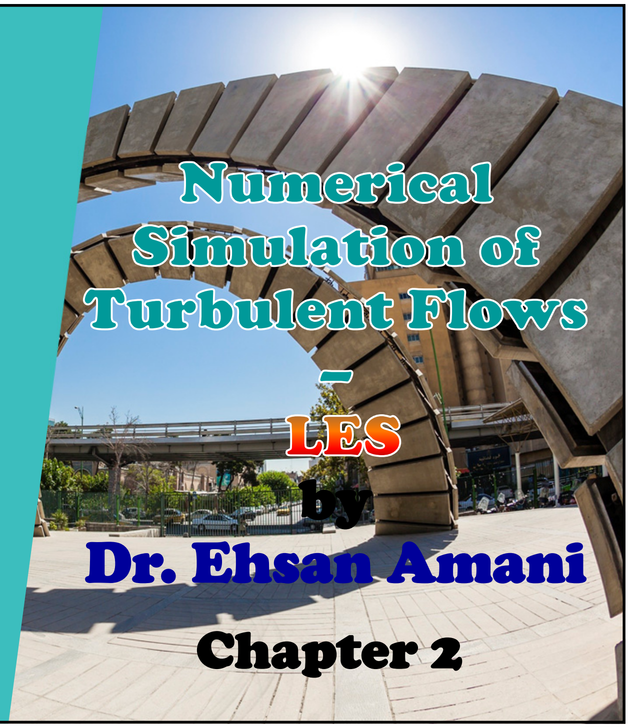

Numerical Simulation of Turbulent Flows – LES

by

Dr. Ehsan Amani

Chapter 2

II.1 A mathematical note

- Vectors and tensors
- Algebra and calculus

⇨ Lecture Notes: 2.1

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II.2 The continuity equation

- **Assuming continuum:**

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{U}) = 0 \qquad (2.1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_j) = 0 \qquad (2.2)$$

  - **For incompressible flows (solenoidal or divergence-free velocity field)**

$$\vec{\nabla}.\vec{U} = 0 \qquad (2.3)$$

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II.3 The momentum equation

- **Assuming continuum and in an inertial frame:**

$$\rho \frac{D\vec{U}}{Dt} = -\underbrace{\vec{\nabla} P}_{\text{Pressure}} + \vec{\nabla}.\underbrace{\underline{\tau}}_{\text{Viscous stress tensor}} + \rho \vec{g} \qquad (2.8)$$

  - **Conservative advection term (using continuity):**

$$\rho \frac{D\vec{U}}{Dt} = \frac{\partial}{\partial t}(\rho \vec{U}) + \vec{\nabla}.(\rho \vec{U}\vec{U}) \qquad (2.9)$$

  - **For a Newtonian fluid (Stokes law):**

$$\underline{\tau} = 2\mu \underline{S} - \frac{2}{3}\mu(\vec{\nabla}.\vec{U})\underline{I} \qquad (2.10)$$

$$\underline{S} = \frac{1}{2}\left[\left(\vec{\nabla}\vec{U}\right)^T + \vec{\nabla}\vec{U}\right] \qquad (2.11) \qquad \underline{\Omega} = \frac{1}{2}\left[\left(\vec{\nabla}\vec{U}\right)^T - \vec{\nabla}\vec{U}\right] \qquad (2.12)$$

Symmetric part of $\vec{\nabla}\vec{U}$or $\left(\vec{\nabla}\vec{U}\right)^T$: Rate-of-strain tensor

Antisymmetric part of $\left(\vec{\nabla}\vec{U}\right)^T$: Rate-of-rotation tensor

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II.3 The momentum equation

- **Exercise:**

$$\left(\vec{\nabla}\vec{U}\right)^T = \underline{S} + \underline{\Omega} \qquad (2.13) \quad \text{: Pope's definition of grad}(\vec{U})$$

$$\vec{\nabla}\vec{U} = \underline{S} - \underline{\Omega} \qquad (2.14) \quad \text{: Present definition of grad}(\vec{U})$$

- **Exercise: For incompressible, Newtonian flows with constant properties ($\nu = \mu/\rho =$ cte):**

$$\frac{D\vec{U}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{U} \qquad (2.15)$$

Modified Pressure: $p = P + \rho\psi, \qquad \vec{g} = \vec{\nabla}\psi$

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II.4 The mechanical energy equation

⇨ **Lecture Notes: 2.4**

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II.5 Pressure Poisson's equation

- **Exercise: For incompressible, Newtonian flows with constant properties ($\nu = \mu/\rho =$ cte):**

$$\vec{\nabla}^2 p = -\rho \vec{\nabla}\vec{U} : \vec{\nabla}\vec{U} = -\rho \vec{\nabla}.\left(\vec{\nabla}.\vec{U}\vec{U}\right) \qquad (2.21)$$

$$\frac{\partial^2 p}{\partial x_j \partial x_j} = -\rho \frac{\partial U_j}{\partial x_i}\frac{\partial U_i}{\partial x_j}$$

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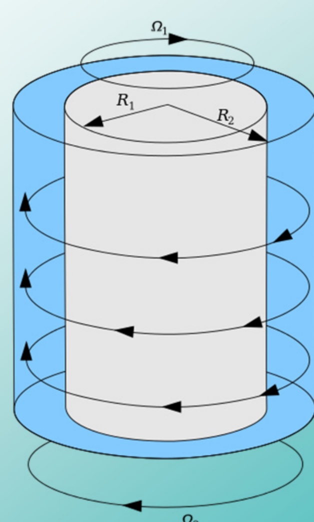
II.6 The non-linearity and chaos

- **The Taylor – Benard experiment**



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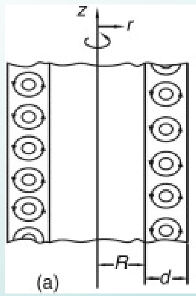
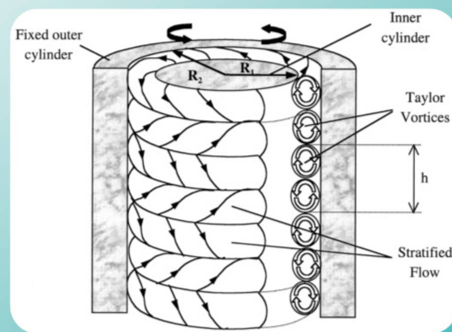
II.6 The non-linearity and chaos

- **The Taylor – Benard experiment**





Taylor vortices ►

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II.6 The non-linearity and chaos

- **Chaos :**
  - ✓ **Acute sensitivity of a system to initial condition, boundary condition, or state**
  - ✓ **Cause: non-liner terms in governing equations**

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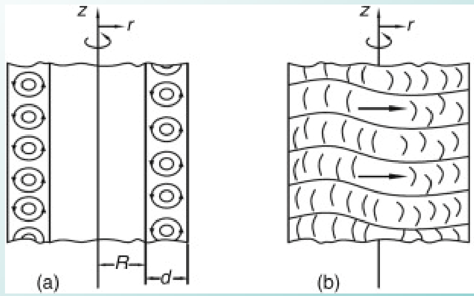
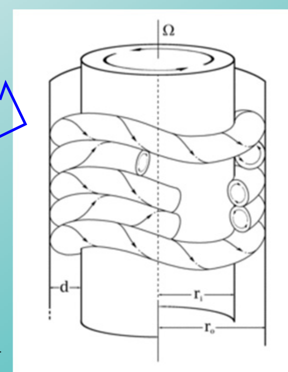
II.6 The non-linearity and chaos

- **The Taylor – Benard experiment**



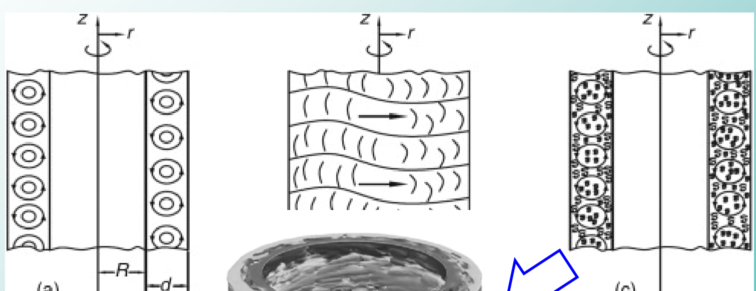
Taylor vortices



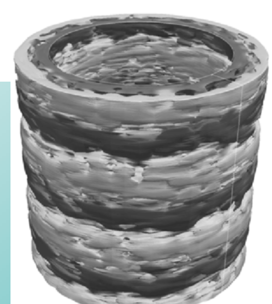
Wavy Taylor vortices ►

II.6 The non-linearity and chaos

- **The Taylor – Benard experiment**



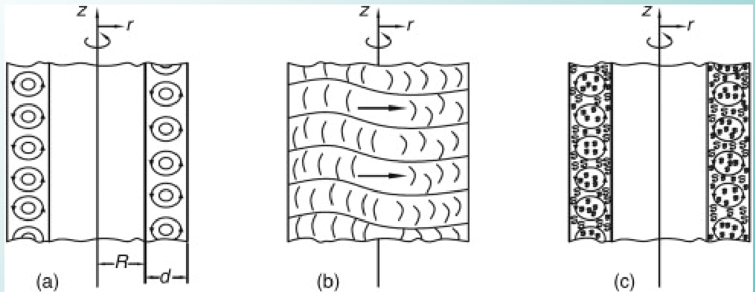
Taylor vortices



◄ Turbulent Taylor vortices

II.6 The non-linearity and chaos

- **The Taylor – Benard experiment**



Taylor vortices | **Wavy Taylor vortices** | **Turbulent Taylor vortices**

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II.6 The non-linearity and chaos

- **Turbulence:**
  - ✓ **A minute unpredictable perturbation in initial condition, boundary condition, or state produces a large change in the subsequent motion.**
  - ✓ **Fully chaotic and unpredictable**

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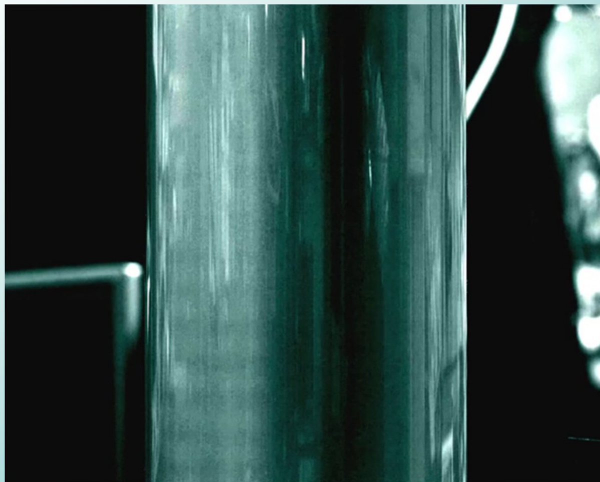
II.6 The non-linearity and chaos

- The Taylor – Benard experiment



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II.6 The non-linearity and chaos

- Chaos:
  - ✓ Acute sensitivity of a system to initial condition, boundary condition, or state
  - ✓ Cause: non-liner terms in governing equations
- Turbulence:
  - ✓ A minute unpredictable perturbation in initial condition, boundary condition, or state produces a large change in the subsequent motion.
  - ✓ Fully chaotic and unpredictable

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Examples of chaotic behavior

Lorenz equation

- Exercise, see section 1.3 [1]

Logistic equation

- Exercise, search Wikipedia: Logistic map

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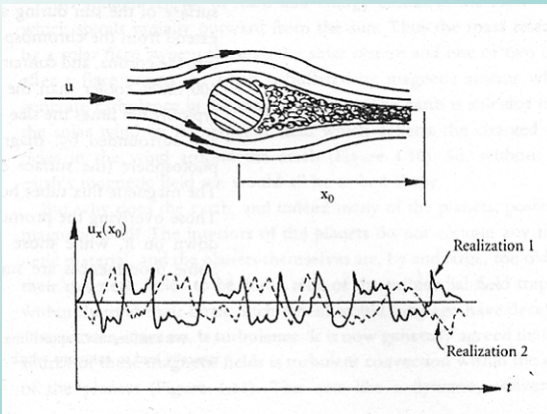
The statistical approach

- Realizations and mean
  - Stationary



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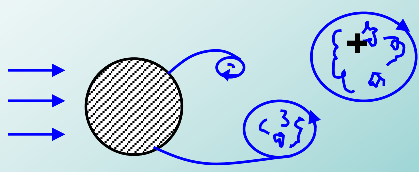
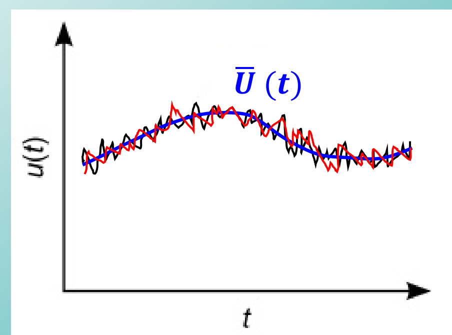
The statistical approach

- **Realizations and mean**
  - Stationary
  - Instationary





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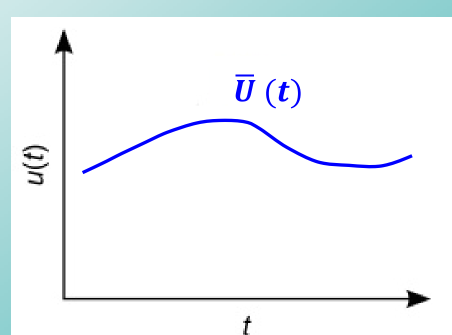
The statistical approach

- **Realizations and mean**
  - Stationary
  - Instationary



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The statistical approach

- **Averaging types**
  - **Mathematical expectation**

$$\langle U(\vec{x},t)\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} U^{(n)}(\vec{x},t) \qquad \textbf{(2.24)}$$

Nth experiment

Number of experiments

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The statistical approach

- **Averaging types**

Average	Definition	Relation to $\langle U\rangle$
Ensemble average	$\langle U(\vec{x},t)\rangle_N = \frac{1}{N}\sum_{n=1}^{N} U^{(n)}(\vec{x},t)$	$\lim_{N\to\infty}\langle U\rangle_N = \langle U\rangle$
Time average	$\overline{U} = \langle U(\vec{x},t)\rangle_T = \frac{1}{T}\int_t^{t+T} U(\vec{x},t')dt'$	For stationary flows: $\lim_{T\to\infty}\langle U\rangle_T = \langle U\rangle$
Spatial average	$\langle U(\vec{x},t)\rangle_L = \frac{1}{L^3}\iiint U(\vec{x}',t)dx_1'dx_2'dx_3'$	For homogeneous flows: $\lim_{L\to\infty}\langle U\rangle_L = \langle U\rangle$

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Hands-on practice

- **HW#1:**
  - ✓ **Fluent installation and preliminary practice**

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