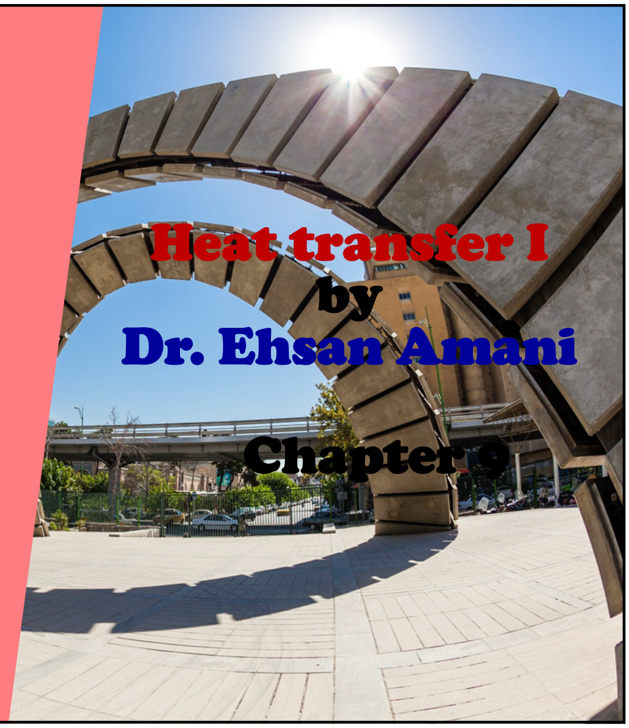


Heat transfer I

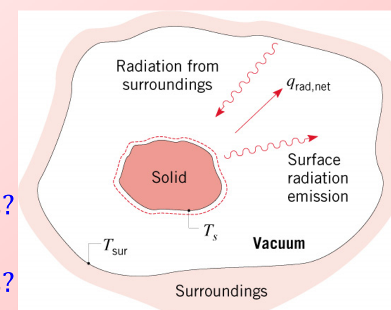
by

Dr. Ehsan Amani

Chapter 9

1.9 Radiation: Introduction

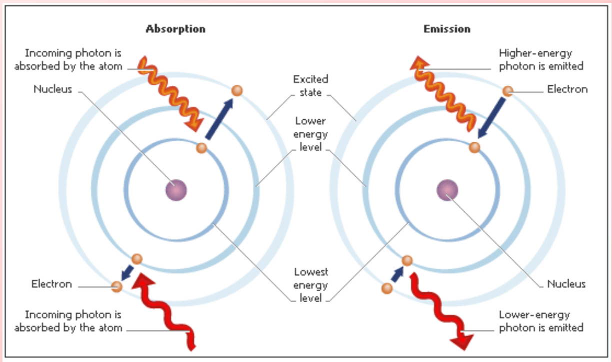
- **Evidence**
  - Consider a solid of temperature T_s in an evacuated enclosure whose walls are at a fixed temperature T_{surr}
  - What changes occur if $T_s > T_{\text{surr}}$? Why?
  - What changes occur if $T_s < T_{\text{surr}}$? Why?



- **Radiation**
  - Emission corresponds to heat transfer from the matter and hence to a reduction in its thermal energy
  - Radiation may also be intercepted and absorbed by matter, resulting in the increase in its thermal energy.

1.9 Radiation: Introduction

- **Physics**
  - Emission is due to oscillations and transitions of the many electrons that comprise matter, which are, in turn, sustained by the thermal energy of the matter
  - The physical mechanism underlying this transfer is complex and not fully understood



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1.9 Radiation: Introduction

- **Modeling**
  - Quantum mechanics (microscopic)
  - Wave mechanics (macroscopic)
  - Both (wave–particle duality)

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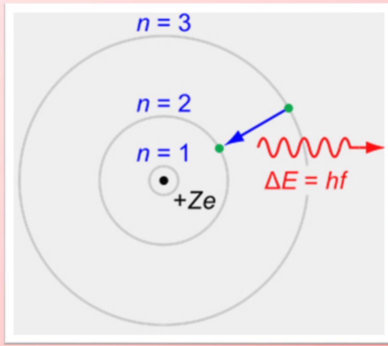
2.9 Radiation modeling

- **Wave–particle duality**
  - Photon: An elementary particle, pack of energy
  - Each wave or photon carries with it an amount of energy, ε, determined from quantum mechanics as:

$$\epsilon = h\nu, \qquad h = 6.626 \times 10^{-34}\ \text{J s}, \qquad (1.9)$$

Wave frequency **Planck's constant**



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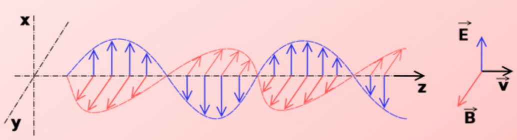
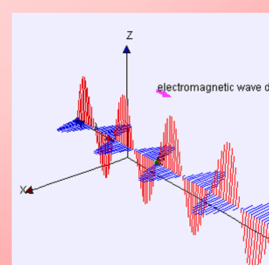
2.9 Radiation modeling

- **Wave**
  - In physics, a wave is an oscillation of a physical quantity accompanied by a transfer of energy that travels through space or mass
- **Electromagnetic waves**
  - Oscillations of electric and magnetic fields that propagate at the speed of light through a vacuum





  - Produced whenever charged particles are accelerated, and these waves can subsequently interact with any charged particles

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2.9 Radiation modeling

- **Electromagnetic waves**
  - Characterized by a wavelength λ and frequency ν which are related through the speed c at which radiation propagates in the medium

$$\lambda = \frac{c}{\nu} \qquad (2.9)$$

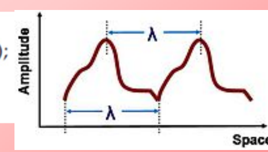
EM wave speed in vacuum

$$c = \frac{c_0}{n} \qquad (3.9) \qquad c_0 = 2.998 \ x \ 10^8 \ \text{m/s}$$

Refractive index of the medium (1; vacuum, ~1 gases; …)

  - Each wave may be identified either by its:

frequency, ν	(measured in cycles/s = s^{-1} = Hz);
wavelength, λ	(measured in μm = 10^{-6} m or nm = 10^{-9} m);
wavenumber, η	(measured in cm^{-1}); or
angular frequency, ω	(measured in radians/s = s^{-1}).



  - All four quantities are related:

$$\nu = \frac{\omega}{2\pi} = \frac{c}{\lambda} = c\eta.$$

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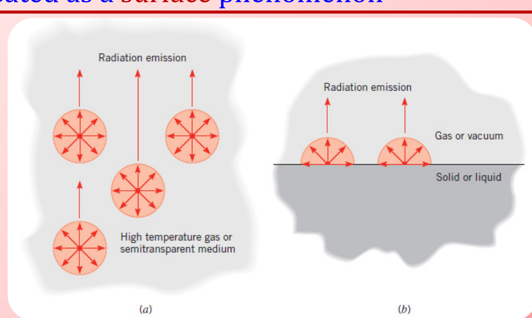
2.9 Radiation modeling

- **Volumetric versus surface phenomenon**
  - Emission from a gas or a semitransparent solid or liquid is a volumetric phenomenon
  - For an opaque solid or liquid, emission originates from atoms and molecules within 1 μm of the surface
  - Emission from an opaque solid or liquid, except in nanoscale, is treated as a surface phenomenon



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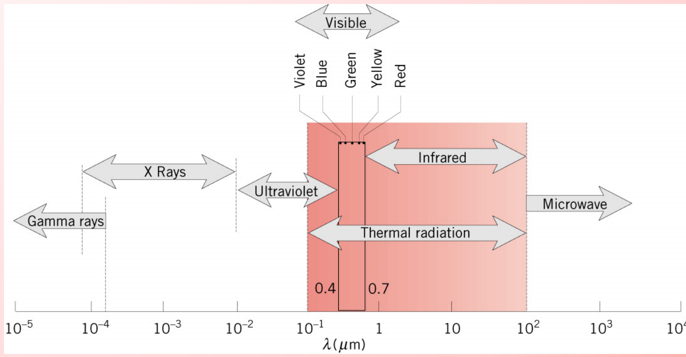
2.9 Radiation modeling

- **The electromagnetic spectrum**



  - Thermal radiation is confined to the infrared, visible and ultraviolet regions of the spectrum $(0.1 < \lambda < 100\mu m)$
  - These wavelengths can affect the temperature of the material

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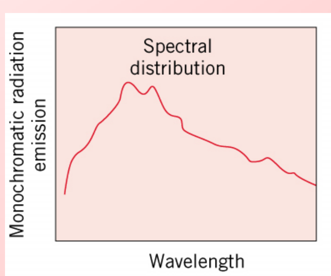
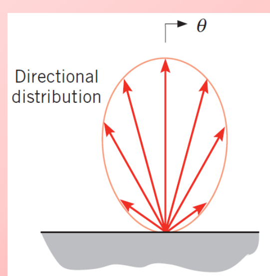
2.9 Radiation modeling

- **Directional and spectral consideration**
  - The amount of radiation emitted by an opaque surface varies with wavelength
  - The spectral distribution over all wavelengths
  - Radiation emitted by a surface will be in all directions associated with a hypothetical hemisphere about the surface and is characterized by a directional distribution.





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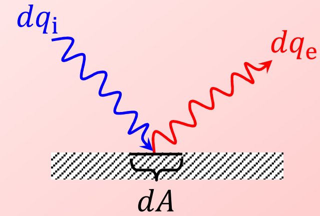
3.9 Radiation heat flux

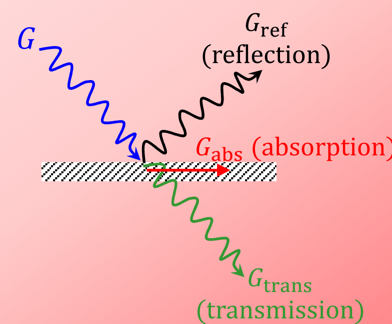
- **Important definitions**

emissive power $\left(\frac{W}{m^2}\right)$ ← $E = \frac{dq_e}{dA}$



irradiation $\left(\frac{W}{m^2}\right)$ ← $G = \frac{dq_i}{dA}$

$$G = \underbrace{G_a}_{\alpha G} + \underbrace{G_r + G_t}_{(1-\alpha)G} \qquad (3.9)$$



absorptivity $(-)$ ← $\alpha = \frac{G_a}{G}$

reflectivity $(-)$ ← $\rho = \frac{G_r}{G} \qquad (4.9)$

transmissivity $(-)$ ← $\tau = \frac{G_t}{G}$

$$\rho + \alpha + \tau = 1 \qquad (5.9)$$

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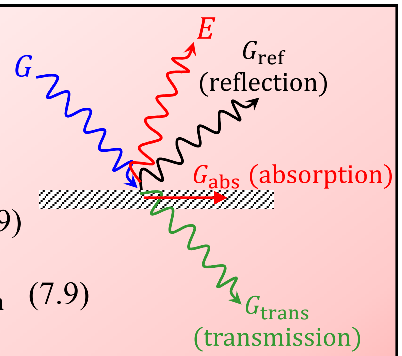
3.9 Radiation heat flux

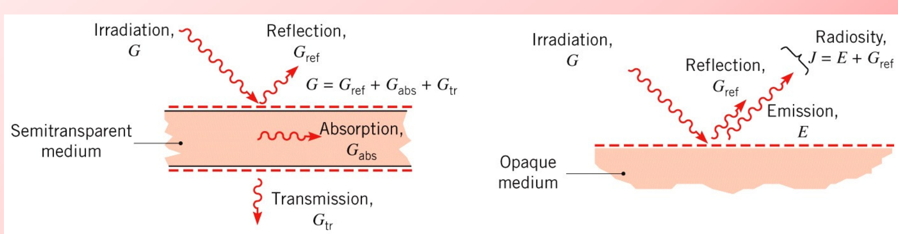


- **Important definitions**

radiosity $\left(\frac{W}{m^2}\right)$ ← $J = E + G_r + G_t \qquad (6.9)$

Net radiative flux $\left(\frac{W}{m^2}\right)$ ← $q''_{rad} = J - G = E - G_a \qquad (7.9)$

- Opaque surface ($\tau = 0$)



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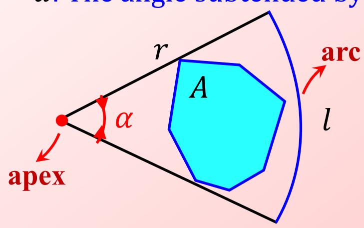
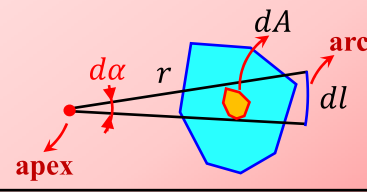
4.9 Directional and spectral considerations

- **Angle**
  - Plane angle: measure of the ratio of a circle arc to its radius
  - α: The angle subtended by the object A or arc l



$$\underbrace{\alpha}_{\text{(rad)}} \equiv \frac{l}{r} \qquad 0 \le \alpha \le 2\pi$$

  - $d\alpha$: The differential angle subtended by the infinitesimal object dA or infinitesimal arc dl



$$d\alpha = \frac{dl}{r}$$

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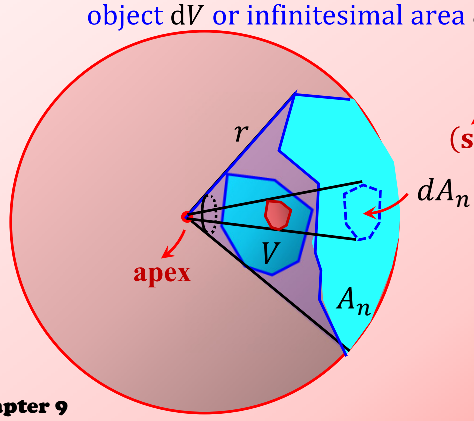
4.9 Directional and spectral considerations

- **Solid angle**
  - Extension of angle to 3D space
  - ω: The angle subtended by the object V or spherical surface A_n
  - $d\omega$: The differential solid angle subtended by the infinitesimal object dV or infinitesimal area dA_n



$$\underbrace{\omega}_{\text{(sr)}} \equiv \frac{A_n}{r^2} \qquad 0 \le \omega \le \underbrace{4\pi}_{\substack{2\pi \\ \text{(for a plane surface)}}}$$

$$d\omega \equiv \frac{dA_n}{r^2} \qquad (8.9)$$

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4.9 Directional and spectral considerations

- **Solid angle**
  - Using spherical coordinates (elements)

$$dA_n = r^2 \sin\theta d\theta d\phi \longrightarrow d\omega = \frac{dA_n}{r^2} = \sin\theta d\theta d\phi \qquad (9.9)$$

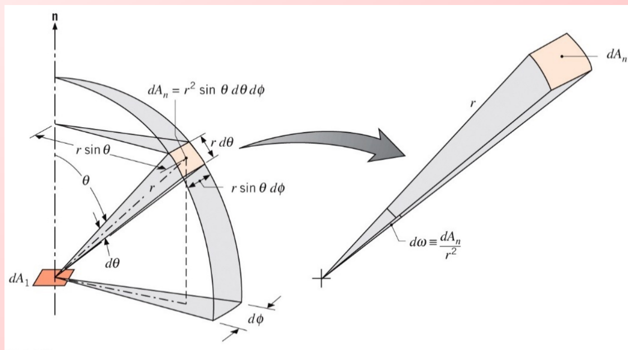

Figure 12.7

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4.9 Directional and spectral considerations

- **Spectral intensity**
  - Radiation emitted by a surface will be in all directions associated with a hypothetical hemisphere about the surface

Emission from surface element dA_1 in directions $(\omega + d\omega)$ and with wavelengths $(\lambda + d\lambda)$

$\boldsymbol{f(T, \text{surface}, \lambda, \theta, \phi)}$

$$I_{\lambda,e}(\lambda,\theta,\phi) \equiv \frac{dq_e}{\underbrace{(dA_1\cos\theta)}_{\text{Project surface area perpendicular to the direction}} \cdot d\omega \cdot d\lambda} \qquad (10.9)$$

Spectral intensity ($\frac{W}{m^2 sr \mu m}$)

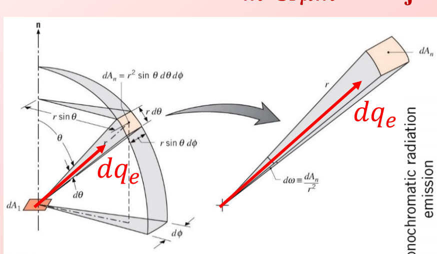
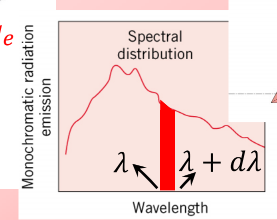
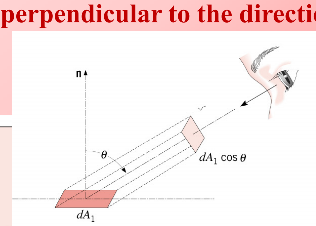


Figure 12.7





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4.9 Directional and spectral considerations

- **Spectral emissive power**
  - Radiation emitted with wavelengths $(\lambda + d\lambda)$ over all directions

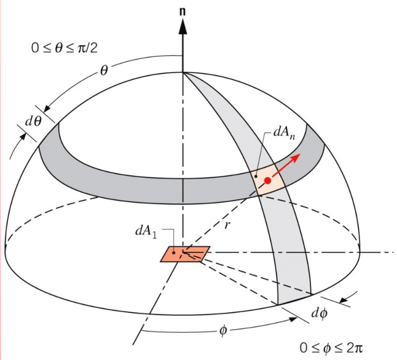
$$E_\lambda(\lambda) = q''_\lambda(\lambda) = \int_{dir} \frac{dq_e}{dA\,d\lambda} = \int_0^{2\pi} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\, d\omega \qquad (9.9)$$

$$E_\lambda(\lambda) = q''_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi \qquad (11.9)$$

Spectral emissive power ($\frac{W}{m^2\mu m}$)

total emissive power ($\frac{W}{m^2}$)

$$E = q''_e = \int_0^\infty E_\lambda(\lambda)d\lambda = \int_0^\infty\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi\, d\lambda \qquad (12.9)$$



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4.9 Directional and spectral considerations

- **Diffuse emitter**
  - Radiation intensity is independent of direction

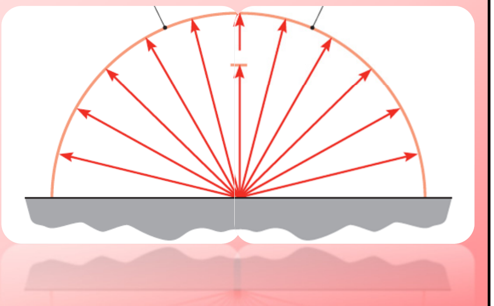
$$I_{\lambda,e}(\lambda,\theta,\phi) = I_{\lambda,e}(\lambda) \qquad (13.9)$$

  - Exercise: Show that for a diffuse surface

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda)$$
$$E = \pi I_e \qquad (14.9)$$

$$I_e = \int_0^\infty I_{\lambda,e}(\lambda)d\lambda \qquad (15.9)$$

Total intensity ($\frac{W}{m^2 sr}$)



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4.9 Directional and spectral considerations

- **Other radiation intensities**
  - Similarly,

Incident radiation intensity $(\frac{W}{m^2 sr \mu m})$

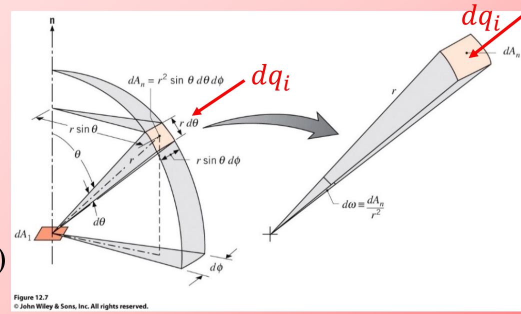
$$I_{\lambda,i}(\lambda,\theta,\phi) \equiv \frac{dq_i}{(dA_1\cos\theta)\cdot d\omega \cdot d\lambda} \qquad (16.9)$$

$$G_\lambda(\lambda) = \int_0^{2\pi} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta d\omega \qquad (17.9)$$

spectral irradiation $(\frac{W}{m^2 \mu m})$

$$G = \int_0^\infty G_\lambda(\lambda) d\lambda \qquad (18.9)$$

Total irradiation $(\frac{W}{m^2})$



  - Diffuse irradiation

$$G_\lambda(\lambda) = \pi I_{\lambda,i}(\lambda) \qquad (19.9)$$

$$G = \pi I_i$$

$$I_i = \int_0^\infty I_{\lambda,i}(\lambda) d\lambda \qquad (20.9)$$

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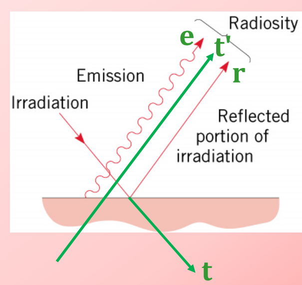
4.9 Directional and spectral considerations

- **Other radiation intensities**
  - Similarly,

Radiosity intensity $(\frac{W}{m^2 sr \mu m})$

$$I_{\lambda,e+r+t'}(\lambda,\theta,\phi) \equiv I_{\lambda,e}(\lambda,\theta,\phi) + I_{\lambda,r}(\lambda,\theta,\phi) + I_{\lambda,t'}(\lambda,\theta,\phi)$$



$$J_\lambda(\lambda) = \int_0^{2\pi} I_{\lambda,e+r+t'}(\lambda,\theta,\phi)\cos\theta d\omega \qquad (21.9)$$

spectral radiosity $(\frac{W}{m^2 \mu m})$

$$J = q''_{rad} = \int_0^\infty J_\lambda(\lambda) d\lambda \qquad (18.9)$$

Total radiosity $(\frac{W}{m^2})$

  - Diffuse surfaces (diffusive emitter + reflector + transmitter)

$$J_\lambda(\lambda) = \pi I_{\lambda,e+r+t'}(\lambda) = \pi I_{\lambda,e+r+t}(\lambda) \qquad J = \pi I_{e+r+t} \qquad (23.9)$$

$$I_{e+r+t} = \int_0^\infty I_{\lambda,e+r+t}(\lambda) d\lambda \qquad (24.9)$$

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5.9 Blackbody radiation

- **The blackbody**
  - Ideal absorber: Absorbs all incident radiation, regardless of direction or wavelength ($\alpha = 1$)
  - Ideal emitter: For a prescribed temperature and wavelength, no surface can emit more radiation than a blackbody
  - Diffuse emitter
- **Spectral distribution**
  - With the aid of quantum mechanics, Planck obtained:

Planck's constant

$$I_{\lambda,e} = I_{\lambda,b}(\lambda,T) = \frac{2hc_0^2}{\lambda^5\left[\exp(hc_0/\lambda k_B T)-1\right]} \quad k_B = 1.3805\times10^{-23}\,J/K \qquad (25.9)$$

Boltzmann constant

- Therefore,

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]} \qquad (26.9)$$

Chapter 9 $C_1 = 3.742\,x10^8\,\mathrm{W}\cdot\mu\mathrm{m}^4/\mathrm{m}^2$ $C_2 = 1.439\,x10^4\,\mu\mathrm{m}\cdot\mathrm{K}$ By E. Amani

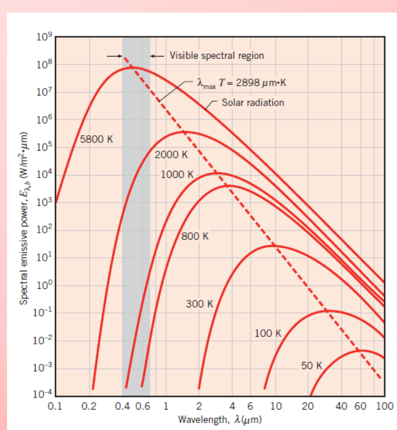
5.9 Blackbody radiation

- **Spectral distribution**

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]} \qquad (26.9)$$

$C_1 = 3.742\,x10^8\,\mathrm{W}\cdot\mu\mathrm{m}^4/\mathrm{m}^2$

$C_2 = 1.439\,x10^4\,\mu\mathrm{m}\cdot\mathrm{K}$



- Wien's displacement law:

$$dE_{\lambda,b}/d\lambda = 0 \longrightarrow$$

$$\lambda_{\max}T = C_3 = 2898\ \mu\mathrm{m}\cdot\mathrm{K} \qquad (27.9)$$

- Total emissive power:

$$E_b = \pi I_b = \int_0^\infty E_{\lambda,b}\,d\lambda = \sigma T^4 \qquad (29.9)$$

$$\sigma = 5.670\,x10^{-8}\ \mathrm{W/m^2\cdot K^4}$$

Stefan-Boltzmann constant

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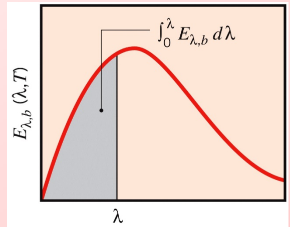
5.9 Blackbody radiation



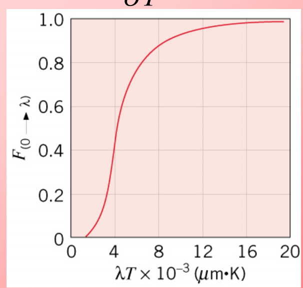
- **Spectral distribution**
  - The fraction of total blackbody emission that is in a prescribed wavelength interval or band $\lambda_1 < \lambda < \lambda_2$ is:

$$F_{(\lambda_1-\lambda_2)} = F_{(0-\lambda_2)} - F_{(0-\lambda_1)} = \frac{\int_0^{\lambda_2} E_{\lambda,b}d\lambda - \int_o^{\lambda_1} E_{\lambda,b}d\lambda}{\sigma T^4}$$

$$F_{(0-\lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b}d\lambda}{\sigma T^4} = f(\lambda T)$$

TABLE 12.2 Blackbody Radiation Functions

λT (μm·K)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ (μm·K·sr)$^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373



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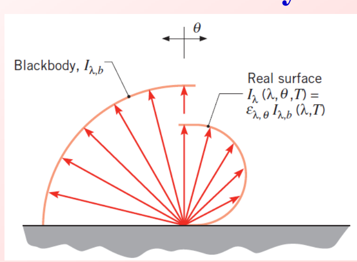
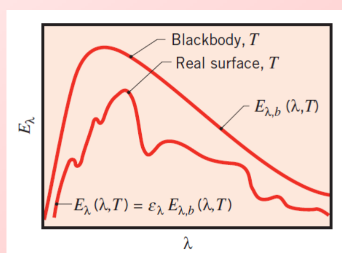
6.9 Real surface radiation

- **Surface Emissivity**





spectral, directional emissivity
$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)} \qquad (31.9)$$

Spectral emissivity
$$\varepsilon_\lambda(\lambda,T) \equiv \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)} = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta}\cos\theta\sin\theta d\theta d\phi \qquad (32.9)$$

?

total emissivity
$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)} \qquad (33.9)$$

$$E(T) = \varepsilon(T)\sigma T^4 \qquad (34.9)$$

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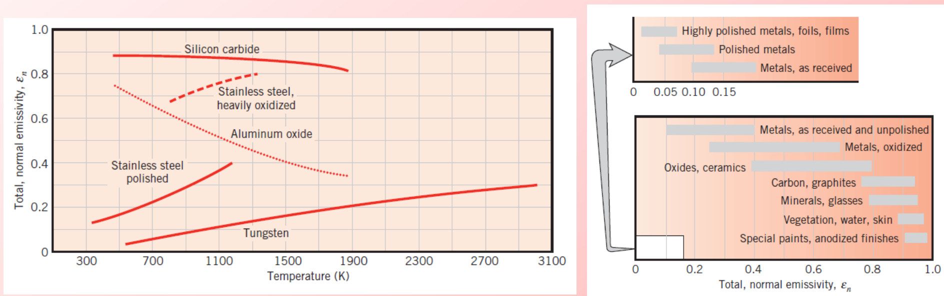
6.9 Real surface radiation

- **Surface Emissivity**

$$E(T) = \varepsilon(T)\sigma T^4 \qquad (34.9)$$

f**(Material, polish,** T**)**

➢ To a reasonable approximation: $\varepsilon \approx \varepsilon_n$, $\varepsilon_\lambda \approx \varepsilon_{\lambda,n}$ $\quad \varepsilon_n = \varepsilon(\theta = 0)$



➢ Low emissivity of polished metals and increasing emissivity for unpolished and oxidized surfaces

➢ Comparatively large emissivities of nonconductors

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6.9 Real surface radiation

- **Surface absorption, reflection, and transmission**

➢ Similarly,

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,a}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)} \quad (36.9) \qquad \alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,a}(\lambda)}{G_\lambda(\lambda)} \qquad \alpha \equiv \frac{G_a}{G}$$

spectral, directional absorptivity — **Spectral absorptivity** — **(total) absorptivity**

$$\rho_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,r}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)} \quad (37.9) \qquad \rho_\lambda(\lambda) \equiv \frac{G_{\lambda,r}(\lambda)}{G_\lambda(\lambda)} \qquad \rho \equiv \frac{G_r}{G}$$

spectral, directional reflectivity — **Spectral reflectivity** — **(total) reflectivity**

$$\tau_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,t}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)} \quad (38.9) \qquad \tau_\lambda(\lambda) \equiv \frac{G_{\lambda,t}(\lambda)}{G_\lambda(\lambda)} \qquad \tau \equiv \frac{G_t}{G}$$

spectral, directional transmissivity — **Spectral transmissivity** — **(total) transmissivity**

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6.9 Real surface radiation

- **Surface absorption, reflection, and transmission**

$$\begin{aligned}\rho_{\lambda,\theta}+\alpha_{\lambda,\theta}+\tau_{\lambda,\theta}&=1\\ \rho_{\lambda}+\alpha_{\lambda}+\tau_{\lambda}&=1\\ \rho+\alpha+\tau&=1\end{aligned}\qquad(39.9)$$

  - Opaque surface:

$$\tau_{\lambda,\theta}=\tau_{\lambda}=\tau=0$$

  - Diffuse reflector:

$$I_{\lambda,i,r}(\lambda,\theta,\phi)=I_{\lambda,i,r}(\lambda)$$

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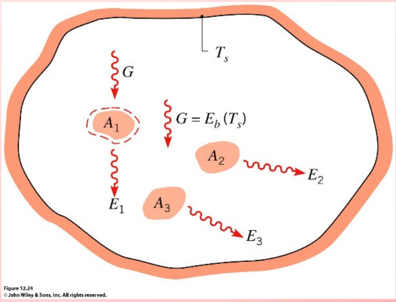
7.9 Kirchhoff's law

- **For proof, see reference [4]**
  - Corollary 1: In a large, isothermal enclosure of surface temperature T_s, the emission power of enclosure surface and irradiation received by a small object is diffuse and equal to blackbody emission power



$$G=E_{\mathrm{b}}(T_{\mathrm{s}})$$

  - Corollary 2: For an arbitrary surface

$$\varepsilon_{\lambda,\theta}=\alpha_{\lambda,\theta}\qquad(40.9)$$

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7.9 Kirchhoff's law

- **What about ε_λ and α_λ?**

$$\varepsilon_\lambda = \frac{E_\lambda}{E_b} = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi} \overset{?}{=} \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi} = \frac{G_{\lambda,a}}{G_\lambda} = \alpha_\lambda$$

- **Therefore $\varepsilon_\lambda = \alpha_\lambda$, if either of the following conditions is met:**
  - A diffuse surface ($\varepsilon_{\lambda,\theta}$ and $\alpha_{\lambda,\theta}$ are independent of θ and ϕ): non-conductive materials
  - The diffuse irradiation ($I_{\lambda,i}$ is independent of θ and ϕ)

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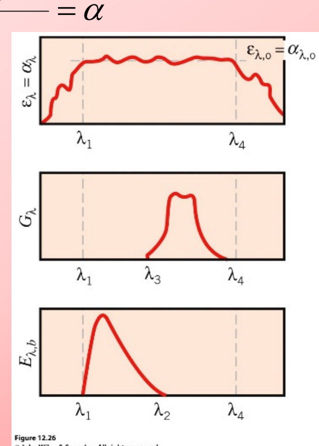
7.9 Kirchhoff's law

- **What about ε and α?**

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda) d\lambda}{E_b(T)} \overset{?}{=} \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{G} = \alpha$$

- **Therefore $\varepsilon = \alpha$, if $\varepsilon_\lambda = \alpha_\lambda$ and either of the following conditions is met:**
  - The spectral irradiation is equal to the blackbody irradiation at the surface temperature, i.e., $G_\lambda = E_{b,\lambda}(\lambda, T)$
  - The gray surface (ε_λ and α_λ are independent of λ): Problem dependent



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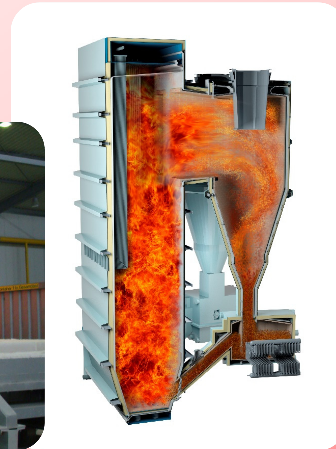
8.9 Sample problems

Furnace



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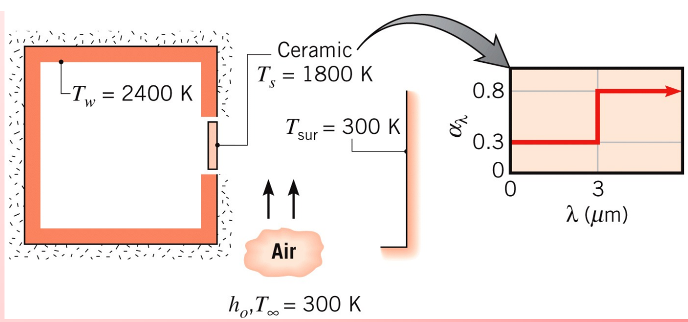
8.9 Sample problems

A thin-walled plate separates the interior of a large furnace from surroundings at 300 K. The plate is fabricated from a ceramic material for which diffuse surface behavior may be assumed and the exterior surface is air cooled. With the furnace operating at 2400 K, convection at the interior surface may be neglected.

(a) If the temperature of the ceramic plate is not to exceed 1800 K, what is the minimum value of the outside convection coefficient, h_o, that must be maintained by the air-cooling system?

(b) Compute and plot the plate temperature as a function of ho for $50 < h_o < 250$ $W/m^2.K$?



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The end of chapter 9