CSE 317 July 2022
Artificial Intelligence

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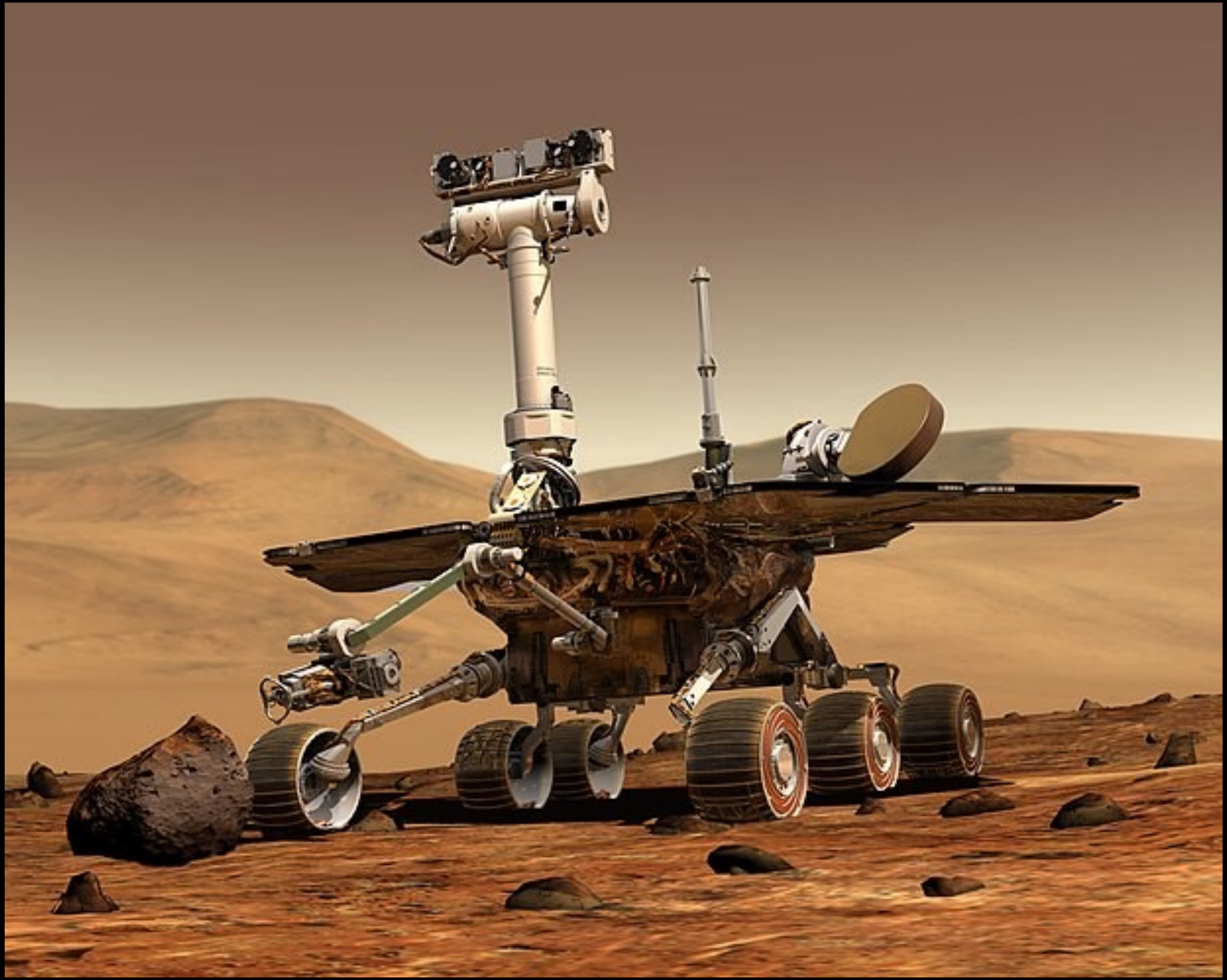
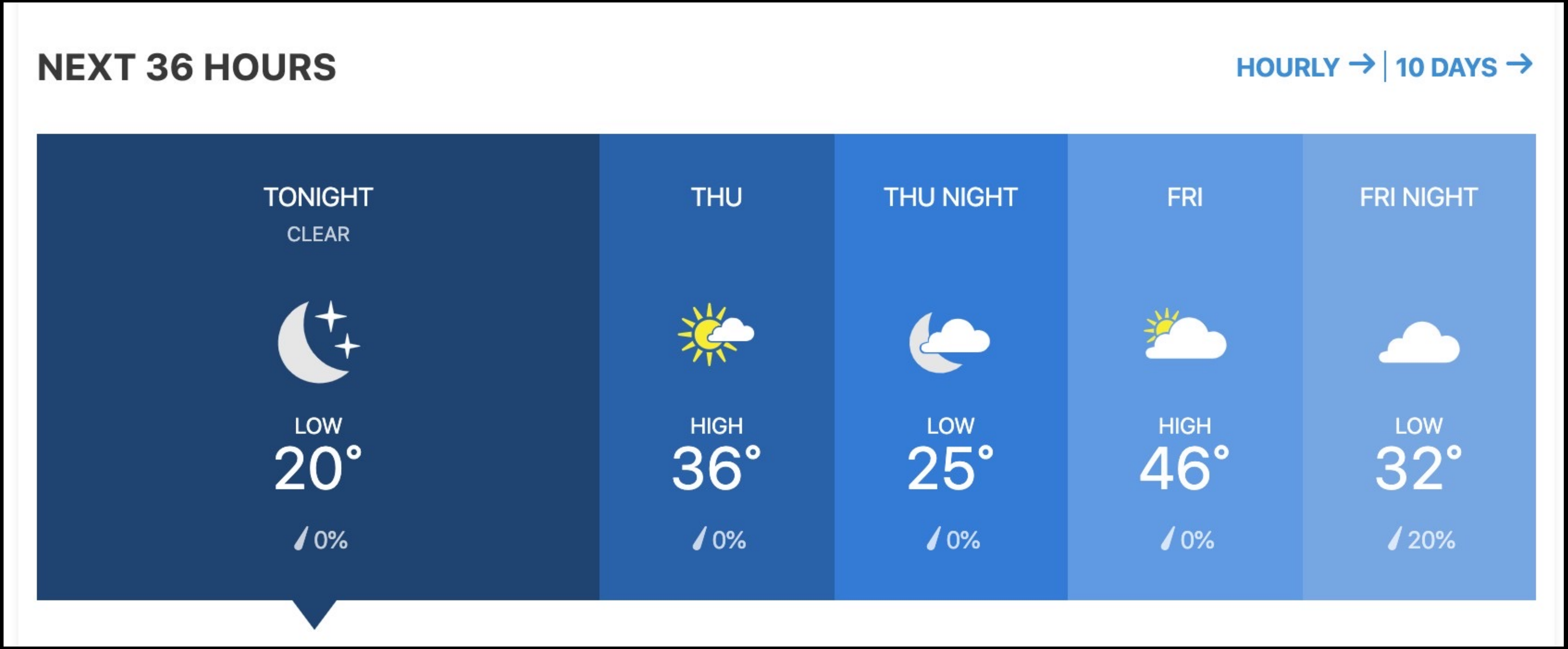
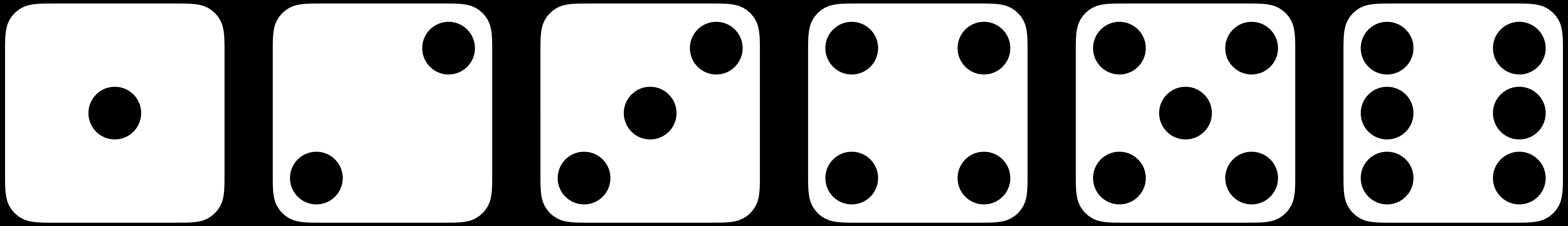
Uncertainty




NEXT 36 HOURS
HOURLY → | 10 DAYS →
TONIGHT
CLEAR
LOW
20°
0%
THU
HIGH
36°
0%
THU NIGHT
LOW
25°
0%
FRI
HIGH
46°
0%
FRI NIGHT
LOW
32°
20%



Probability

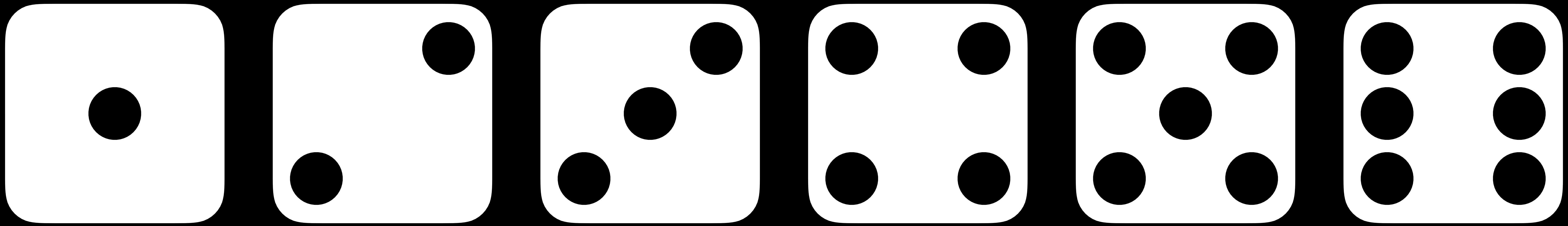
Possible Worlds

$$\omega$$

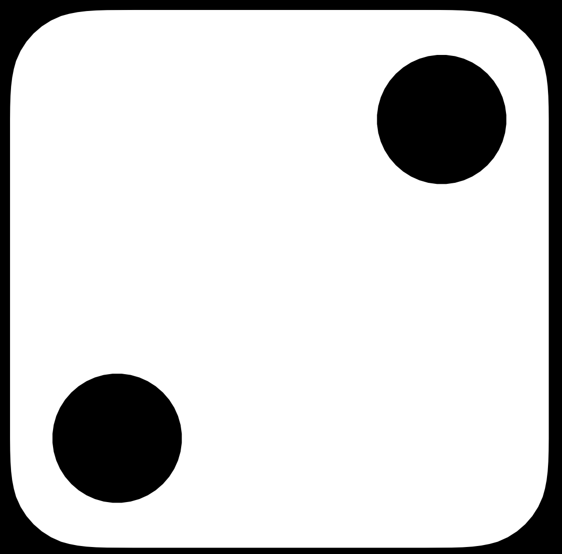
$$P(\omega)$$

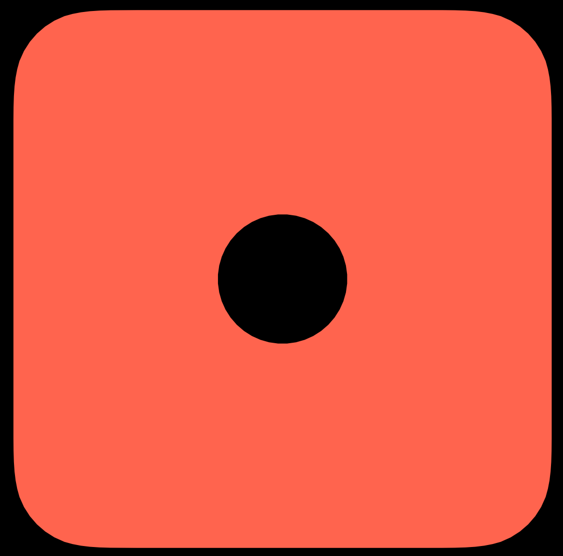
$$0 \leq P(\omega) \leq 1$$

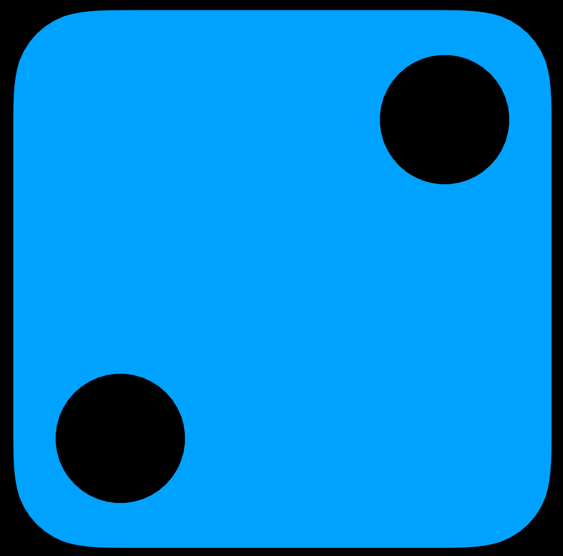
$$\sum_{\omega \in \Omega} P(\omega) = 1$$

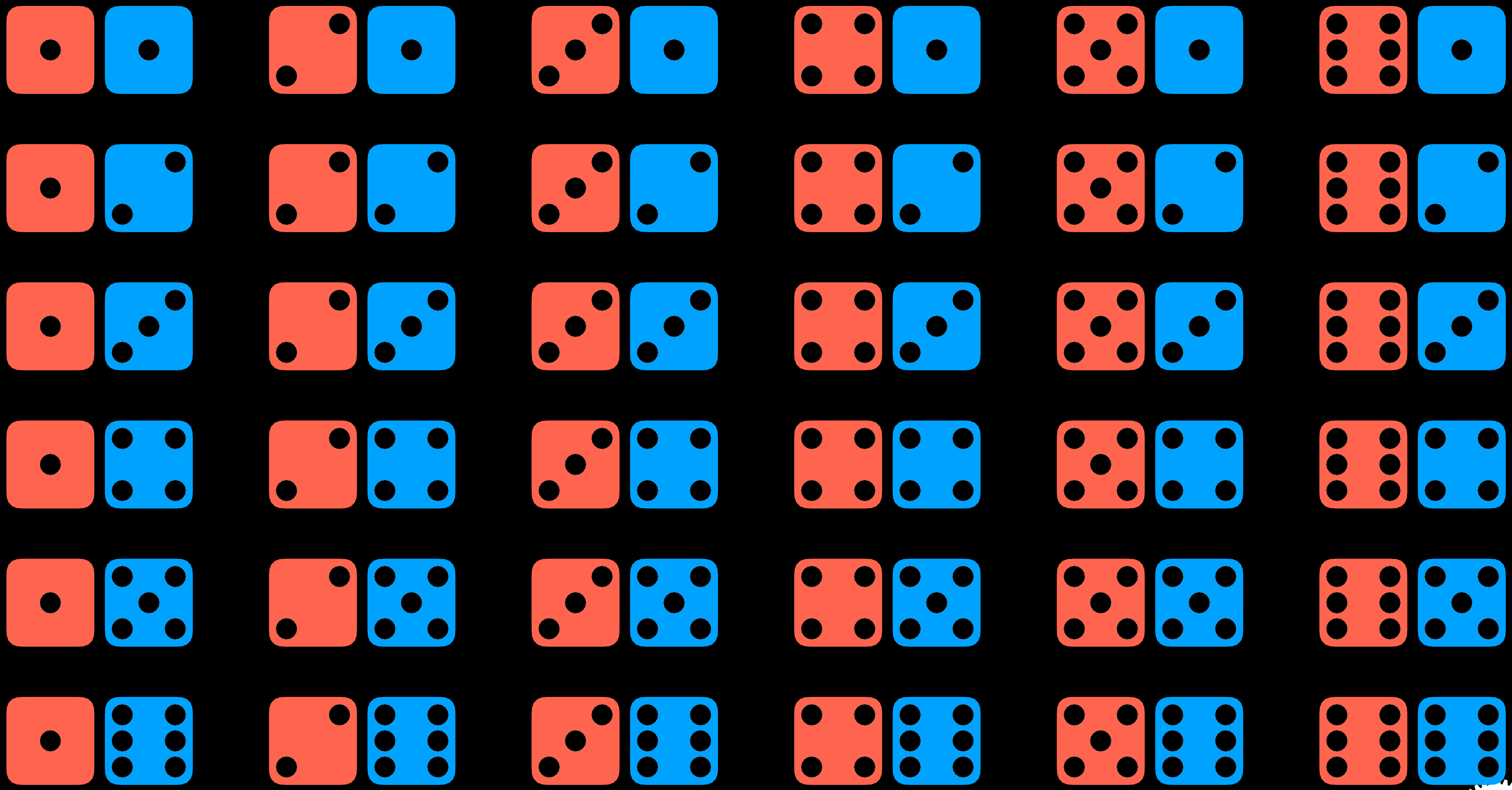


$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

$P($  $) = \dfrac{1}{6}$







2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	**12**

$$P(sum\ to\ 12) = \frac{1}{36}$$

$$P(sum\ to\ 7) = \frac{6}{36} = \frac{1}{6}$$

unconditional probability

degree of belief in a proposition
in the absence of any other evidence

conditional probability

degree of belief in a proposition given some evidence that has already been revealed conditional probability

$$P(a \mid b)$$

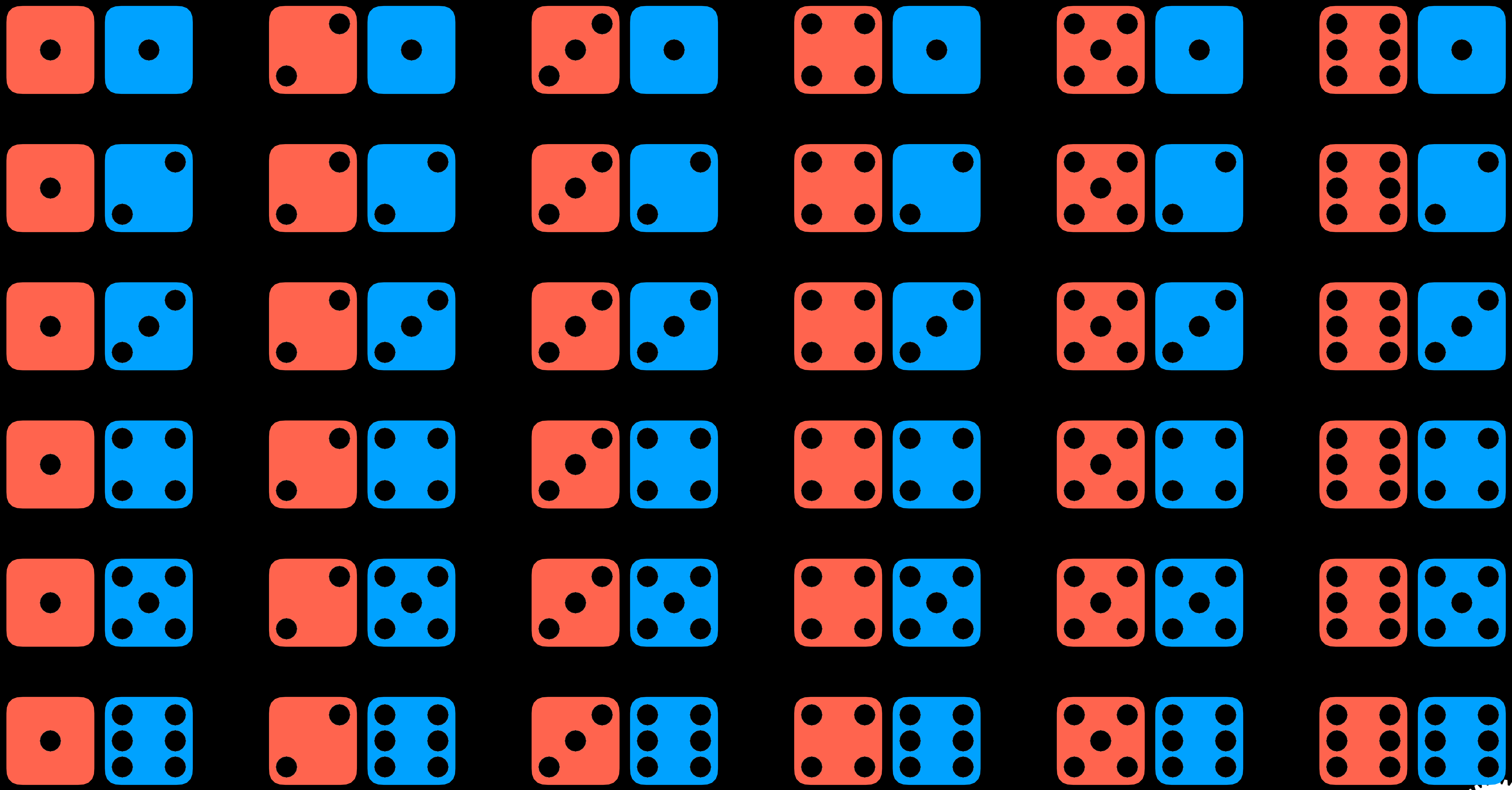
$$P(rain\ today \mid rain\ yesterday)$$

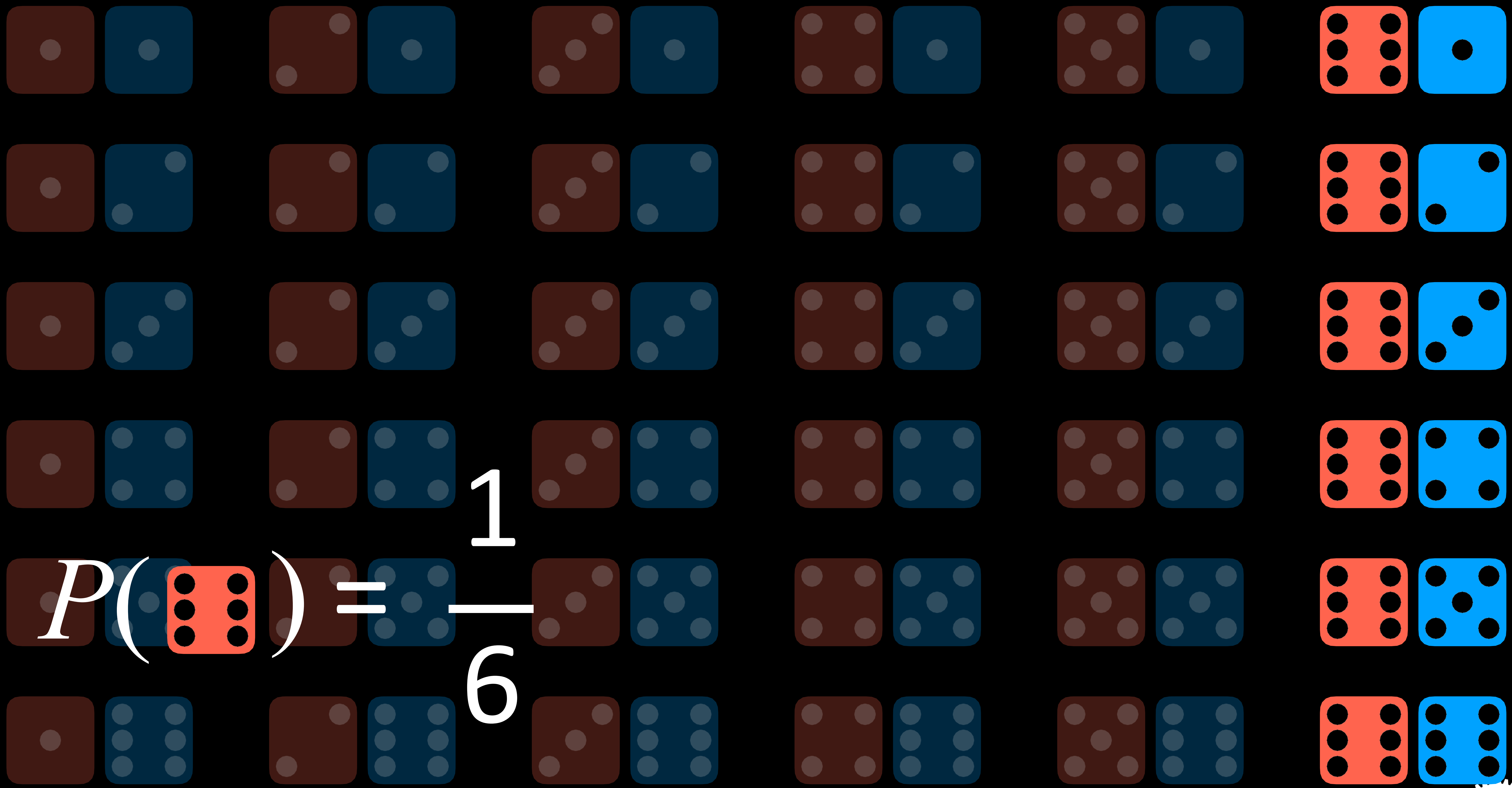
$$P(route\ change \mid traffic\ conditions)$$

$$P(disease \mid test\ results)$$

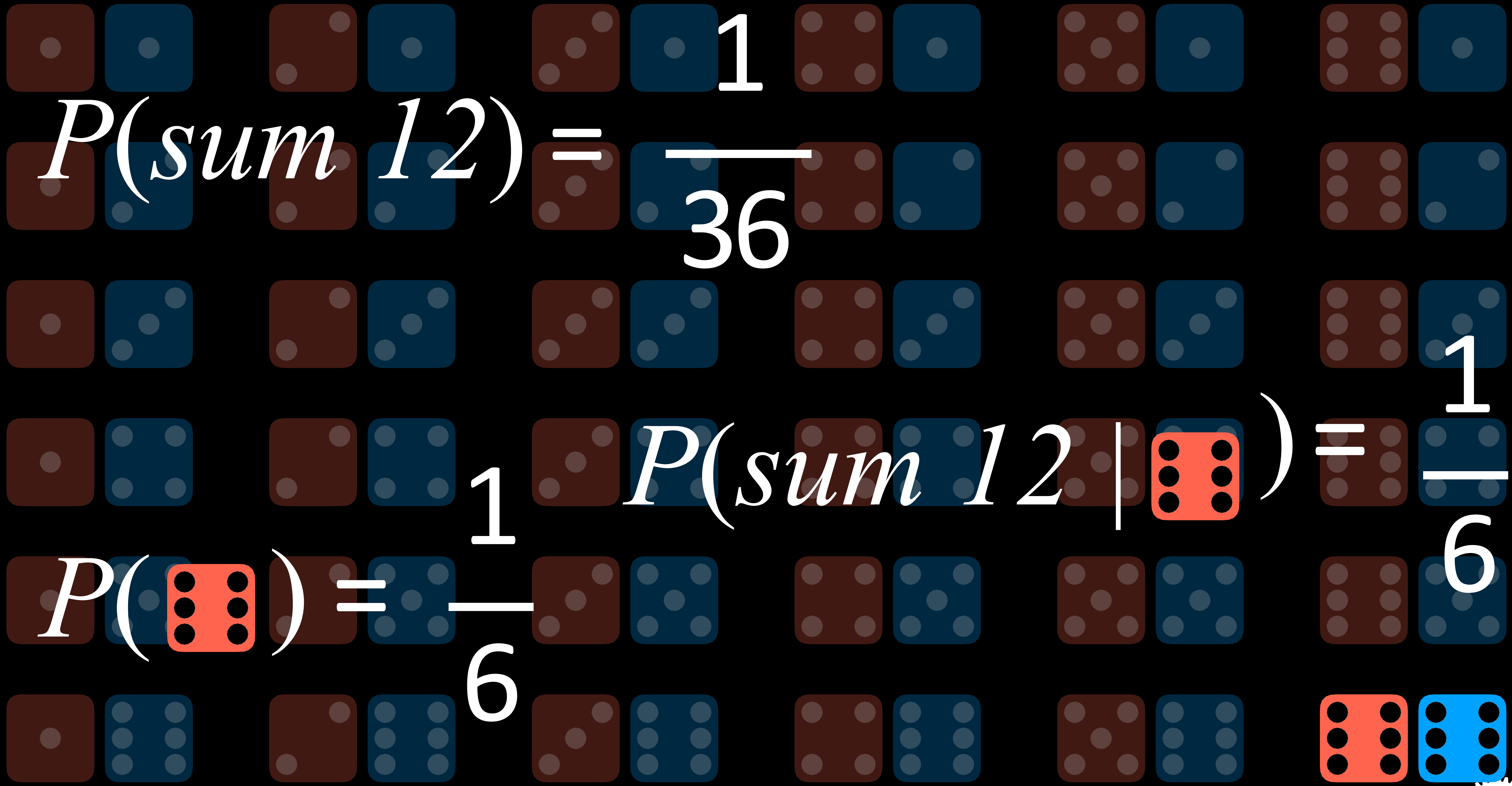
$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(sum\ 12 \mid \text{⚅})$$





$$P(⚅) = \frac{1}{6}$$



$$P(\text{sum } 12) = \frac{1}{36}$$

$$P(\text{sum } 12 \mid \quad) = \frac{1}{6}$$

$$P(\quad) = \frac{1}{6}$$

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(a \wedge b) = P(b)P(a \mid b)$$

$$P(a \wedge b) = P(a)P(b \mid a)$$

random variable

a variable in probability theory with a domain of possible values it can take on

random variable

$Roll$

$\{1, 2, 3, 4, 5, 6\}$

random variable

$$Weather$$

$$\{sun, cloud, rain, wind, snow\}$$

random variable

$Traffic$

$\{none, light, heavy\}$

random variable

$$Flight$$

$$\{on\ time,\ delayed,\ cancelled\}$$

probability distribution

$$P(Flight = on\ time) = 0.6$$
$$P(Flight = delayed) = 0.3$$
$$P(Flight = cancelled) = 0.1$$

probability distribution

$$\mathbf{P}(Flight) = \langle 0.6, 0.3, 0.1 \rangle$$

independence

the knowledge that one event occurs does not affect the probability of the other event independence

$$P(a \wedge b) = P(a)P(b \mid a)$$

independence

$$P(a \wedge b) = P(a)P(b)$$

independence

$$P(⚅\,⚅) = P(⚅)P(⚅)$$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

independence

$$P(\text{⚅⚃}) \neq P(\text{⚅})\,P(\text{⚃})$$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

independence

$$P(\text{⚅⚃}) = P(\text{⚅})P(\text{⚃} \mid \text{⚅})$$

$$= \frac{1}{6} \cdot 0 = 0$$

Bayes' Rule

$$P(a \wedge b) = P(b)\ P(a \mid b)$$

$$P(a \wedge b) \qquad P(a)\ P(b \mid a)$$

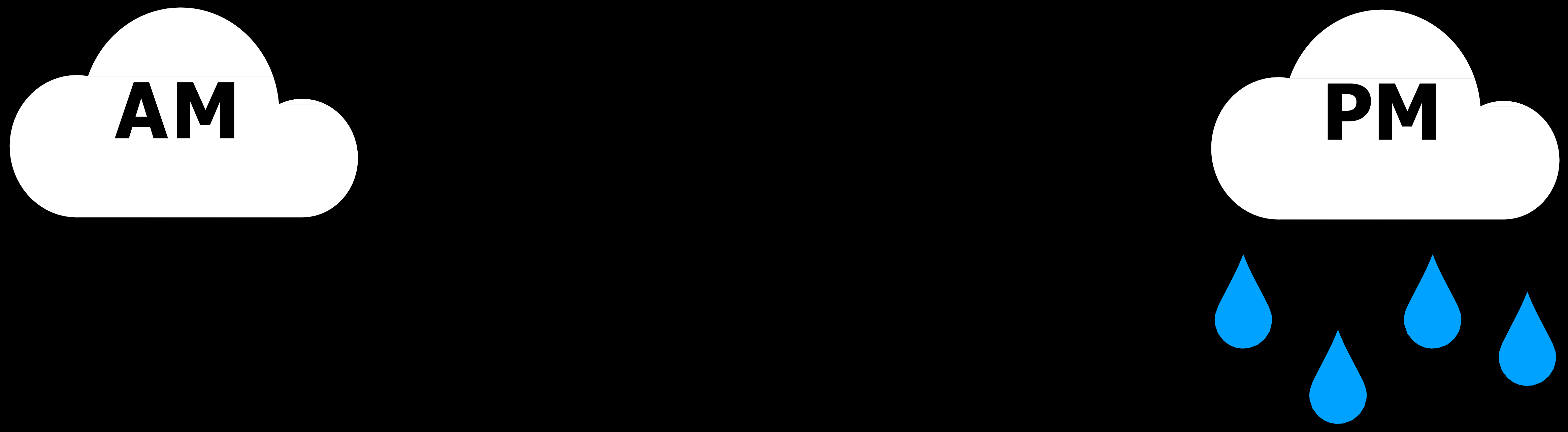
$$P(a)\;P(b \mid a) = P(b)\;P(a \mid b)$$

Bayes' Rule

$$P(b \mid a) = \frac{P(b)\ P(a \mid b)}{P(a)}$$

Bayes' Rule

$$P(b \mid a) = \frac{P(a \mid b)\, P(b)}{P(a)}$$



Given clouds in the morning,
what's the probability of rain in the afternoon?

- 80% of rainy afternoons start with cloudy mornings.
- 40% of days have cloudy mornings.
- 10% of days have rainy afternoons.

$$P(rain \mid clouds) = \frac{P(clouds \mid rain)P(rain)}{P(clouds)}$$

$$= \frac{(.8)(.1)}{.4}$$

$$= 0.2$$

Knowing

$$P(cloudy\ morning \mid rainy\ afternoon)$$

we can calculate

$$P(rainy\ afternoon \mid cloudy\ morning)$$

Knowing

$$P(visible\ effect \mid unknown\ cause)$$

we can calculate

$$P(unknown\ cause \mid visible\ effect)$$

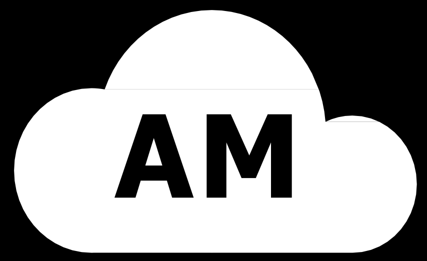
Knowing

$$P(\textit{medical test result} \mid \textit{disease})$$

we can calculate

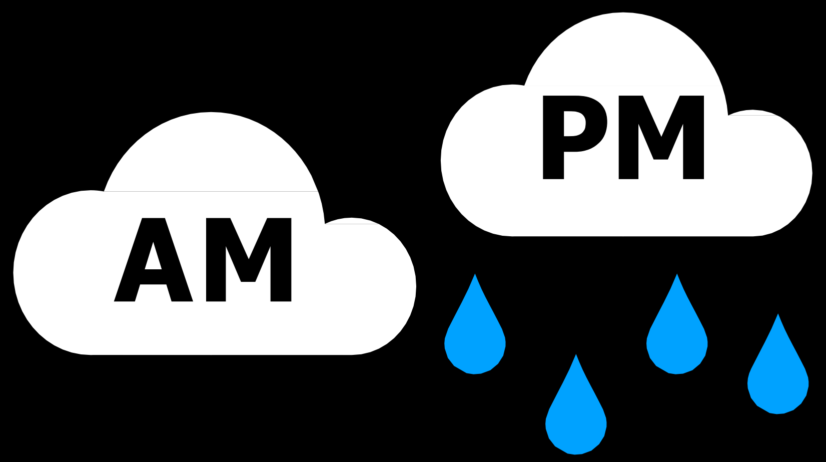
$$P(\textit{disease} \mid \textit{medical test result})$$

Joint Probability



$C = cloud$	$C = \neg cloud$
0.4	0.6

$R = rain$	$R = \neg rain$
0.1	0.9



	$R = rain$	$R = \neg rain$
$C = cloud$	0.08	0.32
$C = \neg cloud$	0.02	0.58

$\mathbf{P}(C \mid \mathit{rain})$

$$\mathbf{P}(C \mid \mathit{rain}) = \frac{\mathbf{P}(C, \mathit{rain})}{P(\mathit{rain})} = \alpha\mathbf{P}(C, \mathit{rain})$$

$$= \alpha\langle 0.08, 0.02\rangle = \langle 0.8, 0.2\rangle$$

	$R = \mathit{rain}$	$R = \neg\mathit{rain}$
$C = \mathit{cloud}$	0.08	0.32
$C = \neg\mathit{cloud}$	0.02	0.58

Probability Rules

Negation

$$P(\neg a) = 1 - P(a)$$

Inclusion-Exclusion

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

Marginalization

$$P(a) = P(a, b) + P(a, \neg b)$$

Marginalization

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

Marginalization

	$R = rain$	$R = \neg rain$
$C = cloud$	0.08	0.32
$C = \neg cloud$	0.02	0.58

$$P(C = cloud)$$
$$= P(C = cloud, R = rain) + P(C = cloud, R = \neg rain)$$
$$= 0.08 + 0.32$$
$$= 0.40$$

Conditioning

$$P(a) = P(a \mid b)P(b) + P(a \mid \neg b)P(\neg b)$$

Conditioning

$$P(X = x_i) = \sum_j P(X = x_i \mid Y = y_j)P(Y = y_j)$$

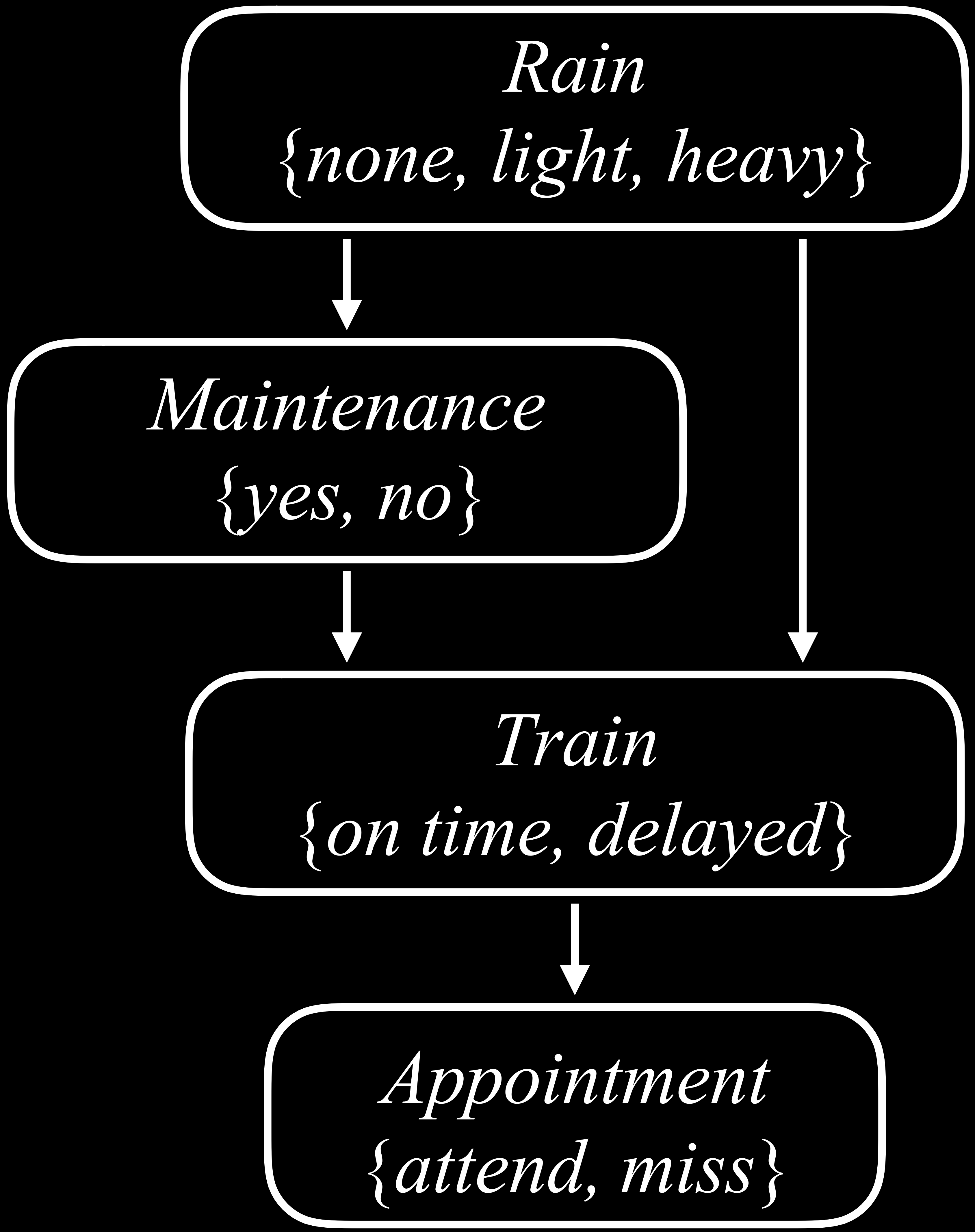
Bayesian Networks

Bayesian network

data structure that represents the dependencies among random variables

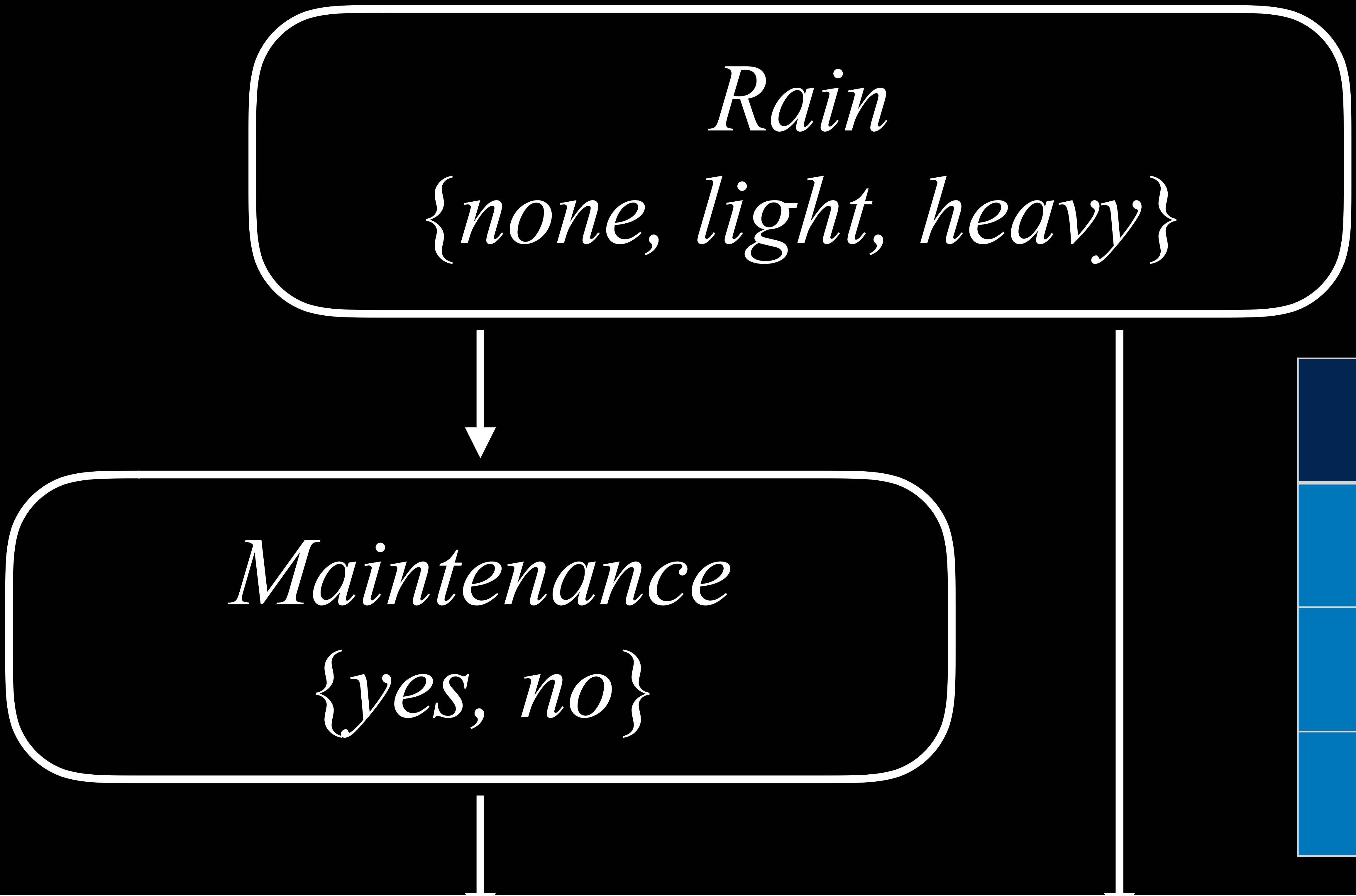
Bayesian network

- directed graph
- each node represents a random variable
- arrow from X to Y means X is a parent of Y
- each node X has conditional probability distribution $\mathbf{P}(X \mid Parents(X))$

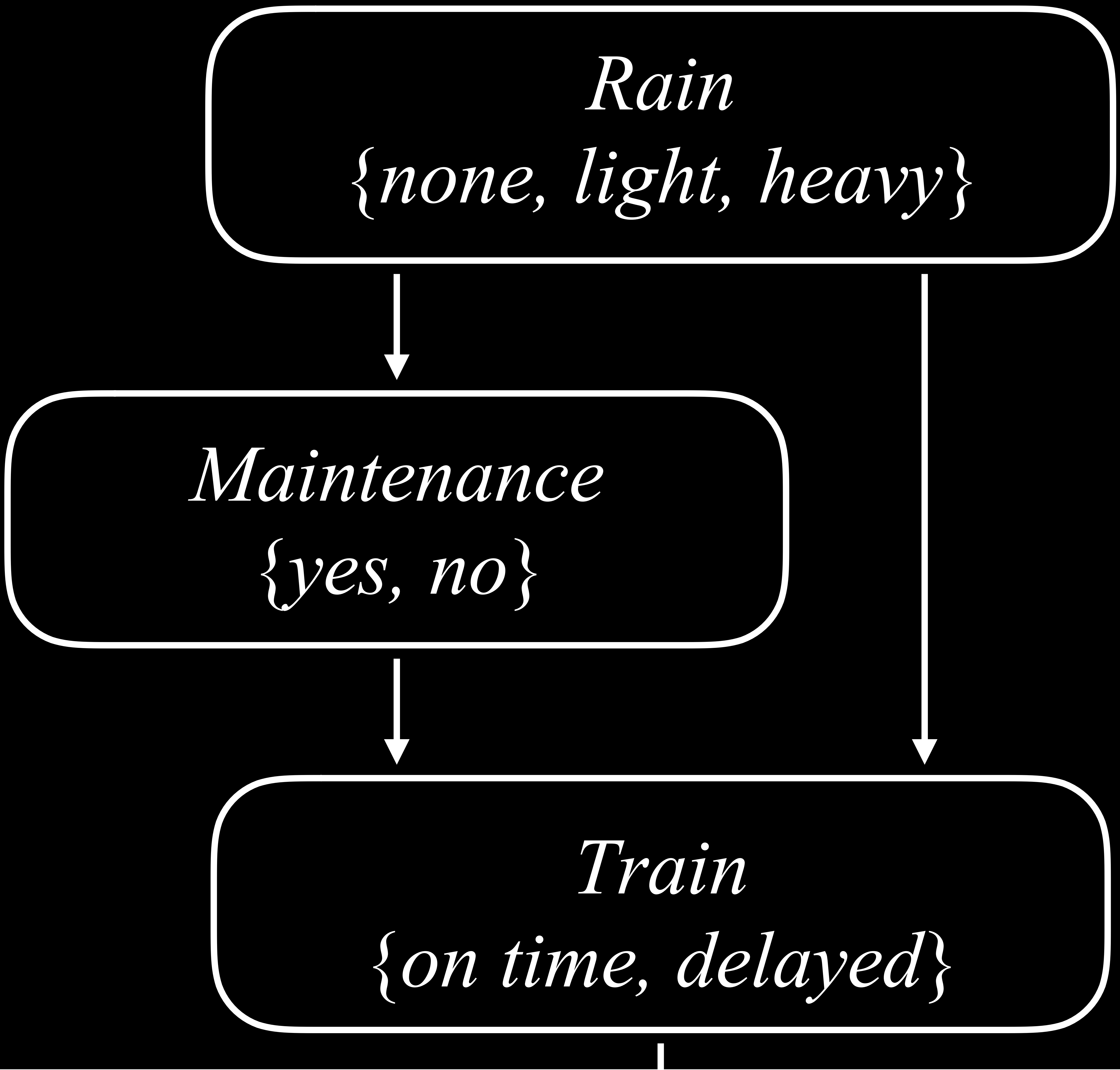

Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment
{attend, miss}

Rain
{none, light, heavy}

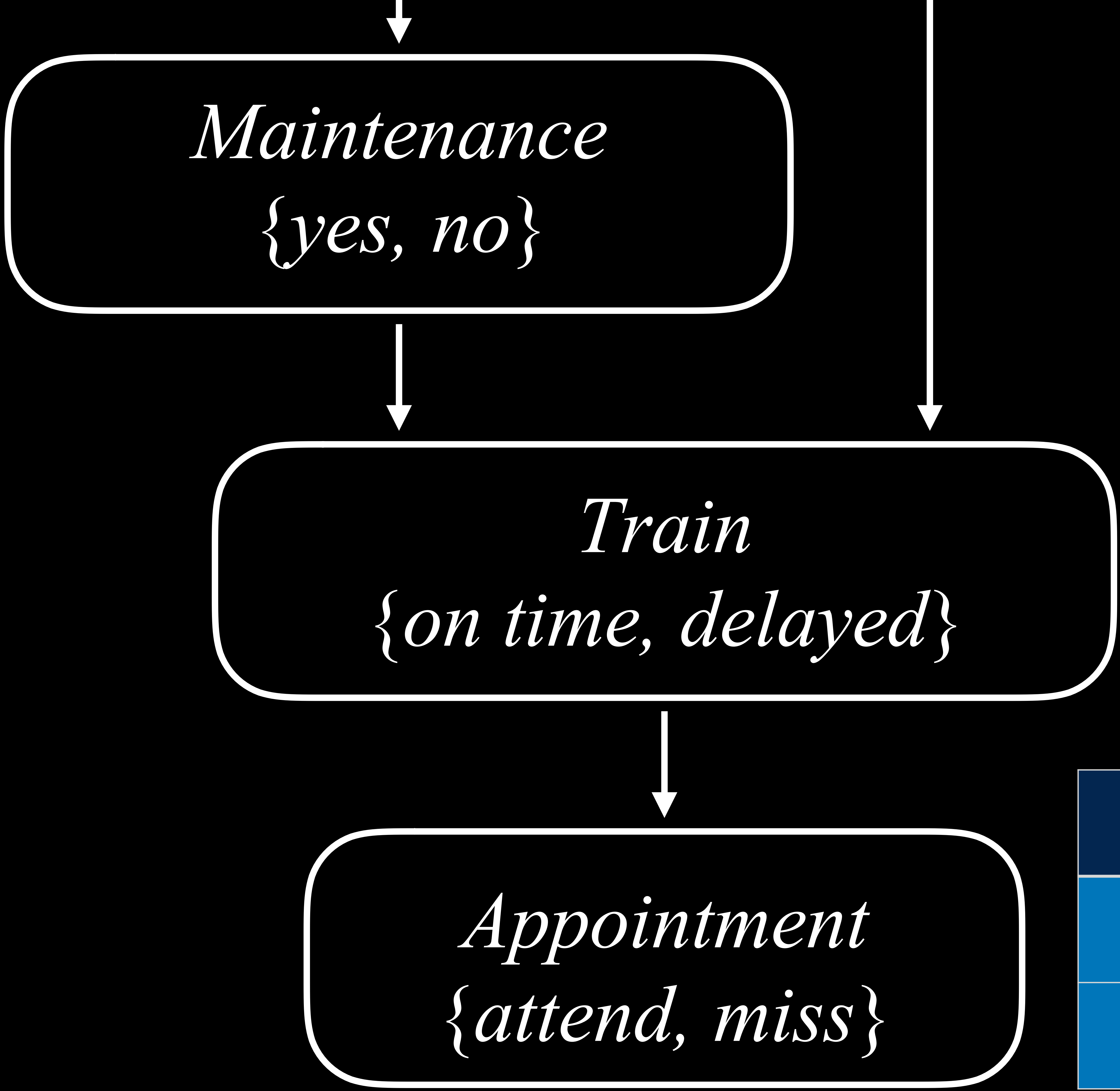
none	*light*	*heavy*
0.7	0.2	0.1



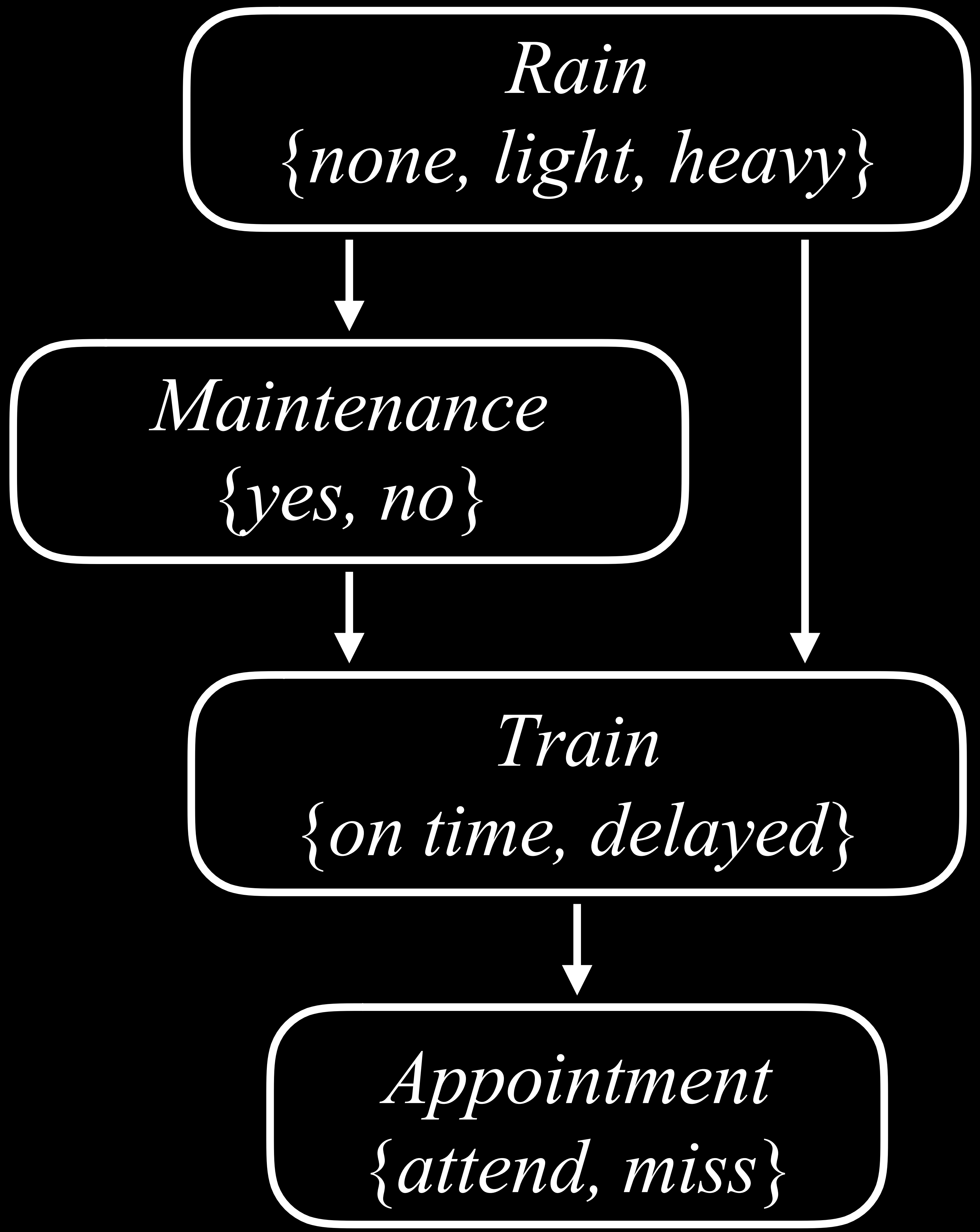
R	*yes*	*no*
none	0.4	0.6
light	0.2	0.8
heavy	0.1	0.9



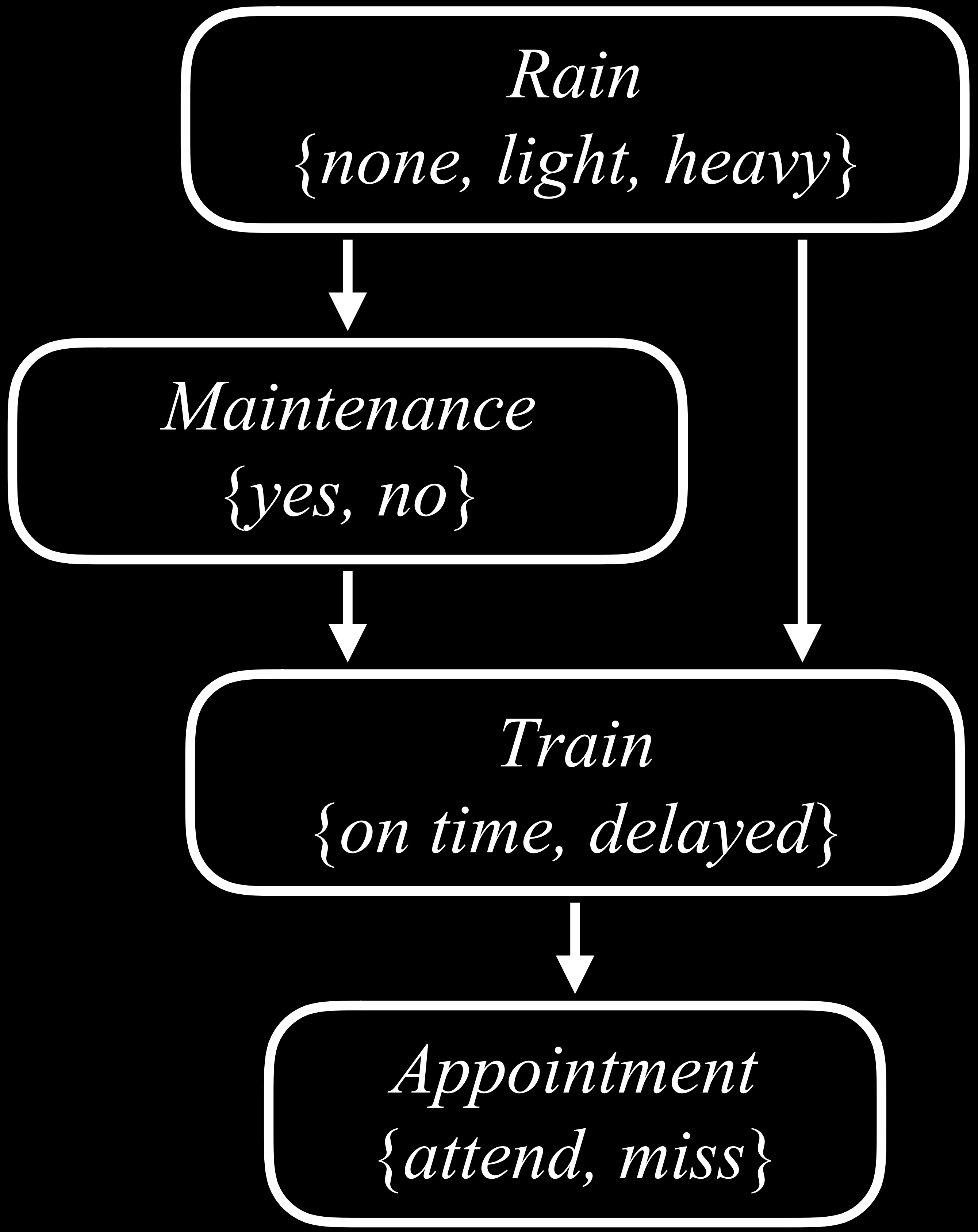
R	***M***	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6
heavy	*no*	0.5	0.5



T	*attend*	*miss*
on time	0.9	0.1
delayed	0.6	0.4


Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment
{attend, miss}

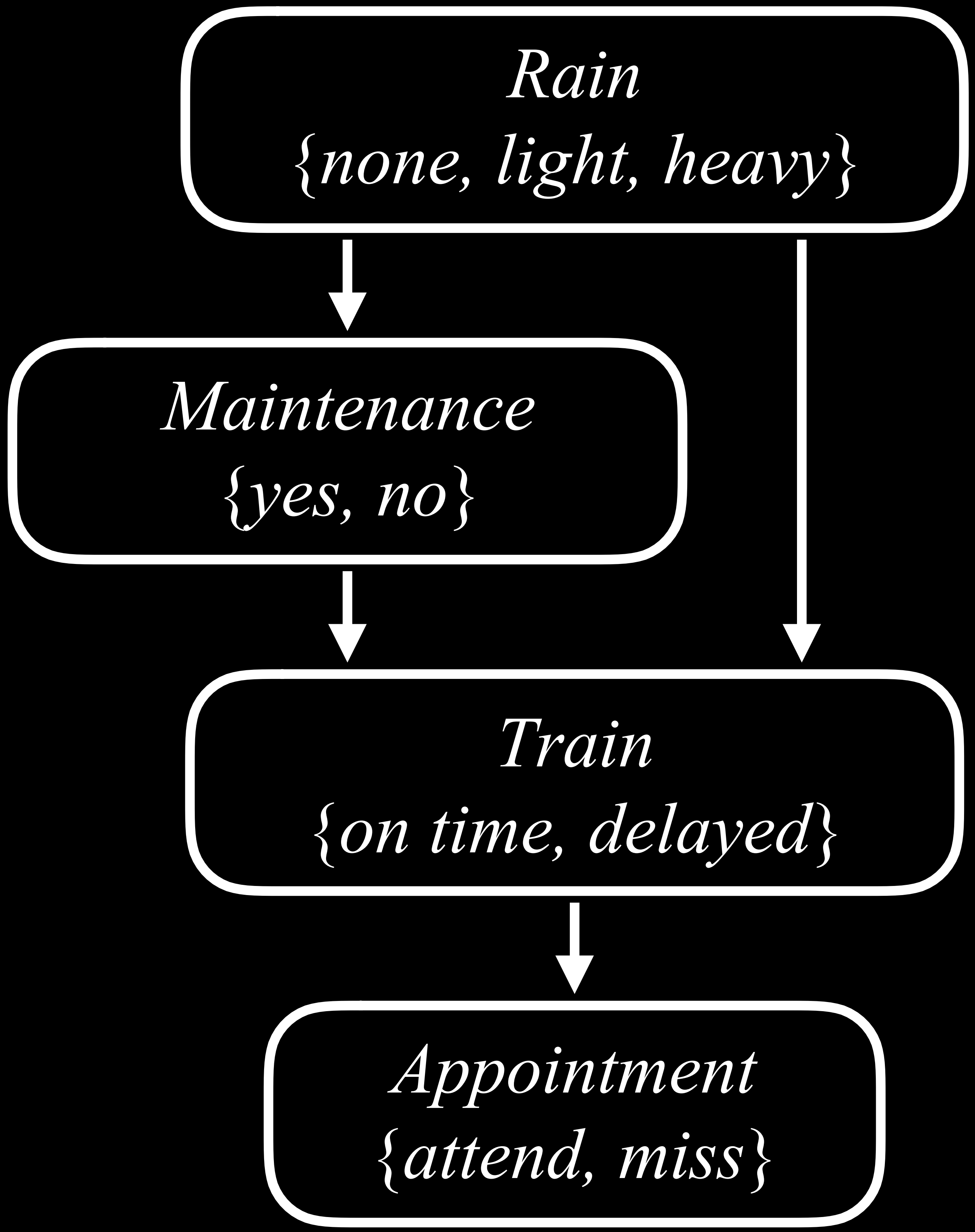
Computing Joint Probabilities



$$P(light)$$

$$P(light)$$

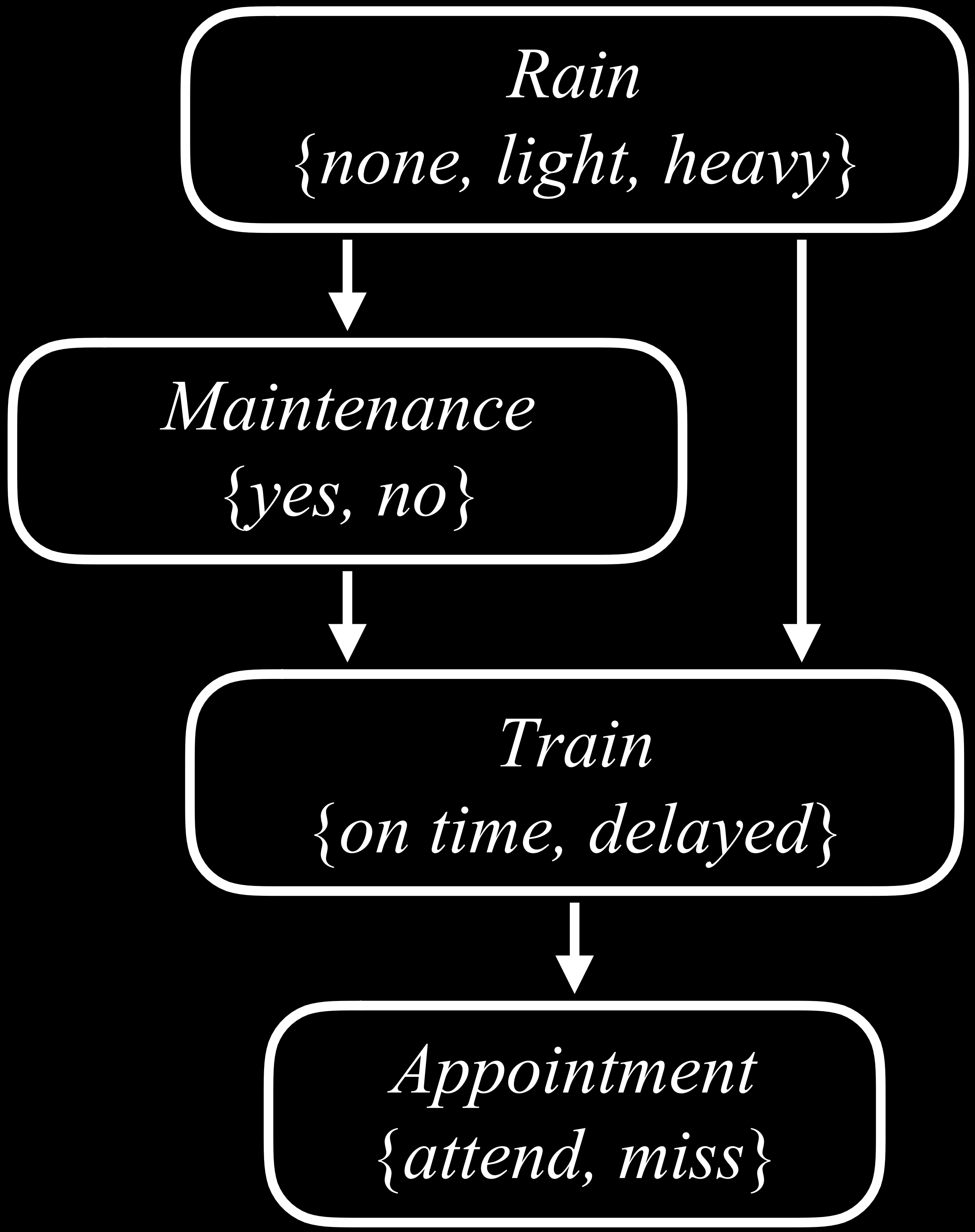
Computing Joint Probabilities



$$P(light, no)$$

$$P(light)\, P(no \mid light)$$

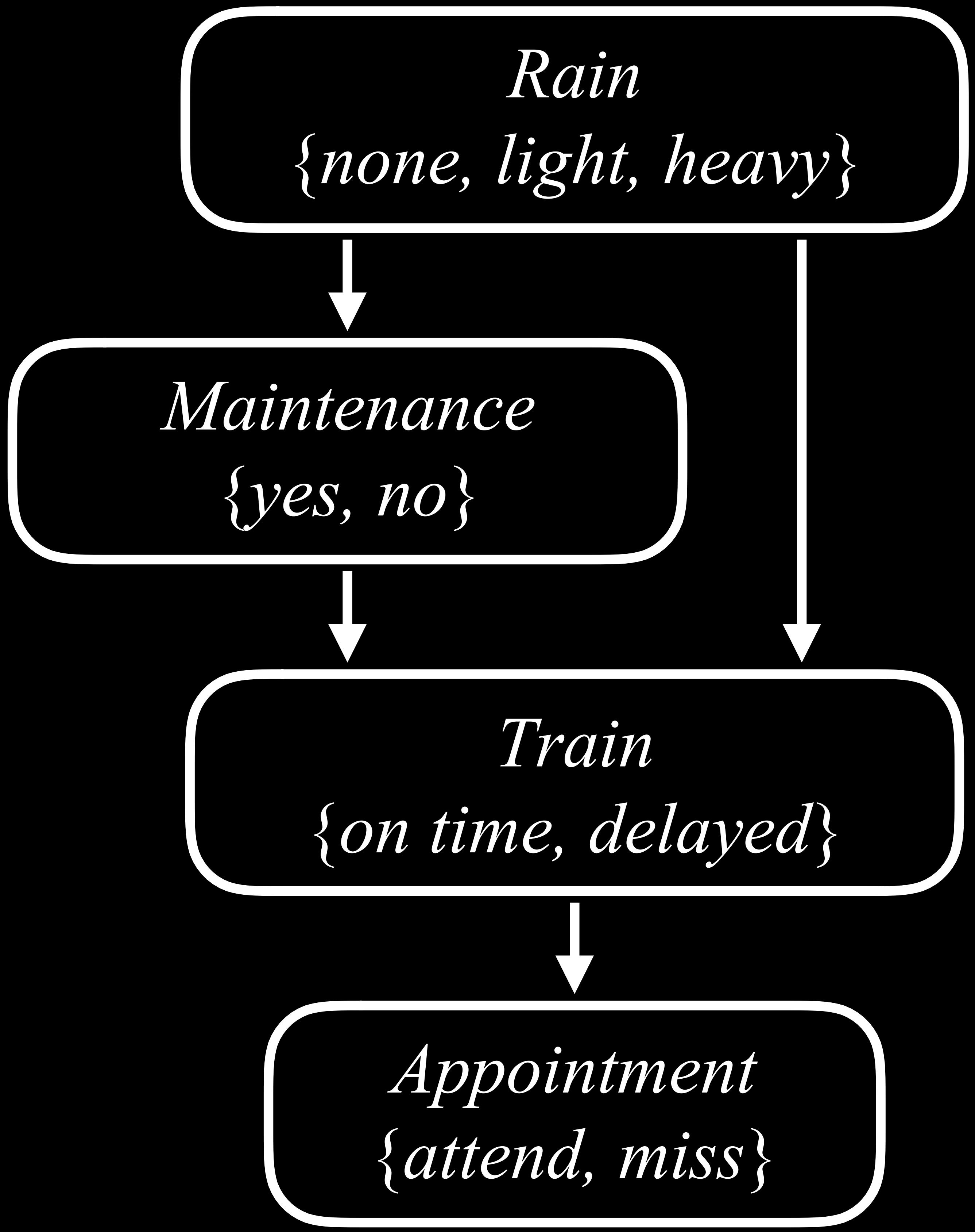
Computing Joint Probabilities



$$P(light, no, delayed)$$

$$P(light)\ P(no \mid light)\ P(delayed \mid light, no)$$

Computing Joint Probabilities



$$P(light, no, delayed, miss)$$

$$P(light)\ P(no \mid light)\ P(delayed \mid light, no)\ P(miss \mid delayed)$$

Inference

Inference

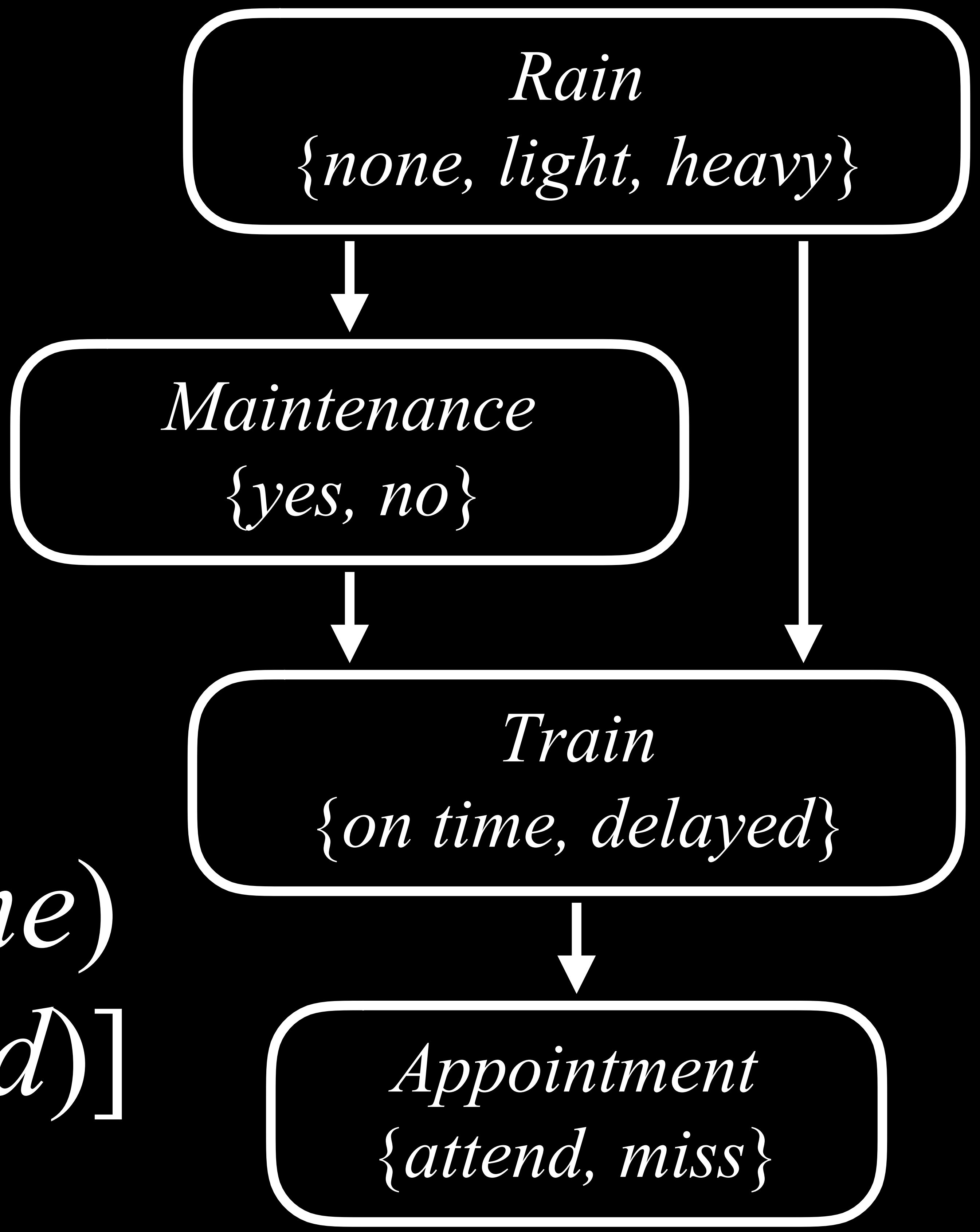
- Query $\mathbf{X}$: variable for which to compute distribution
- Evidence variables $\mathbf{E}$: observed variables for event $\mathbf{e}$
- Hidden variables $\mathbf{Y}$: non-evidence, non-query variable.

- Goal: Calculate $\mathbf{P}(X \mid \mathbf{e})$

P(Appointment | *light, no*)

= α P(Appointment, *light, no*)

= α [P(Appointment, *light, no, on time*)
+ P(Appointment, *light, no, delayed*)]



Inference by Enumeration

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

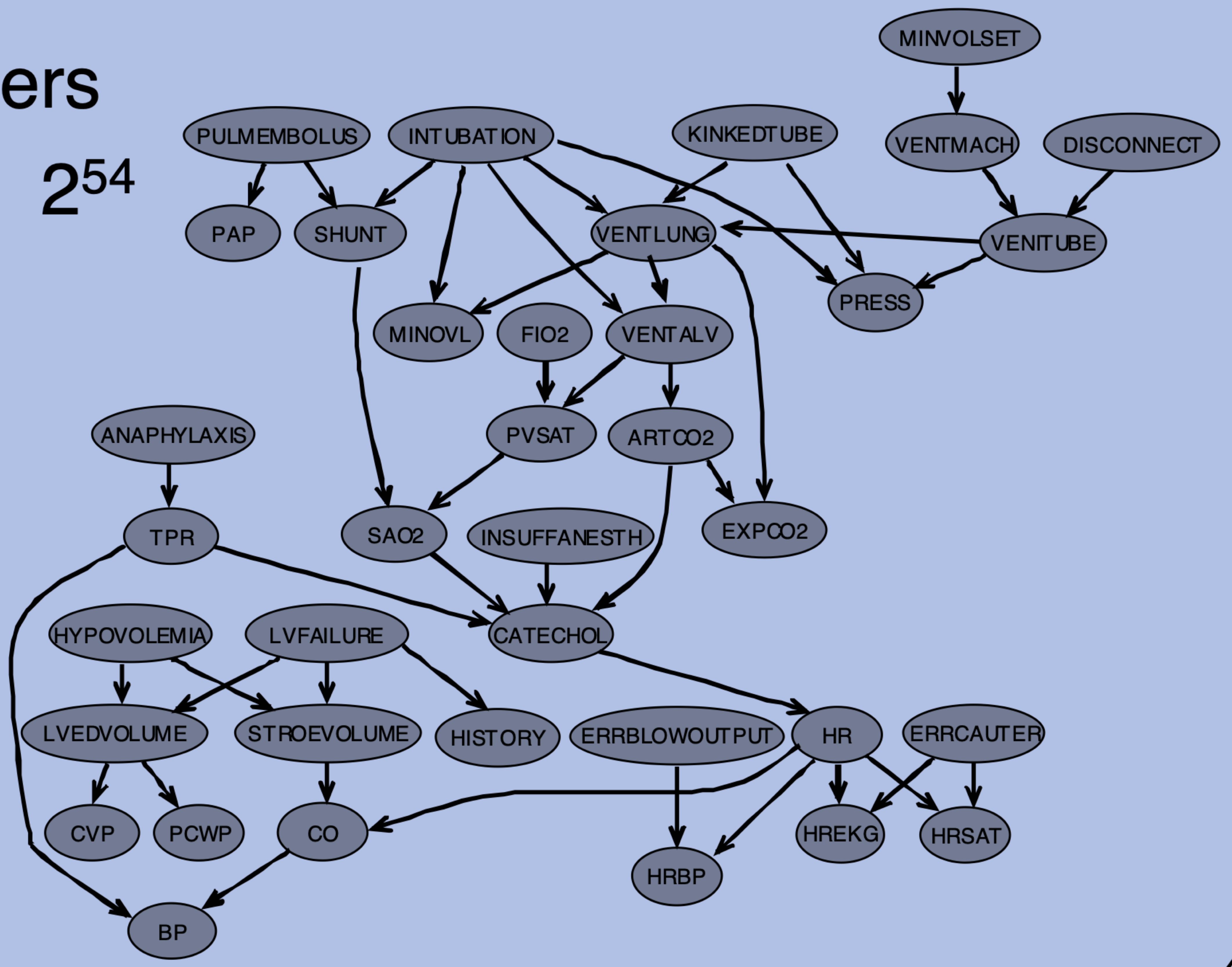
X is the query variable.
$\mathbf{e}$ is the evidence.
$\mathbf{y}$ ranges over values of hidden variables.
α normalizes the result.

Example: “ICU Alarm” network

Domain: Monitoring Intensive-Care Patients

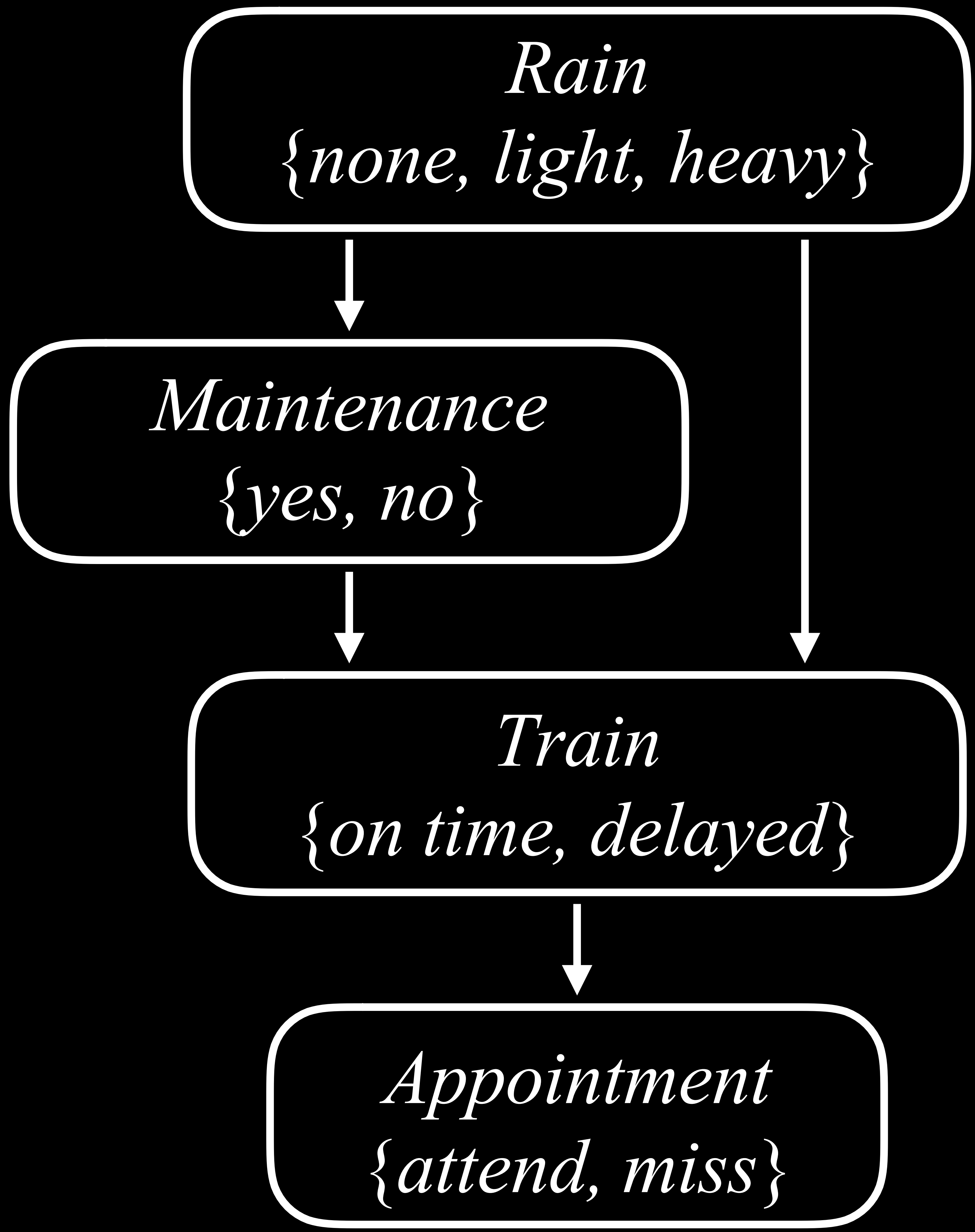
- 37 variables
- 509 parameters

  …instead of 2^{54}



Approximate Inference

Sampling

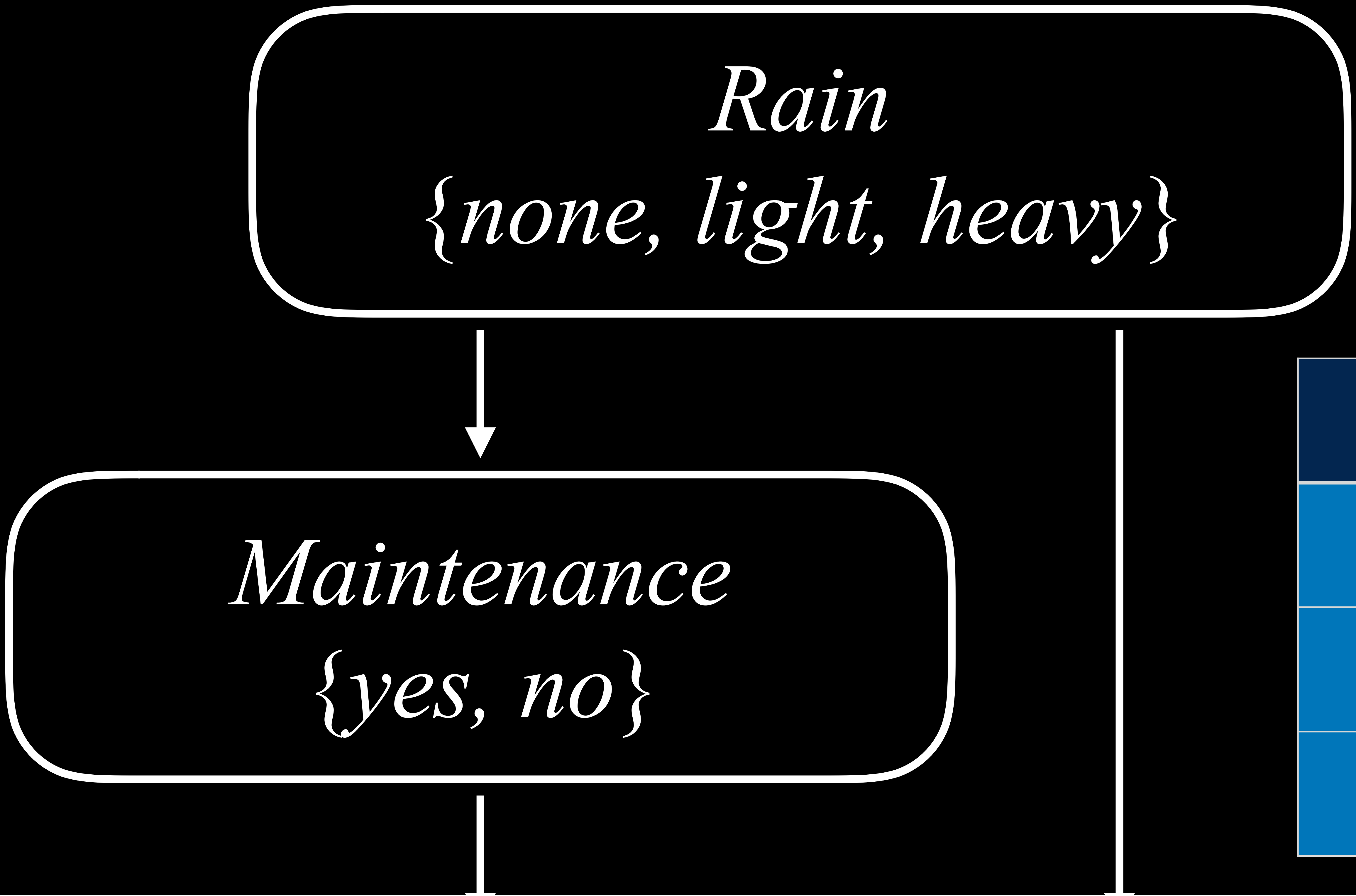

Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment
{attend, miss}

R = *none*

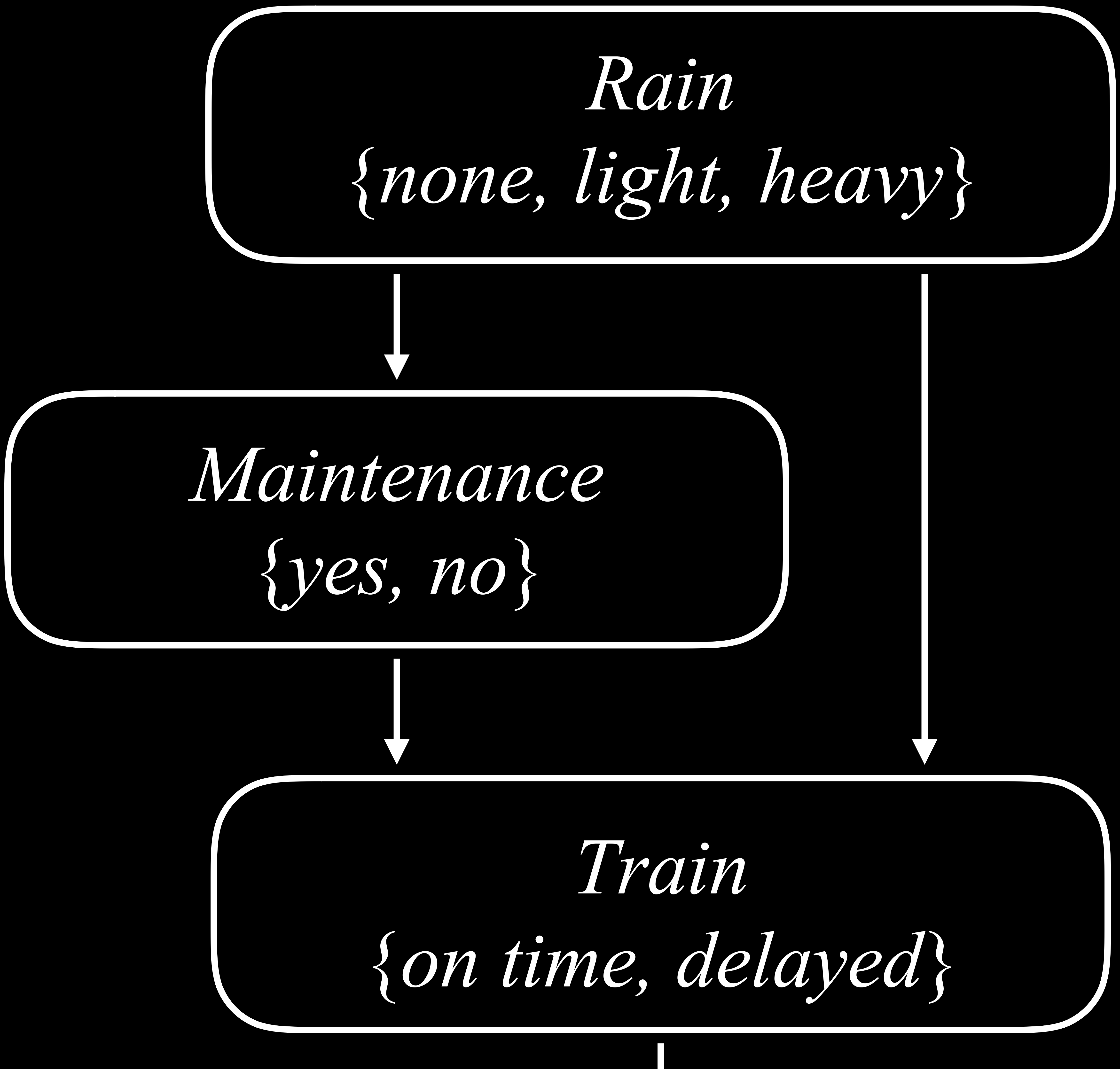
Rain
{*none*, *light*, *heavy*}

none	*light*	*heavy*
0.7	0.2	0.1

R = *none*
M = *yes*

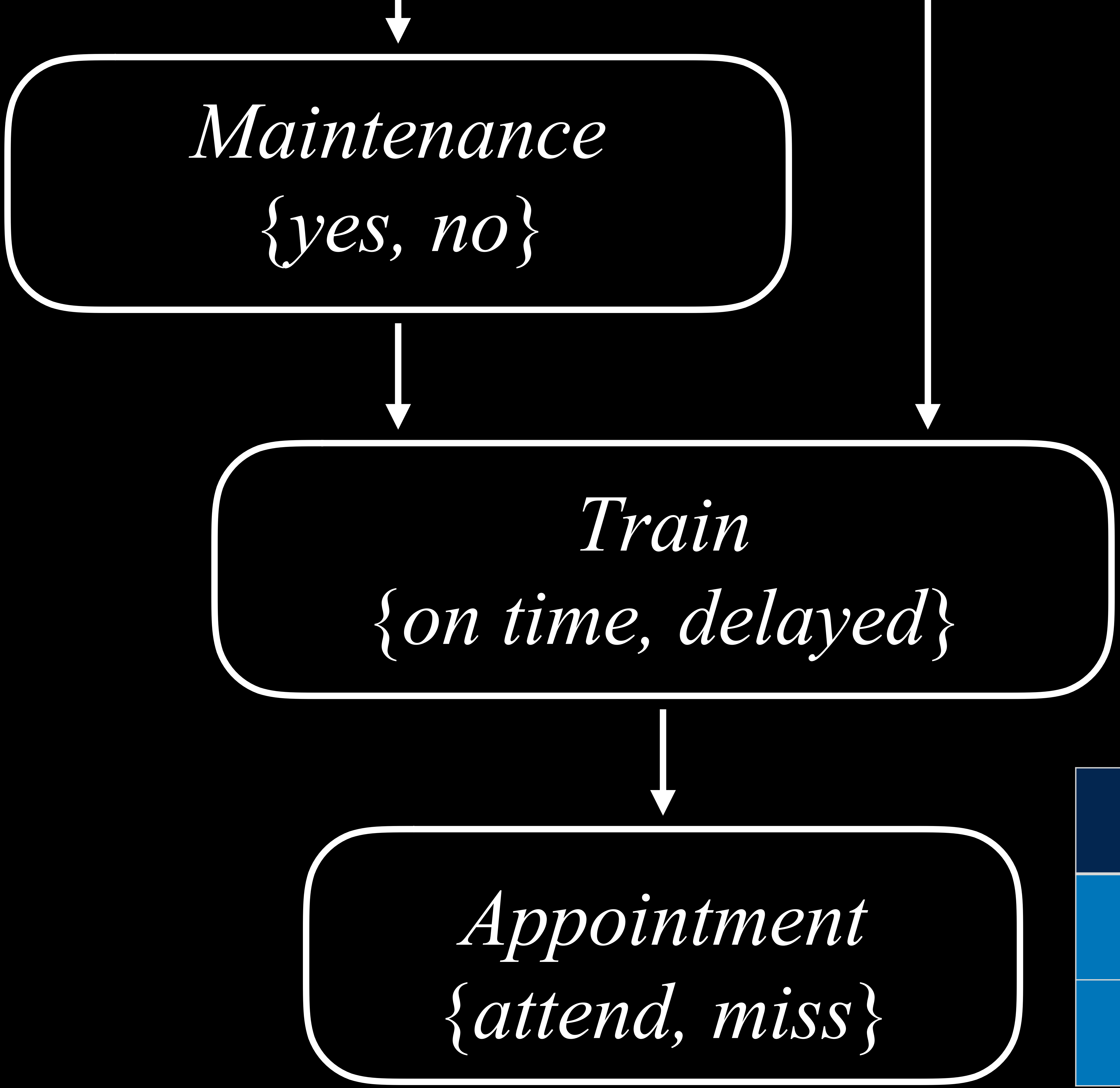


R	*yes*	*no*
none	0.4	0.6
light	0.2	0.8
heavy	0.1	0.9



R = *none*
M = *yes*
T = *on time*

R	*M*	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6
heavy	*no*	0.5	0.5



R = *none*
M = *yes*
T = *on time*
A = *attend*

T	*attend*	*miss*
on time	0.9	0.1
delayed	0.6	0.4

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *light*
M = *no*
T = *on time*
A = *miss*

R = *light*
M = *yes*
T = *delayed*
A = *attend*

R = *none*
M = *no*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *heavy*
M = *no*
T = *delayed*
A = *miss*

R = *light*
M = *no*
T = *on time*
A = *attend*

$$\mathrm{P}(\mathit{Train} = \mathit{on\ time})\ ?$$

R = *light*
M = *no*
T = *on time*
A = *miss*

R = *light*
M = *yes*
T = *delayed*
A = *attend*

R = *none*
M = *no*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *heavy*
M = *no*
T = *delayed*
A = *miss*

R = *light*
M = *no*
T = *on time*
A = *attend*

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

$$P(\text{Rain} = \textit{light} \mid \text{Train} = \textit{on time})\ ?$$

R = *light*
M = *no*
T = *on time*
A = *miss*

R = *light*
M = *yes*
T = *delayed*
A = *attend*

R = *none*
M = *no*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *none*
M = *yes*
T = *on time*
A = *attend*

R = *heavy*
M = *no*
T = *delayed*
A = *miss*

R = *light*
M = *no*
T = *on time*
A = *attend*

R = *light*	R = *light*	R = *none*	R = *none*
M = *no*	M = *yes*	M = *no*	M = *yes*
T = *on time*	T = *delayed*	T = *on time*	T = *on time*
A = *miss*	A = *attend*	A = *attend*	A = *attend*

R = *none*	R = *none*	R = *heavy*	R = *light*
M = *yes*	M = *yes*	M = *no*	M = *no*
T = *on time*	T = *on time*	T = *delayed*	T = *on time*
A = *attend*	A = *attend*	A = *miss*	A = *attend*

R = *light* M = *no* T = *on time* A = *miss*	R = *light* M = *yes* T = *delayed* A = *attend*	R = *none* M = *no* T = *on time* A = *attend*	R = *none* M = *yes* T = *on time* A = *attend*
R = *none* M = *yes* T = *on time* A = *attend*	R = *none* M = *yes* T = *on time* A = *attend*	R = *heavy* M = *no* T = *delayed* A = *miss*	R = *light* M = *no* T = *on time* A = *attend*

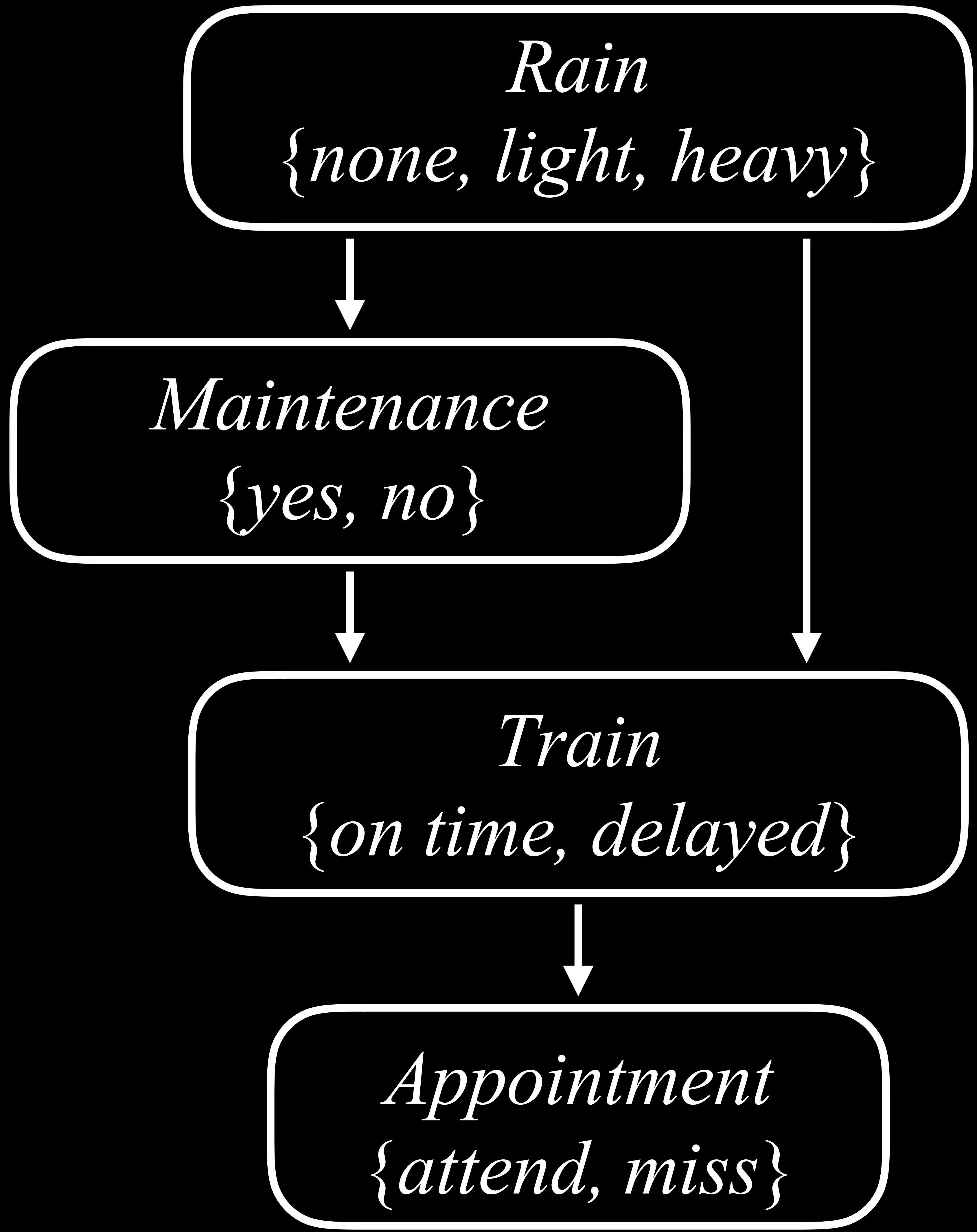
Rejection Sampling

Likelihood Weighting

Likelihood Weighting

- Start by fixing the values for evidence variables.
- Sample the non-evidence variables using conditional probabilities in the Bayesian Network.
- Weight each sample by its likelihood: the probability of all of the evidence.

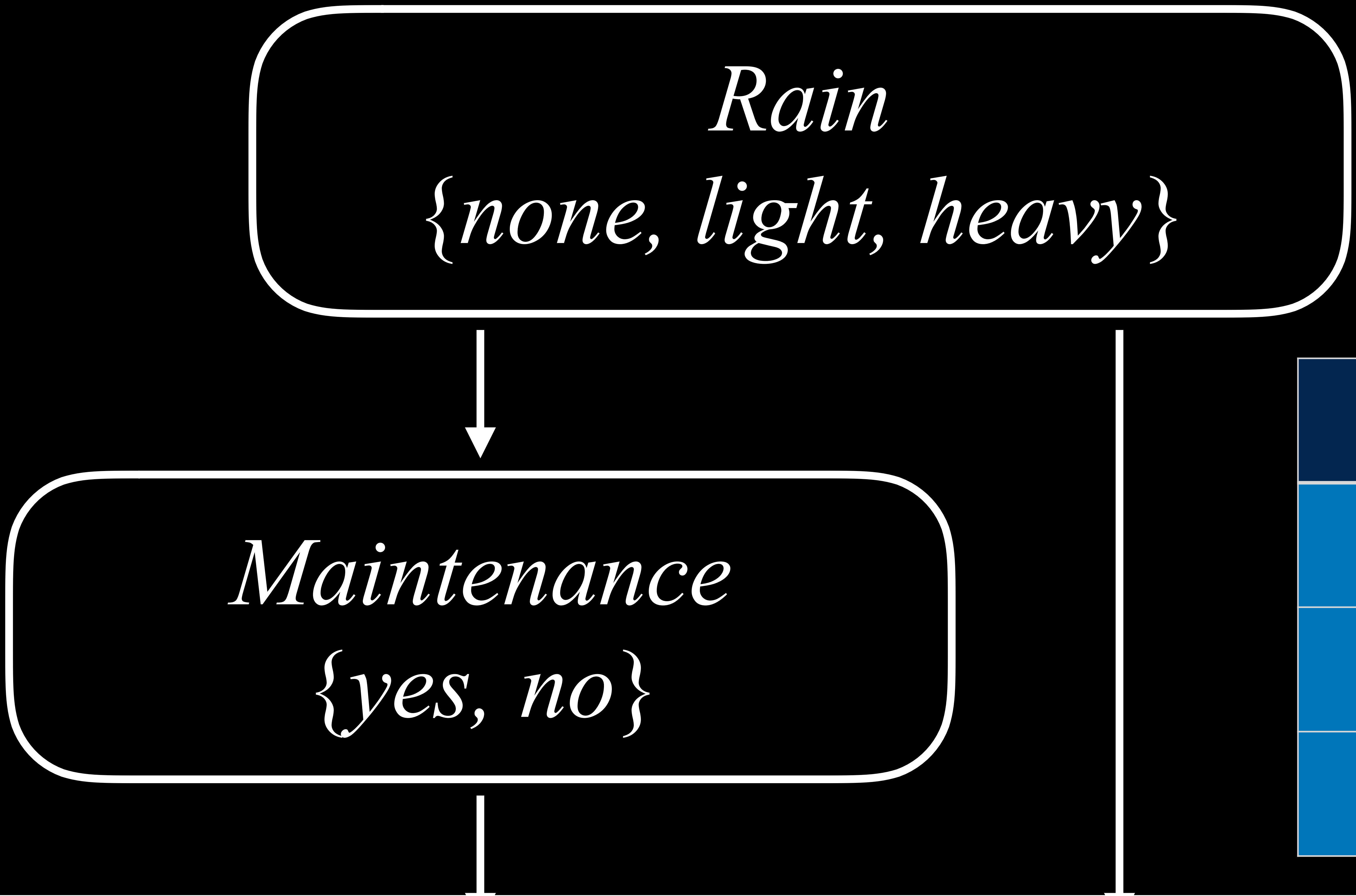
$$P(\text{Rain} = \textit{light} \mid \text{Train} = \textit{on time})\ ?$$


Rain
{none, light, heavy}
Maintenance
{yes, no}
Train
{on time, delayed}
Appointment
{attend, miss}

R = *light*
T = *on time*

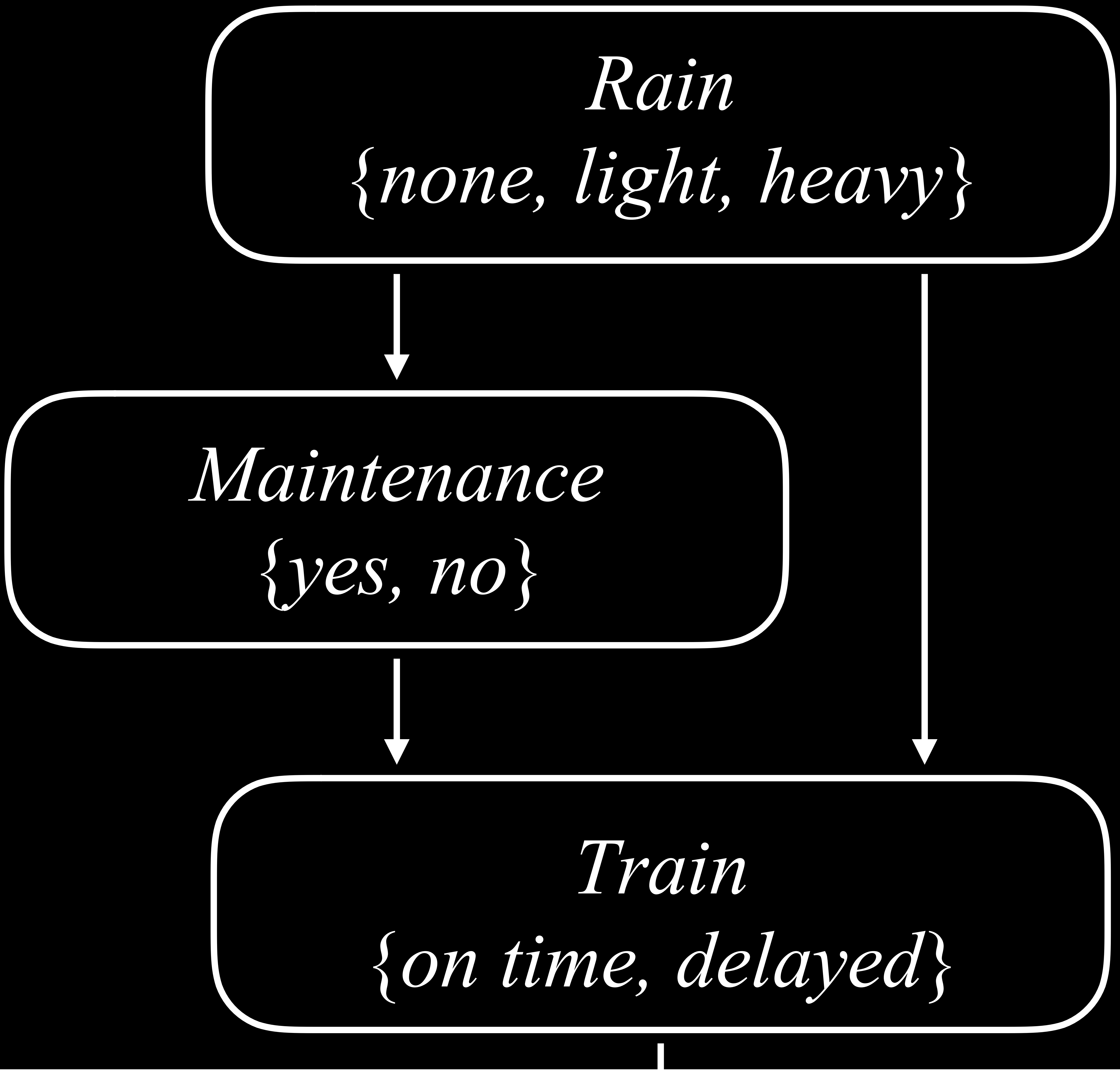
Rain
{*none, light, heavy*}

none	*light*	*heavy*
0.7	0.2	0.1



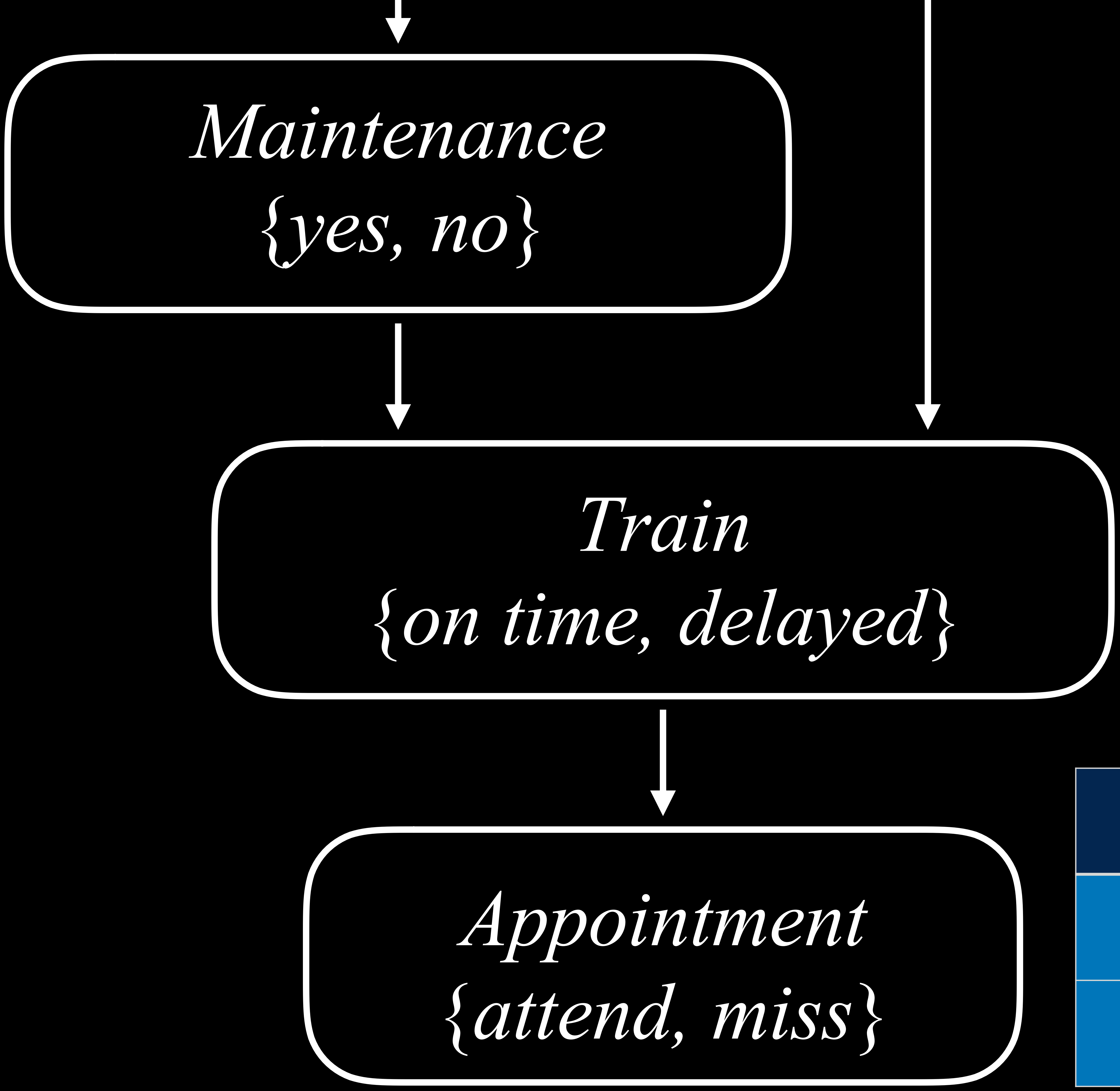
R = *light*
M = *yes*
T = *on time*

R	*yes*	*no*
none	0.4	0.6
light	0.2	0.8
heavy	0.1	0.9



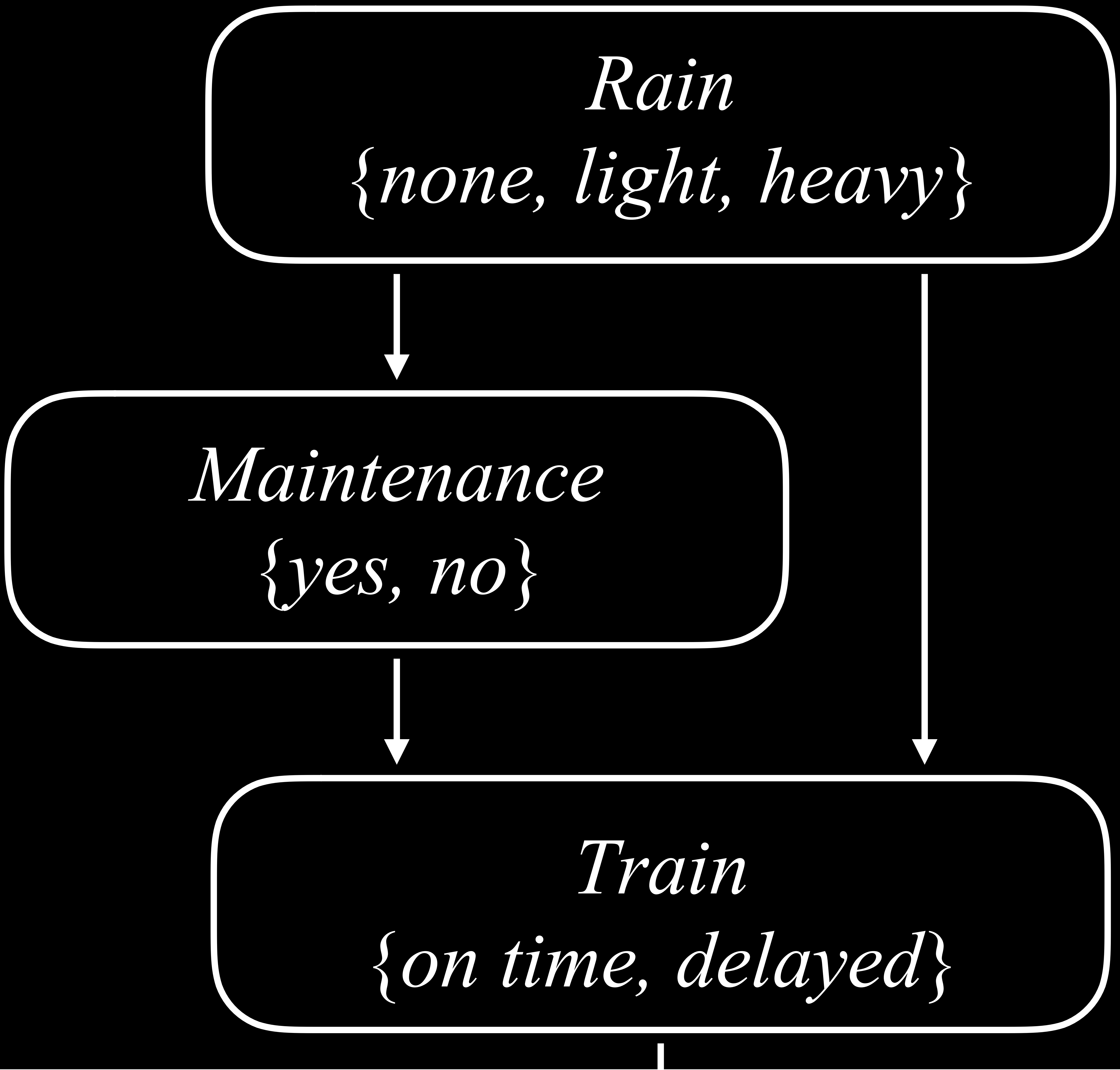
R = *light*
M = *yes*
T = *on time*

R	***M***	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6
heavy	*no*	0.5	0.5



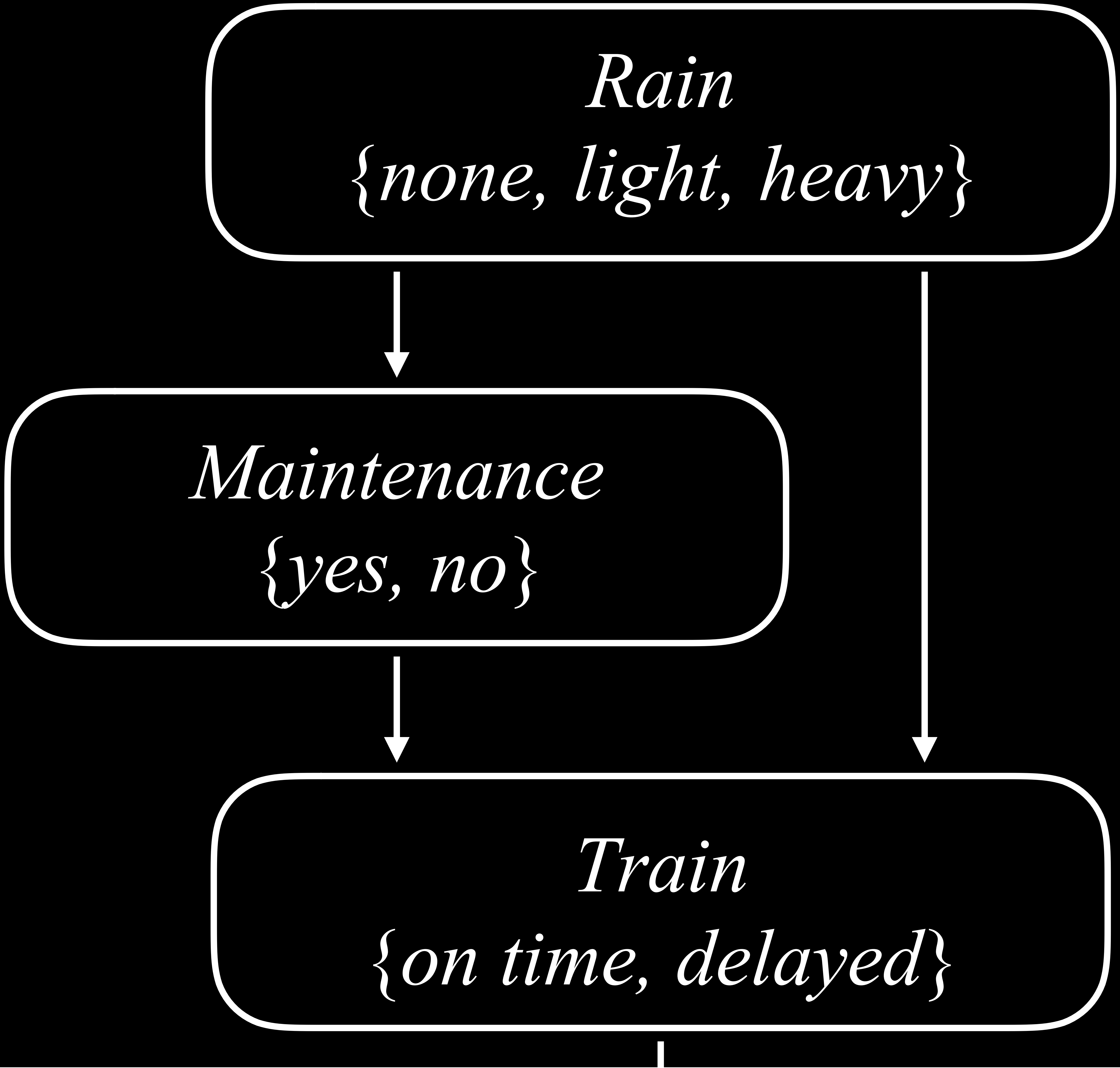
R = *light*	
M = *yes*	
T = *on time*	
A = *attend*	

T	*attend*	*miss*
on time	0.9	0.1
delayed	0.6	0.4



R = *light*
M = *yes*
T = *on time*
A = *attend*

R	***M***	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6
heavy	*no*	0.5	0.5



R = *light*
M = *yes*
T = *on time*
A = *attend*

R	***M***	*on time*	*delayed*
none	*yes*	0.8	0.2
none	*no*	0.9	0.1
light	*yes*	0.6	0.4
light	*no*	0.7	0.3
heavy	*yes*	0.4	0.6
heavy	*no*	0.5	0.5

CT-2 Review

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.
Answer to Q1: here we are gaining knowledge about the elements presented in array for future conclusion presented to deduce the goal (how the symbol will be annotated) goal is
Q2) Express the knowledge encoded by lines 19-25 using a single line of text.
Answer to Q2: we are creating implications to reach a conclusion based on the array implements where the (4x4-4) =12 implications are made to create symbol of colors.
Q3) Express the knowledge encoded by lines 27-33 using a single line of text.
Answer to Q3: Here we are creating symbol implications based on the colors that are presented on previous but not in 2nd statement.
Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle?
Answer to Q4: Yes, according to the last knowledge gained, the given new symbol
Q5) Write the output of running the given source code?
Answer to Q4: It will give output an array of symbols ~~that~~ by the knowledge it gained using the implications.

→ will deduce to a conclusion which is the

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.
Answer to Q1: $\forall c : S(c,0) \vee S(c,1) \vee S(c,2) \vee S(c,3)$
Q2) Express the knowledge encoded by lines 19-25 using a single line of text.
Answer to Q2: $\forall c : x : S(c,x) \rightarrow \forall y : (x \neq y) : \neg S(c,y)$
Q3) Express the knowledge encoded by lines 27-33 using a single line of text.
Answer to Q3: $\forall x : \forall c_1 : S(c_1,x) \rightarrow \forall c_2 : (c_1 \neq c_2) : \neg S(c_2,x)$
Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle?
Answer to Q4: No it is not sufficient
Q5) Write the output of running the given source code?
Answer to Q4: prints the ~~solution of~~ correct assignment of symbols that solves the game

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.
<u>Answer to Q1:</u> For all colors, each cell will be occupied by only one color.
Q2) Express the knowledge encoded by lines 19-25 using a single line of text.
<u>Answer to Q2:</u> If a color has already occupied a cell, it will not occupy any other cell.
Q3) Express the knowledge encoded by lines 27-33 using a single line of text.
<u>Answer to Q3:</u> If a cell is already occupied by a color, no other color will occupy it.
Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle? <u>Answer to Q4:</u> Yes
Q5) Write the output of running the given source code?
<u>Answer to Q4:</u> Symbol("red0"), Symbol("blue1"), Symbol("yellow2"), Symbol("green3")

al
4= 16 symbols

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.

Answer to Q1: The four symbols at indices 0 to 3 are given 8 ~~specific~~ colors. all colors to assign a symbol for each possible color. [Symbol (red0), Sym (blue0), Sym (green0), Sym (Yellow0) and so on]

Q2) Express the knowledge encoded by lines 19-25 using a single line of text.

Answer to Q2: For any two distinct balls, the colors of the ball are different. ~~ie At Yc~~ If the first ball is red, the second one can't be red.

Q3) Express the knowledge encoded by lines 27-33 using a single line of text.

Answer to Q3: A ball has only one unique color, it cannot have multiple colors

Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle?

Answer to Q4: Yes, Ball 3 is yellow, Ball 4 is green (we know, colors are distinct and the only remaining colors are yellow, green.

Q5) Write the output of running the given source code?

Answer to Q4: If knowledge base is true, then red0, blue1 are true and green2, yellow3 are false.
output: ~~red0, blue 1, yellow 2~~, green3.
Symbol(red0), Symbol(blue1), Symbol (Yellow2), Symbol (Green3)

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.
Answer to Q1: Each of the foun positions will have one of the foun colons (ned, blue, green on yellow).
Q2) Express the knowledge encoded by lines 19-25 using a single line of text.
Answer to Q2: Every position's colon must be unique. Means we cannot repeat any assigned colons.
Q3) Express the knowledge encoded by lines 27-33 using a single line of text.
Answer to Q3: One position cannot be colored with more than one color.
Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle? **Answer to Q4:** Yes, it is possible to derive a solution.
Q5) Write the output of running the given source code?
Answer to Q4: [red0, blue1, yellow2, green3]. This is the output, because given the knowledge red and blue are right in place, we only need to swap yellow and green which the model checking should derive.

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.
<u>Answer to Q1:</u>
Q2) Express the knowledge encoded by lines 19-25 using a single line of text.
<u>Answer to Q2:</u>
Q3) Express the knowledge encoded by lines 27-33 using a single line of text.
<u>Answer to Q3:</u>
Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle? <u>Answer to Q4:</u>
Q5) Write the output of running the given source code?
<u>Answer to Q4:</u>

Answers are provided in the next page

Now answer the following five questions (Q1-Q5) based on the above code:

Q1) Express the knowledge encoded by lines 11-17 using a single line of text.

Answer to Q1: each color can be. in either of the four cells.

Q2) Express the knowledge encoded by lines 19-25 using a single line of text.

Answer to Q2: same color cannot be in multiple cells.

Q3) Express the knowledge encoded by lines 27-33 using a single line of text.

Answer to Q3: same cell cannot have multiple colors

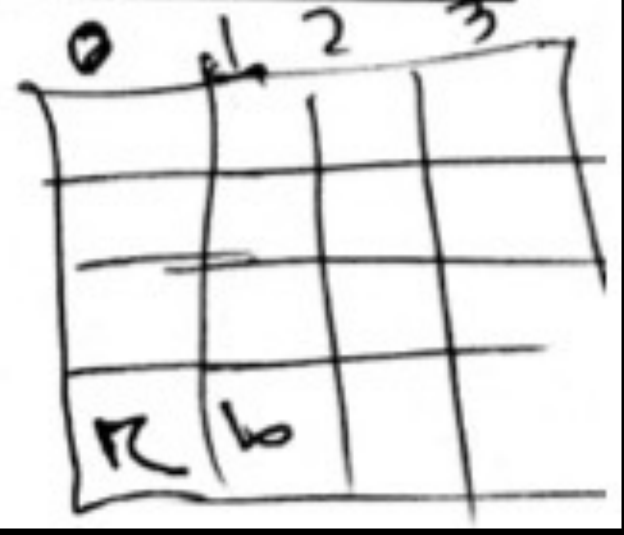
Q4) Is the knowledge encoded by lines 35-37 sufficient to derive a solution for this puzzle?

Answer to Q4: Yes

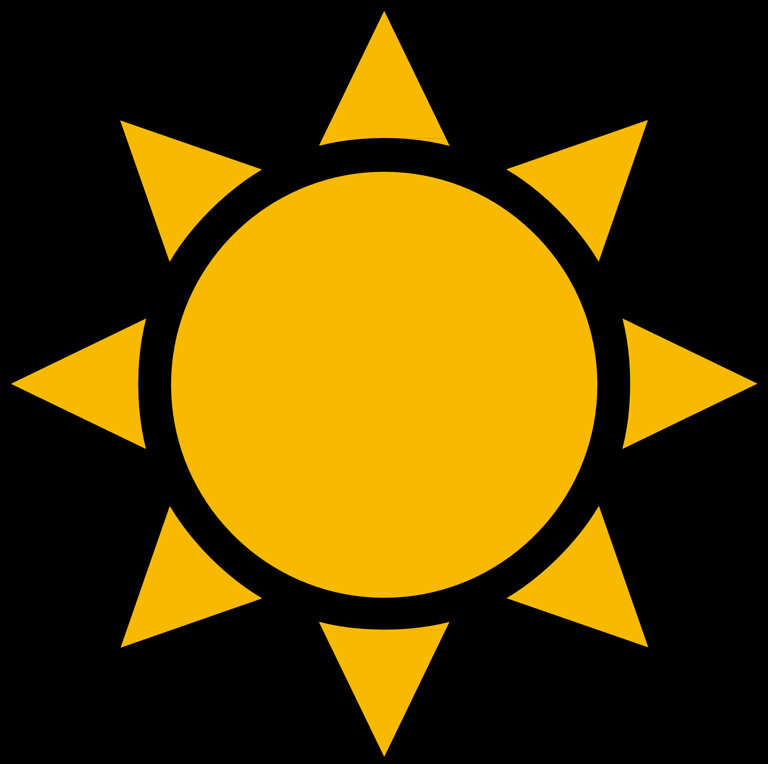
Q5) Write the output of running the given source code?

Answer to Q4:

{red, blue, yellow, green}



Uncertainty over Time



X_t: Weather at time t

Markov assumption

the assumption that the current state depends on only a finite fixed number of previous states

Markov Chain

Markov chain

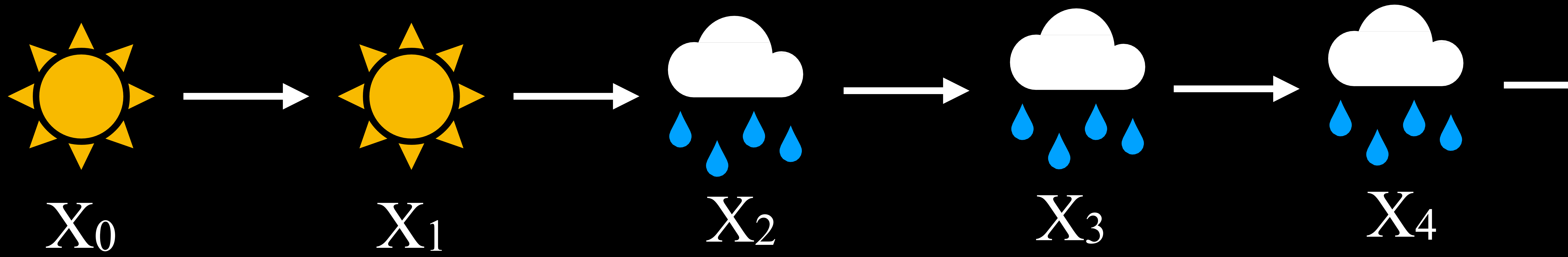
a sequence of random variables where the distribution of each variable follows the Markov assumption

Transition Model

Tomorrow (X_{t+1})

Today (X_t)

		
	0.8	0.2
	0.3	0.7


X_0
X_1
X_2
X_3
X_4

Sensor Models

Hidden State	Observation
robot's position	robot's sensor data
words spoken	audio waveforms
user engagement	website or app analytics
weather	umbrella

Hidden Markov Models

Hidden Markov Model

a Markov model for a system with hidden states that generate some observed event

Sensor Model

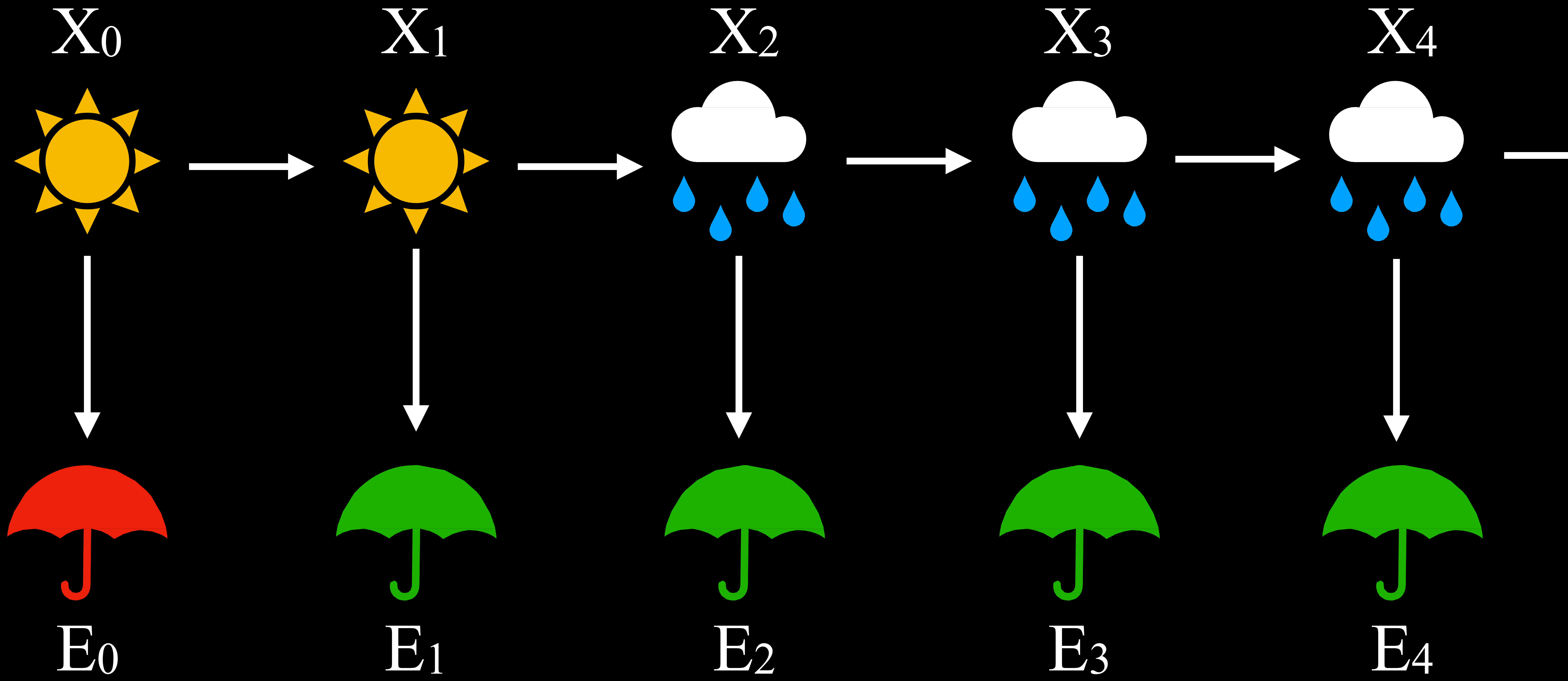
Observation (E_t)

State (X_t)

		
	0.2	0.8
	0.9	0.1

sensor Markov assumption

the assumption that the evidence variable depends only the corresponding state


X_0
X_1
X_2
X_3
X_4
E_0
E_1
E_2
E_3
E_4

Task	Definition
filtering	given observations from start until now, calculate distribution for **current** state
prediction	given observations from start until now, calculate distribution for a **future** state
smoothing	given observations from start until now, calculate distribution for **past** state
most likely explanation	given observations from start until now, calculate most likely **sequence** of states

Uncertainty