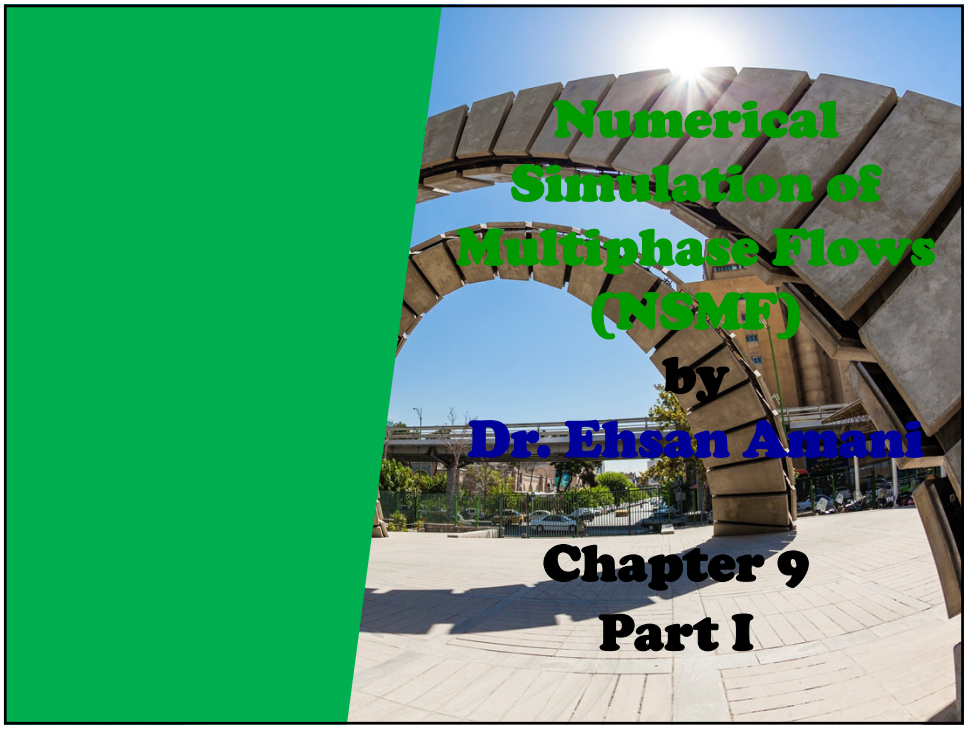
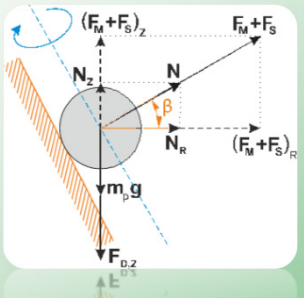


IX. Fluid-particle interaction (FPI)



IX.1 Fluid-to-particle transfer rates

IX.1.1 Momentum transfer – forces and moments on a particle

IX.1.2 Heat transfer

IX.1.3 Particle tracking numerics

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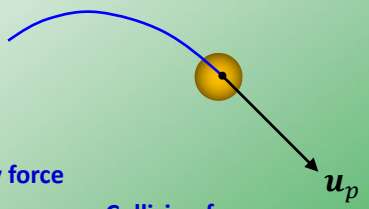
Momentum transfer – particle tracking

- For each particle, p:



$$\frac{dx_p}{dt} = u_p$$

Fluid-particle interaction force

Body force

Collision force

$$F = m_p \frac{du_p}{dt}; \quad F = F_{\text{FPI}} + F_{\text{B}} + F_{\text{col}}$$

$$T = I_p \frac{d\omega_p}{dt}; \quad T = T_{\text{FPI}} + T_{\text{col}}$$

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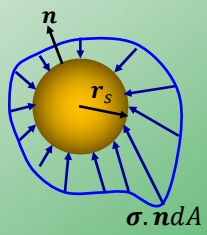
FPI forces and moments

- DNS formulation:



$$F_{\text{FPI}} = \int_{A_p} \sigma.n dA = \int_{A_p} (-pn + \tau.n) dA$$

$$T_{\text{FPI}} = \int_{A_p} r_s \times (n.\tau) dA$$

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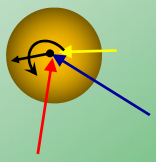
FPI forces and moments

- Point-particle formulation:
  - Simplifying assumptions
  - Semi-analytical solutions
  - Superposition



Drag force (streamwise steady viscous+pressure)

Virtual (or added) mass force (unsteady pressure)

Wall (lubrication) force

Thermophoretic force

$$\boldsymbol{F}_{\mathrm{FPI}} = \boldsymbol{F}_{\mathrm{D}} + \boldsymbol{F}_{\mathrm{L}} + \boldsymbol{F}_{\mathrm{vm}} + \boldsymbol{F}_{\mathrm{H}} + \boldsymbol{F}_{0} + \boldsymbol{F}_{\mathrm{W}} + \boldsymbol{F}_{\mathrm{B}} + \boldsymbol{F}_{\mathrm{T}}$$

Lift force (transverse steady pressure)

History (or Basset) force (unsteady viscous)

Undisturbed flow force

Brownian force

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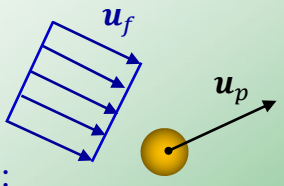
Drag force



- Simplifying assumptions:
  - Steady-state
  - Stokes regime ($Re_p < 1$) ⇨ **Re number relaxation**
  - Spherical particle ⇨ **Equivalent diameter !**
  - No evaporation/condensation ⇨ **Blowing effect**
  - Uniform flow ⇨ **Seen velocity (u_s) definition**
  - Incompressible low-Mach flow ⇨ **Compressibility effect**
  - No internal flow ⇨ **Internal flow relaxation**
  - No rotation

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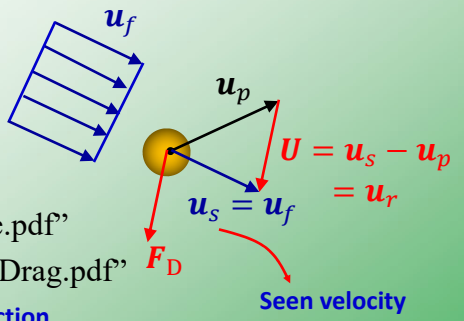
Drag force



- Derivation:
  - See my notes: "dragForce.pdf"
  - For more details: "StokesDrag.pdf"
- Result:

Pressure drag / Friction drag

$$F_D = \underbrace{2\pi\mu U a}_{\text{Pressure drag}} + \underbrace{4\pi\mu U a}_{\text{Friction drag}} = 6\pi\mu U a$$

- The drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 \underbrace{A_p}_{\pi a^2}} = \frac{6\pi\mu U a}{\frac{1}{2}\rho U^2 \pi a^2} \Rightarrow C_D = \frac{24}{Re_p};\ Re_p = \frac{2a\rho U}{\mu} = \frac{|\boldsymbol{u}_r| d_p}{\upsilon_f}$$

Particle Reynolds number

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Drag force

- Re number relaxation:
  - Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
  - Drag coefficient and friction factor ⇨ **Notes**

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Drag force

- Re number relaxation:
  - Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
  - Drag coefficient and friction factor

⇨ **Notes: Chap9.1.1.1**

$$\boldsymbol{F}_{\mathrm{D}} = m_p \frac{\boldsymbol{u}_r}{\tau_p};\ \ \tau_p = \tau_v = \frac{1}{f_D} \frac{\rho_p d_p^2}{18 \rho_f \upsilon_f}$$

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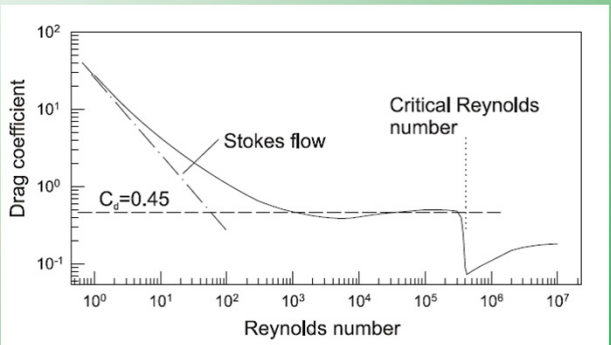
Drag force

- Re number relaxation:
  - Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
  - Drag coefficient and friction factor
  - Experimental measurements



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Drag force

- Re number relaxation:
  - Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
  - Drag coefficient and friction factor
  - Experimental measurements
  - A curve-fit, semi-empirical correlation by Schiller and Naumann [1]:

$$f_D = \begin{cases} 1 & Re_p \le 0.1 \\ 1 + \frac{1}{6} Re_p^{2/3} & 0.1 < Re_p \le 1000 \end{cases}$$

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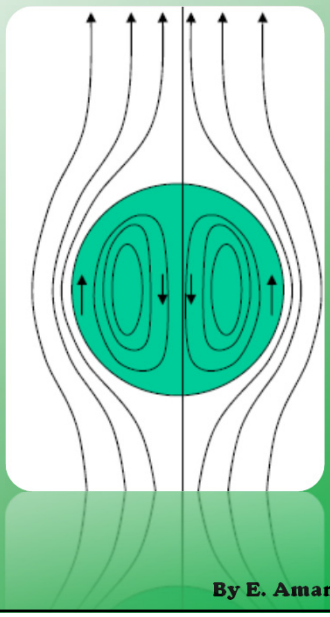
Drag force

- Internal flow relaxation
- …



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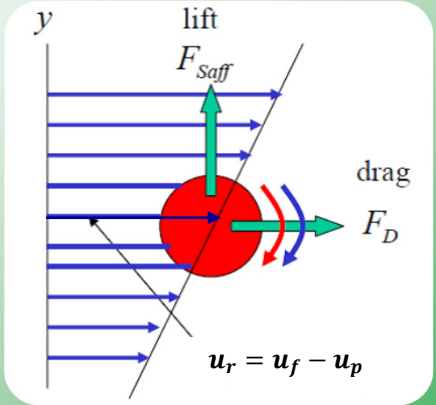
Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field



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Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field
- A semi-analytical solution [6]:

$$\boldsymbol{F}_{\mathrm{L}} = 1.61 C_s \, \rho_f d_p^2 \left(\frac{2\upsilon_f}{|\boldsymbol{\omega}_f|}\right)^{\frac{1}{2}} \boldsymbol{u}_r \times \boldsymbol{\omega}_f; \quad \boldsymbol{\omega}_f = 0.5 \, \nabla \times \boldsymbol{u}_f$$

$$C_s = \begin{cases} \left(1 - 0.3314\lambda_v^{\frac{1}{2}}\right) \exp\left(-\frac{Re_p}{10}\right) + 0.3314\lambda_v^{\frac{1}{2}} & ; Re_p \le 40 \\ 0.0524\left(\lambda_v Re_p\right)^{1/2} & ; Re_p > 40 \end{cases}, \quad \lambda_v = d_p \frac{|\boldsymbol{\omega}_f|}{|\boldsymbol{u}_r|}$$

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Saffman Lift force

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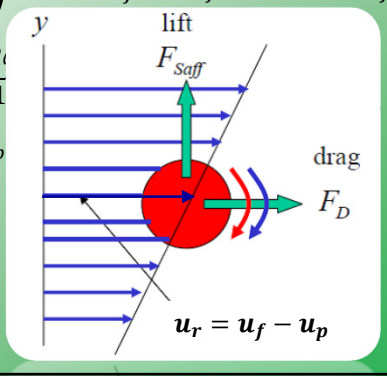

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- Important for large ρ_f/ρ_p or large particles

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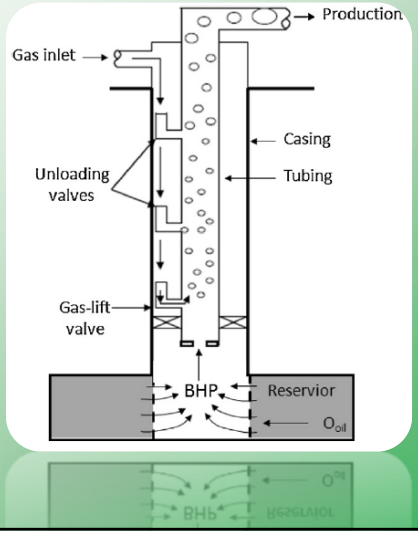
Saffman Lift force

- Application: Gas-lift [15]

Gas-lift in oil wells ▶



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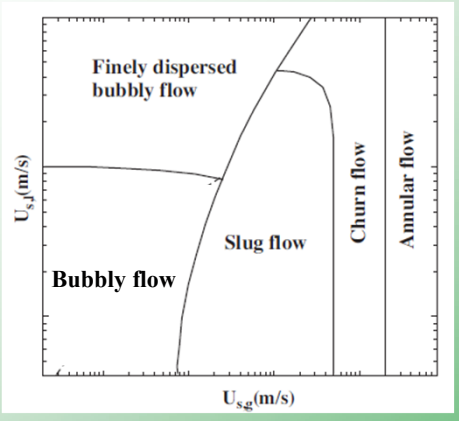
Saffman Lift force

- Application: Gas-lift [15]



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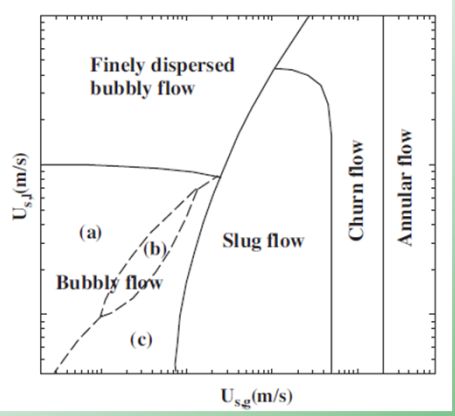
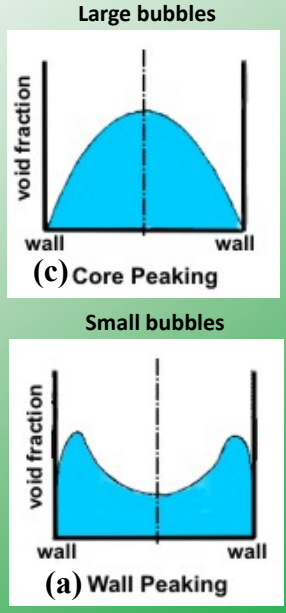
Saffman Lift force

- Application: Gas-lift [15]





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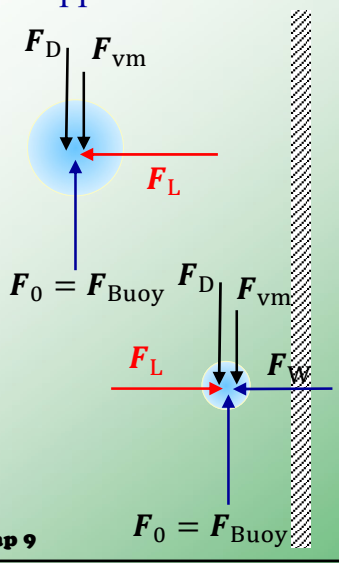
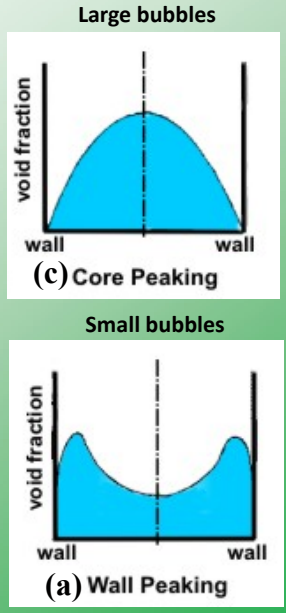
Saffman Lift force

- Application: Gas-lift [15]





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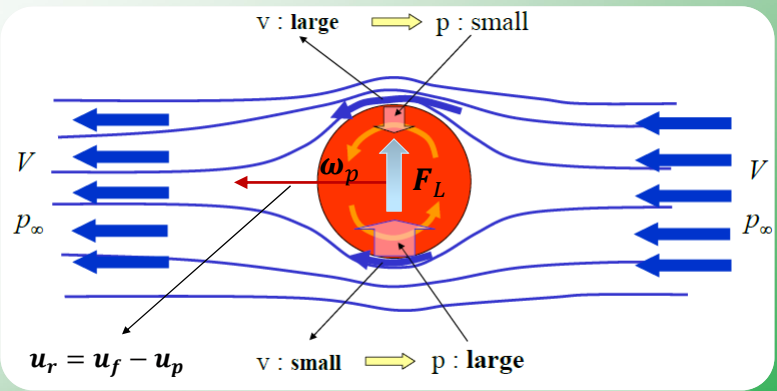
Saffman Lift force

- Application: Gas-lift [15]
- More physical discussion and applications: exercise

Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation



Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation
- A semi-analytical solution [7, 8]:

$$\boldsymbol{F}_{\mathrm{L}} = 0.5 C_{LR}\, \rho_f A_p \frac{|\boldsymbol{u}_r|}{|\boldsymbol{\omega}_r|}(\boldsymbol{u}_r \times \boldsymbol{\omega}_r); \quad \boldsymbol{\omega}_r = \boldsymbol{\omega}_p - \boldsymbol{\omega}_f$$

$$C_{\mathrm{LR}} = \begin{cases} \dfrac{Re_\omega}{Re_p} & ; Re_p \le 1 \\ 0.45 + \left(\dfrac{Re_\omega}{Re_p} - 0.45\right)\exp\left(-0.05684 Re_\omega^{0.4} Re_p^{0.3}\right) & ; Re_p < 2000 \end{cases}$$

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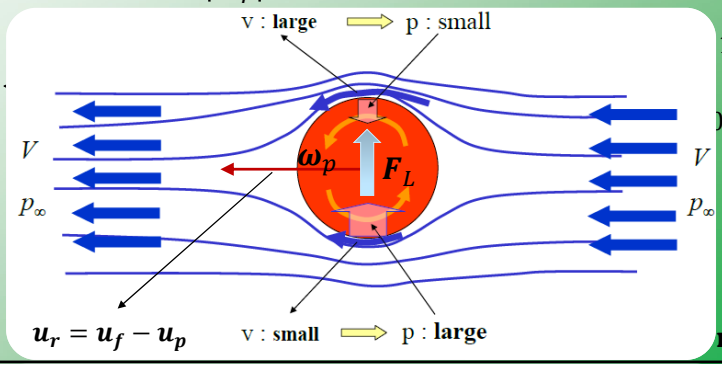
Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation
- A semi-analytical solution [7, 8]:

$$\boldsymbol{F}_{\mathrm{L}} = 0.5 C_{LR}\, \rho_f A_p \frac{|\boldsymbol{u}_r|}{|\boldsymbol{\omega}_r|}(\boldsymbol{u}_r \times \boldsymbol{\omega}_r); \quad \boldsymbol{\omega}_r = \boldsymbol{\omega}_p - \boldsymbol{\omega}_f$$

$C_{\mathrm{LR}} =$ 1 ; $Re_p < 2000$



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Magnus Lift force

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- Important for large ρ_f/ρ_p or large rotating particles

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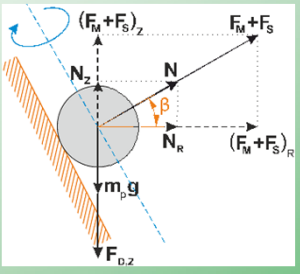
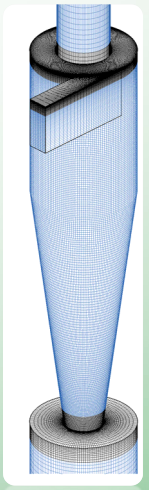
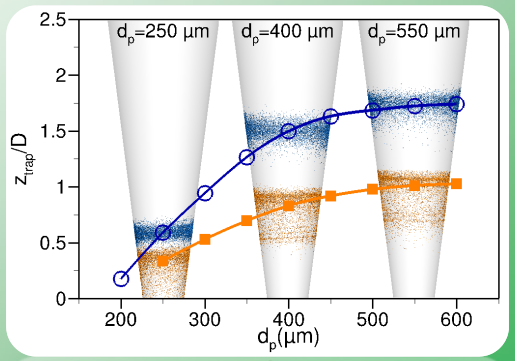
Magnus Lift force

- Application: cyclone erosion [16]







Neglecting lift forces for large particles: filled red squares

◄

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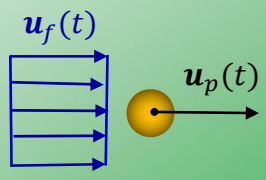
Magnus Lift force

- Application: cyclone erosion [16]
- More physical discussion and applications: exercise

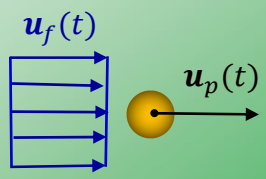
Virtual (added) mass force

- Evidence: A spherical particle in an unsteady inviscid flow experiences a force
- Can be viewed as increased inertia force
- Accelerating a part of fluid with the particle



Virtual (added) mass force

- For a basic derivation, consult classic aerodynamics books (e.g. Principles of Ideal-Fluid aerodynamics by K. Karamcheti).
- The analytical solution [3]:



$$\boldsymbol{F}_{\mathrm{vm}} = C_{vm}\frac{\rho_f \forall_p}{2}\left(\frac{D\boldsymbol{u}_f}{Dt} - \frac{d\boldsymbol{u}_p}{dt}\right);\ C_{vm} = 1$$

- Scales as: (ρ_f/ρ_p)
- More physical discussion and applications: exercise

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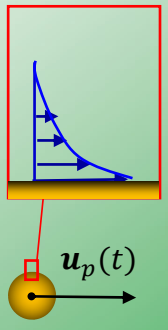
History (Basset) force

- Accelerating a part of fluid with the particle through viscous force
- A semi-analytical solution [4, 5]:



$$\boldsymbol{F}_{\mathrm{H}} = C_B\frac{3}{2}d_p^2\sqrt{\pi\mu_f\rho_f}\left[\int_0^t \frac{\frac{d\boldsymbol{u}_r}{dt'}}{\sqrt{t-t'}}dt' + \frac{\boldsymbol{u}_{r,0}}{\sqrt{t}}\right]$$

$$C_B = 1 - 0.527\left[1 - \exp\left(-0.14Re_pSl^{0.82}\right)^{2.5}\right];\quad Sl = \frac{1}{2\pi f\tau_p}$$

- Scales as: (ρ_f/ρ_p)
- More physical discussion and applications: exercise

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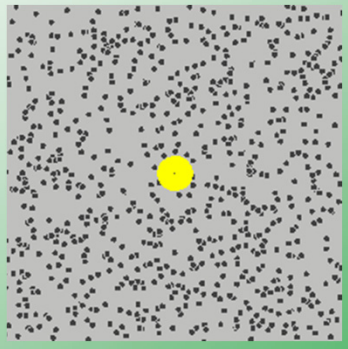
Brownian Force

- For sub-micron particles
- Random force



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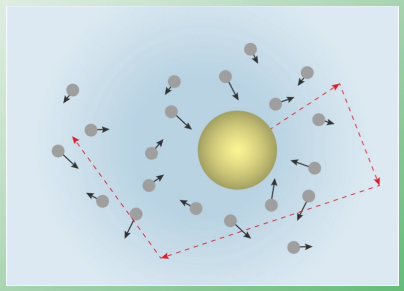
Brownian Force

- For sub-micron particles
- Random force
- A stochastic process [9, 10]:



$$\boldsymbol{F}_{\mathrm{B}} = m_p \sqrt{\frac{\pi S_0}{\delta t}} |\zeta| \boldsymbol{\eta}; \quad S_0 = \frac{216 \mu_f K_B T_f}{\pi^2 d_p^5 \rho_p^2 C_c}$$

$$C_c = 1 + \frac{2\lambda}{d_p}\left[1.257 + 0.4 \exp\left(-\frac{1.1 d_p}{2\lambda}\right)\right]$$

$\boldsymbol{\eta} = a\, cos(\theta)\, \boldsymbol{i} + a\, sin(\theta)\, \boldsymbol{j} + u\boldsymbol{k}, \theta = 2\pi X, u = 2Y - 1, a = \sqrt{1-u^2},$
$X, Y \in U(0,1), \zeta \in N(0,1)$

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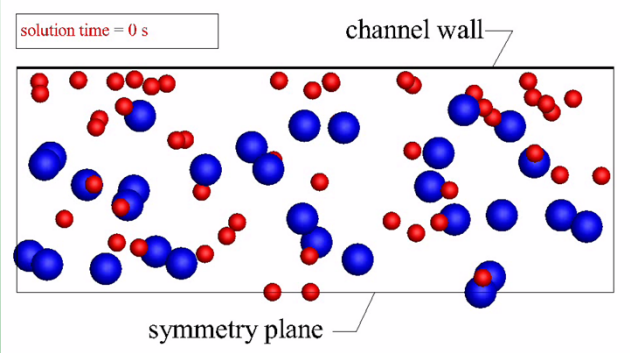
Brownian Force

- Application: Electroosmotic flow in nano-channels [17]

Na^+ (red spheres) and Cl^- (blue spheres) ion transport in a nano-channel ▶



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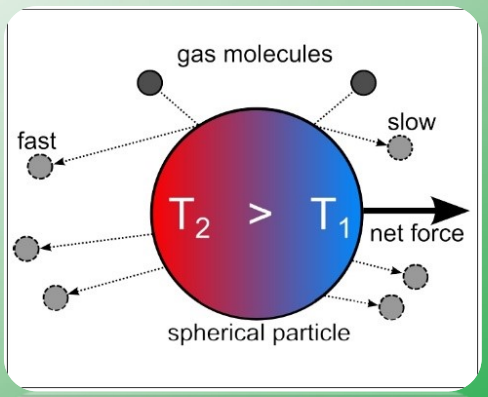
Thermophoretic Force

- For sub-micron particles in a temperature gradient
- A closure [11]:

$$\boldsymbol{F}_{\mathrm{T}} = -D_{T,p}\frac{\nabla T}{T}$$



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Case study #3: Conveying T-junction

- **Pneumatic conveying T-junction**
  - Eulerian-Lagrangian simulation
    1. Preliminary analysis
    2. Solution using ANSYS Fluent

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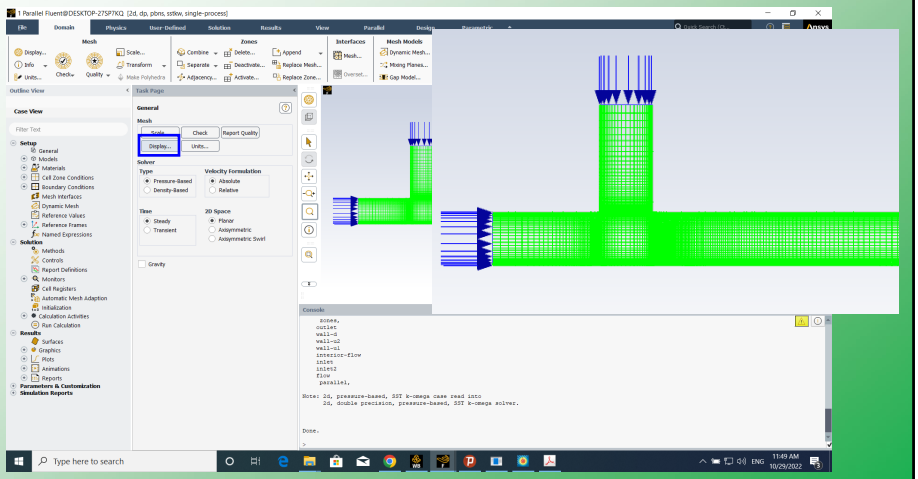
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Open: 1.cas



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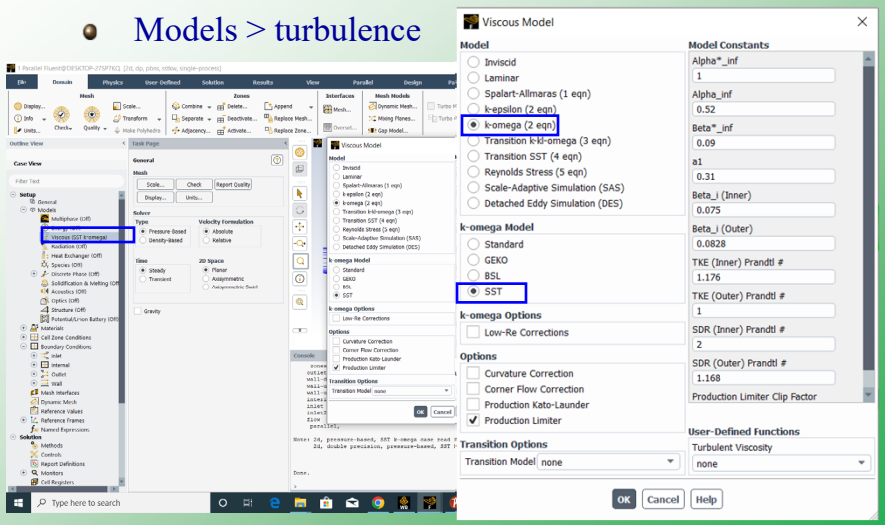
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Models > turbulence



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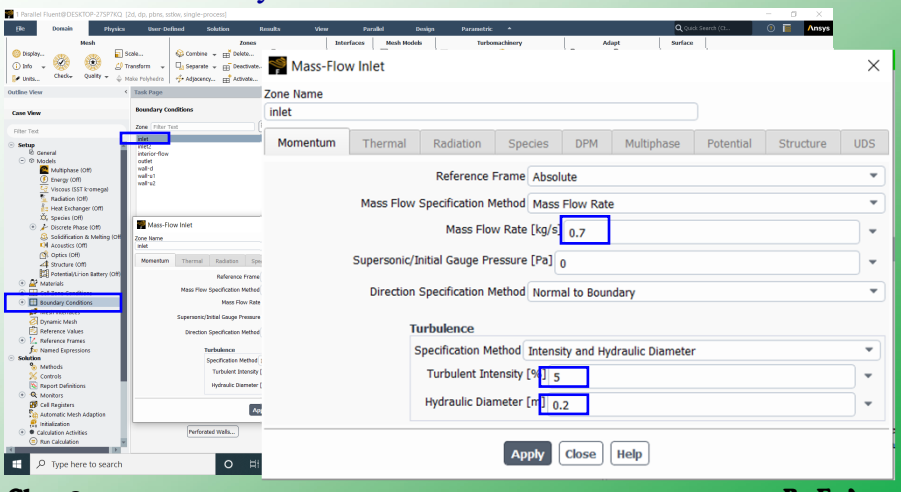
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Boundary condition



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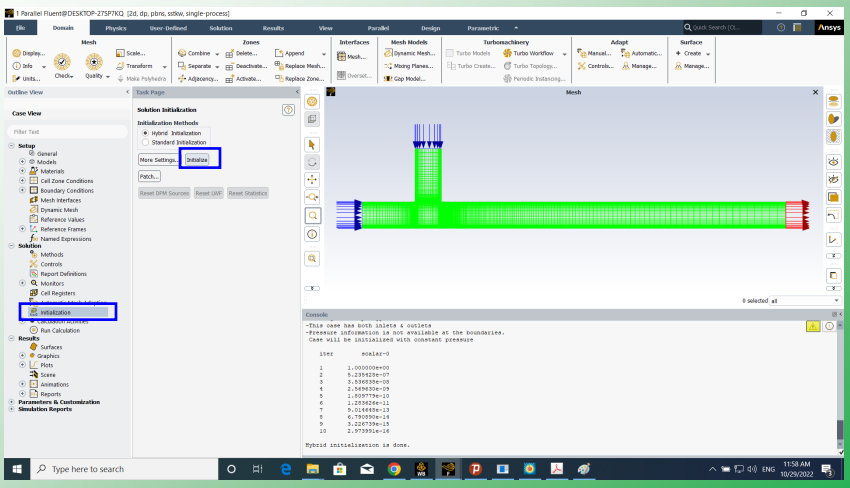
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Initialization



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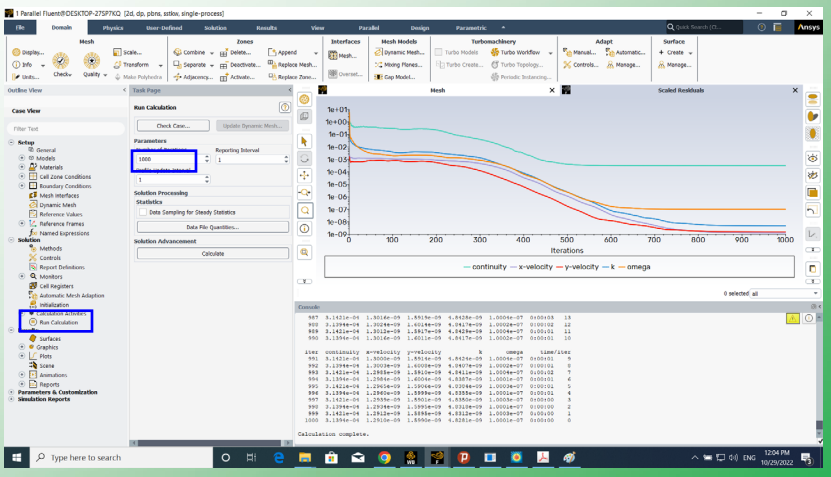
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Run (1000 iterations)



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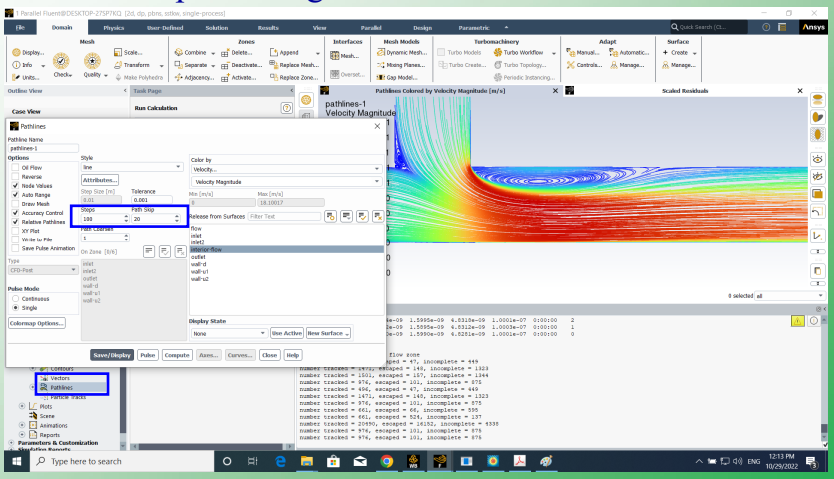
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Post processing: Pathlines



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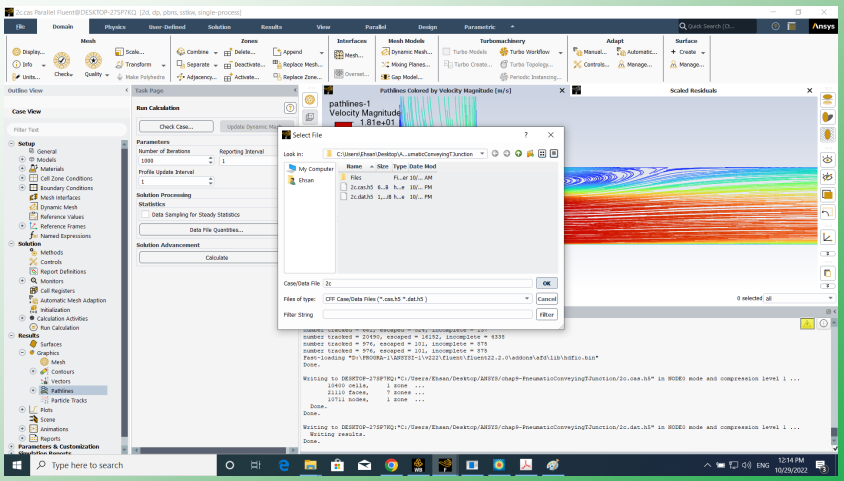
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
  - Save case and data: 2c



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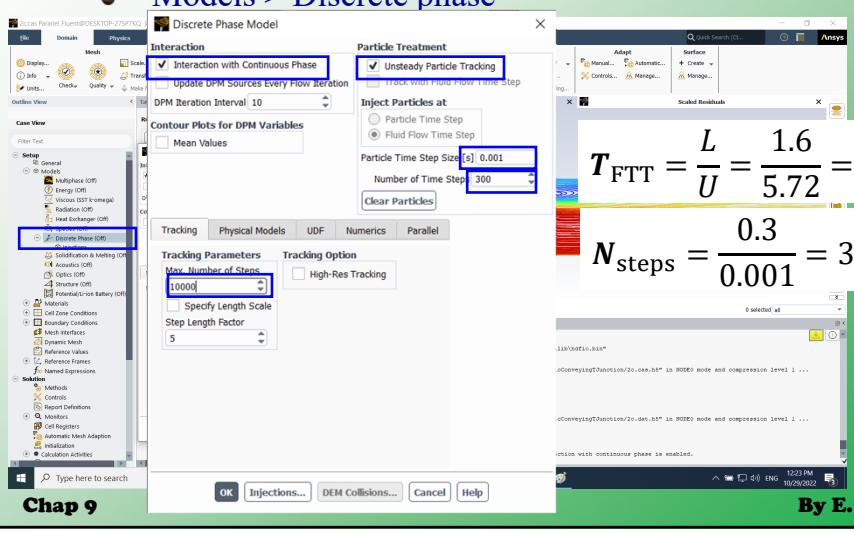
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase



$$T_{\text{FTT}} = \frac{L}{U} = \frac{1.6}{5.72} = 0.3\ s$$

$$N_{\text{steps}} = \frac{0.3}{0.001} = 300$$

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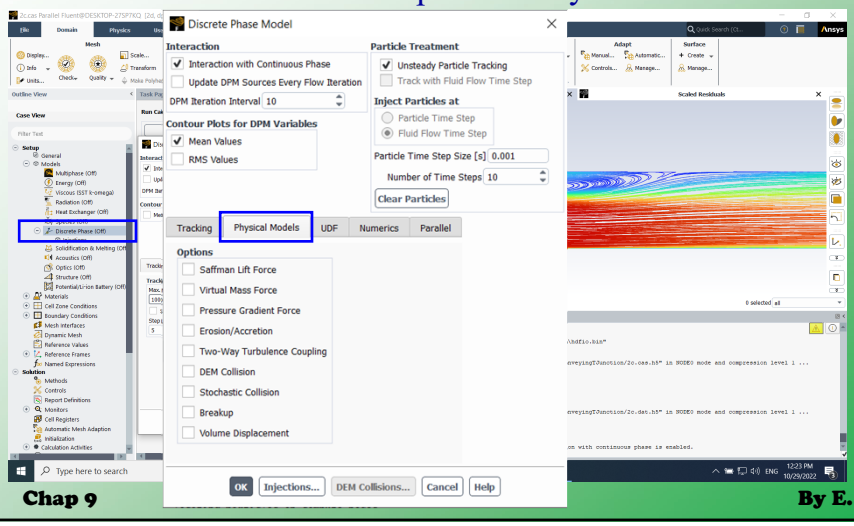
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase > Physical models



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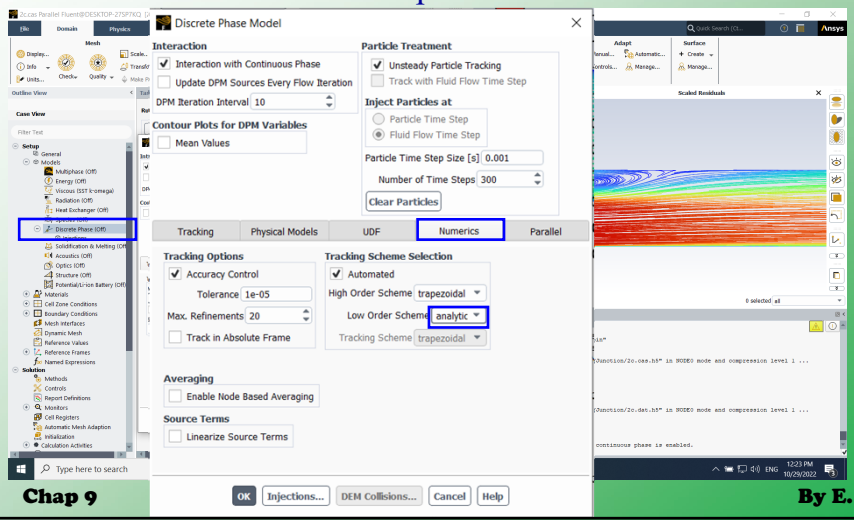
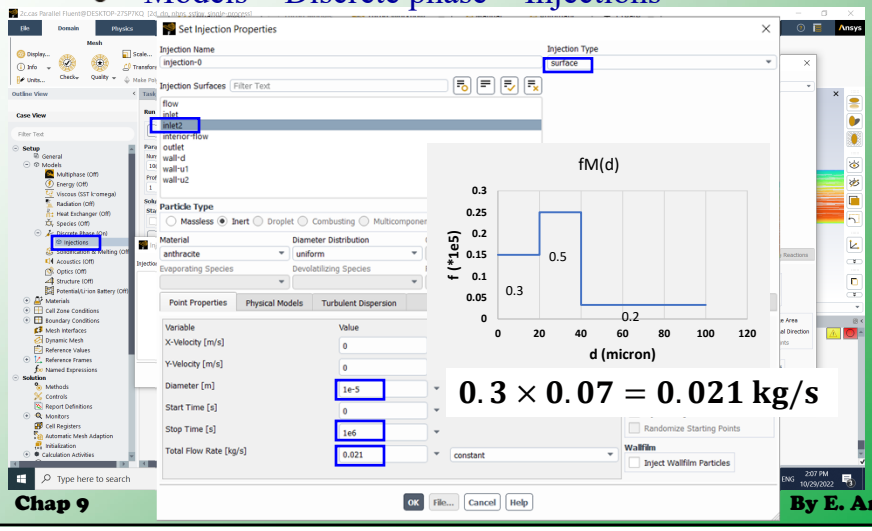
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase > Numerics



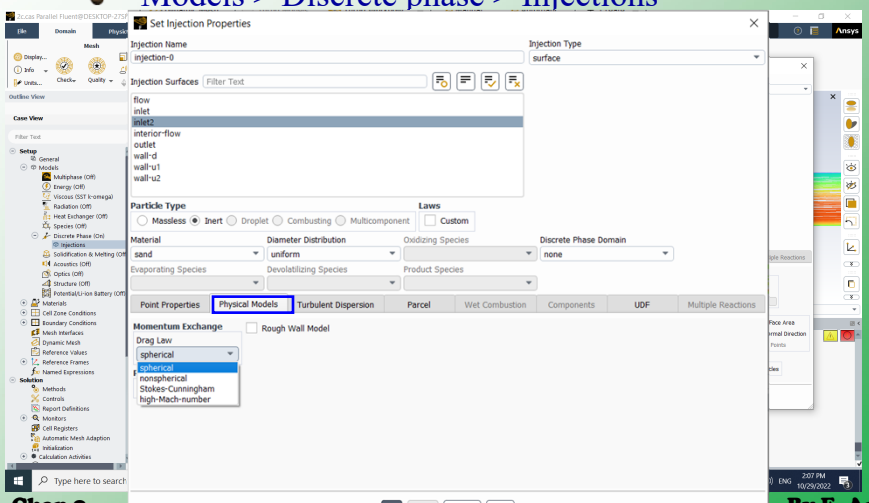
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase > Injections



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase > Injections



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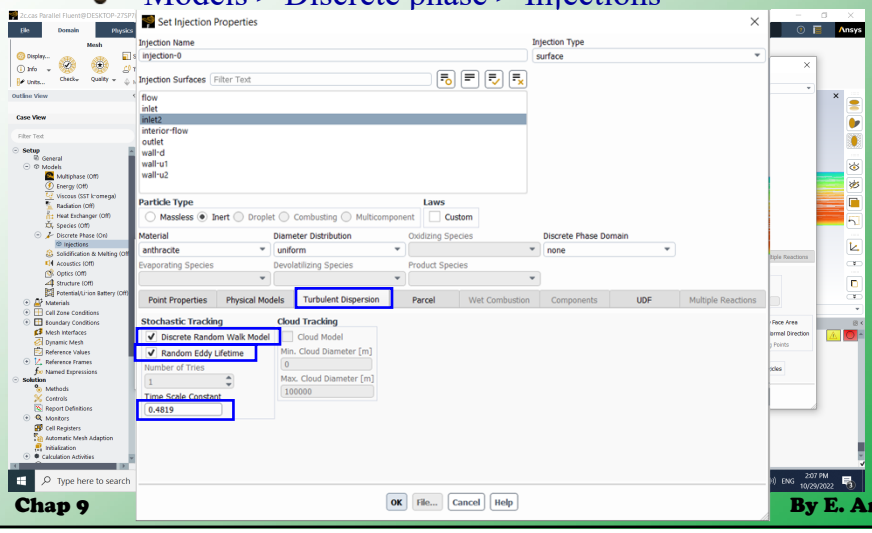
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase > Injections



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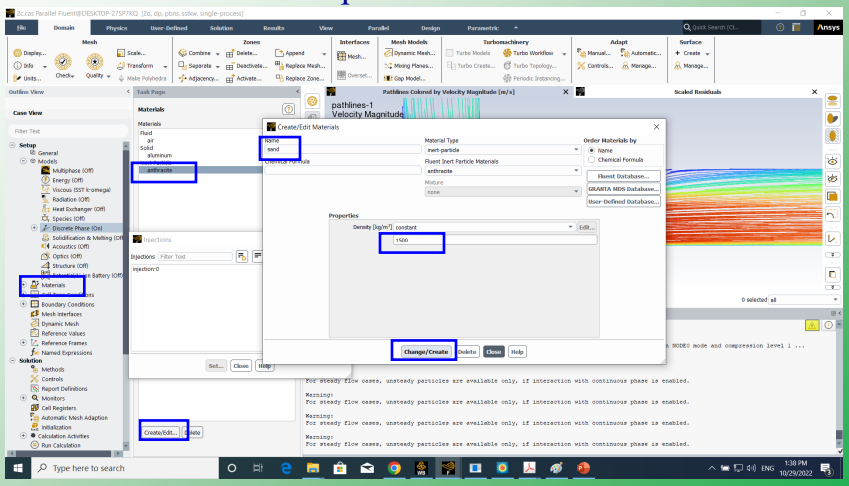
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Materials > Inert particle



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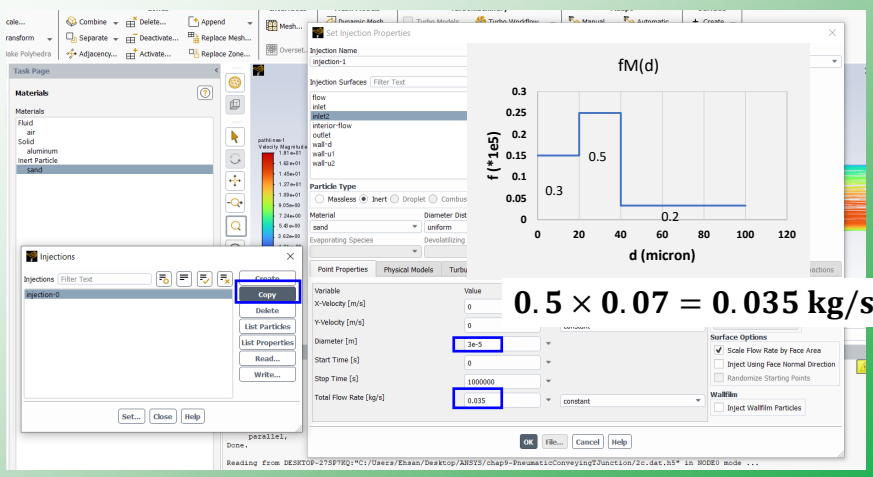
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Models > Discrete phase > Injections



$0.5 \times 0.07 = 0.035$ kg/s

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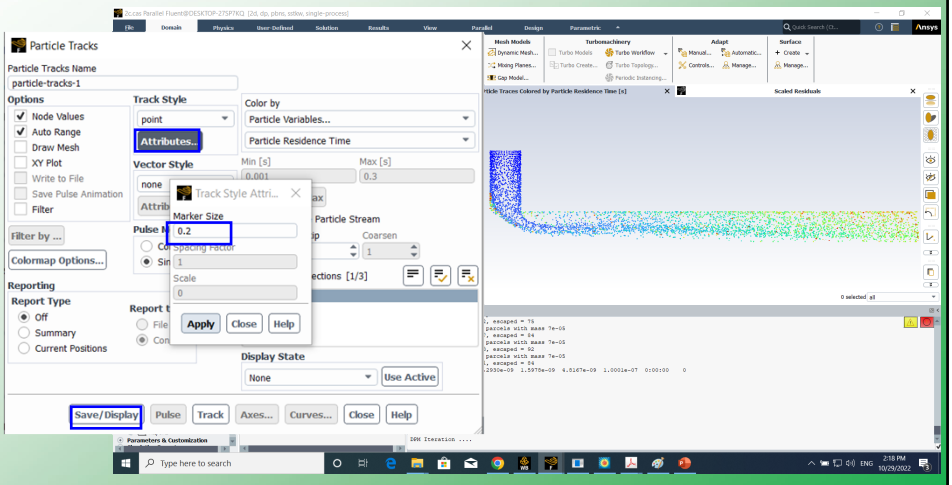
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Run (10 iterations)
  - Save case and data: 3c

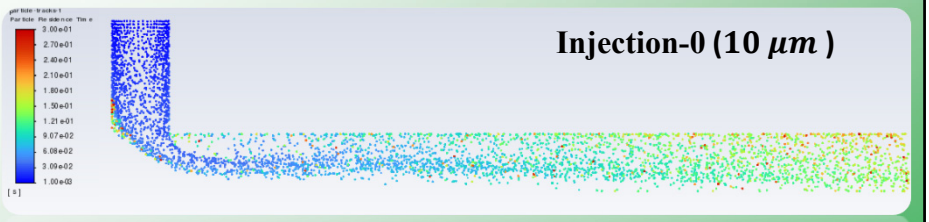
Case study #3: Conveying T-junction

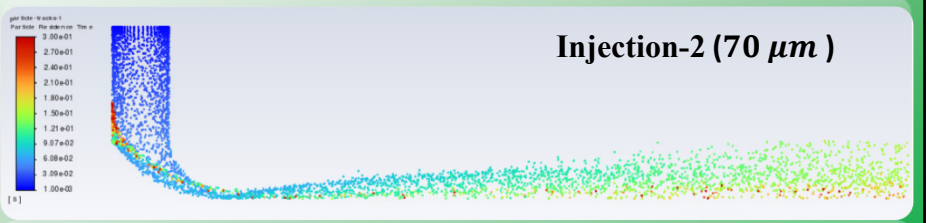
- Solution using ANSYS Fluent (one-way)
  - Results > Particle tracks



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
  - Results > Particle tracks





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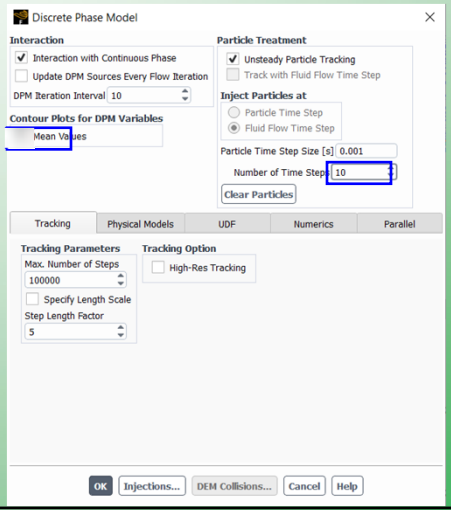
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
  - Models > Discrete phase



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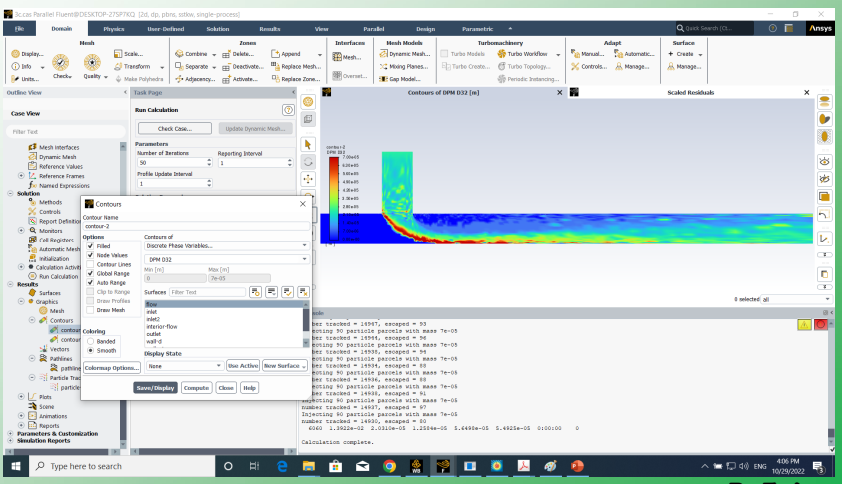
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
  - Run (5000 iterations)
  - Save case and data: 5c

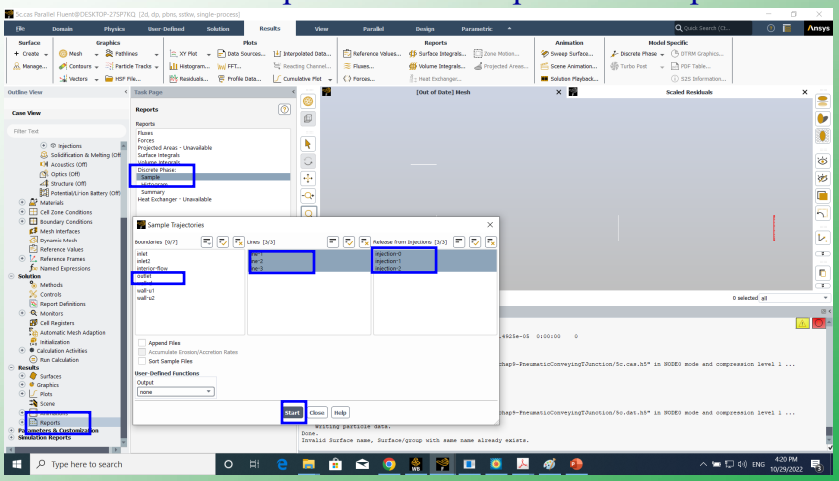
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
  - Results > Graphics > Contours



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
  - Results > Reports > Discrete phase > Sample



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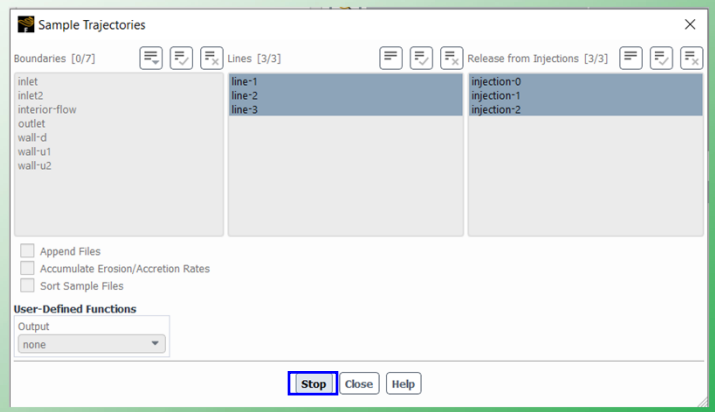
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
  - Run (?? iterations)
  - Save case and data: 6c



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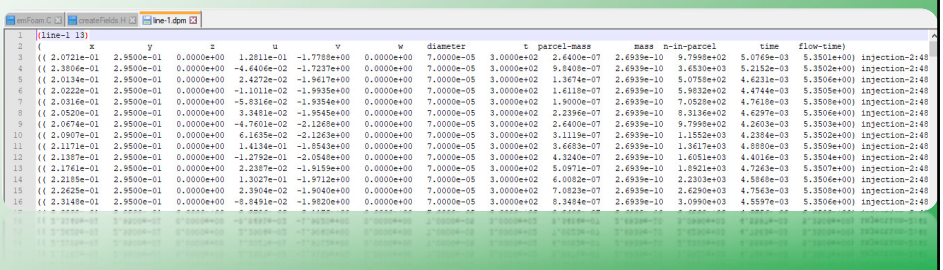
Case study #3: Conveying T-junction

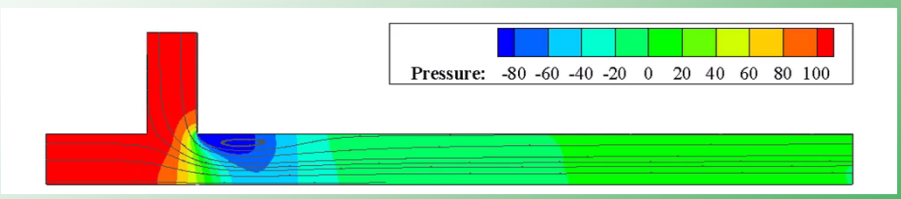
- Solution using ANSYS Fluent (two-way)
  - Run (?? iterations)
  - Save case and data: 6c
  - See files: line-1.dpm, line-2.dpm, line-3.dpm, and outlet.dpm



```
(1line-1 13)
(       x          y          z          u          v          w       diameter          t  parcel-mass       mass  n-in-parcel       time    flow-time)
(( 2.0721e-01  2.9500e-01  0.0000e+00  1.2811e-01 -1.7788e+00  0.0000e+00  7.0000e-05  3.0000e+02  2.6400e-07  2.6939e-10  9.7998e+02  5.0769e-03  5.3501e+00) injection-2:48
(( 2.3806e-01  2.9500e-01  0.0000e+00 -4.6406e-02 -1.7237e+00  0.0000e+00  7.0000e-05  3.0000e+02  9.8408e-07  2.6939e-10  3.6530e+03  5.2152e-03  5.3502e+00) injection-2:48
(( 2.0134e-01  2.9500e-01  0.0000e+00  2.4272e-02 -1.9617e+00  0.0000e+00  7.0000e-05  3.0000e+02  1.3674e-07  2.6939e-10  5.0758e+02  4.6231e-03  5.3506e+00) injection-2:48
(( 2.0222e-01  2.9500e-01  0.0000e+00 -1.1011e-02 -1.9935e+00  0.0000e+00  7.0000e-05  3.0000e+02  1.6118e-07  2.6939e-10  5.9832e+02  4.4744e-03  5.3505e+00) injection-2:48
(( 2.0316e-01  2.9500e-01  0.0000e+00 -5.8316e-02 -1.9354e+00  0.0000e+00  7.0000e-05  3.0000e+02  1.9000e-07  2.6939e-10  7.0528e+02  4.7618e-03  5.3508e+00) injection-2:48
(( 2.0520e-01  2.9500e-01  0.0000e+00  3.3481e-02 -1.9545e+00  0.0000e+00  7.0000e-05  3.0000e+02  2.2396e-07  2.6939e-10  8.3136e+02  4.6297e-03  5.3506e+00) injection-2:48
(( 2.0674e-01  2.9500e-01  0.0000e+00 -4.7601e-02 -2.1269e+00  0.0000e+00  7.0000e-05  3.0000e+02  2.6400e-07  2.6939e-10  9.7998e+02  4.2603e-03  5.3503e+00) injection-2:48
(( 2.0907e-01  2.9500e-01  0.0000e+00  6.1635e-02 -2.1263e+00  0.0000e+00  7.0000e-05  3.0000e+02  3.1119e-07  2.6939e-10  1.1552e+03  4.2384e-03  5.3502e+00) injection-2:48
(( 2.1171e-01  2.9500e-01  0.0000e+00  1.4134e-01 -1.8543e+00  0.0000e+00  7.0000e-05  3.0000e+02  3.6683e-07  2.6939e-10  1.3617e+03  4.8880e-03  5.3509e+00) injection-2:48
(( 2.1387e-01  2.9500e-01  0.0000e+00 -1.2792e-01 -2.0548e+00  0.0000e+00  7.0000e-05  3.0000e+02  4.3240e-07  2.6939e-10  1.6051e+03  4.4016e-03  5.3504e+00) injection-2:48
(( 2.1761e-01  2.9500e-01  0.0000e+00  2.2387e-02 -1.9159e+00  0.0000e+00  7.0000e-05  3.0000e+02  5.0971e-07  2.6939e-10  1.8921e+03  4.7263e-03  5.3507e+00) injection-2:48
(( 2.2185e-01  2.9500e-01  0.0000e+00  1.3027e-01 -1.9712e+00  0.0000e+00  7.0000e-05  3.0000e+02  6.0082e-07  2.6939e-10  2.2303e+03  4.5869e-03  5.3506e+00) injection-2:48
(( 2.2625e-01  2.9500e-01  0.0000e+00  2.3904e-02 -1.9040e+00  0.0000e+00  7.0000e-05  3.0000e+02  7.0823e-07  2.6939e-10  2.6290e+03  4.7563e-03  5.3508e+00) injection-2:48
(( 2.3148e-01  2.9500e-01  0.0000e+00 -8.8491e-02 -1.9820e+00  0.0000e+00  7.0000e-05  3.0000e+02  8.3484e-07  2.6939e-10  3.0990e+03  4.5597e-03  5.3506e+00) injection-2:48
```


Case study #3: Conveying T-junction

- Solution using ANSYS Fluent
  - Making animations using Tecplot



Other topics

- Simplified relation between FPI force in ESS and ELSS ⇨ **Notes:Chap9.1.1.10**
- See "FPISummary.pdf" for a summary of FPI force formulation.

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