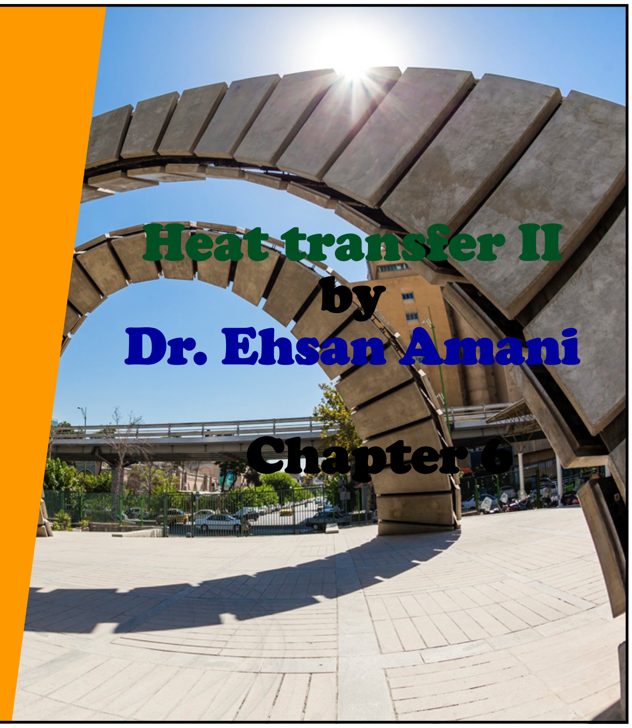


Heat transfer II

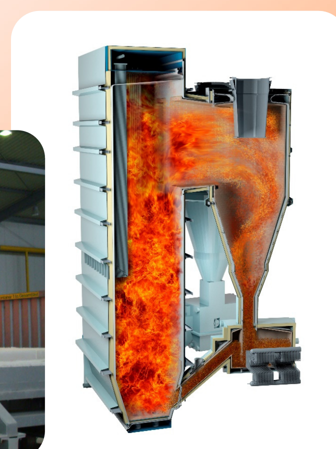
by

Dr. Ehsan Amani

Chapter 6

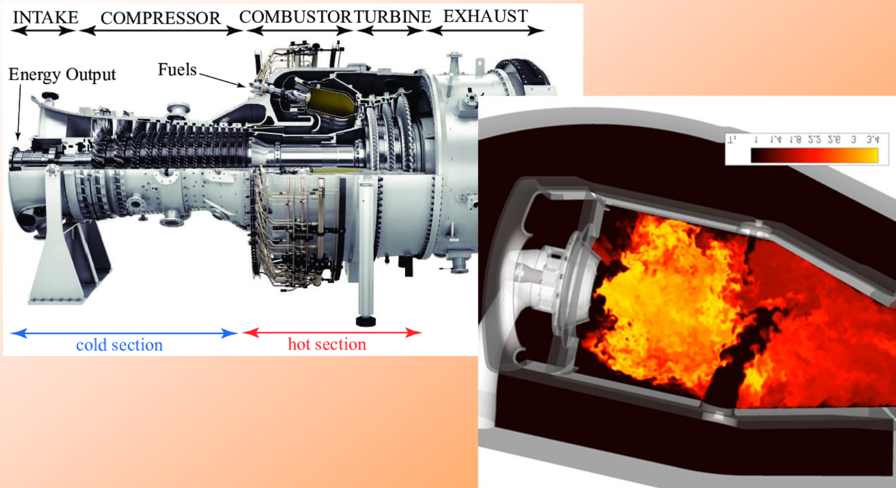
Volumetric radiation: Applications

- **Furnace**



Volumetric radiation: Applications

- **Combustion chambers**



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Volumetric radiation: Applications

- **Combustion chambers**

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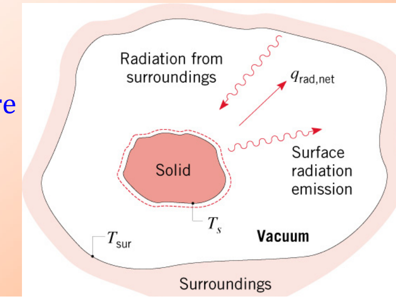
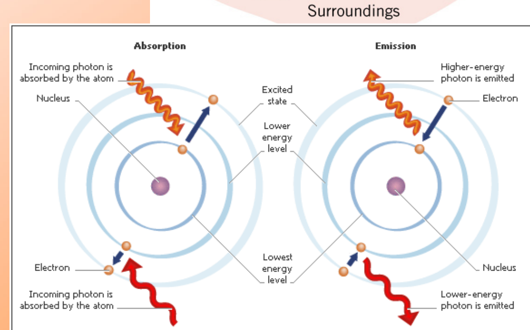
Review on HT_I

- **Evidence**
  - Consider a solid of temperature T_s in an evacuated enclosure whose walls are at a fixed temperature T_{surr}



- **Physics**
  - Emission is due to oscillations and transitions of the many electrons that comprise matter, which are, in turn, sustained by the thermal energy of the matter



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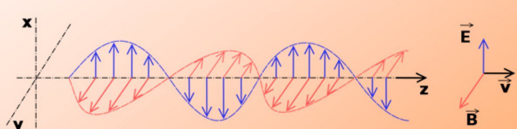
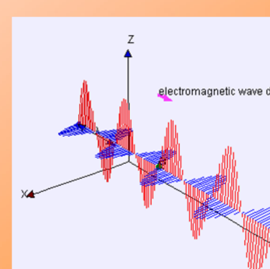
Review on HT_I

- **Electromagnetic waves**
  - Oscillations of electric and magnetic fields (Produced whenever charged particles are accelerated) that propagate at the speed of light through a vacuum





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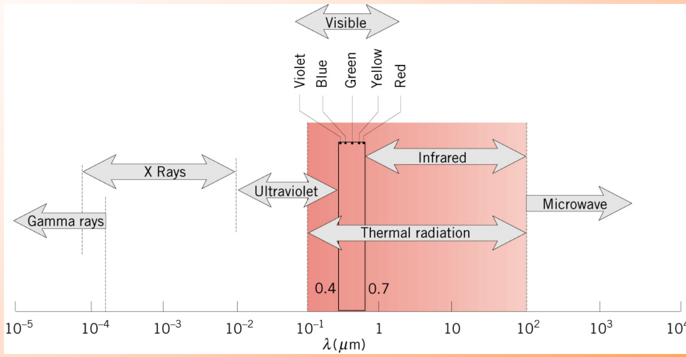
Review on HT_I

- **The electromagnetic spectrum**



- Thermal radiation is confined to the infrared, visible and ultraviolet regions of the spectrum $(0.1 < \lambda < 100\mu m)$
- These wavelengths can affect the temperature of the material

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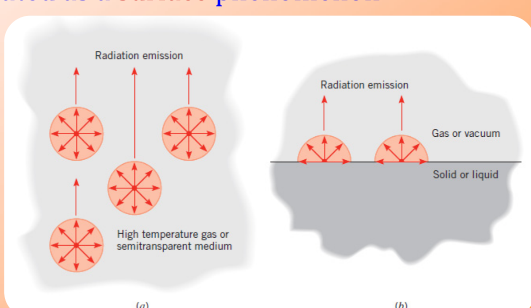
Review on HT_I

- **Volumetric versus surface phenomenon**
  - Emission from a gas or a semitransparent solid or liquid is a volumetric phenomenon
  - For an opaque solid or liquid, emission originates from atoms and molecules within 1 μm of the surface
  - Emission from an opaque solid or liquid, except in nanoscale, is treated as a surface phenomenon



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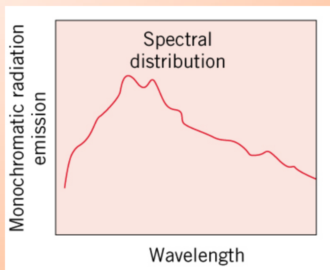
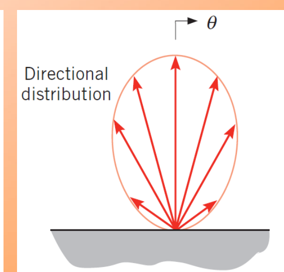
Review on HT_I

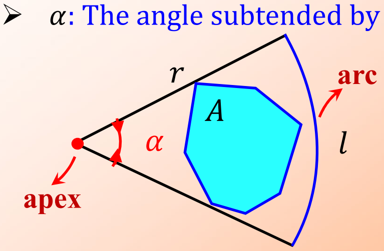
- **Directional and spectral consideration**
  - The amount of radiation emitted by an opaque surface varies with wavelength
  - The spectral distribution over all wavelengths
  - Radiation emitted by a surface will be in all directions associated with a hypothetical hemisphere about the surface and is characterized by a directional distribution.





Review on HT_I

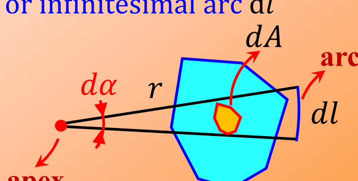
- **Angle**
  - Plane angle: measure of the ratio of a circle arc to its radius
  - α: The angle subtended by the object A or arc l



$$\alpha \equiv \frac{l}{r} \qquad 0 \le \alpha \le 2\pi$$

(rad)

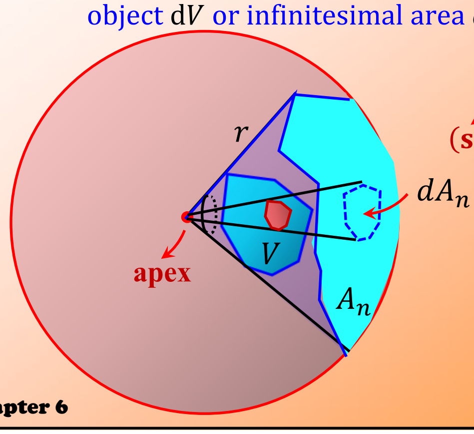
  - $d\alpha$: The differential angle subtended by the infinitesimal object dA or infinitesimal arc dl



$$d\alpha = \frac{dl}{r}$$

Review on HT_I

- **Solid angle**
  - Extension of angle to 3D space
  - ω: The angle subtended by the object V or spherical surface A_n
  - $d\omega$: The differential solid angle subtended by the infinitesimal object dV or infinitesimal area dA_n



$$\omega \equiv \frac{A_n}{r^2} \qquad 0 \le \omega \le 4\pi$$

(sr)

2π **(for a plane surface)**

$$d\omega \equiv \frac{dA_n}{r^2} \qquad (8.9)$$

Chapter 6 **By E. Amani**

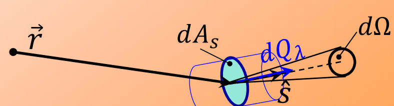
Radiation intensity in a medium

- **Spectral directional intensity at each point and time:**

Space locations **directions** **Radiation energy flow in the positive $\hat{s}$ direction within dΩ and $d\lambda$**

$$I_\lambda(\vec{r},\hat{s},t) \equiv \frac{dQ_\lambda(\hat{s})}{dA_s\, d\lambda\, d\Omega} \qquad (1.6)$$

Spectral directional intensity **A surface element normal to $\hat{s}$**



Chapter 6 **By E. Amani**

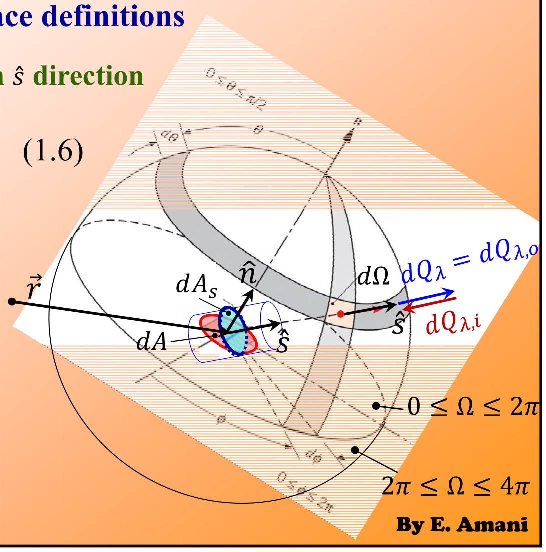
Radiation intensity in a medium

- **Spectral directional intensity at each point and time:**
  - **Connection to surface definitions**

Positive in $\hat{s}$ direction

$$I_\lambda(\vec{r},\hat{s},t) \equiv \frac{dQ_\lambda(\hat{s})}{dA(\hat{n}.\hat{s})d\lambda d\Omega} \quad (1.6)$$

$$\hat{n}=\hat{s} \quad = \frac{dQ_\lambda(\hat{s})}{dA_s d\lambda d\Omega}$$



  - **See appendix A for the proof**

Radiation intensity in a medium

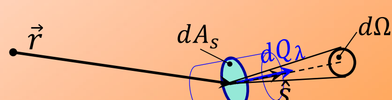
- **Spectral directional intensity at each point and time:**

Space locations; directions; Radiation energy flow in the positive $\hat{s}$ direction within $d\Omega$ and $d\lambda$

$$I_\lambda(\vec{r},\hat{s},t) \equiv \frac{dQ_\lambda(\hat{s})}{dA_s d\lambda d\Omega} \quad (1.6)$$

Spectral directional intensity; A surface element normal to $\hat{s}$

$$\vec{q}''_\lambda \equiv \int_{4\pi} \hat{s}\left(\frac{dQ_\lambda(\hat{s})}{dA_s d\lambda}\right)$$

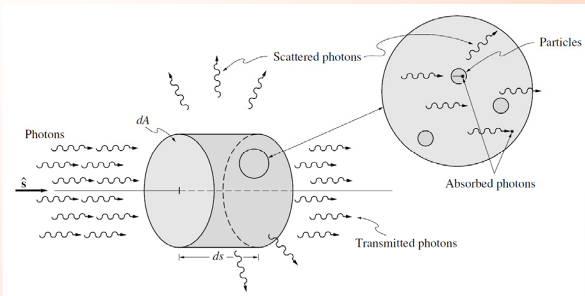


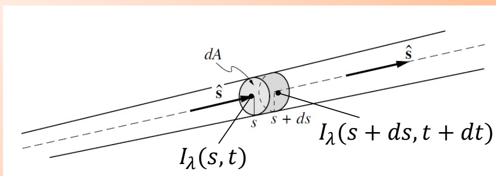
Spectral radiation Heat flux $\quad \vec{q}''_\lambda = \int_{4\pi} I_\lambda \hat{s} d\Omega \quad (2.6)$

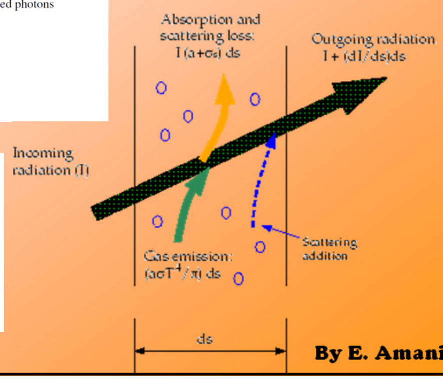
Total radiation heat flux $\quad \vec{q}''_{\text{rad}} = \int_0^\infty \vec{q}''_\lambda d\lambda \quad (3.6)$

Radiation in a participating medium

- **Absorption, transmission, and scattering**







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Radiation in a participating medium

- **Absorption loss**

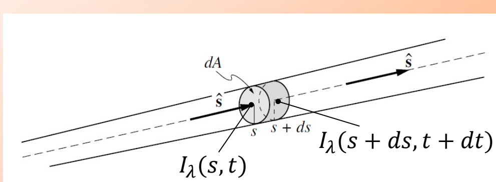
(linear) absorption coefficient ← $K_\lambda \equiv \frac{-(dI_\lambda)_a/ds}{I_\lambda}$ (3.6) $(dI_\lambda)_a = -K_\lambda I_\lambda ds$

Intensity reduction due to absorption in $\hat{s}$ direction

- **Scattering loss (out-scattering)**

(linear) scattering coefficient ← $\sigma_{s\lambda} \equiv \frac{-(dI_\lambda)_{s^-}/ds}{I_\lambda}$ (4.6) $(dI_\lambda)_{s^-} = -\sigma_{s\lambda} I_\lambda ds$

Intensity reduction due to scattering in $\hat{s}$ direction



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Radiation in a participating medium

- **Emission gain**

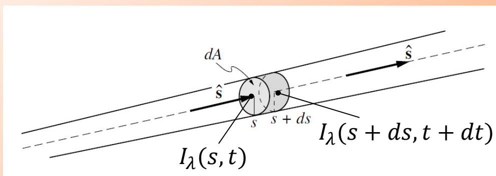
Intensity increase due to emission in $\hat{s}$ direction

(linear) emission coefficient

$$e_\lambda \equiv \frac{+(dI_\lambda)_e/ds}{I_{b\lambda}} \qquad (5.6) \qquad (dI_\lambda)_e = e_\lambda I_{b\lambda} ds$$

  - **Assuming local thermodynamic equilibrium and Kirchhoff's law ($e_\lambda = K_\lambda$)**

$$(dI_\lambda)_e = K_\lambda I_{b\lambda} ds \qquad (6.6)$$



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Radiation in a participating medium

- **Scattering gain (in-scattering)**
  - **See appendix A for the proof**

Intensity increase due to scattering into $\hat{s}$ direction from other directions

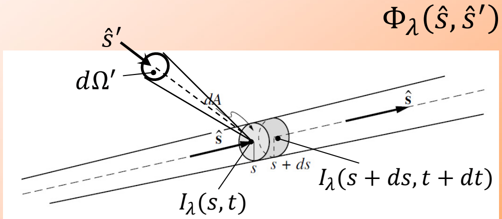
$$(dI_\lambda)_{s^+} = \frac{\sigma_{s\lambda}}{4\pi} ds \int_{4\pi} I_\lambda(\hat{s}') \Phi_\lambda(\hat{s}, \hat{s}') d\Omega' \qquad (9.6)$$

Scattering phase function

$$\int_{4\pi} \Phi_\lambda(\hat{s}, \hat{s}') d\Omega' = 4\pi$$

  - **For diffuse scattering**

$$\Phi_\lambda(\hat{s}, \hat{s}') = 1 \qquad (8.6)$$



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Radiation Transport Equation (RTE)

- **Assumptions**
  - **A constant refractive index**
  - **Local thermodynamic equilibrium**
  - **Flow speeds are small compared to the speed of light**

$$\underbrace{I_\lambda(s+ds,t+dt)-I_\lambda(s,t)}_{\frac{\partial I_\lambda}{\partial s}ds+\frac{\partial I_\lambda}{\partial t}dt} = (dI_\lambda)_e+(dI_\lambda)_a+(dI_\lambda)_{s^+}+(dI_\lambda)_{s^-} \quad (10.6)$$

$$dt=\frac{ds}{c}$$

$$\underbrace{\frac{1}{c}\frac{\partial I_\lambda}{\partial t}}_{\text{negligible}}+\frac{\partial I_\lambda}{\partial s}=K_\lambda I_{b\lambda}-(K_\lambda+\sigma_{s\lambda})I_\lambda+\frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi}I_\lambda(\hat{s}')\Phi_\lambda(\hat{s},\hat{s}')d\Omega' \quad (13.6)$$

Thermal energy balance

- **The source term due to radiation (net radiation inflow):** ⇨ **Lecture Notes: VI.3**

$$\dot{q}_{\text{rad}}=-\vec{\nabla}.\vec{q}''_{\text{rad}}=-\int_0^\infty K_\lambda(4\pi I_{b\lambda}-G_\lambda)d\lambda \quad (14.6)$$

Incident radiation function

$$G_\lambda(\vec{r},t)=\int_{4\pi}I_\lambda(\vec{r},\hat{s},t)d\Omega \quad (15.6)$$

  - **Exercise: Why is the direct term due to the scattering vanished in the thermal energy balance?**
  - **Exercise: For a gray medium ($K_\lambda=K$), show that**

$$\dot{q}_{\text{rad}}=-K(4\sigma T^4-G) \quad (16.6)$$

$$G=\int_0^\infty G_\lambda d\lambda \quad (17.6)$$

Optical thickness

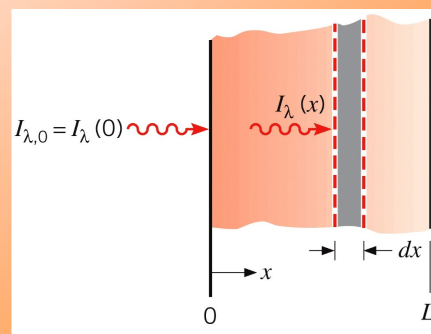
- **Assuming negligible scattering:**

$$(13.6)\ \frac{dI_\lambda}{ds} = K_\lambda(I_{b\lambda} - I_\lambda) \longrightarrow \frac{I_\lambda(s) - I_{b\lambda}}{I_\lambda(0) - I_{b\lambda}} = -\overbrace{\int_0^s K_\lambda ds'}^{\tau_\lambda(s)} \quad (18.6)$$

$$\longrightarrow I_\lambda(s) = I_{b\lambda}\left(1 - e^{-\tau_\lambda(s)}\right) + I_\lambda(0)e^{-\tau_\lambda(s)} \quad (19.6)$$

$$I_\lambda(L) = I_{b\lambda}(1 - e^{-\tau_\lambda}) + I_\lambda(0)e^{-\tau_\lambda}$$

Optical thickness $\leftarrow$ $\tau_\lambda \equiv \int_0^L K_\lambda dx \quad (20.6)$



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Simplified RTE models

- **Assumptions:**
  - **Large optical thickness of the medium**
  - **Linear scattering phase function ($\Phi_\lambda(\hat{s}, \hat{s}') = 1 + C\hat{s}.\hat{s}'$)**

$$\vec{\nabla}.\left(\Gamma_\lambda \vec{\nabla} G_\lambda\right) - K_\lambda G_\lambda + 4\pi I_{b\lambda} = 0 \quad (21.6)$$

$$\Gamma_\lambda = \frac{1}{3(K_\lambda + \sigma_{s\lambda}) - C\sigma_{s\lambda}} \quad (22.6)$$

  - **This model is known as "P1 model"**
  - **No directional dependence**

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The end of chapter 6

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