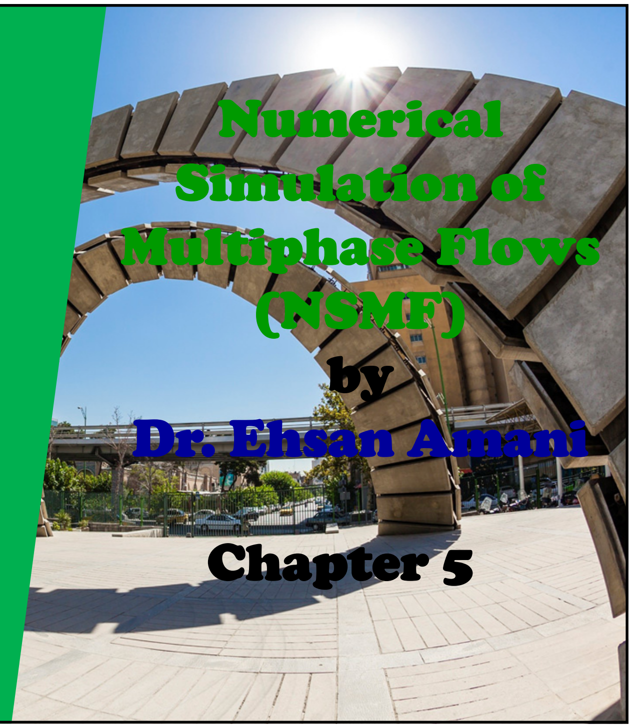

Numerical Simulation of Multiphase Flows (NSMF)

by

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Chapter 5

Eulerian averaged equations and models

- **Key relations of averaging**

⇨ Lecture Notes: V.1

- **The multifluid model**
  - **Ensemble averaging the multifluid instantaneous equations**

$$\langle(5.3a)\rangle \rightarrow \frac{\partial}{\partial t}\langle\rho_k\chi_k\rangle + \frac{\partial}{\partial x_j}\langle\rho_k U_{k,j}\chi_k\rangle = \left\langle S_m^{(I_k)}\right\rangle$$

(7.2) (10.2)

$$\frac{\partial\bar{\rho}_k}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho}_k\widetilde{U}_{k,j}\right) = \left\langle S_m^{(I_k)}\right\rangle \equiv \underbrace{\sum_{q=1}^{N}\left(\dot{m}_{qk} - \dot{m}_{kq}\right)}_{\text{Interphase mass coupling: needs modeling}} \quad (13.5)$$

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- **The multifluid model**
  - **Usually p and T are assumed at (phase) equilibrium, which is called mechanical and thermal (local) homogeneity, and are the same for all phases. However, velocity is determined per phase**

$$\langle(5.3b)\rangle \longrightarrow \frac{\partial}{\partial t}\underbrace{\langle\rho_k\chi_k U_{k,i}\rangle}_{(10.2)} + \frac{\partial}{\partial x_j}\underbrace{\langle\rho_k U_{k,j} U_{k,i}\chi_k\rangle}_{(6.5)} = \frac{\partial}{\partial x_j}\underbrace{\langle\sigma_{k,ij}\chi_k\rangle}_{(7.2)} + \underbrace{\langle\rho_k\chi_k\rangle}_{(7.2)} g_i + \left\langle S_{U_i}^{(I_k)}\right\rangle + \left\langle U_{k,i} S_m^{(I_k)}\right\rangle$$

$$\frac{\partial}{\partial t}\left(\bar{\rho}_k \widetilde{U}_{k,i}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}_k \widetilde{U}_{k,j}\widetilde{U}_{k,i}\right) = \frac{\partial \bar{\sigma}_{k,ij}}{\partial x_j} + \bar{\rho}_k g_i - \frac{\partial}{\partial x_j}\left(\overbrace{\bar{\rho}_k \widetilde{U''_{k,j}U''_{k,i}}}^{\text{Reynolds stress: needs modeling}}\right) + \underbrace{\left\langle S_{U_i}^{(I_k)}\right\rangle + \left\langle U_{k,i} S_m^{(I_k)}\right\rangle}_{\text{Interphase momentum coupling: need modeling}} \quad (15.5)$$

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- **The multifluid model**
  - **Unclosed terms**

$$\left\langle \boldsymbol{S}_U^{(I_k)}\right\rangle = \sum_{q=1}^{N} \boldsymbol{F}_{qk} = \sum_{q=1}^{N}\left(\overset{\text{Drag force}}{\boldsymbol{F}_{qk}^{\text{drag}}} + \overset{\text{Lift force}}{\boldsymbol{F}_{qk}^{\text{lift}}} + \underset{\text{Turbulent dispersion force}}{\boldsymbol{F}_{qk}^{\text{td}}} + \overset{\text{Wall lubrication force}}{\boldsymbol{F}_{qk}^{\text{wall}}} + \underset{\text{Virtual mass force}}{\boldsymbol{F}_{qk}^{\text{vm}}} + \overset{\text{Undisturbed flow force}}{\boldsymbol{F}_{qk}^{0}} + \cdots\right) \quad (16.5)$$

$$\left\langle \boldsymbol{U}_k S_m^{(I_k)}\right\rangle = \sum_{q=1}^{N}\left(\dot{m}_{qk}\boldsymbol{U}_k - \dot{m}_{kq}\boldsymbol{U}_q\right) \quad (17.5)$$

  - **Note 1: $\boldsymbol{F}_{kq} = -\boldsymbol{F}_{qk}$**
  - **Note 2: For dilute flows where the presence of a phase (carrier-phase) is dominant $\sum_{q=1}^{N}\boldsymbol{F}_{qk} = \boldsymbol{F}_{cq}$**

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Eulerian averaged equations and models

- **The multifluid model**
  - **Note 1:** $\boldsymbol{F}_{kq} = -\boldsymbol{F}_{qk}$
  - **Note 2: For dilute flows where the presence of a phase (carrier-phase) is dominant** $\sum_{q=1}^{N} \boldsymbol{F}_{qk} = \boldsymbol{F}_{cq}$
  - **Note 3: The modeling of interphase coupling terms are highly problem dependent. This will be discussed in chapters 6 and 9.**
  - **Note 4: The 1st and 4th terms of the RHS Eq. (15.5) can be recast as (see Capecelatro (2013) for a proof)**

$$\frac{\partial \overbrace{\bar{\sigma}_{k,ij}}^{\alpha_k \langle \sigma_{ij} \rangle_{|k}}}{\partial x_j} + \left\langle S_{U_i}^{(I_k)} \right\rangle = \alpha_k \frac{\partial \langle \sigma_{ij} \rangle_{|k}}{\partial x_j} + \underbrace{\left\langle S_{U_i}^{\prime (I_k)} \right\rangle}_{\text{The same as Eq. (16.5) excluding } \boldsymbol{F}_{qk}^{0}} \qquad (18.5)$$

Eulerian averaged equations and models

- **The multifluid model**
  - **Similarly, for scalar transport equations, starting from Eq. (5.3c),**

$$\frac{\partial}{\partial t}\left(\bar{\rho}_k \tilde{Q}_k\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}_k \tilde{U}_{k,j} \tilde{Q}_k\right) = \frac{\partial \bar{J}_{Q_k,j}}{\partial x_j} - \frac{\partial}{\partial x_j}\overbrace{\left(\bar{\rho}_k \widetilde{U_{k,j}'' Q_k''}\right)}^{\text{Reynolds flux: needs modeling}} + \bar{\rho}_k \underbrace{\tilde{S}_{Q_k}}_{\text{Averaged source term: usually needs modeling}}$$
$$\underbrace{\left\langle S_{Q_k}^{(I_k)} \right\rangle + \left\langle Q_k S_m^{(I_k)} \right\rangle}_{\text{Interphase scalar coupling: need modeling}} \qquad (19.5)$$

  - **Summary**

$$\sum_{k=1}^{N} \alpha_k = 1 \qquad (20.5)$$

Unknowns	Equations
α_k (N)	Eq. (13.5) (N)
$\tilde{U}_{k,i}$ (3N)	Eq. (15.5) (3N)
$\bar{p}$ (1)	Eq. (20.5) (1)
4N+1	4N+1

Eulerian averaged equations and models

- **The drift-flux model**
  - **Exercise: Show that summing Eqs. (13.5) and (15.5) over all phases yields**

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho_m U_{m,j}\right) = 0 \qquad (21.5)$$

$$\frac{\partial}{\partial t}\left(\rho_m U_{m,i}\right) + \frac{\partial}{\partial x_j}\left(\rho_m U_{m,j} U_{m,i}\right) = \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} + \rho_m g_i - \frac{\partial}{\partial x_j}\sum_{q=1}^{N}\left(\bar{\rho}_k \underbrace{U_{km,j}}_{\text{Drift velocity: needs modeling}} U_{km,i}\right) - \frac{\partial}{\partial x_j}\underbrace{\left(\rho_m \widetilde{U''_{m,j} U''_{m,i}}\right)}_{\text{Reynolds stress: needs modeling}} + \underbrace{\langle F_{\sigma_i} \rangle}_{\text{Averaged surface tension: needs modeling}} \qquad (22.5)$$

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Eulerian averaged equations and models

- **The drift-flux model**
  - **Summary:**

Unknowns	Equations
α_k (N)	Eq. (21.5) (1), Eq. (13.5) (N-1)
$U_{m,i}$ (3)	Eq. (22.5) (3)
$\bar{p}$ (1)	Eq. (20.5) (1)
N+4	N+4

  - **Note 1: In many cases, $\langle F_{\sigma_i} \rangle$ is neglected.**
  - **Note 2: If $\boldsymbol{U}_{km} = 0$, the model is called the homogeneous model and there is no relative velocity between phases.**
- **The Population Balance Model (PBM)**
  - **Trade-off between the multifluid and drift-flux**

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