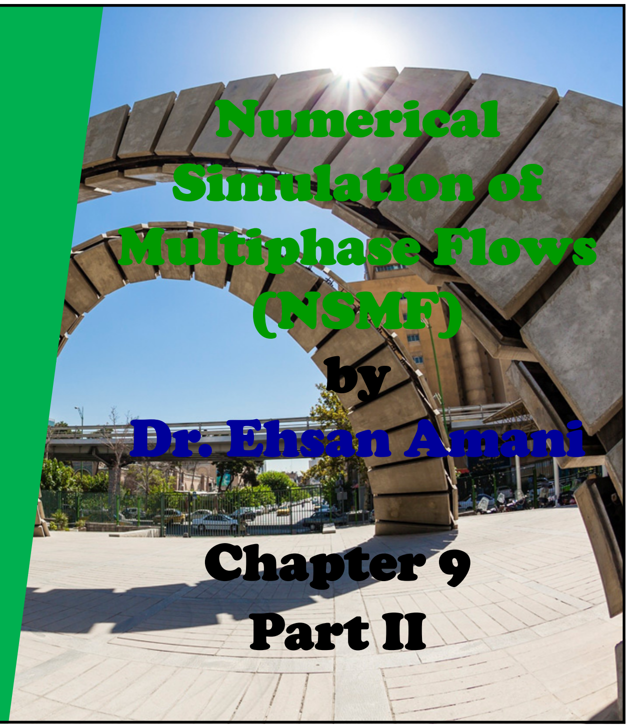


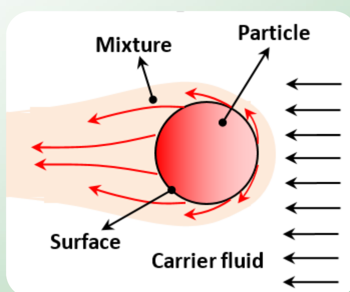
Numerical Simulation of Multiphase Flows (NSMF)

by

Dr. Ehsan Amani

Chapter 9
Part II

IX. Fluid-particle interaction (FPI)

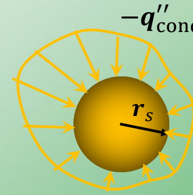


IX.1 Fluid-to-particle transfer rates

- IX.1.1 Momentum transfer – forces and moments on a particle
- IX.1.2 Heat transfer
- IX.1.3 Particle tracking numerics

Heat transfer

- DNS formulation:



$$\overbrace{m_p \frac{d\bar{\imath}_p}{dt}}^{q_{in}}$$

$$= \underbrace{\int_{A_p} -\boldsymbol{q}''_{\text{cond}}.\,\boldsymbol{n}dA}_{q_{\text{conv}}} + \dot{m}_p\left(h_{v,\text{surf}} - \bar{h}_p + w'\,w'/2\right) + q_R$$

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Heat transfer

- Point-particle formulation:
  - Assuming uniform temperature inside droplet:

$$q_{\text{in}} = m_p \frac{d\bar{\imath}_p}{dt} = m_p \frac{di_p}{dt} = m_p c_p \frac{dT_p}{dt}$$

  - Convection modeling:

$$q_{\text{conv}} = h_m \underbrace{A_p}_{\pi d_p^2} (T_s - T_{\text{surf}}) = \pi d_p \lambda_m \text{Nu}_m (T_s - T_{\text{surf}})$$

$$\text{Nu}_m = \frac{h_m d_p}{\lambda_m}$$

  - Therefore,

$$m_p c_p \frac{dT_p}{dt} = \pi d_p \lambda_m \text{Nu}_m (T_s - \overbrace{T_{\text{surf}}}^{T_p}) + \overset{-\frac{m_p}{\tau_M}}{\dot{m}_p}(\overbrace{h_{v,\text{surf}} - \bar{h}_p}^{h_{fg,v,\text{surf}} = L_{v,\text{surf}}} + \overset{\text{neglected}}{\cancel{w'w'/2}}) + q_R$$

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Heat transfer

- Finally:

$$\frac{dT_p}{dt} = \frac{T_s - T_p}{\tau_T} - \frac{1}{\tau_M}\frac{L_{v,\mathrm{surf}}}{c_p} + \frac{q_R}{m_p c_p}; \quad \tau_T = \frac{\rho_p c_p d_p^2}{6\lambda_m \mathrm{Nu}_m}$$

Heat transfer time scale

- Different models, different correlations for Nu_m, e.g. [13, 14]
- See “FPISummary.pdf” for a short summary.

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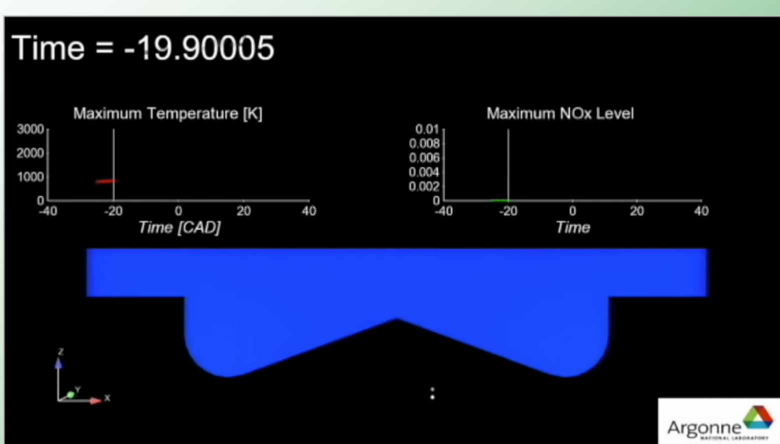
Heat transfer

- Application: internal combustion engines



▲ **CI engine simulation- fuel atomization, evaporation, mixing, and combustion**

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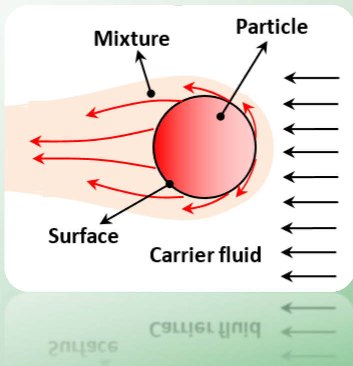
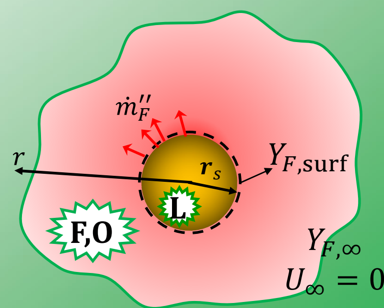
Other topics

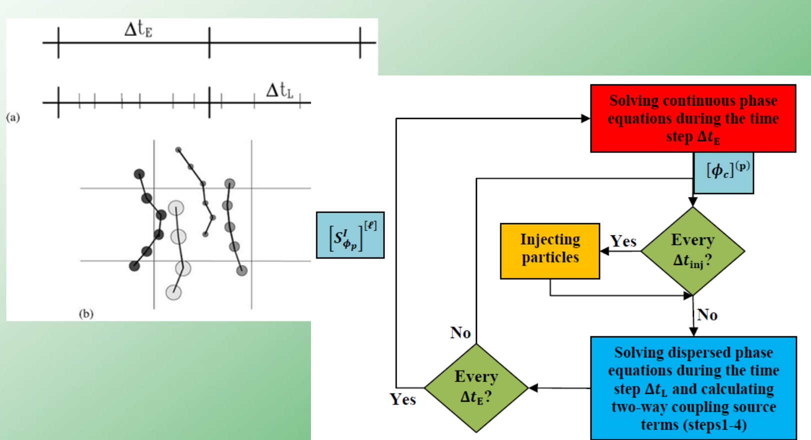
- Mass transfer





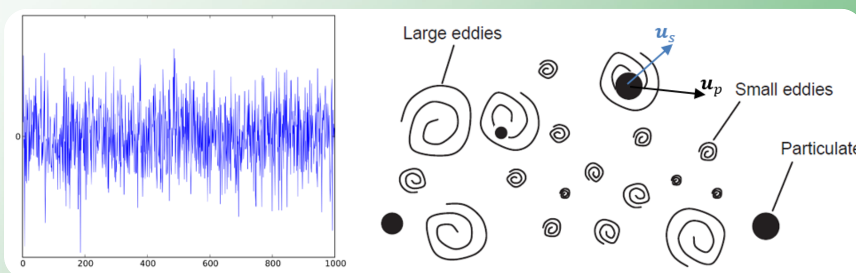
Other topics

- Two-way coupling



Other topics

- Dispersion



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Numerics of particle tracking

The general form of ODE Lagrangian equations is:

$$\frac{d\Phi_p}{dt} = \frac{\Phi_s - \Phi_p}{\tau_\Phi} + a \qquad (74.9)$$

By proper selection of Φ, τ_Φ and a, the Lagrangian governing equations of a particle are obtained.

Several approaches have been proposed for the ~~into~~ numerical solution of this equation

+ Implicit Euler: (at n+1) $O(\Delta t)$

$$\frac{\Phi_p^{n+1} - \Phi_p^{n}}{\Delta t} = \frac{\Phi_s^{n+1} - \Phi_p^{n+1}}{\tau_\Phi^{n+1}} + a^{n+1} \rightarrow$$

$$\Phi_p^{n+1} = \left[\Phi_p^{n} + \Delta t \left(a^{n+1} + \Phi_s^{n+1}/\tau_\Phi^{n+1}\right)\right] / \left(1 + \Delta t/\tau_\Phi^{n+1}\right) \qquad (75.9)$$

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Numerics of particle tracking

+ Trapezoidal at $n+\frac{1}{2}$ $O(\Delta t^2)$

$$\frac{\varphi_p^{n+1}-\varphi_p^{n}}{\Delta t}=\frac{\varphi^{n+\frac{1}{2}}-\varphi_p^{n+\frac{1}{2}}}{\tau_\varphi^{n+\frac{1}{2}}}+a^{n+\frac{1}{2}} \longrightarrow \qquad \varphi_p^{n+\frac{1}{2}} \to \frac{1}{2}\left(\varphi_p^{n}+\varphi_p^{n+1}\right)$$

$$\varphi_p^{n+1}=\left[\varphi_p^{n}\left(1-\frac{\Delta t}{2\tau_\varphi^{n+\frac{1}{2}}}\right)+\frac{\Delta t}{\tau_\varphi^{n+\frac{1}{2}}}\varphi_s^{n+\frac{1}{2}}+\Delta t\, a^{n+\frac{1}{2}}\right]\Big/\left(1+\frac{\Delta t}{2\tau_\varphi^{n+\frac{1}{2}}}\right) \quad (76.9)$$

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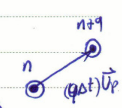
Numerics of particle tracking

Note 1: φ_s^{n+q} is estimated by one of the following formula:



a) $\varphi_s^{n+q} \simeq \varphi_s^{n}$ (77.9)

b) $\varphi_s^{n+q} \simeq \varphi_s^{n}+(q\Delta t)\vec{U}_p\cdot(\vec{\nabla}\varphi_s)^{n}$ (78.9) needs $\vec{\nabla}\varphi_s$

c) $\varphi_s^{n+q} \simeq \varphi_s(\vec{x}_p^{\,n+q})$, $\vec{x}_p^{\,n+q} \simeq \vec{x}_p^{\,n}+q\Delta t\,\vec{U}_p^{(n)}$ (79.9) inaccurate near the boundaries needs extrapolation outside domain

$\hookrightarrow$ approximation of particle location at $n+q$

For a^{n+q} and τ_φ^{n+q}, the approximation like Eq. (77.9) is usually used.

Note 2: The position equation in these two methods is advanced by

$$\left[\frac{d\vec{x}_p}{dt}=\vec{U}_p\right]^{n+\frac{1}{2}} \longrightarrow \frac{\vec{x}_p^{\,n+1}-\vec{x}_p^{\,n}}{\Delta t}=\vec{U}_p^{\,n+\frac{1}{2}}=\frac{1}{2}\left(\vec{U}_p^{\,n}+\vec{U}_p^{\,n+1}\right) \longrightarrow$$

$$\vec{x}_p^{\,n+1}=\vec{x}_p^{\,n}+\frac{\Delta t}{2}\left(\vec{U}_p^{\,n}+\vec{U}_p^{\,n+1}\right) \quad (80.9)$$

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Numerics of particle tracking

+ Analytic method $O(\Delta t)$

Assuming τ_p, φ_s, and a to be constant at $t = \oplus$ where $\oplus$ can be n, $n+\frac{1}{2}$, or $n+1$, Eq. (74.9) can be integrated analytically between t_n to t_{n+1} to yield (exercise)

$$\varphi_p^{n+1} - \varphi_p^{n} = \left[\varphi_s^{\oplus} - \varphi_p^{n} + a^{\oplus}\tau_p^{\oplus}\right]\left[1 - \exp\left(-\frac{\Delta t}{\tau_p^{\oplus}}\right)\right] \quad (81.9)$$

then (exercise), inserting into $d\vec{x}_p/dt = \vec{U}_p$ results in

$$\vec{x}_p^{\,n+1} = \vec{x}_p^{\,n} + \Delta t\left(\varphi_s^{\oplus} + a^{\oplus}\tau_p^{\oplus}\right) - \tau_p^{\oplus}\left(\varphi_p^{n+1} - \varphi_p^{n}\right) \quad (82.9)$$

the values at $\oplus$ can be estimated by Eqs. (77.9)-(79.9).

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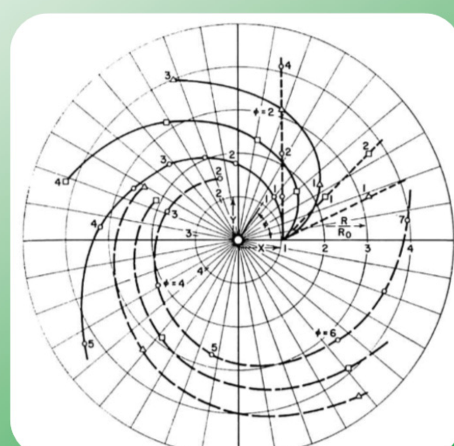
Project#2: ELSS of multiphase flows

Part I

- Lagrangian particle tracking
- Python coding
- Validation against a simple case
- Particle in 2D vortex
- Error analysis of discretization schemes



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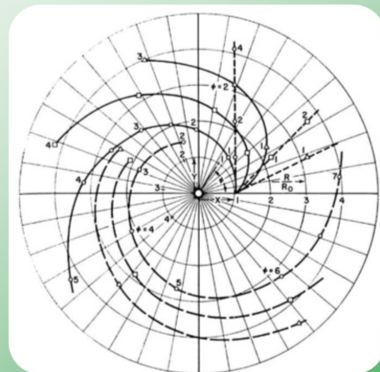
Hands-on practice

- **HW#9:**
  - Some useful analytical solutions for particle tracking
  - Test of FLUENT Lagrangian particle tracking



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