The implementation of quadrature based moment methods in twoPhaseEulerFoam

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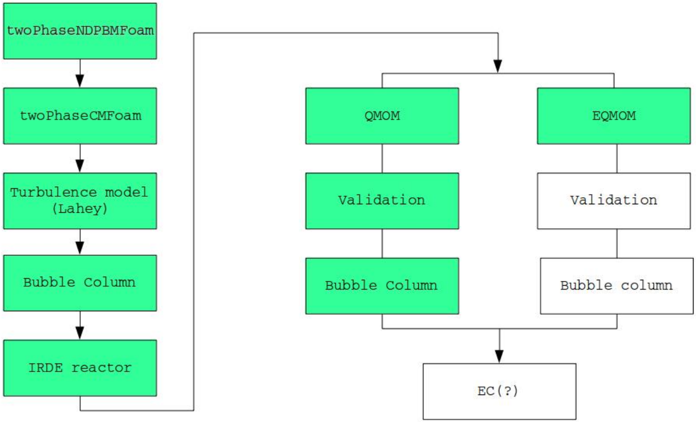
Outline

The updated plan of project



Population balance methods

- Number density approach
- Interfacial Area Transport Equation (IATE)
- Class Method (CM)
- Quadrature Method of Moments (QMOM)

$$n(L; \mathbf{x}, t) = \sum_{i=1}^{N} W(i) \times \delta(L - L_i)$$

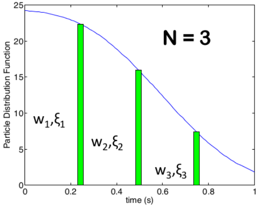
$$m_k = \int_0^{+\infty} n(L; \mathbf{x}, t) L^k dL$$

Quadrature method of moments, McGraw (1997)

$$m_k = \int_0^{+\infty} n(L; \mathbf{x}, t) L^k dL$$

$$m_k = \sum_{i=1}^{N} L_i^k w_i(t, x), \qquad k = 0, 1, 2, ..., 2N - 1$$

$$\frac{\partial m_k(t, x)}{\partial t} + \nabla.(\mathbf{U}^{(k)} m_k(t, x)) = \overline{S}_k$$



Source terms in QMOM

$$\overline{S}_k = \overline{B}_{ag,k} - \overline{D}_{ag,k} + \overline{B}_{br,k} - \overline{D}_{br,k}$$

$$\overline{B}_{ag,k} = \frac{1}{2} \sum_{i}^{N} \sum_{j}^{N} W_i W_j (L_i^3 + L_j^3)^{k/3} a_{ij}$$

$$\overline{D}_{ag,k} = \sum_{i}^{N} \sum_{j}^{N} W_i W_j L_i^k a_{ij}$$

$$\overline{B}_{br,k} = \sum_{i}^{N} W_i \overline{b}_i^{(k)} \beta_i$$

$$\overline{D}_{br,k} = \sum_{i}^{N} L_i^k W_i \beta_i$$

QMOM Validation (D. L. Marchisio et al. (2003))

$$\frac{\partial m_k(t,x)}{\partial t} = \overline{B}_{ag,k} - \overline{D}_{ag,k} + \overline{B}_{br,k} - \overline{D}_{br,k}$$

- Aggregation kernel:

$$a_{ij} = a(L_i, L_j) = 0.02$$

- Breakage kernel:

$$\beta_i = \beta(L_i) = 1$$

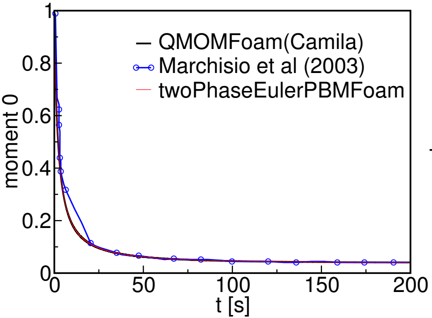
- Daughter distribution:

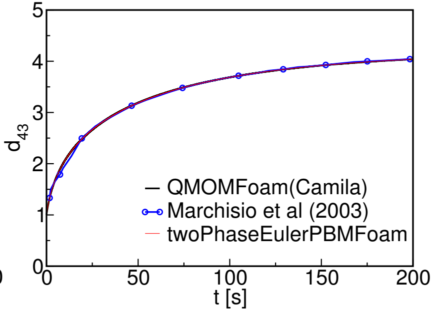
$$\overline{b}_i^{(k)} = 2^{(3-k)/3} L_i^k$$

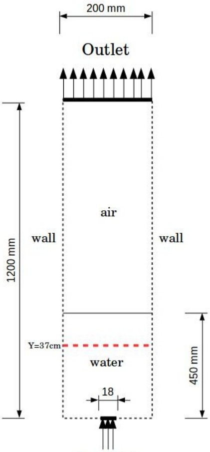
- Moment initialization:

$$m_k = 0, k = 0, 1, ..., 5$$

Comparison


QMOMFoam(Camila)
Marchisio et al (2003)
twoPhaseEulerPBMFoam
moment 0
t [s]
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200


QMOMFoam(Camila)
Marchisio et al (2003)
twoPhaseEulerPBMFoam
d43
t [s]
0
1
2
3
4
5
0
50
100
150
200



- Liquid (water) : continuous phase
- Gas (air) : dispersed phase
- $d \Rightarrow QMOM$ $\boxed{d = \frac{m3}{m2}}$
- Liquid: Turbulent
- Gas: Laminar
- Incompressible fluids
- PDE for moments:
  fvm::ddt(m[i])+ fvm::div(phia, m[i], fScheme)-fvm::Sp(fvc::div(phia), m[i])

Coupling QMOM and twoPhaseEulerPBMFoam (2)

$$\frac{\partial m_k(t,x)}{\partial t} + \nabla.(\mathbf{U}^{(k)} m_k(t,x)) = \overline{B}_{ag,k} - \overline{D}_{ag,k} + \overline{B}_{br,k} - \overline{D}_{br,k}$$

- Aggregation kernel:

$$a_{ij} = a(L_i, L_j) = 0.02$$

- Breakage kernel:

$$\beta_i = \beta(L_i) = 1$$

- Daughter distribution:

$$\overline{b}_i^{(k)} = 2^{(3-k)/3} L_i^k$$

$$\boxed{Divergence}$$

Coupling QMOM and twoPhaseEulerPBMFoam (3)

$$\frac{\partial m_k(t,x)}{\partial t} + \nabla.(\mathbf{U}^{(k)} m_k(t,x)) = \overline{B}_{ag,k} - \overline{D}_{ag,k} + \overline{B}_{br,k} - \overline{D}_{br,k}$$

- Aggregation kernel:

$$a_{ij} = a(L_i, L_j) \Rightarrow \text{Luo aggregation kernel}$$

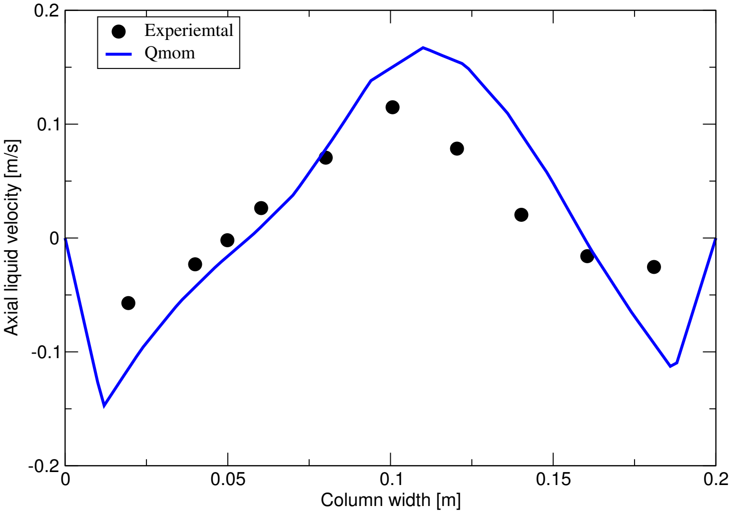
- Breakage kernel: Luo breakage kernel

$$\beta_i = \beta(L_i) \Rightarrow \text{Luo breakage kernel}$$

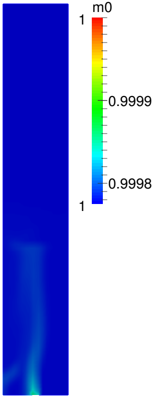
- Daughter distribution:

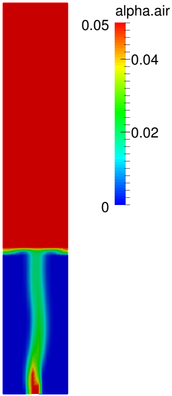
$$\overline{b}_i^{(k)} = 2^{(3-k)/3} L_i^k$$

Validation QMOM+twoPhaseEulerFOAM



Result QMOM+twoPhaseEulerFOAM





Extended Quadrature Method of Moments (EQMOM), C. Yuan et al. (2012)

- Quadrature method of moments (QMOM)

$$n(L;\mathbf{x},t) = \sum_{i=1}^{N} W(i) \times \delta_\sigma(L - L_i)$$

- Extended Quadrature Method of Moments (EQMOM)

$$\lim_{\delta \to 0} \delta_\sigma(L, L_i) = \delta(L - L_i)$$

$$n(L;\mathbf{x},t) = \sum_{i=1}^{N} W(i) \times \delta_\sigma(L, L_i)$$

Source terms in EQMOM

$$\overline{S}_k = \overline{B}_{ag,k} - \overline{D}_{ag,k} + \overline{B}_{br,k} - \overline{D}_{br,k}$$

$$\overline{B}_{ag,k} = \frac{1}{2} \sum_{\alpha_1=1}^{N} \sum_{\beta_1=1}^{N_\alpha} W_{\alpha_1} W_{\alpha_1\beta_1} \sum_{\alpha_2=1}^{N} \sum_{\beta_2=1}^{N_\alpha} W_{\alpha_2} W_{\alpha_2\beta_2} \left(L^3_{\alpha_1\beta_1} + L^3_{\alpha_2\beta_2}\right)^{k/3} a_{\alpha_1\beta_1\alpha_2\beta}$$

$$\overline{D}_{ag,k} = \sum_{\alpha_1=1}^{N} \sum_{\beta_1=1}^{N_\alpha} L^k_{\alpha_1\beta_1} W_{\alpha_1} W_{\alpha_1\beta_1} \sum_{\alpha_2=1}^{N} \sum_{\beta_2=1}^{N_\alpha} W_{\alpha_2} W_{\alpha_2\beta_2} a_{\alpha_1\beta_1\alpha_2\beta_2}$$

$$\overline{B}_{br,k} = \sum_{\alpha_1=1}^{N} \sum_{\beta_1=1}^{N_\alpha} W_{\alpha} W_{\alpha\beta} \overline{b}^{(k)}_{\alpha\beta} \beta_{\alpha\beta}$$

$$\overline{B}_{br,k} = \sum_{\alpha_1=1}^{N} \sum_{\beta_1=1}^{N_\alpha} W_{\alpha} W_{\alpha\beta} L^k_{\alpha\beta} \beta_{\alpha\beta}$$

EQMOM algorithm

1. Guess σ, compute the $2n$ moments m_k^* for $k = 0, ..., 2n-1$
2. Use PD or Wheeler algorithm with m_k^* for for $k = 0, ..., 2n-1$ to find n weights and n abscissas L_α
3. Compute m_{2n}^* using w_α and L_α
4. Construct $J_n(\sigma)$ from m^* and σ
5. Find root to achieve

EQMOM and Normal distribution

$$m_0^* = m_0$$

$$m_1^* = m_1$$

$$m_2^* = m_2 - m_0^* \sigma^2$$

$$m_3^* = m_3 - 3 m_1^* \sigma^2$$

$$m_4^* = m_4 - 6 m_2^* \sigma^2 - 3 m_0^* \sigma^2$$

$$m_5^* = m_5 - 10 m_3^* \sigma^2 - 15 m_1^* \sigma^4$$

$$m_6^* = m_6 - 15 m_4^* \sigma^2 - 45 m_2^* \sigma^4 - 15 m_0^* \sigma^6$$

EQMOM and Normal distribution $N = 1$, $\sigma = 1$, $\mu = 5$

$$m_0 = 1.0,\ m_1 = 5.0,\ m_2 = 33.33$$

$$\Downarrow$$

$$L_1 = 5 \text{ and } w_1 = 1$$

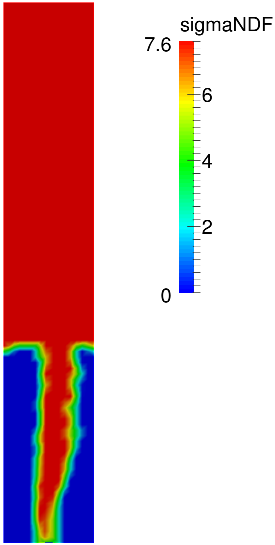
$$m_2^* = \sum_{\alpha=1}^{N} W_\alpha L_\alpha^2 = w_1 \times L_1^2$$

$$m_2^* = m_2 - m_0^* \sigma^2$$

$$J_n(\sigma) = w_1 \times L_1^2 - m_2 - m_0^* \sigma^2$$

$$\boxed{\sigma^2 = 8.33}$$

EQMOM+twoPhaseEulerFoam



The conclusion

- Done:
    - twoPhaseEulerPBMFoam was updated with QMOM and EQMOM
    - QMOM validation
    - twoPhaseEulerFoam+QMOM Validation
    - Initial EQMOM library
- Coming:
    - Apply EQMOM source terms
    - EQMOM validation with **"Ehsan Madadi and Alberto Passalacqua (2015)"**

Thank you for your attention!

Appendix

- Luo Breakage kernels

$$\Omega_{br}(V, V') = k \int_{\zeta}^{1} \frac{(1+\zeta)^2}{\zeta^n} exp(-b\zeta^m) d\zeta$$

- Luo aggregation kernels

$$\Omega_{ag}(V_i, V_j) = \omega_{ag} \times P_{ag}(V_i, V_j)$$