ELSS – Eulerian continuous-phase equations

Summary:
Transport equations, Eqs. (8.8)-(10.8)
+
The volume fraction, Eq. (11.8)
+
closures for interface transfer (two-way coupling) source terms, $\langle S_m^{(I)} \rangle$, $\langle S_{U_i}^{(I)} \rangle$, $\langle U_{I,i} S_m^{(I)} \rangle$ etc. satisfying averaged jump condition constraints ⟨Eqs. (1.8b)⟩
+
Closures for Reynolds stresses and fluxes, $\widetilde{u_i'' u_j''}$ and $\widetilde{u_j'' Q''}$

Comments:
- Ensemble averaging Eqs. (4.8), dropping the index k assuming one continuous phase, for the derivation see references [1, 2]
- Fluctuation ($Q'' \equiv Q - \tilde{Q}$)

The continuous phase continuity:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{U}_j\right) = \langle S_m^{(I)} \rangle \tag{8.8}$$

The continuous phase momentum:

$$\frac{\partial}{\partial t}\left(\bar{\rho}\tilde{U}_i\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{U}_j\tilde{U}_i\right) = -\frac{\partial \bar{p}}{\partial x_j} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \bar{\rho} g_i - \frac{\partial}{\partial x_j}\left(\bar{\rho}\widetilde{u_i'' u_j''}\right) + \langle S_{U_i}^{(I)} \rangle + \langle U_{I,i} S_m^{(I)} \rangle \tag{9.8}$$

The continuous phase scalars:

$$\frac{\partial}{\partial t}\left(\bar{\rho}\tilde{Q}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{U}_j\tilde{Q}\right) = -\frac{\partial \bar{J}_{Q,j}}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\bar{\rho}\widetilde{u_j'' Q''}\right) + \bar{\rho}\tilde{S}_Q + \langle S_Q^{(I)} \rangle + \langle Q_I S_m^{(I)} \rangle \tag{10.8}$$

Volume fraction by sampling particles:

$$\alpha_c = 1 - \alpha_d \tag{11.8}$$

References

1. Michaelides, E., C.T. Crowe, and J.D. Schwarzkopf, *Multiphase flow handbook*. 2016: CRC Press.
2. Naud, B., *PDF modeling of turbulent sprays and flames using a particle stochastic approach.* 2003.