Table with the del operator in cartesian, cylindrical and spherical coordinates

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, φ, z)	Spherical coordinates (r, θ, φ), where θ is the polar and φ is the azimuthal angle[α]
A vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\varphi\hat{\boldsymbol{\varphi}}$
Gradient ∇f[1]	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$
Divergence $\nabla \cdot \mathbf{A}$[1]	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$
Curl $\nabla \times \mathbf{A}$[1]	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\varphi}}$
Laplace operator $\nabla^2 f \equiv \Delta f$[1]	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$
Vector Laplacian $\nabla^2\mathbf{A} \equiv \Delta\mathbf{A}$	$\nabla^2 A_x\hat{\mathbf{x}} + \nabla^2 A_y\hat{\mathbf{y}} + \nabla^2 A_z\hat{\mathbf{z}}$	— View by clicking [show] — $\left(\nabla^2 A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial A_\varphi}{\partial\varphi}\right)\hat{\boldsymbol{\rho}} + \left(\nabla^2 A_\varphi - \frac{A_\varphi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\varphi}\right)\hat{\boldsymbol{\varphi}} + \nabla^2 A_z\hat{\mathbf{z}}$	— View by clicking [show] — $\left(\nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}\right)\hat{\mathbf{r}} + \left(\nabla^2 A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\varphi}{\partial\varphi}\right)\hat{\boldsymbol{\theta}} + \left(\nabla^2 A_\varphi - \frac{A_\varphi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\varphi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\varphi}\right)\hat{\boldsymbol{\varphi}}$
Material derivative[α][2] $(\mathbf{A}\cdot\nabla)\mathbf{B}$	$\mathbf{A}\cdot\nabla B_x\hat{\mathbf{x}} + \mathbf{A}\cdot\nabla B_y\hat{\mathbf{y}} + \mathbf{A}\cdot\nabla B_z\hat{\mathbf{z}}$	$\left(A_\rho\frac{\partial B_\rho}{\partial\rho} + \frac{A_\varphi}{\rho}\frac{\partial B_\rho}{\partial\varphi} + A_z\frac{\partial B_\rho}{\partial z} - \frac{A_\varphi B_\varphi}{\rho}\right)\hat{\boldsymbol{\rho}} + \left(A_\rho\frac{\partial B_\varphi}{\partial\rho} + \frac{A_\varphi}{\rho}\frac{\partial B_\varphi}{\partial\varphi} + A_z\frac{\partial B_\varphi}{\partial z} + \frac{A_\varphi B_\rho}{\rho}\right)\hat{\boldsymbol{\varphi}} + \left(A_\rho\frac{\partial B_z}{\partial\rho} + \frac{A_\varphi}{\rho}\frac{\partial B_z}{\partial\varphi} + A_z\frac{\partial B_z}{\partial z}\right)\hat{\mathbf{z}}$	— View by clicking [show] — $\left(A_r\frac{\partial B_r}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_r}{\partial\theta} + \frac{A_\varphi}{r\sin\theta}\frac{\partial B_r}{\partial\varphi} - \frac{A_\theta B_\theta + A_\varphi B_\varphi}{r}\right)\hat{\mathbf{r}} + \left(A_r\frac{\partial B_\theta}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\theta}{\partial\theta} + \frac{A_\varphi}{r\sin\theta}\frac{\partial B_\theta}{\partial\varphi} + \frac{A_\theta B_r}{r} - \frac{A_\varphi B_\varphi\cot\theta}{r}\right)\hat{\boldsymbol{\theta}} + \left(A_r\frac{\partial B_\varphi}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\varphi}{\partial\theta} + \frac{A_\varphi}{r\sin\theta}\frac{\partial B_\varphi}{\partial\varphi} + \frac{A_\varphi B_r}{r} + \frac{A_\varphi B_\theta\cot\theta}{r}\right)\hat{\boldsymbol{\varphi}}$
Tensor divergence $\nabla\cdot\mathbf{T}$	— View by clicking [show] — $\left(\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{yx}}{\partial y} + \frac{\partial T_{zx}}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{zy}}{\partial z}\right)\hat{\mathbf{y}} + \left(\frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} + \frac{\partial T_{zz}}{\partial z}\right)\hat{\mathbf{z}}$	— View by clicking [show] — $\left[\frac{\partial T_{\rho\rho}}{\partial\rho} + \frac{1}{\rho}\frac{\partial T_{\varphi\rho}}{\partial\varphi} + \frac{\partial T_{z\rho}}{\partial z} + \frac{1}{\rho}(T_{\rho\rho} - T_{\varphi\varphi})\right]\hat{\boldsymbol{\rho}} + \left[\frac{\partial T_{\rho\varphi}}{\partial\rho} + \frac{1}{\rho}\frac{\partial T_{\varphi\varphi}}{\partial\varphi} + \frac{\partial T_{z\varphi}}{\partial z} + \frac{1}{\rho}(T_{\rho\varphi} + T_{\varphi\rho})\right]\hat{\boldsymbol{\varphi}} + \left[\frac{\partial T_{\rho z}}{\partial\rho} + \frac{1}{\rho}\frac{\partial T_{\varphi z}}{\partial\varphi} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{\rho z}}{\rho}\right]\hat{\mathbf{z}}$	— View by clicking [show] — $\left[\frac{\partial T_{rr}}{\partial r} + 2\frac{T_{rr}}{r} + \frac{1}{r}\frac{\partial T_{\theta r}}{\partial\theta} + \frac{\cot\theta}{r}T_{\theta r} + \frac{1}{r\sin\theta}\frac{\partial T_{\varphi r}}{\partial\varphi} - \frac{1}{r}(T_{\theta\theta} + T_{\varphi\varphi})\right]\hat{\mathbf{r}} + \left[\frac{\partial T_{r\theta}}{\partial r} + 2\frac{T_{r\theta}}{r} + \frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial\theta} + \frac{\cot\theta}{r}T_{\theta\theta} + \frac{1}{r\sin\theta}\frac{\partial T_{\varphi\theta}}{\partial\varphi} + \frac{T_{\theta r}}{r} - \frac{\cot\theta}{r}T_{\varphi\varphi}\right]\hat{\boldsymbol{\theta}} + \left[\frac{\partial T_{r\varphi}}{\partial r} + 2\frac{T_{r\varphi}}{r} + \frac{1}{r}\frac{\partial T_{\theta\varphi}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial T_{\varphi\varphi}}{\partial\varphi} + \frac{T_{\varphi r}}{r} + \frac{\cot\theta}{r}(T_{\theta\varphi} + T_{\varphi\theta})\right]\hat{\boldsymbol{\varphi}}$