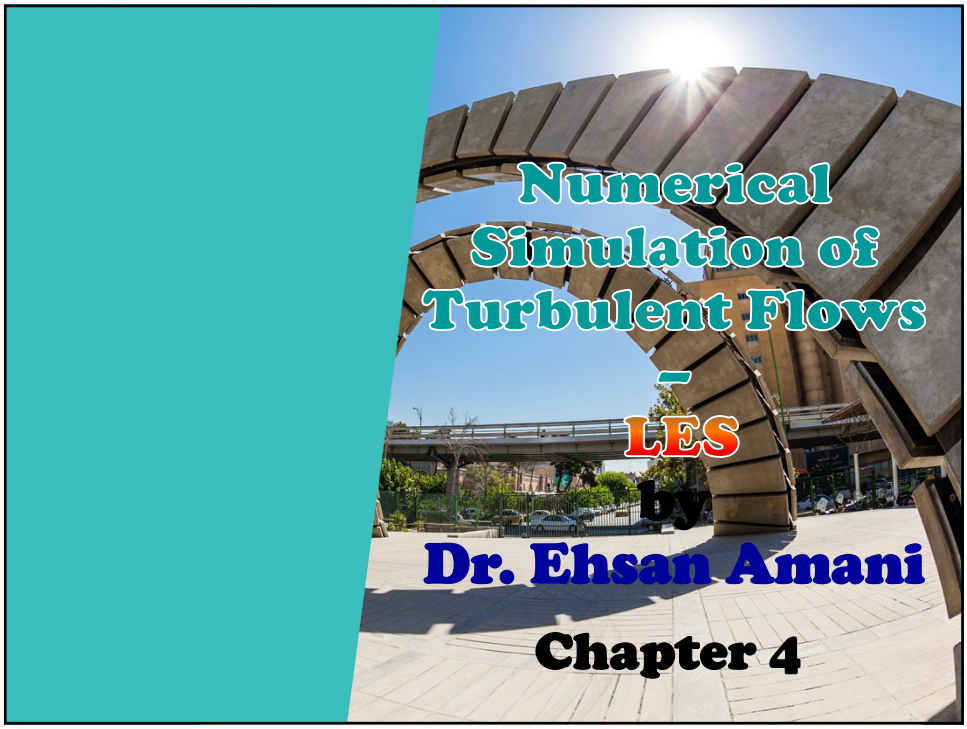
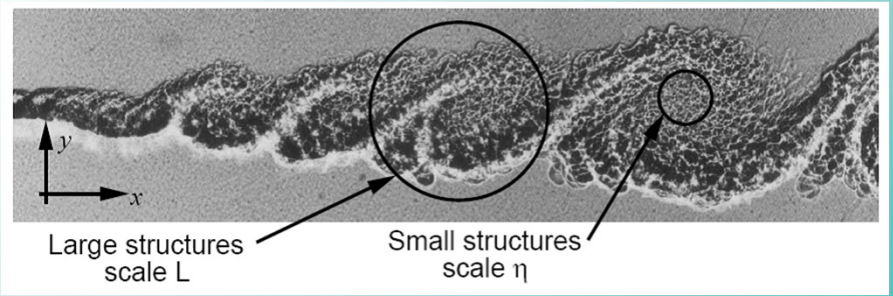


VI.1 The energy cascade

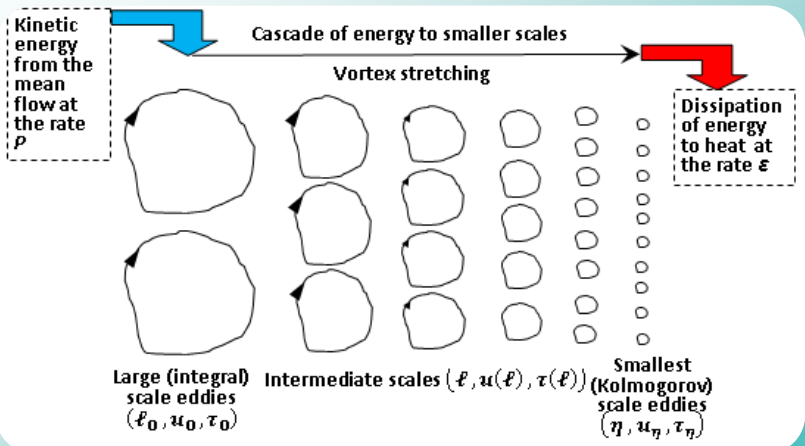
A broad range of scales



▲ Visualization of the flow in a mixing layer (from Brown & Roshko 1974)

VI.1 The energy cascade

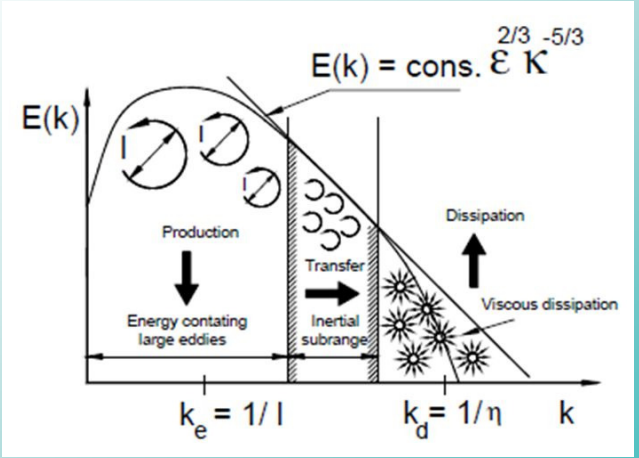
Richardson's view of energy cascade



Chap 4 By E. Amani

VI.1 The energy cascade

Energy spectrum



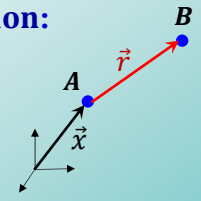
Chap 4 By E. Amani

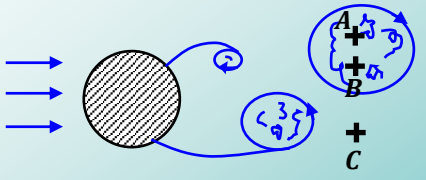
VI.2 Two-point correlation

- **(Spatial) correlation function definition:**

$$\underline{R} \equiv \langle \vec{u}_A \vec{u}_B \rangle = \langle u_{A,i} u_{B,j} \rangle \hat{e}_i \hat{e}_j \quad (4.27)$$

$$R_{ij}(\vec{r}, \vec{x}, t) = \langle u_i(\vec{x}, t) u_j(\vec{x} + \vec{r}, t) \rangle$$



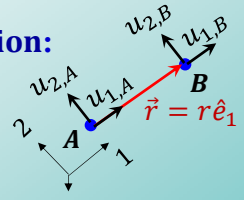


$$\langle \vec{u}_A \vec{u}_B \rangle \neq 0$$
$$\langle \vec{u}_A \vec{u}_C \rangle \approx 0$$

VI.2 Two-point correlation

- **(Spatial) correlation function definition:**
  - **Assuming homogeneity**

$$R_{ij}(\vec{r}, \vec{x}, t) = R_{ij}(\vec{r}, t)$$



Longitudinal correlation

$$f(r) = \frac{R_{11}(r\hat{e}_1)}{\langle u_1 u_1 \rangle}$$

Transverse correlation

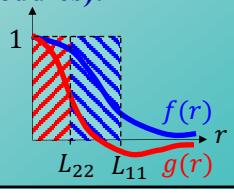
$$g(r) = \frac{R_{22}(r\hat{e}_1)}{\langle u_2 u_2 \rangle}$$

(4.29)′

  - **Integral length scales (large eddies):**

$$L_{11} = \int_0^\infty f(r)dr$$

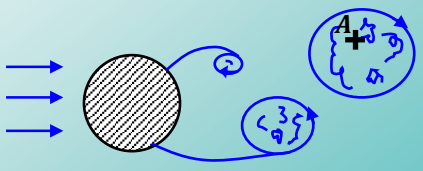
$$L_{22} = \int_0^\infty g(r)dr \quad (4.30)$$



VI.2 Two-point correlation

- **(Temporal) correlation (autocorrelation):**

$$B_{ij}(\tau, \vec{x}, t) \equiv \langle u_i(\vec{x}, t) u_j(\vec{x}, t+\tau) \rangle$$



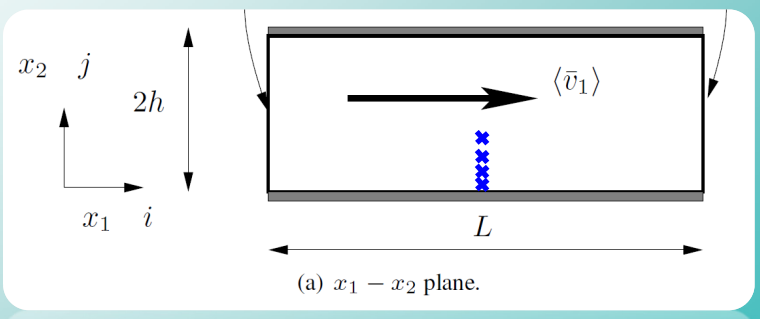
$$B_{ij}^{\text{norm}}(\tau, \vec{x}, t) \equiv \frac{B_{(i)(j)}}{\langle u_{(i)} u_{(j)} \rangle}$$

- **Integral time scales (large eddies):**

$$T_{ij} = \int_0^\infty B_{ij}^{\text{norm}}(\tau) d\tau$$

Hands-on practice

- **HW#3:**
  - **Python installation**
  - **CFD with python (preliminary experience)**
  - **Calculating different statistics using DNS data**
  - **Finding resources on the Net**



(a) $x_1 - x_2$ plane.