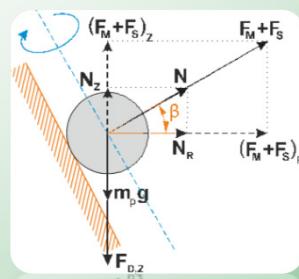


IX. Fluid-particle interaction (FPI)



IX.1 Fluid-to-particle transfer rates

IX.1.1 Momentum transfer – forces and moments on a particle

IX.1.2 Heat transfer

IX.1.3 Particle tracking numerics

Momentum transfer – particle tracking

- For each particle, p:

$\frac{dx_p}{dt} = u_p$ Fluid-particle interaction force
 $F = m_p \frac{d\mathbf{u}_p}{dt}; \quad F = F_{\text{FPI}} + F_B + F_{\text{col}}$ Body force
 $T = I_p \frac{d\omega_p}{dt}; \quad T = T_{\text{FPI}} + T_{\text{col}}$ Collision force

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FPI forces and moments

- DNS formulation:

$$\mathbf{F}_{\text{FPI}} = \int_{A_p} \boldsymbol{\sigma} \cdot \mathbf{n} dA = \int_{A_p} (-p\mathbf{n} + \boldsymbol{\tau} \cdot \mathbf{n}) dA$$

$$\mathbf{T}_{\text{FPI}} = \int_{A_p} \mathbf{r}_s \times (\mathbf{n} \cdot \boldsymbol{\tau}) dA$$

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FPI forces and moments

- Point-particle formulation:

- Simplifying assumptions
- Semi-analytical solutions
- Superposition

$$\mathbf{F}_{\text{FPI}} = \mathbf{F}_D + \mathbf{F}_L + \mathbf{F}_{vm} + \mathbf{F}_H + \mathbf{F}_0 + \mathbf{F}_W + \mathbf{F}_B + \mathbf{F}_T$$

Drag force Virtual (or added)
 (streamwise steady mass force
 viscous+pressure)(unsteady pressure)

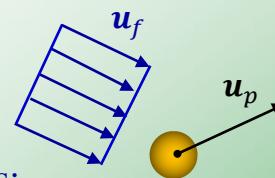
Lift force History (or
 (transverse steady Basset) force
 pressure) (unsteady
 viscous)

Wall (lubrication) force
 Thermophoretic force
 Undisturbed flow force
 Brownian force

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Drag force



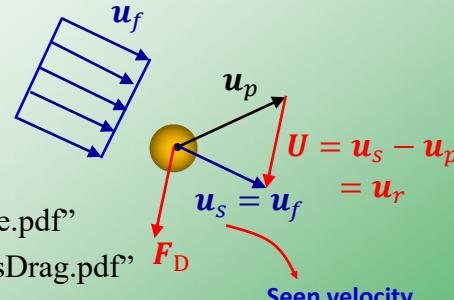
- Simplifying assumptions:

- Steady-state
- Stokes regime ($Re_p < 1$) \rightarrow Re number relaxation
- Spherical particle \rightarrow Equivalent diameter !
- No evaporation/condensation \rightarrow Blowing effect
- Uniform flow \rightarrow Seen velocity (u_s) definition
- Incompressible low-Mach flow \rightarrow Compressibility effect
- No internal flow \rightarrow Internal flow relaxation
- No rotation

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Drag force



- Derivation:

- See my notes: "dragForce.pdf"
- For more details: "StokesDrag.pdf"

- Result:

$$F_D = \underbrace{2\pi\mu U a}_{\text{Pressure drag}} + \underbrace{4\pi\mu U a}_{\text{Friction drag}} = 6\pi\mu U a$$

- The drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A_p} = \frac{6\pi\mu U a}{\frac{1}{2}\rho U^2 \pi a^2} \rightarrow C_D = \frac{24}{Re_p}; Re_p = \frac{2a\rho U}{\mu} = \frac{|u_r|d_p}{v_f}$$

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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor

→ Notes: Chap9.1.1.1

$$F_D = m_p \frac{u_r}{\tau_p}; \tau_p = \tau_v = \frac{1}{f_D} \frac{\rho_p d_p^2}{18 \rho_f v_f}$$

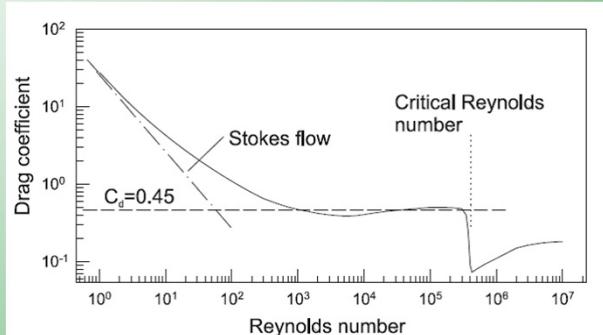
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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor
- Experimental measurements



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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor
- Experimental measurements
- A curve-fit, semi-empirical correlation by Schiller and Naumann [1]:

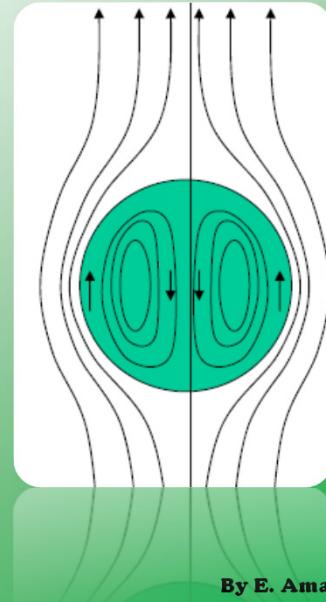
$$f_D = \begin{cases} 1 & Re_p \leq 0.1 \\ 1 + \frac{1}{6} Re_p^{2/3} & 0.1 < Re_p \leq 1000 \end{cases}$$

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Drag force

- Internal flow relaxation
- ...

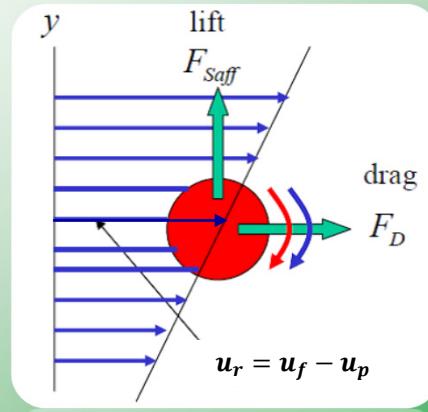


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Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field



Chap 9

$$\vec{u}^* = \vec{u} - \vec{u}_p$$

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Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field
- A semi-analytical solution [6]:

$$\mathbf{F}_L = 1.61 C_s \rho_f d_p^2 \left(\frac{2v_f}{|\omega_f|} \right)^{\frac{1}{2}} \mathbf{u}_r \times \boldsymbol{\omega}_f; \quad \boldsymbol{\omega}_f = 0.5 \nabla \times \mathbf{u}_f$$

$$C_s = \begin{cases} \left(1 - 0.3314\lambda_v^{\frac{1}{2}}\right) \exp\left(-\frac{Re_p}{10}\right) + 0.3314\lambda_v^{\frac{1}{2}} & ; Re_p \leq 40 \\ 0.0524(\lambda_v Re_p)^{1/2} & ; Re_p > 40 \end{cases}, \quad \lambda_v = d_p \frac{|\omega_f|}{|\mathbf{u}_r|}$$

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Saffman Lift force

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$\mathbf{u}_r = \mathbf{u}_f - \mathbf{u}_p$

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Saffman Lift force

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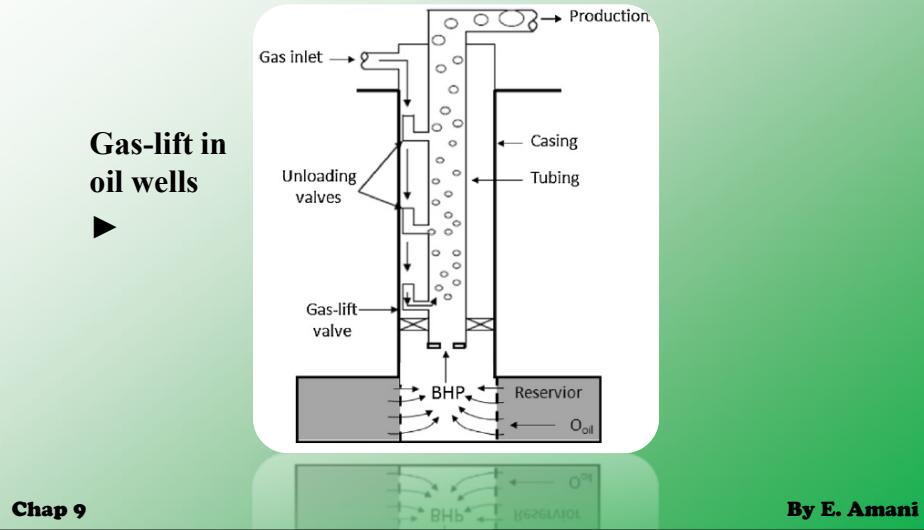
- Important for large ρ_f/ρ_p or large particles

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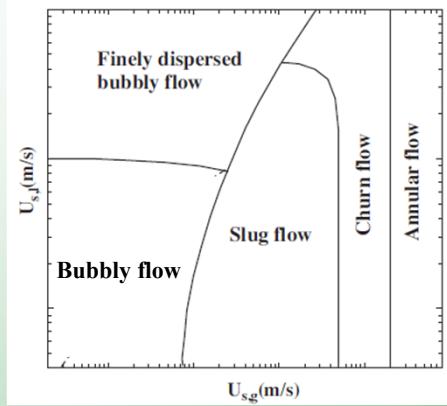
Saffman Lift force

- Application: Gas-lift [15]



Saffman Lift force

- Application: Gas-lift [15]

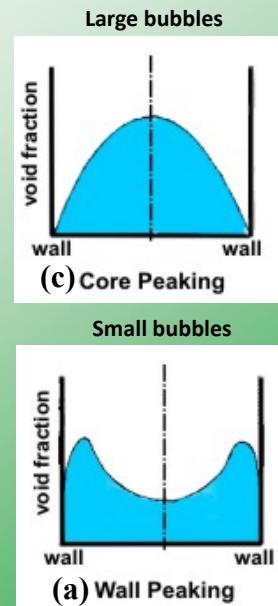
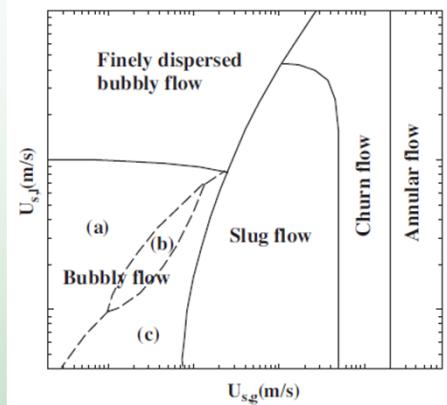


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Saffman Lift force

- Application: Gas-lift [15]

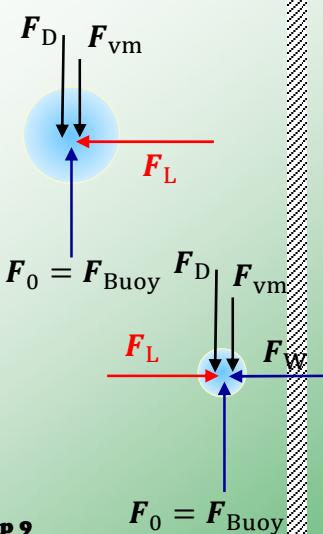


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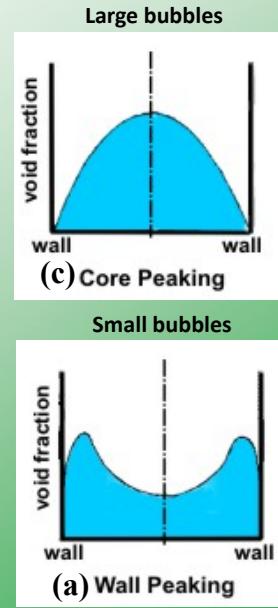
Saffman Lift force

- Application: Gas-lift [15]



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$$F_0 = F_{Buoy}$$



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Saffman Lift force

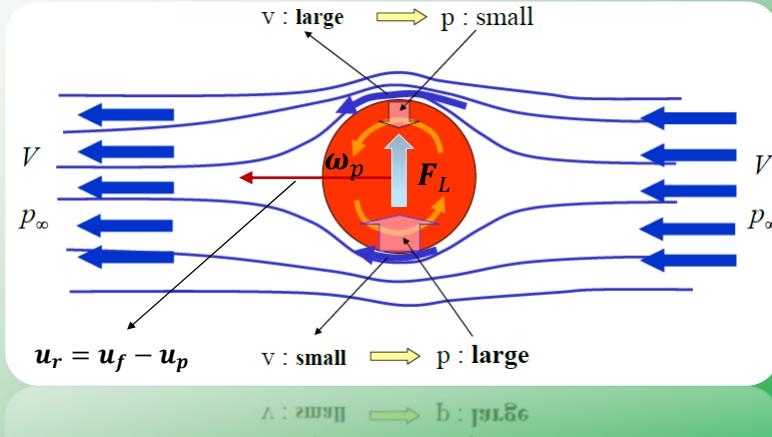
- Application: Gas-lift [15]
- More physical discussion and applications: exercise

Chap 9

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Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation



Chap 9

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Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation
- A semi-analytical solution [7, 8]:

$$\mathbf{F}_L = 0.5 C_{LR} \rho_f A_p \frac{|\mathbf{u}_r|}{|\boldsymbol{\omega}_r|} (\mathbf{u}_r \times \boldsymbol{\omega}_r); \quad \boldsymbol{\omega}_r = \boldsymbol{\omega}_p - \boldsymbol{\omega}_f$$

$$C_{LR} = \begin{cases} \frac{Re_\omega}{Re_p} & ; Re_p \leq 1 \\ 0.45 + \left(\frac{Re_\omega}{Re_p} - 0.45 \right) \exp(-0.05684 Re_\omega^{0.4} Re_p^{0.3}) & ; Re_p < 2000 \end{cases}$$

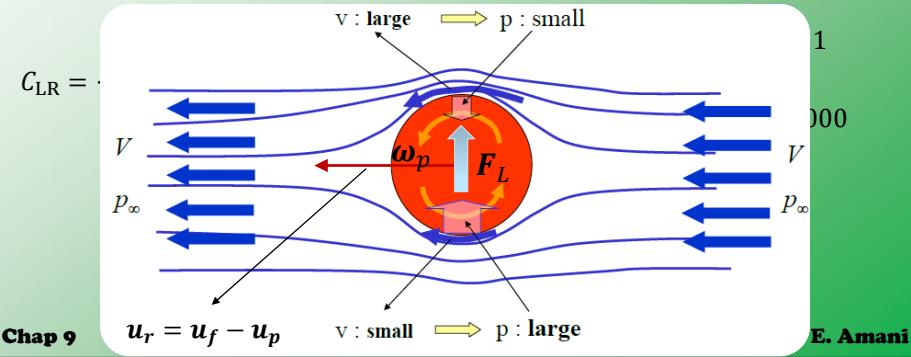
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Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation
- A semi-analytical solution [7, 8]:

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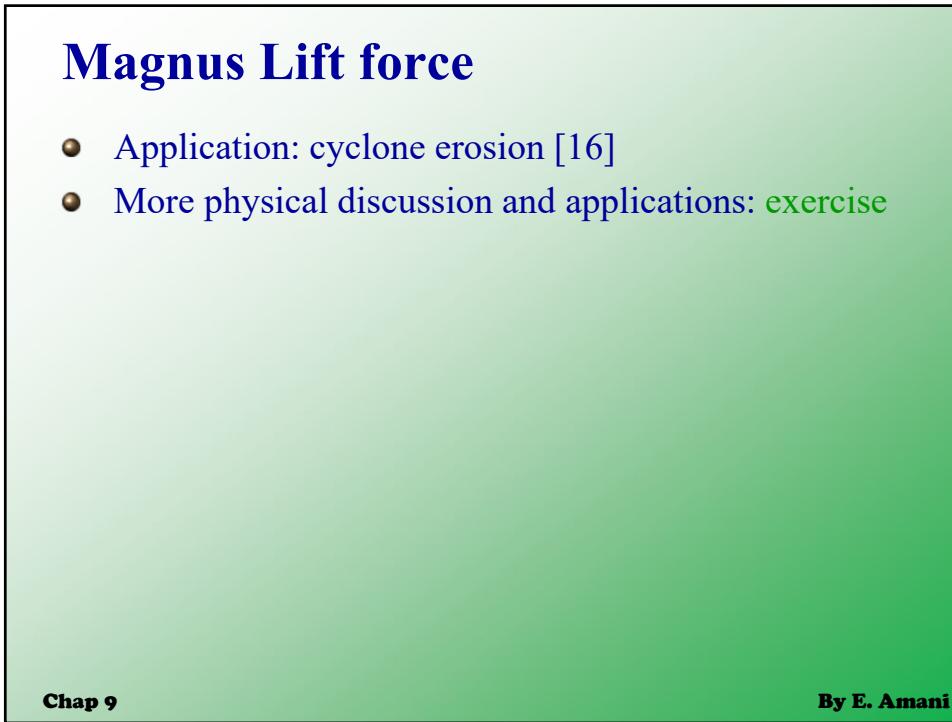
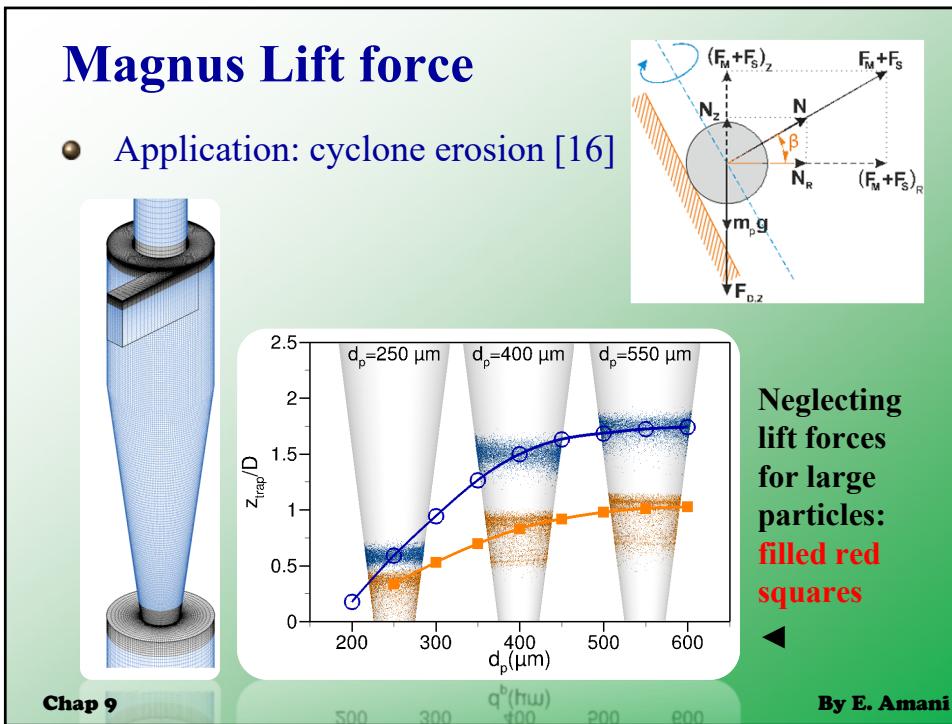
Magnus Lift force

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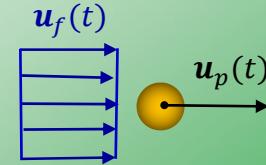
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- Important for large ρ_f/ρ_p or large rotating particles



Virtual (added) mass force

- Evidence: A spherical particle in an unsteady inviscid flow experiences a force
- Can be viewed as increased inertia force
- Accelerating a part of fluid with the particle



Chap 9

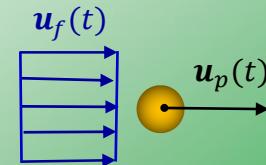
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Virtual (added) mass force

- For a basic derivation, consult classic aerodynamics books (e.g. Principles of Ideal-Fluid aerodynamics by K. Karamcheti).
- The analytical solution [3]:

$$\mathbf{F}_{vm} = C_{vm} \frac{\rho_f}{2} \nabla_p \left(\frac{D\mathbf{u}_f}{Dt} - \frac{d\mathbf{u}_p}{dt} \right); \quad C_{vm} = 1$$

- Scales as: (ρ_f / ρ_p)
- More physical discussion and applications: exercise



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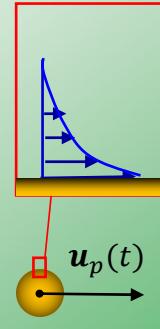
History (Basset) force

- Accelerating a part of fluid with the particle through viscous force
- A semi-analytical solution [4, 5]:

$$\mathbf{F}_H = C_B \frac{3}{2} d_p^2 \sqrt{\pi \mu_f \rho_f} \left[\int_0^t \frac{d\mathbf{u}_r}{dt'} dt' + \frac{\mathbf{u}_{r,0}}{\sqrt{t}} \right]$$

$$C_B = 1 - 0.527 \left[1 - \exp(-0.14 Re_p Sl^{0.82})^{2.5} \right]; \quad Sl = \frac{1}{2\pi f \tau_p}$$

- Scales as: (ρ_f / ρ_p)
- More physical discussion and applications: [exercise](#)

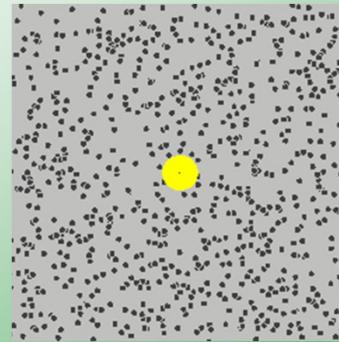


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Brownian Force

- For sub-micron particles
- Random force

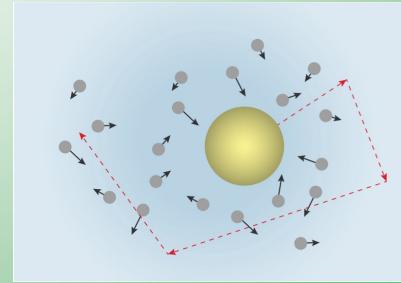


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Brownian Force

- For sub-micron particles
- Random force
- A stochastic process [9, 10]:



$$\mathbf{F}_B = m_p \sqrt{\frac{\pi S_0}{\delta t}} |\zeta| \boldsymbol{\eta}; \quad S_0 = \frac{216 \mu_f K_B T_f}{\pi^2 d_p^5 \rho_p^2 C_c}$$

$$C_c = 1 + \frac{2\lambda}{d_p} \left[1.257 + 0.4 \exp\left(-\frac{1.1d_p}{2\lambda}\right) \right]$$

$$\boldsymbol{\eta} = a \cos(\theta) \mathbf{i} + a \sin(\theta) \mathbf{j} + u \mathbf{k}, \theta = 2\pi X, u = 2Y - 1, a = \sqrt{1 - u^2}, \\ X, Y \in U(0,1), \zeta \in N(0,1)$$

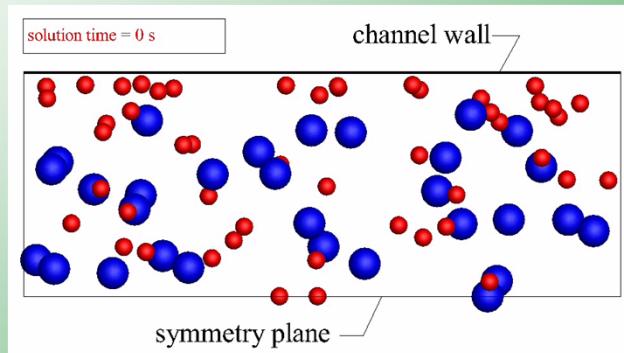
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Brownian Force

- Application: Electroosmotic flow in nano-channels [17]

Na^+ (red spheres) and Cl^- (blue spheres) ion transport in a nano-channel
►



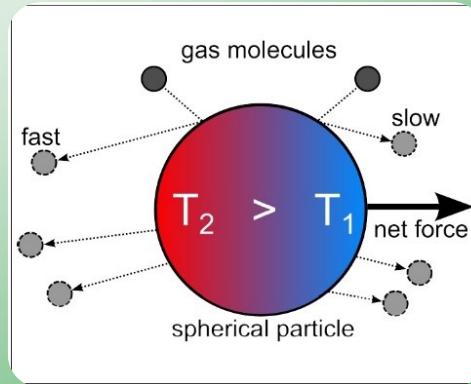
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Thermophoretic Force

- For sub-micron particles in a temperature gradient
- A closure [11]:

$$F_T = -D_{T,p} \frac{\nabla T}{T}$$



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Case study #3: Conveying T-junction

- Pneumatic conveying T-junction
 - Eulerian-Lagrangian simulation
 - Preliminary analysis
 - Solution using ANSYS Fluent

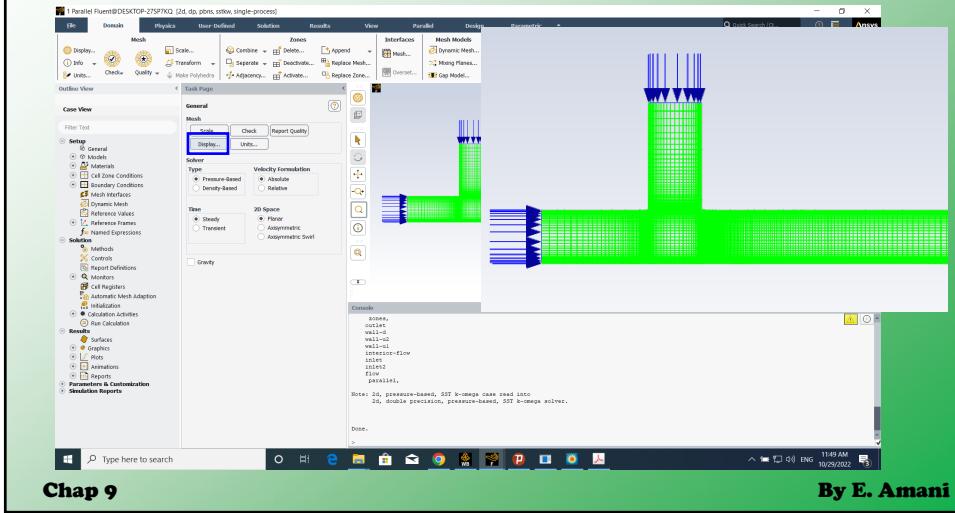
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)

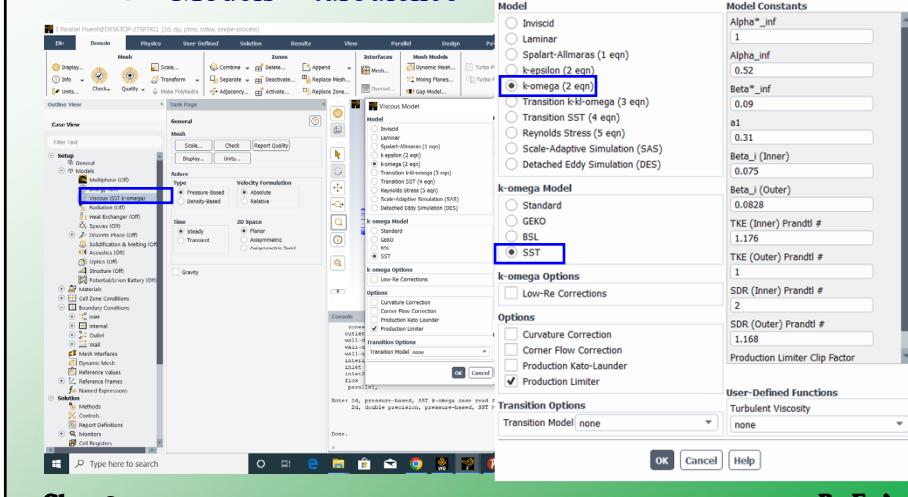
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Case study #3: Conveying T-junction

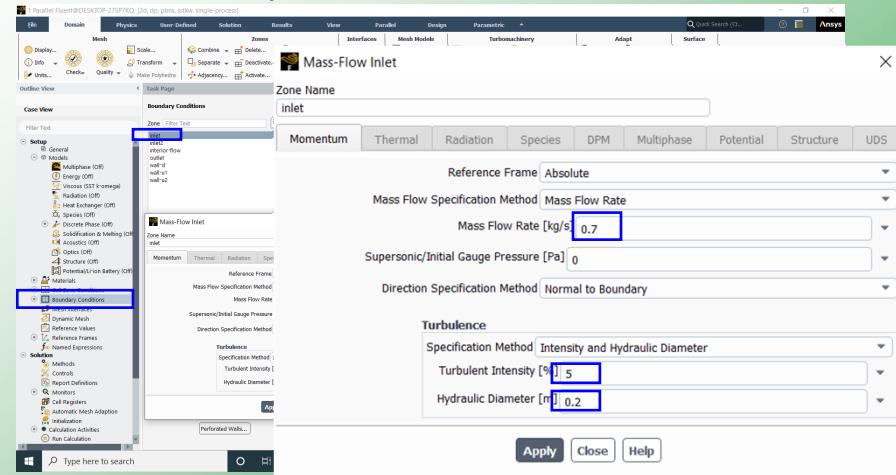
- Solution using ANSYS Fluent (single phase)

• Models > turbulence



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
Boundary condition

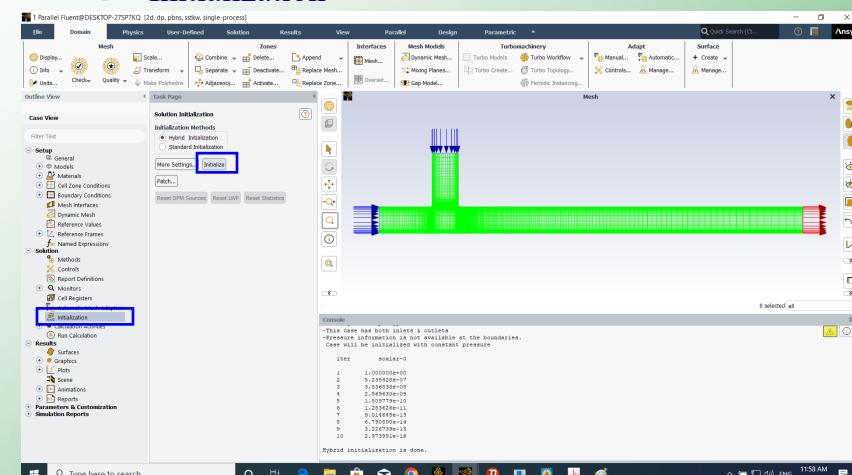


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
Initialization

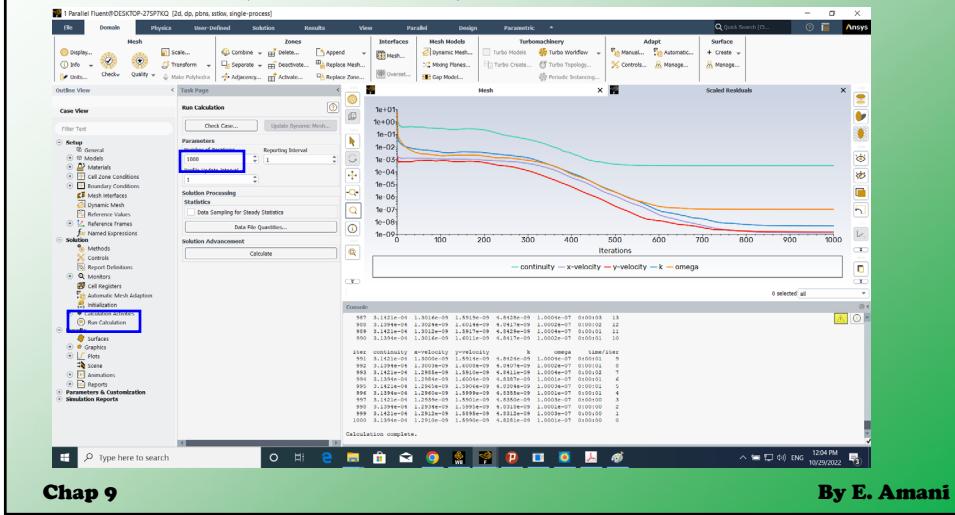


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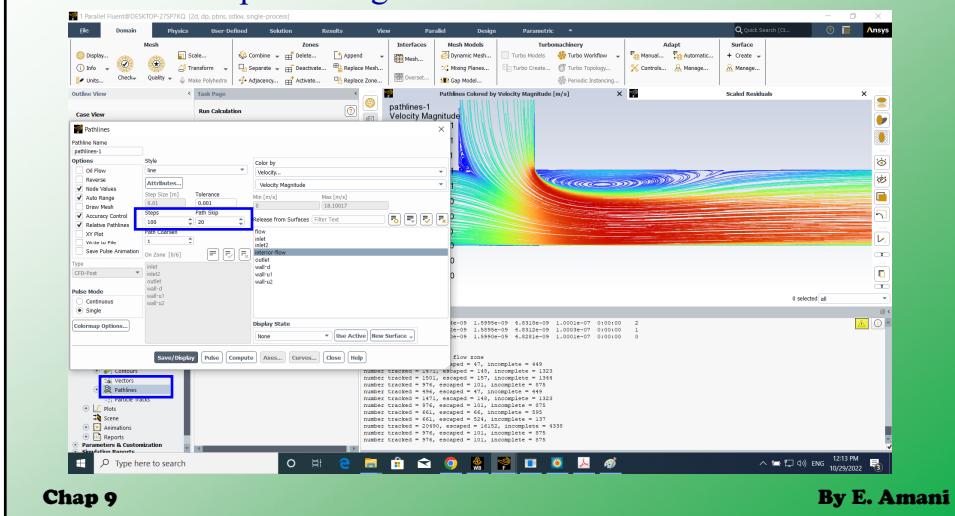
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
- Run (1000 iterations)



Case study #3: Conveying T-junction

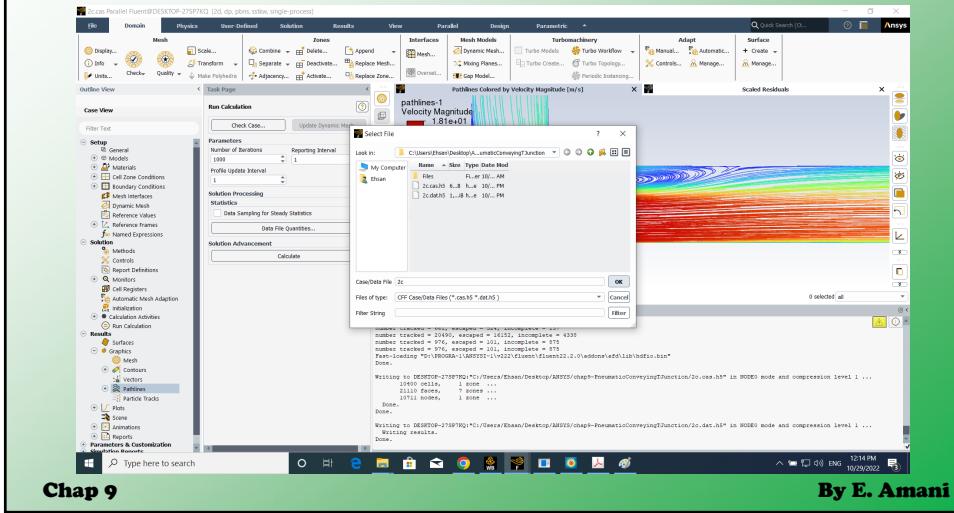
- Solution using ANSYS Fluent (single phase)
- Post processing: Pathlines



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)

• Save case and data: 2c



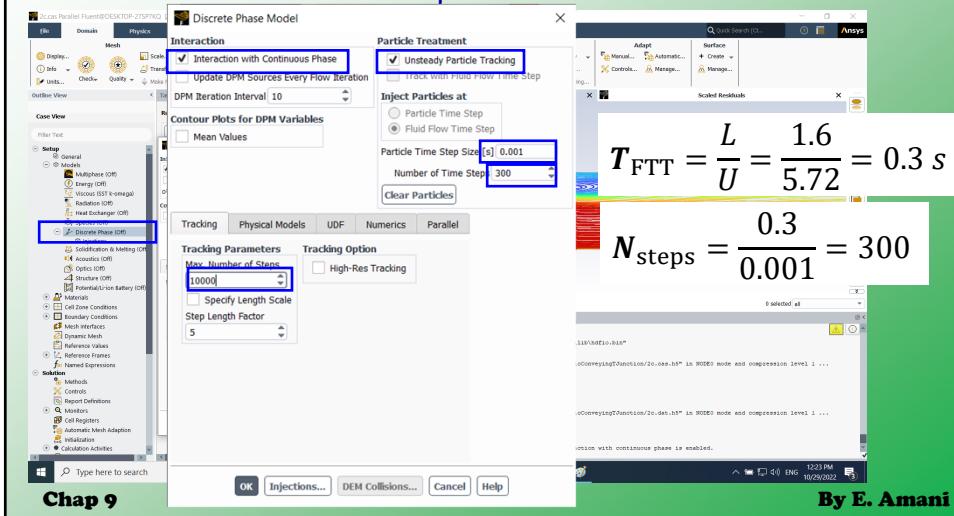
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

• Models > Discrete phase

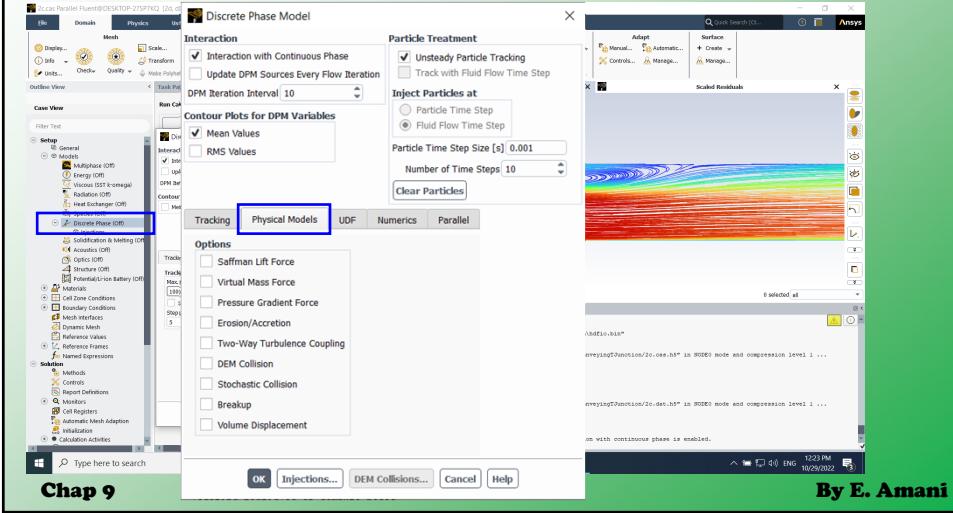


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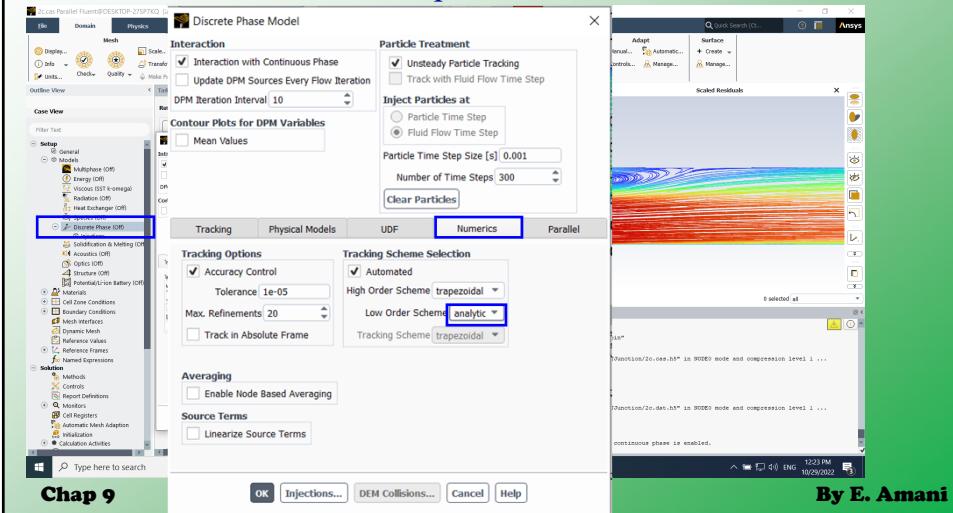
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
- Models > Discrete phase > Physical models



Case study #3: Conveying T-junction

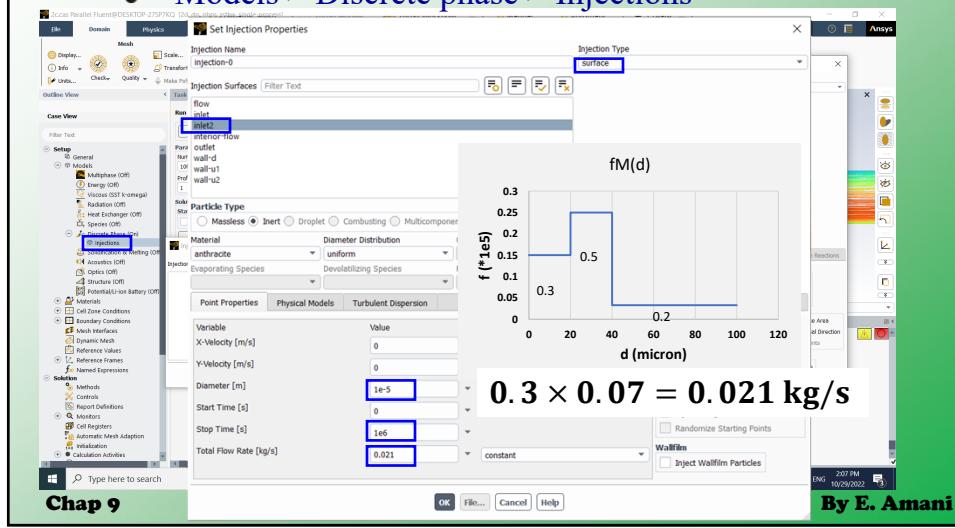
- Solution using ANSYS Fluent (one-way)
- Models > Discrete phase > Numerics



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

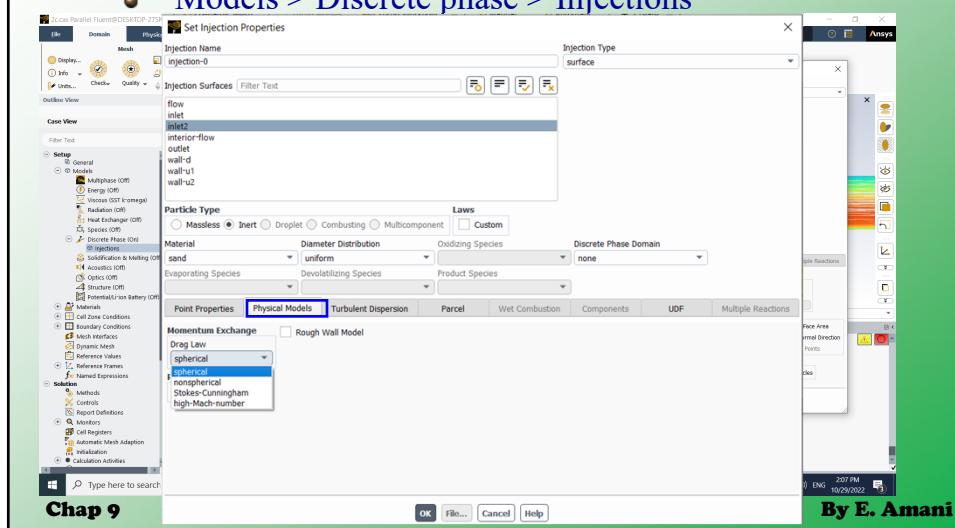
Models > Discrete phase > Injections



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

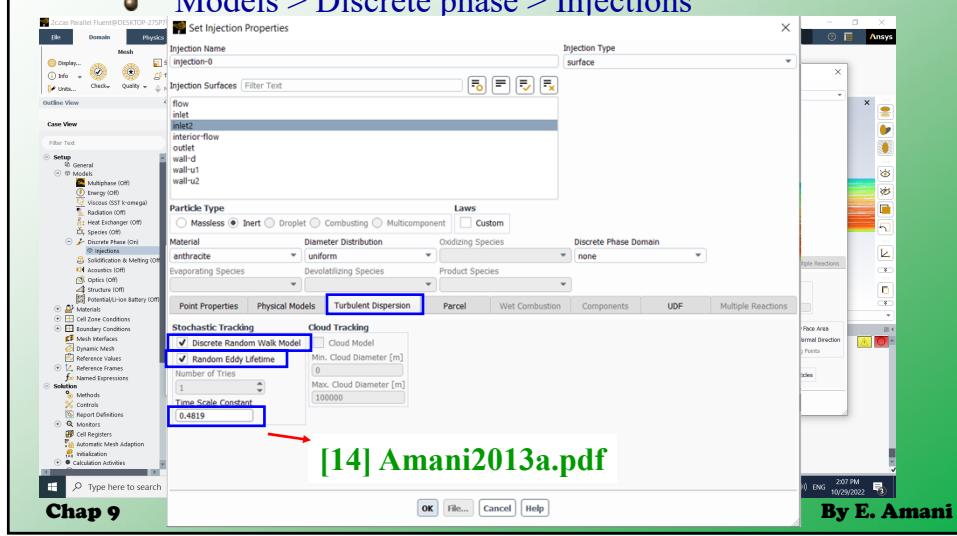
Models > Discrete phase > Injections



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

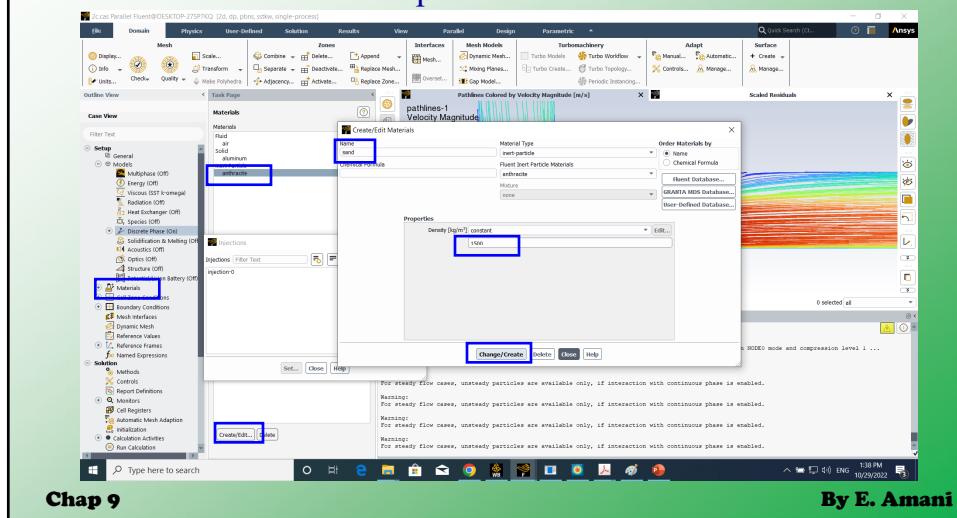
Models > Discrete phase > Injections



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

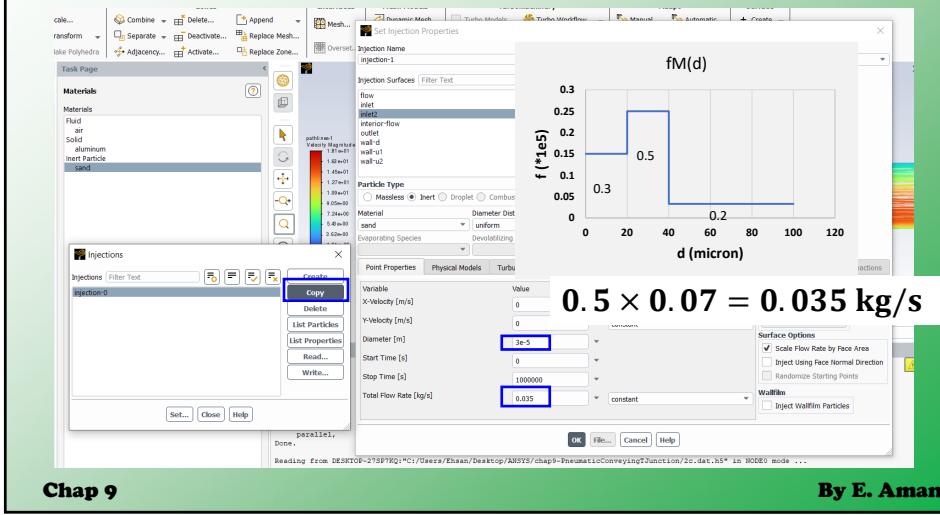
- Materials > Inert particle



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

- Models > Discrete phase > Injections



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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

- Run (10 iterations)
- Save case and data: 3c

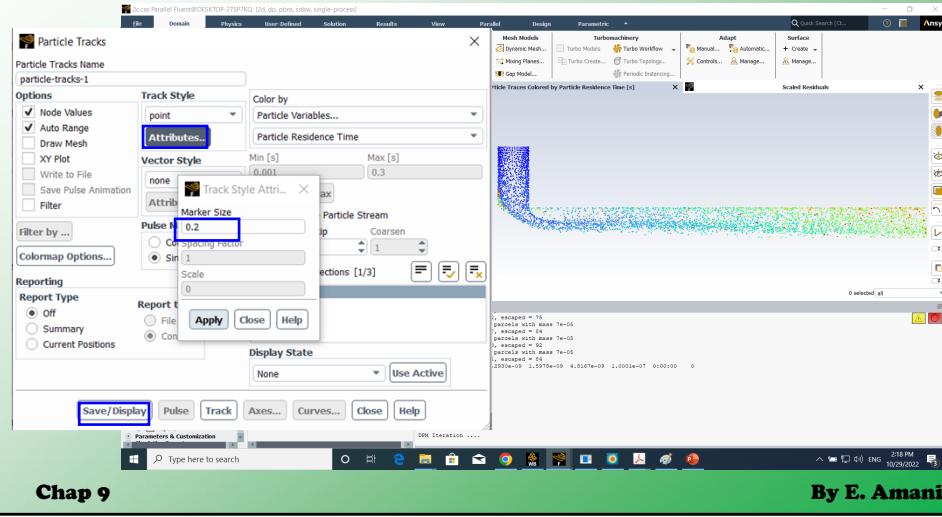
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

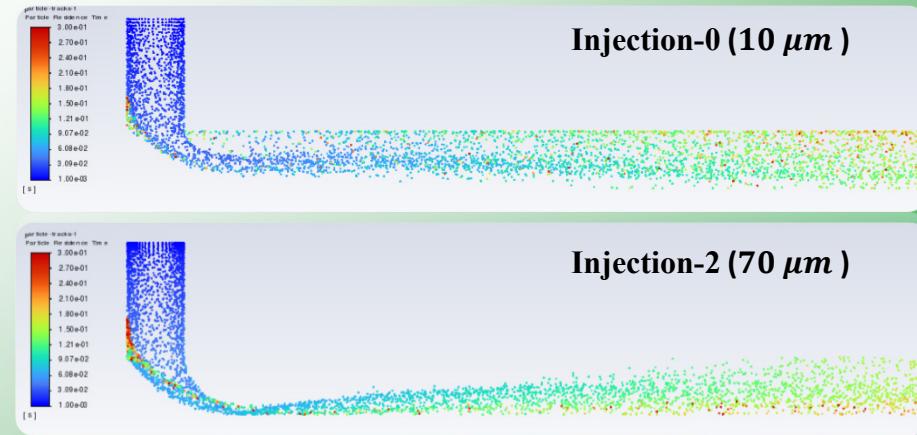
- Results > Particle tracks



Case study #3: Conveying T-junction

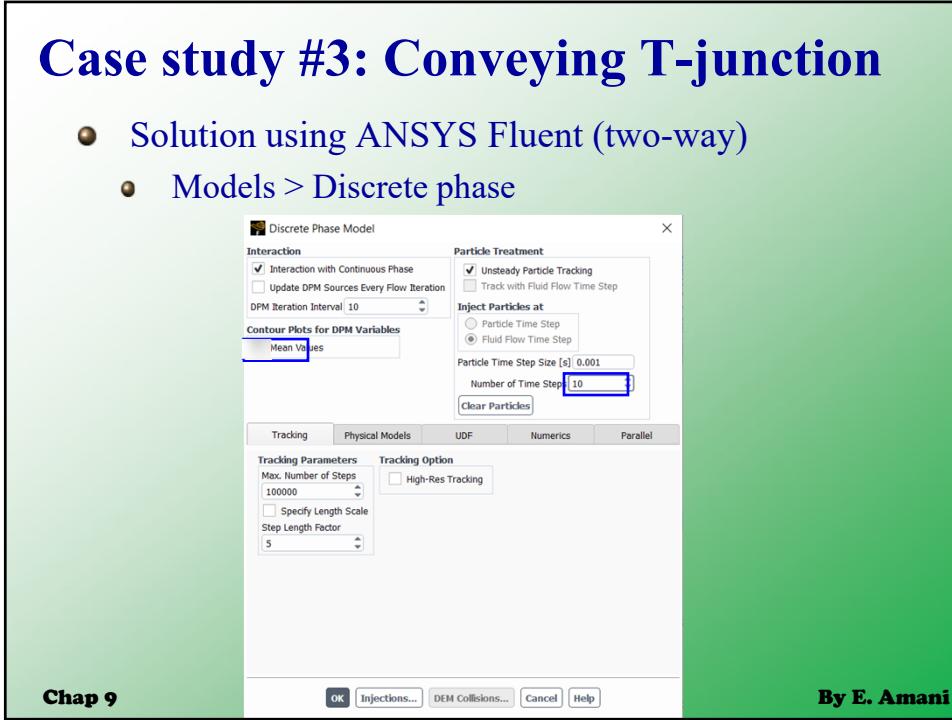
- Solution using ANSYS Fluent (one-way)

- Results > Particle tracks



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
 - Models > Discrete phase



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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
 - Run (5000 iterations)
 - Save case and data: 5c

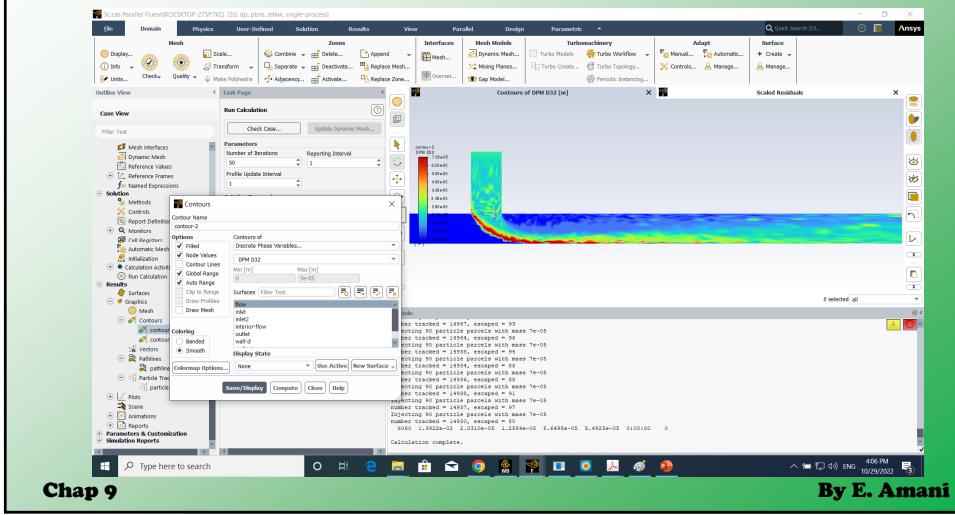
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)

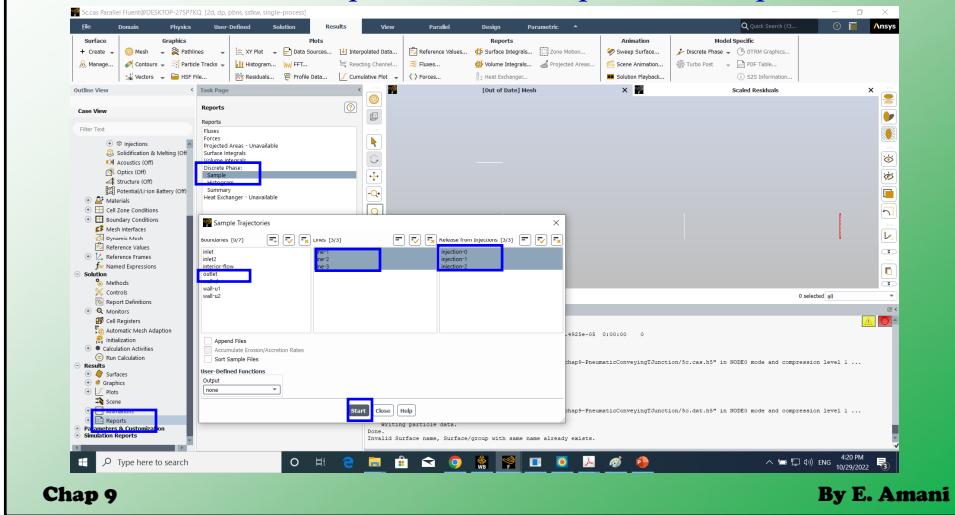
- Results > Graphics > Contours



Case study #3: Conveying T-junction

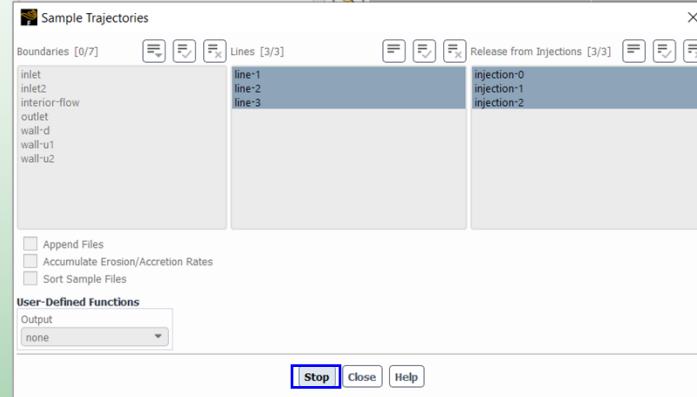
- Solution using ANSYS Fluent (two-way)

- Results > Reports > Discrete phase > Sample



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
 - Run (?? iterations)
 - Save case and data: 6c



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Case study #3: Conveying T-junction

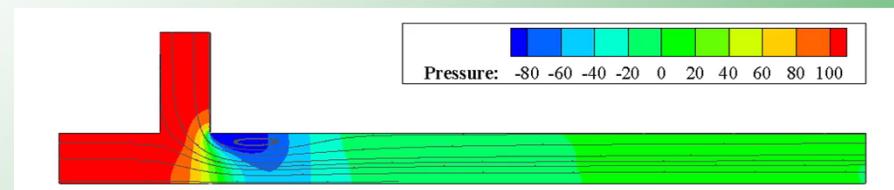
- Solution using ANSYS Fluent (two-way)
 - Run (?? iterations)
 - Save case and data: 6c
 - See files: line-1.dpm, line-2.dpm, line-3.dpm, and outlet.dpm

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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent
- Making animations using Tecplot



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Other topics

- Simplified relation between FPI force in ESS and ELSS \Rightarrow [Notes:Chap9.1.1.10](#)
- See “[FPISummary.pdf](#)” for a summary of FPI force formulation.

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