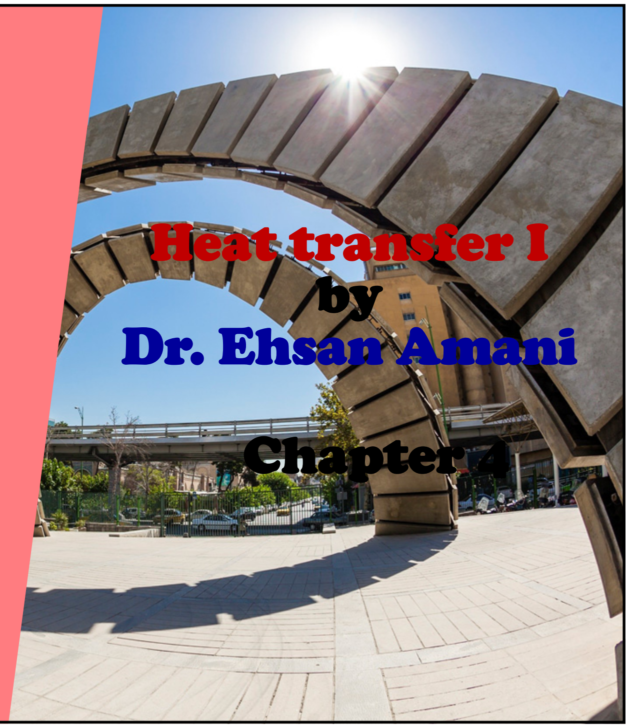
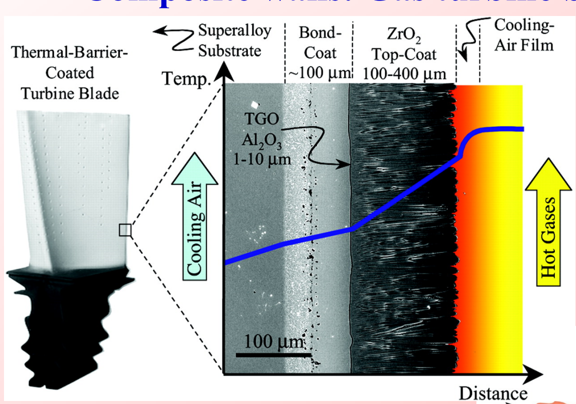


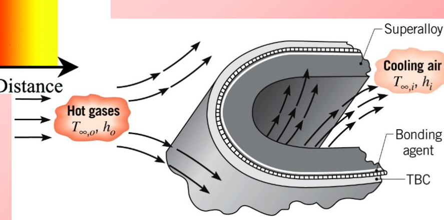
Heat transfer I
by
Dr. Ehsan Amani

Chapter 4

1D heat conduction: Applications

- **Composite walls: Gas turbine blade TBC**

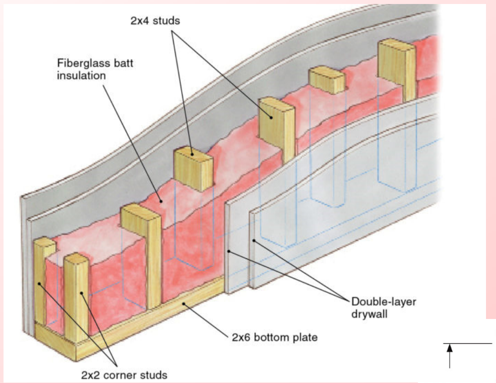


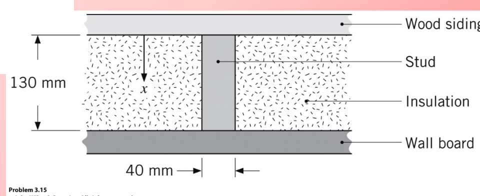


Chapter 4 By E. Amani

1D heat conduction: Applications

- **Composite walls: Building composite wall**

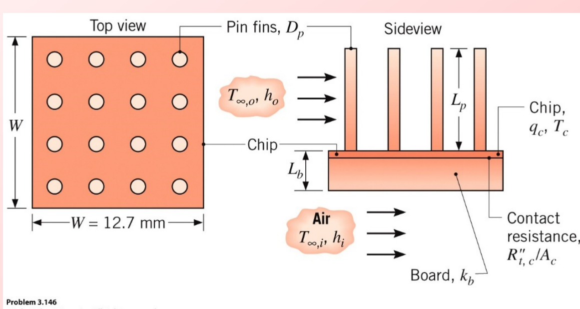


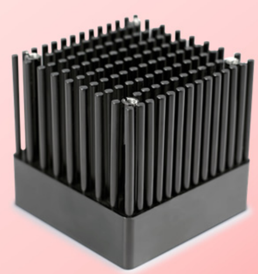


Chapter 4 By E. Amani

1D heat conduction: Applications

- **Fins: Electronic chip cooling**

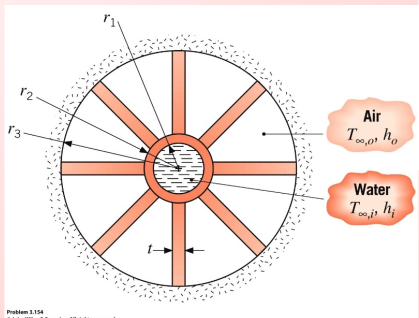


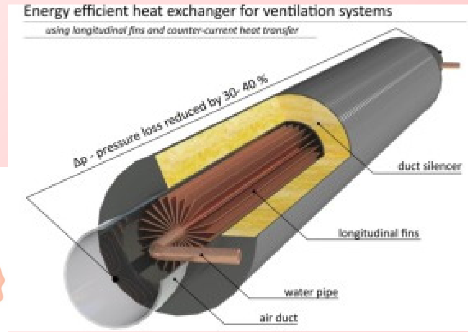


Chapter 4 By E. Amani

1D heat conduction: Applications

- **Fins: Heat exchangers (air heater)**

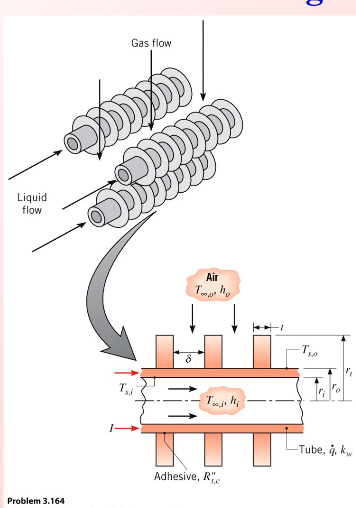




Chapter 4 By E. Amani

1D heat conduction: Applications

- **Fins: Heat exchangers (condenser)**



Chapter 4 By E. Amani

1.4 Thermal resistance

- **Assumptions to use thermal resistance:**
  - 1D conduction
  - Steady
  - No thermal energy generation
  - Constant conductivity

1.4 Thermal resistance

- **Plane wall (1D cartesian):**
  - 1D conduction

Steady

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)}+\cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}+\dot{q}=\cancel{\rho c_v\frac{\partial T}{\partial t}}$$

1D

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\dot{q}=0$$

⇨ **Lecture Notes**

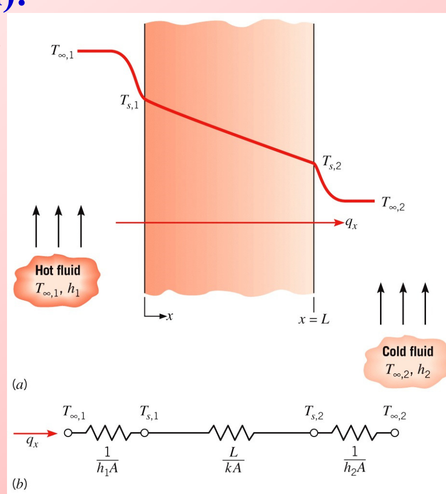


Figure 3.1

1.4 Thermal resistance

- **Contact resistance:**
  - Conduction + convection + radiation
  - Temperature jump

$$R_{t,c} = \frac{T_A - T_B}{q_x}$$

Contact resistance $\left(\frac{K}{W}\right)$

$$R_{t,c} = \frac{R''_{t,c}}{A_c}$$

$$R''_{t,c} = \frac{T_A - T_B}{q''_x}$$

***f*(materials of two surfaces and trapped fluid, surface roughness or contact pressure)**

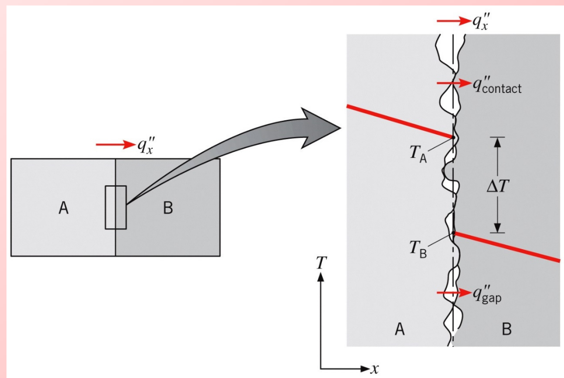


Figure 3.4

1.4 Thermal resistance

- **Contact resistance:**
  - Conduction + convection + radiation
  - Temperature jump
  - Tables

TABLE 3.1 Thermal contact resistance for (*a*) metallic interfaces under vacuum conditions and (*b*) aluminum interface (10-μm surface roughness, 10^5 N/m^2) with different interfacial fluids [1]

Thermal Resistance, $R''_{t,c} \times 10^4$ (m$^2 \cdot$K/W)

(*a*) Vacuum Interface			(*b*) Interfacial Fluid	
Contact pressure	100 kN/m^2	10,000 kN/m^2	Air	2.75
Stainless steel	6–25	0.7–4.0	Helium	1.05
Copper	1–10	0.1–0.5	Hydrogen	0.720
Magnesium	1.5–3.5	0.2–0.4	Silicone oil	0.525
Aluminum	1.5–5.0	0.2–0.4	Glycerine	0.265

Table 3.1

1.4 Thermal resistance

- **Contact resistance:**
  - Conduction + convection + radiation
  - Temperature jump
  - Tables

TABLE 3.2 Thermal resistance of representative solid/solid interfaces

Interface	$R''_{t,c} \times 10^4$ ($m^2 \cdot K/W$)	Source
Silicon chip/lapped aluminum in air (27–500 kN/m^2)	0.3–0.6	[2]
Aluminum/aluminum with indium foil filler (~100 kN/m^2)	~0.07	[1, 3]
Stainless/stainless with indium foil filler (~3500 kN/m^2)	~0.04	[1, 3]
Aluminum/aluminum with metallic (Pb) coating	0.01–0.1	[4]
Aluminum/aluminum with Dow Corning	~0.07	[1, 3]

1.4 Thermal resistance

- **Composite walls:**
  - 1D?!!!
  - Estimation:
    1. Isothermal surfaces normal to x direction
    2. Adiabatic surfaces parallel to x direction

$$R_{tot,1} = R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_c + R_4$$

$$R_{tot,2} = \frac{R_5 R_6}{R_5 + R_6}$$

$$R_5 = 2R_1 + R_2 + 2R_c + 2R_4$$

$$R_6 = 2R_1 + R_3 + 2R_c + 2R_4$$

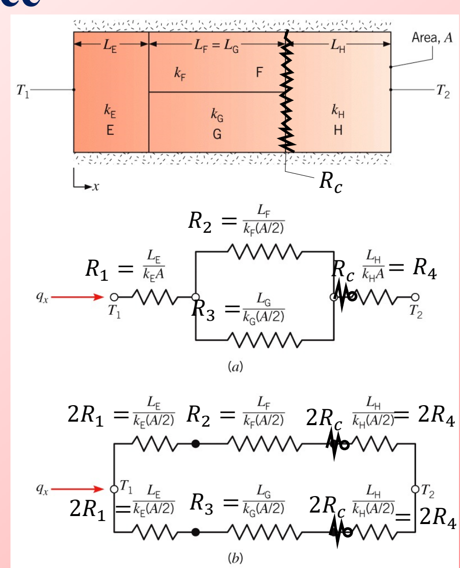
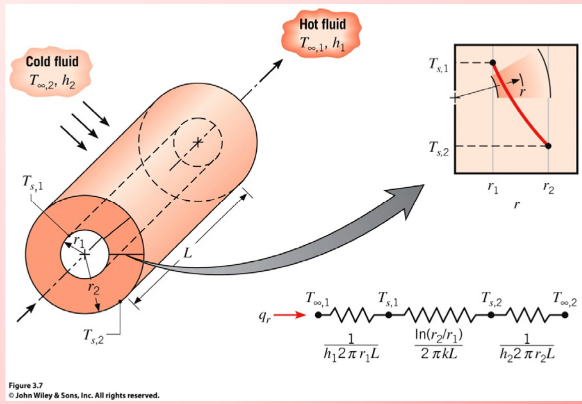


Figure 3.3

1.4 Thermal resistance

- **Radial systems:**

⇨ **Lecture Notes**



➢ Exercise: what is the critical radius for a cylindrical or spherical wall?

Chapter 4 **By E. Amani**

1.4 Thermal resistance

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\mathrm{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

➢ Note: $\Delta T = T_{s,1} - T_{s,2}$ is defined in the direction of q.

Chapter 4 **By E. Amani**

1.4 Thermal resistance

- **Sample problem:**

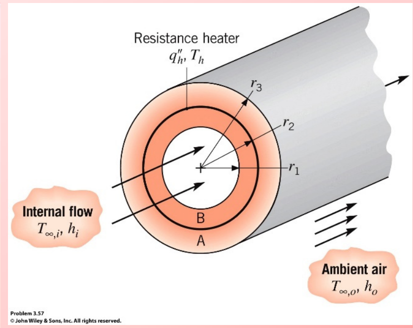
3.57 A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B, which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible.

Liquid pumped through the tube is at a temperature $T_{\infty,i}$ and provides a convection coefficient h_i at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{\infty,o}$ and provides a convection coefficient of h_o. Under steady-state conditions, a uniform heat flux of q''_h is dissipated by the heater.

(a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.

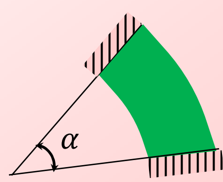
(b) Obtain an expression that may be used to determine the heater temperature, T_h.

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, q'_o/q'_i. How might the variables of the problem be adjusted to minimize this ratio?



1.4 Thermal resistance

- **Exercise: Show that**



$$R_{\text{cond}} = \frac{2\pi}{\alpha} R_{\text{cond,fc}} \qquad (17.4)$$

(rad) **Full cylinder**

2.4 The energy balance for a 1D differential element

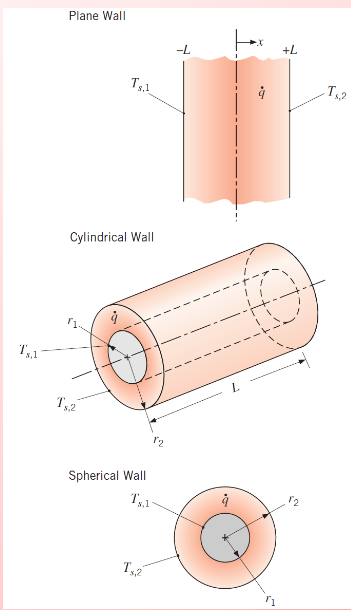
- **When?**
  - Violating assumptions 1-4
  - General element shape
  - Heat transfer in other directions in addition to the x-direction

⇨ **Lecture Notes**

- **Special case:**
  - Assumptions 1 (1D), 2 (St.), 4 ($k = cte$)
  - $+ \dot{q} = cte$
  - Solutions:

Chapter 4 By E. Amani

2.4 The energy balance for a 1D differential element



Chapter 4 By E. Amani

2.4 The energy balance for a 1D differential element

Table C.1 One-Dimensional, Steady-State Solutions to the Heat Equation for Plane, Cylindrical, and Spherical Walls with Uniform Generation and Asymmetrical Surface Conditions

Temperature Distribution

Plane Wall
$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \quad \text{(C.1)}$$

Cylindrical Wall
$$T(r) = T_{s,2} + \frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]\frac{\ln(r_2/r)}{\ln(r_2/r_1)} \quad \text{(C.2)}$$

Spherical Wall
$$T(r) = T_{s,2} + \frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]\frac{(1/r) - (1/r_2)}{(1/r_1) - (1/r_2)} \quad \text{(C.3)}$$

Heat Flux

Plane Wall
$$q''(x) = \dot{q}x - \frac{k}{2L}(T_{s,2} - T_{s,1}) \quad \text{(C.4)}$$

Cylindrical Wall
$$q''(r) = \frac{\dot{q}r}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r\ln(r_2/r_1)} \quad \text{(C.5)}$$

Spherical Wall
$$q''(r) = \frac{\dot{q}r}{3} - \frac{k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r^2[(1/r_1) - (1/r_2)]} \quad \text{(C.6)}$$

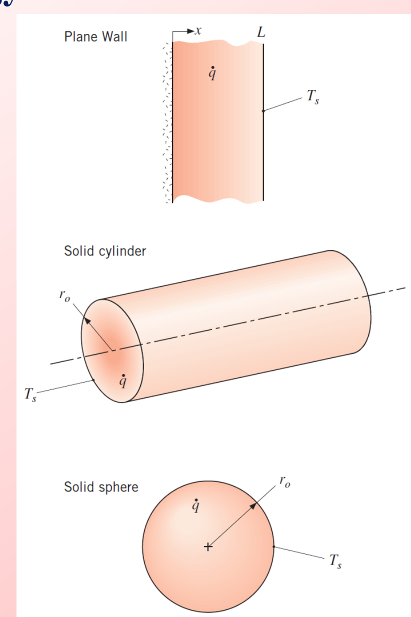
Heat Rate

Plane Wall
$$q(x) = \left[\dot{q}x - \frac{k}{2L}(T_{s,2} - T_{s,1})\right]A_x \quad \text{(C.7)}$$

Cylindrical Wall
$$q(r) = \dot{q}\pi L r^2 - \frac{2\pi L k}{\ldots} \cdot \left[\frac{\dot{q}r_2^2}{\ldots}\left(1 - \frac{r_1^2}{\ldots}\right) + (T_{s,2} - T_{s,1})\right] \quad \text{(C.8)}$$

Chapter 4 **By E. Amani**

2.4 The energy balance for a 1D differential element



Chapter 4 **By E. Amani**

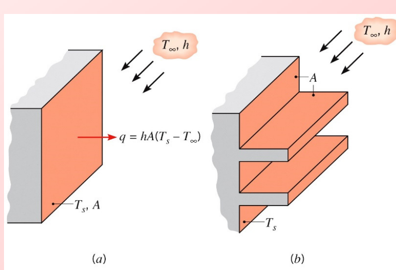
2.4 The energy balance for a 1D differential element

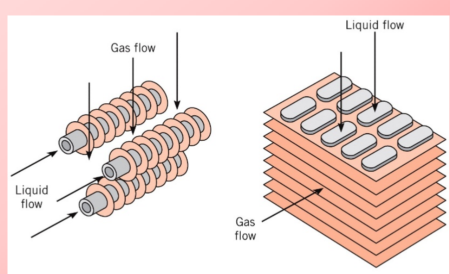
TABLE C.3 One-Dimensional, Steady-State Solutions to the Heat Equation for Uniform Generation in a Plane Wall with One Adiabatic Surface, a Solid Cylinder, and a Solid Sphere

	Temperature Distribution	
Plane Wall	$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$	(C.22)
Circular Rod	$T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.23)
Sphere	$T(r) = \frac{\dot{q}r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.24)
	Heat Flux	
Plane Wall	$q''(x) = \dot{q}x$	(C.25)
Circular Rod	$q''(r) = \frac{\dot{q}r}{2}$	(C.26)
Sphere	$q''(r) = \frac{\dot{q}r}{3}$	(C.27)
	Heat Rate	
Plane Wall	$q(x) = \dot{q}xA_x$	(C.28)
Circular Rod	$q(r) = \dot{q}\pi L r^2$	(C.29)
Sphere	$q(r) = \frac{\dot{q}4\pi r^3}{3}$	(C.30)

3.4 Extended surfaces

- **Heat transfer enhancement**
  - Surface thermal resistance is dominant in many cases
  - $T_s - T_\infty$ is usually constant (design constraint)
  - Increasing h rises the pumping cost
  - A? -> Fins $\quad q'' = \frac{T_s - T_\infty}{R_{\text{conv}}} = \frac{T_s - T_\infty}{1/hA}$





3.4 Extended surfaces

- **Fin types**
  - Straight fin of uniform cross section
  - Straight fin of nonuniform cross section
  - Annular fin
  - Pin fin

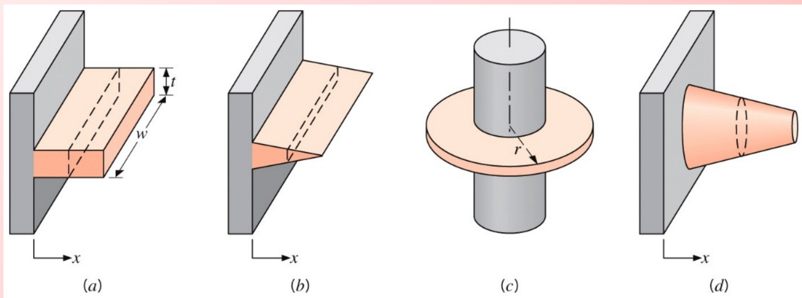


Figure 3.15

Chapter 4 By E. Amani

3.4 Extended surfaces

- **Fin analysis using Eq. (18.4)**
  - Fins of uniform cross section

⇨ **Lecture Notes**

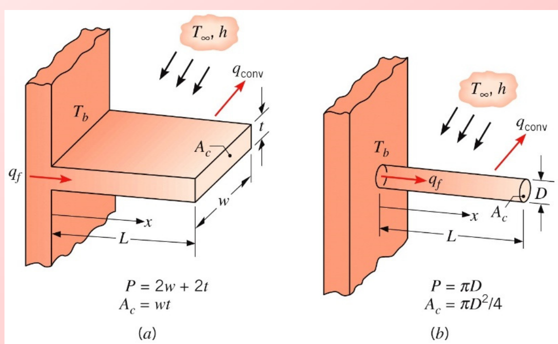


Figure 3.17

Chapter 4 By E. Amani

3.4 Extended surfaces

- **Fin analysis using Eq. (18.4)**
  - Fins of uniform cross section

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

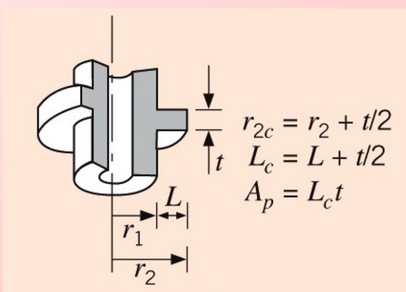
Chapter 4 By E. Amani

3.4 Extended surfaces

- **Fin analysis using Eq. (18.4)**
  - Annular fins with a rectangular cross-section
    - ✓ See reference [1] for derivation

$$\frac{\theta}{\theta_b} = \frac{I_0(mr)K_1(mr_2) + K_0(mr)I_1(mr_2)}{I_0(mr_1)K_1(mr_2) + K_0(mr_1)I_1(mr_2)}$$

    - ✓ Needs Bessel functions (Appendix B [1])



Chapter 4 By E. Amani

3.4 Extended surfaces

- **Fin analysis using Eq. (18.4)**
  - Annular fins with a rectangular cross-section
  - For other fin profiles, see Table 3.5 (discussed later)

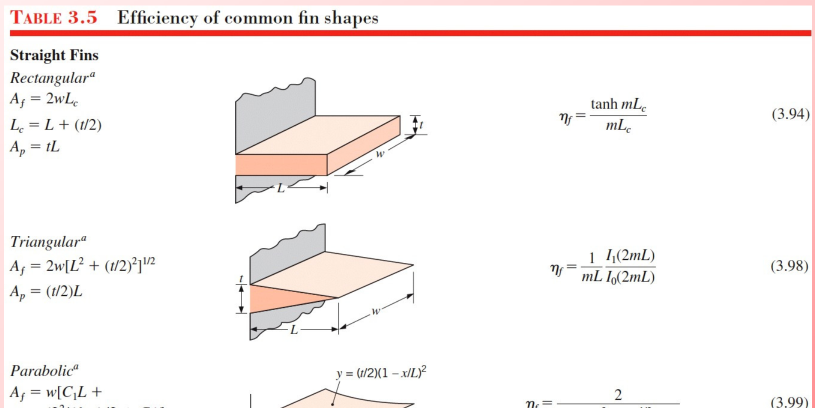


TABLE 3.5 Efficiency of common fin shapes

Straight Fins

Rectangular[a] $A_f = 2wL_c$, $L_c = L + (t/2)$, $A_p = tL$	$\eta_f = \dfrac{\tanh mL_c}{mL_c}$	(3.94)
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$, $A_p = (t/2)L$	$\eta_f = \dfrac{1}{mL}\dfrac{I_1(2mL)}{I_0(2mL)}$	(3.98)
Parabolic[a] $A_f = w[C_1 L +$	$\eta_f = \dfrac{2}{\ldots}$	(3.99)

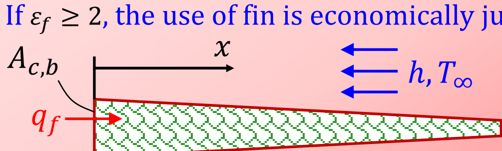
3.4 Extended surfaces

- **Fin performance**
  - Extra conduction thermal resistance
  - Does not necessarily result in enhancement
  - Fin performance measures:

$$\varepsilon_f \equiv \frac{\text{fin heat transfer}}{\text{heat transfer without fin}} = \frac{q_f}{hA_{c,b}\theta_b} \qquad (37.4)$$

Fin effectiveness

- ✓ The same h is used for calculating both nominator and denominator of Eq. (37.4)
- ✓ If $\varepsilon_f \geq 2$, the use of fin is economically justified



3.4 Extended surfaces

- **Fin efficiency example**
  - For straight fin of uniform cross-section

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
	$\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\theta_b$

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} \longrightarrow \underset{L \to \infty}{\varepsilon_{f,\max}} = \left(\frac{kP}{hA_c}\right)^{1/2} \qquad (38.4)$$

- Conclusions:
  - ✓ $k \uparrow \Rightarrow \varepsilon_f \uparrow$: The fin material is usually copper or aluminum
  - ✓ $h \uparrow \Rightarrow \varepsilon_f \downarrow$: The fin is in a gas environment or for natural convection
  - ✓ $P/A_c \uparrow \Rightarrow \varepsilon_f \uparrow$: A large number of thin fins are superior

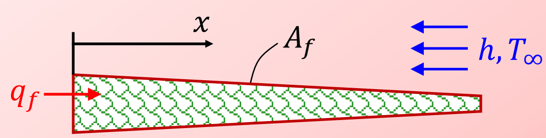
Chapter 4 By E. Amani

3.4 Extended surfaces

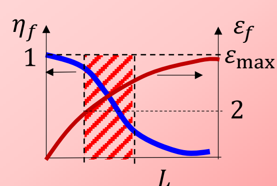
- **Fin performance measure**

$$\eta_f \equiv \frac{\text{fin heat transfer}}{\text{fin heat transfer at the ideal condition } (T_b: k \to \infty)} = \frac{q_f}{q_{f,\max}} = \frac{q_f}{hA_f\theta_b} \qquad (39.4)$$

Fin efficiency



- Exercise: How does η_f change with k, h, and P/A_c?
- Design criteria

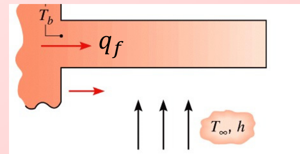


Chapter 4 By E. Amani

3.4 Extended surfaces

- **Fin thermal resistance**

$$R_{t,f} \equiv \frac{T_b - T_\infty}{q_f} = \frac{\theta_b}{q_f} \quad (40.4)$$



  - Exercise: Show that

$$\varepsilon_f \equiv \frac{R_{t,b}}{R_{t,f}}, \quad R_{t,b} = \frac{1}{hA_{c,b}} \quad (41.4)$$

$$R_{t,f} = \frac{1}{hA_f \eta_f} \quad (42.4)$$

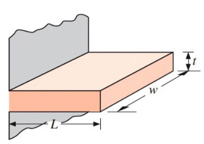
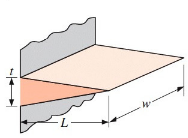
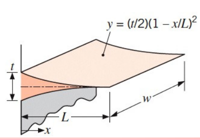
  - Given θ_b and fin geometrical parameters (hA_f or hA_c), the four quantities q_f, $R_{t,f}$, η_f, and ε_f are dependent and can be calculated knowing one of them.

Chapter 4 **By E. Amani**

3.4 Extended surfaces

- **Fin tables:**

TABLE 3.5 Efficiency of common fin shapes

Straight Fins			
Rectangular[a] $A_f = 2wL_c$ $L_c = L + (t/2)$ $A_p = tL$		$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.94)
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$ $A_p = (t/2)L$		$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$	(3.98)
Parabolic[a] $A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$ $C_1 = [1 + (t/L)^2]^{1/2}$ $A_p = (t/3)L$		$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$	(3.99)

Chapter 4 **By E. Amani**

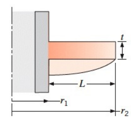
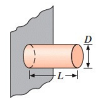
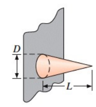
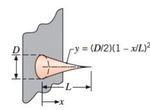
3.4 Extended surfaces

- **Fin tables:**

For adiabatic tip:
$L_c = L, r_c = r$

For active tip:
$L_c, r_c \rightarrow$ See table 3.5

TABLE 3.5 *Continued*

Annular Fin *Rectangular*[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$ $V = \pi(r_2^2 - r_1^2)t$		$\eta_f = C_2 \dfrac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \dfrac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$	(3.96)
Pin Fins *Rectangular*[b] $A_f = \pi D L_c$ $L_c = L + (D/4)$ $V = (\pi D^2/4)L$		$\eta_f = \dfrac{\tanh mL_c}{mL_c}$	(3.100)
Triangular[b] $A_f = \dfrac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$ $V = (\pi/12)D^2 L$		$\eta_f = \dfrac{2}{mL}\dfrac{I_2(2mL)}{I_1(2mL)}$	(3.101)
Parabolic[b] $A_f = \dfrac{\pi L^3}{8D}\{C_3 C_4 - \dfrac{L}{2D}\ln[(2DC_4/L) + C_3]\}$ $C_3 = 1 + 2(D/L)^2$ $C_4 = [1 + (D/L)^2]^{1/2}$ $V = (\pi/20)D^2 L$		$\eta_f = \dfrac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$	(3.102)

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

A_p: **profile area** $\quad V = A_p w$: **fin volume**

3.4 Extended surfaces

- **Overall surface resistance and efficiency**
  - Exercise: Show that (for an array of similar fins)

Overall surface efficiency

$$\eta_{o(c)} \equiv \frac{q_t}{q_{\max}} = \frac{Nq_f + q_b}{hA_t\theta_b} \quad (44.4)$$

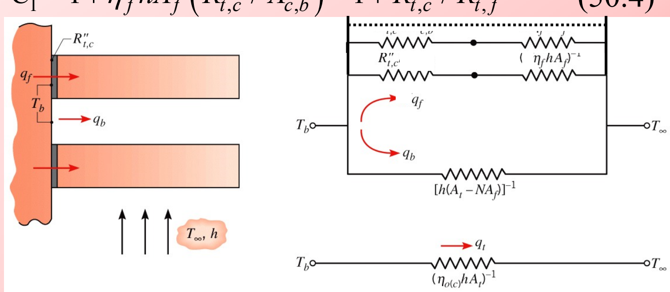
$$A_t = NA_f + A_b \quad (45.4)$$

Number of fins **Exposed base area**

$$R_{t,o(c)} = \frac{1}{\eta_{o(c)} hA_t} \quad (48.4)$$

$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right) \quad (49.4)$$

$$C_1 = 1 + \eta_f hA_f\left(R''_{t,c} / A_{c,b}\right) = 1 + R_{t,c} / R_{t,f} \quad (50.4)$$



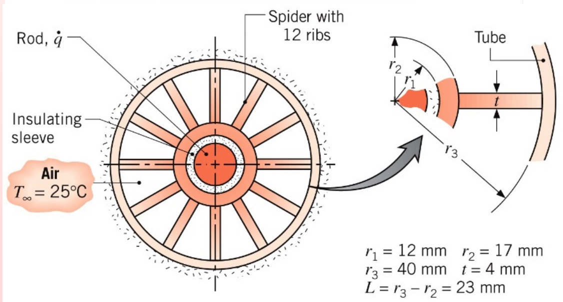
4.4 Sample problems

3.153 A long rod of 20-mm diameter and a thermal conductivity of 1.5 W/m·K has a uniform internal volumetric thermal energy generation of 10^6 W/m^3. The rod is covered with an electrically insulating sleeve of 2-mm thickness and thermal conductivity of 0.5 W/m·K. A spider with 12 ribs and dimensions as shown in the sketch has a thermal conductivity of 175 W/m·K, and is used to support the rod and to maintain concentricity with an 80-mm-diameter tube. Air at T_∞ = 25°C passes over the spider surface, and the convection coefficient is 20 W/m^2·K. The outer surface of the tube is well insulated.

Assume that the tube wall is an insulator.

(a) Sketch the thermal circuit required to compute the rod surface temperature and compute this temperature.

(b) What is the temperature of center of the rod?



Chapter 4 By E. Amani

4.4 Sample problems

TABLE 3.5 Efficiency of common fin shapes

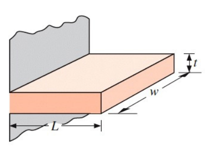
Straight Fins

Rectangular[a]

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$



$$\eta_f = \frac{\tanh mL_c}{mL_c} \qquad (3.94)$$

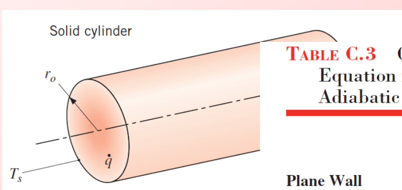


TABLE C.3 One-Dimensional, Steady-State Solutions to the Heat Equation for Uniform Generation in a Plane Wall with One Adiabatic Surface, a Solid Cylinder, and a Solid Sphere

	Temperature Distribution	
Plane Wall	$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$	(C.22)
Circular Rod	$T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.23)
Sphere	$T(r) = \frac{\dot{q}r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.24)

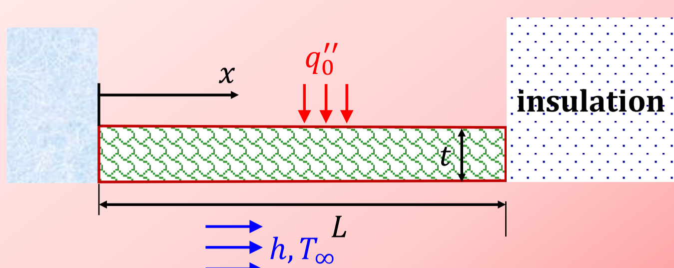
Chapter 4 By E. Amani

3.6 Summary

- **The formulation used for all or a part of a problem (1D, steady):**
  1. Assumptions ($\dot{q} = 0, k = cte$): Thermal resistance
  2. Planar/cylindrical/spherical walls with $\dot{q} = cte, k = cte$: (Appendix C)
  3. Fins ($\dot{q} = 0$, uniform convective surface): Thermal resistance + Tables 3.4 and 3.5
  4. Except for conditions of section 5.3: Simplifying general conduction equation
  5. Uniform convective surface, uniform q_0'', $\dot{q} = cte$: Equation (18.4)
  6. Energy balance for a general differential element

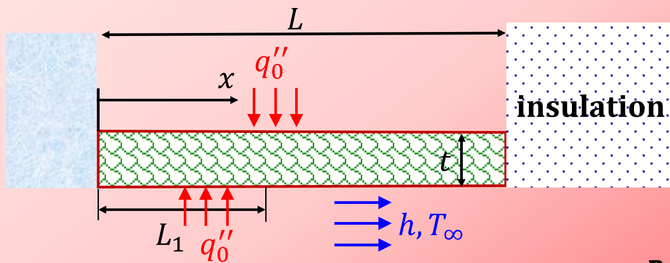
4.4 Sample problems

Challenge. (Mixed surface condition): Determine the temperature distribution and base heat transfer for the 2D profile shown in the figure (fin width: w). Solve this problem with as many methods as you can.



3.6 Summary

- **The formulation used for all or a part of a problem (1D, steady)**
- **Additional tricks:**
  1. Mixed uniform surface condition: Equivalent pure convection (fins)
  2. Discontinuous condition along x-direction: Decomposition into sub-problems



Chapter 4 By E. Amani

The end of chapter 4

Chapter 4 By E. Amani