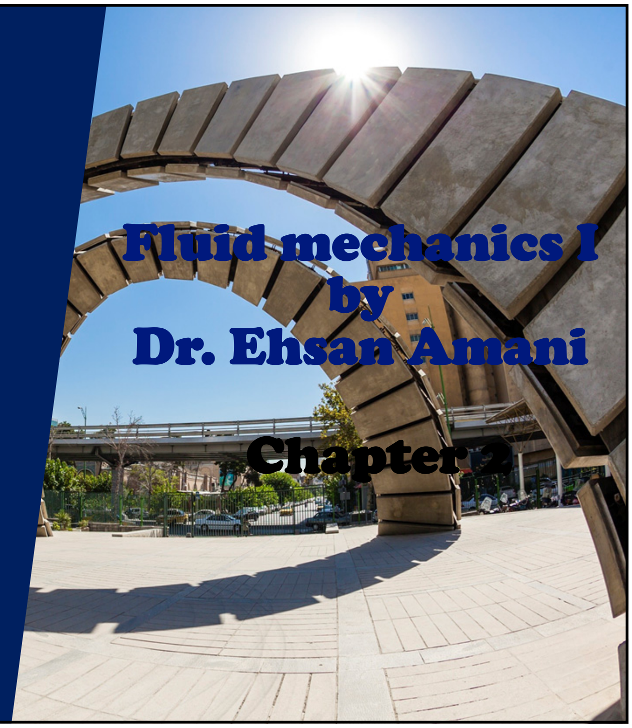


Fluid mechanics I
by
Dr. Ehsan Amani

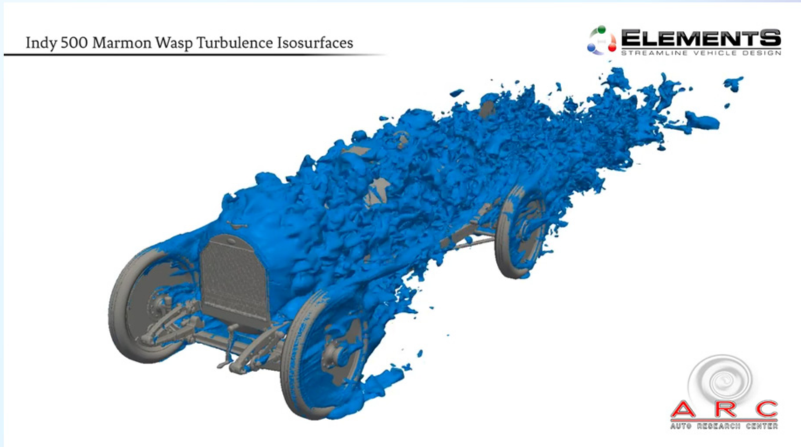
Chapter 2

Fluid mechanics applications

- Very large scale (Geophysical flows)

Fluid mechanics applications

- **Large scale (vehicles)**



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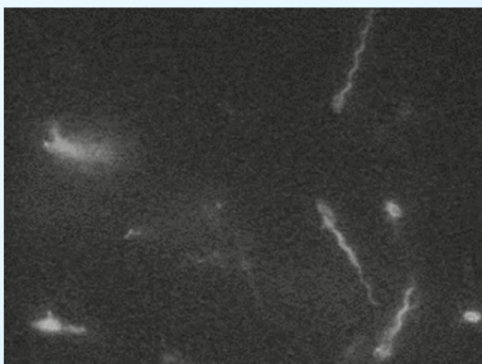
Fluid mechanics applications

- **Small scale (E coli bacteria)**



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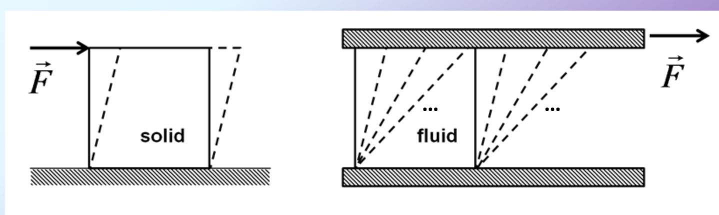
Basic definitions

- **Fluid**
  - **"Soft" and easily deformable?!**
  - **Completely fills the volume of any container?!**
  - **Deforms continuously under a shearing stress of any magnitude (flow)?!**
  - **Finite vs. infinite strain**
  - **Liquid or gas**



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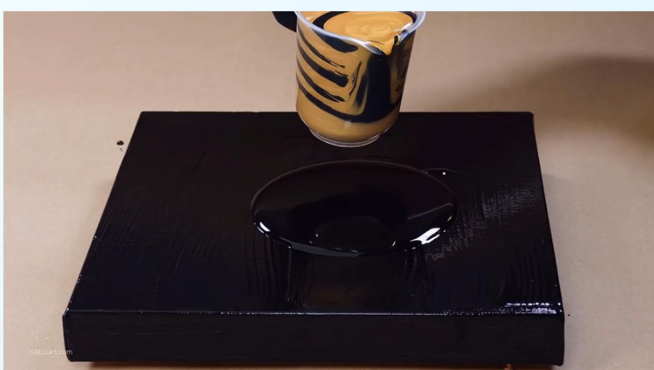
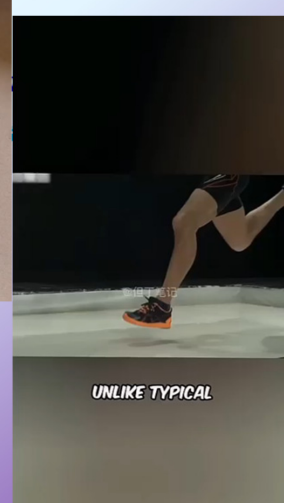
Basic definitions





- **Semi-solid (Soft solid)**
  - **Some Non-Newtonian fluids**
  - **Some polymers**
  - **…**

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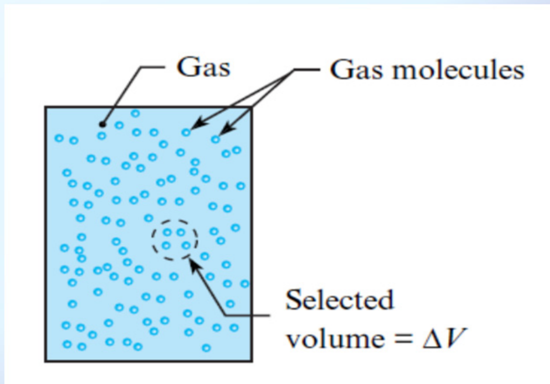
Mechanics study approaches

- Discrete particels
- Continuum mechanics



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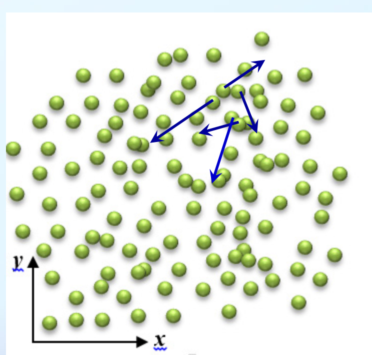
Continuum mechanics

- Discrete particles mechanics
  - Lagrangian: Molecule particles



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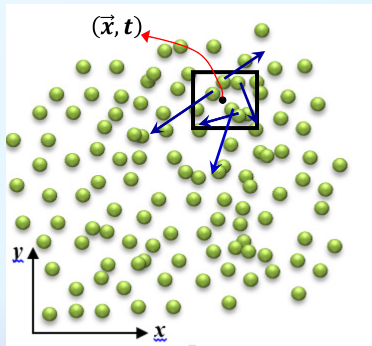
Continuum mechanics

- Discrete particles mechanics vs. continuum



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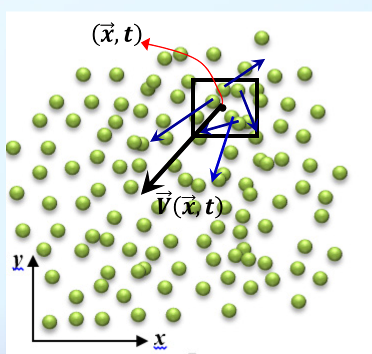
Continuum mechanics

- Discrete particles mechanics vs. continuum



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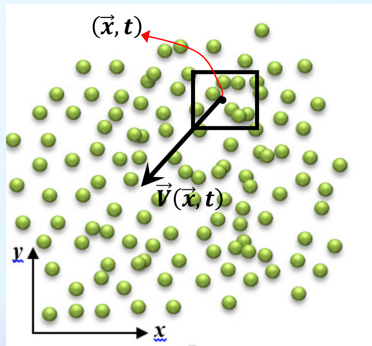
Continuum mechanics

- **Discrete particles mechanics vs. continuum**
  - **Fluid element or macroscopic properties**



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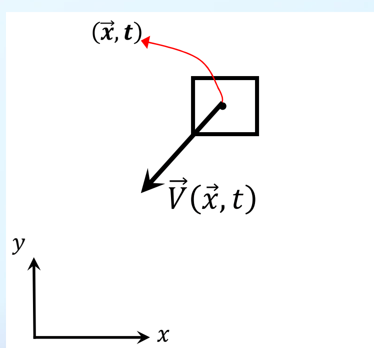
Continuum mechanics

- **Continuum mechanics**
  - Fluid element properties



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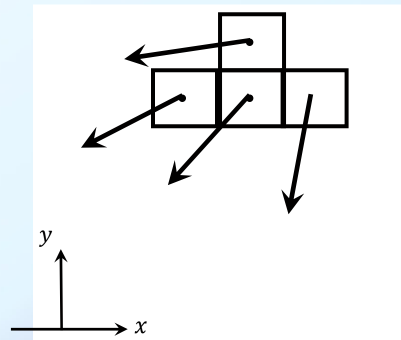
Continuum mechanics

- **Continuum mechanics**
  - Fluid element properties



Field (dependent variable)

$$\vec{V}\underbrace{(x, y, z, t)}_{\text{Independent variables}} = u(x, y, z, t)\hat{\imath} + v(x, y, z, t)\hat{\jmath} + w(x, y, z, t)\hat{k}$$

Chapter 2 **By E. Amani**

Continuum mechanics

- **Continuum mechanics**
  - **(Piecewise) continuous fields of fluid (macroscopic) properties** $\vec{V}(x, y, z, t)$, …
  - **Exercise: What is the advantage and limitation of continuum approach? See continuum hypothesis section of fluid mechanics page in wikipedia**
  - **Homogeneous system:** (t) **instead of** (x, y, z, t)
  - **Fluid (macroscopic) properties:**
    - **Primary: Determined by basic principles, e.g., Velocity** $(\vec{V}(x, y, z, t))$**, pressure** $(p(x, y, z, t))$**, temperature** $(T(x, y, z, t))$**,** …
    - **Thermophysical: Determined using empirical and semi-empirical laws, e.g., viscosity** $(\mu(x, y, z, t))$**, specific heat capacity** $(c_p(x, y, z, t))$**,** …

Chapter 2 **By E. Amani**

Dimensions and units

- **Description of a property**

$$\mathit{length}\ (L) \xleftarrow{\textbf{qualitative}} D \xrightarrow{\textbf{quantitative}} 10\ (in)$$

$$\mathit{dimension} \longleftarrow \mathit{Property} \longrightarrow \mathit{value}\ (\mathit{unit})$$

- **Dimension**
  - **Qualitative description (type)**
  - **Primary: Length (L), time (T), mass (M) or force (F), temperature (Θ), …**
  - **Secondary: Combinations of primary**
- **Dimensional homogrnity**
  - **Determination of the dimension of a new property**
  - **Necessary condition for the validity of a physical relation**

Dimensions and units

Table 1.1

Dimensions Associated with Common Physical Quantities

	FLT System	*MLT* System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}

⇨ Lecture Notes

Density and specific weight

- **Density: Mass per unit volume**

  (kg/m^3) ← ρ

- **Specific weight: Weight per unit volume**

  (N/m^3) ← $\gamma = \rho g$

- **Specific gravity: Relative density**

  $(-)$ ← $SG = \dfrac{\rho}{\rho_{H2O@4℃}}$ (1.2)

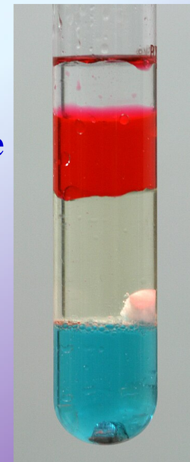
  $1000\ (kg/m^3) = 1.94\ (slug/ft^3)$ ← $\rho_{H2O@4℃}$

- **Data?**
  - **Pure material:** $\rho = \rho(p, T)$
  - **Ideal gas:** $\rho = \dfrac{p}{RT}$ (2.2) $R = \dfrac{R_u}{W}, R_u = \bar{R} = 8.314\ \dfrac{kJ}{kmol.K}$ (3.2)

gas constant $\left(\frac{kJ}{kmol.K}\right)$

Universal gas constant

Molecular weight (kg/mol)

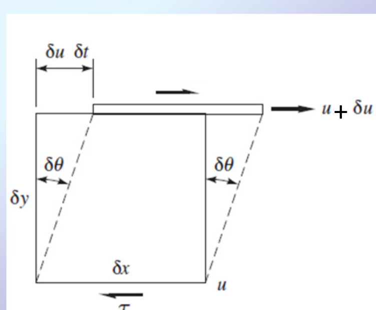


Chapter 2 By E. Amani

Viscosity

- **Newtonian law of viscosity (Stokes empirical law)**
  - **Laminar flow**
  - **Straight parallel flow**
  - **Newtonian fluids**

deformation rate ← $\dfrac{\delta\theta}{\delta t} = \dfrac{\tan\delta\theta}{\delta t} = \dfrac{[(u+\delta u)\delta t - u\delta t]/\delta y}{\delta t} = \dfrac{\delta u}{\delta y}$



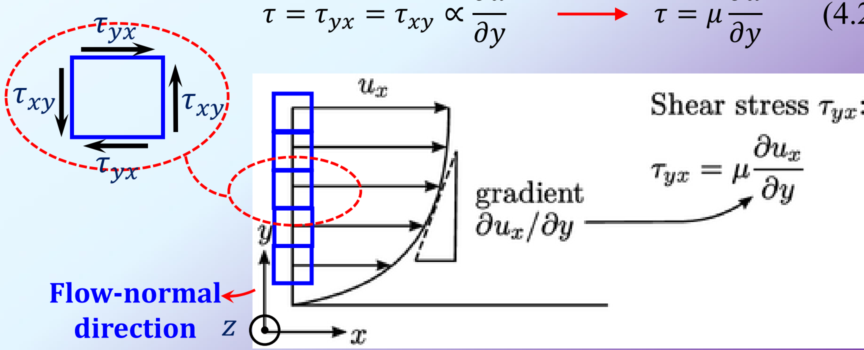
Chapter 2 By E. Amani

Viscosity

- **Newtonian law of viscosity (Stokes empirical law)**
  - **Laminar flow**
  - **Straight parallel flow**
  - **Newtonian fluids**

(dynamic) viscosity $\frac{FT}{L^2}$ **($Pa.s$)**

$$\tau = \tau_{yx} = \tau_{xy} \propto \frac{\partial u}{\partial y} \longrightarrow \tau = \mu \frac{\partial u}{\partial y} \qquad (4.2)$$



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Viscosity

- **Physical interpretation: fluid resistance to deformation**

$$\tau = \overset{cte}{\mu} \underbrace{\overset{\uparrow \ \downarrow}{\frac{\partial u}{\partial y}}}_{\text{Deformation rate}}$$

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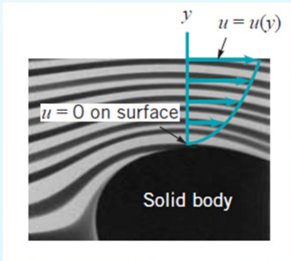
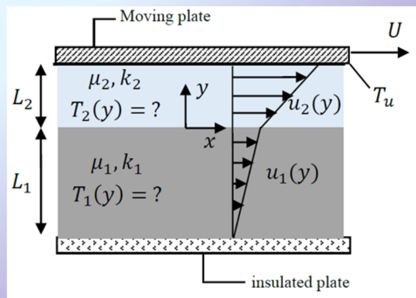
Viscosity

- **Data?**
  - **For most of fluids: Empirical tables $\mu = \mu(T)$**
  - **See Ref. [1], Appendix B, or Ref. [2], Appendix A**
- **No-slip condition**





$\vec{V} = \vec{V}_s$; on the interface (5.2) $\vec{V}_1 = \vec{V}_2$; on the interface

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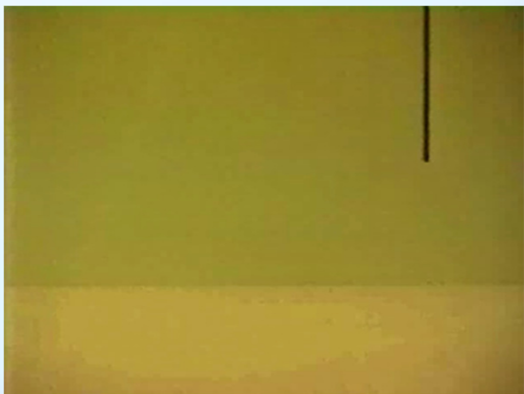
Viscosity

- **No-slip condition**



- **No-slip condition violation:**
  - **Rearfied gases, ideal fluid, …**

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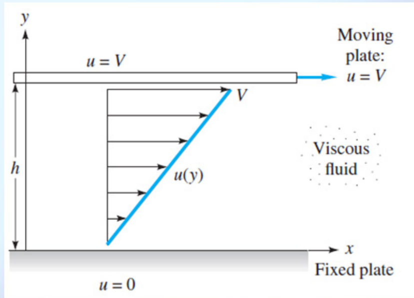
Newtonian law of viscosity

- **Example**

Consider a plate moving with velocity of V on the top of a thin fluid layer (film) of thickness h. Assuming a linear flow velocity profile, determine the force required to move the plate.

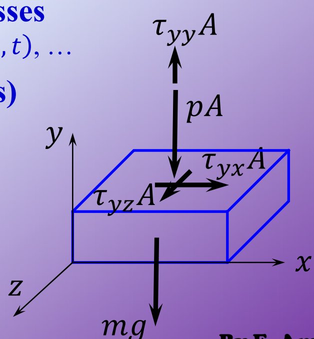


⇨ **Lecture Notes**

Chapter 2 By E. Amani

Internal force of a fluid flow

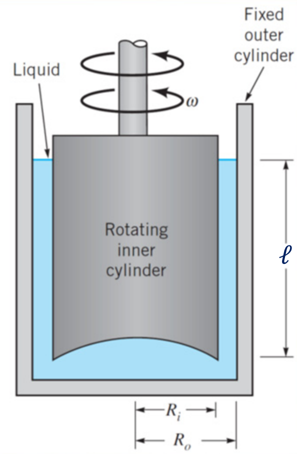
- **Body forces (per unit mass)**
  - **Gravity $\vec{g}$**
  - **Electromagnetic**
  - **…**
- **Surface force (per unit area: stress)**
  - **Viscous shear (and normal) stresses** $\tau_{xy}(x,y,z,t), \tau_{xz}(x,y,z,t), \ldots, \tau_{xx}(x,y,z,t), \ldots$
  - **Pressure** $p(x,y,z,t)$ **(normal stress)**
- **Line force (per unit length)**
  - **Surface tension** $\sigma(x,y,z,t)$
- **How about other forces like buoyancy, drag, …?**



Chapter 2 By E. Amani

Sample problems

The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in the figure. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque Γ required to develop ω is measured and the viscosity is calculated from these two measurements. Develop an equation relating $\mu, \omega, \Gamma, \ell, R_o$, and R_i. Neglect end effects and assume the velocity distribution in the gap is linear.



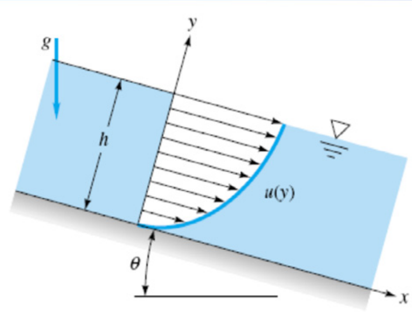
Lecture Notes

Chapter 2 By E. Amani

Sample problems

A thin layer of glycerin flows down an inclined, wide plate with the velocity distribution shown in the Figure. For h=0.3 in. and θ=20°, determine the surface velocity, U. Note that the velocity is constant in tangential direction.

$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$$

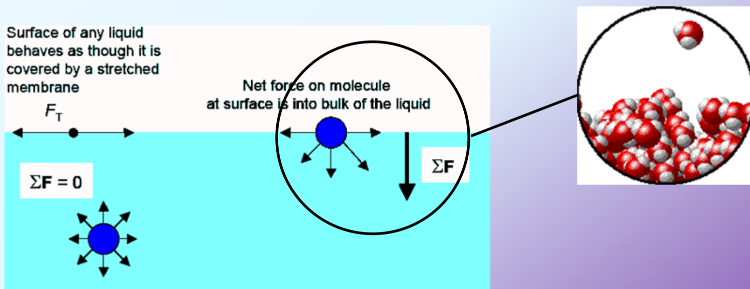


Lecture Notes

Chapter 2 By E. Amani

Surface tension

- Microscopic view point
  - The intermolecular cohesive force near the fluid-fluid interface is anisotropic



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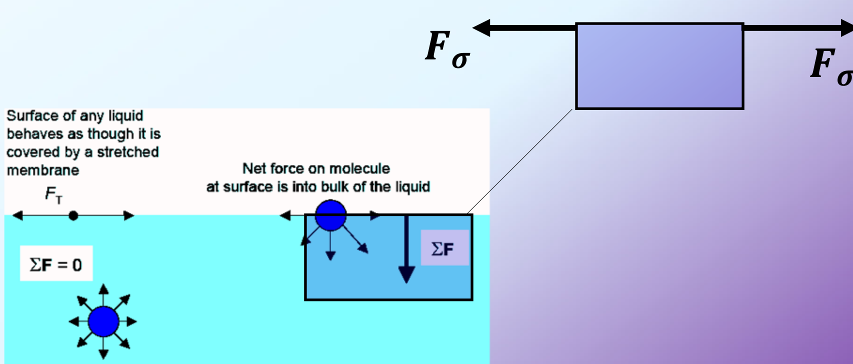
Surface tension

- Macroscopic view point



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Surface tension

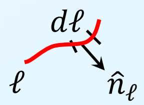
- Demonstration

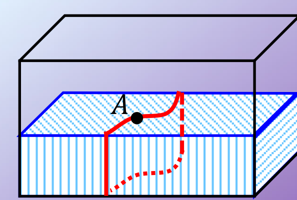
Surface tension

- Macroscopic effects
  - Distributed very close to a fluid-fluid interface
  - Considered only when the interface is cut in the free-body diagram
  - Exerted as the resultant force on the interface fluid element
  - Force per unit length (surface tension coefficient, σ (N/m)) on a curve on the interface
  - Tangent to the intefrace
  - Normal to the chosen curve
  - σ depends on the material of the two fluids in contact, temperature, and pressure

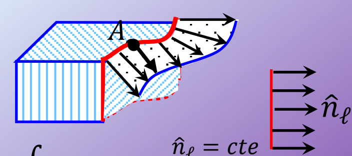
Surface tension

- **Determining the direction of the surface tension at each point A of a curve on the interface**
  - **Within the tangent plane to the interface at A**
  - **Normal to the chosen curve**
  - **Causes attraction (tensile)**





$$d\vec{F}_\sigma = (\sigma d\ell)\hat{n}_\ell$$

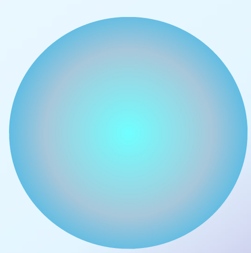


$$\vec{F}_\sigma = \int_\ell \sigma \hat{n}_\ell d\ell \xrightarrow{\sigma = cte} \vec{F}_\sigma = \sigma \int_\ell \hat{n}_\ell d\ell \xrightarrow{\hat{n}_\ell = cte} \vec{F}_\sigma = \sigma \ell \hat{n}_\ell$$

Surface tension

- **Example**

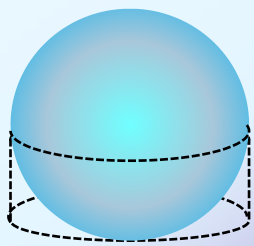
Calculate the difference between the inner and outer pressures of a soap bubble.

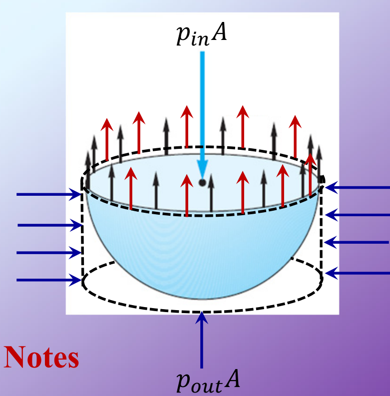


Surface tension

- **Example**

Calculate the difference between the inner and outer pressures of a soap bubble.





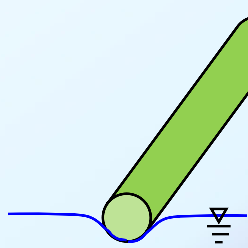
⇨ **Lecture Notes**

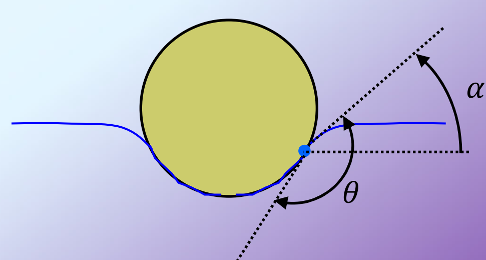
Chapter 2 By E. Amani

Surface tension

- **Example**

Calculate the angle alpha shown in the figure for a needle on the top surface of a water in a glass.



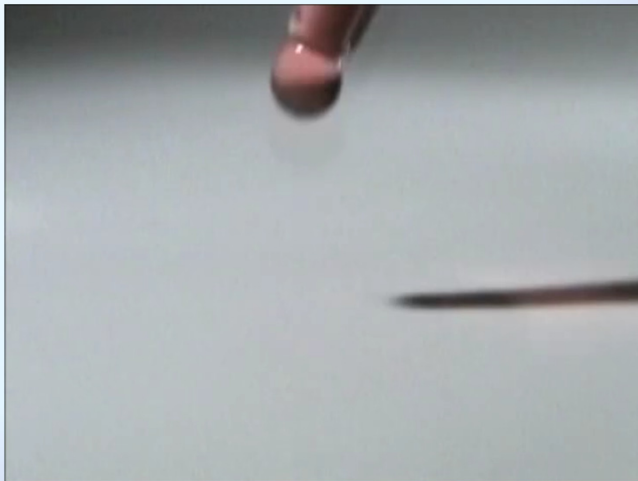


⇨ **Lecture Notes**

Chapter 2 By E. Amani

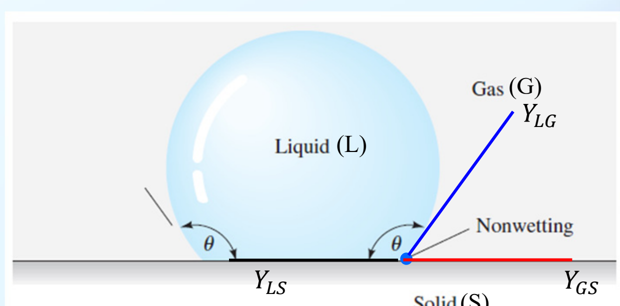
Surface tension

- Contact angle



Surface tension

- Contact angle

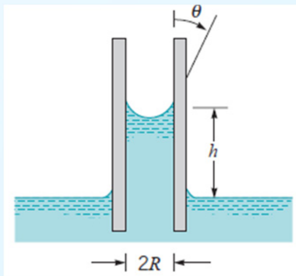


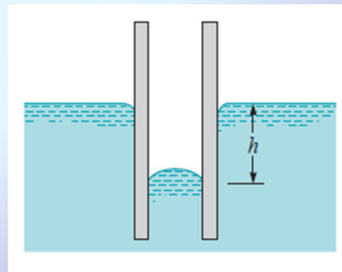
- θ depends on the three materials in contact, temperature, and pressure

Contact angle

- **Example**

Calculate the height of the liquid in a capillary tube.





Lecture Notes

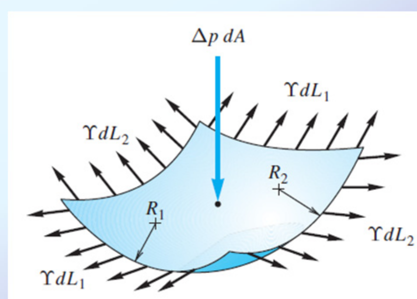
Chapter 2 By E. Amani

Surface tension

- **Pressure jump across a fluid-fluid interface**

$$\Delta p = p^{+} - p^{-} = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (9.2)$$

principal radii of curvature



- **Exercise: Using Eq. (9.2), calculate the pressure difference across a soap bubble.**
- **For a large free surface:** $R_1 = R_2 \to \infty; p^{+} - p^{-} \to 0$

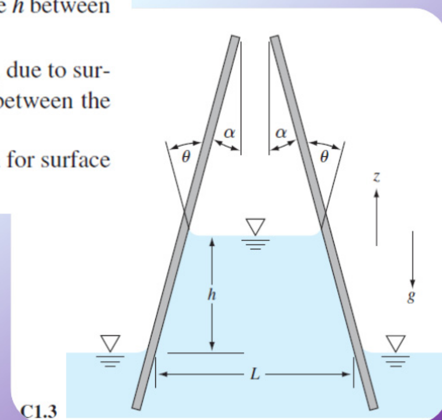
Chapter 2 By E. Amani

Sample problems

Two thin flat plates, tilted at an angle α, are placed in a tank of liquid of known surface tension Υ and contact angle θ, as shown in Fig. C1.3. At the free surface of the liquid in the tank, the two plates are a distance L apart and have width b into the page. The liquid rises a distance h between the plates, as shown.

(*a*) What is the total upward (z-directed) force, due to surface tension, acting on the liquid column between the plates?

(*b*) If the liquid density is ρ, find an expression for surface tension Υ in terms of the other variables.



C1.3

⇨ **Lecture Notes**

The end of chapter 2