HW 1:

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2.4) $Rec \to R^d$ $x \in R^d$

$F_{d\text{-}rec} = \{ f_{(a_1, b_1, a_2, b_2, \dots, a_d, b_d)}(x_1, \dots, x_d) : a_i \leq b_i, i \in \{1, \dots d\} \}$

$$= \begin{cases} 1 & \text{if } a_i \leq x_i \leq b_i, \ i \in \{1, \dots, d\} \\ -1 & \text{otherwise} \end{cases}$$

a) we define $B_1, B_2, B_3 \dots B_{2d}$. each rectangle has 2 dimensions and mass probability $\frac{\varepsilon}{2d}$.

b) set S contains ⊕ examples $\Rightarrow$ for B_i s we have $R(f_S^{ERM}) \leq \varepsilon$

c) similar to the revious section

$$\underbrace{R(f_S^{ERM})}_{\text{Probability sample belong } B^* - B} \leq \underbrace{\cancel{2d}\left(\frac{\varepsilon}{\cancel{2d}}\right)}_{\text{Probility of sample Belong to } B_1 \cup \dots \cup B_d} = \varepsilon$$

d) $P(\text{Sample} \notin \text{each } B_i) = 1 - \frac{\varepsilon}{2d}$

e) expanding d for all rectangles

$$P(\text{Sample} \notin \text{any rect } B_i) = \left(1 - \frac{\varepsilon}{2d}\right)^n \leq e^{\frac{n\varepsilon}{2d}}$$

f) calculating the prob of $R(f_S^{ERM}) \leq \epsilon$

$$P(R(f_S^{ERM}) \leq \epsilon) = 1 - P(\text{sample is bad})$$

$$\geq 1 - \sum_{1}^{2d} e^{-n\epsilon/2d}$$

$$\geq 1 - 2d\left(e^{-\frac{n\epsilon}{2d}}\right)$$

g) we want to show $P(R(f_S^{ERM}) \leq \epsilon)$ to be at least $1-\delta$

$$\cancel{1} - 2d\left(e^{-\frac{n\epsilon}{2d}}\right) \geq \cancel{1} - \delta \quad \text{multiply by } -1 \text{ on both sides}$$

$$\delta \geq 2d\left(e^{-\frac{n\epsilon}{2d}}\right)$$

$$e^{\frac{n\epsilon}{2d}} \geq \frac{\delta}{2d} \quad \text{take } \ln(..) \text{ " " "}$$

$$\ln(2d/\delta) \leq \frac{n\epsilon}{2d} \quad \text{rearrange}$$

$$\therefore \; n \geq \frac{2d \ln(2d/\delta)}{\epsilon}$$