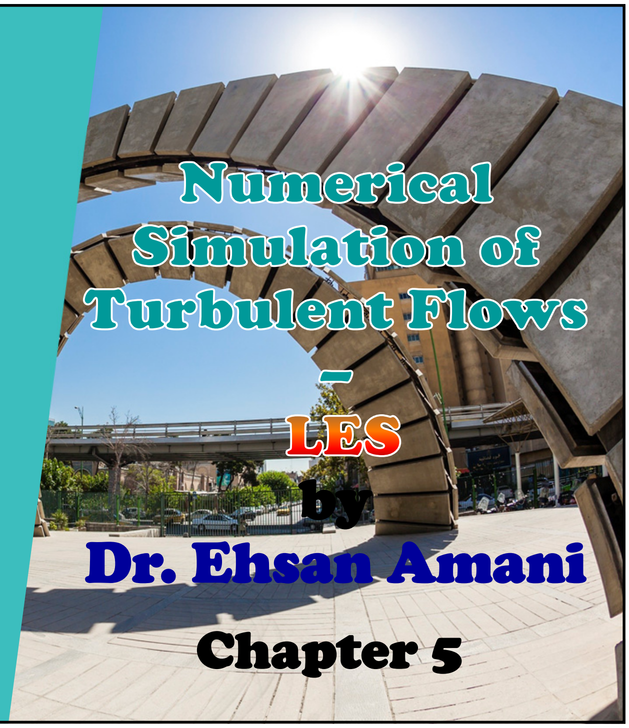


Numerical Simulation of Turbulent Flows – LES

by

Dr. Ehsan Amani

Chapter 5

Introduction

Reynolds-Averaged-Numerical-Simulation or Reynolds-Averaged-Navier-Stokes (RANS)

- The ensemble averaged momentum equation:

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_i \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} = \nu \frac{\partial^2 \langle U_i \rangle}{\partial x_j \partial x_j} - \frac{\partial \langle u_i u_j \rangle}{\partial x_j} - \frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i}$$

  - RANS: For stationary flows, the ensemble average, $\langle . \rangle$, can be regarded as the time average, $\langle . \rangle_T$.
  - Closure problem: Six additional unknowns $\langle u_i u_j \rangle$

Chap 5 By E. Amani

V.1 Turbulent viscosity models

Boussinesq's gradient diffusion assumption

- Based on the similarity between turbulent and viscous stress:

$$-\langle\vec{u}\vec{u}\rangle = 2\nu_T\langle\underline{S}\rangle - \frac{2}{3}k\underline{I} \qquad (5.11)$$

$$\langle u_i u_j\rangle = -\underbrace{\nu_T}_{\text{Turbulent viscosity}}\left(\frac{\partial\langle U_i\rangle}{\partial x_j} + \frac{\partial\langle U_j\rangle}{\partial x_i}\right) + \frac{2}{3}k\delta_{ij}$$

  - The closure is reduced to one variable, i.e., ν_T.
  - Exercise: What are the values of normal Reynolds stresses based on this assumption? Does it impose any limitation to the generality of the model?
  - A large group of models → ...

Chap 5 By E. Amani

V.2 Two-equation models

$k - \varepsilon$ model

- Modeling:
  - Since k and ε are key parameters in turbulent flows, assume that: $\nu_T = f(k,\varepsilon)$
  - Based on a simple dimensional analysis (exercise):

$$\nu_T = \underbrace{C_\mu}_{\text{Empirical model constant}} k^2/\varepsilon \qquad (5.13)$$

  - Transport equation of k, Eq. (3.21):

$$\frac{\bar{D}k}{\bar{D}t} + \vec{\nabla}.\vec{T}' = \mathcal{P} - \varepsilon$$

  - Exercise: Substituting Eq. (5.11) into Eq. (3.19), derive $\mathcal{P}$ in the Cartesian coordinates system and show that this term is not unknown anymore.

$$\mathcal{P} = -\langle u_i u_j\rangle\frac{\partial\langle U_i\rangle}{\partial x_j} \qquad (3.19)$$

Chap 5 By E. Amani

V.2 Two-equation models

$k-\varepsilon$ model

- Modeling:
  - A gradient diffusion assumption for $\vec{T}'$:

$$\vec{T}' = -\frac{\nu_T}{\sigma_k}\vec{\nabla}k \qquad (5.14)$$

$\sigma_k \rightarrow$ **Empirical model constant**

  - The modeled (closed) transport equation for k:

$$\frac{\overline{D}k}{\overline{D}t} = \vec{\nabla}.\left(\frac{\nu_T}{\sigma_k}\vec{\nabla}k\right) + \mathcal{P} - \varepsilon \qquad (5.15)$$

  - An alternative form, assuming $\varepsilon \sim \tilde{\varepsilon}$ (HW#2):

$$\frac{\overline{D}k}{\overline{D}t} = \vec{\nabla}.\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\vec{\nabla}k\right] + \mathcal{P} - \varepsilon \qquad (5.15)'$$

  - Similarly, a modeled equation for ε can be derived.

Chap 5 By E. Amani

V.2 Two-equation models

$k-\varepsilon$ model

- Final model:

$$\nu_T = C_\mu k^2/\varepsilon \qquad (5.11)$$

$$\frac{\overline{D}k}{\overline{D}t} = \vec{\nabla}.\left(\frac{\nu_T}{\sigma_k}\vec{\nabla}k\right) + \mathcal{P} - \varepsilon \qquad (5.15)$$

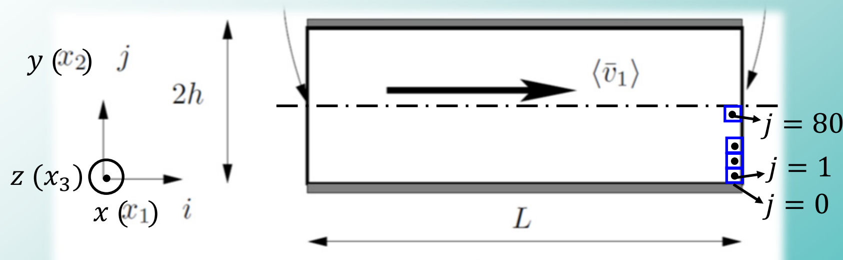
$$\frac{\overline{D}\varepsilon}{\overline{D}t} = \vec{\nabla}.\left(\frac{\nu_T}{\sigma_\varepsilon}\vec{\nabla}\varepsilon\right) + C_{\varepsilon 1}\frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \qquad (5.16)$$

  - 5 model constants: C_μ, σ_k, σ_ε, $C_{\varepsilon 1}$, and $C_{\varepsilon 2}$
  - To be tuned experimentally.
  - May be case dependent or even variable in a single problem (generality!!)

Chap 5 By E. Amani

Hands-on practice

- **HW#3 (continue):**
  - Calculating different RANS statistics using DNS data



(a) $x_1 - x_2$ plane.

  - DNS with a $192 \times 160 \times 192$ grid, however, data on:

$$192 \times 81 \times 192 \text{ nodes}$$

$$1 + 160/2$$

$$y = 0$$

  - For this problem, consider $\langle . \rangle$ as the spatial average in (x, z) plane rather than the time average, $\langle . \rangle_T$.

Chap 5 **By E. Amani**

Hands-on practice

- **HW#3 (continue): :**
  - DNS data variables (superscript "*" indicates dimensional variables):

$$x_i = \frac{x_i^*}{h}, U_i = \frac{U_i^*}{u_\tau}, p = \frac{p^*}{\rho u_\tau^2}$$

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Wall shear stress

Friction velocity

  - In your report, you need the wall unit:

$$x_i^+ = \frac{u_\tau x_i^*}{\nu} = Re_\tau x_i, U_i^+ = \frac{U_i^*}{u_\tau} = U_i$$

$$Re_\tau = \frac{u_\tau h}{\nu}$$

Friction Reynolds number

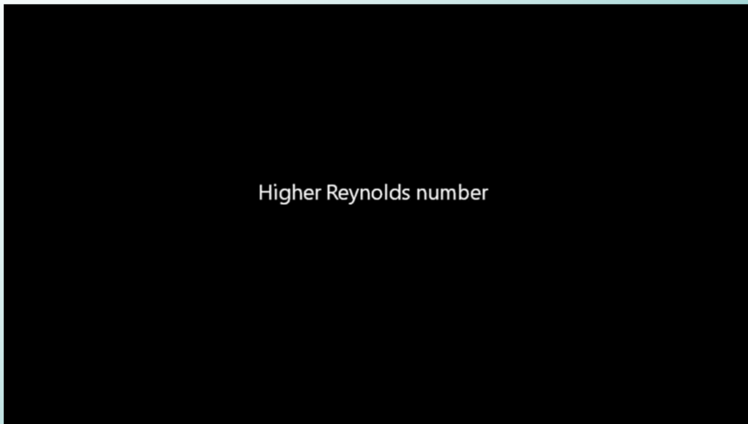
  - The channel Reynolds number vs. friction Reynolds number:

$$\mathrm{Re} = \frac{U_b^* h}{\nu} = \frac{U_b^*}{u_\tau} u_\tau \frac{h}{\nu} = Re_\tau U_b$$

Chap 5 **By E. Amani**

V.3 Other RANS models and numerical considerations

Boundary conditions



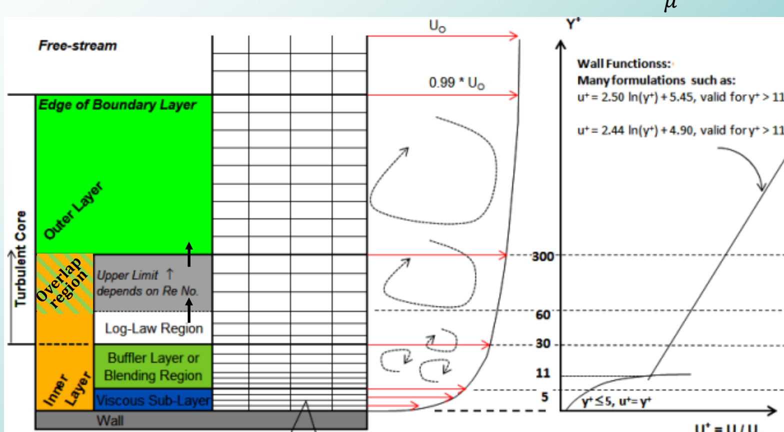
▲ Turbulent boundary layer (TBL)

Chap 5 By E. Amani

V.3 Other RANS models and numerical considerations

Boundary conditions

$$y^+ = \frac{\rho u_\tau y}{\mu} \qquad u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$



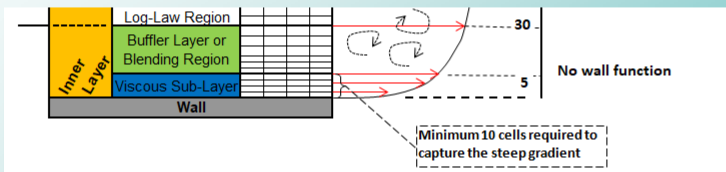
▲ Turbulent boundary layer regions

Chap 5 By E. Amani

Boundary conditions

(Near) wall treatment

- Wall-resolved modeling



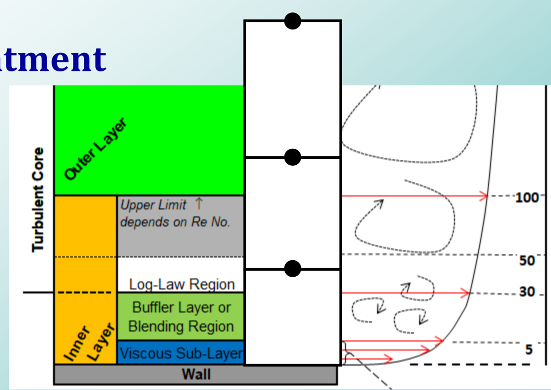
  - First node $y^+ \sim 1-5$
  - 6 to 10 cells covering $y^+ < 10$
  - Grid expansion ratio < 1.2

Chap 5 By E. Amani

Boundary conditions

(Near) wall treatment

- Wall function



  - First node can have much larger y^+
  - Approximation where important physical phenomena are away from the wall
  - For the formulations, consult ANSYS Fluent theory guide!
  - Check the range of validity !
    - $y^+ > 30$ (standard wall function), $y^+ < 300$

Chap 5 By E. Amani

Boundary conditions

(Near) wall treatment

- Post-processing y^+ check

$$y^+ = \frac{\rho u_\tau y}{\mu} \qquad u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- Pre-processing

$$y^+ = \frac{\rho u_\tau y}{\mu} \Leftrightarrow y = \frac{\mu y^+}{\sqrt{\rho \tau_w}}$$

  - Approximation,
    - ✓ Flat plate TBL:

$$\tau_w = \frac{1}{2} C_f \rho U_\infty^2 \qquad C_f = 0.058\, Re_L^{-0.2}$$

    - ✓ Pipe flow:

$$\tau_w = \frac{1}{8} f \rho U_b^2 \qquad f = [0.79 \ln(Re_D) - 1.64]^{-2}$$

    - ✓ Flow between parallel plates (height $2h$):

$$D_h = 4h \qquad Re_{D_h} = 4Re_b$$

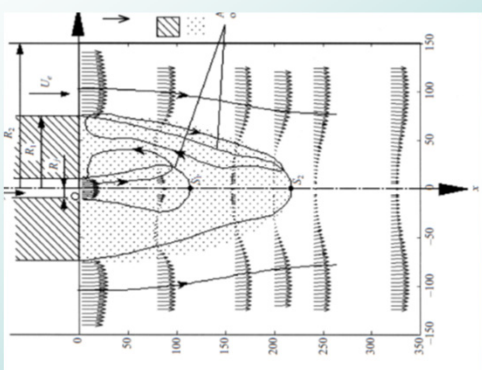
$$\tau_w = \frac{1}{8} f \rho U_b^2 \quad f = \left[0.79 \ln\left(0.64 Re_{D_h}\right) - 1.64\right]^{-2}$$

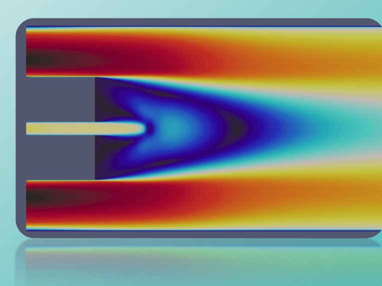
Chap 5 By E. Amani

Boundary conditions

Inlet condition

- Extended inlet





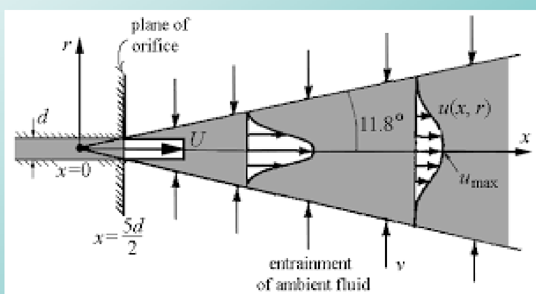
- Precursor simulation

Chap 5 By E. Amani

Other RANS models

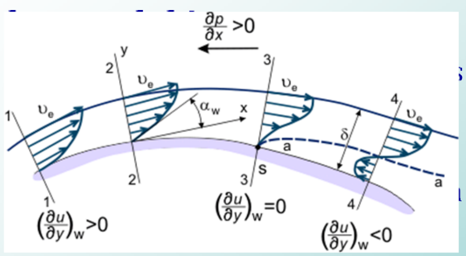
$k-\varepsilon$ model issues

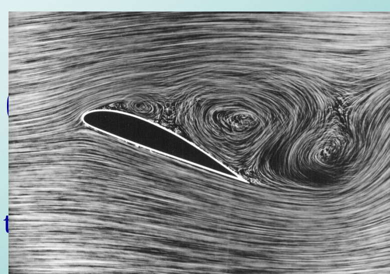
- Valid for fully turbulent regimes (needs near-wall modifications)
  - Low-Re damping term
  - Tow-layer modeling (switch to other models for y^+<30)
- Round-jet anomaly



Chap 5 By E. Amani

Other RANS models





- Poor performance for
  - large/adverse pressure gradients

Chap 5 By E. Amani

Other RANS models

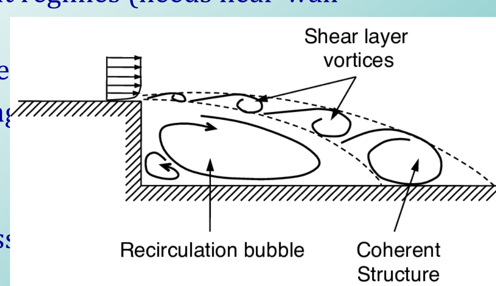
k-ε model issues

- Valid for fully turbulent regimes (needs near-wall modifications)
  - Low-Re damping term
  - Tow-layer modeling (switch to other models for y^+<30)
- Round-jet anomaly
- Poor performance for
  - large/adverse press
  - separating flows

Other RANS models

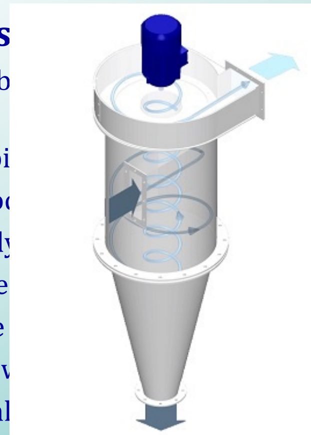
k-ε model issues

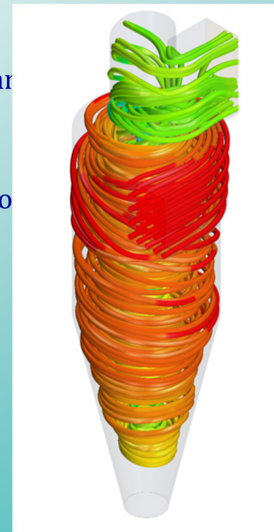
- Valid for fully turbulent regimes (needs near-wall modifications)
  - Low-Re damping te
  - Tow-layer modeling
- Round-jet anomaly
- Poor performance for
  - large/adverse press
  - separating flows
  - strong streamline curvatures



Other RANS models

k-ε model issues





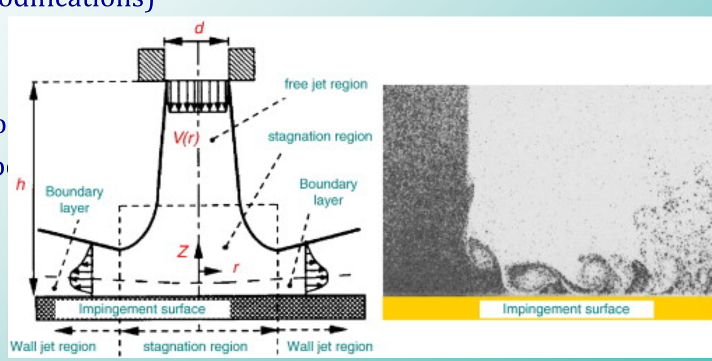
- Valid for fully turb ar modifications)
  - Low-Re dampi
  - Tow-layer mo o
- Round-jet anomaly
- Poor performance
  - large/adverse
  - separating flov
  - strong streaml
  - swirling flows

Chap 5 By E. Amani

Other RANS models

k-ε model issues

- Valid for fully turbulent regimes (needs near-wall modifications)
  -
  -
- Ro
- Po
  -
  -
  -
  -
  - stagnation flows

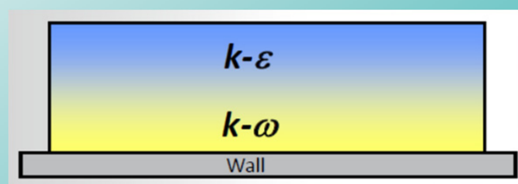


Chap 5 By E. Amani

Other RANS models

More advanced two-equation closures

- RNG k-ε
- Realizable k-ε (recommended k-ε variant)
- k-ω SST (recommended)
  - Switches to k-ε in free stream, k-ω in BL
  - The best 2-eq. closure for resolved wall treatment
  - Force on the wall
  - Heat transfer to the wall

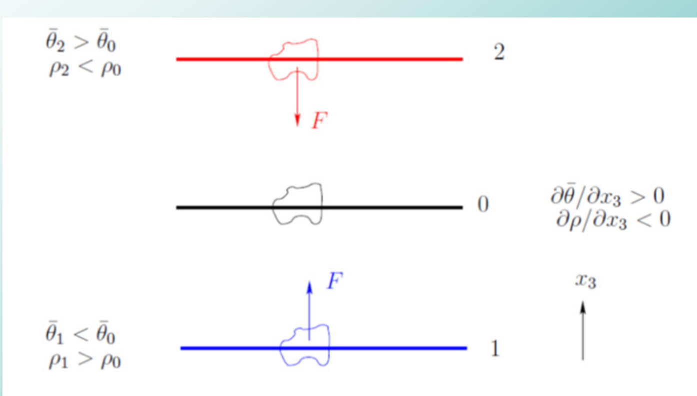


Chap 5 By E. Amani

Other RANS models

Eddy viscosity model shortcomings

- Buoyancy (stable stratification)



Chap 5 By E. Amani

Other RANS models

Eddy viscosity model shortcomings

- Buoyancy (stable stratification)
- Normal stress prediction (anisotropy)
- Swirling and recirculating flows (curvature effect)
- Stagnation flows (irrotational strain)

- Avoiding eddy viscosity assumption:
  - Reynolds Stress Model (RSM)

$$\frac{\bar{D}\langle u_i u_j\rangle}{\bar{D}t} = \nu \frac{\partial^2 \langle u_i u_j\rangle}{\partial x_k \partial x_k} - \frac{\partial T_{kij}}{\partial x_k} + P_{ij} + R_{ij} - \tilde{\varepsilon}_{ij} \qquad (5.17)$$

+ The epsilon or omega equation (7 equations)

Chap 5 By E. Amani

Other RANS models

RSM model

- Closed production term:

Production rate tensor

$$\mathcal{P}_{ij} = -\left[\langle u_i u_k\rangle \frac{\partial \langle U_j\rangle}{\partial x_k} + \langle u_j u_k\rangle \frac{\partial \langle U_i\rangle}{\partial x_k}\right] \qquad (5.18)$$

  - Exercise: What is the relation of $\mathcal{P}_{ij}$ with $\mathcal{P}$ in k equation? Why?
- Closures:
  - The local isotropy assumption for $\tilde{\varepsilon}_{ij}$:

$$\tilde{\varepsilon}_{ij} \sim \tilde{\varepsilon}_{ij} = \frac{2}{3}\varepsilon\delta_{ij} \qquad (5.22)$$

  - A gradient-diffusion assumption for T_{kij}:

$$T_{kij} = -\frac{\nu_T}{\sigma_k}\frac{\partial \langle u_i u_j\rangle}{\partial x_k} \qquad (5.23)$$

Chap 5 By E. Amani

Other RANS models

RSM model

- Closures:
  - There is more involved in the modeling of the pressure-strain term, R_{ij}.
  - In addition to k and ε; $\langle S_{ij}\rangle$, $\langle \Omega_{ij}\rangle$, and b_{ij} (the anisotropy tensor) are important, where:

$$b_{ij} = \frac{\langle u_i u_j\rangle}{2k} - \frac{1}{3}\delta_{ij} \qquad (5.26)$$

  - Quadratic pressure-strain model:

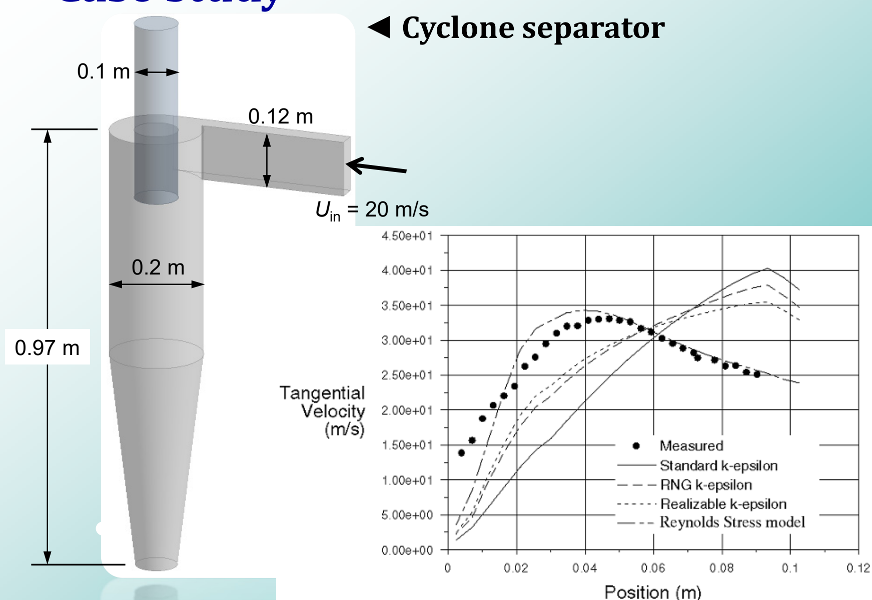
$$R_{ij} = -C_1\varepsilon b_{ij} + C_2\varepsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{mn}b_{mn}\delta_{ij}\right) + C_3 k\langle S_{ij}\rangle + C_4 k\left(b_{ik}\langle S_{jk}\rangle + b_{jk}\langle S_{ik}\rangle - \frac{2}{3}b_{mn}\langle S_{mn}\rangle\delta_{ij}\right) + C_5 k\left(b_{ik}\langle \Omega_{jk}\rangle + b_{jk}\langle \Omega_{ik}\rangle\right) \qquad (5.22)$$

  - 9 model constants: σ_k, σ_ε, $C_{\varepsilon 1}$,$C_{\varepsilon 2}$, and C_1 to C_5

Chap 5 **By E. Amani**

Case study

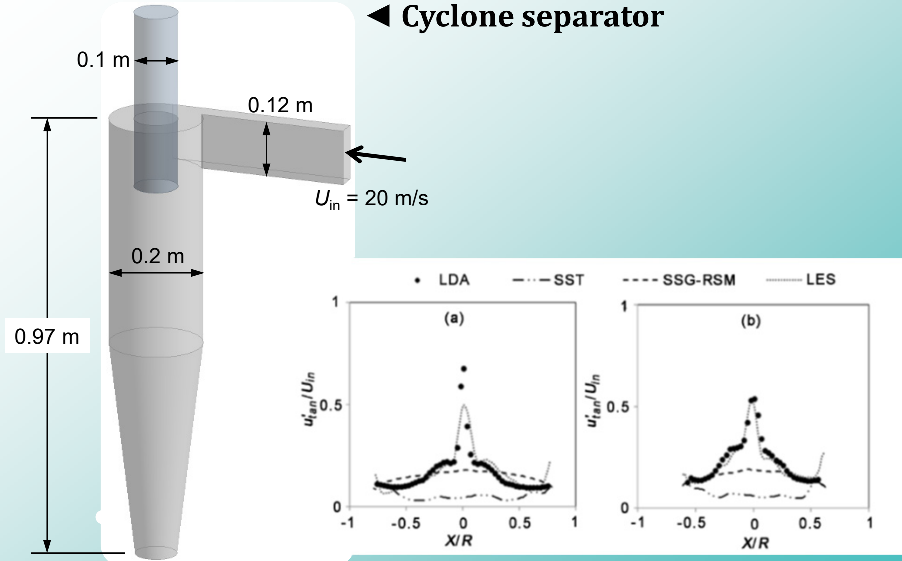
◄ **Cyclone separator**



Chap 5 **By E. Amani**

Case study

◄ **Cyclone separator**



Chap 5 **By E. Amani**

Unsteady RANS (URANS)

- Adding unsteady terms (simple URANS)
  - For example, k-ω SST:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho u_j k)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+\tilde{P}-\beta^*\rho k\omega$$

$$\frac{\partial(\rho \omega)}{\partial t}+\frac{\partial(\rho u_j \omega)}{\partial x_j}=\frac{\alpha\tilde{P}}{\nu_t}-\beta\rho\omega^2+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+2(1-F_1)\rho\sigma_{\omega_2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

- However, RANS is too dissipative!
- Reducing μ_t locally and allowing unsteadiness to grow (improved URANS)
  - For example, scale-adaptive simulation (SAS)
    - k-ω SST SAS: additional source term in ω equation

$$\frac{\partial(\rho \omega)}{\partial t}+\frac{\partial(\rho u_j \omega)}{\partial x_j}=\frac{\alpha\tilde{P}}{\nu_t}-\beta\rho\omega^2+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+2(1-F_1)\rho\sigma_{\omega_2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}+\boldsymbol{Q_{SAS}}$$

    - $\mu_t \sim \rho k/\omega$: $\omega\uparrow \rightarrow \mu_t\downarrow$ (depending on grid size)

Chap 5 **By E. Amani**

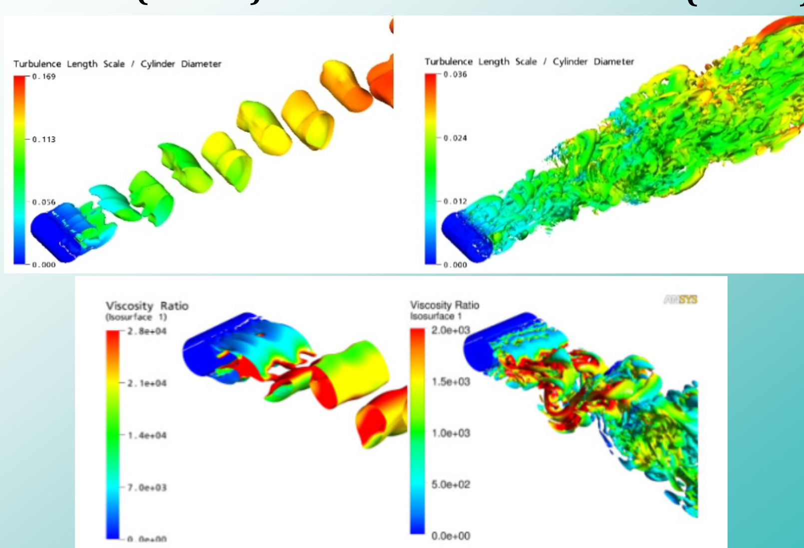
Unsteady RANS (URANS)

Reference LES solution▼

Chap 5 By E. Amani

Unsteady RANS (URANS)

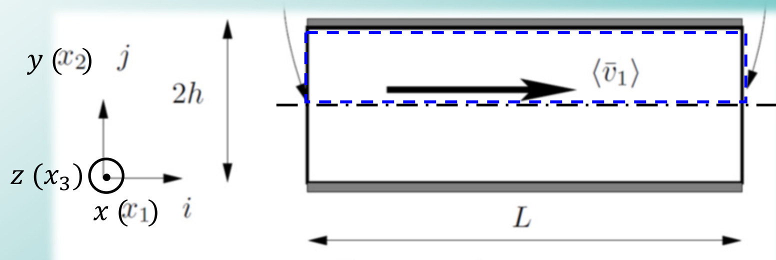
k-ω SST (URANS)▼ **▼k-ω SST SAS (URANS)**



Chap 5 By E. Amani

Hands-on practice

- **HW#4:**
  - Precursor simulation
  - ANSYS Fluent practice
  - 2-equation, 4-equation, and 7-equation models
  - The importance of wall functions (enhanced wall treatment)
  - RANS models comparison



(a) $x_1 - x_2$ plane.

Chap 5 By E. Amani