Fluid-particle momentum interaction summary

<table>
<tr><th>Interaction</th><th>Formulation</th></tr>
<tr><td>Drag Force [1]</td><td>

$$\boldsymbol{F}_\mathrm{D} = m_p \frac{\boldsymbol{u}_r}{\tau_p}, \tau_p = \tau_v = \frac{\tau_{v,\mathrm{St}}}{f_D}, \tau_{v,\mathrm{St}} = \frac{\rho_p d_p^2}{18\rho_f \upsilon_f}, \quad f_D = \begin{cases} 1 & Re_p \le 0.1 \\ 1 + \frac{1}{6} Re_p^{\frac{2}{3}} & 0.1 < Re_p \le 1000 \end{cases}$$

$$\boldsymbol{u}_r = \boldsymbol{u}_s - \boldsymbol{u}_p = \boldsymbol{u}_f - \boldsymbol{u}_p, \qquad Re_p = \frac{|\boldsymbol{u}_r| d_p}{\upsilon_f}, \; s \leftrightarrow f \leftrightarrow c$$

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<tr><td>Drag Torque [2]</td><td>

$$\boldsymbol{T}_\mathrm{D} = \begin{cases} -\pi \rho_f \upsilon_f d_p^3 \boldsymbol{\omega}_r & ; Re_\omega \le 1 \\ -\frac{C_R}{64} \rho_f d_p^5 |\boldsymbol{\omega}_r| \boldsymbol{\omega}_r & ; Re_\omega > 1 \end{cases}, \; C_R = \left(\frac{C_1}{Re_\omega^{0.5}} + \frac{C_2}{Re_\omega}\right), \boldsymbol{\omega}_r = \boldsymbol{\omega}_p - \boldsymbol{\omega}_f, \; \boldsymbol{\omega}_f = 0.5\, \boldsymbol{\nabla} \times \boldsymbol{u}_f$$

$$Re_\omega = \frac{|\boldsymbol{\omega}_r| d_p^2}{\upsilon_f} \; ; \quad C_1 \text{ and } C_2 \; \rightarrow$$

Dennis et al.(1980)

Re_ω	C_1	C_2
$Re_\omega < 32$	0	64π
$32 < Re_\omega < 1000$	12.9	128.4

Takagi

Re_ω	C_1	C_2
$10 < Re_\omega < 20$	5.32	37.2
$20 < Re_\omega < 50$	6.44	32.2
$50 < Re_\omega < 100$	6.45	32.1

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<tr><td>Virtual Mass Force [3]</td><td>

$$\boldsymbol{F}_\mathrm{vm} = C_{vm} \frac{\rho_f \forall_p}{2} \left(\frac{D\boldsymbol{u}_f}{Dt} - \frac{d\boldsymbol{u}_p}{dt}\right), \qquad C_{vm} = 1$$

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<tr><td>History (Basset) Force [4, 5]</td><td>

$$\boldsymbol{F}_\mathrm{H} = C_B \frac{3}{2} d_p^2 \sqrt{\pi \mu_f \rho_f} \left[\int_0^t \frac{\frac{d\boldsymbol{u}_r}{dt'}}{\sqrt{t - t'}} dt' + \frac{\boldsymbol{u}_{r,0}}{\sqrt{t}}\right], \; C_B = 1 - 0.527 \left[1 - \exp\left(-0.14 Re_p Sl^{0.82}\right)^{2.5}\right],$$

$$Sl = \frac{1}{2\pi f \tau_p}$$

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<tr><td>Saffman Lift force [6]</td><td>

$$\boldsymbol{F}_\mathrm{L} = 1.61 C_s\, \rho_f d_p^2 \left(\frac{2\upsilon_f}{|\boldsymbol{\omega}_f|}\right)^{\frac{1}{2}} \boldsymbol{u}_r \times \boldsymbol{\omega}_f \,,$$

$$C_s = \begin{cases} \left(1 - 0.3314 \lambda_v^{\frac{1}{2}}\right) \exp\left(-\frac{Re_p}{10}\right) + 0.3314 \lambda_v^{\frac{1}{2}} & ; Re_p \le 40 \\ 0.0524 \left(\lambda_v Re_p\right)^{1/2} & ; Re_p > 40 \end{cases}, \quad \lambda_v = d_p \frac{|\boldsymbol{\omega}_f|}{|\boldsymbol{u}_r|}$$

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<tr><td>Magnus Lift force [7, 8]</td><td>

$$\boldsymbol{F}_\mathrm{L} = 0.5 C_{LR}\, \rho_f A_p \frac{|\boldsymbol{u}_r|}{|\boldsymbol{\omega}_r|} (\boldsymbol{u}_r \times \boldsymbol{\omega}_r) \,,$$

$$C_\mathrm{LR} = \begin{cases} \frac{Re_\omega}{Re_p} & ; Re_p \le 1 \\ 0.45 + \left(\frac{Re_\omega}{Re_p} - 0.45\right) \exp\left(-0.05684 Re_\omega^{0.4} Re_p^{0.3}\right) & ; Re_p < 2000 \end{cases}$$

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<tr><td>Brownian Force [9, 10]</td><td>

$$\boldsymbol{F}_\mathrm{B} = m_p \sqrt{\frac{\pi S_0}{\delta t}} |\zeta| \boldsymbol{\eta}, S_0 = \frac{216 \mu_f K_B T_f}{\pi^2 d_p^5 \rho_p^2 C_c}, C_c = 1 + \frac{2\lambda}{d_p} \left[1.257 + 0.4 \exp\left(-\frac{1.1 d_p}{2\lambda}\right)\right]$$

$$\boldsymbol{\eta} = a \cos(\theta)\, \boldsymbol{i} + a \sin(\theta)\, \boldsymbol{j} + u\boldsymbol{k}, \theta = 2\pi X, u = 2Y - 1, a = \sqrt{1 - u^2}\,, X, Y \in U(0,1), \zeta \in N(0,1)$$

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<tr><td>Thermophoretic Force [11]</td><td>

$$\boldsymbol{F}_\mathrm{T} = -D_{T,p} \frac{\boldsymbol{\nabla} T}{T}, D_{T,p} = \frac{6\pi d_p \mu_f^2 C_s \left(\frac{\lambda_f}{\lambda_p} + C_t \frac{2\lambda}{d_p}\right)}{\rho_f \left(1 + 3C_m \frac{2\lambda}{d_p}\right)\left(1 + 2\frac{\lambda_f}{\lambda_p} + 2C_t \frac{2\lambda}{d_p}\right)}, C_c = 1.17, C_t = 2.18, C_m = 1.14$$

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<tr><td>Wall Force [12]</td><td>

$$\boldsymbol{F}_\mathrm{W} = C_w \frac{\pi}{12} \rho_f d_p^4 |\boldsymbol{u}_r|^2 \left(\frac{1}{y_w^2} - \frac{1}{(D - y_w)^2}\right) \boldsymbol{n}$$

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Heat and Mass transfer Summary

$$\dot{m}_p = \frac{dm_p}{dt} = -\frac{m_p}{\tau_M};\ \tau_M = \frac{\rho_p d_p^2}{6\rho_m D_m \mathrm{Sh}^* \ln(1+B_M)} \tag{46.9}$$

$$\frac{dT_p}{dt} = \frac{T_s - T_p}{\tau_T} - \frac{1}{\tau_M}\frac{L_{v,\mathrm{surf}}}{c_p} + \frac{q_R}{m_p c_p};\ \ \tau_T = \frac{\rho_p c_p d_p^2}{6\lambda_m \mathrm{Nu}_m} \tag{48.9}$$

Heat and Mass transfer correlation [13, 14]

$$\mathrm{Sh}^* = \frac{2 + 0.87\mathrm{Re}_p^{1/2}\mathrm{Sc}_m^{1/3}}{(1+B_M)^{0.7}};\ Sc_m = \mu_m/\rho_m D_m$$

$$\mathrm{Nu}_m = \frac{ln\ (1+B_T)}{B_T}\mathrm{Nu}_{0,m} \qquad B_T = \frac{-\dot{m}_p C_v}{\pi d_p \lambda_m \mathrm{Nu}_m}$$

$$\mathrm{Nu}_{0,m} = \begin{cases} 2 + 0.552\mathrm{Re}_p^{1/2}\mathrm{Pr}_m^{1/3} & ;\ \mathrm{Re}_p > 10 \\ 1 + (1+\mathrm{Re}_p\mathrm{Pr}_m)^{\frac{1}{3}}\, max\left(1, \mathrm{Re}_p^{0.077}\right) & ;\ \text{otherwise} \end{cases} \qquad \mathrm{Pr}_m = \mu_m C_m/\lambda_m$$

References

1. Schiller, L. and A. Naumann, *A drag coefficient correlation. Vdi Zeitung, 77: 318–320, 1935.* Cité page. **38**.
2. Sommerfeld, M., *Theoretical and experimental modelling of particulate flows.* Lecture series, 2000. **6**: p. 3-7.
3. Auton, T., J. Hunt, and M. Prud'Homme, *The force exerted on a body in inviscid unsteady non-uniform rotational flow.* Journal of Fluid Mechanics, 1988. **197**: p. 241-257.
4. Reeks, M. and S. McKee, *The dispersive effects of Basset history forces on particle motion in a turbulent flow.* The Physics of fluids, 1984. **27**(7): p. 1573-1582.
5. Michaelides, E.E. and A. Roig, *A reinterpretation of the Odar and Hamilton data on the unsteady equation of motion of particles.* AIChE Journal, 2011. **57**(11): p. 2997-3002.
6. Mei, R. and J. Klausner, *Shear lift force on spherical bubbles.* International journal of heat and fluid flow, 1994. **15**(1): p. 62-65.
7. Rubinow, S. and J.B. Keller, *The transverse force on a spinning sphere moving in a viscous fluid.* Journal of Fluid Mechanics, 1961. **11**(3): p. 447-459.
8. Oesterle, B. and T.B. Dinh, *Experiments on the lift of a spinning sphere in a range of intermediate Reynolds numbers.* Experiments in Fluids, 1998. **25**(1): p. 16-22.
9. Li, A. and G. Ahmadi, *Dispersion and deposition of spherical particles from point sources in a turbulent channel flow.* Aerosol science and technology, 1992. **16**(4): p. 209-226.
10. Ounis, H., G. Ahmadi, and J.B. McLaughlin, *Brownian particle deposition in a directly simulated turbulent channel flow.* Physics of Fluids A: Fluid Dynamics, 1993. **5**(6): p. 1427-1432.
11. Brock, J.R., *On the theory of thermal forces acting on aerosol particles.* Journal of Colloid Science, 1962. **17**(8): p. 768-780.
12. Tomiyama, A., *Struggle with computational bubble dynamics.* Multiphase Science and Technology, 1998. **10**(4): p. 369-405.
13. Amani, E. and M. Nobari, *A calibrated evaporation model for the numerical study of evaporation delay in liquid fuel sprays.* International journal of heat and mass transfer, 2013. **56**(1): p. 45-58.
14. Amani, E. and M. Nobari, *Systematic tuning of dispersion models for simulation of evaporating sprays.* International Journal of Multiphase Flow, 2013. **48**: p. 11-31.