Conservation of mass (continuity)

- **For a general control volume**

$$\frac{Dm_s}{Dt}=0 \quad (1.7) \qquad\qquad B=m \longrightarrow b\equiv\frac{dB}{dm}=\frac{dm}{dm}=1$$

$$\xrightarrow[\text{1st RTT}]{(7.6)} \frac{Dm_s}{Dt}=\frac{d}{dt}\int_{\mho_\sigma}\rho \overset{1}{b}\, d\mho+\int_A \rho \overset{1}{b}\overbrace{\left(\vec{V}-\vec{V}_\sigma\right)}^{\vec{V}_r}.\vec{n}dA \overset{(1.7)}{=} 0$$

$$\longrightarrow \underbrace{\frac{d}{dt}\iiint_{\mho_\sigma}\rho d\mho}_{\text{Rate of change of mass in the control volume}}+\underbrace{\oiint_A \rho\vec{V}_r.\vec{n}dA}_{\text{Net outlet mass flowrate from the control surface}}=0 \quad (2.7) \qquad \frac{dm_\sigma}{dt}+\sum_{\text{out}}\dot{m}-\sum_{\text{in}}\dot{m}=0$$

- **For a fixed Control Volume (C.V.)** ($\vec{V}_\sigma=0$)

$$\xrightarrow{(2.7)} \frac{\partial}{\partial t}\iiint_{\mho_\sigma}\rho d\mho+\oiint_A \rho\vec{V}.\vec{n}dA=0 \quad (3.7)$$

Chapter 7 By E. Amani

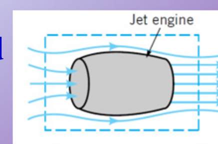
Conservation of mass (continuity)

- **Simplified forms**
  1. **For a fixed C.V. in a steady flow:**

$$\frac{\partial}{\partial t}\iiint_{\mho_\sigma}\rho d\mho=\iiint_{\mho_\sigma}\cancel{\frac{\partial\rho}{\partial t}}^{0}d\mho=0 \xrightarrow{(3.7)}$$

$$\oiint_A \rho\vec{V}.\vec{n}dA=0 \quad (4.7) \quad \sum_{\text{out}}\dot{m}=\sum_{\text{in}}\dot{m}$$

  2. **For a moving, non-deforming C.V. and a steady flow (in the C.V.-attached reference frame)**



$$\xrightarrow{(2.7)} \oiint_A \rho\vec{V}_r.\vec{n}dA=0 \quad (4.7)'$$

  3. **For an incompressible flow (and single-phase):**

$$\xrightarrow[\rho=cte]{(2.7)} \rho\frac{d}{dt}\iiint_{\mho_\sigma}d\mho+\rho\oiint_A \vec{V}_r.\vec{n}dA=0 \longrightarrow \frac{d\mho_\sigma}{dt}+\underbrace{\oiint_A \vec{V}_r.\vec{n}dA}_{\sum_{\text{out}}Q-\sum_{\text{in}}Q}=0 \quad (4.7)''$$

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Conservation of mass (continuity)

- **Simplified forms**
  4. **For a moving, non-deforming C.V. and an incompressible flow (in the C.V.-attached reference frame)**

$$\xrightarrow{(4.7)''} \oiint_A \vec{V}_r.\vec{n}dA=0 \quad (5.7) \quad \sum_{\text{out}}Q=\sum_{\text{in}}Q$$

  5. **For a 1D inlet or outlet normal to the flow**

$$\dot{m}=\iint \rho\vec{V}_r.\vec{n}dA=\rho V_r A \qquad Q=\iint \vec{V}_r.\vec{n}dA=V_r A$$

- **Note: It can be shown that Eqs. (4.7)" and (5.7) also valid for incompressible multiphase flows ($D\rho/Dt=0$ instead of $\rho=cte$).**

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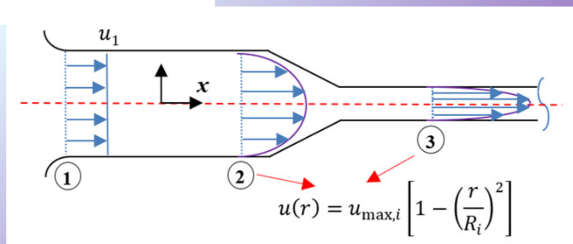
Conservation of mass (continuity)

- **Sample problem**

A steady-state incompressible flow develops in a pipe system. The velocity profiles are shown at three different sections. Determine the maximum velocity at sections 2 and 3 as a function of the inlet velocity, u_1, and pipe sizing ($R_1 = R_2 = 2R_3$). The flow is assumed parallel, in the x direction.

⇨ **Lecture Notes**



Chapter 7 By E. Amani

The (linear) momentum equation

- **For a general control volume**

⇨ **Lecture Notes**

$$\underbrace{\sum \vec{F}_\sigma}_{\text{Resultant external force acting on the control volume}} = \underbrace{\frac{d}{dt}\iiint_{\mho_\sigma} \vec{V}\rho d\mho}_{\text{Rate of change of momentum in the control volume}} + \underbrace{\oiint_A \vec{V}(\rho\vec{V}_r.\vec{n})dA}_{\text{Net momentum outflowrate from the control surface}} \qquad (10.7)$$

- **Simplified forms**

1. **For an inertial C.V. (moving with a constant velocity)**

$$\xrightarrow[\vec{V}\to\vec{V}_r]{(10.7)} \quad \sum \vec{F}_\sigma = \frac{d}{dt}\iiint_{\mho_\sigma} \vec{V}_r\rho d\mho + \oiint_A \vec{V}_r(\rho\vec{V}_r.\vec{n})dA \qquad (11.7)$$

✓ **Exercise: For an inertial C.V. both Eqs. (10.7) and (11.7) are valid. Therefore, the right-hand-side of these equations should be equal. Show this equality with the aid of the continuity.**

Chapter 7 By E. Amani

The (linear) momentum equation

- **Simplified forms**
  2. **For an inertial C.V. and a steady flow in the C.V.-attached reference frame**

Non-deforming

$$\frac{d}{dt}\iiint_{\mho_\sigma} \vec{V}_r \rho d\mho = \iiint_{\mho_\sigma} \underbrace{\frac{d}{dt}[\rho(\vec{V} - \vec{V}_\sigma)]}_{0} d\mho$$

(11.7) $\rightarrow$ $$\sum \vec{F}_\sigma = \oiint_A \vec{V}_r(\rho \vec{V}_r . \vec{n}) dA \qquad (12.7)$$

  3. **For 1D inlets and outlets:**

(10.7) $\rightarrow$ $$\sum \vec{F}_\sigma = \frac{d\vec{P}_\sigma}{dt} + \sum_{\text{out}} \dot{m}\vec{V} - \sum_{\text{in}} \dot{m}\vec{V} \qquad (10.7)'$$

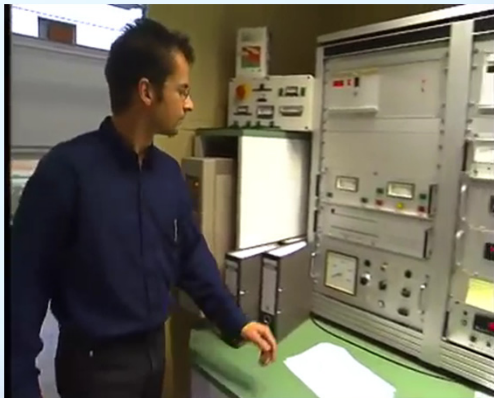
(11.7) $\rightarrow$ $$\sum \vec{F}_\sigma = \frac{d\vec{P}_{\sigma,r}}{dt} + \sum_{\text{out}} \dot{m}\vec{V}_r - \sum_{\text{in}} \dot{m}\vec{V}_r \qquad (11.7)'$$

The (linear) momentum equation

- **Example: Pelton turbine**

The (linear) momentum equation

- **Example: Pelton turbine**



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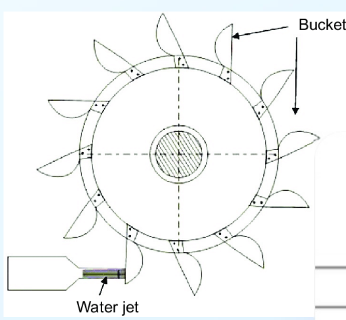
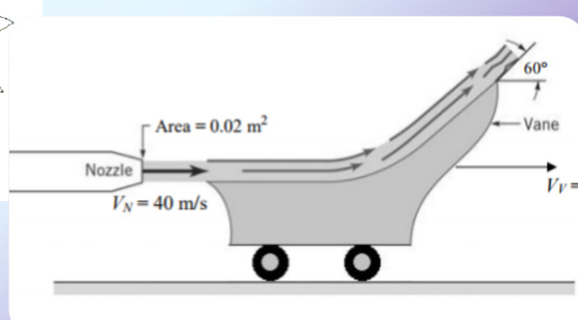
The (linear) momentum equation

- **A simplified model: vane on wheels**





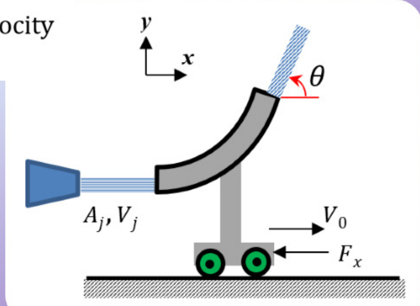
Chapter 7 By E. Amani

The (linear) momentum equation

- **Sample problem**

A water jet impinges upon a vane on wheel as shown in the figure. Calculate the horizontal force, F_x, exerted on the vane system as a function of the given parameters, stating all assumptions you make, when

a) The system is immobile.
b) The system moves with a fixed velocity of V_0.

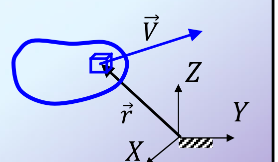


⇨ Lecture Notes

The angular momentum equation

- **For a general control volume**

⇨ Lecture Notes



$\vec r$ is the location vector from a fixed point in the inertial frame

$$\overbrace{\sum(\vec{r}\times\vec{F}_\sigma)}^{\sum \vec{M}_\sigma} = \overbrace{\frac{d}{dt}\iiint_{\mho_\sigma}(\vec{r}\times\vec{V})\rho\, d\mho}^{\vec{H}_\sigma} + \oint\!\!\!\oint_A (\vec{r}\times\vec{V})(\rho\vec{V}_r.\vec{n})dA \qquad (17.7)$$

- **Simplified forms**

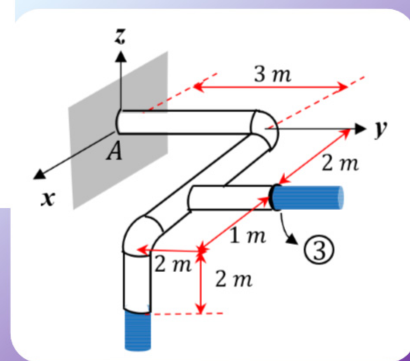
1. **For an inertial C.V.**

$$\xrightarrow[\vec{V}\to\vec{V}_r]{(17.7)} \quad \sum(\vec{r}\times\vec{F}_\sigma) = \frac{d}{dt}\iiint_{\mho_\sigma}(\vec{r}\times\vec{V}_r)\rho\, d\mho + \oint\!\!\!\oint_A (\vec{r}\times\vec{V}_r)(\rho\vec{V}_r.\vec{n})dA \qquad (18.7)$$

The angular momentum equation

- **Sample problem**

The piping system shown in the figure has been attached to the structure at section A. Water flows steadily in the system. The water flow rate and pressure at section A are $Q_A = 0.01\, m^3/s$ and $p_A = 300\, kPa$. Calculate the resultant force and moment exerted from the water and piping system on the wall structure. Water outflow at section 3 is $Q_3 = 0.004\, m^3/s$, the internal pipe cross-sectional area is $A_{in} = 2580\, mm^2$, the specific weight of the system (pipe and water) per unit length of pipes is $W' = 300\, N/m$, and $\rho_{water} = 1000\, kg/m^3$.



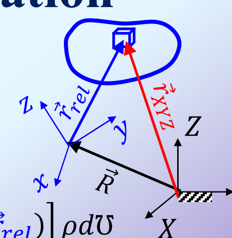
⇨ **Lecture Notes**

Chapter 7 By E. Amani

The (linear) momentum equation

- **Using a noninertial reference frame**

⇨ **Lecture Notes**



$$\sum \vec{F}_\sigma - \iiint_{\mho_\sigma} \left[\ddot{\vec{R}} + 2\vec{\omega} \times \vec{V}_{rel} + \dot{\vec{\omega}} \times \vec{r}_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{rel})\right] \rho d\mho$$

$$= \frac{d}{dt} \iiint_{\mho_\sigma} \vec{V}_{rel} \rho d\mho + \oiint_A \vec{V}_{rel} (\rho \vec{V}_r . \vec{n}) dA \qquad (14.7)$$

$$\vec{V}_{rel} = \vec{V}_{XYZ} - (\dot{\vec{R}} + \vec{\omega} \times \vec{r}_{rel}) \qquad \vec{V}_r = \vec{V}_{XYZ} - \vec{V}_\sigma$$

- **Simplified forms (noninertial reference frame)**

1. **For a nondeforming (rigid-body-like moving) C.V. and the C.V.-attached local reference frame**

$$\vec{V}_{rel} = \vec{V}_r$$

Chapter 7 By E. Amani

The (linear) momentum equation

- **Simplified forms (noninertial reference frame)**
  2. **For a nondeforming C.V. in a linear motion (uniform $\ddot{\vec{R}}$ and $\vec{\omega} = \dot{\vec{\omega}} = 0$) and the C.V.-attached local reference frame**

$$\iiint_{\mho_\sigma} \ddot{\vec{R}} \rho d\mho = \ddot{\vec{R}} \iiint_{\mho_\sigma} \rho d\mho = \ddot{\vec{R}} m_\sigma \longrightarrow$$

$$\sum \vec{F}_\sigma - m_\sigma \ddot{\vec{R}} = \frac{d}{dt} \iiint_{\mho_\sigma} \vec{V}_r \rho d\mho + \oiint_A \vec{V}_r (\rho \vec{V}_r . \vec{n}) dA \qquad (15.7)$$

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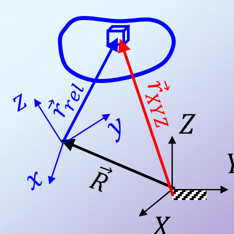
The angular momentum equation

- **Using a noninertial reference frame**
  - **Exercise: Show that**



$$\sum (\vec{r}_{rel} \times \vec{F}_\sigma) - \iiint_{\mho_\sigma} \vec{r}_{rel} \times \left[\ddot{\vec{R}} + 2\vec{\omega} \times \vec{V}_{rel} + \dot{\vec{\omega}} \times \vec{r}_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{rel}) \right] \rho d\mho =$$

$$\frac{d}{dt} \iiint_{\mho_\sigma} (\vec{r}_{rel} \times \vec{V}_{rel}) \rho d\mho + \oiint_A (\vec{r}_{rel} \times \vec{V}_{rel}) (\rho \vec{V}_r . \vec{n}) dA \qquad (21.7)$$

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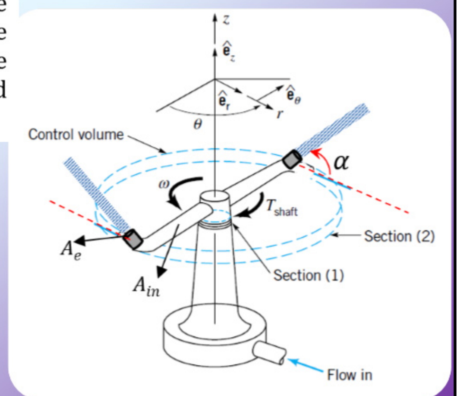
The angular momentum equation

- **Sample problem**

The rotary water sprinkler shown in the figure rotates with the constant angular velocity of ω. Calculate the frictional torque on the axis of the sprinkler as a function of the other parameters. The water inflow is q, the length each is arm $\ell/2$, and A_{in} and A_e are the inner cross-sectional area of the arm and ejector exit.



⇨ **Lecture Notes**

Chapter 7 By E. Amani

Turbomachinery

- **Radial-flow: centrifugal pump**

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Turbomachinery

- **Radial-flow: centrifugal pump**

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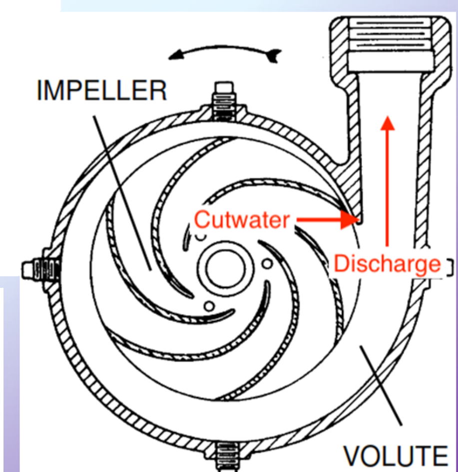
Turbomachinery

- **Radial-flow: centrifugal pump**



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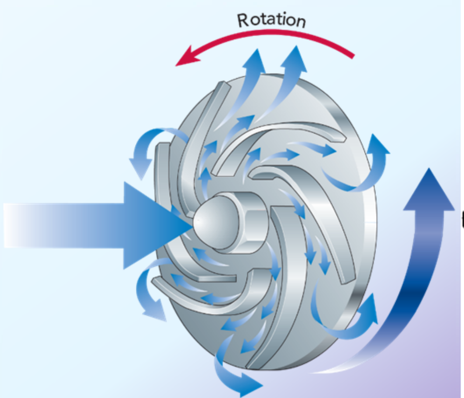
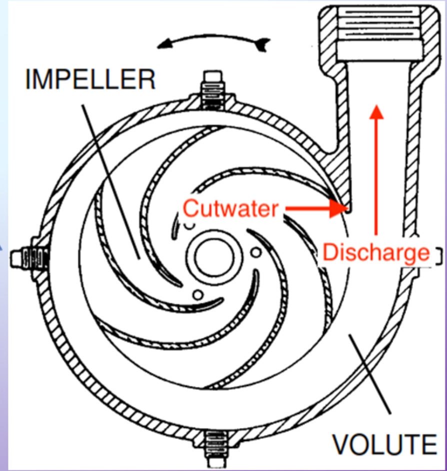
Turbomachinery

- **Radial-flow: centrifugal pump**





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Turbomachinery

- **Radial-flow: centrifugal pump**

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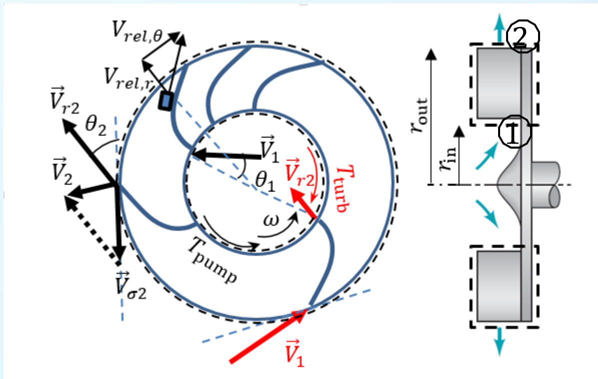
Turbomachinery

- **Radial-flow**



$$\vec{V}_r = \vec{V} - \vec{V}_\sigma \qquad \left.\begin{aligned} \vec{V}_r &\to \vec{W} \\ \vec{V}_\sigma &\to \vec{U} = r\omega\hat{e}_{\boldsymbol{\theta}} \end{aligned}\right\} \longrightarrow W_r = V_r$$

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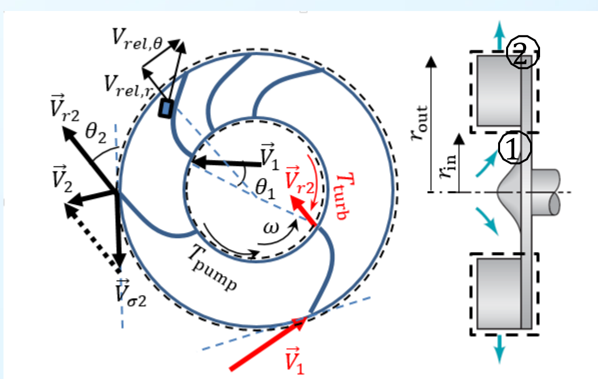
Turbomachinery

- **Radial-flow**



$$T_{\text{shaft}} = (-\dot{m}_{in})(r_{in}V_{\theta,in}) + (\dot{m}_{out})(r_{out}V_{\theta,out})$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega$$

- **Proof: Appendix A**

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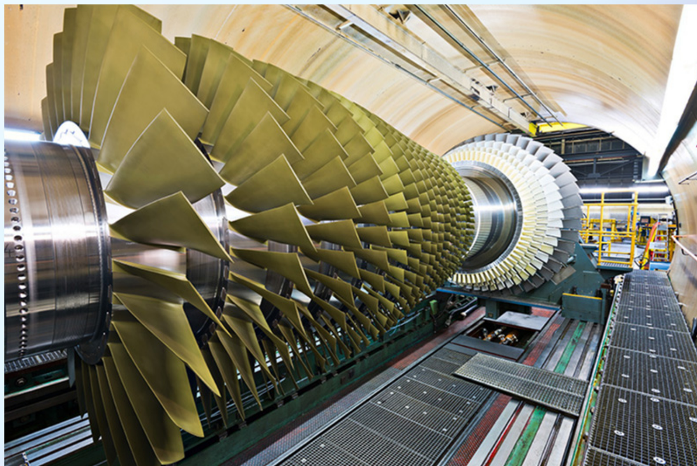
Turbomachinery

- **Axial-flow: axial compressor**



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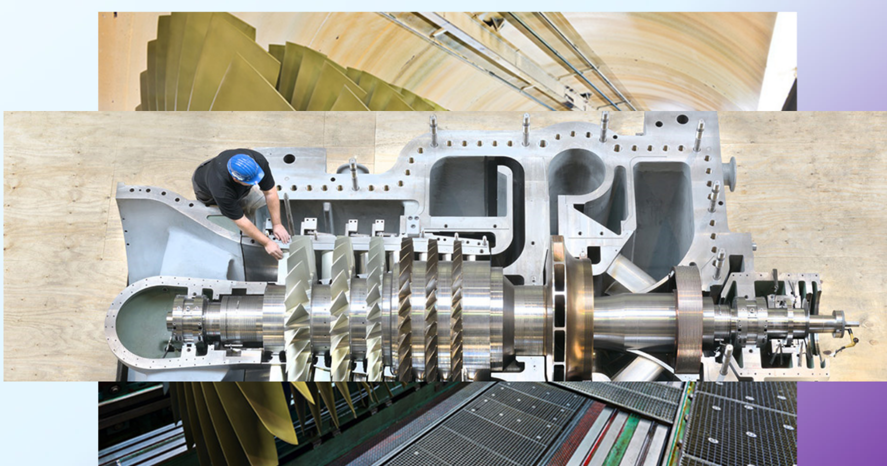
Turbomachinery

- **Axial-flow: axial compressor**



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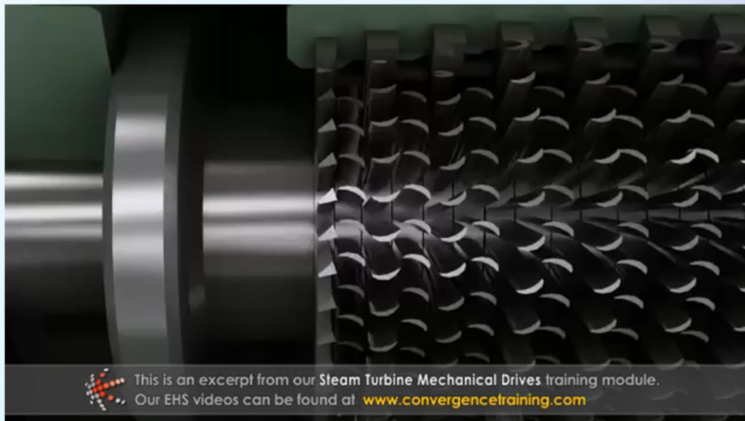
Turbomachinery

- **Axial-flow: axial compressor**



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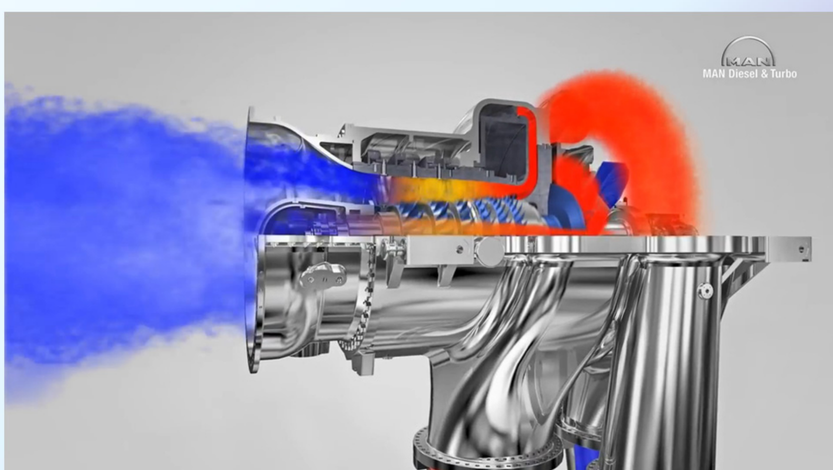
Turbomachinery

- **Axial-flow: axial compressor**



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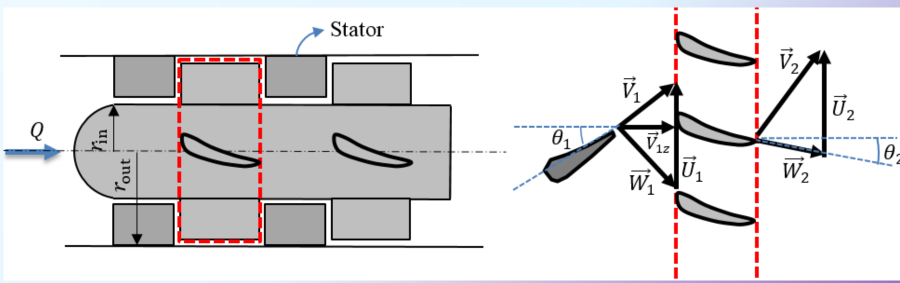
Turbomachinery

- **Axial-flow**



$$V_{1z} = \frac{Q}{A_{out} - A_{in}} \qquad U = \bar{r}\omega = \left(\frac{r_{out} + r_{in}}{2}\right)\omega$$

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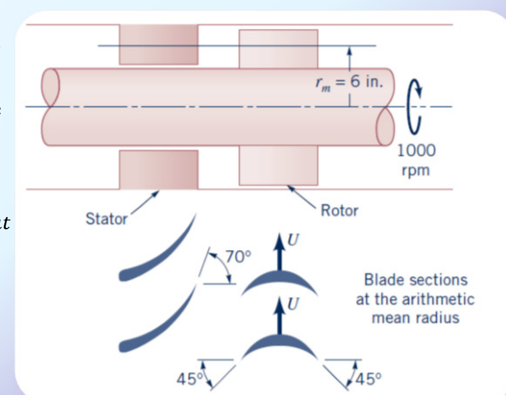
Turbomachinery

- **Sample problem**

The sketch of the arithmetic mean radius blade sections of an axial-flow water turbine is shown in the figure. The rotor speed is 1000 rpm. The inner and outer radius of the blades are $r_{in} = 4\ in$ and $r_{out} = 8\ in$. The water flowrate within the turbine is $Q = 0.5\ ft^3/s$. Calculate the output power of the turbine. The water density is assumed as 1.94 $slug/ft^3$.



Lecture Notes

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The end of chapter 7