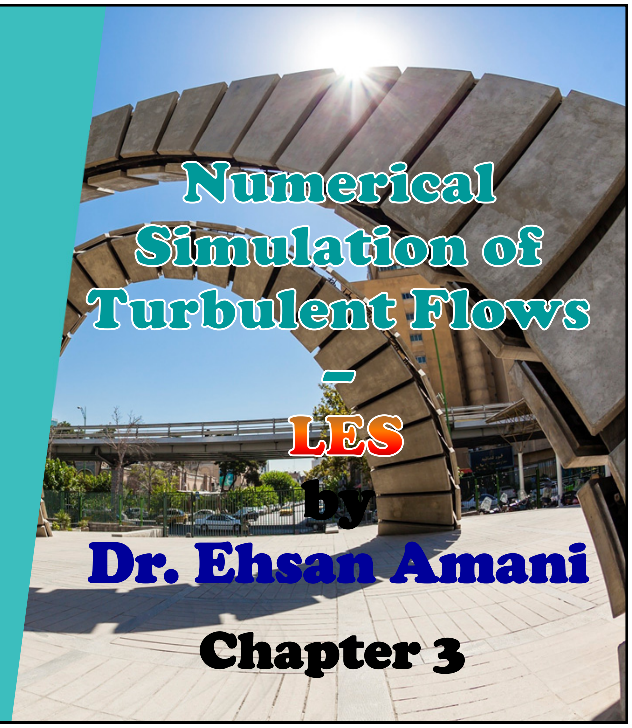


Numerical Simulation of Turbulent Flows
–
LES

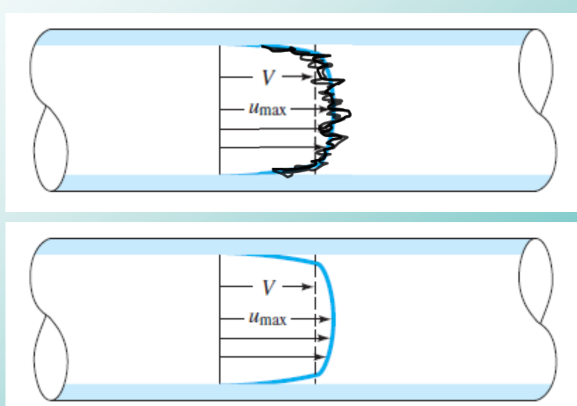
by

Dr. Ehsan Amani

Chapter 3

Introduction

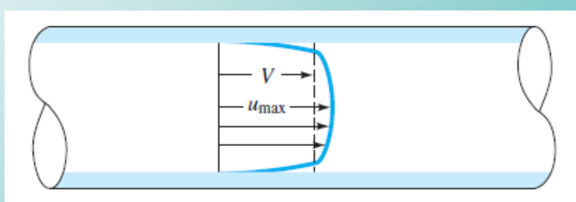
The concept of averaging



Introduction

The concept of averaging

- The mean (averaged) velocity can be captured by a much coarser grid and much lower computational cost (+)
- Therefore, the governing equations of the mean properties (RANS equations) can be governed (the goal of this chapter) and solved.



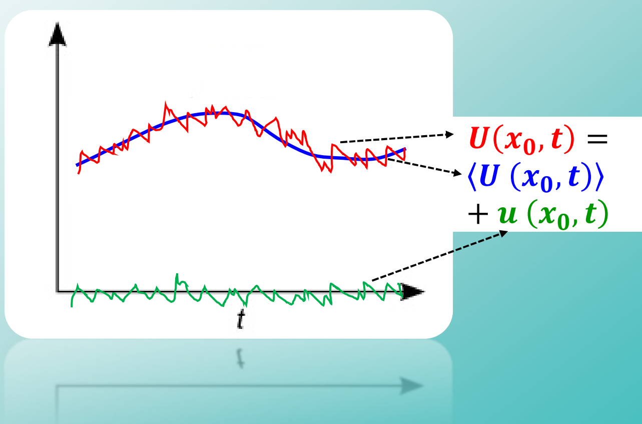
Introduction

The concept of averaging

- The mean (averaged) velocity can be captured by a much coarser grid and much lower computational cost (+)
- Therefore, the governing equations of the mean properties (RANS equations) can be governed (the goal of this chapter) and solved.
- In most engineering applications, we seek to find the mean properties.
- Therefore, when a direct numerical simulation (DNS) is applied, the solution is averaged to find the mean flow.
- A question arises: what is the benefit of DNS or LES? Or what is the limitation of RANS?

III.1 Reynolds averaging

Mean and fluctuation



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III.1 Reynolds averaging

⇨ Lecture Notes: 3.1

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III.2 The decomposition of the kinetic energy

- **The mean of kinetic energy can be decomposed into two parts:**

$$\xrightarrow{(3.7)} \langle E(\vec{x},t)\rangle = \left\langle \frac{1}{2}\vec{U}.\vec{U} \right\rangle = \underbrace{\frac{1}{2}\langle\vec{U}\rangle.\langle\vec{U}\rangle}_{\bar{E}(\vec{x},t)} + \underbrace{\frac{1}{2}\langle\vec{u}.\vec{u}\rangle}_{k(\vec{x},t)}$$

$$\langle E(\vec{x},t)\rangle = \bar{E}(\vec{x},t) + k(\vec{x},t) \quad (3.10)$$

Kinetic energy of the mean flow **Turbulent kinetic energy**

$$\bar{E}(\vec{x},t) = \frac{1}{2}\langle\vec{U}\rangle.\langle\vec{U}\rangle \quad (3.11)$$

$$k(\vec{x},t) = \frac{1}{2}\langle\vec{u}.\vec{u}\rangle = \frac{1}{2}\langle u_i u_i\rangle = \frac{1}{2}tr\{\langle u_i u_j\rangle\} \quad (3.12)$$

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III.2 The decomposition of the kinetic energy

- **Averaging kinetic energy transport equation, Eq. (2.17):**

$$\underbrace{\left\langle \frac{DE}{Dt} \right\rangle}_{(3.6)} + \vec{\nabla}.\left\langle \underbrace{\frac{p\vec{U}}{\rho} - 2\nu\underline{S}.\vec{U}}_{\vec{T}} \right\rangle = -2\nu\underbrace{\langle\underline{S}:\underline{S}\rangle}_{(3.7)}$$

$$\frac{\bar{D}\langle E\rangle}{\bar{D}t} + \vec{\nabla}.\langle\vec{u}e\rangle \qquad \langle\underline{S}\rangle:\langle\underline{S}\rangle + \langle\underline{s}:\underline{s}\rangle$$

$$\frac{\bar{D}\langle E\rangle}{\bar{D}t} + \vec{\nabla}.\left[\langle\vec{u}e\rangle + \langle\vec{T}\rangle\right] = -\bar{\varepsilon} - \varepsilon \quad (3.13)$$

$\bar{\varepsilon} = 2\nu\langle\underline{S}\rangle:\langle\underline{S}\rangle$ (3.14) **: dissipation due to the mean flow**

$\varepsilon = 2\nu\langle\underline{s}:\underline{s}\rangle$ (3.15) **: turbulent dissipation**

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III.2 The decomposition of the kinetic energy

- **Knowing:**

$$\langle \underline{S} \rangle = \left\langle \frac{1}{2}\left[\left(\vec{\nabla}\vec{U}\right)^T + \vec{\nabla}\vec{U}\right]\right\rangle = \frac{1}{2}\left[\left(\vec{\nabla}\langle\vec{U}\rangle\right)^T + \vec{\nabla}\langle\vec{U}\rangle\right] \quad (3.16)$$

$$\underline{s} = \underline{S} - \langle \underline{S} \rangle = \frac{1}{2}\left[\left(\vec{\nabla}\vec{u}\right)^T + \vec{\nabla}\vec{u}\right] \quad (3.17)$$

$$\vec{\nabla}.\underline{S} = \frac{1}{2}\nabla^2\vec{U} \quad (3.17)'$$

$$\vec{\nabla}.\underline{\Omega} = -\frac{1}{2}\nabla^2\vec{U} \quad (3.17)''$$

Proof: Appendix

III.2 The decomposition of the kinetic energy

- **Taking the dot product of $\langle\vec{U}\rangle$ and mean momentum equation, Eq. (3.4)' (Proof: appendix):**

$$\frac{\bar{D}\bar{E}}{\bar{D}t} + \vec{\nabla}.\left[\langle\vec{U}\rangle.\langle\vec{u}\vec{u}\rangle + \frac{\langle p\rangle\langle\vec{U}\rangle}{\rho} - 2\nu\langle\vec{U}\rangle.\langle\underline{S}\rangle\right] = \underbrace{\langle\vec{u}\vec{u}\rangle : \vec{\nabla}\langle\vec{U}\rangle}_{-\mathcal{P}} - \bar{\varepsilon} \quad (3.18)$$

$$\mathcal{P} = -\langle\vec{u}\vec{u}\rangle : \vec{\nabla}\langle\vec{U}\rangle = -\langle u_i u_j\rangle \frac{\partial\langle U_i\rangle}{\partial x_j} \quad (3.19)$$

- **Exercise: Subtracting Eq. (3.18) from Eq. (3.13), show that:**

$$\frac{\bar{D}k}{\bar{D}t} + \vec{\nabla}.\left[\frac{1}{2}\langle\vec{u}\vec{u}.\vec{u}\rangle + \frac{\langle p'\vec{u}\rangle}{\rho} - 2\nu\langle\vec{u}.\underline{s}\rangle\right] = \mathcal{P} - \varepsilon \quad (3.21)$$

III.2 The decomposition of the kinetic energy

$$\frac{\bar{D}\bar{E}}{\bar{D}t} + \vec{\nabla}.\left[\langle\vec{U}\rangle.\langle\vec{u}\vec{u}\rangle + \frac{\langle p\rangle\langle\vec{U}\rangle}{\rho} - 2\nu\langle\vec{U}\rangle.\langle\underline{S}\rangle\right] = -\mathcal{P} - \bar{\varepsilon} \qquad (3.18)$$

$$\frac{\bar{D}k}{\bar{D}t} + \vec{\nabla}.\left[\frac{1}{2}\langle\vec{u}\vec{u}.\vec{u}\rangle + \frac{\langle p'\vec{u}\rangle}{\rho} - 2\nu\langle\vec{u}.\underline{s}\rangle\right] = \mathcal{P} - \varepsilon \qquad (3.21)$$

$$\mathcal{P} = -\langle u_i u_j\rangle\frac{\partial\langle U_i\rangle}{\partial x_j} \qquad (3.19)$$

- **Experiment: $\mathcal{P}$ is almost always a positive quantity (Production of turbulent kinetic energy).**
- **The action of the mean velocity gradients working against the Reynolds stresses**
- **Removes energy from the mean flow and transfers it to the fluctuating field**

Homework

- **HW#2:**
  - **The derivation of some important relations**
  - **A practice on working with tensorial as well as suffix notation**