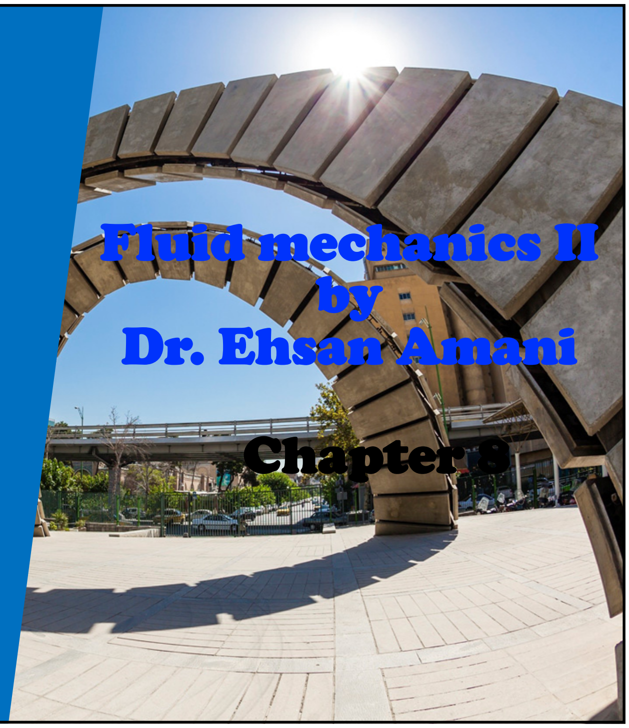
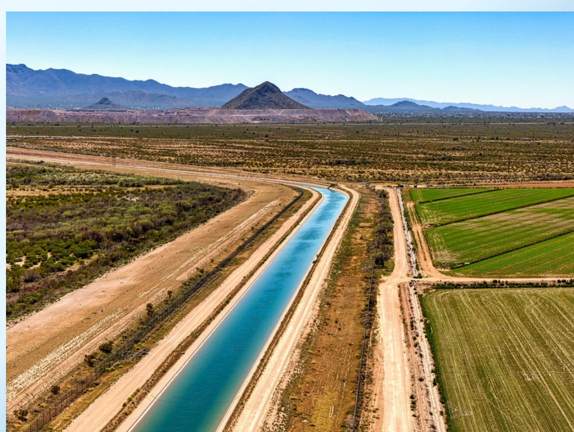


Fluid mechanics II
by
Dr. Ehsan Amani

Chapter 8

Open-channel flow applications

- Canals



irrigation canal ▲

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Open-channel flow applications

- Canals
- Culverts

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Open-channel flow applications

- Canals
- Culverts
- Spillways

Bell-mouth spillway ▲

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Open-channel flow applications

- Canals
- Culverts
- Spillways

Bell-mouth spillway ▲

Open-channel flow applications

- Canals
- Culverts
- Spillways

Ogee spillway ▲

Open-channel flow applications

- Canals
- Culverts
- Spillways
- Weirs

Triangular sharp-crested weir▲

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Open-channel flow applications

- Canals
- Culverts
- Spillways
- Weirs
- Sluice gates

Dam sluice gates▲

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Open-channel flow applications

- Canals
- Culverts
- Spillways
- Weirs
- Sluice gates
- Sewers

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Open-channel flow applications

- Canals
- Culverts
- Spillways
- Weirs
- Sluice gate
- Sewers
- Dams
- …

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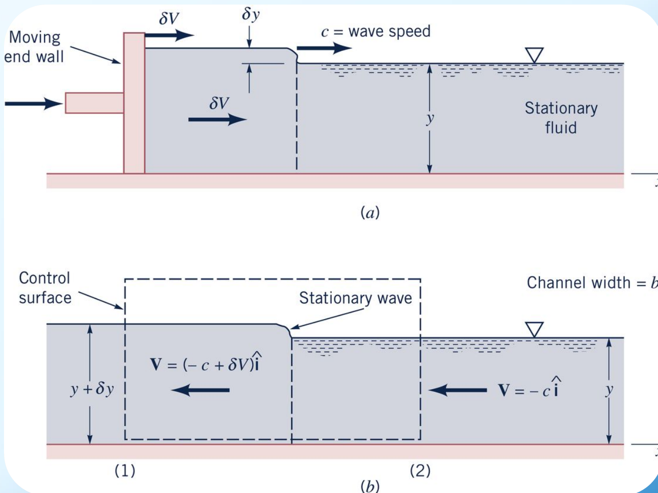
Gravity wave and Froude number



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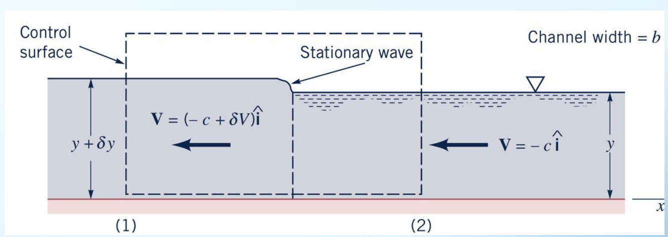
Gravity wave and Froude number



- **1D flow assumption + rectangular channel:**
- Mass: $\rho c b y = \rho(c - \delta V)(y + \delta y)b$
- Momentum: $-\frac{1}{2}\rho g b[(y + \delta y)^2 - y^2] = \rho c b y(c - \delta V - c)$
  - Exercise: Using Bernoulli equation normal to streamline, show that the pressure hydrostatically changes
- Removing δV and solving for c: $c = \sqrt{gy\left(1 + \frac{\delta y}{y}\right)\left(1 + \frac{\frac{1}{2}\delta y}{y}\right)}$

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Gravity wave and Froude number

- Large-amplitude single-wave:

$$c = \sqrt{gy\left(1+\frac{\delta y}{y}\right)\left(1+\frac{\frac{1}{2}\delta y}{y}\right)}$$

$\delta y \uparrow \Longrightarrow c \uparrow$

- Small-amplitude elementary single-wave:

$$c = \sqrt{gy}$$

- Froude number:

$$\mathrm{Fr} \equiv \frac{V}{c} = \frac{V}{\sqrt{gL}}, L = y$$

For rectangular channel

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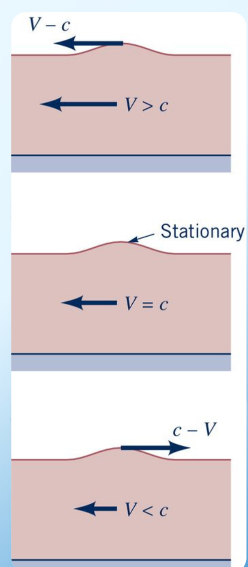
Gravity wave and Froude number

- **Regimes:**

❑ Supercritical ($\mathrm{Fr} > 1$):

❑ Critical ($\mathrm{Fr} = 1$):

❑ Subcritical ($\mathrm{Fr} < 1$):



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Forces

- **Gravity ✓**
- **Inertia ✓**
- **Friction ✓×**
- **Surface tension ✓×**
- **pressure ×**

$$\mathrm{Fr} \sim \frac{\text{Inertia}}{\text{Gravity}}$$

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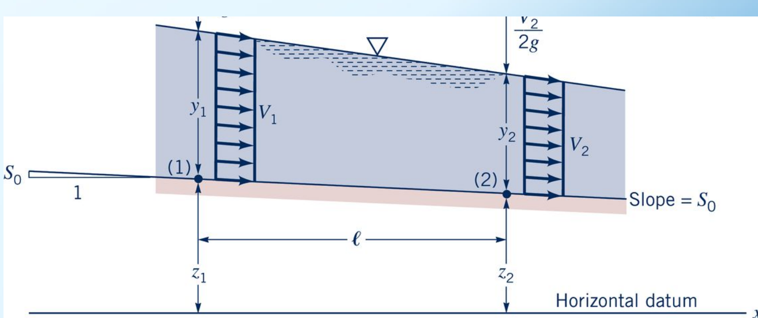
Classification

- **Based on depth variation:**

Normal depth

1. Uniform Flow (UF): $dy/dx = 0, \quad y = y_n$
2. Gradually Varying Flow (GVF): $dy/dx \ll 1$
3. Rapidly Varying Flow (RVF): $dy/dx = O(1)$



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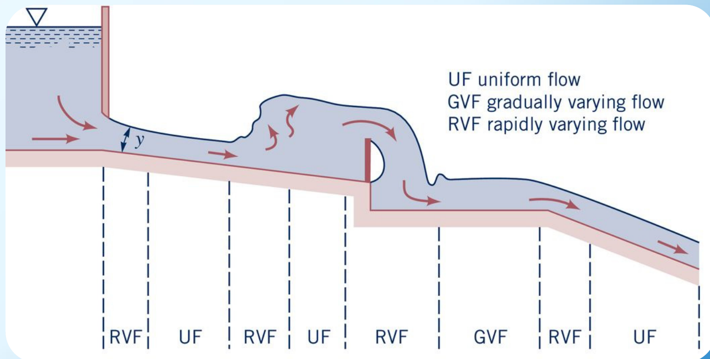

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Classification

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  3. Rapidly Varying Flow (RVF): $dy/dx = O(1)$
- **Laminar/turbulent:**
  1. Laminar (**Re < 500**)
  2. Turbulent (**Re > 12500**)

Hydraulic radius

$\mathrm{Re} = VR_h/\nu$

Flow of

50 °F water ($\nu = 1.41 \times 10^{-5}\ ft/s$), $V = 1\ ft/s$, and $R_h = 10\ ft$

⇒ $\mathrm{Re} = 7.1 \times 10^{+5}$

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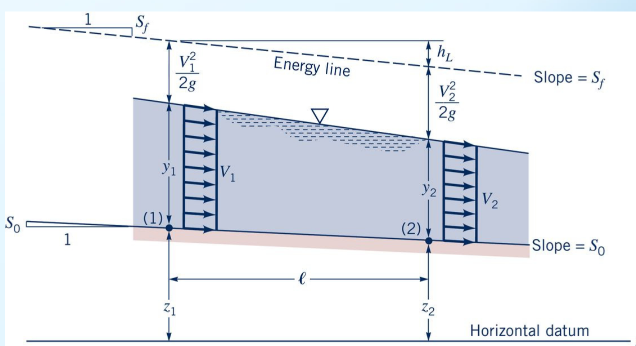
Classification

- **Based on depth variation:**
  1. Uniform Flow (UF): $dy/dx = 0, \quad y = y_n$
  2. Gradually Varying Flow (GVF): $dy/dx \ll 1$
  3. Rapidly Varying Flow (RVF): $dy/dx = O(1)$
- **Laminar/turbulent:**
  1. Laminar (**$\mathbf{Re} < 500$**)
  2. Turbulent (**$\mathbf{Re} > 12500$**)
- **Wave propagation characteristics:**
  1. Subcritical (**$\mathbf{Fr} < 1$**)
  2. Critical (**$\mathbf{Fr} = 1$**)
  3. Supercritical (**$\mathbf{Fr} > 1$**)

Specific energy diagram

- **Assumptions:**



- 1D flow
- Steady flow
- Constant bottom slope ($S_0 = (z_1 - z_2)/\ell$)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$p_1/\gamma = y_1, p_2/\gamma = y_2$

Specific energy diagram

- **Leads to:**

$$y_1 + \frac{V_1^2}{2g} + S_0\ell = y_2 + \frac{V_2^2}{2g} + h_L$$

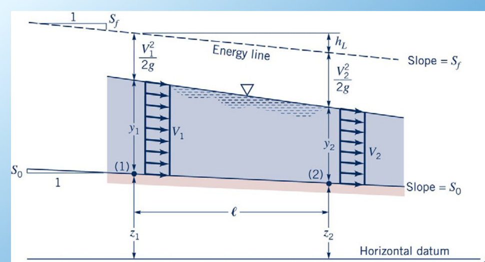
- **The specific energy (head) definition:**

$$E \equiv y + \frac{V^2}{2g}$$

- **and friction slope ($S_f = h_L/\ell$) results in:**

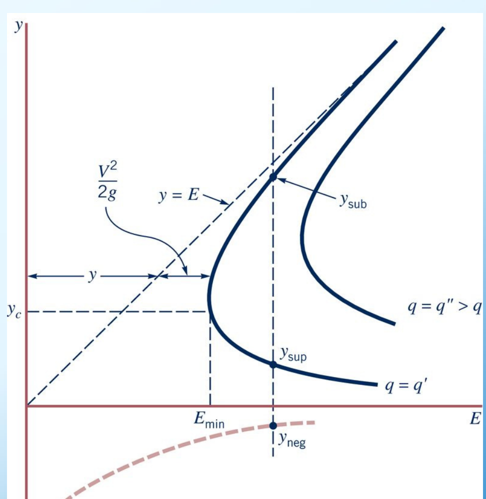
$$E_1 = E_2 + \underbrace{(S_f - S_0)\ell}_{(z_2 - z_1) + h_L}$$

For non-constant bottom slope



Specific energy diagram

- **diagram**



Specific energy diagram

- **For rectangular channel of width b**

($q = Q/b = Vyb/b = Vy$):

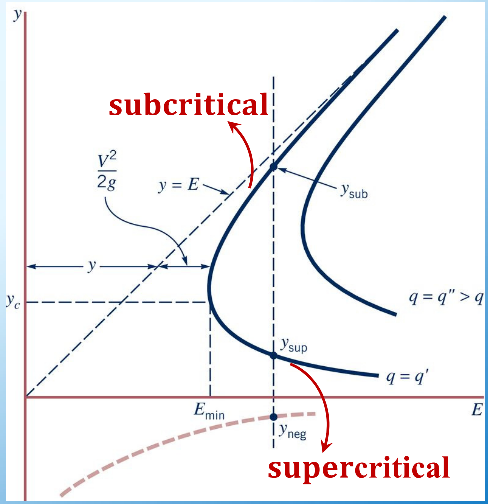
$$E = y + \frac{q^2}{2gy^2}$$

$$\rightarrow y^3 - Ey^2 + (q^2/2g) = 0$$

$$dE/dy = 0 \rightarrow \quad y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$E_{\min} = \frac{3y_c}{2}$$

$$V_c = \frac{q}{y_c} = \sqrt{gy_c} \rightarrow \mathrm{Fr}_c = \frac{V_c}{c} = 1$$



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Specific energy diagram

- **For non-rectangular channel,**

for a given Q, $A(y)$,

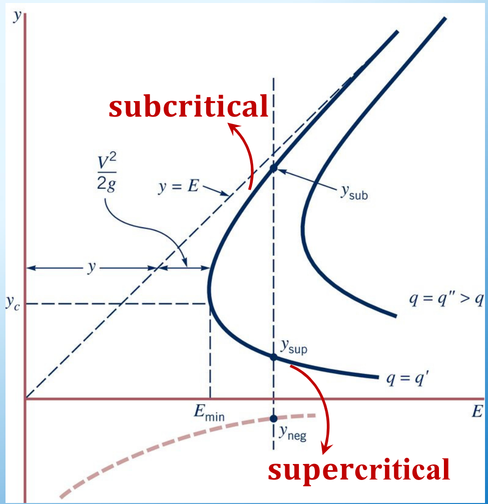
and $b_0(y)$ [2]:

$$A(y_c) = \left(\frac{b_0(y_c)Q^2}{g}\right)^{1/3}$$

Should be solved by an trial and error approach for y_c and:

$$V_c = c = \frac{Q}{A_c} = \left(\frac{gA(y_c)}{b_0(y_c)}\right)^{1/2}$$

$$\mathrm{Fr} = \frac{V}{c} = \frac{V}{\sqrt{gL}} \rightarrow L = \frac{A(y_c)}{b_0(y_c)}$$



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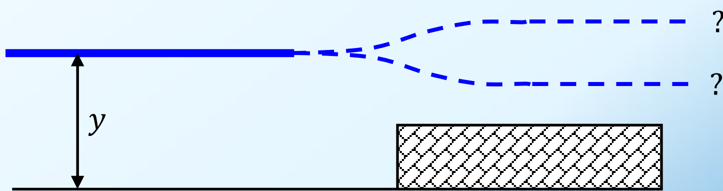
Flow over a bump



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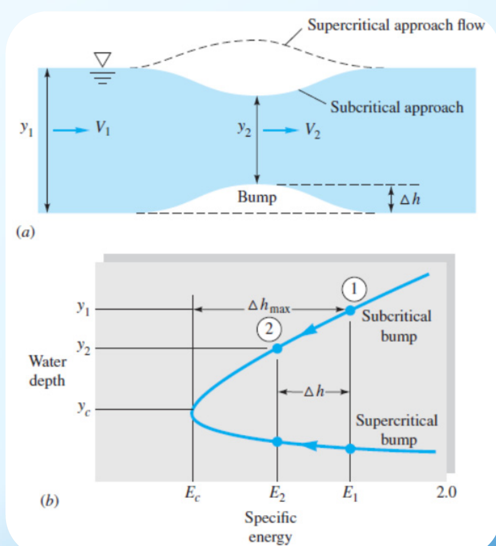
Flow over a bump

$$E_1 = E_2 + (z_2 - z_1) + h_L$$

$$z_2 > z_1$$

$$E_2 < E_1$$



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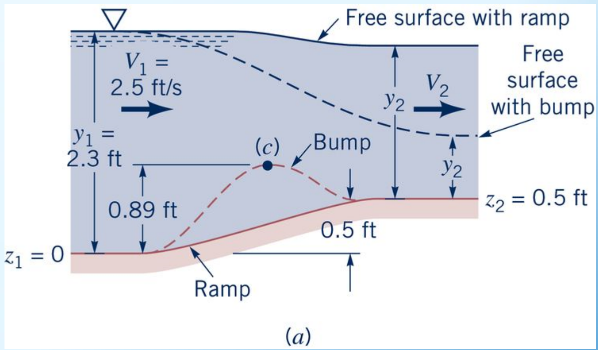
Example

- **Rectangular channel, water, neglecting friction, $y_2 = ?$**



(a)

Notes: VIII.1

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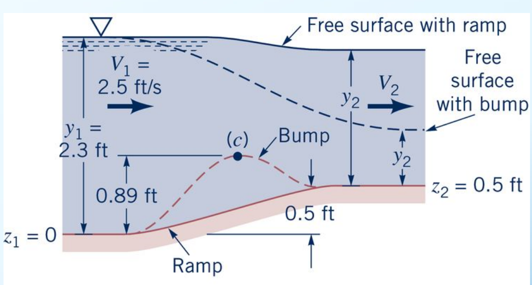
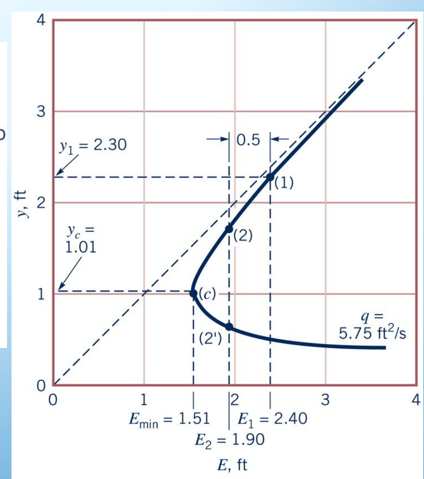
Example

- **Rectangular channel, water, neglecting friction, $y_2 = ?$**



(a)



(b)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = 1.01\ ft \qquad E_{\min} = \frac{3y_c}{2} = 1.51\ \text{ft}$$

$$E_1 = E_{\min} + z_c - z_1$$
$$\rightarrow z_c - z_1 = 0.89\ ft$$

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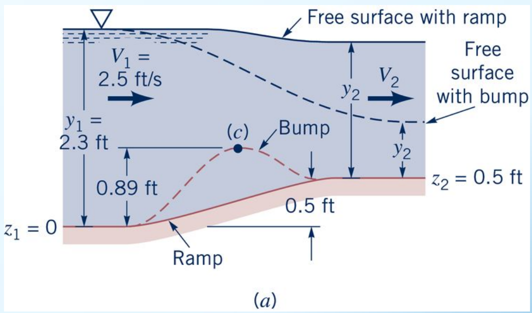
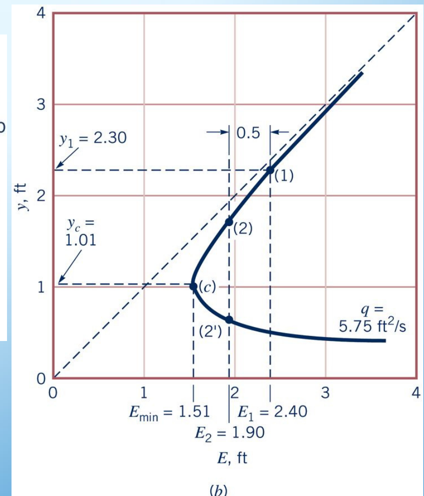
Question

- **What happens if the bump level gets higher than 0.89 ft?**
  - **Choking**



(a)



(b)

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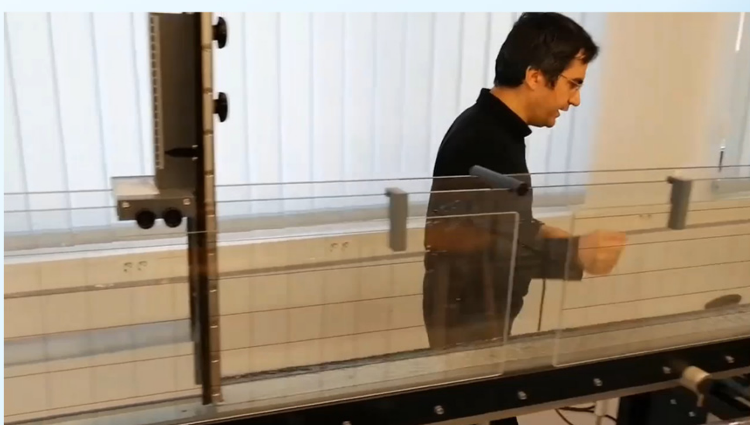
Question

- **Choking**



  - **See also:** https://en.wikiversity.org/wiki/Classic_energy_problem_in_open-channel_flow

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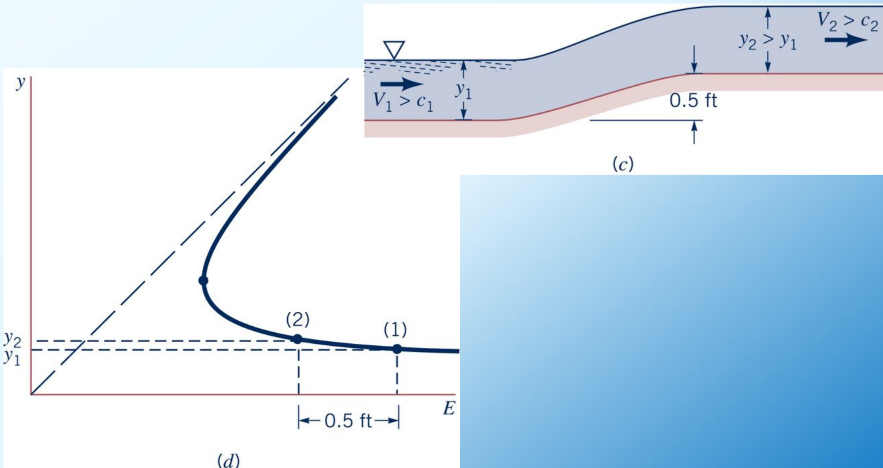
Example

- **Rectangular channel, neglecting friction, $y_2 = ?$**



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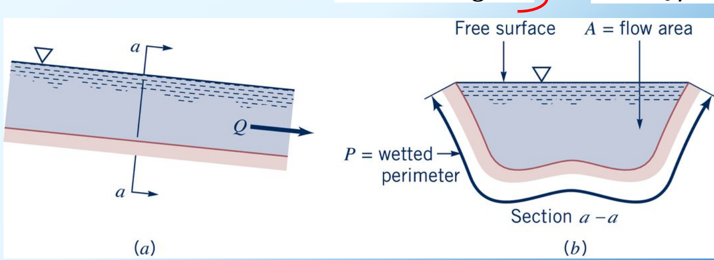
Uniform flow

- **Bottom slope cancels out friction slope (balance between gravity and friction)**
- **For constant area and shape channels:**

$$\left.\begin{array}{r} E_1 = E_2 + (S_f - S_0)\ell \\ \text{if } S_0 = S_f \end{array}\right\} \quad \left.\begin{array}{l} E_1 = E_2 \\ E = y + \dfrac{Q^2}{2gA^2} \end{array}\right\} \quad A = cte \Rightarrow \begin{array}{l} y = cte \\ V = Q/A = cte \end{array} \rightarrow \frac{dy}{dx} = 0$$



Open-channel flow counterpart of fully-developed internal flow

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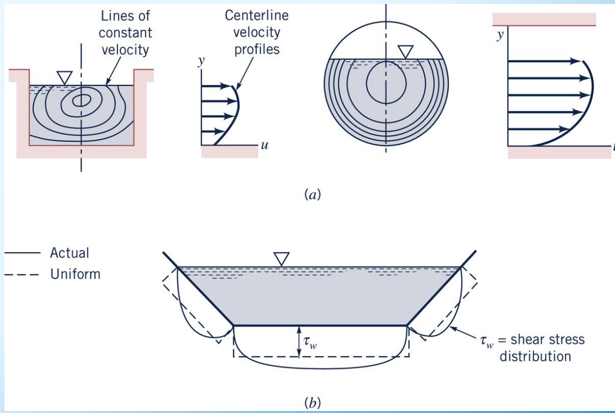
Uniform flow

- **Assumptions:**
  - **Steady flow**
  - **1D flow (uniform velocity and shear stress profiles)**
  - **Constant-area and -shape channel**



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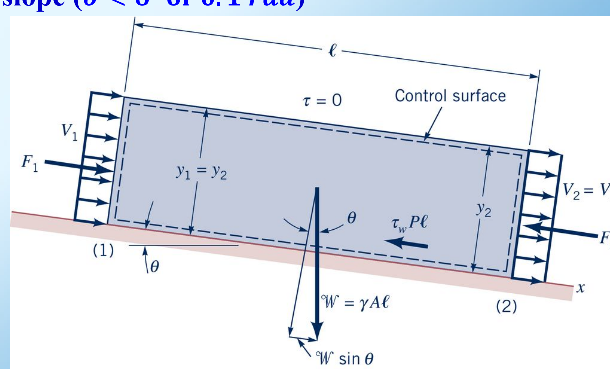
Uniform flow

- **Assumptions:**
  - **Steady flow**
  - **1D flow (uniform velocity and shear stress profiles)**
  - **Constant-area and -shape channel**
  - **Small bottom slope ($\theta < 6°$ or $0.1\ rad$)**



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Uniform flow

- **Momentum balance**

$\sin\theta \approx \tan\theta = S_0$, $\mathcal{W} = \gamma A \ell$,

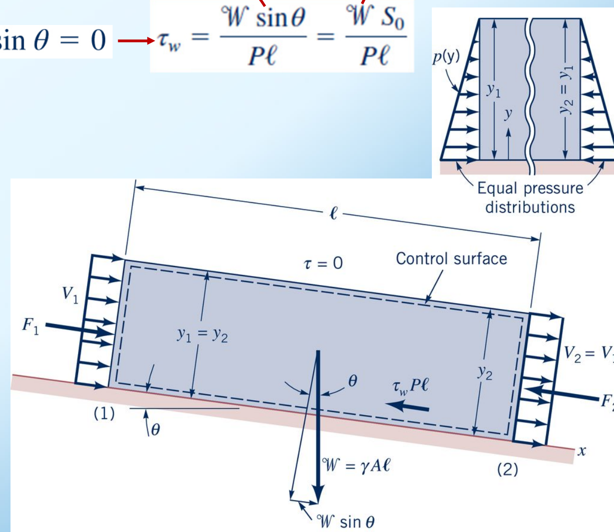
$$F_1 - F_2 - \tau_w P\ell + \mathcal{W}\sin\theta = 0 \rightarrow \tau_w = \frac{\mathcal{W}\sin\theta}{P\ell} = \frac{\mathcal{W}\,S_0}{P\ell}$$

$$\tau_w = \frac{\gamma A \ell S_0}{P\ell} = \gamma R_h S_0$$

$$R_h \equiv \frac{A}{P} = \frac{1}{4} D_h$$

Hydraulic radius

Wetted perimeter



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Uniform flow

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$$\tau_w = \frac{\gamma A \ell S_0}{P\ell} = \gamma R_h S_0$$

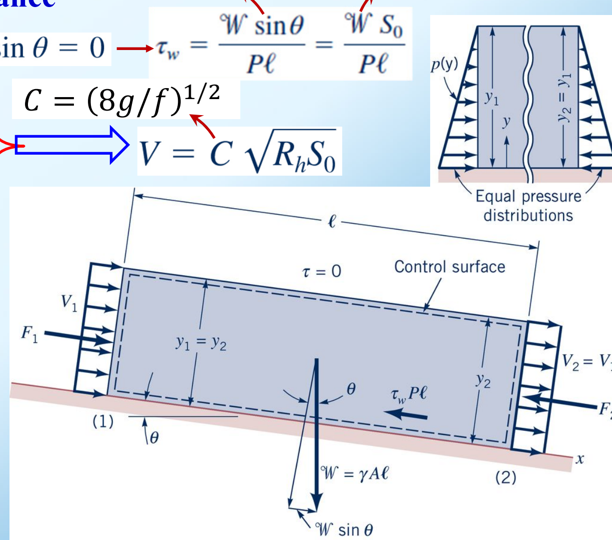
(31.4) $\tau_W = \frac{1}{8}\rho V^2 f$

$$C = (8g/f)^{1/2}$$

$$\Rightarrow V = C\sqrt{R_h S_0}$$

Assuming turbulent fully-developed pipe flow for wholly turbulent regime

$f = f(\varepsilon/D)$



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Uniform flow

- **Manning equation**

Manning equation

Manning resistance coefficient (dimensional!!)

$$\underbrace{V = C\sqrt{R_h S_0}}_{\text{Chezy equation}} \xrightarrow{\text{Experiment}} V = \frac{R_h^{2/3} S_0^{1/2}}{n}$$

where $\kappa = 1$ if SI units are used, and $\kappa = 1.49$ if BG units are used.

$$\boxed{V = \frac{\kappa}{n} R_h^{2/3} S_0^{1/2}}$$

$$\boxed{Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}}$$

for water

Values of the Manning Coefficient, *n* (Ref. 6)

Wetted Perimeter	*n*	Wetted Perimeter	*n*
A. Natural channels		***D. Artificially lined channels***	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
B. Floodplains		Steel, painted	0.014
		Steel, riveted	0.015
Pasture, farmland	0.035	Cast iron	0.013
Light brush	0.050	Concrete, finished	0.012
Heavy brush	0.075	Concrete, unfinished	0.014
Trees	0.15	Planed wood	0.012
C. Excavated earth channels		Clay tile	0.014
Clean	0.022	Brickwork	0.015
Gravelly	0.025	Asphalt	0.016
Weedy	0.030	Corrugated metal	0.022
Stony, cobbles	0.035	Rubble masonry	0.025

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Uniform flow

- **Normal depth (y_N):**

$$Q = \frac{\kappa}{n} A \left(\underbrace{\frac{A}{P}}_{R_h}\right)^{2/3} S_0^{1/2} = \frac{\kappa}{n}\frac{A^{5/3} S_0^{1/2}}{P^{2/3}} \rightarrow \underbrace{A(y_N) = \left(\frac{nQ}{\kappa S_0^{1/2}}\right)^{3/5} P^{2/5}(y_N)}_{\text{solved for } y_N}$$

- **Best hydraulic cross section ($P_{\min}$ or $A_{\min}$) for given Q, S_0, and material (n):**

$$\frac{dP}{dy} = 0 = \frac{dA}{dy}$$

- **Best hydraulic cross section gives the maximum flow rate for given A, S_0, and material (n)**

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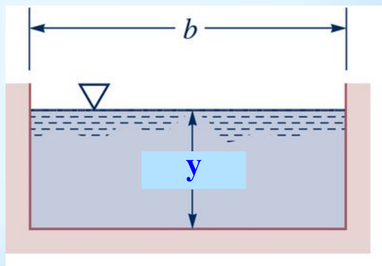
Example

- For a rectangular cross-section open-channel, given Q, S_0, and material (n): a) Determine the best hydraulic cross section. b) Calculate the normal depth in this channel as a function of other parameters.



Notes: VIII.2

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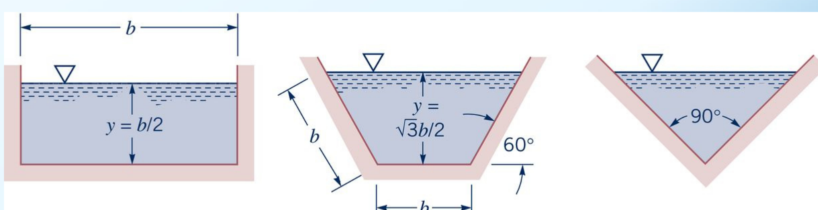
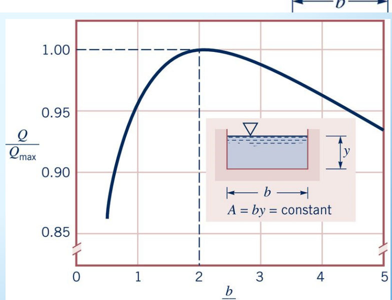
Uniform flow

- Best hydraulic cross section ($P_{\min}$) for given Q, S_0, and material (n):





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Gradually Varied Flow (GVF)

- **Factors leading to GVF:**
  - Varying bottom slope ($S_0 \neq cte$)
  - Varying cross-sectional area or shape ($A \neq cte$)
  - Obstruction (developing flow)
- **Governing equation:**
  - Momentum (Bernoulli) + continuity [2]:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}, \mathrm{Fr}^2 = \frac{V^2 b_0}{gA}$$

$$V = \frac{\kappa}{n} R_h^{2/3} S_{0n}^{1/2} \rightarrow S_f = S_{0n} = \frac{n^2}{\kappa^2} \frac{V^2}{R_h^{4/3}}$$

Assuming Manning equation locally

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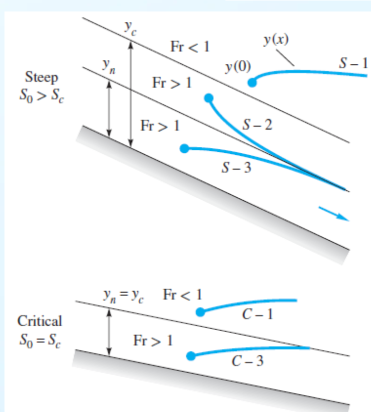
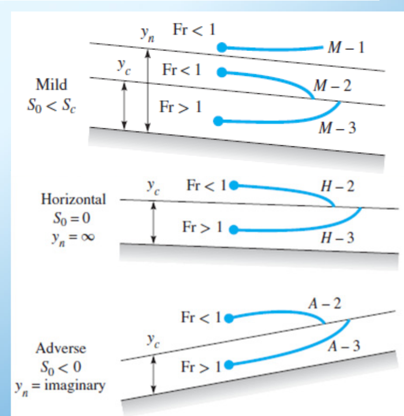
Gradually Varied Flow (GVF)

- **12 basic solution curves:**

Should be solved numerically

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}, \mathrm{Fr}^2 = \frac{Q^2 b_0}{gA^3}, S_f = \frac{n^2}{\kappa^2} \frac{Q^2}{R_h^{4/3} A^2}$$





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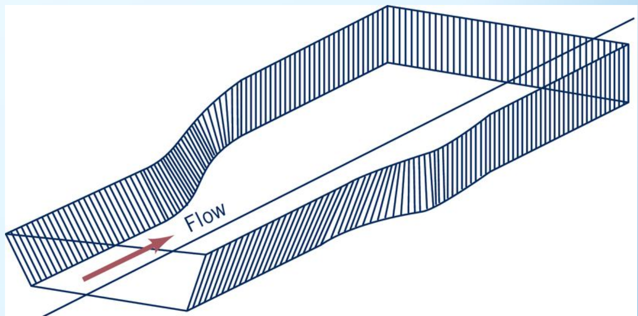
Rapidly Varied Flow (RVF)

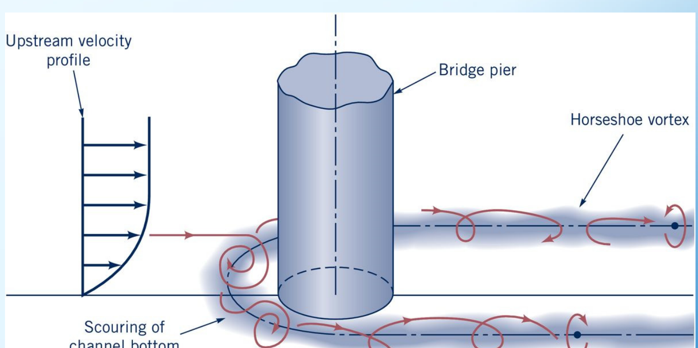
- 2D or 3D



channel transition section ▲

Rapidly Varied Flow (RVF)

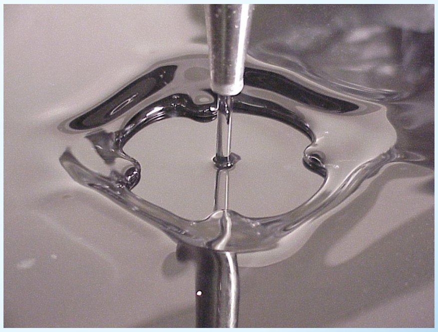
- 2D or 3D



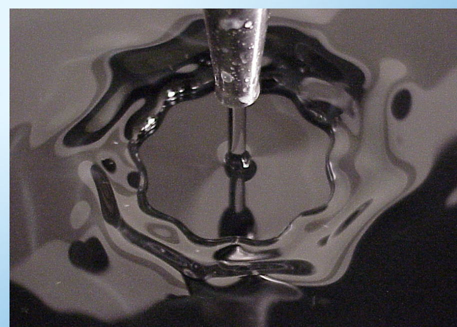
The flow around a bridge pier ▲

Rapidly Varied Flow (RVF)

- 2D or 3D

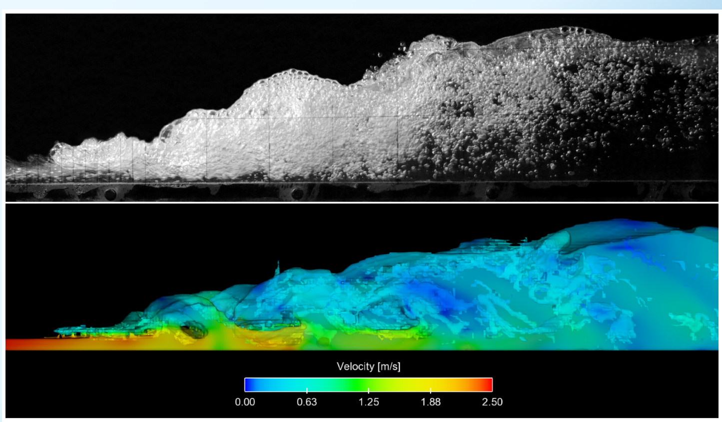


asymmetric circular ▲ ▶
hydraulic jump



Rapidly Varied Flow (RVF)

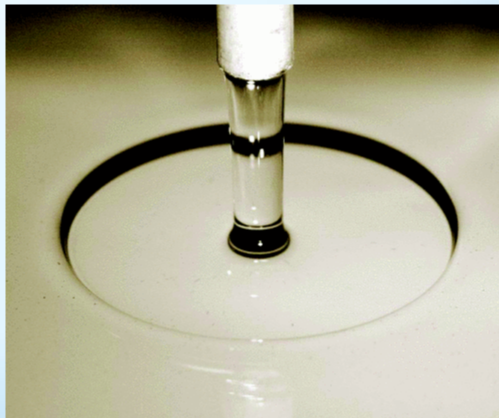
- 2D or 3D
- 1D



Planar hydraulic jump ▲

Rapidly Varied Flow (RVF)

- 2D or 3D
- 1D



Symmetric circular hydraulic jump ▲

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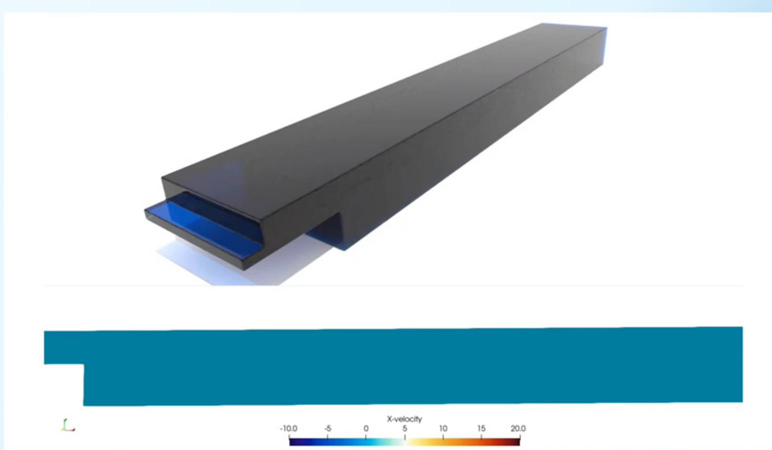
Rapidly Varied Flow (RVF)

- CFD



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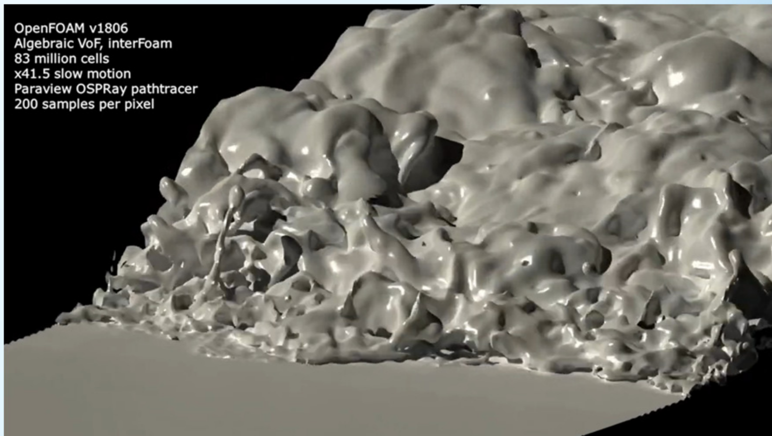
Rapidly Varied Flow (RVF)

- **CFD**



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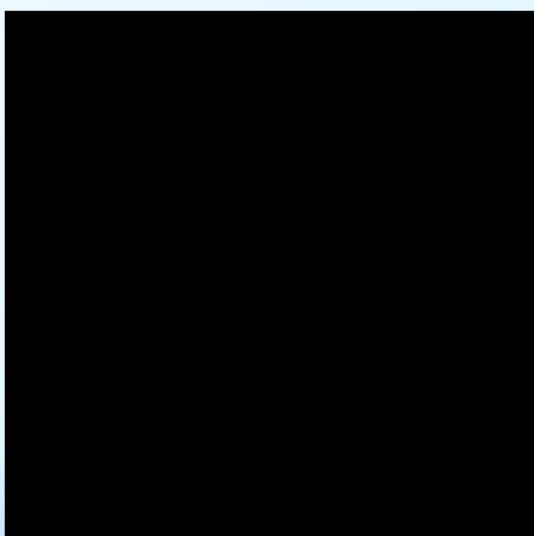
Hydraulic jump



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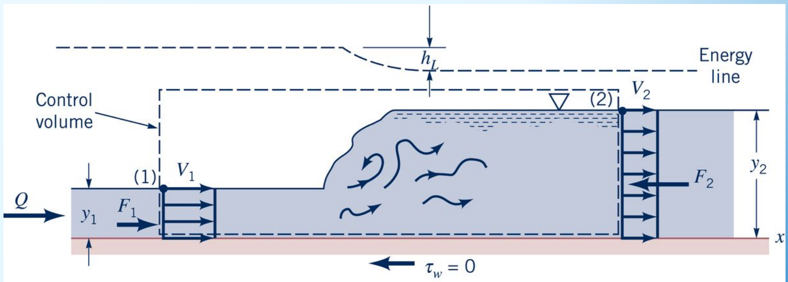
Hydraulic jump

- A mechanism for supercritical upstream flow transition to subcritical downstream flow



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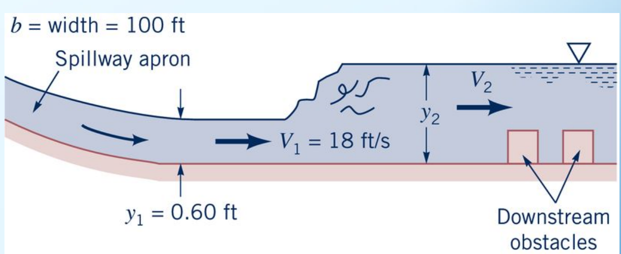
Hydraulic jump

- **Situations leading to hydraulic jump:**
- Spillway + obstacle



(a)

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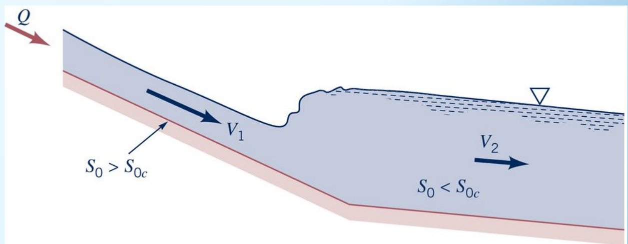
Hydraulic jump

- **Situations leading to hydraulic jump:**
- Spillway + obstacle
- Sudden change in channel slope



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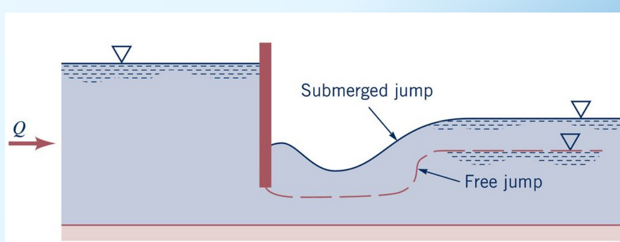
Hydraulic jump

- **Situations leading to hydraulic jump:**
- Spillway + obstacle
- Sudden change in channel slope
- Sluice gate
- …



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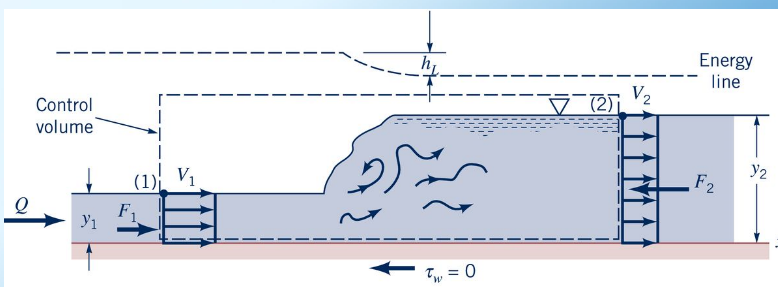
1D hydraulic jump

- **Assumptions:**
  - **Sections 1 and 2: Steady and 1D flow**
  - **Rectangular channel of constant width (b)**



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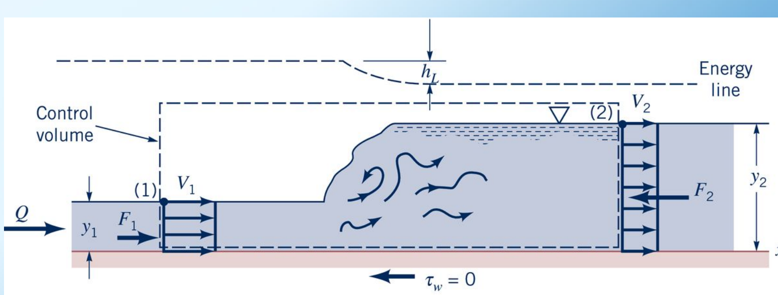
1D hydraulic jump

- **Finite control volume approach:**
  - Momentum:

$$F_1 - F_2 = \rho Q(V_2 - V_1) = \rho V_1 y_1 b(V_2 - V_1)$$

$F_1 = p_{c1}A_1 = \gamma y_1^2 b/2$ and $F_2 = p_{c2}A_2 = \gamma y_2^2 b/2$, where $p_{c1} = \gamma y_1/2$ and $p_{c2} = \gamma y_2/2$

$$\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1)$$



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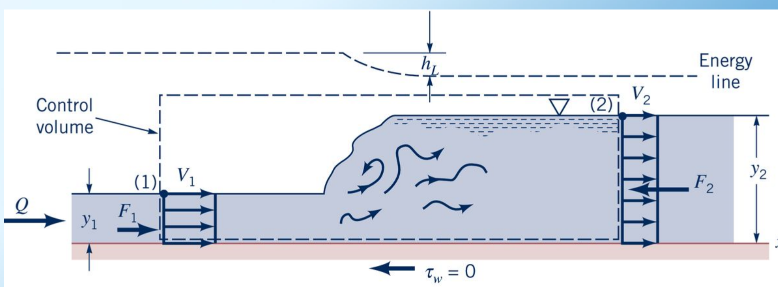
1D hydraulic jump

- **Finite control volume approach:**
- Momentum: $\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1)$
- Mass: $y_1 b V_1 = y_2 b V_2 = Q$
- Energy: $y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$
- **3 unknowns:** y_2, V_2, h_L



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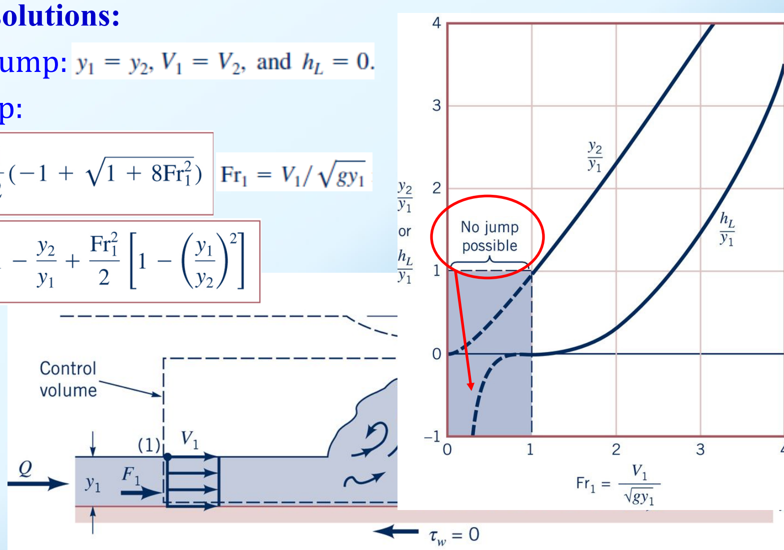
1D hydraulic jump

- **2 solutions:**
- No jump: $y_1 = y_2$, $V_1 = V_2$, and $h_L = 0$.
- Jump:

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right)} \quad \mathrm{Fr}_1 = V_1/\sqrt{g y_1}$$

$$\boxed{\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\mathrm{Fr}_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right]}$$



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1D hydraulic jump

- **Exercise: Show that the Hydraulic jump relation can be written as a function of upstream Froude number as:**

$\mathrm{Fr}_1 = V_1/\sqrt{g y_1}$

$\mathrm{Fr}_2 = V_2/\sqrt{g y_2}$

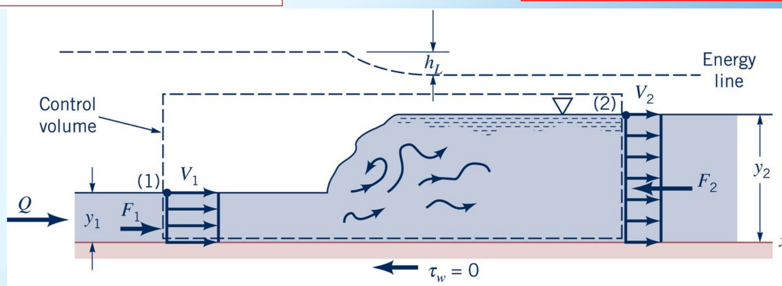
$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right)$$

$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\mathrm{Fr}_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right]$$

$\mathrm{Fr}_2/\mathrm{Fr}_1 = (y_1/y_2)^{3/2}$ $\Rightarrow$

$$\frac{y_2}{y_1} = \frac{1}{4}\left(\frac{1 + \sqrt{1 + 8\mathrm{Fr}_2^2}}{\mathrm{Fr}_2^2}\right)$$

$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\mathrm{Fr}_2^2}{2}\left[\left(\frac{y_2}{y_1}\right)^3 - \frac{y_2}{y_1}\right]$$



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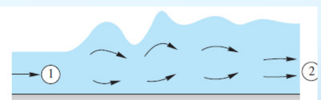
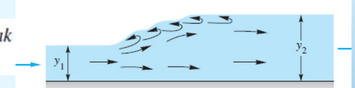
Hydraulic jump classification

Standing-wave or *undular jump* about $4y_2$ long; low dissipation, less than 5 percent.



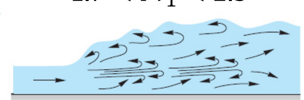
$1.0 < Fr_1 < 1.7$

Smooth surface rise with small rollers, known as a *weak jump;* dissipation 5 to 15 percent.



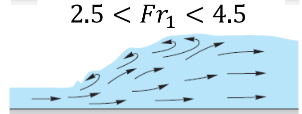
$1.7 < Fr_1 < 2.5$

Unstable, *oscillating jump;* each irregular pulsation creates a large wave that can travel downstream for miles, damaging earth banks and other structures. Not recommended for design conditions. Dissipation 15 to 45 percent.



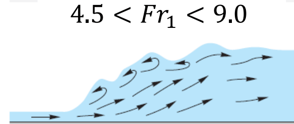
$2.5 < Fr_1 < 4.5$

Stable, well-balanced, *steady jump;* best performance and action, insensitive to downstream conditions. Best design range. Dissipation 45 to 70 percent.



$4.5 < Fr_1 < 9.0$

Rough, somewhat intermittent *strong jump,* but good performance. Dissipation 70 to 85 percent.



$Fr_1 > 9.0$

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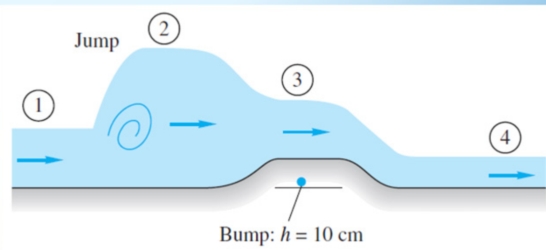
Example

P10.95 A 10-cm-high bump in a wide horizontal water channel creates a hydraulic jump just upstream and the flow pattern in Fig. P10.95. Neglecting losses except in the jump, for the case $y_3 = 30$ cm, estimate (*a*) V_4, (*b*) y_4, (*c*) V_1, and (*d*) y_1.



P10.95

Notes: VIII.3

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Analogy between Compressible and Open-Channel Flows

Open-channel flow		Compressible flow
Small-amplitude gravity waves	⟷	Sound waves
Froude number $\mathrm{Fr} = V/c_{oc}$	⟷	Mach number $\mathrm{Ma} = V/c$
$ybV = cte$	⟷	$\rho AV = cte$
depth y	⟷	density ρ
width b	⟷	area A

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The end of chapter 8

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