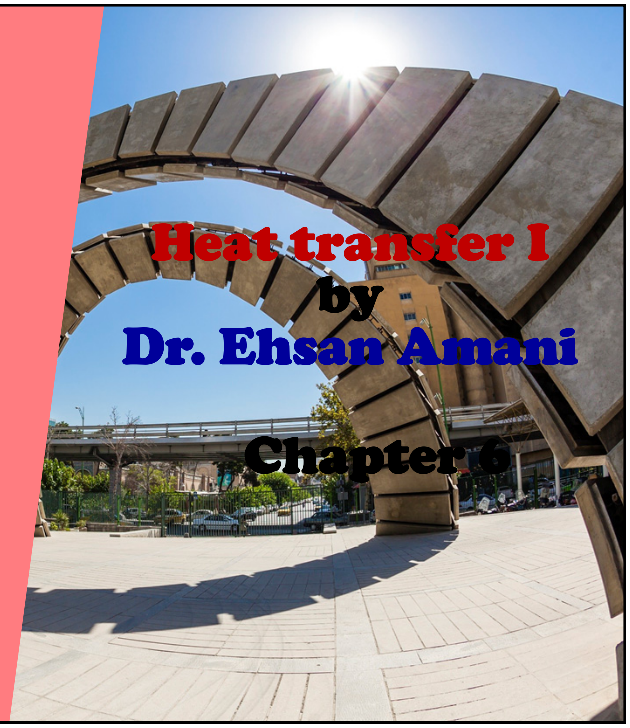


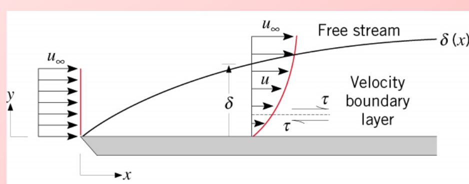
Heat transfer I
by
Dr. Ehsan Amani

Chapter 6

1.6 Objective parameters in HT engineering

- **The wall shear stress**
  - Pressure drop in internal flows
  - Drag force in external flows
  - 2D:

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \qquad (1.6)$$



  - 3D (exercise):

$$\tau_s = \mu\left[\left(\frac{\partial u}{\partial y}\Big|_{y=0}\right)^2 + \left(\frac{\partial w}{\partial z}\Big|_{y=0}\right)^2\right]^{1/2} \qquad (2.6)$$

  - Needs $u(x, y, z, t)$, $v(x, y, z, t)$, ...?

Chapter 6 By E. Amani

1.6 Objective parameters in HT engineering

- **The wall shear stress**
  - Non-dimensional parameters:

Friction factor $c_f \equiv \dfrac{\tau_s}{\rho V^2/2}$ (3.6)

Characteristic velocity

For external flows: $V \equiv u_\infty(x=0) \equiv u_{\infty,0}$ (4.6)

  - Area-averaged parameters:

Area-averaging operator $\bar{c}_f = \dfrac{1}{A_s}\int_{A_s} c_f dA$ (5.6)

$$\bar{c}_f = \frac{1}{A_s}\int_{A_s}\frac{\tau_s}{\rho V^2/2}dA \xrightarrow{\text{If } V=cte} = \frac{1}{\rho V^2/2}\underbrace{\frac{1}{A_s}\int_{A_s}\tau_s dA}_{\bar{\tau}_s} = \frac{\bar{\tau}_s}{\rho V^2/2} \quad (6.6)$$

$\hat{e}_\tau$

$$\vec{F}_\tau = \int_{A_s}\tau_s \hat{e}_\tau dA \xrightarrow{\text{If } \hat{e}_\tau = cte} = \hat{e}_\tau\int_{A_s}\tau_s dA = \hat{e}_\tau \bar{\tau}_s A_s$$

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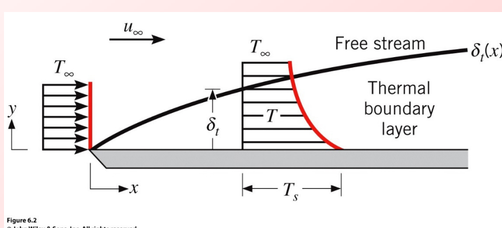
1.6 Objective parameters in HT engineering

- **Convection heat transfer coefficient**



Lecture Notes

- **To compute τ_s and h, the fields $\vec{V}(x,y,z,t)$ and $T(x,y,z,t)$ should be known a priori.**
- **Flow equations + heat transfer equation**

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2.6 Flow and heat transfer governing equations

- **Fluid flow equations (Fluid mechanics II)**
  - Summary from fluid mechanics II
  - Continuity (chap6-table6-1.pdf)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{U}) = 0 \qquad (16.6)$$

Cartesian $\vec{U} = u_x \hat{\imath} + u_y \hat{\jmath} + u_z \hat{k}$ or $\vec{U} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (16.6)'$$

Incompressible $\vec{\nabla}.\vec{U} = 0 \qquad (19.6)$

Cartesian $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (19.6)'$$

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2.6 Flow and heat transfer governing equations

- **Fluid flow equations (Fluid mechanics II)**
  - Summary from fluid mechanics II
  - Continuity (chap6-table6-1.pdf)

Incompressible $\vec{\nabla}.\vec{U} = 0 \qquad (19.6)$

Cartesian $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (19.6)'$$

§B.1 NEWTON'S LAW OF VISCOSITY

$[\boldsymbol{\tau} = +\mu(\nabla \mathbf{v} + (\nabla \mathbf{v})^\dagger) + (\frac{2}{3}\mu + \kappa)(\nabla \cdot \mathbf{v})\boldsymbol{\delta}]$

Cartesian coordinates (x, y, z):

$$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates (r, θ, z):

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

Spherical coordinates (r, θ, ϕ):

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2.6 Flow and heat transfer governing equations

- **Fluid flow equations (Fluid mechanics II)**
  - Summary from fluid mechanics II
  - Continuity (chap6-table6-1.pdf)
  - Momentum (chap6-table6-1.pdf, chap6-table6-2.pdf)

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Fluid flow equations (Fluid mechanics II)**

§B.5 THE EQUATION OF MOTION IN TERMS OF τ

$$[\rho D\mathbf{v}/Dt = -\nabla p + [\nabla \cdot \boldsymbol{\tau}] + \rho \mathbf{g}]$$

Cartesian coordinates (x, y, z):[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x \quad \text{(B.5-1)}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y \quad \text{(B.5-2)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z \quad \text{(B.5-3)}$$

[a] These equations have been written without making the assumption that τ is symmetric. This means, for example, that when the usual assumption is made that the stress tensor is symmetric, τ_{xy} and τ_{yx} may be interchanged.

Cylindrical coordinates (r, θ, z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r \quad \text{(B.5-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta \quad \text{(B.5-5)}$$

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Fluid flow equations (Fluid mechanics II)**

§B.1 NEWTON'S LAW OF VISCOSITY

$$[\boldsymbol{\tau} = +\mu(\nabla\mathbf{v} + (\nabla\mathbf{v})^\dagger) + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})\boldsymbol{\delta}]$$

Cartesian coordinates (x, y, z):

$$\tau_{xx} = +\mu\left[2\frac{\partial v_x}{\partial x}\right] + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{yy} = +\mu\left[2\frac{\partial v_y}{\partial y}\right] + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{zz} = +\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{xy} = \tau_{yx} = +\mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]$$

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = +\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{\theta\theta} = +\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{zz} = +\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu + \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{r\theta} = \tau_{\theta r} = +\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = +\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]$$

Chapter 6 **By E. Amani**

2.6 Flow and heat transfer governing equations

- **Fluid flow equations (Fluid mechanics II)**

§B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \quad \text{(B.6-1)}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \quad \text{(B.6-2)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-3)}$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \quad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \quad \text{(B.6-5)}$$

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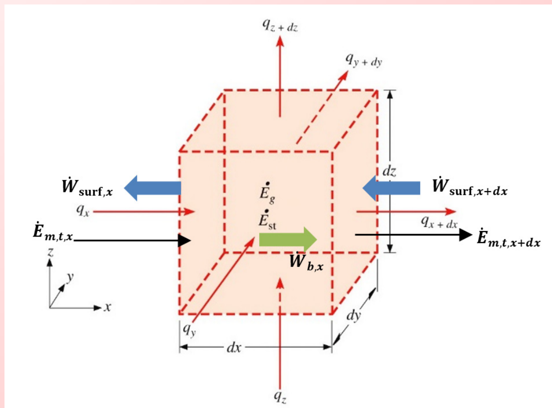
2.6 Flow and heat transfer governing equations

- **Energy equation**
  - ➢ Continuum
  - ➢ Single-component fluid



⇨ **Lecture Notes**

Chapter 6 **By E. Amani**

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - ➢ Continuum
  - ➢ Single-component fluid

$$\overbrace{\frac{\partial}{\partial t}\left[\rho\left(u_t+\frac{V^2}{2}\right)\right]}^{\text{rate of (energy) change at a fixed point}}+\overbrace{\frac{\partial}{\partial x}\left[\rho u_x\left(u_t+\frac{V^2}{2}\right)\right]+\frac{\partial}{\partial y}\left[\rho u_y\left(u_t+\frac{V^2}{2}\right)\right]+\frac{\partial}{\partial z}\left[\rho u_z\left(u_t+\frac{V^2}{2}\right)\right]}^{\text{net (energy) advection}}$$

$$=\overbrace{-\frac{\partial q''_x}{\partial x}-\frac{\partial q''_y}{\partial y}-\frac{\partial q''_z}{\partial z}}^{\text{net conduction}}\overbrace{-\frac{\partial p u_x}{\partial x}-\frac{\partial p u_y}{\partial y}-\frac{\partial p u_z}{\partial z}}^{\text{net pressure work (flow work)}}$$

$$\overbrace{+\frac{\partial}{\partial x}\left[\tau_{xx}u_x+\tau_{xy}u_y+\tau_{xz}u_z\right]+\frac{\partial}{\partial y}\left[\tau_{yx}u_x+\tau_{yy}u_y+\tau_{yz}u_z\right]+\frac{\partial}{\partial z}\left[\tau_{zx}u_x+\tau_{zy}u_y+\tau_{zz}u_z\right]}^{\text{net viscous work}}$$

$$\overbrace{+f_x u_x+f_y u_y+f_z u_z}^{\text{body force work}}+\overbrace{\dot{q}}^{\text{thermal energy generation}} \qquad (21.6)$$

Chapter 6 **By E. Amani**

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Differential form vs. integral form

$$\dot{E}_{\text{st}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g$$

$$\underbrace{\left(\dot{U}_t + \dot{KE}\right.}_{\text{rate of (energy) change}} + \underbrace{\left.\dot{PE}\right)_{\text{st}}}_{\text{body force work}} = \int_{\text{Net:in}} \left(\underbrace{pv}_{\text{flow work}} + \underbrace{u_t + \frac{v^2}{2}}_{\text{net (energy) advection}} + \underbrace{gz}_{\text{body force work}}\right) d\dot{m}$$

$$+ \int_{\text{Net:in}} \left(\underbrace{q''_{\text{cond}}}_{\text{net conduction}} + \underbrace{q''_{\text{conv}}}_{?} + \underbrace{q''_{\text{rad}}}_{?}\right) dA$$

$$+\underbrace{\dot{W}_s}_{?} + \underbrace{\dot{W}_{\text{flow},\tau}}_{\text{net viscous work}} \qquad (21.6)'$$

$$+ \underbrace{\int_V \dot{q}\, dV}_{\text{thermal energy generation}}$$

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Continuum
  - Single-component fluid

$$\frac{\partial}{\partial t}\left[\rho \overbrace{\left(\underbrace{u_t}_{?} + \frac{V^2}{2}\right)}^{e \text{ (total specific energy)}}\right] + \frac{\partial}{\partial x}\left[\rho u_x\left(u_t + \frac{V^2}{2}\right)\right] + \frac{\partial}{\partial y}\left[\rho u_y\left(u_t + \frac{V^2}{2}\right)\right] + \frac{\partial}{\partial z}\left[\rho u_z\left(u_t + \frac{V^2}{2}\right)\right]$$

$$= -\frac{\partial q''_x}{\partial x} - \frac{\partial q''_y}{\partial y} - \frac{\partial q''_z}{\partial z} - \frac{\partial p u_x}{\partial x} - \frac{\partial p u_y}{\partial y} - \frac{\partial p u_z}{\partial z}$$

$$+ \frac{\partial}{\partial x}\left[\tau_{xx}u_x + \tau_{xy}u_y + \tau_{xz}u_z\right] + \frac{\partial}{\partial y}\left[\tau_{yx}u_x + \tau_{yy}u_y + \tau_{yz}u_z\right] + \frac{\partial}{\partial z}\left[\tau_{zx}u_x + \tau_{zy}u_y + \tau_{zz}u_z\right]$$

$$+ f_x u_x + f_y u_y + f_z u_z + \dot{q} \qquad (21.6)$$

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2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Material derivative form

$$(21.6)-\left(u_t+\frac{V^2}{2}\right)\times(16.6)' \longrightarrow$$

$$\overbrace{\rho\frac{\partial}{\partial t}\left(u_t+\frac{V^2}{2}\right)+\left[\rho u_x\frac{\partial}{\partial x}\left(u_t+\frac{V^2}{2}\right)+\rho u_y\frac{\partial}{\partial y}\left(u_t+\frac{V^2}{2}\right)+\rho u_z\frac{\partial}{\partial z}\left(u_t+\frac{V^2}{2}\right)\right]}^{\rho\frac{D}{Dt}\left(u_t+\frac{V^2}{2}\right)}$$

$$=-\frac{\partial q''_x}{\partial x}-\frac{\partial q''_y}{\partial y}-\frac{\partial q''_z}{\partial z}-\frac{\partial pu_x}{\partial x}-\frac{\partial pu_y}{\partial y}-\frac{\partial pu_z}{\partial z}$$

$$+\frac{\partial}{\partial x}[\tau_{xx}u_x+\tau_{xy}u_y+\tau_{xz}u_z]+\frac{\partial}{\partial y}[\tau_{yx}u_x+\tau_{yy}u_y+\tau_{yz}u_z]+\frac{\partial}{\partial z}[\tau_{zx}u_x+\tau_{zy}u_y+\tau_{zz}u_z]$$

$$+f_xu_x+f_yu_y+f_zu_z+\dot{q} \qquad (22.6)$$

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Kinetic energy equation

$$u_x.(\mathrm{mom}_x)+u_y.(\mathrm{mom}_y)+u_z.(\mathrm{mom}_z)=$$
$$u_x.(B.5-1)+u_y.(B.5-2)+u_z.(B.5-3) \longrightarrow$$

$$\rho\frac{D(V^2/2)}{Dt}=-u_x\frac{\partial p}{\partial x}-u_y\frac{\partial p}{\partial y}-u_z\frac{\partial p}{\partial z}$$

$$+\frac{\partial}{\partial x}[\tau_{xx}u_x+\tau_{xy}u_y+\tau_{xz}u_z]+\frac{\partial}{\partial y}[\tau_{yx}u_x+\tau_{yy}u_y+\tau_{yz}u_z]+\frac{\partial}{\partial z}[\tau_{zx}u_x+\tau_{zy}u_y+\tau_{zz}u_z]$$

$$-\overbrace{\left[\tau_{xx}\frac{\partial u_x}{\partial x}+\tau_{xy}\frac{\partial u_y}{\partial x}+\tau_{xz}\frac{\partial u_z}{\partial x}+\tau_{yx}\frac{\partial u_x}{\partial y}+\tau_{yy}\frac{\partial u_y}{\partial y}+\tau_{yz}\frac{\partial u_z}{\partial y}+\tau_{zx}\frac{\partial u_x}{\partial z}+\tau_{zy}\frac{\partial u_y}{\partial z}+\tau_{zz}\frac{\partial u_z}{\partial z}\right]}^{\mu\Phi(>0)\,=\,\textbf{mechanical energy to thermal energy (viscous dissipation, irreversible)}}$$

$$+f_xu_x+f_yu_y+f_zu_z \qquad (23.6)$$

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Thermal energy equation

(22.6) - (23.6) $\longrightarrow$

$\rho \frac{Du_t}{Dt} \equiv$ rate of change at a fixed point + net advection

$$\underbrace{\rho\frac{\partial u_t}{\partial t}+\left[\rho u_x\frac{\partial u_t}{\partial x}+\rho u_y\frac{\partial u_t}{\partial y}+\rho u_z\frac{\partial u_t}{\partial z}\right]}_{\text{rate of change at a fixed point + net advection}}=\underbrace{-\frac{\partial q''_x}{\partial x}-\frac{\partial q''_y}{\partial y}-\frac{\partial q''_z}{\partial z}}_{\text{net conduction}}\underbrace{-p\frac{\partial u_x}{\partial x}-p\frac{\partial u_y}{\partial y}-p\frac{\partial u_z}{\partial z}}_{\text{pressure work to thermal energy (reversible)}}$$

$$+\underbrace{\left[\tau_{xx}\frac{\partial u_x}{\partial x}+\tau_{xy}\frac{\partial u_y}{\partial x}+\tau_{xz}\frac{\partial u_z}{\partial x}+\tau_{yx}\frac{\partial u_x}{\partial y}+\tau_{yy}\frac{\partial u_y}{\partial y}+\tau_{yz}\frac{\partial u_z}{\partial y}+\tau_{zx}\frac{\partial u_x}{\partial z}+\tau_{zy}\frac{\partial u_y}{\partial z}+\tau_{zz}\frac{\partial u_z}{\partial z}\right]}_{\mu\Phi(>0)\,=\,\text{mechanical energy to thermal energy (viscous dissipation, irreversible)}}$$

$$+\underbrace{\dot{q}}_{\text{thermal energy generation}} \qquad (24.6)$$

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - $\mu\Phi(>0)$: The viscous dissipation
    - ✓ Assuming Newtonian fluids for τ_{ij} (B.1)

$$\Phi=2\left[\left(\frac{\partial u_x}{\partial x}\right)^2+\left(\frac{\partial u_y}{\partial y}\right)^2+\left(\frac{\partial u_z}{\partial z}\right)^2\right]$$

$$+\left[\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)^2+\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right)^2+\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)^2\right]-\frac{2}{3}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)^2$$

(25.6)

- ✓ A source in the thermal energy equation
- ✓ A sink in the kinetic energy equation
- ✓ Converts energy from the kinetic to thermal form

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Incompressible flow
  - Fourier's law for conduction
  - No phase change

$$du_t = du_s = c_v dT = c_p dT \longrightarrow \frac{\partial u_t}{\partial t} = \frac{du_t}{dT}\frac{\partial T}{\partial t} = c_p \frac{\partial T}{\partial t}, \frac{\partial u_t}{\partial x} = \cdots \longrightarrow$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z}\right] = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \mu\Phi + \dot{q}$$

(26.6)

  - Exercise: Under which conditions is Eq. (26.6) simplified to Eq. (9.3) (Conduction equation)?

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Exercise: show that the thermal energy equation can be written as (enthalpy form): (hint: $u_t = h_t - pv$)

$$\overbrace{\rho\frac{\partial h_t}{\partial t} + \left[\rho u_x \frac{\partial h_t}{\partial x} + \rho u_y \frac{\partial h_t}{\partial y} + \rho u_z \frac{\partial h_t}{\partial z}\right]}^{\rho\frac{Dh_t}{Dt} \equiv \text{ rate of change at a fixed point + net advection}} \overbrace{-\frac{\partial p}{\partial t}}^{\text{from the rate of change at a fixed point}} = \overbrace{-\frac{\partial q''_x}{\partial x} - \frac{\partial q''_y}{\partial y} - \frac{\partial q''_z}{\partial z}}^{\text{net conduction}}$$

$$\overbrace{+u_x \frac{\partial p}{\partial x} + u_y \frac{\partial p}{\partial y} + u_z \frac{\partial p}{\partial z}}^{\text{pressure work to kinetic energy with minus sign}}$$

$$\overbrace{+\mu\Phi}^{\text{viscous dissipation}} + \overbrace{\dot{q}}^{\text{thermal energy generation}}$$

(24.6)h

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Energy equation**
  - Ideal gas $dh_t = c_p dT$
  - Fourier's law for conduction
  - No phase change
  - Exercise: Using Eq. (24.6)h, show that:

$$\rho c_p \left[\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z}\right] = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \frac{Dp}{Dt} + \mu\Phi + \dot{q} \quad (26.6)\text{i}$$

Chapter 6 By E. Amani

2.6 Flow and heat transfer governing equations

- **Computational Fluid Dynamics (CFD)**
  - Numerical solution of fluid flow equations: detailed design
    - ✓ Incompressible flows:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0 \quad (19.6)'$$

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \quad (\text{B.6-1})$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \quad (\text{B.6-2})$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad (\text{B.6-3})$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z}\right] = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \mu\Phi + \dot{q} \quad (26.6)$$

    - ✓ 5 equations and 5 unknowns ($u_x, u_y, u_z, p,$ and T)
    - ✓ Exercise: How about an ideal gas?

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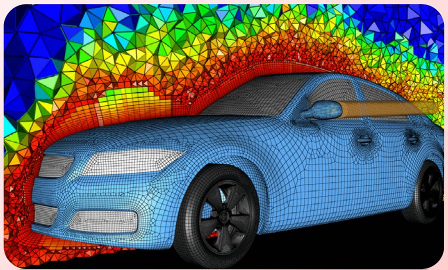
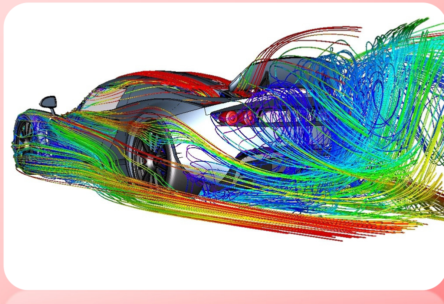
2.6 Flow and heat transfer governing equations

- **Computational Fluid Dynamics (CFD)**
  - Numerical solution of fluid flow equations: detailed design
    - ✓ Flow over a bluff body





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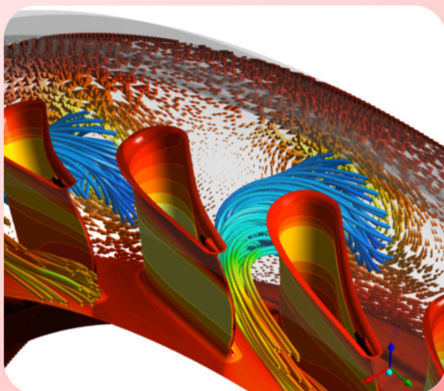
2.6 Flow and heat transfer governing equations

- **Computational Fluid Dynamics (CFD)**
  - Numerical solution of fluid flow equations: detailed design
    - ✓ Gas turbines



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2.6 Flow and heat transfer governing equations

- **Computational Fluid Dynamics (CFD)**
  - Numerical solution of fluid flow equations: detailed design
    - ✓ Gas turbines

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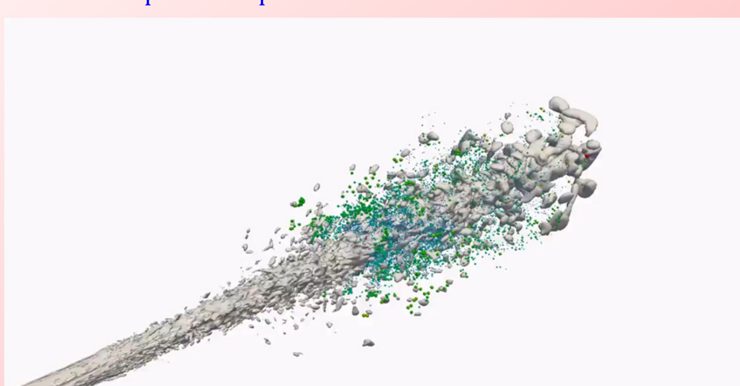
2.6 Flow and heat transfer governing equations

- **Computational Fluid Dynamics (CFD)**
  - Numerical solution of fluid flow equations: detailed design
    - ✓ Complex multiphase flows



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2.6 Flow and heat transfer governing equations

- **Computational Fluid Dynamics (CFD)**
  - Numerical solution of fluid flow equations: detailed design
    - ✓ Complex multiphase flows
    - ✓ For more examples, visit: https://sites.google.com/view/dramani

Chapter 6 By E. Amani

3.6 Boundary Layer (BL)

- **Importance**
  - A relevant topic in most of engineering problems
  - The governing equations are simpler than a general flow
  - Still shares many phenomena with general flows
  - Its dimensional analysis can be extended to general flows

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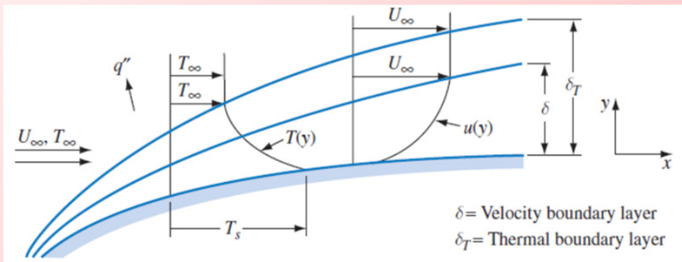
3.6 Boundary Layer (BL)

- **Hydrodynamic BL**
  - Large velocity gradients

Free-stream velocity

$$y=\delta(x): u(x,y)=0.99u_\infty(x) \qquad (28.6)$$



- **Thermal BL**
  - Large temperature gradients

$$y=\delta_T(x): \frac{T_s-T(x,y)}{T_s-T_\infty(x)}=0.99 \qquad (29.6)$$

3.6 Boundary Layer (BL)

- **Laminar vs. Turbulent**

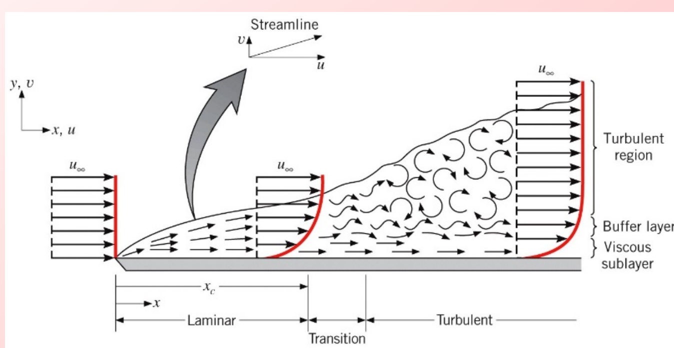


Figure 6.4

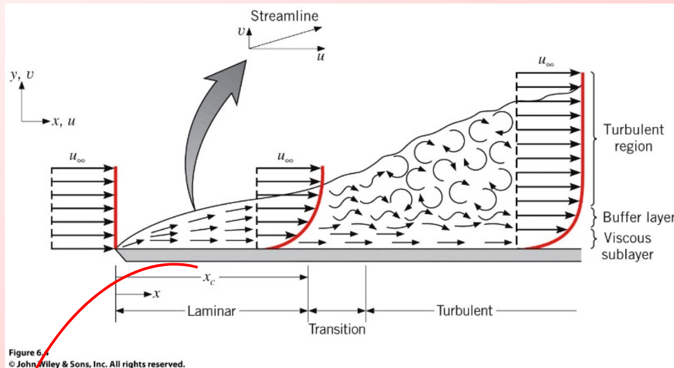
- Critical Reynolds number

Characteristic velocity; Characteristic length

$$Re \equiv \frac{\rho V L}{\mu} \quad (30.6) \xrightarrow{\text{For BL}} \begin{matrix} V\equiv u_\infty(x) \\ L\equiv x \end{matrix} \longrightarrow Re_x \equiv \frac{\rho u_\infty x}{\mu} = \frac{u_\infty x_c}{\nu} \quad (31.6)$$

3.6 Boundary Layer (BL)

- **Laminar vs. Turbulent**



f(geomety, surface roughness, freestream turbulence, ...)

- ➢ Critical Reynolds number

$$Re_{x,c} \equiv \frac{u_\infty x_c}{\nu} \quad (32.6) \quad \xrightarrow{\textbf{For BL}} \quad Re_{x,c} \sim 5\times10^5 \quad (33.6)$$

on a smooth flat plate

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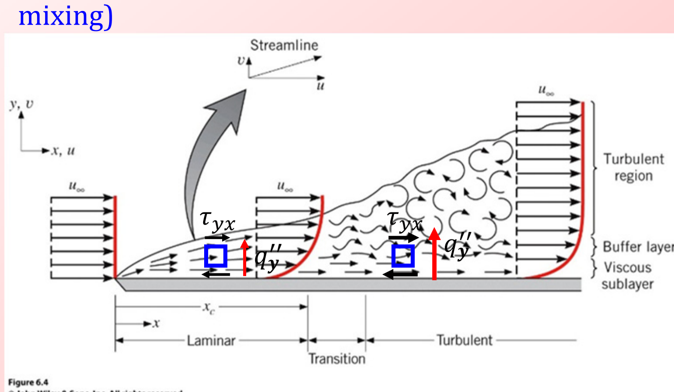
3.6 Boundary Layer (BL)

- **Laminar vs. Turbulent**
  - ➢ Effective shear stress (viscous stress $\mu \partial u/\partial y$+ turbulent mixing)



  - ➢ Heat transfer augmentation (conduction $k\partial T/\partial y$+ turbulent mixing)

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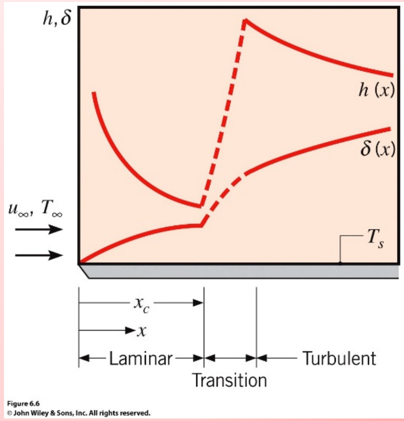
3.6 Boundary Layer (BL)

- **Laminar vs. Turbulent**
  - The variation of h along BL



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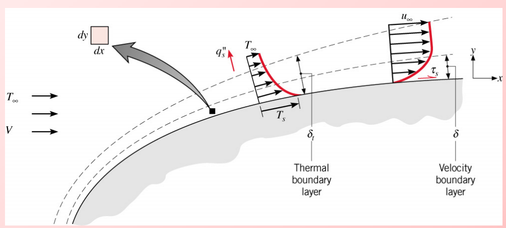
3.6 Boundary Layer (BL)



- **BL equations**
- **BL approximation**
  - Incompressible flow
  - Steady
  - 2D
  - Constant properties ($\mu, k = cte$)
  - Negligible body-force effect ($f_x \sim f_y \sim 0$)
  - Negligible thermal energy generation ($\dot{q} \cong 0$)
  - Thin boundary layer ($\delta(x) \ll L$)

$$\frac{\partial p}{\partial y} \sim 0 \longrightarrow p = p(x) = p_\infty(x) \longrightarrow \frac{\partial p}{\partial x} = \frac{dp_\infty}{dx}$$

$$u \gg v \qquad \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} \qquad \frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial x^2} \qquad (34.6)$$

$$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x} \qquad \frac{\partial^2 T}{\partial y^2} \gg \frac{\partial^2 T}{\partial x^2}$$

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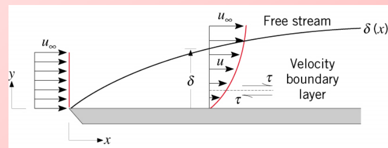
3.6 Boundary Layer (BL)

- **BL equations**
  - Applying BL approximation, Eq. (34.6), to Navier-Stokes, Eq. (B.6-1), and energy, Eq. (26.6):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (35.6)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp_\infty}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (36.6)$$



$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (37.6)$$

  - Boundary conditions:

$$u(x,0)=v(x,0)=0 \qquad (38.6)$$

$$u(x,\infty)=u_\infty(x) \qquad (39.6)$$

$$T(x,0)=T_s \qquad (40.6) \quad \text{or} \quad -k\left.\frac{\partial T}{\partial y}\right|_{y=0}=q_s'' \qquad (40.6)'$$

$$T(x,\infty)=T_\infty \qquad (41.6)$$

Chapter 6 **By E. Amani**

3.6 Boundary Layer (BL)

- **BL equations**
  - Coupling with the free-stream flow through dp_∞/dx, u_∞, and T_∞
  - Free-stream equations:
    1. Potential flow equations:

$$-\frac{dp_\infty}{dx}=\rho u_\infty\frac{du_\infty}{dx}, u_\infty(x=0)=u_{\infty,0} \qquad \textbf{Bernoulli's equation} \quad (42.6)$$

$$\nabla^2\Phi=0, \text{B.C.}(L), \vec{V}=\vec{\nabla}\Phi \qquad \boldsymbol{\Phi}\textbf{ :Velocity potential}$$

    2. Flat plate:

$$u_\infty(x)=u_{\infty,0}=cte \qquad \frac{dp_\infty}{dx}=0 \qquad (42.6\,)'$$

    3. General case: Navier-Stokes

Chapter 6 **By E. Amani**

4.6 Dimensional analysis and similitude

- **Constant wall temperature**
  - Non-dimensionalizing BL equations, Eqs. (35.6)-(41.6)
  - Steps:
    1. Dependent variables: u, v, T, p_∞
    2. Independent variables: x, y
    3. Non-dimensional variables:

$$x^* = \frac{x}{x_0}, y^* = \frac{y}{y_0}, u^* = \frac{u}{u_0}, v^* = \frac{v}{v_0}, p^* = \frac{p_\infty}{p_0}, T^* = \frac{\overbrace{T - T_s}^{\theta}}{\theta_0}$$

    4. Characteristic parameters:

$$x_0 = y_0 = L$$
$$u_0 = v_0 = u_{\infty,0} = V$$
$$\theta_0 = \theta_\infty = T_\infty - T_s$$

Lecture Notes

4.6 Dimensional analysis and similitude

- **Constant wall temperature**

TABLE 6.1 The boundary layer equations and their y-direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation	Boundary Conditions: Wall	Boundary Conditions: Free Stream	Similarity Parameter(s)
Velocity	$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$ (6.21)′ $u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.21)	$u^*(x^*, 0) = 0$	$u^*(x^*, \infty) = \frac{u_\infty(x^*)}{V}$ (6.23)	$Re_L = \frac{VL}{\nu}$ (6.25)
Thermal	$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Ec}{Re_L}\left(\frac{\partial u^*}{\partial y^*}\right)^2$ (6.22)	$T_s = cte$: $T^*(x^*, 0) = 0$ $q''_s = cte$: $\left.\frac{\partial T^*}{\partial y^*}\right\vert_{y^*=0} = -1$	$T^*(x^*, \infty) = 1$ (6.24) $T^*(x^*, \infty) = 0$	$Re_L, Pr = \frac{\nu}{\alpha}$ (6.26) $Ec = \frac{V^2}{c_p\theta_0}$

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, u^* = \frac{u}{V}, v^* = \frac{v}{V}, p^* = \frac{p_\infty}{\rho V^2}, T^* = \frac{T - T_s}{\theta_0} \quad (43.6)$$

$$\theta_0 = T_\infty - T_s$$

  - The governing (non-dimensional) parameters:

$$\mathrm{Re}_L \equiv \frac{VL}{\nu} \quad (47.6) \qquad \mathrm{Pr} = \frac{\nu}{\alpha} \quad (45.6) \qquad \mathrm{Ec} = \frac{V^2}{c_p\theta_0} \quad (46.6)$$

Prandtl number **Eckert number**

4.6 Dimensional analysis and similitude

- **Constant wall heat flux**
  - Exercise: Show that Table 6.1 is valid + Eq. (43.6)-(46.6), with: $\theta_0 = \dfrac{q_s'' L}{k}$ (43.6)′
- **Corollary**

$$v^*, u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right) \qquad (47.6)$$

$$T^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}, Pr, Ec\right) \qquad (48.6)$$

Geometry effect **High Mach numbers or highly viscous flows**

  - Exercise: Using a dimensional analysis, prove Eqs. (47.6) and (48.6), starting from

$$u = f\left(x, y, \rho, \upsilon, \frac{dp_\infty}{dx}, L, V, L\right), T - T_s = f\left(x, y, \rho, \upsilon, \frac{dp_\infty}{dx}, L, V, \alpha, c_p, T_\infty - T_s\right)$$

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4.6 Dimensional analysis and similitude

- **Corollary**

$$v^*, u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right) \qquad T^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}, Pr, Ec\right)$$

  - Objective parameters:

⇨ **Lecture Notes**

TABLE 6.3 Functional relations pertinent to the Reynolds analogy

Fluid Flow		Heat Transfer	
$u^* = f\left(x^*, y^*, Re_L, \dfrac{dp^*}{dx^*}\right)$	(6.27)	$T^* = f\left(x^*, y^*, Re_L, Pr, \dfrac{dp^*}{dx^*}\right)$	(6.30)
$C_f = \dfrac{2}{Re_L}\dfrac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.28)	$Nu = \dfrac{hL}{k} = +\dfrac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.31) **(T_s= cte)**
$C_f = \dfrac{2}{Re_L} f(x^*, Re_L)$	(6.29)	$Nu = f(x^*, Re_L, Pr)$	(6.32)
		$\overline{Nu} = f(Re_L, Pr)$	(6.33)

Chapter 6 **By E. Amani**

5.6 Physical meaning of non-dimensional parameters

- **Reynolds number**
  - The ratio of inertial to viscous forces:

$$\frac{F_I}{F_v}=\frac{u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}}{\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}}=Re_L f\left(u^*,v^*,\frac{\partial u^*}{\partial y^*},\ldots\right)\propto Re_L$$

TABLE 6.1 The boundary layer equations and their y-dire

Boundary Layer	Conservation Equation
Velocity	$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$ (6.21)′ $u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.21)

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5.6 Physical meaning of non-dimensional parameters

- **Prandtl number**
  - The ratio of (non-dimensional) momentum to energy diffusions:

$$\frac{\delta}{\delta_T}\sim\frac{Diff_v}{Diff_e}=\frac{\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}}{\frac{1}{Re_L \Pr}\frac{\partial^2 T^*}{\partial y^{*2}}}=\Pr f\left(\frac{\partial^2 u^*}{\partial y^{*2}},\frac{\partial^2 T^*}{\partial y^{*2}}\right)\propto \Pr \longrightarrow \frac{\delta}{\delta_T}\sim \Pr$$

$n>0$ **: empirical constant**

$$\left(\frac{\delta}{\delta_T}\right)^n \simeq \Pr \qquad (56.6)$$

Boundary Layer	Conservation Equation
Velocity	$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$ (6.21)′ $u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.21)
Thermal	$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{Ec}{Re_L}\left(\frac{\partial u^*}{\partial y^*}\right)^2$ (6.22)

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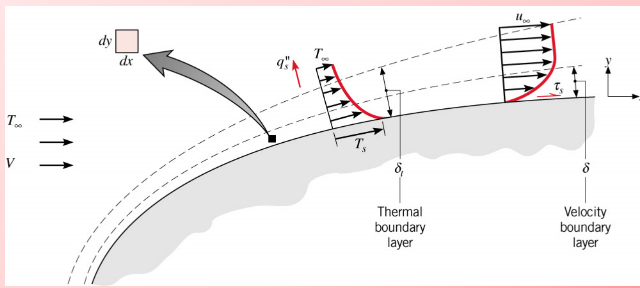
5.6 Physical meaning of non-dimensional parameters

- **Prandtl number**
  - Thermal vs. hydrodynamic BL: $\left(\frac{\delta}{\delta_T}\right)^n \simeq \mathrm{Pr}$
    - ✓ Gases: $\mathrm{Pr} \sim O(1) \longrightarrow \delta_T \approx \delta$
    - ✓ Liquid metals: $\mathrm{Pr} \ll 1 \longrightarrow \delta_T \gg \delta$
    - ✓ Oils: $\mathrm{Pr} \gg 1 \longrightarrow \delta_T \ll \delta$



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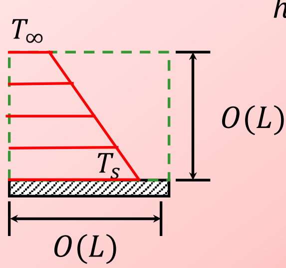
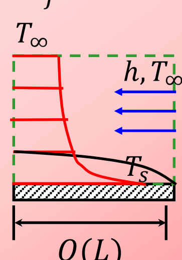
5.6 Physical meaning of non-dimensional parameters

- **Nusselt number**
  - Heat transfer enhancement by convection
  - The ratio of convection to pure conduction:

$$\frac{q''_{\mathrm{conv}}}{q''_{\mathrm{cond}}} = \frac{R_{\mathrm{cond}(ext)}}{R_{\mathrm{conv}}} \sim \frac{\frac{L}{k_f A}}{\frac{1}{hA}} = \frac{hL}{k_f} = \mathrm{Nu}$$





  - Exercise: What is the difference between Nu and Bi?

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5.6 Physical meaning of non-dimensional parameters

- **Important non-dimensional parameters of fluid mechanics**

TABLE 6.2 Selected dimensionless groups of heat transfer

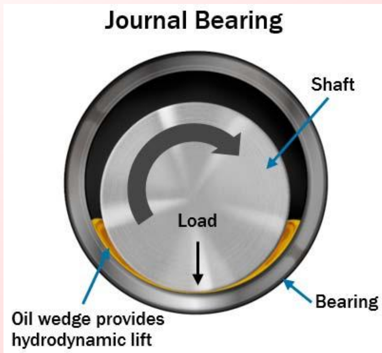
Group	Definition	Interpretation
Biot number (Bi)	$\frac{hL}{k_s}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance
Bond number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress
Eckert number (Ec)	$\frac{V^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference
Fourier number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time
Friction factor (f)	$\frac{\Delta p}{(L/D)(\rho u_m^2/2)}$	Dimensionless pressure drop for internal flow
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	Measure of the ratio of buoyancy forces to viscous forces
Colburn j factor	[illegible]	Dimensionless heat transfer coefficient

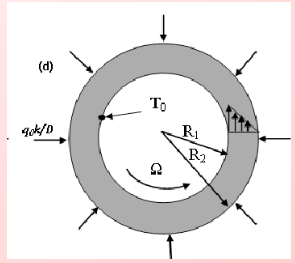
6.6 The similarity between momentum and heat transfer

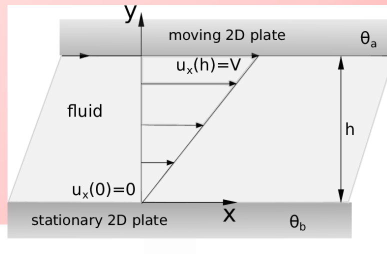
- **Exercise: ?**

7.6 Simple flows: analytical solutions

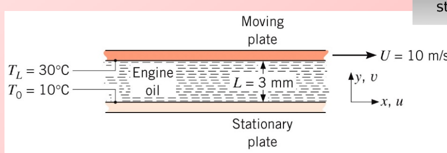
- **Couette flow**







  - See EXAMPLE 6S.1:



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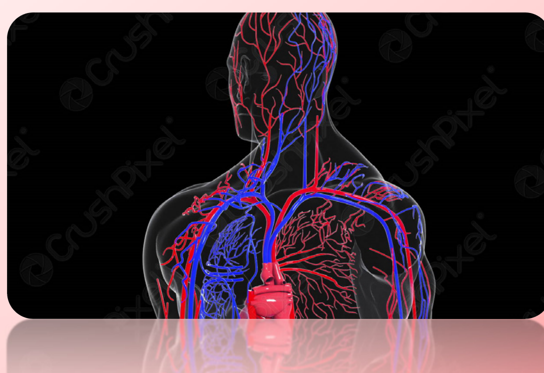
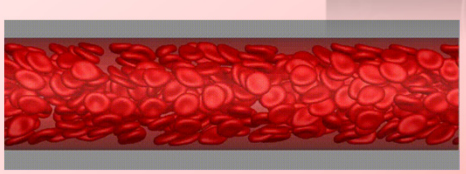
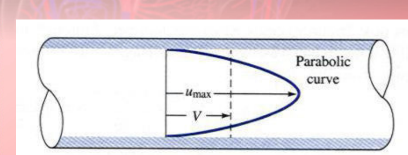
7.6 Simple flows: analytical solutions

- **Poiseuille flow**
  - Blood flow in some capillaries – drug delivery







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7.6 Simple flows: analytical solutions

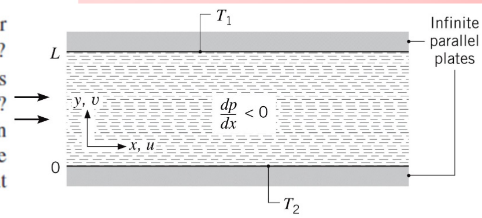
Sample problem

6S.10 Consider the problem of steady, incompressible laminar flow between two stationary, infinite parallel plates maintained at different temperatures.

Referred to as *Poiseuille flow* with heat transfer, this special case of parallel flow is one for which the x velocity component is finite, but the y- and z-components (v and w) are zero.

(a) What is the form of the continuity equation for this case? In what way is the flow *fully developed*?

(b) What forms do the x- and y-momentum equations take? What is the form of the velocity profile? Note that, unlike Couette flow, fluid motion between the plates is now sustained by a finite pressure gradient. How is this pressure gradient related to the maximum fluid velocity?



(c) Assuming viscous dissipation to be significant and recognizing that conditions must be thermally fully developed, what is the appropriate form of the energy equation? Solve this equation for the temperature distribution. What is the heat flux at the upper ($y = L$) surface?

The end of chapter 6