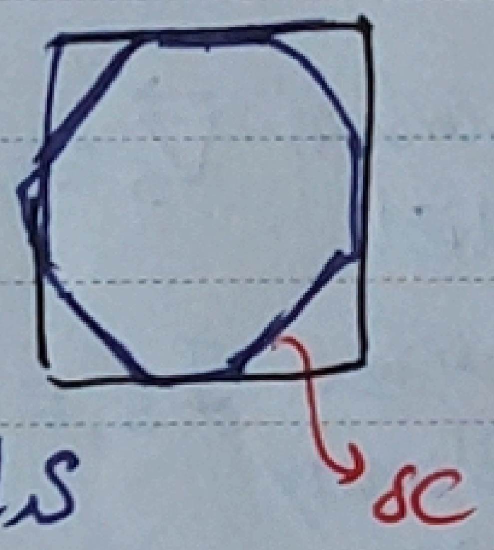
$$\vec{\nabla}_s \cdot \vec{n} = [(\underline{I} - \hat{n}\hat{n}) \cdot \vec{\nabla}] \cdot \hat{n} = \vec{\nabla} \cdot \hat{n} - \hat{n}\underbrace{(\hat{n} \cdot \vec{\nabla}) \cdot \hat{n}}_{\substack{(D.77) \\ \frac{1}{2}(\hat{n} \cdot \vec{\nabla})\cancel{(\hat{n} \cdot \hat{n})} \to 0 \\ 1}} = \vec{\nabla} \cdot \hat{n}$$

The surface tension tensor (stress) $(\underline{T}_s^\sigma)$ is defined as (like stress tensor $\underline{\sigma} \cdot \hat{n}$ = force per unit surface of $\hat{n}$)



$$\underbrace{\underline{T}_s^\sigma \cdot \hat{P}}_{\substack{\text{force per} \\ \text{unit length} \\ \text{of } c}} \equiv \sigma \hat{P} \qquad (9.B3)$$

$$\underset{\substack{\downarrow \\ \text{parallel} \\ \text{to } S}}{\hat{P}} = \underline{I}_s \cdot \hat{P} \xrightarrow{(9.B3)} \underline{T}_s^\sigma = \sigma \underline{I}_s \qquad (10.B3)$$



SDT: $\vec{F} = \underline{T}_s^\sigma$
surface divergence theorem

$$\vec{F}_\sigma = \underline{\int_{\delta S} \vec{f}_\sigma \, dS} = \oint_{\delta c} \underline{T}_s^\sigma \cdot \hat{P} \, dc = \underline{\int_{\delta s} \vec{\nabla}_s \cdot \underline{T}_s^\sigma \, dS} + \int_S K \underbrace{\underline{T}_s^\sigma \cdot \hat{n}}_{\downarrow 0} \, dS$$

$$\sigma \underline{I}_s \cdot \hat{n} = \sigma(\underline{I} - \hat{n}\hat{n}) \cdot \hat{n} = \sigma(\hat{n} - \hat{n}) = 0$$

surface divergence theorem [7] (A.36), [Lemen]

$$\int_S \vec{\nabla}_s \cdot \vec{F} \, dS = \oint_c \vec{F} \cdot \hat{P} \, dc - \int_S K \vec{F} \cdot \vec{n} \, ds$$

$$\vec{f}_\sigma = \vec{\nabla}_s \cdot \underline{T}_s^\sigma \xrightarrow{(10.B3)} \vec{\nabla}_s \cdot (\sigma \underline{I}_s) = \sigma \underbrace{\vec{\nabla}_s \cdot \underline{I}_s}_{K\hat{n}} + \underbrace{\vec{\nabla}_s \sigma \cdot \underline{I}_s}_{\vec{\nabla}_s \sigma}$$

$$\vec{\nabla}_s \cdot \underline{T}_s^\sigma = \vec{\nabla} \cdot \underline{T}_s^\sigma$$

proof:

SVD $\vec{F} = \underline{I}_s$

$$\int_S \vec{\nabla}_s \cdot \vec{I}_s \, ds = \oint_c \underbrace{\underline{I}_s \cdot \hat{P}}_{\hat{P}} dc - \int K \cancel{\underline{I}_s \cdot \hat{n}} \, ds \longrightarrow \vec{\nabla}_s \cdot \underline{I}_s = K\hat{n}$$

$[7](A.38) \int K\hat{n} \, dS$; $\to 0$ (7.B3)

and finaly, to show that $\vec{\nabla}_s \cdot \underline{T}_s^\sigma = \vec{\nabla} \cdot \underline{T}_s^\sigma$:

$$\vec{\nabla}_s \cdot \underline{T}_s^\sigma = [(\underline{I} - \hat{n}\hat{n}) \cdot \vec{\nabla}] \cdot (\sigma \underline{I}_s) = \vec{\nabla} \cdot (\sigma \underline{I}_s) - \hat{n}(\hat{n} \cdot \vec{\nabla}) \cdot \sigma \underline{I}_s$$

$\sigma \hat{n} \cdot \vec{\nabla} \hat{n}$; $\vec{\nabla} \times (\hat{n} \cdot \sigma \underline{I}_s) \overset{\uparrow (D.82)}{=} \sigma \vec{\nabla}\hat{n} \cdot \underline{I}_s$; $\underline{I}_s \cdot \vec{\nabla}\hat{n}$; $(\underline{I} - \hat{n}\hat{n}) \cdot \vec{\nabla}\hat{n}$; (D.8) ; D.77 ; $\frac{1}{2}(\hat{n} \cdot \vec{\nabla})(\hat{n} \cdot \hat{n})$; $\to 0$

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(p. 73)

$$\vec{F}_\sigma = (\vec{\nabla}\cdot T_s^\sigma)\,\delta_s = \vec{\nabla}\cdot(T_s^\sigma\,\delta_s) - \underbrace{T_s^\sigma\cdot\vec{\nabla}\delta_s}_{\sigma\,\underbrace{I_s\cdot\vec{\nabla}\delta_s}_{\cancel{\vec{\nabla}_s\delta_s}\ \to\ 0}} = \vec{\nabla}\cdot(\vec{T}_s\,\delta_s)$$

Similarly

$$\vec{\nabla}_s\cdot T_s^\sigma\,\delta_s = \vec{\nabla}_s\cdot(T_s^\sigma\,\delta_s) - T_s^\sigma\cdot\overset{0}{\cancel{\vec{\nabla}_s\delta_s}} = \vec{\nabla}_s\cdot(T_s^\sigma\,\delta_s)$$