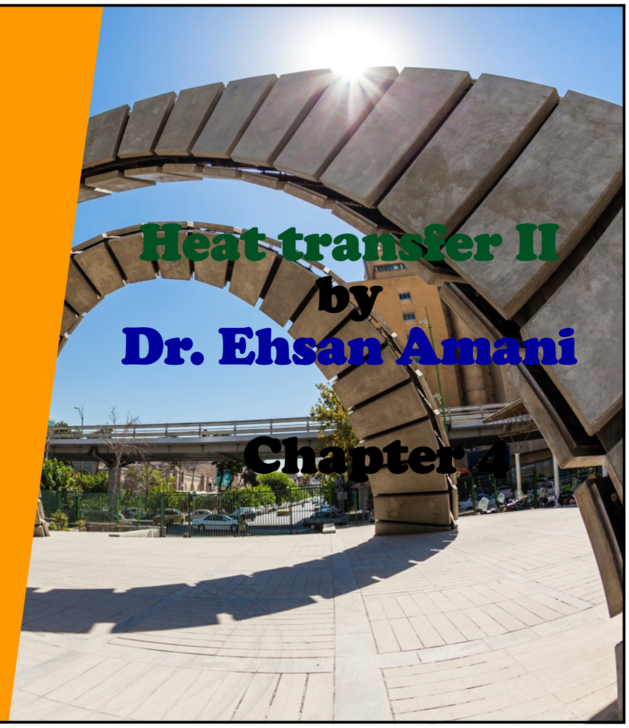


# Heat transfer II

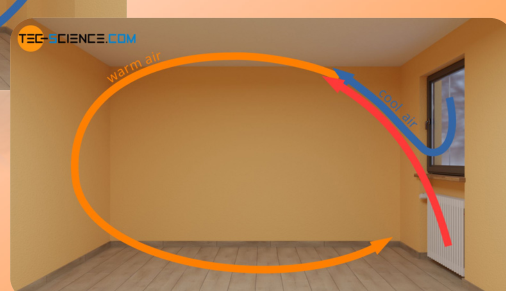
by

**Dr. Ehsan Amani**

## Chapter 4

## Free convection: Applications

- **Air conditioning**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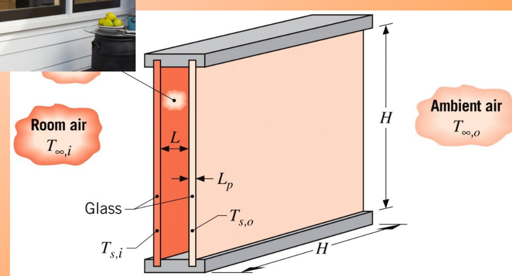
## Free convection: Applications

- **Multi-pane windows**

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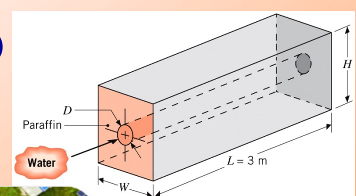
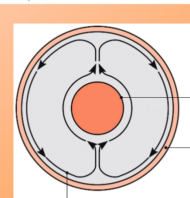
## Free convection: Applications

- **Phase Change Material (PCM)**



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## Free (or natural) convection

- Fluid motion induced by buoyancy forces
- Buoyancy forces: Density gradients + a body force proportional to density
- In heat transfer: density gradients are due to temperature gradients and the body force is gravitational


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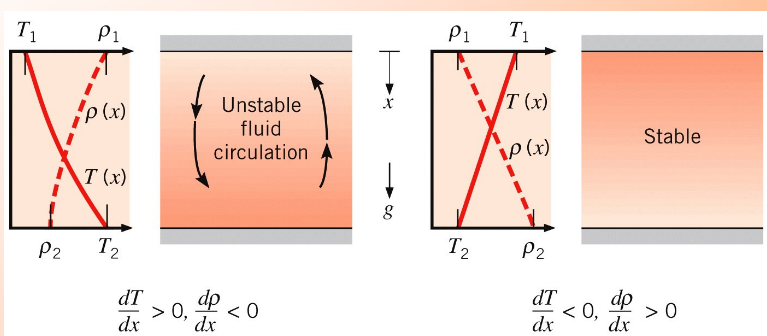
## Free (or natural) convection

- Stable and Unstable Temperature Gradients

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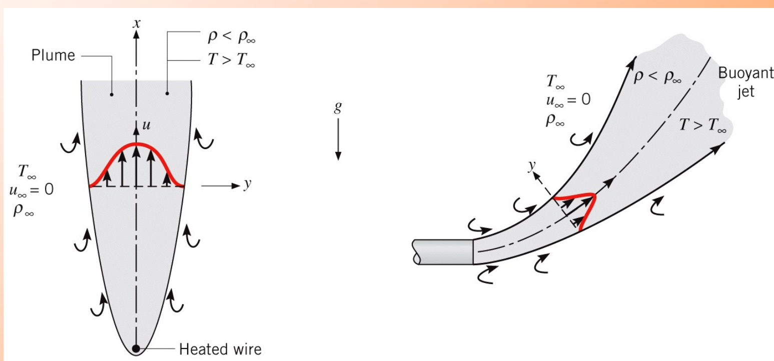
# Free convection classification

- **Free Boundary Flows**
  - **In an extensive (infinite), quiescent (motionless at locations far from the source of buoyancy) fluid**
  - **Examples: Plumes and Buoyant Jets**

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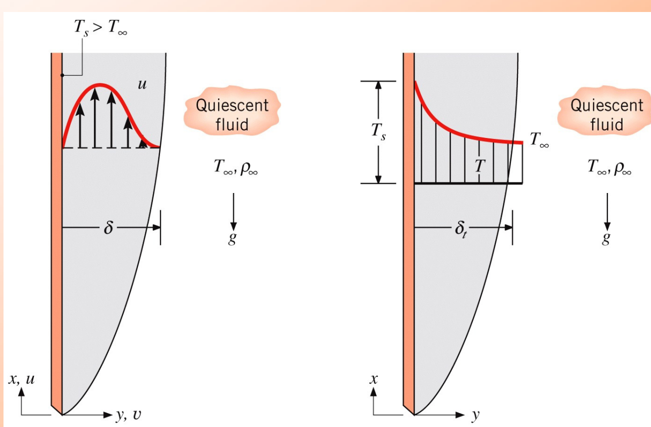
# Free convection classification

- **Free Convection Boundary Layers**
  - **Boundary layer flow on a hot or cold wall induced by buoyancy**

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## Governing equations

- **From Heat Transfer I (No phase change and $\dot{q} = 0$)**

Coupled equations

**Continuity:**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (1.4)'$$

**Momentum:**

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x \qquad (2.4)'$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y \qquad (3.4)'$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + f_z \qquad (4.4)'$$

**Energy:**

$$\rho\left(\frac{\partial u_s}{\partial t} + u\frac{\partial u_s}{\partial x} + v\frac{\partial u_s}{\partial y} + w\frac{\partial u_s}{\partial z}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + \mu\Phi \qquad (5.4)'$$

**2 state equations, e.g.:**

$$u_s = \int_{T_{\mathrm{ref}}}^{T} c_v dT \quad (6.4)' \qquad \rho = \rho(p,T) \quad (7.4)'$$

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## Governing equations

- **Boussinesq approximation**
  - For not too-large temperature gradients
  - Considering a constant density except in the (driving) buoyancy and gravity force terms in the momentum equation
  - Criterion: $\frac{|\Delta\rho|}{\rho_0} \ll 1$

Reference temperature

Reference density $\rho_0 \equiv \rho(T_0)$

$$\Delta\rho \equiv \rho - \rho_0 \qquad (1.4)$$

Volumetric thermal expansion coefficient ($K^{-1}$): Tables A.5 and A.6

$$\beta \equiv -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \approx cte \approx -\frac{1}{\rho_0(1+\Delta\rho/\rho_0)}\frac{\rho - \rho_0}{T - T_0} \xrightarrow{|\Delta\rho|/\rho_0 \ll 1}$$

$$\rho - \rho_0 = \Delta\rho = -\beta\rho_0(T - T_0) \quad (6.4)$$

$$\beta|T - T_0| \ll 1 \qquad (8.4)$$

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# Governing equations

- **Boussinesq approximation**
  - **Simplifying:**

Hydrostatic reference pressure $\frac{\partial p_0}{\partial x_i} \equiv \rho_0 g_i$ (3.4)

Modified pressure $P \equiv p - p_0$ (4.4)

$$\rho_0(1+\underbrace{\frac{\Delta\rho}{\rho_0}}_{|\Delta\rho|/\rho_0 \ll 1})\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial(P+\cancel{p_0})}{\partial x}+\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}+(\cancel{\rho_0}+\underbrace{\Delta\rho}_{-\beta\rho_0(T-T_0)})g_x$$

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# Governing equations

- **Boussinesq approximation**

**Continuity:** $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ (1.4)$''$

**Momentum:**

$$\rho_0\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\rho_0 g_x\beta(T-T_0) \quad (2.4)''$$

$$\rho_0\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\rho_0 g_y\beta(T-T_0) \quad (3.4)''$$

$$\rho_0\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\rho_0 g_z\beta(T-T_0) \quad (4.4)''$$

**Energy:**

$$\rho_0\left(\frac{\partial u_s}{\partial t}+u\frac{\partial u_s}{\partial x}+v\frac{\partial u_s}{\partial y}+w\frac{\partial u_s}{\partial z}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\mu\Phi \quad (5.4)''$$

**1 state equation** $u_s=\int_{T_{\text{ref}}}^{T} c_v dT$ (6.4)$''$

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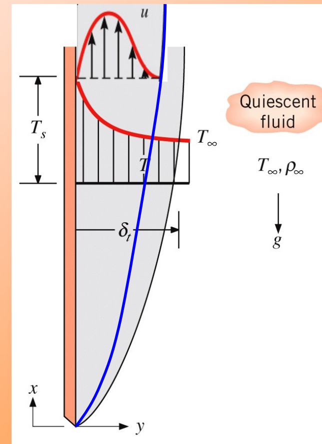
## Governing equations

- **Boussinesq + Boundary Layer (BL) approximations**
  - **2D + Steady + laminar**
  - $\mu, k, c_v = cte$
  - **Vertical plane wall ($g_x = -g$)**
  - **BL approximations**




$$\frac{\partial p}{\partial y} \sim 0 \longrightarrow p = p(x) = p_\infty(x) \longrightarrow \frac{\partial p}{\partial x} = \frac{dp_\infty}{dx}$$

$$u \gg v \qquad \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} \qquad \frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x} \qquad \frac{\partial^2 T}{\partial y^2} \gg \frac{\partial^2 T}{\partial x^2}$$

$$\rho_0 = \rho_\infty \quad T_0 = T_\infty$$

$$(4.4) \longrightarrow \frac{\partial P}{\partial x} = \frac{\partial p}{\partial x} - \frac{\partial p_0}{\partial x} \overset{(3.4)}{=} \frac{dp_\infty}{dx} + \rho_0 g = -\rho_\infty g + \rho_\infty g = 0$$

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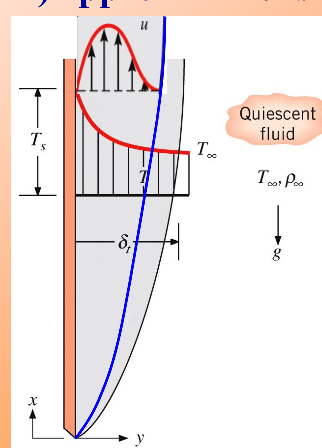
## Governing equations

- **Boussinesq + Boundary Layer (BL) approximations**
  - **Equations**




$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \qquad (9.4)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

  - **Boundary conditions:**

$x = 0; u = 0, T = T_\infty \; (T - T_\infty = 0)$

$y = 0; u = v = 0, T = T_s \; (T - T_\infty = T_s - T_\infty)$

$y \to \infty; u = 0, T = T_\infty \; (T - T_\infty = 0) \qquad (10.4)$

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# Dimensional analysis

- **Non-dimensionalization**
  - **Therefore,**

$$u = u(x, y, g\beta, \nu, \alpha, T_s - T_\infty, L)$$

**For generalization to other geometries**

$$T - T_\infty = u(x, y, g\beta, \nu, \alpha, T_s - T_\infty, L) \qquad (11.4)$$

  - **Exercise: Starting from the parametric relations, Eq. (11.4), show that:**

$$\frac{T - T_\infty}{T_s - T_\infty} = f(\underbrace{\frac{x}{L}}_{x^*}, \underbrace{\frac{y}{L}}_{y^*}, \underbrace{\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}}_{\mathrm{Gr}_L}, \mathrm{Pr}) \qquad (11.4)'$$

$$\mathrm{Nu}_x = f(x^*, \mathrm{Gr}_L, \mathrm{Pr}) \qquad (16.4)'$$

$$\overline{\mathrm{Nu}}_L = f(\mathrm{Gr}_L, \mathrm{Pr}) \qquad (16.4)$$

  - **For a flat plate, removing $L$, Eq. $(16.4)'$ is simplified to:**

$$\mathrm{Nu}_x = f(\mathrm{Gr}_x, \mathrm{Pr}) \quad (16.4)' \qquad \mathrm{Gr}_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

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# Dimensional analysis

- **Non-dimensionalizing equations**
  - Steps:
    1. Dependent variables: $u, v, T$
    2. Independent variables: $x, y$
    3. Non-dimensional variables:

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, u^* = \frac{u}{u_0}, v^* = \frac{v}{u_0}, T^* = \frac{T - T_s}{\theta_0}$$

    4. Characteristic parameters:

$$\theta_0 = T_\infty - T_s$$

    5. Substituting in the governing equations:

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# Dimensional analysis

- **Non-dimensionalizing equations**
  - Steps:
    5. Substituting in the governing equations:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\nu}{u_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}+\underbrace{\frac{g\beta(T_s-T_\infty)L}{u_0^2}}_{1}T^* \qquad (12.4)'$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\alpha}{u_0 L}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$x^*=0; u^*=0, T^*=0$$

$$y^*=0; u^*=v^*=0, T^*=1$$

$$y^*\to\infty; u^*=0, T^*=0$$

    6. Choosing $u_0$ to minimize the governing parameters

$$u_0=[g\beta(T_s-T_\infty)L]^{1/2}$$

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# Dimensional analysis

- **Non-dimensionalizing equations**

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{1}{\mathrm{Gr}_L^{1/2}}\frac{\partial^2 u^*}{\partial y^{*2}}+T^* \qquad (12.4)$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{\mathrm{Gr}_L^{1/2}\mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\mathrm{Gr}_L=\frac{g\beta(T_s-T_\infty)L^3}{\nu^2} \qquad (14.4)$$

Grashof number

$$x^*=0; u^*=0, T^*=0$$

$$y^*=0; u^*=v^*=0, T^*=1 \qquad (13.4)$$

$$y^*\to\infty; u^*=0, T^*=0$$

  - Therefore,

$u^*, T^*=f(x^*, y^*, \mathrm{Gr}_L, \mathrm{Pr})$ $\quad$ $\mathrm{Nu}_x=f(x^*, \mathrm{Gr}_L, \mathrm{Pr})$ $\quad$ $\overline{\mathrm{Nu}}_L=f(\mathrm{Gr}_L, \mathrm{Pr})$

  - What is the ratio of buoyancy to viscous force?

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# Dimensional analysis

- **Non-dimensionalizing equations**
  - Corollary:

$$\overline{\mathrm{Nu}}_L = f(\mathrm{Ra}_L, \mathrm{Pr}) \qquad (17.4)$$

**Rayleigh number**

$$\mathrm{Ra}_L = \mathrm{Gr}_L \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} \qquad (18.4)$$

  - For liquid metals:

$$\mathrm{Gr}_L^{1/2} \gg 1 \xrightarrow{(12.4)} \overline{\mathrm{Nu}}_L = f(\mathrm{Gr}_L^{1/2}\mathrm{Pr}) = f(\mathrm{Ra}_L^{1/2}\mathrm{Pr}^{1/2}) \longrightarrow$$

$$\overline{\mathrm{Nu}}_L = f(\mathrm{Ra}_L \mathrm{Pr}) \quad (19.4)$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \qquad u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Gr}_L^{1/2}}\frac{\partial^2 u^*}{\partial y^{*2}} + T^* \qquad u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Gr}_L^{1/2}\mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}}$$

(12.4)

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# Dimensional analysis

- **Mixed (or combined free and force) convection**
  - **Boussinesq + Boundary Layer (BL) approximations**
  - In Eq. $(2.4)''$, $\frac{\partial P}{\partial x} \neq 0$.
  - Show that Eqs. (12.4) and (13.4) are extended to:

$$\boxed{u_0 = u_\infty, \ p^* = \frac{P}{\rho_0 u_\infty^2}}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

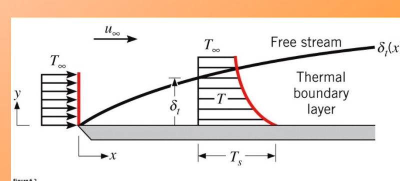
$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\mathrm{Gr}_L}{\mathrm{Re}_L^2}T^* - \frac{dp^*}{dx^*} \qquad (21.4)$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$x^* = 0; u^* = 1, T^* = 0$$

$$y^* = 0; u^* = v^* = 0, T^* = 1 \qquad (22.4)$$

$$y^* \to \infty; u^* = 1, T^* = 0$$

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## Dimensional analysis

- **Mixed (or combined free and force) convection**
  - Therefore,

$$u^*, T^* = f(x^*, y^*, \mathrm{Gr}_L, \mathrm{Re}_L, \mathrm{Pr}) \qquad (23.4)$$

$$\mathrm{Nu}_x = f(x^*, \mathrm{Gr}_L, \mathrm{Re}_L, \mathrm{Pr}) \qquad \overline{\mathrm{Nu}}_L = f(\mathrm{Gr}_L, \mathrm{Re}_L, \mathrm{Pr}) \qquad (24.4)$$

  - The ratio of buoyancy to (forced convection) inertial force: $\mathrm{Gr}_L/\mathrm{Re}_L^2 \ll 1$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\mathrm{Gr}_L}{\mathrm{Re}_L^2}T^* - \frac{dp^*}{dx^*}$$

  - The free convection is omitted when: $\boxed{\mathrm{Gr}_L/\mathrm{Re}_L^2 \ll 1}$
  - The forced convection is omitted when: $\boxed{\mathrm{Gr}_L/\mathrm{Re}_L^2 \gg 1}$

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## Laminar free convection on a vertical plate

- **Semi-analytical solution: The similarity solution**
- **Details of derivation: Appendix A**
  - With a change of variables:

$$\eta = \frac{y}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} \quad (32.4) \qquad \mathrm{Gr}_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2} \quad (31.4)$$

$$f'(\eta) = \frac{u}{2\nu}\frac{x}{\sqrt{\mathrm{Gr}_x}} \quad \text{or} \quad u = \frac{2\nu}{x}\sqrt{\mathrm{Gr}_x}f'(\eta) \quad (33.4)$$

$$\frac{T - T_\infty}{T_s - T_\infty} = T^*(\eta) \quad (34.4)$$

  - Eqs. (12.4) and (13.4) are simplified to

$$\begin{aligned} &f''' + 3ff'' - 2f'^2 + T^* = 0 \\ &T^{*\prime\prime} + 3\mathrm{Pr}fT^{*\prime} = 0 \end{aligned} \qquad (27.4)$$

$$\begin{aligned} &\eta = 0; f = f' = 0, T^* = 1 \\ &\eta \to \infty; f' = 0, T^* = 0 \end{aligned} \qquad (30.4)$$

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# Laminar free convection on a vertical plate

- **Semi-analytical solution: The similarity solution**
  - Numerical solution of Eqs. (27.4) and (30.4):

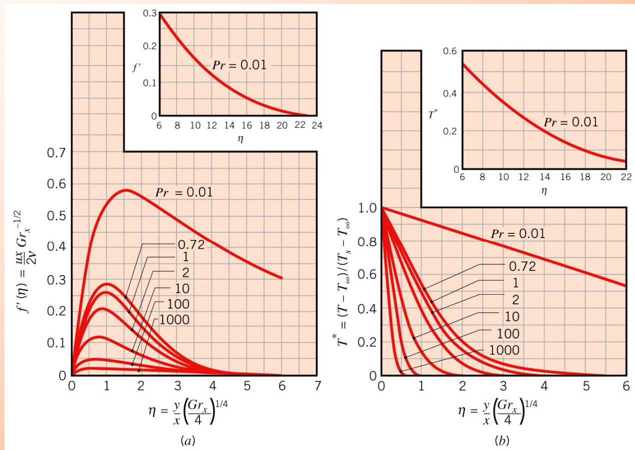


Figure 9.4


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# Laminar free convection on a vertical plate

- **Semi-analytical solution: The similarity solution**
  - Show that:

$$\mathrm{Nu}_x \equiv \frac{hx}{k} = -\left.\frac{dT^*}{d\eta}\right|_{\eta=0} \left(\frac{d\eta}{dy}\right) x = \left(\frac{\mathrm{Gr}_x}{4}\right)^{\frac{1}{4}} \overbrace{T^{*\prime}(\eta=0)}^{g(\mathrm{Pr})}$$

$$\mathrm{Nu}_x = \left(\frac{\mathrm{Gr}_x}{4}\right)^{\frac{1}{4}} g(\mathrm{Pr}) \qquad (35.4)$$

  - By curve fitting to the numerical solution:

$$g(Pr) = \frac{0.75\, Pr^{1/2}}{\left(0.609 + 1.221\, Pr^{1/2} + 1.238\, Pr\right)^{1/4}} \qquad (0 < Pr < \infty) \qquad (36.4)$$

  - For a plate of height $L$:

(35.4)

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \int_0^1 h(x^*)\,dx^* = \frac{k}{L}\int_0^1 \mathrm{Nu}_x(x^*)\,dx^* = \frac{4}{3}\left(\frac{\mathrm{Gr}_L}{4}\right)^{\frac{1}{4}} g(\mathrm{Pr}) \longrightarrow$$

$$\overline{\mathrm{Nu}}_L = \frac{4}{3}\mathrm{Nu}_x(x = L) = \frac{4}{3}\mathrm{Nu}_L \qquad (37.4)$$

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## Laminar free convection on a vertical plate

- **Semi-analytical solution: The similarity solution**
  - Therefore,

$$\overline{\mathrm{Nu}}_L = \frac{4}{3}\left(\frac{\mathrm{Gr}_L}{4}\right)^{\frac{1}{4}} \frac{0.75\ \mathrm{Pr}^{1/2}}{(0.609 + 1.221\mathrm{Pr}^{1/2} + 1.238\mathrm{Pr})^{1/4}} = \frac{0.707\ \mathrm{Ra}_L{}^{1/4}}{(0.609\mathrm{Pr}^{-1} + 1.221\mathrm{Pr}^{-1/2} + 1.238)^{1/4}} \qquad (37.4)$$

  - Alternatively, a purely empirical correlation suggests that:

$$\boxed{\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}} \qquad (9.27)[1]$$

  - Exercise: Are Eq. (37.4) and (9.27)[1] consistent for $\mathrm{Ra}_L \to 0$ and $\mathrm{Ra}_L \to \infty$? Why? Hint: Which condition leads to the violation of BL assumptions, i.e., the thin BL?

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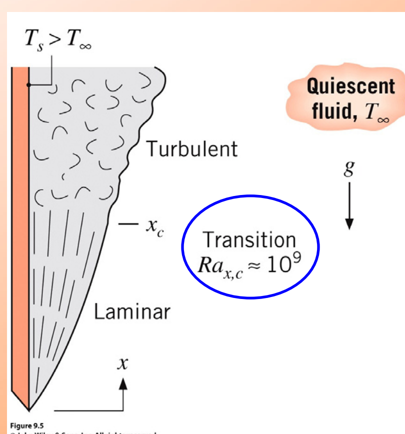
## Turbulent free convection on a vertical plate

- **Transition to turbulence**

$$\mathrm{Ra}_x = \mathrm{Gr}_x\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \qquad (38.4)$$

  - Empirical correlation:

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad (9.26)[1]$$

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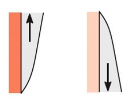
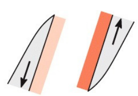
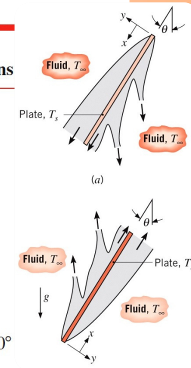
# Empirical correlations

- **External free convection: Table 1 in "chap4-summary.pdf"**

**TABLE 9.2** Summary of free convection empirical correlations for immersed geometries

| Geometry | Recommended Correlation | Restrictions |
|---|---|---|
| 1. Vertical plates[a]  | Equation 9.26 | None |
| 2. Inclined plates Cold surface up or hot surface down  | Equation 9.26 $g \rightarrow g \cos\theta$ | $0 \lesssim \theta \lesssim 60°$ |

[a] The correlation may be applied to a vertical cylinder if $(D/L) \gtrsim (35/Gr_L^{1/4})$.

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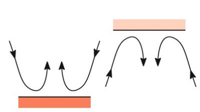
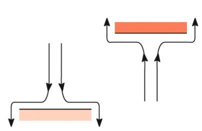
# Empirical correlations

- **External free convection: Table 1 in "chap4-summary.pdf"**

**TABLE 9.2** Summary of free convection empirical correlations for immersed geometries

| Geometry | Recommended Correlation | Restrictions |
|---|---|---|
| 3. Horizontal plates (a) Hot surface up or cold surface down  | Equation 9.30<br>Equation 9.31 | $10^4 \lesssim Ra_L \lesssim 10^7$, $Pr \gtrsim 0.7$<br>$10^7 \lesssim Ra_L \lesssim 10^{11}$ |
| (b) Cold surface up or hot surface down  | Equation 9.32 | $10^4 \lesssim Ra_L \lesssim 10^9$, $Pr \gtrsim 0.7$ |

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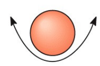
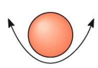
# Empirical correlations

- **External free convection: Table 1 in "chap4-summary.pdf"**

**TABLE 9.2** Summary of free convection empirical correlations for immersed geometries

| Geometry | Recommended Correlation | Restrictions |
|---|---|---|
| 4. Horizontal cylinder  | Equation 9.34 | $Ra_D \lesssim 10^{12}$ |
| 5. Sphere  | Equation 9.35 | $Ra_D \lesssim 10^{11}$<br>$Pr \gtrsim 0.7$ |

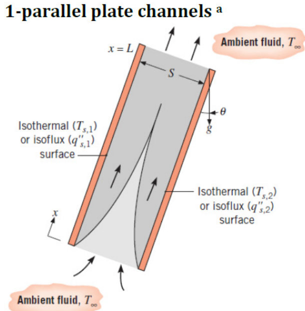
- **Notes:**
  - Film temperature: $T_f = \dfrac{T_s + T_\infty}{2}$
  - For $q_s'' = cte$, use $T_s = T_s(L/2)$ $\quad q_s'' = \bar{h}\left(T_s(\frac{L}{2}) - T_\infty\right)$



# Empirical correlations

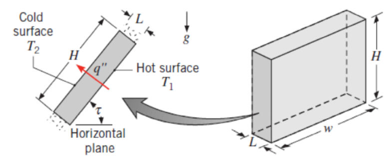
- **Internal free convection: Table 2 in "chap4-summary.pdf"**

**Table 2: Internal free convection**

| Geometry | Recommended Correlation | Restrictions |
|---|---|---|
| 1-parallel plate channels [a]   | **Vertical**<br>(9.45),(9.37),(9.38)<br>Table 9.3 1st and 3rd rows<br><br>(9.46),(9.41),(9.42)<br>Table 9.3 2nd and 4th rows<br><br>**Inclined**<br>(9.47) | Constant Temperature<br><br>Constant flux<br><br>Constant Temperature<br>$0 \le \theta \le 45°$<br>$Ra_S(S/L) > 200$<br>fluid: water |
| 2-Rectangular cavities ($W/L \gg 1$) [b] | (9.48) | |

# Empirical correlations

- **Internal free convection: Table 2 in "chap4-summary.pdf"**

| | | |
|---|---|---|
| **2-Rectangular cavities ($W/L \gg 1$)** [b]<br><br>$L_e = L$ | (9.48)<br><br>**Horizontal ($\tau = 0$)**<br>(9.49)<br><br>**Vertical ($\tau = 90$)**<br>(9.50)-(9.53)<br><br>**Inclined**<br>$\tau^* \rightarrow$ Table 9.4<br>(9.54),(9.55)<br>(9.56),(9.57) | <br><br>($Ra_{L,cr} = 3\times10^5$ $\boxed{Ra_{L,cr2} = 1708}$)<br>$3\times10^5 \lesssim Ra_L \lesssim 7\times10^9$<br><br>$\boxed{(Ra_{L,cr2} = 1000)}$<br>See each correlation<br><br>$\boxed{(Ra_{L,cr2} = 1708/\cos\tau)}$<br>$\tau \le \tau^*$<br>$\tau > \tau^*$ |
| **3-Concentric cylinders and spheres** [b] | | $(Ra_{L,cr2} = 100)$ |

a $T_f = (T_s(x = L) + T_f)/2$

b $T_f = \frac{T_1+T_2}{2}$, $\mathrm{Ra}_L = \frac{g\beta(T_1-T_2)L_e^3}{\alpha\upsilon}$, and $\mathrm{Ra}_{L,cr2}$ indicates the transition from conduction to free convection.

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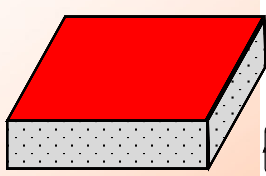
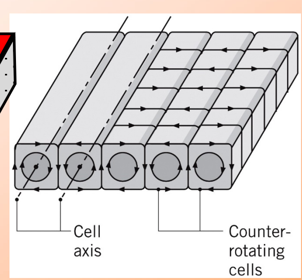
# Empirical correlations

- **Internal free convection: Table 2 in "chap4-summary.pdf"**

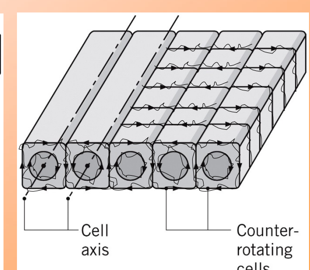


$\mathrm{Ra}_L < \mathrm{Ra}_{L,\mathrm{cr2}} = 1708$

$\overline{\mathrm{Nu}}_L = 1$




$\mathrm{Ra}_{L,\mathrm{cr2}} < \mathrm{Ra}_L < \mathrm{Ra}_{L,\mathrm{cr}}$




$\mathrm{Ra}_L > \mathrm{Ra}_{L,\mathrm{cr}}$

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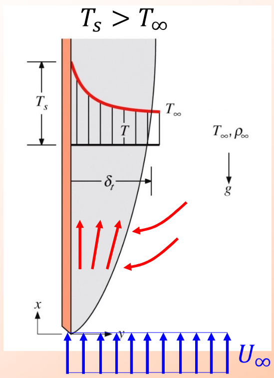
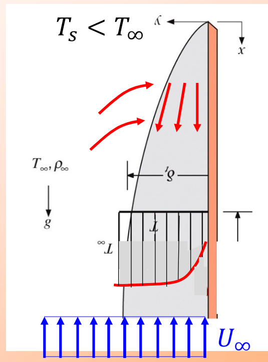
# Empirical correlations

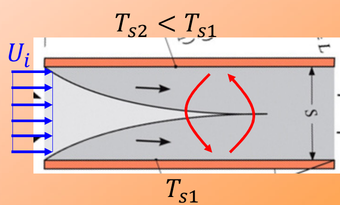
- **Mixed convection**




Assisting flow




Apposing flow




Transverse flow

Assisting or transverse flow

$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n \qquad (39.4)$$

Apposing flow

$$n = \begin{cases} 3.5 & \text{Transverse flow on a horizontal flat surface} \\ 4 & \text{Transverse flow on a sphere or cylinder} \\ 3 & \text{General} \end{cases}$$

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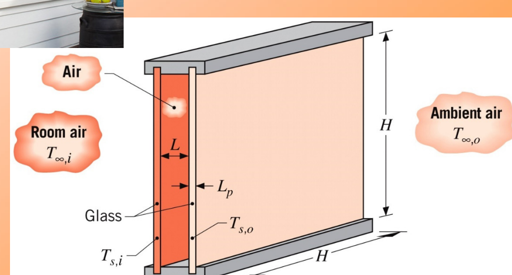
# Sample problems

## Two-pane window

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# Sample problems

## Two-pane window

**9.78** Consider two long vertical plates maintained at uniform temperatures $T_{s,1} > T_{s,2}$. The plates are open at their ends and are separated by the distance $2L$.

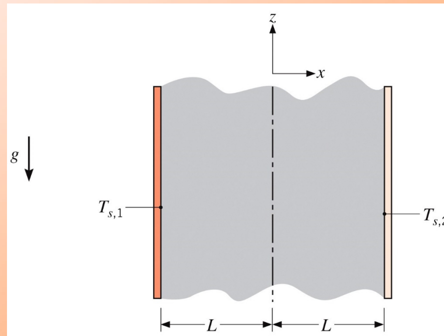
(a) Sketch the velocity distribution in the space between the plates.

(b) Write appropriate forms of the continuity, momentum, and energy equations for laminar flow between the plates.

(c) Evaluate the temperature distribution, and express your result in terms of the mean temperature, $T_m = (T_{s,1} + T_{s,2})/2$.

(d) Estimate the vertical pressure gradient by assuming the density to be a constant $\rho_m$ corresponding to $T_m$. Substituting from the Boussinesq approximation, obtain the resulting form of the momentum equation.

(e) Determine the velocity distribution.




Lecture Notes: IV.10.1

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# Sample problems

## Two-pane window

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1.4)''$$

$$\rho_0\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\rho_0 g_x\beta(T-T_0) \qquad (2.4)''$$

$$\rho_0\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\rho_0 g_y\beta(T-T_0) \qquad (3.4)''$$

$$\rho_0\left(\frac{\partial u_s}{\partial t}+u\frac{\partial u_s}{\partial x}+v\frac{\partial u_s}{\partial y}+w\frac{\partial u_s}{\partial z}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\mu\Phi \qquad (5.4)''$$

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# Sample problems

## Automobile rear window heater

**9.27** The vertical rear window of an automobile is of thickness $L = 8$ mm and height $H = 0.5$ m and contains fine-meshed heating wires that can induce nearly uniform heat flux on the internal surface of the window.

(a) Consider steady-state conditions for which the interior surface of the window is exposed to quiescent air at 10°C, while the exterior surface is exposed to air at −10°C moving in parallel flow over the surface with a velocity of 20 m/s. Determine the heat flux needed to maintain the interior window surface at $T_{s,i} = 15$°C.


Lecture Notes: IV.10.2



# Sample problems

## Automobile rear window heater

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2 \qquad (9.26)$$

$$\overline{Nu}_L = 0.68 + \frac{0.670\, Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \qquad Ra_L \lesssim 10^9 \qquad (9.27)$$

The end of chapter 4