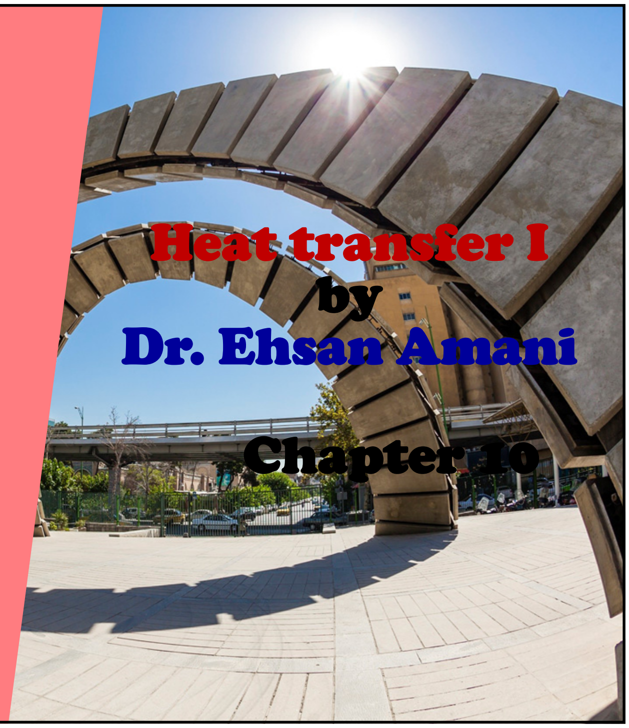
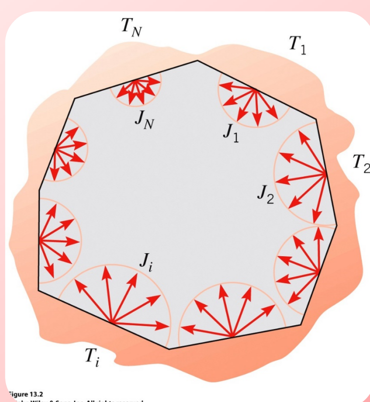


Heat transfer I
by
Dr. Ehsan Amani

Chapter 10

X Radiation between surfaces

- **Assumptions**
  1. There is a non-participating medium between surfaces, e.g. vacuum or most of gases
  2. The surfaces are opaque and diffuse



Chapter 10 By E. Amani

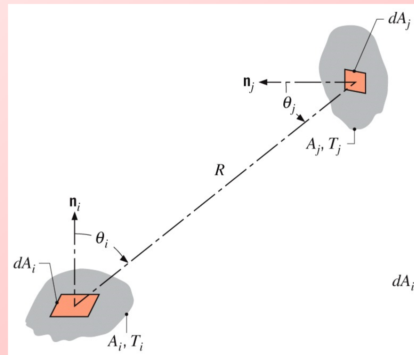
X.1 The view factor

- **Definitions**
  - The fraction of the radiation leaving surface i that is intercepted by surface j

$$F_{ij} = \frac{q_{i\to j}}{q_i} = \frac{q_{i\to j}}{\int_{A_i} J_i dA_i} \qquad (1.10)$$

3. If the radiosity is uniform on surface i:

$$F_{ij} = \frac{q_{i\to j}}{J_i A_i} \qquad (2.10)$$



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X.1 The view factor

- **Formulation** **Diffuse:** $I_{\lambda,e+r,i} = \dfrac{J_{\lambda,i}}{\pi}$

(10.9) or (22.9)

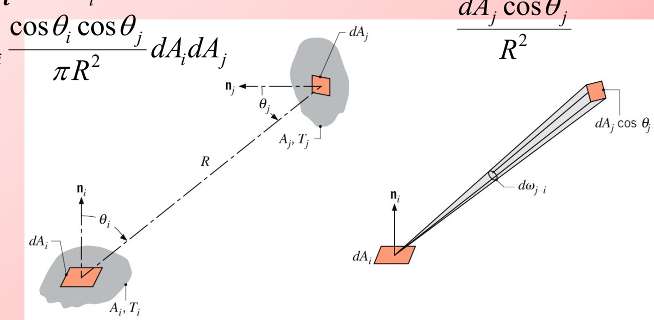
$$dq_{\lambda,i\to j} = \overbrace{I_{\lambda,\theta,e+r,i}} d\lambda \cos\theta_i dA_i d\omega_{j-i}$$

$$dq_{i\to j} = \int_0^\infty dq_{\lambda,i\to j} = \frac{1}{\pi}\underbrace{\left(\int_0^\infty J_{\lambda,i} d\lambda\right)}_{J_i}\cos\theta_i dA_i d\omega_{j-i} = \frac{J_i}{\pi}\cos\theta_i dA_i \underbrace{d\omega_{j-i}}_{(8.9)\ \frac{dA_j\cos\theta_j}{R^2}}$$

Uniform J_i

$$q_{i\to j} = \int dq_{i\to j} = \int_{A_j}\int_{A_i} J_i \frac{\cos\theta_i\cos\theta_j}{\pi R^2} dA_i dA_j$$

$$F_{ij} = \frac{q_{i\to j}}{J_i A_i}$$



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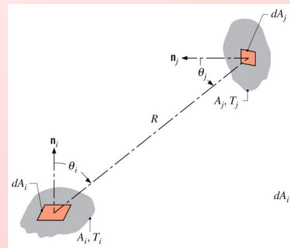
X.1 The view factor

- **Calculation**

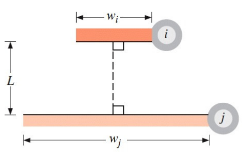
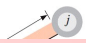
A pure geometrical parameter

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}\,dA_j\,dA_i \qquad (3.10)$$



- For some surface configurations, the analytical integration of Eq. (3.10) has been reported:

TABLE 13.1 View Factors for Two-Dimensional Geometries [4]

Geometry	Relation
Parallel Plates with Midlines Connected by Perpendicular 	$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$ $W_i = w_i/L,\ W_j = w_j/L$
Inclined Parallel Plates of Equal Width and a Common Edge 	$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$

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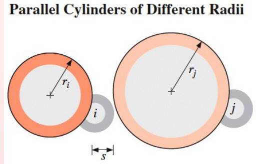
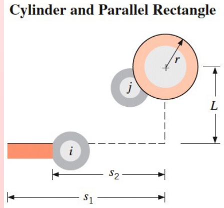
X.1 The view factor

- **Calculation**

TABLE 13.1 *Continued*

Geometry	Relation
Parallel Cylinders of Different Radii 	$F_{ij} = \frac{1}{2\pi}\Big\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\Big\}$ $R = r_j/r_i,\ S = s/r_i$ $C = 1 + R + S$
Cylinder and Parallel Rectangle 	$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$

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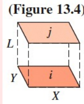
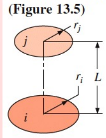
X.1 The view factor

- **Calculation**

Table 13.2 View Factors for Three-Dimensional Geometries [4]

Geometry	Relation
Aligned Parallel Rectangles (Figure 13.4) 	$\bar{X} = X/L, \bar{Y} = Y/L$ $$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$
Coaxial Parallel Disks (Figure 13.5) 	$R_i = r_i/L, R_j = r_j/L$ $$S = 1 + \frac{1+R_j^2}{R_i^2}$$ $$F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$$
Perpendicular Rectangles with a Common Edge (Figure 13.6)	$H = Z/X, W = Y/X$ $$F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{\ldots}\right.$$

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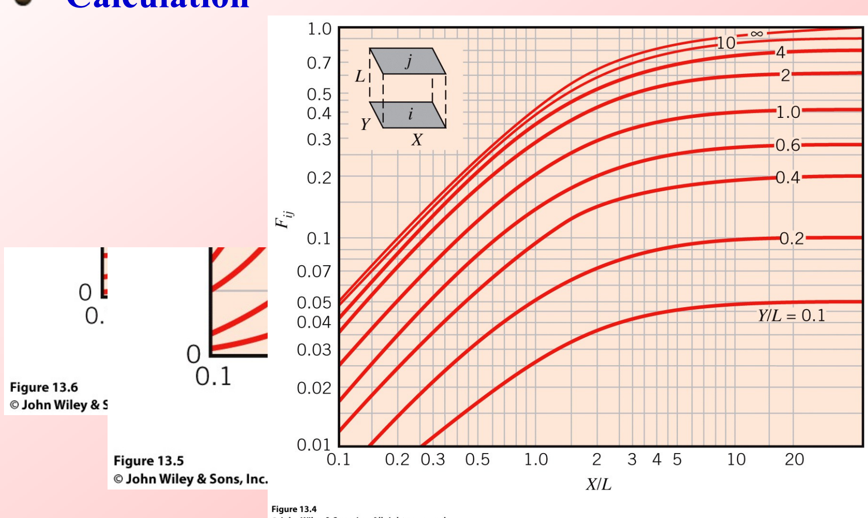
X.1 The view factor

- **Calculation**



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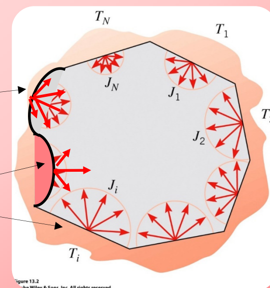
X.1 The view factor

- **Important relations**
  - Reciprocity Relation (proof: exercise)

$$A_i F_{ij} = A_j F_{ji} \qquad (4.10)$$

  - Summation rule: For an enclosure of N surfaces with conditions 1-3 (proof: exercise)

$$\sum_{j=1}^{N} F_{ij} = 1 \qquad (5.10)$$



  - For a concave surface:

$$0 < F_{ii} \leq 1$$

  - For a convex or flat surface:

$$F_{ii} = 0$$

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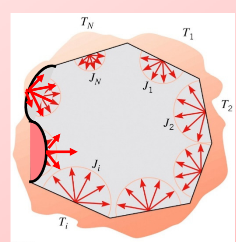
X.1 The view factor

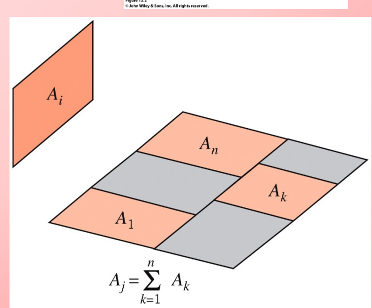
- **Important relations**
  - For the enclosure, from N^2 view factors, $N(N-1)/2$ are independent:



$$N^2 - \underbrace{N}_{\substack{(5.10)\\ i=1,\dots,N}} - \underbrace{\frac{N^2 - N}{2}}_{(4.10)} = \frac{N^2 - N}{2} = \frac{N(N-1)}{2}$$

  - Compound receiver surface

$$F_{i(j)} = \sum_{k=1}^{N} F_{ik} \qquad (6.10)$$



  - Compound emitter surface

$$\xrightarrow[(4.10)]{(6.10)} \frac{A_j F_{(j)i}}{A_i} = \sum_{k=1}^{N} \frac{A_k F_{ki}}{A_i} \longrightarrow F_{(j)i} = \frac{\sum_{k=1}^{N} A_k F_{ki}}{A_j} \qquad (7.10)$$

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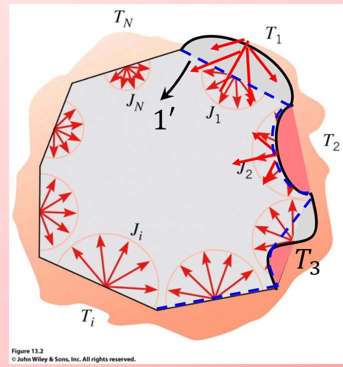
X.1 The view factor

- **Important relations**
  - For the projection of a surface, provided that the projected surface fully covers the original surface:

$$F_{j1} = F_{j1'} \qquad \underset{j=2,\dots,N}{(8.10)}$$

$$F_{11'} = F_{12} + F_{13} + \dots + F_{1N} \qquad (9.10)$$



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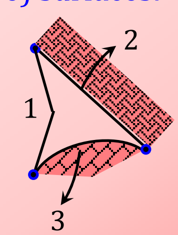
X.1 The view factor

- **Additional techniques (2D geometries)**
  - Enclosures with 3 flat/convex (non-concave) surfaces:



$$F_{11} = F_{22} = F_{33} = 0$$

$$\left.\begin{array}{l} (5.10)\ F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{12} + F_{13} = 1 \\ (5.10)\ F_{21} + F_{22} + F_{23} = 1 \xrightarrow{(4.10)} \frac{A_1}{A_2}F_{12} + F_{23} = 1 \\ (5.10)\ F_{31} + F_{32} + F_{33} = 1 \xrightarrow{(4.10)} \frac{A_1}{A_3}F_{13} + \frac{A_2}{A_3}F_{23} = 1 \end{array}\right\} \xrightarrow{\text{3 unknown, 3 equations}}$$

$$F_{ij} = \frac{A_i + A_j - A_k}{2A_i}; \quad i \neq j \neq k = 1,2,3 \qquad (11.10)$$

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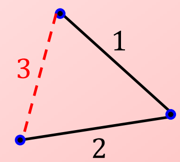
X.1 The view factor

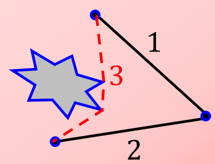
- **Additional techniques (2D geometries)**
  - Enclosures with 3 flat/convex (non-concave) surfaces
  - Intersecting non-concave surfaces:

$F_{12} = ?$



  - Obstacle effect:

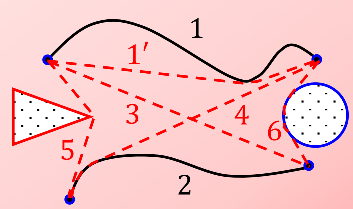
$F_{12} = ?$



X.1 The view factor

- **Additional techniques (2D geometries)**
  - Enclosures with 3 flat/convex (non-concave) surfaces
  - Intersecting non-concave surfaces
  - Obstacle effect
  - Hottel's crossed-string method:

$F_{12} = ?$

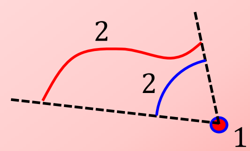


$$F_{12} = \frac{A_4 + A_3 - A_6 - A_5}{2A_1} \qquad (12.10)$$

X.1 The view factor

- **Additional techniques (2D geometries)**
  - Enclosures with 3 flat/convex (non-concave) surfaces
  - Intersecting non-concave surfaces
  - Obstacle effect
  - Hottel's crossed-string method
  - A point source and a surface:

F_{12} =?



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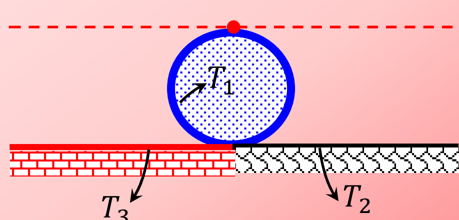
X.1 The view factor

- **Additional techniques (2D geometries)**
  - Enclosures with 3 flat/convex (non-concave) surfaces
  - Intersecting non-concave surfaces
  - Obstacle effect
  - Hottel's crossed-string method
  - A point source and a surface
  - Geometrical symmetry:

F_{13} =?



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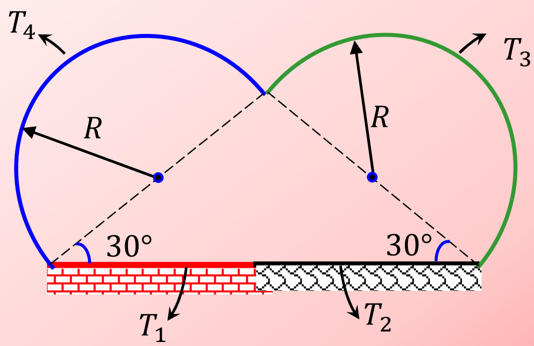
X.1 The view factor

- **Exercise:** F_{13} =?



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X.2 Blackbody Radiation Exchange

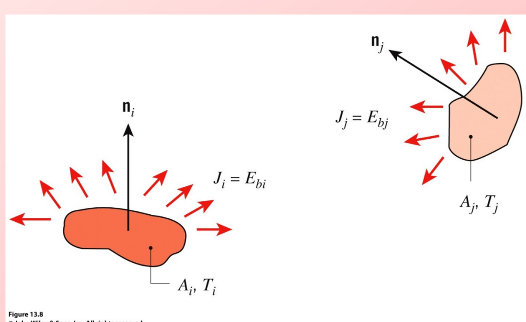
- **Two black-body surfaces:**

$$\xrightarrow{(2.10)} q_{i\to j} = A_i J_i F_{ij} \quad , J_i = E_{bi} = \sigma T_i^4$$

$$q_{ij} = q_{i\to j} - q_{j\to i} \quad q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj} \xrightarrow{(4.10)}$$

Net radiation exchange from i to j

$$q_{ij} = A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right) \qquad (13.10)$$



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X.3 Enclosures with opaque, diffuse, gray surfaces

- **Surface radiative resistance**

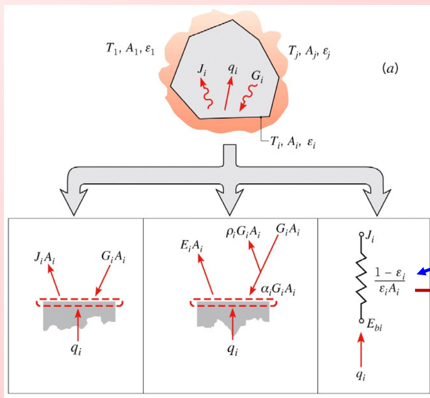
$$\xrightarrow{(7.9)} \quad q_{i,rad} = A_i\left(J_i - G_i\right)$$

$$J_i = E_i + \rho_i G_i = \varepsilon_i E_{bi} + (1-\varepsilon_i)G_i \quad \longrightarrow \quad G_i = \frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i}$$

$$\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$$

$$q_i = \frac{E_{bi} - J_i}{\left(1-\varepsilon_i\right)/\varepsilon_i A_i} \quad (14.10)$$

Suggests a surface radiative resistance of the form



$\varepsilon_i \to 1$ Or $A_i \to \infty$ $\Rightarrow$ $J_i = E_{bi}$ black-body

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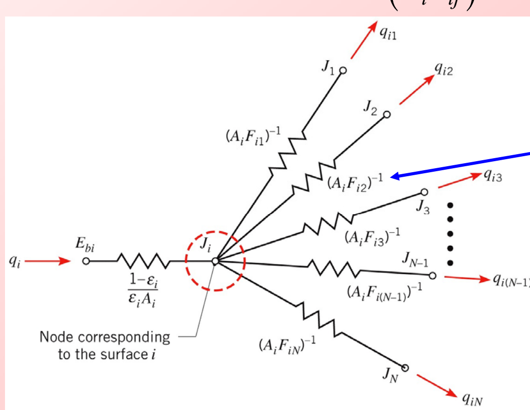
X.3 Enclosures with opaque, diffuse, gray surfaces

- **Geometrical resistance**

$$q_{i\to j} = A_i J_i F_{ij} \qquad q_{j\to i} = A_j J_j F_{ji}$$

$$q_{ij} = q_{i\to j} - q_{j\to i} = A_i F_{ij}\left(J_i - J_j\right) = \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \quad (15.10)$$

Suggests a space or geometrical resistance of the form



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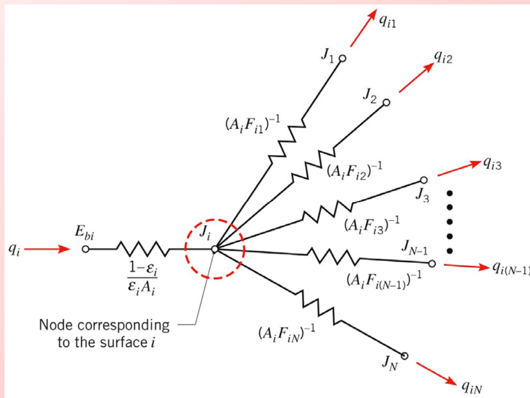
X.3 Enclosures with opaque, diffuse, gray surfaces

- **For an enclosure**

$$q_i = q_{i,rad} = \sum_{j=1}^{N} q_{ij} = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (16.10)$$



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X.3 Enclosures with opaque, diffuse, gray surfaces

- **Methodology of analysis**
  - If q_i is given for surface i, use Eq. (16.10):

$$q_{i,rad} = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (16.10)$$

  - If T_i is given for surface i, use Eq. (17.10):

$$\xrightarrow[(16.10)]{(14.10)} \quad \frac{E_{bi} - J_i}{\left(1-\varepsilon_i\right)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (17.10)$$

  - N equations for N unknowns $(J_i; i = 1, \ldots, N)$

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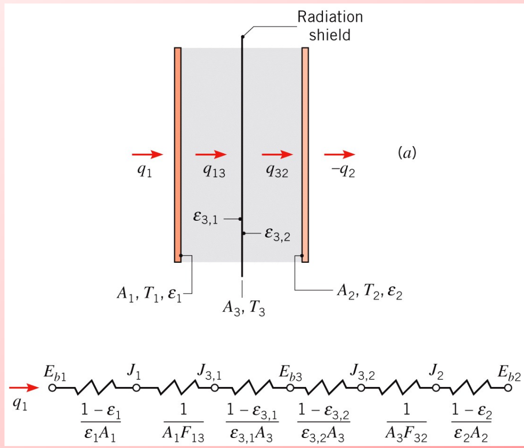
X.4 Radiation shield

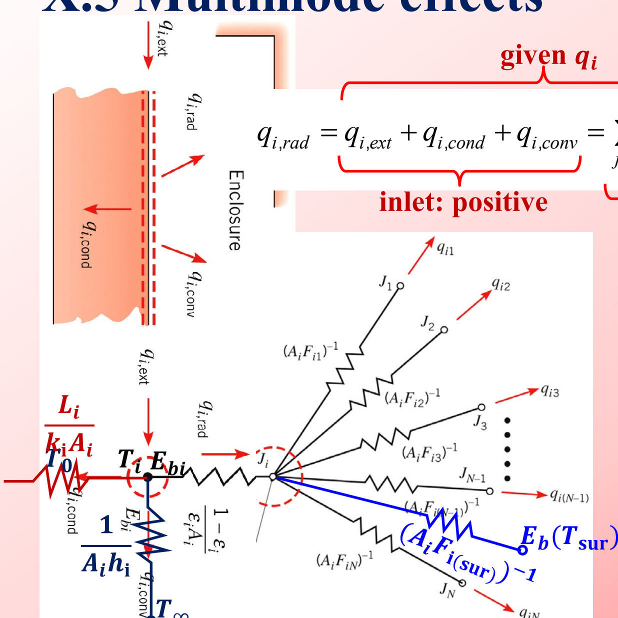
- **Reduction in radiation exchange between two surfaces**
- **High reflectivity (low $\alpha = \varepsilon$) surface(s)**



Chapter 10 **By E. Amani**

X.5 Multimode effects

$$q_{i,rad} = \underbrace{\overbrace{q_{i,ext} + q_{i,cond} + q_{i,conv} = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}}}^{\text{given } q_i}}_{\text{inlet: positive}} = \underbrace{\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i}}_{\text{given } T_i}$$



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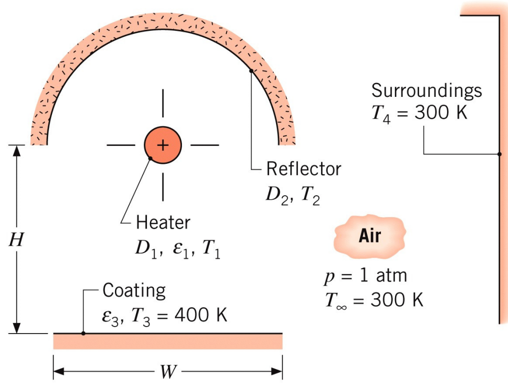
6.10 Sample problems

A long rod heater of diameter D_1 = 10 mm and emissivity ε_1 = 1.0 is coaxial with a well-insulated, semicylindrical reflector of diameter D_2 = 1 m. A long panel of width W = 1m is aligned with the reflector and is separated from the heater by a distance of H = 1m. The panel is coated with a special paint (ε_3 = 0.7), which is cured by maintaining it at 400 K. The panel is well insulated on its back side, and the entire system is located in a large room where the walls and the atmospheric, quiescent air are at 300 K. Heat transfer by convection may be neglected for the reflector surface.

(a) Sketch the equivalent thermal circuit for the system and label all pertinent resistances and potentials?

(b) Expressing your results in terms of appropriate variables, write the system of equations needed to determine the heater and reflector temperatures, T_1 and T_2, respectively. Determine these temperatures for the prescribed conditions?

(c) Determine the rate at which electrical power must be supplied per unit length of the rod heater.



The end of chapter 10