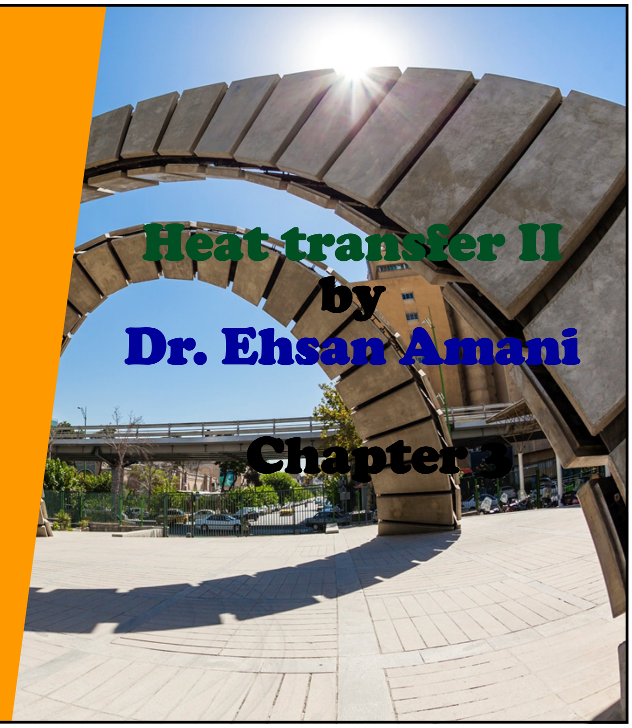
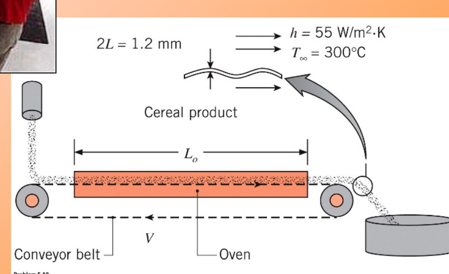


Heat transfer II
by
Dr. Ehsan Amani

Chapter 3

Transient heat conduction: Applications

- **Food industry**



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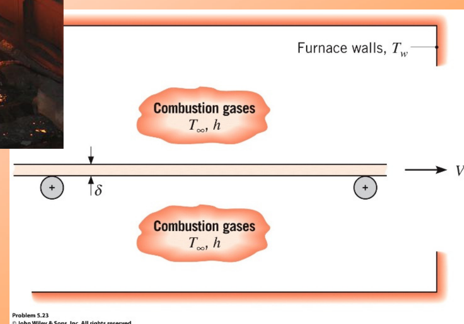
Transient heat conduction: Applications

- **Annealing**



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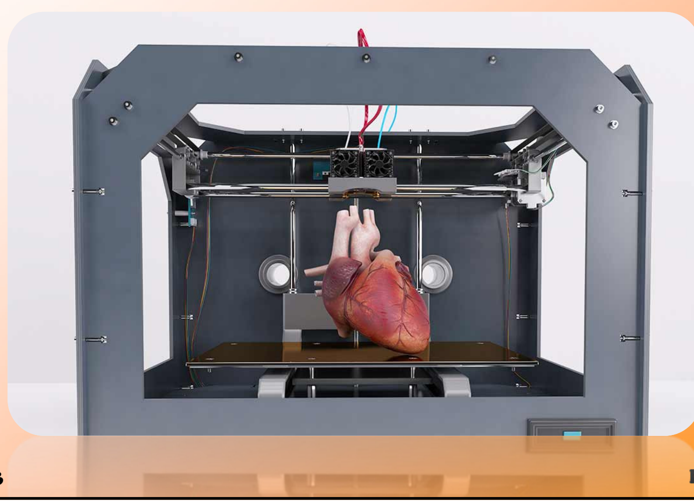
Transient heat conduction: Applications

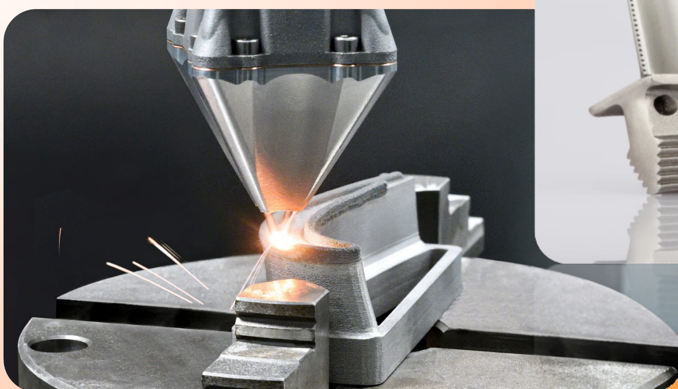
- **3D printing**
  - **Tissue engineering**



Chapter 3 **By E. Amani**

Transient heat conduction: Applications

- 3D printing
  - Tissue engineering
  - 3D metal printing



Chapter 3 By E. Amani

Transient heat conduction: Applications

- 3D printing
  - Tissue engineering
  - 3D metal printing

Chapter 3 By E. Amani

Lumped capacitance method

- **Spatially-uniform (homogeneous) temperature**
- **Criterion** $\boxed{Bi < 0.1}$

Biot number **Characteristic length**

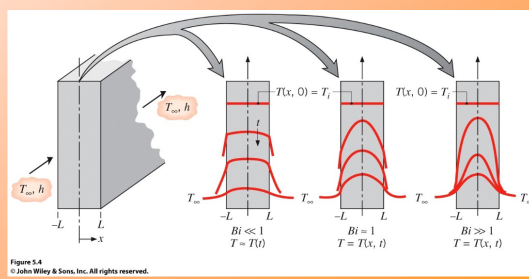
$$Bi \equiv \frac{hL_c}{k} \propto \frac{R_{\text{int}}}{R_{\text{ext}}} = \frac{R_{\text{cond(int)}}}{R_{\text{conv}}} \qquad (1.3)$$

- **For the lumped capacitance method**

Body volume **Convective surface area**

$$L_c \equiv \frac{V}{A_s} \qquad (2.3)$$

$$\text{e.g., } L_c \equiv \frac{2LA}{2A} = L$$



Chapter 3 **By E. Amani**

Lumped capacitance method

- **From Heat Transfer I**

$$\theta \equiv T - T_\infty$$

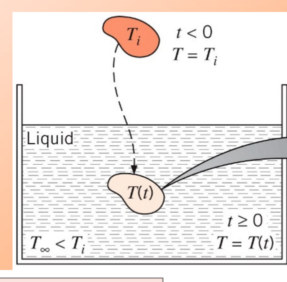
$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{t}{\tau_t}\right] = \exp\left[-\text{Bi.Fo}\right] \qquad (3.3)$$

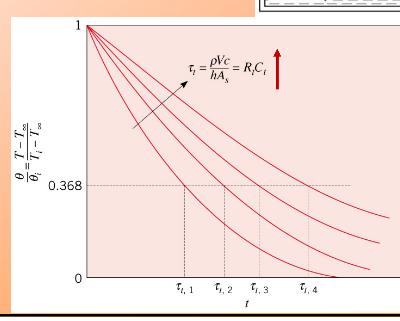
$$\tau_t \equiv \underbrace{\left(\frac{1}{hA_s}\right)}_{R_{\text{conv}}}\underbrace{(\rho\forall c)}_{\text{Lumped Thermal Capacitance, } C_t} = R_{\text{conv}} C_t \qquad (1.3)$$

Thermal response time

$$\text{Fo} \equiv \frac{\alpha t}{L_c^2} \qquad (8.3)$$

Fourier number





Chapter 3 **By E. Amani**

Lumped capacitance method

- **Dimensionless heat rate**

Heat rate to the body

$$q^* \equiv \frac{\overbrace{(q/A_s)}^{q_s''} L_c}{k\underbrace{\Delta T}_{\equiv T_s - T_i}} = \frac{q_s'' L_c}{k(T_s - T_i)} \qquad (5.3)$$

Dimensionless heat rate $(-)$

Total heat to the body from *0* to *t*

$$Q = \Delta E_s = \int_0^t q\,dt$$

Dimensionless heat

$$\frac{Q}{Q_0} \equiv \frac{Q}{\rho c \forall (T_i - T_\infty)} \qquad (7.3)$$

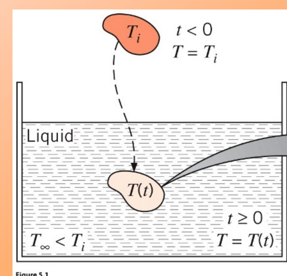


Figure 5.1

Chapter 3 By E. Amani

Lumped capacitance method

- **From Heat Transfer I**

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{t}{\tau_t}\right] = \exp\left[-\mathrm{Bi.Fo}\right] \qquad (3.3)$$

  - **Therefore, for the lumped capacitance method**

$$q^* = \frac{q_s'' L_c}{k(T_s - T_i)} = \frac{h\overbrace{(T_\infty - T)}^{-\theta} L_c}{k\underbrace{(T - T_i)}_{T - T_\infty - (T_i - T_\infty)}}$$

$$q^* = \mathrm{Bi}\frac{\theta/\theta_i}{1 - \theta/\theta_i} \qquad (6.3)$$

$$\frac{Q}{Q_0} = 1 - \exp(-\mathrm{Bi.Fo}) \qquad (7.3)'$$

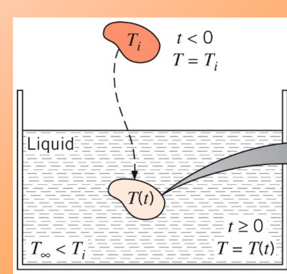
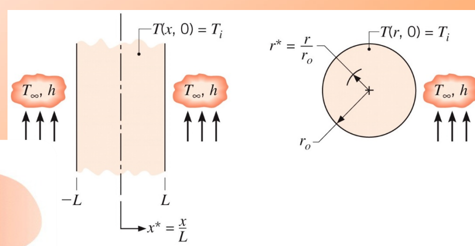


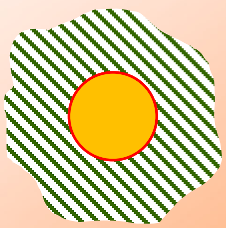
Figure 5.1

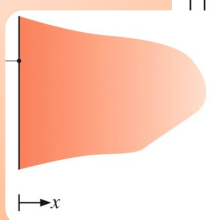
Chapter 3 By E. Amani

Spatial effects

- **How about Bi > 0.1?**
- **Analytical solutions and preliminary design tables are available for**
  - **Three kinds of geometries:**
    1. **Interior cases**
    2. **Semi-infinite solid**
    3. **Exterior cases**



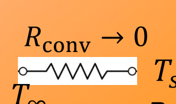




Chapter 3 **By E. Amani**

Spatial effects

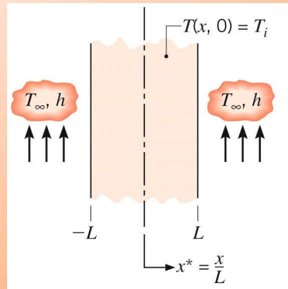
- **How about Bi > 0.1?**
- **Analytical solutions and preliminary design tables are available for**
  - **Three kinds of geometries:**
  - **Three kinds of boundary conditions:**
    1. **Constant temperature** ($T_s = cte$)
    2. **Constant heat flux** ($q_s'' = cte$)
    3. **Convective** (h, T_∞)
  - **Other boundary conditions may be described, e.g., equivalent convection, or approximated by available solutions**
  - $T_s = cte$ **is special case of** (h, T_∞) **when** $(h \to \infty, T_\infty = T_s)$



Chapter 3 **By E. Amani**

Interior cases – convective BC

- **Non-dimensionalization**
  - **1D conduction in a plane wall**
  - $k = cte$
  - $\dot{q} = 0$



$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$T(x,0) = T_i \qquad (10.3)$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad -k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$$

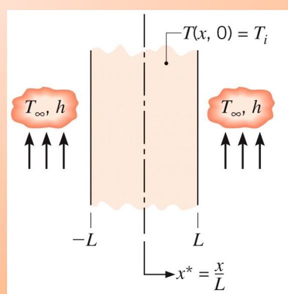
  - **Therefore,**

$$T = T(x,t,T_i,T_\infty,L,k,\alpha,h)$$

Chapter 3 By E. Amani

Interior cases – convective BC

- **Non-dimensionalization**
  - **With homogenization**



$$\theta \equiv T - T_\infty$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

$$\theta(x,0) = \theta_i = T_i - T_\infty \qquad (11.3)$$

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = 0, \quad -k\left.\frac{\partial \theta}{\partial x}\right|_{x=L} = h\theta(L,t)$$

  - **Therefore,**

$$\theta = \theta(x,t,L,k,\alpha,h,\theta_i) \qquad (11.3)'$$

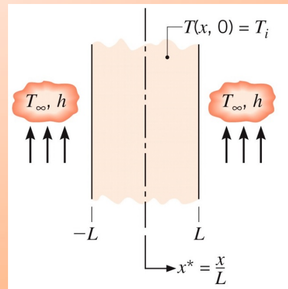
Lecture Notes: III.2.1

Chapter 3 By E. Amani

Interior cases – convective BC

- **Non-dimensionalization** (13.3)

$$x^* = \frac{x}{L_c} = \frac{x}{L}, \theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}, t^* = \frac{t}{L_c^2/\alpha} = \mathrm{Fo}$$



$$\frac{\partial^2\theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial t^*}$$

$$\theta^*(x^*,0) = 1 \qquad (14.3)$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0, \quad \left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -\mathrm{Bi}\,\theta^*(1,t^*)$$

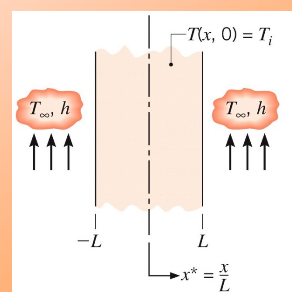
  - **Therefore,**

$$\theta^* = \theta^*(x^*, t^*, \mathrm{Bi}) \qquad (14.3)'$$

  - **Exercise: Show that with non-dimensionalizing the parametric relation Eq. (11.3)′, the same conclusion as Eq. (14.3)′ can be retrieved.**

Interior cases – convective BC

- **Analytical solution**
  - **Eqs. (14.3) are linear and homogeneous; Their exact analytical solution can be obtained by the separation of variables method [2].**
  - **Exact solution:**



$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo)\cos(\zeta_n x^*) \qquad \text{(5.42 a) [1]}$$

$$C_n = \frac{4\sin\zeta_n}{2\zeta_n + \sin(2\zeta_n)} \qquad \text{(5.42 b) [1]}$$

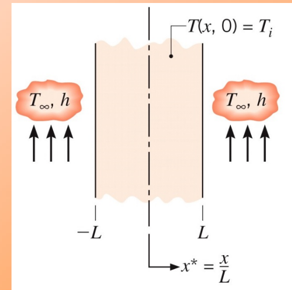
$$\zeta_n \tan\zeta_n = Bi \qquad \text{(5.42 c) [1]} + \text{Appendix B.3}$$

B.3 *The First Four Roots of the Transcendental Equation, $\xi_n \tan \xi_n = Bi$, for Transient Conduction in a Plane Wall*

$Bi = \frac{hL}{k}$	ξ_1	ξ_2	ξ_3	ξ_4
0	0	3.1416	6.2832	9.4248
0.001	0.0316	3.1419	6.2833	9.4249
0.002	0.0447	3.1422	6.2835	9.4250
0.004	0.0632	3.1429	6.2838	9.4252

Interior cases – convective BC

- **Analytical solution**
  - **Eqs. (14.3) are linear and homogeneous; Their exact analytical solution can be obtained by the separation of variables method [2].**
  - **Exact solution:**
  - **Approximate solution ($\mathbf{Fo \geq 0.2}$):**



$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right)\cos\left(\zeta_1 x^*\right) \qquad (5.43\text{ a})\ [1]$$

$$\theta^* = \theta_o^* \cos\left(\zeta_1 x^*\right) \qquad (5.43\text{ b})\ [1]$$

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right) \quad (5.43\text{ c})\ [1]$$ + Table 5.1

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090

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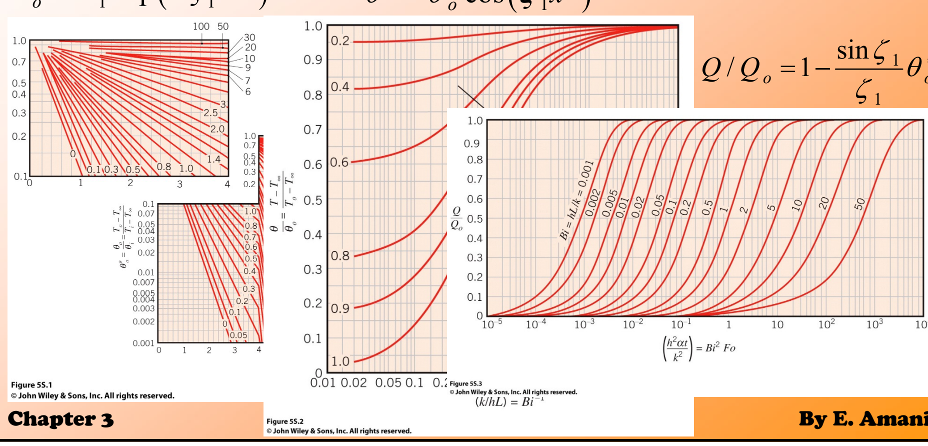
Interior cases – convective BC

- **Analytical solution**
  - **Graphs for the approximate solution: The Heisler Charts (Figures 5S.1-5S.9)**

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right) \qquad \theta^* = \theta_o^* \cos\left(\zeta_1 x^*\right)$$

$$Q/Q_o = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^*$$



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Chapter 3 By E. Amani

Semi-infinite solids

- **Analytical solution**
  - **With the similarity solution method [1,2]**
  - **See Appendix A for case (1)**

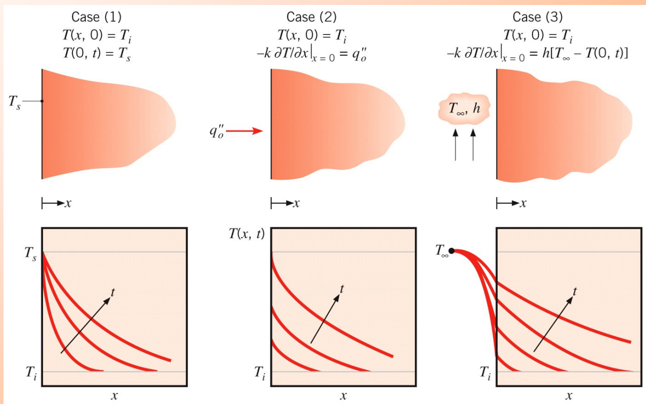

Figure 5.7

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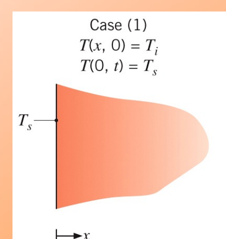
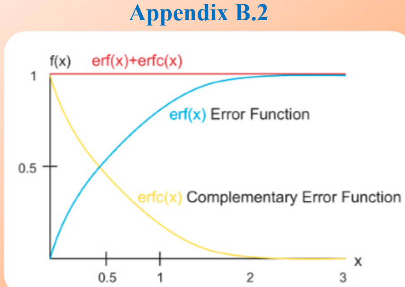
Semi-infinite solids

- **Analytical solution**
  - **With the similarity solution method [1,2]**
  - **See Appendix A for case (1)**



$$\frac{T(x,t)-T_s}{T_i-T_s} = \underbrace{\left(\frac{2}{\pi^{1/2}}\right)\int_0^{\eta=\frac{x}{2\sqrt{\alpha t}}} \exp(-u^2)\,du}_{\text{Gaussian error function: } \mathrm{erf}(\eta)}$$

Appendix B.2



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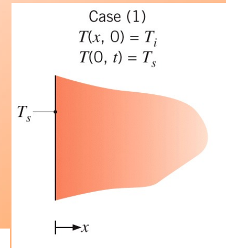
Semi-infinite solids

- **Analytical solution**
  - **With the similarity solution method [1,2]**
  - **See Appendix A for case (1)**



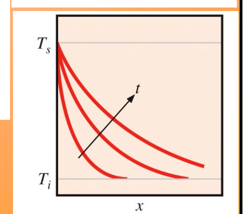
$$\frac{T(x,t)-T_s}{T_i-T_s}=\underbrace{\left(\frac{2}{\pi^{1/2}}\right)\int_0^{\eta=\frac{x}{2\sqrt{\alpha t}}}\exp\left(-u^2\right)du}_{\text{Gaussian error function: erf}(\eta)}$$

Gaussian error function: erf(η)

Appendix B.2

B.2 *Gaussian Error Function*[1]

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679



$$\frac{T(x,t)-T_s}{T_i-T_s}=\text{erf}\,\eta=\text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad (20.3)$$

Chapter 3 **By E. Amani**

Semi-infinite solids

- **Analytical solution**
  - **With the similarity solution method [1,2]**
  - **See Appendix A for case (1)**

$$\frac{T(x,t)-T_s}{T_i-T_s}=\text{erf}\,\eta=\text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad (20.3)$$

  - **When no solution is available for interior or exterior cases and if Fo ≤ 0.2, the semi-infinite solution can be used as an acceptable approximation.**

Chapter 3 **By E. Amani**

Summary: The solution algorithm

1. **Calculate Bi (based on lumped capacitance).**
2. **If $\mathrm{Bi} \leq 0.1$, use the lumped capacitance method, otherwise consider the spatial effects based on the algorithm given in "chap3-summary.pdf":**

Boundary	geometry	Temperature distribution	q_s'' or q^*	Q/Q_0
$T_s = cte$	Semi-infinite	(5.60)	Table 5.2a	← Integrate
	interior	Fo < 0.2: †, Fo ≥ 0.2: Table 5.2c		Fo < 0.2: ← Integrate, Fo ≥ 0.2: Table 5.2c
	exterior	Fo < 0.2: †, Fo ≥ 0.2: ?		← Integrate
$q_s'' = cte$	Semi-infinite	(5.62)	Table 5.2b	$q_s'' t/Q_0$
	interior	Fo < 0.2: †, Fo ≥ 0.2: ?		
	exterior	Fo < 0.2: †, Fo ≥ 0.2: ?		
$\boxed{h, T_\infty}$	Semi-infinite	Table 5.2c	Table 5.2c	Table 5.2c
	$\boxed{\text{interior}}$	Fo < 0.2: †, Fo ≥ 0.2: Table 5.2c	Fo < 0.2: †, Fo ≥ 0.2: Table 5.2c	Fo < 0.2: ← Integrate, Fo ≥ 0.2: Table 5.2c
	exterior	Fo < 0.2: †, Fo ≥ 0.2: ?	Fo < 0.2: †, Fo ≥ 0.2: ?	Fo < 0.2: ← Integrate, Fo ≥ 0.2: ?

† indicates approximation with the semi-infinite solution for Fo < 0.2 where no approximate correlation is available.

Chapter 3 **By E. Amani**

Summary: The solution algorithm

Table 5.2c: Summary of transient heat transfer results for convective boundary condition. Note: for constant surface temperature cases, set $(h \to \infty, T_\infty = T_s, Bi \to \infty)$.

	L_c	Temperature distribution		q_s'' or q^*		Q/Q_0		Note
Geometry		**Exact solution**	**Approximation**	**Exact solution**	**Approximation**	**Exact solution**	**Approximation**	
Semi-infinite	L (arbitrary)	(5.63) Or Figure 5.8	Use exact solution	$h(T_\infty - T(0,t))$	Use exact solution	← Integrate	Use exact solution	
interior								
Plane wall of thickness $2L$	L	(5.42) $\zeta_n = ?$	(Fo ≥ 0.2) (5.43) + Table 5.1 Or Figure 5S.1 and 5S.2	← Differentiate	← Differentiate	← Integrate	(Fo ≥ 0.2) (5.49) Or Figure 5S.3	
Infinite cylinder	r_o	(5.50) $\zeta_n = ?$	(Fo ≥ 0.2) (5.52) + Table 5.1 Or Figure 5S.4 and 5S.5	← Differentiate	← Differentiate	← Integrate	(Fo ≥ 0.2) (5.54) Or Figure 5S.6	
sphere	r_o	(5.51) $\zeta_n = ?$	(Fo ≥ 0.2) (5.53) + Table 5.1 Or Figure 5S.7 and 5S.8	← Differentiate	← Differentiate	← Integrate	(Fo ≥ 0.2) (5.55) Or Figure 5S.9	

$q^* = q_s'' L_c/(k(T_s - T_i))$, $\mathrm{Fo} = \alpha t/L_c^2$, $\mathrm{Bi} = hL_c/k$ and where L_c is the length scale given in the table, T_s is the object surface temperature, and T_i is (a) the initial object temperature for the interior cases and (b) the temperature of the infinite medium for the exterior cases.

Chapter 3 **By E. Amani**

Summary: The solution algorithm

TABLE 5.2a Summary of transient heat transfer results for constant surface temperature cases[a] [8]

Geometry	Length Scale, L_c	$q^*(Fo)$ Exact Solutions	Approximate Solutions $Fo < 0.2$	Approximate Solutions $Fo \geq 0.2$	Maximum Error (%)
Semi-infinite	L (arbitrary)	$\frac{1}{\sqrt{\pi Fo}}$	Use exact solution.		None
Interior Cases					
Plane wall of thickness $2L$	L	$2\sum_{n=1}^{\infty}\exp(-\zeta_n^2 Fo)$ $\zeta_n = (n - \frac{1}{2})\pi$	$\frac{1}{\sqrt{\pi Fo}}$	$2\exp(-\zeta_1^2 Fo)$ $\zeta_1 = \pi/2$	1.7
Infinite cylinder	r_o	$2\sum_{n=1}^{\infty}\exp(-\zeta_n^2 Fo)$ $J_0(\zeta_n) = 0$	$\frac{1}{\sqrt{\pi Fo}} - 0.50 - 0.65\,Fo$	$2\exp(-\zeta_1^2 Fo)$ $\zeta_1 = 2.4050$	0.8

[a] $q^* \equiv q_s'' L_c / k(T_s - T_i)$ and $Fo \equiv \alpha t / L_c^2$, where L_c is the length scale given in the table, T_s is the object surface temperature, and T_i is (a) the initial object temperature for the interior cases and (b) the temperature of the infinite medium for the exterior cases.

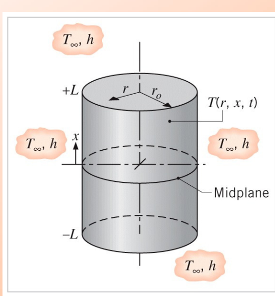
TABLE 5.2b Summary of transient heat transfer results for constant surface heat flux cases[a] [8]

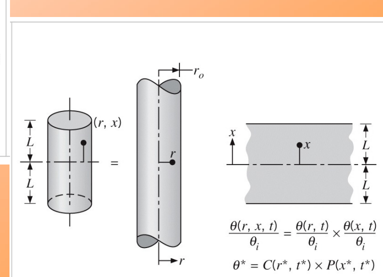
Geometry	Length Scale, L_c	$q^*(Fo)$ Exact Solutions	Approximate Solutions $Fo < 0.2$	Approximate Solutions $Fo \geq 0.2$	Maximum Error (%)
Semi-infinite	L (arbitrary)	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}}$	Use exact solution.		None
Interior Cases					
Plane wall of thickness $2L$	L	$\left[Fo + \frac{1}{3} - 2\sum_{n=1}^{\infty}\frac{\exp(-\zeta_n^2 Fo)}{\zeta_n^2}\right]^{-1}$ $\zeta_n = n\pi$	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}}$	$\left[Fo + \frac{1}{3}\right]^{-1}$	5.3
Infinite cylinder	r [illegible]	$\left[2Fo + \frac{1}{[illegible]} - 2\sum^{\infty}\frac{\exp(-\zeta_n^2 Fo)}{[illegible]}\right]^{-1}$ $J_1(\zeta_n) = 0$	$\frac{1}{[illegible]}\sqrt{\frac{\pi}{[illegible]}} - \frac{\pi}{[illegible]}$	$\left[2Fo + \frac{1}{[illegible]}\right]^{-1}$	2.1

Chapter 3 By E. Amani

Multi-dimensional effects

- **When the equations are linear and homogeneous**
- **When the equations can be decomposed into 1D problems**
  - **Example:**





$$\frac{T(r,x,t) - T_\infty}{T_i - T_\infty} = P(x,t) \times C(r,t)$$

$$= \left.\frac{T(x,t) - T_\infty}{T_i - T_\infty}\right|_{\text{Plane Wall}} \times \left.\frac{T(r,t) - T_\infty}{T_i - T_\infty}\right|_{\text{Infinite Cylinder}}$$

Chapter 3 By E. Amani

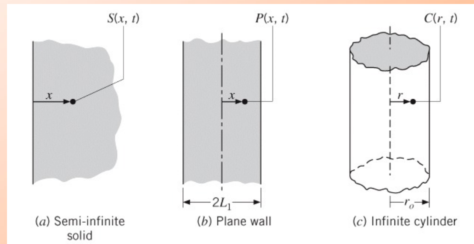
Multi-dimensional effects

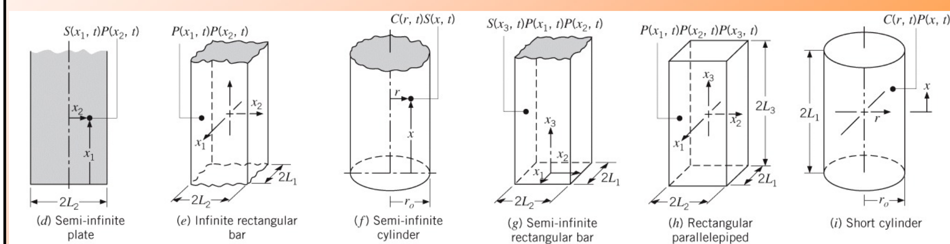
- **Examples (h, T_∞)**

$$S(x,t) = \left.\frac{T(x,t)-T_\infty}{T_i - T_\infty}\right|_{\text{Semi-infinite solid}}$$

$$P(x,t) = \left.\frac{T(x,t)-T_\infty}{T_i - T_\infty}\right|_{\text{Plane Wall}}$$

$$C(r,t) = \left.\frac{T(r,t)-T_\infty}{T_i - T_\infty}\right|_{\text{Infinite Cylinder}}$$





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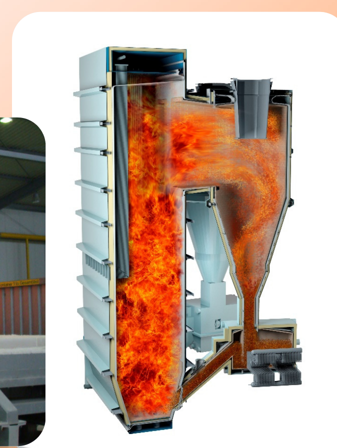
Sample problem

Furnace start-up



Chapter 3 **By E. Amani**

Sample problem

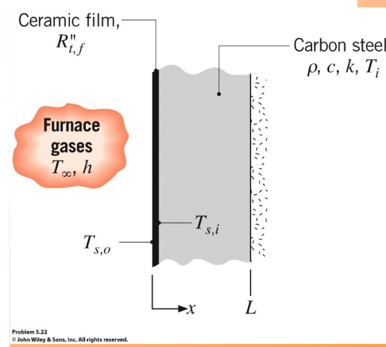
Furnace start-up

5.22 A plane wall of a furnace is fabricated from plain carbon steel ($k = 60\ \mathrm{W/m \cdot K}$, $\rho = 7850\ \mathrm{kg/m^3}$, $c = 430\ \mathrm{J/kg \cdot K}$) and is of thickness $L = 10\ \mathrm{mm}$. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01\ \mathrm{m^2 \cdot K/W}$. The opposite surface is well insulated from the surroundings.

A surface heat flux of $q''_s = 1000\ W/m^2$ is absorbed by the outer surface of the film due to the radiation.

a) At furnace start-up the wall is at an initial temperature of $T_i = 300\ \mathrm{K}$, and combustion gases at $T_\infty = 1300\ \mathrm{K}$ enter the furnace, providing a convection coefficient of $h = 25\ \mathrm{W/m^2 \cdot K}$ at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200\ \mathrm{K}$? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

(b) Solve part (a) assuming that $h = 250\ W/m^2.K$ and $L = 200\ mm$.



Lecture Notes: III.5.2.1

cases,

	L_c	Temperature distribution		q_s^* or q^*		Q/Q_0		Note
Geometry		Exact solution	Approximation	Exact solution	Approximation	Exact solution	Approximation	
Semi-infinite	L (arbitrary)	(5.63) Or Figure 5.8	Use exact solution	$h(T_\infty - T(0,t))$	Use exact solution	← Integrate	Use exact solution	
interior								
Plane wall of thickness $2L$	L	(5.42) $\zeta_n = ?$	(Fo ≥ 0.2) (5.43) + Table 5.1 Or Figure 5S.1 and 5S.2	← Differentiate	← Differentiate	← Integrate	(Fo ≥ 0.2) (5.49) Or Figure 5S.3	
Infinite cylinder	r_o	(5.50) $\zeta_n = ?$	(Fo ≥ 0.2) (5.52) + Table 5.1 Or Figure 5S.4 and 5S.5	← Differentiate	← Differentiate	← Integrate	(Fo ≥ 0.2) (5.54) Or Figure 5S.6	
sphere	r_o	(5.51) $\zeta_n = ?$	(Fo ≥ 0.2) (5.53) + Table 5.1 Or Figure 5S.7 and 5S.8	← Differentiate	← Differentiate	← Integrate	(Fo ≥ 0.2) (5.55) Or Figure 5S.9	

$q^* = q_s^* L_c/(k(T_s - T_i))$, $\mathrm{Fo} = \alpha t/L_c^2$, $\mathrm{Bi} = hL_c/k$ and where L_c is the length scale given in the table, T_s is the object surface temperature, and T_i is (a) the initial object temperature for the interior cases and (b) the temperature of the infinite medium for the exterior cases.

$\boldsymbol{h, T_\infty}$	**Semi-infinite**	Table 5.2c	Table 5.2c	Table 5.2c
	interior	Fo < 0.2: †, Fo ≥ 0.2: Table 5.2c	Fo < 0.2: †, Fo ≥ 0.2: Table 5.2c	Fo < 0.2: ← Integrate, Fo ≥ 0.2: Table 5.2c
	exterior	Fo < 0.2: †, Fo ≥ 0.2: ?	Fo < 0.2: †, Fo ≥ 0.2: ?	Fo < 0.2: ← Integrate, Fo ≥ 0.2: ?

† indicates approximation with the semi-infinite solution for Fo < 0.2 where no approximate correlation is available.

Summary: The solution algorithm

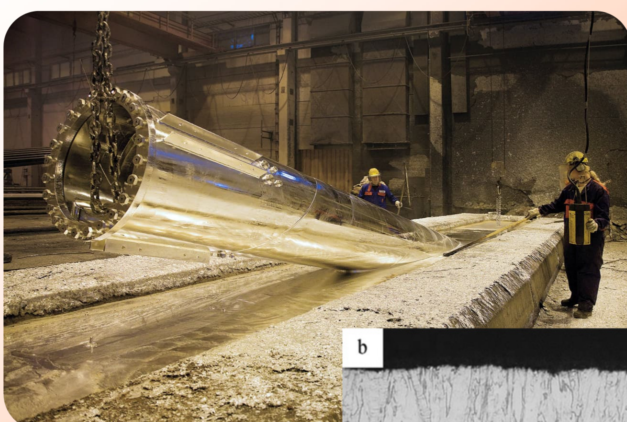
$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right)\cos\left(\zeta_1 x^*\right) \qquad (5.43\text{ a})\ [1]$$

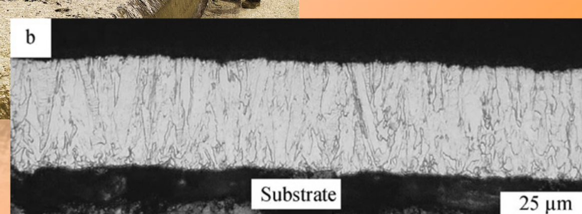
TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090

Sample problem

Solidification: Coating





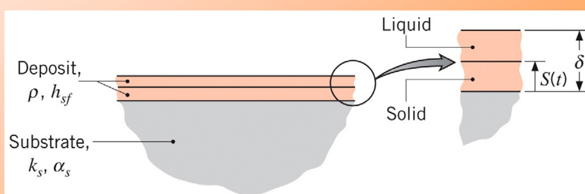
Sample problem

Solidification: Coating

Lecture Notes: III.5.2.2

5.96 Special coatings are often formed by depositing thin layers of a molten material on a solid substrate. Solidification begins at the substrate surface and proceeds until the thickness S of the solid layer becomes equal to the thickness δ of the deposit.

Consider conditions for which molten material at its fusion temperature T_f is deposited on a *large* substrate that is at an initial uniform temperature T_i. With $S = 0$ at $t = 0$, develop an expression for estimating the time t_d required to completely solidify the deposit if it remains at T_f throughout the solidification process. Express your result in terms of the substrate thermal conductivity and thermal diffusivity (k_s, α_s), the density and latent heat of fusion of the deposit (ρ, h_{sf}), the deposit thickness δ, and the relevant temperatures (T_f, T_i).



Summary: The solution algorithm

TABLE 5.2a Summary of transient heat transfer results for constant surface temperature cases[a] [8]

Geometry	Length Scale, L_c	$q^*(Fo)$ Exact Solutions	Approximate Solutions $Fo < 0.2$	Approximate Solutions $Fo \geq 0.2$	Maximum Error (%)
Semi-infinite	L (arbitrary)	$\frac{1}{\sqrt{\pi Fo}}$	Use exact solution.		None
Interior Cases					
Plane wall of thickness $2L$	L	$2\sum_{n=1}^{\infty}\exp(-\zeta_n^2 Fo)$ $\zeta_n = (n - \frac{1}{2})\pi$	$\frac{1}{\sqrt{\pi Fo}}$	$2\exp(-\zeta_1^2 Fo)$ $\zeta_1 = \pi/2$	1.7
Infinite cylinder	r_o	$2\sum_{n=1}^{\infty}\exp(-\zeta_n^2 Fo)$ $J_0(\zeta_n) = 0$	$\frac{1}{\sqrt{\pi Fo}} - 0.50 - 0.65\,Fo$	$2\exp(-\zeta_1^2 Fo)$ $\zeta_1 = 2.4050$	0.8

[a] $q^* \equiv q_s'' L_c / k(T_s - T_i)$ and $Fo \equiv \alpha t / L_c^2$, where L_c is the length scale given in the table, T_s is the object surface temperature, and T_i is (a) the initial object temperature for the interior cases and (b) the temperature of the infinite medium for the exterior cases.

Boundary	geometry	Temperature distribution	q_s'' or q^*	Q/Q_0
$T_s = cte$	**Semi-infinite**	(5.60)	Table 5.2a	← Integrate
	interior	Fo < 0.2: †, Fo ≥ 0.2: Table 5.2c		Fo < 0.2: ← Integrate, Fo ≥ 0.2: Table 5.2c
	exterior	Fo < 0.2: †, Fo ≥ 0.2: ?		← Integrate
$q_s'' = cte$	**Semi-infinite**	(5.62)	Table 5.2b	$q_s'' t/Q_0$
	interior	Fo < 0.2: †, Fo ≥ 0.2: ?		
	exterior	Fo < 0.2: †, Fo ≥ 0.2: ?		
	Semi-infinite	Table 5.2c	Table 5.2c	Table 5.2c

The end of chapter 3