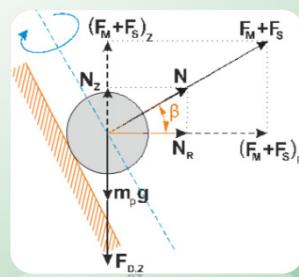


IX. Fluid-particle interaction (FPI)



IX.1 Fluid-to-particle transfer rates

IX.1.1 Momentum transfer – forces and moments on a particle

IX.1.2 Heat transfer

IX.1.3 Particle tracking numerics

Momentum transfer – particle tracking

- For each particle, p:

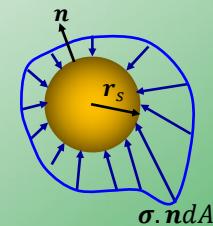
$\frac{dx_p}{dt} = u_p$ Fluid-particle interaction force
 $F = m_p \frac{d\mathbf{u}_p}{dt}; \quad F = F_{\text{FPI}} + F_B + F_{\text{col}}$ Body force
 $T = I_p \frac{d\omega_p}{dt}; \quad T = T_{\text{FPI}} + T_{\text{col}}$ Collision force

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FPI forces and moments

- DNS formulation:



$$\mathbf{F}_{\text{FPI}} = \int_{A_p} \boldsymbol{\sigma} \cdot \mathbf{n} dA = \int_{A_p} (-p\mathbf{n} + \boldsymbol{\tau} \cdot \mathbf{n}) dA$$

$$\mathbf{T}_{\text{FPI}} = \int_{A_p} \mathbf{r}_s \times (\mathbf{n} \cdot \boldsymbol{\tau}) dA$$

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FPI forces and moments

- Point-particle formulation:

- Simplifying assumptions
- Semi-analytical solutions
- Superposition

$$\mathbf{F}_{\text{FPI}} = \mathbf{F}_D + \mathbf{F}_L + \mathbf{F}_{vm} + \mathbf{F}_H + \mathbf{F}_0 + \mathbf{F}_W + \mathbf{F}_B + \mathbf{F}_T$$

Drag force Virtual (or added)
 (streamwise steady mass force
 viscous+pressure)(unsteady pressure)

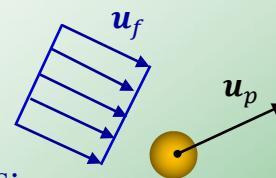
Lift force History (or
 (transverse steadyBasset) force
 pressure) (unsteady
 viscous)

Wall (lubrication) force
 Thermophoretic force
 Undisturbed flow force
 Brownian force

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Drag force



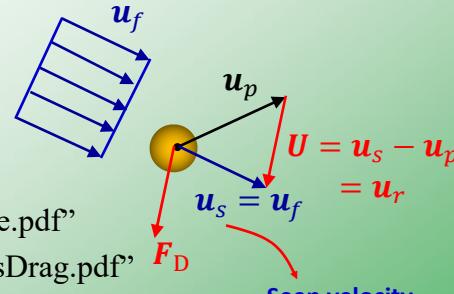
- Simplifying assumptions:

- Steady-state
- Stokes regime ($Re_p < 1$) \rightarrow Re number relaxation
- Spherical particle \rightarrow Equivalent diameter !
- No evaporation/condensation \rightarrow Blowing effect
- Uniform flow \rightarrow Seen velocity (u_s) definition
- Incompressible low-Mach flow \rightarrow Compressibility effect
- No internal flow \rightarrow Internal flow relaxation
- No rotation

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Drag force



- Derivation:

➤ See my notes: "dragForce.pdf"

➤ For more details: "StokesDrag.pdf"

- Result:

$$F_D = \underbrace{2\pi\mu U a}_{\text{Pressure drag}} + \underbrace{4\pi\mu U a}_{\text{Friction drag}} = 6\pi\mu U a$$

- The drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A_p} = \frac{6\pi\mu U a}{\frac{1}{2}\rho U^2 \pi a^2} \rightarrow C_D = \frac{24}{Re_p}; Re_p = \frac{2a\rho U}{\mu} = \frac{|u_r|d_p}{v_f}$$

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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor → Notes

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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor

➡ Notes: Chap9.1.1.1

$$\mathbf{F}_D = m_p \frac{\mathbf{u}_r}{\tau_p}; \quad \tau_p = \tau_v = \frac{1}{f_D} \frac{\rho_p d_p^2}{18 \rho_f v_f}$$

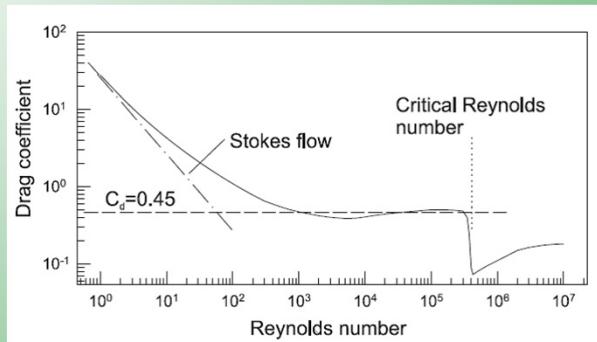
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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor
- Experimental measurements



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Drag force

- Re number relaxation:

- Complex behavior due to separation, laminar-turbulence transition, instabilities (von Karman vortex street)
- Drag coefficient and friction factor
- Experimental measurements
- A curve-fit, semi-empirical correlation by Schiller and Naumann [1]:

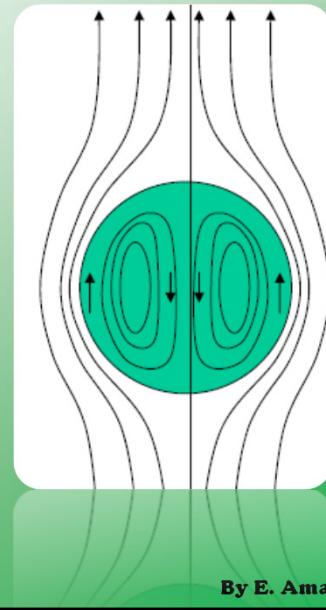
$$f_D = \begin{cases} 1 & Re_p \leq 0.1 \\ 1 + \frac{1}{6} Re_p^{2/3} & 0.1 < Re_p \leq 1000 \end{cases}$$

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Drag force

- Internal flow relaxation
- ...

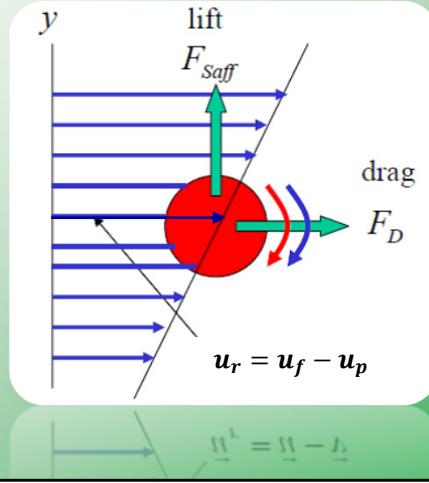


Chap 9

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Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field



Chap 9

$$\vec{W}^L = \vec{W} - \vec{U}$$

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Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field
- A semi-analytical solution [6]:

$$\mathbf{F}_L = 1.61 C_s \rho_f d_p^2 \left(\frac{2v_f}{|\boldsymbol{\omega}_f|} \right)^{\frac{1}{2}} \mathbf{u}_r \times \boldsymbol{\omega}_f; \quad \boldsymbol{\omega}_f = 0.5 \nabla \times \mathbf{u}_f$$

$$C_s = \begin{cases} \left(1 - 0.3314\lambda_v^{\frac{1}{2}}\right) \exp\left(-\frac{Re_p}{10}\right) + 0.3314\lambda_v^{\frac{1}{2}} & ; Re_p \leq 40 \\ 0.0524(\lambda_v Re_p)^{1/2} & ; Re_p > 40 \end{cases}, \quad \lambda_v = d_p \frac{|\boldsymbol{\omega}_f|}{|\mathbf{u}_r|}$$

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Saffman Lift force

- Non-uniform pressure around a particle due to a non-uniform velocity field
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= $d_p \frac{|\omega_f|}{|\mathbf{u}_r|}$

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Saffman Lift force

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- Important for large ρ_f / ρ_p or large particles

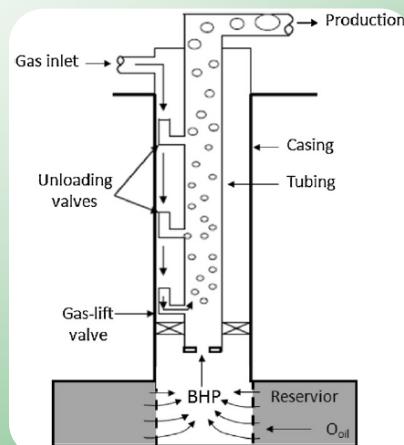
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Saffman Lift force

- Application: Gas-lift [15]

Gas-lift in
oil wells

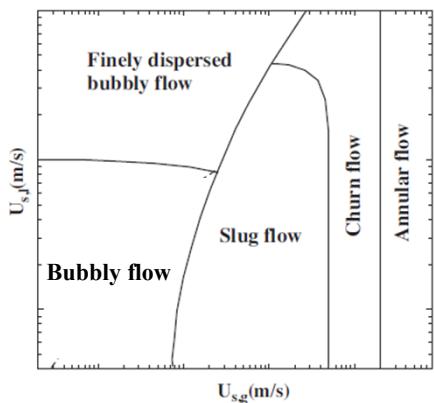


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Saffman Lift force

- Application: Gas-lift [15]

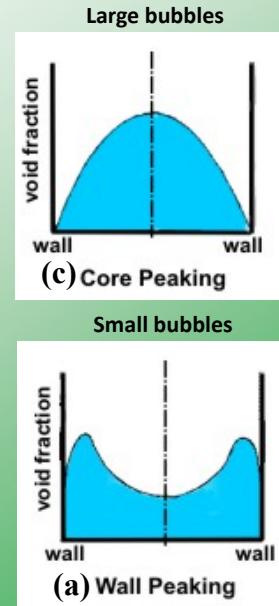
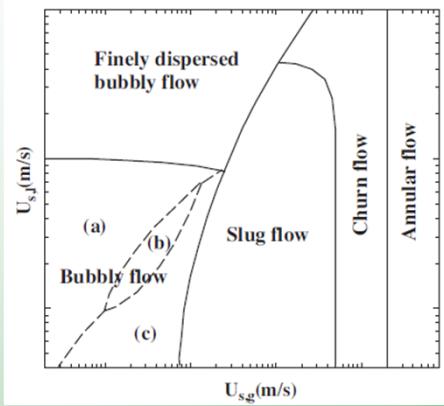


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Saffman Lift force

- Application: Gas-lift [15]

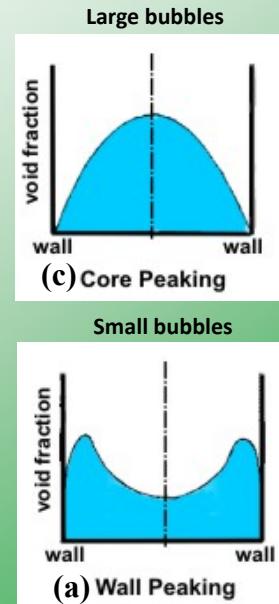
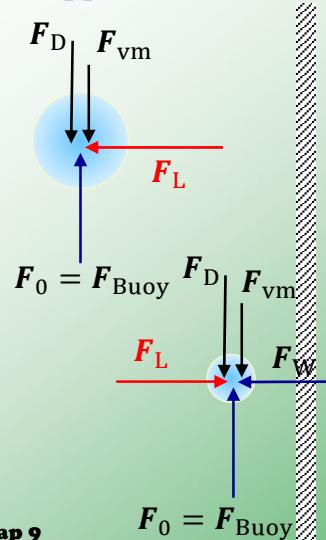


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Saffman Lift force

- Application: Gas-lift [15]



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Saffman Lift force

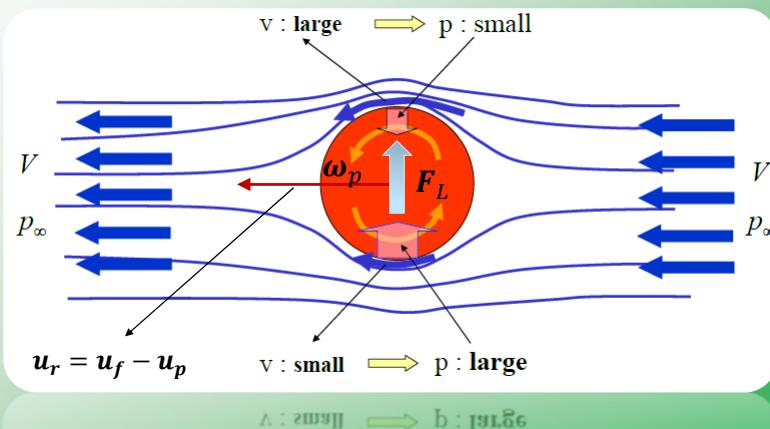
- Application: Gas-lift [15]
- More physical discussion and applications: [exercise](#)

Chap 9

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Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation



Chap 9

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Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation
- A semi-analytical solution [7, 8]:

$$F_L = 0.5 C_{LR} \rho_f A_p \frac{|\mathbf{u}_r|}{|\boldsymbol{\omega}_r|} (\mathbf{u}_r \times \boldsymbol{\omega}_r); \quad \boldsymbol{\omega}_r = \boldsymbol{\omega}_p - \boldsymbol{\omega}_f$$

$$C_{LR} = \begin{cases} \frac{Re_\omega}{Re_p} & ; Re_p \leq 1 \\ 0.45 + \left(\frac{Re_\omega}{Re_p} - 0.45 \right) \exp(-0.05684 Re_\omega^{0.4} Re_p^{0.3}) & ; Re_p < 2000 \end{cases}$$

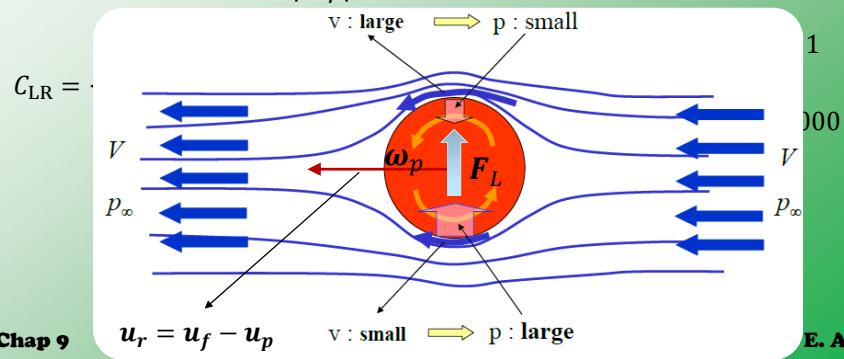
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Magnus Lift force

- Non-uniform pressure around a particle due to particle rotation
- A semi-analytical solution [7, 8]:

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Chap 9

E. Amani

Magnus Lift force

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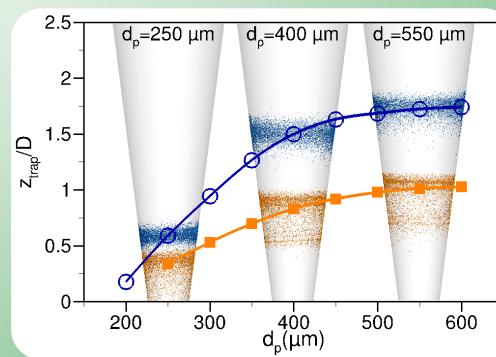
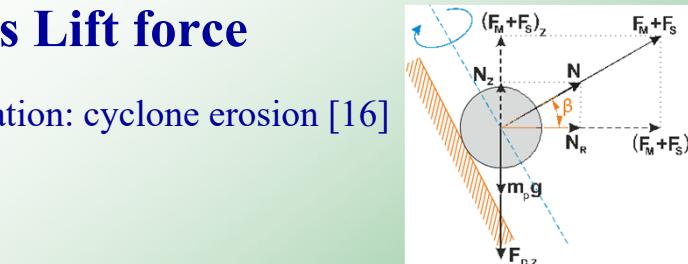
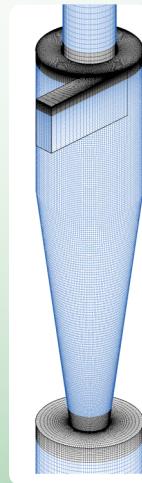
- Important for large ρ_f/ρ_p or large rotating particles

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Magnus Lift force

- Application: cyclone erosion [16]



Neglecting
lift forces
for large
particles:
filled red
squares

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Magnus Lift force

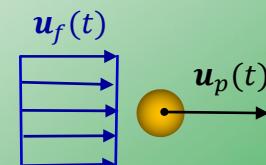
- Application: cyclone erosion [16]
- More physical discussion and applications: exercise

Chap 9

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Virtual (added) mass force

- Evidence: A spherical particle in an unsteady inviscid flow experiences a force
- Can be viewed as increased inertia force
- Accelerating a part of fluid with the particle



Chap 9

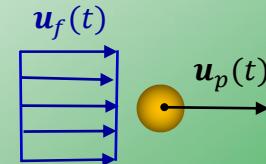
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Virtual (added) mass force

- For a basic derivation, consult classic aerodynamics books (e.g. Principles of Ideal-Fluid aerodynamics by K. Karamcheti).
- The analytical solution [3]:

$$\mathbf{F}_{vm} = C_{vm} \frac{\rho_f V_p}{2} \left(\frac{D\mathbf{u}_f}{Dt} - \frac{d\mathbf{u}_p}{dt} \right); \quad C_{vm} = 1$$

- Scales as: (ρ_f / ρ_p)
- More physical discussion and applications: exercise



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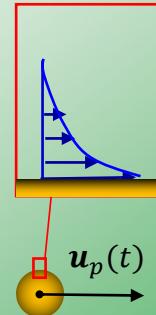
History (Basset) force

- Accelerating a part of fluid with the particle through viscous force
- A semi-analytical solution [4, 5]:

$$\mathbf{F}_H = C_B \frac{3}{2} d_p^2 \sqrt{\pi \mu_f \rho_f} \left[\int_0^t \frac{d\mathbf{u}_r}{dt'} dt' + \frac{\mathbf{u}_{r,0}}{\sqrt{t}} \right]$$

$$C_B = 1 - 0.527 \left[1 - \exp(-0.14 Re_p Sl^{0.82})^{2.5} \right]; \quad Sl = \frac{1}{2\pi f \tau_p}$$

- Scales as: (ρ_f / ρ_p)
- More physical discussion and applications: exercise

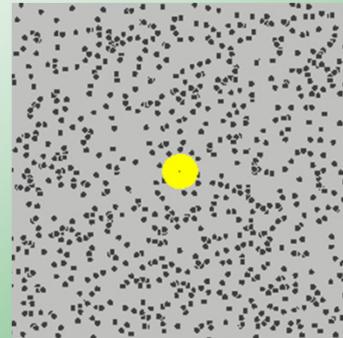


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Brownian Force

- For sub-micron particles
- Random force

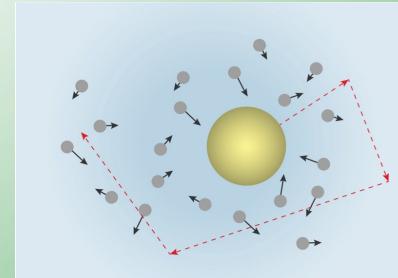


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Brownian Force

- For sub-micron particles
- Random force
- A stochastic process [9, 10]:



$$\mathbf{F}_B = m_p \sqrt{\frac{\pi S_0}{\delta t}} |\zeta| \boldsymbol{\eta}; \quad S_0 = \frac{216 \mu_f K_B T_f}{\pi^2 d_p^5 \rho_p^2 C_c}$$

$$C_c = 1 + \frac{2\lambda}{d_p} \left[1.257 + 0.4 \exp\left(-\frac{1.1d_p}{2\lambda}\right) \right]$$

$$\boldsymbol{\eta} = a \cos(\theta) \mathbf{i} + a \sin(\theta) \mathbf{j} + u \mathbf{k}, \theta = 2\pi X, u = 2Y - 1, a = \sqrt{1 - u^2}, \\ X, Y \in U(0,1), \zeta \in N(0,1)$$

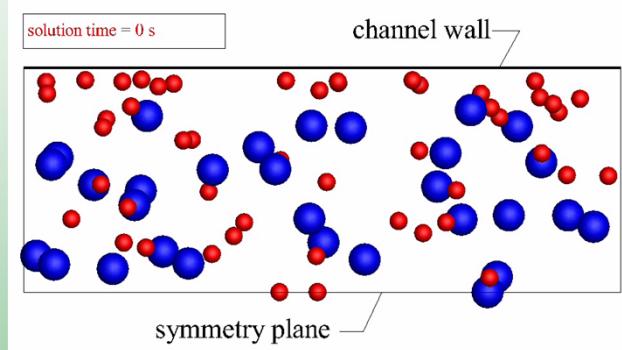
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Brownian Force

- Application: Electroosmotic flow in nano-channels [17]

Na^+ (red spheres) and Cl^- (blue spheres) ion transport in a nano-channel



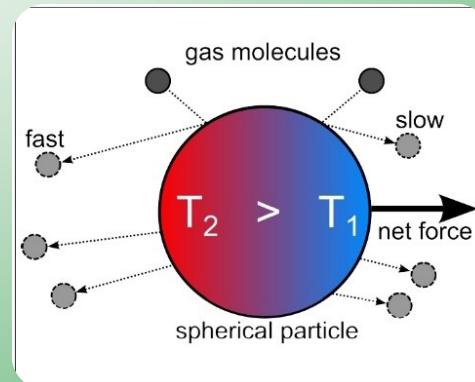
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Thermophoretic Force

- For sub-micron particles in a temperature gradient
- A closure [11]:

$$F_T = -D_{T,p} \frac{VT}{T}$$



Chap 9

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Case study #3: Conveying T-junction

- Pneumatic conveying T-junction

- Eulerian-Lagrangian simulation
 1. Preliminary analysis
 2. Solution using ANSYS Fluent

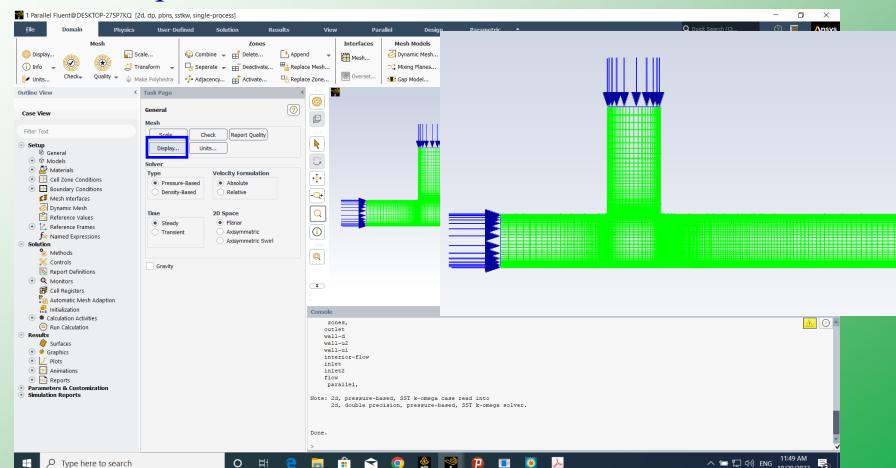
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)

- Open: 1.cas

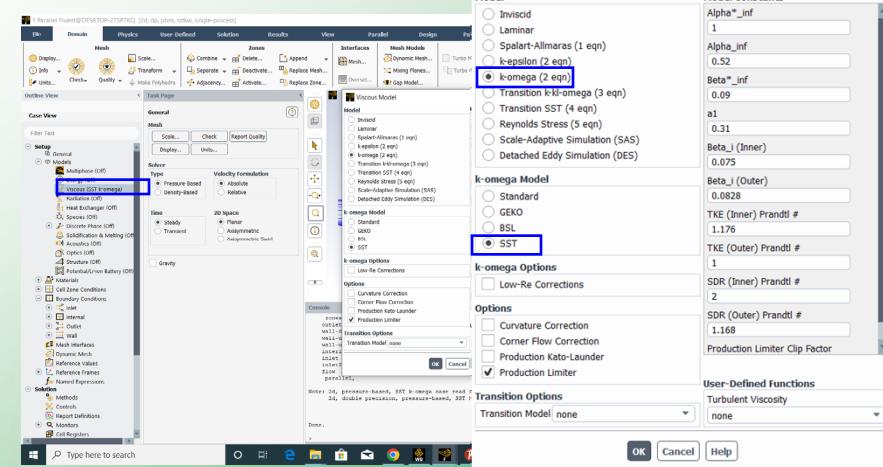


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
- Models > turbulence

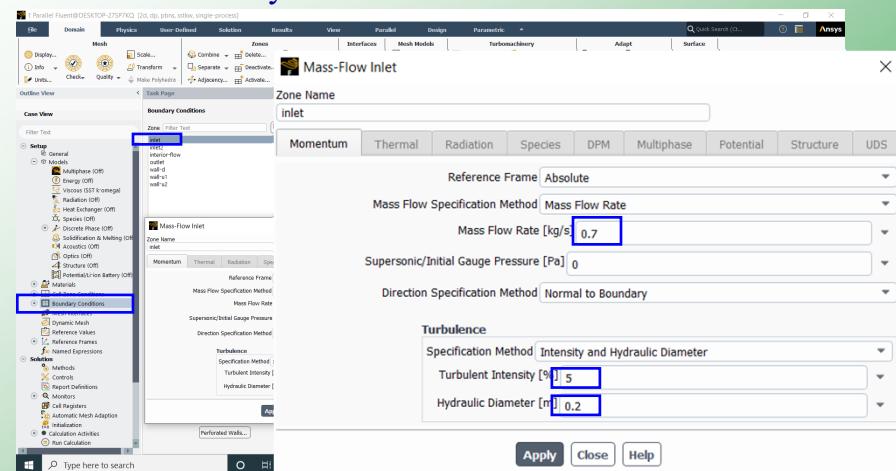


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
- Boundary condition



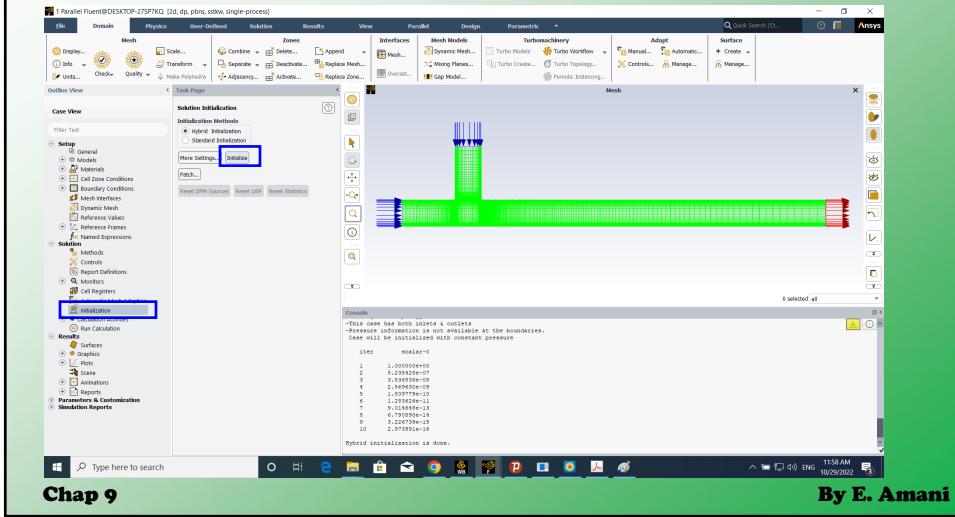
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)

- Initialization



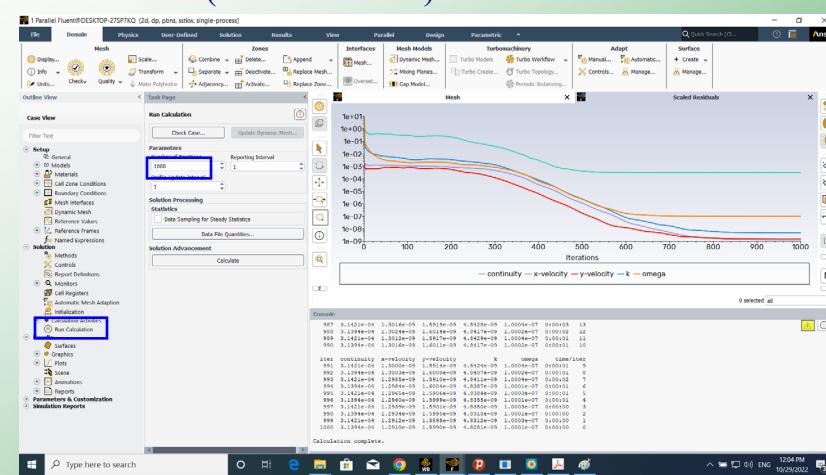
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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)

- Run (1000 iterations)

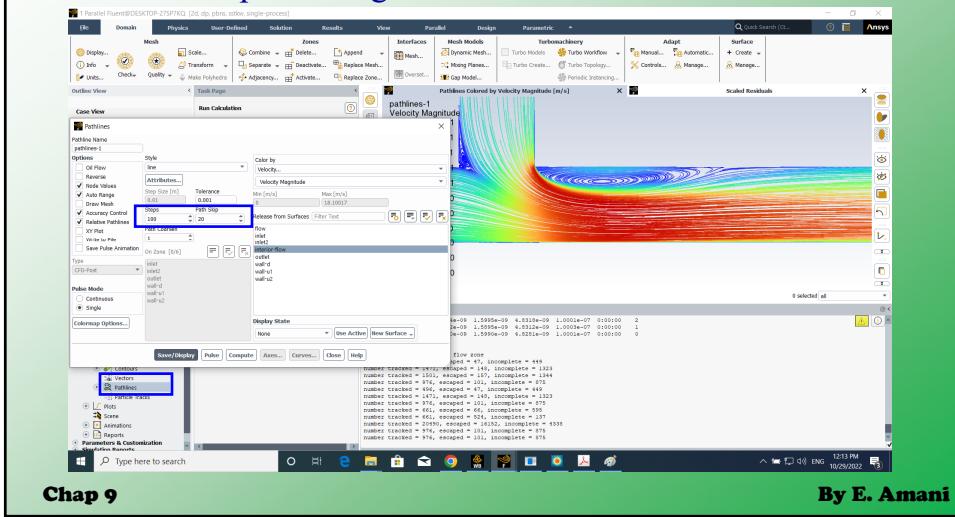


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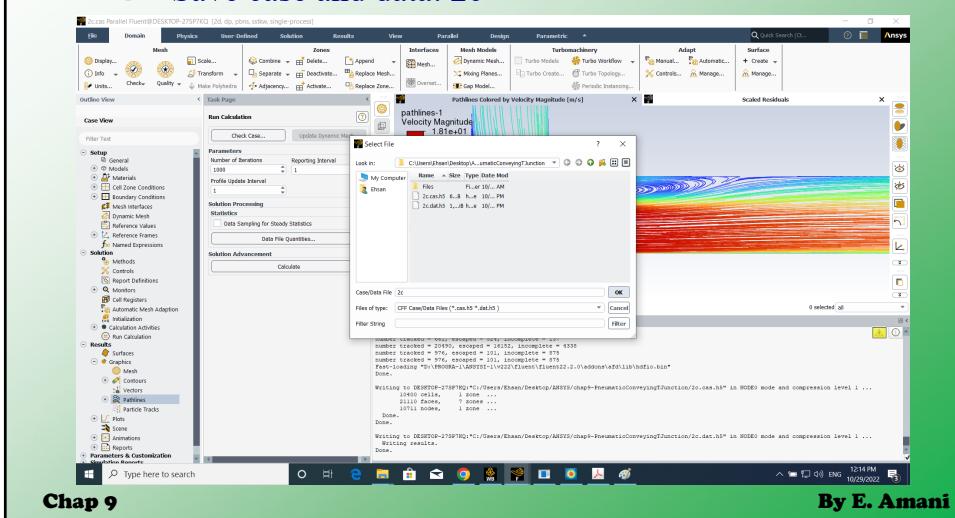
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (single phase)
- Post processing: Pathlines



Case study #3: Conveying T-junction

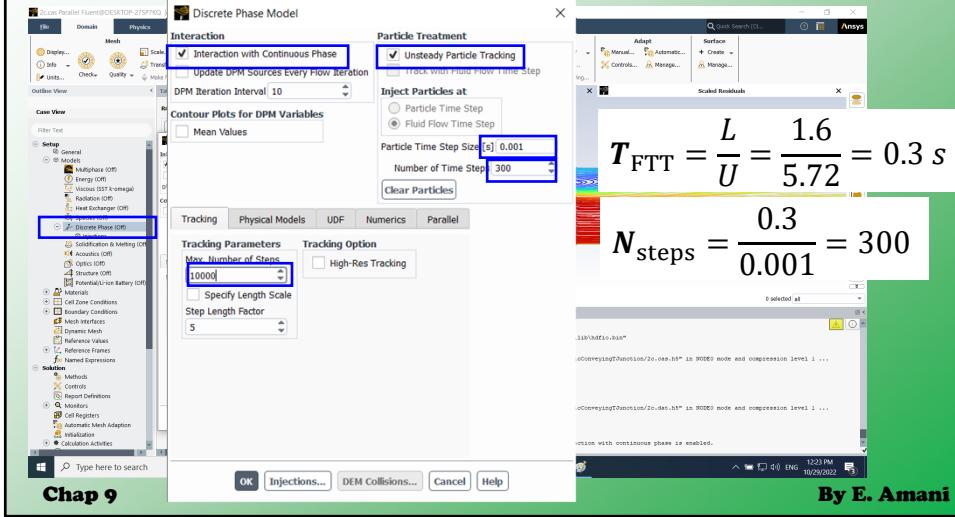
- Solution using ANSYS Fluent (single phase)
- Save case and data: 2c



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

Models > Discrete phase

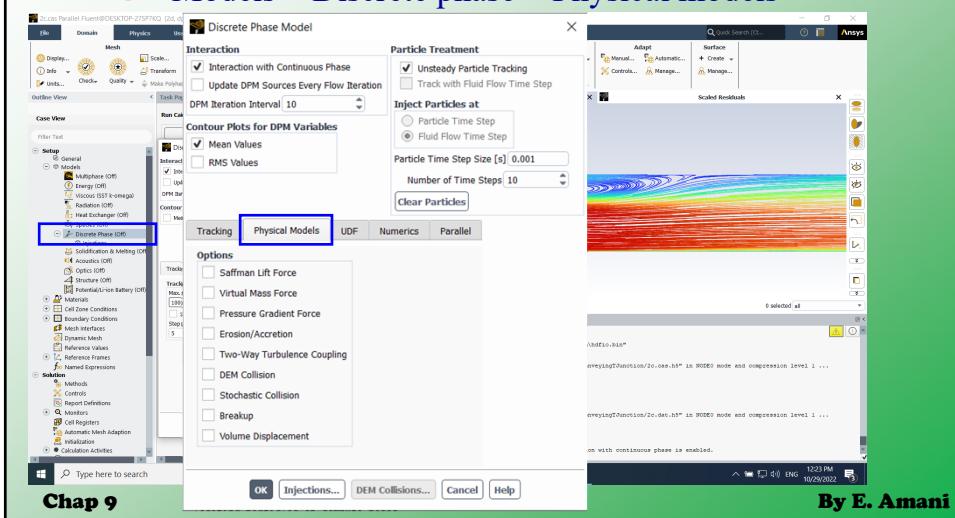


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

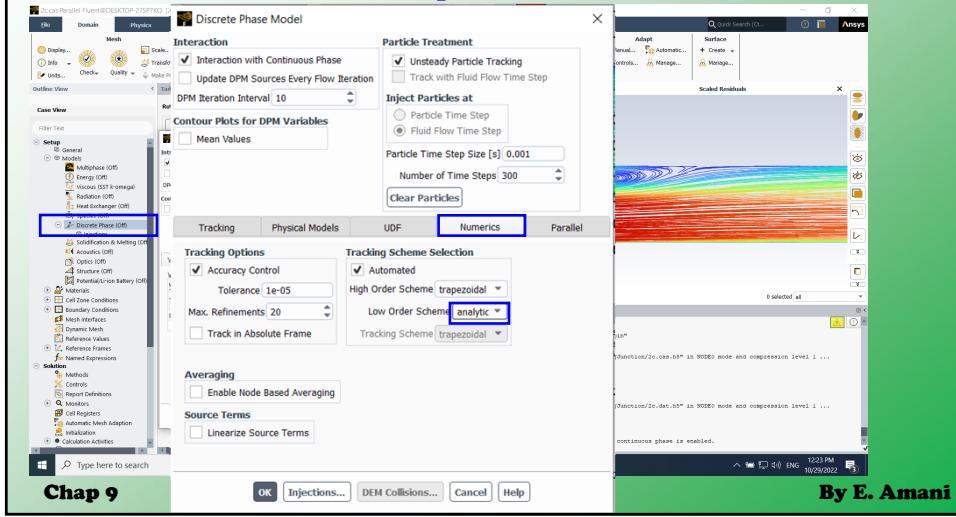
Models > Discrete phase > Physical models



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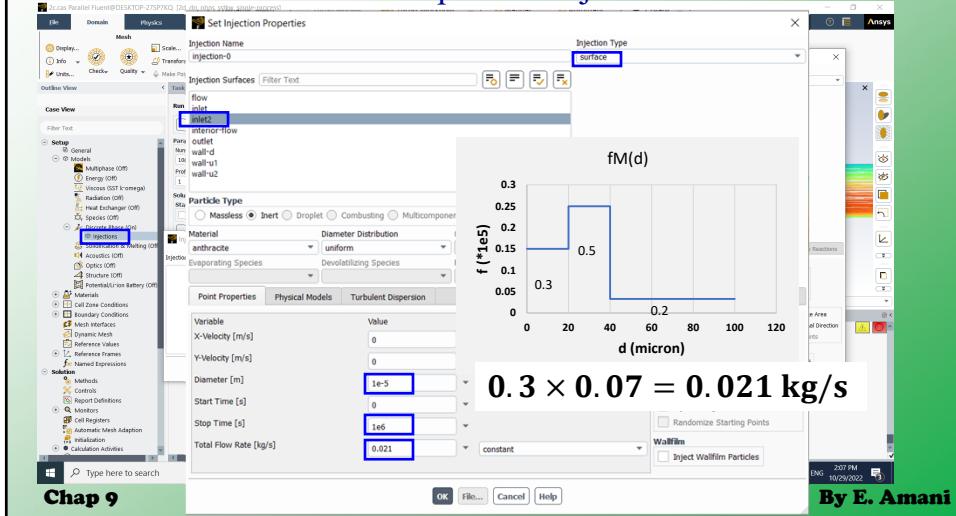
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
Models > Discrete phase > Numerics



Case study #3: Conveying T-junction

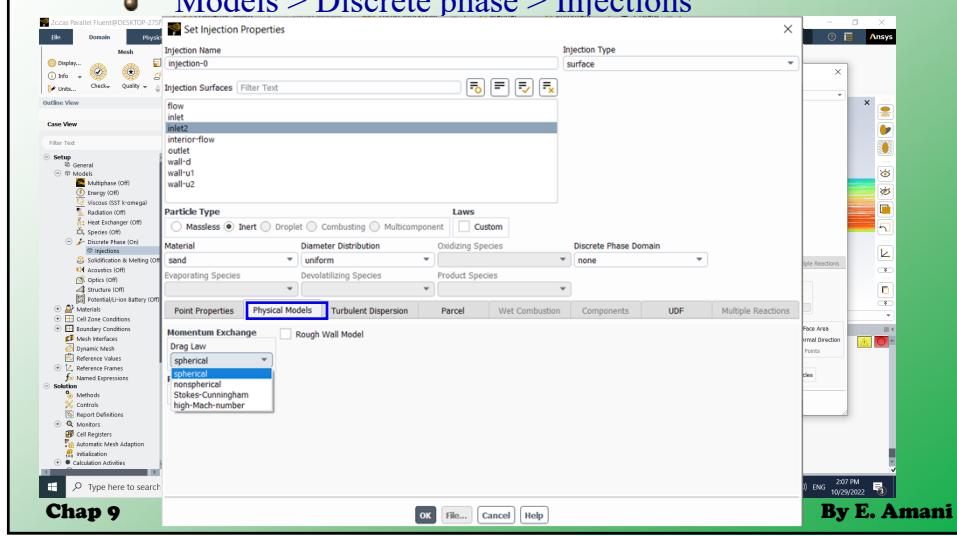
- Solution using ANSYS Fluent (one-way)
Models > Discrete phase > Injections



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

Models > Discrete phase > Injections

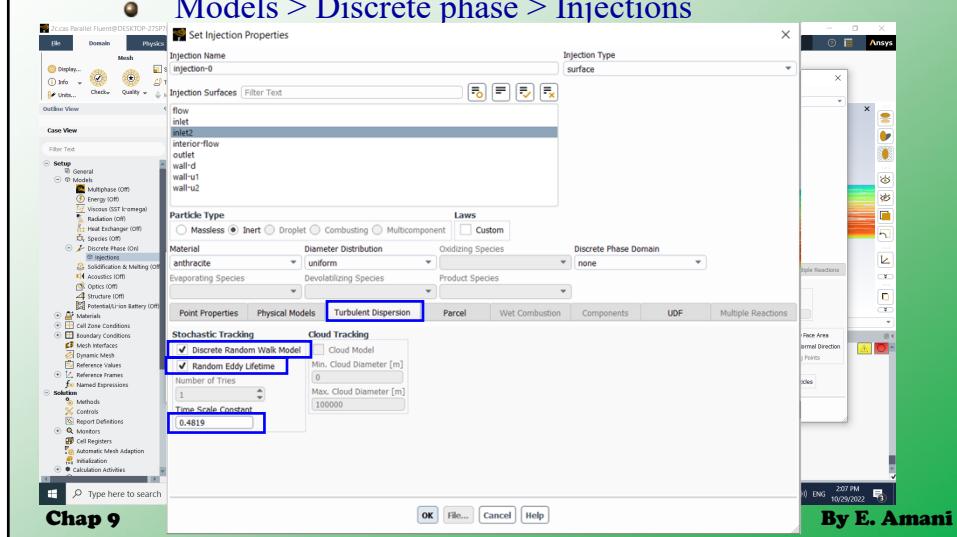


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

Models > Discrete phase > Injections

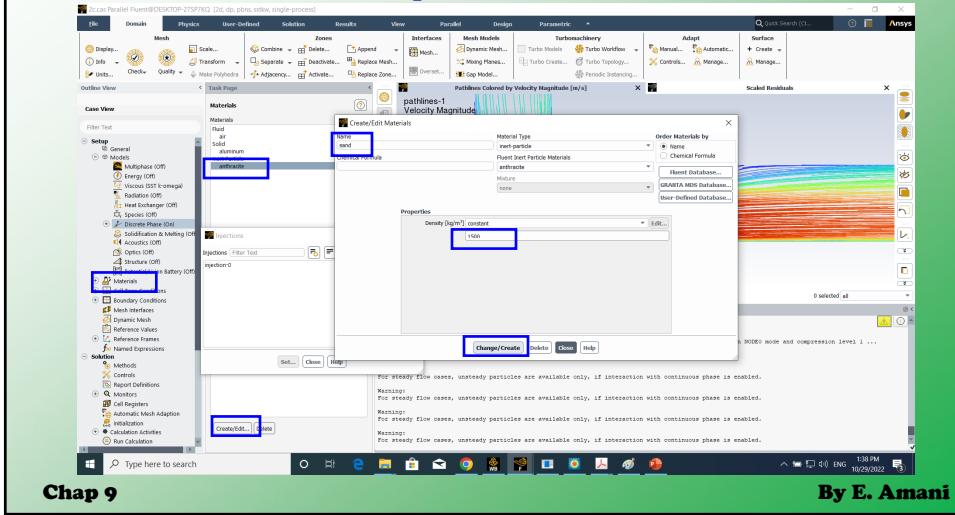


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

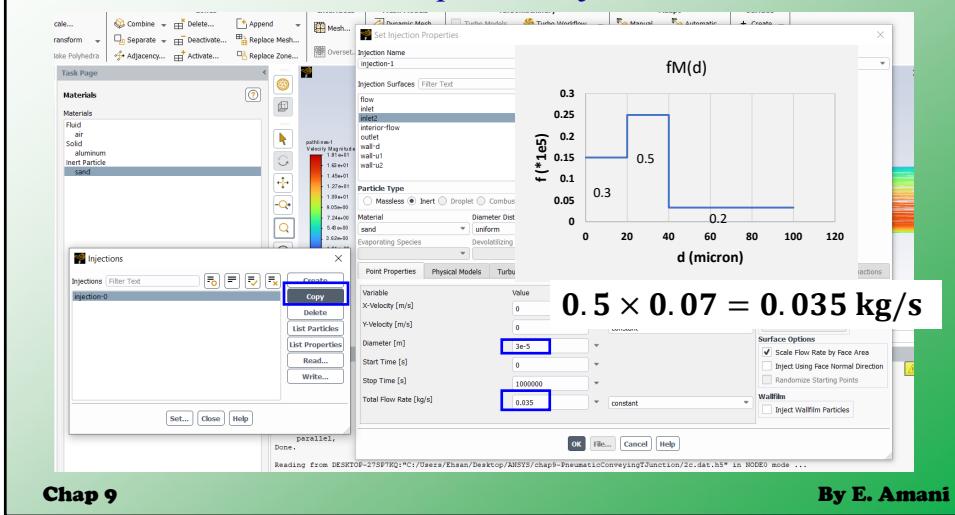
- Materials > Inert particle



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)

- Models > Discrete phase > Injections



Case study #3: Conveying T-junction

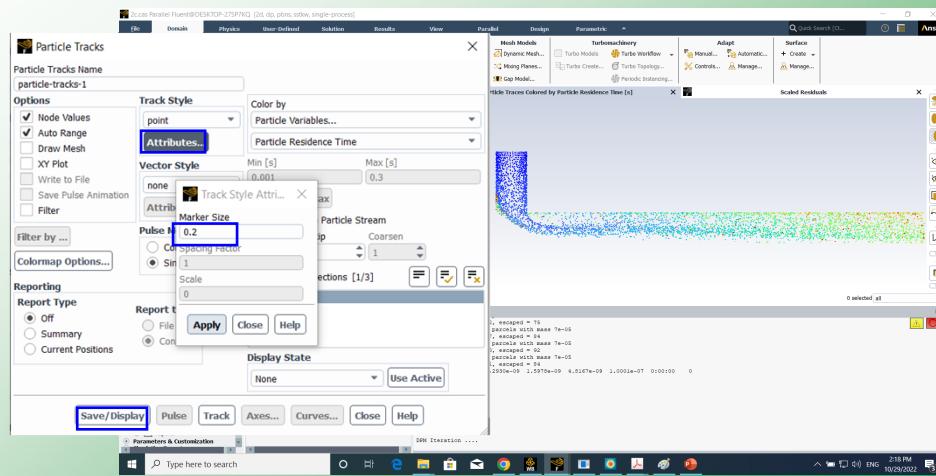
- Solution using ANSYS Fluent (one-way)
 - Run (10 iterations)
 - Save case and data: 3c

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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
 - Results > Particle tracks

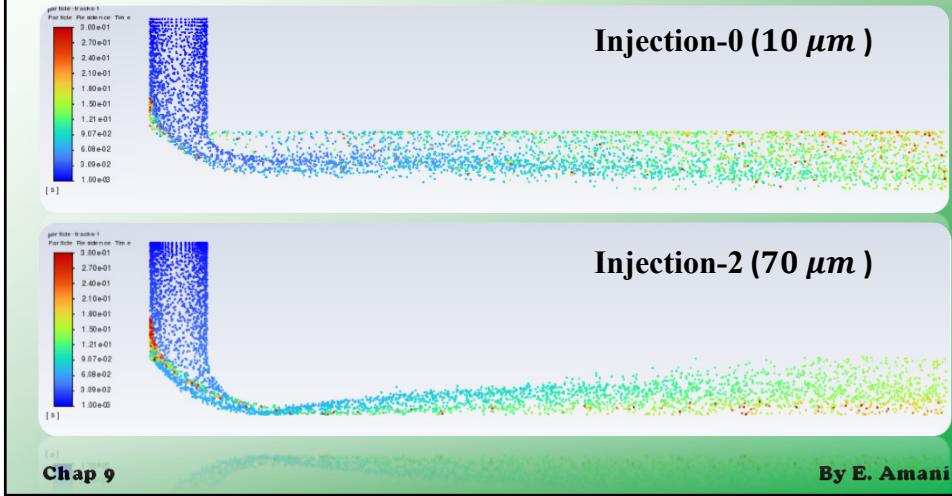


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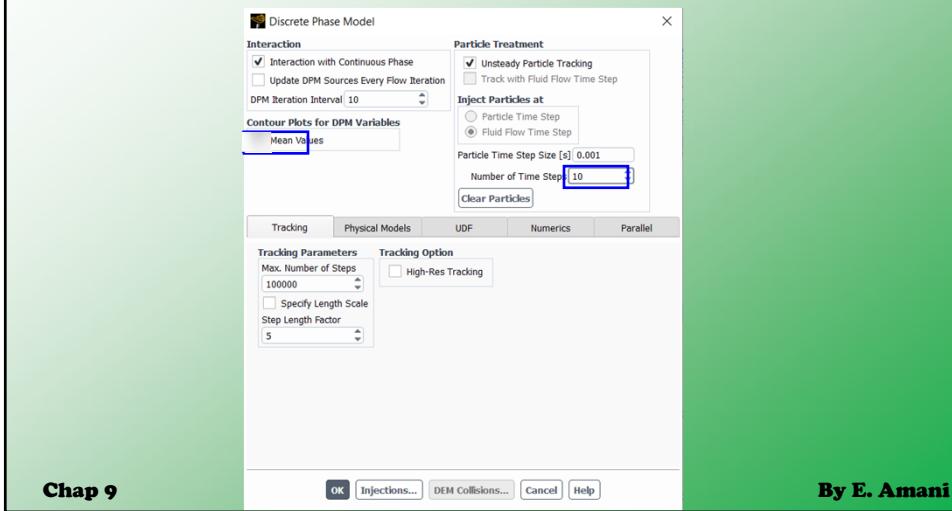
Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (one-way)
- Results > Particle tracks



Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
- Models > Discrete phase



Case study #3: Conveying T-junction

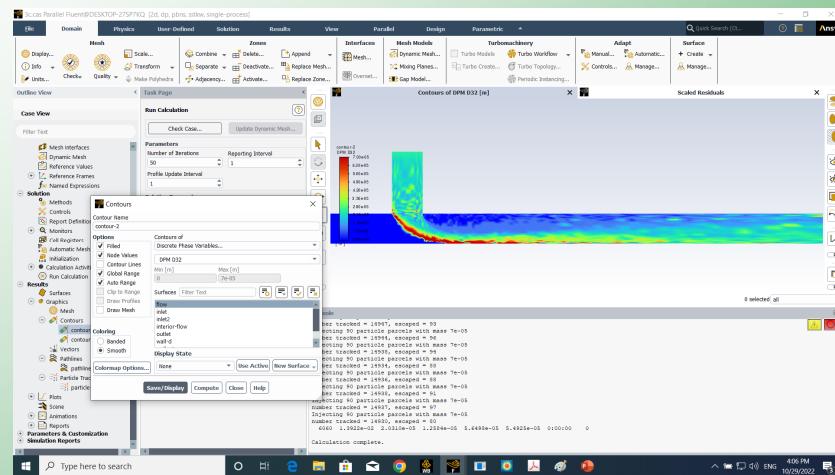
- Solution using ANSYS Fluent (two-way)
 - Run (5000 iterations)
 - Save case and data: 5c

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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
 - Results > Graphics > Contours

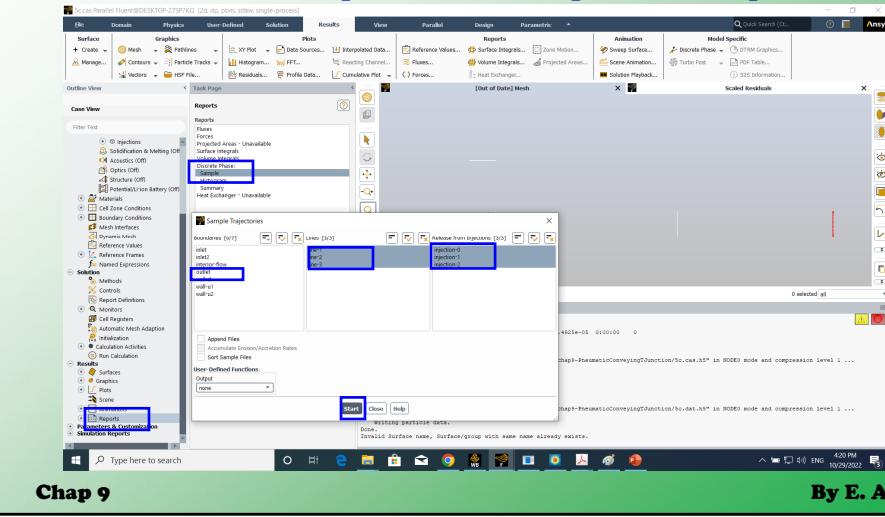


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
- Results > Reports > Discrete phase > Sample

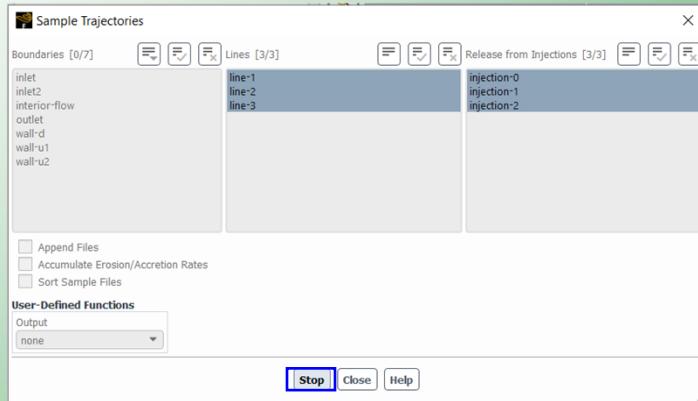


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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
- Run (?? iterations)
- Save case and data: 6c



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Case study #3: Conveying T-junction

- Solution using ANSYS Fluent (two-way)
- Run (?? iterations)
- Save case and data: 6c
- See files: line-1.dpm, line-2.dpm, line-3.dpm, and outlet.dpm

	x	y	z	u	v	w	diameter	t	parcel-mass	mass	n-lin-particle	time	file-time		
1	(line-1.13)														
2															
3	2.0721e-01	2.9500e-01	0.0000e+00	1.2011e-01	-1.7788e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.6400e-07	2.6939e+02	5.0740e-03	5.3501e+001	injection-2:48		
4	((2.3806e-01	2.9500e-01	0.0000e+00	-4.4606e-02	-1.7237e+00	0.0000e+00	7.0000e-05	3.0000e+02	9.8408e-07	2.6939e+02	3.6538e+03	5.2152e-03	5.3502e+001	injection-2:48	
5	((2.0134e-01	2.9500e-01	0.0000e+00	2.4272e-02	-1.9617e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.3674e-07	2.6939e+02	5.0788e+02	4.6231e-03	5.3506e+001	injection-2:48	
6	((2.0222e-01	2.9500e-01	0.0000e+00	-1.1011e-02	-1.9935e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.6118e-07	2.6939e+02	5.9832e+02	4.4744e-03	5.3505e+001	injection-2:48	
7	((2.0087e-01	2.9500e-01	0.0000e+00	-5.8130e-02	-1.9935e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.0398e-07	2.6939e+02	7.0528e+02	4.7447e-03	5.3506e+001	injection-2:48	
8	((2.0520e-01	2.9500e-01	0.0000e+00	3.3451e-02	-1.9645e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.4398e-07	2.6939e+02	4.6237e+03	5.3506e+001	injection-2:48		
9	((2.0674e-01	2.9500e-01	0.0000e+00	-4.7601e-02	-2.1282e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.6400e-07	2.6939e+02	9.7999e+02	4.2603e-03	5.3503e+001	injection-2:48	
10	((2.0907e-01	2.9500e-01	0.0000e+00	6.1635e-02	-2.1263e+00	0.0000e+00	7.0000e-05	3.0000e+02	3.1119e-07	2.6939e+02	1.1552e+03	4.2394e-03	5.3502e+001	injection-2:48	
11	((2.1171e-01	2.9500e-01	0.0000e+00	1.4134e-01	-1.8543e+00	0.0000e+00	7.0000e-05	3.0000e+02	3.6653e-07	2.6939e+02	1.3617e+03	4.8890e-03	5.3509e+001	injection-2:48	
12	((2.1445e-01	2.9500e-01	0.0000e+00	-2.0513e-01	-1.8543e+00	0.0000e+00	7.0000e-05	3.0000e+02	4.2703e-07	2.6939e+02	1.4418e+03	5.3507e+001	injection-2:48		
13	((2.1719e-01	2.9500e-01	0.0000e+00	2.2357e-02	-1.9159e+00	0.0000e+00	7.0000e-05	3.0000e+02	5.0971e-07	2.6939e+02	1.5921e+03	4.7262e-03	5.3507e+001	injection-2:48	
14	((2.2185e-01	2.9500e-01	0.0000e+00	1.3027e-01	-1.9712e+00	0.0000e+00	7.0000e-05	3.0000e+02	6.0082e-07	2.6939e+02	2.2303e+03	4.5966e-03	5.3506e+001	injection-2:48	
15	((2.2625e-01	2.9500e-01	0.0000e+00	2.3904e-02	-1.9040e+00	0.0000e+00	7.0000e-05	3.0000e+02	7.0823e-07	2.6939e+02	2.6290e+03	4.7563e-03	5.3500e+001	injection-2:48	
16	((2.3149e-01	2.9500e-01	0.0000e+00	-8.8491e-02	-1.9820e+00	0.0000e+00	7.0000e-05	3.0000e+02	8.3484e-07	2.6939e+02	3.0990e+03	4.5597e-03	5.3500e+001	injection-2:48	
17	((2.3724e-01	2.9500e-01	0.0000e+00	-1.8897e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	9.3618e-07	2.6939e+02	3.2686e+03	4.4024e-03	5.3500e+001	injection-2:48	
18	((2.4399e-01	2.9500e-01	0.0000e+00	-2.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.0323e-07	2.6939e+02	3.4289e+03	4.2269e-03	5.3500e+001	injection-2:48	
19	((2.5074e-01	2.9500e-01	0.0000e+00	-3.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.1025e-07	2.6939e+02	3.5893e+03	4.0534e-03	5.3500e+001	injection-2:48	
20	((2.5749e-01	2.9500e-01	0.0000e+00	-4.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.1727e-07	2.6939e+02	3.7497e+03	3.8800e-03	5.3500e+001	injection-2:48	
21	((2.6424e-01	2.9500e-01	0.0000e+00	-5.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.2429e-07	2.6939e+02	3.9001e+03	3.7066e-03	5.3500e+001	injection-2:48	
22	((2.7099e-01	2.9500e-01	0.0000e+00	-6.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.3131e-07	2.6939e+02	4.0505e+03	3.5332e-03	5.3500e+001	injection-2:48	
23	((2.7774e-01	2.9500e-01	0.0000e+00	-7.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.3833e-07	2.6939e+02	4.2059e+03	3.3598e-03	5.3500e+001	injection-2:48	
24	((2.8449e-01	2.9500e-01	0.0000e+00	-8.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.4535e-07	2.6939e+02	4.3623e+03	3.1864e-03	5.3500e+001	injection-2:48	
25	((2.9124e-01	2.9500e-01	0.0000e+00	-9.8894e-01	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.5237e-07	2.6939e+02	4.5187e+03	2.9130e-03	5.3500e+001	injection-2:48	
26	((2.9799e-01	2.9500e-01	0.0000e+00	-1.0889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.5939e-07	2.6939e+02	4.6751e+03	2.6396e-03	5.3500e+001	injection-2:48	
27	((3.0474e-01	2.9500e-01	0.0000e+00	-1.1889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.6641e-07	2.6939e+02	4.8315e+03	2.3662e-03	5.3500e+001	injection-2:48	
28	((3.1149e-01	2.9500e-01	0.0000e+00	-1.2889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.7343e-07	2.6939e+02	5.0379e+03	2.0929e-03	5.3500e+001	injection-2:48	
29	((3.1824e-01	2.9500e-01	0.0000e+00	-1.3889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.8045e-07	2.6939e+02	5.2443e+03	1.8196e-03	5.3500e+001	injection-2:48	
30	((3.2499e-01	2.9500e-01	0.0000e+00	-1.4889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.8747e-07	2.6939e+02	5.4507e+03	1.5463e-03	5.3500e+001	injection-2:48	
31	((3.3174e-01	2.9500e-01	0.0000e+00	-1.5889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	1.9449e-07	2.6939e+02	5.6571e+03	1.2730e-03	5.3500e+001	injection-2:48	
32	((3.3849e-01	2.9500e-01	0.0000e+00	-1.6889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.0151e-07	2.6939e+02	5.8635e+03	1.0000e-03	5.3500e+001	injection-2:48	
33	((3.4524e-01	2.9500e-01	0.0000e+00	-1.7889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.0853e-07	2.6939e+02	6.0699e+03	7.2667e-04	5.3500e+001	injection-2:48	
34	((3.5199e-01	2.9500e-01	0.0000e+00	-1.8889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.1555e-07	2.6939e+02	6.2763e+03	4.5334e-04	5.3500e+001	injection-2:48	
35	((3.5874e-01	2.9500e-01	0.0000e+00	-1.9889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.2257e-07	2.6939e+02	6.4827e+03	1.8001e-04	5.3500e+001	injection-2:48	
36	((3.6549e-01	2.9500e-01	0.0000e+00	-2.0889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.2959e-07	2.6939e+02	6.6891e+03	1.5268e-04	5.3500e+001	injection-2:48	
37	((3.7224e-01	2.9500e-01	0.0000e+00	-2.1889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.3661e-07	2.6939e+02	6.8955e+03	1.2535e-04	5.3500e+001	injection-2:48	
38	((3.7899e-01	2.9500e-01	0.0000e+00	-2.2889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.4363e-07	2.6939e+02	7.1019e+03	9.8022e-05	5.3500e+001	injection-2:48	
39	((3.8574e-01	2.9500e-01	0.0000e+00	-2.3889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.5065e-07	2.6939e+02	7.3083e+03	7.0689e-05	5.3500e+001	injection-2:48	
40	((3.9249e-01	2.9500e-01	0.0000e+00	-2.4889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.5767e-07	2.6939e+02	7.5147e+03	4.3356e-05	5.3500e+001	injection-2:48	
41	((3.9924e-01	2.9500e-01	0.0000e+00	-2.5889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.6469e-07	2.6939e+02	7.7211e+03	1.6023e-05	5.3500e+001	injection-2:48	
42	((4.0600e-01	2.9500e-01	0.0000e+00	-2.6889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.7171e-07	2.6939e+02	7.9275e+03	1.3290e-05	5.3500e+001	injection-2:48	
43	((4.1274e-01	2.9500e-01	0.0000e+00	-2.7889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.7873e-07	2.6939e+02	8.1339e+03	1.0557e-05	5.3500e+001	injection-2:48	
44	((4.1949e-01	2.9500e-01	0.0000e+00	-2.8889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.8575e-07	2.6939e+02	8.3403e+03	7.8234e-06	5.3500e+001	injection-2:48	
45	((4.2624e-01	2.9500e-01	0.0000e+00	-2.9889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.9277e-07	2.6939e+02	8.5467e+03	5.0901e-06	5.3500e+001	injection-2:48	
46	((4.3300e-01	2.9500e-01	0.0000e+00	-3.0889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	2.9979e-07	2.6939e+02	8.7531e+03	2.3668e-06	5.3500e+001	injection-2:48	
47	((4.3974e-01	2.9500e-01	0.0000e+00	-3.1889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	3.0681e-07	2.6939e+02	8.9695e+03	1.0935e-06	5.3500e+001	injection-2:48	
48	((4.4649e-01	2.9500e-01	0.0000e+00	-3.2889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	3.1383e-07	2.6939e+02	9.1759e+03	8.2122e-07	5.3500e+001	injection-2:48	
49	((4.5324e-01	2.9500e-01	0.0000e+00	-3.3889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	3.2085e-07	2.6939e+02	9.3823e+03	5.4890e-07	5.3500e+001	injection-2:48	
50	((4.5999e-01	2.9500e-01	0.0000e+00	-3.4889e-00	-7.8632e+00	0.0000e+00	7.0000e-05	3.0000e+02	3.2787e-07						

Other topics

- Simplified relation between FPI force in ESS and ELSS **Notes:Chap9.1.1.10**
- See “[FPISummary.pdf](#)” for a summary of FPI force formulation.

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