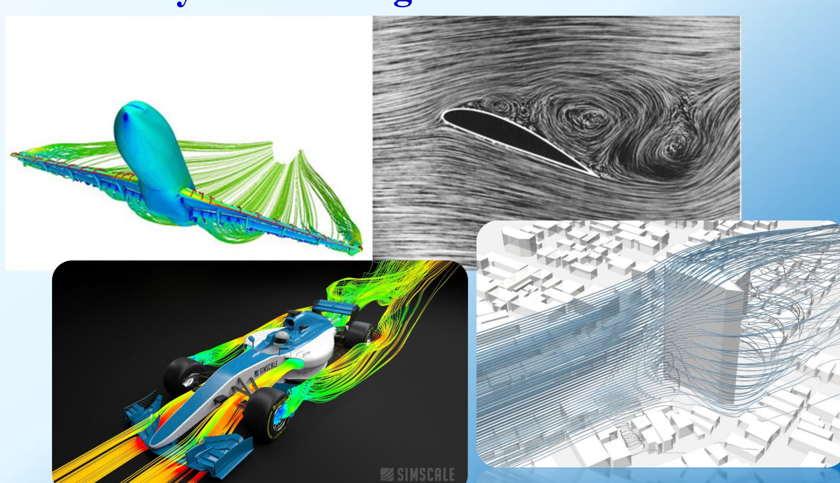
# Fluid mechanics II
by
Dr. Ehsan Amani

Chapter 5

## Drag and lift

- External flows (flow over immersed bodies)
  - Aerodynamics design

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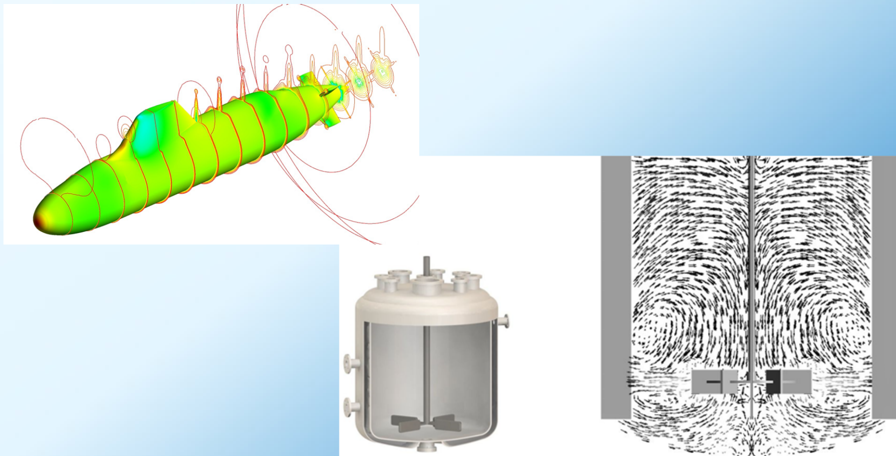
# Drag and lift

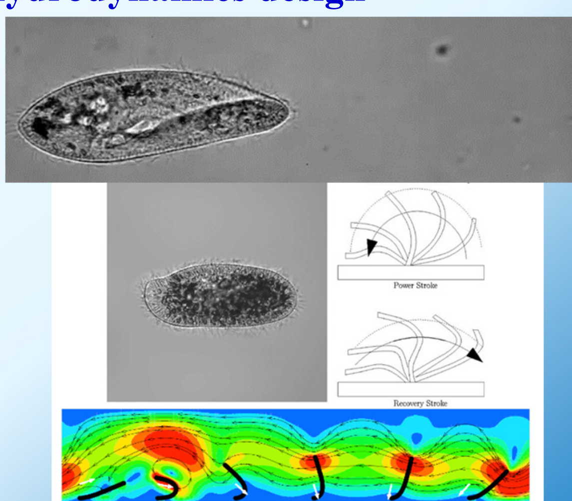
- External flows (flow over immersed bodies)
  - Hydrodynamics design





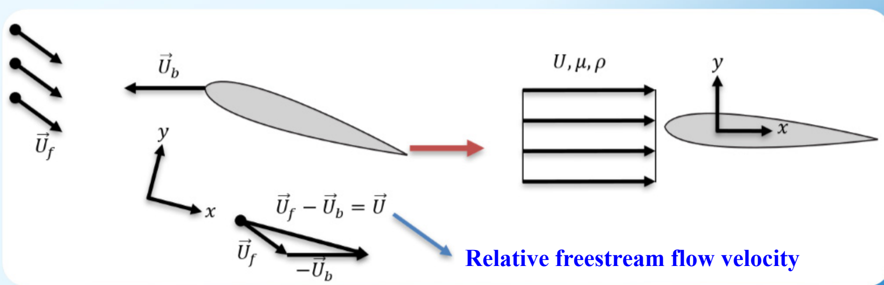
# Drag and lift

- External flows (flow over immersed bodies)
  - Hydrodynamics design



# Drag and lift

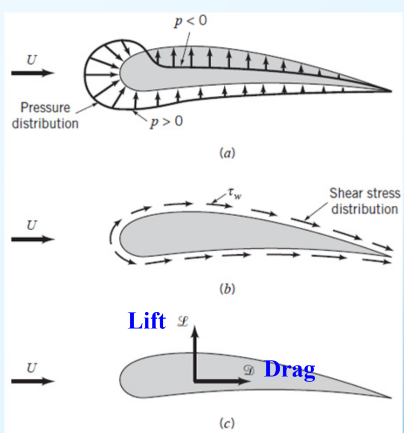
- External flows (flow over immersed bodies)
  - The choice of the reference frame and coordinates system





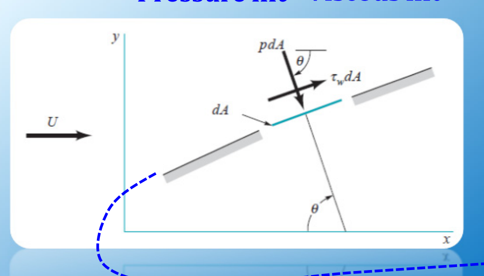

# Drag and lift

- External flows (flow over immersed bodies)
  - Objective variables




$$\mathcal{D} = \int dF_x = \underbrace{\int p\cos\theta\, dA}_{\text{Pressure drag}} + \underbrace{\int \tau_w \sin\theta\, dA}_{\text{Viscous drag}} \quad (1.5)$$

$$\mathcal{L} = \int dF_y = -\underbrace{\int p\sin\theta\, dA}_{\text{Pressure lift}} + \underbrace{\int \tau_w\cos\theta\, dA}_{\text{Viscous lift}} \quad (2.5)$$



# Drag and lift

- External flows (flow over immersed bodies)
  - Objective variables
  - Dimensionless

Drag coefficient $\leftarrow C_D = \dfrac{\mathcal{D}}{\frac{1}{2}\rho U^2 A} \rightarrow$ Characteristic area (3.5)

Lift coefficient $\leftarrow C_{\mathcal{L}} = \dfrac{\mathcal{L}}{\frac{1}{2}\rho U^2 A}$ (4.5)

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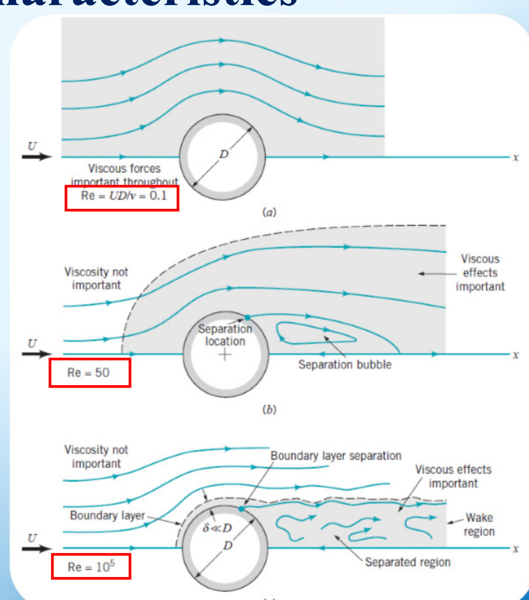
# External flow characteristics

- Viscous flow
  - Boundary layer
  - Separation
  - Wake
  - Outer region
- Inviscid flow
  - Outer region

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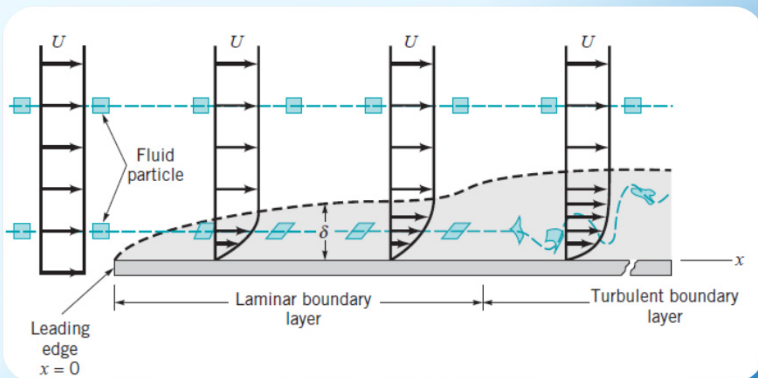
# External flow characteristics

- **Boundary layer**
  - **Thickness, $\delta$**
  - **Thin boundary layer** $Re \gg 1\ (Re > 1000) \longrightarrow \delta \ll L$

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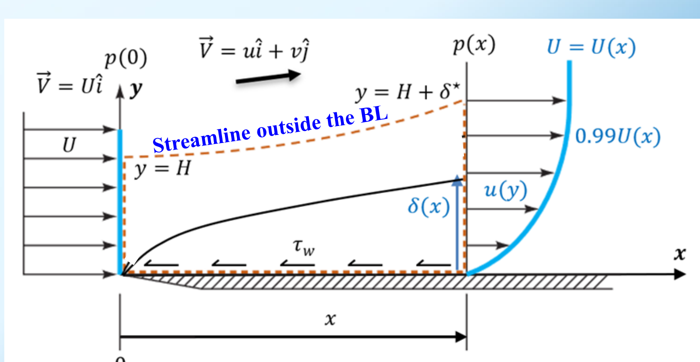
# Boundary Layer (BL)

- **Boundary Layer thickness**

$$y = \delta(x);\ u(x,y) = 0.99U(x) \qquad (5.5)$$

**BL thickness** **Free-stream velocity (outside BL)**

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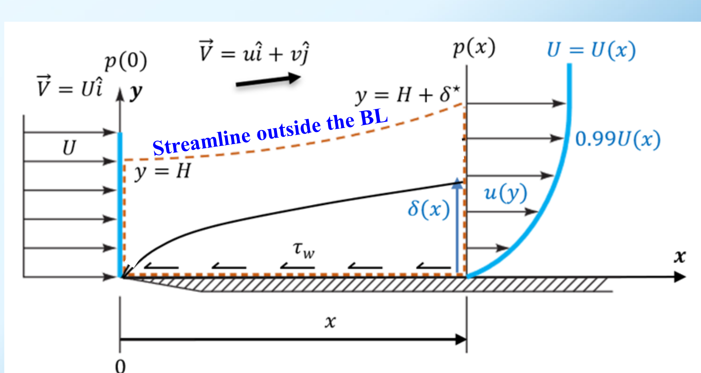
# Boundary Layer (BL)

- **Displacement thickness**

⇨ **Lecture Notes: V.3.1**

**Displacement thickness**

$$\delta^*(x) = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right) dy \qquad (6.5)$$

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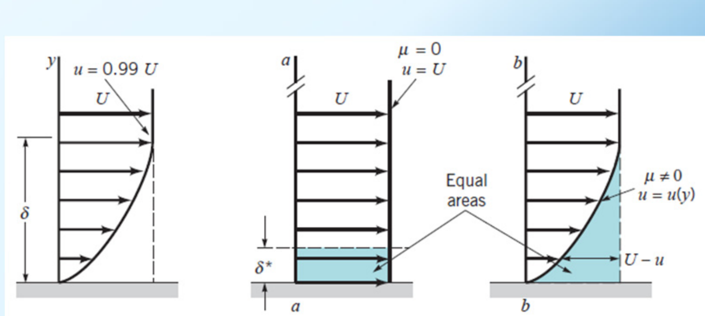
# Boundary Layer (BL)

- **Displacement thickness**
  - **The normal displacement of the free-stream streamlines at each section**
  - **Measures the reduced flow rate near the wall**

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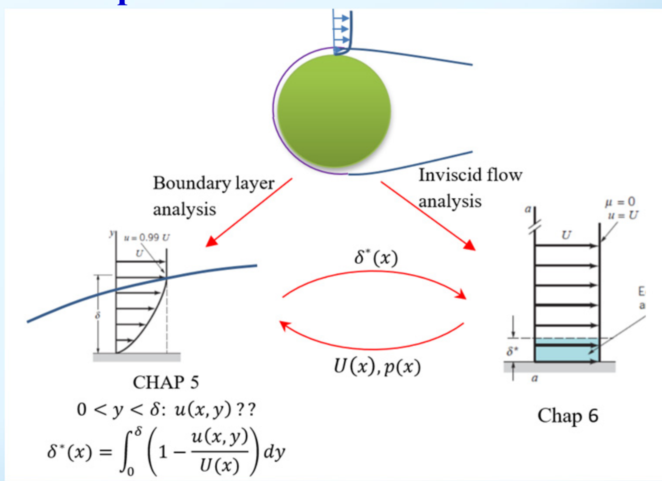
# Boundary Layer (BL)

- **BL/inviscid free-stream flow analysis**
  - **Decoupled method**

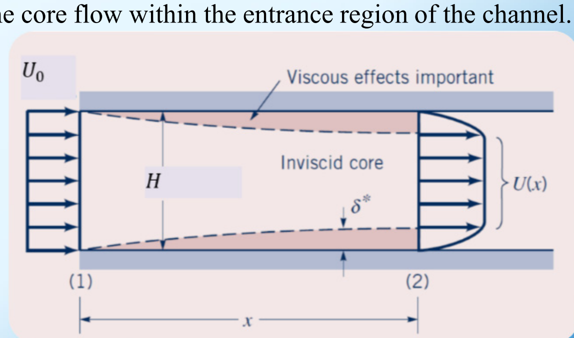





# Boundary Layer (BL)

- **Sample problem**

From boundary layer analysis, the displacement thickness over the walls of a rectangular channel has been determined as $\delta^* = Cx^{1/2}$, where $C$ is a constant coefficient. The channel edge length is $H$ and the flow at its entrance section is uniform with the velocity of $U_0$ and pressure of $p_0$. Estimate the velocity and pressure of the core flow within the entrance region of the channel.



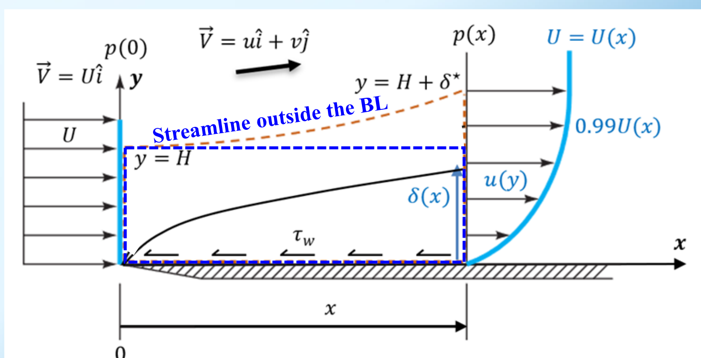

⇨ Lecture Notes: V.3.2

# Boundary Layer (BL)

- **Momentum thickness** ⇨ **Lecture Notes: V.3.3**

**Momentum thickness** ← $\theta(x) = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad (7.5)$

  - **The loss of momentum due to the presence of BL (drag force)**

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# Boundary layer approximation

- **Differential equation analysis**
  - **2D**
  - **Incompressible**
  - **Negligible gravity effect**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right)$$

  - **Thin BL ($\delta \ll L$)**

$$u \gg v \qquad \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} \qquad \frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial x^2}$$

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# Boundary layer approximation

- **BL equations**

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (8.5)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y} \qquad \frac{\partial p}{\partial y}\sim 0 \longrightarrow p=p(x)$$

$$\tau_{yx}=\tau=\begin{cases}\mu\frac{\partial u}{\partial y} & ;\text{laminar flow}\\ \mu\frac{\partial u}{\partial y}-\rho\overline{v'u'} & ;\text{turbulent flow}\end{cases} \qquad (11.5)$$

  - **Boundary conditions**

$$u(x,0)=v(x,0)=0$$
$$u(x,\infty)=U(x) \qquad (9.5)$$

  - **Free-stream analysis**

$$-\frac{\partial p}{\partial x}=\rho\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}\right), U(x=0)=U_0 \quad (10.5)$$ **Bernoulli's equation: Chap 6**

$$\nabla^2\Phi=0, \text{B.C.}(L), \vec{V}=\vec{\nabla}\Phi$$ **Potential flow: Chap 6**

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# Boundary layer approximations

- **Integral equation: Finite control volume**
  - **Derivation using free-body diagram [1-3]**
  - **Derivation using the integration of Eq. (8.5)**

⇨ **Lecture Notes: V.5.1**

$$\frac{\tau_w(x)}{\rho U^2}=\frac{1}{U^2}\frac{\partial}{\partial x}(U\delta^*)+\frac{\partial\theta}{\partial x}+(2\theta+\delta^*)\frac{1}{U}\frac{\partial U}{\partial x} \qquad (12.5)$$ **Von Karman's integral equation**

$$\xrightarrow{\text{steady}} \frac{\tau_w(x)}{\rho U^2}=\frac{d\theta}{dx}+(2+H)\frac{\theta}{U}\frac{dU}{dx}, H=\frac{\delta^*}{\theta} \qquad (14.5)$$

**Note: Friction coefficient** $C_f\equiv\frac{\tau_w(x)}{\rho U^2}$ (13.5)

$$\xrightarrow{+\frac{dP}{dx}=\frac{dU}{dx}=0} \frac{C_f}{2}=\frac{\tau_w(x)}{\rho U^2}=\frac{d\theta}{dx} \qquad (15.5)$$

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# Boundary layer approximate solutions

- **Steady laminar BL on a flat plate**
  - **Sample problem: Von Karman's integral equation**

Using a parabolic velocity profile, $u(y) = c_1 y^2 + c_2 y + c_3$, for laminar boundary layer over a flat plate and Von Karman's integral equation, provide an approximation for the friction and drag coefficients.

**Lecture Notes: V.5.2**

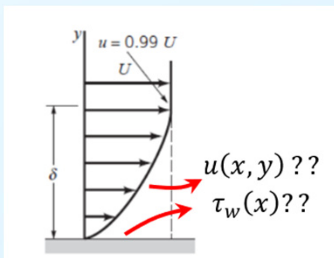


**TABLE 9.2**
**Flat Plate Momentum Integral Results for Various Assumed Laminar Flow Velocity Profiles**

| Profile Character | $\delta Re_x^{1/2}/x$ | $c_f Re_x^{1/2}$ | $C_{Df} Re_\ell^{1/2}$ |
|---|---|---|---|
| a. Blasius solution | 5.00 | 0.664 | 1.328 |
| b. Linear $u/U = y/\delta$ | 3.46 | 0.578 | 1.156 |
| c. Parabolic $u/U = 2y/\delta - (y/\delta)^2$ | 5.48 | 0.730 | 1.460 |
| d. Cubic $u/U = 3(y/\delta)/2 - (y/\delta)^3/2$ | 4.64 | 0.646 | 1.292 |
| e. Sine wave $u/U = \sin[\pi(y/\delta)/2]$ | 4.79 | 0.655 | 1.310 |

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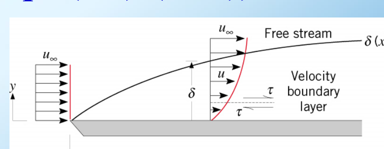
# Boundary layer approximate solutions

- **Steady laminar BL on a flat plate**
  - **Semi-analytical solution for BL differential equations (Simplifying Eqs. (8.5)-(11.5))**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (21.5)$$




$$u(x,0) = v(x,0) = 0, \quad u(x,\infty) = U = cte$$

  - **Similarity solution (change of variable)**
    - **Reducing independent variables**
    - **Finding similarity variable: A detailed mathematical approach [3] (section 3.13)**

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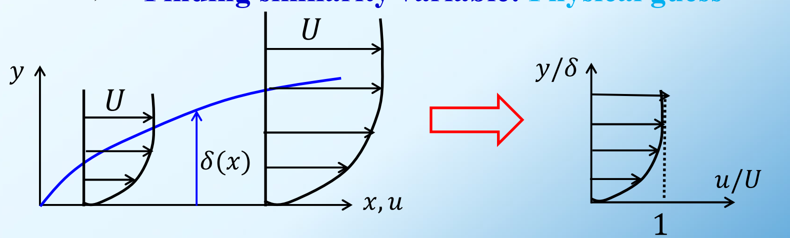
# Boundary layer approximate solutions

- **Steady laminar BL on a flat plate**
  - **Similarity solution (change of variable)**
    - ➢ **Finding similarity variable: Physical guess**




guess

$$\eta = \frac{y}{g(x)} \qquad g(x) \sim \delta(x) \qquad g(x) = \left(\frac{\nu x}{U}\right)^{1/2} \qquad \eta = y\left(\frac{U}{\nu x}\right)^{1/2} \qquad (22.5)$$

  - **See "chap5-BlasiusSimilaritySolution.pdf" for the details of mathematics**

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# Boundary layer approximate solutions

- **Steady laminar BL on a flat plate**
  - **Similarity solution (change of variable)**

$$2f''' + ff'' = 0 \qquad (25.5) \qquad f'(\eta) = \frac{u}{U}$$

$$f(0) = f'(0) = 0, f'(0) = 1 \qquad (26.5)$$

  - **Numerical solution of Eqs. (25.5) and (26.5)**

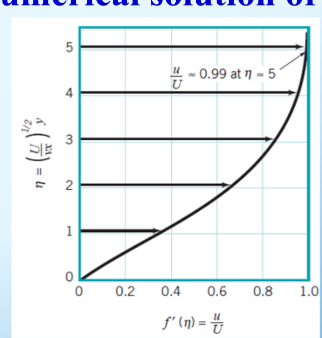


**TABLE 9.1**
**Laminar Flow along a Flat Plate (the Blasius Solution)**

| $\eta = y(U/\nu x)^{1/2}$ | $f'(\eta) = u/U$ | $\eta$ | $f'(\eta)$ |
|---|---|---|---|
| 0 | 0 | 3.6 | 0.9233 |
| 0.4 | 0.1328 | 4.0 | 0.9555 |
| 0.8 | 0.2647 | 4.4 | 0.9759 |
| 1.2 | 0.3938 | 4.8 | 0.9878 |
| 1.6 | 0.5168 | 5.0 | 0.9916 |
| 2.0 | 0.6298 | 5.2 | 0.9943 |
| 2.4 | 0.7290 | 5.6 | 0.9975 |
| 2.8 | 0.8115 | 6.0 | 0.9990 |
| 3.2 | 0.8761 | $\infty$ | 1.0000 |

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# Boundary layer approximate solutions

- **Steady laminar BL on a flat plate**
  - **Similarity solution (change of variable)**

$$\eta = y\left(\frac{U}{\nu x}\right)^{1/2} \longrightarrow \eta = 5 = \delta\left(\frac{U}{\nu x}\right)^{1/2} \longrightarrow \frac{\delta}{x} = \frac{5}{\mathrm{Re}_x^{1/2}} \quad (27.5)$$

$$\frac{u}{U} \xrightarrow{\delta^*(x) = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy} \frac{\delta^*}{x} = \frac{1.721}{\mathrm{Re}_x^{1/2}}, \frac{\theta}{x} = \frac{0.664}{\mathrm{Re}_x^{1/2}} \quad (28.5)$$

$$\tau_w(x) = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu U \left.\frac{\partial f'}{\partial \eta}\frac{\partial \eta}{\partial y}\right|_{y=0} = \mu U f''(0)\left(\frac{U}{\nu x}\right)^{1/2} = 0.332U^{3/2}\rho\left(\frac{\nu}{x}\right)^{1/2} \quad (29.5)$$

$$\longrightarrow C_f = \frac{0.664}{\mathrm{Re}_x^{1/2}}, C_D = \frac{1.328}{\mathrm{Re}_\ell^{1/2}} \quad (30.5)$$

  - **Exercise: For BL on a flat plate $\tau_w$ scales with $U^{3/2}$ while for fully-developed flow in a pipe scales with $U$. Can you justify why the dependence on $U$ is stronger in the former case?**

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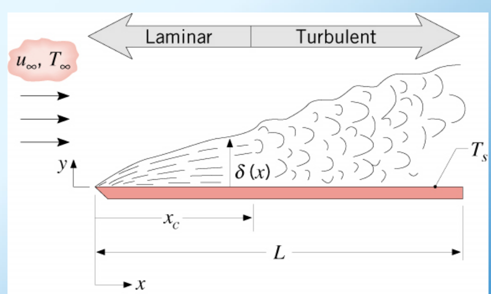
# Boundary layer approximate solutions

- **Steady turbulent BL on a flat plate**
  - **Critical Reynolds number**

$$Re_x = \frac{\rho U x}{\mu}$$

$$Re_{x,cr} = \frac{\rho U x_{cr}}{\mu} = 5 \times 10^5$$

**(Smooth flat plate)**

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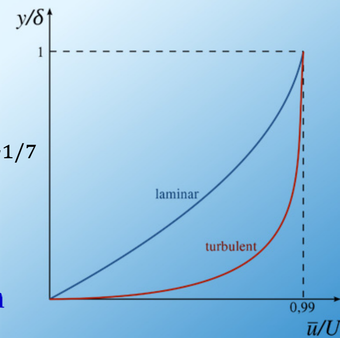
# Boundary layer approximate solutions

## Steady turbulent BL on a flat plate

### Von Karman's integral equation

1. **Choosing a velocity profile**

$$u = c_1 y^{1/7} + c_2$$




2. **Boundary conditions**

$$u(x,0) = 0, u(x,\delta) = U \longrightarrow c_2 = 0, c_1 = U/\delta^{1/7}$$

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \qquad (31.5)$$

3. **Von Karman's integral equation**

$$\frac{\tau_w(x)}{\rho U^2} = \frac{d\theta}{dx}$$

$$\theta(x) = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy = \frac{7}{27}\delta \qquad (32.5)$$



# Boundary layer approximate solutions

## Steady turbulent BL on a flat plate

### Von Karman's integral equation

4. **Estimation of $\tau_w$**

$$\tau_w(x) = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu U \left.\frac{\partial (u/U)}{\partial y}\right|_{y=0} = \frac{1}{7}\frac{\mu U}{\delta^{1/7}} \left. y^{-6/7}\right|_{y=0} \to \infty \text{※}$$

➢ **Alternative estimations:**

- ✓ **Logarithmic profile (exercise)**
- ✓ **Prandtl approximation (exercise)**
- ✓ **Blasius empirical approximation**

$$\frac{\tau_w(x)}{\rho U^2} = 0.0225 \left(\frac{U\delta}{\nu}\right)^{-1/4} \qquad (35.5)$$

$$\longrightarrow 0.0225 \left(\frac{U\delta}{\nu}\right)^{-1/4} = \frac{d\theta}{dx} = \frac{7}{27}\frac{d\delta}{dx} \xrightarrow[\delta(0)=0]{\text{Integration}} \frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}} \qquad (36.5)$$

$$\xrightarrow{(35.5)} \tau_w(x) = \frac{0.0288 \rho U^2}{Re_x^{1/5}} \longrightarrow c_f = \frac{2\tau_w}{\rho U^2} = \frac{0.0576}{Re_x^{1/5}} \qquad (37.5)$$

# Boundary layer approximate solutions

- **Steady turbulent BL on a flat plate**
  - **Analytical relations (smooth wall)**

| Blasius laminar boundary layer solution | Blasius turbulent boundary layer approximation ($x > x_c$) |
|---|---|
| $\frac{\delta}{x} = \frac{5}{\mathrm{Re}_x^{1/2}}$ | $\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}}$ |
| $C_f = \frac{0.664}{\mathrm{Re}_x^{1/2}}$ | $C_f = \frac{0.0576}{\mathrm{Re}_x^{1/5}}$ |
| $C_D = \frac{1.328}{\mathrm{Re}_\ell^{1/2}}$ | $C_D = \frac{0.072}{\mathrm{Re}_\ell^{1/5}}$ |

  - **Empirical diagrams**

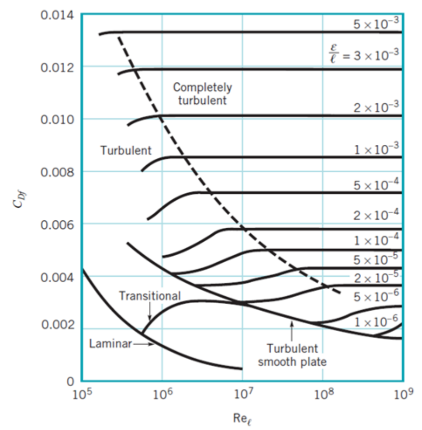



FIGURE 9.15 Friction drag coefficient for a flat plate parallel to the upstream flow (Ref. 18, with permission).

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# Boundary layer separation

- **Pressure gradient effect on BL**

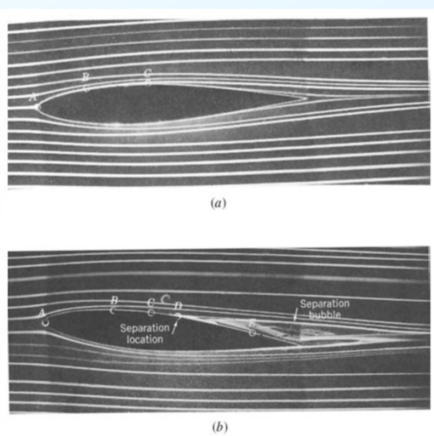



FIGURE 9.18 Flow visualization photographs of flow past an airfoil (the boundary layer velocity profiles for the points indicated are similar to those indicated in Fig. 9.17*b*): (*a*) zero angle of attack, no separation, (*b*) 5° angle of attack, flow separation. Dye in water. (Photograph courtesy of ONERA, France.)

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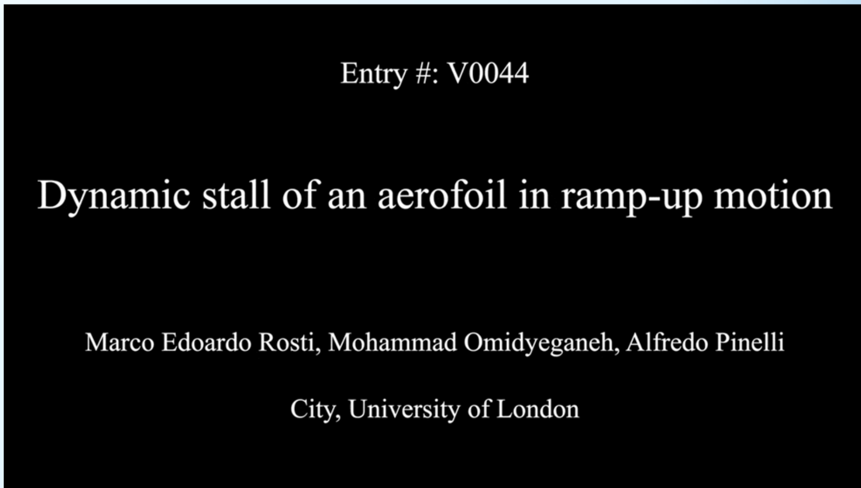
# Boundary layer separation

- **Pressure gradient effect on BL**

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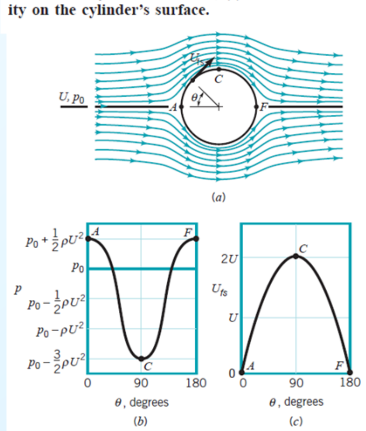
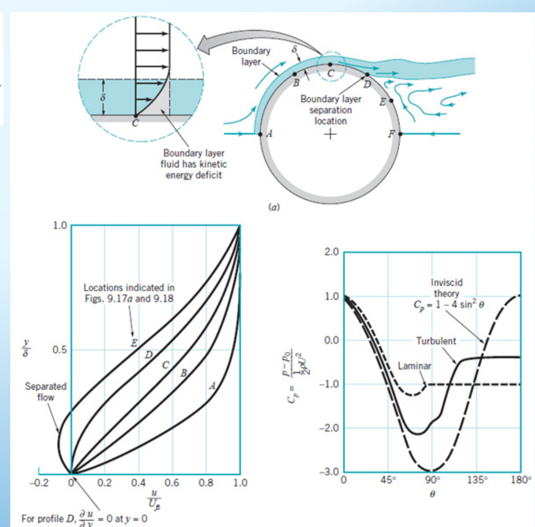
# Boundary layer separation

- **Pressure gradient effect on BL**

FIGURE 9.16 Inviscid flow past a circular cylinder: (*a*) streamlines for the flow if there were no viscous effects, (*b*) pressure distribution on the cylinder's surface, (*c*) free-stream velocity on the cylinder's surface.

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# Boundary layer separation

- **Pressure gradient effect on BL**

$$\xrightarrow{(8.5)} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y} \xrightarrow[u=v=0]{\text{At } y=0;} \frac{\partial \tau_{yx}}{\partial y} = \frac{\partial p}{\partial x}$$

$$\xrightarrow[\overline{v'u'}=0,\ \tau_{yx}=\mu\frac{\partial u}{\partial y}]{\text{At } y=0;} \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \frac{1}{\mu}\frac{\partial p}{\partial x} \overset{(10.5)}{\cong} \frac{\rho}{\mu}U\frac{dU}{dx} \qquad (38.5)$$

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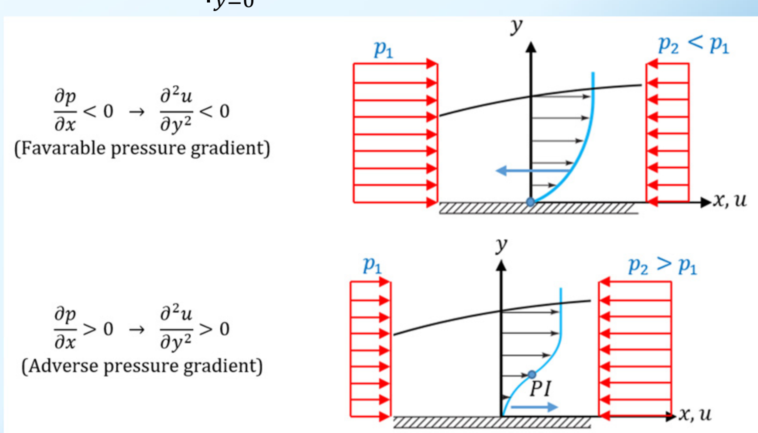
# Boundary layer separation

- **Pressure gradient effect on BL**

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \frac{1}{\mu}\frac{\partial p}{\partial x} \cong \frac{\rho}{\mu}U\frac{dU}{dx} \qquad (38.5)$$

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# Boundary layer separation

- **Pressure gradient effect on BL**

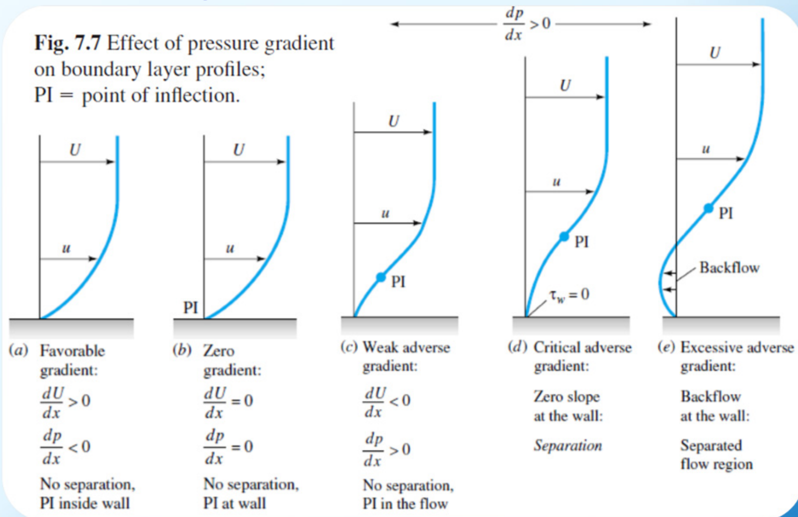


**Fig. 7.7** Effect of pressure gradient on boundary layer profiles; PI = point of inflection.

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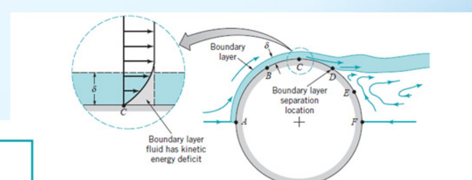
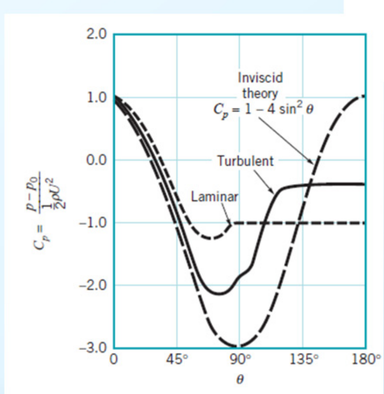
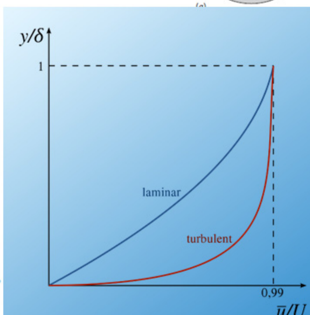
# Boundary layer separation

- **Separation delay by transition to turbulence**

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# Empirical diagrams and tables

- **Drag**

Friction drag, Pressure drag

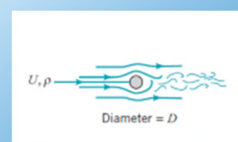
$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A} = \frac{\mathcal{D}_f + \mathcal{D}_p}{\frac{1}{2}\rho U^2 A} = C_{D,f} + C_{D,p}$$

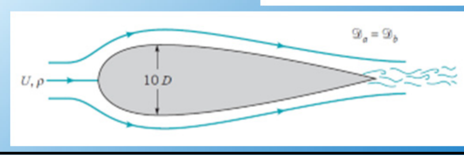
$$C_D = C_D(\text{shape}, Re, Ma, Fr, {}^{\varepsilon}/_{L})$$

Mach number (if $Ma > 0.3$)

Froude number (free-surface flow)

- **Shape effect: separation**
  - **Streamlined:** $C_D \sim C_{D,f}$
  - **Blunt:** $C_D \sim C_{D,p}$

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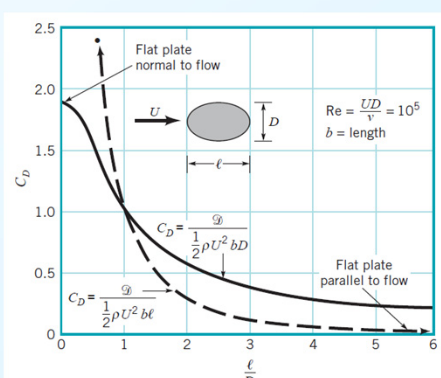
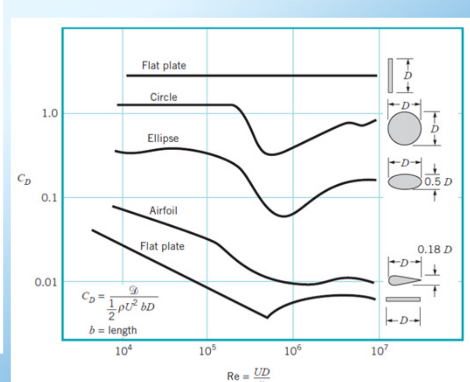
# Empirical diagrams and tables

- **Drag**
  - **Shape effect: separation**

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# Empirical diagrams and tables

- **Drag**
  - **Reynolds number effect**
    1. **Creeping flow ($Re < 1$):**
       - **Inertial force is negligible**
       - **Drag does not depend on $Re$**

$$\mathcal{D} = f(U, L, \mu) \xrightarrow[\text{analysis}]{\text{Dimensional}} \frac{\mathcal{D}}{\mu L U} = C = cte$$

**From experiment**

$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A} = \frac{C\mu L U}{\frac{1}{2}\rho U^2 L^2} \longrightarrow C_D = \frac{2C}{Re}, \qquad Re = \frac{\rho U L}{\mu}$$

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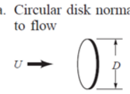
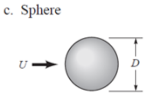
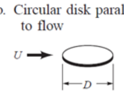
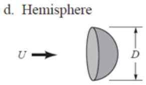
# Empirical diagrams and tables

- **Drag**
  - **Reynolds number effect**
    1. **Creeping flow ($Re < 1$):** $C_D = \dfrac{2C}{Re}$

**TABLE 9.4**
**Low Reynolds Number Drag Coefficients (Ref. 7) ($Re = \rho UD/\mu$, $A = \pi D^2/4$)**

| Object | $C_D = \mathcal{D}/(\rho U^2 A/2)$ (for $Re \lesssim 1$) | Object | $C_D$ |
|---|---|---|---|
| a. Circular disk normal to flow  | 20.4/Re | c. Sphere  | 24.0/Re |
| b. Circular disk parallel to flow  | 13.6/Re | d. Hemisphere  | 22.2/Re |

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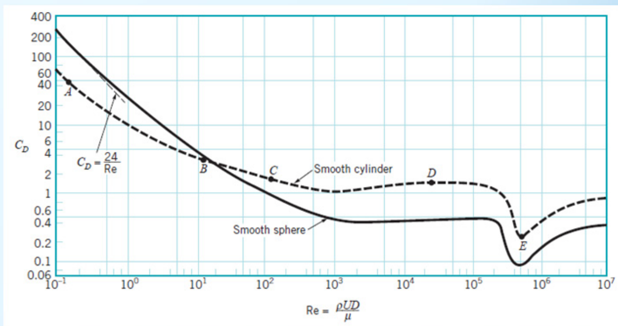
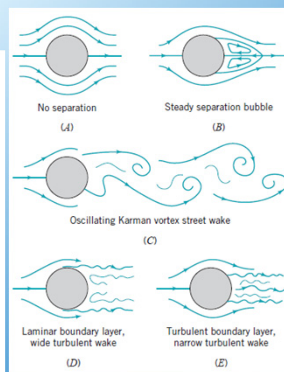
# Empirical diagrams and tables

- **Drag**
  - **Reynolds number effect**
    2. **Moderate Reynolds numbers ($10^3 < Re < 10^5$):**
       - **Flat plate (provided correlations):** $C_D = C_{D,f} \propto Re^{-\frac{1}{2}}$
       - **Blunt:** $C_D \sim C_{D,p} \sim cte$

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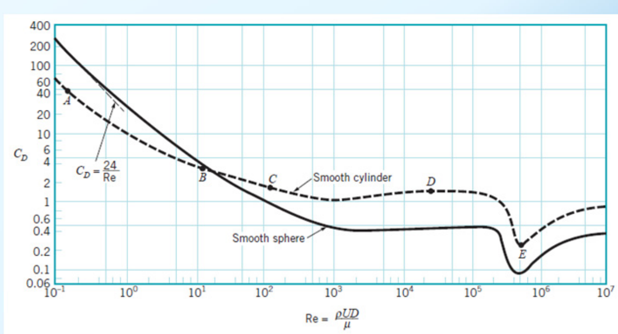
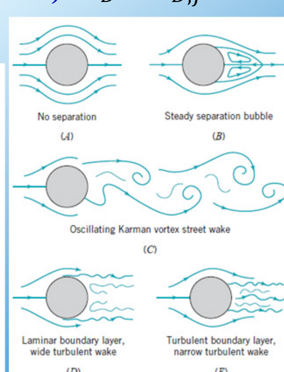
# Empirical diagrams and tables

- **Drag**
  - **Reynolds number effect**
    3. **Large Reynolds numbers ($Re > 10^5$):**
       - **Flat plate (provided correlations):** $C_D = C_{D,f} \propto Re^{-\frac{1}{5}}$

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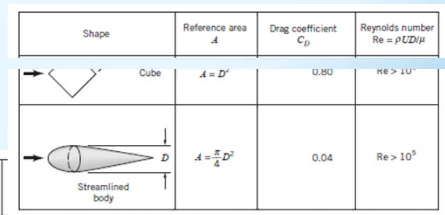
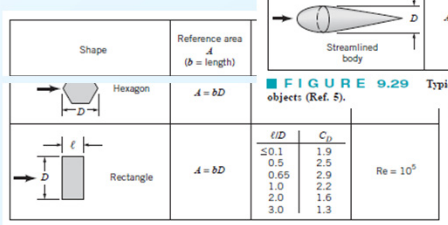
# Empirical diagrams and tables

- **Drag**
  - **Reynolds number effect**
    3. **Large Reynolds numbers: See tables**



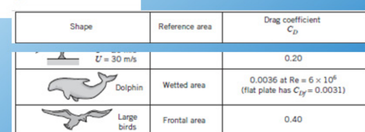
| Shape | Reference area $A$ | Drag coefficient $C_D$ | Reynolds number $Re = \rho UD/\mu$ |
|---|---|---|---|
| Cube | $A = D^2$ | 0.80 | Re > 10[illegible] |
| Streamlined body | $A = \frac{\pi}{4}D^2$ | 0.04 | $Re > 10^5$ |

FIGURE 9.29 Typical drag coefficients for regular three-dimensional objects (Ref. 5).



| Shape | Reference area $A$ ($b$ = length) | | |
|---|---|---|---|
| Hexagon | $A = bD$ | | |
| Rectangle | $A = bD$ | $\ell/D$: ≤0.1, 0.5, 0.65, 1.0, 2.0, 3.0 — $C_D$: 1.9, 2.5, 2.9, 2.2, 1.6, 1.3 | $Re = 10^5$ |

FIGURE 9.28 Typical drag coefficients for regular two-dimensional objects (Refs. 5, 6).



| Shape | Reference area | Drag coefficient $C_D$ |
|---|---|---|
| $U$ = 30 m/s | | 0.20 |
| Dolphin | Wetted area | 0.0036 at $Re = 6 \times 10^6$ (flat plate has $C_{Df} = 0.0031$) |
| Large birds | Frontal area | 0.40 |

FIGURE 9.30 Typical drag coefficients for objects of interest (Refs. 5, 6, 15, 20).

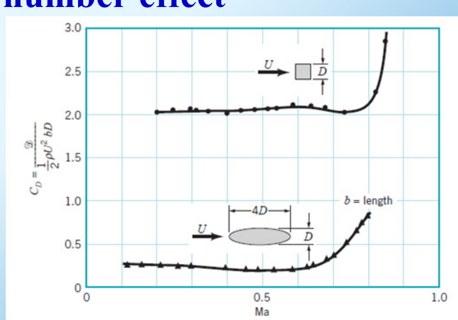


# Empirical diagrams and tables

- **Drag**
  - **Surface roughness effect**
    - **Streamlined** $\varepsilon \uparrow \Longrightarrow C_D = C_{D,f} \uparrow$
    - **Blunt**

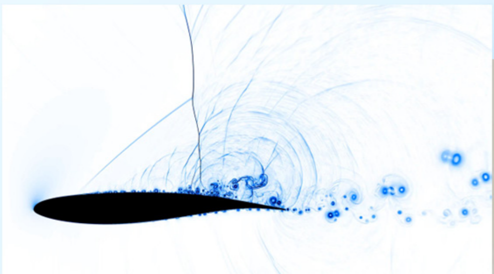
$$\varepsilon \uparrow \Longrightarrow C_D = C_{D,p} \begin{cases} \downarrow & ;\text{causes transition to turbulence} \\ \leftrightarrow & ;\text{otherwise} \end{cases}$$

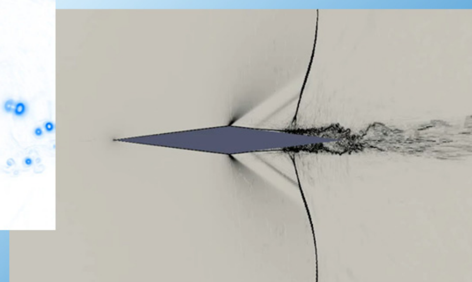
  - **Mach number effect**



# Empirical diagrams and tables

- **Drag**
  - **Mach number effect**
    - **Exercise: Why are the leading edge shape of the airfoil of subsonic and supersonic airplane wings are different?**





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# Empirical diagrams and tables

- **Lift**

Friction Lift Pressure Lift

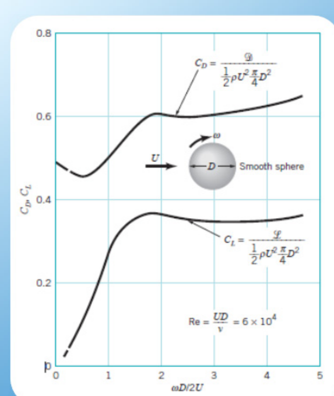
$$C_{\mathcal{L}} = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{\mathcal{L}_f + \mathcal{L}_p}{\frac{1}{2}\rho U^2 A} = C_{\mathcal{L},f} + C_{\mathcal{L},p}$$

Usually $C_{\mathcal{L}} \sim C_{D,p}$

$$C_{\mathcal{L}} = C_{\mathcal{L}}\left(\text{shape}, Re, Ma, Fr, {}^{\varepsilon}/_{L}\right)$$

Strong dependence weak dependence

- **Needs asymmetry**
- **Shape or motion**
- **Discussed further in Chap 6**

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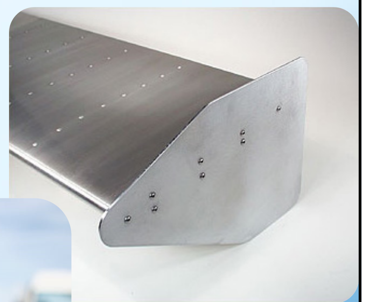
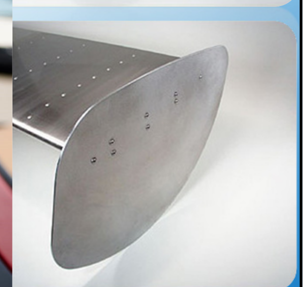
# Sample problem

- Sport car wing tip plate



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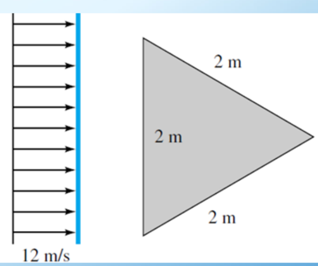
# Sample problem

- Sport car wing tip plate

A thin equilateral-triangle plate is immersed parallel to a 12 m/s stream of water at 20°C, as in Fig. P7.34. Assuming $Re_{tr} = 5 \times 10^5$, estimate the drag of this plate.




⇨ **Lecture Notes: V.9**

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# The end of chapter 5

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