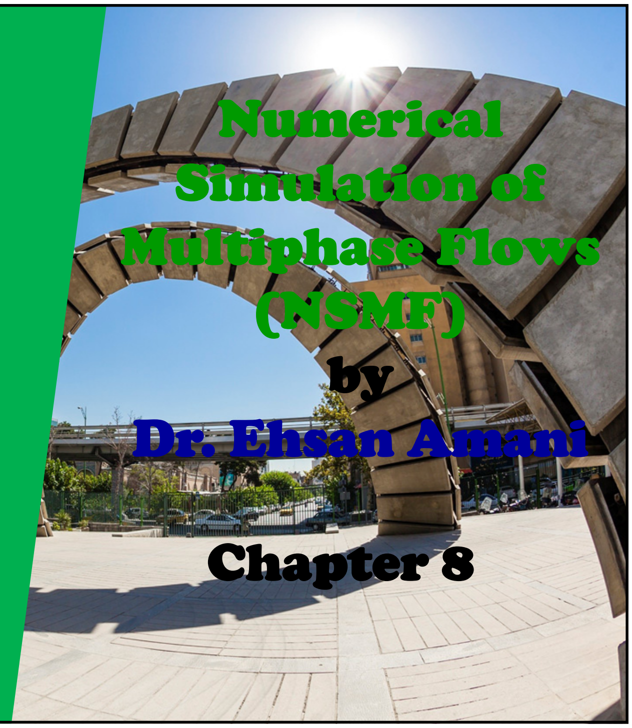


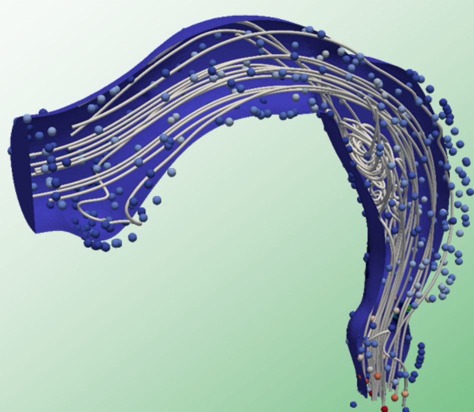
Numerical Simulation of Multiphase Flows (NSMF)

by

Dr. Ehsan Amani

Chapter 8

Averaged or statistical approaches



- **Eulerian-Lagrangian Statistical Simulation (ELSS)**
- …

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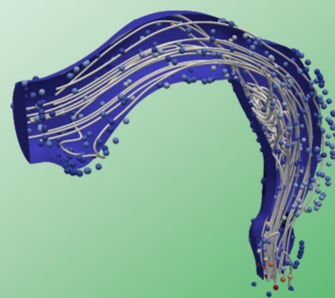
Eulerian-Lagrangian SS

- Ensemble averaging Eqs. of DNS-approach 2, dropping the index k assuming one continuous phase, for the derivation see references [1, 2]
- Fluctuation definition $\left(Q'' \equiv Q - \tilde{Q}\right)$
- Averaging non-linear terms produces additional unclosed terms (closure problem)



Example ▼

$$\frac{\partial}{\partial t}\overbrace{\langle \rho_k \chi_k \rangle}^{\bar{\rho}_k} + \frac{\partial}{\partial x_j}\overbrace{\langle \rho_k U_{k,j} \chi_k \rangle}^{\bar{\rho}_k \tilde{U}_{k,j}} = \left\langle S_m^{(I_k)} \right\rangle$$

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The Eulerian continuous-phase equations

- The continuous phase continuity:

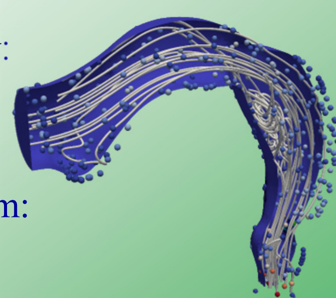
$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{U}_j\right) = \langle S_m^{(I)} \rangle$$

- The continuous phase momentum:

$$\frac{\partial}{\partial t}\left(\bar{\rho}\tilde{U}_i\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{U}_j\tilde{U}_i\right) = -\frac{\partial \bar{p}}{\partial x_j} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \bar{\rho} g_i - \frac{\partial}{\partial x_j}\left(\bar{\rho}\widetilde{u_i'' u_j''}\right) + \langle S_{U_i}^{(I)} \rangle + \langle U_{I,i} S_m^{(I)} \rangle$$

- The continuous phase scalars:

$$\frac{\partial}{\partial t}\left(\bar{\rho}\tilde{Q}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{U}_j\tilde{Q}\right) = -\frac{\partial \bar{J}_{Q,j}}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\bar{\rho}\widetilde{u_j'' Q''}\right) + \bar{\rho}\tilde{S}_Q + \langle S_Q^{(I)} \rangle + \langle Q_I S_m^{(I)} \rangle$$



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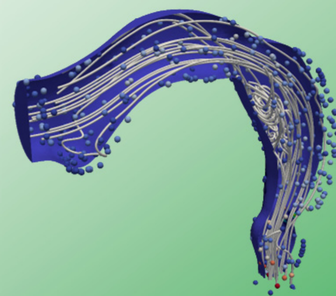
The Eulerian continuous-phase equations

- Volume fraction by sampling particles:

$$\alpha_c = 1 - \alpha_d$$



- For the summary of the Eulerian continuous-phase equations, see file “Chap8-ELSSContinuousPhaseEquations.pdf”.

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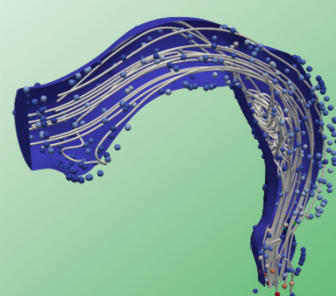
The Eulerian continuous-phase equations

- Closures:

+ closures for interface transfer (two-way coupling) source terms, $\langle S_m^{(I)}\rangle$, $\langle S_{U_i}^{(I)}\rangle$, $\langle U_{I,i} S_m^{(I)}\rangle$ etc. satisfying averaged jump condition constraints

+ closures for Reynolds stresses and fluxes, $\widetilde{u_i'' u_j''}$ and $\widetilde{u_j'' Q''}$



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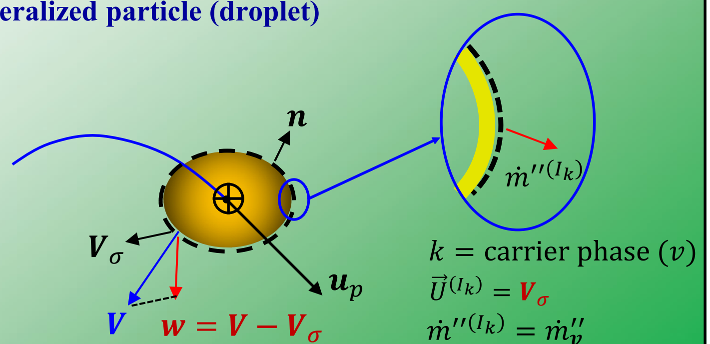
The Lagrangian dispersed-phase equations

- **Exact (instantaneous) equations**
  - **See "ELSSExactLagrangianEquations.pdf" for the derivations**
  - **Important points:**
    - **Generalized particle (droplet)**



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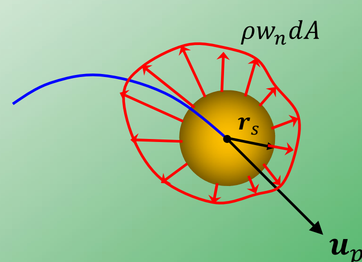
The Lagrangian dispersed-phase equations

- **Exact (instantaneous) equations**
  - **Mass equation**

$$\frac{dm_p}{dt} = -\int_{A_p} \rho w_n dA$$



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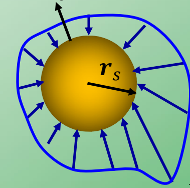
The Lagrangian dispersed-phase equations

- Exact (instantaneous) equations
  - Momentum equation

$\boldsymbol{n}$



$\boldsymbol{\sigma}.\boldsymbol{n}dA$

Center of mass position

Center of mass velocity

$$\frac{d\boldsymbol{x}_p}{dt} = \boldsymbol{u}_p$$

Fluid-particle interaction force

$$\boldsymbol{F} = m_p \frac{d\boldsymbol{u}_p}{dt}; \quad \boldsymbol{F} = \boldsymbol{F}_{\mathrm{FPI}} + \boldsymbol{F}_{\mathrm{B}} + \boldsymbol{F}_{\mathrm{col}}$$

Body force   Collision force

$$\boldsymbol{F}_{\mathrm{FPI}} = \int_{A_p} \boldsymbol{\sigma}.\boldsymbol{n}dA = \int_{A_p} (-p\boldsymbol{n} + \boldsymbol{\tau}.\boldsymbol{n})dA$$

➢ Assumptions:
- Neglecting internal flow effect
- Neglecting tangential outflow effect
- Assuming uniform efflux (zero-thrust)
- Assuming spherical particle

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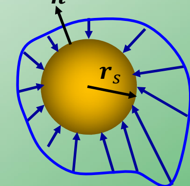
The Lagrangian dispersed-phase equations

- Exact (instantaneous) equations
  - Angular momentum equation

$\boldsymbol{n}$



$\boldsymbol{\sigma}.\boldsymbol{n}dA$

(mass) moment of inertia

Particle angular velocity

$$\boldsymbol{T} = I_p \frac{d\boldsymbol{\omega}_p}{dt}; \quad \boldsymbol{T} = \boldsymbol{T}_{\mathrm{FPI}} + \boldsymbol{T}_{\mathrm{col}}$$

$$\boldsymbol{T}_{\mathrm{FPI}} = \int_{A_p} \boldsymbol{r}_s \times (\boldsymbol{n}.\boldsymbol{\tau})dA$$

➢ Assumptions:
- Neglecting internal flow effect
- Neglecting tangential outflow effect
- Assuming uniform efflux (zero-thrust)
- Assuming spherical particle

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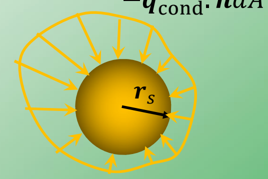
The Lagrangian dispersed-phase equations

- Exact (instantaneous) equations
  - Energy equation



$$\overbrace{m_p \frac{d\bar{\iota}_p}{dt}}^{q_{in}} = \underbrace{\int_{A_p} -\boldsymbol{q}''_{\mathrm{cond}}.\boldsymbol{n}dA}_{q_{\mathrm{conv}}} + \dot{m}_p\left(h_{v,\mathrm{surf}} - \bar{h}_p + w'\,w'/2\right) + q_R$$

➢ Assumptions?

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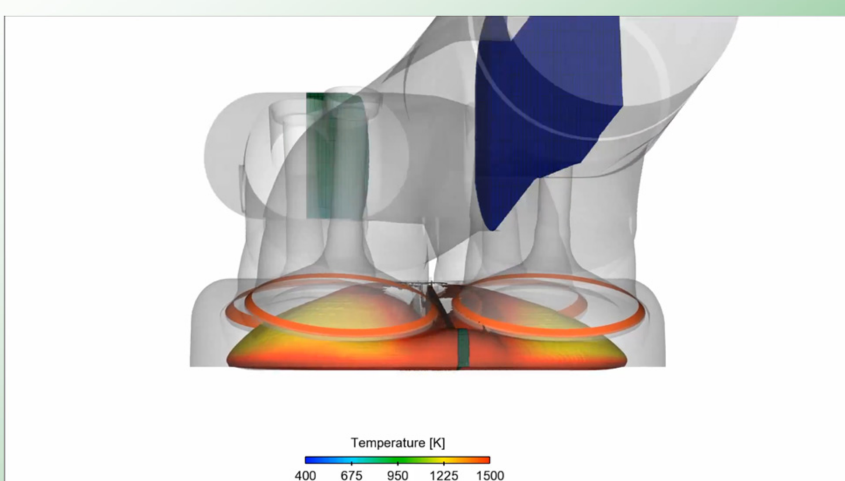
Eulerian-Lagrangian approach



Port fuel injection (PFI) in gasoline engines ▲

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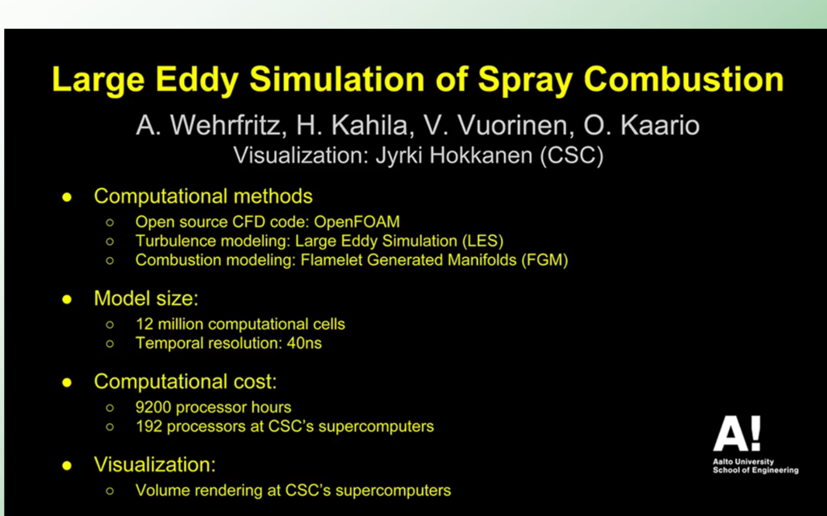
Eulerian-Lagrangian approach



Turbulent spray combustion ▲

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1. **Michaelides, E., C.T. Crowe, and J.D. Schwarzkopf, *Multiphase flow handbook*. 2016: CRC Press.**
2. **Naud, B., *PDF modeling of turbulent sprays and flames using a particle stochastic approach.* 2003.**
3. **Kataoka, I., *Local instant formulation of two-phase flow.* International Journal of Multiphase Flow, 1986. 12(5): p. 745-758.**
4. **Tryggvason, G., R. Scardovelli, and S. Zaleski, *Direct Numerical Simulations of Gas-Liquid Multiphase Flows*. 2011: Cambridge University Press.**

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