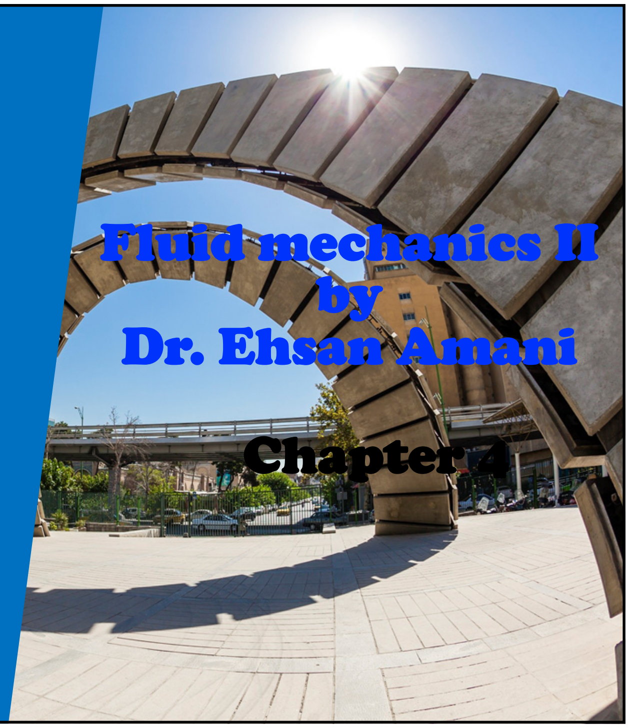


Fluid mechanics II
by
Dr. Ehsan Amani

Chapter 4

Internal flow

- Piping system: Typical components

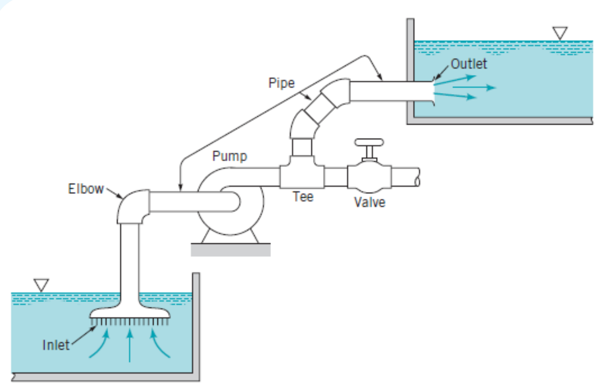
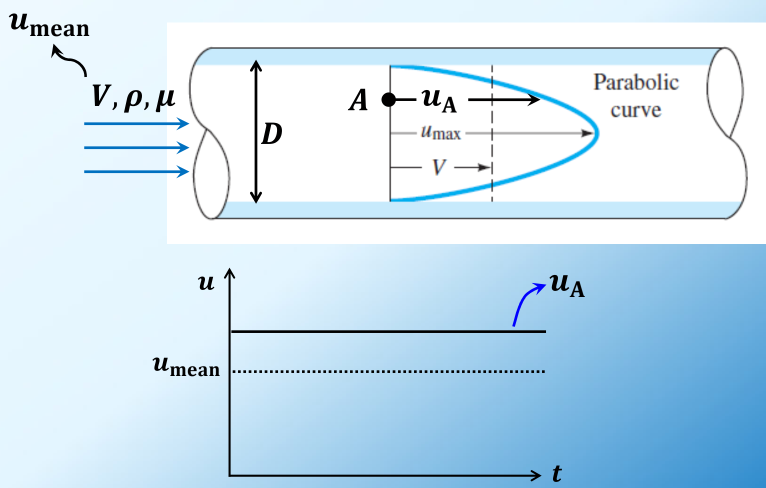


FIGURE 8.1 Typical pipe system components.

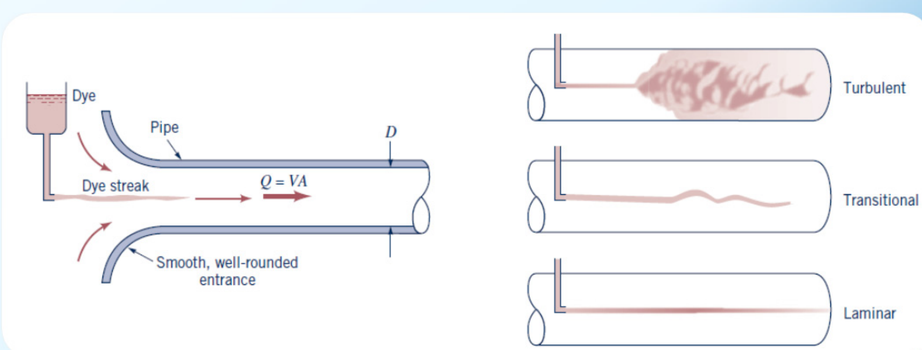
Flow in pipes

- **Flow regime: laminar vs. turbulent flow**



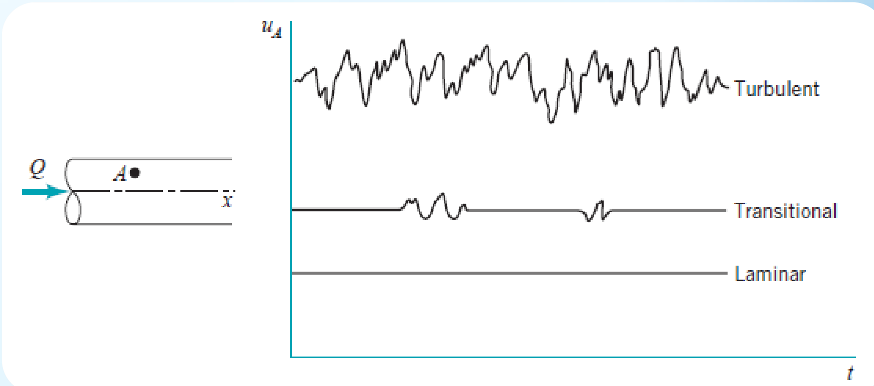
Flow in pipes

- **Flow regime: laminar vs. turbulent flow**



Flow in pipes

- **Flow regime: laminar vs. turbulent flow**



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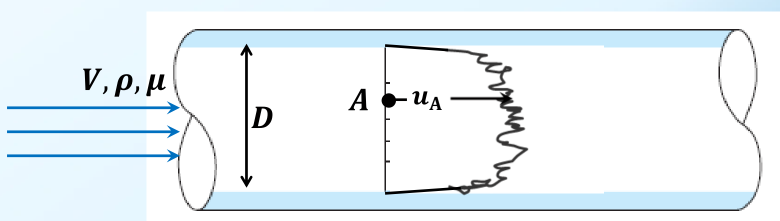
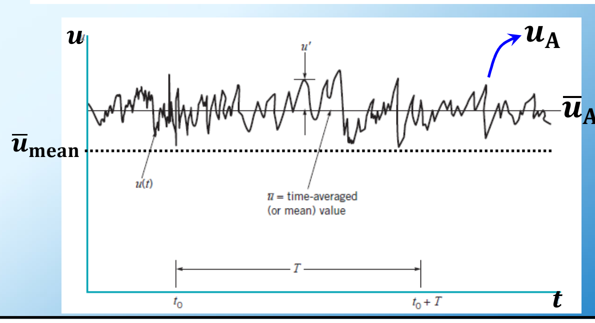
Flow in pipes

- **Flow regime: laminar vs. turbulent flow**





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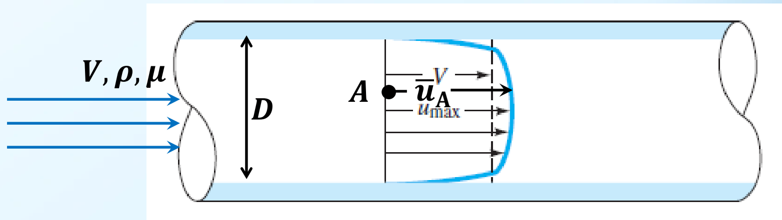
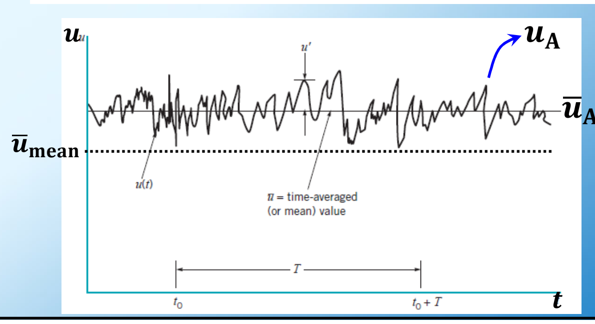
Flow in pipes

- **Flow regime: laminar vs. turbulent flow**





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Flow in pipes

- **Turbulence: non-linear systems**

§B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cartesian coordinates (x, y, z):

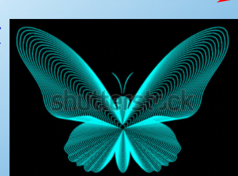
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \quad \text{(B.6-1)}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \quad \text{(B.6-2)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-3)}$$

Fluid flow Inertia is non-linear!

- **Butterfly effect**



Chapter 4 By E. Amani

Flow in pipes

- **Turbulent mean properties**
  - **Time averaging**

Mean (time-averaged) velocity

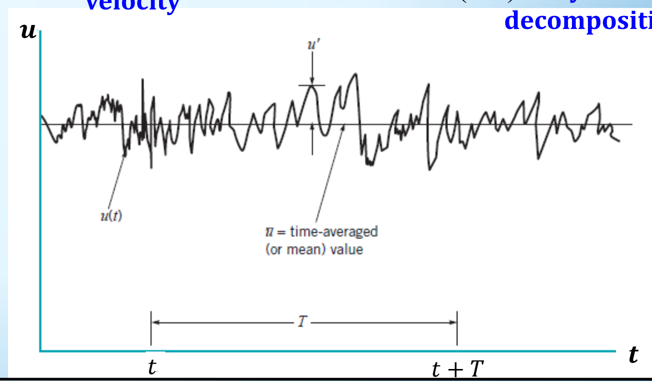
$$\bar{u}(x,y,z,t) = \frac{1}{T}\int_{t}^{t+T} u(x,y,z,t')dt' \qquad (1.4)$$

Fluctuating velocity

$$u' = u - \bar{u} \qquad (2.4)$$

$$u = \bar{u} + u' \qquad (3.4)$$

Reynolds decomposition



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Flow in pipes

- **Turbulent flows**
  - **In the rest of slides, for turbulent flows, we use u instead of $\bar{u}$, unless stated otherwise.**
  - **Fluid mechanics I: Reynolds number**

Characteristic velocity **Characteristic length**

$$Re \sim \frac{\text{Inertia}}{\text{viscous force}} = \frac{\rho V L}{\mu}$$

  - **Most practical applications are turbulent**
  - **Transitional pipe flow occurs at**

Bulk velocity **Pipe diameter**

$$2100 < Re_D = \frac{\rho V D}{\mu} < 4000 \qquad (4.4)$$

  - **For the design purposes:** $Re_{D,cr} \sim 2300$

Critical Reynolds number

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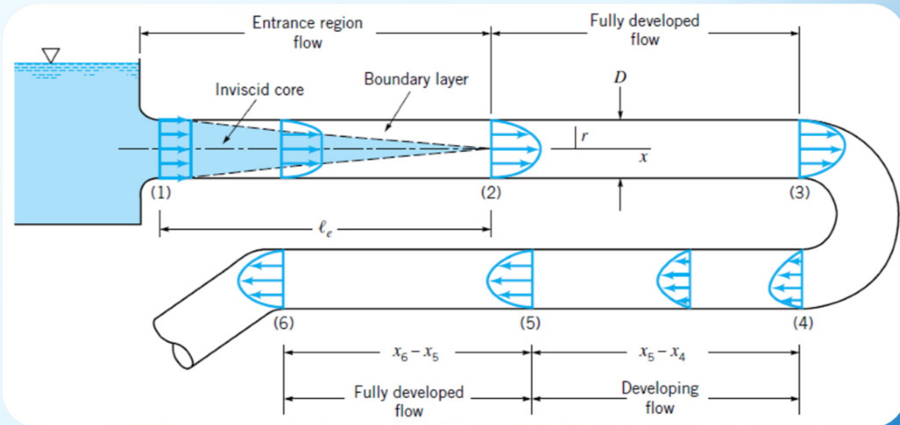
Flow in pipes

- Developing vs. fully-developed flow



Flow in pipes

- Flow analysis tools:
  1. Experiment (+dimensional analysis)
  2. Finite control volume
  3. Differential equations
     - Analytical solution
     - Numerical solution

Flow in pipes

- **Developing vs. fully-developed flow**
  - **Experiment (+dimensional analysis)**

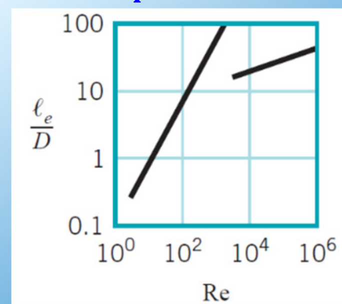
⇨ **Lecture Notes: IV.1.2**

$$\frac{\ell_e}{D} = f(Re_D)$$

$$\frac{\ell_e}{D} = 0.06 \text{ Re for laminar flow}$$

$$\frac{\ell_e}{D} = 4.4\,(\text{Re})^{1/6} \text{ for turbulent flow} \tag{5.4}$$

experiment



Chapter 4 By E. Amani

Fully-developed pipe flow

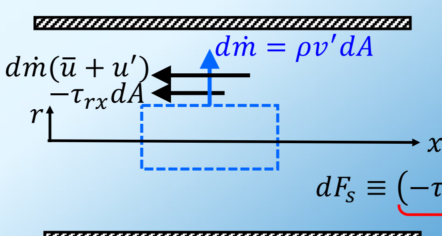
- **Finite control volume analysis**
  - **Fluid mechanics I:**

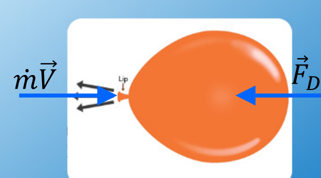
$$\sum \vec{F} = \frac{d\vec{P}_\sigma}{dt} + \underbrace{\int_A \vec{V}(\rho \vec{V}_r . \vec{n})dA}_{\sum_{\text{out}} \dot{m}\vec{V} - \sum_{\text{in}} \dot{m}\vec{V}} \longrightarrow \sum \vec{F} + \sum_{\text{in}} \dot{m}\vec{V} - \sum_{\text{out}} \dot{m}\vec{V} = \frac{d\vec{P}_\sigma}{dt}$$

Inlet: force in the velocity direction

outlet: force opposite to the velocity direction

  - **Turbulent vs. laminar**





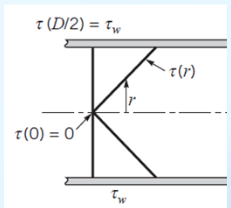
$$dF_s \equiv \underbrace{\left(-\tau_{rx} + \rho v'(\bar{u} + u')\right)}_{\tau} dA = \tau dA$$

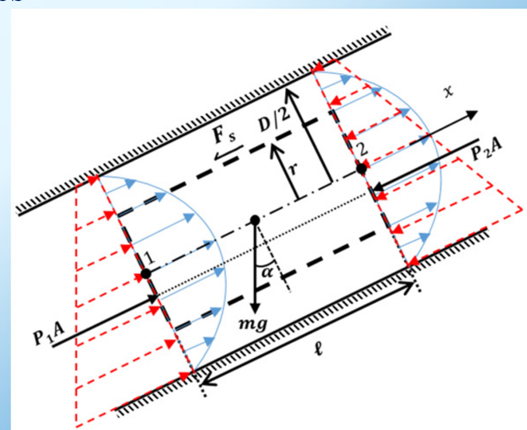
Chapter 4 By E. Amani

Fully-developed pipe flow

- **Finite control volume analysis**
  - **The relation between pressure drop and wall shear stress**

⇨ **Lecture Notes: IV.2.2**





$$h_L = -\frac{\Delta P_e}{\gamma} = \frac{4\ell}{\gamma D}\tau_w \qquad (11.4)$$

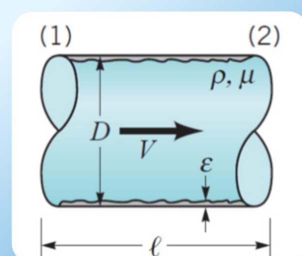
$$\Delta P_e = (P_2 - P_1) + \rho g \ell \sin\alpha \qquad (6.4)$$

Chapter 4 By E. Amani

Fully-developed pipe flow

- **Experiment (+dimensional analysis)**
  - **Calculation of the pressure drop**
  - **From experiments:**

$$\Delta P_e = F(V, D, \ell, \rho, \mu, \varepsilon)$$



  - **Exercise: Using a dimensional analysis, show that** $\frac{\Delta P_e}{\frac{1}{2}\rho V^2} = \tilde{\phi}\,(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D})$
  - **Experimental observations suggests:**

$$\Delta P_e \propto \frac{\ell}{D} \longrightarrow \frac{\Delta P_e}{\frac{1}{2}\rho V^2} = \frac{\ell}{D} f(Re_D, \frac{\varepsilon}{D}) \longrightarrow \Delta P_e = f\frac{\ell}{D}\frac{\rho V^2}{2} \qquad (30.4)$$

Friction factor (from experiment)

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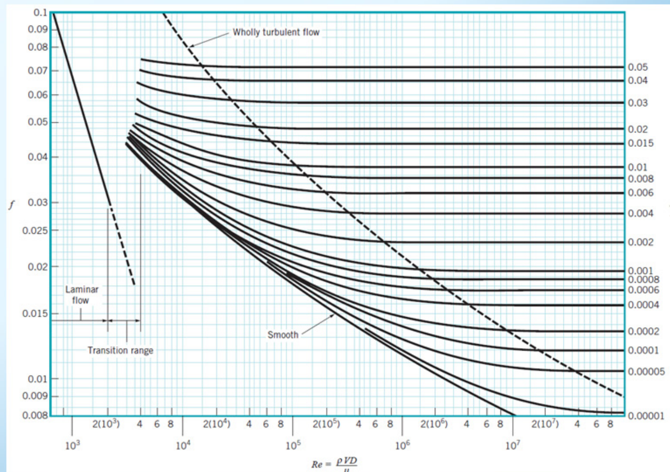
Fully-developed pipe flow

- Experiment (+dimensional analysis)
  - Moody chart

$$\Delta P_e = f\,\frac{\ell}{D}\,\frac{\rho V^2}{2}$$



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Fully-developed pipe flow

- Experiment (+dimensional analysis)
  - Calculation of the pressure drop

$$\Delta P_e = f\,\frac{\ell}{D}\,\frac{\rho V^2}{2}$$

TABLE 8.1
Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Chapter 4 By E. Amani

Fully-developed pipe flow

- **Experiment (+dimensional analysis)**
  - **Calculation of the pressure drop**

$$\Delta P_e = f\frac{\ell}{D}\frac{\rho V^2}{2} \longrightarrow f = \frac{(\Delta P_e/\ell)D}{\frac{1}{2}\rho V^2}$$

  - **In general,**

$$f \equiv \frac{(dP_e/dx)D}{\frac{1}{2}\rho V^2} \quad (31.4)$$

  - **For fully-developed pipe flow, using Eq. (10.4)**

$$f = \frac{8\tau_w}{\rho V^2} \quad (31.4)$$

  - **For laminar fully-developed pipe flow, using Eq. (56.3)**

$$f = \frac{8\frac{8V\mu}{D}}{\rho V^2} = \frac{64}{\frac{\rho VD}{\mu}} = \frac{64}{Re_D} \quad (32.4)$$

Chapter 4 **By E. Amani**

Head loss in piping

- **Piping (systems): Pipes and fittings**

Chapter 4 **By E. Amani**

Head loss in piping

- **Piping systems: Pipes and fittings**
  - **Remember, the extended Bernoulli equation**

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1 = \frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L+h_s \qquad (33.4)$$

$$h_L = \underbrace{h_{L,\text{major}}}_{\text{Fully-developed flow in straight pipes}} + \underbrace{h_{L,\text{minor}}}_{\text{Developing flow, fittings, equipment}}$$

  - **For pipes of constant cross-section**

$$h_L = -\frac{\Delta P_e}{\gamma} = f\frac{\ell}{D}\frac{V^2}{2g} \quad (34.4) \longrightarrow h_{L,\text{major}} = f\frac{\ell}{D}\frac{V^2}{2g}$$

  - **Similarly,** Loss coefficient $K_L = K_L(\text{geometry}, Re)$

$$h_{L,\text{minor}} = -\frac{\Delta P_e}{\gamma} \equiv K_L\frac{V^2}{2g} \qquad (35.4)$$

  - **For large** Re, $K_L = K_L(\text{geometry})$. Why?

Chapter 4 **By E. Amani**

Head loss in piping

- **Loss coefficient**

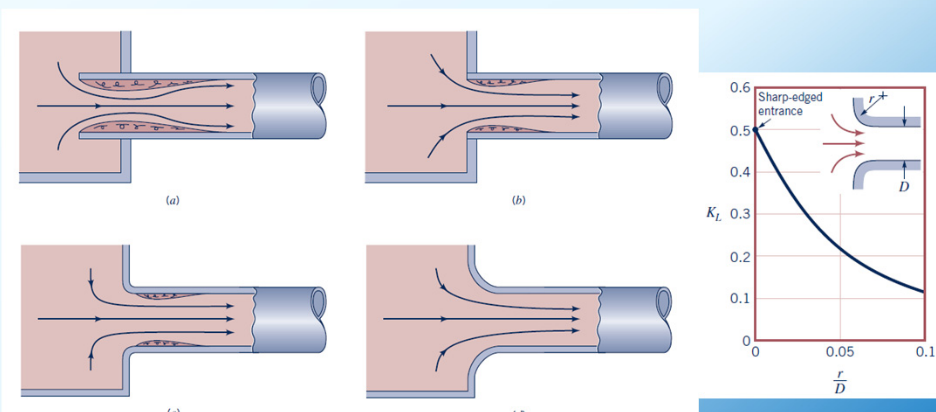
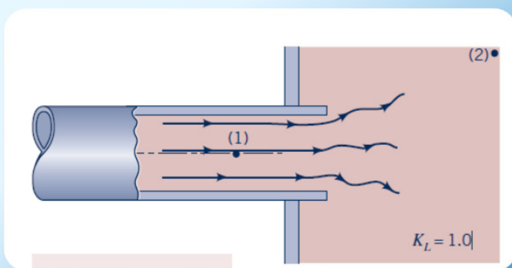


Figure 8.22 Entrance flow conditions and loss coefficient (Data from Refs. 28, 29). (a) Reentrant, $K_L = 0.8$, (b) sharp-edged, $K_L = 0.5$, (c) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (d) well-rounded, $K_L = 0.04$ (see Fig. 8.24).

Chapter 4 **By E. Amani**

Head loss in piping

- **Loss coefficient**
  - **Exercise: Using the extended Bernoulli equation, show that the loss coefficient for a pipe to large tank entrance is 1, regardless of the details of the geometry of the entrance.**



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Head loss in piping

- **Loss coefficient**

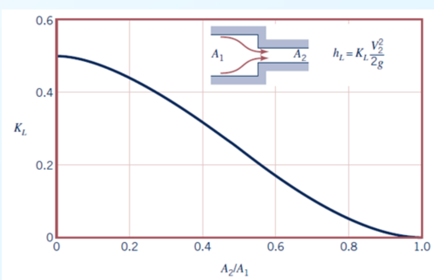

Figure 8.25 Loss coefficient for a sudden contraction (Ref. 10).

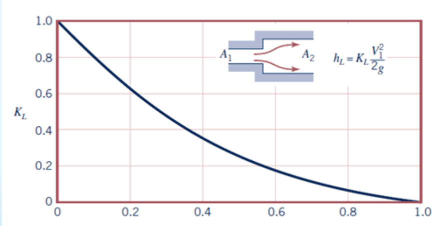

Figure 8.26 Loss coefficient for a sudden expansion (Ref. 10).

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Head loss in piping

- **Loss coefficient**

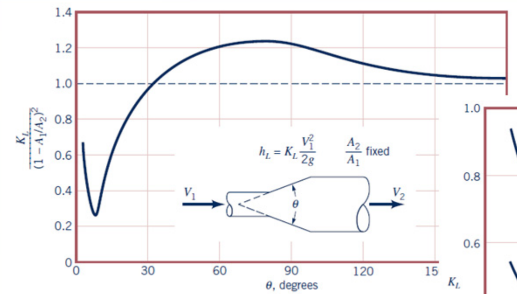


Figure 8.28 Loss coefficient for a typical conical diffuser (Ref. 5).

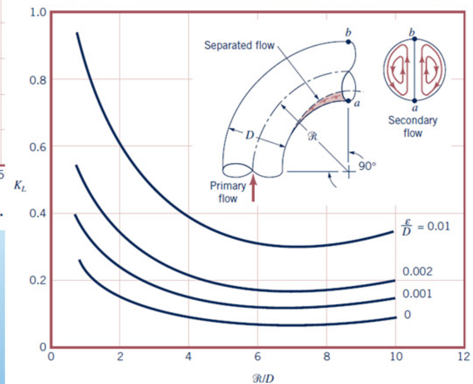


Figure 8.29 Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).

Chapter 4 By E. Amani

Head loss in piping

- **Loss coefficient**

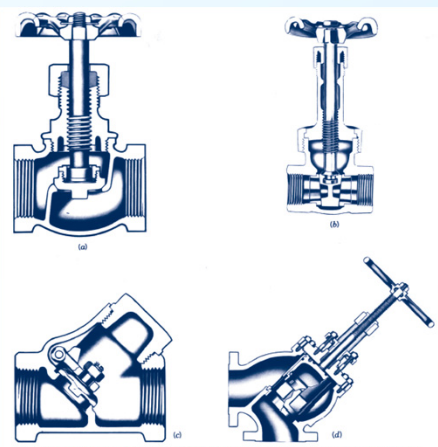


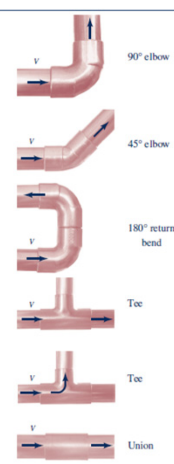
Figure 8.31 Internal structure of various valves: (a) globe valve, (b) gate valve, (c) swing check valve, (d) stop check valve. (Courtesy of Crane Co., Fluid Handling Division.)

Table 8.2

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ (Data from Refs. 5, 10, 27)

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

*See Fig. 8.31 for typical valve geometry.



Chapter 4 By E. Amani

Piping calculation

- **Node & branch**

$$\sum_{i \in Node} Q_i = 0$$

$$\Delta H_{1\to2} = \underbrace{\left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right)}_{H_1} - \underbrace{\left(\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right)}_{H_2} = h_L + h_s \quad (36.4)$$

$\Delta H_{\text{loop}} = 0 = h_L + h_s$

Positive in flow direction

$$h_L = \sum_{i \in major} (\pm) f_i \frac{\ell_i}{D_i} \frac{V_i^2}{2g} + \sum_{i \in minor} (\pm) K_i \frac{V_i^2}{2g} \qquad Q_i = V_i A_i$$

$$= \frac{1}{2g}\left[\sum_{i \in major} (\pm) \frac{f_i \ell_i}{D_i A_i^2} Q_i^2 + \sum_{i \in minor} (\pm) \frac{K_i}{A_i^2} Q_i^2\right] \quad (37.4)$$

No branching ($Q_i = Q$) →

$$h_L = \underbrace{\frac{1}{2g}\left[\sum_{i \in major} \frac{f_i \ell_i}{D_i A_i^2} + \sum_{i \in minor} \frac{K_i}{A_i^2}\right]}_{\tilde{R}} Q^2 = \tilde{R} Q^2 \quad (38.4)$$

n flow direction

Chapter 4 By E. Amani

Piping calculation

- **Node & branch**

Positive in flow direction

$$h_s = (\pm)\frac{dW_s}{gdm} = (\pm)\frac{dW_s/dt}{gdm/dt} = (\pm)\frac{\dot{W}_s}{g\rho Q} \quad (39.4)$$

Useful power

$$\dot{W}_s = \begin{cases} -P_i e & ;\text{Nominal input power: pump, fan, compressor, …} \\ +P_i/e & ;\text{Nominal output power: turbine, …} \end{cases}$$

efficiency

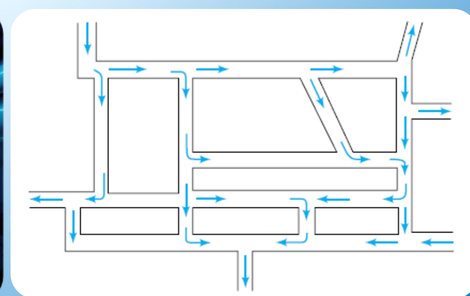
- **Similarity to electric circuit problems**
  - **The node & branch method**
- **Difference with electric circuit problems**
  - **Pipe network branch equation is non-linear and the resistance may be flow-direction-dependent: needs an trial and error solution**

$\Delta V = RI + E \qquad \Delta H = \tilde{R} Q^2 + h_s$

Chapter 4 By E. Amani

Piping calculation

- Complicated practical problems
  - Involves the solution of a system of non-linear equations using efficient trial and error
  - Need special engineering software
  - Pipe network course



Chapter 4 By E. Amani

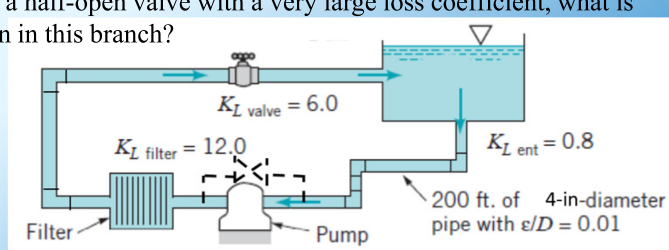
Piping calculation

- Sample problem

Water is circulated from a large tank, through a filter, and back to the tank as shown in the figure. The power added to the water by the pump is 200 ft.lb/s. Some minor loss coefficients are given in this figure and others should be extracted from a database. The elbows are all regular 90°, flanged type.

a) Present the governing equations for calculating the flowrate through the filter.
b) What is the best initial guess for the friction factor in the pipe segments?
c) Solve the equations and determine the flow rate.
d) If a branching is added over the pump as shown with the dashed line in the figure, including a half-open valve with a very large loss coefficient, what is the flow direction in this branch?

⇨ **Lecture Notes: IV.5.1**

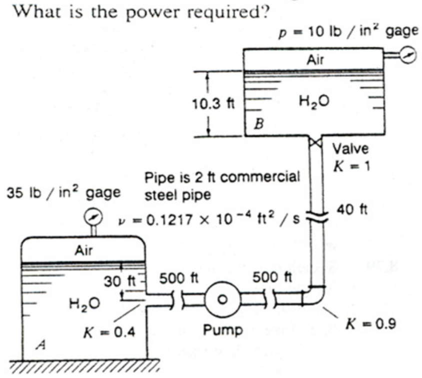


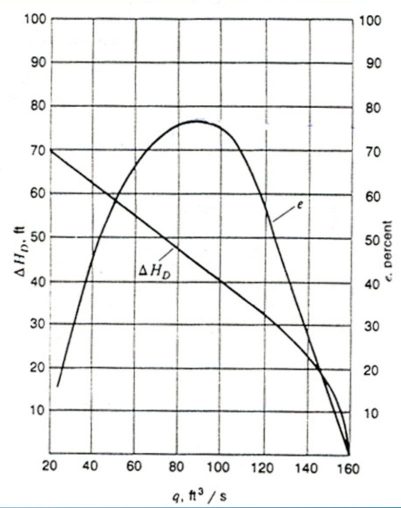
Chapter 4 By E. Amani

Piping calculation

- **Sample problem**

What is the flow q from A to B for the system shown in Fig. ? The pump has the characteristics shown in Fig. What is the power required?



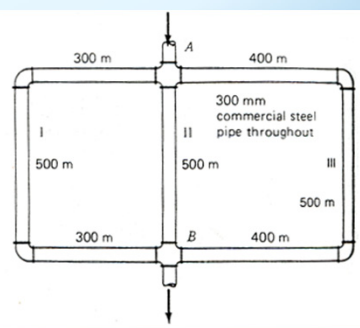


Chapter 4 ⇨ Lecture Notes: IV.5.2 By E. Amani

Piping calculation

- **Sample problem**

A flow q of 800 L/s goes through the pipe system in Fig. . What is the pressure drop between A and B if the elevation of A is 100 m and of B is 200 m? Neglect minor losses here. The water is at 5°C.



⇨ Lecture Notes: IV.5.3

Chapter 4 By E. Amani

Differential equations of turbulent flows

- **The effective stress**
  - **Consider a component of viscous stress, e.g.,**

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad (12.4)$$

  - **Time-averaging Eq. (12.4) yields**

$$\bar{\tau}_{xy} = \frac{1}{T}\int_t^{t+T} \tau_{xy} dt' = \frac{1}{T}\int_t^{t+T} \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) dt'$$
$$= \mu\left[\frac{\partial}{\partial y}\left(\frac{1}{T}\int_t^{t+T} u dt'\right) + \frac{\partial}{\partial x}\left(\frac{1}{T}\int_t^{t+T} v dt'\right)\right] = \mu\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)$$

$$\bar{\tau}_{xy} = \mu\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right) \qquad (14.4)$$

  - **For viscous stress, the relation between mean quantities are the same as the one between instantaneous variables**

Chapter 4 By E. Amani

Differential equations of turbulent flows

- **The effective stress**
  - **How about the other terms?**
  - **Time-averaging the Navier Stokes equation in conservative form (Eq. (33.3)')**

$$\frac{\partial \overline{\rho u}}{\partial t} + \frac{\partial \overline{\rho u u}}{\partial x} + \frac{\partial \overline{\rho v u}}{\partial y} + \frac{\partial \overline{\rho w u}}{\partial z} = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z} + \bar{f}_x$$

  - **For simplicity, assuming stationary ($\frac{\partial \bar{u}}{\partial t} = 0$) incompressible flow**

$$\frac{\partial \rho \overline{u u}}{\partial x} + \frac{\partial \rho \overline{v u}}{\partial y} + \frac{\partial \rho \overline{w u}}{\partial z} = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z} + \bar{f}_x$$

  - **Note the non-linear inertia terms, e.g.,**

$$\overline{v u} \neq \bar{v}\bar{u}$$

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Differential equations of turbulent flows

- **The effective stress**
  - **Using Reynolds decomposition**

$$u=\bar{u}+u' \xrightarrow{\text{Time-averaging}} \bar{u}=\bar{\bar{u}}+\overline{u'} \xrightarrow[\bar{\bar{u}}=\bar{u}\ ?]{\text{Stationary}} \boxed{\overline{u'}=0}$$

$$\overline{vu}=\overline{(\bar{v}+v')(\bar{u}+u')}=\overline{\bar{v}\bar{u}}+\underbrace{\overline{\bar{v}u'}}_{0\ ?}+\underbrace{\overline{v'\bar{u}}}_{0\ ?}+\overline{v'u'}=\bar{v}\bar{u}+\overline{v'u'}$$

$$\overline{vu}=\bar{v}\bar{u}+\overline{v'u'} \qquad (17.4)$$

$$\frac{\partial\rho\bar{u}\bar{u}}{\partial x}+\frac{\partial\rho\bar{v}\bar{u}}{\partial y}+\frac{\partial\rho\bar{w}\bar{u}}{\partial z}=-\left(\frac{\partial\rho\overbrace{\overline{u'u'}}}{\partial x}+\frac{\partial\rho\overbrace{\overline{v'u'}}}{\partial y}+\frac{\partial\rho\overbrace{\overline{w'u'}}}{\partial z}\right)-\frac{\partial\bar{p}}{\partial x}+\frac{\partial\bar{\tau}_{xx}}{\partial x}+\frac{\partial\bar{\tau}_{yx}}{\partial y}+\frac{\partial\bar{\tau}_{zx}}{\partial z}+\bar{f}_x \qquad (18.4)$$

Reynolds stresses

  - **For the mean non-linear inertia terms, we need the information on fluctuating properties.**
  - **The mean momentum equation is unclosed.**

Chapter 4 **By E. Amani**

Differential equations of turbulent flows

- **The effective stress**
  - **Owing to some similarities between viscous stress and Reynold stress, they are usually grouped together**

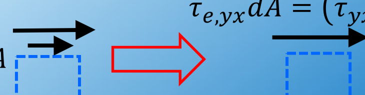
$$\frac{\partial\rho\bar{u}\bar{u}}{\partial x}+\frac{\partial\rho\bar{v}\bar{u}}{\partial y}+\frac{\partial\rho\bar{w}\bar{u}}{\partial z}=-\frac{\partial\bar{p}}{\partial x}+\frac{\partial}{\partial x}\underbrace{(\bar{\tau}_{xx}-\rho\overline{u'u'})}_{\tau_{e,xx}}+\frac{\partial}{\partial y}\underbrace{(\bar{\tau}_{yx}-\rho\overline{v'u'})}_{\tau_{e,yx}}+\frac{\partial}{\partial z}\underbrace{(\bar{\tau}_{zx}-\rho\overline{w'u'})}_{\tau_{e,zx}}+\bar{f}_x \qquad (19.4)$$

$$\tau_{e,ij}=\bar{\tau}_{ij}-\rho\overline{u_i'u_j'}=\tau_{\text{lam},ij}+\tau_{\text{turb},ij} \qquad (20.4)$$

  - **Remember,**

$$\overline{-\rho v'(\bar{u}+u')dA}=-\rho\overline{v'u'}dA$$

$$\bar{\tau}_{yx}dA$$



$$\tau_{e,yx}dA=\left(\bar{\tau}_{yx}-\rho\overline{v'u'}\right)dA$$

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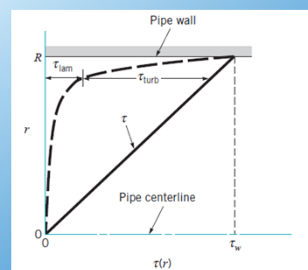
Differential equations of turbulent flows

- **The effective stress**
  - **Dropping overbar**

$$\frac{\partial \rho uu}{\partial x}+\frac{\partial \rho vu}{\partial y}+\frac{\partial \rho wu}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial \tau_{e,xx}}{\partial x}+\frac{\partial \tau_{e,yx}}{\partial y}+\frac{\partial \tau_{e,zx}}{\partial z}+f_x \qquad (22.4)$$

$$\tau_{e,ij}=\begin{cases}\tau_{\mathrm{lam},ij}=\mu\left(\dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial u_j}{\partial x_i}\right) & ;\text{laminar flow}\\ \tau_{\mathrm{lam},ij}+\tau_{\mathrm{turb},ij} & ;\text{turbulent flow}\end{cases} \qquad (23.4)$$

- **Turbulence modeling**
  - **Importance**
  - **Highly complex task**



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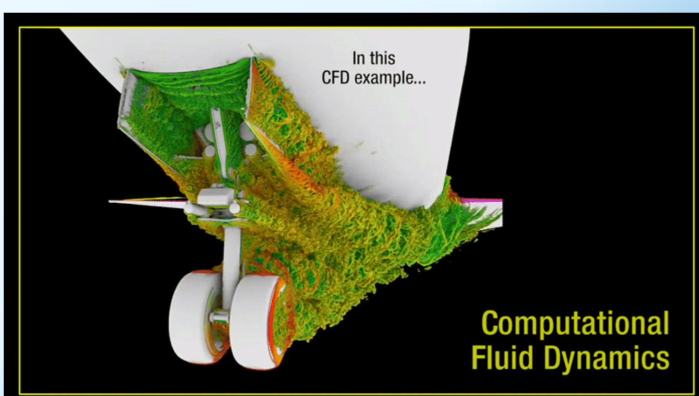
Differential equations of turbulent flows

- **Turbulence modeling**
  - **"Numerical Simulation of turbulent flows" graduate course**



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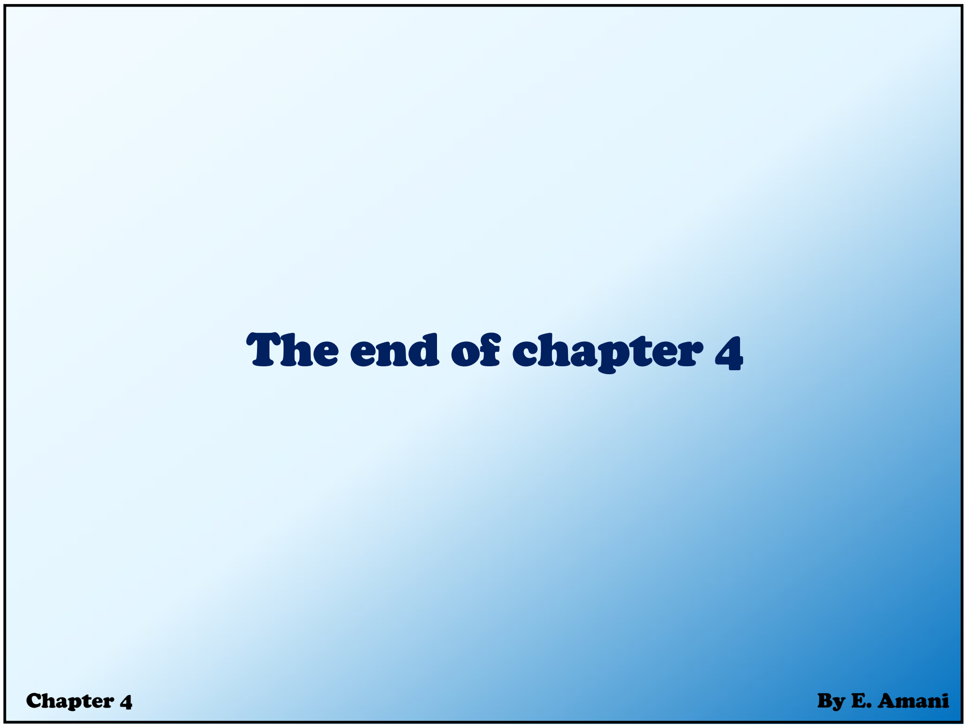
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