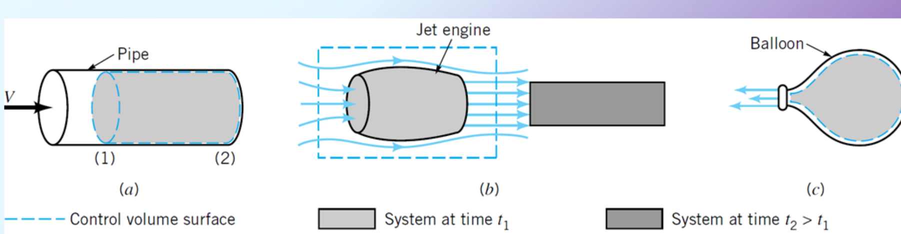
Fluid mechanics I
by
Dr. Ehsan Amani

Chapter 6

Fundamental Principles

- Mass conservation: The continuity
- Newton’s second law: The momentum equation
- Energy conservation
- The second law of thermodynamics

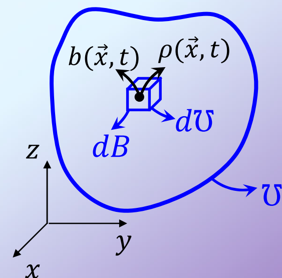
- What is a control volume?



Chapter 6 By E. Amani

Intensive vs. extensive properties

- **For an arbitrary volume in space**



Extensive property $B = \sum_{\mho} dB = \int_{\mho} dB$

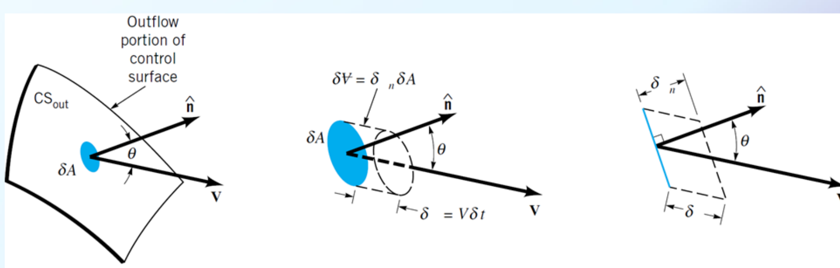
Intensive property (per unit mass) $b \equiv \dfrac{dB}{dm} \leftrightarrow dB = b\overbrace{dm}^{\rho d\mho} = \rho b d\mho \quad (1.6)$

$\rightarrow B = \int_{\mho} dB = \int_{\mho} \rho b d\mho \quad (2.6)$

Chapter 6 By E. Amani

Flow rate

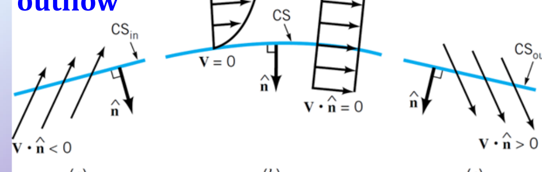
- **Flow rate of a property across a surface**



$d\mho = dA(Vdt)\cos\theta$; $(Vdt)\cos\theta \rightarrow \vec{V}.\vec{n}$

$\rightarrow d\mho = \vec{V}.\vec{n}dAdt \quad (3.6)$

Positive outflow



Chapter 6 By E. Amani

Flow rate

- Flow rate of a property across a surface

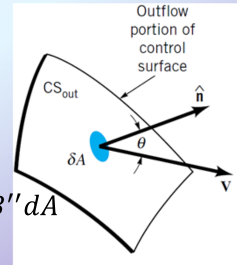
$$dB = bdm = \rho b \underbrace{d\mho}_{\vec{V}.\vec{n}dAdt} = \rho b\vec{V}.\vec{n}dAdt$$

Outflow (rate) of B from surface dA $\rightarrow$ $d\dot{B} = \rho b\vec{V}.\vec{n}dA$ (4.6)

(5.6)

(Net) outflow (rate) of B from surface A $\leftarrow$ $\dot{B} = \int_A d\dot{B} = \int_A \underbrace{\rho b\vec{V}.\vec{n}}_{d\dot{B}'' = d\dot{B}/dA}dA = \int_A d\dot{B}''dA$

(Out)flux of B $\leftarrow$ $d\dot{B}'' = d\dot{B}/dA$



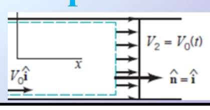
- Example: For mass as an extensive property

$$B = m = \int_\mho dm \rightarrow b \equiv \frac{dB}{dm} = \frac{dm}{dm} = 1 \rightarrow \dot{B} = \dot{m} = \int_A \underbrace{\rho\vec{V}.\vec{n}}_{\text{Mass flux}}dA$$

Mass flux

Mass flow rate

- Example: For 1D outlet

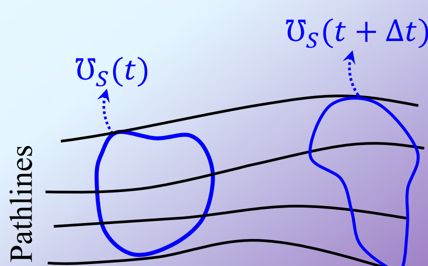


$$\dot{m} = \int_A \rho\vec{V}.\vec{n}dA = \int_A \rho VdA = \rho V\int_A dA = \rho VA$$

Chapter 6 By E. Amani

Control volume types

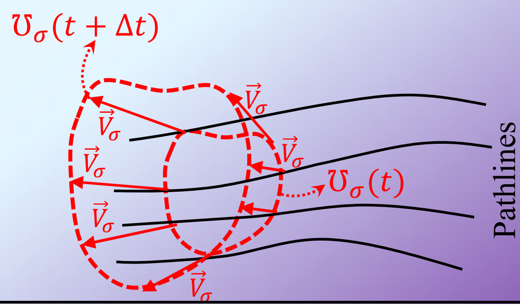
- (Closed) system or control mass
  - A collection of matter of fixed identity (the same atoms or fluid particles)
  - Fixed mass
  - Its shape and volume can change with flow
  - No mass flux across its boundaries



Chapter 6 By E. Amani

Control volume types

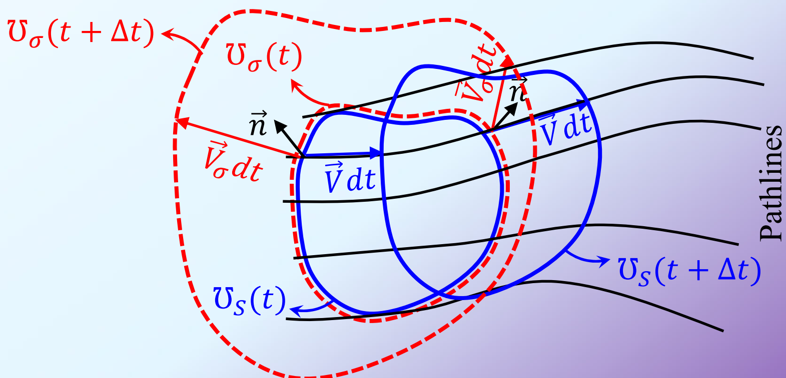
- Control volume
  - An arbitrary volume in space
  - Its boundaries move with a prescribed velocity $\vec{V}_\sigma$
  - Its shape and volume can change as you need (by $\vec{V}_\sigma$)
  - Special case ($\vec{V}_\sigma = 0$): Fixed control volume
  - Special case ($\vec{V}_\sigma = \vec{V}$): Control mass



Chapter 6 By E. Amani

Reynolds transport theorems

- System to control volume



⇨ Lecture Notes

Chapter 6 By E. Amani

Reynolds transport theorems

- **The first Reynolds transport theorem**

$$\frac{DB_S}{Dt} = \frac{dB_\sigma}{dt} + \underbrace{\int_A \rho b(\vec{V} - \vec{V}_\sigma).\vec{n}dA}_{\sum_{\text{out}} \dot{m}b - \sum_{\text{in}} \dot{m}b}$$

$$\underbrace{\frac{D}{Dt}\int_{\mho_S} \rho b d\mho}_{\text{Rate of change of } B \text{ in the system}} = \underbrace{\frac{d}{dt}\int_{\mho_\sigma} \rho b d\mho}_{\text{Rate of change of } B \text{ in the control volume}} + \underbrace{\int_A \rho b\underbrace{(\vec{V} - \vec{V}_\sigma)}_{\vec{V}_r}.\vec{n}dA}_{\text{Net outflow of } B \text{ from the control surface}} \quad (7.6)$$

- **The second Reynolds transport theorem ($\vec{V}_\sigma = 0$)**

$$\frac{DB_S}{Dt} = \frac{\partial B_\sigma}{\partial t} + \int_A \rho b\vec{V}.\vec{n}dA \quad (8.6)$$

- **Resemblance to the material derivative**

$$\frac{Db_p}{Dt} = \frac{\partial b}{\partial t} + (\vec{V}.\vec{\nabla})b \quad (9.6)$$

Chapter 6 By E. Amani

The end of chapter 6

Chapter 6 By E. Amani