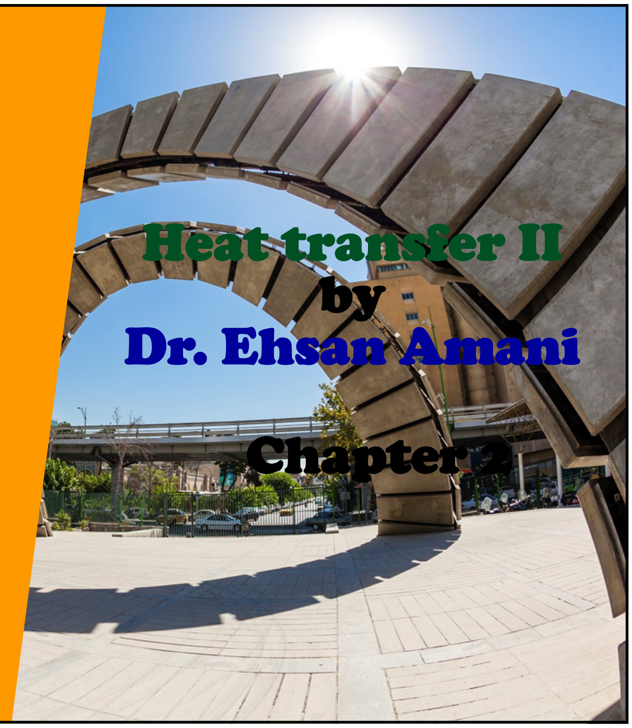
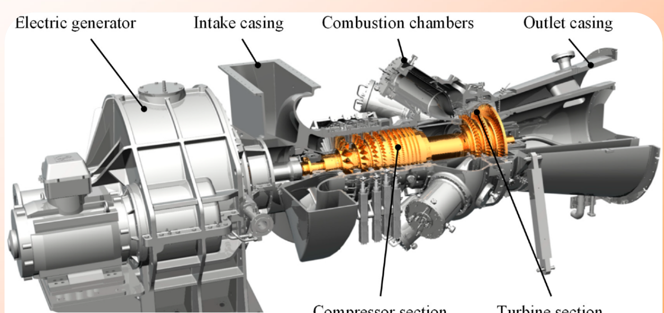


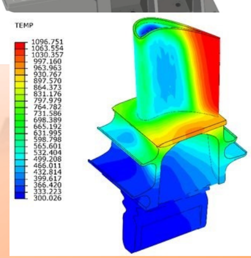
Heat transfer II
by
Dr. Ehsan Amani

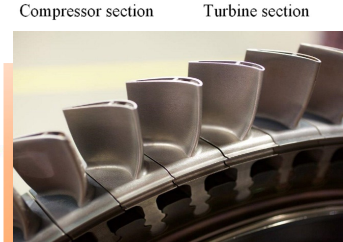
Chapter 2

Temperature distribution: Applications

- **Thermal stress analysis: Gas turbine blade**



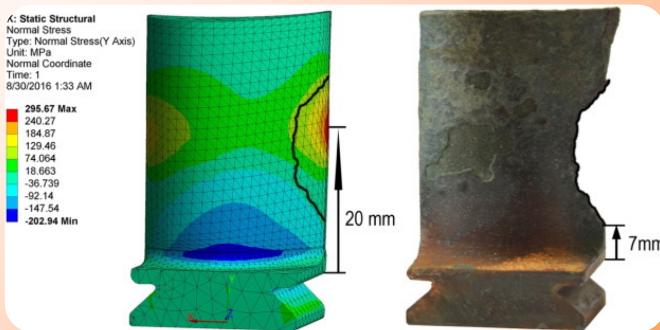




Chapter 2 By E. Amani

Temperature distribution: Applications

- **Thermal stress analysis: Gas turbine blade**



Chapter 2 By E. Amani

Temperature distribution: Applications

- **Thermal stress analysis: Internal combustion engines**

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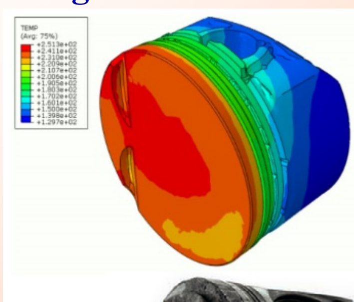
Temperature distribution: Applications

- **Thermal stress analysis: Internal combustion engines**



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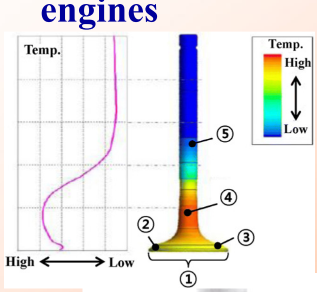
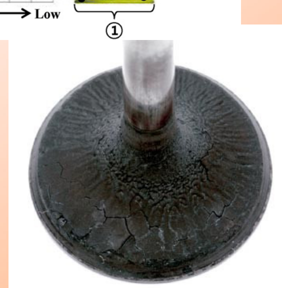
Temperature distribution: Applications

- **Thermal stress analysis: Internal combustion engines**

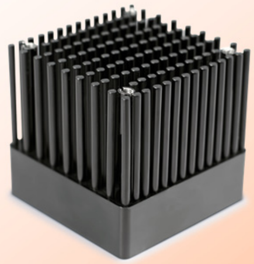


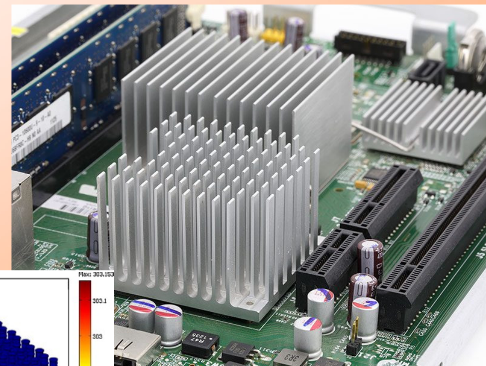


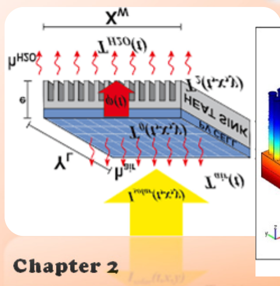

Chapter 2 By E. Amani

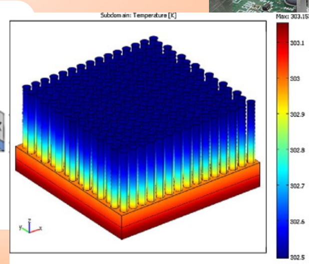
Temperature distribution: Applications

- **Heat transfer across a control surface – heat sink**









Conduction: Review

- **Conduction heat flux and Fourier's law**
  - For 1D problems:

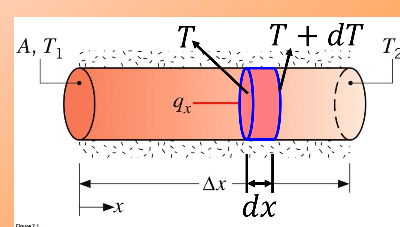
slope of T − x

$$q''_x = -k\frac{dT}{dx} \qquad (1.2)$$

$\boxed{\textbf{Fourier law}}$

Thermal conductivity $\left(\frac{W}{m.K}\right)$

$f(\text{material}, T, p)$



Conduction: Review

- **Conduction heat flux and Fourier's law**
  - For 3D problems: $\vec{q}'' = -k\vec{\nabla}T \quad (2.2)$

$$\vec{q}'' = q_x''\vec{i} + q_y''\vec{j} + q_z''\vec{k} = -k\frac{\partial T}{\partial x}\vec{i} - k\frac{\partial T}{\partial y}\vec{j} - k\frac{\partial T}{\partial z}\vec{k} \qquad (3.2)$$

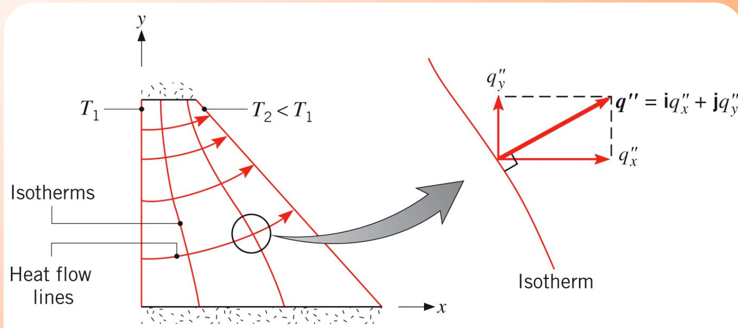


Figure 4.1

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Conduction: Review

- **Heat conduction equation**

1. Neglecting mechanical energy change
2. No phase change
3. Neglecting or no advection (rigid-body motion is allowed)
4. The internal energy depends on temperature only (solids, incompressible liquids, ideal gases)
5. Isotropic conduction (Fourier's law)

$$\vec{\nabla}.\left(k\vec{\nabla}T\right) + \dot{q} = \rho c_v \frac{\partial T}{\partial t} \qquad (3.2)$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_v \frac{\partial T}{\partial t} \qquad (4.2)$$

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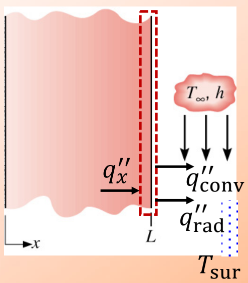
Conduction: Review

- **Boundary and initial conditions**
  - **First-order in time, second-order in space**
  - **Use energy balance at the boundary:**
    - Example:



$$\dot{E}_{in} = \dot{E}_{out} \longrightarrow q''_x(x=L) = q''_{conv} + q''_{rad}$$

$$-k\frac{dT}{dx}\bigg|_{x=L} = h[T(x=L) - T_\infty] + h_r[T(x=L) - T_{sur}]$$

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Conduction: 2D, steady

- **Solution methods**
  - Analytical methods
  - Numerical methods
    - ✓ Grid-based methods
      1. Finite Difference (FD)
      2. Finite Volume (FV)
      3. Finite Elements (FE)
      4. Spectral methods
      5. …
    - ✓ Meshless methods
      1. Discrete Phase Method (DPH)
      2. Discrete Element Method (DEM)
      3. Smoothed Particle Hydrodynamics (SPH)
      4. …

Chapter 2 By E. Amani

Preliminary design

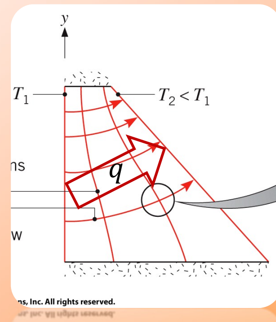
- **The shape factor**

$$S \equiv \frac{q}{k\Delta T} \quad (5.2) \longrightarrow q = Sk\Delta T$$

Heat transfer rate from surface 1 to 2

$= T_1 - T_2$

(conduction) shape factor (m)



- **Dimensionless conduction heat rate**

Active area

Characteristic length

$$q_{ss}^* \equiv \frac{(q/A_s)L_c}{k\Delta T} \quad (6.2) \longrightarrow q = \frac{q_{ss}^* k A_s \Delta T}{L_c}$$

Dimensionless conduction heat rate $(-)$

$$\xrightarrow[(6.2)]{(5.2)} \quad q_{ss}^* = \frac{SL_c}{A_s} \quad (7.2)$$

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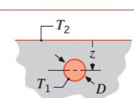
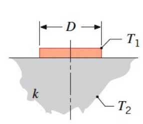
Preliminary design

- **Tables for S and q_{ss}^* :**
  - Calculated from the results of analytical, numerical, or experimental methods

TABLE 4.1 Conduction shape factors and dimensionless conduction heat rates for selected systems.

(a) Shape factors [$q = Sk(T_1 - T_2)$]

System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\frac{2\pi D}{1 - D/4z}$
Case 2			
Case 10 Disk of diameter D and temperature T_1 on a semi-infinite medium of thermal conductivity k and temperature T_2		None	$2D$
Case 11			

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Preliminary design

- **Tables for S and q_{ss}^* :**
  - Calculated from the results of analytical, numerical, or experimental methods

TABLE 4.1 *Continued*

(*b*) **Dimensionless conduction heat rates [$q = q_{ss}^* kA_s(T_1 - T_2)/L_c$; $L_c \equiv (A_s/4\pi)^{1/2}$]**

System	Schematic	Active Area, A_s	q_{ss}^*
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2	T_1, D, T_2	πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2	T_1, D, T_2	$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14			

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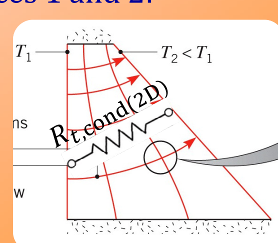
Preliminary design

- **Tables for S and q_{ss}^* :**
  - Calculated from the results of analytical, numerical, or experimental methods
  - For $T_s = cte$ boundary conditions at surfaces 1 and 2
  - For$q_s'' = cte$ can also be used as an approximation
    - ✓ In this case, use $\Delta T = \bar{T}_1 - \bar{T}_2$
  - Thermal resistance between surfaces 1 and 2:

$$R_{t,\mathrm{cond(2D)}} \equiv \frac{\Delta T}{q} = \frac{1}{Sk} = \frac{L_c}{kA_s q_{ss}^*} \qquad (8.2)$$



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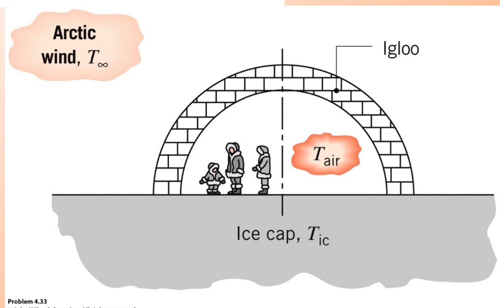
Sample problem

4.33 An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo, the surface heat transfer coefficient is 6 W/m$^2\cdot$K; on the outside, under normal wind conditions, it is 15 W/m$^2\cdot$K. The thermal conductivity of compacted snow is 0.15 W/m$\cdot$K. The temperature of the ice cap on which the igloo sits is −20°C and has the same thermal conductivity as the compacted snow.

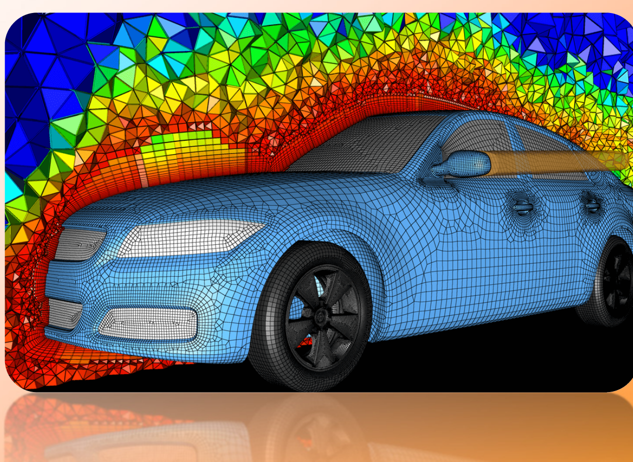
(a) Assuming that the occupants' body heat provides a continuous source of 320 W within the igloo, calculate the inside air temperature when the outside air temperature is $T_\infty = -40$°C. Be sure to consider heat losses through the floor of the igloo.



Lecture Notes: II.4

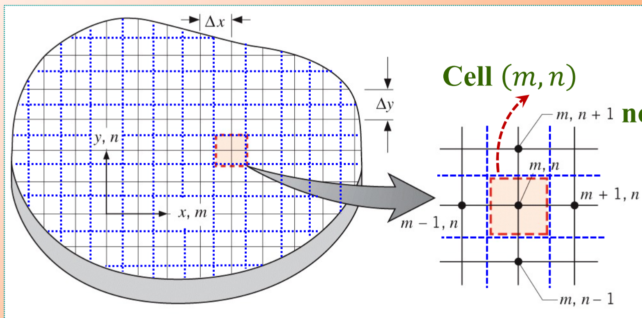
Numerical solution methods

- **Steps**
  1. Generating the mesh or grid (for mesh-based methods):



Numerical solution methods

- **Mesh**
  - Different mesh types are possible
  - The most basic one is cell-centered structured mesh
  - Each node is located at the center of each cell and is identified, in 2D, with 2 integers: (m, n)



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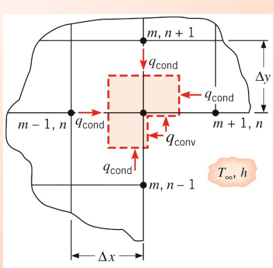
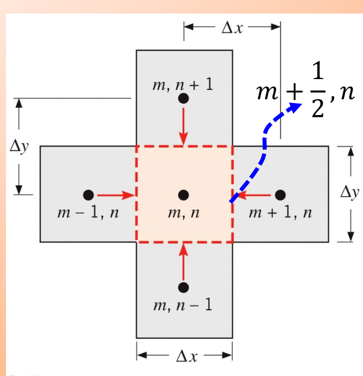
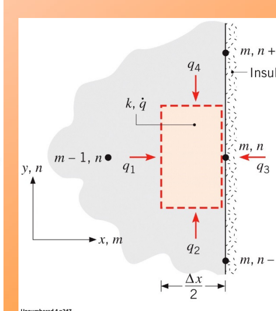
Numerical solution methods

- **Mesh**
  - Internal nodes vs. boundary nodes (at a boundary)
  - Internal cell (has four internal faces) vs. boundary cell
  - The cell faces are shown by $m \pm \frac{1}{2}$ and/or $n \pm \frac{1}{2}$.







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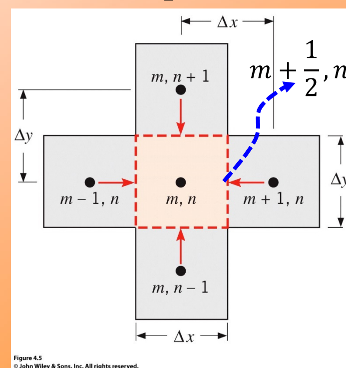
Numerical solution methods

- **Mesh**
  - Internal nodes vs. boundary nodes (at a boundary)
  - Internal cell (has four internal faces) vs. boundary cell
  - The cell faces are shown by $m \pm \frac{1}{2}$ and/or $n \pm \frac{1}{2}$.
  - The cell size is shown by Δx and Δy in x and y directions, respectively.
  - The cell size can be non-uniform (varying size).



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Numerical solution methods

- **Steps**
  1. Generating the mesh or grid (for mesh-based methods)
  2. Solution algorithm:
     - ✓ When a system of coupled PDE equations is solved.
  3. Discretization of the PDE for internal cells:
     - ✓ Finite Difference (FD)
     - ✓ Finite Volume (FV)

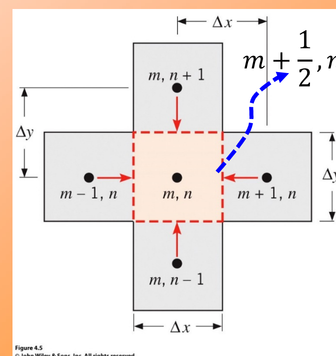
Chapter 2 By E. Amani

Numerical solution methods

- **FD discretization (internal cells)**
  - The numerical approximation of derivatives using Taylor series expansion.
    - ✓ Example: Central Difference (CD) approximation

Order of discretization

$$\left.\frac{\partial T}{\partial x}\right|_{m,n} = \frac{T_{m+1/2,n} - T_{m-1/2,n}}{x_{m+1/2} - x_{m-1/2}} + \underbrace{\mathcal{O}(\Delta x^2)}_{\text{Truncation error}} \qquad (9.2)$$



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Numerical solution methods

- **FD discretization (internal cells)**
  - The numerical approximation of derivatives using Taylor series expansion.
    - ✓ Example: Central Difference (CD) approximation

$$\left.\frac{\partial T}{\partial x}\right|_{m,n} = \frac{T_{m+1/2,n} - T_{m-1/2,n}}{x_{m+1/2} - x_{m-1/2}} + \mathcal{O}(\Delta x^2) \qquad (9.2)$$

- ✓ Proof:

$$T_{m+1/2,n} = T_{m,n} + \left.\frac{\partial T}{\partial x}\right|_{m,n} \frac{\Delta x}{2} + \left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \frac{\Delta x^2}{4} + \mathcal{O}(\Delta x^3)$$

$$T_{m-1/2,n} = T_{m,n} - \left.\frac{\partial T}{\partial x}\right|_{m,n} \frac{\Delta x}{2} + \left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \frac{\Delta x^2}{4} - \mathcal{O}(\Delta x^3)$$

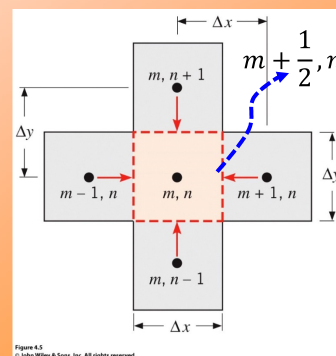
$$T_{m+1/2,n} - T_{m-\frac{1}{2},n} = \left.\frac{\partial T}{\partial x}\right|_{m,n} \Delta x + \mathcal{O}(\Delta x^3) \longrightarrow (9.2)$$

Chapter 2 By E. Amani

Numerical solution methods

- **FD discretization (internal cells)**
  - The numerical approximation of derivatives using Taylor series expansion.
    - ✓ Example: Central Difference (CD) approximation

$$\left.\frac{\partial T}{\partial x}\right|_{m,n} = \frac{T_{m+1/2,n} - T_{m-1/2,n}}{x_{m+1/2} - x_{m-1/2}} + \mathcal{O}(\Delta x^2) \quad (9.2)$$



    - ✓ Exercise: Prove that

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} = \frac{T_{m+1,n} - 2T_{m,n} + T_{m-1,n}}{\Delta x^2} + \mathcal{O}(\Delta x^2) \quad (10.2)$$

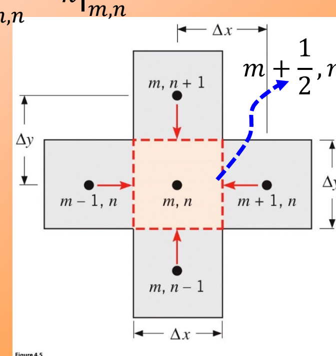
Chapter 2 By E. Amani

Numerical solution methods

- **FD discretization (internal cells)**
  - Discretization of 2D steady heat conduction equation
    - ✓ For each internal cell m, n (assuming constant k)

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\dot{q}}{k} = 0\right)_{m.n} \longrightarrow \left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} + \left.\frac{\partial^2 T}{\partial y^2}\right|_{m,n} + \left.\frac{\dot{q}}{k}\right|_{m,n} = 0$$

$$\frac{T_{m+1,n} - 2T_{m,n} + T_{m-1,n}}{\Delta x^2} + \frac{T_{m,n+1} - 2T_{m,n} + T_{m,n-1}}{\Delta y^2} + \mathcal{O}(\Delta x^2, \Delta y^2) + \frac{\dot{q}(m,n)}{k} = 0$$



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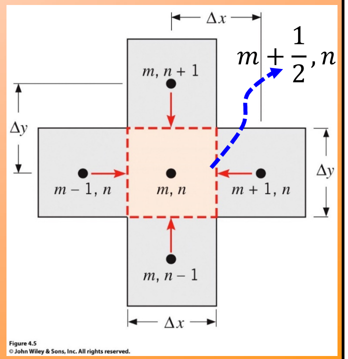
Numerical solution methods

- **FD discretization (internal cells)**
  - Discretization of 2D steady heat conduction equation
    - ✓ For each internal cell m, n (assuming constant k)

$$\left(T_{m-1,n}+T_{m+1,n}\right)+\frac{\Delta x^2}{\Delta y^2}\left(T_{m,n+1}+T_{m,n-1}\right)-2\left(1+\frac{\Delta x^2}{\Delta y^2}\right)T_{m,n}=-\frac{\dot{q}(m,n)}{k} \qquad (11.2)$$



Chapter 2 By E. Amani

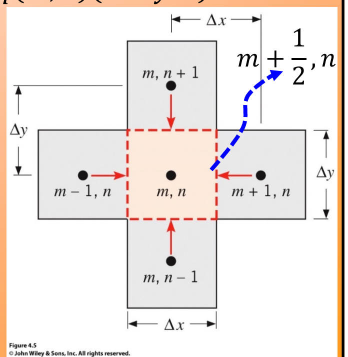
Numerical solution methods

- **FV discretization (internal cells)**
  - The integration of PDE or Integral form of equation over each cell m, n
    - ✓ For each internal cell m, n (assuming constant k)

$$\dot{E}_{\text{in}}-\overset{0}{\cancel{\dot{E}_{\text{out}}}}+\dot{E}_g=\overset{0}{\cancel{\dot{E}_{\text{st}}}}\longrightarrow\underbrace{\sum_{nb}}_{\text{Neighbor cells}} q_{nb\to m,n}+\dot{q}(m,n)(\Delta x\Delta y.1)=0$$

$$q_{m+1,n\to m,n}=-q_x\Big|_{m+1/2,n}=-\left(-kA\frac{dT}{dx}\right)\Bigg|_{m+\frac{1}{2},n}=kA\frac{dT}{dx}\Bigg|_{m+1/2,n}=k(\Delta y.1)\frac{T_{m+1,n}-T_{m,n}}{\Delta x}$$

$$q_{m+1,n\to m,n}=k(\Delta y.1)\frac{T_{m+1,n}-T_{m,n}}{\Delta x}$$



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Numerical solution methods

- **FV discretization (internal cells)**
  - The integration of PDE or Integral form of equation over each cell m, n
    - ✓ For each internal cell m, n (assuming constant k)

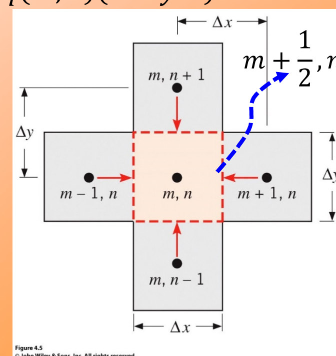
$$\dot{E}_{\text{in}} - \cancelto{0}{\dot{E}_{\text{out}}} + \dot{E}_g = \cancelto{0}{\dot{E}_{\text{st}}} \longrightarrow \sum_{nb} q_{nb \to m,n} + \dot{q}(m,n)(\Delta x \Delta y.1) = 0$$

$$q_{m+1,n\to m,n} = k\Delta y \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$q_{m-1,n\to m,n} = k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_{m,n+1\to m,n} = k\Delta x \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$q_{m,n-1\to m,n} = k\Delta x \frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$



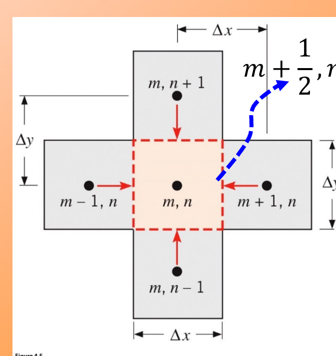
- ✓ Therefore,

Numerical solution methods

- **FV discretization (internal cells)**
  - The integration of PDE or Integral form of equation over each cell m, n
    - ✓ For each internal cell m, n (assuming constant k)

$$\left(T_{m-1,n} + T_{m+1,n}\right) + \frac{\Delta x^2}{\Delta y^2}\left(T_{m,n+1} + T_{m,n-1}\right) - 2\left(1 + \frac{\Delta x^2}{\Delta y^2}\right) T_{m,n} = -\frac{\dot{q}(m,n)}{k} \qquad (11.2)$$

- ✓ FD and FV discretization are not always the same.



Numerical solution methods

- **Steps**
  1. Generating the mesh or grid (for mesh-based methods)
  2. Solution algorithm
  3. Discretization of the PDE for internal cells
  4. Discretization of the PDE for boundary cells
     - ✓ Finite Difference (FD)
     - ✓ Finite Volume (FV)

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Numerical solution methods

- **FV discretization (boundary cells)**
  - ➢ The integration of PDE or Integral form of equation over each boundary cell m, n
    - ✓ Example: For the boundary cell m, n (assuming constant k)

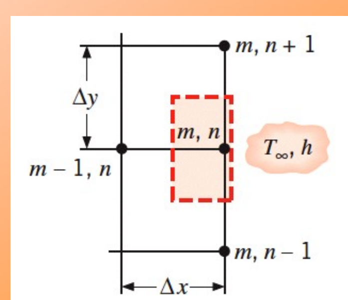
$$\dot{E}_{\text{in}} + \dot{E}_g = 0 \longrightarrow \sum_{nb} q_{nb\to m,n} + \dot{q}\left(\frac{\Delta x}{2}\Delta y\right) = 0$$

$$q_{m-1,n\to m,n} = k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_{m,n+1\to m,n} = k\frac{\Delta x}{2}\frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$q_{m,n-1\to m,n} = k\frac{\Delta x}{2}\frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$

$$q_{m+1,n\to m,n} = h\Delta y\left(T_\infty - T_{m,n}\right)$$



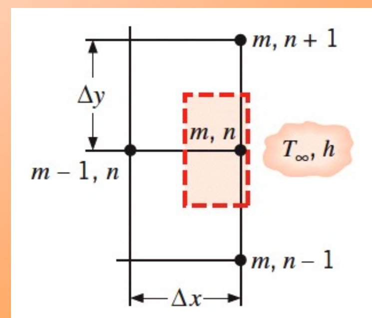
$\longrightarrow$

Chapter 2 By E. Amani

Numerical solution methods

- **FV discretization (boundary cells)**
  - The integration of PDE or Integral form of equation over each boundary cell m,n
    - ✓ Example: For the boundary cell m,n (assuming constant k)

$$T_{m-1,n}+\frac{\Delta x^2}{2\Delta y^2}\left(T_{m,n+1}+T_{m,n-1}\right)-\left(1+\frac{\Delta x^2}{\Delta y^2}+\frac{h\Delta x}{k}\right)T_{m,n}=-\left(\frac{\dot{q}\Delta x^2}{2k}+\frac{h\Delta x T_\infty}{k}\right) \quad (12.2)$$

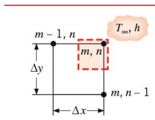
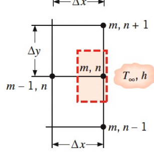


Chapter 2 By E. Amani

Numerical solution methods

- **FV discretization (boundary cells)**
  - The integration of PDE or Integral form of equation over each boundary cell m,n
    - ✓ Table 4.2 [1]: For the boundary cell with $\Delta x=\Delta y$ and $\dot{q}=0$

TABLE 4.2 Summary of nodal finite-difference equations

Configuration	Finite-Difference Equation for $\Delta x=\Delta y$	
	$(T_{m,n-1}+T_{m-1,n})+2\frac{h\,\Delta x}{k}T_\infty-2\left(\frac{h\,\Delta x}{k}+1\right)T_{m,n}=0$	(4.43)
	Case 4. Node at an external corner with convection	
	$(2T_{m-1,n}+T_{m,n+1}+T_{m,n-1})+\frac{2h\,\Delta x}{k}T_\infty-2\left(\frac{h\,\Delta x}{k}+2\right)T_{m,n}=0$	(4.42)[a]
	Case 3. Node at a plane surface with convection	

[a,b] To obtain the finite-difference equation for an adiabatic surface (or surface of symmetry), simply set h or q'' equal to zero.

- ✓ Exercise: Compare (12.2) with (4.42)[1]

Chapter 2 By E. Amani

Numerical solution methods

- **FV discretization (boundary cells)**
  - Method of equivalent thermal resistances (recommended)
    - ✓ Example: For the boundary cell m, n (assuming constant k)

$$\dot{E}_{\text{in}} + \dot{E}_g = 0 \longrightarrow \sum_{nb} q_{nb\to m,n} + \dot{q}\left(\frac{3}{4}\Delta x \Delta y\right) = 0$$

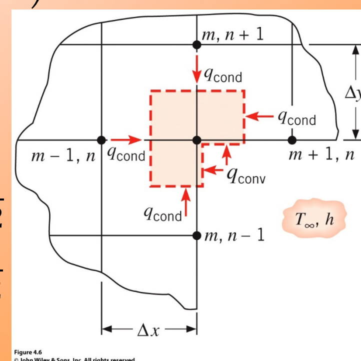
$$q_{m-1,n\to m,n} = \frac{T_{m-1,n} - T_{m,n}}{R_W} \qquad R_W = \frac{\Delta x}{k\Delta y}$$

$$q_{m,n+1\to m,n} = \frac{T_{m,n+1} - T_{m,n}}{R_N} \qquad R_N = \frac{\Delta y}{k\Delta x}$$

$$q_{m,n-1\to m,n} = \frac{T_{m,n-1} - T_{m,n}}{R_S} \qquad R_S = \frac{\Delta y}{k\Delta x/2}$$

$$q_{m+1,n\to m,n} = \frac{T_{m,n-1} - T_{m,n}}{R_E} \qquad R_E = \frac{\Delta x}{k\Delta y/2}$$

$$q_{\infty\to m,n} = \frac{T_\infty - T_{m,n}}{R_{\text{conv}}} \quad R_{\text{conv}} = \frac{1}{h\left(\frac{\Delta x}{2} + \frac{\Delta y}{2}\right)}$$



Chapter 2 By E. Amani

Numerical solution methods

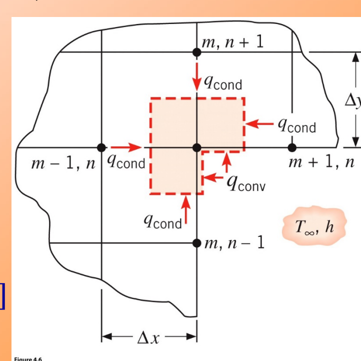
- **FV discretization (boundary cells)**
  - Method of equivalent thermal resistances (recommended)
    - ✓ Example: For the boundary cell m, n (assuming constant k)

$$2T_{m-1,n} + 2\frac{\Delta x^2}{\Delta y^2}T_{m,n+1} + T_{m+1,n} + \frac{\Delta x^2}{\Delta y^2}T_{m,n-1}$$

$$-\left[3\left(1 + \frac{\Delta x^2}{\Delta y^2}\right) + \frac{h\Delta x}{k}\left(1 + \frac{\Delta x}{\Delta y}\right)\right]T_{m,n}$$

$$= -\left[\frac{3}{2}\Delta x^2 \dot{q} + \frac{h\Delta x}{k}\left(1 + \frac{\Delta x}{\Delta y}\right)T_\infty\right] \tag{13.2}$$

✓ Exercise: Compare (13.2) with (4.41)[1]



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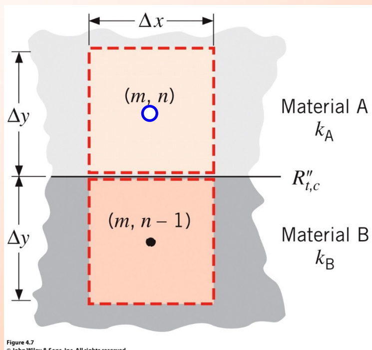
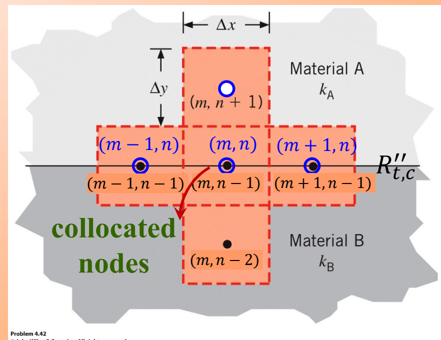
Numerical solution methods

- **FV discretization (interface cells)**
  - Exercise: Find the equation for nodes $m, n-1$ and m, n in the following two grid arrangements





  - Which grid arrangement is better?

Chapter 2 By E. Amani

Numerical solution methods

- **Steps**
  1. Generating the mesh or grid (for mesh-based methods)
  2. Solution algorithm
  3. Discretization of the PDE for internal cells
  4. Discretization of the PDE for boundary cells
  5. The solution of the system of linear algebraic equations

Chapter 2 By E. Amani

Numerical solution methods

- **Linear solvers**
  - The simplest and classic method is called Gauss-Seidel method

Algorithm 1: Gauss-Seidel

Inputs: Parameter definitions

```
for each m,n ∈ nodeList do                                   ▷ Initialization
    T_{m,n} = T^{kp1}_{m,n} = T^0_{m,n}
end for
repeat
    for each m,n ∈ nodeList do        ▷ Calculate T^{kp1}_{m,n} from the discretized equation given its neighbor values
        T^{kp1}_{m,n} = f(T^{kp1}_{nb})
    end for
    ε = max_{m,n} |(T^{kp1}_{m,n} − T_{m,n}) / T^{kp1}_{m,n}|   ▷ Convergence error calculation
    if ε < 10^{-6} then exit                                      ▷ Convergence check
    for each m,n ∈ nodeList do                                   ▷ Preparing for the next iteration
        T_{m,n} = T^{kp1}_{m,n}
    end for
until convergence
```

Internal nodes: $\left(T^{k+1}_{m-1,,n} + T^{k+1}_{m+1,,n}\right) + \dfrac{\Delta x^2}{\Delta y^2}\left(T^{k+1}_{m,n+1} + T^{k+1}_{m,n-1}\right) - 2\left(1 + \dfrac{\Delta x^2}{\Delta y^2}\right) T^{k+1}_{m,,n} = -\dfrac{\dot{q}(m,n)}{k}$

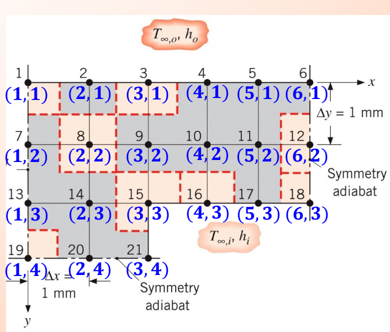
Boundary nodes: $T^{k+1}_{m,,n} = \cdots$

Chapter 2 **By E. Amani**

Numerical solution methods

- **Linear solvers**
  - More efficient solvers (computational linear algebra)
  - Coefficient matrix

$$T \equiv \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}, \quad T_1 \rightarrow T_{1,1}, \quad T_2 \rightarrow T_{2,1}$$



$(m,n) \leftrightarrow k$

Equation for T_1: $a_{11}T_1 + a_{12}T_2 + \cdots + a_{1N}T_N = C_1$

Equation for T_2: $a_{21}T_1 + a_{22}T_2 + \cdots + a_{2N}T_N = C_2$

$\vdots \qquad \vdots$

Equation for T_N: $a_{N1}T_1 + a_{N2}T_2 + \cdots + a_{NN}T_N = C_N$

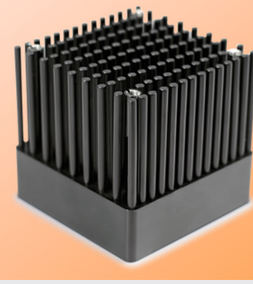
$$\boldsymbol{AT = C}$$

$$A \equiv \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}, \quad T \equiv \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}, \quad C \equiv \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}$$

Chapter 2 **By E. Amani**

Heat transfer enhancement (augmentation)

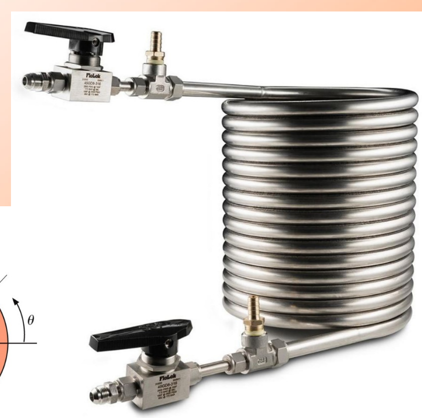
- Fins

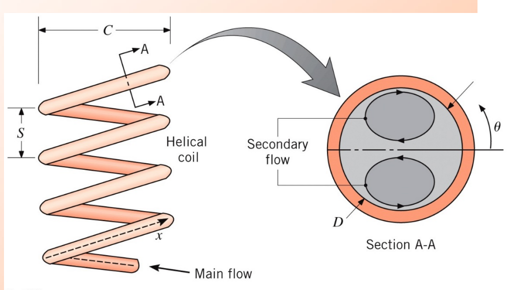


Chapter 2 By E. Amani

Heat transfer enhancement (augmentation)

- Fins
- Secondary flow

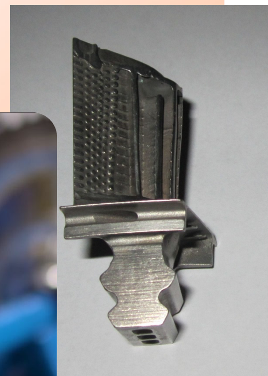


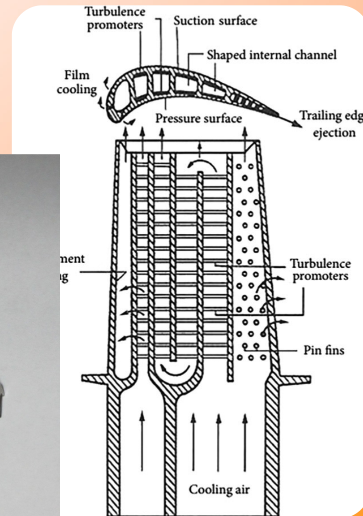


Chapter 2 By E. Amani

Heat transfer enhancement (augmentation)

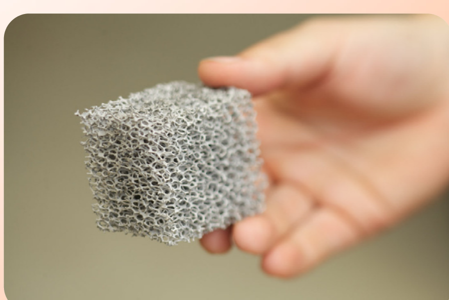
- Fins
- Secondary flow
- Turbulence generation

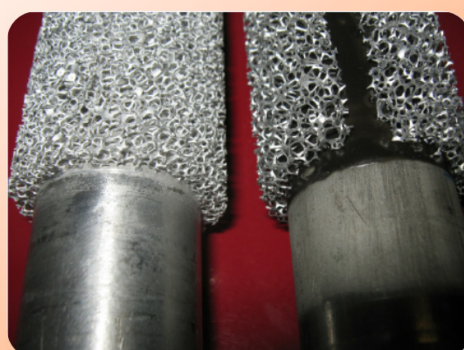




Heat transfer enhancement (augmentation)

- Fins
- Secondary flow
- Turbulence generation
- Porous media





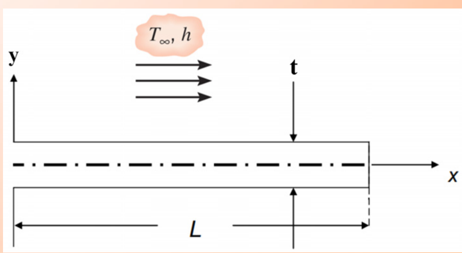
Heat transfer enhancement (augmentation)

- **Fins**
- **Secondary flow**
- **Turbulence generation**
- **Porous media**
- **Phase change**
- **Nanofluid**
- **…**

Chapter 2 By E. Amani

Project: Composite fin

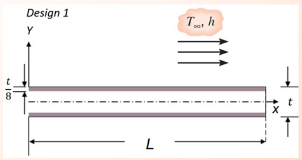
- Objective: Optimal usage of superconductors

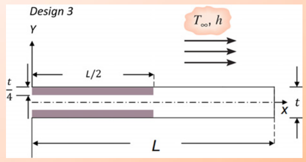


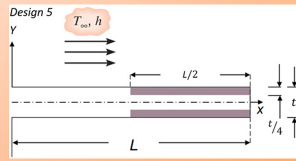
Chapter 2 By E. Amani

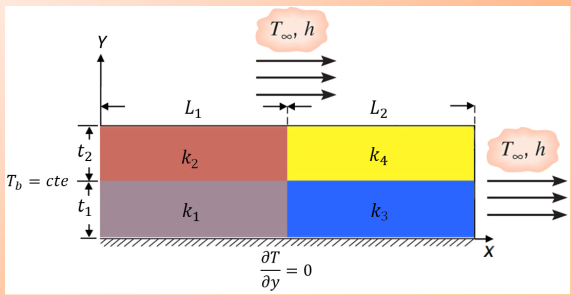
Project: Composite fin

- Objective: Optimal usage of superconductors
- Many problems (designs) with one code









Chapter 2 By E. Amani

Project: Composite fin

- Objective: Optimal usage of superconductors
- Many problems (designs) with one code
- Hands-on practice of the steps of numerical solution

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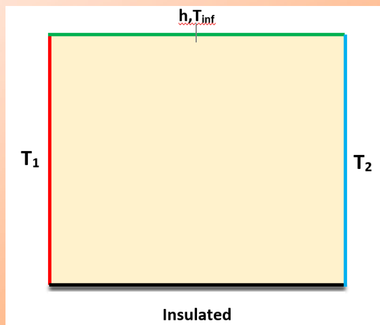
Project: Composite fin

- Objective: Optimal usage of superconductors
- Many problems (designs) with one code
- Hands-on practice of the steps of numerical solution
- A preliminary FV code development experience starting from samples (3 samples in “FVCodeSamples”)



Chapter 2 By E. Amani

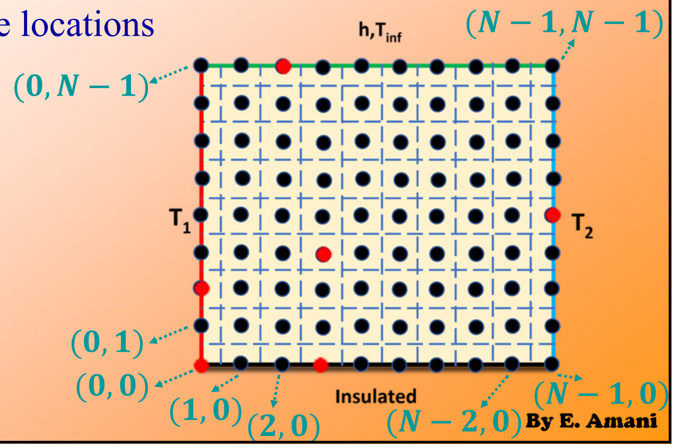
Project: Composite fin

- Objective: Optimal usage of superconductors
- Many problems (designs) with one code
- Hands-on practice of the steps of numerical solution
- A preliminary FV code development experience starting from samples (3 samples in “FVCodeSamples”)
- Steps of a numerical study or detailed design (validation, grid-independence, …)
- Writing a scientific report

Chapter 2 By E. Amani

Code: simpleSample.py

- Sketch the grid configuration
- Structured cell-centered grid
- Uniform grid with $\Delta x = \Delta y$
- Precise node locations

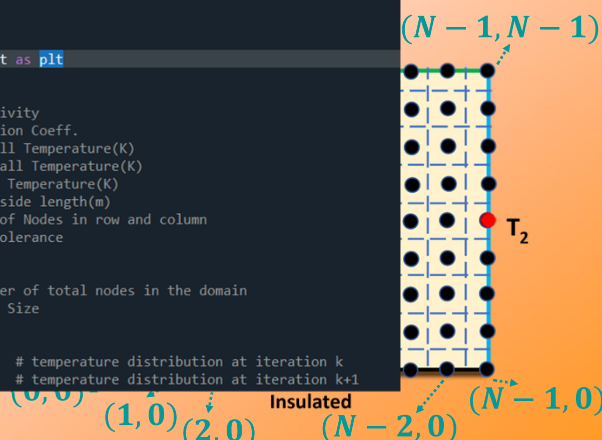


Chapter 2 By E. Amani

Code: simpleSample.py

simpleSample.py

```python
# -*- coding: utf-8 -*-
"""
Heat Transfer II
simpleSample.py
"""

import numpy as np
import matplotlib.pyplot as plt

# Inputs
k = 237         # Conductivity
h = 100         # Convection Coeff.
T1 = 1000       # Left Wall Temperature(K)
T2 = 500        # Right Wall Temperature(K)
Tinf = 300      # Ambient Temperature(K)
L = 1           # Square side length(m)
N = 51          # Number of Nodes in row and column
erTol = 1e-6    # Error tolerance

# Grid
ntot = N * N      # Number of total nodes in the domain
dx = L / (N - 1) # Mesh Size

# Defining variables
Tk = np.zeros((N, N))     # temperature distribution at iteration k
Tkp1 = np.zeros((N, N))   # temperature distribution at iteration k+1
```

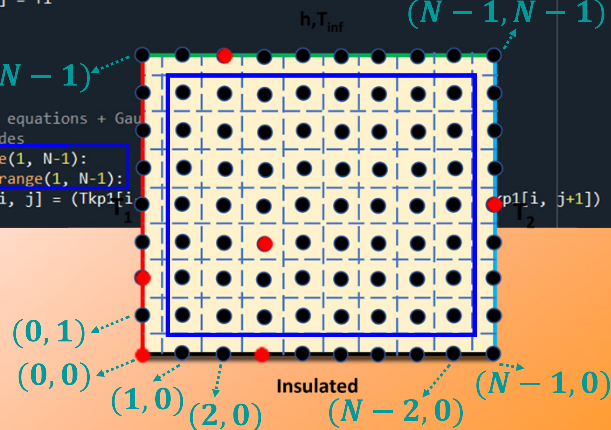


Chapter 2 By E. Amani

Code: simpleSample.py

simpleSample.py

```python
# Initialization
for j in range(N):
    for i in range(N):
        Tk[i, j] = T1
        Tkp1[i, j] = T1

# main loop
while True:
    C = 0

    # Discretized equations + Gau
    # Interior nodes
    for j in range(1, N-1):
        for i in range(1, N-1):
            Tkp1[i, j] = (Tkp1[i                p1[i, j+1]) / 4
```



Code: simpleSample.py

simpleSample.py

```python
# Initialization
for j in range(N):
    for i in range(N):
        Tk[i, j] = T1
        Tkp1[i, j] = T1

# main loop
while True:
    C = 0

    # Discretized equations + Gauss-Siedel
    # Interior nodes
    for j in range(1, N-1):
        for i in range(1, N-1):
            Tkp1[i, j] = (Tkp1[i-1, j] + Tkp1[i+1, j] + Tkp1[i, j-1] + Tkp1[i, j+1]) / 4
```

$$(T_{m-1,n}+T_{m+1,n}) + \frac{\Delta x^2}{\Delta y^2}(T_{m,n+1}+T_{m,n-1}) - 2\left(1+\frac{\Delta x^2}{\Delta y^2}\right)T_{m,n} = -\frac{\dot{q}(m,n)}{k} \quad (11.2)$$

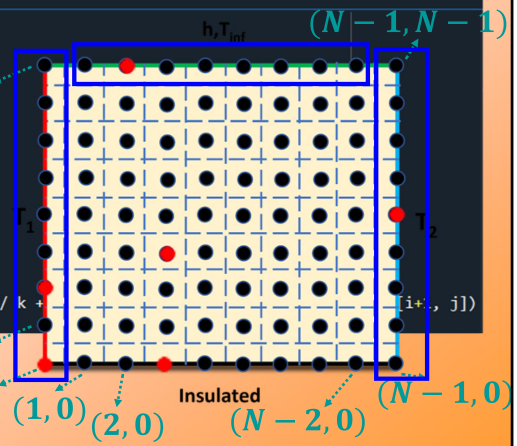
$$\xrightarrow[\Delta x = \Delta y,\, \dot{q} = 0]{} (T_{m-1,n}+T_{m+1,n}) + (T_{m,n+1}+T_{m,n-1}) - 4T_{m,n} = 0$$

$$\xrightarrow[\text{Gauss}-\text{Seidel}]{} (T^{k+1}_{m-1,,n}+T^{k+1}_{m+1,,n}) + (T^{k+1}_{m,n+1}+T^{k+1}_{m,n-1}) - 4T^{k+1}_{m,,n} = 0$$

Code: simpleSample.py

simpleSample.py

```
    # boundary conditions
    # Left isothermal Wall
    i = 0
    for j in range(N):
        Tkp1[i, j] = T1

    # Right isothermal Wall
    i = N-1
    for j in range(N):
        Tkp1[i, j] = T2

    # Top wall (convection)
    j = N-1
    for i in range(1, N-1):
        Tkp1[i, j] = (2 * h * dx * Tinf / k +
```



Chapter 2 **By E. Amani**

Code: simpleSample.py

simpleSample.py

```
    # boundary conditions
    # Left isothermal Wall
    i = 0
    for j in range(N):
        Tkp1[i, j] = T1

    # Right isothermal Wall
    i = N-1
    for j in range(N):
        Tkp1[i, j] = T2

    # Top wall (convection)
    j = N-1
    for i in range(1, N-1):
        Tkp1[i, j] = (2 * h * dx * Tinf / k + 2 * Tkp1[i, j-1] + Tkp1[i-1, j] + Tkp1[i+1, j])
```

? (Exercise)

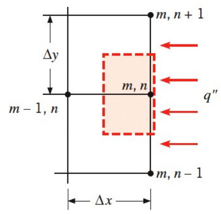
Chapter 2 **By E. Amani**

Code: simpleSample.py

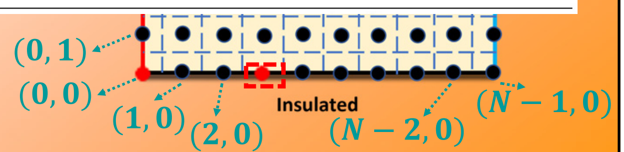
simpleSample.py

```
    # Bottom adiabatic wall
    j = 0
    for i in range(1, N-1):
        Tkp1[i, j] = (2 * Tkp1[i, j+1] + Tkp1[i-1, j] + Tkp1[i+1, j]) / 4
```

TABLE 4.2 Summary of nodal finite-difference equations

Configuration	Finite-Difference Equation for $\Delta x = \Delta y$
	$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2q''\Delta x}{k} - 4T_{m,n} = 0 \quad (4.44)^b$$
	Case 5. Node at a plane surface with uniform heat flux

$$\Delta x \to \Delta y, q'' = 0 \longrightarrow (2T_{i,j+1} + T_{i-1,j} + T_{i+1,j}) - 4T_{i,j} = 0$$



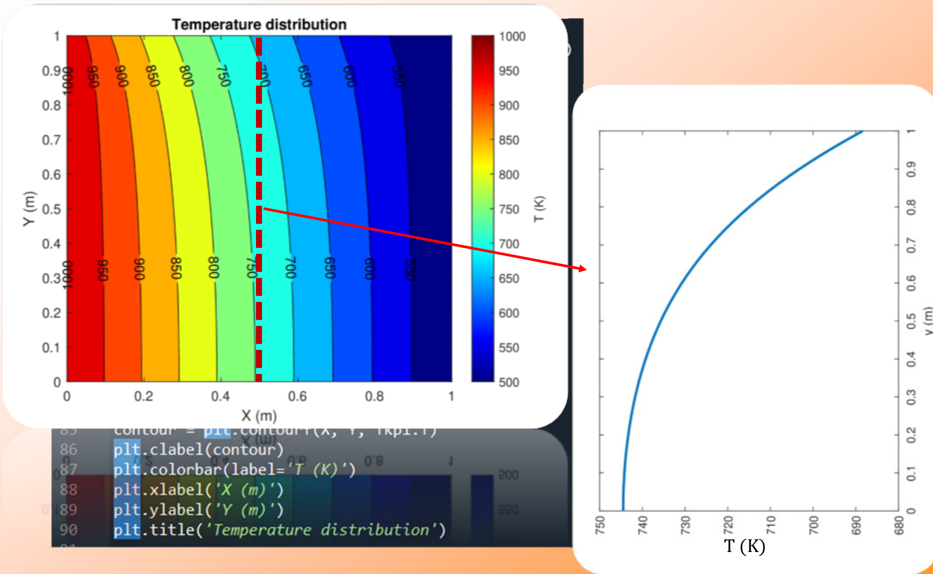
Chapter 2 By E. Amani

Code: simpleSample.py

```
    # error
    er = np.max(np.max(np.abs((Tkp1 - Tk) / Tkp1)))

    # convergence criterion
    if er <= erTol:
        break

    # preparing for the next iteration
    Tk = Tkp1.copy()

T_mid = Tkp1[((N-1)//2), :]

# Post-processing
# Drawing the 2d temperature Distribution
x = np.linspace(0, L, N)
y = np.linspace(0, L, N)

# Figure 1: Contour plot
plt.figure(1)
X, Y = np.meshgrid(x, y)
contour = plt.contourf(X, Y, Tkp1.T)
plt.clabel(contour)
plt.colorbar(label='T (K)')
plt.xlabel('X (m)')
plt.ylabel('Y (m)')
plt.title('Temperature distribution')
```

Chapter 2 By E. Amani

Code: simpleSample.py



Chapter 2 By E. Amani

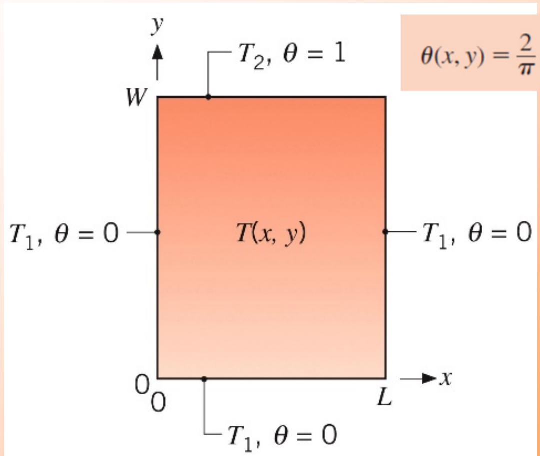
Conduction: 2D, steady

- **Solution methods**
  - Numerical methods
  - Analytical methods
    1. Separation of variables
    2. The method of characteristics
    3. Integral transforms (Fourier, Hankel, Laplace, ...)
    4. Green's function
    5. ...

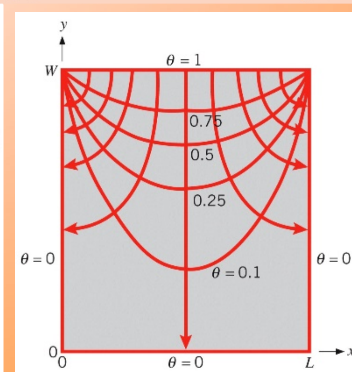
Chapter 2 By E. Amani

Analytical solution methods

- **Separation of variables**
  - Appendix A: The details of the method with an example



$$\theta(x, y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin \frac{n\pi x}{L} \frac{\sinh (n\pi y/L)}{\sinh (n\pi W/L)}$$



Chapter 2 By E. Amani

The end of chapter 2

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