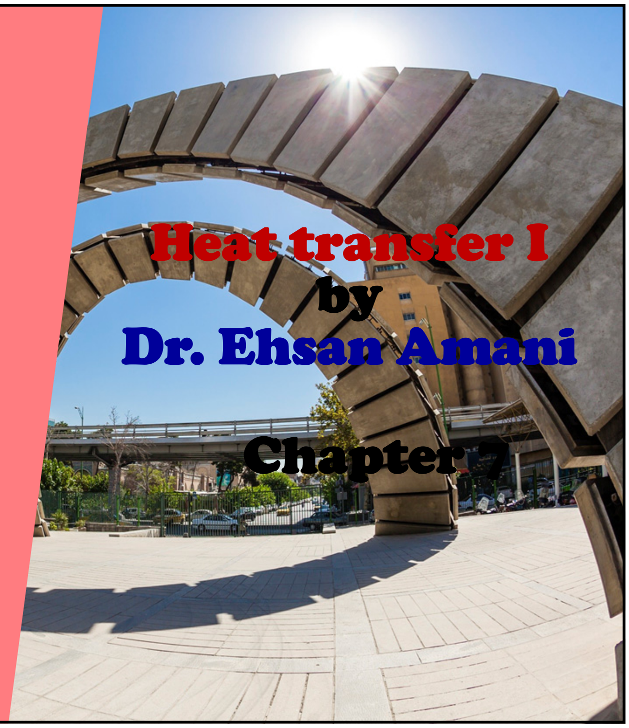

Heat transfer I
by
Dr. Ehsan Amani

Chapter 7

1.7 Objectives, definitions, and assumptions

- **Preliminary design**
  - Chapters 7 and 8
  - Simplified solutions or empirical relations rather than numerical solutions (detailed design)
- **External flows:** The Boundary Layer (BL) on a wall grows freely untouched by the presence of the other wall BLs
- **Assumptions**
  - Forced convection
  - Low Mach numbers (negligible viscous dissipation)
  - Steady
  - No phase-change
  - Constant properties: At film temperature, unless stated otherwise

$$T_f = \frac{T_s + T_\infty}{2}$$

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1.7 Objectives, definitions, and assumptions

- **Objective parameters**

$$c_f \rightarrow \tau_s$$

$$\overline{c}_f \rightarrow \overline{\tau}_s$$

$$\mathrm{Nu} \rightarrow h \quad q''_s = h(T_s - T_\infty) \quad T_s = cte : q''_s = \checkmark \quad q''_s = cte : T_s = \checkmark$$

$$\overline{\mathrm{Nu}} \rightarrow h_{\text{ave}} \quad (11.6)\ T_s = cte : q_s = \checkmark \quad (14.6)\ q''_s = cte : \overline{T}_s = \checkmark$$

➢ For external flows, Re_x, Nu_x, and $\overline{\mathrm{Nu}}_x$ are usually used instead of Re_L, Nu, and $\overline{\mathrm{Nu}}$

$$\mathrm{Re}_x \equiv \frac{Vx}{\nu} \quad (1.7) \qquad \mathrm{Nu}_x \equiv \frac{hx}{k} \quad (2.7)$$

Average over $x = 0$ to x

$$\overline{\mathrm{Nu}}_x \equiv \frac{h_{\text{ave},x}\,x}{k} \quad (3.7)$$

Chapter 7 **By E. Amani**

2.7 The flat plate in parallel flow

- **Laminar flow over an isothermal plate**
  - ➢ For incompressible, constant properties flows, the Navier-Stokes equations are decoupled from energy
  - ➢ Using the similarity solution method, the semi-analytical solution of the BL equations, Eqs. (6.21), (6.21)′, and (6.23), were given in fluid mechanics II

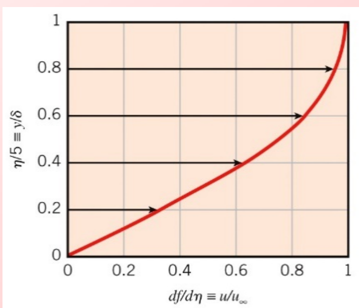

TABLE 7.7 Summary of convectio	
Correlation	
$\delta = 5x\,Re_x^{-1/2}$	(7.17)
$C_{f,x} = 0.664\,Re_x^{-1/2}$	(7.18)
$\overline{C}_{f,x} = 1.328\,Re_x^{-1/2} = 2C_{f,x}$	(7.24)

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2.7 The flat plate in parallel flow

- **Laminar flow over an isothermal plate**
  - For incompressible, constant properties flows, the Navier-Stokes equations are decoupled from energy
  - Using the similarity solution method, the semi-analytical solution of the BL equations, Eqs. (6.21), (6.21)′, and (6.23), were given in fluid mechanics II
  - Given the velocity profile, the BL energy equation with BC, Eqs. (6.22) and (6.24), can be solved using the similarity solution method, see reference [1].

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2.7 The flat plate in parallel flow

- **Laminar flow over an isothermal plate**
  - The results are as follows:

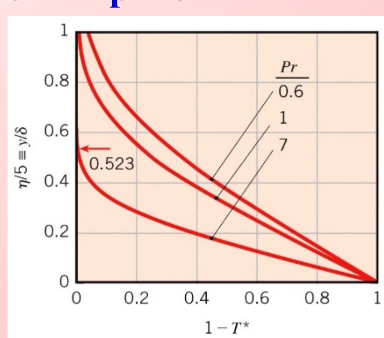
Generally, be cautious about the range of validity of each correlation

$$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3} \qquad \boxed{Pr \gtrsim 0.6^{d}} \qquad (7.21)$$

$$\delta_t = \delta\, Pr^{-1/3} \qquad (7.22)$$

$$\overline{Nu}_x = 0.664\, Re_x^{1/2}\, Pr^{1/3} = 2Nu_x \qquad (7.25)$$

$$\boxed{Pr \gtrsim 0.6^{d}}$$



  - For liquid metals (Pr ≪ 1), Eqs. (7.21) and (7.25) are not valid. Instead, assuming $u = u_\infty = cte$ across thermal BL, another semi-analytical solution is obtained as:

$$Nu_x = 0.564\, Pe_x^{1/2} \qquad (7.26)$$

$$\boxed{Pr \lesssim 0.05,\ Pe_x \gtrsim 100}$$

$$\mathrm{Pe}_x \equiv \mathrm{Re}_x \mathrm{Pr} \qquad (11.7)$$

Peclet number

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2.7 The flat plate in parallel flow

- **Laminar flow over an isothermal plate**
  - Exercise: Using equations of Table 6.1, discuss why for liquid metals, the functional form of the Nusselt number is simplified to:

$$\mathrm{Nu} = f(x^*, \mathrm{Re}_L, \mathrm{Pr}) \longrightarrow \mathrm{Nu} = f(x^*, \mathrm{Pe}_L)$$

  - Which is in accordance with:

$$Nu_x = 0.564\, Pe_x^{1/2} \qquad (7.26)$$

$$Pr \lesssim 0.05,\ Pe_x \gtrsim 100$$

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2.7 The flat plate in parallel flow

- **Turbulent flow over an isothermal plate**
  - Empirical correlations (table 7.7):

$C_{f,x} = 0.0592\, Re_x^{-1/5}$	(7.28) (exp.)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\, Re_x^{-1/5} \approx \delta_t$	(7.29) (exp.)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3}$	(7.30) (Chilton-Colburn)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074\, Re_L^{-1/5} - 1742\, Re_L^{-1}$	(7.33)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, e $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037\, Re_L^{4/5} - 871) Pr^{1/3}$	(7.31)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, e $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$

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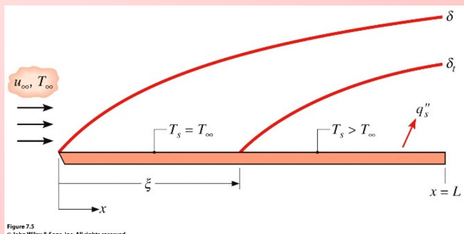
2.7 The flat plate in parallel flow

- **Flow over a plate with uniform heat flux**
  - Exercise: Is it a reasonable approximation to use isothermal correlations for uniform surface heat flux? Local laminar, average laminar, local turbulent, and average turbulent? See, reference [1]

- **Flow over a plate with adiabatic starting length**
  - see, reference [1], section 7.2.4



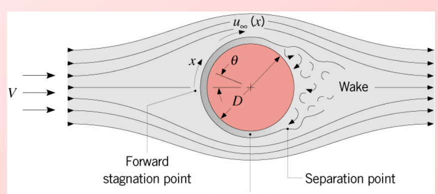
Chapter 7 By E. Amani

3.7 The cylinder in cross flow

- **Relevant Reynolds number**

$$u_{\infty,0} = u_{\text{in}}$$

$$\mathrm{Re}_D \equiv \frac{VD}{\nu} \qquad (16.7)$$



- **Drag coefficient: Non-dimensional drag force**

Drag force (pressure + viscous)

$$C_D = \frac{F_D}{A_f\left(\rho V^2/2\right)} \qquad (17.7)$$

Streamwise projected area

  - For bluff bodies, C_D is a useful parameter rather than $\bar{C}_f$

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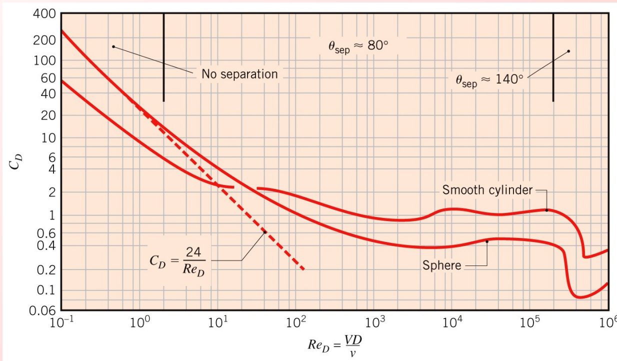
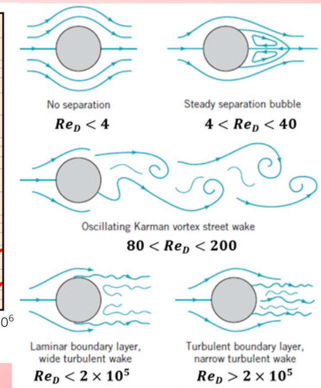
3.7 The cylinder in cross flow

- **Drag coefficient: Non-dimensional drag force**
  - Experimental measurements:





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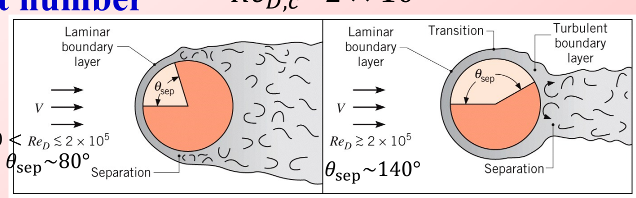
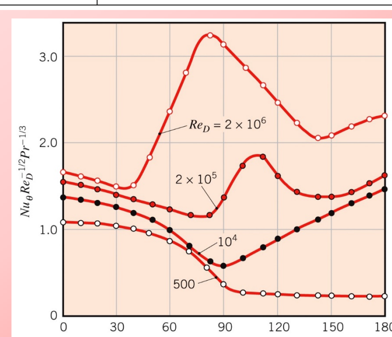
3.7 The cylinder in cross flow

- **Local Nusselt number** $Re_{D,c} \sim 2 \times 10^5$

$$\mathrm{Nu} = \mathrm{Nu}_D \equiv \frac{h(\theta)D}{k}$$



- Nu Variation
  1. BL growth till the separation point?
  2. BL transition to turbulence?
  3. Separation? BL growth vs. mixing



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3.7 The cylinder in cross flow

- **Average Nusselt number**
  - Experimental measurements:

$$\overline{\mathrm{Nu}}_D \equiv \frac{h_{\mathrm{ave}} D}{k}$$

$\overline{Nu}_D = C\, Re_D^m\, Pr^n (Pr/Pr_s)^{1/4}$ (Table 7.4)	(7.45) (exp.)	Cylinder	Average, T_∞, $1 \lesssim Re_D \lesssim 10^6$, $0.7 \lesssim Pr \lesssim 500$
$\overline{Nu}_D = 0.3 + [0.62\, Re_D^{1/2}\, Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4}] \times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	(exp.) (7.46)	Cylinder	Average, T_f, $Re_D\, Pr \gtrsim 0.2$

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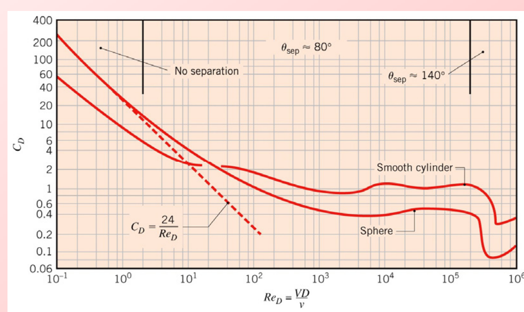
4.7 Flow over a rigid sphere

- **Drag coefficient**



  - Curve fit: Schiller and Naumann correlation for sphere

$$C_D = \begin{cases} \dfrac{24}{\mathrm{Re}_D} & ; \mathrm{Re}_D \leq 1 \\ \dfrac{24}{\mathrm{Re}_D}\left(1 + 0.15\mathrm{Re}_D^{0.687}\right) & ; \mathrm{Re}_D < 1000 \\ 0.445 & ; 1000 < \mathrm{Re}_D < 2 \times 10^5 \end{cases} \quad (18.7)$$

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4.7 Flow over a rigid sphere

- **Average Nusselt number**
  - Experimental measurements:

$$\overline{\mathrm{Nu}}_D \equiv \frac{h_{\mathrm{ave}} D}{k}$$

$\overline{Nu}_D = 2 + (0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}) Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	(7.48) (exp.)	Sphere (solid)	Average, T_∞, $3.5 \lesssim Re_D \lesssim 7.6 \times 10^4$, $0.71 \lesssim Pr \lesssim 380$, $1.0 \lesssim (\mu/\mu_S) \lesssim 3.2$
$\overline{Nu}_D = 2 + 0.6\, Re_D^{1/2}\, Pr^{1/3}$	(7.49) (exp.)	Falling drop	Average, T_∞

Chapter 7 By E. Amani

5.7 Summary of correlations

- **See, file "chap7-summary.pdf"**

TABLE 7.7 Summary of convection heat transfer correlations for external flow[a]

Correlation		Geometry	Conditions[b]
$\delta = 5x\, Re_x^{-1/2}$	(7.17) (Ana.)	Flat plate	Laminar, T_f
$C_{f,x} = 0.664\, Re_x^{-1/2}$	(7.18) (Ana.)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$	(7.21)(Semi-Ana.)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$[d]
$+ 0.06\, Re_D^{2/3}) Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	(7.48)	(solid)	$0.71 \lesssim Pr \lesssim 380$, $1.0 \lesssim (\mu/\mu_S) \lesssim 3.2$
$\overline{Nu}_D = 2 + 0.6\, Re_D^{1/2}\, Pr^{1/3}$	(7.49) (exp.)	Falling drop	Average, T_∞
$\overline{Nu}_D = C_1 C_2\, Re_{D,\max}^m\, Pr^{0.36} (Pr/Pr_s)^{1/2}$ (Tables 7.5, 7.6)	(7.50), (7.51) (exp.)	Tube bank[c]	Average, $\overline{T}$, $10 \lesssim Re_D \lesssim 2 \times 10^6$, $0.7 \lesssim Pr \lesssim 500$

[a]Correlations in this table pertain to isothermal surfaces; for special cases involving an unheated starting length or a uniform surface heat flux, see Section 7.2.4 or 7.2.5.

[b]The temperature listed under "Conditions" is the temperature at which properties should be evaluated.

[c]For tube banks and packed beds, properties are evaluated at the average fluid temperature, $\overline{T} = (T_i + T_o)/2$.

$Re_{D,\max} = \frac{V_{\max} D}{\nu}$

$V_{\max} = \begin{cases} (7.52) & \text{Aligned} \\ \max(7.52, 7.53) & \text{Staggered} \end{cases}$

[d]For 0.05<Pr<0.6, use (7.27) and $\overline{Nu}_x = 2Nu_x$.

[e]For other values of $Re_{x,c}$, use (7.31)-(7.33).

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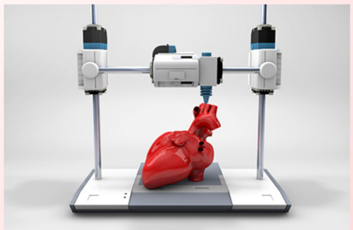
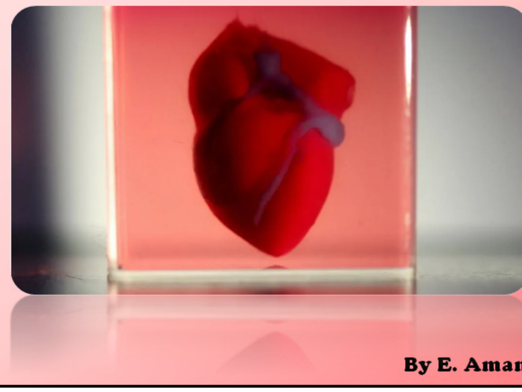
6.7 Sample problem: 3D printing

- **3D printing**





6.7 Sample problem: 3D printing

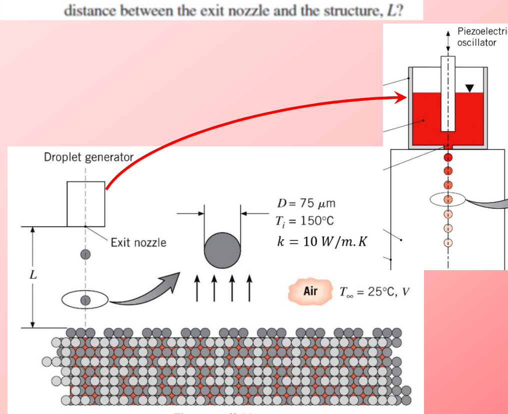
Sample problem

7.84 Tissue engineering involves the development of biological substitutes that restore or improve tissue function. Once manufactured, engineered organs can be implanted and grow within the patient, obviating chronic shortages of natural organs that arise when traditional organ transplant procedures are used. Artificial organ manufacture involves two major steps. First, a porous *scaffold* is fabricated with a specific pore size and pore distribution, as well as overall shape and size. Second, the top surface of the scaffold is seeded with human cells that grow into the pores of the scaffold. The scaffold material is biodegradable and is eventually replaced with healthy tissue. The artificial organ is then ready to be implanted in the patient.

The complex pore shapes, small pore sizes, and unusual organ shapes preclude use of traditional manufacturing methods to fabricate the scaffolds. A method that has been used with success is a *solid freeform fabrication* technique whereby small spherical drops are directed to a substrate. The drops are initially molten and solidify when they impact the room-temperature substrate. By controlling the location of the droplet deposition, complex scaffolds can be built up, one drop at a time. A device similar to that of Problem 7.78 is used to generate uniform, 75-μm-diameter drops at an initial temperature of $T_i = 150°C$. The particles are sent through quiescent air at $T_\infty = 25°C$. The droplet properties are $\rho = 2200\ kg/m^3$, $c = 700\ J/kg \cdot K$.

(a) It is desirable for the droplets to exit the nozzle at their terminal velocity. Determine the terminal velocity of the drops.

(b) It is desirable for the droplets to impact the structure at a temperature of $T_2 = 120°C$. What is the required distance between the exit nozzle and the structure, L?



The end of chapter 7