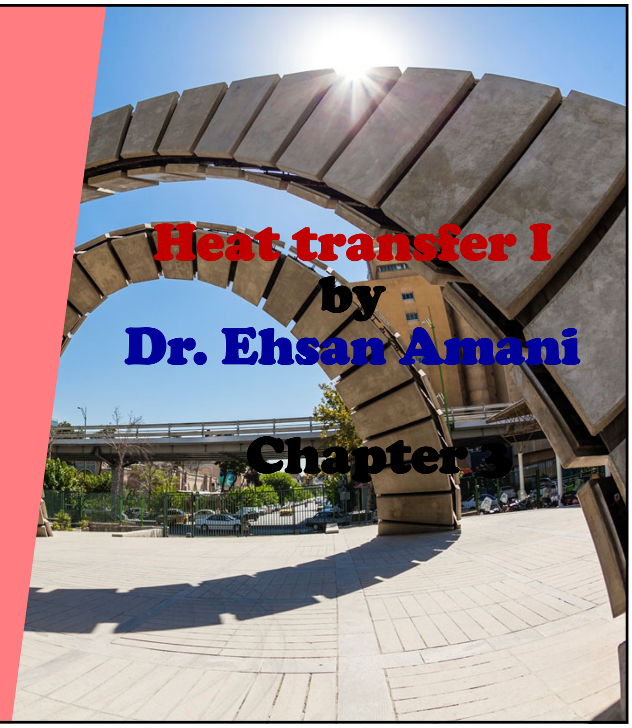

Heat transfer I
by
Dr. Ehsan Amani

Chapter 3

Objectives

- **Fourier's law:**
  - Experiment: for almost all materials (solids, liquids, gases):

$$q''_x = -k\frac{dT}{dx} \qquad (1.3)$$

  - What is the extension to 3D?
  - Other coordinates systems?
- **Heat conduction equation:**
  - Temperature distribution in solid and stagnant fluids
  - Applications?

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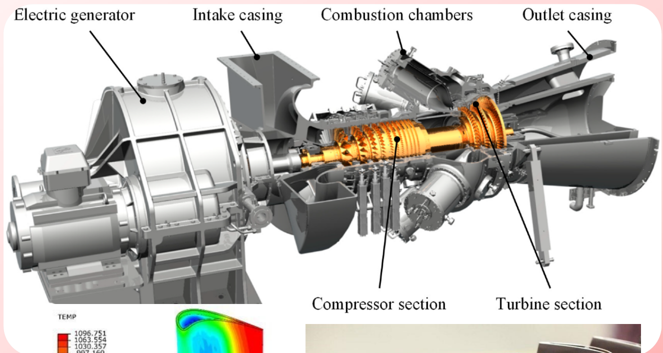
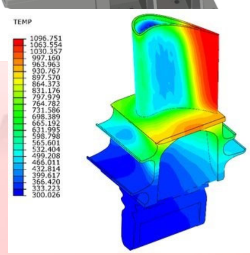
Temperature distribution: Applications

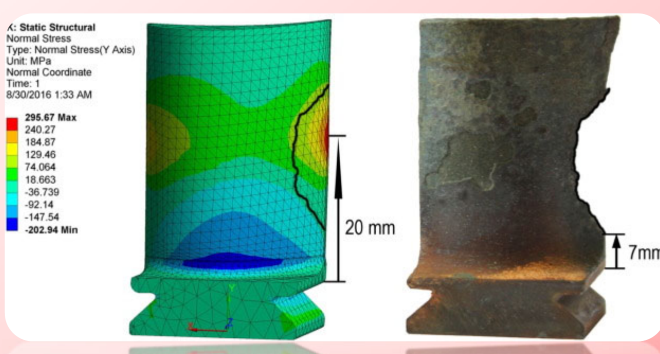
- **Stress and strain analysis: Gas turbine blade**





Temperature distribution: Applications

- **Stress and strain analysis: Gas turbine blade**



Temperature distribution: Applications

- Stress and strain analysis: Internal combustion engine

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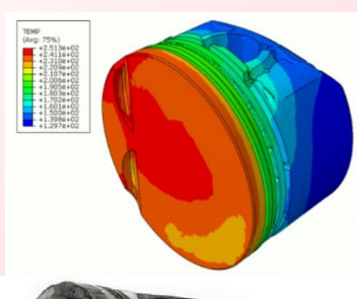
Temperature distribution: Applications

- Stress and strain analysis: Internal combustion engine



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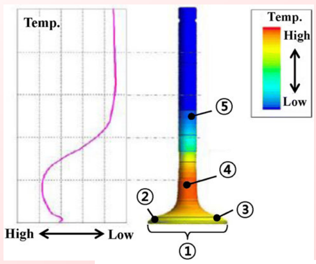
Temperature distribution: Applications

- **Stress and strain analysis: Internal combustion engine**

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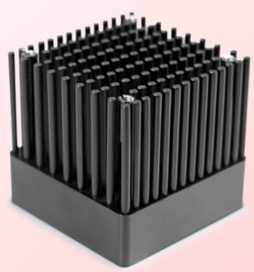
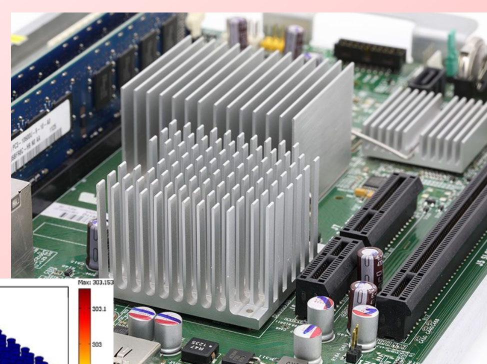
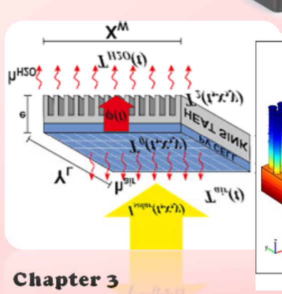
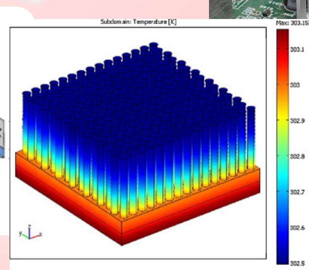
Temperature distribution: Applications

- **Heat transfer across a control surface – heat sink**

Chapter 3 By E. Amani

1.3 Conduction heat flux and Fourier’s law

- **Extending Eq. (1.3) to 3D:**
  - Assuming isotropic conduction

$$\vec{q}'' = q''_x \vec{i} + q''_y \vec{j} + q''_z \vec{k} = -k\frac{\partial T}{\partial x}\vec{i} - k\frac{\partial T}{\partial y}\vec{j} - k\frac{\partial T}{\partial z}\vec{k} \qquad (2.3)$$

  - Cartesian coordinates: Gradient vector

$$\vec{\nabla}\Phi = \frac{\partial \Phi}{\partial x}\vec{i} + \frac{\partial \Phi}{\partial y}\vec{j} + \frac{\partial \Phi}{\partial z}\vec{k} \qquad (3.3)$$

$$\vec{q}'' = -k\vec{\nabla}T \qquad (4.3)$$ Fourier's law (3D)

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1.3 Conduction heat flux and Fourier’s law

- **Fourier’s law:** $\vec{q}'' = -k\vec{\nabla}T \qquad (4.3)$

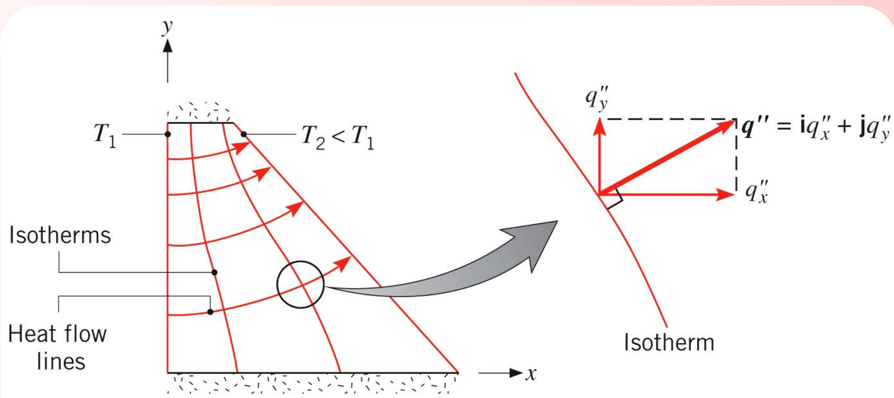

Figure 4.1

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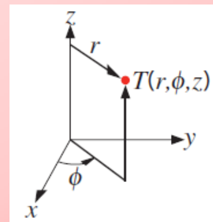
1.3 Conduction heat flux and Fourier's law

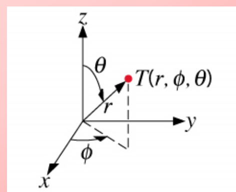
- **Knowing the gradient operator in other coordinates (see Appendix A):**
  - Cylindrical coordinates: (r, ϕ, z)

$$\vec{\nabla} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\phi \frac{\partial}{r\partial \phi} + \vec{e}_z \frac{\partial}{\partial z} \qquad (6.3)$$

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial r}}_{q''_r}\vec{e}_r \underbrace{-k\frac{\partial T}{r\partial \phi}}_{q''_\phi}\vec{e}_\phi \underbrace{-k\frac{\partial T}{\partial z}}_{q''_z}\vec{e}_z \qquad (7.3)$$



  - Spherical coordinates?



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1.3 Conduction heat flux and Fourier's law

- **Conductivity:**
  - Transport property vs. thermodynamic property:
  - Calculation:
    - ✓ Microscopic relations: See section 2.2 [1]
    - ✓ Empirical correlations
    - ✓ Empirical tables (see Appendix A.1 to A.3)

TABLE A.1 Thermophysical Properties of Selected Metallic Solids[a]

		Properties at 300 K				Properties at Various Temperatures (K) k (W/m · K)/c_p (J/kg · K)									
Composition	Melting Point (K)	ρ (kg/m³)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m²/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Aluminum															
Pure	933	2702	903	237	97.1	302	237	240	231	218					
						482	798	949	1033	1146					
Alloy 2024-T6 (4.5% Cu, 1.5% Mg, 0.6% Mn)	775	2770	875	177	73.0	65	163	186	186						
						473	787	925	1042						
Alloy 195, Cast (4.5% Cu)		2790	883	168	68.2			174	185						
								—	—						
Beryllium	1550	1850	1825	200	59.2	990	301	161	126	106	90.8	78.7			
						203	1114	2191	2604	2823	3018	3227	3519		
Bismuth	545	9780	122	7.86	6.59	16.5	9.69	7.04							
						112	120	127							

Appendix A ■ Thermophysical P

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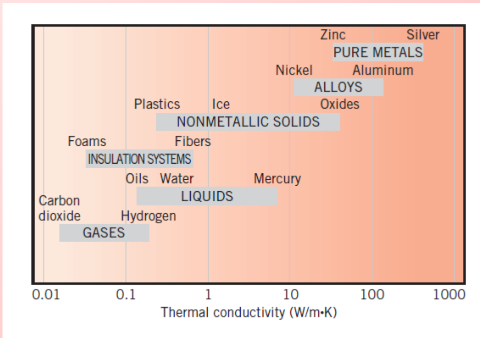
1.3 Conduction heat flux and Fourier's law

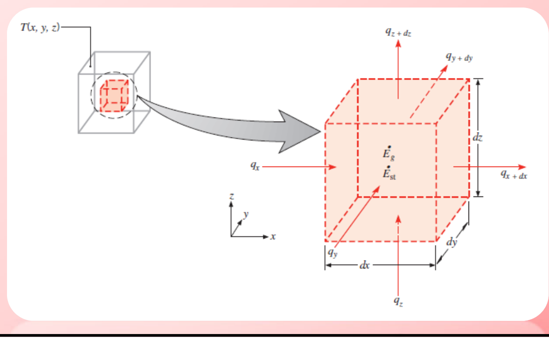
- **Conductivity:**
  - Transport property vs. thermodynamic property
  - Calculation:
  - Key factors:
    - ✓ Molecular distance and configuration: Solids>liquids>gases?
    - ✓ Electric conductivity: conductors>insulators?



Chapter 3 By E. Amani

2.3 The heat conduction equation

- **Assumptions:**
  1. Neglecting mechanical energy change
  2. No phase change
  3. Neglecting or no advection (rigid-body motion is allowed)
  4. The internal energy depends on temperature only (solids, incompressible liquids, ideal gases)



Lecture Notes

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2.3 The heat conduction equation

- **Assumptions:**
  1. Neglecting mechanical energy change
  2. No phase change
  3. Neglecting or no advection (rigid-body motion is allowed)
  4. The internal energy depends on temperature only (solids, incompressible liquids, ideal gases)

$$-\frac{\partial q''_x}{\partial x}-\frac{\partial q''_y}{\partial y}-\frac{\partial q''_z}{\partial z}+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (8.3)$$

  5. Isotropic conduction (Fourier's law)

Per unit volume

$$\underbrace{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)}_{\text{Net inflow of thermal energy into the control volume in x-direction}}+\underbrace{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)}_{\text{Net ... in y-direction}}+\underbrace{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{\text{Net ... in z-direction}}+\underbrace{\dot{q}}_{\text{Thermal energy generation}}=\underbrace{\rho c_v\frac{\partial T}{\partial t}}_{\text{Thermal energy change of the material element}} \qquad (9.3)$$

Chapter 3 By E. Amani

2.3 The heat conduction equation

- **Assumptions:**
  1. Neglecting mechanical energy change
  2. No phase change
  3. Neglecting or no advection (rigid-body motion is allowed)
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$$-\frac{\partial q''_x}{\partial x}-\frac{\partial q''_y}{\partial y}-\frac{\partial q''_z}{\partial z}+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (8.3)$$

  5. Isotropic conduction (Fourier's law)

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (9.3)$$

  6. For solid or incompressible gas

$$c_v=c_p=c$$

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2.3 The heat conduction equation

- **Tensorial form:**

$$-\vec{\nabla}.\vec{q}''+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (11.3)$$

$$\vec{\nabla}.\left(k\vec{\nabla}T\right)+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (11.3)'$$

- **Cylindrical coordinates (Appendix A):**

$$\frac{1}{r}\frac{\partial}{\partial r}(rq''_r)+\frac{1}{r}\frac{\partial q''_\phi}{\partial \phi}+\frac{\partial q''_z}{\partial z}+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (12.3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_v\frac{\partial T}{\partial t} \qquad (13.3)$$

- **Spherical coordinates (Appendix A)?**
- **Exercise: Derive Eq. (13.3) using the energy balance on a cylindrical coordinate element.**

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2.3 The heat conduction equation

- **Assumptions:**
  - 1-6 +
  7. Homogeneous k ($k = k(t)$):

$$\nabla^2 T+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (14.3)$$

A measure of thermal energy conductivity

$$\alpha=\frac{k}{\rho c_p} \qquad (15.3)$$

Thermal diffusivity $\left(\frac{m^2}{s}\right)$

A measure of thermal energy storage

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2.3 The heat conduction equation

- **Physical interpretation:**

$\dot{q} = 0$
applied $\vec{\nabla} T$ $\longrightarrow$ $\nabla^2 T = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

$\alpha \uparrow \longrightarrow \dfrac{\partial T}{\partial t} \uparrow$ **Fast temperature change**

$\alpha \downarrow \longrightarrow \dfrac{\partial T}{\partial t} \downarrow$ **Slow temperature change**

Chapter 3 **By E. Amani**

3.3 Boundary and initial conditions

- **First-order in time, second-order in space:**

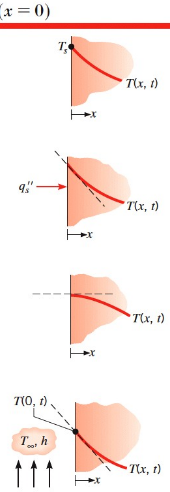
TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

$$T(0, t) = T_s \quad (2.31)$$

2. Constant surface heat flux

(a) Finite heat flux

$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = q_s'' \quad (2.32)$$

(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\Big|_{x=0} = 0 \quad (2.33)$$

3. Convection surface condition

$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = h[T_\infty - T(0, t)] \quad (2.34)$$

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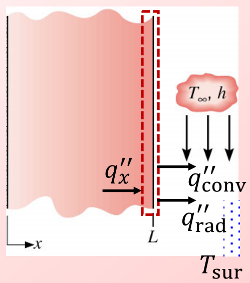
3.3 Boundary and initial conditions

- **First-order in time, second-order in space**
- **Use energy balance at the boundary (recommended):**
  - Example:



$$\dot{E}_{in} = \dot{E}_{out} \longrightarrow q''_x(x=L) = q''_{\text{conv}} + q''_{\text{rad}}$$

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h[T(x=L) - T_\infty] + h_r[T(x=L) - T_{\text{sur}}]$$

4.3 Sample problems

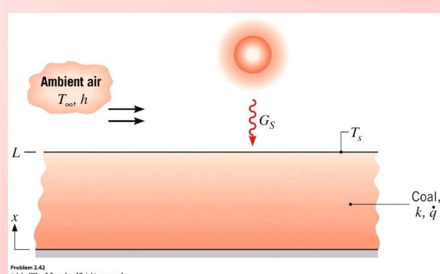
2.42 A plane layer of coal of thickness $L = 1$ m experiences uniform volumetric generation at a rate of $\dot{q} = 20\ \text{W/m}^3$ due to slow oxidation of the coal particles. Averaged over a daily period, the top surface of the layer transfers heat by convection to ambient air for which $h = 5\ \text{W/m}^2\cdot\text{K}$ and $T_\infty = 25°\text{C}$, while receiving solar irradiation in the amount $G_S = 400\ \text{W/m}^2$. Irradiation from the atmosphere may be neglected. The solar absorptivity and emissivity of the surface are each $\alpha_S = \varepsilon = 0.95$.

(a) Write the steady-state form of the heat diffusion equation for the layer of coal. Verify that this equation is satisfied by a temperature distribution of the form

$$T(x) = T_s + \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

From this distribution, what can you say about conditions at the bottom surface ($x = 0$)? Sketch the temperature distribution and label key features.

(b) Obtain an expression for the rate of heat transfer by conduction per unit area at $x = L$. Applying an energy balance to a control surface about the top surface of the layer, obtain an expression for T_s. Evaluate T_s and $T(0)$ for the prescribed conditions.

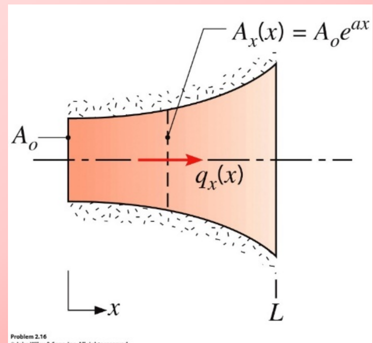


4.3 Sample problems

2.16 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.

(a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where $\dot{q}_o$ is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.



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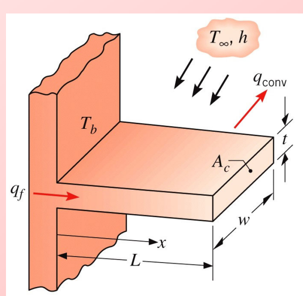
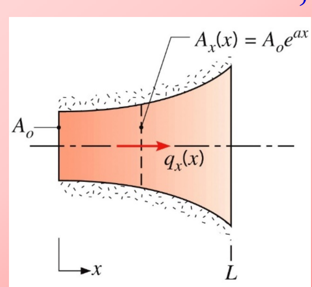
5.3 1D Temperature distribution

- **Do not use 1D heat equation in the Cartesian coordinates when:**
  1. A body with a variable cross-section (why?)
  2. Boundary heat transfer in other directions (why?)
- **Use direct energy balance for differential elements, instead.**





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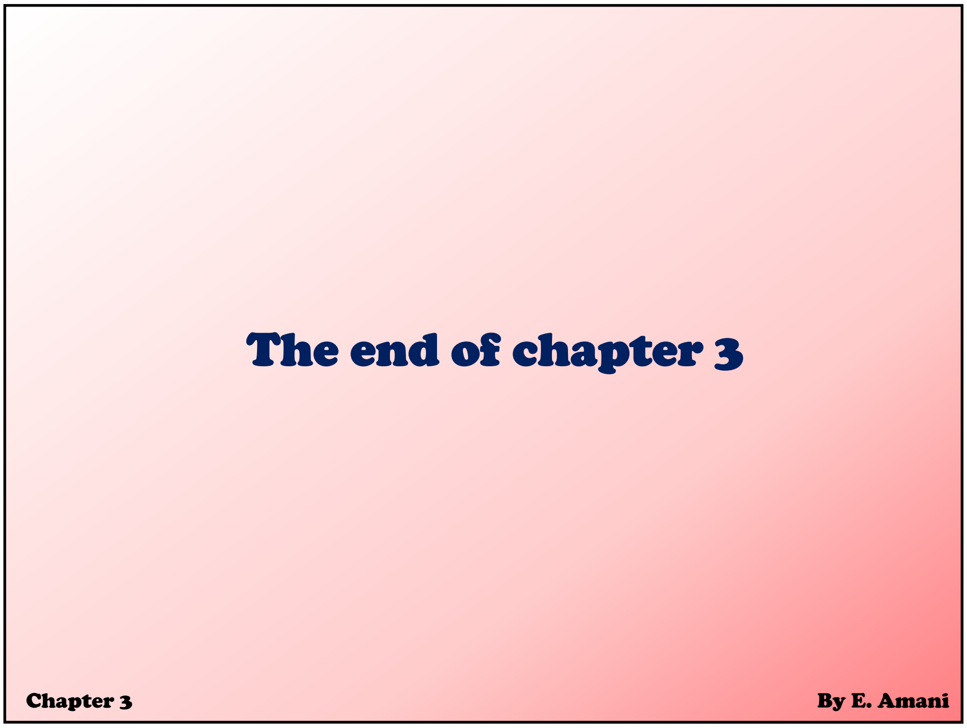
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